Syllabus of an Aperitif of the Rigour of Mathematics

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June 14, 2019

To deliver each concept, I would go from concrete to abstract to concrete, so I would often begin with real life situations which would lead to abstract definitions after which I would provide many examples. I would also include many extensions and historical contexts, hopefully they can enable the students to realize the importance of those concepts and learn to appreciate them. I try to reduce the load of homework. Most of the homework are designed to ensure students' understanding of my teaching. After each class, I will stay for twenty to thirty minutes to answer students' question.

1 Day 1: Distance

Before we start

I will provide a brief introduction of the course, choose a leader to help me on the field trip (as I am not local), introduce myself and let the students introduce themselves.

1.1 Warm Up

Draw a grid on the black/white board with points on it. Find the distance between the points. Thus introducing the three properties of distance function (or metric)

1.2 Definition

Provide a rigorous definition of distance.

1.3 Examples

Examples of distance on R^n, on the set of converging sequences(we will know what this means later), on continuous(we will know what this means later) functions on (0,1).

1.4 Extension

Why do we define it like this? The uniqueness of the limit of a sequence.(we will know what this means later) Why this concept is really important? Normed linear spaces, L^p spaces, and inner product spaces as metric spaces.

1.5 Homework

1. Provide an example of metric space, and verify that the distance function satisfies indeed the three properties.

2. Read the preface of *What is Mathematics?*

2 Day 2: Real Numbers

2.1 Warm Up

1. Recall what we have learned about real numbers in our first classes in middle school.

2. The story of primitive people who wants to compare the number of coconuts and spears

2.2 Properties

1. Prove that $\sqrt{2}$ is irrational.

2. the existence of rational number between any two rational numbers.

3. exists no biggest rational number smaller than $\sqrt{2}$, and no smallest number bigger than it.

2.3 Definition

A not so formal explanation of Dedekind cut to define real numbers.

2.4 Interesting Facts

Comparing the cardinal numbers of natural numbers, rational numbers and real numbers. Not all infinities are equal!

2.5 Extension

1. A glimpse of history about numbers: from Pythagoras to Cantor.
2. Different points of view emerging at the end of 19th century.
3. Why do we need this definition?

2.6 Homework

1. Read chapter 29 of *Men of Mathematics*
2. Read chapter 2 part 2 of *What is Mathematics?*. You may skip 2.5.

3 Day 3: Limit 1

3.1 Warm Up

A debate about whether 0.99999... equals to 1. What does 0.99999... actually mean?

3.2 Definition

Provide a descriptive definition of limit, and a formal one with the inequalities involving ε and δ. Describe this definition intuitively using the example of hitting a target with arrows.

3.3 Examples

I would provide two to three examples of limits of sequences and two to three of limits of series.

3.4 Extension

1. Some paradoxes about limit.
2. Using Limit to define continuity and differentiation.

I wish that the students can do it by themselves.

3.5 Homework

1. Read Part 2.1 of Chapter 4 of *What is Mathematics?*
2. Find a sequence that diverges, a series that diverges, a function that is continuous at 0, but is not differentiable at 0.

4 Day 4: Limit 2

4.1 Review

Review the definitions of limit, continuity and differentiation.

4.2 theorem

Bolzano's theorem: an intuitive lead in, its proof, and an application of it.

4.3 History

1. The Ancient Greek's encounter of infinitesimal.
2. Berkeley's doubts against fluxion.
3. Cauchy's achievement
4. Weierstrass's achievement.

4.4 Homework

1. Read chapter 5 of*The Gallery of Calculus*
2. Draw a chronology of the development of limit.

5 Day 5: Field Trip

6 Day 6: Area 1

6.1 Warm Up

Find the area of triangle, of an oil stain, the area under the graph of a function.

6.2 definition

Define area in terms of limit: Riemann integral.

6.3 Examples

Three examples that it works and one that fails.

History

1. The story of Archimedes
2. The story of Isaac Newton
3. The story of Riemann

6.4 Homework

1. Chapter 8, 1.1, 1.2, 1.3 of *What is Mathematics?*
2. Read the part about Riemann integral of Chapter 7 of R*The Gallery of Calculus*.
3. Prepare an evaluation of my teaching.

7 Day 7: Area 2

7.1 Limitations of Riemann Integral

7.2 Introducing Lebesgue Integral

7.3 Example Lebesgue Integral

7.4 A conclusion

I will first review all the definitions that I have mentioned and reiterate their importance. I will than suggest my students several further readings. I will talk with the students to evaluate my ways of teaching during the past week and the improvement that I need to make and discuss what is the best way to teach mathematics.