Contents

| For | reword | | <i>page</i> xi |
|-----|--------|--|----------------|
| Pre | eface | | xiii |
| 1 | Intro | duction – a Tour of Multiple View Geometry | 1 |
| | 1.1 | Introduction – the ubiquitous projective geometry | 1 |
| | 1.2 | Camera projections | 6 |
| | 1.3 | Reconstruction from more than one view | 10 |
| | 1.4 | Three-view geometry | 12 |
| | 1.5 | Four view geometry and <i>n</i> -view reconstruction | 13 |
| | 1.6 | Transfer | 14 |
| | 1.7 | Euclidean reconstruction | 16 |
| | 1.8 | Auto-calibration | 17 |
| | 1.9 | The reward I: 3D graphical models | 18 |
| | 1.10 | The reward II: video augmentation | 19 |
| PA | RT 0: | The Background: Projective Geometry, Transformations an | d Esti- |
| | tion | The Buenground Trojective Geometry, Trumstormutions un | 23 |
| | Outlin | e | 24 |
| 2 | Proie | ective Geometry and Transformations of 2D | 25 |
| | 2.1 | Planar geometry | 25 |
| | 2.2 | The 2D projective plane | 26 |
| | 2.3 | Projective transformations | 32 |
| | 2.4 | A hierarchy of transformations | 37 |
| | 2.5 | The projective geometry of 1D | 44 |
| | 2.6 | Topology of the projective plane | 46 |
| | 2.7 | Recovery of affine and metric properties from images | 47 |
| | 2.8 | More properties of conics | 58 |
| | 2.9 | Fixed points and lines | 61 |
| | 2.10 | Closure | 62 |
| 3 | Proje | ective Geometry and Transformations of 3D | 65 |
| | 3.1 | Points and projective transformations | 65 |
| | 3.2 | Representing and transforming planes, lines and quadrics | 66 |

vi Contents

| 2.4 The letered Control | |
|--|-------------------------|
| 3.4 The hierarchy of transformations | 77 |
| 3.5 The plane at infinity | 79 |
| 3.6 The absolute conic | 81 |
| 3.7 The absolute dual quadric | 83 |
| 3.8 Closure | 85 |
| 4 Estimation – 2D Projective Transformations | 87 |
| 4.1 The Direct Linear Transformation (DLT) al | gorithm 88 |
| 4.2 Different cost functions | 93 |
| 4.3 Statistical cost functions and Maximum Lik | telihood estimation 102 |
| 4.4 Transformation invariance and normalization | on 104 |
| 4.5 Iterative minimization methods | 110 |
| 4.6 Experimental comparison of the algorithms | 115 |
| 4.7 Robust estimation | 116 |
| 4.8 Automatic computation of a homography | 123 |
| 4.9 Closure | 127 |
| 5 Algorithm Evaluation and Error Analysis | 132 |
| 5.1 Bounds on performance | 132 |
| 5.2 Covariance of the estimated transformation | 138 |
| 5.3 Monte Carlo estimation of covariance | 149 |
| 5.4 Closure | 150 |
| PART I: Camera Geometry and Single View Geome | try 151 |
| Outline | 152 |
| 6 Camera Models | 153 |
| 6.1 Finite cameras | 153 |
| 6.2 The projective camera | 158 |
| 6.3 Cameras at infinity | 166 |
| 6.4 Other camera models | 174 |
| 6.5 Closure | 176 |
| 7 Computation of the Camera Matrix P | 178 |
| 7.1 Basic equations | 178 |
| 7.2 Geometric error | 180 |
| 7.3 Restricted camera estimation | 184 |
| 7.4 Radial distortion | 189 |
| 7.5 Closure | 193 |
| 8 More Single View Geometry | 195 |
| 8.1 Action of a projective camera on planes, lin | |
| 8.2 Images of smooth surfaces | 200 |
| 8.3 Action of a projective camera on quadrics | 201 |
| 8.4 The importance of the camera centre | 202 |
| 8.5 Camera calibration and the image of the ab | |

| | | Contents | vii |
|-----|----------|--|-----|
| | 8.6 | Vanishing points and vanishing lines | 213 |
| | 8.7 | Affine 3D measurements and reconstruction | 220 |
| | 8.8 | Determining camera calibration K from a single view | 223 |
| | 8.9 | Single view reconstruction | 229 |
| | 8.10 | The calibrating conic | 231 |
| | 8.11 | Closure | 233 |
| PA] | RT II: T | Гwo-View Geometry | 237 |
| | Outline | • | 238 |
| 9 | Epipo | lar Geometry and the Fundamental Matrix | 239 |
| | 9.1 | Epipolar geometry | 239 |
| | 9.2 | The fundamental matrix F | 241 |
| | 9.3 | Fundamental matrices arising from special motions | 247 |
| | 9.4 | Geometric representation of the fundamental matrix | 250 |
| | 9.5 | Retrieving the camera matrices | 253 |
| | 9.6 | The essential matrix | 257 |
| | 9.7 | Closure | 259 |
| 10 | 3D Re | econstruction of Cameras and Structure | 262 |
| | 10.1 | Outline of reconstruction method | 262 |
| | 10.2 | Reconstruction ambiguity | 264 |
| | 10.3 | The projective reconstruction theorem | 266 |
| | 10.4 | Stratified reconstruction | 267 |
| | 10.5 | Direct reconstruction – using ground truth | 275 |
| | 10.6 | Closure | 276 |
| 11 | Comp | outation of the Fundamental Matrix F | 279 |
| | 11.1 | Basic equations | 279 |
| | 11.2 | The normalized 8-point algorithm | 281 |
| | 11.3 | The algebraic minimization algorithm | 282 |
| | 11.4 | Geometric distance | 284 |
| | 11.5 | Experimental evaluation of the algorithms | 288 |
| | 11.6 | Automatic computation of F | 290 |
| | 11.7 | Special cases of F-computation | 293 |
| | 11.8 | Correspondence of other entities | 294 |
| | 11.9 | Degeneracies | 295 |
| | | A geometric interpretation of F-computation | 297 |
| | | The envelope of epipolar lines | 298 |
| | | Image rectification | 302 |
| | | Closure | 308 |
| 12 | 1 | | 310 |
| | 12.1 | Problem statement | 310 |
| | 12.2 | Linear triangulation methods | 312 |
| | 12.3 | Geometric error cost function | 313 |
| | 12.4 | Sampson approximation (first-order geometric correction) | 314 |

viii Contents

| | 12.5 | An optimal solution | 315 |
|----|---------|--|-----|
| | 12.6 | Probability distribution of the estimated 3D point | 321 |
| | 12.7 | Line reconstruction | 321 |
| | 12.8 | Closure | 323 |
| 13 | Scene | planes and homographies | 325 |
| | 13.1 | Homographies given the plane and vice versa | 326 |
| | 13.2 | Plane induced homographies given F and image correspondences | 329 |
| | 13.3 | Computing F given the homography induced by a plane | 334 |
| | 13.4 | The infinite homography H_{∞} | 338 |
| | 13.5 | Closure | 340 |
| 14 | Affin | e Epipolar Geometry | 344 |
| | 14.1 | Affine epipolar geometry | 344 |
| | 14.2 | The affine fundamental matrix | 345 |
| | 14.3 | Estimating F _A from image point correspondences | 347 |
| | 14.4 | Triangulation | 353 |
| | 14.5 | Affine reconstruction | 353 |
| | 14.6 | Necker reversal and the bas-relief ambiguity | 355 |
| | | Computing the motion | 357 |
| | 14.8 | Closure | 360 |
| PA | RT III: | Three-View Geometry | 363 |
| | Outlin | · | 364 |
| 15 | The T | Trifocal Tensor | 365 |
| | | The geometric basis for the trifocal tensor | 365 |
| | 15.2 | The trifocal tensor and tensor notation | 376 |
| | 15.3 | Transfer | 379 |
| | 15.4 | The fundamental matrices for three views | 383 |
| | 15.5 | Closure | 387 |
| 16 | Comp | outation of the Trifocal Tensor ${\mathcal T}$ | 391 |
| | 16.1 | Basic equations | 391 |
| | 16.2 | The normalized linear algorithm | 393 |
| | 16.3 | The algebraic minimization algorithm | 395 |
| | 16.4 | Geometric distance | 396 |
| | 16.5 | Experimental evaluation of the algorithms | 399 |
| | 16.6 | Automatic computation of \mathcal{T} | 400 |
| | 16.7 | Special cases of \mathcal{T} -computation | 404 |
| | 16.8 | Closure | 406 |
| PA | RT IV: | N-View Geometry | 409 |
| | Outlin | • | 410 |
| 17 | N-Li | nearities and Multiple View Tensors | 411 |
| - | 17.1 | Bilinear relations | 411 |
| | | Trilinear relations | 414 |

| | | Contents | ix |
|----|--------|---|-----|
| | 17.3 | Quadrilinear relations | 418 |
| | 17.4 | Intersections of four planes | 421 |
| | 17.5 | Counting arguments | 422 |
| | 17.6 | Number of independent equations | 428 |
| | 17.7 | Choosing equations | 431 |
| | 17.8 | Closure | 432 |
| 18 | N-Vie | ew Computational Methods | 434 |
| | 18.1 | Projective reconstruction – bundle adjustment | 434 |
| | 18.2 | Affine reconstruction – the factorization algorithm | 436 |
| | 18.3 | Non-rigid factorization | 440 |
| | 18.4 | Projective factorization | 444 |
| | 18.5 | Projective reconstruction using planes | 447 |
| | 18.6 | Reconstruction from sequences | 452 |
| | 18.7 | Closure | 456 |
| 19 | Auto- | Calibration | 458 |
| | 19.1 | Introduction | 458 |
| | 19.2 | Algebraic framework and problem statement | 459 |
| | 19.3 | Calibration using the absolute dual quadric | 462 |
| | 19.4 | The Kruppa equations | 469 |
| | 19.5 | A stratified solution | 473 |
| | 19.6 | Calibration from rotating cameras | 481 |
| | 19.7 | Auto-calibration from planes | 485 |
| | 19.8 | Planar motion | 486 |
| | 19.9 | Single axis rotation – turntable motion | 490 |
| | 19.10 | Auto-calibration of a stereo rig | 493 |
| | 19.11 | Closure | 497 |
| 20 | Dualit | tv | 502 |
| | | Carlsson–Weinshall duality | 502 |
| | | Reduced reconstruction | 508 |
| | 20.3 | Closure | 513 |
| 21 | Cheir | ality | 515 |
| -1 | 21.1 | Quasi-affine transformations | 515 |
| | 21.2 | Front and back of a camera | 518 |
| | 21.2 | Three-dimensional point sets | 519 |
| | 21.4 | Obtaining a quasi-affine reconstruction | 520 |
| | 21.5 | Effect of transformations on cheirality | 521 |
| | 21.6 | Orientation | 523 |
| | 21.7 | The cheiral inequalities | 525 |
| | | - 110 The fire qualities | 525 |

528

530

531

21.8 Which points are visible in a third view

21.9 Which points are in front of which

21.10 Closure

x Contents

| 22 Dogg | navata Canfigurations | 522 |
|---|---|------------|
| | nerate Configurations | 533 |
| 22.1 | Camera resectioning | 533 |
| 22.2 | Degeneracies in two views | 539 |
| 22.3 | Carlsson–Weinshall duality | 546 |
| 22.4 | Three-view critical configurations | 553 |
| 22.5 | Closure | 558 |
| PART V | Appendices | 561 |
| Appendix 1Tensor NotationAppendix 2Gaussian (Normal) and χ^2 Distributions | | 562 565 |
| | | |
| Appendix | 4 Matrix Properties and Decompositions | 578 |
| Appendix | 5 Least-squares Minimization | 588 |
| Appendix | 6 Iterative Estimation Methods | 597 |
| Appendix | 7 Some Special Plane Projective Transformations | 628 |
| Bibliograp | phy | 634 |
| Index | | 646 |