

Q1.

$$\begin{aligned} \max \quad & 4x_1 + 3x_2 \\ \text{st} \quad & x_1 + 4x_2 \leq 160 \\ & 2x_1 + x_2 \leq 80 \\ & x_2 \leq 30 \\ & x_1, x_2 \geq 0. \end{aligned}$$

a) Already in standard form:

$$b = \begin{pmatrix} 160 \\ 80 \\ 30 \end{pmatrix}, \quad c = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \quad A = \begin{pmatrix} 1 & 4 \\ 2 & 1 \\ 0 & 1 \end{pmatrix}, \quad x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

Dual: $\min b^T y$
 $\text{S.t. } A^T y \geq c$
 $y \geq 0$

$\min 160y_1 + 80y_2 + 30y_3$
$\text{S.t. } y_1 + 2y_2 \geq 4$
$4y_1 + y_2 + y_3 \geq 3$
$y_1, y_2, y_3 \geq 0$

b) $\max 4x_1 + 3x_2 = u$
 $\text{S.t. } x_1 + 4x_2 + x_3 = 160$
 $2x_1 + x_2 + x_4 = 80$
 $x_2 + x_5 = 30$
 $x_1, x_2, x_3, x_4, x_5 \geq 0$

$$\min(160/1, 80/2) = 40 \Rightarrow x_1 \text{ entering, } x_4 \text{ leaving.}$$

$$\begin{aligned} \max \quad & x_2 - 2x_4 = u - 160 \\ \text{S.t.} \quad & \frac{7}{2}x_2 + x_3 - \frac{1}{2}x_4 = 120 \\ & x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_4 = 40 \\ & x_2 + x_5 = 30 \\ & x_1, x_2, x_3, x_4, x_5 \geq 0 \end{aligned}$$

$$\min\left(\frac{120}{\frac{7}{2}}, \frac{1}{30}\right) = \frac{1}{30} \Rightarrow x_2 \text{ entering, } x_5 \text{ leaving.}$$

$$\begin{aligned} \max \quad & -2x_4 - x_5 = u - 190 \\ \text{S.t.} \quad & x_3 - \frac{1}{2}x_4 - \frac{7}{2}x_5 = 15 \\ & x_1 + \frac{1}{2}x_4 - \frac{1}{2}x_5 = 25 \\ & x_2 + x_5 = 30 \\ & x_1, x_2, x_3, x_4, x_5 \geq 0 \end{aligned}$$

It results $x_1=25, x_2=30$ for original solution,
 and $x_1=25, x_2=30, x_3=15, x_4=x_5=0$ for simplex
 method solution. Optimal value of objective = 190.

c). $y_1=0, y_2=2, y_3=1$ since $y_1, y_2, y_3 =$
 negative coefficient of x_3, x_4, x_5 , and objective
 value $\Rightarrow Z=190 \Rightarrow Z=190$.

d). $x_1=25, x_2=30$

$$\text{max. } 4x_1 + 3x_2$$

$$\text{s.t. } x_1 + 4x_2 \leq 160$$

$$2x_1 + x_2 \leq 80$$

$$x_2 \leq 30$$

$$x_1, x_2 \geq 0$$

$$\text{obj} = 25 \times 4 + 30 \times 3 = 90$$

$$25 + 4 \times 30 = 145 \leq 160 \checkmark$$

$$25 \times 2 + 30 = 80 \leq 80 \checkmark$$

$$30 \leq 30 \checkmark$$

$$25, 30 \geq 0 \checkmark$$

$$\text{obj} = 0 + 80 \times 2 + 30 = 190 \checkmark$$

$$\text{min } 160y_1 + 80y_2 + 30y_3$$

$$\text{s.t. } y_1 + 2y_2 \geq 4$$

$$4y_1 + y_2 + y_3 \geq 3$$

$$y_1, y_2, y_3 \geq 0$$

$$0 + 2 \times 2 = 4 \geq 4 \checkmark$$

$$0 + 2 + 1 = 3 \geq 3 \checkmark$$

$$0, 2, 1 \geq 0 \checkmark$$

∴ it is feasible to Q c)'s answer.

e). $\text{max } 4x_1 + 3x_2 = u$

$$\text{s.t. } x_1 + 4x_2 + x_3 = 160$$

$$2x_1 + x_2 + x_4 = 80 + \epsilon$$

$$x_2 + x_5 = 30$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

$$\min(160/1, 80/2) = 40 \Rightarrow x_1 \text{ entering, } x_4 \text{ leaving.}$$

$$\text{max } x_2 - 2x_4 = u - 160 - 2\epsilon$$

$$\text{s.t. } \frac{7}{2}x_2 + x_3 - \frac{1}{2}x_4 = 120 - \frac{1}{2}\epsilon$$

$$x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_4 = 40 + \frac{1}{2}\epsilon$$

$$x_2 + x_5 = 30$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

$\min(120/\frac{1}{2}, 1/30) = 1/30 \Rightarrow x_2$ entering, x_5 leaving.

$$\begin{array}{ll} \text{max} & -2x_4 - x_5 = n - 190 - 2\varepsilon \\ \text{s.t.} & x_3 - \frac{1}{2}x_4 - \frac{1}{2}x_5 = 15 - \frac{1}{2}\varepsilon \\ x_1 & + \frac{1}{2}x_4 - \frac{1}{2}x_5 = 25 + \frac{1}{2}\varepsilon \\ x_2 & + x_5 = 30 \\ x_1, x_2, x_3, x_4, x_5 & \geq 0 \end{array}$$

thus $x_1 = 25 + \frac{1}{2}\varepsilon$, $x_2 = 30$, obj for original question = $190 + 2\varepsilon$.

(x_1 can increase by $\frac{1}{2}\varepsilon$, obj increase by 2ε , x_2 not change)

f). $\delta = 2$, from above implementation, x_2 will not be affected since it already reaches boundary. x_2 will only increase since 2nd constraint has positive coefficient of ε . only consider x_3 since it has negative coefficient of ε , since $x_3 \geq 0 \Rightarrow x_3 = 15 - \frac{1}{2}\varepsilon \geq 0 \quad \varepsilon \leq 30$.