Atmospheric Monitoring

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1 Introduction

The usual approach in photometric calibration in a filter band b is presented in this section. Let us consider an object above the atmosphere with a specific flux[1] $F_\nu(\lambda)$. Its flux at telescope pupil $F_\nu(\lambda,t)^{pupil}(\lambda, az, alt, t)$ (before entering the telescope) is :

$$F_\nu(\lambda,t)^{pupil}(\lambda, az, alt, t) = T_{atm}(\lambda, az, alt, t) \cdot F_\nu(\lambda) \tag{1}$$

where $T_{atm}(\lambda, az, alt, t)$ is the atmospheric transmission which has the following form:

$$T_{atm}(\lambda, az, alt, t) = e^{-\tau(\lambda, az, alt, t)} \tag{2}$$

The flux is observed through a passband b of the telescope and an ADU[2] counting rate C_b is recorded by the CCD after conversion of the photons into photoelectrons in the pixel of coordinates (x, y) :

$$C_b(alt, az, t, x, y) = C \int F_\nu^{pupil}(\lambda, az, alt, t) \cdot S_b^{syst}(\lambda, t, x, y) \frac{d\lambda}{\lambda} \tag{3}$$

where the $1/\lambda$ comes for the conversion of energy into a photon.

The constant C is :

$$C = \frac{\pi D^2}{4gh} \tag{4}$$

where g is the electronic conversion of the number of photoelectrons per ADU count ($g > 1$).

Le us defines the effective normalised passband $\Phi^{obs}(\lambda, t)$:

$$\boxed{\Phi_b^{obs}(\lambda,t) = \frac{T_{atm}(\lambda, az, alt, t) \cdot S_b^{syst}(\lambda, t, x, y) \cdot \frac{1}{\lambda}}{\int_b T_{atm}(\lambda, az, alt, t) \cdot S_b^{syst}(\lambda, t, x, y) \frac{d\lambda}{\lambda}}} \tag{5}$$

Note above the (x, y) pixel variation should have been corrected. Then

$$\int_b \Phi_b^{obs}(\lambda, t) d\lambda = 1 \tag{6}$$

Note any grey attenuation component in the atmospheric transmission is removed in the definition of $\Phi_b^{obs}(\lambda, t)$.

The observed flux $F_b^{obs}(t)$ in the band b reads:

$$F_b^{obs}(t) = \int F_\nu(\lambda) \Phi_b^{obs}(\lambda, t) d\lambda \propto C_b(alt, az, t) \tag{7}$$

[1] A specific flux is usually expressed in Jansky unit where 1Jy=1 $erg/s/cm^2/Hz$.
[2] ADU means ADC units.

Even if the source has a steady flux $F_\nu(\lambda)$, $F_b^{obs}(t)$ varies in time due to atmospheric variation (even one could include instrument non-corrected variation). However in a catalogue we should report a constant value independent of atmospheric condition.

Thus one should provide a standardized flux F_b^{std} observed through a standardized effective normalized passband $\Phi_b^{std}(\lambda)$:

$$F_b^{std} = \int F_\nu(\lambda)\Phi_b^{std}(\lambda)d\lambda \tag{8}$$

The flux are given in "natural" magnitude unit :

$$m_b^{nat} = -2.5\log_{10}\left(\frac{F_b^{obs}}{F_{AB}}\right) \tag{9}$$

where the observed flux is reported relative to an unobserved ideal AB source through the same similar effective filter.

An ideal AB source is $F_\nu(\lambda) = 3631 Jy$.

The natural magnitudes can be translated into a standardized magnitudes as follow :

$$\begin{aligned}
m_b^{nat} &= -2.5\log_{10}\left(\frac{F_b^{obs}}{F_{AB}}\right) && (10)\\
&= -2.5\log_{10}\left(\frac{\int F_\nu(\lambda)\Phi_b^{obs}(\lambda,t)d\lambda}{F_{AB}}\right) && (11)\\
&= -2.5\log_{10}\left(\frac{\int F_\nu(\lambda)\Phi_b^{obs}(\lambda,t)d\lambda}{\int F_\nu(\lambda)\Phi_b^{std}(\lambda)d\lambda}\right) - 2.5\log_{10}\left(\frac{\int F_\nu(\lambda)\Phi_b^{std}(\lambda)d\lambda}{F_{AB}}\right) && (12)
\end{aligned}$$

$$m_b^{nat} = \Delta m_b^{obs} + m_b^{std} \tag{13}$$

where the ratio

$$\boxed{\Delta m_b^{obs} = -2.5\log_{10}\left(\frac{\int F_\nu(\lambda)\Phi_b^{obs}(\lambda,t)d\lambda}{\int F_\nu(\lambda)\Phi_b^{std}(\lambda)d\lambda}\right)} \tag{14}$$

which depends on the wavelength dependence of $F_\nu(\lambda)$ is a calibration factor.

m_b^{std} is the magnitude to be quoted in the catalog in addition to $\Phi_b^{std}(\lambda)$. Then magnitudes between experiment can be compared and even translated in another magnitude unit.

In general, a standard calibration procedure consists in giving the standardized magnitude or measured objects:

$$\boxed{m_b^{std} = -2.5\log_{10}(C_b(alt, az, t)) - \Delta m_b^{obs} + Z_b(t)} \tag{15}$$

where:

- $C_b(alt, az, t)$ is the measured experimentally measured in ADU unit,
- Δm_b^{obs} is the color correction in that band for that particular object with a particular wavelength dependence $F_\nu(\lambda)$, (not including any grey - flat wavelength attenuation),
- Z_b is defined as the zero point includes correction wavelength independent that are similar for all sources observed in the field of view of the telescope at the same time.

Z_b in addition to the time varying grey attenuation, the zero point includes constants related to the definition of the reference system, here the AB source reference.

$$\boxed{Z_b^{obs}(t) = 2.5 \log_{10}\left(CF_{AB} \int_0^{\infty} S^{atm}(\lambda, alt, az, t) S_b^{syst}(\lambda, x, y, t) \frac{d\lambda}{\lambda} \right)} \tag{16}$$

2 Method

In the following method, we develop at first order the wavelength dependence of the SED and also the atmospheric transmission wavelength dependence.

Let us define for some reference time t_0 :

- $\mathcal{F}_S(\lambda, t_0) \simeq \mathcal{F}_S(\lambda)$ the Spectral Energy Distribution (SED) of some reference star supposed to be steady in Luminosity (in units of photons or photoelectrons per wavelength unit),
- $T(\lambda, \hat{z}, t_0)$ the atmospheric transmission at airmass $\hat{z}$ corresponding at Auxtel pointing at time t_0, which is the quantity we want to monitor,
- $I_V(\lambda, t_0)$, the total instrumental transmission supposed to be constant or corrected such :

$$I_V(\lambda, t_0) = V(\lambda, t_0) \cdot \epsilon_{CCD}(\lambda, t_0) \cdot O(\lambda, t_0) \tag{17}$$

  where $V(\lambda, t_0)$ is the filter passband (ex. the LSST Filter band : V= U,G,R,I,Z,Y), $\epsilon_{CCD}(\lambda, t_0)$ is the quantum efficiency, $O(\lambda, t_0)$ is the optical throughput.

We define the varying function :

$$\tau(\lambda, t) = T(\lambda, \hat{z}, t) \cdot I_V(\lambda, t) \tag{18}$$

The reference spectrum $S_V(t_0)$ measured in the telescope at reference time t_0 is :

$$S_V(t_0) = A \int_V \mathcal{F}_S(\lambda, t_0) T(\lambda, \hat{z}, t_0) I_V(\lambda, t_0) d\lambda \tag{19}$$

where A is an known constant.

We can rewrite the measured spectrum $S_V(t)$ at any time t in a more compact form :

$$S_V(t) = A \int_V \mathcal{F}_S(\lambda) \tau(\lambda, t) d\lambda \tag{20}$$

We assume an ideal AB source which spectrum is fully defined $\mathcal{F}_{AB}$.

For that particular ideal source, let us define the following ratio :

$$k(t) = \frac{\int \tau(\lambda, t) \mathcal{F}_{AB} d\lambda}{\int \tau(\lambda, t_0) \mathcal{F}_{AB} d\lambda} \tag{21}$$

$k(t)$ encapsulate time dependence of grey attenuation. Let us assume this $k(t)$ to be fully estimated.

Let us defines the grey attenuation corrected quantities :

$$\left\{ \begin{array}{lcl} S'_V(t) & = & \frac{S_V(t)}{k(t)} \\ \tau'(\lambda, t) & = & \frac{\tau(\lambda,t)}{k(t)} \end{array} \right. \tag{22}$$

Then observing a reference star S of flux $\mathcal{F}_S$ for which the equation **??** is well established at time t_0.

$$\frac{S'_V(t) - S_V(t_0)}{S_V(t_0)} = \frac{A}{S_V(t_0)} \int_V \left(\tau'(\lambda, t) - \tau(\lambda, t_0)\right) \mathcal{F}_S(\lambda) d\lambda \tag{23}$$

Because we know quite well the flux of the reference star in the V band, we assume we can approximate $\mathcal{F}_S(\lambda)$ at first order in the band V with λ_V is the V-band center as follow:

$$\mathcal{F}_S(\lambda)|_V = \mathcal{F}_{AB}(\lambda_V) \left(1 + \alpha_S(\lambda - \lambda_V)\right) \tag{24}$$

$$\frac{S'_V(t) - S_V(t_0)}{S_V(t_0)} = \frac{A}{S_V(t_0)} \int_V \left(\tau'(\lambda, t) - \tau(\lambda, t_0)\right) \mathcal{F}_{AB}(\lambda_V) \left(1 + \alpha_S(\lambda - \lambda_V)\right) d\lambda \tag{25}$$

We make a second assumption[3] :

$$\tau'(\lambda, t) - \tau(\lambda, t_0) = \tau(\lambda, t_0) \left(\frac{1}{k(t)} \frac{\tau(\lambda, t)}{\tau(\lambda, t_0)} - 1 \right) \simeq \tau(\lambda, t_0) a(t) (\lambda - \lambda_V) \tag{26}$$

The Magnitude variation of the source S in the V band is:

$$dM^S_{V_{AB}} = -\frac{2.5}{\ln(10)} \frac{dF^S_{V_{AB}}}{F_{V_{AB}}} \tag{27}$$

Then

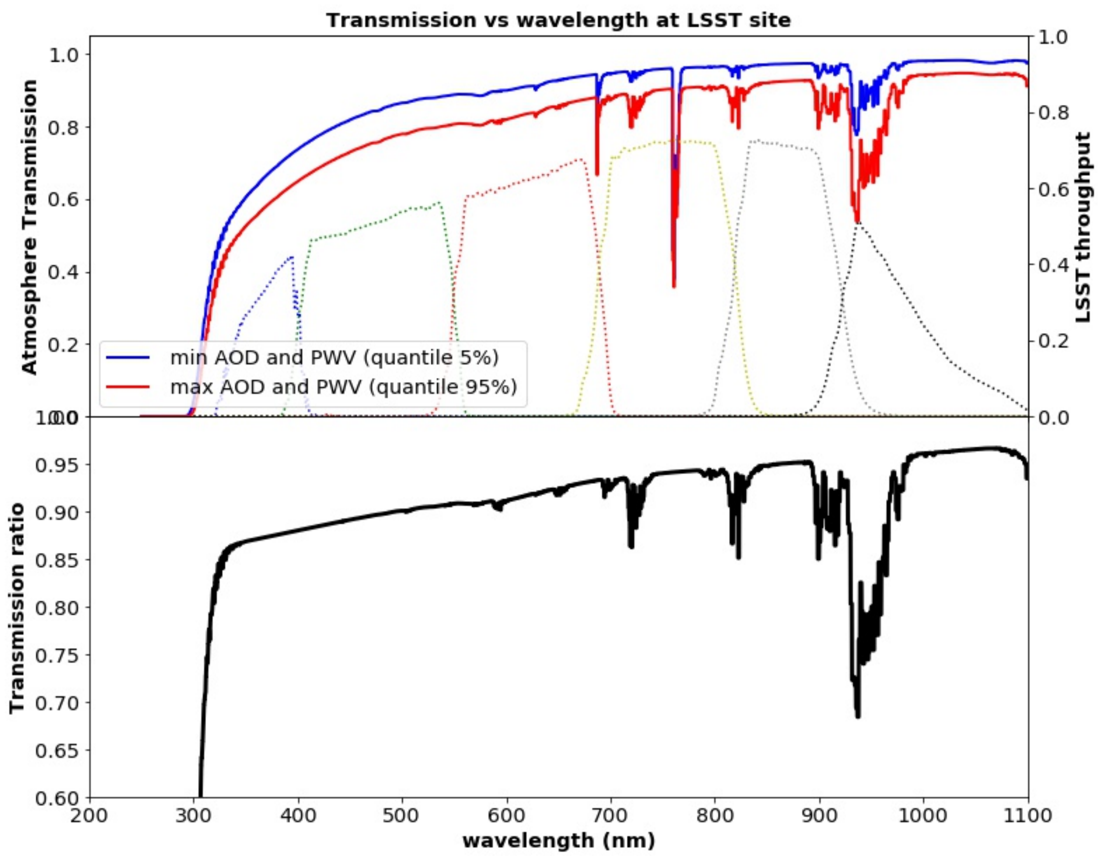
$$-\frac{\ln(10)}{2.5} \left(M^S_{V_{AB}}(t) - M^S_{V_{AB}}(t_0)\right) \simeq \frac{A\mathcal{F}_{AB}(\lambda_V)\tau(\lambda, t_0)}{S_V(t_0)} a(t) \int_{\lambda_V - \Delta\lambda/2}^{\lambda_V + \Delta\lambda/2} (\lambda - \lambda_V) \left(1 + \alpha_S(\lambda - \lambda_V)\right) d\lambda \tag{28}$$

We defines $\Delta\lambda$:

$$\Delta\lambda = \frac{S_V(t_0)}{A\mathcal{F}_{AB}(\lambda_V)\tau(\lambda, t_0)} \tag{29}$$

$$0.921 \left(M^S_{V_{AB}}(t) - M^S_{V_{AB}}(t_0)\right) = \frac{a(t)\alpha_S}{12} (\Delta\lambda)^2 \tag{30}$$

[3] I don't understand this approximation is true.



(a) Quantiles of the atmospheric transmission profile and their ratio.

Appendix

A Magnitudes in AB system

The flux reference in a AB system is the flat SED in Jansky units such :

$$\mathcal{F}^E_\nu = \frac{dP}{d\nu dS} = 3631 Jy \tag{31}$$

where $dP = \frac{dE}{dt}$ is the power emitted by the source and ν the frequency and dS a surface element.

$$1 Jy = 10^{-26} W \cdot Hz^{-1} \cdot m^{-2} = 10^{-23} erg \cdot s^{-1} \cdot Hz^{-1} \cdot cm^{-2} \tag{32}$$

The energy, frequency and wavelength are related by:

$$E = h\nu = \frac{hc}{\lambda} \tag{33}$$

Given that

$$\frac{dP}{dS} = \mathcal{F}^E_\nu d\nu = \mathcal{F}^E_\lambda d\lambda \tag{34}$$

$$\mathcal{F}^E_\lambda = \mathcal{F}^E_\nu \frac{d\nu}{d\lambda} = \frac{c}{\lambda^2} \mathcal{F}^E_\nu \tag{35}$$

To convert this power dP into a number a photon rate dn_γ :

$$\frac{dn_\gamma}{dS} = \frac{\lambda}{hc} \frac{dP}{dS} \tag{36}$$

Then the photon flux $\mathcal{F}^{n_\gamma}_\lambda$ is given by

$$\frac{dn_\gamma}{dS} = \mathcal{F}^{n_\gamma}_\lambda d\lambda = \frac{dn_\gamma}{dP} \mathcal{F}^E_\lambda d\lambda = \frac{\lambda}{hc} \frac{c}{\lambda^2} \mathcal{F}^E_\nu d\lambda \tag{37}$$

Finally

$$\mathcal{F}^{n_\gamma}_\lambda = \frac{1}{h\lambda} \mathcal{F}^E_\nu \tag{38}$$

The SED for an ideal perfect AB source is,

$$\mathcal{F}^{n_\gamma}_{\lambda AB} = \frac{3631 Jy}{h\lambda} = \frac{3631 \times 10^{-26} W/Hz/m^2 \times 10^{-4} m^2/cm^2}{6.26 \cdot 10^{-34} J.s / 550nm} \left(\frac{550nm}{\lambda_{nm}}\right) \tag{39}$$

$$= 9972 \; phot/s/nm/cm^2 \left(\frac{550nm}{\lambda_{nm}}\right) \tag{40}$$

It has an $1/\lambda$ photon distribution.

B Approximation of a source flux in a band filter

B.1 Power law

Let us assume in a band V width the Flux of a star $\mathcal{F}_S(\lambda)$ can be approximated at first order by a power law if index α_S as follow :

$$\mathcal{F}_S(\lambda) \simeq \mathcal{F}_S(\lambda_V)(\lambda - \lambda_V)^{\alpha_S} \tag{41}$$

This expression can be developed as follow :

$$\begin{aligned} \mathcal{F}_S(\lambda) &\simeq \mathcal{F}_S(\lambda_V) + \left(\frac{d\mathcal{F}_S}{d\lambda}\right)(\lambda_V)(\lambda - \lambda_V) && (42) \\ &\simeq \mathcal{F}_S(\lambda_V)\alpha_S(\lambda - \lambda_V) && (43) \end{aligned}$$

B.2 Exponential Law

If the flux is expressed as :

$$\mathcal{F}_S(\lambda) \simeq \mathcal{F}_S(\lambda_V)e^{\alpha_S(\lambda - \lambda_V)} \tag{44}$$

then we still have the same approximation at first order:

$$\begin{aligned} \mathcal{F}_S(\lambda) &\simeq \mathcal{F}_S(\lambda_V) + \left(\frac{d\mathcal{F}_S}{d\lambda}\right)(\lambda_V)(\lambda - \lambda_V) && (45) \\ &\simeq \mathcal{F}_S(\lambda_V)\alpha_S(\lambda - \lambda_V) && (46) \end{aligned}$$

C Function $\tau(\lambda, t)$

$$\tau'(\lambda, t) - \tau(\lambda, t_0) = \tau(\lambda, t_0)\left(\frac{1}{k(t)}\frac{\tau(\lambda, t)}{\tau(\lambda, t_0)} - 1\right) = \tau(\lambda, t_0)\left(\frac{T(\lambda, t)}{T(\lambda, t_0)} - 1\right) \tag{47}$$

The atmospheric transmission can be parametrized as :

$$T(\lambda, t, z) = \exp(-K(\lambda, t)z) \tag{48}$$

$K(\lambda, t)$ is the atmospheric extinction coefficient.

$$K(\lambda, t) = \frac{K_r^{scatt}(t)}{\lambda^4} + \frac{K_a^{scatt}(t)}{\lambda^{\beta(t)}} + \sum_i K_i^{abs}(\lambda)a_i(t) \tag{49}$$

- $\frac{K_r^{scatt}(t)}{\lambda^4}$ is the extinction coefficient for Rayleigh, where $K_r^{scatt}(t) = K_r^{scatt}(t_0)\frac{P(t)}{P(t_0)}$ scattering,

- $\frac{K_a^{scatt}(t)}{\lambda^{\beta(t)}}$ is the extinction coefficient for aerosol scattering,
- $K_i^{abs}(\lambda)a_i(t)$ is the extinction coefficient for molecular absorption of O_2, O_3 and precipitable water vapor.

For all interaction processes, the wavelength part is separated from the time dependant part except for the aerosols for which the exponent depends on time.

The Rayleigh contribution is supposed perfectly known because the pressure $P(t)$ is measured and the absorption band K_i^{abs} is constant and known. The varying parameters, the aerosol parameters $K_a^{scatt}(t), \beta(t)$, the absorption coefficient $a_i(t)$[4] for PWV require to be monitored.

If atmospheric variations are small enough,

$$\begin{aligned} R(\lambda,t) &= \frac{T(\lambda,t,z)}{T(\lambda,t_0,z)} - 1 = \exp\left\{-\left(K(\lambda,t) - K(\lambda,t_0)\right)z - 1\right\} && (50) \\ &\simeq \left(K(\lambda,t) - K(\lambda,t_0)\right)z && (51) \end{aligned}$$

In a band V, $K(\lambda,t)$ can be expressed at first order as :

$$K(\lambda,t) = K(\lambda_V,t) + \frac{\partial K}{\partial \lambda}_{\lambda=\lambda_V}(\lambda - \lambda_V) \tag{52}$$

If $K \simeq \frac{K_0}{\lambda^\delta}$, $\frac{\partial K}{\partial \lambda} \simeq -\delta \frac{K_0}{\lambda^{\delta+1}} \simeq -\delta \frac{K}{\lambda}$

- For Rayleigh scattering , $K_r^{scatt}(\lambda,t) = K_r^{scatt}(\lambda_V,t)(1 - 4\frac{\lambda-\lambda_V}{\lambda_V})$,
- For Aerosols, $K_a^{scatt}(\lambda,t) = K_a^{scatt}(\lambda_V,t)(1 - \beta(t)\frac{\lambda-\lambda_V}{\lambda_V})$,

We do not develop the $K_i^{abs}(\lambda)$ with respect to $\lambda = \lambda_V$, because the $K_i^{abs}(\lambda)$ quickly vary with λ within the wavelength band. This contribution will be absorbed in the in band constant $a_i(t)$.

$$\begin{aligned} \frac{R(\lambda,t)}{z} &= \left(K_r^{scatt}(\lambda_V,t) - K_r^{scatt}(\lambda_V,t_0)\right)(1 - 4\frac{\lambda-\lambda_V}{\lambda_V}) && (53) \\ &+ \left(K_a^{scatt}(\lambda_V,t) - K_a^{scatt}(\lambda_V,t_0)\right) - (K_a^{scatt}(\lambda_V,t_0)((\beta(t) - \beta(t_0))\frac{\lambda-\lambda_V}{\lambda_V}) && (54) \\ &+ \sum_i \overline{K_i^{abs}}\left(a_i(t) - a_i(t_0)\right) && (55) \end{aligned}$$

This term may be written in a compact form :

$$\frac{R(\lambda,t)}{z} = K_r^0 \frac{\delta P(t)}{P}(1 - 4\frac{\lambda-\lambda_V}{\lambda_V}) + \Delta K_a(t) - K_a^0 \Delta\beta(t)\frac{\lambda-\lambda_V}{\lambda_V} + \sum_i \overline{K_i^{abs}}\Delta a_i(t) \tag{56}$$

We can even simplify this equation :

$$\frac{R(\lambda,t)}{z} = A(t) + B(t)\frac{\lambda-\lambda_V}{\lambda_V} + \sum_i C_i^{abs}(t) \tag{57}$$

The expression above show color neutral terms and color dependant terms (weighted by $\frac{\lambda-\lambda_V}{\lambda_V}$).

[4] $a_i(t)$ for Ozone will be monitored by Satellite data

The monitoring variables to estimate from AuxTel data are $A_V(t), B_V(t), C_{iV}^{abs}(t)$.

The $A(t)$ term may be absorbed when subtracting the grey component.But this grey component is present in the Rayleigh and Aerosol simulation. $B(t)$ may be present in all bands. C_{O^3} is present in G and R band, C_{O^2} in I band (may be also some C_{H2O} in I band), and C_{PWV} in Y band.

D Old personal notes

D.1 Spectral Energy Distribution

The spectrum is a Spectral Energy Density (SED) written as $S^E_\lambda(\lambda)$ (The E in the superscript refers to the energy or power distribution which is not the number of photons distribution). For example for the CALSPEC database, the spectra unit is and erg.s^{-1}.cm^{-2-1} , the wavelength is in given Angstroms(Å).

D.2 Measurement

However the CCD make some measurement of the number of photo-electrons $n_e(\lambda)d\lambda$ (number of photo-electrons per second in $d\lambda$ unit wavelength) or number of photons $n_\gamma(\lambda)d\lambda$ (number of photons per second in $d\lambda$ unit wavelength).

The relation between the number of electrons $dN_e = n_e(\lambda) \cdot d\lambda$, the number of photons $dN_\gamma = n_\gamma(\lambda) \cdot d\lambda$ and the ADU count numbers $dN_{ADU} = n_{ADU}(\lambda) \cdot d\lambda$ (CCD signal digitization units) in wavelength unit $d\lambda$ per unit of time dt (one second) is:

$$dN_e = n_e(\lambda) \cdot d\lambda = \epsilon_{CCD}(\lambda) \cdot n_\gamma(\lambda) \cdot d\lambda = g_{el} \cdot n_{ADU}(\lambda) \cdot d\lambda$$

where

- $\epsilon_{CCD}(\lambda)$: CCD quantum efficiency, no unit,
- g_{el} : electronic gain used to convert ADU into electrons, it is in unit of electron per ADU,
- $n_e(\lambda)$: number of electrons in a pixel per wavelength unit $d\lambda$ and per time unit dt,
- $S^E_\lambda(\lambda)$: SED : energy per wavelength unit, collection surface unit, detection time unit (exposure), and wavelength unit (in present CALSEC case, erg per cm^2 per second per Angstrom).

D.3 Spectrum in photon unit

The CCD measures a number of photons that induce photoelectrons, not the incident energy. Thus we have to convert the spectral energy density of the source $S^E_\lambda(\lambda) \cdot d\lambda$ into a number of photon energy density $S^{N_\gamma}_\lambda(\lambda) \cdot d\lambda$:

$$S^{N_\gamma}_\lambda(\lambda) \cdot d\lambda = \frac{S^E_\lambda(\lambda) \cdot d\lambda}{hc/\lambda}$$

- The number of photons density from the source is then expressed as $dN_\gamma(\lambda)/d\lambda$:

$$\frac{dN_\gamma(\lambda)}{d\lambda} = S^{N_\gamma}_\lambda(\lambda) = \frac{\lambda}{hc} \cdot S^E_\lambda(\lambda)$$

- Note in some textbook, the spectral energy density $S^E_\nu(\nu)$ is tabulated in frequency unit.

Given

$$dE = S^E_\nu(\nu) \cdot d\nu = S^E_\lambda(\lambda) \cdot d\lambda$$

$$S^E_\lambda(\lambda) = S^E_\nu(\nu) \cdot |\frac{d\nu}{d\lambda}| = \frac{c}{\lambda^2} \cdot S^E_\nu(\nu)$$

thus we can also find the expression for the number of photons in the detector:

$$\frac{dN_\gamma(\lambda)}{d\lambda} = \frac{1}{h\lambda} \cdot S^E_\nu(\nu(\lambda))$$

Or if we wanted to express $dN_\gamma(\lambda)$ as a function of ν :

$$dN_\gamma(\nu) = dN_\gamma(\lambda) \cdot |\frac{d\lambda}{d\nu}| = \frac{c}{\nu^2} \cdot dN_\gamma(\lambda)$$

... to be continued to show:

D.4 The flux in a wavelength bandwidth

Transmission in atmosphere $T^{atm}(\lambda)$, detector filters $T^{filt}(\lambda)$, or grating $T^{grat}(\lambda)$, optics (mirrors and lenses) $T^{opt}(\lambda)$, detector (CCD) $\epsilon_{CCD}(\lambda)$, focusing efficiency ϵ_{PSF} are often expressed as a function of the wavelength λ and the SED either using $S^E_\lambda(\lambda)$ or using $S^E_\nu(\nu(\lambda))$: ($T^{grat}(\lambda)$ is for auxiliary telescope.) The flux (in ADU) in a pixel in $\Delta\lambda$ is expressed as:

$$\begin{aligned}
F^{ADU}_{\Delta\lambda} &= \frac{\pi D^2}{4g_{el}} \int_{\Delta\lambda} T^{atm}(\lambda) \cdot T^{grat}(\lambda) \cdot T^{opt}(\lambda) \cdot T^{filt}(\lambda) \cdot \epsilon_{PSF} \cdot \epsilon_{CCD}(\lambda) \cdot \frac{dN_\gamma(\lambda)}{d\lambda} \cdot d\lambda && (58) \\
F^{ADU}_{\Delta\lambda} &= \frac{\pi D^2}{4g_{el}hc} \int_{\Delta\lambda} T^{atm}(\lambda) \cdot T^{grat}(\lambda) \cdot T^{opt}(\lambda) \cdot T^{filt}(\lambda) \cdot \epsilon_{PSF} \cdot \epsilon_{CCD}(\lambda) \cdot S^E_\lambda(\lambda) \cdot \lambda \cdot d\lambda && (59) \\
F^{ADU}_{\Delta\lambda} &= \frac{\pi D^2}{4g_{el}h} \int_{\Delta\lambda} T^{atm}(\lambda) \cdot T^{grat}(\lambda) \cdot T^{opt}(\lambda) \cdot T^{filt}(\lambda) \cdot \epsilon_{PSF} \cdot \epsilon_{CCD}(\lambda) \cdot S^E_\nu(\lambda) \cdot \frac{d\lambda}{\lambda} && (60) \\
& && (61)
\end{aligned}$$

- we should not forget the λ term,
- we should not forget the $T^{grat}(\lambda)$ term. I would guess the following approximation $T^{grat}(\lambda) \propto 1/\lambda^\alpha$ (α is an unknown index power) and that the term $\lambda \cdot T^{grat}(\lambda)$ is quite constant from 600 nm to 1000 nm in our case.
- ϵ_{PSF} : Is the fraction of light selected by the aperture. Because the light of a punctual object is spread over the PSF (standing for Point Spread Function), the aperture may be smaller than the whole extend of the PSF. So $\epsilon_{PSF} < 1$ must be known.

D.5 Application to LSST

LSST measures the flux F^{ADU}_{filt} independently into six filters F, where $F = U, G, R, I, Z, Y4$. Each Filter is a passband of transmission function $T_F(\lambda)$. We skip $^{grat}(\lambda)$ and ϵ_{PSF} for simplicity. The following table summarise how the Magnitude M_{filt} is computed.

$$F_{filt}^{ADU}(code) = \frac{\pi D^2}{4 g_{el} h c} \int_{\lambda.in.filt} T_{code}^{atm}(\lambda) \cdot T^{opt}(\lambda) \cdot T^{filt}(\lambda) \cdot \epsilon_{CCD}(\lambda) \cdot S_{\lambda}^{E}(\lambda) \cdot \lambda \cdot d\lambda$$

F_{filt}^{ADU}	:	Flux measured in LSST Filter "filt"
$T_{code}^{atm}(\lambda)$	:	Atmospheric transparency either from Modtran ($code = MT$) or LibRadTran ($code = RT$)
$T^{opt}(\lambda$	:	Optical througput
$T^{filt}(\lambda)$	:	Filter transmission
$\epsilon_{CCD}(\lambda)$	:	Quantum efficiency of the CCD
$S_{\lambda}^{E}(\lambda)$	:	SED of astrophysical object
g_{el}	:	Electronic gain : number of electrons per ADU
D	:	Telescope diameter
h, c	:	Fundamental constants (Planck and speed of light constants)

$$\begin{aligned} M_{filt}(code) &= -2.5 \cdot \log_{10} F_{filt}^{ADU}(code) \\ \Delta M_{filt} &= M_{filt}(RT) - M_{filt}(MT) \end{aligned}$$

References

[1] Jones 2011
Level 2 Photometric Calibration for the LSST Survey R. Lynne Jones et al. http://citeseerx.ist.psu.edu/viewdoc/download;jsessionid=92EF3F4CE327997EF4F70A8B4848591E?doi=10.1.1.476.9938&rep=rep1&type=pdf