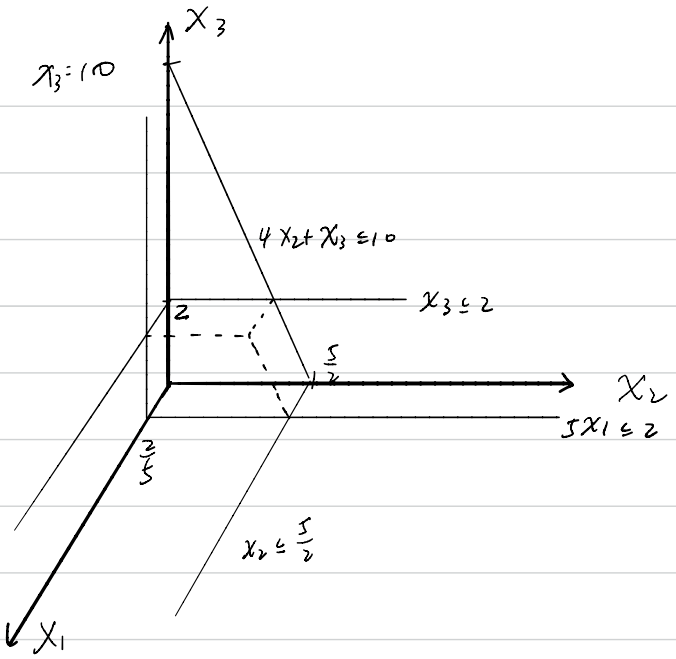
Q1.

a).



b)

Coordinates (x_1, x_2, x_3)	x_1	x_2	x_3	objective function value	
$(\frac{2}{5}, 0, 0)$	$2/5$	0	0	$\frac{2}{5}$	
$(\frac{2}{5}, 0, 2)$	$2/5$	0	2	$\frac{12}{5}$	
$(0, 0, 0)$	0	0	0	0	
$(0, 0, 2)$	0	0	2	2	
$(0, 2, 2)$	0	2	2	4	
$(\frac{2}{5}, 2, 2)$	$2/5$	2	2	$\frac{22}{5}$	← Max objective function
$(0, \frac{5}{2}, 0)$	0	$5/2$	0	$\frac{5}{2}$	
$(\frac{2}{5}, \frac{5}{2}, 0)$	$2/5$	$5/2$	0	$\frac{29}{10}$	

c) Optimal Solution = $(\frac{2}{5}, 2, 2)$, objective function = $\frac{22}{5}$

Since Max/min only occur at a vertex. so we simply draw feasible region, list all vertex & their objective function value and pick the max.

Q2.

$$\begin{pmatrix} 2 & 6 & -2 \\ 1 & 1 & 1 \\ 0 & 3 & -2 \end{pmatrix}^{-1} = \left(\begin{array}{ccc|ccc} 2 & 6 & -2 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 3 & -2 & 0 & 0 & 1 \end{array}\right) \quad ②\times 2$$

$$= \left(\begin{array}{ccc|ccc} 2 & 6 & -2 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 2 & 0 \\ 0 & 3 & -2 & 0 & 0 & 1 \end{array}\right) \overset{②-①}{=} \left(\begin{array}{ccc|ccc} 2 & 6 & -2 & 1 & 0 & 0 \\ 0 & -4 & 4 & -1 & 2 & 0 \\ 0 & 3 & -2 & 0 & 0 & 1 \end{array}\right)$$

$② \times -\frac{1}{4}$
$③ \times \frac{1}{3}$ $= \left(\begin{array}{ccc|ccc} 2 & 6 & -2 & 1 & 0 & 0 \\ 0 & 1 & -1 & \frac{1}{4} & -\frac{1}{2} & 0 \\ 0 & 1 & -\frac{2}{3} & 0 & 0 & \frac{1}{3} \end{array}\right) \overset{③-②}{=} \left(\begin{array}{ccc|ccc} 2 & 6 & -2 & 1 & 0 & 0 \\ 0 & 1 & -1 & \frac{1}{4} & -\frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} & -\frac{1}{4} & \frac{1}{2} & \frac{1}{3} \end{array}\right)$

$① / 2$
$③ \times 3$ $= \left(\begin{array}{ccc|ccc} 1 & 3 & -1 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & -1 & \frac{1}{4} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{3}{4} & \frac{3}{2} & 1 \end{array}\right) \overset{②+③}{=} \left(\begin{array}{ccc|ccc} 1 & 3 & -1 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & -\frac{1}{2} & 1 & 1 \\ 0 & 0 & 1 & -\frac{3}{4} & \frac{3}{2} & 1 \end{array}\right)$

$① - 3②$ $= \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 2 & -3 & -3 \\ 0 & 1 & 0 & -\frac{1}{2} & 1 & 1 \\ 0 & 0 & 1 & -\frac{3}{4} & \frac{3}{2} & 1 \end{array}\right) \overset{①+③}{=} \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & \frac{5}{4} & -\frac{3}{2} & -2 \\ 0 & 1 & 0 & -\frac{1}{2} & 1 & 1 \\ 0 & 0 & 1 & -\frac{3}{4} & \frac{3}{2} & 1 \end{array}\right)$

$$\Rightarrow \begin{pmatrix} 2 & 6 & -2 \\ 1 & 1 & 1 \\ 0 & 3 & -2 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{5}{4} & -\frac{3}{2} & -2 \\ -\frac{1}{2} & 1 & 1 \\ -\frac{3}{4} & \frac{3}{2} & 1 \end{pmatrix}$$

Q3: Max. $S_1 + S_2 + S_3$

Constrains: $5X_1 + S_1 = 2$

$4X_2 + X_3 + S_2 = 10$

$X_3 + S_3 = 2$

$X_1, X_2, X_3 \geq 0$

$S_1 = 2, \quad S_2 = 10 \quad S_3 = 2. \quad X_1 = X_2 = X_3 = 0.$

①

	X_1	X_2	X_3	S_1	S_2	S_3
$S_1 = 2$	5	0	0	1	0	0
$S_2 = 10$	0	4	1	0	1	0
$S_3 = 2$	0	0	1	0	0	1

⇒ Initial point $(X_1, X_2, X_3) = (0, 0, 0)$

②

	X_1	X_2	X_3	S_1	S_2	S_3
$X_1 = 2/5$	1	0	0	1/5	0	0
$S_2 = 10$	0	4	1	0	1	0
$S_3 = 2$	0	0	1	0	0	1

⇒ Next point $(X_1, X_2, X_3) = (2/5, 0, 0)$

③

	X_1	X_2	X_3	S_1	S_2	S_3
$X_1 = 2/5$	1	0	0	1/5	0	0
$X_2 = 5/2$	0	1	1/4	0	1/4	0
$S_3 = 2$	0	0	1	0	0	1

⇒ Next point $(X_1, X_2, X_3) = (2/5, 5/2, 0)$

④

	x_1	x_2	x_3	s_1	s_2	s_3
$x_1 = 2/5$	1	0	0	1/5	0	0
$x_2 = 2$	0	1	0	0	1/4	-1/4
$x_3 = 2$	0	0	1	0	0	1

⇒ Next point $(x_1, x_2, x_3) = (2/5, 2, 2)$

Since all pivot columns have been cleared out, it reaches optimal solution, $(2/5, 2, 2)$, with objective $= 22/5$