

Let E be the arithmetic mean of a_i : $E = \frac{1}{n} \sum_{i=1}^n a_i$. Also let

$$V = \frac{1}{n} \sum_{i=1}^n (a_i - E)^2 = \frac{1}{n} \sum_{i=1}^n a_i^2 - \frac{2}{n} \sum_{i=1}^n a_i E + n \cdot \frac{E^2}{n} = \frac{1}{n} \sum_{i=1}^n a_i^2 + E(E - \frac{2}{n} \sum_{i=1}^n a_i) = \frac{1}{n} \sum_{i=1}^n a_i^2 - E^2.$$

Let's divide initial inequality by n^2 and substitute the above notation:

$$\frac{1}{n} \sum_{i \in A} a_i^2 + 4 \left(\frac{\sum_{i=1}^n a_i}{n} \right)^2 \leq 4 = \frac{4}{n} \sum_{i=1}^n a_i^2$$

It's sufficient to prove the following inequality:

$$\frac{1}{n} \sum_{i \in A} \frac{a_i^2}{2} \leq V$$

From inequality between quadratic and arithmetic mean we know that $\sqrt{\frac{\sum_{i=1}^n a_i^2}{n}} \geq \frac{\sum_{i=1}^n a_i}{n} \implies \sum_{i=1}^n a_i \leq n$. Moreover, since $a \geq 2$, then $a - 1 \geq \frac{a}{2}$. However,

$$V = \frac{1}{n} \sum_{i=1}^n (a_i - E)^2 \geq \frac{1}{n} \sum_{i \in A} (a_i - E)^2 \geq \frac{1}{n} \sum_{i \in A} (a_i - 1)^2 \geq \frac{1}{n} \sum_{i \in A} \left(\frac{a_i}{2} \right)^2,$$

which ends the proof.