



Implemented as simple rule



Implemented as programmed rule



Implicitly covered by other rules

## CHAPTER IV

### LAPLACE TRANSFORMS

#### 4.1. General formulas

	$f(t)$	$g(p) = \int_0^{\infty} e^{-pt} f(t) dt$
(1)	$\frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{zt} g(z) dz$	$g(p)$
(2)	$f(t+a) = f(t)$	$(1-e^{-ap})^{-1} \int_0^a e^{-pt} f(t) dt$
(3)	$f(t+a) = -f(t)$	$(1+e^{-ap})^{-1} \int_0^a e^{-pt} f(t) dt$
(4)	$0 \quad t < ba^{-1}$ $f(at-b) \quad t > ba^{-1}$ $a, b > 0$	$a^{-1} e^{-ba^{-1} p} g(a^{-1} p)$
(5)	$e^{-at} f(t)$	$g(p+a)$
(6)	$t^n f(t)$	$(-1)^n \frac{d^n g(p)}{dp^n}$
(7)	$t^{-n} f(t)$	$\int_p^{\infty} \cdots \int_p^{\infty} g(p) (dp)^n$ $n$ -th repeated integral
(8)	$f^{(n)}(t)$	$p^n g(p) - p^{n-1} f(0) - p^{n-2} f'(0) - \dots - f^{(n-1)}(0)$

## General formulas (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(9)	$\int_0^t \cdots \int_0^t f(t) (dt)^n$	$p^{-n} g(p)$
(10)	$\left( t \frac{d}{dt} \right)^n f(t)$ where e.g. $\left( t \frac{d}{dt} \right)^2 f(t) = t \frac{d}{dt} \left\{ t \frac{d}{dt} [f(t)] \right\}$	$\left( -\frac{d}{dp} p \right)^n g(p)$ $\left( \frac{d}{dp} p \right)^2 g(p) = \frac{d}{dp} \left\{ p \frac{d}{dp} [pg(p)] \right\}$
(11)	$\left( \frac{d}{dt} t \right)^n f(t)$	$\left( -p \frac{d}{dp} \right)^n g(p)$
(12)	$\left( t^{-1} \frac{d}{dt} \right)^n f(t)$ if $\left( \frac{1}{t} \frac{d}{dt} \right)^k f(t) = 0$ for $t = 0, k = 0, \dots, n-1$	$\int_p^\infty p \int_p^\infty \cdots p \int_p^\infty pg(p) (dp)^n$
(13)	$t^m f^{(n)}(t)$	$m \geq n$ $\left( -\frac{d}{dp} \right)^n [p^n g(p)]$
(14)	$t^m f^{(n)}(t)$	$m < n$ $\begin{aligned} & \left( -\frac{d}{dp} \right)^n [p^n g(p)] + (-1)^{n-m} \\ & \times \left[ \frac{(n-1)!}{(n-m-1)!} p^{n-m-1} f(0) \right. \\ & + \frac{(n-2)!}{(n-m-2)!} p^{n-m-2} f'(0) \\ & \left. + \cdots + m! f^{(n-m-1)}(0) \right] \end{aligned}$

## General formulas (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(15)	$\frac{d^n}{dt^n} [t^m f(t)] \quad m \geq n$	$(-1)^m p^n g^{(m)}(p)$
(16)	$\frac{d^n}{dt^n} [t^m f(t)] \quad m < n$	$(-1)^m p^n g^{(m)}(p) - m! p^{n-m-1} f(0)$ $- \frac{(m+1)!}{1!} p^{n-m-2} f'(0)$ $- \dots - \frac{(n-1)!}{(n-m-1)!} f^{(n-m-1)}(0)$
(17)	$\left( e^t \frac{d}{dt} \right)^n f(t)$ provided that $f^{(k)}(0) = 0$ for $k = 0, 1, \dots, n-1$	$(p-1)\dots(p-n) g(p-n)$
(18)	$\int_0^t t^{-1} f(t) dt$	$p^{-1} \int_p^\infty g(p) dp$
(19)	$\int_t^\infty t^{-1} f(t) dt$	$p^{-1} \int_0^p g(p) dp$
(20)	$\int_0^t f_1(u) f_2(t-u) du$	$g_1(p) g_2(p)$
(21)	$f_1(t) f_2(t)$	$\frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} g_1(z) g_2(p-z) dz$
(22)	$f(t^2)$	$\pi^{-\frac{1}{2}} \int_0^\infty e^{-\frac{1}{4}p^2 u^{-2}} g(u^2) du$
(23)	$t^n f(t^2)$	$2^{-\frac{1}{2}n} \pi^{-\frac{1}{2}} \int_0^\infty u^{n-2} e^{-\frac{1}{4}p^2 u^2} \times \text{He}_n(2^{-\frac{1}{2}} pu) g(u^{-2}) du$

## General formulas (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(24)	$t^\nu f(t^2)$	$2^{-\frac{\nu}{2}} \pi^{-\frac{1}{2}} \int_0^\infty u^{\nu-2} e^{-\frac{1}{4}p^2 u^2} \times D_\nu(pu) g(\frac{1}{2}u^{-2}) du$
(25)	$t^{\nu-1} f(t^{-1})$	$\text{Re } \nu > -1$ $p^{-\frac{\nu}{2}} \nu \int_0^\infty u^{\frac{\nu}{2}-1} J_\nu(2u^{\frac{1}{2}} p^{\frac{1}{2}}) g(u) du$
(26)	$f(ae^{t-a})$	$a > 0$ $[a \Gamma(p+1)]^{-1} \int_0^\infty e^{-u} u^p g(u/a) du$
(27)	$f(a \sinh t)$	$a > 0$ $\int_0^\infty J_p(au) g(u) du$
(28)	$\sum_{n=1}^{\infty} n^{-1} f(n^{-1}t)$	$\int_0^\infty (e^{pu} - 1)^{-1} f(u) du$
(29)	$\int_0^\infty \frac{t^{u-1}}{\Gamma(u)} f(u) du$	$g(\log p)$
(30)	$\int_0^\infty u^{-\frac{\nu}{2}} \sin(2u^{\frac{1}{2}} t^{\frac{1}{2}}) f(u) du$	$\pi^{1/2} p^{-3/2} g(p^{-1})$
(31)	$t^{-\frac{\nu}{2}} \int_0^\infty \cos(2u^{\frac{1}{2}} t^{\frac{1}{2}}) f(u) du$	$\pi^{\frac{\nu}{2}} p^{-\frac{\nu}{2}} g(p^{-1})$
(32)	$t^\nu \int_0^\infty J_{2\nu}(2u^{\frac{1}{2}} t^{\frac{1}{2}}) u^{-\nu} f(u) du$	$p^{-2\nu-1} g(p^{-1})$
(33)	$t^{-\frac{\nu}{2}} \int_0^\infty e^{-\frac{1}{4}u^2/t} f(u) du$	$\pi^{\frac{\nu}{2}} p^{-\frac{\nu}{2}} g(p^{\frac{1}{2}})$
(34)	$t^{-\frac{\nu}{2}n-\frac{\nu}{2}} \int_0^\infty e^{-\frac{1}{4}u^2/t}$ $\times \text{He}_n(2^{-\frac{\nu}{2}} ut^{-\frac{1}{2}}) f(u) du$	$2^{\frac{\nu}{2}n} \pi^{\frac{\nu}{2}n-\frac{\nu}{2}} g(p^{\frac{1}{2}})$

## General formulas (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(35)	$t^{-\nu} \int_0^\infty e^{-u^2/(8t)} \times D_{2\nu-1}(2^{-\frac{u}{4t}} ut^{-\frac{\nu}{2}}) f(u) du$	$2^{\nu-\frac{1}{2}} \pi^{\frac{\nu}{2}} p^{\nu-1} g(p^{\frac{\nu}{2}})$
(36)	$\int_0^t \left(\frac{t-u}{au}\right)^\nu \times J_{2\nu}[2(aut - au^2)^{\frac{\nu}{2}}] f(u) du$	$p^{-2\nu-1} g(p + ap^{-1})$
(37)	$\int_0^t J_0[(t^2 - u^2)^{\frac{\nu}{2}}] f(u) du$	$(p^2 + 1)^{-\frac{\nu}{2}} g[(p^2 + 1)^{\frac{\nu}{2}}]$
(38)	$f(t) - \int_0^t J_1(u) f[(t^2 - u^2)^{\frac{\nu}{2}}] du$	$g[(p^2 + 1)^{\frac{\nu}{2}}]$
(39)	$\int_0^t \left(\frac{t-u}{t+u}\right)^\nu \times J_{2\nu}[(t^2 - u^2)^{\frac{\nu}{2}}] f(u) du$	$(p^2 + 1)^{-\frac{\nu}{2}} [(p^2 + 1)^{\frac{\nu}{2}} + p]^{-2\nu} \times g[(p^2 + 1)^{\frac{\nu}{2}}]$

## 4.2. Algebraic functions

(1)	1	$p^{-1}$	$\operatorname{Re} p > 0$
(2)	0 $0 < t < a$ 1 $a < t < b$ 0 $t > b$	$p^{-1}(e^{-ap} - e^{-bp})$	
(3)	$t^n$	$n! p^{-n-1}$	$\operatorname{Re} p > 0$
(4)	0 $0 < t < b$ $t^n$ $t > b$	$e^{-bp} \sum_{m=0}^n \frac{n!}{m!} \frac{b^m}{p^{n-m+1}}$	$\operatorname{Re} p > 0$

## Algebraic functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(5)	$t^n$ 0 $t > b$	$\frac{n!}{p^{n+1}} - e^{-bp} \sum_{m=0}^n \frac{n!}{m!} \frac{b^m}{p^{n-m+1}}$
(6)	0 $(t+a)^{-1}$ $t > b$ $ \arg(a+b)  < \pi$	$-e^{ap} \operatorname{Ei}[-(a+b)p] \quad \operatorname{Re} p > 0$
(7)	0 $(t+a)^{-1}$ 0 $t > c$ $-a \text{ not between } b \text{ and } c$	$e^{ap} \{ \operatorname{Ei}[-(a+c)p] - \operatorname{Ei}[-(a+b)p] \}$
(8)	$(t-a)^{-1}$ $a \geq 0$	$-e^{-ap} \overline{\operatorname{Ei}}(ap) \quad \operatorname{Re} p > 0$ The integral is a Cauchy Principal value
(9)	$\frac{1}{(t+a)^n}$ $n \geq 2, \quad  \arg a  < \pi$	$\sum_{m=1}^{n-1} \frac{(m-1)!}{(n-1)!} \frac{(-p)^{n-m-1}}{a^m}$ $- \frac{(-p)^{n-1}}{(n-1)!} e^{ap} \operatorname{Ei}(-ap)$ $\operatorname{Re} p \geq 0$
(10)	0 $(t+a)^{-n}$ $t > b$ $ \arg(a+b)  < \pi, \quad n \geq 2$	$e^{-bp} \sum_{m=1}^{n-1} \frac{(m-1)!}{(n-1)!} \frac{(-p)^{n-m-1}}{(a+b)^m}$ $- \frac{(-p)^{n-1}}{(n-1)!} e^{ap} \operatorname{Ei}[-(a+b)p]$ $\operatorname{Re} p > 0$
For further formulas of a similar type see Bierens de Haan,D., 1867: <i>Nouvelles tables d'intégrales définies</i> , 727 p.		

## Algebraic functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(11)	$t^n (t+a)^{-1} \quad n \geq 1, \quad  \arg a  < \pi$	$(-)^{n-1} a^n e^{\alpha p} \operatorname{Ei}(-ap) + \sum_{m=1}^n (m-1)! (-a)^{n-m} p^{-m}$ $\operatorname{Re} p > 0$
(12)	$(At+B\alpha)(t^2-\alpha^2)^{-1} \quad  \arg(\pm i\alpha)  < \pi$	$-\frac{1}{2}(A-B)e^{\alpha p} \operatorname{Ei}(-ap) - \frac{1}{2}(A+B)e^{-\alpha p} \operatorname{Ei}(ap) \quad \operatorname{Re} p > 0$
(13)	$(At+B\alpha)(t^2-\alpha^2)^{-1} \quad a > 0$ Cauchy Principal value	$-\frac{1}{2}(A-B)e^{-\alpha p} \operatorname{Ei}(-ap) - \frac{1}{2}(A+B)e^{-\alpha p} \overline{\operatorname{Ei}}(ap) \quad \operatorname{Re} p > 0$
(14)	$(At+B\alpha)(t^2+\alpha^2)^{-1} \quad  \arg(\pm i\alpha)  < \pi$	$(A \cos ap - B \sin ap) \operatorname{ci}(ap) - (A \sin ap + B \cos ap) \operatorname{si}(ap)$ $\operatorname{Re} p > 0$
(15)	$0 \quad t^{-\frac{1}{2}}$	$\pi^{\frac{1}{2}} p^{-\frac{1}{2}} \operatorname{Erfc}(b^{\frac{1}{2}} p^{\frac{1}{2}}) \quad \operatorname{Re} p > 0$
(16)	$t^{-\frac{1}{2}} \quad 0$	$\pi^{\frac{1}{2}} p^{-\frac{1}{2}} \operatorname{Erf}(b^{\frac{1}{2}} p^{\frac{1}{2}})$
(17)	$t^{n-\frac{1}{2}}$	$\pi^{\frac{1}{2}} 1/2 (3/2) \dots (n-1/2) p^{-n-\frac{1}{2}} \quad \operatorname{Re} p > 0$
(18)	$(t+a)^{-\frac{1}{2}} \quad  \arg a  < \pi$	$\pi^{\frac{1}{2}} p^{-\frac{1}{2}} e^{\alpha p} \operatorname{Erfc}(\alpha^{\frac{1}{2}} p^{\frac{1}{2}}) \quad \operatorname{Re} p > 0$
(19)	$0 \quad t^{-3/2}$	$2b^{-\frac{1}{2}} e^{-bp} - 2\pi^{\frac{1}{2}} p^{\frac{1}{2}} \operatorname{Erfc}(b^{\frac{1}{2}} p^{\frac{1}{2}}) \quad \operatorname{Re} p \geq 0$
(20)	$(t+a)^{-3/2} \quad  \arg a  < \pi$	$2a^{-\frac{1}{2}} - 2\pi^{\frac{1}{2}} p^{\frac{1}{2}} e^{\alpha p} \operatorname{Erfc}(\alpha^{\frac{1}{2}} p^{\frac{1}{2}}) \quad \operatorname{Re} p \geq 0$

## Algebraic functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(22)	$t^{\frac{1}{2}}(t+a)^{-1}$ $ arg a  < \pi$	$(\pi/p)^{\frac{1}{2}} - \pi a^{\frac{1}{2}} e^{\alpha p} \operatorname{Erfc}(a^{\frac{1}{2}} p^{\frac{1}{2}})$ $\operatorname{Re} p > 0$
(23)	$0$ $t^{-1}(t-b)^{\frac{1}{2}}$ $0 < t < b$ $t > b$	$(\pi/p)^{\frac{1}{2}} e^{-bp} - \pi b^{\frac{1}{2}} \operatorname{Erfc}(b^{\frac{1}{2}} p^{\frac{1}{2}})$ $\operatorname{Re} p > 0$
(24)	$t^{-\frac{1}{2}}(1+2\alpha t)$	$\pi^{1/2} p^{-3/2} (p+a)$ $\operatorname{Re} p > 0$
(25)	$t^{-\frac{1}{2}}(t+a)^{-1}$ $ arg a  < \pi$	$\pi a^{-\frac{1}{2}} e^{\alpha p} \operatorname{Erfc}(a^{\frac{1}{2}} p^{\frac{1}{2}})$ $\operatorname{Re} p \geq 0$
(26)	$0$ $t^{-1}(t-b)^{-\frac{1}{2}}$ $0 < t < b$ $t > b$	$\pi b^{-\frac{1}{2}} \operatorname{Erfc}(b^{\frac{1}{2}} p^{\frac{1}{2}})$ $\operatorname{Re} p \geq 0$
(27)	$t(t^2 + a^2)^{-\frac{1}{2}}$ $ arg a  < \pi/2$	$\frac{1}{2}\pi a [\mathbf{H}_1(ap) - Y_1(ap)] - a \operatorname{Re} p > 0$
(28)	$t(b^2 - t^2)^{-\frac{1}{2}}$ $0 < t < b$ $0$ $t > b$	$\frac{1}{2}\pi b [\mathbf{L}_1(bp) - I_1(bp)] + b \operatorname{Re} p > 0$
(29)	$0$ $t(t^2 - b^2)^{-\frac{1}{2}}$ $0 < t < b$ $t > b$	$b K_1(bp)$ $\operatorname{Re} p > 0$
(30)	$(t^2 + 2\alpha t)^{-\frac{1}{2}}(t+a)$ $ arg a  < \pi$	$\alpha e^{\alpha p} K_1(ap)$ $\operatorname{Re} p > 0$
(31)	$(2bt - t^2)^{-\frac{1}{2}}(b-t)$ $0 < t < 2b$ $0$ $t > 2b$	$\pi b e^{-bp} I_1(bp)$ $\operatorname{Re} p > 0$
(32)	$[t + (t^2 + a^2)^{\frac{1}{2}}]^{-1}$ $ arg a  < \pi/2$	$\frac{1}{2}\pi a^{-1} p^{-1} [\mathbf{H}_1(ap) - Y_1(ap)] - a^{-2} p^{-2}$ $\operatorname{Re} p > 0$
(33)	$\sin \theta (1+t+\cos \theta)^{-1} (t^2 + 2t)^{-\frac{1}{2}}$	$\exp[2p \cos^2(\frac{1}{2}\theta)]$ $\times [\theta - \sin \theta \int_0^p K_0(v) e^{-v \cos \theta} dv]$ $\operatorname{Re} p > 0$

**Algebraic functions (cont'd)**

	$f(t)$	$g(y) = \int_0^\infty e^{-pt} f(t) dt$
(34)	$[t + (1+t^2)^{\frac{1}{2}}]^n + [t - (1+t^2)^{\frac{1}{2}}]^n$	$2O_n(p)$ $\operatorname{Re} p > 0$
(35)	$[t + (1+t^2)^{\frac{1}{2}}]^n (1+t^2)^{-\frac{n}{2}}$	$\frac{1}{2}[S_n(p) - \pi E_n(p) - \pi Y_n(p)]$ $\operatorname{Re} p > 0$
(36)	$[t - (1+t^2)^{\frac{1}{2}}]^n (1+t^2)^{-\frac{n}{2}}$	$-\frac{1}{2}[S_n(p) + \pi E_n(p) + \pi Y_n(p)]$ $\operatorname{Re} p > 0$

**4.3. Powers with an arbitrary index**

(1)	$t^\nu$	$\operatorname{Re} \nu > -1$	$\Gamma(\nu+1)p^{-\nu-1}$	$\operatorname{Re} p > 0$
(2)	$0$ $t^\nu$	$0 < t < b$ $t > b$	$p^{-\nu-1}\Gamma(\nu+1, bp)$	$\operatorname{Re} p > 0$
(3)	$t^\nu$ $0$	$0 < t < b$ $t > b$ $\operatorname{Re} \nu > -1$	$p^{-\nu-1}\gamma(\nu+1, bp)$	
(4)	$(t+a)^\nu$	$ \arg a  < \pi$	$p^{-\nu-1}e^{\alpha p}\Gamma(\nu+1, \alpha p)$	$\operatorname{Re} p > 0$
(5)	$0$ $(t-b)^\nu$	$0 < t < b$ $t > b$ $\operatorname{Re} \nu > -1$	$\Gamma(\nu+1)p^{-\nu-1}e^{-bp}$	$\operatorname{Re} p > 0$
(6)	$(b-t)^\nu$ $0$	$0 < t < b$ $t > b$ $\operatorname{Re} \nu > -1$	$p^{-\nu-1}e^{-bp}\gamma(\nu+1, -bp)$	
(7)	$t^\nu(t+a)^{-1}$	$ \arg a  < \pi, \quad \operatorname{Re} \nu > -1$	$\Gamma(\nu+1)\alpha^\nu e^{\alpha p}\Gamma(-\nu, \alpha p)$	$\operatorname{Re} p > 0$

## Arbitrary powers (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(8)	$0 \quad 0 < t < b$ $t^{-1}(t-b)^\nu \quad t > b$ $\text{Re } \nu > -1$	$\Gamma(\nu+1)b^\nu \Gamma(-\nu, bp) \quad \text{Re } p > 0$
(9)	$t^{\nu-1}(1+t^2)^{-1} \quad \text{Re } \nu > 0$	$\pi \csc(\nu\pi) V_\nu(2p, 0) \quad \text{Re } p > 0$
(10)	$(1+t^2)^{\nu-\frac{1}{2}}$	$2^{\nu-1} \pi^{\frac{1}{2}} \Gamma(\nu+\frac{1}{2}) p^{-\nu} [\mathbf{H}_\nu(p) - Y_\nu(p)] \quad \text{Re } p > 0$
(11)	$0 \quad 0 < t < b$ $(t^2-b^2)^{\nu-\frac{1}{2}} \quad t > b$ $\text{Re } \nu > -\frac{1}{2}$	$\pi^{-\frac{1}{2}} \Gamma(\nu+\frac{1}{2})(2b/p)^\nu K_\nu(bp) \quad \text{Re } p > 0$
(12)	$(b^2-t^2)^{\nu-\frac{1}{2}} \quad 0 < t < b$ $0 \quad t > b$ $\text{Re } \nu > -\frac{1}{2}$	$\frac{1}{2}\pi^{\frac{1}{2}} \Gamma(\nu+\frac{1}{2})(2b/p)^\nu [I_\nu(bp) - \mathbf{L}_\nu(bp)]$
(13)	$(t^2+2at)^{\nu-\frac{1}{2}} \quad  \arg a  < \pi, \quad \text{Re } \nu > -\frac{1}{2}$	$\pi^{-\frac{1}{2}} \Gamma(\nu+\frac{1}{2})(2a/p)^\nu e^{ap} K_\nu(ap) \quad \text{Re } p > 0$
(14)	$(2bt-t^2)^{\nu-\frac{1}{2}} \quad 0 < t < 2b$ $0 \quad t > 2b$ $\text{Re } \nu > -\frac{1}{2}$	$\pi^{\frac{1}{2}} \Gamma(\nu+\frac{1}{2})(2b/p)^\nu e^{-bp} I_\nu(bp)$
(15)	$(t^2+it)^{\nu-\frac{1}{2}} \quad \text{Re } \nu \neq -\frac{1}{2}$	$-\frac{1}{2}i \pi^{\frac{1}{2}} \Gamma(\nu+\frac{1}{2}) p^{-\nu} e^{\frac{1}{2}ip} H_\nu^{(2)}(\frac{1}{2}p) \quad \text{Re } p > 0$
(16)	$(t^2-it)^{\nu-\frac{1}{2}} \quad \text{Re } \nu > -\frac{1}{2}$	$\frac{1}{2}i \pi^{\frac{1}{2}} \Gamma(\nu+\frac{1}{2}) p^{-\nu} e^{-\frac{1}{2}ip} H_\nu^{(1)}(\frac{1}{2}p) \quad \text{Re } p > 0$

## Arbitrary powers (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(17)	$0 \quad 0 < t < 2b$ $(t+2\alpha)^\nu (t-2b)^{-\nu} \quad t > 2b$ $ \arg(\alpha+b)  < \pi, \quad \operatorname{Re} \nu < 1$	$\nu \pi \csc(\nu \pi) p^{-1} e^{-(\alpha+b)p} K_{2\nu}[(\alpha+b)p]$ $\operatorname{Re} p > 0$
(18)	$0 \quad 0 < t < b$ $(t-b)^{\nu-1} (t+b)^{-\nu+\frac{1}{2}} \quad t > b$ $\operatorname{Re} \nu > 0$	$2^{\nu-\frac{1}{2}} \Gamma(\nu) p^{-\frac{1}{2}} D_{1-2\nu}(2b^{\frac{1}{2}} p^{\frac{1}{2}})$ $\operatorname{Re} p > 0$
(19)	$0 \quad 0 < t < b$ $(t-b)^{\nu-1} (t+b)^{-\nu-\frac{1}{2}} \quad t > b$ $\operatorname{Re} \nu > 0$	$2^{\nu-\frac{1}{2}} \Gamma(\nu) b^{-\frac{1}{2}} D_{-2\nu}(2b^{\frac{1}{2}} p^{\frac{1}{2}})$ $\operatorname{Re} p \geq 0$
(20)	$t^{\nu-1} (t+a)^{-\nu+\frac{1}{2}}$ $\operatorname{Re} \nu > 0, \quad  \arg a  < \pi$	$2^{\nu-\frac{1}{2}} \Gamma(\nu) p^{-\frac{1}{2}} e^{\frac{1}{2} \alpha p} D_{1-2\nu}(2^{\frac{1}{2}} \alpha^{\frac{1}{2}} p^{\frac{1}{2}})$ $\operatorname{Re} p > 0$
(21)	$t^{\nu-1} (t+a)^{-\nu-\frac{1}{2}}$ $\operatorname{Re} \nu > 0, \quad  \arg \alpha  < \pi$	$2^\nu \Gamma(\nu) \alpha^{-\frac{1}{2}} e^{\frac{1}{2} \alpha p} D_{-2\nu}(2^{\frac{1}{2}} \alpha^{\frac{1}{2}} p^{\frac{1}{2}})$ $\operatorname{Re} p \geq 0$
(22)	$0 \quad 0 < t < b$ $(t+a)^{2\mu-1} (t-b)^{2\nu-1} \quad t > b$ $\operatorname{Re} \nu > 0, \quad  \arg(\alpha+b)  < \pi$	$\Gamma(2\nu)(\alpha+b)^{\mu+\nu-1} p^{-\mu-\nu} e^{\frac{1}{2} p(\alpha-b)}$ $\times W_{\mu-\nu, \mu+\nu-\frac{1}{2}}(ap+bp) \quad \operatorname{Re} p > 0$
(23)	$0 \quad 0 < t < a$ $(t-a)^{2\mu-1} (b-t)^{2\nu-1} \quad a < t < b$ $0 \quad t > b$ $\operatorname{Re} \mu > 0, \quad \operatorname{Re} \nu > 0$	$B(2\mu, 2\nu) (b-a)^{\mu+\nu-1} p^{-\mu-\nu} e^{-\frac{1}{2} p(a+b)}$ $\times M_{\mu-\nu, \mu+\nu-\frac{1}{2}}(bp-ap)$
(24)	$t^{\alpha-1} (1-t)^{\beta-1} (1-\sigma t)^{-\gamma} \quad 0 < t < 1$ $0 \quad t > 1$ $\operatorname{Re} \alpha > 0, \quad \operatorname{Re} \beta > 0$ $ \arg(1-\sigma)  < \pi$	$B(\alpha, \beta) \Phi_1(\alpha, \gamma, \alpha+\beta; \sigma, -p)$

## Arbitrary powers (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(25)	$[(t^2 + 1)^{\frac{\nu}{2}} + t]^\nu$	$p^{-1} S_{1,\nu}(p) + \nu p^{-1} S_{0,\nu}(p)$ $\text{Re } p > 0$
(26)	$[(t^2 + 1)^{\frac{\nu}{2}} - t]^\nu$	$p^{-1} S_{1,\nu}(p) - \nu p^{-1} S_{0,\nu}(p)$ $\text{Re } p > 0$
(27)	$(t^2 + 1)^{-\frac{\nu}{2}} [(t^2 + 1)^{\frac{\nu}{2}} + t]^\nu$	$\pi \csc(\nu\pi) [J_{-\nu}(p) - J_{-\nu}(p)]$ $\text{Re } p > 0$
(28)	$(t^2 + 1)^{-\frac{\nu}{2}} [(t^2 + 1)^{\frac{\nu}{2}} - t]^\nu$	$S_{0,\nu}(p) - \nu S_{-1,\nu}(p)$ $\text{Re } p > 0$
(29)	$0 \quad 0 < t < 1$ $\frac{[(t^2 - 1)^{\frac{\nu}{2}} + t]^\nu + [(t^2 - 1)^{\frac{\nu}{2}} + t]^{-\nu}}{(t^2 - 1)^{\frac{\nu}{2}}} \quad t > 1$	$2K_\nu(p)$ $\text{Re } p > 0$
(30)	$[(t + 2a)^{\frac{\nu}{2}} + t^{\frac{\nu}{2}}]^{2\nu}$ $-[(t + 2a)^{\frac{\nu}{2}} - t^{\frac{\nu}{2}}]^{2\nu} \quad  \arg a  < \pi$	$2^{\nu+1} \nu a^\nu p^{-1} e^{ap} K_\nu(ap) \quad \text{Re } p > 0$
(31)	$0 \quad 0 < t < b$ $[(t+b)^{\frac{\nu}{2}} + (t-b)^{\frac{\nu}{2}}]^{2\nu}$ $-[(t+b)^{\frac{\nu}{2}} - (t-b)^{\frac{\nu}{2}}]^{2\nu} \quad t > b$	$2^{\nu+1} \nu b^\nu p^{-1} K_\nu(bp) \quad \text{Re } p > 0$
(32)	$t^{-\nu-1} (t^2 + 1)^{-\frac{\nu}{2}}$ $\times [1 + (t^2 + 1)^{\frac{\nu}{2}}]^{\nu+\frac{1}{2}} \quad \text{Re } \nu < 0$	$2^{\frac{\nu}{2}} \Gamma(-\nu) D_\nu[(2ip)^{\frac{\nu}{2}}] D_\nu[(-2ip)^{\frac{\nu}{2}}]$ $\text{Re } p \geq 0$
(33)	$(2a)^{2\nu} [t + (t^2 + 4a^2)^{\frac{\nu}{2}}]^{2\nu}$ $\times (t^3 + 4a^2 t)^{-\frac{\nu}{2}} \quad \text{Re } a > 0$	$(\frac{1}{2}\pi)^{3/2} p^{1/2} [J_{\nu+\frac{1}{2}}(ap) Y_{\nu-\frac{1}{2}}(ap)$ $- J_{\nu-\frac{1}{2}}(ap) Y_{\nu+\frac{1}{2}}(ap)] \quad \text{Re } p > 0$
(34)	$0 \quad 0 < t < 1$ $t^{-\frac{\nu}{2}} (t^2 - 1)^{-\frac{\nu}{2}} \{ [t + (t^2 - 1)^{\frac{\nu}{2}}]^{2\nu}$ $+ [t - (t^2 - 1)^{\frac{\nu}{2}}]^{2\nu} \} \quad t > 1$	$(2p/\pi)^{\frac{\nu}{2}} K_{\nu+\frac{1}{2}}(\frac{1}{2}p) K_{\nu-\frac{1}{2}}(\frac{1}{2}p)$ $\text{Re } p > 0$

**4.4. Step - , jump - , and other sectionally rational functions**  
 $n = 0, 1, 2, \dots$

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(1)	0 $2nb < t < (2n+1)b$ 1 $(2n-1)b < t < 2nb$	$p^{-1}(e^{bp} + 1)^{-1}$ $\operatorname{Re} p > 0$
(2)	0 $(4n-1)b < t < (4n+1)b$ 2 $(4n+1)b < t < (4n+3)b$	$p^{-1} \operatorname{sech}(bp)$ $\operatorname{Re} p > 0$
(3)	$\frac{1}{2}$ $2nb < t < (2n+1)b$ $-\frac{1}{2}$ $(2n-1)b < t < 2nb$	$\frac{1}{2}p^{-1} \tanh(\frac{1}{2}bp)$ $\operatorname{Re} p > 0$
(4)	$n$ $nb < t < (n+1)b$	$p^{-1}(e^{bp} - 1)^{-1}$ $\operatorname{Re} p > 0$
(5)	$n+1$ $nb < t < (n+1)b$	$p^{-1}(1 - e^{-bp})^{-1}$ $\operatorname{Re} p > 0$
(6)	$2n+1$ $2nb < t < 2(n+1)b$	$p^{-1} \operatorname{ctnh}(bp)$ $\operatorname{Re} p > 0$
(7)	0 $0 < t < b$ $2n$ $(2n-1)b < t < (2n+1)b$	$p^{-1} \operatorname{csch}(bp)$ $\operatorname{Re} p > 0$
(8)	$n$ $b\pi^2 n^2 < t < b\pi^2(n+1)^2$	$\frac{1}{2}p^{-1} [\theta_3(0 i\pi bp) - 1]$ $\operatorname{Re} p > 0$
(9)	$n$ $\log n < t < \log(n+1)$	$p^{-1} \zeta(p)$ $\operatorname{Re} p > 0$
(10)	$\sum_{0 \leq \log n \leq t} (t - \log n)^{\alpha-1}$ $\operatorname{Re} \alpha > 0$	$\Gamma(\alpha) p^{-\alpha} \zeta(p)$ $\operatorname{Re} p > 0$
(11)	$(1-\alpha)^{-1} (1-\alpha^n)$ $nb < t < (n+1)b$	$p^{-1} (e^{bp} - \alpha)^{-1}$ $\operatorname{Re} p > 0, \quad b \operatorname{Re} p > \operatorname{Re}(\log \alpha)$
(12)	$\binom{n}{m}$ $nb < t < (n+1)b$	$\frac{e^{-bp}}{p(e^{bp} - 1)^m}$ $\operatorname{Re} p > 0$

## Sectionally rational functions (cont'd)

	$f(t)$	$g(y) = \int_0^\infty e^{-pt} f(t) dt$
(13)	$n^{\frac{1}{n}}$ $nb < t < (n+1)b$	$\frac{1-e^{-bp}}{(-b)^{\frac{1}{n}} p} \frac{d^{\frac{1}{n}}}{dp^{\frac{1}{n}}} (1-e^{-bp})^{-1}$ $\text{Re } p > 0$
(14)	$t$ $1$ $t > 1$	$p^{-2}(1-e^{-p})$ $\text{Re } p > 0$
(15)	$t$ $2-t$ $0$ $0 < t < 1$ $1 < t < 2$ $t > 2$	$p^{-2}(1-e^{-p})^2$
(16)	$a(t-nb)$ $nb < t < (n+1)b$	$ap^{-2} - \frac{1}{2} abp^{-1} [\operatorname{ctnh}(\frac{1}{2} bp) - 1]$ $= ap^{-2} (e^{bp} - 1)^{-1} (e^{bp} - bp - 1)$ $\text{Re } p > 0$
(17)	$\frac{1-\alpha^n}{1-\alpha} t - b \frac{1-(n+1)\alpha^n + n\alpha^{n+1}}{(1-\alpha)^2}$ $nb < t < (n+1)b$	$\frac{1}{p^2(e^{bp}-\alpha)}$ $\text{Re } p > 0, \quad b \text{Re } p > \text{Re } \log \alpha$
(18)	$(2n+1)t - 2bn(n+1)$ $2nb < t < 2(n+1)b$	$p^{-2} \operatorname{ctnh}(bp)$ $\text{Re } p > 0$
(19)	$b - (-1)^n(2bn + b - t)$ $2nb < t < 2(n+1)b$	$p^{-2} \tanh(bp)$ $\text{Re } p > 0$
(20)	$0$ $t - (-1)^n(t - 2nb)$ $(2n-1)b < t < (2n+1)b$ $n \geq 1$	$p^{-2} \operatorname{sech}(bp)$ $\text{Re } p > 0$

## Sectionally rational functions (cont'd)

	$f(t)$	$g(y) = \int_0^\infty e^{-pt} f(t) dt$	
(21)	$0 \quad 0 < t < b$ $2n(t - bn) \quad (2n-1)b < t < (2n+1)b$ $n \geq 1$	$p^{-2} \operatorname{csch}(bp)$	$\operatorname{Re} p > 0$
(22)	$\frac{1}{4} [1 - (-1)^n] (2t - b) + \frac{1}{2} (-1)^n bn \quad nb < t < (n+1)b$	$p^{-2} (e^{bp} + 1)^{-1}$	$\operatorname{Re} p > 0$
(23)	$0 \quad 0 < t < b$ $nt - \frac{1}{2}bn(n+1) \quad nb < t < (n+1)b$ $n \geq 1$	$p^{-2} (e^{bp} - 1)^{-1}$	$\operatorname{Re} p > 0$
(24)	$\frac{1}{2}t^2 \quad 0 < t < 1$ $1 - \frac{1}{2}(t-2)^2 \quad 1 < t < 2$ $1 \quad t > 2$	$p^{-3} (1 - e^{-p})^2$	$\operatorname{Re} p > 0$
(25)	$\frac{1}{2}t^2 \quad 0 < t < 1$ $3/4 - (t-3/2)^2 \quad 1 < t < 2$ $\frac{1}{2}(t-3)^2 \quad 2 < t < 3$ $0 \quad t > 3$	$p^{-3} (1 - e^{-p})^3$	
(26)	$(t-nb)^2 \quad nb < t < (n+1)b$	$2p^{-3} - p^{-2}(b^2 + 2bp)(e^{bp} - 1)^{-1}$	$\operatorname{Re} p > 0$
For further similar integrals see Gardner, M. F. and J. L. Barnes, 1942: <i>Transients in linear systems</i> , I, Wiley.			

## 4.5. Exponential functions

(1)	$e^{-\alpha t}$	$(p + \alpha)^{-1}$	$\operatorname{Re} p > -\operatorname{Re} \alpha$
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## Exponential functions (cont'd)

	$f(t)$	$\mathcal{G}(p) = \int_0^\infty e^{-pt} f(t) dt$
(2)	$te^{-\alpha t}$	$(p + \alpha)^{-2}$ $\operatorname{Re} p > -\operatorname{Re} \alpha$
(3)	$t^{\nu-1} e^{-\alpha t}$ $\operatorname{Re} \nu > 0$	$\Gamma(\nu)(p + \alpha)^{-\nu}$ $\operatorname{Re} p > -\operatorname{Re} \alpha$
(4)	$t^{-1}(e^{-\alpha t} - e^{-\beta t})$	$\log(p + \beta) - \log(p + \alpha)$ $\operatorname{Re} p > -\operatorname{Re} \alpha, -\operatorname{Re} \beta$
(5)	$t^{-2}(1 - e^{-\alpha t})^2$	$(p + 2\alpha)\log(p + 2\alpha) + p \log p$ $- 2(p + \alpha)\log(p + \alpha)$ $\operatorname{Re} p \geq 0, -\operatorname{Re} 2\alpha$
(6)	$t^{-1} - \frac{1}{2}t^{-2}(t+2)(1-e^{-t})$	$-1 + (p + \frac{1}{2})\log(1 + 1/p)$ $\operatorname{Re} p > 0$
(7)	$(1 + e^{-t})^{-1}$	$\frac{1}{2}\psi(\frac{1}{2}p + \frac{1}{2}) - \frac{1}{2}\psi(\frac{1}{2}p)$ $\operatorname{Re} p > 0$
(8)	$(1 - e^{-t/\alpha})^{\nu-1}$ $\operatorname{Re} \alpha > 0, \operatorname{Re} \nu > 0$	$\alpha B(\alpha p, \nu)$ $\operatorname{Re} p > 0$
(9)	$t^n(1 - e^{-t/\alpha})^{-1}$ $\operatorname{Re} \alpha > 0$	$(-\alpha)^{n+1} \psi^{(n)}(\alpha p)$ $\operatorname{Re} p > 0$ $\psi^{(n)}(z) = \frac{d^n}{dz^n} \psi(z)$
(10)	$t^{\nu-1}(1 - e^{-t/\alpha})^{-1}$ $\operatorname{Re} \nu > 1$	$\alpha^\nu \Gamma(\nu) \zeta(\nu, \alpha p)$ $\operatorname{Re} p > 0$
(11)	$t^{-1}(1 - e^{-t})^{-1} - t^{-2} - \frac{1}{2}t^{-1}$	$p + \log \Gamma(p) - p \log p + \frac{1}{2} \log(\frac{1}{2}p/\pi)$ $\operatorname{Re} p > 0$
(12)	$(1 - e^{-\alpha t})(1 - e^{-t})^{-1}$	$\psi(p + \alpha) - \psi(p)$ $\operatorname{Re} p > 0, -\operatorname{Re} \alpha$
(13)	$\frac{1 - e^{-\alpha t}}{t(1 + e^{-t})}$	$\log \frac{\Gamma(\frac{1}{2}p) \Gamma(\frac{1}{2}\alpha + \frac{1}{2}p + \frac{1}{2})}{\Gamma(\frac{1}{2}p + \frac{1}{2}) \Gamma(\frac{1}{2}\alpha + \frac{1}{2}p)}$ $\operatorname{Re} p > 0, -\operatorname{Re} \alpha$

## Exponential functions (cont'd)

	$f(t)$	$\mathcal{G}(p) = \int_0^\infty e^{-pt} f(t) dt$
(14)	$(1-e^{-t})^{\nu-1} (1-ze^{-t})^{-\mu}$ $\text{Re } \nu > 0, \quad  \arg(1-z)  < \pi$	$B(p, \nu) {}_2F_1(\mu, p; p+\nu; z)$ $\text{Re } p > 0$
(15)	$(1-e^{-t})^{-1} (1-e^{-\alpha t})(1-e^{-\beta t})$	$\psi(p+\alpha) + \psi(p+\beta)$ $-\psi(p+\alpha+\beta) - \psi(p)$ $\text{Re } p > 0, \quad -\text{Re } \alpha$ $\text{Re } p > -\text{Re } \beta, \quad -\text{Re } (\alpha + \beta)$
(16)	$\frac{(1-e^{-\alpha t})(1-e^{-\beta t})}{t(1-e^{-t})}$	$\log \frac{\Gamma(p)\Gamma(p+\alpha+\beta)}{\Gamma(p+\alpha)\Gamma(p+\beta)}$ $\text{Re } p > 0, \quad -\text{Re } \alpha$ $\text{Re } p > -\text{Re } \beta, \quad -\text{Re } (\alpha + \beta)$
(17)	$\frac{(1-e^{-\alpha t})(1-e^{-\beta t})(1-e^{-\gamma t})}{t(1-e^{-t})}$	$\log \frac{\Gamma(p)\Gamma(p+\beta+\gamma)\Gamma(p+\alpha+\gamma)\Gamma(p+\alpha+\beta)}{\Gamma(p+\alpha)\Gamma(p+\beta)\Gamma(p+\gamma)\Gamma(p+\alpha+\beta+\gamma)}$ $2 \text{Re } p >  \text{Re } \alpha  +  \text{Re } \beta  +  \text{Re } \gamma $
(18)	$\frac{[\alpha + (1-e^{-t})^{\frac{1}{2}}]^{-\nu} + [\alpha - (1-e^{-t})^{\frac{1}{2}}]^{-\nu}}{(1-e^{-t})^{\frac{1}{2}}}$	$2^{p+1} e^{(p-\nu)\pi i} \frac{\Gamma(p)}{\Gamma(\nu)} (\alpha^2 - 1)^{\frac{1}{2}p - \frac{1}{2}\nu}$ $\times Q_{p-1}^{\nu-p}(\alpha) \quad \text{Re } p > 0$
(19)	$0 \quad 0 < t < b$ $(1-e^{-2t})^{-\frac{1}{2}} [e^{-b} (1-e^{-2t})^{\frac{1}{2}} - e^{-t} (1-e^{-2b})^{\frac{1}{2}}]^{\nu} \quad t > b$ $\text{Re } \nu > -1$	$\frac{(\pi)^{\frac{1}{2}} \Gamma(p) \Gamma(\nu+1)}{2^{\frac{1}{2}p + \frac{1}{2}\nu} \Gamma(\frac{1}{2}p + \frac{1}{2}\nu + \frac{1}{2})} e^{-\frac{1}{2}b(p+\nu)}$ $\times P_{-\frac{1}{2}p + \frac{1}{2}\nu}^{-\frac{1}{2}p - \frac{1}{2}\nu} [(1-e^{-2b})^{\frac{1}{2}}] \quad \text{Re } p > 0$
(20)	$e^{(\mu-1)t} (1-e^{-t})^{\mu-\frac{1}{2}} [(1-e^{-t}) \sin \theta - i(1-e^{-t}) \cos \theta]^{\mu-\frac{1}{2}}$ $\text{Re } \mu > -\frac{1}{2}$	$\frac{2^{\mu-1} \Gamma(\mu+\frac{1}{2}) \Gamma(p-\mu+1)}{\pi^{\frac{1}{2}} \Gamma(p+\mu+1)} \sin^\mu \theta$ $\times e^{(p+\frac{1}{2})i\theta + (\frac{1}{2}\mu - \frac{1}{2})\pi i}$ $\times [\pi P_\nu^\mu(\cos \theta) + 2i Q_\nu^\mu(\cos \theta)] \quad \text{Re } p > \text{Re } \mu - 1$

Another formula may be derived from this by changing  $i$  into  $-i$  throughout.

## Exponential functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(21)	$0 \quad 0 < t < b$ $e^{-\frac{1}{4}t^2/\alpha} \quad t > b$ $\text{Re } \alpha > 0$	$\pi^{\frac{1}{2}} \alpha^{\frac{1}{2}} e^{\alpha p^2} \text{Erfc}(\alpha^{\frac{1}{2}} p + \frac{1}{2} \alpha^{\frac{1}{2}} b)$
(22)	$t e^{-\frac{1}{4}t^2/\alpha}$ $\text{Re } \alpha > 0$	$2\alpha - 2\pi^{1/2} \alpha^{3/2} p e^{\alpha p^2} \text{Erfc}(\alpha^{1/2} p)$
(23)	$t^{-\frac{1}{2}} e^{-\frac{1}{4}t^2/\alpha}$ $\text{Re } \alpha > 0$	$\alpha^{\frac{1}{2}} p^{\frac{1}{2}} e^{\frac{1}{2}\alpha p^2} K_{\frac{1}{2}}(\frac{1}{2} \alpha p^2)$
(24)	$t^{\nu-1} e^{-t^2/(8\alpha)}$ $\text{Re } \alpha > 0, \quad \text{Re } \nu > 0$	$\Gamma(\nu) 2^\nu \alpha^{\frac{1}{2}\nu} e^{\alpha p^2} D_{-\nu}(2p \alpha^{\frac{1}{2}})$
(25)	$e^{-\frac{1}{4}\alpha/t}$ $\text{Re } \alpha \geq 0$	$\alpha^{\frac{1}{2}} p^{-\frac{1}{2}} K_1(\alpha^{\frac{1}{2}} p^{\frac{1}{2}}) \quad \text{Re } p > 0$
(26)	$t^{\frac{1}{2}} e^{-\frac{1}{4}\alpha/t}$ $\text{Re } \alpha \geq 0$	$\frac{1}{2} \pi^{1/2} p^{-3/2} (1 + \alpha^{1/2} p^{1/2}) e^{-\alpha^{\frac{1}{2}} p^{\frac{1}{2}}} \quad \text{Re } p > 0$
(27)	$t^{-\frac{1}{2}} e^{-\frac{1}{4}\alpha/t}$ $\text{Re } \alpha \geq 0$	$\pi^{\frac{1}{2}} p^{-\frac{1}{2}} e^{-\alpha^{\frac{1}{2}} p^{\frac{1}{2}}} \quad \text{Re } p > 0$
(28)	$t^{-3/2} e^{-\frac{1}{4}\alpha/t}$ $\text{Re } \alpha > 0$	$2\pi^{\frac{1}{2}} \alpha^{-\frac{1}{2}} e^{-\alpha^{\frac{1}{2}} p^{\frac{1}{2}}} \quad \text{Re } p \geq 0$
(29)	$t^{\nu-1} e^{-\frac{1}{4}\alpha/t}$ $\text{Re } \alpha > 0$	$2(\frac{1}{4}\alpha/p)^{\frac{1}{2}\nu} K_\nu(\alpha^{\frac{1}{2}} p^{\frac{1}{2}}) \quad \text{Re } p > 0$
(30)	$t^{-\frac{1}{2}} (e^{-\frac{1}{4}\alpha/t} - 1)$ $\text{Re } \alpha \geq 0$	$\pi^{\frac{1}{2}} p^{-\frac{1}{2}} (e^{-\alpha^{\frac{1}{2}} p^{\frac{1}{2}}} - 1) \quad \text{Re } p \geq 0$
(31)	$e^{-2\alpha^{\frac{1}{2}} t^{\frac{1}{2}}}$ $ \arg \alpha  < \pi$	$p^{-1} - \pi^{1/2} \alpha^{1/2} p^{-3/2} e^{\alpha/p} \times \text{Erfc}(\alpha^{1/2} p^{-1/2}) \quad \text{Re } p > 0$
(32)	$t^{\frac{1}{2}} e^{-2\alpha^{\frac{1}{2}} t^{\frac{1}{2}}}$ $ \arg \alpha  < \pi$	$-\alpha^{1/2} p^{-2} + \pi^{1/2} p^{-5/2} (\alpha + \frac{1}{2} p) e^{\alpha/p} \times \text{Erfc}(\alpha^{1/2} p^{-1/2}) \quad \text{Re } p > 0$

**Exponential functions (cont'd)**

	$f(t)$	$\mathcal{L}(f) = \int_0^\infty e^{-pt} f(t) dt$
(33)	$t^{-\frac{\nu}{2}} e^{-2\alpha^{\frac{1}{2}} t^{\frac{1}{2}}}$ $ \arg \alpha  < \pi$	$\pi^{\frac{1}{2}} p^{-\frac{\nu}{2}} e^{\alpha/p} \text{Erfc}(\alpha^{\frac{1}{2}} p^{-\frac{1}{2}})$ $\text{Re } p > 0$
(34)	$(2t)^{-\frac{\nu}{2}} e^{-2\alpha^{\frac{1}{2}} t^{\frac{1}{2}}}$ $ \arg \alpha  < \pi$	$(\frac{1}{2} \alpha/p)^{\frac{\nu}{2}} e^{\frac{\nu}{2} \alpha/p} K_{\frac{\nu}{2}}(\frac{1}{2} \alpha/p)$ $\text{Re } p > 0$
(35)	$(2t)^{\nu-1} e^{-2\alpha^{\frac{1}{2}} t^{\frac{1}{2}}}$ $\text{Re } \nu > 0$	$\Gamma(2\nu) p^{-\nu} e^{\frac{\nu}{2} \alpha/p} D_{-2\nu}[(2\alpha/p)^{\frac{1}{2}}]$ $\text{Re } p > 0$
(36)	$\exp(-\alpha e^{-t})$	$\alpha^{-p} \gamma(p, \alpha)$ $\text{Re } p > 0$
(37)	$\exp(-\alpha e^t)$ $\text{Re } \alpha > 0$	$\alpha^p \Gamma(-p, \alpha)$
(38)	$(1-e^{-t})^{\nu-1} \exp(\alpha e^{-t})$ $\text{Re } \nu > 0$	$\frac{\Gamma(\nu) \Gamma(p)}{\Gamma(\nu+p)} \alpha^{-\frac{\nu}{2} \nu - \frac{1}{2} p} e^{\frac{\nu}{2} \alpha}$ $\times M_{\frac{\nu}{2}, \nu - \frac{1}{2} p, \frac{\nu}{2}, \nu + \frac{1}{2} p - \frac{1}{2}}(\alpha)$ $\text{Re } p > 0$
(39)	$(1-e^{-t})^{\nu-1} \exp(-\alpha e^t)$ $\text{Re } \alpha > 0, \text{ Re } \nu > 0$	$\Gamma(\nu) \alpha^{\frac{\nu}{2} p - \frac{1}{2}} e^{-\frac{\nu}{2} \alpha} W_{\frac{\nu}{2} - \frac{1}{2} p - \nu, -\frac{1}{2} p}(\alpha)$
(40)	$(1-e^{-t})^{\nu-1} (1-\lambda e^{-t})^{-\mu}$ $\times \exp(\alpha e^{-t})$ $\text{Re } \nu > 0,  \arg(1-\lambda)  < \pi$	$\frac{\Gamma(\nu) \Gamma(p)}{\Gamma(\nu+p)} \Phi_1(p, \mu, \nu; \lambda, \alpha)$ $\text{Re } p > 0$
(41)	$(e^t - 1)^{\nu-1} \exp[-\alpha/(e^t - 1)]$ $\text{Re } \alpha > 0$	$\Gamma(p - \nu + 1) e^{\frac{\nu}{2} \alpha} \alpha^{\frac{\nu}{2} \nu - \frac{1}{2}}$ $\times W_{\frac{\nu}{2} \nu - \frac{1}{2} - p, \frac{\nu}{2} \nu}(\alpha)$ $\text{Re } p > \text{Re } \nu - 1$

## 4.6. Logarithmic functions

	$f(t)$	$\mathcal{G}(p) = \int_0^\infty e^{-pt} f(t) dt$
(1)	$\log t$	beware: here, gamma = exp(EulerGamma) $-p^{-1} \log(\gamma p)$ Re $p > 0$
(2)	$0$ $\log t$	$0 < t < b$ $t > b$ $p^{-1} [e^{-bp} \log b - \text{Ei}(-bp)]$ Re $p > 0$
(3)	$0$ $\log(t/b)$	$0 < t < b$ $t > b$ $-p^{-1} \text{Ei}(-bp)$ Re $p > 0$
(4)	$\log(1+\alpha t)$	$ \arg \alpha  < \pi$ $-p^{-1} e^{p/\alpha} \text{Ei}(-p/\alpha)$ Re $p > 0$
(5)	$\log(t+\alpha)$	$ \arg \alpha  < \pi$ $p^{-1} [\log \alpha - e^{\alpha p} \text{Ei}(-\alpha p)]$ Re $p > 0$
(6)	$\log b-t $	$b > 0$ $p^{-1} [\log b - e^{-bp} \overline{\text{Ei}}(bp)]$ Re $p > 0$
(7)	$t^n \log t$	$\frac{n!}{p^{n+1}} \left[ 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} - \log(\gamma p) \right]$ $\text{Re } p > 0$
(8)	$0$ $t^{-1} \log(2t-1)$	$0 < t < 1$ $t > 1$ $\frac{1}{2} [\text{Ei}(-\frac{1}{2}p)]^2$ Re $p > 0$
(9)	$t^{-\frac{n}{2}} \log t$	$-\pi^{\frac{n}{2}} p^{-\frac{n}{2}} \log(4\gamma p)$ Re $p > 0$
(10)	$t^{n-\frac{1}{2}} \log t$	$n \geq 1$ $\pi^{\frac{n}{2}} \frac{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)}{p^{n+\frac{1}{2}} 2^n} \left[ 2 \left( 1 + \frac{1}{3} + \frac{1}{5} + \dots + \frac{1}{2n-1} \right) - \log(4\gamma p) \right]$ $\text{Re } p > 0$
(11)	$t^{\nu-1} \log t$	$\text{Re } \nu > 0$ $\Gamma(\nu) p^{-\nu} [\psi(\nu) - \log p]$ Re $p > 0$

**Logarithmic functions (cont'd)**

	$f(t)$	$\mathcal{L}(f) = \int_0^\infty e^{-pt} f(t) dt$
(12)	$t^{\nu-1} [\psi(\nu) - \log t]$ $\operatorname{Re} \nu > 0$	$\Gamma(\nu) p^{-\nu} \log p$ $\operatorname{Re} p > 0$
(13)	$(\log t)^2$	$p^{-1} \{ \pi^2/6 + [\log(\gamma p)]^2 \}$ $\operatorname{Re} p > 0$
(14)	$\log  t^2 - a^2 $ $a > 0$	$p^{-1} [\log a^2 - e^{ap} \operatorname{Ei}(-ap) - e^{-ap} \overline{\operatorname{Ei}}(ap)]$ $\operatorname{Re} p > 0$
(15)	$\log(t^2 - a^2)$ $ \operatorname{Im} a  > 0$	$p^{-1} [\log a^2 - e^{ap} \operatorname{Ei}(-ap) - e^{-ap} \operatorname{Ei}(ap)]$ $\operatorname{Re} p > 0$
(16)	$\log(t^2 + a^2)$	$2p^{-1} [\log a - \operatorname{ci}(ap) \cos(ap) - \operatorname{si}(ap) \sin(ap)]$ $\operatorname{Re} p > 0$
(17)	$t^{-1} [\log(t^2 + a^2) - \log a^2]$	$[\operatorname{ci}(ap)]^2 + [\operatorname{si}(ap)]^2$ $\operatorname{Re} p > 0$
(18)	$0 \quad 0 < t < b$ $\log \frac{(t+b)^{\frac{1}{2}} + (t-b)^{\frac{1}{2}}}{2^{\frac{1}{2}} b^{\frac{1}{2}}} \quad t > b$	$\frac{1}{2} p^{-1} K_0(bp)$ $\operatorname{Re} p > 0$
(19)	$\log \frac{t^{\frac{1}{2}} + (t+2a)^{\frac{1}{2}}}{2^{\frac{1}{2}} a^{\frac{1}{2}}} \quad  \arg a  < \pi$	$\frac{1}{2} p^{-1} e^{ap} K_0(ap)$ $\operatorname{Re} p > 0$
(20)	$\log \frac{(t+ib)^{\frac{1}{2}} + (t-ib)^{\frac{1}{2}}}{2^{\frac{1}{2}} b^{\frac{1}{2}}} \quad b > 0$	$\frac{1}{4} \pi p^{-1} [\mathbf{H}_0(bp) - Y_0(bp)]$ $\operatorname{Re} p > 0$
(21)	$\frac{\log[4t(2b-t)/b^2]}{t^{\frac{1}{2}}(2b-t)^{\frac{1}{2}}} \quad 0 < t < 2b$ $0 \quad t > 2b$	$\pi e^{-bp} [\frac{1}{2} \pi Y_0(ibp) - \log(\frac{1}{2}\gamma) J_0(ibp)]$

## 4.7. Trigonometric functions

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(1)	$\sin(at)$	$a(p^2 + a^2)^{-1}$ $\text{Re } p >  \text{Im } a $
(2)	$ \sin(at) $ $a > 0$	$a(p^2 + a^2)^{-1} \operatorname{ctnh}(\tfrac{1}{2}\pi a^{-1} p)$ $\text{Re } p > 0$
(3)	$\sin^{2n}(at)$	$\frac{(2n)! a^{2n}}{p[p^2 + (2a)^2][p^2 + (4a)^2] \cdots [p^2 + (2na)^2]}$ $\text{Re } p > 2n  \text{Im } a $
(4)	$0$ $\sin^{2n} t$ $0 < t < \pi/2$ $t > \pi/2$	$\frac{(2n)! e^{-\frac{1}{2}\pi p}}{p(2^2 + p^2)(4^2 + p^2) \cdots (4n^2 + p^2)}$ $\times \left\{ 1 + \frac{p^2}{2!} + \frac{p^2(2^2 + p^2)}{4!} \right.$ $\left. + \cdots + \frac{p^2(2^2 + p^2) \cdots [4(n-1)^2 + p^2]}{(2n)!} \right\}$ $\text{Re } p > 0$
(5)	$\sin^{2n} t$ $0$ $0 < t < \pi/2$ $t > \pi/2$	$\frac{(2n)! e^{-\frac{1}{2}\pi p}}{p(2^2 + p^2)(4^2 + p^2) \cdots (4n^2 + p^2)}$ $\times \left\{ e^{\frac{1}{2}\pi p} - 1 - \frac{p^2}{2!} \right.$ $\left. - \cdots - \frac{p^2(2^2 + p^2) \cdots [4(n-1)^2 + p^2]}{(2n)!} \right\}$
(6)	$\sin^{2n} t$ $0$ $0 < t < m\pi$ $t > m\pi$ $m = 1, 2, 3, \dots$	$\frac{(2n)! (1 - e^{-\pi p \pi})}{p(2^2 + p^2)(4^2 + p^2) \cdots (4n^2 + p^2)}$
(7)	$\sin^{2n+1}(at)$	$\frac{(2n+1)! a^{2n+1}}{(p^2 + a^2)[p^2 + (3a)^2] \cdots [p^2 + [(2n+1)a]^2]}$ $\text{Re } p > (2n+1)  \text{Im } a $

## Trigonometric functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(8)	$\begin{cases} 0 & 0 < t < \pi/2 \\ \sin^{2n+1} t & t > \pi/2 \end{cases}$	$\frac{(2n+1)! p e^{-\frac{\pi}{2} \pi p}}{(1^2+p^2)(3^2+p^2)\dots[(2n+1)^2+p^2]} \times \left\{ 1 + \frac{1^2+p^2}{3!} + \frac{(1^2+p^2)(3^2+p^2)}{5!} \right.$ $\left. + \dots + \frac{(1^2+p^2)(3^2+p^2)\dots[(2n-1)^2+p^2]}{(2n+1)!} \right\}$ <p style="text-align: right;"><math>\text{Re } p &gt; 0</math></p>
(9)	$\begin{cases} \sin^{2n+1} t & 0 < t < \pi/2 \\ 0 & t > \pi/2 \end{cases}$	$\frac{(2n+1)! p e^{-\frac{\pi}{2} \pi p}}{(1^2+p^2)(3^2+p^2)\dots[(2n+1)^2+p^2]} \times \left\{ \frac{e^{\frac{\pi}{2} \pi p}}{p} - 1 - \frac{1^2+p^2}{3!} \right.$ $\left. - \dots - \frac{(1^2+p^2)(3^2+p^2)\dots[(2n-1)^2+p^2]}{(2n+1)!} \right\}$
(10)	$\begin{cases} \sin^{2n+1} t & 0 < t < m\pi \\ 0 & t > m\pi \end{cases}$ <p style="text-align: center;"><math>m = 1, 2, 3, \dots</math></p>	$\frac{(2n+1)! [1 - (-1)^m e^{-m\pi p}]}{(1^2+p^2)(3^2+p^2)\dots[(2n+1)^2+p^2]}$
(11)	$ \sin(at) ^{2\nu}$ $a > 0, \quad \text{Re } \nu > -\frac{1}{2}$	$\frac{B(1+\frac{1}{2}ip/a, 1-\frac{1}{2}ip/a)}{(2\nu+1)2^{2\nu}p B(\nu+1+\frac{1}{2}ip/a, \nu+1-\frac{1}{2}ip/a)}$ <p style="text-align: right;"><math>\text{Re } p &gt; 0</math></p>
(12)	$\begin{cases} 0 & 0 < t < \pi/2 \\ t \sin t & t > \pi/2 \end{cases}$	$\frac{e^{-\frac{\pi}{2} \pi p}}{(1+p^2)^2} [\frac{1}{2}p \pi (1+p^2) + p^2 - 1]$ <p style="text-align: right;"><math>\text{Re } p &gt; 0</math></p>
(13)	$\begin{cases} t \sin t & 0 < t < \pi/2 \\ 0 & t > \pi/2 \end{cases}$	$(1+p^2)^{-2} \{ 2p - e^{-\frac{\pi}{2} \pi p} \times [\frac{1}{2}p \pi (1+p^2) + p^2 - 1] \}$

## Trigonometric functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(14)	$t^n \sin(\alpha t)$	$n! \frac{p^{n+1}}{(p^2 + \alpha^2)^{n+1}}$ $\times \sum_{0 \leq 2m \leq n} (-1)^m \binom{n+1}{2m+1} \left(\frac{\alpha}{p}\right)^{2m+1}$ $\text{Re } p >  \text{Im } \alpha $
(15)	$t^{\nu-1} \sin(\alpha t)$	$\text{Re } \nu > -1$ $\frac{1}{2} i \Gamma(\nu) [(p+ia)^{-\nu} - (p-ia)^{-\nu}]$ $= \Gamma(\nu) (p^2 + \alpha^2)^{-\frac{\nu}{2}} \nu \sin[\nu \tan^{-1}(a/p)]$ $\text{Re } p >  \text{Im } \alpha $
(16)	$t^{-1} \sin(\alpha t)$	$\tan^{-1}(a/p)$ $\text{Re } p >  \text{Im } \alpha $
(17)	$t^{-1} \sin^2(\alpha t)$	$\frac{1}{4} \log(1 + 4\alpha^2 p^{-2})$ $\text{Re } p > 2 \text{Im } \alpha $
(18)	$t^{-1} \sin^3(\alpha t)$	$\frac{1}{2} \tan^{-1}(a/p) - \frac{1}{4} \tan^{-1}[2\alpha p/(p^2 + 3\alpha^2)]$ $\text{Re } p > 3 \text{Im } \alpha $
(19)	$t^{-1} \sin^4(\alpha t)$	$\frac{1}{8} \log \frac{(p^2 + 4\alpha^2)^2}{p^3} - \frac{1}{16} \log(p^2 + 16\alpha^2)$ $\text{Re } p > 4 \text{Im } \alpha $
(20)	$t^{-2} \sin^2(\alpha t)$	$\alpha \tan^{-1}(2\alpha/p) - \frac{1}{4} p \log(1 + 4\alpha^2 p^{-2})$ $\text{Re } p \geq 2 \text{Im } \alpha $
(21)	$t^{-2} \sin^3(\alpha t)$	$\frac{1}{4} p \tan^{-1}(3\alpha/p) - \frac{3}{4} p \tan^{-1}(a/p)$ + $(3\alpha/8) \log[(p^2 + 3\alpha^2)/(p^2 + \alpha^2)]$ $\text{Re } p \geq 3 \text{Im } \alpha $
(22)	$(e^{-t} - 1)^{-1} \sin(\alpha t)$	$\frac{1}{2} i \psi(p - i\alpha + 1) - \frac{1}{2} i \psi(p + i\alpha + 1)$ $\text{Re } p >  \text{Im } \alpha  - 1$
(23)	$(1 - e^{-t})^{-1} \sin(\alpha t)$	$\frac{1}{2} i \psi(p - i\alpha) - \frac{1}{2} i \psi(p + i\alpha)$ $\text{Re } p >  \text{Im } \alpha $

## Trigonometric functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(24)	$(1-e^{-t})^{\nu-1} \sin(\alpha t)$ $\text{Re } \nu > -1$	$\frac{1}{2}i B(\nu, p+i\alpha) - \frac{1}{2}i B(\nu, p-i\alpha)$ $\text{Re } p >  \text{Im } \alpha $
(25)	$t^{\nu-1} e^{-\frac{1}{2}t^2/\alpha} \sin(\beta t)$ $\text{Re } \alpha > 0, \quad \text{Re } \nu > -1$	$\frac{1}{2}i \Gamma(\nu) \alpha^{\frac{1}{2}\nu} e^{\frac{1}{4}\alpha(p^2-\beta^2)}$ $\times \{e^{\frac{1}{2}ip\alpha\beta} D_{-\nu}[\alpha^{\frac{1}{2}}(p+i\beta)]$ $- e^{-\frac{1}{2}ip\alpha\beta} D_{-\nu}[\alpha^{\frac{1}{2}}(p-i\beta)]\}$
(26)	$\log t \sin(\alpha t)$	$\frac{p \tan^{-1}(a/p) - a \log[\gamma(p^2 + a^2)^{\frac{1}{2}}]}{p^2 + a^2}$ $\text{Re } p >  \text{Im } \alpha $
(27)	$\log t \sin^2(\frac{1}{2}\alpha t)$	$p^{-1} (p^2 + a^2)^{-1} [ap \tan^{-1}(a/p)$ $+ p^2 \log(p^2 + a^2)^{\frac{1}{2}}$ $- (p^2 + a^2) \log p - a^2 \log \gamma]$ $\text{Re } p > 2 \text{Im } \alpha $
(28)	$t^{-1} \log t \sin(\alpha t)$	$-\log[\gamma(p^2 + a^2)^{\frac{1}{2}}] \tan^{-1}(a/p)$ $\text{Re } p >  \text{Im } \alpha $
(29)	$t^{\nu-1} \log t \sin(\alpha t)$ $\text{Re } \nu > -1$	$\Gamma(\nu) (p^2 + a^2)^{-\frac{1}{2}\nu} \sin[\nu \tan^{-1}(a/p)]$ $\times \{ \psi(\nu) - \log(p^2 + a^2)^{\frac{1}{2}}$ $+ \tan^{-1}(a/p) \operatorname{ctn}[\nu \tan^{-1}(a/p)] \}$ $\text{Re } p >  \text{Im } \alpha $
(30)	$\sin(t^2)$ $(\frac{1}{2}\pi)^{\frac{1}{2}} \{ \cos(\frac{1}{4}p^2) [\frac{1}{2} - C(\frac{1}{4}p^2)] + \sin(\frac{1}{4}p^2) [\frac{1}{2} - S(\frac{1}{4}p^2)] \}$	<b>ERRATA</b> $\text{Re } p > 0.$
(31)	$t^{-1} \sin(t^2)$	$\frac{1}{2}\pi [\frac{1}{2} - C(\frac{1}{4}p^2)]^2 + \frac{1}{2}\pi [\frac{1}{2} - S(\frac{1}{4}p^2)]^2$ $\text{Re } p > 0$
(32)	$\sin(2\alpha^{\frac{1}{2}}t^{\frac{1}{2}})$	$\pi^{1/2} \alpha^{1/2} p^{-3/2} e^{-\alpha/p}$ $\text{Re } p > 0$

## Trigonometric functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(33)	$t^n \sin(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$	$(-1)^n 2^{-n-\frac{1}{2}} \pi^{\frac{1}{2}} p^{-n-1} e^{-\alpha/p} \\ \times \text{He}_{2n+1}(2^{\frac{1}{2}} \alpha^{\frac{1}{2}} p^{-\frac{1}{2}}) \quad \text{Re } p > 0$
(34)	$t^{-1} \sin(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$	$\pi \text{Erf}(\alpha^{\frac{1}{2}} p^{-\frac{1}{2}}) \quad \text{Re } p > 0$
(35)	$t^{\frac{1}{2}} \sin(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$	$\alpha^{1/2} p^{-2} - i \pi^{1/2} p^{-5/2} (\frac{1}{2}p - \alpha) e^{-\alpha/p} \\ \times \text{Erf}(i \alpha^{1/2} p^{-1/2}) \quad \text{Re } p > 0$
(36)	$t^{-\frac{1}{2}} \sin(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$	$-i \pi^{\frac{1}{2}} p^{-\frac{1}{2}} e^{-\alpha/p} \text{Erf}(i \alpha^{\frac{1}{2}} p^{-\frac{1}{2}}) \quad \text{Re } p > 0$
(37)	$t^{\nu-1} \sin(2^{\frac{1}{2}} \alpha^{\frac{1}{2}} t^{\frac{1}{2}})$ $\text{Re } \nu > -\frac{1}{2}$	$2^{-\nu-\frac{1}{2}} \pi^{\frac{1}{2}} \sec(\nu\pi) p^{-\nu} e^{-\frac{1}{4}\alpha/p} \\ \times [D_{2\nu-1}(-\alpha^{\frac{1}{2}} p^{-\frac{1}{2}}) - D_{2\nu-1}(\alpha^{\frac{1}{2}} p^{-\frac{1}{2}})] \quad \text{Re } p > 0$
(38)	$0 \quad 0 < t < b \\ \sin[\alpha(t^2 - b^2)^{\frac{1}{2}}] \quad t > b$	$abp^{-1} K_1(bp) \quad \text{Re } p >  \text{Im } \alpha $
(39)	$\sin(\alpha e^{-t})$	$\alpha^{-p} \Gamma(p) [U_p(2\alpha, 0) \sin \alpha \\ - U_{p+1}(2\alpha, 0) \cos \alpha] \quad \text{Re } p > 0$
(40)	$\sin[\alpha(1 - e^{-t})]$	$\alpha^{-p} \Gamma(p) U_{p+1}(2\alpha, 0) \quad \text{Re } p > 0$
(41)	$(e^{-t} - 1)^{-\frac{1}{2}} \sin[\alpha(1 - e^{-t})^{\frac{1}{2}}]$	$\pi^{\frac{1}{2}} \Gamma(p + \frac{1}{2})(2/\alpha)^p \mathbf{H}_p(\alpha) \quad \text{Re } p > -\frac{1}{2}$
(42)	$(1 - e^{-t})^{-\frac{1}{2}} \sin[\alpha(e^{-t} - 1)^{\frac{1}{2}}] \quad \alpha > 0$	$\pi^{\frac{1}{2}} \Gamma(\frac{1}{2} - p)(\frac{1}{2}\alpha)^p [I_p(\alpha) - \mathbf{L}_{-p}(\alpha)] \quad \text{Re } p > -\frac{1}{2}$
(43)	$\cos(\alpha t)$	$p(p^2 + \alpha^2)^{-1} \quad \text{Re } p >  \text{Im } \alpha $

## Trigonometric functions (cont'd)

	$f(t)$	$\mathcal{L}(f) = \int_0^\infty e^{-pt} f(t) dt$
(44)	$ \cos(at) $	$(p^2 + a^2)^{-1} [p + a \coth(\frac{1}{2}\pi a^{-1} p)]$ $\text{Re } p > 0$
(45)	$\cos^2(at)$	$(p^2 + 2a^2)(p^2 + 4a^2 p)^{-1}$ $\text{Re } p > 2 \text{Im } a $
(46)	$\cos^3(at)$	$p(p^2 + 7a^2)(p^2 + a^2)^{-1}(p^2 + 9a^2)^{-1}$ $\text{Re } p > 3 \text{Im } a $
(47)	$\cos^{2n}(at)$	$\frac{(2n)! a^{2n}}{p[p^2+(2a)^2][p^2+(4a)^2]\dots[p^2+(2na)^2]}$ $\times \left\{ 1 + \frac{p^2}{2!a^2} + \frac{p^2[p^2+(2a)^2]}{4!a^4} \right.$ $\left. + \dots + \frac{p^2(p^2+4a^2)\dots[p^2+4(na-a)^2]}{(2n)!a^{2n}} \right\}$ $\text{Re } p > 2n \text{Im } a $
(48)	$0$ $\cos^{2n} t$	$0 < t < \pi/2$ $t > \pi/2$ $\frac{(2n)! e^{-\frac{1}{2}p\pi}}{p(2^2+p^2)(4^2+p^2)\dots(4n^2+p^2)}$ $\text{Re } p > 0$
(49)	$\cos^{2n} t$ $0$	$0 < t < \pi/2$ $t > \pi/2$ $\frac{(2n)!}{p(2^2+p^2)(4^2+p^2)\dots(4n^2+p^2)}$ $\times \left\{ -e^{-\frac{1}{2}p\pi} + 1 + \frac{p^2}{2!} \right.$ $\left. + \dots + \frac{p^2(2^2+p^2)\dots[4(n-1)^2+p^2]}{(2n)!} \right\}$

## Trigonometric functions (cont'd)

	$f(t)$	$\mathcal{G}(p) = \int_0^\infty e^{-pt} f(t) dt$
(50)	$0 \quad 0 < t < \pi/2$ $\cos^{2n} t \quad \pi/2 < t < (m + \frac{1}{2})\pi$ $0 \quad t > (m + \frac{1}{2})\pi$ $m = 1, 2, 3, \dots$	$\frac{(2n)! e^{-\frac{1}{4}p\pi} (1 - e^{-\pi p\pi})}{p(2^2 + p^2)(4^2 + p^2) \dots (4n^2 + p^2)}$
(51)	$\cos^{2n+1}(\alpha t)$	$\frac{(2n+1)! \alpha^{2n} p}{(p^2 + \alpha^2)[p^2 + (3\alpha)^2] \dots [p^2 + (2n\alpha + \alpha)^2]} \left\{ 1 \right.$ $+ \frac{p^2 + \alpha^2}{3! \alpha^2} + \frac{(p^2 + \alpha^2)(p^2 + 9\alpha^2)}{5! \alpha^4}$ $\left. + \dots + \frac{[p^2 + \alpha^2][p^2 + (3\alpha)^2] \dots [p^2 + (2n\alpha - \alpha)^2]}{(2n+1)! \alpha^{2n}} \right\}$ <p style="text-align: right;"><math>\operatorname{Re} p &gt; (2n+1)  \operatorname{Im} \alpha </math></p>
(52)	$0 \quad 0 < t < \pi/2$ $\cos^{2n+1} t \quad t > \pi/2$	$\frac{-(2n+1)! e^{-\frac{1}{4}p\pi}}{(1^2 + p^2)(3^2 + p^2) \dots [(2n+1)^2 + p^2]}$ <p style="text-align: right;"><math>\operatorname{Re} p &gt; 0</math></p>
(53)	$\cos^{2n+1} t \quad 0 < t < \pi/2$ $0 \quad t > \pi/2$	$\frac{(2n+1)! p}{(1^2 + p^2)(3^2 + p^2) \dots [(2n+1)^2 + p^2]}$ $\times \left\{ \frac{e^{-\frac{1}{4}p\pi}}{p} + 1 + \frac{1^2 + p^2}{3!} \right.$ $\left. + \dots + \frac{(1^2 + p^2)(3^2 + p^2) \dots [(2n-1)^2 + p^2]}{(2n+1)!} \right\}$
(54)	$0 \quad 0 < t < \pi/2$ $\cos^{2n+1} t \quad \pi/2 < t < (m + \frac{1}{2})\pi$ $0 \quad t > (m + \frac{1}{2})\pi$ $m = 1, 2, 3, \dots$	$\frac{(2n+1)! e^{-\frac{1}{4}p\pi} (e^{-\pi(p+i)} - 1)}{(1^2 + p^2)(3^2 + p^2) \dots [(2n+1)^2 + p^2]}$

## Trigonometric functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(55)	$0 \quad 0 < t < \pi/2$ $t \cos t \quad t > \pi/2$	$-(1+p^2)^{-2} e^{-\frac{1}{2}p\pi} [\frac{1}{2}\pi(1+p^2) + 2p]$ $\text{Re } p > 0$
(56)	$t \cos t \quad 0 < t < \pi/2$ $0 \quad t > \pi/2$	$\frac{p^2 - 1 + e^{-\frac{1}{2}p\pi} [\frac{1}{2}\pi(1+p^2) + 2p]}{(1+p^2)^2}$
(57)	$t^n \cos(at)$	$\frac{n! p^{n+1}}{(p^2 + a^2)^{n+1}}$ $\times \sum_{0 \leq 2m \leq n+1} (-)^m \binom{n+1}{2m} \left(\frac{a}{p}\right)^{2m}$ $\text{Re } p >  \text{Im } a $
(58)	$t^{\nu-1} \cos(at) \quad \text{Re } \nu > 0$	$\frac{1}{2} \Gamma(\nu) [(p-ia)^{-\nu} + (p+ia)^{-\nu}]$ $= \Gamma(\nu) (p^2 + a^2)^{-\frac{1}{2}\nu}$ $\times \cos[\nu \tan^{-1}(a/p)]$ $\text{Re } p >  \text{Im } a $
(59)	$t^{-1}(1-\cos at)$	$\frac{1}{2} \log(1+a^2/p^2) \quad \text{Re } p >  \text{Im } a $
(60)	$(1-e^{-t})^{\nu-1} \cos(at) \quad \text{Re } \nu > 0$	$\frac{1}{2} B(\nu, p-ia) + \frac{1}{2} B(\nu, p+ia)$ $\text{Re } p >  \text{Im } a $
(61)	$t^{\nu-1} e^{-\frac{1}{2}t^2/\alpha} \cos(\beta t) \quad \text{Re } \alpha > 0, \text{ Re } \nu > 0$	$\frac{1}{2} \Gamma(\nu) \alpha^{\frac{1}{2}\nu} e^{\frac{1}{4}\alpha(p^2 - \beta^2)}$ $\times \{e^{-\frac{1}{2}i\alpha\beta p} D_{-\nu}[\alpha^{\frac{1}{2}}(p-i\beta)]$ $+ e^{\frac{1}{2}i\alpha\beta p} D_{-\nu}[\alpha^{\frac{1}{2}}(p+i\beta)]\}$
(62)	$\log t \cos(at)$	$-\frac{\alpha \tan^{-1}(a/p) + p \log[\gamma(p^2 + a^2)^{\frac{1}{2}}]}{p^2 + a^2}$ $\text{Re } p >  \text{Im } a $

## Trigonometric functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(63)	$t^{\nu-1} \log t \cos(\alpha t) \quad \operatorname{Re} \nu > 0$	$\frac{\Gamma(\nu)}{(p^2 + \alpha^2)^{\frac{\nu}{2}}} \cos[\nu \tan^{-1}(\alpha/p)] \\ \times \{\psi(\nu) - \log(p^2 + \alpha^2)^{\frac{\nu}{2}} \\ - \tan^{-1}(\alpha/p) \tan[\nu \tan^{-1}(\alpha/p)]\} \\ \operatorname{Re} p >  \operatorname{Im} \alpha $
(64)	$\cos(t^2)$	$(\tfrac{1}{2}\pi)^{\frac{1}{2}} [\tfrac{1}{2} - \cos(\tfrac{1}{4}p^2)S(\tfrac{1}{4}p^2)] \\ (\tfrac{1}{2}\pi)^{\frac{1}{2}} \{ \cos(\tfrac{1}{4}p^2) [\tfrac{1}{2} - S(\tfrac{1}{4}p^2)] - \sin(\tfrac{1}{4}p^2) [\tfrac{1}{2} - C(\tfrac{1}{4}p^2)] \} \quad \operatorname{Re} p > 0.$
(65)	$\cos(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$	$p^{-1} + i\pi^{1/2} \alpha^{1/2} p^{-3/2} e^{-\alpha/p} \\ \times \operatorname{Erf}(i\alpha^{1/2} p^{-1/2}) \quad \operatorname{Re} p > 0$
(66)	$t^{\frac{1}{2}} \cos(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$	$\pi^{1/2} p^{-5/2} (\tfrac{1}{2}p - \alpha) e^{-\alpha/p} \quad \operatorname{Re} p > 0$
(67)	$t^{-\frac{1}{2}} \cos(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$	$\pi^{\frac{1}{2}} p^{-\frac{1}{2}} e^{-\alpha/p} \quad \operatorname{Re} p > 0$
(68)	$t^{n-\frac{1}{2}} \cos(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$	$(-2)^{-n} \pi^{\frac{1}{2}} p^{-n-\frac{1}{2}} e^{-\alpha/p} \\ \times \operatorname{He}_{2n}(2^{\frac{1}{2}} \alpha^{\frac{1}{2}} p^{-\frac{1}{2}}) \quad \operatorname{Re} p > 0$
(69)	$t^{\nu-1} \cos(2^{\frac{1}{2}} \alpha^{\frac{1}{2}} t^{\frac{1}{2}}) \quad \operatorname{Re} \nu > 0$	$2^{-\frac{1}{2}-\nu} \pi^{\frac{1}{2}} \csc(\nu\pi) p^{-\nu} e^{-\frac{1}{2}\alpha/p} \\ \times [D_{2\nu-1}(\alpha^{\frac{1}{2}} p^{-\frac{1}{2}}) + D_{2\nu-1}(-\alpha^{\frac{1}{2}} p^{\frac{1}{2}})] \\ \operatorname{Re} p > 0$
(70)	$(2t-t^2)^{-\frac{1}{2}} \cos[\alpha(2t-t^2)^{\frac{1}{2}}] \\ 0 < t < 2 \\ 0 \qquad \qquad \qquad t > 2$	$\pi e^{-p} J_0[(\alpha^2 - p^2)^{\frac{1}{2}}]$
(71)	$t^{-\frac{1}{2}} \sinh \alpha [\cosh \alpha - \cos(t^{\frac{1}{2}})]^{-1} \quad \operatorname{Re} \alpha > 0$	$2\pi e^{\alpha^2 p} [\theta_3(2\alpha p, 4p) + \hat{\theta}_3(2\alpha p, 4p)] \\ - \pi^{\frac{1}{2}} p^{-\frac{1}{2}} \quad \operatorname{Re} p > 0$
(72)	$\cos(\alpha e^{-t})$	$\alpha^{-p} \Gamma(p) [U_p(2\alpha, 0) \cos \alpha \\ + U_{p+1}(2\alpha, 0) \sin \alpha] \quad \operatorname{Re} p > 0$

## Trigonometric functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(73)	$\cos [\alpha(1 - e^{-t})]$	$\alpha^{-p} \Gamma(p) U_p(2\alpha, 0)$ Re $p > 0$
(74)	$(e^t - 1)^{-\frac{1}{2}} \cos [\alpha(e^t - 1)^{\frac{1}{2}}]$	$\pi^{\frac{1}{2}} \Gamma(p + \frac{1}{2})(2/\alpha)^p J_p(\alpha)$ Re $p > -\frac{1}{2}$
(75)	$(1 - e^{-t})^{-\frac{1}{2}} \cos [\alpha(e^t - 1)^{\frac{1}{2}}]$ $\alpha > 0$	$2\pi^{\frac{1}{2}} [\Gamma(p + \frac{1}{2})]^{-1} (\frac{1}{2}\alpha)^p K_p(\alpha)$ Re $p > -\frac{1}{2}$
(76)	$t^{-1} [\cos(\alpha t) - \cos(\beta t)]$	$\frac{1}{2} \log [(p^2 + \beta^2)(p^2 + \alpha^2)^{-1}]$ Re $p >  \operatorname{Im} \alpha ,  \operatorname{Im} \beta $
(77)	$t^{-2} (\cos \alpha t - \cos \beta t)$	$\frac{1}{2} p \log [(p^2 + \alpha^2)(p^2 + \beta^2)^{-1}]$ $+ \beta \tan^{-1}(\beta/p) - \alpha \tan^{-1}(\alpha/p)$ Re $p \geq  \operatorname{Im} \alpha ,  \operatorname{Im} \beta $
(78)	$\sin(\alpha t) \sin(\beta t)$	$\frac{2\alpha\beta p}{[p^2 + (\alpha + \beta)^2][p^2 + (\alpha - \beta)^2]}$ Re $p \geq  \operatorname{Im}(\pm \alpha \pm \beta) $
(79)	$\cos(\alpha t) \sin(\beta t)$	$\frac{\beta(p^2 - \alpha^2 + \beta^2)}{[p^2 + (\alpha + \beta)^2][p^2 + (\alpha - \beta)^2]}$ Re $p >  \operatorname{Im}(\pm \alpha \pm \beta) $
(80)	$\cos(\alpha t) \cos(\beta t)$	$\frac{p(p^2 + \alpha^2 + \beta^2)}{[p^2 + (\alpha + \beta)^2][p^2 + (\alpha - \beta)^2]}$ Re $p >  \operatorname{Im}(\pm \alpha \pm \beta) $
(81)	$t^{-1} \sin(\alpha t) \sin(\beta t)$	$\frac{1}{4} \log \frac{p^2 + (\alpha + \beta)^2}{p^2 + (\alpha - \beta)^2}$ Re $p >  \operatorname{Im}(\pm \alpha \pm \beta) $

## Trigonometric functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(82)	$t^{-1} \sin(\alpha t) \cos(\beta t)$	$\frac{1}{2} \tan^{-1} \frac{2\alpha p}{p^2 - \alpha^2 + \beta^2}$ $\text{Re } p >  \text{Im}(\pm\alpha \pm \beta) $
(83)	$t^{-2} \sin(\alpha t) \sin(\beta t)$	$\frac{1}{2} \alpha \tan^{-1} \frac{2\beta p}{p^2 + \alpha^2 - \beta^2}$ + $\frac{1}{2} \beta \tan^{-1} \frac{2\alpha p}{p^2 - \alpha^2 + \beta^2}$ + $\frac{1}{4} p \log \frac{p^2 + (\alpha - \beta)^2}{p^2 + (\alpha + \beta)^2}$ $\text{Re } p \geq  \text{Im}(\pm\alpha \pm \beta) $
(84)	$\csc t \sin[(2n+1)t]$	$\frac{1}{p} + \sum_{m=1}^n \frac{2p}{p^2 + 4m^2}$ $\text{Re } p > 0$
(85)	$\tan t \cos[(2n+1)t]$	$\frac{2n+1}{p^2 + (2n+1)^2} + 2 \sum_{m=0}^{n-1} \frac{(-1)^m (2m+1)}{p^2 + (2m+1)^2}$ $\text{Re } p > 0$
(86)	$\frac{(2at \cos \alpha t - \sin \alpha t) \sin \alpha t}{t^2}$	$\frac{1}{4} p \log \left( 1 + \frac{4\alpha^2}{p^2} \right)$ $\text{Re } p > 2 \text{Im } \alpha $
(87)	$\frac{\alpha t \cos \alpha t - \sin \alpha t}{t^2}$	$p \tan^{-1} \frac{\alpha}{p} - \alpha$ $\text{Re } p >  \text{Im } \alpha $

## 4.8. Inverse trigonometric functions

(1)	$\sin^{-1} t$	$0 < t < 1$	$\frac{1}{2}\pi p^{-1} [I_0(p) - L_0(p)]$
	0	$t > 1$	

## Inverse trigonometric functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(2)	$t \sin^{-1} t$ 0 $t > 1$	$\frac{1}{2}\pi p^{-2} [\mathbf{L}_0(p) - \mathbf{I}_0(p) + p \mathbf{L}_1(p) - p \mathbf{I}_1(p)] + p^{-1}$
(3)	$\tan^{-1}(t/a)$	$p^{-1} [-\text{ci}(ap)\sin(ap) - \text{si}(ap)\cos(ap)] \quad \text{Re } p > 0$
(4)	$\text{ctn}^{-1}(t/a)$	$p^{-1} [\frac{1}{2}\pi + \text{ci}(ap)\sin(ap) + \text{si}(ap)\cos(ap)] \quad \text{Re } p > 0$
(5)	$t \tan^{-1}(t/a)$	$p^{-2} [-\text{ci}(ap)\sin(ap) - \text{si}(ap)\cos(ap) + ap^{-1}[\text{ci}(ap)\cos(ap) - \text{si}(ap)\sin(ap)]] \quad \text{Re } p > 0$
(6)	$t \text{ctn}^{-1}(t/a)$	$p^{-2} [\frac{1}{2}\pi + \text{ci}(ap)\sin(ap) + \text{si}(ap)\cos(ap) + ap^{-1}[\text{si}(ap)\sin(ap) - \text{ci}(ap)\cos(ap)]] \quad \text{Re } p > 0$
(7)	$t^{\nu-\frac{1}{2}}(1+t^2)^{\frac{1}{2}\nu-\frac{1}{2}}$ $\times e^{-i(\nu-\frac{1}{2})\text{ctn}^{-1}t}$ $\text{Re } \nu > -\frac{1}{2}$	$\frac{1}{2}i\pi^{\frac{1}{2}}\Gamma(\nu+\frac{1}{2})p^{-\nu}e^{-\frac{1}{2}ip}H_{\nu}^{(1)}(\frac{1}{2}p) \quad \text{Re } p > 0$
(8)	$t^{\nu-\frac{1}{2}}(1+t^2)^{\frac{1}{2}\nu-\frac{1}{2}}$ $\times \sin[(\nu-\frac{1}{2})\text{ctn}^{-1}t]$ $\text{Re } \nu > -\frac{1}{2}$	$-\frac{1}{2}\pi^{\frac{1}{2}}\Gamma(\nu+\frac{1}{2})p^{-\nu}[J_{\nu}(\frac{1}{2}p)\cos(\frac{1}{2}p) + Y_{\nu}(\frac{1}{2}p)\sin(\frac{1}{2}p)] \quad \text{Re } p > 0$
(9)	$t^{\nu-\frac{1}{2}}(1+t^2)^{\frac{1}{2}\nu-\frac{1}{2}}$ $\times \cos[(\nu-\frac{1}{2})\text{ctn}^{-1}t]$ $\text{Re } \nu > -\frac{1}{2}$	$\frac{1}{2}\pi^{\frac{1}{2}}\Gamma(\nu+\frac{1}{2})p^{-\nu}[J_{\nu}(\frac{1}{2}p)\sin(\frac{1}{2}p) - Y_{\nu}(\frac{1}{2}p)\cos(\frac{1}{2}p)] \quad \text{Re } p > 0$
(10)	$t^{\nu-\frac{1}{2}}(1+t^2)^{\frac{1}{2}\nu-\frac{1}{2}}$ $\times \sin[\beta - (\nu-\frac{1}{2})\text{ctn}^{-1}t]$ $\text{Re } \nu > -\frac{1}{2}$	$\frac{1}{2}\pi^{\frac{1}{2}}\Gamma(\nu+\frac{1}{2})p^{-\nu}[J_{\nu}(\frac{1}{2}p)\cos(\frac{1}{2}p-\beta) + Y_{\nu}(\frac{1}{2}p)\sin(\frac{1}{2}p-\beta)] \quad \text{Re } p > 0$

## Inverse trigonometric functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(11)	$\frac{\cos[(2n+\frac{1}{2}) \cos^{-1}(\frac{1}{2}t/b)]}{(4b^2t-t^3)^{\frac{1}{2}}}$ $0 < t < 2b$ $0 \quad t > 2b$	$(-1)^n (\frac{1}{2}\pi p)^{\frac{n}{2}} I_n(bp) K_{n+\frac{1}{2}}(bp)$
(12)	$\frac{\cos[\nu \cos^{-1}(\frac{1}{2}t/b)]}{(4b^2t-t^3)^{\frac{1}{2}}}$ $0 < t < 2b$ $0 \quad t > 2b$	$(\frac{1}{2}\pi)^{3/2} p^{1/2} [I_{\frac{1}{2}\nu-\frac{1}{2}}(bp) I_{-\frac{1}{2}\nu-\frac{1}{2}}(bp) - I_{\frac{1}{2}\nu+\frac{1}{2}}(bp) I_{\frac{1}{2}\nu+\frac{1}{2}}(bp)]$
(13)	$0 \quad 0 < t < a$ $\frac{\cos[n \cos^{-1}((2t-a-b)/(b-a))]}{(t-a)^{\frac{1}{2}} (b-t)^{\frac{1}{2}}}$ $a < t < b$ $0 \quad t > b$	$\pi e^{-\frac{1}{2}(a+b)p} I_n[\frac{1}{2}(b-a)p]$
(14)	$[t(t+1)(t+2)]^{-\frac{1}{2}}$ $\times \cos[\nu \cos^{-1}(1+t)^{-1}]$	$\pi^{\frac{1}{2}} e^p D_{\nu-\frac{1}{2}}(2^{\frac{1}{2}} p^{\frac{1}{2}}) D_{-\nu-\frac{1}{2}}(2^{\frac{1}{2}} p^{\frac{1}{2}})$ $\text{Re } p > 0$
(15)	$\frac{\cos[\nu \cos^{-1} e^{-t}]}{(1-e^{-2t})^{\frac{1}{2}}}$	$\frac{\pi 2^{-p}}{p B(\frac{1}{2}p + \frac{1}{2}\nu + \frac{1}{2}, \frac{1}{2}p - \frac{1}{2}\nu + \frac{1}{2})}$ $\text{Re } p > 0$

## 4.9. Hyperbolic functions

(1)	$\sinh(\alpha t)$	$\alpha(p^2 - \alpha^2)^{-1}$	$\text{Re } p >  \text{Re } \alpha $
(2)	$\cosh(\alpha t)$	$p(p^2 - \alpha^2)^{-1}$	$\text{Re } p >  \text{Re } \alpha $
(3)	$\sinh^2(\alpha t)$	$2\alpha^2(p^3 - 4\alpha^2 p)^{-1}$	$\text{Re } p > 2 \text{Re } \alpha $

**Hyperbolic functions (cont'd)**

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(4)	$\cosh^2(\alpha t)$	$(p^2 - 2\alpha^2)(p^3 - 4\alpha^2 p)^{-1}$ $\text{Re } p > 2 \text{Re } \alpha $
(5)	$[\sinh(\alpha t)]^\nu$ $\text{Re } \alpha > 0, \text{ Re } \nu > -1$	$2^{-\nu-1} \alpha^{-1} B(\frac{1}{2}p/\alpha - \frac{1}{2}\nu, \nu+1)$ $\text{Re } p > \text{Re } \nu\alpha$
(6)	$[\cosh(\alpha t) - 1]^\nu$ $\text{Re } \alpha > 0, \text{ Re } \nu > -\frac{1}{2}$	$2^{-\nu-1} \alpha^{-1} B(p/\alpha - \nu, 2\nu + 1)$ $\text{Re } p > \text{Re } \nu\alpha$
(7)	$\operatorname{sech} t$	$\frac{1}{2} \psi(\frac{1}{4}p + \frac{3}{4}) - \frac{1}{2} \psi(\frac{1}{4}p + \frac{1}{4})$ $\text{Re } p > -1$
(8)	$\operatorname{sech}^2 t$	$\frac{1}{2}p [\psi(\frac{1}{4}p + \frac{1}{2}) - \psi(\frac{1}{4}p)] - 1$ $\text{Re } p > -2$
(9)	$\tanh t$	$\frac{1}{2} \psi(\frac{1}{4}p + \frac{1}{2}) - \frac{1}{2} \psi(\frac{1}{4}p) - p^{-1}$ $\text{Re } p > 0$
(10)	$t^{-1} - \operatorname{csch} t$	$\psi(\frac{1}{2}p + \frac{1}{2}) - \log(\frac{1}{2}p)$ $\text{Re } p > 0$
(11)	$t^{-1} - \operatorname{ctnh} t$	$\psi(\frac{1}{2}p) + p^{-1} - \log(\frac{1}{2}p)$ $\text{Re } p > 0$
(12)	$\frac{2}{t} \sinh(\alpha t)$	$\log \frac{p+\alpha}{p-\alpha}$ $\text{Re } p >  \text{Re } \alpha $
(13)	$0 \quad 0 < t < 1$ $\frac{2}{t} \sinh(\alpha t) \quad t > 1$	$-\operatorname{Ei}(\alpha-p) + \operatorname{Ei}(-\alpha-p)$ $\text{Re } p >  \text{Re } \alpha $
(14)	$\frac{2}{t} \sinh(\alpha t) \quad 0 < t < 1$ $0 \quad t > 1$	$\log \frac{p+\alpha}{p-\alpha} + \operatorname{Ei}(\alpha-p) - \operatorname{Ei}(-\alpha-p)$

## Hyperbolic functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(15)	$0 \quad 0 < t < 1$ $\frac{2}{t} \cosh(at) \quad t > 1$	$-Ei(a-p) - Ei(-a-p)$ $Re p >  Re a $
(16)	$\frac{1}{t} \tanh t$	$\log(\frac{1}{4}p) + 2 \log \frac{\Gamma(\frac{1}{4}p)}{\Gamma(\frac{1}{4}p + \frac{1}{2})}$ $Re p > 0$
(17)	$\frac{1}{t} (1 - \operatorname{sech} t)$	$2 \log \frac{\Gamma(\frac{1}{4}p + \frac{3}{4})}{\Gamma(\frac{1}{4}p + \frac{1}{4})} - \log(\frac{1}{4}p)$ $Re p > 0$
(18)	$t^{\nu-1} \sinh(at) \quad Re \nu > -1$	$\frac{1}{2} \Gamma(\nu) [(p-a)^{-\nu} - (p+a)^{-\nu}]$ $Re p >  Re a $
(19)	$t^{\nu-1} \cosh(at) \quad Re \nu > 0$	$\frac{1}{2} \Gamma(\nu) [(p-a)^{-\nu} + (p+a)^{-\nu}]$ $Re p >  Re a $
(20)	$t^{\nu-1} \operatorname{csch} t \quad Re \nu > 1$	$2^{1-\nu} \Gamma(\nu) \zeta(\nu, \frac{1}{2}p + \frac{1}{2})$ $Re p > -1$
(21)	$t^{\nu-1} \operatorname{ctnh} t \quad Re \nu > 1$	$\Gamma(\nu) [2^{1-\nu} \zeta(\nu, \frac{1}{2}p) - p^{-\nu}]$ $Re p > 0$
(22)	$t^{\nu-1} (\operatorname{ctnh} t - 1) \quad Re \nu > 1$	$2^{1-\nu} \Gamma(\nu) \zeta(\nu, \frac{1}{2}p + 1) \quad Re p > -2$
(23)	$0 \quad 0 < t < b$ $(\cosh t - \cosh b)^{\nu-1} \quad t > b$ $Re \nu > 0$	$-i 2^{\frac{\nu}{2}} \pi^{-\frac{\nu}{2}} e^{\nu \pi i} \Gamma(\nu) (\sinh b)^{\nu-\frac{1}{2}}$ $\times Q_{p-\frac{1}{2}}^{\frac{\nu}{2}-\nu}(\cosh b)$ $Re p > Re \nu - 1$
(24)	$\sin(at) \sinh(at)$	$2a^2 p (p^4 + 4a^4)^{-\frac{1}{2}}$ $Re p >  Re a  +  Im a $

## Hyperbolic functions (cont'd)

	$f(t)$	$\mathcal{L}(f) = \int_0^\infty e^{-pt} f(t) dt$
(25)	$\cos(at) \sinh(at)$	$(ap^2 - 2a^3)(p^4 + 4a^4)^{-1}$ $\operatorname{Re} p >  \operatorname{Re} a  +  \operatorname{Im} a $
(26)	$\sin(at) \cosh(at)$	$(ap^2 + 2a^3)(p^4 + 4a^4)^{-1}$ $\operatorname{Re} p > \operatorname{Re}(\pm a \pm i\alpha)$
(27)	$\cos(at) \cosh(at)$	$p^3(p^4 + 4a^4)^{-1}$ $\operatorname{Re} p > \operatorname{Re}(\pm a \pm i\alpha)$
(28)	$e^{-\alpha \sinh t}$ $\operatorname{Re} \alpha > 0$	$\pi \csc(\pi p) [J_p(a) - J_p(-a)]$
(29)	$e^{-\alpha \sinh(t+i\psi)}$ $-\pi/2 < \psi < \pi/2$ $ \arg a  < \pi/2 - \psi$	$\csc(\pi p) [\int_0^\pi e^{i\alpha \sin \theta \cos \theta} \times \cos(p\theta - \alpha \cos \psi \sin \theta) d\theta - \pi e^{ip\psi} J_p(a)]$
(30)	$e^{-\alpha \cosh t}$ $\operatorname{Re} \alpha > 0$	$\csc(\pi p) [\int_0^\pi e^{\alpha \cos \theta} \cos(p\theta) d\theta - \pi I_p(a)]$
(31)	$(\sinh \frac{1}{2}t)^2 \beta e^{-2\alpha \operatorname{ctnh} \frac{1}{2}t}$ $\operatorname{Re} \alpha > 0$	$\frac{1}{2} a^{\frac{1}{2}\beta - \frac{1}{2}} \Gamma(p-\beta) [W_{-p+\frac{1}{2}, \beta}(4a) - (p-\beta) W_{-p-\frac{1}{2}, \beta}(4a)]$ $\operatorname{Re} p > \operatorname{Re} \beta$
(32)	$\log \cosh t$	$\frac{1}{2} p^{-1} [\psi(\frac{1}{4}p + \frac{1}{2}) - \psi(\frac{1}{4}p)] - p^{-2}$ $\operatorname{Re} p > 0$
(33)	$\log(\sinh t) - \log t$	$p^{-1} [\log(\frac{1}{2}p) - \frac{1}{2}p^{-1} - \psi(\frac{1}{2}p)]$ $\operatorname{Re} p > 0$
(34)	$\sinh(2a^{\frac{1}{2}}t^{\frac{1}{2}})$	$\pi^{1/2} a^{1/2} p^{-3/2} e^{\alpha/p}$ $\operatorname{Re} p > 0$

## Hyperbolic functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(35)	$\cosh(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$	$\pi^{1/2} \alpha^{1/2} p^{-3/2} e^{\alpha/p} \operatorname{Erf}(\alpha^{1/2} p^{-1/2}) + p^{-1}$ $\operatorname{Re} p > 0$
(36)	$t^{\frac{1}{2}} \sinh(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$	$\pi^{1/2} p^{-5/2} (\frac{1}{2}p + \alpha) e^{\alpha/p}$ $\times \operatorname{Erf}(\alpha^{1/2} p^{-1/2}) - \alpha^{1/2} p^{-2}$ $\operatorname{Re} p > 0$
(37)	$t^{\frac{1}{2}} \cosh(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$	$\pi^{1/2} p^{-5/2} (\frac{1}{2}p + \alpha) e^{\alpha/p}$ $\operatorname{Re} p > 0$
(38)	$t^{-\frac{1}{2}} \sinh(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$	$\pi^{1/2} p^{-1/2} e^{\alpha/p} \operatorname{Erf}(\alpha^{1/2} p^{-1/2})$ $\operatorname{Re} p > 0$
(39)	$t^{-\frac{1}{2}} \cosh(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$	$\pi^{\frac{1}{2}} p^{-\frac{1}{2}} e^{\alpha/p}$ $\operatorname{Re} p > 0$
(40)	$t^{-\frac{1}{2}} \sinh^2(\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$	$\frac{1}{2} \pi^{\frac{1}{2}} p^{-\frac{1}{2}} (e^{\alpha/p} - 1)$ $\operatorname{Re} p > 0$
(41)	$t^{-\frac{1}{2}} \cosh^2(\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$	$\frac{1}{2} \pi^{\frac{1}{2}} p^{-\frac{1}{2}} (e^{\alpha/p} + 1)$ $\operatorname{Re} p > 0$
(42)	$t^{-3/4} \sinh(2^{3/2} \alpha^{1/2} t^{1/2})$	$\pi(2\alpha)^{\frac{1}{4}} p^{-\frac{1}{4}} e^{\alpha/p} I_{\frac{1}{4}}(\alpha/p)$ $\operatorname{Re} p > 0$
(43)	$t^{-3/4} \cosh(2^{3/2} \alpha^{1/2} t^{1/2})$	$\pi(2\alpha)^{\frac{1}{4}} p^{-\frac{1}{4}} e^{\alpha/p} I_{-\frac{1}{4}}(\alpha/p)$ $\operatorname{Re} p > 0$
(44)	$t^{\nu-1} \sinh(2^{\frac{1}{2}} \alpha^{\frac{1}{2}} t^{\frac{1}{2}})$ $\operatorname{Re} \nu > -\frac{1}{2}$	$\Gamma(2\nu)(2p)^{-\nu} e^{\frac{1}{4}\alpha/p} [D_{-2\nu}(-\alpha^{\frac{1}{2}} p^{-\frac{1}{2}}) - D_{-2\nu}(\alpha^{\frac{1}{2}} p^{-\frac{1}{2}})]$ $\operatorname{Re} p > 0$
(45)	$t^{\nu-1} \cosh(2^{\frac{1}{2}} \alpha^{\frac{1}{2}} t^{\frac{1}{2}})$ $\operatorname{Re} \nu > 0$	$\Gamma(2\nu)(2p)^{-\nu} e^{\frac{1}{4}\alpha/p} [D_{-2\nu}(-\alpha^{\frac{1}{2}} p^{-\frac{1}{2}}) + D_{-2\nu}(\alpha^{\frac{1}{2}} p^{-\frac{1}{2}})]$ $\operatorname{Re} p > 0$
(46)	$\frac{\sin(bt)}{\cos(bt)} \frac{\tanh(at^{\frac{1}{2}})}{\operatorname{ctnh}(at^{\frac{1}{2}})}$	see Mordell, L. J., 1920: <i>Mess. of Math.</i> 49, 65-72

## Hyperbolic functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(47)	$\frac{\sinh[(2\nu+1)\pi^{\frac{1}{2}}t^{\frac{1}{2}}]}{\pi^{\frac{1}{2}}t^{\frac{1}{2}} \sinh(\pi^{\frac{1}{2}}t^{\frac{1}{2}})}$	For this and more generally for the Laplace transform of $\frac{a \cosh(At^{\frac{1}{2}}) + b \cosh(Bt^{\frac{1}{2}})}{t^{\frac{1}{2}}\{a^2 + 2ab \cosh[(A+B)t^{\frac{1}{2}}] + b^2\}}$ see Mordell, L. J., 1933: <i>Acta Math.</i> 61, 323-360 and <i>Quart. J. Math.</i> 1920, 48, 329-342.
(48)	$(e^t - 1)^{-\frac{1}{2}} \sinh[a(1-e^{-t})^{\frac{1}{2}}]$	$\pi^{\frac{1}{2}} \Gamma(p + \frac{1}{2}) 2^p a^{-p} L_p(a)$ $\text{Re } p > -\frac{1}{2}$
(49)	$(e^t - 1)^{-\frac{1}{2}} \cosh[a(1-e^{-t})^{\frac{1}{2}}]$	$\pi^{\frac{1}{2}} \Gamma(p + \frac{1}{2}) 2^p a^{-p} I_p(a)$ $\text{Re } p > -\frac{1}{2}$
(50)	$\tanh[\frac{1}{2}\pi(e^{2t} - 1)^{\frac{1}{2}}]$	$2^{-p} \zeta(p-1)$ $\text{Re } p > 0$

## 4.10. Inverse hyperbolic functions

(1)	$\sinh^{-1} t$	$\frac{1}{2}\pi p^{-1} [\mathbf{H}_0(p) - Y_0(p)]$	$\text{Re } p > 0$
(2)	$0 \quad 0 < t < b$ $\cosh^{-1}(t/b) \quad t > b$	$p^{-1} K_0(bp)$	$\text{Re } p > 0$
(3)	$\cosh^{-1}(1+t/a) \quad  \arg a  < \pi$	$p^{-1} e^{\alpha p} K_0(\alpha p)$	$\text{Re } p > 0$
(4)	$t \sinh^{-1} t$	$\pi \frac{\mathbf{H}_0(p) - Y_0(p)}{2p^2} + \pi \frac{\mathbf{H}_1(p) - Y_1(p)}{2p} - \frac{1}{p}$	$\text{Re } p > 0$
(5)	$\sinh[(2n+1) \sinh^{-1} t]$	$O_{2n+1}(p)$	$\text{Re } p > 0$

## Inverse hyperbolic functions (cont'd)

	$f(t)$	$\int_0^\infty e^{-pt} f(t) dt$
(6)	$\cosh(2n \sinh^{-1} t)$	$O_{2n}(p)$ $\operatorname{Re} p > 0$
(7)	$\sinh(\nu \sinh^{-1} t)$	$\nu p^{-1} S_{0,\nu}(p)$ $\operatorname{Re} p > 0$
(8)	$\cosh(\nu \sinh^{-1} t)$	$p^{-1} S_{1,\nu}(p)$ $\operatorname{Re} p > 0$
(9)	$0 \quad 0 < t < b$ $\sinh[\nu \cosh^{-1}(t/b)] \quad t > b$	$\nu p^{-1} K_\nu(bp)$ $\operatorname{Re} p > 0$
(10)	$\sinh[\nu \cosh^{-1}(1+t/a)]$ $ \arg a  < \pi$	$\nu p^{-1} e^{\alpha p} K_\nu(\alpha p)$ $\operatorname{Re} p > 0$
(11)	$(1+t^2)^{-\frac{n}{2}} \exp(n \sinh^{-1} t)$	$\frac{1}{2}[S_n(p) - \pi E_n(p) - \pi Y_n(p)]$ $\operatorname{Re} p > 0$
(12)	$(1+t^2)^{-\frac{n}{2}} \exp(-n \sinh^{-1} t)$	$\frac{1}{2}(-1)^{n+1}[S_n(p) + \pi E_n(p) + \pi Y_n(p)]$ $\operatorname{Re} p > 0$
(13)	$(1+t^2)^{-\frac{n}{2}} \exp(-\nu \sinh^{-1} t)$	$\pi \csc(\nu\pi)[J_\nu(p) - J_\nu(p)]$ $\operatorname{Re} p > 0$
(14)	$\frac{\sinh(\nu \sinh^{-1} t)}{(t^2+1)^{\frac{n}{2}}}$	$\nu S_{-1,\nu}(p)$ $\operatorname{Re} p > 0$
(15)	$0 \quad 0 < t < b$ $\frac{\cosh(n \sinh^{-1} t)}{(t^2+1)^{\frac{n}{2}}} \quad t > b$	$S_n(\sinh^{-1} b, p)$ $\operatorname{Re} p > 0$
(16)	$\frac{\cosh(\nu \sinh^{-1} t)}{(t^2+1)^{\frac{n}{2}}}$	$S_{0,\nu}(p)$ $\operatorname{Re} p > 0$

## Inverse hyperbolic functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(17)	$0 \quad 0 < t < b$ $\frac{\cosh(n \cosh^{-1} t)}{(t^2 - 1)^{\frac{n}{2}}} \quad t > b > 1$	$C_n(\cosh^{-1} b, p) \quad \operatorname{Re} p > 0$
(18)	$0 \quad 0 < t < b$ $\frac{\cosh[\nu \cosh^{-1}(t/b)]}{(t^2 - b^2)^{\frac{\nu}{2}}} \quad t > b$	$K_\nu(bp) \quad \operatorname{Re} p > 0$
(19)	$\frac{\cosh[\nu \cosh^{-1}(1+t/a)]}{(t^2 + at)^{\frac{\nu}{2}}} \quad  \arg a  < \pi$	$e^{\alpha p} K_\nu(\alpha p) \quad \operatorname{Re} p > 0$
(20)	$(t^3 + 4\alpha^2 t)^{-\frac{\nu}{2}} \times \exp[2\nu \sinh^{-1}(\frac{1}{2}t/a)] \quad \operatorname{Re} \alpha > 0$	$(\frac{1}{2}\pi)^{3/2} p^{1/2} [J_{\nu+\frac{1}{4}}(\alpha p) J_{\nu-\frac{1}{4}}(\alpha p) + Y_{\nu+\frac{1}{4}}(\alpha p) Y_{\nu-\frac{1}{4}}(\alpha p)] \quad \operatorname{Re} p > 0$
(21)	$(t^3 + 4\alpha^2 t)^{-\frac{\nu}{2}} \times \exp[-2\nu \sinh^{-1}(\frac{1}{2}t/a)] \quad \operatorname{Re} \alpha > 0$	$(\frac{1}{2}\pi)^{3/2} p^{1/2} [J_{\nu+\frac{1}{4}}(\alpha p) Y_{\nu-\frac{1}{4}}(\alpha p) - J_{\nu-\frac{1}{4}}(\alpha p) Y_{\nu+\frac{1}{4}}(\alpha p)] \quad \operatorname{Re} p > 0$
(22)	$(t^3 + 4\alpha^2 t)^{-\frac{\nu}{2}} \{ \cos[(\nu + \frac{1}{4})\pi] \times \exp[-2\nu \sinh^{-1}(\frac{1}{2}t/a)] + \sin[(\nu + \frac{1}{4})\pi] \times \exp[2\nu \sinh^{-1}(\frac{1}{2}t/a)] \} \quad \operatorname{Re} \alpha > 0$	$(\frac{1}{2}\pi)^{3/2} p^{1/2} [J_{\frac{1}{4}+\nu}(\alpha p) J_{\frac{1}{4}-\nu}(\alpha p) + Y_{\frac{1}{4}+\nu}(\alpha p) Y_{\frac{1}{4}-\nu}(\alpha p)] \quad \operatorname{Re} p > 0$
(23)	$(t^3 + 4\alpha^2 t)^{-\frac{\nu}{2}} \{ \sin[(\nu + \frac{1}{4})\pi] \times \exp[-2\nu \sinh^{-1}(\frac{1}{2}t/a)] - \cos[(\nu + \frac{1}{4})\pi] \times \exp[2\nu \sinh^{-1}(\frac{1}{2}t/a)] \} \quad \operatorname{Re} \alpha > 0$	$(\frac{1}{2}\pi)^{3/2} p^{1/2} [J_{\frac{1}{4}-\nu}(\alpha p) Y_{\frac{1}{4}-\nu}(\alpha p) - J_{\frac{1}{4}-\nu}(\alpha p) Y_{\frac{1}{4}+\nu}(\alpha p)] \quad \operatorname{Re} p > 0$

## Inverse hyperbolic functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(24)	$\frac{\sinh[2\nu \sinh^{-1}(\frac{1}{2}t/a)]}{(t^3 + 4a^2 t)^{\frac{1}{2}}} \quad  \arg a  < \pi$	$\frac{\pi^{3/2} p^{1/2}}{8i} [e^{\nu\pi i} H_{\frac{1}{2}+\nu}^{(1)}(ap) H_{\frac{1}{2}-\nu}^{(2)}(ap) - e^{-\nu\pi i} H_{\frac{1}{2}-\nu}^{(1)}(ap) H_{\frac{1}{2}+\nu}^{(2)}(ap)]$ $\text{Re } p > 0$
(25)	$\frac{\cosh[2\nu \sinh^{-1}(\frac{1}{2}t/a)]}{(t^3 + 4a^2 t)^{\frac{1}{2}}} \quad  \arg a  < \pi$	$\frac{\pi^{3/2} p^{1/2}}{8} [e^{\nu\pi i} H_{\frac{1}{2}+\nu}^{(1)}(ap) H_{\frac{1}{2}-\nu}^{(2)}(ap) + e^{-\nu\pi i} H_{\frac{1}{2}-\nu}^{(1)}(ap) H_{\frac{1}{2}+\nu}^{(2)}(ap)]$ $\text{Re } p > 0$
(26)	$0 \quad 0 < t < 2b$ $\frac{\cosh[2\nu \cosh^{-1}(\frac{1}{2}t/b)]}{(t^3 - 4b^2 t)^{\frac{1}{2}}} \quad t > 2b$	$\frac{p^{\frac{1}{2}}}{(2\pi)^{\frac{1}{2}}} K_{\nu+\frac{1}{2}}(bp) K_{\nu-\frac{1}{2}}(bp)$ $\text{Re } p > 0$
(27)	$\frac{\cosh[2\nu \cosh^{-1}(1 + \frac{1}{2}t/a)]}{[t(t+2a)(t+4a)]^{\frac{1}{2}}} \quad  \arg a  < \pi$	$\frac{p^{\frac{1}{2}}}{(2\pi)^{\frac{1}{2}}} e^{2\alpha p} K_{\nu+\frac{1}{2}}(ap) K_{\nu-\frac{1}{2}}(ap)$ $\text{Re } p > 0$

## 4.11. Orthogonal polynomials

(1)	$P_n(t)$	Sum of powers with negative indices in the expansion, in ascending powers of $p$ , of $(-1)^n (\frac{1}{2}\pi)^{\frac{1}{2}} p^{-\frac{1}{2}} I_{-n-\frac{1}{2}}(p)$ $\text{Re } p > 0$
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## Orthogonal polynomials (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(2)	$P_n(1-t)$	$\begin{aligned} & e^{-p} p^n \left( \frac{1}{p} \frac{d}{dp} \right)^n \left( \frac{e^p}{p} \right) \\ &= p^n \left( 1 + \frac{1}{2} \frac{d}{dp} \right)^n \left( \frac{1}{p^{n+1}} \right) \\ & \text{Re } p > 0 \end{aligned}$
(3)	$P_n(e^{-t})$	$\begin{aligned} & \frac{(p-1)(p-2)(p-3)\dots(p-n+1)}{(p+n)(p+n-2)\dots(p-n+2)} \\ & \text{Re } p > 0 \end{aligned}$
(4)	$P_{2n}(\cos t)$	$\begin{aligned} & \frac{(p^2+1^2)(p^2+3^2)\dots[p^2+(2n-1)^2]}{p(p^2+2^2)(p^2+4^2)\dots[p^2+(2n)^2]} \\ & \text{Re } p > 0 \end{aligned}$
(5)	$P_{2n+1}(\cos t)$	$\begin{aligned} & \frac{p(p^2+2^2)(p^2+4^2)\dots[p^2+(2n)^2]}{(p^2+1^2)(p^2+3^2)\dots[p^2+(2n+1)^2]} \\ & \text{Re } p > 0 \end{aligned}$
(6)	$P_{2n}(\cosh t)$	$\begin{aligned} & \frac{(p^2-1^2)(p^2-3^2)\dots[p^2-(2n-1)^2]}{p(p^2-2^2)(p^2-4^2)\dots[p^2-(2n)^2]} \\ & \text{Re } p > 2n \end{aligned}$
(7)	$P_{2n+1}(\cosh t)$	$\begin{aligned} & \frac{p(p^2-2^2)(p^2-4^2)\dots[p^2-(2n)^2]}{(p^2-1^2)(p^2-3^2)\dots[p^2-(2n+1)^2]} \\ & \text{Re } p > 2n+1 \end{aligned}$
(8)	$2^\nu i^n (n+\nu) \Gamma(\nu) C_n^\nu(-it)$	$A_{n,\nu}(p) \quad \text{Re } p > 0$
(9)	$\begin{aligned} & [t(2a-t)]^{\nu-\frac{1}{2}} C_n^\nu(t/a-1) \\ & 0 < t < 2a \\ & t > 2a \\ & \text{Re } \nu > -\frac{1}{2} \end{aligned}$	$\begin{aligned} & (-1)^n \frac{\pi \Gamma(2\nu+n)}{n! \Gamma(\nu)} \left( \frac{a}{2p} \right)^\nu e^{-ap} \\ & \times I_{\nu+n}(ap) \end{aligned}$

## Orthogonal polynomials (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(10)	$P_n^{(\alpha, \beta)}(t)$	$2(1-2\mu)_n A_{\kappa, \mu, n}(2p) \quad \operatorname{Re} p > 0$ $\kappa = \frac{1}{2}\alpha - \frac{1}{2}\beta$ $\mu = \frac{1}{2}\alpha + \frac{1}{2}\beta + \frac{1}{2} + n$
(11)	$t^{\alpha-1} \operatorname{He}_n(t)$ $\operatorname{Re} \alpha > \begin{cases} 0 & \text{if } n \text{ is even} \\ -1 & \text{if } n \text{ is odd} \end{cases}$	$\sum_{m=0}^{[n/2]} \frac{n! \Gamma(\alpha+n-2m)}{m! (n-2m)!} (-\frac{1}{2})^m p^{2m-\alpha-n} \quad \operatorname{Re} p > 0$ $[n/2] = \begin{cases} \frac{1}{2}n & \text{if } n \text{ is even} \\ \frac{1}{2}n - \frac{1}{2} & \text{if } n \text{ is odd} \end{cases}$
(12)	$\operatorname{He}_{2n+1}(t^{\frac{1}{2}})$	$\pi^{\frac{1}{2}} 2^{-n-1} \frac{(2n+1)!}{n!} \frac{(\frac{1}{2}-p)^n}{p^{n+3/2}} \quad \operatorname{Re} p > 0$
(13)	$t^{-\frac{1}{2}} \operatorname{He}_{2n}(t^{\frac{1}{2}})$	$\pi^{\frac{1}{2}} 2^{-n} \frac{(2n)!}{n!} \frac{(\frac{1}{2}-p)^n}{p^{n+\frac{1}{2}}} \quad \operatorname{Re} p > 0$
(14)	$t^{\alpha-\frac{1}{2}n-1} \operatorname{He}_n(t^{\frac{1}{2}})$ $\operatorname{Re} \alpha > \begin{cases} \frac{1}{2}n & \text{if } n \text{ is even} \\ \frac{1}{2}n - \frac{1}{2} & \text{if } n \text{ is odd} \end{cases}$	$\Gamma(\alpha) p^{-\alpha} {}_2F_1(-\frac{1}{2}n, \frac{1}{2}-\frac{1}{2}n; 1-\alpha; 2p)$ If $\alpha$ is an integer, take the first $1+[n/2]$ terms of the series. $\operatorname{Re} p > 0$
(15)	$e^{\beta t} \operatorname{He}_{2n+1}[2^{\frac{1}{2}}(\alpha-\beta)^{\frac{1}{2}} t^{\frac{1}{2}}]$	$(-2)^{-n} (\frac{1}{2}\pi)^{\frac{1}{2}} (\alpha-\beta)^{\frac{1}{2}}$ $\times \frac{(2n+1)!}{n!} \frac{(p-\alpha)^n}{(p-\beta)^{n+3/2}}$ $\operatorname{Re} p > \operatorname{Re} \beta$
(16)	$e^{\beta t} t^{-\frac{1}{2}} \operatorname{He}_{2n}[2^{\frac{1}{2}}(\alpha-\beta)^{\frac{1}{2}} t^{\frac{1}{2}}]$	$(-2)^{-n} \pi^{\frac{1}{2}} \frac{(2n)!}{n!} \frac{(p-\alpha)^n}{(p-\beta)^{n+\frac{1}{2}}}$ $\operatorname{Re} p > \operatorname{Re} \beta$

## Orthogonal polynomials (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(17)	$t^{-\frac{1}{2}} \left[ \text{He}_n \left( \frac{x+t^{\frac{1}{2}}}{\lambda} \right) + \text{He}_n \left( \frac{x-t^{\frac{1}{2}}}{\lambda} \right) \right]$	$(2\pi/p)^{\frac{1}{2}} (1 - \frac{1}{2}\lambda^{-2} p^{-1})^{\frac{1}{2}n}$ $\times \text{He}_n \left[ \frac{x}{(\lambda^2 - \frac{1}{2}p^{-1})^{\frac{1}{2}}} \right] \quad \text{Re } p > 0$
(18)	$t^{-\frac{1}{2}(n+1)} e^{-\frac{1}{2}\alpha/t} \text{He}_n(\alpha^{\frac{1}{2}} t^{-\frac{1}{2}})$ $\text{Re } \alpha > 0$	$2^{\frac{1}{2}n} \pi^{\frac{1}{2}} p^{\frac{1}{2}n-\frac{1}{2}} e^{-(2ap)^{\frac{1}{2}}} \quad \text{Re } p > 0$
(19)	$t^{-\frac{1}{2}} \text{He}_{2n}(2^{\frac{1}{2}} \alpha^{\frac{1}{2}} t^{\frac{1}{2}})$ $\times \text{He}_{2n}(2^{\frac{1}{2}} \beta^{\frac{1}{2}} t^{\frac{1}{2}})$	$\frac{\pi^{\frac{1}{2}} (2m+2n)!}{(-2)^{m+n} (m+n)!} \frac{(p-\alpha)^n (p-\beta)^n}{p^{m+n+\frac{1}{2}}}$ $\times {}_2F_1 \left[ \begin{matrix} -m, -n; -m-n+\frac{1}{2}; \\ (p-\alpha)(p-\beta) \end{matrix} \right] \quad \text{Re } p > 0$
(20)	$(\alpha\beta t)^{-\frac{1}{2}} \text{He}_{2n+1}(2^{\frac{1}{2}} \alpha^{\frac{1}{2}} t^{\frac{1}{2}})$ $\times \text{He}_{2n+1}(2^{\frac{1}{2}} \beta^{\frac{1}{2}} t^{\frac{1}{2}})$	$\frac{-\pi^{\frac{1}{2}} (2m+2n+2)!}{(-2)^{m+n+1} (m+n+1)!} \frac{(p-\alpha)^n (p-\beta)^n}{p^{m+n+3/2}}$ $\times {}_2F_1 \left[ \begin{matrix} -m, -n; -m-n-\frac{1}{2}; \\ (p-\alpha)(p-\beta) \end{matrix} \right] \quad \text{Re } p > 0$
(21)	$t^{-\frac{1}{2}} e^{-(\alpha+\beta)t} \text{He}_n(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$ $\times \text{He}_n(2\beta^{\frac{1}{2}} t^{\frac{1}{2}})$	$\pi^{\frac{1}{2}} n! \frac{(\alpha+\beta-p)^{\frac{1}{2}n}}{(\alpha+\beta+p)^{\frac{1}{2}n+\frac{1}{2}}}$ $\times P_n \left\{ \frac{2\alpha^{\frac{1}{2}} \beta^{\frac{1}{2}}}{[(\alpha+\beta)^2 - p^2]^{\frac{1}{2}}} \right\} \quad \text{Re } (\alpha + \beta + p) > 0$
(22)	$t^{-\frac{1}{2}} \left[ \text{He}_n \left( \frac{x+t^{\frac{1}{2}}}{\lambda} \right) \text{He}_n \left( \frac{y+t^{\frac{1}{2}}}{\mu} \right) + \text{He}_n \left( \frac{x-t^{\frac{1}{2}}}{\lambda} \right) \text{He}_n \left( \frac{y-t^{\frac{1}{2}}}{\mu} \right) \right]$	$\frac{2\pi^{\frac{1}{2}} \lambda^{-m} \mu^{-n}}{(2p)^{\frac{1}{2}m+\frac{1}{2}n+\frac{1}{2}}} \sum_{k=0}^{\min(m,n)} \left\{ \binom{m}{k} \binom{n}{k} k! \right.$ $\times (2\lambda^2 p - 1)^{\frac{1}{2}m+\frac{1}{2}k} (2\mu^2 p - 1)^{\frac{1}{2}n+\frac{1}{2}k}$ $\times \text{He}_{m-k} \left[ \frac{x}{(\lambda^2 - \frac{1}{2}p^{-1})^{\frac{1}{2}}} \right]$ $\times \text{He}_{n-k} \left[ \frac{y}{(\mu^2 - \frac{1}{2}p^{-1})^{\frac{1}{2}}} \right] \left. \right\} \quad \text{Re } p > 0$

**Orthogonal polynomials (cont'd)**

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(23)	$(e^{-t}-1)^{-\frac{1}{2}} \text{He}_{2n}[x^{\frac{1}{2}}(1-e^{-t})^{\frac{1}{2}}]$	$\frac{(-2)^n \pi^{\frac{1}{2}} (2n)! \Gamma(p+\frac{1}{2})}{\Gamma(n+p+1)} L_n^p(\frac{1}{2}x)$ $\text{Re } p > -\frac{1}{2}$
(24)	$\text{He}_{2n+1}[x^{\frac{1}{2}}(1-e^{-t})^{\frac{1}{2}}]$	$\frac{(-2)^n \pi^{\frac{1}{2}} (2n+1)! \Gamma(p) x^{\frac{1}{2}}}{\Gamma(n+p+3/2)} L_n^p(\frac{1}{2}x)$ $\text{Re } p > 0$
(25)	$L_n(t)$	$(p-1)^n p^{-n-1}$ $\text{Re } p > 0$
(26)	$t^n L_n(t)$	$n! p^{-n-1} P_n(1-2p^{-1})$ $\text{Re } p > 0$
(27)	$L_n^\alpha(t)$	$\sum_{m=0}^n \binom{\alpha+m-1}{m} \frac{(p-1)^{n-m}}{p^{n-m+1}}$ $\text{Re } p > 0$
(28)	$t^\alpha L_n^\alpha(t)$ $\text{Re } \alpha > -1$	$\frac{\Gamma(\alpha+n+1)}{n!} \frac{(p-1)^n}{p^{\alpha+n+1}}$ $\text{Re } p > 0$
(29)	$t^\beta L_n^\alpha(t)$ $\text{Re } \beta > -1$	$\frac{\Gamma(\beta+n+1)}{n!} \frac{(p-1)^n}{p^{\beta+n+1}}$ $\times {}_2F_1[-n, \alpha-\beta; -\beta-n; p/(p-1)]$ $\text{Re } p > 0$
(30)	$t^{2\alpha} [L_n^\alpha(t)]^2$ $\text{Re } \alpha > -\frac{1}{2}$	$\frac{2^{2\alpha} \Gamma(\alpha+\frac{1}{2}) \Gamma(n+\frac{1}{2})}{\pi (n!)^2 p^{2\alpha+1}}$ $\times {}_2F_1[-n, \alpha+\frac{1}{2}; \frac{1}{2}-n; (1-2/p)^2]$ $\text{Re } p > 0$

## Orthogonal polynomials (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(31)	$t^\alpha e^{\lambda t} L_n^\alpha(\kappa t)$ $\text{Re } \alpha > -1$	$\frac{\Gamma(\alpha+n+1)}{n!} \frac{(p-\kappa-\lambda)^n}{(p-\lambda)^{\alpha+n+1}}$ $\text{Re}(p-\lambda) > 0$
(32)	$e^{-t} \sum_{n=0}^{\infty} a_{nn} L_n(2t)$ where $a_{nn}$ is given by $P_n(z) = \sum_{n=0}^{\infty} a_{nn} z^n$	$\frac{1}{p+1} P_n\left(\frac{p-1}{p+1}\right)$ $\text{Re } p > -1$
(33)	$t^{-n} e^{-\lambda/t} L_n^\alpha(\lambda/t)$ $\text{Re } \lambda > 0$	$(-1)^n (2/n!) \lambda^{-\frac{n}{2}} p^{\frac{n}{2}} K_\alpha(2\lambda^{\frac{1}{2}} p^{\frac{1}{2}})$ $\text{Re } p > 0$
(34)	$L_n(\lambda t) L_n(\kappa t)$	$\frac{(p-\lambda-\kappa)^n}{p^{n+1}} P_n\left[\frac{p^2 - (\lambda+\kappa)p + 2\lambda\kappa}{p(p-\lambda-\kappa)}\right]$ $\text{Re } p > 0$
(35)	$t^\alpha L_n^\alpha(\lambda t) L_n^\alpha(\kappa t)$ $\text{Re } \alpha > -1$	$\frac{\Gamma(m+n+\alpha+1)}{m! n!} \frac{(p-\lambda)^n (p-\kappa)^n}{p^{m+n+\alpha+1}}$ $\times {}_2F_1\left[-m, -n; -m-n-\alpha; \frac{p(p-\lambda-\kappa)}{(p-\lambda)(p-\kappa)}\right]$ $\text{Re } p > 0$
(36)	$t^{2\alpha} L_n^\alpha(\lambda t) L_n^\alpha(\kappa t)$ $\text{Re } \alpha > -\frac{1}{2}$	$\frac{\Gamma(2\alpha+1) \Gamma(n+\alpha+1)}{n! p^{2\alpha+1}}$ $\times \sum_{r=0}^{\infty} \left\{ \frac{(-1)^r [1-(\lambda+\kappa)/(2p)]^{n-r}}{r! \Gamma(\alpha-r+1)} \right.$ $\left. \times C_{n+r}^{\alpha+\frac{1}{2}} \left[ \frac{p^2 - (\lambda+\kappa)p + 2\lambda\kappa}{p(p-\lambda-\kappa)} \right] \right\}$ $\text{Re } p > 0$

## Orthogonal polynomials (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(37)	$t^n p_n(m, t)$	$m! (p-1)^n p^{-n-1} \quad \operatorname{Re} p > 0$
(38)	$t^{\alpha-1} p_n(m, t) \quad \operatorname{Re} \alpha > \min(n, m)$	$\frac{m! \Gamma(\alpha-n)}{(m-n)! p^{\alpha-n}} \\ \times {}_2F_1(-n, \alpha-n; m-n+1; r/p) \quad \operatorname{Re} p > 0$

## 4.12. Gamma function, error function, exponential integral and related functions

(1)	$\binom{t}{n} t^{\nu-1} \quad \operatorname{Re} \nu > 0$	$\frac{\Gamma(\nu)}{p^\nu} \Phi_n \left( \nu, \frac{1}{p} \right) \quad \operatorname{Re} p > 0$ where $\sum_0^\infty h^n \Phi_n(\nu, z) = [1 - z \log(h+1)]^{-\nu}$
(2)	$\operatorname{Erf}(\frac{1}{2}t/a) \quad  \arg a  < \frac{1}{4}\pi$	$p^{-1} e^{a^2 p^2} \operatorname{Erfc}(ap) \quad \operatorname{Re} p > 0$
(3)	$e^{-a^2 t^2} \operatorname{Erf}(iat) \quad  \arg a  < \frac{1}{4}\pi$	$(2ai\pi^{\frac{1}{2}})^{-1} e^{\frac{1}{4}a^{-2} p^2} \operatorname{Ei}(-\frac{1}{4}a^{-2} p^2) \quad \operatorname{Re} p > 0$
(4)	$\operatorname{Erf}(a^{\frac{1}{2}} t^{\frac{1}{2}})$	$a^{\frac{1}{2}} p^{-1} (p+a)^{-\frac{1}{2}} \quad \operatorname{Re} p > 0, -\operatorname{Re} a$
(5)	$e^{\alpha t} \operatorname{Erf}(a^{\frac{1}{2}} t^{\frac{1}{2}})$	$a^{\frac{1}{2}} p^{-\frac{1}{2}} (p-a)^{-1} \quad \operatorname{Re} p > 0, \operatorname{Re} \alpha$
(6)	$\operatorname{Erf}(\frac{1}{2}a^{\frac{1}{2}} t^{-\frac{1}{2}}) \quad \operatorname{Re} \alpha > 0$	$p^{-1} (1 - e^{-a^{\frac{1}{2}} p^{\frac{1}{2}}}) \quad \operatorname{Re} p > 0$

## Gamma function etc. (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(7)	$\text{Erfc}(\frac{1}{2}t/\alpha) \quad  \arg \alpha  < \frac{1}{4}\pi$	$p^{-1} [1 - e^{\alpha^2 p^2} \text{Erfc}(\alpha p)]$
(8)	$e^{-\alpha^2 t^2} \text{Erfc}(i\alpha t)$	$\frac{1}{2}\pi^{\frac{1}{4}} \alpha^{-1} e^{\frac{1}{4}\alpha^{-2} p^2} [\text{Erfc}(\frac{1}{2}\alpha^{-1} p) + i\pi^{-1} \text{Ei}(-\frac{1}{4}\alpha^{-2} p^2)] \quad \text{Re } p > 0$
(9)	$\text{Erfc}(\alpha^{\frac{1}{4}} t^{\frac{1}{4}})$	$p^{-\frac{1}{4}} (p + \alpha)^{-\frac{1}{4}} [(p + \alpha)^{\frac{1}{4}} - \alpha^{\frac{1}{4}}] \quad \text{Re } p > -\text{Re } \alpha$
(10)	$e^{\alpha t} \text{Erfc}(\alpha^{\frac{1}{4}} t^{\frac{1}{4}})$	$p^{-\frac{1}{4}} (p^{\frac{1}{4}} + \alpha^{\frac{1}{4}})^{-1} \quad \text{Re } p > 0$
(11)	$\text{Erfc}(\frac{1}{2}\alpha^{\frac{1}{4}} t^{-\frac{1}{4}}) \quad \text{Re } \alpha > 0$	$p^{-\frac{1}{4}} e^{-\alpha^{\frac{1}{4}} p^{\frac{1}{4}}} \quad \text{Re } p > 0$
(12)	$e^{\alpha t} \text{Erfc}(\alpha^{\frac{1}{4}} t^{\frac{1}{4}} + \frac{1}{2}\beta^{\frac{1}{4}} t^{-\frac{1}{4}}) \quad \text{Re } \beta > 0$	$p^{-\frac{1}{4}} (p^{\frac{1}{4}} + \alpha^{\frac{1}{4}})^{-1} \times \exp(-\alpha^{\frac{1}{4}} \beta^{\frac{1}{4}} - \beta^{\frac{1}{4}} p^{\frac{1}{4}}) \quad \text{Re } p > 0$
(13)	$S(t)$	$\frac{[(p^2 + 1)^{\frac{1}{4}} - p]^{\frac{1}{4}}}{2p(p^2 + 1)^{\frac{1}{4}}} \quad \text{Re } p > 0$
(14)	$C(t)$	$\frac{[(p^2 + 1)^{\frac{1}{4}} - p]^{-\frac{1}{4}}}{2p(p^2 + 1)^{\frac{1}{4}}} \quad \text{Re } p > 0$
(15)	$S(t^{\frac{1}{4}})$	$p^{-\frac{1}{4}} [\frac{1}{2} - \cos(\frac{1}{4}p^2) C(\frac{1}{4}p^2) - \sin(\frac{1}{4}p^2) S(\frac{1}{4}p^2)]$
(16)	$C(t^{\frac{1}{4}})$	$p^{-\frac{1}{4}} [\frac{1}{2} \cos(\frac{1}{2}p^2) - \cos(\frac{1}{4}p^2) S(\frac{1}{4}p^2) + \sin(\frac{1}{4}p^2) C(\frac{1}{4}p^2)]$
(17)	$\text{Si}(t)$	$p^{-1} \text{ctn}^{-1} p \quad \text{Re } p > 0$
(18)	$\text{si}(t)$	$-p^{-1} \tan^{-1} p \quad \text{Re } p > 0$

## Gamma function etc. (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(19)	$\text{Ci}(t) = -\text{si}(t)$	$\frac{1}{2} p^{-1} \log(p^2 + 1) \quad \text{Re } p > 0$
(20)	$\cos t \text{ Si}(t) - \sin t \text{ Ci}(t)$	$(p^2 + 1)^{-1} \log p \quad \text{Re } p > 0$
(21)	$\sin t \text{ Si}(t) + \cos t \text{ Ci}(t)$	$-(p^2 + 1)^{-1} p \log p \quad \text{Re } p > 0$
(22)	$\text{Si}(t^2)$	$\pi [\frac{1}{2} - C(\frac{1}{4} p^2)]^2 + \pi [\frac{1}{2} - S(\frac{1}{4} p^2)]^2 \quad \text{Re } p > 0$
(23)	$\overline{\text{Ei}}(t)$	$-p^{-1} \log(p-1) \quad \text{Re } p > 1$
(24)	$\text{Ei}(-t)$	$-p^{-1} \log(p+1) \quad \text{Re } p > 0$
(25)	$t^{-\frac{1}{2}} \text{ Ei}(-t)$	$-2\pi^{\frac{1}{4}} p^{-\frac{1}{2}} \log[p^{\frac{1}{4}} + (p+1)^{\frac{1}{4}}] \quad \text{Re } p > 0$
(26)	$\sin(\alpha t) \text{ Ei}(-t)$	$-(p^2 + \alpha^2)^{-1} \{ \frac{1}{2} \alpha \log[(p+1)^2 + \alpha^2] - p \tan^{-1}[\alpha/(p+1)] \} \quad \text{Re } p >  \text{Im } \alpha $
(27)	$\cos(\alpha t) \text{ Ei}(-t)$	$-(p^2 + \alpha^2)^{-1} \{ \frac{1}{2} p \log[(p+1)^2 + \alpha^2] + \alpha \tan^{-1}[\alpha/(p+1)] \} \quad \text{Re } p >  \text{Im } \alpha $
(28)	$\text{li}(e^t)$	$-p^{-1} \log(p-1) \quad \text{Re } p > 1$
(29)	$\text{li}(e^{-t})$	$-p^{-1} \log(p+1) \quad \text{Re } p > 0$
(30)	$\Gamma(\nu, at) \quad \text{Re } \nu > -1$	$\Gamma(\nu) p^{-1} [1 - (1+p/a)^{-\nu}] \quad \text{Re } p > -\text{Re } \alpha$
(31)	$e^{\alpha t} \Gamma(\nu, at) \quad \text{Re } \nu > -1$	$\Gamma(\nu) (p-a)^{-1} (1 - a^\nu p^{-\nu}) \quad \text{Re } p > 0$

## Gamma function etc. (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(32)	$\Gamma(\nu, a/t)$ $ arg a  < \frac{1}{2}\pi$	$2a^{\frac{1}{2}\nu} p^{\frac{1}{2}\nu-1} K_\nu(2a^{\frac{1}{2}}p^{\frac{1}{2}}) \quad Re p > 0$
(33)	$t^{\mu-1} e^{at} \Gamma(\nu, a/t)$ $Re(\nu - \mu) < 1, \quad  arg a  < \pi$	$2^{2+\mu-2\nu} \Gamma(1+\mu-\nu) (a/p)^{\frac{1}{2}\mu}$ $\times S_{2\nu-\mu-1, \mu}(2a^{\frac{1}{2}}p^{\frac{1}{2}}) \quad Re p > 0$
(34)	$e^{\beta t} \gamma(\nu, at)$ $Re \nu > -1$	$\Gamma(\nu) a^\nu (p-\beta)^{-1} (p+a-\beta)^{-\nu}$ $Re p > Re \beta, \quad Re(\beta-a)$
(35)	$\gamma(\frac{1}{4}, 2^{-3} a^{-2} t^2)$ $ arg a  < \frac{1}{4}\pi$	$2^{\frac{1}{2}} a^{\frac{1}{2}} p^{-\frac{1}{2}} e^{a^2 p^2} K_{\frac{1}{4}}(a^2 p^2)$ $Re p > 0$
(36)	$\gamma(\nu, 2^{-3} a^{-2} t^2)$ $ arg a  < \frac{1}{4}\pi, \quad Re \nu > -\frac{1}{2}$	$2^{-\nu-1} \Gamma(2\nu) p^{-1} e^{a^2 p^2} D_{-2\nu}(2ap)$ $Re p > 0$
(37)	$e^{-\frac{1}{4}t^2/a} \gamma(\nu, \frac{1}{4}e^{i\pi} t^2/a)$ $ arg a  < \frac{1}{2}\pi, \quad Re \nu > -\frac{1}{2}$	$2^{1-2\nu} \Gamma(2\nu) a^{\frac{1}{2}} e^{ap^2 + \nu\pi i}$ $\times \Gamma(\frac{1}{2}-\nu, ap^2) \quad Re p > 0$

## 4.13. Legendre functions

(1)	$[t(1+t)]^{-\frac{1}{2}\mu} P_\nu^\mu(1+2t)$ $Re \mu < 1$	$\pi^{-\frac{1}{2}} p^{\mu-\frac{1}{2}} e^{\frac{1}{2}p} K_{\nu+\frac{1}{2}}(\frac{1}{2}p)$ $Re p > 0$
(2)	$(1+t^{-1})^{\frac{1}{2}\mu} P_\nu^\mu(1+2t)$ $Re \mu < 1$	$p^{-1} e^{\frac{1}{2}p} W_{\mu, \nu+\frac{1}{2}}(p) \quad Re p > 0$
(3)	$t^{\lambda+\frac{1}{2}\mu-1} (t+2)^{\frac{1}{2}\mu} P_\nu^{-\mu}(1+t)$ $Re(\lambda + \mu) > 0$	$-\pi^{-1} \sin(\nu\pi) p^{-\lambda-\mu}$ $\times E(-\nu, \nu+1, \lambda+\mu; \mu+1; 2p) \quad Re p > 0$
(4)	$t^{\lambda-\frac{1}{2}\mu-1} (t+2)^{-\frac{1}{2}\mu} P_\nu^{-\mu}(1+t)$ $Re \lambda > 0$	$E(\mu+\nu+1, \mu-\nu, \lambda; \mu+1; 2p)$ $\frac{2^\mu}{2^\mu p^\lambda} \Gamma(\mu+\nu+1) \Gamma(\mu-\nu) \quad Re p > 0$

## Legendre functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(5)	$(\alpha+t)^{\frac{1}{2}\nu}(\beta+t)^{\frac{1}{2}\nu}$ $\times P_\nu[2\alpha^{-1}\beta^{-1}(\alpha+t)(\beta+t)-1]$ $ \arg \alpha  < \pi, \quad  \arg \beta  < \pi$	$\pi^{-1} (\alpha\beta)^{\frac{1}{2}\nu+\frac{1}{4}} e^{\frac{1}{4}(\alpha+\beta)p}$ $\times K_{\nu+\frac{1}{4}}(\frac{1}{2}\alpha p) K_{\nu+\frac{1}{4}}(\frac{1}{2}\beta p)$ $ \arg(\alpha p)  < \pi, \quad  \arg(\beta p)  < \pi$
(6)	$(1+t)^{-1} P_\nu[2(1+t)^{-2}-1]$	$p^{-1} e^p W_{\nu+\frac{1}{4}, 0}(p) W_{-\nu-\frac{1}{4}, 0}(p)$ $\text{Re } p > 0$
(7)	$t^{-\frac{1}{2}\mu} P_\nu^\mu[(1+t)^{\frac{1}{2}}] \quad \text{Re } \mu < 1$	$2^\mu p^{\mu/2-5/4} e^{p/2} W_{\frac{1}{2}\mu+\frac{1}{4}, \frac{1}{2}\nu+\frac{1}{4}}(p)$ $\text{Re } p > 0$
(8)	$t^{-\frac{1}{2}\mu} (1+t)^{-\frac{1}{2}} P_\nu^\mu[(1+t)^{\frac{1}{2}}] \quad \text{Re } \mu < 1$	$2^\mu p^{\frac{1}{2}\mu-\frac{1}{2}} p^{\frac{1}{2}p} W_{\frac{1}{2}\mu-\frac{1}{4}, \frac{1}{2}\nu+\frac{1}{4}}(p)$ $\text{Re } p > 0$
(9)	$t^{\frac{1}{2}} P_\nu^{\frac{1}{2}}[(1+t^2)^{\frac{1}{2}}] P_\nu^{-\frac{1}{2}}[(1+t^2)^{\frac{1}{2}}]$	$\frac{1}{2} (\frac{1}{2}\pi/p)^{\frac{1}{2}} H_{\nu+\frac{1}{4}}^{(1)}(\frac{1}{2}p) H_{\nu+\frac{1}{4}}^{(2)}(\frac{1}{2}p)$ $\text{Re } p > 0$
(10)	$(\alpha+t)^{-\frac{1}{2}\nu-\frac{1}{2}}(\beta+t)^{\frac{1}{2}\nu}$ $\times [-1-(\alpha+\beta)/t]/t^{\frac{1}{2}\mu}$ $\times P_\nu^\mu[\alpha^{\frac{1}{2}}\beta^{\frac{1}{2}}(\alpha+t)^{-\frac{1}{2}}(\beta+t)^{-\frac{1}{2}}]$ $ \arg \alpha  < \pi, \quad  \arg \beta  < \pi$ $\text{Re } \mu < 1$	$2^{\frac{1}{2}} p^{-\frac{1}{2}} e^{\frac{1}{4}(\alpha+\beta)p} D_{\mu-\nu-1}(2^{\frac{1}{2}}\alpha^{\frac{1}{2}}p^{\frac{1}{2}})$ $\times D_{\mu+\nu}(2^{\frac{1}{2}}\beta^{\frac{1}{2}}p^{\frac{1}{2}}) \quad \text{Re } p > 0$ $ \arg(\alpha p)  < \pi, \quad  \arg(\beta p)  < \pi$
(11)	$(1-e^{-2t})^{\frac{1}{2}\mu} P_\nu^{-\mu}(e^t) \quad \text{Re } \mu > -1$	$\frac{2^{p-1} \Gamma(\frac{1}{2}p + \frac{1}{2}\nu + \frac{1}{2}) \Gamma(\frac{1}{2}p - \frac{1}{2}\nu)}{\pi^{\frac{1}{2}} \Gamma(p + \mu + 1)}$ $\text{Re } p > \text{Re } \nu, \quad -1 - \text{Re } \nu$
(12)	$\left[ (e^t - 1) \left( \frac{ae^t}{a-2} - 1 \right) \right]^{\frac{1}{2}\mu}$ $\times P_\nu^{-\mu}(ae^t - a + 1) \quad \text{Re } a > 0, \quad \text{Re } \mu > -1$	$\frac{\Gamma(p - \mu + \nu + 1) \Gamma(p - \nu - \mu)}{\Gamma(p + 1)} \left( \frac{a}{a-2} \right)^{\frac{1}{2}p}$ $\times P_\nu^{\mu-p}(a-1) \quad \text{Re } p > \text{Re } (\mu - \nu) - 1, \quad \text{Re } (\mu + \nu)$

## Legendre functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(13)	$(1-z^2 + z^2 e^{-t})^\mu \\ \times \{P_{2\nu}^{2\mu}[z(1-e^{-t})^{\frac{1}{2}}] \\ - P_{2\nu}^{2\mu}[-z(1-e^{-t})^{\frac{1}{2}}]\} \quad  z  < 1$	$\frac{-2^{2\mu+1} \pi z \Gamma(p)}{\Gamma(-\mu-\nu) \Gamma(\frac{1}{2}-\mu+\nu) \Gamma(p+3/2)} \\ \times {}_2F_1(1/2-\mu-\nu, \nu-\mu+1; p+3/2; z^2) \\ \text{Re } p > 0$
(14)	$(1-e^{-t})^{-\frac{1}{2}} (1-z^2 + z^2 e^{-t})^\mu \\ \times \{P_{2\nu}^{2\mu}[z(1-e^{-t})^{\frac{1}{2}}] \\ + P_{2\nu}^{2\mu}[-z(1-e^{-t})^{\frac{1}{2}}]\} \quad  z  < 1$	$\frac{2^{2\mu+1} \pi \Gamma(p)}{\Gamma(\frac{1}{2}-\mu-\nu) \Gamma(1-\mu+\nu) \Gamma(p+\frac{1}{2})} \\ \times {}_2F_1(-\mu-\nu, \frac{1}{2}-\mu+\nu; p+\frac{1}{2}; z^2) \\ \text{Re } p > 0$
(15)	$\sinh^{2\mu}(\frac{1}{2}t) P_{2n}^{-2\mu}[\cosh(\frac{1}{2}t)] \\ \text{Re } \mu > -\frac{1}{2}$	$\frac{\Gamma(2\mu+\frac{1}{2}) \Gamma(p-n-\mu) \Gamma(p+n-\mu+\frac{1}{2})}{4^\mu \pi^{\frac{1}{2}} \Gamma(p+n+\mu+1) \Gamma(p-n+\mu+\frac{1}{2})} \\ \text{Re } p > n + \text{Re } \mu$
(16)	$t^{\lambda+\frac{1}{2}\mu-1} (t+2)^{\frac{1}{2}\mu} Q_\nu^\mu(1+t) \\ \text{Re } \lambda > 0, \quad \text{Re } (\lambda + \mu) > 0$	$\frac{\Gamma(\nu+\mu+1)}{\Gamma(\nu-\mu+1)} \left\{ \frac{\sin(\nu\pi)}{2p^{\lambda+\mu} \sin(\mu\pi)} \right. \\ \times E(-\nu, \nu+1, \lambda+\mu; \mu+1; 2p) \\ - \frac{\sin[(\mu+\nu)\pi]}{2^{1-\mu} p^\lambda \sin(\mu\pi)} \\ \left. \times E(\nu-\mu+1, -\nu-\mu, \lambda; 1-\mu; 2p) \right\} \\ \text{Re } p > 0$
(17)	$t^{\lambda-\frac{1}{2}\mu-1} (t+2)^{\frac{1}{2}\mu} Q_\nu^\mu(1+t) \\ \text{Re } \lambda > 0, \quad \text{Re } (\lambda - \mu) > 0$	$- \frac{\sin(\nu\pi)}{2p^{\lambda-\mu} \sin(\mu\pi)} \\ \times E(-\nu, \nu+1, \lambda-\mu; 1-\mu; 2p) \\ - \frac{\sin[(\mu-\nu)\pi]}{2^{1+\mu} p^\lambda \sin(\mu\pi)} \\ \times E(\mu+\nu+1, \mu-\nu, \lambda; 1+\mu; 2p) \\ \text{Re } p > 0$

**4.14. Bessel functions of arguments  $kt$  and  $kt^{\frac{1}{2}}$** 

	$f(t)$	$\mathcal{G}(p) = \int_0^\infty e^{-pt} f(t) dt$
(1)	$J_\nu(at)$ $\text{Re } \nu > -1$	$r^{-1} (a/R)^\nu = r^{-1} e^{-\nu \sinh^{-1}(p/a)}$ $\text{Re } p >  \text{Im } a $
(2)	$t J_\nu(at)$ $\text{Re } \nu > -2$	$r^{-3} (p + \nu r) (a/R)^\nu$ $\text{Re } p >  \text{Im } a $
(3)	$t^2 J_\nu(at)$ $\text{Re } \nu > -3$	$\left( \frac{\nu^2 - 1}{r^3} + 3p \frac{p + \nu r}{r^5} \right) \left( \frac{a}{R} \right)^\nu$ $\text{Re } p >  \text{Im } a $
(4)	$t^n J_n(at)$	$1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1) a^n r^{-2n-1}$ $\text{Re } p >  \text{Im } a $
(5)	$t^{-1} J_\nu(at)$ $\text{Re } \nu > 0$	$\nu^{-1} (a/R)^\nu$ $\text{Re } p \geq  \text{Im } a $
(6)	$t^{-2} J_\nu(at)$ $\text{Re } \nu > 1$	$\frac{a}{2\nu} \left[ \frac{1}{\nu-1} \left( \frac{a}{R} \right)^{\nu-1} + \frac{1}{\nu+1} \left( \frac{a}{R} \right)^{\nu+1} \right]$ $\text{Re } p \geq  \text{Im } a $
(7)	$t^\nu J_\nu(at)$ $\text{Re } \nu > -\frac{1}{2}$	$2^\nu \pi^{-\frac{\nu}{2}} \Gamma(\nu + \frac{1}{2}) a^\nu r^{-2\nu-1}$ $\text{Re } p >  \text{Im } a $
(8)	$t^{\nu+1} J_\nu(at)$ $\text{Re } \nu > -1$	$2^{\nu+1} \pi^{-\frac{\nu}{2}} \Gamma(\nu + 3/2) a^\nu r^{-2\nu-3} p$ $\text{Re } p >  \text{Im } a $
(9)	$t^\mu J_\nu(at)$ $\text{Re } (\mu + \nu) > -1$	$\Gamma(\mu + \nu + 1) r^{-\mu-1} P_\mu^{-\nu}(p/r)$ $\text{Re } p >  \text{Im } a $
(10)	$t^\mu \sin(at) J_\mu(at)$ $a > 0, \quad \text{Re } \mu > -1$	$\frac{\Gamma(\mu+1) a^{\mu+1}}{2^{\frac{\mu}{2}} \pi} \times \int_0^{\frac{\pi}{2}} \frac{(\cos \theta)^{\mu+\frac{1}{2}} \cos[(\mu-\frac{1}{2})\theta]}{(\frac{1}{4}p^2 + a^2 \cos^2 \theta)^{\mu+1}} d\theta$ $\text{Re } p > 0$

$$r = (p^2 + a^2)^{\frac{1}{2}}, \quad R = p + r$$

Bessel functions of  $kt$  and  $kt^{\frac{1}{2}}$  (cont'd)

	$f(t)$	$\int_0^\infty e^{-pt} f(t) dt$
(11)	$t^{\mu-1} \cos(at) J_\mu(at)$ $a > 0, \quad \operatorname{Re} \mu > 0$	$\frac{\Gamma(\mu) a^\mu}{2^{\frac{\mu}{2}} \pi} \times \int_0^{\frac{\pi}{2}} \frac{(\cos \theta)^{\mu-\frac{1}{2}} \cos[(\mu+\frac{1}{2})\theta]}{(a^2 p^2 + a^2 \cos^2 \theta)^\mu} d\theta$ $\operatorname{Re} p > 0$
(12)	$[J_0^2(\frac{1}{2}at)]^2$	$2\pi^{-1} r^{-1} K(a/r) \quad \operatorname{Re} p >  \operatorname{Im} a $
(13)	$[J_1^2(\frac{1}{2}at)]^2$	$2\pi^{-1} a^{-2} r^{-2} [(2p^2 + a^2) K(a/r) - 2(p^2 + a^2) E(a/r)]$ $\operatorname{Re} p >  \operatorname{Im} a $
(14)	$J_\nu(at) J_\nu(\beta t) \quad \operatorname{Re} \nu > -\frac{1}{2}$	$\frac{1}{\pi a^{\frac{\nu}{2}} \beta^{\frac{\nu}{2}}} Q_{\nu-\frac{1}{2}}\left(\frac{p^2 + a^2 + \beta^2}{2a\beta}\right)$ $\operatorname{Re} p >  \operatorname{Im} a  +  \operatorname{Im} \beta $
(15)	$t J_0(\frac{1}{2}at) J_1(\frac{1}{2}at)$	$2\pi^{-1} a^{-1} r^{-1} [K(a/r) - E(a/r)]$ $\operatorname{Re} p >  \operatorname{Im} a $
(16)	$t J_\nu^2(at) \quad \operatorname{Re} \nu > -1$	$2^{2\nu+1} (\nu + \frac{1}{2}) \pi^{-1} a^{2\nu} p^{-2\nu-2}$ $\times B(\nu + \frac{1}{2}, \nu + \frac{3}{2})$ $\times {}_2F_1(\nu + 1/2, \nu + 3/2; 2\nu + 1; -4a^2/p^2)$ $\operatorname{Re} p > 2 \operatorname{Im} a $
(17)	$t^{-2} J_1^2(t)$	$\frac{1}{2} \pi^{-1} \int_0^{\pi} [(p^2 + 2 - 2 \cos \phi)^{\frac{1}{2}} - p] \times (1 + \cos \phi) d\phi \quad \operatorname{Re} p > 0$
(18)	$t^{\frac{1}{2}} J_\nu^2(\frac{1}{2}at) \quad \operatorname{Re} \nu > -\frac{3}{4}$	$\frac{a \Gamma(2\nu + 3/2)}{2^{\nu+3/2} p^{1/2} r} P_{1/4}^{-\nu}\left(\frac{r}{p}\right) P_{-1/4}^{-\nu}\left(\frac{r}{p}\right)$ $\operatorname{Re} p >  \operatorname{Im} a $

$$r = (p^2 + a^2)^{\frac{1}{2}}, \quad R = p + r$$

Bessel functions of  $kt$  and  $kt^{\frac{1}{2}}$  (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(19)	$t^{-\frac{1}{2}} J_\nu^2(\frac{1}{2}at)$ Re $\nu > -\frac{1}{4}$	$2^{-\nu-\frac{1}{2}} \Gamma(2\nu+\frac{1}{2}) p^{-\frac{1}{2}} [P_{-\frac{\nu}{2}}(r/p)]^2$ Re $p >  \text{Im } a $
(20)	$t^{\frac{1}{2}} J_\nu(\frac{1}{2}at) J_{-\nu}(\frac{1}{2}at)$	$\frac{a\pi^{\frac{1}{2}}}{2p^{\frac{1}{2}} r} [( \nu + \frac{1}{4}) P_{-\frac{\nu}{2}}(r/p) P_{\frac{\nu}{2}}(r/p) - (\nu - \frac{1}{4}) P_{\frac{\nu}{2}}(r/p) P_{-\frac{\nu}{2}}(r/p)]$ Re $p >  \text{Im } a $
(21)	$t^{\frac{1}{2}} J_\nu(\frac{1}{2}at) J_{\nu+1}(\frac{1}{2}at)$ Re $\nu > -5/4$	$\frac{a \Gamma(2\nu+5/2)}{2^{\nu+5/2} p^{1/2} r} P_{-1/4}^{-\nu}\left(\frac{r}{p}\right) P_{-1/4}^{-\nu-1}\left(\frac{r}{p}\right)$ Re $p >  \text{Im } a $
(22)	$t^{-\frac{1}{2}} J_\nu(\frac{1}{2}at) J_{-\nu}(\frac{1}{2}at)$	$2^{-\frac{1}{2}} \pi^{\frac{1}{2}} p^{-\frac{1}{2}} P_{-\frac{\nu}{2}}(r/p) P_{\frac{\nu}{2}}(r/p)$ Re $p >  \text{Im } a $
(23)	$t^{2\nu} J_\nu^2(at)$ Re $\nu > -\frac{1}{4}$	$\frac{2^{4\nu} a^{2\nu} \Gamma(\nu+\frac{1}{2}) \Gamma(2\nu+\frac{1}{2})}{\pi \Gamma(\nu+1) p^{4\nu+1}} \times {}_2F_1(\nu+\frac{1}{2}, 2\nu+\frac{1}{2}; \nu+1; -4a^2/p^2)$ Re $p > 2 \text{Im } a $
(24)	$t^{\mu-1} J_{\nu_1}(\alpha_1 t) \dots J_{\nu_n}(\alpha_n t)$ Re $(\mu+N) > 0$ $N = \nu_1 + \dots + \nu_n$ $\alpha = \alpha_1 + \dots + \alpha_n$	$\frac{2^{-N} \Gamma(\mu+N)}{\Gamma(\nu_1+1) \dots \Gamma(\nu_n+1)} \alpha_1^{\nu_1} \dots \alpha_n^{\nu_n}$ $\times (p+ia)^{-\mu-N} F_A \left( \begin{matrix} \mu+N; \nu_1+\frac{1}{2}, \dots, \nu_n+\frac{1}{2}; \\ 2\nu_1+1, \dots, 2\nu_n+1; \frac{2\alpha_1 i}{p+ia}, \dots, \frac{2\alpha_n i}{p+ia} \end{matrix} \right)$ Re $(p \pm i\alpha_1, \pm \dots \pm i\alpha_n) > 0$ If $n=2$ , $F_A$ is to be replaced by $F_2$

$$r = (p^2 + a^2), \quad R = p + r$$

**Bessel functions of  $kt$  and  $kt^{\frac{1}{2}}$  (cont'd)**

	$f(t)$	$\mathcal{L}(f) = \int_0^\infty e^{-pt} f(t) dt$
(25)	$J_0(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$	$p^{-1} e^{-\alpha/p}$ <span style="float: right;"><math>\operatorname{Re} p &gt; 0</math></span>
(26)	$J_\nu(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$ <span style="float: right;"><math>\operatorname{Re} \nu &gt; -2</math></span>	$\frac{1}{2} \alpha^{1/2} \pi^{1/2} p^{-3/2} e^{-\frac{1}{2}\alpha/p} [I_{\frac{1}{2}\nu-\frac{1}{2}}(\frac{1}{2}\alpha/p) - I_{\frac{1}{2}\nu+\frac{1}{2}}(\frac{1}{2}\alpha/p)]$ <span style="float: right;"><math>\operatorname{Re} p &gt; 0</math></span>
(27)	$t^{\frac{1}{2}} J_1(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$	$\alpha^{\frac{1}{2}} p^{-2} e^{-\alpha/p}$ <span style="float: right;"><math>\operatorname{Re} p &gt; 0</math></span>
(28)	$t^{n-\frac{1}{2}} J_1(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$	$(-1)^{n-1} \alpha^{-\frac{1}{2}} n! p^{-n} e^{-\frac{1}{2}\alpha/p} k_{2n}(\frac{1}{2}\alpha/p)$ <span style="float: right;"><math>\operatorname{Re} p &gt; 0</math></span>
(29)	$t^{-\frac{1}{2}} J_\nu(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$ <span style="float: right;"><math>\operatorname{Re} \nu &gt; -1</math></span>	$\pi^{\frac{1}{2}} p^{-\frac{1}{2}} e^{-\frac{1}{2}\alpha/p} I_{\frac{1}{2}\nu}(\frac{1}{2}\alpha/p)$ <span style="float: right;"><math>\operatorname{Re} p &gt; 0</math></span>
(30)	$t^{\frac{1}{2}\nu} J_\nu(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$ <span style="float: right;"><math>\operatorname{Re} \nu &gt; -1</math></span>	$\alpha^{\frac{1}{2}\nu} p^{-\nu-1} e^{-\alpha/p}$ <span style="float: right;"><math>\operatorname{Re} p &gt; 0</math></span>
(31)	$t^{-\frac{1}{2}\nu} J_\nu(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$	$\frac{e^{i\nu\pi} p^{\nu-1}}{\alpha^{\frac{1}{2}\nu} \Gamma(\nu)} e^{-\alpha/p} \gamma\left(\nu, \frac{\alpha}{p} e^{-i\pi}\right)$ <span style="float: right;"><math>\operatorname{Re} p &gt; 0</math></span>
(32)	$t^{\frac{1}{2}\nu-1} J_\nu(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$ <span style="float: right;"><math>\operatorname{Re} \nu &gt; 0</math></span>	$\alpha^{-\frac{1}{2}\nu} \gamma(\nu, \alpha/p)$ <span style="float: right;"><math>\operatorname{Re} p &gt; 0</math></span>
(33)	$t^{\frac{1}{2}\nu+n} J_\nu(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$ <span style="float: right;"><math>\operatorname{Re} \nu + n &gt; -1</math></span>	$n! \alpha^{\frac{1}{2}\nu} p^{-n-\nu-1} e^{-\alpha/p} L_n^\nu(\alpha/p)$ <span style="float: right;"><math>\operatorname{Re} p &gt; 0</math></span>
(34)	$t^{\mu-\frac{1}{2}} J_{2\nu}(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$ <span style="float: right;"><math>\operatorname{Re} (\mu + \nu) &gt; -\frac{1}{2}</math></span>	$\frac{\Gamma(\mu+\nu+\frac{1}{2})}{\alpha^{\frac{1}{2}} \Gamma(2\nu+1)} p^{-\mu} e^{-\frac{1}{2}\alpha/p} M_{\mu, \nu}(\alpha/p)$ <span style="float: right;"><math>\operatorname{Re} p &gt; 0</math></span>

**Bessel functions of  $kt$  and  $kt^{\frac{1}{2}}$  (cont'd)**

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(35)	$t^{\mu-1} J_{2\nu}(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$ $\text{Re } (\mu + \nu) > 0$	$\frac{\Gamma(\mu + \nu) \alpha^\nu}{\Gamma(2\nu + 1) p^{\mu+\nu}} \\ \times {}_1F_1(\mu + \nu; 2\nu + 1; -\alpha/p)$ $\text{Re } p > 0$
(36)	$t^{\nu-\frac{1}{2}} \{ J_{2\mu}(2t^{\frac{1}{2}}) \cos[(\nu + \mu)\pi] - J_{-2\mu}(2t^{\frac{1}{2}}) \cos[(\nu - \mu)\pi] \}$ $\text{Re } (\nu \pm \mu) > -\frac{1}{2}$	$- \sin(2\mu\pi) p^{-\nu} e^{-\frac{1}{2}p^{-1}} W_{\nu, \mu}(p^{-1})$ $\text{Re } p > 0$
(37)	$t^{\frac{1}{2}\nu} L_n^\nu(t) J_\nu(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$	$\alpha^{\frac{1}{2}\nu} e^{-\alpha/p} \frac{(p-1)^n}{p^{\nu+n+1}} L_n^\nu \left[ \frac{\alpha}{p(1-p)} \right]$ $\text{Re } p > 0$
(38)	$J_\nu(t) J_{2\nu}(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$ $\text{Re } \nu > -\frac{1}{2}$	$\frac{e^{-ap/(p^2+1)}}{(p^2+1)^{\frac{1}{2}}} J_\nu \left( \frac{\alpha}{p^2+1} \right)$ $\text{Re } p > 0$
(39)	$J_\nu(\alpha t^{\frac{1}{2}}) J_\nu(\beta t^{\frac{1}{2}})$ $\text{Re } \nu > -\frac{1}{2}$	$p^{-1} e^{-\frac{1}{2}(\alpha^2 + \beta^2)p} I_\nu(\frac{1}{2}\alpha\beta/p)$ $\text{Re } p > 0$
(40)	$t^{-1} J_\nu^2(2t^{\frac{1}{2}})$ $\text{Re } \nu > 0$	$\nu^{-1} e^{-2/p} [I_\nu(2/p) + 2 \sum_{n=1}^{\infty} I_{\nu+n}(2/p)]$ $\text{Re } p > 0$
(41)	$t^{-\frac{1}{2}} J_\nu(\alpha e^{\frac{1}{4}\pi i} t^{\frac{1}{2}})$ $\times J_\nu(\alpha e^{-\frac{1}{4}\pi i} t^{\frac{1}{2}})$ $\text{Re } \nu > -\frac{1}{2}$	$\frac{p^{\frac{1}{2}} 2^{1-\nu} \Gamma(\nu + \frac{1}{2})}{\alpha^2 [\Gamma(\nu + 1)]^2} M_{\frac{1}{4}, \frac{1}{4}\nu} \left( \frac{\alpha^2}{2p} \right)$ $\times M_{-\frac{1}{4}, \frac{1}{4}\nu} \left( \frac{\alpha^2}{2p} \right)$ $\text{Re } p > 0$

Bessel functions of  $kt$  and  $kt^{\frac{1}{2}}$  (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(42)	$t^{\lambda-1} J_{2\mu}(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}}) J_{2\nu}(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$ $\operatorname{Re}(\lambda + \mu + \nu) > 0$	$\frac{2\Gamma(\lambda + \mu + \nu) \alpha^{\mu+\nu}}{\Gamma(2\mu+1) \Gamma(2\nu+1) p^{\lambda+\mu+\nu}} \\ \times {}_3F_3(\mu + \nu + \frac{1}{2}, \mu + \nu + 1, \lambda + \mu + \nu; \\ 2\mu + 1, 2\nu + 1, 2\mu + 2\nu + 1; -4\alpha/p) \\ \operatorname{Re} p > 0$
(43)	$t^{\nu-1} J_{2\mu_1}(2\alpha_1^{\frac{1}{2}} t^{\frac{1}{2}}) \dots J_{2\mu_n}(2\alpha_n^{\frac{1}{2}} t^{\frac{1}{2}})$ $M = \mu_1 + \dots + \mu_n, \quad \operatorname{Re}(\nu + M) > 0$	$\frac{\Gamma(\nu + M) p^{-\nu-M} \alpha_1^{\mu_1} \dots \alpha_n^{\mu_n}}{\Gamma(2\mu_1 + 1) \dots \Gamma(2\mu_n + 1)} \\ \times \Psi_2(\nu + M; 2\mu_1 + 1, \dots, 2\mu_n + 1; \frac{\alpha_1}{p}, \dots, \frac{\alpha_n}{p}) \\ \operatorname{Re} p > 0$
(44)	$Y_0(at)$	$-2\pi^{-1} r^{-1} \sinh^{-1}(p/a)$ $\operatorname{Re} p >  \operatorname{Im} a $
(45)	$Y_\nu(at)$ $ \operatorname{Re} \nu  < 1$	$a^\nu \operatorname{ctn}(\nu\pi) r^{-1} R^{-\nu - a^{-\nu}} \csc(\nu\pi) r^{-1} R^\nu$ $\operatorname{Re} p >  \operatorname{Im} a $
(46)	$t Y_0(at)$	$2\pi^{-1} r^{-2} [1 - pr^{-1} \log(R/a)]$ $\operatorname{Re} p >  \operatorname{Im} a $
(47)	$t Y_1(at)$	$-2\pi^{-1} r^{-2} [p a^{-1} + ar^{-1} \log(R/a)]$ $\operatorname{Re} p >  \operatorname{Im} a $
(48)	$t^\mu Y_\nu(at)$ $\operatorname{Re}(\mu \pm \nu) > -1$	$r^{-\mu-1} [\Gamma(\mu + \nu + 1) \operatorname{ctn}(\nu\pi) P_\mu^{-\nu}(p/r) \\ - \Gamma(\mu - \nu + 1) \csc(\nu\pi) P_\mu^\nu(p/r)]$ $\operatorname{Re} p >  \operatorname{Im} a $
(49)	$t^{-\frac{1}{2}} Y_{2\nu}(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$ $ \operatorname{Re} \nu  < \frac{1}{2}$	$-\pi^{\frac{1}{2}} p^{-\frac{1}{2}} e^{-\frac{1}{2}\alpha/p} [\tan(\nu\pi) I_\nu(\frac{1}{2}\alpha/p) \\ + \pi^{-1} \sec(\nu\pi) K_\nu(\frac{1}{2}\alpha/p)]$ $\operatorname{Re} p > 0$

$$r = (p^2 + \alpha^2)^{\frac{1}{2}}, \quad R = p + r$$

**Bessel functions of  $kt$  and  $kt^{\frac{1}{2}}$  (cont'd)**

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(50)	$t^{\mu-\frac{1}{2}} Y_{2\nu}(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$ $\operatorname{Re}(\mu \pm \nu) > -\frac{1}{2}$	$\alpha^{-\frac{1}{2}} p^{-\mu} e^{-\frac{1}{2}\alpha/p}$ $\times \left\{ \frac{\tan[(\mu-\nu)\pi]\Gamma(\mu+\nu+\frac{1}{2})}{\Gamma(2\nu+1)} M_{\mu\nu}(a/p) \right.$ $\left. - \sec[(\mu-\nu)\pi] W_{\mu\nu}(a/p) \right\}$ $\operatorname{Re} p > 0$
(51)	$t^{-1} \{ [at^{\frac{1}{2}} J_1(t^{\frac{1}{2}}) + b J_0(t^{\frac{1}{2}})]^2 + [at^{\frac{1}{2}} Y_1(t^{\frac{1}{2}}) + b Y_0(t^{\frac{1}{2}})]^2 \}^{-1}$	$2 I(a, b; p)$ $\operatorname{Re} p > 0$ For expansions of $I(a, b; p)$ and a short numerical table of $I(0, 1; p)$ cf. Jaeger, J. C., and Martha Clarke 1942: <i>Proc. Roy. Soc. Edinburgh Sect. A</i> 61, 229-230.
(52)	$H_0^{(1)}(at)$	$r^{-1} - 2i\pi^{-1} r^{-1} \sinh^{-1}(p/a)$ $\operatorname{Re} p >  \operatorname{Im} a $
(53)	$H_0^{(2)}(at)$	$r^{-1} + 2i\pi^{-1} r^{-1} \sinh^{-1}(p/a)$ $\operatorname{Re} p >  \operatorname{Im} a $
(54)	$H_\nu^{(1)}(at)$ $ \operatorname{Re} \nu  < 1$	$ir^{-1} \csc(\nu\pi)(e^{-i\nu\pi} a^\nu R^{-\nu} - a^{-\nu} R^\nu)$ $\operatorname{Re} p >  \operatorname{Im} a $
(55)	$H_\nu^{(2)}(at)$ $ \operatorname{Re} \nu  < 1$	$ir^{-1} \csc(\nu\pi)(a^{-\nu} R^\nu - e^{i\nu\pi} a^\nu R^{-\nu})$ $\operatorname{Re} p >  \operatorname{Im} a $
(56)	$t H_0^{(1)}(at)$	$\frac{p}{r^3} \left( 1 - \frac{2i}{\pi} \log \frac{R}{a} \right) + \frac{2i}{\pi r^2}$ $\operatorname{Re} p >  \operatorname{Im} a $
(57)	$t H_0^{(2)}(at)$	$\frac{p}{r^3} \left( 1 + \frac{2i}{\pi} \log \frac{R}{a} \right) - \frac{2i}{\pi r^2}$ $\operatorname{Re} p >  \operatorname{Im} a $

$$r = (p^2 + a^2)^{\frac{1}{2}}, \quad R = p + r$$

**Bessel functions of  $kt$  and  $kt^{\frac{1}{2}}$  (cont'd)**

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(58)	$t H_1^{(1)}(at)$	$\frac{a}{r^3} \left( 1 - \frac{2i}{\pi} \log \frac{R}{a} \right) - \frac{2ip}{\pi ar^2}$ $\text{Re } p >  \text{Im } a $
(59)	$t H_1^{(2)}(at)$	$\frac{a}{r^3} \left( 1 + \frac{2i}{\pi} \log \frac{R}{a} \right) + \frac{2ip}{\pi ar^2}$ $\text{Re } p >  \text{Im } a $
(60)	$t^{-\frac{1}{2}} H_{2\nu}^{(1)}(2a^{\frac{1}{2}} t^{\frac{1}{2}})$ $ \text{Re } \nu  < \frac{1}{2}$	$\pi^{\frac{1}{2}} p^{-\frac{1}{2}} \sec(\nu\pi) e^{-\frac{1}{2}\alpha/p}$ $\times [e^{-i\nu\pi} I_\nu(\frac{1}{2}\alpha/p)$ $- i\pi^{-1} K_\nu(\frac{1}{2}\alpha/p)]$ $\text{Re } p > 0$
(61)	$t^{-\frac{1}{2}} H_{2\nu}^{(2)}(2a^{\frac{1}{2}} t^{\frac{1}{2}})$ $ \text{Re } \nu  < \frac{1}{2}$	$\pi^{\frac{1}{2}} p^{-\frac{1}{2}} e^{-\frac{1}{2}\alpha/p} \sec(\nu\pi)$ $\times [e^{i\nu\pi} I_\nu(\frac{1}{2}\alpha/p)$ $+ i\pi^{-1} K_\nu(\frac{1}{2}\alpha/p)]$ $\text{Re } p > 0$
(62)	$t^{-\frac{1}{2}\nu} H_\nu^{(1)}(2a^{\frac{1}{2}} t^{\frac{1}{2}})$ $\text{Re } \nu < 1$	$\frac{p^{\nu-1} e^{-\alpha/p}}{\pi i a^{\frac{1}{2}\nu}} \Gamma(1-\nu) \Gamma(\nu, e^{-i\pi}\alpha/p)$ $\text{Re } p > 0$
(63)	$t^{-\frac{1}{2}\nu} H_\nu^{(2)}(2a^{\frac{1}{2}} t^{\frac{1}{2}})$ $\text{Re } \nu < 1$	$-\frac{p^{\nu-1} e^{-\alpha/p}}{\pi i a^{\frac{1}{2}\nu}} \Gamma(1-\nu) \Gamma(\nu, e^{i\pi}\alpha/p)$ $\text{Re } p > 0$
(64)	$t^{\nu-\frac{1}{2}} H_1^{(1)}(2a^{\frac{1}{2}} t^{\frac{1}{2}})$ $\text{Re } \nu > 0$	$\frac{\Gamma(\nu+1)}{ip^\nu \sin(\nu\pi)} e^{-\frac{1}{2}\alpha/p} k_{-2\nu} \left( \frac{ae^{-\pi i}}{2p} \right)$ $\text{Re } p > 0$
(65)	$t^{\nu-\frac{1}{2}} H_1^{(2)}(2a^{\frac{1}{2}} t^{\frac{1}{2}})$ $\text{Re } \nu > 0$	$\frac{i \Gamma(\nu+1)}{p^\nu \sin(\nu\pi)} e^{-\frac{1}{2}\alpha/p} k_{-2\nu} \left( \frac{ae^{\pi i}}{2p} \right)$ $\text{Re } p > 0$

$$r = (p^2 + \alpha^2)^{\frac{1}{2}}, \quad R = p + r$$

**Bessel functions of  $kt$  and  $kt^{\frac{1}{2}}$  (cont'd)**

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(66)	$t^{\mu-\frac{1}{2}} H_{2\nu}^{(1)}(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$ $\operatorname{Re}(\mu \pm \nu) > -\frac{1}{2}$	$\frac{\Gamma(\mu + \nu + \frac{1}{2}) \Gamma(\mu - \nu + \frac{1}{2})}{\pi \alpha^{\frac{1}{2}} e^{\nu \pi i + \frac{1}{4}\alpha/p} p^\mu}$ $\times W_{-\mu, \nu}(e^{-i\pi} \alpha/p) \quad \operatorname{Re} p > 0$
(67)	$t^{\mu-\frac{1}{2}} H_{2\nu}^{(2)}(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$ $\operatorname{Re}(\mu \pm \nu) > -\frac{1}{2}$	$\frac{\Gamma(\mu + \nu + \frac{1}{2}) \Gamma(\mu - \nu + \frac{1}{2})}{\pi \alpha^{\frac{1}{2}} e^{-\nu \pi i + \frac{1}{4}\alpha/p} p^\mu}$ $\times W_{-\mu, \nu}(e^{i\pi} \alpha/p) \quad \operatorname{Re} p > 0$

**4.15. Bessel functions of other arguments**

(1)	$J_{\nu+\frac{1}{2}}(\frac{1}{2}t^2)$ $\operatorname{Re} \nu > -1$	$\pi^{-\frac{1}{2}} \Gamma(\nu+1) D_{-\nu-1}(pe^{\frac{1}{4}\pi i})$ $\times D_{-\nu-1}(pe^{-\frac{1}{4}\pi i}) \quad \operatorname{Re} p > 0$
(2)	$t^{\frac{1}{2}} J_{\frac{1}{4}}(at^2)$ $a > 0$	$\frac{\pi^{\frac{1}{2}} p^{\frac{1}{2}}}{4a} \left[ H_{\frac{1}{4}}\left(\frac{p^2}{4a}\right) - Y_{\frac{1}{4}}\left(\frac{p^2}{4a}\right) \right]$ $\operatorname{Re} p > 0$
(3)	$t^{\frac{1}{2}} J_{-\frac{1}{4}}(at^2)$ $a > 0$	$\frac{\pi^{\frac{1}{2}} p^{\frac{1}{2}}}{4a} \left[ H_{\frac{1}{4}}\left(\frac{p^2}{4a}\right) - Y_{\frac{1}{4}}\left(\frac{p^2}{4a}\right) \right]$ $\operatorname{Re} p > 0$
(4)	$t^{3/2} J_{-\frac{1}{4}}(at^2)$ $a > 0$	$-\frac{\pi^{1/2} p^{3/2}}{8a^2} \left[ H_{-\frac{1}{4}}\left(\frac{p^2}{4a}\right) - Y_{-\frac{1}{4}}\left(\frac{p^2}{4a}\right) \right]$ $\operatorname{Re} p > 0$
(5)	$t^{3/2} J_{-\frac{1}{4}}(at^2)$ $a > 0$	$\frac{\pi^{1/2} p^{3/2}}{8a^2} \left[ H_{-\frac{1}{4}}\left(\frac{p^2}{4a}\right) - Y_{-\frac{1}{4}}\left(\frac{p^2}{4a}\right) \right]$ $\operatorname{Re} p > 0$

## Bessel functions of other arguments (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(6)	$t^{\frac{1}{8}} J_{1/8}(t^2/16) J_{-1/8}(t^2/16)$	$2^{-\frac{1}{2}} \pi^{\frac{1}{8}} p^{\frac{1}{8}} \sec(\pi/8) \times H_{1/8}^{(1)}(p^2) H_{1/8}^{(2)}(p^2) \quad \text{Re } p > 0$
(7)	$t^{\frac{1}{8}} J_{\nu+1/8}(t^2/16) J_{\nu-1/8}(t^2/16)$ $\text{Re } \nu > -3/8$	$2^{1/2} (\pi p)^{-3/2} \Gamma(\nu+3/8) \Gamma(\nu+5/8) \times W_{-\nu, 1/8}(2e^{\pi i/2} p^2) \times W_{-\nu, 1/8}(2e^{-\pi i/2} p^2) \quad \text{Re } p > 0$
(8)	$t^{-1} J_\nu(t^{-1})$	$2 J_\nu(2^{\frac{1}{2}} p^{\frac{1}{2}}) K_\nu(2^{\frac{1}{2}} p^{\frac{1}{2}}) \quad \text{Re } p > 0$
(9)	$0 \quad 0 < t < b \\ J_0(ay) \quad t > b$	$r^{-1} e^{-br} \quad \text{Re } p >  \text{Im } a $
(10)	$0 \quad 0 < t < b \\ t J_0(ay) \quad t > b$	$pr^{-3} (br+1) e^{-br} \quad \text{Re } p >  \text{Im } a $
(11)	$0 \quad 0 < t < b \\ \frac{J_0(ay)}{t-\lambda} \quad t > b \\  \arg(b-\lambda)  < \pi$	$-e^{-br} \int_0^\infty e^{-u} [u^2 - 2(\lambda p - br)u + \lambda^2(r-p)^2]^{-\frac{1}{2}} du \quad \text{Re } p >  \text{Im } a $
(12)	$0 \quad 0 < t < b \\ y J_1(ay) \quad t > b$	$\alpha r^{-3} (br+1) e^{-br} \quad \text{Re } p >  \text{Im } a $
(13)	$0 \quad 0 < t < b \\ y^{-1} J_1(ay) \quad t > b$	$\alpha^{-1} b^{-1} (e^{-bp} - e^{-br}) \quad \text{Re } p >  \text{Im } a $
(14)	$0 \quad 0 < t < b \\ y^{-1} J_\nu(ay) \quad t > b \\ \text{Re } \nu > -1$	$I_{\frac{1}{2}\nu}[\frac{1}{2}b(r-p)] K_{\frac{1}{2}\nu}[\frac{1}{2}b(r+p)] \quad \text{Re } p >  \text{Im } a $

$$y = (t^2 - b^2)^{\frac{1}{2}}, \quad r = (p^2 + \alpha^2)^{\frac{1}{2}}, \quad R = p + r$$

## Bessel functions of other arguments (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(15)	$0 \quad 0 < t < b$ $ty^{-1} J_1(ay) \quad t > b$	$\alpha^{-1} e^{-bp} - \alpha^{-1} pr^{-1} e^{-br}$ $\text{Re } p >  \text{Im } \alpha $
(16)	$0 \quad 0 < t < b$ $(t-b)^{\frac{1}{2}\nu} (t+b)^{-\frac{1}{2}\nu} J_\nu(ay) \quad t > b$ $\text{Re } \nu > -1$	$\alpha^\nu r^{-1} R^{-\nu} e^{-br}$ $\text{Re } p >  \text{Im } \alpha $
(17)	$0 \quad 0 < t < b$ $y^\nu J_\nu(ay) \quad t > b$ $\text{Re } \nu > -1$	$2^{\frac{1}{2}} \pi^{-\frac{1}{2}} \alpha^\nu b^{\nu+\frac{1}{2}} r^{-\nu-\frac{1}{2}} K_{\nu+\frac{1}{2}}(br)$ $\text{Re } p >  \text{Im } \alpha $
(18)	$0 \quad 0 < t < b$ $y^{2\mu} J_{2\nu}(ay) \quad t > b$ $\text{Re } (\mu + \nu) > -1$	$\sum_{n=0}^{\infty} \frac{(-1)^n (ab)^{2\nu+2n} (2b)^{2\mu+4n} \Gamma(\mu+\nu+n+1)}{\pi^{\frac{1}{2}} n! \Gamma(2\nu+n+1) (2p)^{\mu+\nu+n+\frac{1}{2}}} \times K_{\mu+\nu+n+\frac{1}{2}}(bp)$ $\text{Re } p >  \text{Im } \alpha $
(19)	$J_0[\alpha(t^2 + \beta t)^{\frac{1}{2}}] \quad  \arg \beta  < \pi$	$r^{-1} e^{\frac{1}{2}\beta(p-r)}$ $\text{Re } p >  \text{Im } \alpha $
(20)	$(t^2 + \beta t)^{\frac{1}{2}\nu} J_\nu[\alpha(t^2 + \beta t)^{\frac{1}{2}}]$ $\text{Re } \nu > -1, \quad  \arg \beta  < \pi$	$\pi^{-\frac{1}{2}} (\frac{1}{2}a)^\nu (\beta/r)^{\nu+\frac{1}{2}} e^{\frac{1}{2}\beta p} K_{\nu+\frac{1}{2}}(\frac{1}{2}\beta r)$ $\text{Re } p >  \text{Im } \alpha $
(21)	$t^{\frac{1}{2}\nu} (t+\beta)^{-\frac{1}{2}\nu} J_\nu[\alpha(t^2 + \beta t)^{\frac{1}{2}}]$ $\text{Re } \nu > -1, \quad  \arg \beta  < \pi$	$\alpha^\nu r^{-1} R^{-\nu} e^{\frac{1}{2}\beta(p-r)}$ $\text{Re } p >  \text{Im } \alpha $
(22)	$t^{\frac{1}{2}\nu-1} (t+1)^{-\frac{1}{2}\nu} J_\nu[\alpha(t^2 + t)^{\frac{1}{2}}]$ $\text{Re } \nu > 0$	$2^\nu \alpha^{-\nu} \gamma(\nu, \frac{1}{2}r - \frac{1}{2}p)$ $\text{Re } p >  \text{Im } \alpha $

$$y = (t^2 - b^2)^{\frac{1}{2}}, \quad r = (p^2 + \alpha^2)^{\frac{1}{2}}, \quad R = p + r$$

## Bessel functions of other arguments (cont'd)

	$f(t)$	$\mathcal{L}(f) = \int_0^\infty e^{-pt} f(t) dt$
(22)	$t^{\lambda - \frac{1}{2}\nu - 1} (t+1)^{-\frac{1}{2}\nu} J_\nu[\alpha(t^2 + t)^{\frac{1}{2}}]$ $\text{Re } \nu + 1 > \text{Re } \lambda > 0$	$\frac{2^\nu \alpha^{-\nu}}{\Gamma(\nu - \lambda + 1)} \\ \times \int_0^{\frac{1}{4}r^2 - \frac{1}{4}p} e^{-u} u^{\lambda - 1} (\frac{1}{4}a^2 - pu - u^2)^{\nu - \lambda} du \\ \text{Re } p >  \text{Im } \alpha $
(23)	$(t^2 + 2it)^{\frac{1}{2}\nu} J_\nu[\alpha(t^2 + 2it)^{\frac{1}{2}}]$ $\text{Re } \nu > -1$	$-i 2^{-\frac{1}{2}} \pi^{\frac{1}{2}} \alpha^\nu r^{-\nu - \frac{1}{2}} e^{ip} H_{\nu + \frac{1}{2}}^{(2)}(r) \\ \text{Re } p >  \text{Im } \alpha $
(24)	$(t^2 + 2it)^{\lambda - \frac{1}{2}\nu} J_\nu[\alpha(t^2 + 2it)^{\frac{1}{2}}]$ $\text{Re } \lambda > -1$	$\frac{2^{\lambda - \nu - \frac{1}{2}} \pi^{\frac{1}{2}} e^{ip} \Gamma(\lambda + 1)}{ir^{\lambda + \frac{1}{2}} \Gamma(\nu - \lambda)} \\ \times \sum_{n=0}^{\infty} \frac{\Gamma(\nu - \lambda + n)}{2^n n! \Gamma(\nu + n + 1) r^n} H_{\lambda + n + \frac{1}{2}}^{(2)}(r) \\ \text{Re } p >  \text{Im } \alpha $
(25)	$\exp[i\alpha(1-e^{-t})] J_\nu(\alpha e^{-t})$	$\frac{J_\nu(\alpha)}{\nu + p} + 2 \sum_{n=1}^{\infty} i^n \frac{(\nu - p + 1)_{n-1}}{(\nu + p)_{n+1}} (\nu + n) \\ \times J_{\nu+n}(\alpha) \quad \text{Re } p > -\text{Re } \nu$
(26)	$\sin[\alpha(1-e^{-t})] J_\nu(\alpha e^{-t})$	$2 \sum_{n=0}^{\infty} \frac{(-1)^n (\nu - p + 1)_{2n}}{(\nu + p)_{2n+2}} (\nu + 2n - 1) \\ \times J_{\nu+2n+1}(\alpha) \quad \text{Re } p > -\text{Re } \nu$
(27)	$\cos[\alpha(1-e^{-t})] J_\nu(\alpha e^{-t})$	$\frac{J_\nu(\alpha)}{\nu + p} + \sum_{n=0}^{\infty} 2(-1)^n \frac{(\nu - p + 1)_{2n-1}}{(\nu + p)_{2n+1}} \\ \times (\nu + 2n) J_{\nu+2n}(\alpha) \quad \text{Re } p > -\text{Re } \nu$

$$\gamma = (t^2 - b^2)^{\frac{1}{2}}, \quad r = (p^2 + \alpha^2)^{\frac{1}{2}}, \quad R = p + r$$

## Bessel functions of other arguments (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(28)	$J_\mu(\alpha e^{-t}) J_\nu[\alpha(1-e^{-t})]$ $\text{Re } \nu > -1$	$\left(\frac{2}{\alpha}\right)^p \sum_{n=0}^{\infty} \frac{(-1)^n \Gamma(p+n)}{(\mu+n)n! B(p, \mu+n)}$ $\times J_{\mu+\nu+p+2n}(\alpha) \quad \text{Re } p > -\text{Re } \mu$
(29)	$(1-e^{-t})^{\frac{1}{2}\nu} J_\nu[\alpha(1-e^{-t})^{\frac{1}{2}}]$ $\text{Re } \nu > -1$	$\Gamma(p) (2/\alpha)^p J_{\nu+p}(\alpha) \quad \text{Re } p > 0$
(30)	$(1-e^{-t})^{-\frac{1}{2}\nu} J_\nu[\alpha(1-e^{-t})^{\frac{1}{2}}]$	$\frac{s_{\nu+p-1, p-\nu}(\alpha)}{2^\nu \alpha^p \Gamma(\nu)} \quad \text{Re } p > 0$
(31)	$(e^{-t}-1)^{\frac{1}{2}\nu} J_\nu[2\alpha(e^{-t}-1)^{\frac{1}{2}}]$ $\alpha > 0, \quad \text{Re } \nu > -1$	$\frac{2\alpha^p}{\Gamma(p+1)} K_{\nu-p}(2\alpha)$ $\text{Re } p > \frac{1}{2} \text{Re } \nu - \frac{3}{4}$
(32)	$(e^{-t}-1)^\mu J_{\frac{1}{2}\nu}[2\alpha(e^{-t}-1)^{\frac{1}{2}}]$ $\alpha > 0, \quad \text{Re } (\mu + \nu) > -1$	$\frac{\alpha^{2\nu} B(\mu+\nu+1, p-\mu-\nu)}{\Gamma(2\nu+1)}$ $\times {}_1F_2(\mu+\nu+1; \mu+\nu+1-p; 2\nu+1; \alpha^2)$ $+ \frac{\alpha^{2p-2\mu} \Gamma(\mu+\nu-p)}{\Gamma(\nu-\mu+p+1)}$ $\times {}_1F_2(p+1; p+1+\nu-\mu, p+1-\mu-\nu; \alpha^2)$ $\text{Re } p > \text{Re } \mu - 7/4$
(33)	$J_\nu(2\alpha \sinh t)$ $\text{Re } \nu > -1, \quad \alpha > 0$	$I_{\frac{1}{2}\nu+\frac{1}{2}p}(\alpha) K_{\frac{1}{2}\nu-\frac{1}{2}p}(\alpha) \quad \text{Re } p > -\frac{1}{2}$
(34)	$\operatorname{csch}(t) J_\nu(\alpha \operatorname{csch} t) \quad \alpha > 0$	$\frac{\Gamma(\frac{1}{2}p + \frac{1}{2}\nu + \frac{1}{2})}{\alpha \Gamma(\nu+1)} W_{-\frac{1}{2}p, \frac{1}{2}\nu}(\alpha)$ $\times M_{\frac{1}{2}p, \frac{1}{2}\nu}(\alpha) \quad \text{Re } p > -\text{Re } \nu - 1$

## Bessel functions of other arguments (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(35)	$\csc(\frac{1}{2}t) \exp\left(\frac{\alpha - \beta e^t}{e^t - 1}\right)$ $\times J_{2\nu} \left[ \frac{\alpha^{\frac{1}{2}} \beta^{\frac{1}{2}}}{\sinh(\frac{1}{2}t)} \right]$ $\text{Re } \alpha > 0, \quad \text{Re } \beta > 0$	$\frac{2\Gamma(p + \nu + \frac{1}{2})}{\alpha^{\frac{1}{2}} \beta^{\frac{1}{2}} \Gamma(2\nu + 1)} e^{-\frac{1}{2}(\alpha + \beta)}$ $\times W_{-p, \nu}(\beta) M_{p, \nu}(\alpha)$ $\text{Re } p > -\text{Re } \nu - \frac{1}{2}$
(36)	$t^{-1} Y_\nu(t^{-1})$	$2Y_\nu(2^{\frac{1}{2}} p^{\frac{1}{2}}) K_\nu(2^{\frac{1}{2}} p^{\frac{1}{2}}) \quad \text{Re } p > 0$
(37)	$t^{-1} H_\nu^{(1)}(t^{-1})$	$2H_\nu^{(1)}(2^{\frac{1}{2}} p^{\frac{1}{2}}) K_\nu(2^{\frac{1}{2}} p^{\frac{1}{2}}) \quad \text{Re } p > 0$
(38)	$t^{-1} H_\nu^{(2)}(t^{-1})$	$2H_\nu^{(2)}(2^{\frac{1}{2}} p^{\frac{1}{2}}) K_\nu(2^{\frac{1}{2}} p^{\frac{1}{2}}) \quad \text{Re } p > 0$

4.16. Modified Bessel functions of arguments  $kt$  and  $kt^{\frac{1}{2}}$ 

(1)	$I_\nu(at)$	$\text{Re } \nu > -1$	$a^\nu s^{-1} S^{-\nu}$	$\text{Re } p >  \text{Re } a $
(2)	$t I_\nu(at)$	$\text{Re } \nu > -2$	$a^\nu (p + \nu s) s^{-3} S^{-\nu}$	$\text{Re } p >  \text{Re } a $
(3)	$t^{-1} I_1(at)$		$\frac{(p + a)^{\frac{1}{2}} - (p - a)^{\frac{1}{2}}}{(p + a)^{\frac{1}{2}} + (p - a)^{\frac{1}{2}}}$	$\text{Re } p >  \text{Re } a $
(4)	$t^{-1} I_\nu(at)$	$\text{Re } \nu > 0$	$\nu^{-1} a^\nu S^{-\nu}$	$\text{Re } p >  \text{Re } a $
(5)	$t^{-\frac{1}{2}} I_\nu(t)$	$\text{Re } \nu > -\frac{1}{2}$	$2^{\frac{1}{2}} \pi^{-\frac{1}{2}} Q_{\nu - \frac{1}{2}}(p)$	$\text{Re } p > 1$
(6)	$t^\nu I_\nu(at)$	$\text{Re } \nu > -\frac{1}{2}$	$2^\nu \pi^{-\frac{1}{2}} \Gamma(\nu + \frac{1}{2}) a^\nu s^{-2\nu - 1}$	$\text{Re } p >  \text{Re } a $

$$s = (p^2 - a^2)^{\frac{1}{2}}, \quad S = p + s$$

## Modified Bessel functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(7)	$t^{\nu+1} I_\nu(at) \quad \operatorname{Re} \nu > -1$	$2^{\nu+1} \pi^{-\frac{\nu}{2}} \Gamma(\nu+3/2) a^\nu p s^{-2\nu-3}$ $\operatorname{Re} p >  \operatorname{Re} a $
(8)	$t^\mu I_\nu(at) \quad \operatorname{Re}(\mu + \nu) > -1$	$\Gamma(\mu + \nu + 1) s^{-\mu-1} P_{\mu}^{-\nu}(p/s)$ $\operatorname{Re} p >  \operatorname{Re} a $
(9)	$t^{\mu-\frac{\nu}{2}} I_{\nu+\frac{\nu}{2}}(at) \quad \operatorname{Re}(\mu + \nu) > -1$	$\frac{2^{\frac{\nu}{2}} \sin(\nu\pi) s^{-\mu}}{\pi^{\frac{\nu}{2}} a^{\frac{\nu}{2}} \sin[(\mu+\nu)\pi]} Q_\nu^\mu\left(\frac{p}{a}\right)$ $\operatorname{Re} p >  \operatorname{Re} a $
(10)	$I_0^2(\frac{1}{2}at)$	$2\pi^{-1} p^{-1} E(a/p) \quad \operatorname{Re} p >  \operatorname{Re} a $
(11)	$t I_0(\frac{1}{2}at) I_1(\frac{1}{2}at)$	$\frac{2p E(a/p)}{\pi a(p^2 - a^2)} - \frac{2K(a/p)}{\pi a p}$ $\operatorname{Re} p >  \operatorname{Re} a $
(12)	$t^{-\frac{\nu}{2}} I_\mu(at) I_\nu(bt) \quad \operatorname{Re}(\mu + \nu) > -\frac{1}{2}$	$c^{\frac{\nu}{2}} \Gamma(\mu + \nu + \frac{1}{2}) P_{\nu-\frac{\nu}{2}}^{-\mu}(\cosh a)$ $\times P_{\mu-\frac{\nu}{2}}^{-\nu}(\cosh \beta)$ $\operatorname{Re}(p \pm a \pm b) > 0$ where $\sinh a = ac, \quad \sinh \beta = bc,$ $\cosh a \cosh \beta = pc,$ $ \operatorname{Im} a , \quad  \operatorname{Im} \beta  < \frac{1}{2}\pi$
(13)	$t^{2\lambda-1} I_{2\mu}(at) I_{2\nu}(\beta t) \quad \operatorname{Re}(\lambda + \mu + \nu) > 0$	$\frac{2^{2\lambda-1} a^{2\mu} \beta^{2\nu} \Gamma(\lambda + \mu + \nu) \Gamma(\lambda + \mu + \nu + \frac{1}{2})}{\pi^{\frac{\nu}{2}} p^{2\lambda+2\mu+2\nu} \Gamma(2\mu+1) \Gamma(2\nu+1)}$ $\times F_4(\lambda + \mu + \nu, \lambda + \mu + \nu + \frac{1}{2}; 2\mu+1, 2\nu+1; a^2/p^2, \beta^2/p^2)$ $\operatorname{Re} p >  \operatorname{Re} a  +  \operatorname{Re} \beta $

$$s = (p^2 - a^2)^{\frac{\nu}{2}}, \quad S = p + s$$

## Modified Bessel functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(14)	$I_0(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$	$p^{-1} e^{\alpha/p}$ $\operatorname{Re} p > 0$
(15)	$t^{-1/2} I_0(2^{3/2} \alpha^{1/2} t^{1/2})$	$\pi^{\frac{1}{2}} p^{-\frac{1}{2}} e^{\alpha/p} I_0(\alpha/p)$ $\operatorname{Re} p > 0$
(16)	$t^{-\frac{1}{2}} I_1(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$	$\alpha^{-\frac{1}{2}} (e^{\alpha/p} - 1)$ $\operatorname{Re} p > 0$
(17)	$t^{-1/2} I_\nu(2^{3/2} \alpha^{1/2} t^{1/2})$ $\operatorname{Re} \nu > -1$	$\pi^{\frac{1}{2}} p^{-\frac{1}{2}} e^{\alpha/p} I_{\frac{1}{2}\nu}(\alpha/p)$ $\operatorname{Re} p > 0$
(18)	$t^{\frac{1}{2}\nu} I_\nu(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$ $\operatorname{Re} \nu > -1$	$\alpha^{\frac{1}{2}\nu} p^{-\nu-1} e^{\alpha/p}$ $\operatorname{Re} p > 0$
(19)	$t^{-\frac{1}{2}\nu} I_\nu(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$	$\alpha^{-\frac{1}{2}\nu} [\Gamma(\nu)]^{-1} p^{\nu-1} e^{\alpha/p} \gamma(\nu, \alpha/p)$ $\operatorname{Re} p > 0$
(20)	$t^{\mu-\frac{1}{2}} I_{2\nu}(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$ $\operatorname{Re}(\mu + \nu) > -\frac{1}{2}$	$\frac{\Gamma(\mu + \nu + \frac{1}{2}) e^{\frac{1}{2}\alpha/p}}{\alpha^{\frac{1}{2}} \Gamma(2\nu + 1) p^\mu} M_{-\mu, \nu} \left( \frac{\alpha}{p} \right)$ $\operatorname{Re} p > 0$
(21)	$I_\nu^2(2^{\frac{1}{2}} \alpha^{\frac{1}{2}} t^{\frac{1}{2}})$ $\operatorname{Re} \nu > -1$	$p^{-1} e^{\alpha/p} I_\nu(\alpha/p)$ $\operatorname{Re} p > 0$
(22)	$I_\nu(2^{\frac{1}{2}} \alpha^{\frac{1}{2}} t^{\frac{1}{2}}) I_\nu(2^{\frac{1}{2}} \beta^{\frac{1}{2}} t^{\frac{1}{2}})$ $\operatorname{Re} \nu > -1$	$p^{-1} \exp[\frac{1}{2}(\alpha + \beta)/p] I_\nu(\alpha^{\frac{1}{2}} \beta^{\frac{1}{2}}/p)$ $\operatorname{Re} p > 0$
(23)	$K_0(\alpha t)$	$s^{-1} \log(S/\alpha) = s^{-1} \sinh^{-1}(s/\alpha)$ $\operatorname{Re} p > -\operatorname{Re} \alpha$
(24)	$K_\nu(\alpha t)$ $ \operatorname{Re} \nu  < 1$	$\frac{1}{2} \pi \csc(\nu\pi) s^{-1} [\alpha^{-\nu} S^\nu - \alpha^\nu S^{-\nu}]$ $\operatorname{Re} p > -\operatorname{Re} \alpha$

$$s = (p^2 - \alpha^2)^{\frac{1}{2}}, \quad S = p + s$$

## Modified Bessel functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(25)	$t K_0(at)$	$ps^{-3} \log(S/a) - s^{-2}$ $\text{Re } p > -\text{Re } \alpha$
(26)	$t K_1(at)$	$\alpha^{-1} ps^{-2} - \alpha s^{-3} \log(S/a)$ $\text{Re } p > -\text{Re } \alpha$
(27)	$t^{\mu-\frac{1}{2}} K_{\nu+\frac{1}{2}}(at)$ $\text{Re } (\mu + \nu) > -1$ $\text{Re } (\mu - \nu) > 0$	$2^{-\frac{1}{2}} \alpha^{-\frac{1}{2}} \pi^{\frac{1}{2}} \Gamma(\mu - \nu) \Gamma(\mu + \nu + 1)$ $\times s^{-\mu} P_{\nu}^{-\mu}(p/a)$ $\text{Re } p > -\text{Re } \alpha$
(28)	$t^\mu K_\nu(at)$ $\text{Re } (\mu \pm \nu) > -1$	$\frac{\sin(\mu\pi) \Gamma(\mu - \nu + 1)}{\sin[(\mu + \nu)\pi] s^{\mu+1}} Q_\mu^\nu\left(\frac{p}{s}\right)$ $\text{Re } p > -\text{Re } \alpha$
(29)	$\frac{1}{2t} \exp\left(-\frac{\lambda}{2at}\right) K_\nu(a\lambda t)$ $\text{Re } (\lambda/a) > 0$	$K_\nu(\alpha^{-\frac{1}{2}} \lambda^{\frac{1}{2}} S^{\frac{1}{2}}) K_\nu(\alpha^{\frac{1}{2}} \lambda^{\frac{1}{2}} S^{-\frac{1}{2}})$ $\text{Re } p > -\text{Re } (\alpha\lambda)$
(30)	$t^{-\frac{1}{2}} I_\mu(at) K_\nu(bt)$ $\text{Re } (\mu \pm \nu) > -\frac{1}{2}$	$\frac{c^{\frac{1}{2}} \Gamma(\mu - \nu + \frac{1}{2}) \cos(\mu\pi)}{\cos(\mu + \nu)\pi} P_{\nu-\frac{1}{2}}^{-\mu}(\cosh \alpha)$ $\times Q_{\mu-\frac{1}{2}}^{-\nu}(\cosh \beta)$ $\text{Re } (p \pm a + b) > 0$ for definition of $\alpha$ , $\beta$ , and $c$ see (12) of this section.
(31)	$t^{-\frac{1}{2}} K_\mu(at) K_\nu(bt)$ $ \text{Re } \mu  +  \text{Re } \nu  < \frac{1}{2}$	$\frac{c^{\frac{1}{2}} \Gamma(\frac{1}{2} - \mu - \nu) \cos(\mu\pi) \cos(\nu\pi)}{\cos(\mu + \nu)\pi \cos(\mu - \nu)\pi}$ $\times Q_{\nu+\frac{1}{2}}^{-\mu}(\cosh \alpha) Q_{\mu-\frac{1}{2}}^{-\nu}(\cosh \beta)$ $\text{Re } (p + a + b) > 0$ $\alpha, \beta, c$ defined in (12).

$$s = (p^2 - a^2)^{\frac{1}{2}}, \quad S = p + s$$

## Modified Bessel functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(32)	$K_0(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$	$-\frac{1}{2} p^{-1} e^{\alpha/p} \text{Ei}(-\alpha/p) \quad \text{Re } p > 0$
(33)	$K_1(2^{3/2} \alpha^{1/2} t^{1/2})$	$2^{-3/2} \alpha^{1/2} \pi^{1/2} p^{-3/2} e^{\alpha/p} \times [K_1(\alpha/p) - K_0(\alpha/p)] \quad \text{Re } p > 0$
(34)	$t^{-1/2} K_0(2^{3/2} \alpha^{1/2} t^{1/2})$	$\frac{1}{2} \pi^{\frac{1}{2}} p^{-\frac{1}{2}} e^{\alpha/p} K_0(\alpha/p) \quad \text{Re } p > 0$
(35)	$t^{-1/2} K_\nu(2^{3/2} \alpha^{1/2} t^{1/2})$ $  \text{Re } \nu   < 1$	$\frac{1}{2} \pi^{\frac{1}{2}} p^{-\frac{1}{2}} \sec(\frac{1}{2} \nu \pi) e^{\alpha/p} K_{\frac{1}{2}\nu}(\alpha/p)$ $\text{Re } p > 0$
(36)	$t^{\frac{1}{2}\nu} K_\nu(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}}) \quad \text{Re } \nu > -1$	$\frac{1}{2} \alpha^{\frac{1}{2}\nu} \Gamma(\nu+1) p^{-\nu-1} e^{\alpha/p}$ $\times \Gamma(-\nu, \alpha/p) \quad \text{Re } p > 0$
(37)	$t^{\mu-\frac{1}{2}} K_{2\nu}(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$ $\text{Re}(\mu \pm \nu) > -\frac{1}{2}$	$\frac{\Gamma(\mu+\nu+\frac{1}{2}) \Gamma(\mu-\nu+\frac{1}{2})}{2\alpha^{\frac{1}{2}} p^\mu} e^{\frac{1}{2}\alpha/p}$ $\times W_{-\mu, \nu}(\alpha/p) \quad \text{Re } p > 0$
(38)	$t^{-\frac{1}{2}} K_{2\nu}(2^{\frac{1}{2}} \alpha^{\frac{1}{2}} t^{\frac{1}{2}})$ $\times \{ \sin[(\nu-\frac{1}{4})\pi] J_{2\nu}(2^{\frac{1}{2}} \alpha^{\frac{1}{2}} t^{\frac{1}{2}})$ $+ \cos[(\nu-\frac{1}{4})\pi] Y_{2\nu}(2^{\frac{1}{2}} \alpha^{\frac{1}{2}} t^{\frac{1}{2}}) \}$ $  \text{Re } \nu   < \frac{1}{4}$	$-2^{-3/2} \pi^{-1/2} p^{1/2} \alpha^{-1} \Gamma(\frac{1}{4}+\nu)$ $\times \Gamma(\frac{1}{4}-\nu) W_{\frac{1}{4}, \nu}(e^{\frac{1}{2}\pi i} \alpha/p)$ $\times W_{\frac{1}{4}, \nu}(e^{-\frac{1}{2}\pi i} \alpha/p) \quad \text{Re } p > 0$
(39)	$t^{2\nu} K_{2\nu}(t^{\frac{1}{2}}) I_{2\nu}(t^{\frac{1}{2}})$ $\text{Re } \nu > -\frac{1}{4}$	$\frac{1}{2} \Gamma(2\nu+\frac{1}{2}) p^{-3\nu-\frac{1}{2}} e^{\frac{1}{2}p} p^{-1}$ $\times W_{-\nu, \nu}(p^{-1}) \quad \text{Re } p > 0$

## 4.17. Modified Bessel functions of other arguments

(1)	$\exp\left(-\frac{t^2}{16\alpha}\right) I_0\left(\frac{t^2}{16\alpha}\right)$ $\text{Re } \alpha \geq 0$	$\frac{2^{\frac{1}{2}} \alpha^{\frac{1}{2}}}{\pi^{\frac{1}{2}}} e^{\alpha p^2} K_0(\alpha p^2) \quad \text{Re } p > 0$
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## Modified Bessel functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(2)	$t^{\frac{\alpha}{2}} \exp\left(-\frac{t^2}{8\alpha}\right) I_{\frac{\alpha}{2}}\left(\frac{t^2}{8\alpha}\right)$ $\text{Re } \alpha \geq 0$	$\frac{2}{\Gamma(\frac{1}{4})} \frac{\alpha^{\frac{1}{2}}}{p^{\frac{1}{2}}} e^{ap^2} \Gamma(\frac{1}{4}, ap^2)$ $\text{Re } p > 0$
(3)	$t^{2\nu} \exp\left(-\frac{t^2}{8\alpha}\right) I_\nu\left(\frac{t^2}{8\alpha}\right)$ $\text{Re } \alpha \geq 0, \quad \text{Re } \nu > -\frac{1}{4}$	$\frac{\alpha^{\frac{1}{2}\nu} \Gamma(4\nu+1)}{2^{4\nu} \Gamma(\nu+1)} p^{-\nu-1} e^{\frac{1}{4}ap^2}$ $\times W_{-\frac{3\nu}{2}, \frac{\nu}{2}}(ap^2) \quad \text{Re } p > 0$
(4)	$\frac{1}{t} \exp\left(-\frac{\alpha+\beta}{2t}\right) I_\nu\left(\frac{\alpha-\beta}{2t}\right)$ $\text{Re } \alpha \geq \text{Re } \beta > 0$	$2K_\nu[(\alpha^{\frac{1}{2}} + \beta^{\frac{1}{2}})p^{\frac{1}{2}}] I_\nu[(\alpha^{\frac{1}{2}} - \beta^{\frac{1}{2}})p^{\frac{1}{2}}]$ $\text{Re } p > 0$
(5)	$0 \quad 0 < t < b$ $I_0(\alpha y) \quad t > b$	$s^{-1} e^{-bs} \quad \text{Re } p >  \text{Re } \alpha $
(6)	$0 \quad 0 < t < b$ $t I_0(\alpha y) \quad t > b$	$p(bs^{-2} + s^{-3})e^{-bs} \quad \text{Re } p >  \text{Re } \alpha $
(7)	$0 \quad 0 < t < b$ $y I_1(\alpha y) \quad t > b$	$\alpha(bs^{-2} - s^{-3})e^{-bs} \quad \text{Re } p >  \text{Re } \alpha $
(8)	$0 \quad 0 < t < b$ $y^{-1} I_1(\alpha y) \quad t > b$	$\alpha^{-1} b^{-1} (e^{-bs} - e^{-bp}) \quad \text{Re } p >  \text{Re } \alpha $
(9)	$0 \quad 0 < t < b$ $y^{-1} t I_1(\alpha y) \quad t > b$	$\alpha^{-1} ps^{-1} e^{-bs} - \alpha^{-1} e^{-bp} \quad \text{Re } p >  \text{Re } \alpha $

$$y = (t^2 - b^2)^{\frac{1}{2}} \quad s = (p^2 - \alpha^2)^{\frac{1}{2}}, \quad S = p + s$$

## Modified Bessel functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(10)	$0 \quad 0 < t < b$ $\gamma^\nu I_\nu(\alpha y) \quad t > b$ $\text{Re } \nu > -1$	$2^{\frac{\nu}{2}} \pi^{-\frac{\nu}{2}} \alpha^\nu b^{\nu+\frac{1}{2}} s^{-\nu-\frac{1}{2}} K_{\nu+\frac{1}{2}}(bs)$ $\text{Re } p >  \text{Re } \alpha $
(11)	$0 \quad 0 < t < b$ $(t-b)^{\frac{\nu}{2}} \nu (t+b)^{-\frac{\nu}{2}} \nu I_\nu(\alpha y) \quad t > b$ $\text{Re } \nu > -1$	$\alpha^\nu s^{-1} S^{-\nu} e^{-bs} \quad \text{Re } p >  \text{Re } \alpha $
(12)	$I_0[\alpha(t^2 + \beta t)^{\frac{\nu}{2}}] \quad  \arg \beta  < \pi$	$s^{-1} e^{\frac{\nu}{2}\beta(p-s)} \quad \text{Re } p >  \text{Re } \alpha $
(13)	$(t^2 + \beta t)^{\frac{\nu}{2}} \nu I_\nu[\alpha(t^2 + \beta t)^{\frac{\nu}{2}}] \quad \text{Re } \nu > -1, \quad  \arg \beta  < \pi$	$\pi^{-\frac{\nu}{2}} (\frac{1}{2} \alpha)^\nu (\beta/s)^{\nu+\frac{1}{2}} e^{\frac{\nu}{2}\beta p}$ $\times K_{\nu+\frac{1}{2}}(\frac{1}{2} \beta s) \quad \text{Re } p >  \text{Re } \alpha $
(14)	$t^{\frac{\nu}{2}} \nu (t+\beta)^{-\frac{\nu}{2}} \nu I_\nu[\alpha(t^2 + \beta t)^{\frac{\nu}{2}}] \quad \text{Re } \nu > -1, \quad  \arg \beta  < \pi$	$\alpha^\nu s^{-1} S^{-\nu} e^{\frac{\nu}{2}\beta(p-s)} \quad \text{Re } p >  \text{Re } \alpha $
(15)	$t^{\mu-1} (t+\beta)^{-\mu} I_{2\nu}[\alpha(t^2 + \beta t)^{\frac{\nu}{2}}] \quad \text{Re } (\mu + \nu) > 0, \quad  \arg \beta  < \pi$	$\frac{2\Gamma(\mu+\nu) e^{\frac{\nu}{2}\beta p}}{\alpha\beta\Gamma(2\nu+1)} M_{\frac{\nu}{2}-\mu, \nu} \left( \frac{\alpha^2 \beta}{2S} \right)$ $\times W_{\frac{\nu}{2}-\mu, \nu} \left( \frac{\beta S}{2} \right) \quad \text{Re } p >  \text{Re } \alpha $
For more general formulas see MacRobert, T. M., 1948: <i>Philos. Mag.</i> (7) 39, pp. 466-471.		
(16)	$(2t-t^2)^{\frac{\nu}{2}} \nu^{-\frac{1}{2}} C_n^\nu(t-1) \quad 0 < t < 2$ $\times I_{\nu-\frac{1}{2}}[\alpha(2t-t^2)^{\frac{\nu}{2}}]$ $0 \quad t > 2$ $\text{Re } \nu > -\frac{1}{2}$	$(-1)^n \frac{2^{\frac{\nu}{2}} \pi^{\frac{\nu}{2}} \alpha^{\nu-\frac{1}{2}}}{r^\nu e^p} C_n^\nu \left( \frac{p}{r} \right) I_{\nu+n}(r)$ $r = (p^2 + \alpha^2)^{\frac{1}{2}}$

$$y = (t^2 - b^2)^{\frac{\nu}{2}}, \quad s = (p^2 - \alpha^2)^{\frac{1}{2}}, \quad S = p + s$$

## Modified Bessel functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(17)	$\exp[a(1-e^{-t})] I_\nu(ae^{-t})$	$\frac{I_\nu(a)}{\nu+p} + \sum_{n=1}^{\infty} \frac{(\nu-p+1)_{n-1}}{(\nu+p)_{n+1}} (\nu+n) \\ \times I_{\nu+p}(a) \quad \text{Re } p > -\text{Re } \nu$
(18)	$t^{-\frac{1}{2}} e^{-\alpha/t} K_\nu(a/t) \quad \text{Re } \alpha > 0$	$2\pi^{1/2} p^{-1/2} K_{2\nu}(2^{3/2} \alpha^{1/2} p^{1/2}) \quad \text{Re } p > 0$
(19)	$\frac{1}{t} \exp\left(-\frac{\alpha+\beta}{2t}\right) K_\nu\left(\frac{\alpha-\beta}{2t}\right) \quad \text{Re } \alpha > \text{Re } \beta > 0$	$2K_\nu[(\alpha^{\frac{1}{2}} + \beta^{\frac{1}{2}}) p^{\frac{1}{2}}] K_\nu[(\alpha^{\frac{1}{2}} - \beta^{\frac{1}{2}}) p^{\frac{1}{2}}] \quad \text{Re } p > 0$
(20)	$t^{\mu-1} (t+\beta)^{-\mu} K_{2\nu}[\alpha(t^2 + \beta t)^{\frac{1}{2}}] \quad \text{Re } (\mu \pm \nu) > 0, \quad  \arg \beta  < \pi$	$\alpha^{-1} \beta^{-1} \Gamma(\mu+\nu) \Gamma(\mu-\nu) e^{\frac{1}{2}\beta p} \\ \times W_{\frac{1}{2}-\mu, \nu}(\frac{1}{2}\alpha^2 \beta S^{-1}) \\ \times W_{\frac{1}{2}-\mu, \nu}(\frac{1}{2}\beta S) \quad \text{Re } p >  \text{Re } \alpha $
(21)	$-2\pi^{-1} K_0[2\alpha \sinh(\frac{1}{2}t)] \quad \text{Re } \alpha > 0$	$J_p(a) \frac{\partial Y_p(a)}{\partial p} - Y_p(a) \frac{\partial J_p(a)}{\partial p}$
(22)	$-2\pi^{-1} \cosh t K_0[2\alpha \sinh(\frac{1}{2}t)] \quad \text{Re } \alpha > 0$	$J'_p(a) \frac{\partial Y_p'(a)}{\partial p} - Y'_p(a) \frac{\partial J_p'(a)}{\partial p} \\ + \frac{p^2}{\alpha^2} \left[ J_p(a) \frac{\partial Y_p(a)}{\partial p} \right. \\ \left. - Y_p(a) \frac{\partial J_p(a)}{\partial p} \right] \quad \left[ J'_p = \frac{d J_p}{d \alpha} \right]$
(23)	$2\pi^{-2} \sin(2\nu\pi) \times K_{2\nu}[2\alpha \sinh(\frac{1}{2}t)] \quad \text{Re } \alpha > 0$	$J_{\nu-p}(a) Y_{-\nu-p}(a) - J_{-\nu-p}(a) Y_{\nu-p}(a)$

$$\gamma = (t^2 - b^2)^{\frac{1}{2}}, \quad s = (p^2 - \alpha^2)^{\frac{1}{2}}, \quad S = p + s$$

**Modified Bessel functions (cont'd)**

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(24)	$\operatorname{csch}(\frac{1}{2}t) K_{2\nu}[\alpha \operatorname{csch}(\frac{1}{2}t)]$ $\operatorname{Re} \alpha > 0$	$\alpha^{-1} \Gamma(p + \nu + \frac{1}{2}) \Gamma(p - \nu + \frac{1}{2})$ $\times W_{-p, \nu}(i\alpha) W_{-p, \nu}(-i\alpha)$ $\operatorname{Re}(p \pm \nu) > -1$
(25)	$\frac{1}{\sinh(\frac{1}{2}t)} \exp\left(-\frac{\alpha e^{t+\beta}}{e^t - 1}\right)$ $\times K_{2\nu} \left[ \frac{\alpha^{\frac{1}{2}} \beta^{\frac{1}{2}}}{\sinh(\frac{1}{2}t)} \right]$ $\operatorname{Re} \alpha > 0, \quad \operatorname{Re} \beta > 0$	$\frac{1}{\alpha^{\frac{1}{2}} \beta^{\frac{1}{2}}} \Gamma(p + \nu + \frac{1}{2}) \Gamma(p - \nu + \frac{1}{2})$ $\times e^{-\frac{1}{2}\alpha + \frac{1}{2}\beta} W_{-p, \nu}(\alpha) W_{-p, \nu}(\beta)$ $\operatorname{Re}(p \pm \nu) > -\frac{1}{2}$

**4.18. Kelvin's functions and related functions**

(1)	$\operatorname{ber} t$	$[\frac{1}{2}(p^4 + 1)^{-\frac{1}{4}} + \frac{1}{2}p^2(p^4 + 1)^{-\frac{1}{2}}]^{\frac{1}{2}}$ $\operatorname{Re} p > 2^{-\frac{1}{2}}$
(2)	$\operatorname{bei} t$	$[\frac{1}{2}(p^4 + 1)^{-\frac{1}{4}} - \frac{1}{2}p^2(p^4 + 1)^{-\frac{1}{2}}]^{\frac{1}{2}}$ $\operatorname{Re} p > 2^{-\frac{1}{2}}$
(3)	$\operatorname{ber}(2t^{\frac{1}{2}})$	$p^{-1} \cos p^{-1}$ $\operatorname{Re} p > 0$
(4)	$\operatorname{bei}(2t^{\frac{1}{2}})$	$p^{-1} \sin p^{-1}$ $\operatorname{Re} p > 0$
(5)	$t^{\frac{1}{2}\nu} \operatorname{ber}_\nu(t^{\frac{1}{2}})$ $\operatorname{Re} \nu > -1$	$2^{-\nu} p^{-\nu-1} \cos [\frac{1}{4}(1+3\nu\pi p)/p]$ $\operatorname{Re} p > 0$
(6)	$t^{\frac{1}{2}\nu} \operatorname{bei}_\nu(t^{\frac{1}{2}})$ $\operatorname{Re} \nu > -1$	$2^{-\nu} p^{-\nu-1} \sin [\frac{1}{4}(1+3\nu\pi p)/p]$ $\operatorname{Re} p > 0$
(7)	$0 \quad 0 < t < b$ $\operatorname{ber}(\alpha y) + i \operatorname{bei}(\alpha y) \quad t > b$	$v^{-1} e^{-bv}$ $\operatorname{Re}(p \pm \alpha i^{\frac{1}{2}}) > 0$

$$\gamma = (t^2 - b^2)^{\frac{1}{2}}, \quad v = (p^2 - i\alpha^2)^{\frac{1}{2}}, \quad V = p + v$$

## Kelvin's functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(8)	$0 \quad 0 < t < b$ $t [\text{ber}(\alpha y) + i \text{bei}(\alpha y)] \quad t > b$	$p v^{-3} (b v + 1) e^{-bv}$ $\text{Re}(p \pm \alpha i^{\frac{1}{2}}) > 0$
(9)	$0 \quad 0 < t < b$ $y [\text{ber}_1(\alpha y) + i \text{bei}_1(\alpha y)] \quad t > b$	$\alpha v^{-3} (b v + 1) e^{-bv + \frac{1}{2}\pi i}$ $\text{Re}(p \pm \alpha i^{\frac{1}{2}}) > 0$
(10)	$0 \quad 0 < t < b$ $y^{-1} [\text{ber}_1(\alpha y) + i \text{bei}_1(\alpha y)] \quad t > b$	$\alpha^{-1} b^{-1} e^{-\frac{1}{2}\pi i} (e^{-bp} - e^{-bv})$ $\text{Re}(p \pm \alpha i^{\frac{1}{2}}) > 0$
(11)	$0 \quad 0 < t < b$ $t y^{-1} [\text{ber}_1(\alpha y) + i \text{bei}_1(\alpha y)] \quad t > b$	$\alpha^{-1} e^{-\frac{1}{2}\pi i} (e^{-bp} - p v^{-1} e^{-bv})$ $\text{Re}(p \pm \alpha i^{\frac{1}{2}}) > 0$
(12)	$0 \quad 0 < t < b$ $\left( \frac{t-b}{t+b} \right)^{\frac{1}{2}\nu} [\text{ber}_\nu(\alpha y) + i \text{bei}_\nu(\alpha y)] \quad t > b$ $\text{Re } \nu > -1$	$\alpha^\nu v^{-1} V^{-\nu} e^{\frac{1}{2}\nu\pi i - bv}$ $\text{Re}(p \pm \alpha i^{\frac{1}{2}}) > 0$
(13)	$t [\text{ker}(\alpha t) + i \text{kei}(\alpha t)]$	$p v^{-3} \log(i^{-\frac{1}{2}} V/a) - v^{-2}$ $\text{Re}(p \pm \alpha i^{\frac{1}{2}}) > 0$
(14)	$t [\text{ker}_1(\alpha t) + i \text{kei}_1(\alpha t)]$	$i^{1/2} p \alpha^{-1} v^{-2} + \alpha i^{3/2} v^{-3} \log(i^{-1/2} V/a)$ $\text{Re}(p \pm \alpha i^{\frac{1}{2}}) > 0$
(15)	$0 \quad 0 < t < b$ $\text{ker}(\alpha y) + i \text{kei}(\alpha y) \quad t > b$	$v^{-1} e^{-bv} \log(i^{-\frac{1}{2}} V/a)$ $\text{Re}(p + \alpha i^{\frac{1}{2}}) > 0$

$$\gamma = (t^2 - b^2)^{\frac{1}{2}}, \quad v = (p^2 - i \alpha^2)^{\frac{1}{2}}, \quad V = p + v$$

## Kelvin's functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(16)	$0 \quad 0 < t < b$ $t [\ker(\alpha y) + i \operatorname{kei}(\alpha y)] \quad t > b$	$-v^{-2} e^{-bv} [1 + (bp - pv^{-1}) \log(i^{-\frac{1}{2}} V/\alpha)]$ $\operatorname{Re}(p + \alpha i^{\frac{1}{2}}) > 0$
(17)	$0 \quad 0 < t < b$ $v [\ker_1(\alpha y) + i \operatorname{kei}_1(\alpha y)] \quad t > b$	$v^{-1} e^{-bv} [i^{1/2} p/\alpha + i^{3/2} \alpha (b + 1/v) \times \log(i^{-1/2} V/\alpha)]$ $\operatorname{Re}(p + \alpha i^{\frac{1}{2}}) > 0$
(18)	$0 \quad 0 < t < b$ $\left(\frac{t-b}{t+b}\right)^{\frac{1}{2}\nu} [\ker_\nu(\alpha y) + i \operatorname{kei}_\nu(\alpha y)] \quad t > b$ $ \operatorname{Re} \nu  < 1$	$\frac{\pi e^{-bp-\frac{1}{2}i\nu\pi}}{2\nu \sin(\nu\pi)} \left[ \left(\frac{V}{\alpha i^{\frac{1}{2}}}\right)^\nu - \left(\frac{\alpha i^{\frac{1}{2}}}{V}\right)^\nu \right]$ $\operatorname{Re}(p + \alpha i^{\frac{1}{2}}) > 0$
(19)	$V_\nu^{(b)}(2t^{\frac{1}{2}}) \quad \operatorname{Re} \nu > 0$	$p I_\nu(2p^{-1}) \quad \operatorname{Re} p > 0$
(20)	$t^{\frac{1}{2}} W_\nu^{(b)}(2t^{\frac{1}{2}}) \quad \operatorname{Re} \nu > -2$	$p^{-2} I_\nu(2p^{-1}) \quad \operatorname{Re} p > 0$
(21)	$X_\nu^{(b)}(2t^{\frac{1}{2}}) \quad \operatorname{Re} \nu > -1$	$p^{-1} I_\nu(2p^{-1}) \quad \operatorname{Re} p > 0$
(22)	$t^{-\frac{1}{2}} Z_\nu^{(b)}(2t^{\frac{1}{2}}) \quad \operatorname{Re} \nu > 0$	$I_\nu(2p^{-1}) \quad \operatorname{Re} p > 0$

## 4.19. Functions related to Bessel functions, Struve, Lommel, and Bessel integral functions

(1)	$H_0(at)$	$2\pi^{-1} r^{-1} \log(r/p + \alpha/p)$ $\operatorname{Re} p >  \operatorname{Im} \alpha $
(2)	$H_1(at)$	$\frac{2}{\pi p} - \frac{2p}{\pi a r} \log \frac{r+\alpha}{p}$ $\operatorname{Re} p >  \operatorname{Im} \alpha $

$$\gamma = (t^2 - b^2)^{\frac{1}{2}}, \quad v = (p^2 - i \alpha^2)^{\frac{1}{2}}, \quad V = p + v, \quad r = (p^2 + \alpha^2)^{\frac{1}{2}}$$

## Related functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(3)	$\mathbf{H}_2(at)$	$\frac{2}{\pi} \left( -\frac{2}{a} + \frac{a}{3p^2} + \frac{a^2 + 2p^2}{a^2 r} \log \frac{r+a}{p} \right)$ $\text{Re } p >  \text{Im } a $
(4)	$\mathbf{H}_3(at)$	$2\pi^{-1} p^{-1} (1/3 + 4a^{-2}p^2 + 2a^2p^{-2}/15)$ $- \frac{6a^2p + 8p^3}{\pi a^3 r} \log \frac{r+a}{p}$ $\text{Re } p >  \text{Im } a $
(5)	$\mathbf{H}_{\frac{1}{2}}(at)$ $\text{Re } a > 0$	$(1/2 a p)^{-1/2} - a^{-1/2} R^{1/2}/r$ $\text{Re } p >  \text{Im } a $
(6)	$\mathbf{H}_{-n-\frac{1}{2}}(at)$	$(-1)^n a^{n+1/2} R^{-n-1/2}/r \quad \text{Re } p >  \text{Im } a $
(7)	$t^{-1} \mathbf{H}_1(at)$	$\frac{2}{\pi} \left( -1 + \frac{r}{a} \log \frac{r+a}{p} \right) \quad \text{Re } p >  \text{Im } a $
(8)	$t^{-1} \mathbf{H}_2(at)$	$\frac{2}{\pi} \left( \frac{p}{a} + \frac{a}{3p} - \frac{r}{a} \log \frac{a+r}{p} \right)$ $\text{Re } p >  \text{Im } a $
(9)	$t^{-1} \mathbf{H}_3(at)$	$\frac{2}{\pi} \left( \frac{a^2}{15p^2} - \frac{4p^2}{3a^2} - \frac{7}{9} \right.$ $\left. + \frac{4p^2r + a^2r}{3a^3} \log \frac{r+a}{p} \right)$ $\text{Re } p >  \text{Im } a $
(10)	$t^{1/2} \mathbf{H}_{1/2}(at)$	$2^{1/2} \pi^{-1/2} a^{3/2} p^{-1} r^{-2} \quad \text{Re } p >  \text{Im } a $
(11)	$t^{1/2} \mathbf{H}_{-1/2}(at)$	$2^{1/2} \pi^{-1/2} a^{1/2} r^{-2} \quad \text{Re } p >  \text{Im } a $

$$r = (p^2 + a^2)^{1/2}, \quad R = p + r$$

## Related functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(12)	$t^{\frac{1}{2}} H_{3/2}(at)$	$2^{\frac{1}{2}} \pi^{-\frac{1}{2}} a^{\frac{1}{2}} [\frac{1}{2} p^{-2} - r^{-2} + a^{-2} \log(r/p)]$ $\text{Re } p >  \text{Im } a $
(13)	$t^{\frac{1}{2}} H_{-3/2}(at)$	$2^{1/2} \pi^{-1/2} [a^{-1/2} p r^{-2}$ $- a^{-3/2} \tan^{-1}(a/p)]$ $\text{Re } p >  \text{Im } a $
(14)	$t^{-\frac{1}{2}} H_{\frac{1}{2}}(at)$	$(\frac{1}{2} \pi a)^{-\frac{1}{2}} \log(r/p)$ $\text{Re } p >  \text{Im } a $
(15)	$t^{-\frac{1}{2}} H_{-\frac{1}{2}}(at)$	$(\frac{1}{2} \pi a)^{-\frac{1}{2}} \tan^{-1}(a/p)$ $\text{Re } p >  \text{Im } a $
(16)	$t^{-\frac{1}{2}} H_{3/2}(at)$	$(\frac{1}{2} \pi a)^{-\frac{1}{2}} [\frac{1}{2} a p^{-1} - a^{-1} p \log(r/p)]$ $\text{Re } p >  \text{Im } a $
(17)	$t^{3/2} H_{3/2}(at)$	$2^{1/2} \pi^{-1/2} a^{5/2} (3 p^2 + a^2) p^{-3} r^{-4}$ $\text{Re } p >  \text{Im } a $
(18)	$L_0(at)$	$2 \pi^{-1} s^{-1} \sin^{-1}(a/p)$ $\text{Re } p >  \text{Re } a $
(19)	$L_1(at)$	$2 \pi^{-1} p^{-1} [-1 + a^{-1} p^2 s^{-1} \sin^{-1}(a/p)]$ $\text{Re } p >  \text{Re } a $
(20)	$L_2(at)$	$\frac{2}{\alpha \pi} \left( -2 - \frac{\alpha^2}{3 p^2} + \frac{2 p^2 - \alpha^2}{\alpha s} \sin^{-1} \frac{\alpha}{p} \right)$ $\text{Re } p >  \text{Re } a $
(21)	$L_3(at)$	$\frac{2}{\pi p} \left( \frac{1}{3} - \frac{4 p^2}{\alpha^2} - \frac{2 \alpha^2}{15 p^2}$ $+ \frac{4 p^4 - 3 \alpha p^2}{\alpha^3 s} \sin^{-1} \frac{\alpha}{p} \right)$ $\text{Re } p >  \text{Re } a $

$$r = (p^2 + \alpha^2)^{\frac{1}{2}}, \quad R = p + r, \quad s = (p^2 - \alpha^2)^{\frac{1}{2}}, \quad S = p + s$$

## Related functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(22)	$t^{-1} L_1(at)$	$2\pi^{-1} [1 - \alpha^{-1} s \sin^{-1}(a/p)]$ $\text{Re } p >  \text{Re } \alpha $
(23)	$t^{-2} L_2(at)$	$2\pi^{-1} [p/a - \alpha p^{-1}/3 - \alpha^{-2} ps \sin^{-1}(a/p)]$ $\text{Re } p >  \text{Re } \alpha $
(24)	$t^{-1} L_3(at)$	$\frac{2}{\pi} \left( \frac{4p^2}{3\alpha^2} - \frac{7}{9} - \frac{\alpha^2}{15p^2} - \frac{4p^2 s - \alpha^2 s}{3\alpha^3} \sin^{-1} \frac{\alpha}{p} \right)$ $\text{Re } p >  \text{Re } \alpha $
(25)	$L_{\frac{1}{2}}(at)$	$\alpha^{-\frac{1}{2}} S^{\frac{1}{2}}/s - (\frac{1}{2}\alpha p)^{-\frac{1}{2}}$ $\text{Re } p >  \text{Re } \alpha $
(26)	$L_{-n-\frac{1}{2}}(at)$	$\alpha^{n+\frac{1}{2}} s^{-1} S^{-n-\frac{1}{2}}$ $\text{Re } p >  \text{Re } \alpha $
(27)	$t^{\frac{1}{2}} L_{\frac{1}{2}}(at)$	$2^{1/2} \pi^{-1/2} \alpha^{3/2} p^{-1} s^{-2}$ $\text{Re } p >  \text{Re } \alpha $
(28)	$t^{\frac{1}{2}} L_{-\frac{1}{2}}(at)$	$2^{\frac{1}{2}} \pi^{-\frac{1}{2}} \alpha^{\frac{1}{2}} s^{-2}$ $\text{Re } p >  \text{Re } \alpha $
(29)	$t^{\frac{1}{2}} L_{3/2}(at)$	$2^{\frac{1}{2}} \alpha^{\frac{1}{2}} \pi^{-\frac{1}{2}} [s^{-2} - \frac{1}{2} p^{-2} - \alpha^{-2} \log(s/p)]$ $\text{Re } p >  \text{Re } \alpha $
(30)	$t^{\frac{1}{2}} L_{-3/2}(at)$	$(\frac{1}{2}\pi\alpha)^{-\frac{1}{2}} [ps^{-2} - \alpha^{-1} \operatorname{ctnh}^{-1}(p/\alpha)]$ $\text{Re } p >  \text{Re } \alpha $
(31)	$t^{-\frac{1}{2}} L_{\frac{1}{2}}(at)$	$-(\frac{1}{2}\pi\alpha)^{-\frac{1}{2}} \log(s/p)$ $\text{Re } p >  \text{Re } \alpha $

$$s = (p^2 - \alpha^2)^{\frac{1}{2}}, \quad S = p + s$$

## Related functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(32)	$t^{-\frac{1}{2}} L_{-\frac{1}{2}}(at)$	$(\frac{1}{2}a\pi)^{-\frac{1}{2}} \operatorname{ctnh}^{-1}(p/a)$ $\operatorname{Re} p >  \operatorname{Re} a $
(33)	$t^{-\frac{1}{2}} L_{3/2}(at)$	$(\frac{1}{2}a\pi)^{-\frac{1}{2}} [\alpha^{-1} p \log(s/p) - \frac{1}{2}ap^{-1}]$ $\operatorname{Re} p >  \operatorname{Re} a $
(34)	$t^{3/2} L_{3/2}(at)$	$2^{1/2} \pi^{-1/2} \alpha^{5/2} (3p^2 - \alpha^2)p^{-3}s^{-4}$ $\operatorname{Re} p >  \operatorname{Re} a $
(35)	$t^\nu L_\nu(at)$ $\operatorname{Re} \nu > -\frac{1}{2}$	$\begin{aligned} & \frac{(2a)^\nu \Gamma(\nu + \frac{1}{2})}{\pi^{\frac{1}{2}} s^{2\nu+1}} \\ & - \frac{\Gamma(2\nu+1)(a/p)^\nu}{(\frac{1}{2}\pi p)^{\frac{1}{2}}(a^2 - p^2)^{-\frac{1}{2}\nu - \frac{1}{4}}} P_{-\nu - \frac{1}{2}}^{\frac{1}{2}} \left( \frac{a}{p} \right) \end{aligned}$ $\operatorname{Re} p >  \operatorname{Re} a $
(36)	$t^{\frac{1}{2}\nu} L_\nu(t^{\frac{1}{2}})$ $\operatorname{Re} \nu > -3/2$	$2^{-\nu} p^{-\nu-1} e^{\frac{1}{4}p^{-1}} \operatorname{Erf}(\frac{1}{2}p^{-\frac{1}{2}})$ $\operatorname{Re} p > 0$
(37)	$t^{\frac{1}{2}\nu} L_{-\nu}(t^{\frac{1}{2}})$	$\frac{2^{-\nu} p^{-\nu-1}}{\Gamma(\frac{1}{2}-\nu)} e^{\frac{1}{4}p^{-1}} \gamma(\frac{1}{2}-\nu, \frac{1}{4}p^{-1})$ $\operatorname{Re} p > 0$
(38)	$t^{\frac{1}{2}\mu} S_{\mu, \frac{1}{2}}(\frac{1}{2}t^2)$ $\operatorname{Re} \mu > -\frac{3}{4}$	$2^{-2\mu-1} p^{\frac{1}{2}} \Gamma(2\mu+3/2)$ $\times S_{-\mu-1, \frac{1}{2}}(\frac{1}{2}p^2)$ $\operatorname{Re} p > 0$

For further similar formulas see Meijer, C. S., 1935: *Nederl. Akad. Wetensch., Proc.* 38, 628-634.

(39)	$J i_0(t)$	$-p^{-1} \sinh^{-1} p$ $\operatorname{Re} p > 0$
(40)	$J i_\nu(t)$ $\operatorname{Re} \nu > 0$	$\nu^{-1} p^{-1} [(p^2 + 1)^{\frac{1}{2}} - p]^{\nu} - \nu^{-1} p^{-1}$ $\operatorname{Re} p > 0$
(41)	$J i_0(2t^{\frac{1}{2}})$	$\frac{1}{2}p^{-1} \operatorname{Ei}(-p^{-1})$ $\operatorname{Re} p > 0$

$$s = (p^2 - \alpha^2)^{\frac{1}{2}}, \quad S = p + s$$

## 4.20. Parabolic cylinder functions

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(1)	$t^\nu e^{\frac{1}{4}t^2} D_{-\mu}(t)$ $\operatorname{Re} \nu > -1$	$\begin{aligned} & \frac{\Gamma(\nu+1)}{\Gamma(\mu)} \int_0^\infty x^{\mu-1} (p+x)^{-\nu-1} \\ & \times e^{-\frac{1}{2}x^2} dx \\ & = p^{\mu-\nu-1} \sum_{r=0}^{\infty} (r!)^{-1} (\mu)_{2r} \\ & \times \Gamma(\nu-2r-\mu+1) (-\frac{1}{2}p^2)^r \end{aligned}$ <p style="text-align: right;"><math>\operatorname{Re} p &gt; 0</math></p>
(2)	$\exp\left(-\frac{t^2}{4a}\right) \left[ D_{-2\nu}\left(-\frac{t}{a}\right) - D_{-2\nu}\left(\frac{t}{a}\right) \right]$	$\begin{aligned} & 2^{\frac{\nu}{2}} \pi^{\frac{\nu}{2}} a^{1-2\nu} p^{-2\nu} e^{\frac{1}{2}a^2 p^2} \\ & \times \frac{\Gamma(\nu, \frac{1}{2}a^2 p^2)}{\Gamma(\nu)} \end{aligned}$ <p style="text-align: right;"><math>\operatorname{Re} p &gt; 0</math></p>
(3)	$D_{2n+1}(2^{\frac{\nu}{2}} t^{\frac{\nu}{2}})$	$\begin{aligned} & (-2)^n \Gamma(n+3/2) (p-1/2)^n \\ & \times (p+1/2)^{-n-3/2} \end{aligned}$ <p style="text-align: right;"><math>\operatorname{Re} p &gt; -\frac{1}{2}</math></p>
(4)	$D_{2\nu}(-2a^{\frac{\nu}{2}} t^{\frac{\nu}{2}}) - D_{2\nu}(2a^{\frac{\nu}{2}} t^{\frac{\nu}{2}})$	$\frac{2^{\nu+3/2} \pi a^{1/2} (p-a)^{\nu-1/2}}{\Gamma(-\nu) (p+a)^{\nu+1}}$ <p style="text-align: right;"><math>\operatorname{Re} p &gt;  \operatorname{Re} a </math></p>
(5)	$t^{-\frac{\nu}{2}} D_{2n}(2^{\frac{\nu}{2}} t^{\frac{\nu}{2}})$	$(-2)^n \Gamma(n+\frac{1}{2})(p-\frac{1}{2})^n (p+\frac{1}{2})^{-n-\frac{1}{2}}$ <p style="text-align: right;"><math>\operatorname{Re} p &gt; -\frac{1}{2}</math></p>
(6)	$t^{-\frac{\nu}{2}} [D_{2\nu}(2a^{\frac{\nu}{2}} t^{\frac{\nu}{2}}) + D_{2\nu}(-2a^{\frac{\nu}{2}} t^{\frac{\nu}{2}})]$	$2^{\nu+1} \pi (p-a)^\nu (p+a)^{-\nu-\frac{1}{2}} / \Gamma(\frac{1}{2}-\nu)$ <p style="text-align: right;"><math>\operatorname{Re} p &gt;  \operatorname{Re} a </math></p>
(7)	$t^{-\frac{\nu}{2}\nu-\frac{\nu}{2}} e^{\frac{1}{4}t^2} D_\nu(t^{\frac{\nu}{2}})$ $\operatorname{Re} \nu < 1$	$\pi^{\frac{\nu}{2}} p^{-\frac{\nu}{2}} (1+2^{\frac{\nu}{2}} p^{\frac{\nu}{2}})^\nu$ $\operatorname{Re} p > 0$
(8)	$t^{-\nu/2-3/2} e^{t/4} D_\nu(t^{1/2})$ $\operatorname{Re} \nu < -1$	$-2^{\frac{\nu}{2}} \pi^{\frac{\nu}{2}} (\nu+1)^{-1} (1+2^{\frac{\nu}{2}} p^{\frac{\nu}{2}})^{\nu+1}$ $\operatorname{Re} p > 0$

## Parabolic cylinder functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(9)	$t^{\nu-1} e^{\frac{1}{4}t} D_{2\nu+2n-1}(t^{\frac{1}{2}})$ $\text{Re } \nu > 0$	$\frac{\pi^{\frac{1}{2}} \Gamma(2n+2\nu) (1-2p)^n}{2^{2n-\frac{1}{2}+\nu} n! p^{n+\nu}} \\ \times {}_2F_1(n+\nu, \frac{1}{2}-\nu; n+1; 1-\frac{1}{2}p^{-1})$ $\text{Re } p > 0$
(10)	$t^{\nu-1} e^{\frac{1}{4}t} D_{2\mu-1}(t^{\frac{1}{2}})$ $\text{Re } \nu > 0, \quad \text{Re } (\nu - \mu) > -1$	$2^{\frac{1}{2}} \pi^{\frac{1}{2}} \Gamma(2\nu) (2p)^{-\frac{1}{2}\mu-\frac{1}{2}\nu} \\ \times (2p-1)^{\frac{1}{2}\mu-\frac{1}{2}\nu} P_{\mu+\nu-1}^{\mu-\nu}(2^{-\frac{1}{2}}p^{-\frac{1}{2}})$ $\text{Re } p > 0$
(11)	$[D_{-n-1}(-i 2^{\frac{1}{2}} t^{\frac{1}{2}})]^2$ $- [D_{-n-1}(i 2^{\frac{1}{2}} t^{\frac{1}{2}})]^2$	$\frac{2\pi i}{n! p^{\frac{1}{2}}} \frac{(p-1)^n}{(p+1)^{n+1}}$ $\text{Re } p > 0$
(12)	$t^{-\nu} e^{-\alpha/(8t)} \\ \times D_{2\nu-1}(2^{-\frac{1}{2}} \alpha^{\frac{1}{2}} t^{-\frac{1}{2}})$ $\text{Re } \alpha > 0$	$2^{\nu-\frac{1}{2}} \pi^{\frac{1}{2}} p^{\nu-1} e^{-\alpha^{\frac{1}{2}} p^{\frac{1}{2}}}$ $\text{Re } p > 0$
(13)	$\frac{e^{\frac{1}{4}t}}{(e^t-1)^{\mu+\frac{1}{2}}} \exp\left(-\frac{\alpha}{1-e^{-t}}\right)$ $\times D_{2\mu} \left[ \frac{2\alpha^{\frac{1}{2}}}{(1-e^{-t})^{\frac{1}{2}}} \right]$ $\text{Re } \alpha > 0$	$e^{-\alpha} 2^{p+\mu} \Gamma(p+\mu) D_{-2p}(2\alpha^{\frac{1}{2}})$ $\text{Re } p > -\text{Re } \mu$

## 4.21. Gauss' hypergeometric function

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(1)	$t^{\alpha-1} F(\frac{1}{2}+\nu, \frac{1}{2}-\nu; \alpha; -\frac{1}{2}t)$ $\operatorname{Re} \alpha > 0$	$\pi^{-\frac{1}{2}} \Gamma(\alpha) (2p)^{\frac{1}{2}-\alpha} K_\nu(p)$ $\operatorname{Re} p > 0$
(2)	$t^{\gamma-1} F(\alpha, \beta; \delta; -t)$ $\operatorname{Re} \gamma > 0$	$\frac{\Gamma(\delta) p^{-\gamma}}{\Gamma(\alpha) \Gamma(\beta)} E(\alpha, \beta, \gamma; \delta; p)$ $\operatorname{Re} p > 0$
(3)	$t^{\gamma-1} (1+t)^{\alpha+\beta-\delta} F(\alpha, \beta; \delta; -t)$ $\operatorname{Re} \gamma > 0$	$\frac{\Gamma(\delta) p^{-\gamma}}{\Gamma(\delta-\alpha) \Gamma(\delta-\beta)}$ $\times E(\delta-\alpha, \delta-\beta, \gamma; \delta; p)$ $\operatorname{Re} p > 0$
(4)	$t^{\gamma-1} F(2\alpha, 2\beta; \gamma; -\lambda t)$ $\operatorname{Re} \gamma > 0, \quad  \arg \lambda  < \pi$	$\Gamma(\gamma) p^{-\gamma} (p/\lambda)^{\alpha+\beta-\frac{1}{2}} e^{\frac{1}{2}p/\lambda}$ $\times W_{\frac{1}{2}-\alpha-\beta, \alpha-\beta}(\frac{1}{2}p/\lambda)$ $\operatorname{Re} p > 0$
(5)	$0 \quad 0 < t < 1$ $(t^2-1)^{2\alpha-\frac{1}{2}}$ $\times F(\alpha-\frac{1}{2}\nu, \alpha+\frac{1}{2}\nu; 2\alpha+\frac{1}{2}; 1-t^2)$ $t > 1$ $\operatorname{Re} \alpha > -\frac{1}{4}$	$2^{2\alpha} \pi^{-\frac{1}{2}} p^{-2\alpha} \Gamma(2\alpha+\frac{1}{2}) K_\nu(p)$ $\operatorname{Re} p > 0$
(6)	$[(\alpha+t)(\beta+t)]^{-\frac{1}{2}-\nu}$ $\times F\left[\frac{1}{2}+\nu, \frac{1}{2}+\nu; 1; \frac{t(\alpha+\beta+t)}{(\alpha+t)(\beta+t)}\right]$ $ \arg \alpha  < \pi, \quad  \arg \beta  < \pi$	$\pi^{-1} (\alpha\beta)^{-\nu} e^{\frac{1}{2}(\alpha+\beta)p} K_\nu(\frac{1}{2}\alpha p) K_\nu(\frac{1}{2}\beta p)$ $ \arg \alpha p  < \pi, \quad  \arg \beta p  < \pi$ $\operatorname{Re} p > 0$
(7)	$t^{-\frac{1}{2}} (1+\alpha/t)^\mu (1+\beta/t)^\nu$ $\times F\left[-\mu, -\nu; \frac{1}{2}-\mu-\nu; \frac{t(\alpha+\beta+t)}{(\alpha+t)(\beta+t)}\right]$ $ \arg \alpha  < \pi, \quad  \arg \beta  < \pi$ $\operatorname{Re}(\mu + \nu) < 1$	$2^{-\mu-\nu} \Gamma(\frac{1}{2}-\mu-\nu) p^{-\frac{1}{2}} e^{\frac{1}{2}(a+b)p}$ $\times D_{2\mu}(2^{\frac{1}{2}} a^{\frac{1}{2}} p^{\frac{1}{2}}) D_{2\nu}(2^{\frac{1}{2}} b^{\frac{1}{2}} p^{\frac{1}{2}})$ $ \arg \alpha p  < \pi, \quad  \arg \beta p  < \pi$ $\operatorname{Re} p > 0$

## Gauss' hypergeometric functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(8)	$t^{-\kappa-\lambda} (\alpha+t)^{\kappa-\mu-\frac{1}{2}} (\beta+t)^{\lambda-\mu-\frac{1}{2}}$ $\times {}_x F \left[ \begin{matrix} \frac{1}{2}-\kappa+\mu, \frac{1}{2}-\lambda+\mu; 1-\kappa-\lambda; \\ \end{matrix} \frac{t(\alpha+\beta+t)}{(\alpha+t)(\beta+t)} \right]$ $ \arg \alpha  < \pi, \quad  \arg \beta  < \pi$ $\operatorname{Re}(\kappa + \lambda) < 1$	$\Gamma(1-\kappa-\lambda) (\alpha\beta)^{-\mu-\frac{1}{2}} p^{-1} e^{\frac{1}{2}(\alpha+\beta)p}$ $\times {}_{\kappa, \mu} W_{\kappa, \mu}(\alpha p) {}_{\lambda, \mu} W_{\lambda, \mu}(\beta p) \quad \operatorname{Re} p > 0$ $ \arg \alpha p  < \pi, \quad  \arg \beta p  < \pi$
(9)	$(1-e^{-t})^{\lambda-1} F(\alpha, \beta; \gamma; \delta e^{-t})$ $\operatorname{Re} \lambda > 0, \quad  \arg(1-\delta)  < \pi$	$B(p, \lambda) {}_3 F_2(\alpha, \beta, p; \gamma, p+\lambda; \delta)$ $\operatorname{Re} p > 0$
(10)	$(1-e^{-t})^\mu$ $\times {}_x F(-n, \mu+\beta+n; \beta; e^{-t})$ $\operatorname{Re} \mu > -1$	$B(p, \mu+n+1) B(p, \beta+n-p) / B(p, \beta-p)$ $\operatorname{Re} p > 0$
(11)	$(1-e^{-t})^{\gamma-1} F(\alpha, \beta; \gamma; 1-e^{-t})$ $\operatorname{Re} \gamma > 0$	$\frac{\Gamma(p) \Gamma(\gamma-\alpha-\beta+p) \Gamma(\gamma)}{\Gamma(\gamma-\alpha+p) \Gamma(\gamma-\beta+p)}$ $\operatorname{Re} p > 0, \quad \operatorname{Re} p > \operatorname{Re}(\alpha+\beta-\gamma)$
(12)	$(1-e^{-t})^{\gamma-1}$ $\times {}_x F[\alpha, \beta; \gamma; \delta(1-e^{-t})]$ $\operatorname{Re} \gamma > 0, \quad  \arg(1-\delta)  < \pi$	$B(p, \gamma) F(\alpha, \beta; p+\gamma; \delta) \quad \operatorname{Re} p > 0$
(13)	$(1-e^{-t})^{\lambda-1}$ $\times {}_x F[\alpha, \beta; \gamma; \delta(1-e^{-t})]$ $\operatorname{Re} \lambda > 0, \quad  \arg(1-\delta)  < \pi$	$B(p, \lambda) {}_3 F_2(\alpha, \beta, \lambda; \gamma, p+\lambda; \delta)$ $\operatorname{Re} p > 0$

### 4.22. Confluent hypergeometric functions

**Particular confluent hypergeometric functions occur in sections  
4.11, 4.12, 4.14 - 4.18, 4.20**

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(1)	$k_0(t)$	$(p+1)^{-1}$ $\operatorname{Re} p > -1$
(2)	$k_{2n+2}(t)$	$2(1-p)^n(1+p)^{-n-2}$ $\operatorname{Re} p > -1$
(3)	$k_{2\nu}(t)$	$[2\pi\nu(1-\nu)]^{-1} \sin(\nu\pi)$ $\times {}_2F_1(1, 2; 2-\nu; \frac{1}{2}-\frac{1}{2}p)$ $\operatorname{Re} p > 0$
(4)	$t^{n-\frac{1}{2}} k_{2n+2}(t)$	$(-1)^{n-1} \frac{(2n)! \pi^{\frac{1}{2}}}{(n+1)! 2^{2n+\frac{1}{2}}} (p+1)^{-n-1}$ $\times P_{2n+1}^{\frac{1}{2}} (p-1)^{\frac{1}{2}} (p+1)^{-\frac{1}{2}}$ $\operatorname{Re} p > 0$
(5)	$e^{-t^2} k_{2n}(t^2)$	$(-1)^{n-1} 2^{-1/4-3n/2} p^{n-3/2} e^{p^2/16}$ $\times W_{-\frac{1}{4}-\frac{1}{2}n, \frac{1}{4}-\frac{1}{2}n}(p^2/8)$
(6)	$t^{-\frac{1}{2}} e^{-t^2} k_{2n}(t^{\frac{1}{2}})$	$\sum_{r=0}^{n-1} (-1)^r \binom{n-1}{r} \left(\frac{2}{p}\right)^{\frac{1}{2}(n+1-r)}$ $\times e^{1/(2p)} D_{-n+r-1}(2^{\frac{1}{2}} p^{-\frac{1}{2}})$ $\operatorname{Re} p > 0$
(7)	$t^{-1} k_{2n+2}(\frac{1}{2}t) k_{2n+2}(\frac{1}{2}t)$	$(-p)^{n+n} (p+1)^{-n-n-2}$ $\times {}_2F_1(-m, -n; 2; p^{-2})$ $\operatorname{Re} p > -1$

## Confluent hypergeometric functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(8)	$\frac{e^{\frac{\alpha+\beta}{2}t}}{a\beta t} k_{2m+2}(\frac{1}{2}\alpha t) \times k_{2n+2}(\frac{1}{2}\beta t)$	$\frac{(-1)^{m+n}(m+n+1)!}{(m+1)!(n+1)!} \frac{(p-a)^m(p-\beta)^n}{p^{m+n+2}} {}_2F_1\left[-m, -n; -m-n-1; \frac{p(p-\alpha-\beta)}{(p-a)(p-\beta)}\right]$ $\text{Re } p > 0$
(9)	$t^{\lambda-1} k_{2m_1+2}(\alpha_1 t) \cdots k_{2m_n+2}(\alpha_n t)$ $\text{Re } \lambda + n > 0$	$(-1)^n 2^n \alpha_1 \cdots \alpha_n (p+A)^{-\lambda-n} \Gamma(\lambda+n) {}_xF_A\left(\lambda+n; -m_1, \dots, -m_n; 2, \dots, 2; \frac{2\alpha_1}{p+A}, \dots, \frac{2\alpha_n}{p+A}\right)$ $\text{Re } p > 0$ $M = m_1 + \cdots + m_n$ $A = \alpha_1 + \cdots + \alpha_n$
(10)	$t^{\mu-\frac{1}{2}} M_{\kappa, \mu}(\alpha t)$ $\text{Re } \mu > -\frac{1}{2}$	$\alpha^{\mu+\frac{1}{2}} \Gamma(2\mu+1) \frac{(p-\frac{1}{2}\alpha)^{\kappa-\mu-\frac{1}{2}}}{(p+\frac{1}{2}\alpha)^{\kappa+\mu+\frac{1}{2}}}$ $\text{Re } p > \frac{1}{2}  \text{Re } \alpha $
(11)	$t^{\nu-1} M_{\kappa, \mu}(\alpha t)$ $\text{Re } (\mu + \nu) > -\frac{1}{2}$	$\alpha^{\mu+\frac{1}{2}} \Gamma(\mu+\nu+\frac{1}{2})(p+\frac{1}{2}\alpha)^{-\mu-\nu-\frac{1}{2}} {}_xF_1\left[\mu+\nu+\frac{1}{2}, \mu-\kappa+\frac{1}{2}; 2\mu+1; \alpha/(p+\frac{1}{2}\alpha)\right]$ $\text{Re } p > \frac{1}{2}  \text{Re } \alpha $
(12)	$t^{2\nu-1} e^{-\frac{1}{2}t^2/\alpha} M_{-3\nu, \nu}(t^2/\alpha)$ $\text{Re } \alpha > 0, \quad \text{Re } \nu > -\frac{1}{4}$	$\frac{1}{2} \pi^{-\frac{1}{2}} \Gamma(4\nu+1) \alpha^{-\nu} p^{-4\nu} e^{\alpha p^2/8} K_{2\nu}(\alpha p^2/8)$ $\text{Re } p > 0$
(13)	$t^{2\mu-1} e^{-\frac{1}{2}t^2/\alpha} M_{\kappa, \mu}(t^2/\alpha)$ $\text{Re } \alpha > 0, \quad \text{Re } \mu > -\frac{1}{4}$	$2^{-3\mu-\kappa} \Gamma(4\mu+1) \alpha^{\frac{1}{2}(\kappa+\mu-1)} p^{\kappa-\mu-1} e^{\alpha p^2/8} W_{-\frac{1}{2}(\kappa+3\mu), \frac{1}{2}(\kappa-\mu)}(\frac{1}{4}\alpha p^2)$ $\text{Re } p > 0$

## Confluent hypergeometric functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(14)	$t^{\nu-1} M_{\kappa_1, \mu_1 - \frac{1}{2}}(\alpha_1 t)$ $\cdots M_{\kappa_n, \mu_n - \frac{1}{2}}(\alpha_n t)$ $M = \mu_1 + \cdots + \mu_n$ $\operatorname{Re}(\nu + M) > 0$	$a_1^{\mu_1} \cdots a_n^{\mu_n} (p+A)^{-\nu-M} \Gamma(\nu+M)$ $\times {}_2F_A \left( \begin{matrix} \nu+M; \mu_1 - \kappa_1, \dots, \mu_n - \kappa_n; \\ 2\mu_1, \dots, 2\mu_n; \end{matrix} \frac{\alpha_1}{p+A}, \dots, \frac{\alpha_n}{p+A} \right)$ $A = \frac{1}{2}(\alpha_1 + \cdots + \alpha_n)$ $\operatorname{Re}(p \pm \frac{1}{2}\alpha_1 \pm \cdots \pm \frac{1}{2}\alpha_n) > 0$
(15)	$(e^{-t}-1)^{\mu-\frac{1}{2}} \exp(-\frac{1}{2}\lambda e^t)$ $\times M_{\kappa, \mu}(\lambda e^t - \lambda) \quad \operatorname{Re} \mu > -\frac{1}{2}$	$\frac{\Gamma(2\mu+1)\Gamma(\frac{1}{2}+\kappa-\mu+p)}{\Gamma(p+1)}$ $\times W_{-\kappa-\frac{1}{2}, p, \mu-\frac{1}{2}}(\lambda)$ $\operatorname{Re} p > \operatorname{Re}(\mu - \kappa) - \frac{1}{2}$
(16)	$t^{\nu-1} W_{\kappa, \mu}(at)$ $\operatorname{Re}(\nu \pm \mu) > -\frac{1}{2}$	$\frac{\Gamma(\mu+\nu+\frac{1}{2})\Gamma(\nu-\mu+\frac{1}{2})a^{\mu+\frac{1}{2}}}{\Gamma(\nu-\kappa+1)(p+\frac{1}{2}a)^{\mu+\nu+\frac{1}{2}}}$ $\times {}_2F_1 \left( \begin{matrix} \mu+\nu+\frac{1}{2}, \mu-\kappa+\frac{1}{2}; \\ p-\frac{1}{2}a \end{matrix} \frac{p-\frac{1}{2}a}{p+\frac{1}{2}a} \right)$ $\operatorname{Re}(p+\frac{1}{2}a) > 0$
(17)	$t^{-1} \exp(-\frac{1}{2}a/t) W_{\frac{1}{2}, \mu}(a/t)$ $\operatorname{Re} a > 0$	$2\pi^{-\frac{1}{2}} (2ap)^{\frac{1}{2}} K_{\mu+\frac{1}{2}}(a^{\frac{1}{2}}p^{\frac{1}{2}})$ $\times K_{\mu-\frac{1}{2}}(a^{\frac{1}{2}}p^{\frac{1}{2}}) \quad \operatorname{Re} p > 0$
(18)	$t^{-1} \exp(\frac{1}{2}a/t) W_{-\frac{1}{2}, \mu}(a/t)$ $ \arg a  < \pi$	$\frac{1}{4} \mu^{-1} (a\pi^3 p)^{\frac{1}{2}} [H_{\mu+\frac{1}{2}}^{(1)}(a^{\frac{1}{2}}p^{\frac{1}{2}})$ $\times H_{\mu-\frac{1}{2}}^{(2)}(a^{\frac{1}{2}}p^{\frac{1}{2}})$ $+ H_{\mu-\frac{1}{2}}^{(1)}(a^{\frac{1}{2}}p^{\frac{1}{2}}) H_{\mu+\frac{1}{2}}^{(2)}(a^{\frac{1}{2}}p^{\frac{1}{2}})]$ $\operatorname{Re} p > 0$
(19)	$t^{3\nu-\frac{1}{2}} \exp(\frac{1}{2}a/t) W_{\nu, \nu}(a/t)$ $ \arg a  < \pi, \quad \operatorname{Re} \nu > -\frac{1}{4}$	$\frac{1}{2} \Gamma(2\nu+\frac{1}{2}) a^{\nu+\frac{1}{2}} p^{-2\nu} H_{2\nu}^{(1)}(a^{\frac{1}{2}}p^{\frac{1}{2}})$ $\times H_{2\nu}^{(2)}(a^{\frac{1}{2}}p^{\frac{1}{2}}) \quad \operatorname{Re} p > 0$

## Confluent hypergeometric functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(20)	$t^{-3\nu-\frac{1}{2}} \exp(-\frac{1}{2}\alpha/t) W_{\nu,\nu}(\alpha/t)$ $\text{Re } \alpha > 0$	$2\pi^{-\frac{1}{2}} \alpha^{\frac{1}{2}-\nu} p^{2\nu} [K_{2\nu}(\alpha^{\frac{1}{2}} p^{\frac{1}{2}})]^2$ $\text{Re } p > 0$
(21)	$t^\kappa \exp(\frac{1}{2}\alpha/t) W_{\kappa,\mu}(\alpha/t)$ $ \arg \alpha  < \pi, \quad \text{Re } (\kappa \pm \mu) > -\frac{1}{2}$	$2^{1-2\kappa} \alpha^{\frac{1}{2}} p^{-\kappa-\frac{1}{2}} S_{2\kappa,2\mu}(2\alpha^{\frac{1}{2}} p^{\frac{1}{2}})$ $\text{Re } p > 0$
(22)	$t^{-\kappa} \exp(-\frac{1}{2}\alpha/t) W_{\kappa,\mu}(\alpha/t)$ $\text{Re } \alpha > 0$	$2\alpha^{\frac{1}{2}} p^{\kappa-\frac{1}{2}} K_{2\mu}(2\alpha^{\frac{1}{2}} p^{\frac{1}{2}}) \quad \text{Re } p > 0$
(23)	$(1-e^{-t})^{-\kappa} \exp\left[-\frac{\lambda}{2(e^{-t}-1)}\right] \times W_{\kappa,\mu}\left(\frac{\lambda}{(e^{-t}-1)}\right) \quad \text{Re } \lambda > 0$	$\frac{\Gamma(\frac{1}{2}+\mu+p)\Gamma(\frac{1}{2}-\mu+p)}{\Gamma(1-\kappa+p)} e^{\frac{1}{2}\lambda} W_{-p,\mu}(\lambda) \quad \text{Re } (\frac{1}{2} \pm \mu + p) > 0$
(24)	$\lambda e^{-t} (e^{-t}-1)^{-\kappa-1} \exp\left[-\frac{\lambda}{2(e^{-t}-1)}\right] \times W_{\kappa,\mu}\left(\frac{\lambda}{1-e^{-t}}\right) \quad \text{Re } \lambda > 0$	$\Gamma(\kappa+p) W_{-p,\mu}(\lambda) \quad \text{Re } p > -\text{Re } \kappa$

## 4.23. Generalized hypergeometric series

(1)	$t^{\gamma-1} {}_1F_1(\alpha; \gamma; \lambda t) \quad \text{Re } \gamma > 0$	$\Gamma(\gamma) p^{\alpha-\gamma} (p-\lambda)^{-\alpha} \quad \text{Re } p > 0, \quad \text{Re } \lambda > 0$
(2)	$\frac{t^{\gamma-1} e^{-t}}{(1-\lambda)^2} {}_1F_1\left[\alpha; \gamma; \frac{-4\lambda t}{(1-\lambda)^2}\right] \quad \text{Re } \gamma > 0$	$\frac{\Gamma(\gamma)}{(p+1)^\gamma} \left(1-2\frac{p-1}{p+1}\lambda+\lambda^2\right)^{-\alpha} \quad \text{Re } p > -1$ $\text{Re } p > -\text{Re}[(1+\lambda)^2(1-\lambda)^{-2}] > 0$
(3)	$t^{\alpha+\nu-\frac{1}{2}} \times {}_1F_2(\frac{1}{2}+\nu; 1+2\nu, \frac{1}{2}+\nu+\alpha; -2t) \quad \text{Re } (\alpha+\nu+\frac{1}{2}) > 0$	$2^\nu \Gamma(\nu+1) \Gamma(\alpha+\nu+\frac{1}{2}) p^{-\alpha-\frac{1}{2}} \times e^{-1/p} I_\nu(p^{-1}) \quad \text{Re } p > 0$

## Generalized hypergeometric series (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(4)	$t^{\beta-1} {}_1F_2(-n; \alpha+1, \beta; \lambda t)$ $\text{Re } \beta > 0$	$n! [\Gamma(\beta)/(\alpha+1)_n] p^{-\beta} L_n^\alpha(\lambda/p)$ $\text{Re } p > 0$
(5)	${}_2F_2(-n, n+1; 1, 1; t)$	$p^{-1} P_n(1-2/p)$ $\text{Re } p > 0$
(6)	$t^{\gamma-1} {}_2F_2(-n, n+1; 1, \gamma; t)$ $\text{Re } \gamma > 0$	$\Gamma(\gamma) p^{-\gamma} P_n(1-2/p)$ $\text{Re } p > 0$
(7)	$t^{\gamma-1} {}_2F_2(-n, n; \gamma, \frac{1}{2}; t)$ $\text{Re } \gamma > 0$	$\Gamma(\gamma) p^{-\gamma} \cos[2n \sin^{-1}(p^{-\frac{1}{2}})]$ $\text{Re } p > 0$
(8)	$t^{\gamma-1} {}_2F_2(-n, n+1; \gamma, 3/2; t)$ $\text{Re } \gamma > 0$	$\frac{\Gamma(\gamma)}{(2n+1)p^\gamma} \sin[(2n+1)\sin^{-1}(p^{-\frac{1}{2}})]$ $\text{Re } p > 0$
(9)	$t^{\gamma-1} {}_2F_2(-n, n+2\nu; \nu+\frac{1}{2}, \gamma; t)$ $\text{Re } \gamma > 0$	$n B(n, 2\nu) \Gamma(\gamma) p^{-\gamma} C_n^\nu(1-2/p)$ $\text{Re } p > 0$
(10)	$t^{\gamma-1} {}_2F_2(-n, \alpha+n; \beta, \gamma; t)$ $\text{Re } \gamma > 0$	$\Gamma(\gamma) p^{-\gamma} F(-n, \alpha+n; \beta; p^{-1})$ $\text{Re } p > 0$
(11)	$t^{\mu+\nu-1} e^{-\frac{1}{2}t^2}$ $\times {}_2F_2\left(\mu, \nu; \frac{\mu+\nu}{2}, \frac{1+\mu+\nu}{2}; -\frac{t^2}{4}\right)$ $\text{Re } (\mu + \nu) > 0$	$\Gamma(\mu+\nu) e^{\frac{1}{4}p^2} D_{-\mu}(p) D_{-\nu}(p)$
(12)	$t^{2\alpha-1}$ $\times {}_3F_2(1, \frac{1}{2}-\mu+\nu, \frac{1}{2}-\mu-\nu;$ $\alpha, \alpha+\frac{1}{2}; -\lambda^2 t^2)$ $\text{Re } \lambda > 0, \text{ Re } \alpha > 0$	$\Gamma(2\alpha) \lambda^{2\mu-1} p^{1-2\alpha-2\mu} S_{2\mu, 2\nu}(p/\lambda)$ $\text{Re } p > 0$

## Generalized hypergeometric series (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(13)	$t^{2\alpha-1}$ $\times {}_4F_3(\frac{1}{2}+\mu+\nu, \frac{1}{2}-\mu+\nu, \frac{1}{2}+\mu-\nu, \frac{1}{2}-\mu-\nu; \frac{1}{2}, \alpha, \alpha+\frac{1}{2}; -\lambda^2 t^2/4)$ $\text{Re } \alpha > 0, \quad \text{Re } \lambda > 0$	$\frac{\pi \Gamma(2\alpha)}{4\lambda p^{2\alpha-1}} [e^{(\mu-\nu)\pi i} H_{2\mu}^{(1)}(p/\lambda) H_{2\nu}^{(2)}(p/\lambda) + e^{(\nu-\mu)\pi i} H_{2\mu}^{(2)}(p/\lambda) H_{2\nu}^{(1)}(p/\lambda)]$ $ \arg p  < \frac{1}{2}\pi$
(14)	$t^{2\alpha-1}$ $\times {}_4F_3(1+\mu+\nu, 1-\mu+\nu, 1+\mu-\nu, 1-\mu-\nu; \frac{3}{2}, \alpha, \alpha+\frac{1}{2}; -\lambda^2 t^2/4)$ $\text{Re } \lambda > 0, \quad \text{Re } \alpha > 0$	$\frac{\pi \Gamma(2\alpha) p^{2-2\alpha}}{8i\lambda^2(\mu^2-\nu^2)} [e^{(\mu-\nu)\pi i} H_{2\mu}^{(1)}(p/\lambda) H_{2\nu}^{(2)}(p/\lambda) - e^{(\nu-\mu)\pi i} H_{2\mu}^{(2)}(p/\lambda) H_{2\nu}^{(1)}(p/\lambda)]$ $\text{Re } p > 0$
(15)	$t^{2\alpha-1}$ $\times {}_4F_3(\frac{1}{2}+\mu-\kappa, \frac{1}{2}-\mu-\kappa, \frac{1}{2}-\kappa, 1-\kappa; 1-2\kappa, \alpha, \alpha+\frac{1}{2}; -\lambda^2 t^2)$ $\text{Re } \lambda > 0, \quad \text{Re } \alpha > 0$	$\Gamma(2\alpha) \lambda^{2\kappa} p^{-2\alpha-2\kappa} W_{\kappa, \mu}(ip/\lambda) \\ \times W_{\kappa, \mu}(-ip/\lambda) \quad \text{Re } p > 0$
(16)	$t^{\rho_n-1}$ $\times {}_mF_n(\alpha_1, \dots, \alpha_m; \rho_1, \dots, \rho_n; \lambda t)$ $m \leq n, \quad \text{Re } \rho_n > 0$	$\Gamma(\rho_n) p^{-\rho_n} \\ \times {}_mF_{n-1}(\alpha_1, \dots, \alpha_m; \rho_1, \dots, \rho_{n-1}; \lambda/p)$ $\text{Re } p > 0 \text{ if } m < n$ $\text{Re } p > \text{Re } \lambda \text{ if } m = n$
(17)	$t^{\sigma-1}$ $\times {}_mF_n(\alpha_1, \dots, \alpha_m; \rho_1, \dots, \rho_n; \lambda t)$ $m \leq n, \quad \text{Re } \sigma > 0$	$\Gamma(\sigma) p^{-\sigma} \\ \times {}_{m+1}F_n(\alpha_1, \dots, \alpha_m, \sigma; \rho_1, \dots, \rho_n; \lambda/p)$ $\text{Re } p > 0 \text{ if } m < n$ $\text{Re } p > \text{Re } \lambda \text{ if } m = n$
(18)	$t^{2\sigma-1}$ $\times {}_mF_n(\alpha_1, \dots, \alpha_m; \rho_1, \dots, \rho_n; \lambda^2 t^2)$ $m < n, \quad \text{Re } \sigma > 0$	$\Gamma(2\sigma) p^{-2\sigma} \\ \times {}_{m+2}F_n(\alpha_1, \dots, \alpha_m, \frac{1}{2}\sigma, \frac{1}{2}\sigma + \frac{1}{2}; \rho_1, \dots, \rho_n; 4\lambda^2 p^{-2})$ $\text{Re } p > 0 \text{ if } m < n - 1$ $\text{Re } p >  \text{Re } \lambda  \text{ if } m = n - 1$

## Generalized hypergeometric series (cont'd)

	$f(t)$	$\int_0^\infty e^{-pt} f(t) dt$
(19)	$t^{\sigma-1}$ $\times {}_nF_n[\alpha_1, \dots, \alpha_n; \rho_1, \dots, \rho_n; (\lambda t)^k]$ $m + k \leq n + 1, \quad \operatorname{Re} \sigma > 0$	$\Gamma(\sigma) p^{-\sigma}$ $\times {}_{n+k}F_n \left[ \alpha_1, \dots, \alpha_n, \frac{\sigma}{k}, \frac{\sigma+1}{k}, \dots, \frac{\sigma+k-1}{k}; \right.$ $\left. \rho_1, \dots, \rho_n; \left( \frac{k\lambda}{p} \right)^k \right]$ $\operatorname{Re} p > 0 \text{ if } m + k \leq n$ $\operatorname{Re}(p + k\lambda e^{2\pi i r/k}) > 0 \text{ for } r = 0, 1, \dots, k-1 \text{ if } m + k = n + 1$
(20)	$t^{-\frac{n}{2}}$ $\times {}_{2n}F_{2n} \left( \begin{matrix} \alpha_1, \frac{\alpha_1+1}{2}, \dots, \frac{\alpha_n}{2}, \frac{\alpha_n+1}{2}; \\ \frac{\rho_1}{2}, \frac{\rho_1+1}{2}, \dots, \frac{\rho_n}{2}, \frac{\rho_n+1}{2}; \end{matrix} -2^{n-n-2} \frac{k^2}{t} \right)$ $k > 0, \quad m \leq n$	$\pi^{\frac{n}{2}} p^{-\frac{n}{2}} {}_nF_n(\alpha_1, \dots, \alpha_n; \rho_1, \dots, \rho_n; -kp^{\frac{n}{2}})$ $\operatorname{Re} p > 0$
(21)	$(1-e^{-t})^{\lambda-1}$ $\times {}_nF_n(\alpha_1, \dots, \alpha_n; \rho_1, \dots, \rho_n; \gamma e^{-t})$ $\operatorname{Re} \lambda > 0, \quad m \leq n$ Valid for $m = n + 1$ if $ \gamma  < 1$	$B(\lambda, p)$ $\times {}_{n+1}F_{n+1}(\alpha_1, \dots, \alpha_n, p; \rho_1, \dots, \rho_n, p+\lambda; \gamma)$ $\operatorname{Re} p > 0$
(22)	$(1-e^{-t})^{\lambda-1}$ $\times {}_nF_n[\alpha_1, \dots, \alpha_n; \rho_1, \dots, \rho_n; \gamma(1-e^{-t})]$ $\operatorname{Re} \lambda > 0, \quad m \leq n$ Valid for $m = n + 1$ if $ \gamma  < 1$	$B(\lambda, p)$ $\times {}_{n+1}F_{n+1}(\alpha_1, \dots, \alpha_n, \lambda; \rho_1, \dots, \rho_n, p+\lambda; \gamma)$ $\operatorname{Re} p > 0$
(23)	$t^{\alpha_{n+1}-1} E(m; \alpha_r : n; \beta_s : t^{-1})$ $\operatorname{Re} \alpha_{n+1} > 0$	$p^{-\alpha_{n+1}} E(m+1; \alpha_r : n; \beta_s : p) \quad \operatorname{Re} p > 0$

## Generalized hypergeometric series (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(24)	$(e^t - 1)^{\alpha_m + 1}$ $\times {}_2E_1 \left( m; \alpha_r : n; \beta_s : \frac{\lambda}{1-e^{-t}} \right)$ $\operatorname{Re} \alpha_{m+1} > -1$	$\Gamma(p - \alpha_{m+1}) E(m+1; \alpha_r : n; \beta_s, p : \lambda)$ $\operatorname{Re} p > \operatorname{Re} \alpha_{m+1}$
(25)	$t^{-2\nu} S_1(\nu, \nu - \frac{1}{2}, -\nu - \frac{1}{2}, \nu - \frac{1}{2}; at)$	$2^{-2\nu - \frac{1}{2}} \pi^{-\frac{1}{2}} p^{2\nu - 1} H_{2\nu}(4a/p)$ $\operatorname{Re} p > 0$
(26)	$t^{-2\nu - 1}$ $\times S_1(\nu, \nu - \frac{1}{2}, -\nu - \frac{1}{2}, \nu + \frac{1}{2}; at)$	$2^{-2\nu - 3/2} \pi^{-1/2} p^{2\nu} H_{2\nu}(4a/p)$ $\operatorname{Re} p > 0$
(27)	$t^{-2\lambda - 1}$ $\times S_1(\nu - \frac{1}{2}, -\nu - \frac{1}{2}, \lambda, \lambda + \frac{1}{2}; at)$ $\operatorname{Re}(\nu - \lambda) > 0$	$2^{-2\lambda - 1} \pi^{-\frac{1}{2}} p^{2\lambda} J_{2\nu}(4a/p)$ $\operatorname{Re} p > 0$
(28)	$t^{-2\nu} S_2(\nu, \nu - \frac{1}{2}, -\nu - \frac{1}{2}, \nu - \frac{1}{2}; at)$	$2^{-2\nu} \pi^{\frac{1}{2}} p^{2\nu - 1} [I_{2\nu}(4a/p) - L_{2\nu}(4a/p)]$ $\operatorname{Re} p > 0$
(29)	$t^{-2\nu - 1}$ $\times S_2(\nu, -\nu - \frac{1}{2}, \nu - \frac{1}{2}, \nu + \frac{1}{2}; at)$ $\operatorname{Re} \nu < 0$	$2^{-2\nu - 1} \pi^{\frac{1}{2}} \sec(2\nu\pi) p^{2\nu}$ $\times [I_{-2\nu}(4a/p) - L_{2\nu}(4a/p)]$ $\operatorname{Re} p > 0$
(30)	$t^{-2\nu} S_2(\nu, -\nu - \frac{1}{2}, \nu - \frac{1}{2}, \nu - \frac{1}{2}; at)$ $\operatorname{Re} \nu < \frac{1}{2}$	$2^{-2\nu} \pi^{\frac{1}{2}} \sec(2\nu\pi) p^{2\nu - 1}$ $\times [I_{-2\nu}(4a/p) - L_{2\nu}(4a/p)]$ $\operatorname{Re} p > 0$
(31)	$t^{-2\lambda - 1}$ $\times S_2(\nu - \frac{1}{2}, -\nu - \frac{1}{2}, \lambda + \frac{1}{2}, \lambda; at)$ $\operatorname{Re}(\lambda \pm \nu) < 0$	$2^{-2\lambda} \pi^{-\frac{1}{2}} p^{2\lambda} K_{2\nu}(4a/p)$ $\operatorname{Re} p > 0$
(32)	$t^{-2\nu} S_3(\nu, \nu - \frac{1}{2}, -\nu - \frac{1}{2}, \nu - \frac{1}{2}; at)$ $\operatorname{Re} \nu < \frac{1}{2}$	$2^{-2\nu} \pi^{3/2} \sec(2\nu\pi) p^{2\nu - 1}$ $\times [H_{2\nu}(4a/p) - Y_{2\nu}(4a/p)]$ $\operatorname{Re} p > 0$

## Generalized hypergeometric series (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(33)	$t^{\alpha-1} \times E(a_1, \dots, a_h; \rho_1, \dots, \rho_k; t^{-1})$ $\text{Re } \alpha > 0$	$p^{-\alpha} E(a_1, \dots, a_h, \alpha; \rho_1, \dots, \rho_k; p)$
(34)	$t^{-\alpha} G_{h, k}^{m, n} \left( t \left  \begin{matrix} a_1, \dots, a_h \\ b_1, \dots, b_k \end{matrix} \right. \right)$ $h + k < 2(m + n)$ $\text{Re } \alpha > \text{Re } b_j + 1, \quad j = 1, \dots, m$	$p^{\alpha-1} G_{h+1, k}^{m+n+1} \left( p^{-1} \left  \begin{matrix} a, a_1, \dots, a_h \\ b_1, \dots, b_k \end{matrix} \right. \right)$ $ \arg p  < (m + n - \frac{1}{2}h - \frac{1}{2}k)\pi$
The same formula is valid if $h < k$ (or $h = k$ and $\text{Re } p > 1$ ) and $\text{Re } \alpha < \text{Re } b_j + 1, \quad j = 1, \dots, m$ .		

## 4.24. Hypergeometric functions of several variables

(1)	$t^{\beta'-1} \Phi_1(\alpha, \beta, \gamma; x, yt)$ $\text{Re } \beta' > 0$	$\Gamma(\beta') p^{-\beta'} F_1(\alpha, \beta, \beta', \gamma; x, y/p)$ $\text{Re } p > 0, \quad \text{Re } p > \text{Re } y$
(2)	$t^{\beta-1} \Phi_2(\alpha, \alpha', \gamma; xt, y)$ $\text{Re } \beta > 0$	$\Gamma(\beta) p^{-\beta} \Xi_1(\alpha, \alpha', \beta, \gamma; x/p, y)$ $\text{Re } p > 0, \quad \text{Re } p > \text{Re } x$
(3)	$t^{\gamma-1} \Phi_2(\beta, \beta', \gamma; xt, yt)$ $\text{Re } \gamma > 0$	$\Gamma(\gamma) p^{-\gamma} (1-x/p)^{-\beta} (1-y/p)^{-\beta'}$ $\text{Re } p > 0, \quad \text{Re } x, \quad \text{Re } y$
(4)	$t^{\alpha-1} \Phi_2(\beta, \beta', \gamma; xt, yt)$ $\text{Re } \alpha > 0$	$\Gamma(\alpha) p^{-\alpha} F_1(\alpha, \beta, \beta', \gamma; x/p, y/p)$ $\text{Re } p > 0, \quad \text{Re } x, \quad \text{Re } y$
(5)	$t^{\gamma-1} \Phi_2(\beta_1, \dots, \beta_n; \gamma; \lambda_1 t, \dots, \lambda_n t)$ $\text{Re } \gamma > 0$	$\frac{\Gamma(\gamma)}{p^\gamma} \left(1 - \frac{\lambda_1}{p}\right)^{-\beta_1} \dots \left(1 - \frac{\lambda_n}{p}\right)^{-\beta_n}$ $\text{Re } p > 0, \quad \text{Re } \lambda, \quad m = 1, \dots, n$
(6)	$t^{\alpha-1} \Phi_3(\beta, \gamma; xt, y)$ $\text{Re } \alpha > 0$	$\Gamma(\alpha) p^{-\alpha} \Xi_2(\alpha, \beta, \gamma; x/p, y)$ $\text{Re } p > 0, \quad \text{Re } x$

## Hypergeometric functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(7)	$t^{\beta'-1} \Phi_3(\beta, \gamma; x, yt)$ $\text{Re } \beta' > 0$	$\Gamma(\beta') p^{-\beta'} \Phi_2(\beta, \beta', \gamma; x, y/p)$ $\text{Re } p > 0, \quad \text{Re } y$
(8)	$t^{2\alpha-1} \Phi_3(\beta, \gamma; x, yt^2)$ $\text{Re } \alpha > 0$	$\Gamma(2\alpha) p^{-2\alpha} \Xi_1(\alpha, \beta, \alpha + \frac{1}{2}, \gamma; 4yp^{-2}, x)$ $\text{Re } p > 2 \text{Re } y ^{\frac{1}{2}} $
(9)	$t^{\gamma-1} \Phi_3(\beta, \gamma; xt, yt)$ $\text{Re } \gamma > 0$	$\Gamma(\gamma) p^{-\gamma} (1-x/p)^{-\beta} e^{y/p}$ $\text{Re } p > 0, \quad \text{Re } x$
(10)	$t^{\alpha-1} \Phi_3(\beta, \gamma; xt, yt)$ $\text{Re } \alpha > 0$	$\Gamma(\alpha) p^{-\alpha} \Phi_1(\alpha, \beta, \gamma; x/p, y/p)$ $\text{Re } p > 0, \quad \text{Re } x$
(11)	$t^{\beta'-1} \Psi_1(\alpha, \beta, \gamma, \gamma'; x, yt)$ $\text{Re } \beta' > 0$	$\Gamma(\beta') p^{-\beta'} F_2(\alpha, \beta, \beta', \gamma, \gamma'; x, y/p)$ $\text{Re } p > 0, \quad \text{Re } y$
(12)	$t^{\beta-1} \Psi_2(\alpha, \gamma, \gamma'; xt, y)$ $\text{Re } \beta > 0$	$\Gamma(\beta) p^{-\beta} \Psi_1(\alpha, \beta, \gamma, \gamma'; x/p, y)$ $\text{Re } p > 0, \quad \text{Re } x$
(13)	$t^{\alpha-1} \Psi_2(\beta, \gamma, \gamma'; xt, yt)$ $\text{Re } \alpha > 0$	$\Gamma(\alpha) p^{-\alpha} F_4(\alpha, \beta, \gamma, \gamma'; x/p, y/p)$ $\text{Re } p > 0, \quad \text{Re } x, \quad \text{Re } y$
(14)	$t^{\beta'-1} \Xi_1(\alpha, \alpha', \beta, \gamma; x, yt)$ $\text{Re } \beta' > 0$	$\Gamma(\beta') p^{-\beta'} F_3(\alpha, \alpha', \beta, \beta', \gamma; x, y/p)$ $\text{Re } p > 0, \quad \text{Re } y$
(15)	$t^{\alpha'-1} \Xi_2(\alpha, \beta, \gamma; x, yt)$ $\text{Re } \alpha' > 0$	$\Gamma(\alpha') p^{-\alpha'} \Xi_1(\alpha, \alpha', \beta, \gamma; x, y/p)$ $\text{Re } p > 0, \quad \text{Re } y$
(16)	$t^{2\alpha'-1} \Xi_2(\alpha, \beta, \gamma; x, yt^2)$ $\text{Re } \alpha' > 0$	$\Gamma(2\alpha') p^{-2\alpha'} \\ \times F_3(\alpha, \alpha', \beta, \alpha' + \frac{1}{2}, \gamma; x, 4yp^{-2})$ $\text{Re } p > 2 \text{Re } y ^{\frac{1}{2}} $

## 4.25. Elliptic functions

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(1)	$\theta_4(\frac{1}{2}x/l   i\pi t/l^2)$ $-l \leq x \leq l$	$lp^{-\frac{1}{2}} \cosh(xp^{\frac{1}{2}}) \operatorname{csch}(lp^{\frac{1}{2}})$ $\operatorname{Re} p > 0$
(2)	$\theta_1(\frac{1}{2}x/l   i\pi t/l^2)$ $-l \leq x \leq l$	$-lp^{-\frac{1}{2}} \sinh(xp^{\frac{1}{2}}) \operatorname{sech}(lp^{\frac{1}{2}})$ $\operatorname{Re} p > 0$
(3)	$\theta_2(\frac{1}{2} + \frac{1}{2}x/l   i\pi t/l^2)$ $-l \leq x \leq l$	$-lp^{-\frac{1}{2}} \sinh(xp^{\frac{1}{2}}) \operatorname{sech}(lp^{\frac{1}{2}})$ $\operatorname{Re} p > 0$
(4)	$\theta_3(\frac{1}{2} + \frac{1}{2}x/l   i\pi t/l^2)$ $-l \leq x \leq l$	$lp^{-\frac{1}{2}} \cosh(xp^{\frac{1}{2}}) \operatorname{csch}(lp^{\frac{1}{2}})$ $\operatorname{Re} p > 0$
(5)	$\hat{\theta}_4(\frac{1}{2}x/l   i\pi t/l^2)$ $-l \leq x \leq l$	$-lp^{-\frac{1}{2}} \sinh(xp^{\frac{1}{2}}) \operatorname{csch}(lp^{\frac{1}{2}})$ $\operatorname{Re} p > 0$
(6)	$\hat{\theta}_3(\frac{1}{2} + \frac{1}{2}x/l   i\pi t/l^2)$ $-l \leq x \leq l$	$-lp^{-\frac{1}{2}} \sinh(xp^{\frac{1}{2}}) \operatorname{csch}(lp^{\frac{1}{2}})$ $\operatorname{Re} p > 0$
(7)	$e^{\alpha t} \theta_3(\alpha^{\frac{1}{2}} t   i\pi t)$	$\frac{1}{2}p^{-\frac{1}{2}} [\tanh(p^{\frac{1}{2}} + \alpha^{\frac{1}{2}}) + \tanh(p^{\frac{1}{2}} - \alpha^{\frac{1}{2}})]$ $\operatorname{Re} p > 0$
(8)	$e^{\alpha t} \hat{\theta}_3(\alpha^{\frac{1}{2}} t   i\pi t)$	$\frac{1}{2}p^{-\frac{1}{2}} [\tanh(p^{\frac{1}{2}} + \alpha^{\frac{1}{2}}) - \tanh(p^{\frac{1}{2}} - \alpha^{\frac{1}{2}}) + 2]$ $\operatorname{Re} p > 0$
(9)	$\frac{\partial}{\partial x} \theta_4\left(\frac{x}{2l} \middle  \frac{i\pi t}{l^2}\right)$ $-l < x < l$	$\frac{l \sinh(xp^{\frac{1}{2}})}{\sinh(lp^{\frac{1}{2}})}$ $\operatorname{Re} p > 0$
(10)	$\frac{\partial}{\partial x} \theta_1\left(\frac{x}{2l} \middle  \frac{i\pi t}{l^2}\right)$ $-l < x < l$	$-\frac{l \cosh(xp^{\frac{1}{2}})}{\sinh(lp^{\frac{1}{2}})}$ $\operatorname{Re} p > 0$
(11)	$\frac{\partial}{\partial x} \theta_2\left(\frac{x+l}{2l} \middle  \frac{i\pi t}{l^2}\right)$ $-l < x < l$	$-\frac{l \cosh(xp^{\frac{1}{2}})}{\cosh(lp^{\frac{1}{2}})}$ $\operatorname{Re} p > 0$

**Elliptic functions (cont'd)**

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(12)	$\frac{\partial}{\partial x} \theta_3 \left( \frac{x+l}{2l} \middle  \frac{i\pi t}{l^2} \right)$	$\frac{l \sinh(xp^{1/2})}{\sinh(lp^{1/2})}$ <span style="float: right;"><math>\operatorname{Re} p &gt; 0</math></span>

**4.26. Miscellaneous functions**

(1)	$\nu(t)$	$(p \log p)^{-1}$	$\operatorname{Re} p > 0$
(2)	$(1-e^{-t})^{-1} \nu(t)$	$\int_0^\infty \zeta(u+1, p) du$	$\operatorname{Re} p > 1$
(3)	$t^{-1/2} \nu(2t^{1/2})$	$2\pi^{1/2} p^{-1/2} \nu(p^{-1})$	$\operatorname{Re} p > 0$
(4)	$\nu(e^{-t})$	$\int_0^\infty \frac{du}{(p+u) \Gamma(u+1)}$	$\operatorname{Re} p > 0$
(5)	$\nu(1-e^{-t})$	$\Gamma(p) \nu(1, p)$	$\operatorname{Re} p > 0$
(6)	$\nu(t, a)$ $\operatorname{Re} a > -1$	$p^{-a-1}/\log p$	$\operatorname{Re} p > 1$
(7)	$\nu(2t^{1/2}, 2a)$ $\operatorname{Re} a > -1$	$\frac{1}{2}\pi^{1/2} p^{-3/2} \nu(p^{-1}, a - 1/2)$	$\operatorname{Re} p > 0$
(8)	$t^{-1/2} \nu(2t^{1/2}, 2a)$ $\operatorname{Re} a > -1/2$	$2\pi^{1/2} p^{-1/2} \nu(p^{-1}, a)$	$\operatorname{Re} p > 0$
(9)	$\mu(t, a-1)$ $\operatorname{Re} a > 0$	$\Gamma(a) p^{-1} (\log p)^{-a}$	$\operatorname{Re} p > 1$
(10)	$t^{-1/2} \mu(2t^{1/2}, a)$	$2^{a+1} \pi^{1/2} p^{-1/2} \mu(p^{-1}, a)$	$\operatorname{Re} p > 0$

## Miscellaneous functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t) dt$
(11)	$V_n(t)$ $V_n(t)$ is defined by the generating function $\frac{1}{1-z} \exp\left(-\frac{1+z}{1-z} t\right)$ $= \sum_{n=0}^{\infty} (n + \frac{1}{2}) V_n(t) P_n(z)$	$\frac{2}{p-1} Q_n\left(\frac{p+1}{p-1}\right)$ $\text{Re } p > 0$
(12)	$U^{m,n}(t)$ $U^{m,n}(t)$ is defined by the generating function $\frac{e^{-at} I_0(bt)}{(1-x)(1-y)} = \sum_{m,n=0}^{\infty} x^m y^n U^{m,n}(t)$ where $a+b = \left(\frac{1+x}{1-x}\right)^2, \quad a-b = \left(\frac{1+y}{1-y}\right)^2$	$\frac{1}{p+1} P_m\left(\frac{p-1}{p+1}\right) P_n\left(\frac{p-1}{p+1}\right)$ $\text{Re } p > -1$



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## CHAPTER V

### INVERSE LAPLACE TRANSFORMS

#### 5.1. General formulas

Most general formulas are in section 4.1. The present section contains a few additions.

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(1)	$g(p)$	$f(t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{pt} g(p) dp$
(2)	$g(p+p^{\frac{1}{2}})$	$\frac{1}{2} \pi^{-1/2} \int_0^t u(t-u)^{-3/2} e^{-\frac{1}{4}u^2/(t-u)} \times f(u) du$
(3)	$p^{-\frac{1}{2}} g(p+p^{\frac{1}{2}})$	$\pi^{-\frac{1}{2}} \int_0^t (t-u)^{-\frac{1}{2}} e^{-\frac{1}{4}u^2/(t-u)} f(u) du$
(4)	$(p+a)^{-\nu} g[cp + (p+a)^{\frac{1}{2}}]$ $c > 0$	$2^{\nu-\frac{1}{2}} \pi^{-\frac{1}{2}} \int_0^t (t-cu)^{\nu-1} \exp -a(t-cu) - \frac{u^2}{8(t-cu)} \Big] D_{1-2\nu} \left[ \frac{u}{2^{\frac{1}{2}}(t-cu)^{\frac{1}{2}}} \right] \times f(u) du$
(5)	$g(r)$	$f(t) - a \int_0^t f[(t^2 - u^2)^{\frac{1}{2}}] J_1(au) du$
(6)	$r^{-1} g(r)$	$\int_0^t J_0[a(t^2 - u^2)^{\frac{1}{2}}] f(u) du$

$$r = (p^2 + a^2)^{\frac{1}{2}}, \quad R = p + r$$

## General formulas (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(7)	$pr^{-1} g(r)$	$f(t) - at \int_0^t (t^2 - u^2)^{-\frac{1}{2}} \times J_1[\alpha(t^2 - u^2)^{\frac{1}{2}}] f(u) du$
(8)	$r^{-1} R^{-\nu} g(r)$ $\operatorname{Re} \nu > -1$	$\alpha^{-\nu} \int_0^t \left(\frac{t-u}{t+u}\right)^{\frac{1}{2}\nu} \times J_\nu[\alpha(t^2 - u^2)^{\frac{1}{2}}] f(u) du$
(9)	$g(\beta + r - p) - g(\beta)$	$-at^{-\frac{1}{2}} \int_0^t t^{-\frac{1}{2}} (t + 2u)^{-\frac{1}{2}} \times J_1[\alpha t^{\frac{1}{2}} (t + 2u)^{\frac{1}{2}}] e^{-\beta u} uf(u) du$
(10)	$r^{-1} R^{-\nu} g(r-p)$ $\operatorname{Re} \nu > -1$	$\alpha^{-\nu} t^{\frac{1}{2}\nu} \int_0^\infty (t + 2u)^{-\frac{1}{2}\nu} \times I_\nu[\alpha t^{\frac{1}{2}} (t + 2u)^{\frac{1}{2}}] f(u) du$
(11)	$r^{-1} R^{-\nu} g(p-r)$ $\operatorname{Re} \nu > -1$	$\alpha^{-\nu} t^{\frac{1}{2}\nu} \int_0^\infty (t - 2u)^{-\frac{1}{2}\nu} \times I_\nu[\alpha t^{\frac{1}{2}} (t - 2u)^{\frac{1}{2}}] f(u) du$
(12)	$g(s)$	$f(t) + \alpha \int_0^t f[(t^2 - u^2)^{\frac{1}{2}}] I_1(u) du$
(13)	$s^{-1} g(s)$	$\int_0^t I_0[\alpha(t^2 - u^2)^{\frac{1}{2}}] f(u) du$
(14)	$ps^{-1} g(s)$	$f(t) + at \int_0^t (t^2 - u^2)^{-\frac{1}{2}} \times I_1[\alpha(t^2 - u^2)^{\frac{1}{2}}] f(u) du$
(15)	$s^{-1} S^{-\nu} g(s)$ $\operatorname{Re} \nu > -1$	$\alpha^{-\nu} \int_0^t \left(\frac{t-u}{t+u}\right)^{\frac{1}{2}\nu} \times I_\nu[\alpha(t^2 - u^2)^{\frac{1}{2}}] f(u) du$

$$r = (p^2 + \alpha^2)^{\frac{1}{2}}, \quad R = p + r, \quad s = (p^2 - \alpha^2)^{\frac{1}{2}}, \quad S = v + s$$

**General formulas (cont'd)**

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(16)	$g(\beta + s - p) - g(\beta)$	$at^{-\frac{1}{2}} \int_0^\infty e^{-\beta u} (t+2u)^{-\frac{1}{2}} \times I_1 [at^{\frac{1}{2}}(t+2u)^{\frac{1}{2}}] uf(u) du$
(17)	$s^{-1} S^{-\nu} g(s-p)$ $\operatorname{Re} \nu > -1$	$a^{-\nu} t^{\frac{1}{2}\nu} \int_0^\infty (t+2u)^{-\frac{1}{2}\nu} \times I_\nu [at^{\frac{1}{2}}(t+2u)^{\frac{1}{2}}] f(u) du$
(18)	$s^{-1} S^{-\nu} g(p-s)$ $\operatorname{Re} \nu > -1$	$a^{-\nu} t^{\frac{1}{2}\nu} \int_0^\infty (t-2u)^{-\frac{1}{2}\nu} \times I_\nu [at^{\frac{1}{2}}(t-2u)^{\frac{1}{2}}] f(u) du$

**5.2. Rational functions**

(1)	$(p+a)^{-1}$	$e^{-at}$
(2)	$(\lambda p + \mu)(p+a)^{-2}$	$[\lambda + (\mu - a\lambda)t] e^{-at}$
(3)	$(\lambda p + \mu) [(p+a)^2 - \beta^2]^{-1}$	$\lambda e^{-at} \cosh(\beta t) + \beta^{-1} (\mu - a\lambda) \times e^{-at} \sinh(\beta t)$
(4)	$(\lambda p + \mu) [(p+a)^2 + \beta^2]^{-1}$	$\lambda e^{-at} \cos(\beta t) + \beta^{-1} (\mu - a\lambda) \times e^{-at} \sin(\beta t)$
(5)	$\frac{\lambda p + \mu}{(p+a)(p+\beta)}$	$\frac{a\lambda - \mu}{a - \beta} e^{-at} + \frac{\beta\lambda - \mu}{\beta - a} e^{-\beta t}$
(6)	$(\lambda p^2 + \mu p + \nu)(p+a)^{-3}$	$[\lambda + (\mu - 2a\lambda)t + \frac{1}{2}(a^2\lambda - a\mu + \nu)t^2] \times e^{-at}$

$$s = (p^2 - a^2)^{\frac{1}{2}}, \quad S = p + s$$

## Rational functions (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(7)	$3\alpha^2 \frac{\lambda p^2 + \mu p + \nu}{p^3 + \alpha^3}$	$(\lambda\alpha^2 - \mu\alpha + \nu) e^{-\alpha t}$ $- (2\lambda\alpha^2 - \mu\alpha - \nu) e^{\frac{1}{2}\alpha t} \cos(\frac{1}{2}3^{\frac{1}{2}}\alpha t)$ $+ 3^{\frac{1}{2}}(\mu\alpha - \nu) e^{\frac{1}{2}\alpha t} \sin(\frac{1}{2}3^{\frac{1}{2}}\alpha t)$
(8)	$\frac{\lambda p^2 + \mu p + \nu}{(p + \alpha)^2(p + \beta)}$	$\left[ \frac{\alpha(\alpha - 2\beta)\lambda + \mu\beta - \nu}{(\alpha - \beta)^2} \right] e^{-\alpha t}$ $- \frac{\alpha^2\lambda - \alpha\mu + \nu}{\alpha - \beta} t e^{-\alpha t}$ $+ \frac{\beta^2\lambda - \beta\mu + \nu}{(\alpha - \beta)^2} e^{-\beta t}$
(9)	$\frac{\lambda p^2 + \mu p + \nu}{[(p + \alpha)^2 + \beta^2](p + \gamma)}$	$\left[ \lambda - \frac{\lambda\gamma^2 - \mu\gamma + \nu}{(\alpha - \gamma)^2 + \beta^2} \right] e^{-\alpha t} \cos(\beta t)$ $+ \frac{1}{\beta} \left[ \mu - (\alpha + \gamma) \lambda \right.$ $\left. - (\alpha - \gamma) \frac{\lambda\gamma^2 - \mu\gamma + \nu}{(\alpha - \gamma)^2 + \beta^2} \right] e^{-\alpha t} \sin(\beta t)$ $+ \frac{\lambda\gamma^2 - \mu\gamma + \nu}{(\alpha - \gamma)^2 + \beta^2} e^{-\gamma t}$
(10)	$\frac{\lambda^2 p + \mu p + \nu}{(p + \alpha)(p + \beta)(p + \gamma)}$	$\frac{\lambda\alpha^2 + \mu\alpha + \nu}{(\alpha - \beta)(\alpha - \gamma)} e^{-\alpha t}$ $+ \frac{\lambda\beta^2 - \mu\beta + \nu}{(\beta - \alpha)(\beta - \gamma)} e^{-\beta t}$ $+ \frac{\lambda\gamma^2 - \mu\gamma + \nu}{(\gamma - \alpha)(\gamma - \beta)} e^{-\gamma t}$
(11)	$4\alpha^3 \frac{\lambda p^3 + \mu p^2 + \nu p + \rho}{p^4 + 4\alpha^4}$	$4\alpha^3\lambda \cos(at) \cosh(at)$ $+ (2\alpha^2\mu - \rho) \cos(at) \sinh(at)$ $+ (2\alpha^2\mu + \rho) \sin(at) \cosh(at)$ $+ 2\alpha\nu \sin(at) \sinh(at)$

**Rational functions (cont'd)**

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(12)	$2\alpha^3 \frac{\lambda p^3 + \mu p^2 + \nu p + \rho}{p^4 - \alpha^4}$	$(\alpha^3 \lambda - \alpha \nu) \cos(\alpha t) + (\alpha^2 \mu - \rho) \sin(\alpha t)$ + $(\alpha^3 \lambda + \alpha \nu) \cosh(\alpha t)$ + $(\alpha^2 \mu + \rho) \sinh(\alpha t)$
(13)	$\frac{\lambda p^3 + \mu p^2 + \nu p + \rho}{(p^2 + \alpha^2)^2}$	$\lambda \cos(\alpha t) + \frac{\rho + \alpha^2 \mu}{2\alpha^3} \sin(\alpha t)$ + $\frac{\nu - \alpha^2 \lambda}{2\alpha} t \sin(\alpha t) - \frac{\rho - \alpha^2 \mu}{2\alpha^2} t \cos(\alpha t)$
(14)	$(\beta^2 - \alpha^2) \frac{\lambda p^3 + \mu p^2 + \nu p + \rho}{(p^2 + \alpha^2)(p^2 + \beta^2)}$	$(\nu - \alpha^2 \lambda) \cos(\alpha t) + (\rho/\alpha - \alpha \mu) \sin(\alpha t)$ - $(\nu - \beta^2 \lambda) \cos(\beta t) - (\rho/\beta - \beta \mu) \sin(\beta t)$
For further similar formulas see Gardner, M. F. and J. L. Barnes, 1942 : <i>Transients in linear systems</i> , I, Wiley.		
(15)	$p^{-1}(p^{-1}-1)(p^{-1}-\frac{1}{2})\dots(p^{-1}-1/n)$	$A_n(t)$
(16)	$2(1-p)^n(1+p)^{-n-2}$	$k_{2n+2}(t)$
(17)	$\frac{\lambda_1 p^{n-1} + \lambda_2 p^{n-2} + \dots + \lambda_n}{(p+\alpha)^n}$	$\left\{ \lambda_1 + \left[ \lambda_2 - \binom{n-1}{1} \lambda_1 \alpha \right] t \right.$ $+ \left[ \lambda_3 - \binom{n-2}{1} \lambda_2 \alpha + \binom{n-1}{2} \lambda_1 \alpha^2 \right] \frac{t^2}{2!}$ $+ \left[ \lambda_4 - \binom{n-3}{1} \lambda_3 \alpha + \binom{n-2}{2} \lambda_2 \alpha^2 \right. \left. - \binom{n-1}{3} \lambda_1 \alpha^3 \right] \frac{t^3}{3!}$ $+ \dots + [\lambda_n - \lambda_{n-1} \alpha + \dots + \lambda_1 (-\alpha)^{n-1}] \times \frac{t^{n-1}}{(n-1)!} \right\} e^{-\alpha t}$

## Rational functions (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(18)	$p^{-1}(p+a)^{-n}$	$a^{-n}[1 - e^{-at} e_{n-1}(at)]$ $e_n(z) = 1 + \frac{z}{1!} + \dots + \frac{z^n}{n!}$
(19)	$\frac{\lambda_1 p^{n-1} + \lambda_2 p^{n-2} + \dots + \lambda_n}{(p+a_1)(p+a_2)\dots(p+a_n)}$	$\frac{\lambda_1 (-a_1)^{n-1} + \dots + \lambda_n}{(a_2 - a_1)(a_3 - a_1)\dots(a_n - a_1)} e^{-a_1 t}$ + $n-1$ similar terms obtained by cyclic permutation of $a_1, \dots, a_n$
(20)	$\frac{Q(p)}{P(p)}$ $P(p) = (p-a_1)(p-a_2)\dots(p-a_n)$ $Q(p) = \text{polynomial of degree } \leq n-1$ $a_i \neq a_k, \text{ for } i \neq k$	$\sum_{m=1}^n \frac{Q(a_m)}{P_m(a_m)} e^{\alpha_m t}$ $P_m(p) = \frac{P(p)}{p - a_m}$
(21)	$\frac{Q(p)}{P(p)}$ $P(p) = (p-a_1)^{m_1} \dots (p-a_n)^{m_n}$ $Q(p) = \text{polynomial of degree } < m_1 + \dots + m_n - 1$ $a_i \neq a_k, \text{ for } i \neq k$	$\sum_{k=1}^n \sum_{l=1}^{m_k} \frac{\Phi_{kl}(a_k)}{(m_k - l)! (l-1)!} t^{m_k - l} e^{\alpha_k t}$ $\Phi_{kl}(p) = \frac{d^{l-1}}{dp^{l-1}} \left[ \frac{Q(p)}{P_k(p)} \right]$ $P_k(p) = \frac{P(p)}{(p - a_k)^{m_k}}$
(22)	$\frac{(2n+1)! \alpha^{2n+1}}{(p^2+\alpha^2)(p^2+3^2\alpha^2)\dots[p^2+(2n+1)^2\alpha^2]}$	$\sin^{2n+1}(\alpha t)$
(23)	$\frac{(2n)! \alpha^{2n}}{p(p^2+2^2\alpha^2)(p^2+4^2\alpha^2)\dots(p^2+4n^2\alpha^2)}$	$\sin^{2n}(\alpha t)$

## Rational functions (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(24)	$\frac{p(p^2+2^2\alpha^2)(p^2+4^2\alpha^2)\dots[p^2+(2n)^2\alpha^2]}{(p^2+\alpha^2)(p^2+3^2\alpha^2)\dots[p^2+(2n+1)^2\alpha^2]}$	$P_{2n+1}[\cos(\alpha t)]$
(25)	$\frac{(p^2+\alpha^2)(p^2+2^2\alpha^2)\dots[p^2+(2n-1)^2\alpha^2]}{p(p^2+2^2\alpha^2)(p^2+4^2\alpha^2)\dots[p^2+(2n)^2\alpha^2]}$	$P_{2n}[\cos(\alpha t)]$
(26)	$\frac{Q(p)+p\eta(p)}{P(p)}$ $P(p) = (p^2 + \alpha_1^2) \dots (p^2 + \alpha_n^2)$ $Q(p), \eta(p)$ even polynomials of degree $\leq 2n - 2$ $\alpha_i \neq \alpha_k, \text{ for } i \neq k$	$\sum_{m=1}^n \frac{1}{P_m(i\alpha_m)} [\eta(i\alpha_m) \cos(\alpha_m t) + (\alpha_m)^{-1} Q(i\alpha_m) \sin(\alpha_m t)]$ $P_m(p) = \frac{P(p)}{p^2 + \alpha_m^2}$

## 5.3. Irrational algebraic functions

(1)	$(p-a)^{-1} p^{-\frac{1}{2}}$	$a^{-\frac{1}{2}} e^{\alpha t} \operatorname{Erf}(a^{\frac{1}{2}} t^{\frac{1}{2}})$
(2)	$p^{-1} (p-a)^{-1} p^{-\frac{1}{2}}$	$\alpha^{-3/2} e^{\alpha t} \operatorname{Erf}(a^{1/2} t^{1/2}) - 2 \alpha^{-1} \pi^{-1/2} t^{1/2}$
(3)	$2\pi i (p-1)^n (p+1)^{-n-1} p^{-\frac{1}{2}}$	$n! [D_{-n-1}^2 (-i 2^{\frac{1}{2}} t^{\frac{1}{2}}) - D_{-n-1}^2 (i 2^{\frac{1}{2}} t^{\frac{1}{2}})]$
(4)	$(p^{\frac{1}{2}} + \alpha)^{-1}$	$\pi^{-\frac{1}{2}} t^{-\frac{1}{2}} - \alpha e^{\alpha^2 t} \operatorname{Erfc}(\alpha t^{\frac{1}{2}})$
(5)	$\alpha p^{-1} (p^{\frac{1}{2}} + \alpha)^{-1}$	$1 - e^{\alpha^2 t} \operatorname{Erfc}(\alpha t^{\frac{1}{2}})$
(6)	$(\alpha-\beta)p^{\frac{1}{2}}(p^{\frac{1}{2}} + \alpha^{\frac{1}{2}})^{-1}(p-\beta)^{-1}$	$\alpha e^{\alpha t} \operatorname{Erfc}(\alpha^{\frac{1}{2}} t^{\frac{1}{2}}) + \alpha^{\frac{1}{2}} \beta^{\frac{1}{2}} e^{\beta t} \operatorname{Erfc}(\beta^{\frac{1}{2}} t^{\frac{1}{2}}) - \beta e^{\beta t}$

## Irrational algebraic functions (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(7)	$p^{-\frac{1}{2}} (p^{\frac{1}{2}} + a)^{-1}$	$e^{\alpha^2 t} \operatorname{Erfc}(at^{\frac{1}{2}})$
(8)	$\alpha^2 p^{-3/2} (p^{1/2} + a)^{-1}$ (err.)	$2\pi^{-\frac{1}{2}} at^{\frac{1}{2}} + e^{\alpha^2 t} \operatorname{Erfc}(at^{\frac{1}{2}}) - 1$
(9)	$(\alpha - \beta) \beta^{\frac{1}{2}} (p - \beta)^{-1}$ $\times p^{-\frac{1}{2}} (p^{\frac{1}{2}} + a^{\frac{1}{2}})^{-1}$	$\beta^{\frac{1}{2}} e^{\alpha t} \operatorname{Erfc}(\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$ $+ a^{\frac{1}{2}} e^{\beta t} \operatorname{Erf}(\beta^{\frac{1}{2}} t^{\frac{1}{2}}) - \beta^{\frac{1}{2}} e^{\beta t}$
(10)	$(p^{\frac{1}{2}} + a^{\frac{1}{2}})^{-2}$	$1 - 2\pi^{-\frac{1}{2}} \alpha^{\frac{1}{2}} t^{\frac{1}{2}}$ $+ (1 - 2\alpha t) e^{\alpha t} [\operatorname{Erf}(\alpha^{\frac{1}{2}} t^{\frac{1}{2}}) - 1]$
(11)	$p^{-1} (p^{\frac{1}{2}} + a^{\frac{1}{2}})^{-2}$	$\alpha^{-1} + (2t - \alpha^{-1}) e^{\alpha t} \operatorname{Erfc}(\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$ $- 2\pi^{-\frac{1}{2}} \alpha^{-\frac{1}{2}} t^{\frac{1}{2}}$
(12)	$p^{-\frac{1}{2}} (p^{\frac{1}{2}} + a)^{-2}$	$2\pi^{-\frac{1}{2}} t^{\frac{1}{2}} - 2\alpha t e^{\alpha^2 t} \operatorname{Erfc}(at^{\frac{1}{2}})$
(13)	$(p^{\frac{1}{2}} + a)^{-3}$	$2\pi^{-\frac{1}{2}} (\alpha^2 t + 1) t^{\frac{1}{2}}$ $- \alpha t e^{\alpha^2 t} (2\alpha^2 t + 3) \operatorname{Erfc}(at^{\frac{1}{2}})$
(14)	$p^{\frac{1}{2}} (p^{\frac{1}{2}} + a)^{-3}$	$(2\alpha^4 t^2 + 5\alpha^2 t + 1) e^{\alpha^2 t} \operatorname{Erfc}(at^{\frac{1}{2}})$ $- 2\pi^{-\frac{1}{2}} \alpha (\alpha^2 t + 2) t^{\frac{1}{2}}$
(15)	$p^{-\frac{1}{2}} (p^{\frac{1}{2}} + a)^{-3}$	$(2*\alpha**2*t+1)$ (err.) $(2\alpha t^2 + 1) t e^{\alpha^2 t} \operatorname{Erfc}(at^{1/2}) - 2\pi^{-1/2} \alpha t^{3/2}$
(16)	$3(p^{\frac{1}{2}} + a)^{-4}$	$- 2\pi^{-1/2} \alpha^3 t^{5/2} (2\alpha^2 t + 5)$ $+ t (4\alpha^4 t^2 + 12\alpha^2 t + 3) e^{\alpha^2 t} \operatorname{Erfc}(at^{1/2})$
(17)	$p^{-1} (p^{\frac{1}{2}} - \alpha)(p^{\frac{1}{2}} + a)^{-1}$	$2e^{\alpha^2 t} \operatorname{Erfc}(at^{\frac{1}{2}}) - 1$
(18)	$p^{-1} (p^{\frac{1}{2}} - \alpha)^2 (p^{\frac{1}{2}} + a)^{-2}$	$1 + 8\alpha^2 t e^{\alpha^2 t} \operatorname{Erfc}(at^{\frac{1}{2}}) - 8\pi^{-\frac{1}{2}} \alpha t^{\frac{1}{2}}$

## Irrational algebraic functions (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(19)	$p^{-1} (p^{\frac{1}{2}} - \alpha)^3 (p^{\frac{1}{2}} + \alpha)^{-3}$	$2(8\alpha^4 t^2 + 8\alpha^2 t + 1) e^{\alpha^2 t} \text{Erfc}(\alpha t^{\frac{1}{2}}) - 8\pi^{-\frac{1}{2}} \alpha t^{\frac{1}{2}} (2\alpha^2 t + 1) - 1$
(20)	$(p - \alpha)^{\frac{1}{2}} - (p - \beta)^{\frac{1}{2}}$	$\frac{1}{2} \pi^{-1/2} t^{-3/2} (e^{\beta t} - e^{\alpha t})$
(21)	$(p + \alpha)^{-\frac{1}{2}}$	$\pi^{-\frac{1}{2}} t^{-\frac{1}{2}} e^{-\alpha t}$
(22)	$(p + \beta)^{-1} (p + \alpha)^{\frac{1}{2}}$	$\pi^{-\frac{1}{2}} t^{-\frac{1}{2}} e^{-\alpha t} + (\alpha - \beta)^{\frac{1}{2}} e^{-\beta t} \text{Erf}[(\alpha - \beta)^{\frac{1}{2}} t^{\frac{1}{2}}]$
(23)	$(p + \alpha)^{-1} (p + \beta)^{-\frac{1}{2}}$	$(\beta - \alpha)^{-\frac{1}{2}} e^{-\alpha t} \text{Erf}[(\beta - \alpha)^{\frac{1}{2}} t^{\frac{1}{2}}]$
(24)	$(p + \alpha)^{\frac{1}{2}} (p - \alpha)^{-\frac{1}{2}} - 1$	$\alpha [ I_0(\alpha t) + I_1(\alpha t) ]$
(25)	$(p + \alpha + \beta)^{1/2} (p + \alpha - \beta)^{-3/2}$	$e^{-\alpha t} [(1 + 2\beta t) I_0(\beta t) + 2\beta t I_1(\beta t)]$
(26)	$(p + \alpha + \beta)^{-1/2} (p + \alpha - \beta)^{-3/2}$	$t e^{-\alpha t} [ I_0(\beta t) + I_1(\beta t) ]$
(27)	$p^{-1} (p + \alpha - \beta)^{\frac{1}{2}} (p + \alpha + \beta)^{-\frac{1}{2}}$	$e^{-\alpha t} I_0(\beta t) + (\alpha - \beta) \int_0^t e^{-\alpha u} I_0(\beta u) du$
(28)	$\frac{(p + \alpha)^{\frac{1}{2}} - (p - \alpha)^{\frac{1}{2}}}{(p + \alpha)^{\frac{1}{2}} + (p - \alpha)^{\frac{1}{2}}}$	$t^{-1} I_1(\alpha t)$
Campbell, G. A., and R. M. Foster, 1931: <i>Fourier integrals for practical applications</i> , Bell Telephone Laboratories, New York, contains other similar integrals.		
(29)	$(p + \alpha)^{-n-\frac{1}{2}}$	$\frac{\pi^{-\frac{1}{2}} t^{n-\frac{1}{2}} e^{-\alpha t}}{\frac{1}{2} \cdot \frac{3}{2} \cdots (n - \frac{1}{2})}$

## Irrational algebraic functions (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(30)	$(p - \alpha)^n (p - \beta)^{-n - \frac{1}{2}}$	$\frac{(-2)^n n! e^{\beta t}}{(2n)! \pi^{\frac{1}{2}} t^{\frac{n}{2}}} H e_{2n} [2^{\frac{1}{2}} (\alpha - \beta)^{\frac{1}{2}} t^{\frac{n}{2}}]$
(31)	$(p - \alpha)^n (p - \beta)^{-n - 3/2}$	$\frac{(-2)^n 2^{\frac{n}{2}} n!}{(2n+1)! \pi^{\frac{1}{2}}} e^{\beta t} H e_{2n+1} [2^{\frac{1}{2}} (\alpha - \beta)^{\frac{1}{2}} t^{\frac{n}{2}}]$
(32)	$(p - \alpha)^n (p - \beta)^{-n - n - \frac{3}{2}}$	$\begin{aligned} & \frac{(-1)^n e^{\beta t} 2^{n+k+\frac{1}{2}}}{(\alpha - \beta)^n \pi^{\frac{1}{2}}} \\ & \times \sum_{k=1}^n \binom{m}{k} \frac{(n+k)!}{(2n+2k+1)!} \\ & \times H e_{2n+2k+1} [2^{\frac{1}{2}} (\alpha - \beta)^{\frac{1}{2}} t^{\frac{n}{2}}] \end{aligned}$
(33)	$(p - \alpha)^n (p - \beta)^{-n - n - \frac{1}{2}}$	$\begin{aligned} & \frac{(-1)^n e^{\beta t} 2^{n+k}}{(\alpha - \beta)^n \pi^{\frac{1}{2}} t^{\frac{n}{2}}} \sum_{k=1}^n \binom{m}{k} \frac{(n+k)!}{(2n+2k)!} \\ & \times H e_{2n+2k} [2^{\frac{1}{2}} (\alpha - \beta)^{\frac{1}{2}} t^{\frac{n}{2}}] \end{aligned}$
(34)	$(p^2 + \alpha p + \beta)^{-\frac{1}{2}}$	$e^{-\frac{1}{2} \alpha t} J_0 [(\beta - \frac{1}{4} \alpha^2)^{\frac{1}{2}} t]$
(35)	$r^{-1}$	$J_0(\alpha t)$
(36)	$p^{-1} r^{-3}$	$\frac{1}{2} \pi \alpha^{-2} t [J_1(\alpha t) H_0(\alpha t) - J_0(\alpha t) H_1(\alpha t)]$
(37)	$r^{-1} R^{-\frac{1}{2}}$	$2^{\frac{1}{2}} \pi^{-\frac{1}{2}} t^{-\frac{1}{2}} \cos(\alpha t)$
(38)	$(r-p)^{\frac{1}{2}} = \alpha R^{-\frac{1}{2}}$	$2^{-1/2} \pi^{-1/2} t^{-3/2} \sin(\alpha t)$
(39)	$\alpha r^{-1} R^{-\frac{1}{2}}$	$2^{\frac{1}{2}} \pi^{-\frac{1}{2}} t^{-\frac{1}{2}} \sin(\alpha t)$

$$r = (p^2 + \alpha^2)^{\frac{1}{2}}, \quad R = p + r$$

## Irrational algebraic functions (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(40)	$p^{-1} r^{-1} R^{\frac{1}{2}}$	$2\alpha^{-\frac{1}{2}} C(\alpha t)$
(41)	$p^{-1} r^{-1} R^{-\frac{1}{2}}$	$2\alpha^{-3/2} S(\alpha t)$
(42)	$r^{-2n-1}$	$\frac{\alpha^{-n} t^n J_n(\alpha t)}{1 \cdot 3 \cdot 5 \dots (2n-1)}$
(43)	$R^{-n}$	$n \alpha^{-n} t^{-1} J_n(\alpha t)$
(44)	$s^{-1}$	$I_0(\alpha t)$
(45)	$p^{-1} s^{-3}$	$\frac{1}{2} \pi \alpha^{-2} t [I_1(\alpha t) L_0(\alpha t) - I_0(\alpha t) L_1(\alpha t)]$
(46)	$s^{-1} S^{\frac{1}{2}}$	$2^{\frac{1}{2}} \pi^{-\frac{1}{2}} t^{-\frac{1}{2}} \cosh(\alpha t)$
(47)	$\alpha s^{-1} S^{-\frac{1}{2}}$	$2^{\frac{1}{2}} \pi^{-\frac{1}{2}} t^{-\frac{1}{2}} \sinh(\alpha t)$
(48)	$s^{-2n-1}$	$\frac{t^n I_n(\alpha t)}{1 \cdot 3 \cdot 5 \dots (2n-1) \alpha^n}$
(49)	$S^{-n}$	$n \alpha^{-n} t^{-1} I_n(\alpha t)$
(50)	$[(p^4 + \alpha^4)^{\frac{1}{4}} + p^2]^{\frac{1}{2}} (p^4 + \alpha^4)^{-\frac{1}{2}}$	$2^{\frac{1}{2}} \operatorname{ber}(\alpha t)$
(51)	$[(p^4 + \alpha^4)^{\frac{1}{4}} - p^2]^{\frac{1}{2}} (p^4 + \alpha^4)^{-\frac{1}{2}}$	$2^{\frac{1}{2}} \operatorname{bei}(\alpha t)$

$$r = (p^2 + \alpha^2)^{\frac{1}{4}}, \quad R = p + r, \quad s = (p^2 - \alpha^2)^{\frac{1}{4}}, \quad S = p + s$$

## 5.4. Powers with an arbitrary index

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(1)	$\Gamma(\nu)(p+a)^{-\nu}$ $\text{Re } \nu > 0$	$t^{\nu-1} e^{-at}$
(2)	$p^{-m-1}(p-1)^n$	$\nu^m p_n(m, t)/m!$
(3)	$\Gamma(\nu)(p+a)^{-\nu}(p-\beta)^{-1}$ $\text{Re } \nu > 0$	$(a+\beta)^{-\nu} e^{\beta t} \gamma[\nu, (a+\beta)t]$
(4)	$\Gamma(\nu+1)(p-\lambda)^n(p-\mu)^{-\nu-1}$ $\text{Re } \nu > n-1$	$n! t^{\nu-n} e^{\mu t} L_n^{\nu-n}[(\mu-\lambda)t]$
(5)	$\Gamma(\nu)(p+a)^{-\nu}(p+\beta)^{-\nu}$ $\text{Re } \nu > 0$	$\pi^{1/2} (a-\beta)^{1/2-\nu} t^{\nu-1/2} e^{-1/2(a+\beta)t}$ $\times I_{\nu-1/2} [1/2(a-\beta)t]$
(6)	$(p-a)^\lambda (p-\beta)^{-\lambda-1/2}$	$2^{-\lambda-1} \pi^{-1} \Gamma(1/2-\lambda) t^{-1/2} e^{1/2(a+\beta)t}$ $\times \{D_{2\lambda}[-2^{1/2}(a-\beta)^{1/2}t^{1/2}]$ $+ D_{2\lambda}[2^{1/2}(a-\beta)^{1/2}t^{1/2}]\}$
(7)	$(p-a)^\lambda (p-\beta)^{-\lambda-3/2}$	$2^{-\lambda-3/2} \pi^{-1} (a-\beta)^{-1/2} \Gamma(-1/2-\lambda)$ $\times e^{1/2(a+\beta)t} \{D_{2\lambda+1}[-2^{1/2}(a-\beta)^{1/2}t^{1/2}]$ $- D_{2\lambda+1}[2^{1/2}(a-\beta)^{1/2}t^{1/2}]\}$
(8)	$\Gamma(2\nu-2\lambda)(p-a)^{2\lambda} (p-\beta)^{-2\nu}$ $\text{Re } (\nu-\lambda) > 0$	$t^{2\nu-2\lambda-1} e^{\alpha t} {}_1F_1[2\nu, 2\nu-2\lambda; (\beta-a)t]$ $= (a-\beta)^{\lambda-\nu} t^{\nu-\lambda-1} e^{1/2(a+\beta)t}$ $\times M_{\lambda+\nu, \nu-\lambda-1/2}[(a-\beta)t]$
(9)	$\Gamma(\gamma) p^{-\gamma}$ $\times (1-\lambda_1/p)^{-\beta_1} \dots (1-\lambda_n/p)^{-\beta_n}$ $\text{Re } \gamma > 0$	$t^{\gamma-1} \Phi_2(\beta_1, \dots, \beta_n; \gamma; \lambda_1 t, \dots, \lambda_n t)$
(10)	$p^{-2\lambda} (p^2 + a^2)^{-\nu}$ $\text{Re } (\lambda + \nu) > 0$	$[\Gamma(2\lambda+2\nu)]^{-1} t^{2\lambda+2\nu-1}$ $\times {}_1F_2(\nu; \lambda+\nu, \lambda+\nu+1/2; -1/4 a^2 t^2)$

## Arbitrary powers (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(11)	$p^{-3}\lambda(p^3 + \alpha^3)^{-\nu} \quad \text{Re } (\lambda + \nu) > 0$	$\frac{t^{3\lambda+3\nu-1}}{\Gamma(3\lambda+3\nu)} \times {}_1F_3\left(\nu; \lambda+\nu, \lambda+\nu+\frac{1}{3}, \lambda+\nu+\frac{2}{3}; -\frac{\alpha^3 t^3}{27}\right)$
(12)	$\frac{2\Gamma(\nu)(\lambda p + \mu)}{(p^2 - \beta^2)(p + \alpha)^\nu} \quad \text{Re } \nu > 0$	$\left(\lambda + \frac{\mu}{\beta}\right) e^{\beta t} \frac{\gamma[\nu, (\alpha + \beta)t]}{(\alpha + \beta)^\nu} + \left(\lambda - \frac{\mu}{\beta}\right) e^{-\beta t} \frac{\gamma[\nu, (\alpha - \beta)t]}{(\alpha - \beta)^\nu}$
(13)	$[(p + \alpha)^{\frac{\nu}{2}} + \beta^{\frac{\nu}{2}}]^\nu \quad \text{Re } \nu < 0$	$-2^{\frac{\nu}{2}} \pi^{-\frac{\nu}{2}} \nu(2t)^{-\frac{\nu}{2}\nu-1} e^{(\frac{\nu}{2}\beta-\alpha)t} \times D_{\nu-1}(2^{\frac{\nu}{2}} \beta^{\frac{\nu}{2}} t^{\frac{\nu}{2}})$
(14)	$(p + \alpha)^{-\frac{\nu}{2}} [(p + \alpha)^{\frac{\nu}{2}} + \beta^{\frac{\nu}{2}}]^\nu \quad \text{Re } \nu < 1$	$2^{\frac{\nu}{2}} \pi^{-\frac{\nu}{2}} (2t)^{-\frac{\nu}{2}\nu-\frac{\nu}{2}} e^{(\frac{\nu}{2}\beta-\alpha)t} \times D_\nu(2^{\frac{\nu}{2}} \beta^{\frac{\nu}{2}} t^{\frac{\nu}{2}})$
(15)	$[(p + \alpha)^{\frac{\nu}{2}} + (p + \beta)^{\frac{\nu}{2}}]^{-2\nu} \quad \text{Re } \nu > 0$	$\nu(\alpha - \beta)^{-\nu} t^{-1} e^{-\frac{\nu}{2}(\alpha + \beta)t} I_\nu[\frac{1}{2}(\alpha - \beta)t]$
(16)	$[(p + \alpha)^{\frac{\nu}{2}} + (p - \alpha)^{\frac{\nu}{2}}]^{-2\nu} \times (p + \alpha)^{\frac{\nu}{2}} (p - \alpha)^{-\frac{\nu}{2}} \quad \text{Re } \nu > 0$	$\frac{1}{4}(2\alpha)^{1-\nu} [I_{\nu-1}(at) + 2I_\nu(at) + I_{\nu+1}(at)]$
(17)	$[(p + \alpha)^{\frac{\nu}{2}} + (p + \beta)^{\frac{\nu}{2}}]^{-2\nu} \times (p + \alpha)^{-\frac{\nu}{2}} (p + \beta)^{-\frac{\nu}{2}} \quad \text{Re } \nu > -1$	$(\alpha - \beta)^{-\nu} e^{-\frac{\nu}{2}(\alpha + \beta)t} I_\nu[\frac{1}{2}(\alpha - \beta)t]$
(18)	$\Gamma(\nu + \frac{1}{2}) r^{-2\nu-1} \quad \text{Re } \nu > -\frac{1}{2}$	$\pi^{\frac{\nu}{2}} (2\alpha)^{-\nu} t^\nu J_\nu(at)$
(19)	$\Gamma(\nu + \frac{1}{2}) s^{-2\nu-1} \quad \text{Re } \nu > -\frac{1}{2}$	$\pi^{\frac{\nu}{2}} (2\alpha)^{-\nu} t^\nu I_\nu(at)$

$$r = (p^2 + \alpha^2)^{\frac{\nu}{2}}, \quad R = p + r, \quad s = (p^2 - \alpha^2)^{\frac{\nu}{2}}, \quad S = p + s$$

## Arbitrary powers (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(20)	$\Gamma(\nu + \frac{1}{2}) p r^{-2\nu-1}$ $\operatorname{Re} \nu > 0$	$\pi^{\frac{1}{2}} a(2a)^{-\nu} t^\nu J_{\nu-1}(at)$
(21)	$a^\nu R^{-\nu}$ $\operatorname{Re} \nu > 0$	$\nu t^{-1} J_\nu(at)$
(22)	$a^\nu p R^{-\nu}$ $\operatorname{Re} \nu > 1$	$a\nu t^{-1} J_{\nu-1}(at)$ $- \nu(\nu+1)t^{-2} J_\nu(at)$
(23)	$r^{-1} R^{-\nu}$ $\operatorname{Re} \nu > -1$	$a^{-\nu} J_\nu(at)$
(24)	$a^{\nu-1} p r^{-1} R^{-\nu}$ $\operatorname{Re} \nu > 0$	$\frac{1}{2} J_{\nu-1}(at) - \frac{1}{2} J_{\nu+1}(at)$
(25)	$p^{-1} [a^{-\nu} R^\nu + a^\nu R^{-\nu} \cos(\nu\pi)]$ $ \operatorname{Re} \nu  < 1$	$1 + \cos(\nu\pi) - \nu \sin(\nu\pi)$ $\times \int_t^\infty x^{-1} Y_\nu(ax) dx$
(26)	$(p + \nu r)r^{-3} R^{-\nu}$ $\operatorname{Re} \nu > -2$	$a^{-\nu} t J_\nu(at)$
(27)	$\Gamma(\nu + \frac{1}{2}) p s^{-2\nu-1}$ $\operatorname{Re} \nu > 0$	$\pi^{\frac{1}{2}} a(2a)^{-\nu} t^\nu I_{\nu-1}(at)$
(28)	$a^\nu S^{-\nu}$ $\operatorname{Re} \nu > 0$	$\nu t^{-1} I_\nu(at)$
(29)	$a^\nu p S^{-\nu}$ $\operatorname{Re} \nu > 1$	$a\nu t^{-1} I_{\nu-1}(at)$ $- \nu(\nu+1)t^{-2} I_\nu(at)$
(30)	$a^\nu s^{-1} S^{-\nu}$ $\operatorname{Re} \nu > -1$	$I_\nu(at)$
(31)	$a^{\nu-1} p s^{-1} S^{-\nu}$ $\operatorname{Re} \nu > 0$	$\frac{1}{2} I_{\nu-1}(at) + \frac{1}{2} I_{\nu+1}(at)$
(32)	$s^{-1} (a^{-\nu} S^\nu - a^\nu S^{-\nu})$ $ \operatorname{Re} \nu  < 1$	$2\pi^{-1} \sin(\nu\pi) K_\nu(at)$

$$r = (p^2 + a^2)^{\frac{1}{2}}, \quad R = p + r, \quad s = (p^2 - a^2)^{\frac{1}{2}}, \quad S = p + s$$

**Arbitrary powers (cont'd)**

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$		$f(t)$
(33)	$(p + \nu s) s^{-3} S^{-\nu}$ $\operatorname{Re} \nu > -2$		$a^{-\nu} t I_\nu(at)$
(34)	$a^\nu v^{-1} V^{-\nu}$ $\operatorname{Re} \nu > -1$		$e^{-\frac{v}{2} i \pi \nu} [\operatorname{ber}_\nu(at) + i \operatorname{bei}_\nu(at)]$
(35)	$v^{-1} (a^{-\nu} V^\nu - e^{i \pi \nu} a^\nu V^{-\nu})$ $ \operatorname{Re} \nu  < 1$		$2 \pi^{-1} e^{\frac{v}{2} i \pi \nu} \sin(\nu \pi) [\operatorname{ker}_\nu(at) + i \operatorname{kei}_\nu(at)]$

**5.5. Exponential functions of arguments  $p$  and  $1/p$** 

(1)	$p^{-1} e^{-ap}$	$a > 0$	0 1	$0 < t < a$ $t > a$
(2)	$p^{-1} (1 - e^{-ap})$	$a > 0$	1 0	$0 < t < a$ $t > a$
(3)	$p^{-1} (e^{-ap} - e^{-bp})$	$0 \leq a < b$	0 1 0	$0 < t < a$ $a < t < b$ $t > b$
(4)	$p^{-2} (e^{-ap} - e^{-bp})$	$0 \leq a < b$	0 $t-a$ $b-a$	$0 < t < a$ $a < t < b$ $t > b$
(5)	$p^{-3} (e^{-ap} - e^{-bp})$	$0 \leq a < b$	0 $\frac{1}{2}(t-a)^2$ $t(b-a) + \frac{1}{2}(a^2 - b^2)$	$0 < t < a$ $a < t < b$ $t > b$

$$s = (p^2 - a^2)^{\frac{1}{2}}, \quad S = p + s, \quad v = (p^2 - i a^2)^{\frac{1}{2}}, \quad V = p + v$$

Exponential functions of  $p$  and  $1/p$  (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(6)	$p^{-2} (e^{-ap} - e^{-bp})^2 \quad 0 \leq a < b$	$0 \quad t < 2a$ $t - 2a \quad 2a < t < a + b$ $2b - t \quad a + b < t < 2b$ $0 \quad t > 2b$
(7)	$p^{-3} (e^{-ap} - e^{-bp})^2 \quad 0 \leq a < b$	$0 \quad t < 2a$ $\frac{1}{2}(t - 2a)^2 \quad 2a < t < a + b$ $(b - a)^2 - \frac{1}{2}(t - 2b)^2 \quad a + b < t < 2b$ $(b - a)^2 \quad t > 2b$
(8)	$p^{-3} (e^{-ap} - e^{-bp})^3 \quad 0 \leq a < b$	$0 \quad t < 3a$ $\frac{1}{2}(t - 3a)^2 \quad 3a < t < 2a + b$ $[(3/4)(b - a)^2] - [t - (3/2)(a + b)]^2 \quad 2a + b < t < a + 2b$ $\frac{1}{2}(3b - t)^2 \quad a + 2b < t < 3b$ $0 \quad t > 3b$
(9)	$(p + \beta)^{-1} e^{-ap} \quad a > 0$	$0 \quad 0 < t < a$ $e^{-\beta(t-a)} \quad t > a$
(10)	$(\lambda p + \mu)(p^2 - \beta^2)^{-1} e^{-ap} \quad a > 0$	$0 \quad 0 < t < a$ $\lambda \cosh[\beta(t-a)] + \mu \beta^{-1} \sinh[\beta(t-a)] \quad t > a$
(11)	$(\lambda p + \mu)(p^2 + \beta^2)^{-1} e^{-ap} \quad a > 0$	$0 \quad 0 < t < a$ $\lambda \cos[\beta(t-a)] + \mu \beta^{-1} \sin[\beta(t-a)] \quad t > a$
(12)	$(p^2 + a^2)^{-1} (1 - e^{-2m\pi p/a}) \quad a > 0, \quad m = 1, 2, \dots$	$a^{-1} \sin(at) \quad 0 < t < 2m\pi/a$ $0 \quad t > 2m\pi/a$

Exponential functions of  $p$  and  $1/p$  (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(13)	$p(p^2 + a^2)^{-1} (1 - e^{-2m\pi p/a})$ $a > 0, \quad m = 1, 2, 3, \dots$	$\cos(at) \quad 0 < t < 2m\pi/a$ 0 $t > 2m\pi/a$
(14)	$a^2 p^{-1} (p^2 + a^2)^{-1}$ $\times (1 - e^{-2m\pi p/a})$	$2 \sin^2(\frac{1}{2}at) \quad 0 < t < 2m\pi/a$ 0 $t > 2m\pi/a$
(15)	$p^{-1} (e^{ap} - 1)^{-1} \quad a > 0$	$n \quad na < t < (n+1)a$
(16)	$p^{-2} (e^{ap} - 1)^{-1} \quad a > 0$	$nt - \frac{1}{2}an(n+1) \quad na < t < (n+1)a$
(17)	$p^{-1} (e^{ap} + 1)^{-1} \quad a > 0$	0 $2na < t < (2n+1)a$ 1 $(2n+1)a < t < (2n+2)a$
(18)	$p^{-2} (e^{ap} + 1)^{-1} \quad a > 0$	$\frac{1}{4}[1 - (-1)^n](2t - a) + \frac{1}{2}(-1)^n na$ $na < t < (n+1)a$
(19)	$p^{-1} (e^{ap} - \beta)^{-1} \quad a > 0$	$(1 - \beta^n)/(1 - \beta) \quad na < t < (n+1)a$
(20)	$p^{-2} (e^{ap} - \beta)^{-1} \quad a > 0$	$(1 - \beta^n)(1 - \beta)^{-1} t - a(1 - \beta)^{-2}$ $\times [1 - (n+1)\beta^n + n\beta^{n+1}]$ $na < t < (n+1)a$
(21)	$\frac{1}{(p^2 + c^2)(e^{-ap} + 1)} \quad a > 0$ $c > 0, \quad ac \neq (2n+1)\pi$	$\frac{\sin(ct + \frac{1}{2}ac)}{2c \cos(\frac{1}{2}ac)}$ $+ 2a \sum_{n=0}^{\infty} \frac{\cos[(2n+1)\pi t/a]}{a^2 c^2 - (2n+1)^2 \pi^2}$
(22)	$g(p)(e^{ap} + \beta)^{-c} \quad a, c > 0$	$\sum_{0 \leq n \leq t/a-c} \binom{-c}{n} \beta^n f(t - ac - an)$

Exponential functions of  $p$  and  $1/p$  (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(23)	$g(p)(1 + \beta e^{-ap})^{\nu}$ $a > 0$	$\sum_{0 \leq n \leq t/a} \binom{\nu}{n} \beta^n f(t-an)$
(24)	$a(p^2 + a^2)^{-1} (1 + e^{-2\pi p/a})^{-1}$ $a > 0, m = 1, 2, \dots$	$\sin(at) \quad 2n < at/(2m\pi) < 2n + 1$ 0 $2n + 1 < at/(2m\pi) < 2n + 2$ $n = 0, 1, 2, \dots$
(25)	$p(p^2 + a^2)^{-1} (1 + e^{-2\pi p/a})^{-1}$ $a > 0, m = 1, 2, \dots$	$\cos(at) \quad 2n < at/(2m\pi) < 2n + 1$ 0 $2n + 1 < at/(2m\pi) < 2n + 2$ $n = 0, 1, 2, \dots$
(26)	$a^2 p^{-1} (p^2 + a^2)^{-1}$ $\times (1 + e^{-2\pi p/a})^{-1}$ $a > 0, m = 1, 2, \dots$	$2 \sin^2(\frac{1}{2}at)$ $2n < at/(2m\pi) < 2n + 1$ 0 $2n + 1 < at/(2m\pi) < 2n + 2$ $n = 0, 1, 2, \dots$
(27)	$(p^2 + a^2)^{-1} (1 + e^{-\pi p/a})$ $\times (1 - e^{-\pi p/a})^{-1} \quad a > 0$	$a^{-1}  \sin(at) $
(28)	$p(p^2 + a^2)^{-1} (1 + e^{-\pi p/a})$ $\times (1 - e^{-\pi p/a})^{-1}$	$\cos(at) \quad 2n\pi < at < (2n+1)\pi$ $- \cos(at) \quad (2n+1)\pi < at < (2n+2)\pi$ $n = 0, 1, 2, \dots$
(29)	$g(p)(1 + e^{-ap})(1 - e^{-ap})^{-1} \quad a > 0$	$f(t) + 2 \sum_{1 \leq n < t/a} f(t-an)$
(30)	$p^{-1} e^{-ap} (e^{ap} - 1)^n \quad a > 0$	$\binom{n}{m} \quad na < t < (n+1)a$ $n = 0, 1, 2, \dots$
(31)	$e^{a/p} - 1$	$a^{\frac{1}{2}} t^{-\frac{1}{2}} I_1(2a^{\frac{1}{2}} t^{\frac{1}{2}})$

**Exponential functions of  $p$  and  $1/p$  (cont'd)**

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(32)	$p^{-\frac{1}{2}} e^{\alpha/p}$	$\pi^{-\frac{1}{2}} t^{-\frac{1}{2}} \cosh(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$
(33)	$p^{-3/2} e^{\alpha/p}$	$\pi^{-\frac{1}{2}} \alpha^{-\frac{1}{2}} \sinh(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$
(34)	$p^{-5/2} e^{\alpha/p}$	$\pi^{-1/2} \alpha^{-1} t^{1/2} \cosh(2\alpha^{1/2} t^{1/2})$ $- \frac{1}{2} \pi^{-1/2} \alpha^{-3/2} \sinh(2\alpha^{1/2} t^{1/2})$
(35)	$p^{-\nu-1} e^{\alpha/p}$ Re $\nu > -1$	$\alpha^{-\frac{1}{2}\nu} t^{\frac{1}{2}\nu} I_\nu(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$
(36)	$1 - e^{-\alpha/p}$	$\alpha^{\frac{1}{2}} t^{-\frac{1}{2}} J_1(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$
(37)	$p^{-\frac{1}{2}} e^{-\alpha/p}$	$\pi^{-\frac{1}{2}} t^{-\frac{1}{2}} \cos(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$
(38)	$p^{-3/2} e^{-\alpha/p}$	$\pi^{-\frac{1}{2}} \alpha^{-\frac{1}{2}} \sin(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$
(39)	$p^{-5/2} e^{-\alpha/p}$	$\frac{1}{2} \pi^{-1/2} \alpha^{-3/2} \sin(2\alpha^{1/2} t^{1/2})$ $- \alpha^{-1} \pi^{-1/2} t^{1/2} \cos(2\alpha^{1/2} t^{1/2})$
(40)	$p^{-\nu-1} e^{-\alpha/p}$ Re $\nu > -1$	$\alpha^{-\frac{1}{2}\nu} t^{\frac{1}{2}\nu} J_\nu(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$

**5.6. Exponential functions of other arguments**

(1)	$e^{-\alpha^{\frac{1}{2}} p^{\frac{1}{2}}}$ Re $\alpha > 0$	$\frac{1}{2} \pi^{-1/2} \alpha^{1/2} t^{-3/2} e^{-\frac{1}{4}\alpha/t}$
(2)	$pe^{-\alpha^{\frac{1}{2}} p^{\frac{1}{2}}}$ Re $\alpha > 0$	$\frac{1}{4} \pi^{-1/2} \alpha^{1/2} (\frac{1}{2} \alpha t^{-1} - 3) t^{-5/2} e^{-\frac{1}{4}\alpha/t}$
(3)	$p^{-1} e^{-\alpha^{\frac{1}{2}} p^{\frac{1}{2}}}$ Re $\alpha \geq 0$	$\text{Erfc}(\frac{1}{2} \alpha^{\frac{1}{2}} t^{-\frac{1}{2}})$
(4)	$p^{-2} e^{-\alpha^{\frac{1}{2}} p^{\frac{1}{2}}}$ Re $\alpha \geq 0$	$(t + \frac{1}{2} \alpha) \text{Erfc}(\frac{1}{2} \alpha^{\frac{1}{2}} t^{-\frac{1}{2}})$ $- \pi^{-\frac{1}{2}} \alpha^{\frac{1}{2}} t^{\frac{1}{2}} e^{-\frac{1}{4}\alpha/t}$

## Exponential functions of other arguments (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(5)	$p^{\frac{1}{4}} e^{-\alpha^{\frac{1}{2}} p^{\frac{1}{2}}} \quad \text{Re } \alpha > 0$	$\pi^{-1/2} t^{-5/2} (\frac{1}{4} \alpha - \frac{1}{2} t) e^{-\frac{1}{4} \alpha/t}$
(6)	$p^{-\frac{1}{4}} e^{-\alpha^{\frac{1}{2}} p^{\frac{1}{2}}} \quad \text{Re } \alpha \geq 0$	$\pi^{-\frac{1}{2}} t^{-\frac{1}{2}} e^{-\frac{1}{4} \alpha/t}$
(7)	$p^{-3/2} e^{-\alpha^{\frac{1}{2}} p^{\frac{1}{2}}} \quad \text{Re } \alpha \geq 0$	$2\pi^{-\frac{1}{2}} t^{\frac{1}{2}} e^{-\frac{1}{4} \alpha/t} - \alpha^{\frac{1}{2}} \operatorname{Erfc}(\frac{1}{2} \alpha^{\frac{1}{2}} t^{-\frac{1}{2}})$
(8)	$p^{\frac{1}{2}n-\frac{1}{4}} e^{-\alpha^{\frac{1}{2}} p^{\frac{1}{2}}} \quad \text{Re } \alpha > 0$	$2^{-\frac{1}{2}n} \pi^{-\frac{1}{2}} t^{-\frac{1}{2}n-\frac{1}{2}} e^{-\frac{1}{4} \alpha/t} \times \operatorname{He}_n(2^{-\frac{1}{2}} \alpha^{\frac{1}{2}} t^{-\frac{1}{2}})$
(9)	$p^{\nu-\frac{1}{4}} e^{-\alpha^{\frac{1}{2}} p^{\frac{1}{2}}} \quad \text{Re } \alpha > 0$	$2^{-\nu} \pi^{-\frac{1}{2}} t^{-\nu-\frac{1}{4}} e^{-\alpha t^{-1}/8} \times D_{2\nu}(2^{-\frac{1}{2}} \alpha^{\frac{1}{2}} t^{-\frac{1}{2}})$
(10)	$2(p+\beta)^{-1} e^{-\alpha^{\frac{1}{2}} p^{\frac{1}{2}}} \quad \text{Re } \alpha \geq 0$	$e^{-\beta t} [e^{-i \alpha^{\frac{1}{2}} \beta^{\frac{1}{2}}} \operatorname{Erfc}(\frac{1}{2} \alpha^{\frac{1}{2}} t^{-\frac{1}{2}} - i \beta^{\frac{1}{2}} t^{\frac{1}{2}}) + e^{i \alpha^{\frac{1}{2}} \beta^{\frac{1}{2}}} \operatorname{Erfc}(\frac{1}{2} \alpha^{\frac{1}{2}} t^{-\frac{1}{2}} + i \beta^{\frac{1}{2}} t^{\frac{1}{2}})]$
(11)	$p(p^2 + \beta^2)^{-1} e^{-\alpha^{\frac{1}{2}} p^{\frac{1}{2}}} \quad \text{Re } \alpha \geq 0, \quad \text{Re } \beta \geq 0$	$\exp[-(\frac{1}{2} \alpha \beta)^{\frac{1}{2}}] \cos[\beta t - (\frac{1}{2} \alpha \beta)^{\frac{1}{2}}] - \frac{1}{\pi} \int_0^\infty e^{-ut} \sin(\alpha^{\frac{1}{2}} u^{\frac{1}{2}}) \frac{u}{u^2 + \beta^2} du$
(12)	$(p^{\frac{1}{2}} + \beta)^{-1} e^{-\alpha p^{\frac{1}{2}}} \quad \text{Re } \alpha^2 \geq 0$	$\pi^{-\frac{1}{2}} t^{-\frac{1}{2}} e^{-\frac{1}{4} \alpha^2/t} - \beta e^{\alpha \beta + \beta^2 t} \operatorname{Erfc}(\frac{1}{2} \alpha t^{-\frac{1}{2}} + \beta t^{\frac{1}{2}})$
(13)	$p(p^{\frac{1}{2}} + \beta)^{-1} e^{-\alpha p^{\frac{1}{2}}} \quad \text{Re } \alpha^2 > 0$	$\pi^{-1/2} t^{-3/2} (\frac{1}{4} \alpha^2 t^{-1} - \frac{1}{2} \alpha \beta + \beta^2 t) e^{-\frac{1}{4} \alpha^2/t} - \beta^3 e^{\alpha \beta + \beta^2 t} \operatorname{Erfc}(\frac{1}{2} \alpha t^{-1/2} + \beta t^{1/2})$
(14)	$\beta p^{-1} (p^{\frac{1}{2}} + \beta)^{-1} e^{-\alpha p^{\frac{1}{2}}} \quad \text{Re } \alpha^2 \geq 0$	$\operatorname{Erfc}(\frac{1}{2} \alpha t^{-\frac{1}{2}}) - e^{\alpha \beta + \beta^2 t} \operatorname{Erfc}(\frac{1}{2} \alpha t^{-\frac{1}{2}} + \beta t^{\frac{1}{2}})$

## Exponential functions of other arguments (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(15)	$p^{-\frac{1}{2}}(p^{\frac{1}{2}} + \beta)^{-1} e^{-ap^{\frac{1}{2}}}$ $\operatorname{Re} a^2 \geq 0$	$\pi^{-\frac{1}{2}} t^{-\frac{1}{2}} (\frac{1}{2}at^{-\frac{1}{2}} - \beta t^{\frac{1}{2}}) e^{-\frac{1}{4}a^2/t}$ + $\beta^2 e^{a\beta+\beta^2 t} \operatorname{Erfc}(\frac{1}{2}at^{-\frac{1}{2}} + \beta t^{\frac{1}{2}})$
(16)	$p^{-\frac{1}{2}}(p^{\frac{1}{2}} + \beta)^{-1} e^{-ap^{\frac{1}{2}}}$ $\operatorname{Re} a^2 \geq 0$	$e^{a\beta+\beta^2 t} \operatorname{Erfc}(\frac{1}{2}at^{-\frac{1}{2}} + \beta t^{\frac{1}{2}})$
(17)	$p^{-3/2}(p^{1/2} + \beta)^{-1} e^{-ap^{\frac{1}{2}}}$ $\operatorname{Re} a^2 \geq 0$	$2\pi^{-\frac{1}{2}} \beta^{-1} t^{\frac{1}{2}} e^{-\frac{1}{4}a^2/t}$ - $(\beta^{-2} + a\beta^{-1}) \operatorname{Erfc}(\frac{1}{2}at^{-\frac{1}{2}})$ + $\beta^{-2} e^{a\beta+\beta^2 t} \operatorname{Erfc}(\frac{1}{2}at^{-\frac{1}{2}} + \beta t^{\frac{1}{2}})$
(18)	$p^{-1}(p^{\frac{1}{2}} + \beta)^{-2} e^{-ap^{\frac{1}{2}}}$ $\operatorname{Re} a^2 > 0$	$\beta^{-2} \operatorname{Erfc}(\frac{1}{2}at^{-\frac{1}{2}})$ - $2\pi^{-\frac{1}{2}} \beta^{-1} t^{\frac{1}{2}} e^{-\frac{1}{4}a^2/t}$ + $(2t + a\beta^{-1} - \beta^{-2}) e^{a\beta+\beta^2 t}$ × $\operatorname{Erfc}(\frac{1}{2}at^{-\frac{1}{2}} + \beta t^{\frac{1}{2}})$
(19)	$p^{-\frac{1}{2}}(p^{\frac{1}{2}} + \beta)^{-2} e^{-ap^{\frac{1}{2}}}$ $\operatorname{Re} a^2 > 0$	$2\pi^{-\frac{1}{2}} t^{\frac{1}{2}} e^{-\frac{1}{4}a^2/t} - (2\beta t + a) e^{a\beta+\beta^2 t}$ × $\operatorname{Erfc}(\frac{1}{2}at^{-\frac{1}{2}} + \beta t^{\frac{1}{2}})$
See also Campbell, G. A. and R. M. Foster, 1931: <i>Fourier integrals for practical applications</i> , Bell Telephone Laboratories, New York		
(20)	$(p-1)^{-\frac{1}{2}} e^{-ap^{\frac{1}{2}}}$ $\operatorname{Re} a^2 > 0$	$\pi^{-\frac{1}{2}} t^{-\frac{1}{2}} e^{\frac{1}{4}t} \exp(-\frac{1}{4}a^2/t)$ - $a \int_0^\infty e^{-\frac{1}{4}u^2/t} (u^2 - a^2)^{-\frac{1}{2}}$ × $J_1[(u^2 - a^2)^{\frac{1}{2}}] du \}$
(21)	$p^{-\frac{1}{2}\nu-\frac{1}{2}} e^{-ap^{\frac{1}{2}}}$ $\operatorname{Re} a > 0, \quad \operatorname{Re} \nu > -1$	$\frac{1}{2} \pi^{-1/2} a^{-\nu/2} t^{-3/2}$ × $\int_0^\infty u^{1+\nu/2} e^{-\frac{1}{4}u^2/t}$ × $J_\nu(2a^{1/2}u^{1/2}) du$

## Exponential functions of other arguments (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(22)	$p^{-\nu-1} \exp[-\alpha^{-1} p^{-1} (p^2+1)^{\frac{\nu}{2}}]$ Re $\nu > -1$	$\alpha^{-\frac{1}{2}\nu} t^{\frac{1}{2}\nu} \left\{ J_\nu(2\alpha^{\frac{1}{2}}t^{\frac{1}{2}}) - \alpha \int_0^\infty \frac{J_\nu(2\alpha^{\frac{1}{2}}t^{\frac{1}{2}}) J_1((u^2-\alpha^2)^{\frac{1}{2}})}{(u^2-\alpha^2)^{\frac{1}{2}}} du \right\}$
(23)	$e^{-bp} - e^{-br}$	$b > 0$ 0 $0 < t < b$ $\alpha b y^{-1} J_1(\alpha y)$ $t > b$
(24)	$r^{-1} e^{-br}$	$b > 0$ 0 $0 < t < b$ $J_0(\alpha y)$ $t > b$
(25)	$(1-pr^{-1}) e^{-br}$	$b > 0$ 0 $0 < t < b$ $\alpha \left(\frac{t-b}{t+b}\right)^{\frac{1}{2}} J_1(\alpha y)$ $t > b$
(26)	$r^{-2}(b+r^{-1}) e^{-br}$	$b > 0$ 0 $0 < t < b$ $\alpha^{-1} y J_1(\alpha y)$ $t > b$
(27)	$e^{-bp} - pr^{-1} e^{-br}$	$b > 0$ 0 $0 < t < b$ $\alpha t y^{-1} J_1(\alpha y)$ $t > b$
(28)	$r^{-1} R^{\frac{1}{2}} e^{-br}$	$b > 0$ 0 $0 < t < b$ $2^{\frac{1}{2}} \pi^{-\frac{1}{2}} (t+b)^{-\frac{1}{2}} \cos(\alpha y)$ $t > b$
(29)	$\alpha r^{-1} R^{-\frac{1}{2}} e^{-br}$	$b > 0$ 0 $0 < t < b$ $2^{\frac{1}{2}} \pi^{-\frac{1}{2}} (t+b)^{-\frac{1}{2}} \sin(\alpha y)$ $t > b$

$$y = (t^2 - b^2)^{\frac{1}{2}}, \quad r = (p^2 + \alpha^2)^{\frac{1}{2}}, \quad R = p + r$$

## Exponential functions of other arguments (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(30)	$\alpha^\nu r^{-1} R^{-\nu} e^{-br}$ $b > 0$ $\operatorname{Re} \nu > -1$	$0 \quad 0 < t < b$ $(t-b)^{\frac{1}{2}\nu} (t+b)^{-\frac{1}{2}\nu} J_\nu(\alpha y) \quad t > b$
(31)	$1 - e^{-\beta(r-p)}$	$a\beta(t^2 + 2\beta t)^{-\frac{1}{2}} J_1[a(t^2 + 2\beta t)^{\frac{1}{2}}]$
(32)	$r^{-1} R^{\frac{1}{2}} e^{\beta(p-r)}$	$2^{\frac{1}{2}} \pi^{-\frac{1}{2}} (t+2\beta)^{-\frac{1}{2}} \cos[\alpha(t^2 + 2\beta t)^{\frac{1}{2}}]$
(33)	$\alpha r^{-1} R^{-\frac{1}{2}} e^{\beta(p-r)}$	$2^{\frac{1}{2}} \pi^{-\frac{1}{2}} (t+2\beta)^{-\frac{1}{2}} \sin[\alpha(t^2 + 2\beta t)^{\frac{1}{2}}]$
(34)	$\alpha^\nu r^{-1} R^{-\nu} e^{-\beta(r-p)} \quad \operatorname{Re} \nu > -1$	$t^{\frac{1}{2}\nu} (t+2\beta)^{-\frac{1}{2}\nu} J_\nu[a(t^2 + 2\beta t)^{\frac{1}{2}}]$
(35)	$e^{-bs} - e^{-bp}$ $b > 0$	$0 \quad 0 < t < b$ $a b y^{-1} I_1(\alpha y) \quad t > b$
(36)	$s^{-1} e^{-bs}$ $b > 0$	$0 \quad 0 < t < b$ $I_0(\alpha y) \quad t > b$
(37)	$ps^{-1} e^{-bs} - e^{-bp}$ $b > 0$	$0 \quad 0 < t < b$ $a t y^{-1} I_1(\alpha y) \quad t > b$
(38)	$(ps^{-1} - 1) e^{-bs}$ $b > 0$	$0 \quad 0 < t < b$ $a \left( \frac{t-b}{t+b} \right)^{\frac{1}{2}} I_1(\alpha y) \quad t > b$
(39)	$s^{-2}(b+s^{-1}) e^{-bs}$ $b > 0$	$0 \quad 0 < t < b$ $a^{-1} y I_1(\alpha y) \quad t > b$

$$y = (t^2 - b^2)^{\frac{1}{2}}, \quad r = (p^2 + \alpha^2)^{\frac{1}{2}}, \quad R = p + r,$$

$$s = (p^2 - \alpha^2)^{\frac{1}{2}}, \quad S = p + s$$

## Exponential functions of other arguments (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(40)	$\alpha^\nu s^{-1} S^{-\nu} e^{-bs}$ $\text{Re } \nu > -1$	$0 \quad 0 < t < b$ $(t-b)^{\frac{1}{2}\nu} (t+b)^{-\frac{1}{2}\nu} I_\nu(\alpha y) \quad t > b$
(41)	$e^{\beta(p-s)-1}$	$a\beta (t^2 + 2\beta t)^{-\frac{1}{2}} I_1[a(t^2 + 2\beta t)^{\frac{1}{2}}]$
(42)	$s^{-1} e^{\beta(p-s)}$	$I_0[a(t^2 + 2\beta t)]$
(43)	$1-ps^{-1} e^{\beta(p-s)}$	$-a(t+\beta)(t^2 + 2\beta t)^{-\frac{1}{2}} I_1[a(t^2 + 2\beta t)^{\frac{1}{2}}]$
(44)	$\alpha^\nu s^{-1} S^{-\nu} e^{\beta(p-s)} \quad \text{Re } \nu > -1$	$t^{\frac{1}{2}\nu} (t+2\beta)^{-\frac{1}{2}\nu} I_\nu[a(t^2 + 2\beta t)^{\frac{1}{2}}]$
(45)	$(p+a)^{-\frac{1}{2}} (p+\beta)^{-\frac{1}{2}} [p + \frac{1}{2}(a+\beta) + (p+a)^{\frac{1}{2}} (p+\beta)^{\frac{1}{2}}]^{-\nu} \times \exp[-b(p+a)^{\frac{1}{2}} (p+\beta)^{\frac{1}{2}}] \quad \text{Re } \nu > -1, \quad b > 0$	$0 \quad 0 < t < b$ $[\frac{1}{2}(a-\beta)]^{-\nu} (t-b)^{\frac{1}{2}\nu} (t+b)^{-\frac{1}{2}\nu} \times e^{-\frac{1}{2}(a+\beta)t} I_\nu[\frac{1}{2}(a-\beta)y] \quad t > b$
(46)	$(p+a)^{-\frac{1}{2}} (p+\beta)^{-\frac{1}{2}} [p + \frac{1}{2}(a+\beta) + (p+a)^{\frac{1}{2}} (p+\beta)^{\frac{1}{2}}]^{-\nu} \times \exp[\gamma p - \gamma(p+a)^{\frac{1}{2}} (p+\beta)^{\frac{1}{2}}] \quad \text{Re } \nu > -1$	$[\frac{1}{2}(a-\beta)]^{-\nu} t^{\frac{1}{2}\nu} (t+2\gamma)^{-\frac{1}{2}\nu} \times \exp[-\frac{1}{2}(a+\beta)(t+\gamma)] \times I_\nu[\frac{1}{2}(a-\beta)(t^2 + 2\gamma t)^{\frac{1}{2}}]$

## 5.7. Logarithmic functions

(1)	$p^{-1} \log p$	$-\log(\gamma t)$
(2)	$p^{-n-1} \log p$	$[1 + 1/2 + 1/3 + \dots + 1/n - \log(\gamma t)] t^n / n!$
(3)	$p^{-n-\frac{1}{2}} \log p$	$\frac{2^n t^{n-\frac{1}{2}}}{1 \cdot 3 \cdot 5 \cdots (2n-1) \pi^{\frac{1}{2}}} \left[ 2 \left( \frac{1}{1} + \frac{1}{3} + \dots + \frac{1}{2n-1} \right) - \log(4\gamma t) \right]$

$y = (t^2 - b^2)^{\frac{1}{2}}, \quad s = (p^2 - a^2)^{\frac{1}{2}}, \quad S = p + s$

## Logarithmic functions (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(4)	$\Gamma(\nu)p^{-\nu} \log p$ Re $\nu > 0$	$t^{\nu-1} [\psi(\nu) - \log t]$
(5)	$\frac{\log(p+\beta)}{p+\alpha}$	$e^{-\alpha t} \{ \log(\beta-\alpha) - \text{Ei}[(\alpha-\beta)t] \}$
(6)	$\alpha(p^2 + \alpha^2)^{-1} \log p$	$\cos(\alpha t) \text{Si}(\alpha t) + \sin(\alpha t) [\log \alpha - \text{Ci}(\alpha t)]$
(7)	$p(p^2 + \alpha^2)^{-1} \log p$	$\cos(\alpha t) [\log \alpha - \text{Ci}(\alpha t)] - \sin(\alpha t) \text{Si}(\alpha t)$
(8)	$p^{-1} (\log p)^2$	$[\log(\gamma t)]^2 - \pi^2/6$
(9)	$p^{-1} [\log(\gamma p)]^2$	$(\log t)^2 - \pi^2/6$
(10)	$p^{-2} (\log p)^2$	$t \{ [1 - \log(\gamma t)]^2 + 1 - \pi^2/6 \}$
(11)	$p^{-\alpha} (\log p)^{-1}$ $\alpha \geq 0$	$\nu(t, \alpha-1)$
(12)	$\log \frac{p+\beta}{p+\alpha}$	$\frac{e^{-\alpha t} - e^{-\beta t}}{t}$
(13)	$p \log \frac{p+\alpha}{p+\beta} + \beta - \alpha$	$(\alpha t^{-1} + t^{-2}) e^{-\alpha t} - (\beta t^{-1} + t^{-2}) e^{-\beta t}$
(14)	$p^{-1} \log(p^2 + \alpha^2)$	$2 \text{ci}(\alpha t) + 2 \log \alpha$
(15)	$\log \frac{p^2 + \beta^2}{p^2 + \alpha^2}$	$\frac{2}{t} [\cos(\alpha t) - \cos(\beta t)]$
(16)	$p \log \frac{p^2 + \beta^2}{p^2 + \alpha^2}$	$\frac{2}{t^2} [\cos(\beta t) + \beta t \sin(\beta t) - \cos(\alpha t) - \alpha t \sin(\alpha t)]$

## Logarithmic functions (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(17)	$\log \frac{(p+\alpha)^2 + \lambda^2}{(p+\beta)^2 + \lambda^2}$	$2t^{-1} \cos(\lambda t)(e^{-\beta t} - e^{-\alpha t})$
(18)	$\frac{\log[(p+\alpha)^\frac{1}{2} + (p+\beta)^\frac{1}{2}]}{(p+\beta)^\frac{1}{2}}$	$\frac{e^{-\beta t}}{2\pi^\frac{1}{2} t^\frac{1}{2}} \{\log(\alpha-\beta) - \text{Ei}[(\beta-\alpha)t]\}$
(19)	$p^{-1} \log r$	$\log \alpha + \text{ci}(\alpha t)$
(20)	$p^{-2} \log r$ Re $\alpha > 0$	$t [\log \alpha + \alpha^{-1} t^{-1} \sin(\alpha t) + \text{ci}(\alpha t)]$
(21)	$\alpha r^{-2} \log r$	$\frac{1}{2} \sin(\alpha t) \left[ \log \left( \frac{2\alpha}{\gamma t} \right) - \text{Ci}(2\alpha t) \right] + \frac{1}{2} \cos(\alpha t) \text{Si}(2\alpha t)$
(22)	$pr^{-2} \log r$	$\frac{1}{2} \cos(\alpha t) \left[ \log \frac{2\alpha}{\gamma t} - \text{Ci}(2\alpha t) \right] - \frac{1}{2} \sin(\alpha t) \text{Si}(2\alpha t)$
(23)	$p \log(r/p)$	$t^{-2} [\cos(\alpha t) - 1] + \alpha t^{-1} \sin(\alpha t)$
(24)	$r \log \frac{\alpha+r}{p} - \alpha$	$\frac{1}{2}\pi\alpha t^{-1} H_1(\alpha t)$
(25)	$pr^{-1} \log \frac{\alpha+r}{p}$	$\alpha - \frac{1}{2}\pi\alpha H_1(\alpha t)$
(26)	$r^{-1} \log R$	$\log \alpha J_0(\alpha t) - \frac{1}{2}\pi Y_0(\alpha t)$
(27)	$\alpha r^{-3} \log R$	$t [J_1(\alpha t) \log \alpha - \frac{1}{2}\pi Y_1(\alpha t)] - \alpha^{-1} \cos(\alpha t)$

**Logarithmic functions (cont'd)**

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(28)	$pr^{-3} \log R$	$t \{ J_0(\alpha t) \log \alpha - \frac{1}{2}\pi Y_0(\alpha t) \} + \alpha^{-1} \sin(\alpha t)$
(29)	$p \log(s/p)$	$t^{-2} [\cosh(\alpha t) - 1] - \alpha t^{-1} \sinh(\alpha t)$
(30)	$s^{-1} \log S$	$I_0(\alpha t) \log \alpha + K_0(\alpha t)$
(31)	$\alpha s^{-3} \log S$	$t [I_1(\alpha t) \log \alpha - K_1(\alpha t)] + \alpha^{-1} \cosh(\alpha t)$
(32)	$ps^{-3} \log S$	$t [I_0(\alpha t) \log \alpha + K_0(\alpha t)] + \alpha^{-1} \sinh(\alpha t)$
(33)	$2(A^2 - 1)^{-\frac{1}{2}}(B^2 - 1)^{-\frac{1}{2}}$ $\times \log \frac{(A+1)(B+1) + (A^2-1)^{\frac{1}{2}}(B^2-1)^{\frac{1}{2}}}{(A+1)(B+1) - (A^2-1)^{\frac{1}{2}}(B^2-1)^{\frac{1}{2}}}$ where $A^2 = p + \beta, \quad B^2 = p - \beta$	$-e^t I_0(\beta t) \operatorname{Ei}(-t)$

**5.8. Trigonometric functions**

(1)	$p^{-1} \sin(\alpha p^{-1})$	$\operatorname{bei}(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$
(2)	$p^{-1} \cos(\alpha p^{-1})$	$\operatorname{ber}(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$
(3)	$p^{-\frac{1}{2}} \sin(\alpha p^{-1})$	$\pi^{-\frac{1}{2}} t^{-\frac{1}{2}} \sinh(2^{\frac{1}{2}} \alpha^{\frac{1}{2}} t^{\frac{1}{2}}) \sin(2^{\frac{1}{2}} \alpha^{\frac{1}{2}} t^{\frac{1}{2}})$
(4)	$p^{-\frac{1}{2}} \cos(\alpha p^{-1})$	$\pi^{-\frac{1}{2}} t^{-\frac{1}{2}} \cosh(2^{\frac{1}{2}} \alpha^{\frac{1}{2}} t^{\frac{1}{2}}) \cos(2^{\frac{1}{2}} \alpha^{\frac{1}{2}} t^{\frac{1}{2}})$
(5)	$p^{-3/2} \sin(\alpha p^{-1})$	$\pi^{-\frac{1}{2}} \alpha^{-\frac{1}{2}} \cosh(2^{\frac{1}{2}} \alpha^{\frac{1}{2}} t^{\frac{1}{2}}) \sin(2^{\frac{1}{2}} \alpha^{\frac{1}{2}} t^{\frac{1}{2}})$

## Trigonometric functions (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(6)	$p^{-3/2} \cos(ap^{-1})$	$\pi^{-\frac{1}{4}} a^{-\frac{1}{4}} \sinh(2^{\frac{1}{4}} a^{\frac{1}{4}} t^{\frac{1}{4}}) \cos(2^{\frac{1}{4}} a^{\frac{1}{4}} t^{\frac{1}{4}})$
(7)	$p^{-\nu-1} \sin(ap^{-1}) \quad \text{Re } \nu > -2$	$(t/a)^{\frac{1}{4}\nu} [\cos(\frac{3}{4}\pi\nu) \operatorname{bei}_{\nu}(2a^{\frac{1}{4}}t^{\frac{1}{4}}) - \sin(\frac{3}{4}\pi\nu) \operatorname{ber}_{\nu}(2a^{\frac{1}{4}}t^{\frac{1}{4}})]$
(8)	$p^{-\nu-1} \cos(ap^{-1}) \quad \text{Re } \nu > -1$	$(t/a)^{\frac{1}{4}\nu} [\cos(\frac{3}{4}\pi\nu) \operatorname{ber}_{\nu}(2a^{\frac{1}{4}}t^{\frac{1}{4}}) + \sin(\frac{3}{4}\pi\nu) \operatorname{bei}_{\nu}(2a^{\frac{1}{4}}t^{\frac{1}{4}})]$
(9)	$p^{-\frac{1}{4}} e^{-a^{\frac{1}{4}} p^{\frac{1}{4}}} \sin(a^{\frac{1}{4}} p^{\frac{1}{4}})$	$\pi^{-\frac{1}{4}} t^{-\frac{1}{4}} \sin(\frac{1}{2}\alpha t^{-1})$
(10)	$p^{-\frac{1}{4}} e^{-a^{\frac{1}{4}} p^{\frac{1}{4}}} \cos(a^{\frac{1}{4}} p^{\frac{1}{4}})$	$\pi^{-\frac{1}{4}} t^{-\frac{1}{4}} \cos(\frac{1}{2}\alpha t^{-1})$
(11)	$p^{-\mu-\frac{1}{4}} \sin(a^{-\frac{1}{4}} p^{-\frac{1}{4}}) \quad \text{Re } \mu > -1$	$a^{-\frac{1}{4}} [\Gamma(\mu+1)]^{-1} t^\mu {}_0F_2(\mu+1, 3/2; -\frac{1}{4}t/a)$
(12)	$p^{-\mu-1} \cos(a^{-\frac{1}{4}} p^{-\frac{1}{4}}) \quad \text{Re } \mu > -1$	$[\Gamma(\mu+1)]^{-1} t^\mu {}_0F_2(\mu+1, \frac{1}{2}; -\frac{1}{4}t/a)$
(13)	$\Gamma(\nu+\frac{1}{2}) p^{-\nu} \sin[(2n+1)\sin^{-1}(p^{-\frac{1}{2}})] \quad \text{Re } \nu > -\frac{1}{2}$	$t^{\nu-\frac{1}{2}} (2n+1) {}_2F_2(-n, n; \nu+1/2, 3/2; t)$
(14)	$\Gamma(\nu) p^{-\nu} \cos[2n \sin^{-1}(p^{-\frac{1}{2}})] \quad \text{Re } \nu > 0$	$t^{\nu-1} {}_2F_2(-n, n; \nu, \frac{1}{2}; t)$
(15)	$p^{-1} \tan^{-1} p$	$-\operatorname{si}(t)$
(16)	$p^{-1} \operatorname{ctn}^{-1} p$	$\operatorname{Si}(t)$
(17)	$\tan^{-1}(ap^{-1})$	$t^{-1} \sin(at)$

**Trigonometric functions (cont'd)**

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(18)	$p \tan^{-1}(ap^{-1}) - a$	$t^{-2} [at \cos(at) - \sin(at)]$
(19)	$\log(p^2 + a^2)^{\frac{1}{2}} \tan^{-1}(ap^{-1})$	$-t^{-1} \log(\gamma t) \sin(at)$
(20)	$\frac{\sin[\beta + \tan^{-1}(ap^{-1})]}{(p^2 + a^2)^{\frac{1}{2}}}$	$\sin(at + \beta)$
(21)	$\frac{\cos[\beta + \tan^{-1}(ap^{-1})]}{(p^2 + a^2)^{\frac{1}{2}}}$	$\cos(at + \beta)$
(22)	$\tan^{-1}[2ap(p^2 + \beta^2)^{-1}]$	$2t^{-1} \sin(at) \cos[(\alpha^2 + \beta^2)^{\frac{1}{2}} t]$

**5.9. Hyperbolic functions**

(1)	$p^{-1} \operatorname{sech}(ap)$	$a > 0$	0 2	$4n - 1 < t/a < 4n + 1$ $4n + 1 < t/a < 4n + 3$
(2)	$p^{-2} \operatorname{sech}(ap)$	$a > 0$	$t - (-1)^n(t - 2an)$	$2n - 1 < t/a < 2n + 1$
(3)	$p^{-1} \operatorname{csch}(ap)$	$a > 0$	$2n$	$2n - 1 < t/a < 2n + 1$
(4)	$p^{-2} \operatorname{csch}(ap)$	$a > 0$	$2n(t - an)$	$2n - 1 < t/a < 2n + 1$
(5)	$a(p^2 + a^2)^{-1} \operatorname{csch}(\frac{1}{2}\pi p/a)$	$a > 0$	$ \cos(at)  - \cos(at)$	
(6)	$p^{-1} \operatorname{tanh}(pa)$	$a > 0$	$(-1)^{n-1}$	$n - 1 < \frac{1}{2}t/a < n$
(7)	$p^{-2} \operatorname{tanh}(ap)$	$a > 0$	$a + (-1)^n(2an - a - t)$	$n - 1 < \frac{1}{2}t/a < n$

## Hyperbolic functions (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(8)	$p^{-1} \operatorname{ctnh}(ap)$ $a > 0$	$2n-1$ $n-1 < \frac{1}{2}t/a < n$
(9)	$p^{-2} \operatorname{ctnh}(ap)$ $a > 0$	$(2n-1)t - 2an(n-1)$ $n-1 < \frac{1}{2}t/a < n$
(10)	$a(p^2 + a^2)^{-1} \operatorname{ctnh}(\frac{1}{2}\pi p/a)$ $a > 0$	$ \sin(at) $
(11)	$p^{-\frac{1}{2}} \sinh(ap^{-1})$	$\frac{1}{2}\pi^{-\frac{1}{2}} t^{-\frac{1}{2}} [\cosh(2a^{\frac{1}{2}}t^{\frac{1}{2}}) - \cos(2a^{\frac{1}{2}}t^{\frac{1}{2}})]$
(12)	$p^{-3/2} \sinh(ap^{-1})$	$\frac{1}{2}a^{-\frac{1}{2}}\pi^{-\frac{1}{2}} [\sinh(2a^{\frac{1}{2}}t^{\frac{1}{2}}) - \sin(2a^{\frac{1}{2}}t^{\frac{1}{2}})]$
(13)	$p^{-5/2} \sinh(ap^{-1})$	$\frac{1}{2}a^{-1}\pi^{-1/2}t^{1/2} [\cosh(2a^{1/2}t^{1/2}) + \cos(2a^{1/2}t^{1/2})] - \frac{1}{4}a^{-3/2}\pi^{-1/2} \times [\sinh(2a^{1/2}t^{1/2}) + \sin(2a^{1/2}t^{1/2})]$
(14)	$p^{-\nu-1} \sinh(ap^{-1})$ $\operatorname{Re } \nu > -2$	$\frac{1}{2}a^{-\frac{1}{2}\nu} t^{\frac{1}{2}\nu} [I_\nu(2a^{\frac{1}{2}}t^{\frac{1}{2}}) - J_\nu(2a^{\frac{1}{2}}t^{\frac{1}{2}})]$
(15)	$p^{-\frac{1}{2}} \cosh(ap^{-1})$	$\frac{1}{2}\pi^{-\frac{1}{2}} t^{-\frac{1}{2}} [\cos(2a^{\frac{1}{2}}t^{\frac{1}{2}}) + \cosh(2a^{\frac{1}{2}}t^{\frac{1}{2}})]$
(16)	$p^{-3/2} \cosh(ap^{-1})$	$\frac{1}{2}a^{-\frac{1}{2}}\pi^{-\frac{1}{2}} [\sinh(2a^{\frac{1}{2}}t^{\frac{1}{2}}) + \sin(2a^{\frac{1}{2}}t^{\frac{1}{2}})]$
(17)	$p^{-5/2} \cosh(ap^{-1})$	$\frac{1}{2}a^{-1}\pi^{-1/2}t^{1/2} [\cosh(2a^{1/2}t^{1/2}) - \cos(2a^{1/2}t^{1/2})] - \frac{1}{4}a^{-3/2}\pi^{-1/2} \times [\sinh(2a^{1/2}t^{1/2}) - \sin(2a^{1/2}t^{1/2})]$
(18)	$p^{-\nu-1} \cosh(ap^{-1})$ $\operatorname{Re } \nu > -1$	$\frac{1}{2}a^{-\frac{1}{2}\nu} t^{\frac{1}{2}\nu} [I_\nu(2a^{\frac{1}{2}}t^{\frac{1}{2}}) + J_\nu(2a^{\frac{1}{2}}t^{\frac{1}{2}})]$

## Hyperbolic functions (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(19)	$\operatorname{sech}(p^{\frac{v}{2}})$	$-\left[ \frac{\partial}{\partial v} \theta_1(\tfrac{1}{2}v  i\pi t) \right]_{v=0}$
(20)	$p^{-\frac{v}{2}} \operatorname{sech}(p^{\frac{v}{2}})$	$\hat{\theta}_2(\tfrac{1}{2}  i\pi t)$
(21)	$\operatorname{csch}(p^{\frac{v}{2}})$	$-\left[ \frac{\partial}{\partial v} \hat{\theta}_4(\tfrac{1}{2}v  i\pi t) \right]_{v=0}$
(22)	$p^{-\frac{v}{2}} \operatorname{csch}(p^{\frac{v}{2}})$	$\theta_4(0  i\pi t)$
(23)	$p^{-1} \tanh(p^{\frac{v}{2}})$	$\int_0^1 \hat{\theta}_2(\tfrac{1}{2}v  i\pi t) dv$
(24)	$p^{-\frac{v}{2}} \tanh(p^{\frac{v}{2}})$	$\theta_2(0  i\pi t)$
(25)	$p^{-\frac{v}{2}} \tanh(p^{\frac{v}{2}} + a)$	$e^{at^2} [\theta_3(at  i\pi t) + \hat{\theta}_3(at  i\pi t)] - \pi^{-\frac{v}{2}} t^{-\frac{v}{2}}$
(26)	$p^{-\frac{v}{2}} \operatorname{ctnh}(p^{\frac{v}{2}})$	$\theta_3(0  i\pi t)$
(27)	$\frac{\sinh(xp)}{p \cosh(ap)}$ $0 \leq x \leq a$	$\frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n - \frac{1}{2}} \sin[(n - \frac{1}{2})\pi x/a] \times \sin[(n - \frac{1}{2})\pi t/a]$
(28)	$\frac{\cosh(xp)}{p \cosh(ap)}$ $-a \leq x \leq a$	$1 + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n - \frac{1}{2}} \cos[(n - \frac{1}{2})\pi x/a] \times \cos[(n - \frac{1}{2})\pi t/a]$

## Hyperbolic functions (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(29)	$\frac{\sinh(xp)}{p^2 \cosh(ap)}$ $0 \leq x \leq a$	$x + \frac{2a}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{(n - \frac{1}{2})^2} \sin[(n - \frac{1}{2})\pi x/a]$ $\times \cos[(n - \frac{1}{2})\pi t/a]$
(30)	$\frac{\cosh(xp)}{p^2 \cosh(ap)}$ $-a \leq x \leq a$	$t + \frac{2a}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{(n - \frac{1}{2})^2} \cos[(n - \frac{1}{2})\pi x/a]$ $\times \sin[(n - \frac{1}{2})\pi t/a]$
(31)	$\frac{\sinh(xp^{\frac{1}{2}})}{\sinh(lp^{\frac{1}{2}})}$ $-l < x < l$	$\frac{1}{l} \frac{\partial}{\partial x} \theta_4\left(\frac{x}{2l} \middle  \frac{i\pi t}{l^2}\right)$
(32)	$\frac{\sinh(xp^{\frac{1}{2}})}{p^{\frac{1}{2}} \sinh(lp^{\frac{1}{2}})}$ $-l \leq x \leq l$	$-\frac{1}{l} \hat{\theta}_4\left(\frac{x}{2l} \middle  \frac{i\pi t}{l^2}\right)$
(33)	$\frac{\sinh(xp^{\frac{1}{2}})}{\cosh(lp^{\frac{1}{2}})}$ $-l < x < l$	$-\frac{1}{l} \frac{\partial}{\partial x} \hat{\theta}_1\left(\frac{x}{2l} \middle  \frac{i\pi t}{l^2}\right)$
(34)	$\frac{\sinh(xp^{\frac{1}{2}})}{p^{\frac{1}{2}} \cosh(lp^{\frac{1}{2}})}$ $-l \leq x \leq l$	$-\frac{1}{l} \theta_1\left(\frac{x}{2l} \middle  \frac{i\pi t}{l^2}\right)$
(35)	$\frac{\cosh(xp^{\frac{1}{2}})}{\sinh(lp^{\frac{1}{2}})}$ $-l \leq x \leq l$	$-\frac{1}{l} \frac{\partial}{\partial x} \hat{\theta}_4\left(\frac{x}{2l} \middle  \frac{i\pi t}{l^2}\right)$
(36)	$\frac{\cosh(xp^{\frac{1}{2}})}{p^{\frac{1}{2}} \sinh(lp^{\frac{1}{2}})}$ $-l \leq x \leq l$	$\frac{1}{l} \theta_4\left(\frac{x}{2l} \middle  \frac{i\pi t}{l^2}\right)$
(37)	$\frac{\cosh(xp^{\frac{1}{2}})}{\cosh(lp^{\frac{1}{2}})}$ $-l \leq x \leq l$	$-\frac{1}{l} \frac{\partial}{\partial x} \theta_1\left(\frac{x}{2l} \middle  \frac{i\pi t}{l^2}\right)$

## Hyperbolic functions (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(38)	$\frac{\cosh(xp^{\frac{1}{2}})}{p^{\frac{1}{2}} \cosh(lp^{\frac{1}{2}})}$ $-l \leq x \leq l$	$-\frac{1}{l} \hat{\theta}_1\left(\frac{x}{2l}, \frac{t}{l^2}\right)$
(39)	$\frac{1}{p-i\omega} \frac{\sinh(xp^{\frac{1}{2}})}{\sinh(lp^{\frac{1}{2}})}$ $l \geq x > 0$	$\frac{\sinh(xi^{\frac{1}{2}}\omega^{\frac{1}{2}})}{\sinh(li^{\frac{1}{2}}\omega^{\frac{1}{2}})} e^{i\omega t}$ $+ 2\pi \sum_{n=1}^{\infty} \frac{n(-1)^n \sin(n\pi x/l)}{n^2\pi^2 + i\omega l^2}$ $\times e^{-n^2\pi^2 t/l^2}$
(40)	$\frac{1}{p-i\omega} \frac{\cosh(xp^{\frac{1}{2}})}{\cosh(lp^{\frac{1}{2}})}$	$\frac{\cosh(xi^{\frac{1}{2}}\omega^{\frac{1}{2}})}{\cosh(xi^{\frac{1}{2}}\omega^{\frac{1}{2}})} e^{i\omega t}$ $- 2p \sum_{n=0}^{\infty} \frac{(n+\frac{1}{2})(-1)^n \cos[(n+\frac{1}{2})\pi x/l]}{(n+\frac{1}{2})^2\pi^2 + i\omega l^2}$ $\times e^{-(n+\frac{1}{2})^2\pi^2 t/l^2}$
(41)	$p^{-1} \sinh^{-1}(p/a)$	$-Ji_0(at)$
(42)	$(p^2 + a^2)^{-\frac{1}{2}} \sinh^{-1}(p/a)$	$-\frac{1}{2} \pi Y_0(at)$
(43)	$(p^2 - a^2)^{-\frac{1}{2}} \cosh^{-1}(p/a)$	$K_0(at)$
(44)	$\operatorname{ctnh}^{-1}(p/a)$	$t^{-1} \sin(at)$
(45)	$p^{-1} (\sinh^{-1} p)^2$	$\int_t^\infty r^{-1} Y_0(r) dr$

## 5.10. Orthogonal polynomials

(1)	$(p+\beta)^{-n-1} P_n\left(\frac{p+\alpha}{p+\beta}\right)$	$\frac{t^n}{n!} e^{-\beta t} L_n[\frac{1}{2}(\beta-\alpha)t]$
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## Orthogonal polynomials (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(2)	$(p+\beta)^{-\nu} P_n \left( \frac{p+\alpha}{p+\beta} \right)$ Re $\nu > 0$	$\frac{t^{\nu-1}}{\Gamma(\nu)} e^{-\beta t} {}_2F_2[-n, n+1; 1, \nu; \frac{1}{2}(\beta-\alpha)t]$
(3)	$\frac{(p-\alpha-\beta)^n}{p^{n+1}} \\ \times P_n \left[ \frac{p^2 - (\alpha+\beta)p + 2\alpha\beta}{p(p-\alpha-\beta)} \right]$	$L_n(\alpha t) L_n(\beta t)$
(4)	$n! p^{-\frac{n}{2}} P_n(p^{-1})$	$i^{-n} \pi^{-\frac{n}{2}} t^{-\frac{n}{2}} \text{He}_n(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}}) \text{He}_n(i 2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$
(5)	$\frac{(\alpha+\beta-p)^{\frac{n}{2}}}{(\alpha+\beta+p)^{\frac{n}{2}+\frac{1}{2}}} \\ \times P_n \left\{ \frac{2\alpha^{\frac{1}{2}} \beta^{\frac{1}{2}}}{[(\alpha+\beta)^2 - p^2]^{\frac{1}{2}}} \right\}$	$\frac{e^{-2\alpha t} \text{He}_n(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}}) \text{He}_n(2\beta^{\frac{1}{2}} t^{\frac{1}{2}})}{n! \pi^{\frac{n}{2}} t^{\frac{n}{2}}}$
(6)	$(p+\beta)^{-\mu} C_n^\nu \left( \frac{p+\alpha}{p+\beta} \right)$ Re $\mu > 0$ , Re $\nu > 0$	$\frac{t^{\mu-1} e^{-\beta t}}{n B(n, 2\nu) \Gamma(\mu)} \\ \times {}_2F_2[-n, n+2\nu; \mu, \nu+\frac{1}{2}; \frac{1}{2}(\beta-\alpha)t]$
(7)	$p^{-n-\frac{1}{2}} e^{-\alpha/p} \text{He}_{2n}(2\alpha^{\frac{1}{2}} p^{-\frac{1}{2}})$	$(-2)^n \pi^{-\frac{n}{2}} t^{n-\frac{1}{2}} \cos(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$
(8)	$p^{-n-1} e^{\alpha/p} \text{He}_{2n+1}(2\alpha^{\frac{1}{2}} p^{-\frac{1}{2}})$	$(-1)^n 2^{n+\frac{1}{2}} \pi^{-\frac{n}{2}} t^n \sin(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$
(9)	$p^{-\beta} L_n^\alpha \left( \frac{\lambda}{p} \right)$ Re $\beta > 0$	$\frac{t^{\beta-1} {}_1F_2(-n; \alpha+1, \beta; \lambda t)}{n \Gamma(\beta) B(n, \alpha+1)}$
(10)	$n! p^{-n-\alpha-1} e^{-\lambda/p} L_n^\alpha(\lambda p^{-1})$ Re $\alpha > -n - 1$	$\lambda^{-\frac{n}{2}} t^{\frac{n}{2}\alpha+n} J_\alpha(2\lambda^{\frac{1}{2}} t^{\frac{1}{2}})$

### Orthogonal polynomials (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(11)	$\frac{(p-1)^n e^{-\lambda/p}}{p^{n+\alpha+1}} L_n^\alpha \left[ \frac{\lambda}{p(1-p)} \right]$ $\text{Re } \alpha > -1$	$\lambda^{-\frac{n}{2}\alpha} t^{\frac{n}{2}\alpha} L_n^\alpha(t) J_\alpha(2\lambda^{\frac{1}{2}} t^{\frac{1}{2}})$
(12)	$n! B(n + \frac{1}{2}, p + \frac{1}{2}) L_n^p(\lambda)$	$(-2)^{-n} (e^t - 1)^{-\frac{n}{2}} H_{2n} \{ [2\lambda(1-e^{-t})]^{\frac{n}{2}} \}$
(13)	$n! B(n + 3/2, p) L_n^p(\lambda)$	$(-1)^n 2^{-n-\frac{1}{2}} \lambda^{-\frac{n}{2}} \pi^{-\frac{1}{2}}$ $\times H_{2n+1} \{ [2\lambda(1-e^{-t})]^{\frac{n}{2}} \}$

### 5.11. Gamma function, incomplete gamma functions, zeta function and related functions

(1)	$\Gamma(\nu) \Gamma(\alpha p)/\Gamma(\alpha p + \nu)$ $\text{Re } \alpha > 0, \quad \text{Re } \nu > 0$	$\alpha^{-1} (1-e^{-t/\alpha})^{\nu-1}$
(2)	$2^{1-2p} \Gamma(2p) [\Gamma(p + \lambda + \frac{1}{2})$ $\times \Gamma(p - \lambda + \frac{1}{2})]^{-1}$	$\pi^{-1} (1-e^{-t})^{-\frac{1}{2}} \cos[2\lambda \cos^{-1}(e^{-\frac{1}{2}t})]$
(3)	$\frac{2^{p-1} \Gamma(\frac{1}{2}p + \frac{1}{2}\nu + \frac{1}{2}) \Gamma(\frac{1}{2}p - \frac{1}{2}\nu)}{\pi^{\frac{1}{2}} \Gamma(p + \mu + 1)}$ $\text{Re } \mu > -\frac{1}{2}$	$(1-e^{-2t})^{\frac{1}{2}\mu} P_\nu^{-\mu}(e^{-t})$
(4)	$2^{-p} \Gamma(p) [\Gamma(\frac{1}{2}p + \frac{1}{2}n + \frac{1}{2})$ $\times \Gamma(\frac{1}{2}p - \frac{1}{2}n + \frac{1}{2})]^{-1}$	$\pi^{-1} (1-e^{-2t})^{-\frac{1}{2}} T_n(e^{-t})$
(5)	$\frac{\Gamma(p+\alpha)}{\Gamma(p+\beta)} (y+p)_n \quad \text{Re } (\beta - \alpha) > n$	$\frac{e^{-\alpha t}}{\Gamma(\beta - \alpha - n)} (1-e^{-t})^{\beta - \alpha - n - 1}$ $\times {}_2F_1(-n, \beta - \gamma - n; \beta - \alpha - n; 1 - e^{-t})$

## Gamma functions etc. (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(6)	$\frac{\Gamma[\frac{1}{2}(p-n-\mu)] \Gamma[\frac{1}{2}(p+n-\mu+1)]}{\Gamma[\frac{1}{2}(p+n+\mu)+1] \Gamma[\frac{1}{2}(p-n+\mu+1)]}$ $\text{Re } \mu > -\frac{1}{2}$	$\frac{2^{\mu+1} \pi^{\frac{n}{2}}}{\Gamma(\mu + \frac{1}{2})} \sinh^\mu t P_n^{-\mu}(\cosh t)$
(7)	$\frac{\Gamma(p+\alpha) \Gamma(p+\beta)}{\Gamma(p+\gamma) \Gamma(p+\delta)}$ $\text{Re } (\gamma + \delta - \alpha - \beta) > 0$	$\frac{e^{-\alpha t} (1-e^{-t})^{\gamma+\delta-\alpha-\beta-1}}{\Gamma(\gamma+\delta-\alpha-\beta)}$ $\times {}_2F_1(\delta-\beta, \gamma-\beta; \gamma+\delta-\alpha-\beta; 1-e^{-t})$
(8)	$\log \frac{e^p \Gamma(p)}{2^{\frac{n}{2}} \pi^{\frac{n}{2}} p^{p-\frac{n}{2}}}$	$\frac{1}{t} \left( \frac{1}{1-e^{-t}} - \frac{1}{t} - \frac{1}{2} \right)$
(9)	$\log \frac{(p+\alpha)^{\frac{n}{2}} \Gamma(p+\alpha)}{\Gamma(p+\alpha+\frac{1}{2})}$	$\frac{1}{2} t^{-1} e^{-\alpha t} \tanh(\frac{1}{4}t)$
(10)	$\log \frac{\Gamma(p+\alpha+\frac{3}{4})}{(p+\alpha)^{\frac{n}{2}} \Gamma(p+\alpha+\frac{1}{4})}$	$\frac{1}{2} t^{-1} e^{-\alpha t} [1 - \operatorname{sech}(\frac{1}{4}t)]$
(11)	$\log \frac{\Gamma(p+\alpha) \Gamma(p+\beta+\frac{1}{2})}{\Gamma(p+\alpha+\frac{1}{2}) \Gamma(p+\beta)}$	$\frac{e^{-\alpha t} - e^{-\beta t}}{t(1+e^{-\frac{n}{2}t})}$
(12)	$\log \frac{\Gamma(p+\alpha) \Gamma(p+\beta+\gamma)}{\Gamma(p+\alpha+\gamma) \Gamma(p+\beta)}$	$\frac{(e^{-\alpha t} - e^{-\beta t})(1-e^{-\gamma t})}{t(1-e^{-t})}$
(13)	$p^{-1} \psi(ap) \quad \text{Re } \alpha > 0$	$-\log[\gamma(e^{t/\alpha}-1)]$
(14)	$\psi(\frac{1}{2}p + \frac{1}{2}) - \psi(\frac{1}{2}p)$	$2(1+e^{-t})^{-1}$
(15)	$p^{-1} [\psi(\frac{1}{2}p + \frac{1}{2}) - \psi(\frac{1}{2}p)]$	$2 \log(\frac{1}{2} + \frac{1}{2}e^{-t})$

## Gamma functions etc. (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(16)	$\psi(ap + \beta) - \psi(ap + \gamma)$ Re $a > 0$	$a^{-1}(e^{-\gamma t/a} - e^{-\beta t/a})(1 - e^{-t/a})^{-1}$
(17)	$\psi(p + \alpha) + \psi(p + \beta) - \psi(p)$ $- \psi(p + \alpha + \beta)$	$(1 - e^{-\alpha t})(1 - e^{-\beta t})(1 - e^{-t})^{-1}$
(18)	$p^{-1} [\psi(p) - \log p]$	$\log[t(e^t - 1)^{-1}]$
(19)	$\psi(p) - \log p$	$t^{-1} - (1 - e^{-t})^{-1}$
(20)	$\frac{\Gamma(p)\Gamma(a)}{\Gamma(p+a)} [\psi(p+a) - \psi(p)]$ Re $a > 0$	$t(1 - e^{-t})^{a-1}$
(21)	$\log \frac{\Gamma(ap + \beta)}{\Gamma(ap + \lambda)} + (\lambda - \beta) \psi(ap + \delta)$ Re $a > 0$	$(1 - e^{-t/a})^{-1} [t^{-1}(e^{-\beta t/a} - e^{-\lambda t/a}) + a^{-1}(\beta - \lambda)e^{-\delta t/a}]$
(22)	$\psi^{(n)}(ap)$	$(-a)^{-n-1} t^n (1 - e^{-t/a})^{-1}$
(23)	$p^{-\nu} \gamma(\nu, bp)$ Re $\nu > 0$ , $b > 0$	$t^{\nu-1} \quad 0 < t < b$ 0 $\quad t > b$
(24)	$p^{-\nu} e^{-bp} \gamma(\nu, -bp)$ Re $\nu > 0$ , $b > 0$	$(b-t)^{\nu-1} \quad 0 < t < b$ 0 $\quad t > b$
(25)	$\gamma(\nu, a/p)$ Re $\nu > 0$	$a^{\frac{\nu}{2}} t^{\frac{\nu}{2}-1} J_\nu(2a^{\frac{\nu}{2}} t^{\frac{1}{2}})$
(26)	$p^{\nu-1} e^{\alpha/p} \gamma(\nu, a/p)$ Re $\nu > 0$	$\Gamma(\nu) a^{\frac{\nu}{2}} t^{-\frac{\nu}{2}} I_\nu(2a^{\frac{\nu}{2}} t^{\frac{1}{2}})$
(27)	$p^{\nu-3/2} e^{\alpha/p} \gamma(\nu, a/p)$	$\Gamma(\nu) (t/a)^{\frac{\nu}{2}-\frac{3}{2}} L_{\nu-\frac{1}{2}}(2a^{\frac{\nu}{2}} t^{\frac{1}{2}})$

## Gamma functions etc. (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(28)	$p^\mu \gamma(\nu, a/p)$ $\operatorname{Re} \nu > 0, \quad \operatorname{Re}(\nu - \mu) > 0$	$t^{-\mu-1} \int_0^{at} u^{\frac{\nu}{2}\nu + \frac{\mu}{2}\mu - \frac{1}{2}} J_{\nu-\mu-1}(2u^{\frac{1}{2}}) du$
(29)	$\nu \Gamma(\nu - \mu) p^\mu e^{a/p} \gamma(\nu, a/p)$ $\operatorname{Re} \nu > 0, \quad \operatorname{Re} \mu > 0$	$a^\nu t^{\nu-\mu-1} {}_1F_2(1; \nu+1, \nu-\mu; at)$
(30)	$\gamma[\nu, \frac{1}{2}(p^2 + a^2)^{\frac{1}{2}} - \frac{1}{2}p]$ $\operatorname{Re} \nu > 0$	$t^{\frac{1}{2}\nu-1} (t+1)^{-\frac{1}{2}\nu} J_\nu[at^{\frac{1}{2}}(t+1)^{\frac{1}{2}}]$
(31)	$a^{-p} \gamma(p, a)$	$\exp(-ae^{-t})$
(32)	$\Gamma(\nu, bp)$ $b > 0, \quad \operatorname{Re} \nu < 1$	$0 \quad 0 < t < b$ $\frac{b^\nu}{\Gamma(1-\nu)t(t-b)} \quad t > b$
(33)	$\Gamma(1-\nu)e^{\alpha p} \Gamma(\nu, \alpha p)$ $\operatorname{Re} \nu < 1$	$\alpha^\nu (t+a)^{-1} t^{-\nu}$
(34)	$p^{-\nu} \Gamma(\nu, bp)$ $b > 0$	$0 \quad 0 < t < b$ $t^{\nu-1} \quad t > b$
(35)	$p^{-\nu} e^{\alpha p} \Gamma(\nu, \alpha p)$	$(t+a)^{\nu-1}$
(36)	$\alpha^{\frac{\nu}{2}} \Gamma(1-2\nu) e^{\alpha p^2} \Gamma(\nu, \alpha p^2)$ $\operatorname{Re} \alpha > 0, \quad \operatorname{Re} \nu < 1$	$4^\nu e^{i\pi(\nu-\frac{1}{2}) - \frac{1}{4}t^2/\alpha}$ $\times \gamma(\frac{1}{2}-\nu, \frac{1}{4}e^{i\pi} t^2/\alpha)$
(37)	$p^{-\frac{\nu}{2}} e^{\frac{1}{4}\alpha p^2} \Gamma(\frac{1}{4}, \frac{1}{4}\alpha p^2)$ $\operatorname{Re} \alpha > 0$	$\Gamma(\frac{1}{4}) \alpha^{-\frac{\nu}{2}} t^{\frac{\nu}{2}} e^{-\frac{1}{2}t^2/\alpha} I_{\frac{\nu}{2}}(\frac{1}{2}t^2/\alpha)$
(38)	$p^{-2\nu} e^{\frac{1}{2}\alpha^2 p^2} \Gamma(\nu, \frac{1}{2}\alpha^2 p^2)$	$2^{-\frac{\nu}{2}} \pi^{-\frac{\nu}{2}} \Gamma(\nu) \alpha^{2\nu-1} e^{-\frac{1}{4}t^2/\alpha^2}$ $\times [D_{-2\nu}(-t/\alpha) - D_{-2\nu}(t/\alpha)]$

## Gamma functions etc. (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(39)	$\Gamma(1-\nu) p^{\nu-1} e^{\alpha/p} \Gamma(\nu, \alpha/p)$ Re $\nu < 1$	$2\alpha^{\frac{\nu}{2}} t^{-\frac{\nu}{2}} \nu K_\nu(2\alpha^{\frac{\nu}{2}} t^{\frac{\nu}{2}})$
(40)	$p^{\nu-3/2} e^{\alpha/p} \Gamma(\nu, \alpha/p)$ Re $\nu < 3/2$	$\Gamma(\nu) (\alpha/t)^{\frac{\nu}{2}} \nu^{-\frac{1}{2}} [ I_{\frac{\nu}{2}-\nu}(2\alpha^{\frac{\nu}{2}} t^{\frac{\nu}{2}}) - L_{\nu-\frac{1}{2}}(2\alpha^{\frac{\nu}{2}} t^{\frac{\nu}{2}}) ]$
(41)	$p^\mu \Gamma(\nu, \alpha/p)$ Re $(\mu + \nu) < -\frac{1}{2}$ , Re $\mu > 0$	$t^{-\mu-1} \int_{\alpha t}^\infty u^{\frac{\nu}{2} \mu + \frac{1}{2} \nu - \frac{1}{2}} J_{\nu-\mu-1}(2u^{\frac{\nu}{2}}) du$
(42)	$\alpha^p \Gamma(-p, \alpha)$ Re $\alpha > 0$	$e^{-\alpha e^{-t}}$
(43)	$p^{-1} \zeta(p)$	$n$ $\log n < t \leq \log(n+1)$
(44)	$p^{-1} \zeta(p+a)$	$\sum_{1 \leq n \leq \exp t} n^{-a}$
(45)	$\Gamma(a) p^{-a} \zeta(p)$ Re $a > 0$	$\sum_{1 \leq n \leq \exp t} (t - \log n)^{a-1}$
(46)	$p^{-1} \zeta'(p)/\zeta(p)$	$-\psi(e^{-t})$
(47)	$\Gamma(a) \zeta(a, \beta p)$ Re $a > 1$ , Re $\beta > 0$	$\beta^{-a} t^{a-1} (1 - e^{-t/\beta})^{-1}$

## 5.12. Error function, exponential integral and related functions

(1)	$e^{\alpha p^2} \operatorname{Erfc}(\alpha^{\frac{1}{2}} p)$ Re $\alpha > 0$	$\pi^{-\frac{1}{2}} \alpha^{-\frac{1}{2}} e^{-\frac{1}{4}t^2/\alpha}$
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## Error function etc. (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(2)	$e^{\alpha p^2} \operatorname{Erfc}(\alpha^{\frac{1}{2}} p + \frac{1}{2} \alpha^{-\frac{1}{2}} b)$ $\operatorname{Re} \alpha > 0, \quad b > 0$	$0 \quad 0 < t < b$ $\pi^{-\frac{1}{2}} \alpha^{-\frac{1}{2}} e^{-\frac{1}{4}t^2/\alpha} \quad t > b$
(3)	$p^{-1} e^{\alpha p^2} \operatorname{Erfc}(\alpha^{\frac{1}{2}} p)$ $\operatorname{Re} \alpha > 0$	$\operatorname{Erf}(\frac{1}{2} \alpha^{-\frac{1}{2}} t)$
(4)	$(p-a)^{-1} e^{ap^2} \operatorname{Erfc}(p)$	$e^{\alpha(t+a)} [\operatorname{Erf}(\frac{1}{2}t+a) - \operatorname{Erf}(a)]$
(5)	$1 - \alpha^{\frac{1}{2}} \pi^{\frac{1}{2}} p e^{\alpha p^2} \operatorname{Erfc}(\alpha^{\frac{1}{2}} p)$ $\operatorname{Re} \alpha > 0$	$\frac{1}{2} \alpha^{-1} t e^{-\frac{1}{4}t^2/\alpha}$
(6)	$p^{-1} (p+1)^{-1} e^{\frac{1}{4}p^2} \operatorname{Erfc}(\frac{1}{2}p)$	$e^{t+\frac{1}{2}} [\operatorname{Erf}(t+\frac{1}{2}) - \operatorname{Erf}(\frac{1}{2})]$
(7)	$p^{-1} e^{p^2} [\operatorname{Erf}(p) - \operatorname{Erf}(p+b)]$ $b > 0$	$\operatorname{Erf}(\frac{1}{2}t) \quad 0 < t < 2b$ $\operatorname{Erf}(b) \quad t > 2b$
(8)	$\operatorname{Erfc}(b^{\frac{1}{2}} p^{\frac{1}{2}})$ $b > 0$	$0 \quad 0 < t < b$ $\pi^{-1} b^{\frac{1}{2}} t^{-1} (t-b)^{-\frac{1}{2}} \quad t > b$
(9)	$e^{-bp} - \pi^{\frac{1}{2}} b^{\frac{1}{2}} p^{\frac{1}{2}} \operatorname{Erfc}(b^{\frac{1}{2}} p^{\frac{1}{2}})$ $b > 0$	$0 \quad 0 < t < b$ $\frac{1}{2} b^{1/2} t^{-3/2} \quad t > b$
(10)	$p^{-\frac{1}{2}} \operatorname{Erf}(b^{\frac{1}{2}} p^{\frac{1}{2}})$ $b > 0$	$\pi^{-\frac{1}{2}} t^{-\frac{1}{2}} \quad 0 < t < b$ $0 \quad t > b$
(11)	$p^{-\frac{1}{2}} \operatorname{Erfc}(b^{\frac{1}{2}} p^{\frac{1}{2}})$ $b > 0$	$0 \quad 0 < t < b$ $\pi^{-\frac{1}{2}} t^{-\frac{1}{2}} \quad t > b$
(12)	$e^{\alpha p} \operatorname{Erfc}(\alpha^{\frac{1}{2}} p^{\frac{1}{2}})$ $ \arg \alpha  < \pi$	$\pi^{-1} \alpha^{\frac{1}{2}} (t+\alpha)^{-1} t^{-\frac{1}{2}}$

## Error function etc. (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(13)	$1 - (\pi a p)^{\frac{1}{2}} e^{\alpha p} \operatorname{Erfc}(\alpha^{\frac{1}{2}} p^{\frac{1}{2}})$ $ \arg \alpha  < \pi$	$\frac{1}{2} \alpha^{1/2} (t+a)^{-3/2}$
(14)	$p^{-\frac{1}{2}} e^{\alpha p} \operatorname{Erfc}(\alpha^{\frac{1}{2}} p^{\frac{1}{2}})$ $ \arg \alpha  < \pi$	$\pi^{-\frac{1}{2}} (t+a)^{-\frac{1}{2}}$
(15)	$\Gamma(\nu + \frac{1}{2}) p^{-\nu} e^{\alpha p} \operatorname{Erfc}(\alpha^{\frac{1}{2}} p^{\frac{1}{2}})$ $\operatorname{Re} \nu > -\frac{1}{2}, \quad  \arg \alpha  < \pi$	$\pi^{-\frac{1}{2}} \alpha^{-\frac{1}{2}} t^{\nu-\frac{1}{2}} {}_2F_1(1, \frac{1}{2}; \nu+\frac{1}{2}; -t/\alpha)$
(16)	$\operatorname{Erf}(\alpha^{\frac{1}{2}} p^{-\frac{1}{2}})$	$\pi^{-1} t^{-1} \sin(2 \alpha^{\frac{1}{2}} t^{\frac{1}{2}})$
(17)	$p^{-\frac{1}{2}} e^{\alpha/p} \operatorname{Erf}(\alpha^{\frac{1}{2}} p^{-\frac{1}{2}})$	$\pi^{-\frac{1}{2}} t^{-\frac{1}{2}} \sinh(2 \alpha^{\frac{1}{2}} t^{\frac{1}{2}})$
(18)	$p^{-3/2} e^{\alpha/p} \operatorname{Erf}(\alpha^{1/2} p^{-1/2})$	$\pi^{-1/2} \alpha^{-1/2} [\cosh(2 \alpha^{\frac{1}{2}} t^{\frac{1}{2}}) - 1]$
(19)	$\pi^{1/2} p^{-5/2} e^{\alpha/p} \operatorname{Erf}(\alpha^{1/2} p^{-1/2})$	$\alpha^{-1} t^{1/2} \sinh(2 \alpha^{1/2} t^{1/2}) - \alpha^{-1/2} t^{-\frac{1}{2}} \alpha^{-3/2} [\cosh(2 \alpha^{1/2} t^{1/2}) - 1]$
(20)	$p^{-\frac{1}{2}} e^{\alpha/p} \operatorname{Erfc}(\alpha^{\frac{1}{2}} p^{-\frac{1}{2}})$	$\pi^{-\frac{1}{2}} t^{-\frac{1}{2}} e^{-2 \alpha^{\frac{1}{2}} t^{\frac{1}{2}}}$
(21)	$p^{-3/2} e^{\alpha/p} \operatorname{Erfc}(\alpha^{1/2} p^{-1/2})$	$\alpha^{-\frac{1}{2}} \pi^{-\frac{1}{2}} (1 - e^{-2 \alpha^{\frac{1}{2}} t^{\frac{1}{2}}})$
(22)	$p^{-\nu-1} e^{\alpha/p} \operatorname{Erfc}(\alpha^{\frac{1}{2}} p^{-\frac{1}{2}})$ $\operatorname{Re} \nu > -1$	$(t/\alpha)^{\frac{1}{2}\nu} [I_\nu(2 \alpha^{\frac{1}{2}} t^{\frac{1}{2}}) - L_\nu(2 \alpha^{\frac{1}{2}} t^{\frac{1}{2}})]$
(23)	$p^{-\nu-1} e^{\alpha/p} \operatorname{Erf}(\alpha^{\frac{1}{2}} p^{-\frac{1}{2}})$ $\operatorname{Re} \nu > -1$	$(t/\alpha)^{\frac{1}{2}\nu} L_\nu(2 \alpha^{\frac{1}{2}} t^{\frac{1}{2}})$
(24)	$\operatorname{Ei}(-bp)$ $b > 0$	$0 \quad 0 < t < b$ $-1/t \quad t > b$
(25)	$p^{-1} \operatorname{Ei}(-bp)$ $b > 0$	$0 \quad 0 < t < b$ $\log(b/t) \quad t > 0$

## Error function etc. (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(26)	$p^{-1} \operatorname{Ei}[-b(p+a)]$ $b > 0, \quad a \neq 0$	$0 \quad 0 < t < b$ $\operatorname{Ei}(-ba) - \operatorname{Ei}(-at) \quad t > b$
(27)	$p^{-1} [\operatorname{Ei}(-bp) - \log(\gamma p)]$ $b > 0$	$\log t \quad 0 < t < b$ $\log b \quad t > b$
(28)	$e^{\alpha p} \operatorname{Ei}(-\alpha p)$ $ \arg \alpha  < \pi$	$-(t+\alpha)^{-1}$
(29)	$p^{-1} e^{\alpha p} \operatorname{Ei}(-\alpha p)$ $ \arg \alpha  < \pi$	$-\log(1+t/\alpha)$
(30)	$-p^{-1} e^{-bp} \overline{\operatorname{Ei}}(bp)$ $b > 0$	$\log  1-t/b $
(31)	$p e^{\alpha p} \operatorname{Ei}(-\alpha p) + \alpha^{-1}$ $ \arg \alpha  < \pi$	$(t+\alpha)^{-2}$
(32)	$e^{-bp} + bp \operatorname{Ei}(-bp)$ $b > 0$	$0 \quad 0 < t < b$ $bt^{-2} \quad t > b$
(33)	$[\operatorname{Ei}(-\frac{1}{2}p)]^2$	$0 \quad 0 < t < 1$ $2t^{-1} \log(2t-1) \quad t > 1$
(34)	$\operatorname{Ei}(-ap) \operatorname{Ei}(-bp)$ $a, b > 0$	$0 \quad 0 < t < a+b$ $t^{-1} \log[a^{-1} b^{-1} (t-a)(t-b)] \quad t > a+b$
(35)	$\overline{\operatorname{Ei}}(p) \operatorname{Ei}(-p)$	$t^{-1} \log  1-t^2 $
(36)	$e^{bp} [\operatorname{Ei}(-bp)]^2$ $b > 0$	$0 \quad 0 < t < b$ $2(t+b)^{-1} \log(t/b) \quad t > b$
(37)	$e^{bp} \{ [\operatorname{Ei}(-bp)]^2 - 2 \log b \operatorname{Ei}(-2bp) \}$ $b > 0$	$0 \quad 0 < t < b$ $2(t+b)^{-1} \log t \quad t > b$

## Error function etc. (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(38)	$e^{(\alpha+\beta)p} \operatorname{Ei}(-\alpha p) \operatorname{Ei}(-\beta p)$ $ \arg(\alpha + \beta)  < \pi$	$(t + \alpha + \beta)^{-1}$ $\times \log[\alpha^{-1} \beta^{-1} (t + \alpha)(t + \beta)]$
(39)	$e^{(\alpha+\beta)p} [\operatorname{Ei}(-\alpha p) \operatorname{Ei}(-\beta p)$ $- \log(\alpha\beta) \operatorname{Ei}(-\alpha p - \beta p)]$ $ \arg(\alpha + \beta)  < \pi$	$(t + \alpha + \beta)^{-1} \log[(t + \alpha)(t + \beta)]$
(40)	$\exp(\frac{1}{4}\alpha^{-2}p^2) \operatorname{Ei}(-\frac{1}{4}\alpha^{-2}p^2)$ $ \arg \alpha  < \frac{1}{4}\pi$	$2i\pi^{-\frac{1}{2}} \alpha e^{-\alpha^2 t^2} \operatorname{Erf}(i\alpha t)$
(41)	$p^{-1} \operatorname{Ei}(-p^{-1})$	$2Ji_0(2t^{\frac{1}{2}})$
(42)	$p^{-\nu-1} \operatorname{Ei}(-\alpha p^{-1})$ $\operatorname{Re} \nu > -1, \quad \operatorname{Re} \alpha > 0$	$2t^\nu \int_{\infty}^{\alpha^{\frac{1}{2}} t^{\frac{1}{2}}} u^{-\nu-1} J_\nu(2u) du$
(43)	$-p^{-1} e^{\alpha/p} \operatorname{Ei}(-\alpha p^{-1})$	$2K_0(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$
(44)	$p^{-\nu-1} e^{\alpha/p} \operatorname{Ei}(-\alpha p^{-1})$ $\operatorname{Re} \nu > -1$	$t^\nu \int_{\infty}^{\alpha t} u^{-\frac{1}{2}\nu-1} J_\nu[2(u-\alpha t)^{\frac{1}{2}}] du$
(45)	$\operatorname{ci}(\alpha p) \cos(\alpha p) - \operatorname{si}(\alpha p) \sin(\alpha p)$ $\operatorname{Re} \alpha > 0$	$t(t^2 + \alpha^2)^{-1}$
(46)	$\operatorname{ci}(\alpha p) \sin(\alpha p) + \operatorname{si}(\alpha p) \cos(\alpha p)$ $\operatorname{Re} \alpha > 0$	$-\alpha(t^2 + \alpha^2)^{-1}$
(47)	$p^{-1} [\operatorname{ci}(\alpha p) \cos(\alpha p)$ $- \operatorname{si}(\alpha p) \sin(\alpha p)] \quad \operatorname{Re} \alpha > 0$	$\frac{1}{2} \log(1 + t^2/\alpha^2)$
(48)	$p^{-1} [\operatorname{ci}(\alpha p) \sin(\alpha p)$ $+ \operatorname{si}(\alpha p) \cos(\alpha p)] \quad \operatorname{Re} \alpha > 0$	$-\tan^{-1}(t/\alpha)$

## Error function etc. (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(49)	$[\text{ci}(ap)]^2 + [\text{si}(ap)]^2$	$t^{-1} \log(1+t^2/a^2)$
(50)	$\frac{1}{2} - \cos(\frac{1}{4}p^2) C(\frac{1}{4}p^2) - \sin(\frac{1}{4}p^2) S(\frac{1}{4}p^2)$	$2^{\frac{1}{2}} \pi^{-\frac{1}{2}} \sin(t^2)$
(51)	$\frac{1}{2} \cos(\frac{1}{2}p^2) - \cos(\frac{1}{4}p^2) S(\frac{1}{4}p^2) + \sin(\frac{1}{4}p^2) C(\frac{1}{4}p^2)$	$2^{\frac{1}{2}} \pi^{-\frac{1}{2}} \cos(t^2)$
(52)	$[\frac{1}{2} - C(\frac{1}{4}p^2)]^2 + [\frac{1}{2} - S(\frac{1}{4}p^2)]^2$	$2\pi^{-1} t^{-1} \sin(t^2)$
(53)	$C_n(a, p)$	$a > 0$ 0 $0 < t < \cosh a$ $(t^2 - 1)^{-\frac{1}{2}} \cosh(n \cosh^{-1} t)$ $t > \cosh a$
(54)	$S_n(a, p)$	$a > 0$ 0 $0 < t < \sinh a$ $(t^2 - 1)^{-\frac{1}{2}} \cosh(n \sinh^{-1} t)$ $t > \sinh a$

## 5.13. Legendre functions

(1)	$\pi p^{-1} P_\nu(p)$	$0 < \text{Re } \nu < 1$	$-t^{-1} \sin(\nu\pi) W_{0, \nu+\frac{1}{2}}(2t)$
(2)	$s^\mu P_\nu^\mu\left(\frac{p}{a}\right)$	$\text{Re } \mu - 1 < \text{Re } \nu < -\text{Re } \mu$	$\frac{2^{\frac{1}{2}} a^{\frac{1}{2}} t^{-\mu-\frac{1}{2}} K_{\nu+\frac{1}{2}}(at)}{\pi^{\frac{1}{2}} \Gamma(-\mu+\nu+1) \Gamma(-\mu-\nu)}$
(3)	$s^{-\mu} Q_\nu^\mu\left(\frac{p}{a}\right)$	$\text{Re}(\mu + \nu) > -1$	$\frac{\pi^{\frac{1}{2}} a^{\frac{1}{2}} \sin[(\mu+\nu)\pi]}{2^{\frac{1}{2}} \sin(\nu\pi)} t^{\mu-\frac{1}{2}} I_{\nu+\frac{1}{2}}(at)$
(4)	$Q_\nu\left(\frac{p^2 + a^2 + \beta^2}{2ab}\right)$	$\text{Re } \nu > -1$	$\pi a^{\frac{1}{2}} \beta^{\frac{1}{2}} J_{\nu+\frac{1}{2}}(at) J_{\nu+\frac{1}{2}}(\beta t)$

$$s = (p^2 - a^2)^{\frac{1}{2}}$$

## Legendre functions (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(5)	$\Gamma(-2\nu) p^\nu (\alpha^2 - p^2)^{\frac{\nu}{2}} P_\nu^\nu(\alpha/p)$ $\operatorname{Re} \nu < 0$	$2^{-\frac{\nu}{2}} \pi^{\frac{\nu}{2}} (t/\alpha)^{-\nu-\frac{1}{2}} [I_{-\nu-\frac{1}{2}}(\alpha t) - L_{-\nu-\frac{1}{2}}(\alpha t)]$
(6)	$\Gamma(-2\nu) p^{\nu+1} (\alpha^2 - p^2)^{\frac{\nu}{2}} P_\nu^\nu(\alpha/p)$ $\operatorname{Re} \nu < -\frac{1}{2}$	$2^{-\frac{\nu}{2}} \pi^{\frac{\nu}{2}} \alpha (t/\alpha)^{-\nu-\frac{1}{2}} [I_{-\nu-3/2}(\alpha t) - L_{-\nu-3/2}(\alpha t)]$
(7)	$p^{-\frac{\nu}{2}} \nu^{-\frac{1}{2}} (p-\alpha)^{\frac{\nu}{2}} \mu P_\nu^\mu(\alpha^{\frac{1}{2}} p^{-\frac{1}{2}})$ $\operatorname{Re} \mu < 1, \quad \operatorname{Re}(\nu - \mu) > -1$	$\frac{t^{\frac{\nu}{2}(\nu-\mu-1)} e^{\frac{\nu}{2}\alpha t} D_{\mu+\nu}(2^{\frac{\nu}{2}} \alpha^{\frac{1}{2}} t^{\frac{1}{2}})}{\pi^{\frac{\nu}{2}} 2^{\frac{\nu}{2}(\mu-\nu-1)} \Gamma(\nu-\mu+1)}$
(8)	$s^{-\nu-1} P_\nu^\mu(p/s)$ $\operatorname{Re}(\nu - \mu) > -1$	$[\Gamma(\nu-\mu+1)]^{-1} t^\nu I_{-\mu}(\alpha t)$
(9)	$s^{-\nu-1} Q_\nu^\mu(p/s)$ $\operatorname{Re}(\nu \pm \mu) > -1$	$\frac{\sin(\mu+\nu)\pi}{\sin(\nu\pi)} \frac{t^\nu K_\mu(\alpha t)}{\Gamma(\nu-\mu+1)}$
(10)	$p^{-\lambda} Q_{2\nu}(p^{\frac{1}{2}})$ $\operatorname{Re}(\lambda + \nu) > -\frac{1}{2}$	$\frac{\pi^{\frac{\nu}{2}} \Gamma(2\nu+1) t^{\lambda+\nu-\frac{1}{2}}}{2^{2\nu+1} \Gamma(2\nu+3/2) \Gamma(\lambda+\nu+1/2)} \\ \times {}_2F_2\left(\nu+\frac{1}{2}, \nu+1; 2\nu+\frac{3}{2}, \lambda+\nu+\frac{1}{2}; t\right)$
(11)	$p^{-\frac{\nu}{2}} [P_{-\frac{\nu}{2}}^\mu(r/p)]^2$ $\operatorname{Re} \mu < \frac{1}{4}$	$2^{\frac{\nu}{2}-\mu} [\Gamma(\frac{1}{2}-2\mu)]^{-1} t^{-\frac{\nu}{2}} \\ \times [J_{-\mu}(\frac{1}{2}\alpha t)]^2$
(12)	$2^{-\frac{\nu}{2}} \pi^{\frac{\nu}{2}} p^{-\frac{\nu}{2}} P_{-\frac{\nu}{2}}^\mu(r/p) P_{-\frac{\nu}{2}}^{-\mu}(r/p)$	$t^{-\frac{\nu}{2}} J_\mu(\frac{1}{2}\alpha t) J_{-\mu}(\frac{1}{2}\alpha t)$
(13)	$\alpha r^{-1} p^{-\frac{\nu}{2}} P_{\frac{\nu}{2}}^\mu(r/p) P_{-\frac{\nu}{2}}^\mu(r/p)$ $\operatorname{Re} \mu < \frac{3}{4}$	$2^{3/2-\mu} [\Gamma(3/2-2\mu)]^{-1} t^{1/2} \\ \times [J_{-\mu}(\frac{1}{2}\alpha t)]^2$

$$s = (p^2 - \alpha^2)^{\frac{1}{2}}, \quad r = (p^2 + \alpha^2)^{\frac{1}{2}}$$

## Legendre functions (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(14)	$\frac{\Gamma(p-\mu+\nu+1)\Gamma(p-\mu-\nu)}{\Gamma(p+1)}$ $\times \left(\frac{a}{a-2}\right)^{\frac{\mu}{2}} P_{\nu}^{\mu-p}(a-1)$ $\text{Re } \alpha > 0, \quad \text{Re } \mu > -1$	$\left[ (e^{-t}-1) \left( \frac{ae^{-t}}{a-2} - 1 \right) \right]^{\frac{\mu}{2}}$ $\times P_{\nu}^{\mu-p}(ae^{-t}+1-a)$
(15)	$\Gamma(\frac{1}{2}-\mu) Q_{p-\frac{1}{2}}^{\mu}(\cosh a)$ $\text{Re } \mu < \frac{1}{2}, \quad a > 0$	$0 \quad 0 < t < a$ $2^{-\frac{\mu}{2}} \pi^{\frac{1}{2}} e^{\mu \pi i} (\sinh a)^{\mu}$ $\times (\cosh t - \cosh a)^{-\mu-\frac{1}{2}} \quad t > a$
(16)	$\Gamma(\frac{1}{2}-\mu) e^{\alpha p} Q_{p-\frac{1}{2}}^{\mu}(\cosh a)$ $\text{Re } \mu < \frac{1}{2}, \quad  \arg \alpha  < \pi$	$\pi^{\frac{1}{2}} 2^{-\mu-1} e^{\mu \pi i} \sinh^{\mu} a$ $\times [\sinh(\frac{1}{2}t) \sinh(a + \frac{1}{2}t)]^{-\mu-\frac{1}{2}}$
(17)	$2^{p+1} e^{(p-\alpha)\pi i} (\mu^2-1)^{\frac{1}{4}(p-\alpha)}$ $\times \Gamma(p) Q_{p-1}^{\alpha-p}(\mu)$	$\Gamma(\alpha) (1-e^{-t})^{-\frac{1}{2}} \{ [\mu + (1-e^{-t})^{\frac{1}{2}}]^{-\alpha}$ $+ [\mu - (1-e^{-t})^{\frac{1}{2}}]^{-\alpha} \}$
(18)	$\pi^{\frac{1}{2}} 2^{p+\frac{1}{2}} \Gamma(p) (\mu^2-1)^{\frac{1}{4}-\frac{1}{2}p}$ $\times P_{\alpha+p-\frac{1}{2}}^{\frac{1}{2}-p}(\mu)$	$(1-e^{-t})^{-\frac{1}{2}} \{ [\mu + (\mu^2-1)^{\frac{1}{2}}(1-e^{-t})^{\frac{1}{2}}]^{\alpha}$ $+ [\mu - (\mu^2-1)^{\frac{1}{2}}(1-e^{-t})^{\frac{1}{2}}]^{\alpha} \}$
(19)	$\frac{\pi^{\frac{1}{2}} \Gamma(2p) \Gamma(2\nu+1)}{2^{p+\nu-1} \Gamma(p+\nu+\frac{1}{2})} e^{-pa}$ $\times P_{\nu-p}^{-\nu-p}[(1-e^{-2a})^{\frac{1}{2}}]$ $\text{Re } \nu > -\frac{1}{2}, \quad a > 0$	$0 \quad 0 < t < 2a$ $e^{\nu a} (1-e^{-t})^{-\frac{1}{2}} [e^{-a}(1-e^{-t})^{\frac{1}{2}} - e^{-\frac{1}{2}t}(1-e^{-2a})^{\frac{1}{2}}]^{2\nu} \quad t > 2a$

## 5.14. Bessel functions

(1)	$\pi e^{-ap} [\frac{1}{2} \pi Y_0(iap) - J_0(iap) \log(\frac{1}{2}\gamma)] \quad a > 0$	$\frac{\log[4t(2a-t)/a^2]}{t^{\frac{1}{2}}(2a-t)^{\frac{1}{2}}} \quad 0 < t < 2a$ $0 \quad t > 2a$
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## Bessel functions (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(2)	$\cos(ap) J_0(ap) + \sin(ap) Y_0(ap)$ Re $a > 0$	$\frac{2^{3/2} a}{\pi} \frac{[t + (t^2 + 4a^2)^{1/2}]^{-1/2}}{t^{1/2} (t^2 + 4a^2)^{1/2}}$
(3)	$\sin(ap) J_0(ap) - \cos(ap) Y_0(ap)$ Re $a > 0$	$\frac{2^{1/2}}{\pi} \frac{[t + (t^2 + 4a^2)^{1/2}]^{1/2}}{t^{1/2} (t^2 + 4a^2)^{1/2}}$
(4)	$\cos(ap) J_1(ap) + \sin(ap) Y_1(ap)$ Re $a > 0$	$-\frac{2^{5/2} a^2}{\pi} \frac{[t + (t^2 + 4a^2)^{1/2}]^{-3/2}}{t^{1/2} (t^2 + 4a^2)^{1/2}}$
(5)	$\sin(ap) J_1(ap) - \cos(ap) Y_1(ap)$ Re $a > 0$	$\frac{1}{2^{1/2} \pi a} \frac{[t + (t^2 + 4a^2)^{1/2}]^{3/2}}{t^{1/2} (t^2 + 4a^2)^{1/2}}$
(6)	$p^{-\nu} [\cos(ap) J_\nu(ap) + \sin(ap) Y_\nu(ap)]$ Re $\nu > -\frac{1}{2}$ , Re $a > 0$	$-\frac{2t^{\nu-\frac{1}{2}} (t^2 + 4a^2)^{\frac{1}{2}\nu-\frac{1}{2}}}{\pi^{\frac{1}{2}} (2a)^\nu \Gamma(\nu + \frac{1}{2})} \times \sin[(\nu - \frac{1}{2}) \operatorname{ctn}^{-1}(\frac{1}{2}t/a)]$
(7)	$p^{-\nu} [\sin(ap) J_\nu(ap) - \cos(ap) Y_\nu(ap)]$ Re $\nu > -\frac{1}{2}$ , Re $a > 0$	$\frac{2t^{\nu-\frac{1}{2}} (t^2 + 4a^2)^{\frac{1}{2}\nu-\frac{1}{2}}}{\pi^{\frac{1}{2}} (2a)^\nu \Gamma(\nu + \frac{1}{2})} \times \cos[(\nu - \frac{1}{2}) \operatorname{ctn}^{-1}(\frac{1}{2}t/a)]$
(8)	$p^{-\nu} [\cos(ap - \beta) J_\nu(ap) + \sin(ap - \beta) Y_\nu(ap)]$ Re $\nu > -\frac{1}{2}$ , Re $a > 0$	$\frac{2t^{\nu-\frac{1}{2}} (t^2 + 4a^2)^{\frac{1}{2}\nu-\frac{1}{2}}}{\pi^{\frac{1}{2}} (2a)^\nu \Gamma(\nu + \frac{1}{2})} \times \sin[(\frac{1}{2} - \nu) \operatorname{ctn}^{-1}(\frac{1}{2}t/a) + \beta]$
(9)	$p^{-\nu} e^{-iax} H_\nu^{(1)}(ap)$ Re $\nu > -\frac{1}{2}$ $-\pi/2 < \arg a < 3\pi/2$	$-i \frac{2(t^2 - 2ait)^{\nu-\frac{1}{2}}}{\pi^{\frac{1}{2}} (2a)^\nu \Gamma(\nu + \frac{1}{2})}$

## Bessel functions (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(10)	$\Gamma(\nu + \frac{1}{2}) p^{-\nu} e^{i\alpha p} H_\nu^{(2)}(\alpha p)$ $\text{Re } \nu > -\frac{1}{2}, \quad -3\pi/2 < \arg \alpha < \pi/2$	$i \pi^{-\frac{1}{2}} 2^{1-\nu} \alpha^{-\nu} (t^2 + 2\alpha i t)^{\nu - \frac{1}{2}}$
(11)	$p^{-1} J_\nu(2\alpha/p)$ $\text{Re } \nu > -1$	$J_\nu(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}}) I_\nu(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$
(12)	$\Gamma(\nu + 1) \Gamma(\lambda) p^{\nu - \lambda} J_\nu(4\alpha/p)$ $\text{Re } \lambda > 0$	$(2\alpha)^\nu t^{\lambda - 1} {}_0F_3(\nu + 1, \frac{1}{2}\lambda, \frac{1}{2}\lambda + \frac{1}{2}; -\alpha^2 t^2)$
(13)	$p^{-1} e^{(\alpha^2 - \beta^2)\nu p} J_\nu(2\alpha\beta/p)$ $\text{Re } \nu > -1$	$J_\nu(2\beta t^{\frac{1}{2}}) I_\nu(2\alpha t^{\frac{1}{2}})$
(14)	$(p^2 + 1)^{-\frac{1}{2}} e^{-\alpha p/(p^2 + 1)}$ $\times J_\nu\left(\frac{\alpha}{p^2 + 1}\right)$ $\text{Re } \nu > -\frac{1}{2}$	$J_\nu(t) J_{2\nu}(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$
(15)	$p^{-\mu} J_\nu(p^{-\frac{1}{2}})$ $\text{Re } (\mu + \frac{1}{2}\nu) > 0$	$\frac{t^{\mu + \frac{1}{2}\nu - 1} {}_0F_2(\mu + \frac{1}{2}\nu, \nu + 1; -\frac{1}{4}t)}{2^\nu \Gamma(\mu + \frac{1}{2}\nu) \Gamma(\nu + 1)}$
(16)	$(p^2 + \alpha^2)^{-\frac{1}{2}\nu} e^{ip} H_\nu^{(2)}[(p^2 + \alpha^2)^{\frac{1}{2}}]$ $\text{Re } \nu > -\frac{1}{2}$	$i 2^{\frac{1}{2}} \pi^{-\frac{1}{2}} \alpha^{\frac{1}{2}-\nu} (t^2 + 2it)^{\frac{1}{2}\nu - \frac{1}{2}}$ $\times J_{\nu - \frac{1}{2}}[\alpha(t^2 + 2it)^{\frac{1}{2}}]$
(17)	$\Gamma(p + \frac{1}{2})(\frac{1}{2}\alpha)^{-p} J_p(\alpha)$	$\pi^{-\frac{1}{2}} (e^t - 1)^{-\frac{1}{2}} \cos[\alpha(1 - e^{-t})^{\frac{1}{2}}]$
(18)	$\Gamma(p)(\frac{1}{2}\alpha)^{-p} J_{p+\mu}(\alpha)$ $\text{Re } \mu > -1$	$(1 - e^{-t})^{\frac{1}{2}\mu} J_\mu[\alpha(1 - e^{-t})^{\frac{1}{2}}]$
(19)	$p^{\frac{1}{2}} [J_{\nu + \frac{1}{2}}(\alpha p) J_{\nu - \frac{1}{2}}(\alpha p)$ $+ Y_{\nu + \frac{1}{2}}(\alpha p) Y_{\nu - \frac{1}{2}}(\alpha p)]$ $\text{Re } \alpha > 0$	$(\frac{1}{2}\pi)^{-3/2} (t^3 + 4\alpha^2 t)^{-1/2}$ $\times e^{2\nu \sinh^{-1}(\frac{1}{2}t/\alpha)}$

## Bessel functions (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(20)	$p^{\frac{1}{4}} [J_{\frac{1}{4}+\nu}(ap) J_{\frac{1}{4}-\nu}(ap) + Y_{\frac{1}{4}+\nu}(ap) Y_{\frac{1}{4}-\nu}(ap)]$ $\text{Re } \alpha > 0$	$(\frac{1}{2}\pi)^{-3/2} (t^3 + 4\alpha^2 t)^{-1/2}$ $\times \{ \cos[(\nu + \frac{1}{4})\pi] e^{-2\nu \sinh^{-1}(\frac{1}{2}t/\alpha)} + \sin[(\nu + \frac{1}{4})\pi] e^{2\nu \sinh^{-1}(\frac{1}{2}t/\alpha)} \}$
(21)	$p^{\frac{1}{4}} [J_{\nu+\frac{1}{4}}(ap) Y_{\nu-\frac{1}{4}}(ap) - J_{\nu-\frac{1}{4}}(ap) Y_{\nu+\frac{1}{4}}(ap)]$ $\text{Re } \alpha > 0$	$(\frac{1}{2}\pi)^{-3/2} (t^3 + 4\alpha^2 t)^{-1/2}$ $\times e^{-2\nu \sinh^{-1}(\frac{1}{2}t/\alpha)}$
(22)	$p^{\frac{1}{4}} [J_{\frac{1}{4}+\nu}(ap) Y_{\frac{1}{4}-\nu}(ap) - J_{\frac{1}{4}-\nu}(ap) Y_{\frac{1}{4}+\nu}(ap)]$ $\text{Re } \alpha > 0$	$(\frac{1}{2}\pi)^{-3/2} (t^3 + 4\alpha^2 t)^{-1/2}$ $\times \{ \sinh[(\nu + \frac{1}{4})\pi] e^{-2\nu \sinh^{-1}(\frac{1}{2}t/\alpha)} - \cos[(\nu + \frac{1}{4})\pi] e^{2\nu \sinh^{-1}(\frac{1}{2}t/\alpha)} \}$
(23)	$J_{\nu-p}(a) Y_{-\nu-p}(a) - J_{-\nu-p}(a) Y_{\nu-p}(a)$ $\text{Re } \alpha > 0, \quad  \text{Re } \nu  < \frac{1}{2}$	$2\pi^{-2} \sin(2\nu\pi) K_{2\nu}[2\alpha \sinh(\frac{1}{2}t)]$
(24)	$J_p(a) \frac{\partial Y_p(a)}{\partial p} - Y_p(a) \frac{\partial J_p(a)}{\partial p}$ $\text{Re } \alpha > 0$	$-\frac{2}{\pi} K_0[2\alpha \sinh(\frac{1}{2}t)]$
(25)	$p^{\frac{1}{4}} H_{\frac{1}{8}}^{(1)}\left(\frac{p^2}{a}\right) H_{\frac{1}{8}}^{(2)}\left(\frac{p^2}{a}\right)$ $a > 0$	$a \cos\left(\frac{\pi}{8}\right) \frac{(2t)^{\frac{1}{4}}}{\pi^{\frac{1}{4}}} J_{1/8}\left(\frac{at^2}{16}\right)$ $\times J_{-1/8}\left(\frac{at^2}{16}\right)$
(26)	$p^{-\frac{1}{4}} H_\nu^{(1)}(\frac{1}{2}p/a) H_\nu^{(2)}(\frac{1}{2}p/a)$	$2a(2t/\pi)^{\frac{1}{4}} P_{\nu-\frac{1}{4}}^{\frac{1}{4}}[(1+a^2 t^2)^{\frac{1}{4}}]$ $\times P_{\nu-\frac{1}{4}}^{-\frac{1}{4}}[(1+a^2 t^2)^{\frac{1}{4}}]$

## Bessel functions (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(27)	$p^{\frac{1}{4}} H_{\frac{1}{4}+\nu}^{(1)}(ap) H_{\frac{1}{4}-\nu}^{(2)}(ap)$ $\text{Re } \alpha > 0$	$4[\pi^3 t(t^2 + 4a^2)]^{-\frac{1}{4}} e^{-\nu\pi i}$ $\times \{ \cosh[2\nu \sinh^{-1}(\frac{1}{2}t/a)]$ $+ i \sinh[2\nu \sinh^{-1}(\frac{1}{2}t/a)] \}$
(28)	$\pi \Gamma(2\lambda+2) e^{(\mu-\nu)\pi i} p^{-2\lambda}$ $\times H_{\frac{1}{2}\mu}^{(1)}(p/a) H_{2\nu}^{(2)}(p/a)$ $\text{Re } \lambda > -\frac{1}{2}$	$2(2\lambda+1) \alpha t^{2\lambda}$ $\times {}_4F_3(\frac{1}{2}+\mu+\nu, \frac{1}{2}-\mu+\nu, \frac{1}{2}+\mu-\nu, \frac{1}{2}-\mu-\nu;$ $\frac{1}{2}, \lambda+\frac{1}{2}, \lambda+1; -\frac{1}{4}\alpha^2 t^2)$ $+ i 4\alpha^2 (\mu^2 - \nu^2) t^{2\lambda+1}$ $\times {}_4F_3(1+\mu+\nu, 1+\nu-\mu, 1-\mu-\nu, 1+\mu-\nu;$ $3/2, \lambda+1, \lambda+3/2; -\frac{1}{4}\alpha^2 t^2)$
(29)	$\Gamma(2\nu+\frac{1}{2}) p^{-2\nu} H_{2\nu}^{(1)}(\alpha^{\frac{1}{2}} p^{\frac{1}{2}})$ $\times H_{2\nu}^{(2)}(\alpha^{\frac{1}{2}} p^{\frac{1}{2}})$ $\text{Re } \nu > -\frac{1}{4}$	$2\alpha^{-\nu-\frac{1}{2}} t^{3\nu-\frac{1}{2}} e^{\frac{1}{2}\alpha/t} W_{\nu, \nu}(a/t)$
(30)	$p^{\frac{1}{2}} [H_\nu^{(1)}(\alpha^{\frac{1}{2}} p^{\frac{1}{2}}) H_{\nu+1}^{(2)}(\alpha^{\frac{1}{2}} p^{\frac{1}{2}})$ $+ H_{\nu+1}^{(1)}(\alpha^{\frac{1}{2}} p^{\frac{1}{2}}) H_\nu^{(2)}(\alpha^{\frac{1}{2}} p^{\frac{1}{2}})]$	$\bar{\alpha}^{1/2} \pi^{-3/2} (4\nu+2) e^{\frac{1}{2}\alpha/t} W_{-\frac{1}{2}, \nu+\frac{1}{2}}(a/t)$

5.15. Modified Bessel functions of arguments  $kp$  and  $kp^2$ 

(1)	$e^{-bp} I_0(bp)$	$b > 0$	$\pi^{-1} (2bt - t^2)^{-\frac{1}{2}}$ 0	$0 < t < 2b$ $t > 2b$
(2)	$\pi b e^{-bp} I_1(bp)$	$b > 0$	$(b-t)(2bt - t^2)^{-\frac{1}{2}}$ 0	$0 < t < 2b$ $t > 2b$
(3)	$e^{-\frac{1}{2}(a+b)p} I_n[\frac{1}{2}(b-a)p]$	$b > a \geq 0$	$0$ $\frac{\cos(n \cos^{-1} \frac{2t-a-b}{b-a})}{\pi(t-a)^{\frac{1}{2}} (b-t)^{\frac{1}{2}}}$ 0	$0 < t < a$ $a < t < b$ $t > b$

**Modified Bessel functions of  $kp$  and  $kp^2$  (cont'd)**

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(4)	$\frac{\pi^{3/2} e^{-bp/2} p^\nu}{\Gamma(\nu + \frac{1}{2}) b^\nu} I_\nu(\frac{1}{2} bp)$ $\text{Re } \nu < \frac{1}{2}, \quad b > 0$	$\cos(2\pi\nu)(bt - t^2)^{-\nu - \frac{1}{2}} \quad 0 < t < b$ $-\sin(2\pi\nu)(t^2 - bt)^{-\nu - \frac{1}{2}} \quad t > b$
(5)	$\pi^{\frac{1}{2}} \Gamma(\nu + \frac{1}{2}) e^{-\frac{1}{2}bp} b^\nu p^{-\nu} I_\nu(\frac{1}{2} bp)$ $\text{Re } \nu > -\frac{1}{2}, \quad b > 0$	$(bt - t^2)^{\nu - \frac{1}{2}} \quad 0 < t < b$ 0 $\quad t > b$
(6)	$\Gamma(2\nu + n) e^{-\frac{1}{2}bp} b^\nu p^{-\nu} I_{\nu+n}(\frac{1}{2} bp)$ $\text{Re } \nu > -\frac{1}{2}, \quad b > 0$	$\frac{(-1)^n n! \Gamma(\nu) 2^{2\nu}}{\pi (bt - t^2)^{\frac{1}{2} - \nu}} C_n^\nu(2t/b - 1)$ 0 $\quad 0 < t < b$ 0 $\quad t > b$
(7)	$\Gamma(2\nu) p^{-\nu} \operatorname{csch}(ap) I_\nu(ap)$ $a > 0, \quad \text{Re } \nu > -\frac{1}{2}$	$\pi^{-1} 2^\nu a^{-\nu} \Gamma(\nu) [2a(t - 2ak) - (t - 2ak)^2]^{\nu - \frac{1}{2}} \quad k = 0, 1, 2, \dots$ $2ak < t < 2a(k+1)$
(8)	$K_0(bp) \quad b > 0$	0 $\quad 0 < t < b$ $y^{-1} \quad t > b$
(9)	$p^{-1} K_0(bp) \quad b > 0$	0 $\quad 0 < t < b$ $\cosh^{-1}(t/b) \quad t > b$
(10)	$K_1(bp) \quad b > 0$	0 $\quad 0 < t < b$ $b^{-1} t y^{-1} \quad t > b$
(11)	$p^{-1} K_1(bp) \quad b > 0$	0 $\quad 0 < t < b$ $b^{-1} y \quad t > b$
(12)	$K_\nu(bp) \quad b > 0$	0 $\quad 0 < t < b$ $y^{-1} \cosh[\nu \cosh^{-1}(t/b)] \quad t > b$

$$y = (t^2 - b^2)^{\frac{1}{2}}$$

**Modified Bessel functions of  $kp$  and  $kp^2$  (cont'd)**

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(13)	$p^{-1} K_\nu(bp)$ $b > 0$	$0 \quad 0 < t < b$ $\nu^{-1} \sinh [\nu \cosh^{-1}(t/b)] \quad t > b$
(14)	$\Gamma(\nu + \frac{1}{2}) p^{-\nu} K_\nu(bp)$ $b > 0, \quad \operatorname{Re} \nu > -\frac{1}{2}$	$0 \quad 0 < t < b$ $2^{-\nu} \pi^{\frac{1}{2}} b^{-\nu} y^{2\nu-1} \quad t > b$
(15)	$2^{2\mu} \Gamma(2\mu + \frac{1}{2})(p/b)^{-2\mu} K_{2\nu}(bp)$ $\operatorname{Re} \mu > -\frac{1}{4}, \quad b > 0$	$0 \quad 0 < t < b$ $\pi^{\frac{1}{2}} y^{4\mu-1} \times {}_2F_1(\mu-\nu, \mu+\nu; 2\mu+\frac{1}{2}; 1-t^2/b^2) \quad t > b$
(16)	$e^{\alpha p} K_0(ap) \quad  \arg \alpha  < \pi$	$(t^2 + 2\alpha t)^{-\frac{1}{2}}$
(17)	$e^{\alpha p} K_1(ap) \quad  \arg \alpha  < \pi$	$\alpha^{-1} (t+a)(t^2 + 2\alpha t)^{-\frac{1}{2}}$
(18)	$e^{\alpha p} K_\nu(ap) \quad  \arg \alpha  < \pi$	$(t^2 + 2\alpha t)^{-\frac{1}{2}} \cosh [\nu \cosh^{-1}(1+t/a)]$
(19)	$p^{-1} e^{\alpha p} K_0(ap) \quad  \arg \alpha  < \pi$	$\cosh^{-1}(1+t/a)$
(20)	$p^{-1} e^{\alpha p} K_1(ap) \quad  \arg \alpha  < \pi$	$\alpha^{-1} (t^2 + 2\alpha t)^{-\frac{1}{2}}$ <span style="color: blue;">Errata: +1/2</span>
(21)	$p^{-1} e^{\alpha p} K_\nu(ap) \quad  \arg \alpha  < \pi$	$\nu^{-1} \sinh [\nu \cosh^{-1}(1+t/a)]$
(22)	$p^{-\nu} e^{\alpha p} K_\nu(ap)$ $\operatorname{Re} \nu > -\frac{1}{2}, \quad  \arg \alpha  < \pi$	$\pi^{\frac{1}{2}} [\Gamma(\nu + \frac{1}{2})]^{-1} (2\alpha)^{-\nu} (t^2 + 2\alpha t)^{\nu - \frac{1}{2}}$
(23)	$p^\mu e^{\alpha p} K_\nu(ap)$ $\operatorname{Re} \mu < \frac{1}{2}, \quad  \arg \alpha  < \pi$	$2^{-\frac{1}{2}} \pi^{\frac{1}{2}} \alpha^{-\frac{1}{2}} (t^2 + 2\alpha t)^{-\frac{1}{2}\mu - \frac{1}{2}} \times P_{\nu - \frac{1}{2}}^{\mu + \frac{1}{2}}(1+t/a)$

$$y = (t^2 - b^2)^{\frac{1}{2}}$$

Modified Bessel functions of  $kp$  and  $kp^2$  (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(24)	$e^{\alpha p^2} K_0(\alpha p^2)$ Re $\alpha > 0$	$2^{-\frac{1}{2}} \alpha^{-\frac{1}{2}} \pi^{\frac{1}{2}} \exp\left(-\frac{t^2}{16\alpha}\right) I_0\left(\frac{t^2}{16\alpha}\right)$
(25)	$p^{\frac{1}{2}} e^{\alpha p^2} K_{\frac{1}{2}}(\alpha p^2)$ Re $\alpha > 0$	$(2\alpha t)^{-\frac{1}{2}} \exp\left(-\frac{t^2}{8\alpha}\right)$
(26)	$p^{-\frac{1}{2}} e^{\alpha p^2} K_{\frac{1}{2}}(\alpha p^2)$ Re $\alpha > 0$	$(8\alpha)^{-\frac{1}{2}} \gamma\left(\frac{1}{4}, \frac{t^2}{8\alpha}\right)$
(27)	$\Gamma(4\nu+1) p^{-4\nu} e^{\alpha p^2} K_{2\nu}(\alpha p^2)$ Re $\nu > -\frac{1}{4}$ , Re $\alpha > 0$	$2^{3\nu+1} \pi^{\frac{1}{2}} \alpha^\nu t^{2\nu-1} \exp\left(-\frac{t^2}{16\alpha}\right)$ $\times M_{-3\nu, \nu}\left(\frac{t^2}{8\alpha}\right)$

## 5.16. Modified Bessel functions of other arguments

(1)	$I_\nu(2\alpha/p)$ Re $\nu > 0$	$\alpha^{\frac{1}{2}} t^{-\frac{1}{2}} Z_\nu^{(b)}(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$
(2)	$p I_\nu(2\alpha/p)$ Re $\nu > 1$	$\alpha^2 V_\nu^{(b)}(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$
(3)	$p^{-1} I_\nu(2\alpha/p)$ Re $\nu > -1$	$X_\nu^{(b)}(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$
(4)	$p^{-2} I_\nu(2\alpha/p)$ Re $\nu > -2$	$\alpha^{-\frac{1}{2}} t^{\frac{1}{2}} W_\nu^{(b)}(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$
(5)	$p^{-\lambda} I_\nu\left(\frac{2\alpha}{p}\right)$ Re $(\lambda + \nu) > 0$	$\frac{\alpha^\nu t^{\lambda+\nu-1}}{\Gamma(\nu+1)\Gamma(\lambda+\nu)}$ $\times {}_0F_3\left(\nu+1, \frac{\lambda+\nu}{2}, \frac{\lambda+\nu+1}{2}; \frac{1}{4}a^2 t^2\right)$

## Modified Bessel functions of other arguments (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(6)	$p^{-\frac{1}{4}} e^{\alpha/p} I_{\frac{1}{4}}(a/p)$	$\pi^{-1} (2a)^{-\frac{1}{4}} t^{-\frac{1}{4}} \sinh[(8at)^{\frac{1}{2}}]$
(7)	$p^{-\frac{1}{4}} e^{-\alpha/p} I_{\frac{1}{4}}(a/p)$	$\pi^{-1} (2a)^{-\frac{1}{4}} t^{-\frac{1}{4}} \sin[(8at)^{\frac{1}{2}}]$
(8)	$p^{-\frac{1}{4}} e^{\alpha/p} I_{-\frac{1}{4}}(a/p)$	$\pi^{-1} (2a)^{-\frac{1}{4}} t^{-\frac{1}{4}} \cosh[(8at)^{\frac{1}{2}}]$
(9)	$p^{-\frac{1}{4}} e^{-\alpha/p} I_{-\frac{1}{4}}(a/p)$	$\pi^{-1} (2a)^{-\frac{1}{4}} t^{-\frac{1}{4}} \cos[(8at)^{\frac{1}{2}}]$
(10)	$\pi p^{-\frac{1}{4}} e^{\alpha/p} I_{\frac{1}{4}}(a/p)$	$2^{-1/4} a^{-1/4} t^{-3/4} \cosh[(8at)^{1/2}]$ $- 2^{-7/4} a^{-3/4} t^{-5/4} \sinh[(8at)^{1/2}]$
(11)	$p^{-\frac{1}{4}} e^{-\alpha/p} I_{\frac{1}{4}}(a/p)$	$2^{-7/4} a^{-3/4} t^{-5/4} \sin[(8at)^{1/2}]$ $- 2^{-1/4} a^{-1/4} t^{-3/4} \cos[(8at)^{1/2}]$
(12)	$\pi p^{-\frac{1}{4}} e^{\alpha/p} I_{-\frac{1}{4}}(a/p)$	$2^{-1/4} a^{-1/4} t^{-3/4} \sinh[(8at)^{1/2}]$ $- 2^{-7/4} a^{-3/4} t^{-5/4} \cosh[(8at)^{1/2}]$
(13)	$\pi p^{-\frac{1}{4}} e^{-\alpha/p} I_{-\frac{1}{4}}(a/p)$	$- 2^{-1/4} a^{-1/4} t^{-3/4} \sin[(8at)^{1/2}]$ $- 2^{-7/4} a^{-3/4} t^{-5/4} \cos[(8at)^{1/2}]$
(14)	$p^{-1} e^{\alpha/p} I_\nu(a/p) \quad \text{Re } \nu > -1$	$\{I_\nu[(2at)^{\frac{1}{2}}]\}^2$
(15)	$p^{-1} e^{-\alpha/p} I_\nu(a/p) \quad \text{Re } \nu > -1$	$\{J_\nu[(2at)^{\frac{1}{2}}]\}^2$
(16)	$p^{-\frac{1}{2}} e^{\alpha/p} I_\nu(a/p) \quad \text{Re } \nu > -\frac{1}{2}$	$(\pi t)^{-\frac{1}{2}} I_{2\nu}[(8at)^{\frac{1}{2}}]$
(17)	$p^{-\frac{1}{2}} e^{-\alpha/p} I_\nu(a/p) \quad \text{Re } \nu > -\frac{1}{2}$	$(\pi t)^{-\frac{1}{2}} J_{2\nu}[(8at)^{\frac{1}{2}}]$

**Modified Bessel functions of other arguments (cont'd)**

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(18)	$p^{-\lambda} e^{\alpha/p} I_\nu(a/p)$ $\operatorname{Re}(\lambda + \nu) > 0$	$\frac{2^{-\nu} a^\nu t^{\lambda+\nu-1}}{\Gamma(\nu+1)\Gamma(\lambda+\nu)} \\ \times {}_1F_2(\nu+\frac{1}{2}; 2\nu+1, \lambda+\nu; 2\alpha t)$
(19)	$p^{-\lambda} e^{-\alpha/p} I_\nu(a/p)$ $\operatorname{Re}(\lambda + \nu) > 0$	$\frac{2^{-\nu} a^\nu t^{\lambda+\nu-1}}{\Gamma(\nu+1)\Gamma(\lambda+\nu)} \\ \times {}_1F_2(\nu+\frac{1}{2}; 2\nu+1, \lambda+\nu; -2\alpha t)$
(20)	$p^{-\frac{\nu}{2}} \sinh(a/p) I_\nu(a/p)$ $\operatorname{Re} \nu > -\frac{1}{2}$	$\frac{1}{2} \pi^{-\frac{\nu}{2}} t^{-\frac{\nu}{2}} \{ I_{2\nu}[(8\alpha t)^{\frac{\nu}{2}}] \\ - J_{2\nu}[(8\alpha t)^{\frac{\nu}{2}}] \}$
(21)	$p^{-\frac{\nu}{2}} \cosh(a/p) I_\nu(a/p)$ $\operatorname{Re} \nu > -\frac{1}{2}$	$\frac{1}{2} \pi^{-\frac{\nu}{2}} t^{-\frac{\nu}{2}} \{ I_{2\nu}[(8\alpha t)^{\frac{\nu}{2}}] \\ + J_{2\nu}[(8\alpha t)^{\frac{\nu}{2}}] \}$
(22)	$p^{-1} e^{(\alpha^2 + \beta^2)/p} I_\nu(2\alpha\beta/p)$ $\operatorname{Re} \nu > -1$	$I_\nu(2\alpha t^{\frac{\nu}{2}}) I_\nu(2\beta t^{\frac{\nu}{2}})$
(23)	$p^{-1} e^{-(\alpha^2 + \beta^2)/p} I_\nu(2\alpha\beta/p)$ $\operatorname{Re} \nu > -1$	$J_\nu(2\alpha t^{\frac{\nu}{2}}) J_\nu(2\beta t^{\frac{\nu}{2}})$
(24)	$2p^{-1} \sinh[(\alpha^2 + \beta^2)/p]$ $\times I_\nu(2\alpha\beta/p)$ $\operatorname{Re} \nu > -1$	$I_\nu(2\alpha t^{\frac{\nu}{2}}) I_\nu(2\beta t^{\frac{\nu}{2}})$ $- J_\nu(2\alpha t^{\frac{\nu}{2}}) J_\nu(2\beta t^{\frac{\nu}{2}})$
(25)	$2p^{-1} \cosh[(\alpha^2 + \beta^2)/p]$ $\times I_\nu(2\alpha\beta/p)$ $\operatorname{Re} \nu > -1$	$I_\nu(2\alpha t^{\frac{\nu}{2}}) I_\nu(2\beta t^{\frac{\nu}{2}})$ $+ J_\nu(2\alpha t^{\frac{\nu}{2}}) J_\nu(2\beta t^{\frac{\nu}{2}})$

## Modified Bessel functions of other arguments (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(26)	$2^{\frac{v}{2}} \pi^{\frac{v}{2}} r^{-\nu} e^{-p} C_n^\nu(p/r)$ $\times I_{\nu+n}(r) \quad r = (p^2 + a^2)^{\frac{1}{2}}$	$(-1)^n a^{\frac{v}{2}-\nu} (2t-t^2)^{\frac{v}{2}\nu-\frac{v}{2}}$ $\times I_{\nu-\frac{v}{2}}[\alpha(2t-t^2)^{\frac{v}{2}}] \quad 0 < t < 2$ 0 $t > 2$
(27)	$\pi e^{-p} I_0[(p^2 - a^2)^{\frac{1}{2}}]$	$(2t-t^2)^{-\frac{v}{2}} \cos[\alpha(2t-t^2)^{\frac{v}{2}}] \quad 0 < t < 2$ 0 $t > 2$
(28)	$p^{\frac{v}{2}} [I_{\nu-\frac{v}{2}}(bp) I_{-\nu-\frac{v}{2}}(bp)$ $- I_{\nu+\frac{v}{2}}(bp) I_{-\nu+\frac{v}{2}}(bp)]$	$\frac{2^{3/2} \cos[2\nu \cos^{-1}(\frac{1}{2}t/b)]}{\pi^{3/2} (4b^2 t - t^3)^{1/2}} \quad 0 < t < 2b$ 0 $t > 2b$
(29)	$p^{-\frac{v}{2}} \sinh(a/p) K_0(a/p)$	$\pi^{-\frac{v}{2}} t^{-\frac{v}{2}} K_0[(8at)^{\frac{v}{2}}]$ $+ \frac{1}{2} \pi^{\frac{v}{2}} t^{-\frac{v}{2}} Y_0[(8at)^{\frac{v}{2}}]$
(30)	$p^{-\frac{v}{2}} \cosh(a/p) K_0(a/p)$	$\pi^{-\frac{v}{2}} t^{-\frac{v}{2}} K_0[(8at)^{\frac{v}{2}}]$ $- \frac{1}{2} \pi^{\frac{v}{2}} t^{-\frac{v}{2}} Y_0[(8at)^{\frac{v}{2}}]$
(31)	$a^{\frac{v}{2}} p^{-\frac{v}{2}} e^{\alpha/p} K_{\frac{v}{2}}(a/p)$	$(2t)^{-\frac{v}{2}} e^{-(8at)^{\frac{v}{2}}}$
(32)	$\pi^{-\frac{v}{2}} p^{2\lambda} K_{2\nu}(2a/p)$ $\text{Re } (\lambda \pm \nu) < 0$	$2^{2\lambda} t^{-2\lambda-1}$ $\times S_2(\nu-\frac{1}{2}, -\nu-\frac{1}{2}, \lambda+\frac{1}{2}, \lambda; \frac{1}{2}at)$
(33)	$p^{-\frac{v}{2}} e^{\alpha/p} K_\nu(a/p) \quad  \text{Re } \nu  < \frac{1}{2}$	$2\pi^{-\frac{v}{2}} t^{-\frac{v}{2}} \cos(\nu\pi) K_{2\nu}[(8at)^{\frac{v}{2}}]$
(34)	$\pi^{-\frac{v}{2}} p^{-\frac{v}{2}} e^{-\alpha/p} K_\nu(a/p)$ $ \text{Re } \nu  < \frac{1}{2}$	$-t^{-\frac{v}{2}} \sin(\nu\pi) J_{2\nu}[(8at)^{\frac{v}{2}}]$ $-t^{-\frac{v}{2}} \cos(\nu\pi) Y_{2\nu}[(8at)^{\frac{v}{2}}]$

## Modified Bessel functions of other arguments (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(35)	$K_0(\alpha^{\frac{1}{2}} p^{\frac{1}{2}}) \quad \operatorname{Re} \alpha \geq 0, \quad \alpha \neq 0$	$\frac{1}{2} t^{-\frac{1}{2}} e^{-\frac{1}{2} \alpha/t}$
(36)	$\alpha^{\frac{1}{2}} p^{-\frac{1}{2}} K_1(\alpha^{\frac{1}{2}} p^{\frac{1}{2}}) \quad \operatorname{Re} \alpha \geq 0$	$e^{-\frac{1}{2} \alpha/t}$
(37)	$\alpha^{-\frac{1}{2}} p^{\frac{1}{2}} K_1(\alpha^{\frac{1}{2}} p^{\frac{1}{2}}) \quad \operatorname{Re} \alpha > 0$	$\frac{1}{4} t^{-\frac{1}{2}} e^{-\frac{1}{2} \alpha/t}$
(38)	$p^{-\frac{1}{2}} K_\nu(2\alpha^{\frac{1}{2}} p^{\frac{1}{2}}) \quad \operatorname{Re} \alpha > 0$	$\frac{1}{2} \pi^{-\frac{1}{2}} t^{-\frac{1}{2}} e^{-\frac{1}{2} \alpha/t} K_{\frac{1}{2}\nu}(\frac{1}{2} \alpha/t)$
(39)	$\alpha^{\frac{1}{2}\nu} p^{-\frac{1}{2}\nu} K_\nu(2\alpha^{\frac{1}{2}} p^{\frac{1}{2}}) \quad \operatorname{Re} \alpha > 0$	$\frac{1}{2} t^{\nu-\frac{1}{2}} e^{-\alpha/t}$
(40)	$\alpha^{-\frac{1}{2}\nu} p^{\frac{1}{2}\nu} K_\nu(2\alpha^{\frac{1}{2}} p^{\frac{1}{2}}) \quad \operatorname{Re} \alpha > 0$	$\frac{1}{2} t^{-\nu-\frac{1}{2}} e^{-\alpha/t}$
(41)	$p^{\frac{1}{2}\nu-\frac{1}{2}} K_\nu(2\alpha^{\frac{1}{2}} p^{\frac{1}{2}}) \quad \operatorname{Re} \alpha > 0$	$\frac{1}{2} \alpha^{-\frac{1}{2}\nu} \Gamma(\nu, \alpha/t)$
(42)	$p^{\frac{1}{2}\nu+n} K_\nu(2\alpha^{\frac{1}{2}} p^{\frac{1}{2}}) \quad \operatorname{Re} \alpha > 0$	$\frac{1}{2} (-1)^n n! \alpha^{\frac{1}{2}\nu} t^{-n} e^{-\alpha/t} L_n^\nu(\alpha/t)$
(43)	$2\alpha^{\frac{1}{2}} p^{\mu-1} K_{2\nu}(2\alpha^{\frac{1}{2}} p^{\frac{1}{2}}) \quad \operatorname{Re} \alpha > 0$	$t^{\frac{1}{2}-\mu} e^{-\frac{1}{2} \alpha/t} W_{\mu-\frac{1}{2}, \nu}(\alpha/t)$
(44)	$r^{-1} K_1(br) \quad b > 0$	$0 \quad 0 < t < b$ $\alpha^{-1} b^{-1} \sin \alpha y \quad t > b$
(45)	$2^{\frac{1}{2}} \pi^{-\frac{1}{2}} r^{-\nu} K_\nu(br) \quad \operatorname{Re} \nu > -\frac{1}{2}$	$0 \quad 0 < t < b$ $\alpha^{\frac{1}{2}-\nu} b^{-\nu} y^{\nu-\frac{1}{2}} J_{\nu-\frac{1}{2}}(\alpha y) \quad t > b$
(46)	$2^{\frac{1}{2}} \pi^{-\frac{1}{2}} r^{-\nu} e^{\beta p} K_\nu(\beta r) \quad \operatorname{Re} \nu > -\frac{1}{2}, \quad  \arg \beta  < \pi$	$\alpha^{\frac{1}{2}-\nu} \beta^{-\nu} (\beta^2 + 2\beta t)^{\frac{1}{2}\nu-\frac{1}{2}}$ $\times J_{\nu-\frac{1}{2}}[\alpha(\beta^2 + 2\beta t)^{\frac{1}{2}}]$

$$r = (p^2 + \alpha^2)^{\frac{1}{2}}$$

## Modified Bessel functions of other arguments (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(47)	$2^{\frac{v}{2}} \pi^{-\frac{v}{2}} s^{-v} K_v(bs)$ $\operatorname{Re} v > -\frac{1}{2}$	$0 \quad 0 < t < b$ $a^{\frac{v}{2}-v} b^{-v} \gamma^{v-\frac{v}{2}} I_{v-\frac{v}{2}}(ay) \quad t > b$
(48)	$2^{\frac{v}{2}} \pi^{-\frac{v}{2}} s^{-v} e^{\beta p} K_v(\beta s)$ $\operatorname{Re} v > -\frac{1}{2}, \quad  \arg \beta  < \pi$	$a^{\frac{v}{2}-v} \beta^{-v} (t^2 + 2\beta t)^{\frac{v}{2}v-\frac{v}{2}}$ $\times I_{v-\frac{v}{2}}[a(t^2 + 2\beta t)^{\frac{v}{2}}]$
(49)	$\frac{(\frac{1}{2}c)^p K_p(c)}{\Gamma(p+\frac{1}{2})}$ $c > 0$	$\frac{\cos[c(e^t-1)^{\frac{1}{2}}]}{2\pi^{\frac{1}{2}}(1-e^{-t})^{\frac{1}{2}}}$
(50)	$\frac{a^p K_{v-p}(a)}{\Gamma(p+1)}$ $\operatorname{Re} v > -1, \quad a > 0$	$\frac{1}{2}(e^t-1)^{\frac{1}{2}v} J_v[2a(e^t-1)^{\frac{1}{2}}]$
(51)	$J_v(a^{\frac{v}{2}} p^{\frac{v}{2}}) K_v(a^{\frac{v}{2}} p^{\frac{v}{2}})$ $a > 0$	$\frac{1}{2} t^{-1} J_v(\frac{1}{2}a/t)$
(52)	$Y_v(a^{\frac{v}{2}} p^{\frac{v}{2}}) K_v(a^{\frac{v}{2}} p^{\frac{v}{2}})$ $a > 0$	$\frac{1}{2} t^{-1} Y_v(\frac{1}{2}a/t)$
(53)	$H_v^{(1)}(a^{\frac{v}{2}} p^{\frac{v}{2}}) K_v(a^{\frac{v}{2}} p^{\frac{v}{2}})$ $a > 0$	$\frac{1}{2} t H_v^{(1)}(\frac{1}{2}a/t)$
(54)	$H_v^{(2)}(a^{\frac{v}{2}} p^{\frac{v}{2}}) K_v(a^{\frac{v}{2}} p^{\frac{v}{2}})$ $a > 0$	$\frac{1}{2} t^{-1} H_v^{(2)}(\frac{1}{2}a/t)$
(55)	$p^{\frac{v}{2}} I_n(bp) K_{n+\frac{v}{2}}(bp)$ $b > 0$	$\frac{(-1)^n \cos[(2n+\frac{1}{2}) \cos^{-1}(\frac{1}{2}t/b)]}{[\frac{1}{2}\pi(4b^2t-t^3)]^{\frac{v}{2}}}$ $0 < t < 2b$ $0 \quad t > 2b$
(56)	$K_v(a^{\frac{v}{2}} p^{\frac{v}{2}} + \beta^{\frac{v}{2}} p^{\frac{v}{2}})$ $\times I_v(a^{\frac{v}{2}} p^{\frac{v}{2}} - \beta^{\frac{v}{2}} p^{\frac{v}{2}})$ $\operatorname{Re} \alpha > 0, \quad \operatorname{Re} \beta > 0$	$\frac{1}{2} t^{-1} e^{-\frac{v}{2}(\alpha+\beta)/t} I_v[\frac{1}{2}(\alpha-\beta)/t]$

$$s = (p^2 - a^2)^{\frac{v}{2}}, \quad y = (t^2 - b^2)^{\frac{v}{2}}$$

## Modified Bessel functions of other arguments (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(57)	$I_\nu[\frac{1}{2}b(r-p)]K_\nu[\frac{1}{2}b(r+p)]$ $r = (p^2 + \alpha^2)^{\frac{1}{4}}, \quad \operatorname{Re} \nu > -\frac{1}{2}$	$0 \quad 0 < t < b$ $(t^2 - b^2)^{-\frac{1}{4}} J_{2\nu}[\alpha(t^2 - b^2)^{\frac{1}{4}}] \quad t > b$
(58)	$I_{\nu+p}(c)K_{\nu-p}(c)$ $c > 0, \quad \operatorname{Re} \nu > -\frac{1}{2}$	$\frac{1}{2}J_{2\nu}(2c \sinh \frac{1}{2}t)$
(59)	$p^{2\nu}[K_{2\nu}(\alpha^{\frac{1}{4}}p^{\frac{1}{4}})]^2 \quad \operatorname{Re} \alpha > 0$	$\frac{1}{2}\pi^{\frac{1}{4}}\alpha^{\nu-\frac{1}{4}}t^{-3\nu-\frac{1}{4}}e^{-\frac{1}{4}\alpha/t}W_{\nu,\nu}(\alpha/t)$
(60)	$e^{\frac{1}{4}(\alpha+\beta)p}K_{2\nu}(\frac{1}{2}\alpha p)K_{2\nu}(\frac{1}{2}\beta p)$ $ \arg \alpha  < \pi, \quad  \arg \beta  < \pi$	$\pi(\alpha\beta)^{\nu-\frac{1}{4}}(\alpha+t)^{-\nu-\frac{1}{4}}(\beta+t)^{-\nu-\frac{1}{4}}$ $\times P_{2\nu-\frac{1}{2}}[2\alpha^{-1}\beta^{-1}(\alpha+t)(\beta+t)-1]$
(61)	$p^{\frac{1}{4}}K_{\nu+\frac{1}{4}}(\alpha^{\frac{1}{4}}p^{\frac{1}{4}})K_{\nu-\frac{1}{4}}(\alpha^{\frac{1}{4}}p^{\frac{1}{4}})$ $\operatorname{Re} \alpha > 0$	$\frac{1}{2}(2\alpha)^{-\frac{1}{4}}\pi^{\frac{1}{4}}t^{-1}e^{-\frac{1}{4}\alpha/t}W_{\frac{1}{2},\nu}(\alpha/t)$
(62)	$p^{\frac{1}{4}}K_{\nu+\frac{1}{4}}(bp)K_{\nu-\frac{1}{4}}(bp) \quad b > 0$	$0 \quad 0 < t < 2b$ $\frac{2^{\frac{1}{4}}\pi^{\frac{1}{4}}\cosh[2\nu\cosh^{-1}(\frac{1}{2}t/b)]}{(t^3 - 4b^2t)^{\frac{1}{4}}} \quad t > 2b$
(63)	$p^{\frac{1}{4}}e^{2\alpha p}K_{\nu+\frac{1}{4}}(\alpha p)K_{\nu-\frac{1}{4}}(\alpha p)$ $ \arg \alpha  < \pi$	$(2\pi)^{\frac{1}{4}}[t(t+2\alpha)(t+4\alpha)]^{-\frac{1}{2}}$ $\times \cosh[2\nu\cosh^{-1}(1+\frac{1}{2}t/\alpha)]$
(64)	$K_\nu(\alpha^{\frac{1}{4}}p^{\frac{1}{4}} + \beta^{\frac{1}{4}}p^{\frac{1}{4}})$ $\times K_\nu(\alpha^{\frac{1}{4}}p^{\frac{1}{4}} - \beta^{\frac{1}{4}}p^{\frac{1}{4}})$ $\operatorname{Re} \alpha > 0, \quad \operatorname{Re} \beta > 0$	$\frac{1}{2}t^{-1}e^{-\frac{1}{4}(\alpha+\beta)/t}K_\nu[\frac{1}{2}(\alpha-\beta)/t]$
(65)	$K_\nu[p^{\frac{1}{4}} + (p-1)^{\frac{1}{4}}]K_\nu[p^{\frac{1}{4}} - (p-1)^{\frac{1}{4}}]$	$\frac{1}{2}t^{-1}e^{\frac{1}{4}t-1/t}K_\nu(\frac{1}{2}t)$
(66)	$K_\nu[(\lambda S/\alpha)^{\frac{1}{4}}]K_\nu[(\lambda\alpha/S)^{\frac{1}{4}}]$ $S = (p^2 - \alpha^2)^{\frac{1}{4}} + p$ $\operatorname{Re}(\lambda/\alpha) > 0$	$\frac{1}{2}t^{-1}e^{-\frac{1}{2}\lambda\alpha^{-1}t^{-1}}K_\nu(\alpha\lambda t)$

**5.17. Functions related to Bessel functions**

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(1)	$J_\nu(p) - J_\nu(p)$	$\pi^{-1} \sin(\nu\pi) (t^2 + 1)^{-\frac{\nu}{2}} [(t^2 + 1)^{\frac{\nu}{2}} - t]^\nu$
(2)	$\csc(\pi p) [J_p(a) - J_p(a)]$ $\text{Re } a \geq 0$	$\pi^{-1} e^{-a \sinh t}$
(3)	$E_\nu(p) + Y_\nu(p)$	$-(t^2 + 1)^{-\frac{\nu}{2}} \{[(t^2 + 1)^{\frac{\nu}{2}} + t]^\nu + \cos(\nu\pi) [(t^2 + 1)^{\frac{\nu}{2}} - t]^\nu\}$
(4)	$p^{-1} [H_0(ap) - Y_0(ap)]$	$2\pi^{-1} \sinh^{-1}(t/a)$
(5)	$\frac{1}{2}\pi [H_1(ap) - Y_1(ap)] - 1$ $ \arg a  < \frac{1}{2}\pi$	$a^{-1} t (t^2 + a^2)^{-\frac{\nu}{2}}$
(6)	$p^{-\nu} [H_\nu(ap) - Y_\nu(ap)]$ $\text{Re } a > 0$	$\frac{2^{1-\nu} a^{-\nu}}{\pi^{\frac{\nu}{2}} \Gamma(\nu + \frac{1}{2})} (t^2 + a^2)^{\nu - \frac{\nu}{2}}$
(7)	$p^{\frac{\nu}{2}} [H_{\frac{\nu}{2}}(p^2/a) - Y_{\frac{\nu}{2}}(p^2/a)]$ $a > 0$	$a\pi^{-\frac{\nu}{2}} t^{\frac{\nu}{2}} J_{-\frac{\nu}{2}}(\frac{1}{4}at^2)$
(8)	$p^{\frac{\nu}{2}} [H_{-\frac{\nu}{2}}(p^2/a) - Y_{-\frac{\nu}{2}}(p^2/a)]$ $a > 0$	$a\pi^{-\frac{\nu}{2}} t^{\frac{\nu}{2}} J_{\frac{\nu}{2}}(\frac{1}{4}at^2)$
(9)	$p^{3/2} [H_{-\frac{\nu}{2}}(p^2/a) - Y_{-\frac{\nu}{2}}(p^2/a)]$ $a > 0$	$-\frac{1}{2}a^2 \pi^{-1/2} t^{3/2} J_{-\frac{\nu}{2}}(\frac{1}{4}at^2)$
(10)	$p^{3/2} [H_{-\frac{\nu}{2}}(p^2/a) - Y_{-\frac{\nu}{2}}(p^2/a)]$ $a > 0$	$\frac{1}{2}a^2 \pi^{-1/2} t^{3/2} J_{-\frac{\nu}{2}}(\frac{1}{4}at^2)$
(11)	$p^{-\lambda} H_\nu\left(\frac{2a}{p}\right)$ $\text{Re } (\lambda + \nu) > -1$	$\frac{2\pi^{-\frac{\nu}{2}} a^{\nu+1} t^{\lambda+\nu}}{\Gamma(\nu+3/2) \Gamma(\lambda+\nu+1)} \\ \times {}_1F_4\left(1; \frac{3}{2}, \nu + \frac{3}{2}, \frac{\lambda+\nu+1}{2}, \frac{\lambda+\nu}{2} + 1; -\frac{a^2 t^2}{4}\right)$

## Functions related to Bessel functions (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(12)	$p^{-\frac{1}{2}} [\mathbf{H}_0(2ap^{\frac{1}{2}}) - Y_0(2ap^{\frac{1}{2}})]$	$2\pi^{-3/2} t^{-1/2} e^{\frac{1}{2}a^2/t} K_0(\frac{1}{2}a^2/t)$
(13)	$p^{-\frac{1}{2}\nu} [\mathbf{H}_{-\nu}(ap^{\frac{1}{2}}) - Y_{-\nu}(ap^{\frac{1}{2}})]$ $\text{Re } \nu > -\frac{1}{2}$	$2^\nu \pi^{-1} a^{-\nu} \cos(\nu\pi) t^{\nu-1} \exp(\frac{1}{4}a^2 t^{-1})$ $\times \text{Erfc}(\frac{1}{2}at^{-\frac{1}{2}})$
(14)	$p^{-\frac{1}{2}\nu-\frac{1}{2}} [\mathbf{H}_\nu(ap^{\frac{1}{2}}) - Y_\nu(ap^{\frac{1}{2}})]$	$2\pi^{-\frac{1}{2}} a^{-1} [\Gamma(\frac{1}{2}+\nu)]^{-1} t^{-\frac{1}{2}\nu}$ $\times \exp\left(\frac{a^2}{8t}\right) W_{\frac{1}{2}\nu, \frac{1}{2}\nu}\left(\frac{a^2}{4t}\right)$
(15)	$\Gamma(p + \frac{1}{2}) 2^p a^{-p} \mathbf{H}_p(a)$	$\pi^{-\frac{1}{2}} (e^{t-1})^{-\frac{1}{2}} \sin[a(1-e^{-t})^{\frac{1}{2}}]$
(16)	$p^{-1} [I_0(bp) - \mathbf{L}_0(bp)]$ $b > 0$	$2\pi^{-1} \sin^{-1}(t/b)$ 0 $0 < t < b$ $t > b$
(17)	$\frac{1}{2}\pi [L_1(bp) - I_1(bp)] + 1$ $b > 0$	$b^{-1} t (b^2 - t^2)^{-\frac{1}{2}}$ 0 $0 < t < b$ $t > b$
(18)	$\pi^{\frac{1}{2}} \Gamma(\nu + \frac{1}{2}) p^{-\nu} [I_\nu(bp) - \mathbf{L}_\nu(bp)]$ $\text{Re } \nu > -\frac{1}{2}, \quad b > 0$	$2^{1-\nu} b^{-\nu} (b^2 - t^2)^{\nu-\frac{1}{2}}$ 0 $0 < t < b$ $t > b$
(19)	$\pi^{\frac{1}{2}} (2b)^\nu \Gamma(\nu + \frac{1}{2}) p^{-\nu} e^{-bp} \mathbf{L}_\nu(bp)$ $\text{Re } \nu > -\frac{1}{2}, \quad b > 0$	$(2bt - t^2)^{\nu-\frac{1}{2}}$ $-(2bt - t^2)^{\nu-\frac{1}{2}}$ 0 $0 < t < b$ $b < t < 2b$ $t > 2b$
(20)	$\frac{1}{2}\pi^{\frac{1}{2}} \Gamma(\nu + \frac{1}{2}) p^{-\nu} \operatorname{csch} p \mathbf{L}_\nu(p)$ $\text{Re } \nu > -\frac{1}{2}$	$[2(t-2k) - (t-2k)^2]^{\nu-\frac{1}{2}}$ $2k < t < 2k+1$ $-[2(t-2k) - (t-2k)^2]^{\nu-\frac{1}{2}}$ $2k+1 < t < 2k+2$

## Functions related to Bessel functions (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(21)	$\Gamma(\nu + \frac{1}{2}) p^{-\nu} \operatorname{csch}(\frac{1}{2}p) [I_\nu(p) - L_\nu(p)]$ $\operatorname{Re} \nu > -\frac{1}{2}$	$0 \quad 0 < t < 1/2$ $4 \pi^{-\frac{1}{2}} [ \frac{3}{4} + t - k - (t-k)^2 ]^{\nu - \frac{1}{2}}$ $k + 1/2 < t < k + 3/2$ $k = 0, 1, 2, \dots$
(22)	$p^{-\lambda} L_\nu(2a/p)$ $\operatorname{Re}(\lambda + \nu) > -1$	$\frac{2\pi^{-\frac{1}{2}} a^{\nu+1} t^{\lambda+1}}{\Gamma(\nu+3/2) \Gamma(\lambda+\nu+1)}$ $\times {}_1F_4 \left( 1; \frac{3}{2}, \nu + \frac{3}{2}, \frac{\lambda+\nu+1}{2}, \frac{\lambda+\nu}{2} + 1; \frac{a^2 t^2}{4} \right)$
(23)	$p^{-\frac{1}{2}} [I_0(2ap^{\frac{1}{2}}) - L_0(2ap^{\frac{1}{2}})]$ $\operatorname{Re} \alpha > 0$	$(\pi t)^{-\frac{1}{2}} e^{-\frac{1}{2}a^2/t} I_0(\frac{1}{2}a^2/t)$
(24)	$p^{-\frac{1}{2}\nu} [L_{-\nu}(ap^{\frac{1}{2}}) - I_\nu(ap^{\frac{1}{2}})]$ $\operatorname{Re} \nu > -\frac{1}{2}$	$i \pi^{-1} 2^\nu a^{-\nu} \cos(\nu\pi) t^{\nu-1}$ $\times \exp(\frac{1}{4}a^2 t^{-1}) \operatorname{Erf}(\frac{1}{2}iat^{-\frac{1}{2}})$
(25)	$\Gamma(\frac{1}{2}-p)(\frac{1}{2}b)^p [I_p(b) - L_{-p}(b)]$ $b > 0$	$\pi^{-\frac{1}{2}} (1-e^{-t})^{-\frac{1}{2}} \sin[b(e^t-1)^{\frac{1}{2}}]$
(26)	$S_{0,\nu}(p)$	$(1+t^2)^{-\frac{1}{2}} \cosh(\nu \sinh^{-1} t)$
(27)	$S_{-1,\nu}(p)$	$\nu^{-1} (1+t^2)^{-\frac{1}{2}} \sinh(\nu \sinh^{-1} t)$
(28)	$p^{-1} S_{0,\nu}(p)$	$\nu^{-1} \sinh(\nu \sinh^{-1} t)$
(29)	$p^{-1} S_{1,\nu}(p)$	$\cosh(\nu \sinh^{-1} t)$
(30)	$p^{1-2\lambda-\mu} S_{\mu,\nu} \left( \frac{p}{a} \right)$ $\operatorname{Re} \lambda > 0, \quad \operatorname{Re} \alpha > 0$	$\frac{a^{1-\mu} t^{2\lambda-1}}{\Gamma(2\lambda)}$ $\times {}_3F_2 \left( 1, \frac{1-\mu+\nu}{2}, \frac{1-\mu-\nu}{2}; \lambda, \lambda + \frac{1}{2}; -a^2 t^2 \right)$

## Functions related to Bessel functions (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(31)	$p^{\frac{1}{4}} S_{-\mu-1, \frac{1}{4}}(\frac{1}{2}p^2) \quad \operatorname{Re} \mu > -\frac{3}{4}$	$2^{2\mu+1} [\Gamma(2\mu+3/2)]^{-1} t^{\frac{1}{4}} \times s_{\mu, \frac{1}{4}}(\frac{1}{2}t^2)$
Further similar formulas may be found in <i>Nederl. Akad. Wetensch. Proc.</i> , 1935: 38, Part II, p. 629.		
(32)	$p^{-\mu-\frac{1}{4}} S_{2\mu, 2\nu}(2\alpha^{\frac{1}{4}} p^{\frac{1}{4}}) \quad \operatorname{Re}(\mu \pm \nu) > -\frac{1}{2}, \quad  \arg \alpha  < \pi$	$2^{2\mu-1} \alpha^{-\frac{1}{4}} t^{\mu} e^{\frac{1}{4}\alpha/t} W_{\mu, \nu}(a/t)$
(33)	$p^{-\frac{1}{4}\nu} S_{\mu, \nu}(2\alpha^{\frac{1}{4}} p^{\frac{1}{4}}) \quad \operatorname{Re}(\mu - \nu) < 1, \quad  \arg \alpha  < \pi$	$2^{\mu-1} \alpha^{-\frac{1}{4}\nu} t^{\nu-1} e^{\alpha/t} \times \Gamma(\frac{1}{2}\mu + \frac{1}{2}\nu + \frac{1}{2}, a/t)$
(34)	$p^{-1} S_{2, \nu}(ap) - a$	$(\nu - 1/\nu) \sinh[\nu \sinh^{-1}(t/a)]$
(35)	$2 O_n(p)$	$[t + (1+t^2)^{\frac{1}{4}}]^n + [t - (1+t^2)^{\frac{1}{4}}]^n$
(36)	$S_n(p)$	$\frac{[t + (1+t^2)^{\frac{1}{4}}]^n - [t - (1+t^2)^{\frac{1}{4}}]^n}{(1+t^2)^{\frac{1}{4}}}$

## 5.18. Parabolic cylinder functions

(1)	$\Gamma(\nu) e^{\frac{1}{4}\alpha^2 p^2} D_{-\nu}(ap) \quad \operatorname{Re} \nu > 0, \quad  \arg \alpha  < \frac{1}{4}\pi$	$\alpha^{-\nu} t^{\nu-1} e^{-\frac{1}{4}t^2/\alpha^2}$
(2)	$\Gamma(2\nu) p^{-1} e^{\frac{1}{4}\alpha^2 p^2} D_{-2\nu}(ap) \quad \operatorname{Re} \nu > 0, \quad  \arg \alpha  < \frac{1}{4}\pi$	$2^{\nu+1} \gamma(\nu, \frac{1}{2}t^2/\alpha^2)$
(3)	$D_{-2\nu}(2b^{\frac{1}{4}} p^{\frac{1}{4}}) \quad \operatorname{Re} \nu > 0, \quad b > 0$	$\begin{cases} 0 & 0 < t < b \\ \frac{2^{\frac{1}{2}-\nu} b^{\frac{1}{4}}}{\Gamma(\nu)} \frac{(t-b)^{\nu-1}}{(t+b)^{\nu+\frac{1}{4}}} & t > b \end{cases}$

## Parabolic cylinder functions (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(4)	$p^{-\frac{1}{2}} D_{1-2\nu}(2^{\frac{1}{2}} p^{\frac{1}{2}})$ $\text{Re } \nu > 0, \quad b > 0$	$0 \quad 0 < t < b$ $\frac{2^{\frac{1}{2}-\nu}(t-b)^{\nu-1}}{\Gamma(\nu)(t+b)^{\nu-\frac{1}{2}}} \quad t > b$
(5)	$\Gamma(\nu) e^{\frac{1}{2}\alpha p} D_{-2\nu}(2^{\frac{1}{2}} \alpha^{\frac{1}{2}} p^{\frac{1}{2}})$ $\text{Re } \nu > 0, \quad  \arg \alpha  < \pi$	$2^{-\nu} \alpha^{\frac{1}{2}} t^{\nu-1} (t+\alpha)^{-\nu-\frac{1}{2}}$
(6)	$\Gamma(\nu) p^{-\frac{1}{2}} e^{\frac{1}{2}\alpha p} D_{1-2\nu}(2^{\frac{1}{2}} \alpha^{\frac{1}{2}} p^{\frac{1}{2}})$ $\text{Re } \nu > 0, \quad  \arg \alpha  < \pi$	$2^{\frac{1}{2}-\nu} t^{\nu-1} (t+\alpha)^{\frac{1}{2}-\nu}$
(7)	$\Gamma(2\nu) p^{-\nu} e^{\frac{1}{2}\alpha/p} D_{-2\nu}(\alpha^{\frac{1}{2}} p^{-\frac{1}{2}})$ $\text{Re } \nu > 0$	$(2t)^{\nu-1} e^{-2^{\frac{1}{2}} \alpha^{\frac{1}{2}} t^{\frac{1}{2}}}$
(8)	$p^{-\nu} e^{-\frac{1}{2}\alpha/p} D_{2\nu-1}(\alpha^{\frac{1}{2}} p^{-\frac{1}{2}})$ $\text{Re } \nu > 0$	$2^{\nu+\frac{1}{2}} \pi^{-\frac{1}{2}} t^{\nu-1} \sin(\nu\pi - 2^{\frac{1}{2}} \alpha^{\frac{1}{2}} t^{\frac{1}{2}})$
(9)	$2^{p+\nu} \Gamma(p+\nu) D_{-2p}(\alpha)$ $ \arg \alpha  < \frac{1}{4}\pi$	$e^{\frac{1}{2}t}(e^t-1)^{-\nu-\frac{1}{2}} \exp\left[-\frac{\alpha^2 e^{-t}}{4(1-e^{-t})}\right]$ $\times D_{2\nu}\left[\frac{\alpha}{(1-e^{-t})^{\frac{1}{2}}}\right]$
(10)	$\Gamma(\nu+1) D_{-\nu-1}(pe^{\frac{1}{2}\pi i})$ $\times D_{-\nu-1}(pe^{-\frac{1}{2}\pi i}) \quad \text{Re } \nu > -1$	$\pi^{\frac{1}{2}} J_{\nu+\frac{1}{2}}(\frac{1}{2}t^2)$
(11)	$2^{\frac{1}{2}} \Gamma(\nu) D_{-\nu}(2^{\frac{1}{2}} p^{\frac{1}{2}} e^{\frac{1}{2}\pi i})$ $\times D_{-\nu}(2^{\frac{1}{2}} p^{\frac{1}{2}} e^{-\frac{1}{2}\pi i}) \quad \text{Re } \nu > 0$	$t^{\nu-1} (1+t^2)^{-\frac{1}{2}} [1+(1+t^2)^{\frac{1}{2}}]^{\frac{1}{2}-\nu}$
(12)	$e^p D_{\nu-\frac{1}{2}}(2^{\frac{1}{2}} p^{\frac{1}{2}}) D_{-\nu-\frac{1}{2}}(2^{\frac{1}{2}} p^{\frac{1}{2}})$	$\frac{\cos \nu \cos^{-1}[(1+t)^{-1}]}{[\pi t(t+1)(t+2)]^{\frac{1}{2}}}$

**Parabolic cylinder functions (cont'd)**

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(13)	$p^{-\frac{1}{4}} e^{\frac{1}{4}(a+\beta)p} D_{4\mu} (2^{\frac{1}{2}} a^{\frac{1}{2}} p^{\frac{1}{2}})$ $\times D_{4\nu} (2^{\frac{1}{2}} \beta^{\frac{1}{2}} p^{\frac{1}{2}})$ $\operatorname{Re}(\mu + \nu) < \frac{1}{4}$ $ \arg a  < \pi, \quad  \arg \beta  < \pi$	$2^{-\frac{1}{2}} t^{-\mu-\nu-\frac{1}{4}} (t+a)^{\mu-\nu-\frac{1}{4}}$ $\times (t+\beta)^{\nu-\mu-\frac{1}{4}} (-t-a-\beta)^{\mu+\nu+\frac{1}{4}}$ $\times P_{2\nu-2\mu-\frac{1}{2}}^{\nu+\mu+\frac{1}{4}} [a^{\frac{1}{2}} \beta^{\frac{1}{2}} (t+a)^{-\frac{1}{2}} (t+\beta)^{-\frac{1}{2}}]$

**5.19. Gauss' hypergeometric function**

(1)	$F(a, \beta; \gamma; \frac{1}{2}-p/\lambda)$ $\operatorname{Re} a > 0, \quad \operatorname{Re} \beta > 0$	$\frac{\lambda \Gamma(\gamma)}{\Gamma(a) \Gamma(\beta)} (\lambda t)^{\frac{1}{2}(a+\beta-3)}$ $\times W_{\frac{1}{2}(a+\beta+1)-\gamma, \frac{1}{2}(a-\beta)}(\lambda t)$
(2)	$\Gamma(a) p^{-a} F(a, \beta; \gamma; \lambda/p)$ $\operatorname{Re} a > 0$	$\lambda^{-\frac{1}{2}\gamma} t^{a-\frac{1}{2}\gamma-1} M_{\frac{1}{2}\gamma-\beta, \frac{1}{2}\gamma-\frac{1}{2}}(\lambda t)$
(3)	$p^{\gamma-1} (p-1)^n F[-n, a; \gamma; p/(p-1)]$ $\operatorname{Re} \gamma < 1-n$ $\operatorname{Re}(a-\gamma) > n-1$	$n! [\Gamma(1-\gamma)]^{-1} t^{-\gamma-n} L_n^{\alpha-\gamma-n}(t)$
(4)	$p^{n+n} (1+p)^{-n-n-2}$ $\times F(-m, -n; 2; p^{-2})$	$(-1)^{n+n} t^{-1} k_{2n+2}(\frac{1}{2}t) k_{2n+2}(\frac{1}{2}t)$
(5)	$(p+1)^{-2\alpha} F\left[-n, a; \frac{1}{2}-\nu; \left(\frac{p-1}{p+1}\right)^2\right]$ $\operatorname{Re} a > 0$	$\frac{(n!)^2 \pi 2^{1-\alpha}}{\Gamma(a) \Gamma(\frac{1}{2}+n)} t^{2\alpha-1} [L_n^{\alpha-\frac{1}{2}}(t)]^2$
(6)	$(p^2 + \alpha^2)^{-\lambda}$ $\times F\left(\lambda, \mu; \lambda + \mu + \frac{1}{2}; \frac{\alpha^2}{p^2 + \alpha^2}\right)$ $\operatorname{Re} \lambda > 0$	$\frac{\Gamma(\lambda + \mu + \frac{1}{2})}{\Gamma(2\lambda)} \left(\frac{\alpha}{2}\right)^{\frac{1}{2}-\lambda-\mu}$ $\times t^{\lambda-\mu-\frac{1}{2}} J_{\lambda+\mu-\frac{1}{2}}(\alpha t)$

## Gauss' hypergeometric function (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(7)	$(p-a)^n (p-\beta)^m p^{-n-n-2} \\ \times F \left[ \begin{matrix} -m, -n; -m-n-1; \\ \frac{p(p-a-\beta)}{(p-a)(p-\beta)} \end{matrix} \right]$	$\frac{(m+1)!(n+1)!}{(m+n+1)!} \frac{(-1)^{m+n}}{\alpha\beta t} e^{\frac{\alpha+\beta}{2}t} \\ \times k_{2n+2}(\frac{1}{2}\alpha t) k_{2m+2}(\frac{1}{2}\beta t)$
(8)	$(p-a)^n (p-\beta)^m p^{-n-n-\frac{1}{2}} \\ \times F \left[ \begin{matrix} -m, -n; -m-n+\frac{1}{2}; \\ \frac{p(p-a-\beta)}{(p-a)(p-\beta)} \end{matrix} \right]$	$\frac{(-2)^{m+n} (m+n)!}{(2m+2n)! \pi^{\frac{1}{2}} t^{\frac{1}{2}}} e^{\frac{\alpha+\beta}{2}t} \\ \times D_{2n}(2^{\frac{1}{2}}\alpha^{\frac{1}{2}}t^{\frac{1}{2}}) D_{2m}(2^{\frac{1}{2}}\beta^{\frac{1}{2}}t^{\frac{1}{2}})$
(9)	$(p-a)^n (p-\beta)^m p^{-n-n-3/2} \\ \times F \left[ \begin{matrix} -m, -n; -m-n-\frac{1}{2}; \\ \frac{p(p-a-\beta)}{(p-a)(p-\beta)} \end{matrix} \right]$	$-\frac{(-1)^{m+n} (-2)^{m+n+1} (m+n+1)!}{(2m+2n+2)! (\pi\alpha\beta t)^{\frac{1}{2}}} \\ \times e^{\frac{\alpha+\beta}{2}t} D_{2n+1}(2^{\frac{1}{2}}\alpha^{\frac{1}{2}}t^{\frac{1}{2}}) \\ \times D_{2m+1}(2^{\frac{1}{2}}\beta^{\frac{1}{2}}t^{\frac{1}{2}})$
(10)	$(p-a)^n (p-\beta)^m p^{-n-n-\lambda-1} \\ \times F \left[ \begin{matrix} -m, -n; -m-n-\lambda; \\ \frac{p(p-a-\beta)}{(p-a)(p-\beta)} \end{matrix} \right] \quad \text{Re } \lambda > -1$	$\frac{m! n! t^\lambda}{\Gamma(m+n+\lambda+1)} L_n^\lambda(\alpha t) L_m^\lambda(\beta t)$
(11)	$B(p, \gamma) F(a, \beta; \gamma+p; z)$ $\text{Re } \gamma > 0, \quad  \arg(z-1)  < \pi$	$(1-e^{-t})^{\gamma-1} F[a, \beta; \gamma; z(1-e^{-t})]$
(12)	$\frac{\Gamma(p)}{\Gamma(p+\frac{1}{2})} \\ \times F(-\mu-\nu, \frac{1}{2}-\mu+\nu; p+\frac{1}{2}; z^2) \\  z  < 1$	$\frac{\Gamma(\frac{1}{2}-\mu-\nu)\Gamma(\frac{1}{2}-\mu+\nu)}{2^{2\mu+1} \pi (1-e^{-t})^{\frac{1}{2}}} \\ \times (1-z^2+z^2 e^{-t})^\mu \\ \times \{P_{2\nu}^{2\mu}[z(1-e^{-t})]^{\frac{1}{2}} \\ + P_{2\nu}^{2\mu}[-z(1-e^{-t})]^{\frac{1}{2}}\}$

## Gauss' hypergeometric function (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(13)	$\frac{\Gamma(p)}{\Gamma(p+3/2)} \times F(\tfrac{1}{2}-\mu-\nu, 1-\mu+\nu; p+3/2; z^2)$ $ z  < 1$	$-\frac{\Gamma(-\mu-\nu)\Gamma(\tfrac{1}{2}-\mu+\nu)}{4^{\mu+\frac{1}{2}} \pi z}$ $\times (1-z^2 + z^2 e^{-t})^\mu$ $\times \{P_{2\nu}^{2\mu}[z(1-e^{-t})^{\frac{1}{2}}] - P_{2\nu}^{2\mu}[-z(1-e^{-t})^{\frac{1}{2}}]\}$
(14)	$B(p, \nu) F(a, p; p+\nu; z)$ $\text{Re } \nu > 0, \quad  \arg(z-1)  < \pi$	$(1-e^{-t})^{\nu-1} (1-ze^{-t})^{-a}$

## 5.20. Confluent hypergeometric functions

(1)	$p^{-\mu-\frac{1}{2}} e^{-\frac{1}{2}(a+b)p} M_{\kappa, \mu}[(b-a)p]$ $\text{Re } (\mu \pm \kappa) > -\frac{1}{2}, \quad b > a \geq 0$	$0 \quad 0 < t < a$ $\frac{(b-a)^{\frac{1}{2}-\mu}}{B(\frac{1}{2}+\kappa+\mu, \frac{1}{2}-\kappa+\mu)} \frac{(t-a)^{\kappa+\mu-\frac{1}{2}}}{(b-t)^{\kappa-\mu+\frac{1}{2}}} \quad a < t < b$ $0 \quad t > b$
(2)	$p^\kappa e^{\frac{1}{2}\alpha/p} M_{\kappa, \mu}(\alpha/p)$ $\text{Re } (\kappa - \mu) < \frac{1}{2}$	$\frac{\alpha^{\frac{1}{2}} \Gamma(2\mu+1)}{\Gamma(\mu-\kappa+\frac{1}{2})} t^{-\kappa-\frac{1}{2}} I_{2\mu}(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$
(3)	$p^{\frac{1}{2}} M_{\frac{1}{2}, \nu}(\alpha/p) M_{-\frac{1}{2}, \nu}(\alpha/p)$ $\text{Re } \nu > -\frac{1}{4}$	$2^{2\nu} \alpha t^{-\frac{1}{2}} \frac{[\Gamma(2\nu+1)]^2}{\Gamma(2\nu+\frac{1}{2})}$ $\times J_{2\nu}[e^{\frac{1}{2}\pi i}(2\alpha t)^{\frac{1}{2}}]$ $\times J_{2\nu}[e^{-\frac{1}{2}\pi i}(2\alpha t)^{\frac{1}{2}}]$
(4)	$p^{-\mu-\frac{1}{2}} W_{\kappa, \mu}(p)$ $\text{Re } (\mu - \kappa) > \frac{1}{2}$	$0 \quad 0 < t < \frac{1}{2}$ $\frac{1}{\Gamma(\mu+\frac{1}{2}-\kappa)} \frac{(t-\frac{1}{2})^{\mu-\kappa-\frac{1}{2}}}{(t+\frac{1}{2})^{-\mu-\kappa+\frac{1}{2}}} \quad t > \frac{1}{2}$

## Confluent hypergeometric functions (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(5)	$p^{-1} e^{\frac{1}{2}\alpha p} W_{\kappa,\mu}(ap)$ $ \arg a  < \pi, \quad \operatorname{Re} \kappa < 1$	$(1 + \alpha t^{-1})^{\frac{1}{2}\kappa} P_{\mu-\frac{1}{2}}^\kappa(1 + 2t/\alpha)$
(6)	$p^{-\mu-\frac{1}{2}} e^{\frac{1}{2}\alpha p} W_{\kappa,\mu}(ap)$ $ \arg a  < \pi$ $\operatorname{Re}(\frac{1}{2} - \kappa + \mu) > 0$	$\frac{\alpha^{\frac{1}{2}-\mu} t^{\mu-\kappa-\frac{1}{2}} (\alpha+t)^{\mu+\kappa-\frac{1}{2}}}{\Gamma(\frac{1}{2} - \kappa + \mu)}$
(7)	$p^{\kappa-\frac{1}{2}} e^{\frac{1}{2}p} W_{\kappa,\mu}(p)$ $\operatorname{Re} \kappa < \frac{1}{4}$	$2^{-2\kappa-\frac{1}{2}} t^{-\kappa-\frac{1}{2}} (1+t)^{-\frac{1}{2}}$ $\times P_{\frac{1}{2}\mu-\frac{1}{2}}^{2\kappa+\frac{1}{2}}[(1+t)^{\frac{1}{2}}]$
(8)	$p^{\kappa-1} e^{\frac{1}{2}p} W_{\kappa,\mu}(p)$ $\operatorname{Re} \kappa < \frac{3}{4}$	$2^{-2\kappa+\frac{1}{2}} t^{-\kappa+\frac{1}{2}} P_{\frac{1}{2}\mu-\frac{1}{2}}^{2\kappa-\frac{1}{2}}[(1+t)^{\frac{1}{2}}]$
(9)	$p^{-\sigma} e^{\frac{1}{2}p/\alpha} W_{\kappa,\mu}(p/a)$ $ \arg a  < \pi, \quad \operatorname{Re}(\sigma - \kappa) > 0$	$\alpha^{-\kappa} [\Gamma(\sigma - \kappa)]^{-1} t^{\sigma - \kappa - 1}$ $\times {}_2F_1(\frac{1}{2} - \kappa + \mu, \frac{1}{2} - \kappa - \mu; \sigma - \kappa; -at)$
(10)	$p^{-2\mu-1} e^{\frac{1}{2}\alpha p^2} W_{-3\mu,\mu}(ap^2)$ $\operatorname{Re} \alpha > 0, \quad \operatorname{Re} \mu > -1/8$	$2^{8\mu} \alpha^{-\mu} \frac{\Gamma(2\mu+1)}{\Gamma(8\mu+1)} t^{4\mu}$ $\times \exp\left(-\frac{t^2}{8\alpha}\right) I_{2\mu}\left(\frac{t^2}{8\alpha}\right)$
(11)	$p^{-2\mu-1} e^{\frac{1}{2}\alpha p^2} W_{\kappa,\mu}(ap^2)$ $\operatorname{Re} \alpha > 0, \quad \operatorname{Re}(\kappa - \mu) < \frac{1}{2}$	$\frac{2^{1-\kappa+\mu} \alpha^{\frac{1}{2}(\mu+\kappa+1)}}{\Gamma(1-2\kappa+2\mu)} t^{\mu-\kappa-1} \exp\left(-\frac{t^2}{8\alpha}\right)$ $\times M_{-\frac{1}{2}(\kappa+3\mu), \frac{1}{2}(\mu-\kappa)}\left(\frac{t^2}{4\alpha}\right)$
(12)	$p^\kappa W_{\kappa,\mu}(a/p)$ $\operatorname{Re}(\kappa \pm \mu) < \frac{1}{2}$	$\frac{2\alpha^{\frac{1}{2}} t^{-\kappa-\frac{1}{2}} K_{2\mu}(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})}{\Gamma(\frac{1}{2} - \kappa + \mu) \Gamma(\frac{1}{2} - \kappa - \mu)}$



## Confluent hypergeometric functions (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(20)	$p^{\frac{1}{2}} W_{\frac{1}{4}, \nu}(i\alpha/p) W_{\frac{1}{4}, \nu}(-i\alpha/p)$ $ Re \nu  < \frac{1}{4}$	$-\frac{4\alpha(\frac{1}{2}\pi/t)^{\frac{1}{2}} K_{2\nu}[(2\alpha t)^{\frac{1}{2}}]}{\Gamma(\frac{1}{4}+\nu)\Gamma(\frac{1}{4}-\nu)}$ $\times \{J_{2\nu}[(2\alpha t)^{\frac{1}{2}}] \sin [(\nu - \frac{1}{4})\pi]$ $+ Y_{2\nu}[(2\alpha t)^{\frac{1}{2}}] \cos [(\nu - \frac{1}{4})\pi]\}$
(21)	$p^{-3/2} W_{\kappa, 1/8}(\frac{1}{4}ip^2/a)$ $\times W_{\kappa, 1/8}(-\frac{1}{4}ip^2/a)$ $Re \kappa < 3/8, \quad a > 0$	$(\frac{1}{2}\pi^3 t)^{\frac{1}{2}} \frac{J_{-\kappa+1/8}(\frac{1}{2}at^2) J_{-\kappa-1/8}(\frac{1}{2}at^2)}{\Gamma(3/8-\kappa)\Gamma(5/8-\kappa)}$
(22)	$\frac{\Gamma(\frac{1}{2}-\kappa+\mu+p)}{\Gamma(1+2\mu+p)} a^{-\frac{1}{2}(1+2\mu+p)}$ $\times M_{\kappa-\frac{1}{2}p, \mu+\frac{1}{2}p}(a)$ $Re(\frac{1}{2}+\kappa+\mu) > 0$	$\frac{e^{-(\frac{1}{2}+\kappa+\mu)t}}{\Gamma(\frac{1}{2}+\kappa+\mu)} (1-e^{-t})^{\kappa+\mu-\frac{1}{2}}$ $\times \exp[-\alpha(\frac{1}{2}-e^{-t})]$
(23)	$\Gamma(\frac{1}{2}-\kappa-\mu+p) W_{\kappa-p, \mu}(a)$ $Re \alpha > 0$	$a^{\frac{1}{2}-\mu} (e^t-1)^{2\mu-1} \exp[-\frac{1}{2}\alpha$ $+ (\frac{1}{2}-\kappa-\mu)t - \alpha/(e^t-1)]$
(24)	$\frac{\Gamma(\frac{1}{2}+\mu+p)\Gamma(\frac{1}{2}-\mu+p)}{\Gamma(1-\kappa+p)} W_{-p, \mu}(a)$ $ arg a  < \pi$	$(1-e^{-t})^{-\kappa} \exp\left[-\frac{\frac{1}{2}\alpha}{(1-e^{-t})}\right]$ $\times W_{\kappa, \mu}\left[\frac{a}{(e^t-1)}\right]$
(25)	$\frac{\Gamma(\frac{1}{2}-\kappa-\mu+p)}{\Gamma(1+p)} W_{\kappa-\frac{1}{2}p, \mu-\frac{1}{2}p}(a)$ $Re \mu > -\frac{1}{2}$	$\frac{1}{\Gamma(2\mu+1)} (e^t-1)^{\mu-\frac{1}{2}} \exp(-\frac{1}{2}\alpha e^t)$ $\times M_{-\kappa, \mu}[a(e^t-1)]$
(26)	$a^{\mu-\frac{1}{2}+\frac{1}{2}p} W_{\kappa-\frac{1}{2}p, \mu+\frac{1}{2}p}(a)$ $Re(\mu + \kappa) < \frac{1}{2}, \quad Re \alpha > 0$	$[\Gamma(\frac{1}{2}-\mu-\kappa)]^{-1} (e^t-1)^{-\frac{1}{2}-\mu-\kappa}$ $\times \exp[-(\frac{1}{2}-\mu+\kappa)t - \alpha(e^t-\frac{1}{2})]$

## Confluent hypergeometric functions (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(27)	$\Gamma(\frac{1}{2} + \mu + p) M_{p,\mu}(a) W_{-p,\mu}(\beta)$ $\text{Re } \alpha > 0, \quad \text{Re } \beta > 0$	$\frac{1}{2} \Gamma(2\mu + 1) \alpha^{\frac{1}{2}} \beta^{\frac{1}{2}} \operatorname{csch}(\frac{1}{2}t)$ $\times e^{\frac{1}{2}(\alpha - \beta)t} \operatorname{ctnh}(\frac{1}{2}t)$ $\times J_{2\mu}[\alpha^{\frac{1}{2}} \beta^{\frac{1}{2}} \operatorname{csch}(\frac{1}{2}t)]$
(28)	$\Gamma(\frac{1}{2} + \mu + p) \Gamma(\frac{1}{2} - \mu + p)$ $\times W_{-p,\mu}(a) W_{+p,\mu}(\beta)$ $\text{Re } \alpha > 0, \quad \text{Re } \beta > 0$	$\frac{1}{2} \alpha^{\frac{1}{2}} \beta^{\frac{1}{2}} \operatorname{csch}(\frac{1}{2}t) e^{-\frac{1}{2}(\alpha + \beta)t} \operatorname{ctnh}(\frac{1}{2}t)$ $\times K_{2\mu}[\alpha^{\frac{1}{2}} \beta^{\frac{1}{2}} \operatorname{csch}(\frac{1}{2}t)]$
(29)	$p^{-\nu} e^{-1/(8p)} k_{2n}(2^{-3}p^{-1})$	$\frac{1}{2} (-1)^{n-1} (t^{n-\frac{1}{2}}/n!) J_1(t^{\frac{1}{2}})$
(30)	$\Gamma(\nu+1) p^{-\nu} e^{-\frac{1}{2}\alpha/p}$ $\times k_{-2\nu}(\frac{1}{2}e^{\pi i} \alpha/p) \quad \text{Re } \nu > 0$	$-i \sin(\nu\pi) t^{\nu-\frac{1}{2}} H_1^{(2)}(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$
(31)	$\Gamma(\nu+1) p^{-\nu} e^{-\frac{1}{2}\alpha/p}$ $\times k_{-2\nu}(\frac{1}{2}e^{-\pi i} \alpha/p) \quad \text{Re } \nu > 0$	$i \sin(\nu\pi) t^{\nu-\frac{1}{2}} H_1^{(1)}(2\alpha^{\frac{1}{2}} t^{\frac{1}{2}})$

## 5.21. Generalized hypergeometric functions

(1)	$\Gamma(\sigma) p^{-\sigma}$ $\times {}_m F_n(a_1, \dots, a_m; \rho_1, \dots, \rho_n; \lambda/p)$ $m \leq n + 1, \quad \text{Re } \sigma > 0$	$t^{\sigma-1} {}_m F_{n+1}(a_1, \dots, a_m; \rho_1, \dots, \rho_n, \sigma; \lambda t)$
(2)	$\Gamma(2\sigma) p^{-2\sigma}$ $\times {}_m F_n(a_1, \dots, a_m; \rho_1, \dots, \rho_n; \lambda^2/p^2)$ $m \leq n + 1 \quad \text{Re } \sigma > 0$	$t^{2\sigma-1}$ $\times {}_m F_{n+2}(a_1, \dots, a_m; \rho_1, \dots, \rho_n, \sigma, \sigma + \frac{1}{2};$ $\frac{1}{4} \lambda^2 t^2)$
(3)	$\Gamma(k\sigma) p^{-k\sigma}$ $\times {}_m F_n(a_1, \dots, a_m; \rho_1, \dots, \rho_n; \lambda^k/p^k)$ $m \leq n + 1, \quad \text{Re } \sigma > 0$	$t^{k\sigma-1} {}_m F_{n+k}(a_1, \dots, a_m;$ $\rho_1, \dots, \rho_n, \sigma, \sigma + 1/k, \dots, \sigma + k - 1/k;$ $\lambda^k t^k / k^k)$

## Generalized hypergeometric functions (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(4)	$p^{-\frac{1}{2}} \times {}_nF_n(\alpha_1, \dots, \alpha_n; \rho_1, \dots, \rho_n; -\lambda p^{\frac{1}{2}})$ $m \leq n$	$\pi^{-\frac{1}{2}} t^{-\frac{1}{2}} {}_{2n}F_{2n}(\frac{1}{2}\alpha_1, \frac{1}{2}\alpha_1 + \frac{1}{2}, \dots, \frac{1}{2}\alpha_n, \frac{1}{2}\alpha_n + \frac{1}{2}; \frac{1}{2}\rho_1, \frac{1}{2}\rho_1 + \frac{1}{2}, \dots, \frac{1}{2}\rho_n, \frac{1}{2}\rho_n + \frac{1}{2}; -2^{m-n-2} \lambda^2/t)$
(5)	$B(p, \lambda)$ $\times {}_3F_2(\alpha, \beta, p; \gamma, p+\lambda; z)$ $\operatorname{Re} \lambda > 0,  \arg(z-1)  < \pi$	$(1-e^{-t})^{\lambda-1} F(\alpha, \beta; \gamma; ze^{-t})$
(6)	$B(p, \lambda)$ $\times {}_3F_2(\alpha, \beta, \lambda; \gamma, p+\lambda; z)$ $\operatorname{Re} \lambda > 0,  \arg(z-1)  < \pi$	$(1-e^{-t})^{\lambda-1} F(\alpha, \beta; \gamma; z(1-e^{-t}))$
(7)	$2^{2p+a} B(p, p+a)$ $\times {}_4F_3(-n, n+1, p+a; 1, 2p+a; 1)$	$\theta^{-1} [(1-\theta)^a + (-1)^n (1+\theta)^a] P_n(\theta)$ $\theta = (1-e^{-t})^{\frac{1}{2}}$
(8)	$B(p, \sigma)$ $\times {}_{n+1}F_{n+1}(\alpha_1, \dots, \alpha_n, p; \rho_1, \dots, \rho_n, p+\sigma; z)$ $\operatorname{Re} \sigma > 0, m \leq n+1$ $ z  < 1 \text{ if } m = n+1$	$(1-e^{-t})^{\sigma-1}$ $\times {}_nF_n(\alpha_1, \dots, \alpha_n; \rho_1, \dots, \rho_n; ze^{-t})$
(9)	$B(p, \sigma)$ $\times {}_{n+1}F_{n+1}(\alpha_1, \dots, \alpha_n, \sigma; \rho_1, \dots, \rho_n, p+\sigma; z)$ $\operatorname{Re} \sigma > 0, m \leq n+1$ $ z  < 1 \text{ if } m = n+1$	$(1-e^{-t})^{\sigma-1}$ $\times {}_nF_n(\alpha_1, \dots, \alpha_n; \rho_1, \dots, \rho_n; z(1-e^{-t}))$
(10)	$p^{-\lambda-\mu}$ $\times E(-\nu, \nu+1, \lambda+\mu; \mu+1; 2p)$ $\operatorname{Re}(\lambda+\mu) > 0$	$-\pi \csc(\nu\pi) t^{\lambda+\frac{1}{2}\mu-1} (t+2)^{\frac{1}{2}\mu}$ $\times P_{\nu}^{-\mu}(t+1)$

**Generalized hypergeometric functions (cont'd)**

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(11)	$p^{-\lambda} E(\mu+\nu+1, \mu-\nu, \lambda; \mu+1; 2p)$ $\text{Re } \lambda > 0$	$2^\mu \Gamma(\mu+\nu+1) \Gamma(\mu-\nu) t^{\lambda-\frac{1}{2}\mu-1} \times (t+2)^{-\frac{1}{2}\mu} P_{\nu}^{-\mu}(t+1)$
(12)	$p^{-\gamma} E(a, \beta, \gamma; \delta; p)$ $\text{Re } \gamma > 0$	$\frac{\Gamma(a)\Gamma(\beta)}{\Gamma(\delta)} t^{\gamma-1} F(a, \beta; \delta; -t)$
(13)	$p^{-\alpha_m} E(m; \alpha_r : n; \beta_s : p)$ $\text{Re } \alpha_m > 0$	$t^{\alpha_m - 1} E(m-1; \alpha_r : n; \beta_s : 1/t)$
(14)	$\Gamma(p - \alpha_m)$ $\times E(m; \alpha_r : n+1; \beta_1, \dots, \beta_n, p : z)$ $\text{Re } \alpha_m > 0$	$(e^t - 1)^{\alpha_m}$ $\times E[m-1; \alpha_1, \dots, \alpha_{m-1} : n;$ $\beta_1, \dots, \beta_n : z / (1 - e^{-t})]$

**5.22. Elliptic functions and theta functions**

(1)	$p^{-1} K(a/p)$	$\frac{1}{2} \pi I_0^2(\frac{1}{2} at)$
(2)	$K(a/p) - \frac{1}{2} \pi$	$\frac{1}{2} \pi a I_0(\frac{1}{2} \pi t) I_1(\frac{1}{2} at)$
(3)	$pK(a/p) - \frac{1}{2} \pi p$	$\frac{1}{4} \pi a^2 \{ \frac{1}{2} [I_0(\frac{1}{2} at)]^2 + [I_1(\frac{1}{2} at)]^2 + \frac{1}{2} I_0(\frac{1}{2} at) I_2(\frac{1}{2} at) \}$
(4)	$\frac{1}{2} \pi p - pE(a/p)$	$\frac{1}{2} \pi a t^{-1} I_0(\frac{1}{2} at) I_1(\frac{1}{2} at)$
(5)	$p[K(a/p) - E(a/p)]$	$\frac{1}{4} \pi a^2 [I_0^2(\frac{1}{2} at) + I_1^2(\frac{1}{2} at)]$
(6)	$p(p^2 - a^2)^{-1} E(a/p)$	$\frac{1}{2} \pi I_0(\frac{1}{2} at) [I_0(\frac{1}{2} at) + at I_1(\frac{1}{2} at)]$

## Elliptic functions and theta functions (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(7)	$r^{-1} E(a/r)$	$\frac{1}{2}\pi J_0(\frac{1}{2}at)[J_0(\frac{1}{2}at) - at J_1(\frac{1}{2}at)]$
(8)	$r^{-1} K(a/r)$	$\frac{1}{2}\pi J_0^2(\frac{1}{2}at)$
(9)	$(1 - \frac{1}{2}\alpha^2 r^{-2})K(a/r) - E(a/r)$	$\frac{1}{4}\pi\alpha^2 J_1^2(\frac{1}{2}at)$
(10)	$r^{-1}[K(a/r) - E(a/r)]$	$\frac{1}{2}\pi\alpha t J_0(\frac{1}{2}at) J_1(\frac{1}{2}at)$
(11)	$p^{-\frac{1}{2}} \theta_1(a, i\pi p)$	$\pi^{-\frac{1}{2}} \sum_{n=-\infty}^{\infty} (-1)^n J_0[2(\alpha+n-\frac{1}{2})t^{\frac{1}{2}}]$
(12)	$p^{-\frac{1}{2}} \theta_2(a, i\pi p)$	$\pi^{-\frac{1}{2}} \sum_{n=-\infty}^{\infty} (-1)^n J_0[2(\alpha+n)t^{\frac{1}{2}}]$
(13)	$p^{-\frac{1}{2}} \theta_3(a, i\pi p)$	$\pi^{-\frac{1}{2}} \sum_{n=-\infty}^{\infty} J_0[2(\alpha+n)t^{\frac{1}{2}}]$
(14)	$p^{-\frac{1}{2}} \theta_4(a, i\pi p)$	$\pi^{-\frac{1}{2}} \sum_{n=-\infty}^{\infty} J_0[2(\alpha+n+\frac{1}{2})t^{\frac{1}{2}}]$
(15)	$p^{-1} \theta_1(a, i\pi p)$ $ Re a  < \frac{1}{2}$	$\pi^{-1} \sum_{n=-\infty}^{\infty} \frac{(-1)^n \sin[2(\alpha+n-\frac{1}{2})t^{\frac{1}{2}}]}{\alpha+n-\frac{1}{2}}$
(16)	$p^{-1} \theta_2(a, i\pi p)$ $0 < Re a < 1$	$\pi^{-1} \sum_{n=-\infty}^{\infty} \frac{(-1)^n \sin[2(\alpha+n)t^{\frac{1}{2}}]}{\alpha+n}$
(17)	$p^{-1} \theta_3(a, i\pi p)$ $0 < Re a < 1$	$\pi^{-1} \sum_{n=-\infty}^{\infty} \frac{\sin[2(\alpha+n)t^{\frac{1}{2}}]}{\alpha+n}$

$$r = (p^2 + \alpha^2)^{\frac{1}{2}}$$

## Elliptic functions and theta functions (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t) dt$	$f(t)$
(18)	$p^{-1} \theta_4(a, i\pi p)$ $ Re a  < \frac{1}{2}$	$\pi^{-1} \sum_{n=-\infty}^{\infty} \frac{\sin [2(a+n+\frac{1}{2})t^{\frac{1}{2}}]}{a+n+\frac{1}{2}}$
(19)	$p^{-1} \theta_2(0, i\pi p)$	$0 \quad 0 < t < \frac{1}{4}\pi^2$ $2n+2$ $\pi^2(n+1/2)^2 < t < \pi^2(n+3/2)^2$
(20)	$p^{-1} \theta_3(0, i\pi p)$	$2n+1 \quad \pi^2 n^2 < t < \pi^2(n+1)^2$
(21)	$p^{-1} \theta_4(0, i\pi p)$	$1 \quad (2k)^2 \pi^2 < t < (2k+1)^2 \pi^2$ $-1 \quad (2k+1)^2 \pi^2 < t < (2k+2)^2 \pi^2$
(22)	$p^{-\nu} \theta_1(a, i\pi p)$ $Re \nu \geq \frac{1}{2}, \quad  Re a  < \frac{1}{2}$	$\pi^{-\frac{1}{2}} t^{\frac{1}{2}\nu - \frac{1}{4}}$ $\times \sum_{n=-\infty}^{\infty} \frac{(-1)^n J_{\nu-\frac{1}{2}}[2(a+n-\frac{1}{2})t^{\frac{1}{2}}]}{(a+n-\frac{1}{2})^{\nu-\frac{1}{2}}}$
(23)	$p^{-\nu} \theta_2(a, i\pi p)$ $Re \nu \geq \frac{1}{2}, \quad 0 < Re a < 1$	$\pi^{-\frac{1}{2}} t^{\frac{1}{2}\nu - \frac{1}{4}} \sum_{n=-\infty}^{\infty} \frac{(-1)^n J_{\nu-\frac{1}{2}}[2(a+n)t^{\frac{1}{2}}]}{(a+n)^{\nu-\frac{1}{2}}}$
(24)	$p^{-\nu} \theta_3(a, i\pi p)$ $Re \nu \geq \frac{1}{2}, \quad 0 < Re a < 1$	$\pi^{-\frac{1}{2}} t^{\frac{1}{2}\nu - \frac{1}{4}} \sum_{n=-\infty}^{\infty} \frac{J_{\nu-\frac{1}{2}}[2(a+n)t^{\frac{1}{2}}]}{(a+n)^{\nu-\frac{1}{2}}}$
(25)	$p^{-\nu} \theta_4(a, i\pi p)$ $Re \nu > \frac{1}{2}, \quad  Re a  < \frac{1}{2}$	$\pi^{-\frac{1}{2}} t^{\frac{1}{2}\nu - \frac{1}{4}} \sum_{n=-\infty}^{\infty} \frac{J_{\nu-\frac{1}{2}}[2(a+n+\frac{1}{2})t^{\frac{1}{2}}]}{(a+n+\frac{1}{2})^{\nu-\frac{1}{2}}}$