

# Part III

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## Three-View Geometry



*Lord Shiva, c. 1920-40 (print).*

Shiva is depicted as having three eyes. The third eye in the centre of the forehead symbolizes spiritual knowledge and power.

Image courtesy of <http://www.healthyplanetonline.com>

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## Outline

This part contains two chapters on the geometry of three-views. The scene is imaged with three cameras perhaps simultaneously in a trinocular rig, or sequentially from a moving camera.

Chapter 15 introduces a new multiple view object – the trifocal tensor. This has analogous properties to the fundamental matrix of two-view geometry: it is independent of scene structure depending only on the (projective) relations between the cameras. The camera matrices may be retrieved from the trifocal tensor up to a common projective transformation of 3-space, and the fundamental matrices for view-pairs may be retrieved uniquely.

The new geometry compared with the two-view case is the ability to *transfer* from two views to a third: given a point correspondence over two views the position of the point in the third view is determined; and similarly, given a *line* correspondence over two views the position of the line in the third view is determined. This transfer property is of great benefit when establishing correspondences over multiple views.

If the essence of the epipolar constraint over two views is that rays back-projected from corresponding points are coplanar, then the essence of the trifocal constraint over three views is the geometry of a point–line–line correspondence arising from the image of a point on a line in 3-space: corresponding image lines in two views back-project to planes which intersect in a line in 3-space, and the ray back-projected from a corresponding image point in a third view must intersect this line.

Chapter 16 describes the computation of the trifocal tensor from point and *line* correspondences over three-views. Given the tensor, and thus the retrieved camera matrices, a projective reconstruction may be computed from correspondences over multiple views. The reconstruction may be upgraded to similarity or metric as additional information is provided in the same manner as in the two view case.

It is in reconstruction that there is another gain over two-view geometry. Given the cameras, in the two-view case each point correspondence provided four measurements on the three degrees of freedom (the position) of the point in 3-space. In three views there are six measurements on, again, three degrees of freedom. However, it is for lines that there is the more significant gain. In two-views the number of measurements equals the number of degrees of freedom of the line in 3-space, namely four. Consequently, there is no possibility of removing the effects of measurement errors. However, in three views there are six measurements on four degrees of freedom, so a scene line is over-determined and can be estimated by a suitable minimization over measurement errors.