Q1. $x_1 = x_1^+ - x_1^-$; $x_2 = -x_2'$

$\Rightarrow$ max $-2(x_1^+ - x_1^-) - 3x_2' + 8x_3$

s.t. $2(x_1^+ - x_1^-) + x_3 + 5x_4 \le 4$

$7(x_1^+ - x_1^-) + x_2' + x_4 \le -3$

$5(x_1^+ - x_1^-) - 7x_3 = -2$

$x_1^+, x_1^-, x_2', x_3, x_4 \ge 0$

$\Rightarrow$ max $-2x_1^+ + 2x_1^- - 3x_2' + 8x_3$

s.t. $2x_1^+ - 2x_1^- + x_3 + 5x_4 \le 4$

$7x_1^+ - 7x_1^- + x_2' + x_4 \le -3$

$5x_1^+ - 5x_1^- - 7x_3 = -2$

$x_1^+, x_1^-, x_2', x_3, x_4 \ge 0$

$$b = \begin{pmatrix} 4 \\ -3 \\ -2 \end{pmatrix} \quad c = \begin{pmatrix} -2 \\ 2 \\ -3 \\ 8 \\ 0 \end{pmatrix} \quad A = \begin{pmatrix} 2 & -2 & 0 & 1 & 5 \\ 7 & -7 & 1 & 0 & 1 \\ 5 & -5 & 0 & -7 & 0 \end{pmatrix}$$

$$x = \begin{pmatrix} x_1^+ \\ x_1^- \\ x_2' \\ x_3 \\ x_4 \end{pmatrix} \quad y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

$\Rightarrow$ Dual Problem : min $b^t y$

s.t $A^t y \ge c$

$y \ge 0$

$\Rightarrow$ min $4y_1 - 3y_2 - 2y_3$

s.t $2y_1 + 7y_2 + 5y_3 \ge -2$

$-2y_1 - 7y_2 - 5y_3 \ge 2$

$y_2 \ge -3$

$y_1 - 7y_3 \ge 8$

$5y_1 + y_2 \ge 0$

$y_1, y_2 \ge 0$, y_3 free.