CHAPTER IV

LAPLACE TRANSFORMS

4.1. General formulas

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(1)	$\dfrac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{zt} g(z)\,dz$	$g(p)$
(2)	$f(t+a)=f(t)$	$(1-e^{ap})^{-1}\int_0^a e^{-pt} f(t)\,dt$
(3)	$f(t+a)=-f(t)$	$(1+e^{-ap})^{-1}\int_0^a e^{-pt} f(t)\,dt$
(4)	$0 \qquad t<ba^{-1}$ $f(at-b) \qquad t>ba^{-1}$ $a,\ b>0$	$a^{-1}e^{-ba^{-1}p} g(a^{-1}p)$
(5)	$e^{-at} f(t)$	$g(p+a)$
(6)	$t^n f(t)$	$(-1)^n \dfrac{d^n g(p)}{dp^n}$
(7)	$t^{-n} f(t)$	$\int_p^\infty \cdots \int_p^\infty g(p)(dp)^n$ n-th repeated integral
(8)	$f^{(n)}(t)$	$p^n g(p)-p^{n-1}f(0)$ $-p^{n-2}f'(0)-\cdots-f^{(n-1)}(0)$

General formulas (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(9)	$\int_0^t\cdots\int_0^t f(t)(dt)^n$	$p^{-n}g(p)$
(10)	$\left(t\dfrac{d}{dt}\right)^n f(t)$ where e.g. $\left(t\dfrac{d}{dt}\right)^2 f(t)=t\dfrac{d}{dt}\left\{t\dfrac{d}{dt}[f(t)]\right\}$	$\left(-\dfrac{d}{dp}p\right)^n g(p)$ $\left(\dfrac{d}{dp}p\right)^2 g(p)=\dfrac{d}{dp}\left\{p\dfrac{d}{dp}[pg(p)]\right\}$
(11)	$\left(\dfrac{d}{dt}t\right)^n f(t)$	$\left(-p\dfrac{d}{dp}\right)^n g(p)$
(12)	$\left(t^{-1}\dfrac{d}{dt}\right)^n f(t)$ if $\left(\dfrac{1}{t}\dfrac{d}{dt}\right)^k f(t)=0$ for $t=0,\quad k=0,\ldots,n-1$	$\int_p^\infty p\int_p^\infty\cdots p\int_p^\infty pg(p)(dp)^n$
(13)	$t^m f^{(n)}(t)$ $\quad m\geq n$	$\left(-\dfrac{d}{dp}\right)^m [p^n g(p)]$
(14)	$t^m f^{(n)}(t)$ $\quad m<n$	$\left(-\dfrac{d}{dp}\right)^m [p^n g(p)]+(-1)^{m-1}$ $\times\left[\dfrac{(n-1)!}{(n-m-1)!}p^{n-m-1}f(0)\right.$ $+\dfrac{(n-2)!}{(n-m-2)!}p^{n-m-2}f'(0)$ $\left.+\cdots+m!\,f^{(n-m-1)}(0)\right]$

General formulas (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(15)	$\dfrac{d^n}{dt^n}[t^m f(t)]$ $\quad m\geq n$	$(-1)^m p^n g^{(m)}(p)$
(16)	$\dfrac{d^n}{dt^n}[t^m f(t)]$ $\quad m<n$	$(-1)^m p^n g^{(m)}(p)-m!\,p^{n-m-1}f(0)$ $-\dfrac{(m+1)!}{1!}p^{n-m-2}f'(0)$ $-\cdots-\dfrac{(n-1)!}{(n-m-1)!}f^{(n-m-1)}(0)$
(17)	$\left(e^t\dfrac{d}{dt}\right)^n f(t)$ provided that $f^{(k)}(0)=0$ for $k=0,1,\ldots,n-1$	$(p-1)\cdots(p-n)\,g(p-n)$
(18)	$\int_0^t t^{-1}f(t)\,dt$	$p^{-1}\int_p^\infty g(p)\,dp$
(19)	$\int_t^\infty t^{-1}f(t)\,dt$	$p^{-1}\int_0^p g(p)\,dp$
(20)	$\int_0^t f_1(u)f_2(t-u)\,du$	$g_1(p)g_2(p)$
(21)	$f_1(t)\,f_2(t)$	$\dfrac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}g_1(z)g_2(p-z)\,dz$
(22)	$f(t^2)$	$\pi^{-1/2}\int_0^\infty e^{-\frac{1}{4}p^2u^{-2}}g(u^2)\,du$
(23)	$t^n f(t^2)$	$2^{-\frac{1}{2}n}\pi^{-1/2}\int_0^\infty u^{n-2}e^{-\frac{1}{4}p^2u^2}$ $\times\mathrm{He}_n(2^{-1/2}pu)\,g(u^{-2})\,du$

General formulas (cont'd)

	$f(t)$		$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(24)	$t^{\nu}f(t^2)$		$2^{-1/2}\pi^{-1/2}\int_0^\infty u^{\nu-2}e^{-1/4p^2u^2}$ $\times D_{\nu}(pu)g(\tfrac{1}{2}u^{-2})\,du$
(25)	$t^{\nu-1}f(t^{-1})$	$\operatorname{Re}\nu>-1$	$p^{-1/2\nu}\int_0^\infty u^{1/2\nu}J_{\nu}(2u^{1/2}p^{1/2})g(u)\,du$
(26)	$f(ae^t-a)$	$a>0$	$[a\Gamma(p+1)]^{-1}\int_0^\infty e^{-u}u^p g(u/a)\,du$
(27)	$f(a\sinh t)$	$a>0$	$\int_0^\infty J_p(au)g(u)\,du$
(28)	$\sum_{n=1}^{\infty} n^{-1}f(n^{-1}t)$		$\int_0^\infty (e^{pu}-1)^{-1}f(u)\,du$
(29)	$\int_0^\infty \frac{t^{u-1}}{\Gamma(u)}f(u)\,du$		$g(\log p)$
(30)	$\int_0^\infty u^{-1/2}\sin(2u^{1/2}t^{1/2})f(u)\,du$		$\pi^{1/2}p^{-3/2}g(p^{-1})$
(31)	$t^{-1/2}\int_0^\infty \cos(2u^{1/2}t^{1/2})f(u)\,du$		$\pi^{1/2}p^{-1/2}g(p^{-1})$
(32)	$t^{\nu}\int_0^\infty J_{2\nu}(2u^{1/2}t^{1/2})u^{-\nu}f(u)\,du$		$p^{-2\nu-1}g(p^{-1})$
(33)	$t^{-1/2}\int_0^\infty e^{-1/4u^2/t}f(u)\,du$		$\pi^{1/2}p^{-1/2}g(p^{1/2})$
(34)	$t^{-1/2n-1/2}\int_0^\infty e^{-1/4u^2/t}$ $\times \operatorname{He}_n(2^{-1/2}ut^{-1/2})f(u)\,du$		$2^{1/2n}\pi^{1/2}p^{1/2n-1/2}g(p^{1/2})$

General formulas (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(35)	$t^{-\nu}\int_0^\infty e^{-u^2/(8t)} \times D_{2\nu-1}(2^{-1/2}ut^{-1/2})f(u)\,du$	$2^{\nu-1/2}\pi^{1/2}p^{\nu-1}g(p^{1/2})$
(36)	$\int_0^t \left(\frac{t-u}{au}\right)^{\nu} \times J_{2\nu}[2(aut-au^2)^{1/2}]f(u)\,du$	$p^{-2\nu-1}g(p+ap^{-1})$
(37)	$\int_0^t J_0[(t^2-u^2)^{1/2}]f(u)\,du$	$(p^2+1)^{-1/2}g[(p^2+1)^{1/2}]$
(38)	$f(t)-\int_0^t J_1(u)f[(t^2-u^2)^{1/2}]\,du$	$g[(p^2+1)^{1/2}]$
(39)	$\int_0^t \left(\frac{t-u}{t+u}\right)^{\nu} \times J_{2\nu}[(t^2-u^2)^{1/2}]f(u)\,du$	$(p^2+1)^{-1/2}[(p^2+1)^{1/2}+p]^{-2\nu} \times g[(p^2+1)^{1/2}]$

4.2. Algebraic functions

(1)	1		p^{-1}	$\mathrm{Re}\, p>0$
(2)	0 1 0	$0<t<a$ $a<t<b$ $t>b$	$p^{-1}(e^{-ap}-e^{-bp})$	
(3)	t^n		$n!p^{-n-1}$	$\mathrm{Re}\, p>0$
(4)	0 t^n	$0<t<b$ $t>b$	$e^{-bp}\sum_{m=0}^{n}\frac{n!}{m!}\frac{b^m}{p^{n-m+1}}$	$\mathrm{Re}\, p>0$

Algebraic functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t)\,dt$
(5)	$t^n \quad 0 < t < b$ $0 \quad t > b$	$\dfrac{n!}{p^{n+1}} - e^{-bp} \sum_{m=0}^{n} \dfrac{n!}{m!} \dfrac{b^m}{p^{n-m+1}}$
(6)	$0 \quad 0 < t < b$ $(t+a)^{-1} \quad t > b$ $\lvert\arg(a+b)\rvert < \pi$	$-e^{ap}\,\mathrm{Ei}[-(a+b)p] \qquad \mathrm{Re}\, p > 0$
(7)	$0 \quad 0 < t < b$ $(t+a)^{-1} \quad b < t < c$ $0 \quad t > c$ $-a$ not between b and c	$e^{ap}\{\mathrm{Ei}[-(a+c)p] - \mathrm{Ei}[-(a+b)p]\}$
(8)	$(t-a)^{-1} \quad a \geq 0$	$-e^{-ap}\,\overline{\mathrm{Ei}}(ap) \qquad \mathrm{Re}\, p > 0$ The integral is a Cauchy Principal value
(9)	$\dfrac{1}{(t+a)^n} \quad n \geq 2, \quad \lvert\arg a\rvert < \pi$	$\sum_{m=1}^{n-1} \dfrac{(m-1)!}{(n-1)!} \dfrac{(-p)^{n-m-1}}{a^m} - \dfrac{(-p)^{n-1}}{(n-1)!} e^{ap}\,\mathrm{Ei}(-ap)$ $\mathrm{Re}\, p \geq 0$
(10)	$0 \quad 0 < t < b$ $(t+a)^{-n} \quad t > b$ $\lvert\arg(a+b)\rvert < \pi, \quad n \geq 2$	$e^{-bp} \sum_{m=1}^{n-1} \dfrac{(m-1)!}{(n-1)!} \dfrac{(-p)^{n-m-1}}{(a+b)^m} - \dfrac{(-p)^{n-1}}{(n-1)!} e^{ap}\,\mathrm{Ei}[-(a+b)p]$ $\mathrm{Re}\, p > 0$

For further formulas of a similar type see Bierens de Haan, D., 1867: *Nouvelles tables d'intégrales définies*, 727 p.

Algebraic functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)dt$
(11)	$t^n(t+a)^{-1}$ $n\geq 1,$ $\lvert\arg a\rvert<\pi$	$(-)^{n-1}a^n e^{ap}\,\mathrm{Ei}(-ap)$ $+\sum_{m=1}^{n}(m-1)!(-a)^{n-m}p^{-m}$ $\mathrm{Re}\,p>0$
(12)	$(At+Ba)(t^2-a^2)^{-1}$ $\lvert\arg(\pm a)\rvert<\pi$	$-\tfrac{1}{2}(A-B)e^{ap}\,\mathrm{Ei}(-ap)$ $-\tfrac{1}{2}(A+B)e^{-ap}\,\mathrm{Ei}(ap)$ $\mathrm{Re}\,p>0$
(13)	$(At+Ba)(t^2-a^2)^{-1}$ $a>0$ Cauchy Principal value	$-\tfrac{1}{2}(A-B)e^{-ap}\,\mathrm{Ei}(-ap)$ $-\tfrac{1}{2}(A+B)e^{-ap}\,\overline{\mathrm{Ei}}(ap)$ $\mathrm{Re}\,p>0$
(14)	$(At+Ba)(t^2+a^2)^{-1}$ $\lvert\arg(\pm ia)\rvert<\pi$	$(A\cos ap-B\sin ap)\,\mathrm{ci}(ap)$ $-(A\sin ap+B\cos ap)\,\mathrm{si}(ap)$ $\mathrm{Re}\,p>0$
(15)	0 $0<t<b$ $t^{-1/2}$ $t>b$	$\pi^{1/2}p^{-1/2}\,\mathrm{Erfc}(b^{1/2}p^{1/2})$ $\mathrm{Re}\,p>0$
(16)	$t^{-1/2}$ $0<t<b$ 0 $t>b$	$\pi^{1/2}p^{-1/2}\,\mathrm{Erf}(b^{1/2}p^{1/2})$
(17)	$t^{n-1/2}$	$\pi^{1/2}\,1/2(3/2)\cdots(n-1/2)\,p^{-n-1/2}$ $\mathrm{Re}\,p>0$
(18)	$(t+a)^{-1/2}$ $\lvert\arg a\rvert<\pi$	$\pi^{1/2}p^{-1/2}e^{ap}\,\mathrm{Erfc}(a^{1/2}p^{1/2})$ $\mathrm{Re}\,p>0$
(19)	0 $0<t<b$ $t^{-3/2}$ $t>b$	$2b^{-1/2}e^{-bp}-2\pi^{1/2}p^{1/2}\,\mathrm{Erfc}(b^{1/2}p^{1/2})$ $\mathrm{Re}\,p\geq 0$
(20)	$(t+a)^{-3/2}$ $\lvert\arg a\rvert<\pi$	$2a^{-1/2}-2\pi^{1/2}p^{1/2}e^{ap}\,\mathrm{Erfc}(a^{1/2}p^{1/2})$ $\mathrm{Re}\,p\geq 0$

Algebraic functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(22)	$t^{1/2}(t+a)^{-1}$ $\quad |\arg a|<\pi$	$(\pi/p)^{1/2}-\pi a^{1/2}e^{ap}\,\mathrm{Erfc}\,(a^{1/2}p^{1/2})$ $\quad \mathrm{Re}\,p>0$
(23)	0 $\quad 0<t<b$ $t^{-1}(t-b)^{1/2}$ $\quad t>b$	$(\pi/p)^{1/2}e^{-bp}-\pi b^{1/2}\,\mathrm{Erfc}\,(b^{1/2}p^{1/2})$ $\quad \mathrm{Re}\,p>0$
(24)	$t^{-1/2}(1+2at)$	$\pi^{1/2}p^{-3/2}(p+a)$ $\quad \mathrm{Re}\,p>0$
(25)	$t^{-1/2}(t+a)^{-1}$ $\quad |\arg a|<\pi$	$\pi a^{-1/2}e^{ap}\,\mathrm{Erfc}\,(a^{1/2}p^{1/2})$ $\quad \mathrm{Re}\,p\geq 0$
(26)	0 $\quad 0<t<b$ $t^{-1}(t-b)^{-1/2}$ $\quad t>b$	$\pi b^{-1/2}\,\mathrm{Erfc}\,(b^{1/2}p^{1/2})$ $\quad \mathrm{Re}\,p\geq 0$
(27)	$t(t^2+a^2)^{-1/2}$ $\quad |\arg a|<\pi/2$	$\frac{1}{2}\pi a[\mathbf{H}_1(ap)-Y_1(ap)]-a$ $\quad \mathrm{Re}\,p>0$
(28)	$t(b^2-t^2)^{-1/2}$ $\quad 0<t<b$ 0 $\quad t>b$	$\frac{1}{2}\pi b[\mathbf{L}_1(bp)-I_1(bp)]+b$ $\quad \mathrm{Re}\,p>0$
(29)	0 $\quad 0<t<b$ $t(t^2-b^2)^{-1/2}$ $\quad t>b$	$bK_1(bp)$ $\quad \mathrm{Re}\,p>0$
(30)	$(t^2+2at)^{-1/2}(t+a)$ $\quad |\arg a|<\pi$	$ae^{ap}K_1(ap)$ $\quad \mathrm{Re}\,p>0$
(31)	$(2bt-t^2)^{-1/2}(b-t)$ $\quad 0<t<2b$ 0 $\quad t>2b$	$\pi be^{-bp}I_1(bp)$ $\quad \mathrm{Re}\,p>0$
(32)	$[t+(t^2+a^2)^{1/2}]^{-1}$ $\quad |\arg a|<\pi/2$	$\frac{1}{2}\pi a^{-1}p^{-1}[\mathbf{H}_1(ap)-Y_1(ap)]-a^{-2}p^{-2}$ $\quad \mathrm{Re}\,p>0$
(33)	$\sin\theta(1+t+\cos\theta)^{-1}(t^2+2t)^{-1/2}$	$\exp[2p\cos^2(\frac{1}{2}\theta)]$ $\times[\theta-\sin\theta\int_0^p K_0(v)e^{-v\cos\theta}dv]$ $\quad \mathrm{Re}\,p>0$

Algebraic functions (cont'd)

	$f(t)$	$g(y)=\int_0^\infty e^{-pt}f(t)\,dt$
(34)	$[t+(1+t^2)^{1/2}]^n+[t-(1+t^2)^{1/2}]^n$	$2O_n(p)$ $\qquad$ $\mathrm{Re}\,p>0$
(35)	$[t+(1+t^2)^{1/2}]^n(1+t^2)^{-1/2}$	$\tfrac{1}{2}[S_n(p)-\pi\mathbf{E}_n(p)-\pi Y_n(p)]$ $\mathrm{Re}\,p>0$
(36)	$[t-(1+t^2)^{1/2}]^n(1+t^2)^{-1/2}$	$-\tfrac{1}{2}[S_n(p)+\pi\mathbf{E}_n(p)+\pi Y_n(p)]$ $\mathrm{Re}\,p>0$

4.3. Powers with an arbitrary index

(1)	t^ν $\qquad$ $\mathrm{Re}\,\nu>-1$	$\Gamma(\nu+1)p^{-\nu-1}$ $\qquad$ $\mathrm{Re}\,p>0$
(2)	0 $\qquad$ $0<t<b$ t^ν $\qquad$ $t>b$	$p^{-\nu-1}\Gamma(\nu+1,bp)$ $\qquad$ $\mathrm{Re}\,p>0$
(3)	t^ν $\qquad$ $0<t<b$ 0 $\qquad$ $t>b$ $\mathrm{Re}\,\nu>-1$	$p^{-\nu-1}\gamma(\nu+1,bp)$
(4)	$(t+a)^\nu$ $\qquad$ $\lvert\arg a\rvert<\pi$	$p^{-\nu-1}e^{ap}\Gamma(\nu+1,ap)$ $\qquad$ $\mathrm{Re}\,p>0$
(5)	0 $\qquad$ $0<t<b$ $(t-b)^\nu$ $\qquad$ $t>b$ $\mathrm{Re}\,\nu>-1$	$\Gamma(\nu+1)p^{-\nu-1}e^{-bp}$ $\qquad$ $\mathrm{Re}\,p>0$
(6)	$(b-t)^\nu$ $\qquad$ $0<t<b$ 0 $\qquad$ $t>b$ $\mathrm{Re}\,\nu>-1$	$p^{-\nu-1}e^{-bp}\gamma(\nu+1,-bp)$
(7)	$t^\nu(t+a)^{-1}$ $\lvert\arg a\rvert<\pi,\quad \mathrm{Re}\,\nu>-1$	$\Gamma(\nu+1)\,a^\nu e^{ap}\Gamma(-\nu,ap)$ $\mathrm{Re}\,p>0$

Arbitrary powers (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(8)	$0 \qquad 0<t<b$ $t^{-1}(t-b)^{\nu} \qquad t>b$ $\mathrm{Re}\,\nu>-1$	$\Gamma(\nu+1)\,b^{\nu}\,\Gamma(-\nu,\,bp) \qquad \mathrm{Re}\,p>0$
(9)	$t^{\nu-1}(1+t^2)^{-1} \qquad \mathrm{Re}\,\nu>0$	$\pi\csc(\nu\pi)\,V_{\nu}(2p,\,0) \qquad \mathrm{Re}\,p>0$
(10)	$(1+t^2)^{\nu-\frac{1}{2}}$	$2^{\nu-1}\,\pi^{\frac{1}{2}}\,\Gamma(\nu+\tfrac{1}{2})\,p^{-\nu}[\mathbf{H}_{\nu}(p)-Y_{\nu}(p)]$ $\mathrm{Re}\,p>0$
(11)	$0 \qquad 0<t<b$ $(t^2-b^2)^{\nu-\frac{1}{2}} \qquad t>b$ $\mathrm{Re}\,\nu>-\tfrac{1}{2}$	$\pi^{-\frac{1}{2}}\,\Gamma(\nu+\tfrac{1}{2})\,(2b/p)^{\nu}K_{\nu}(bp)$ $\mathrm{Re}\,p>0$
(12)	$(b^2-t^2)^{\nu-\frac{1}{2}} \qquad 0<t<b$ $0 \qquad t>b$ $\mathrm{Re}\,\nu>-\tfrac{1}{2}$	$\tfrac{1}{2}\pi^{\frac{1}{2}}\,\Gamma(\nu+\tfrac{1}{2})\,(2b/p)^{\nu}[I_{\nu}(bp)$ $-\mathbf{L}_{\nu}(bp)]$
(13)	$(t^2+2at)^{\nu-\frac{1}{2}}$ $\lvert\arg a\rvert<\pi, \qquad \mathrm{Re}\,\nu>-\tfrac{1}{2}$	$\pi^{-\frac{1}{2}}\,\Gamma(\nu+\tfrac{1}{2})\,(2a/p)^{\nu}e^{ap}K_{\nu}(ap)$ $\mathrm{Re}\,p>0$
(14)	$(2bt-t^2)^{\nu-\frac{1}{2}} \qquad 0<t<2b$ $0 \qquad t>2b$ $\mathrm{Re}\,\nu>-\tfrac{1}{2}$	$\pi^{\frac{1}{2}}\,\Gamma(\nu+\tfrac{1}{2})\,(2b/p)^{\nu}e^{-bp}I_{\nu}(bp)$
(15)	$(t^2+it)^{\nu-\frac{1}{2}} \qquad \mathrm{Re}\,\nu>-\tfrac{1}{2}$	$-\tfrac{1}{2}i\,\pi^{\frac{1}{2}}\,\Gamma(\nu+\tfrac{1}{2})\,p^{-\nu}e^{\frac{1}{2}ip}H_{\nu}^{(2)}(\tfrac{1}{2}p)$ $\mathrm{Re}\,p>0$
(16)	$(t^2-it)^{\nu-\frac{1}{2}} \qquad \mathrm{Re}\,\nu>-\tfrac{1}{2}$	$\tfrac{1}{2}i\,\pi^{\frac{1}{2}}\,\Gamma(\nu+\tfrac{1}{2})\,p^{-\nu}e^{-\frac{1}{2}ip}H_{\nu}^{(1)}(\tfrac{1}{2}p)$ $\mathrm{Re}\,p>0$

Arbitrary powers (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(17)	$0 \quad 0<t<2b$ $(t+2a)^{\nu}(t-2b)^{-\nu} \quad t>2b$ $\lvert\arg(a+b)\rvert<\pi, \quad \mathrm{Re}\,\nu<1$	$\nu\pi\csc(\nu\pi)p^{-1}e^{-(a+b)p}k_{2\nu}[(a+b)p]$ $\mathrm{Re}\,p>0$
(18)	$0 \quad 0<t<b$ $(t-b)^{\nu-1}(t+b)^{-\nu+\frac{1}{2}} \quad t>b$ $\mathrm{Re}\,\nu>0$	$2^{\nu-\frac{1}{2}}\Gamma(\nu)p^{-\frac{1}{2}}D_{1-2\nu}(2b^{\frac{1}{2}}p^{\frac{1}{2}})$ $\mathrm{Re}\,p>0$
(19)	$0 \quad 0<t<b$ $(t-b)^{\nu-1}(t+b)^{-\nu-\frac{1}{2}} \quad t>b$ $\mathrm{Re}\,\nu>0$	$2^{\nu-\frac{1}{2}}\Gamma(\nu)b^{-\frac{1}{2}}D_{-2\nu}(2b^{\frac{1}{2}}p^{\frac{1}{2}})$ $\mathrm{Re}\,p\geq 0$
(20)	$t^{\nu-1}(t+a)^{-\nu+\frac{1}{2}}$ $\mathrm{Re}\,\nu>0, \quad \lvert\arg a\rvert<\pi$	$2^{\nu-\frac{1}{2}}\Gamma(\nu)p^{-\frac{1}{2}}e^{\frac{1}{2}ap}D_{1-2\nu}(2^{\frac{1}{2}}a^{\frac{1}{2}}p^{\frac{1}{2}})$ $\mathrm{Re}\,p>0$
(21)	$t^{\nu-1}(t+a)^{-\nu-\frac{1}{2}}$ $\mathrm{Re}\,\nu>0, \quad \lvert\arg a\rvert<\pi$	$2^{\nu}\Gamma(\nu)a^{-\frac{1}{2}}e^{\frac{1}{2}ap}D_{-2\nu}(2^{\frac{1}{2}}a^{\frac{1}{2}}p^{\frac{1}{2}})$ $\mathrm{Re}\,p\geq 0$
(22)	$0 \quad 0<t<b$ $(t+a)^{2\mu-1}(t-b)^{2\nu-1} \quad t>b$ $\mathrm{Re}\,\nu>0, \quad \lvert\arg(a+b)\rvert<\pi$	$\Gamma(2\nu)(a+b)^{\mu+\nu-1}p^{-\mu-\nu}e^{\frac{1}{2}p(a-b)}$ $\times W_{\mu-\nu,\,\mu+\nu-\frac{1}{2}}(ap+bp) \quad \mathrm{Re}\,p>0$
(23)	$0 \quad 0<t<a$ $(t-a)^{2\mu-1}(b-t)^{2\nu-1} \quad a<t<b$ $0 \quad t>b$ $\mathrm{Re}\,\mu>0, \quad \mathrm{Re}\,\nu>0$	$\mathrm{B}(2\mu,2\nu)\,(b-a)^{\mu+\nu-1}p^{-\mu-\nu}e^{-\frac{1}{2}p(a+b)}$ $\times M_{\mu-\nu,\,\mu+\nu-\frac{1}{2}}(bp-ap)$
(24)	$t^{\alpha-1}(1-t)^{\beta-1}(1-\sigma t)^{-\gamma}$ $0<t<1$ $0 \quad t>1$ $\mathrm{Re}\,\alpha>0, \quad \mathrm{Re}\,\beta>0$ $\lvert\arg(1-\sigma)\rvert<\pi$	$\mathrm{B}(\alpha,\beta)\,\Phi_1(\alpha,\gamma,\alpha+\beta;\sigma,-p)$

Arbitrary powers (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(25)	$[(t^2+1)^{1/2}+t]^\nu$	$p^{-1} S_{1,\nu}(p)+\nu p^{-1} S_{0,\nu}(p)$ $\quad \operatorname{Re} p>0$
(26)	$[(t^2+1)^{1/2}-t]^\nu$	$p^{-1} S_{1,\nu}(p)-\nu p^{-1} S_{0,\nu}(p)$ $\quad \operatorname{Re} p>0$
(27)	$(t^2+1)^{-1/2}[(t^2+1)^{1/2}+t]^\nu$	$\pi \csc(\nu\pi)[\mathbf{J}_{-\nu}(p)-J_{-\nu}(p)]$ $\quad \operatorname{Re} p>0$
(28)	$(t^2+1)^{-1/2}[(t^2+1)^{1/2}-t]^\nu$	$S_{0,\nu}(p)-\nu S_{-1,\nu}(p)$ $\quad \operatorname{Re} p>0$
(29)	$0 \quad 0<t<1$ $\dfrac{[(t^2-1)^{1/2}+t]^\nu+[(t^2-1)^{1/2}+t]^{-\nu}}{(t^2-1)^{1/2}} \quad t>1$	$2K_\nu(p)$ $\quad \operatorname{Re} p>0$
(30)	$[(t+2a)^{1/2}+t^{1/2}]^{2\nu}$ $-[(t+2a)^{1/2}-t^{1/2}]^{2\nu} \quad \lvert\arg a\rvert<\pi$	$2^{\nu+1}\nu a^\nu p^{-1} e^{ap} K_\nu(ap)$ $\quad \operatorname{Re} p>0$
(31)	$0 \quad 0<t<b$ $[(t+b)^{1/2}+(t-b)^{1/2}]^{2\nu}$ $-[(t+b)^{1/2}-(t-b)^{1/2}]^{2\nu} \quad t>b$	$2^{\nu+1}\nu b^\nu p^{-1} K_\nu(bp)$ $\quad \operatorname{Re} p>0$
(32)	$t^{-\nu-1}(t^2+1)^{-1/2}$ $\times[1+(t^2+1)^{1/2}]^{\nu+1/2} \quad \operatorname{Re}\nu<0$	$2^{1/2}\Gamma(-\nu)D_\nu[(2ip)^{1/2}]D_\nu[(-2ip)^{1/2}]$ $\quad \operatorname{Re} p\ge 0$
(33)	$(2a)^{2\nu}[t+(t^2+4a^2)^{1/2}]^{2\nu}$ $\times(t^3+4a^2t)^{-1/2} \quad \operatorname{Re} a>0$	$(\tfrac{1}{2}\pi)^{3/2} p^{1/2}[J_{\nu+1/4}(ap)Y_{\nu-1/4}(ap)$ $-J_{\nu-1/4}(ap)Y_{\nu+1/4}(ap)]$ $\quad \operatorname{Re} p>0$
(34)	$0 \quad 0<t<1$ $t^{-1/2}(t^2-1)^{-1/2}\{[t+(t^2-1)^{1/2}]^{2\nu}$ $+[t-(t^2-1)^{1/2}]^{2\nu}\} \quad t>1$	$(2p/\pi)^{1/2}K_{\nu+1/4}(\tfrac{1}{2}p)K_{\nu-1/4}(\tfrac{1}{2}p)$ $\quad \operatorname{Re} p>0$

4.4. Step-, jump-, and other sectionally rational functions

$n = 0, 1, 2, \ldots$

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(1)	$0 \qquad 2nb < t < (2n+1)b$ $1 \qquad (2n-1)b < t < 2nb$	$p^{-1}(e^{bp}+1)^{-1} \qquad \mathrm{Re}\, p > 0$
(2)	$0 \qquad (4n-1)b < t < (4n+1)b$ $2 \qquad (4n+1)b < t < (4n+3)b$	$p^{-1} \operatorname{sech}(bp) \qquad \mathrm{Re}\, p > 0$
(3)	$\tfrac{1}{2} \qquad 2nb < t < (2n+1)b$ $-\tfrac{1}{2} \qquad (2n-1)b < t < 2nb$	$\tfrac{1}{2} p^{-1} \tanh(\tfrac{1}{2} bp) \qquad \mathrm{Re}\, p > 0$
(4)	$n \qquad nb < t < (n+1)b$	$p^{-1}(e^{bp}-1)^{-1} \qquad \mathrm{Re}\, p > 0$
(5)	$n+1 \qquad nb < t < (n+1)b$	$p^{-1}(1-e^{-bp})^{-1} \qquad \mathrm{Re}\, p > 0$
(6)	$2n+1 \qquad 2nb < t < 2(n+1)b$	$p^{-1} \operatorname{ctnh}(bp) \qquad \mathrm{Re}\, p > 0$
(7)	$0 \qquad 0 < t < b$ $2n \qquad (2n-1)b < t < (2n+1)b$	$p^{-1} \operatorname{csch}(bp) \qquad \mathrm{Re}\, p > 0$
(8)	$n \qquad b\pi^2 n^2 < t < b\pi^2 (n+1)^2$	$\tfrac{1}{2} p^{-1} [\theta_3(0 \vert i\pi bp) - 1] \qquad \mathrm{Re}\, p > 0$
(9)	$n \qquad \log n < t < \log(n+1)$	$p^{-1} \zeta(p) \qquad \mathrm{Re}\, p > 0$
(10)	$\sum_{0 \le \log n \le t} (t - \log n)^{\alpha-1}$ $\mathrm{Re}\, \alpha > 0$	$\Gamma(\alpha)\, p^{-\alpha} \zeta(p) \qquad \mathrm{Re}\, p > 0$
(11)	$(1-a)^{-1}(1-a^n)$ $nb < t < (n+1)b$	$p^{-1}(e^{bp}-a)^{-1}$ $\mathrm{Re}\, p > 0, \quad b\, \mathrm{Re}\, p > \mathrm{Re}(\log a)$
(12)	$\binom{n}{m} \qquad nb < t < (n+1)b$	$\dfrac{e^{-bp}}{p(e^{bp}-1)^m} \qquad \mathrm{Re}\, p > 0$

Sectionally rational functions (cont'd)

	$f(t)$	$g(y)=\int_0^\infty e^{-pt} f(t)\,dt$
(13)	n^m $\quad nb<t<(n+1)b$	$\dfrac{1-e^{-bp}}{(-b)^m p}\dfrac{d^m}{dp^m}(1-e^{-bp})^{-1}$ $\quad \mathrm{Re}\,p>0$
(14)	t $\quad 0<t<1$ 1 $\quad t>1$	$p^{-2}(1-e^{-p})$ $\quad \mathrm{Re}\,p>0$
(15)	t $\quad 0<t<1$ $2-t$ $\quad 1<t<2$ 0 $\quad t>2$	$p^{-2}(1-e^{-p})^2$
(16)	$a(t-nb)$ $\quad nb<t<(n+1)b$	$ap^{-2}-\tfrac{1}{2}abp^{-1}[\mathrm{ctnh}(\tfrac{1}{2}bp)-1]$ $=ap^{-2}(e^{bp}-1)^{-1}(e^{bp}-bp-1)$ $\quad \mathrm{Re}\,p>0$
(17)	$\dfrac{1-a^n}{1-a}t-b\dfrac{1-(n+1)a^n+na^{n+1}}{(1-a)^2}$ $nb<t<(n+1)b$	$\dfrac{1}{p^2(e^{bp}-a)}$ $\mathrm{Re}\,p>0,\quad b\,\mathrm{Re}\,p>\mathrm{Re}\log a$
(18)	$(2n+1)t-2bn(n+1)$ $2nb<t<2(n+1)b$	$p^{-2}\,\mathrm{ctnh}(bp)$ $\quad \mathrm{Re}\,p>0$
(19)	$b-(-1)^n(2bn+b-t)$ $2nb<t<2(n+1)b$	$p^{-2}\tanh(bp)$ $\quad \mathrm{Re}\,p>0$
(20)	0 $\quad 0<t<b$ $t-(-1)^n(t-2nb)$ $(2n-1)b<t<(2n+1)b$ $n\geq 1$	$p^{-2}\,\mathrm{sech}(bp)$ $\quad \mathrm{Re}\,p>0$

Sectionally rational functions (cont'd)

	$f(t)$	$g(y)=\int_0^\infty e^{-pt} f(t)\,dt$
(21)	$0 \qquad 0<t<b$ $2n(t-bn)$ $(2n-1)b<t<(2n+1)b$ $n\geq 1$	$p^{-2}\operatorname{csch}(bp) \qquad \operatorname{Re} p>0$
(22)	$\tfrac{1}{4}[1-(-1)^n](2t-b)+\tfrac{1}{2}(-1)^n bn$ $nb<t<(n+1)b$	$p^{-2}(e^{bp}+1)^{-1} \qquad \operatorname{Re} p>0$
(23)	$0 \qquad 0<t<b$ $nt-\tfrac{1}{2}bn(n+1)$ $nb<t<(n+1)b$ $n\geq 1$	$p^{-2}(e^{bp}-1)^{-1} \qquad \operatorname{Re} p>0$
(24)	$\tfrac{1}{2}t^2 \qquad 0<t<1$ $1-\tfrac{1}{2}(t-2)^2 \qquad 1<t<2$ $1 \qquad t>2$	$p^{-3}(1-e^{-p})^2 \qquad \operatorname{Re} p>0$
(25)	$\tfrac{1}{2}t^2 \qquad 0<t<1$ $3/4-(t-3/2)^2 \qquad 1<t<2$ $\tfrac{1}{2}(t-3)^2 \qquad 2<t<3$ $0 \qquad t>3$	$p^{-3}(1-e^{-p})^3$
(26)	$(t-nb)^2 \qquad nb<t<(n+1)b$	$2p^{-3}-p^{-2}(b^2+2bp)(e^{bp}-1)^{-1}$ $\operatorname{Re} p>0$
	For further similar integrals see Gardner, M. F. and J. L. Barnes, 1942: *Transients in linear systems*, I, Wiley.	

4.5. Exponential functions

(1)	$e^{-\alpha t}$	$(p+\alpha)^{-1} \qquad \operatorname{Re} p>-\operatorname{Re}\alpha$

Exponential functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(2)	$te^{-\alpha t}$	$(p+\alpha)^{-2}$ $\quad$ $\operatorname{Re} p>-\operatorname{Re}\alpha$
(3)	$t^{\nu-1}e^{-\alpha t}$ $\quad$ $\operatorname{Re}\nu>0$	$\Gamma(\nu)(p+\alpha)^{-\nu}$ $\quad$ $\operatorname{Re} p>-\operatorname{Re}\alpha$
(4)	$t^{-1}(e^{-\alpha t}-e^{-\beta t})$	$\log(p+\beta)-\log(p+\alpha)$ $\operatorname{Re} p>-\operatorname{Re}\alpha,\ -\operatorname{Re}\beta$
(5)	$t^{-2}(1-e^{-\alpha t})^2$	$(p+2\alpha)\log(p+2\alpha)+p\log p$ $-2(p+\alpha)\log(p+\alpha)$ $\operatorname{Re} p\geq 0,\ -\operatorname{Re} 2\alpha$
(6)	$t^{-1}-\frac{1}{2}t^{-2}(t+2)(1-e^{-t})$	$-1+(p+\frac{1}{2})\log(1+1/p)$ $\quad$ $\operatorname{Re} p>0$
(7)	$(1+e^{-t})^{-1}$	$\frac{1}{2}\psi(\frac{1}{2}p+\frac{1}{2})-\frac{1}{2}\psi(\frac{1}{2}p)$ $\quad$ $\operatorname{Re} p>0$
(8)	$(1-e^{-t/\alpha})^{\nu-1}$ $\operatorname{Re}\alpha>0,\quad \operatorname{Re}\nu>0$	$\alpha\,\mathrm{B}(\alpha p,\nu)$ $\operatorname{Re} p>0$
(9)	$t^n(1-e^{-t/\alpha})^{-1}$ $\quad$ $\operatorname{Re}\alpha>0$	$(-\alpha)^{n+1}\psi^{(n)}(\alpha p)$ $\quad$ $\operatorname{Re} p>0$ $\psi^{(n)}(z)=\dfrac{d^n}{dz^n}\psi(z)$
(10)	$t^{\nu-1}(1-e^{-t/\alpha})^{-1}$ $\quad$ $\operatorname{Re}\nu>1$	$\alpha^\nu\Gamma(\nu)\,\zeta(\nu,\alpha p)$ $\quad$ $\operatorname{Re} p>0$
(11)	$t^{-1}(1-e^{-t})^{-1}-t^{-2}-\frac{1}{2}t^{-1}$	$p+\log\Gamma(p)-p\log p+\frac{1}{2}\log(\frac{1}{2}p/\pi)$ $\operatorname{Re} p>0$
(12)	$(1-e^{-\alpha t})(1-e^{-t})^{-1}$	$\psi(p+\alpha)-\psi(p)$ $\quad$ $\operatorname{Re} p>0,\ -\operatorname{Re}\alpha$
(13)	$\dfrac{1-e^{-\alpha t}}{t(1+e^{-t})}$	$\log\dfrac{\Gamma(\frac{1}{2}p)\Gamma(\frac{1}{2}\alpha+\frac{1}{2}p+\frac{1}{2})}{\Gamma(\frac{1}{2}p+\frac{1}{2})\Gamma(\frac{1}{2}\alpha+\frac{1}{2}p)}$ $\operatorname{Re} p>0,\ -\operatorname{Re}\alpha$

Exponential functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(14)	$(1-e^{-t})^{\nu-1}(1-ze^{-t})^{-\mu}$ $\operatorname{Re}\nu>0, \quad \lvert\arg(1-z)\rvert<\pi$	$B(p,\nu)\ {}_2F_1(\mu,p;p+\nu;z)$ $\operatorname{Re}p>0$
(15)	$(1-e^{-t})^{-1}(1-e^{-\alpha t})(1-e^{-\beta t})$	$\psi(p+\alpha)+\psi(p+\beta)$ $-\psi(p+\alpha+\beta)-\psi(p)$ $\operatorname{Re}p>0, \quad -\operatorname{Re}\alpha$ $\operatorname{Re}p>-\operatorname{Re}\beta, \quad -\operatorname{Re}(\alpha+\beta)$
(16)	$\dfrac{(1-e^{-\alpha t})(1-e^{-\beta t})}{t(1-e^{-t})}$	$\log\dfrac{\Gamma(p)\,\Gamma(p+\alpha+\beta)}{\Gamma(p+\alpha)\Gamma(p+\beta)}$ $\operatorname{Re}p>0, \quad -\operatorname{Re}\alpha$ $\operatorname{Re}p>-\operatorname{Re}\beta, \quad -\operatorname{Re}(\alpha+\beta)$
(17)	$\dfrac{(1-e^{-\alpha t})(1-e^{-\beta t})(1-e^{-\gamma t})}{t(1-e^{-t})}$	$\log\dfrac{\Gamma(p)\Gamma(p+\beta+\gamma)\Gamma(p+\alpha+\gamma)\Gamma(p+\alpha+\beta)}{\Gamma(p+\alpha)\Gamma(p+\beta)\Gamma(p+\gamma)\Gamma(p+\alpha+\beta+\gamma)}$ $2\operatorname{Re}p>\lvert\operatorname{Re}\alpha\rvert+\lvert\operatorname{Re}\beta\rvert+\lvert\operatorname{Re}\gamma\rvert$
(18)	$\dfrac{[a+(1-e^{-t})^{1/2}]^{-\nu}+[a-(1-e^{-t})^{1/2}]^{-\nu}}{(1-e^{-t})^{1/2}}$	$2^{p+1}e^{(p-\nu)\pi i}\dfrac{\Gamma(p)}{\Gamma(\nu)}(a^2-1)^{\frac12p-\frac12\nu}$ $\times Q_{p-1}^{\nu-p}(a) \qquad \operatorname{Re}p>0$
(19)	$0 \qquad 0<t<b$ $(1-e^{-2t})^{-1/2}[e^{-b}(1-e^{-2t})^{1/2}$ $-e^{-t}(1-e^{-2b})^{1/2}]^{\nu} \qquad t>b$ $\operatorname{Re}\nu>-1$	$\dfrac{(\pi)^{1/2}\Gamma(p)\Gamma(\nu+1)}{2^{\frac12p+\frac12\nu}\Gamma(\frac12p+\frac12\nu+\frac12)}e^{-\frac12b(p+\nu)}$ $\times P_{-\frac12p+\frac12\nu}^{-\frac12p-\frac12\nu}[(1-e^{-2b})^{1/2}]$ $\operatorname{Re}p>0$
(20)	$e^{(\mu-1)t}(1-e^{-t})^{\mu-1/2}[(1-e^{-t})\sin\theta$ $-i(1-e^{-t})\cos\theta]^{\mu-1/2}$ $\operatorname{Re}\mu>-\frac12$	$\dfrac{2^{\mu-1}\Gamma(\mu+\frac12)\Gamma(p-\mu+1)}{\pi^{1/2}\Gamma(p+\mu+1)}\sin^{\mu}\theta$ $\times e^{(p+\frac12)i\theta+(\frac12\mu-\frac14)\pi i}$ $\times[\pi P_{\nu}^{\mu}(\cos\theta)+2i\,Q_{\nu}^{\mu}(\cos\theta)]$ $\operatorname{Re}p>\operatorname{Re}\mu-1$
	Another formula may be derived from this by changing i into $-i$ throughout.	

Exponential functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(21)	$0 \quad 0<t<b$ $e^{-\frac{1}{4}t^2/a} \quad t>b$ $\operatorname{Re} a>0$	$\pi^{\frac{1}{2}} a^{\frac{1}{2}} e^{ap^2} \operatorname{Erfc}(a^{\frac{1}{2}} p+\frac{1}{2} a^{-\frac{1}{2}} b)$
(22)	$te^{-\frac{1}{4}t^2/a} \quad \operatorname{Re} a>0$	$2a-2\pi^{1/2} a^{3/2} pe^{ap^2} \operatorname{Erfc}(a^{1/2} p)$
(23)	$t^{-\frac{1}{2}} e^{-\frac{1}{4}t^2/a} \quad \operatorname{Re} a>0$	$a^{\frac{1}{4}} p^{\frac{1}{2}} e^{\frac{1}{2}ap^2} K_{\frac{1}{4}}(\frac{1}{2} ap^2)$
(24)	$t^{\nu-1} e^{-t^2/(8a)}$ $\operatorname{Re} a>0, \quad \operatorname{Re} \nu>0$	$\Gamma(\nu) 2^{\nu} a^{\frac{1}{2}\nu} e^{ap^2} D_{-\nu}(2p a^{\frac{1}{2}})$
(25)	$e^{-\frac{1}{4}a/t} \quad \operatorname{Re} a \geq 0$	$a^{\frac{1}{2}} p^{-\frac{1}{2}} K_1(a^{\frac{1}{2}} p^{\frac{1}{2}}) \quad \operatorname{Re} p>0$
(26)	$t^{\frac{1}{2}} e^{-\frac{1}{4}a/t} \quad \operatorname{Re} a \geq 0$	$\frac{1}{2}\pi^{1/2} p^{-3/2}(1+a^{1/2} p^{1/2}) e^{-a^{\frac{1}{2}} p^{\frac{1}{2}}}$ $\operatorname{Re} p>0$
(27)	$t^{-\frac{1}{2}} e^{-\frac{1}{4}a/t} \quad \operatorname{Re} a \geq 0$	$\pi^{\frac{1}{2}} p^{-\frac{1}{2}} e^{-a^{\frac{1}{2}} p^{\frac{1}{2}}} \quad \operatorname{Re} p>0$
(28)	$t^{-3/2} e^{-\frac{1}{4}a/t} \quad \operatorname{Re} a>0$	$2\pi^{\frac{1}{2}} a^{-\frac{1}{2}} e^{-a^{\frac{1}{2}} p^{\frac{1}{2}}} \quad \operatorname{Re} p \geq 0$
(29)	$t^{\nu-1} e^{-\frac{1}{4}a/t} \quad \operatorname{Re} a>0$	$2(\frac{1}{4}a/p)^{\frac{1}{2}\nu} K_{\nu}(a^{\frac{1}{2}} p^{\frac{1}{2}}) \quad \operatorname{Re} p>0$
(30)	$t^{-\frac{1}{2}}(e^{-\frac{1}{4}a/t}-1) \quad \operatorname{Re} a \geq 0$	$\pi^{\frac{1}{2}} p^{-\frac{1}{2}}(e^{-a^{\frac{1}{2}} p^{\frac{1}{2}}}-1) \quad \operatorname{Re} p \geq 0$
(31)	$e^{-2a^{\frac{1}{2}} t^{\frac{1}{2}}} \quad \lvert\arg a\rvert<\pi$	$p^{-1}-\pi^{1/2} a^{1/2} p^{-3/2} e^{a/p}$ $\times \operatorname{Erfc}(a^{1/2} p^{-1/2}) \quad \operatorname{Re} p>0$
(32)	$t^{\frac{1}{2}} e^{-2a^{\frac{1}{2}} t^{\frac{1}{2}}} \quad \lvert\arg a\rvert<\pi$	$-a^{1/2} p^{-2}+\pi^{1/2} p^{-5/2}(a+\frac{1}{2}p) e^{a/p}$ $\times \operatorname{Erfc}(a^{1/2} p^{-1/2}) \quad \operatorname{Re} p>0$

Exponential functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(33)	$t^{-1/2}e^{-2a^{1/2}t^{1/2}}$ $\quad\lvert\arg a\rvert<\pi$	$\pi^{1/2}p^{-1/2}e^{a/p}\,\mathrm{Erfc}(a^{1/2}p^{-1/2})$ $\quad\mathrm{Re}\,p>0$
(34)	$(2t)^{-3/4}e^{-2a^{1/2}t^{1/2}}$ $\quad\lvert\arg a\rvert<\pi$	$(\tfrac{1}{2}a/p)^{1/2}e^{\frac{1}{2}a/p}K_{1/4}(\tfrac{1}{2}a/p)$ $\quad\mathrm{Re}\,p>0$
(35)	$(2t)^{\nu-1}e^{-2a^{1/2}t^{1/2}}$ $\quad\mathrm{Re}\,\nu>0$	$\Gamma(2\nu)\,p^{-\nu}e^{\frac{1}{2}a/p}D_{-2\nu}[(2a/p)^{1/2}]$ $\quad\mathrm{Re}\,p>0$
(36)	$\exp(-ae^{-t})$	$a^{-p}\,\gamma(p,a)$ $\quad\mathrm{Re}\,p>0$
(37)	$\exp(-ae^{t})$ $\quad\mathrm{Re}\,a>0$	$a^{p}\Gamma(-p,a)$
(38)	$(1-e^{-t})^{\nu-1}\exp(ae^{-t})$ $\quad\mathrm{Re}\,\nu>0$	$\dfrac{\Gamma(\nu)\Gamma(p)}{\Gamma(\nu+p)}\,a^{-\frac{1}{2}\nu-\frac{1}{2}p}\,e^{\frac{1}{2}a}$ $\times M_{\frac{1}{2}\nu-\frac{1}{2}p,\,\frac{1}{2}\nu+\frac{1}{2}p-\frac{1}{2}}(a)$ $\quad\mathrm{Re}\,p>0$
(39)	$(1-e^{-t})^{\nu-1}\exp(-ae^{t})$ $\quad\mathrm{Re}\,a>0,\quad\mathrm{Re}\,\nu>0$	$\Gamma(\nu)\,a^{\frac{1}{2}p-\frac{1}{2}}e^{-\frac{1}{2}a}W_{\frac{1}{2}-\frac{1}{2}p-\nu,\,-\frac{1}{2}p}(a)$
(40)	$(1-e^{-t})^{\nu-1}(1-\lambda e^{-t})^{-\mu}$ $\times\exp(ae^{-t})$ $\quad\mathrm{Re}\,\nu>0,\quad\lvert\arg(1-\lambda)\rvert<\pi$	$\dfrac{\Gamma(\nu)\Gamma(p)}{\Gamma(\nu+p)}\,\Phi_1(p,\mu,\nu;\lambda,a)$ $\quad\mathrm{Re}\,p>0$
(41)	$(e^{t}-1)^{\nu-1}\exp[-a/(e^{t}-1)]$ $\quad\mathrm{Re}\,a>0$	$\Gamma(p-\nu+1)\,e^{\frac{1}{2}a}a^{\frac{1}{2}\nu-\frac{1}{2}}$ $\times W_{\frac{1}{2}\nu-\frac{1}{2}-p,\,\frac{1}{2}\nu}(a)$ $\quad\mathrm{Re}\,p>\mathrm{Re}\,\nu-1$

4.6. Logarithmic functions

	$f(t)$		$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	
(1)	$\log t$		$-p^{-1}\log(\gamma p)$	$\mathrm{Re}\, p>0$
(2)	0 $\log t$	$0<t<b$ $t>b$	$p^{-1}[e^{-bp}\log b-\mathrm{Ei}(-bp)]$	$\mathrm{Re}\, p>0$
(3)	0 $\log(t/b)$	$0<t<b$ $t>b$	$-p^{-1}\,\mathrm{Ei}(-bp)$	$\mathrm{Re}\, p>0$
(4)	$\log(1+\alpha t)$	$\lvert\arg\alpha\rvert<\pi$	$-p^{-1}e^{p/\alpha}\mathrm{Ei}(-p/\alpha)$	$\mathrm{Re}\, p>0$
(5)	$\log(t+\alpha)$	$\lvert\arg\alpha\rvert<\pi$	$p^{-1}[\log\alpha-e^{\alpha p}\,\mathrm{Ei}(-\alpha p)]$	$\mathrm{Re}\, p>0$
(6)	$\log\lvert b-t\rvert$	$b>0$	$p^{-1}[\log b-e^{-bp}\,\overline{\mathrm{Ei}}(bp)]$	$\mathrm{Re}\, p>0$
(7)	$t^n\log t$		$\dfrac{n!}{p^{n+1}}\left[1+\dfrac{1}{2}+\dfrac{1}{3}+\cdots+\dfrac{1}{n}-\log(\gamma p)\right]$	$\mathrm{Re}\, p>0$
(8)	0 $t^{-1}\log(2t-1)$	$0<t<1$ $t>1$	$\tfrac{1}{2}[\mathrm{Ei}(-\tfrac{1}{2}p)]^2$	$\mathrm{Re}\, p>0$
(9)	$t^{-1/2}\log t$		$-\pi^{1/2}p^{-1/2}\log(4\gamma p)$	$\mathrm{Re}\, p>0$
(10)	$t^{n-1/2}\log t$	$n\geq 1$	$\pi^{1/2}\dfrac{1\cdot 3\cdot 5\cdot\cdots\cdot(2n-1)}{p^{n+1/2}\,2^n}\left[2\left(1+\dfrac{1}{3}+\dfrac{1}{5}+\cdots+\dfrac{1}{2n-1}\right)-\log(4\gamma p)\right]$	$\mathrm{Re}\, p>0$
(11)	$t^{\nu-1}\log t$	$\mathrm{Re}\,\nu>0$	$\Gamma(\nu)p^{-\nu}[\psi(\nu)-\log p]$	$\mathrm{Re}\, p>0$

Logarithmic functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(12)	$t^{\nu-1}[\psi(\nu)-\log t]$ $\quad$ $\mathrm{Re}\,\nu>0$	$\Gamma(\nu)p^{-\nu}\log p$ $\quad$ $\mathrm{Re}\,p>0$
(13)	$(\log t)^2$	$p^{-1}\{\pi^2/6+[\log(\gamma p)]^2\}$ $\quad$ $\mathrm{Re}\,p>0$
(14)	$\log\lvert t^2-a^2\rvert$ $\quad$ $a>0$	$p^{-1}[\log a^2-e^{ap}\,\mathrm{Ei}(-ap)$ $-e^{-ap}\,\overline{\mathrm{Ei}}(ap)]$ $\quad$ $\mathrm{Re}\,p>0$
(15)	$\log(t^2-a^2)$ $\quad$ $\lvert\mathrm{Im}\,a\rvert>0$	$p^{-1}\,[\log a^2-e^{ap}\,\mathrm{Ei}(-ap)$ $-e^{-ap}\,\mathrm{Ei}(ap)]$ $\quad$ $\mathrm{Re}\,p>0$
(16)	$\log(t^2+a^2)$	$2p^{-1}[\log a-\mathrm{ci}(ap)\cos(ap)$ $-\mathrm{si}(ap)\sin(ap)]$ $\quad$ $\mathrm{Re}\,p>0$
(17)	$t^{-1}[\log(t^2+a^2)-\log a^2]$	$[\mathrm{ci}(ap)]^2+[\mathrm{si}(ap)]^2$ $\quad$ $\mathrm{Re}\,p>0$
(18)	0 $\quad$ $0<t<b$ $\log\dfrac{(t+b)^{1/2}+(t-b)^{1/2}}{2^{1/2}\,b^{1/2}}$ $\quad$ $t>b$	$\tfrac{1}{2}p^{-1}K_0(bp)$ $\quad$ $\mathrm{Re}\,p>0$
(19)	$\log\dfrac{t^{1/2}+(t+2a)^{1/2}}{2^{1/2}\,a^{1/2}}$ $\quad$ $\lvert\arg a\rvert<\pi$	$\tfrac{1}{2}p^{-1}\,e^{ap}K_0(ap)$ $\quad$ $\mathrm{Re}\,p>0$
(20)	$\log\dfrac{(t+ib)^{1/2}+(t-ib)^{1/2}}{2^{1/2}\,b^{1/2}}$ $\quad$ $b>0$	$\tfrac{1}{4}\pi p^{-1}[\mathbf{H}_0(bp)-Y_0(bp)]$ $\quad$ $\mathrm{Re}\,p>0$
(21)	$\dfrac{\log[4t(2b-t)/b^2]}{t^{1/2}(2b-t)^{1/2}}$ $\quad$ $0<t<2b$ 0 $\quad$ $t>2b$	$\pi e^{-bp}[\tfrac{1}{2}\pi Y_0(ibp)-\log(\tfrac{1}{2}\gamma)J_0(ibp)]$

4.7. Trigonometric functions

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(1)	$\sin(at)$	$a(p^2+a^2)^{-1}$ $\quad$ $\operatorname{Re} p > \lvert\operatorname{Im} a\rvert$
(2)	$\lvert\sin(at)\rvert$ $\quad$ $a>0$	$a(p^2+a^2)^{-1}\operatorname{ctnh}(\tfrac{1}{2}\pi a^{-1}p)$ $\quad$ $\operatorname{Re} p>0$
(3)	$\sin^{2n}(at)$	$\dfrac{(2n)!\,a^{2n}}{p[p^2+(2a)^2][p^2+(4a)^2]\cdots[p^2+(2na)^2]}$ $\quad$ $\operatorname{Re} p > 2n\,\lvert\operatorname{Im} a\rvert$
(4)	0 $\quad$ $0<t<\pi/2$ $\sin^{2n} t$ $\quad$ $t>\pi/2$	$\dfrac{(2n)!\,e^{-\frac{1}{2}\pi p}}{p(2^2+p^2)(4^2+p^2)\cdots(4n^2+p^2)}$ $\times\left\{1+\dfrac{p^2}{2!}+\dfrac{p^2(2^2+p^2)}{4!}+\cdots+\dfrac{p^2(2^2+p^2)\cdots[4(n-1)^2+p^2]}{(2n)!}\right\}$ $\quad$ $\operatorname{Re} p>0$
(5)	$\sin^{2n} t$ $\quad$ $0<t<\pi/2$ 0 $\quad$ $t>\pi/2$	$\dfrac{(2n)!\,e^{-\frac{1}{2}\pi p}}{p(2^2+p^2)(4^2+p^2)\cdots(4n^2+p^2)}$ $\times\left\{e^{\frac{1}{2}\pi p}-1-\dfrac{p^2}{2!}-\cdots-\dfrac{p^2(2^2+p^2)\cdots[4(n-1)^2+p^2]}{(2n)!}\right\}$
(6)	$\sin^{2n} t$ $\quad$ $0<t<m\pi$ 0 $\quad$ $t>m\pi$ $m=1,2,3,\ldots$	$\dfrac{(2n)!\,(1-e^{-mp\pi})}{p(2^2+p^2)(4^2+p^2)\cdots(4n^2+p^2)}$
(7)	$\sin^{2n+1}(at)$	$\dfrac{(2n+1)!\,a^{2n+1}}{(p^2+a^2)[p^2+(3a)^2]\cdots\{p^2+[(2n+1)a]^2\}}$ $\quad$ $\operatorname{Re} p>(2n+1)\,\lvert\operatorname{Im} a\rvert$

Trigonometric functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(8)	$0 \qquad 0<t<\pi/2$ $\sin^{2n+1} \qquad t>\pi/2$	$\dfrac{(2n+1)!\,pe^{-\frac{1}{2}\pi p}}{(1^2+p^2)(3^2+p^2)\cdots[(2n+1)^2+p^2]} \times \left\{1+\dfrac{1^2+p^2}{3!}+\dfrac{(1^2+p^2)(3^2+p^2)}{5!} +\cdots+\dfrac{(1^2+p^2)(3^2+p^2)\cdots[(2n-1)^2+p^2]}{(2n+1)!}\right\}$ $\mathrm{Re}\, p>0$
(9)	$\sin^{2n+1} t \qquad 0<t<\pi/2$ $0 \qquad t>\pi/2$	$\dfrac{(2n+1)!\,pe^{-\frac{1}{2}\pi p}}{(1^2+p^2)(3^2+p^2)\cdots[(2n+1)^2+p^2]} \times \left\{\dfrac{e^{\frac{1}{2}\pi p}}{p}-1-\dfrac{1^2+p^2}{3!} -\cdots-\dfrac{(1^2+p^2)(3^2+p^2)\cdots[(2n-1)^2+p^2]}{(2n+1)!}\right\}$
(10)	$\sin^{2n+1} t \qquad 0<t<m\pi$ $0 \qquad t>m\pi$ $m=1, 2, 3, \ldots$	$\dfrac{(2n+1)!\,[1-(-1)^m e^{-mp\pi}]}{(1^2+p^2)(3^2+p^2)\cdots[(2n+1)^2+p^2]}$
(11)	$\|\sin(at)\|^{2\nu}$ $a>0, \qquad \mathrm{Re}\,\nu>-\frac{1}{2}$	$\dfrac{\mathrm{B}(1+\frac{1}{2}ip/a,\ 1-\frac{1}{2}ip/a)}{(2\nu+1)2^{2\nu}p\,\mathrm{B}(\nu+1+\frac{1}{2}ip/a,\ \nu+1-\frac{1}{2}ip/a)}$ $\mathrm{Re}\, p>0$
(12)	$0 \qquad 0<t<\pi/2$ $t \sin t \qquad t>\pi/2$	$\dfrac{e^{-\frac{1}{2}\pi p}}{(1+p^2)^2}[\frac{1}{2}p\,\pi(1+p^2)+p^2-1]$ $\mathrm{Re}\, p>0$
(13)	$t \sin t \qquad 0<t<\pi/2$ $0 \qquad t>\pi/2$	$(1+p^2)^{-2}\{2p-e^{-\frac{1}{2}\pi p}$ $\times[\frac{1}{2}p\,\pi(1+p^2)+p^2-1]\}$

Trigonometric functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(14)	$t^n \sin(at)$	$n!\,\dfrac{p^{n+1}}{(p^2+a^2)^{n+1}} \times \sum_{0\le 2m\le n} (-1)^m \binom{n+1}{2m+1}\left(\dfrac{a}{p}\right)^{2m+1}$ Re $p>\lvert\text{Im}\,a\rvert$
(15)	$t^{\nu-1}\sin(at)$ Re $\nu>-1$	$\tfrac{1}{2}i\Gamma(\nu)[(p+ia)^{-\nu}-(p-ia)^{-\nu}]$ $=\Gamma(\nu)(p^2+a^2)^{-\frac{1}{2}\nu}\sin[\nu\tan^{-1}(a/p)]$ Re $p>\lvert\text{Im}\,a\rvert$
(16)	$t^{-1}\sin(at)$	$\tan^{-1}(a/p)$ Re $p>\lvert\text{Im}\,a\rvert$
(17)	$t^{-1}\sin^2(at)$	$\tfrac{1}{4}\log(1+4a^2p^{-2})$ Re $p>2\lvert\text{Im}\,a\rvert$
(18)	$t^{-1}\sin^3(at)$	$\tfrac{1}{2}\tan^{-1}(a/p)-\tfrac{1}{4}\tan^{-1}[2ap/(p^2+3a^2)]$ Re $p>3\lvert\text{Im}\,a\rvert$
(19)	$t^{-1}\sin^4(at)$	$\dfrac{1}{8}\log\dfrac{(p^2+4a^2)^2}{p^3}-\dfrac{1}{16}\log(p^2+16a^2)$ Re $p>4\lvert\text{Im}\,a\rvert$
(20)	$t^{-2}\sin^2(at)$	$a\tan^{-1}(2a/p)-\tfrac{1}{4}p\log(1+4a^2p^{-2})$ Re $p\ge 2\lvert\text{Im}\,a\rvert$
(21)	$t^{-2}\sin^3(at)$	$\tfrac{1}{4}p\tan^{-1}(3a/p)-\tfrac{3}{4}p\tan^{-1}(a/p)$ $+(3a/8)\log[(p^2+3a^2)/(p^2+a^2)]$ Re $p\ge 3\,\text{Im}\,a\rvert$
(22)	$(e^t-1)^{-1}\sin(at)$	$\tfrac{1}{2}i\psi(p-ia+1)-\tfrac{1}{2}i\psi(p+ia+1)$ Re $p>\lvert\text{Im}\,a\rvert-1$
(23)	$(1-e^{-t})^{-1}\sin(at)$	$\tfrac{1}{2}i\psi(p-ia)-\tfrac{1}{2}i\psi(p+ia)$ Re $p>\lvert\text{Im}\,a\rvert$

Trigonometric functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(24)	$(1-e^{-t})^{\nu-1}\sin(at)$ $\quad \operatorname{Re}\nu>-1$	$\tfrac{1}{2}i\,\mathrm{B}(\nu,p+ia)-\tfrac{1}{2}i\,\mathrm{B}(\nu,p-ia)$ $\quad \operatorname{Re}p>\lvert\operatorname{Im}a\rvert$
(25)	$t^{\nu-1}e^{-\frac{1}{2}t^2/\alpha}\sin(\beta t)$ $\quad \operatorname{Re}\alpha>0,\quad \operatorname{Re}\nu>-1$	$\tfrac{1}{2}i\,\Gamma(\nu)\,\alpha^{\frac{1}{2}\nu}e^{\frac{1}{4}\alpha(p^2-\beta^2)}$ $\times\{e^{\frac{1}{2}ip\alpha\beta}D_{-\nu}[\alpha^{\frac{1}{2}}(p+i\beta)]$ $-e^{-\frac{1}{2}ip\alpha\beta}D_{-\nu}[\alpha^{\frac{1}{2}}(p-i\beta)]\}$
(26)	$\log t\sin(at)$	$\dfrac{p\tan^{-1}(a/p)-a\log[\gamma(p^2+a^2)^{\frac{1}{2}}]}{p^2+a^2}$ $\quad \operatorname{Re}p>\lvert\operatorname{Im}a\rvert$
(27)	$\log t\sin^2(\tfrac{1}{2}at)$	$p^{-1}(p^2+a^2)^{-1}[ap\tan^{-1}(a/p)$ $+p^2\log(p^2+a^2)^{\frac{1}{2}}$ $-(p^2+a^2)\log p-a^2\log\gamma]$ $\quad \operatorname{Re}p>2\lvert\operatorname{Im}a\rvert$
(28)	$t^{-1}\log t\sin(at)$	$-\log[\gamma(p^2+a^2)^{\frac{1}{2}}]\tan^{-1}(a/p)$ $\quad \operatorname{Re}p>\lvert\operatorname{Im}a\rvert$
(29)	$t^{\nu-1}\log t\sin(at)$ $\quad \operatorname{Re}\nu>-1$	$\Gamma(\nu)(p^2+a^2)^{-\frac{1}{2}\nu}\sin[\nu\tan^{-1}(a/p)]$ $\times\{\psi(\nu)-\log(p^2+a^2)^{\frac{1}{2}}$ $+\tan^{-1}(a/p)\operatorname{ctn}[\nu\tan^{-1}(a/p)]\}$ $\quad \operatorname{Re}p>\lvert\operatorname{Im}a\rvert$
(30)	$\sin(t^2)$	$(\tfrac{1}{2}\pi)^{\frac{1}{2}}[\tfrac{1}{2}-\cos(\tfrac{1}{4}p^2)C(\tfrac{1}{4}p^2)$ $-\sin(\tfrac{1}{4}p^2)S(\tfrac{1}{4}p^2)]$ $\quad \operatorname{Re}p>0$
(31)	$t^{-1}\sin(t^2)$	$\tfrac{1}{2}\pi[\tfrac{1}{2}-C(\tfrac{1}{4}p^2)]^2+\tfrac{1}{2}\pi[\tfrac{1}{2}-S(\tfrac{1}{4}p^2)]^2$ $\quad \operatorname{Re}p>0$
(32)	$\sin(2a^{\frac{1}{2}}t^{\frac{1}{2}})$	$\pi^{1/2}a^{1/2}p^{-3/2}e^{-a/p}$ $\quad \operatorname{Re}p>0$

Trigonometric functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(33)	$t^n \sin(2a^{1/2}t^{1/2})$	$(-1)^n 2^{-n-1/2}\pi^{1/2}p^{-n-1}e^{-a/p}$ $\times \mathrm{He}_{2n+1}(2^{1/2}a^{1/2}p^{-1/2})$ $\quad \mathrm{Re}\, p>0$
(34)	$t^{-1}\sin(2a^{1/2}t^{1/2})$	$\pi\, \mathrm{Erf}(a^{1/2}p^{-1/2})$ $\quad \mathrm{Re}\, p>0$
(35)	$t^{1/2}\sin(2a^{1/2}t^{1/2})$	$a^{1/2}p^{-2}-i\pi^{1/2}p^{-5/2}(\tfrac{1}{2}p-a)e^{-a/p}$ $\times \mathrm{Erf}(i\,a^{1/2}p^{-1/2})$ $\quad \mathrm{Re}\, p>0$
(36)	$t^{-1/2}\sin(2a^{1/2}t^{1/2})$	$-i\pi^{1/2}p^{-1/2}e^{-a/p}\mathrm{Erf}(i\,a^{1/2}p^{-1/2})$ $\quad \mathrm{Re}\, p>0$
(37)	$t^{\nu-1}\sin(2^{1/2}a^{1/2}t^{1/2})$ $\quad \mathrm{Re}\,\nu>-\tfrac{1}{2}$	$2^{-\nu-1/2}\pi^{1/2}\sec(\nu\pi)p^{-\nu}e^{-1/4\, a/p}$ $\times[D_{2\nu-1}(-a^{1/2}p^{-1/2})-D_{2\nu-1}(a^{1/2}p^{-1/2})]$ $\quad \mathrm{Re}\, p>0$
(38)	0 $\quad 0<t<b$ $\sin[a(t^2-b^2)^{1/2}]$ $\quad t>b$	$abp^{-1}K_1(bp)$ $\quad \mathrm{Re}\, p>\lvert\mathrm{Im}\, a\rvert$
(39)	$\sin(ae^{-t})$	$a^{-p}\Gamma(p)[U_p(2a,0)\sin a$ $-U_{p+1}(2a,0)\cos a]$ $\quad \mathrm{Re}\, p>0$
(40)	$\sin[a(1-e^{-t})]$	$a^{-p}\Gamma(p)\,U_{p+1}(2a,0)$ $\quad \mathrm{Re}\, p>0$
(41)	$(e^t-1)^{-1/2}\sin[a(1-e^{-t})^{1/2}]$	$\pi^{1/2}\Gamma(p+\tfrac{1}{2})(2/a)^p\,\mathbf{H}_p(a)$ $\quad \mathrm{Re}\, p>-\tfrac{1}{2}$
(42)	$(1-e^{-t})^{-1/2}\sin[a(e^t-1)^{1/2}]$ $\quad a>0$	$\pi^{1/2}\Gamma(\tfrac{1}{2}-p)(\tfrac{1}{2}a)^p[I_p(a)-\mathbf{L}_{-p}(a)]$ $\quad \mathrm{Re}\, p>-\tfrac{1}{2}$
(43)	$\cos(at)$	$p(p^2+a^2)^{-1}$ $\quad \mathrm{Re}\, p>\lvert\mathrm{Im}\, a\rvert$

Trigonometric functions (cont'd)

	$f(t)$		$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(44)	$\lvert\cos(at)\rvert$	$a>0$	$(p^2+a^2)^{-1}[p+a\operatorname{csch}(\tfrac{1}{2}\pi a^{-1}p)]$ $\operatorname{Re} p>0$
(45)	$\cos^2(at)$		$(p^2+2a^2)(p^3+4a^2p)^{-1}$ $\operatorname{Re} p>2\lvert\operatorname{Im} a\rvert$
(46)	$\cos^3(at)$		$p(p^2+7a^2)(p^2+a^2)^{-1}(p^2+9a^2)^{-1}$ $\operatorname{Re} p>3\lvert\operatorname{Im} a\rvert$
(47)	$\cos^{2n}(at)$		$\dfrac{(2n)!\,a^{2n}}{p[p^2+(2a)^2][p^2+(4a)^2]\cdots[p^2+(2na)^2]}$ $\times\left\{1+\dfrac{p^2}{2!a^2}+\dfrac{p^2[p^2+(2a)^2]}{4!\,a^4}+\cdots+\dfrac{p^2(p^2+4a^2)\cdots[p^2+4(na-a)^2]}{(2n)!\,a^{2n}}\right\}$ $\operatorname{Re} p>2n\lvert\operatorname{Im} a\rvert$
(48)	0 $\cos^{2n} t$	$0<t<\pi/2$ $t>\pi/2$	$\dfrac{(2n)!\,e^{-\frac{1}{2}p\pi}}{p(2^2+p^2)(4^2+p^2)\cdots(4n^2+p^2)}$ $\operatorname{Re} p>0$
(49)	$\cos^{2n} t$ 0	$0<t<\pi/2$ $t>\pi/2$	$\dfrac{(2n)!}{p(2^2+p^2)(4^2+p^2)\cdots(4n^2+p^2)}$ $\times\left\{-e^{-\frac{1}{2}p\pi}+1+\dfrac{p^2}{2!}+\cdots+\dfrac{p^2(2^2+p^2)\cdots[4(n-1)^2+p^2]}{(2n)!}\right\}$

Trigonometric functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(50)	$0 \qquad 0<t<\pi/2$ $\cos^{2n} t \qquad \pi/2<t<(m+\frac{1}{2})\pi$ $0 \qquad t>(m+\frac{1}{2})\pi$ $m=1,2,3,\ldots$	$\dfrac{(2n)!\,e^{-\frac{1}{2}p\pi}(1-e^{-mp\pi})}{p(2^2+p^2)(4^2+p^2)\cdots(4n^2+p^2)}$
(51)	$\cos^{2n+1}(at)$	$\dfrac{(2n+1)!\,a^{2n}p}{(p^2+a^2)[p^2+(3a)^2]\cdots[p^2+(2na+a)^2]}\Bigg\{1+\dfrac{p^2+a^2}{3!\,a^2}+\dfrac{(p^2+a^2)(p^2+9a^2)}{5!\,a^4}+\cdots+\dfrac{[p^2+a^2][p^2+(3a)^2]\cdots[p^2+(2na-a)^2]}{(2n+1)!\,a^{2n}}\Bigg\}$ $\operatorname{Re} p>(2n+1)\,\lvert\operatorname{Im} a\rvert$
(52)	$0 \qquad 0<t<\pi/2$ $\cos^{2n+1} t \qquad t>\pi/2$	$\dfrac{-(2n+1)!\,e^{-\frac{1}{2}p\pi}}{(1^2+p^2)(3^2+p^2)\cdots[(2n+1)^2+p^2]}$ $\operatorname{Re} p>0$
(53)	$\cos^{2n+1} t \qquad 0<t<\pi/2$ $0 \qquad t>\pi/2$	$\dfrac{(2n+1)!\,p}{(1^2+p^2)(3^2+p^2)\cdots[(2n+1)^2+p^2]}\times\Bigg\{\dfrac{e^{-\frac{1}{2}p\pi}}{p}+1+\dfrac{1^2+p^2}{3!}+\cdots+\dfrac{(1^2+p^2)(3^2+p^2)\cdots[(2n-1)^2+p^2]}{(2n+1)!}\Bigg\}$
(54)	$0 \qquad 0<t<\pi/2$ $\cos^{2n+1} t \qquad \pi/2<t<(m+\frac{1}{2})\pi$ $0 \qquad t>(m+\frac{1}{2})\pi$ $m=1,2,3,\ldots$	$\dfrac{(2n+1)!\,e^{-\frac{1}{2}p\pi}(e^{-m\pi(p+i)}-1)}{(1^2+p^2)(3^2+p^2)\cdots[(2n+1)^2+p^2]}$

Trigonometric functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(55)	$0 \qquad 0<t<\pi/2$ $t\cos t \qquad t>\pi/2$	$-(1+p^2)^{-2} e^{-\frac{1}{2}p\pi}[\frac{1}{2}\pi(1+p^2)+2p]$ $\mathrm{Re}\, p>0$
(56)	$t\cos t \qquad 0<t<\pi/2$ $0 \qquad t>\pi/2$	$\dfrac{p^2-1+e^{-\frac{1}{2}p\pi}[\frac{1}{2}\pi(1+p^2)+2p]}{(1+p^2)^2}$
(57)	$t^n\cos(at)$	$\dfrac{n!p^{n+1}}{(p^2+a^2)^{n+1}}$ $\times \sum_{0\leq 2m\leq n+1} (-)^m \binom{n+1}{2m}\left(\dfrac{a}{p}\right)^{2m}$ $\mathrm{Re}\, p>\lvert\mathrm{Im}\, a\rvert$
(58)	$t^{\nu-1}\cos(at) \qquad \mathrm{Re}\,\nu>0$	$\frac{1}{2}\Gamma(\nu)[(p-ia)^{-\nu}+(p+ia)^{-\nu}]$ $=\Gamma(\nu)(p^2+a^2)^{-\frac{1}{2}\nu}$ $\times\cos[\nu\tan^{-1}(a/p)]$ $\mathrm{Re}\, p>\lvert\mathrm{Im}\, a\rvert$
(59)	$t^{-1}(1-\cos at)$	$\frac{1}{2}\log(1+a^2/p^2) \qquad \mathrm{Re}\, p>\lvert\mathrm{Im}\, a\rvert$
(60)	$(1-e^{-t})^{\nu-1}\cos(at) \qquad \mathrm{Re}\,\nu>0$	$\frac{1}{2}\mathrm{B}(\nu, p-ia)+\frac{1}{2}\mathrm{B}(\nu, p+ia)$ $\mathrm{Re}\, p>\lvert\mathrm{Im}\, a\rvert$
(61)	$t^{\nu-1} e^{-\frac{1}{2}t^2/a}\cos(\beta t)$ $\mathrm{Re}\, a>0, \quad \mathrm{Re}\,\nu>0$	$\frac{1}{2}\Gamma(\nu)\, a^{\frac{1}{2}\nu} e^{\frac{1}{4}a(p^2-\beta^2)}$ $\times\{e^{-\frac{1}{2}ia\beta p} D_{-\nu}[a^{\frac{1}{2}}(p-i\beta)]$ $+e^{\frac{1}{2}ia\beta p} D_{-\nu}[a^{\frac{1}{2}}(p+i\beta)]\}$
(62)	$\log t\cos(at)$	$-\dfrac{a\tan^{-1}(a/p)+p\log[\gamma(p^2+a^2)^{\frac{1}{2}}]}{p^2+a^2}$ $\mathrm{Re}\, p>\lvert\mathrm{Im}\, a\rvert$

Trigonometric functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(63)	$t^{\nu-1}\log t\cos(at)$ $\quad\mathrm{Re}\,\nu>0$	$\dfrac{\Gamma(\nu)}{(p^2+a^2)^{\frac{1}{2}\nu}}\cos[\nu\tan^{-1}(a/p)]$ $\times\{\psi(\nu)-\log(p^2+a^2)^{\frac{1}{2}}$ $-\tan^{-1}(a/p)\tan[\nu\tan^{-1}(a/p)]\}$ $\quad\mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(64)	$\cos(t^2)$	$(\frac{1}{2}\pi)^{\frac{1}{2}}[\frac{1}{2}-\cos(\frac{1}{4}p^2)S(\frac{1}{4}p^2)$ $+\sin(\frac{1}{4}p^2)C(\frac{1}{4}p^2)]$ $\quad\mathrm{Re}\,p>0$
(65)	$\cos(2a^{\frac{1}{2}}t^{\frac{1}{2}})$	$p^{-1}+i\pi^{1/2}a^{1/2}p^{-3/2}e^{-a/p}$ $\times\mathrm{Erf}(ia^{1/2}p^{-1/2})$ $\quad\mathrm{Re}\,p>0$
(66)	$t^{\frac{1}{2}}\cos(2a^{\frac{1}{2}}t^{\frac{1}{2}})$	$\pi^{1/2}p^{-5/2}(\frac{1}{2}p-a)e^{-a/p}$ $\quad\mathrm{Re}\,p>0$
(67)	$t^{-\frac{1}{2}}\cos(2a^{\frac{1}{2}}t^{\frac{1}{2}})$	$\pi^{\frac{1}{2}}p^{-\frac{1}{2}}e^{-a/p}$ $\quad\mathrm{Re}\,p>0$
(68)	$t^{n-\frac{1}{2}}\cos(2a^{\frac{1}{2}}t^{\frac{1}{2}})$	$(-2)^{-n}\pi^{\frac{1}{2}}p^{-n-\frac{1}{2}}e^{-a/p}$ $\times\mathrm{He}_{2n}(2^{\frac{1}{2}}a^{\frac{1}{2}}p^{-\frac{1}{2}})$ $\quad\mathrm{Re}\,p>0$
(69)	$t^{\nu-1}\cos(2^{\frac{1}{2}}a^{\frac{1}{2}}t^{\frac{1}{2}})$ $\quad\mathrm{Re}\,\nu>0$	$2^{-\frac{1}{2}-\nu}\pi^{\frac{1}{2}}\csc(\nu\pi)p^{-\nu}e^{-\frac{1}{4}a/p}$ $\times[D_{2\nu-1}(a^{\frac{1}{2}}p^{-\frac{1}{2}})+D_{2\nu-1}(-a^{\frac{1}{2}}/p^{\frac{1}{2}})]$ $\quad\mathrm{Re}\,p>0$
(70)	$(2t-t^2)^{-\frac{1}{2}}\cos[a(2t-t^2)^{\frac{1}{2}}]$ $\quad 0<t<2$ 0 $\quad t>2$	$\pi e^{-p}J_0[(a^2-p^2)^{\frac{1}{2}}]$
(71)	$t^{-\frac{1}{2}}\sinh a[\cosh a-\cos(t^{\frac{1}{2}})]^{-1}$ $\quad\mathrm{Re}\,a>0$	$2\pi e^{a^2p}[\theta_3(2ap,4p)+\hat{\theta}_3(2ap,4p)]$ $-\pi^{\frac{1}{2}}p^{-\frac{1}{2}}$ $\quad\mathrm{Re}\,p>0$
(72)	$\cos(ae^{-t})$	$a^{-p}\Gamma(p)[U_p(2a,0)\cos a$ $+U_{p+1}(2a,0)\sin a]$ $\quad\mathrm{Re}\,p>0$

Trigonometric functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(73)	$\cos[\alpha(1-e^{-t})]$	$\alpha^{-p}\,\Gamma(p)\,U_p(2\alpha,0)$ $\quad$ $\operatorname{Re} p>0$
(74)	$(e^{t}-1)^{-1/2}\cos[\alpha(1-e^{-t})^{1/2}]$	$\pi^{1/2}\,\Gamma(p+\tfrac{1}{2})\,(2/\alpha)^p\,J_p(\alpha)$ $\quad$ $\operatorname{Re} p>-\tfrac{1}{2}$
(75)	$(1-e^{-t})^{-1/2}\cos[a(e^{t}-1)^{1/2}]$ $\quad$ $a>0$	$2\pi^{1/2}[\Gamma(p+\tfrac{1}{2})]^{-1}(\tfrac{1}{2}a)^p\,K_p(a)$ $\quad$ $\operatorname{Re} p>-\tfrac{1}{2}$
(76)	$t^{-1}[\cos(\alpha t)-\cos(\beta t)]$	$\tfrac{1}{2}\log[(p^2+\beta^2)(p^2+\alpha^2)^{-1}]$ $\quad$ $\operatorname{Re} p>\lvert\operatorname{Im}\alpha\rvert,\ \lvert\operatorname{Im}\beta\rvert$
(77)	$t^{-2}(\cos\alpha t-\cos\beta t)$	$\tfrac{1}{2}p\log[(p^2+\alpha^2)(p^2+\beta^2)^{-1}]$ $+\beta\tan^{-1}(\beta/p)-\alpha\tan^{-1}(\alpha/p)$ $\quad$ $\operatorname{Re} p\geq\lvert\operatorname{Im}\alpha\rvert,\ \lvert\operatorname{Im}\beta\rvert$
(78)	$\sin(\alpha t)\sin(\beta t)$	$\dfrac{2\alpha\beta p}{[p^2+(\alpha+\beta)^2][p^2+(\alpha-\beta)^2]}$ $\quad$ $\operatorname{Re} p\geq\lvert\operatorname{Im}(\pm\alpha\pm\beta)\rvert$
(79)	$\cos(\alpha t)\sin(\beta t)$	$\dfrac{\beta(p^2-\alpha^2+\beta^2)}{[p^2+(\alpha+\beta)^2][p^2+(\alpha-\beta)^2]}$ $\quad$ $\operatorname{Re} p>\lvert\operatorname{Im}(\pm\alpha\pm\beta)\rvert$
(80)	$\cos(\alpha t)\cos(\beta t)$	$\dfrac{p(p^2+\alpha^2+\beta^2)}{[p^2+(\alpha+\beta)^2][p^2+(\alpha-\beta)^2]}$ $\quad$ $\operatorname{Re} p>\lvert\operatorname{Im}(\pm\alpha\pm\beta)\rvert$
(81)	$t^{-1}\sin(\alpha t)\sin(\beta t)$	$\tfrac{1}{4}\log\dfrac{p^2+(\alpha+\beta)^2}{p^2+(\alpha-\beta)^2}$ $\quad$ $\operatorname{Re} p>\lvert\operatorname{Im}(\pm\alpha\pm\beta)\rvert$

Trigonometric functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(82)	$t^{-1}\sin(at)\cos(\beta t)$	$\tfrac{1}{2}\tan^{-1}\dfrac{2ap}{p^2-a^2+\beta^2}$ $\operatorname{Re} p > \lvert\operatorname{Im}(\pm a \pm\beta)\rvert$
(83)	$t^{-2}\sin(at)\sin(\beta t)$	$\tfrac{1}{2}a\tan^{-1}\dfrac{2\beta p}{p^2+a^2-\beta^2}$ $+\tfrac{1}{2}\beta\tan^{-1}\dfrac{2ap}{p^2-a^2+\beta^2}$ $+\tfrac{1}{4}p\log\dfrac{p^2+(a-\beta)^2}{p^2+(a+\beta)^2}$ $\operatorname{Re} p \geq \lvert\operatorname{Im}(\pm a \pm\beta)\rvert$
(84)	$\csc t\,\sin[(2n+1)t]$	$\dfrac{1}{p}+\sum_{m=1}^{n}\dfrac{2p}{p^2+4m^2}$ $\qquad \operatorname{Re} p > 0$
(85)	$\tan t\,\cos[(2n+1)t]$	$\dfrac{2n+1}{p^2+(2n+1)^2}+2\sum_{m=0}^{n-1}\dfrac{(-1)^m(2m+1)}{p^2+(2m+1)^2}$ $\operatorname{Re} p > 0$
(86)	$\dfrac{(2at\cos at-\sin at)\sin at}{t^2}$	$\tfrac{1}{4}p\log\left(1+\dfrac{4a^2}{p^2}\right)$ $\operatorname{Re} p > 2\lvert\operatorname{Im} a\rvert$
(87)	$\dfrac{at\cos at-\sin at}{t^2}$	$p\tan^{-1}\dfrac{a}{p}-a$ $\qquad \operatorname{Re} p > \lvert\operatorname{Im} a\rvert$

4.8. Inverse trigonometric functions

(1)	$\sin^{-1} t$	$0<t<1$	$\tfrac{1}{2}\pi p^{-1}[I_0(p)-\mathbf{L}_0(p)]$
	0	$t>1$	

Inverse trigonometric functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(2)	$t\sin^{-1} t \qquad 0<t<1$ $0 \qquad t>1$	$\frac{1}{2}\pi p^{-2}[\mathbf{L}_0(p)-I_0(p)$ $+p\,\mathbf{L}_1(p)-p\,I_1(p)]+p^{-1}$
(3)	$\tan^{-1}(t/a)$	$p^{-1}[-\mathrm{ci}(ap)\sin(ap)$ $-\mathrm{si}(ap)\cos(ap)] \qquad \mathrm{Re}\, p>0$
(4)	$\mathrm{ctn}^{-1}(t/a)$	$p^{-1}[\frac{1}{2}\pi+\mathrm{ci}(ap)\sin(ap)$ $+\mathrm{si}(ap)\cos(ap)] \qquad \mathrm{Re}\, p>0$
(5)	$t\tan^{-1}(t/a)$	$p^{-2}[-\mathrm{ci}(ap)\sin(ap)$ $-\mathrm{si}(ap)\cos(ap)]$ $+ap^{-1}[\mathrm{ci}(ap)\cos(ap)$ $-\mathrm{si}(ap)\sin(ap)] \qquad \mathrm{Re}\, p>0$
(6)	$t\,\mathrm{ctn}^{-1}(t/a)$	$p^{-2}[\frac{1}{2}\pi+\mathrm{ci}(ap)\sin(ap)$ $+\mathrm{si}(ap)\cos(ap)]$ $+ap^{-1}[\mathrm{si}(ap)\sin(ap)$ $-\mathrm{ci}(ap)\cos(ap)] \qquad \mathrm{Re}\, p>0$
(7)	$t^{\nu-\frac{1}{2}}(1+t^2)^{\frac{1}{2}\nu-\frac{1}{4}}$ $\times e^{-i(\nu-\frac{1}{2})\,\mathrm{ctn}^{-1} t}$ $\mathrm{Re}\,\nu>-\frac{1}{2}$	$\frac{1}{2}i\pi^{\frac{1}{2}}\Gamma(\nu+\frac{1}{2})p^{-\nu}e^{-\frac{1}{2}ip}H_\nu^{(1)}(\frac{1}{2}p)$ $\mathrm{Re}\, p>0$
(8)	$t^{\nu-\frac{1}{2}}(1+t^2)^{\frac{1}{2}\nu-\frac{1}{4}}$ $\times\sin[(\nu-\frac{1}{2})\,\mathrm{ctn}^{-1} t]$ $\mathrm{Re}\,\nu>-\frac{1}{2}$	$-\frac{1}{2}\pi^{\frac{1}{2}}\Gamma(\nu+\frac{1}{2})p^{-\nu}[J_\nu(\frac{1}{2}p)\cos(\frac{1}{2}p)$ $+Y_\nu(\frac{1}{2}p)\sin(\frac{1}{2}p)] \qquad \mathrm{Re}\, p>0$
(9)	$t^{\nu-\frac{1}{2}}(1+t^2)^{\frac{1}{2}\nu-\frac{1}{4}}$ $\times\cos[(\nu-\frac{1}{2})\,\mathrm{ctn}^{-1} t]$ $\mathrm{Re}\,\nu>-\frac{1}{2}$	$\frac{1}{2}\pi^{\frac{1}{2}}\Gamma(\nu+\frac{1}{2})p^{-\nu}[J_\nu(\frac{1}{2}p)\sin(\frac{1}{2}p)$ $-Y_\nu(\frac{1}{2}p)\cos(\frac{1}{2}p)] \qquad \mathrm{Re}\, p>0$
(10)	$t^{\nu-\frac{1}{2}}(1+t^2)^{\frac{1}{2}\nu-\frac{1}{4}}$ $\times\sin[\beta-(\nu-\frac{1}{2})\,\mathrm{ctn}^{-1} t]$ $\mathrm{Re}\,\nu>-\frac{1}{2}$	$\frac{1}{2}\pi^{\frac{1}{2}}\Gamma(\nu+\frac{1}{2})p^{-\nu}[J_\nu(\frac{1}{2}p)\cos(\frac{1}{2}p-\beta)$ $+Y_\nu(\frac{1}{2}p)\sin(\frac{1}{2}p-\beta)] \qquad \mathrm{Re}\, p>0$

Inverse trigonometric functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(11)	$\dfrac{\cos[(2n+\frac{1}{2})\cos^{-1}(\frac{1}{2}t/b)]}{(4b^2t-t^3)^{\frac{1}{2}}}$ $\quad 0<t<2b$ $0 \quad t>2b$	$(-1)^n(\frac{1}{2}\pi p)^{\frac{1}{2}} I_n(bp)K_{n+\frac{1}{2}}(bp)$
(12)	$\dfrac{\cos[\nu\cos^{-1}(\frac{1}{2}t/b)]}{(4b^2t-t^3)^{\frac{1}{2}}}$ $\quad 0<t<2b$ $0 \quad t>2b$	$(\frac{1}{2}\pi)^{3/2}p^{1/2}[I_{\frac{1}{2}\nu-\frac{1}{4}}(bp)I_{-\frac{1}{2}\nu-\frac{1}{4}}(bp)$ $-I_{\frac{1}{2}\nu+\frac{1}{4}}(bp)I_{-\frac{1}{2}\nu+\frac{1}{4}}(bp)]$
(13)	$0 \quad 0<t<a$ $\dfrac{\cos\{n\cos^{-1}[(2t-a-b)/(b-a)]\}}{(t-a)^{\frac{1}{2}}(b-t)^{\frac{1}{2}}}$ $\quad a<t<b$ $0 \quad t>b$	$\pi e^{-\frac{1}{2}(a+b)p} I_n[\frac{1}{2}(b-a)p]$
(14)	$[t(t+1)(t+2)]^{-\frac{1}{2}}$ $\times\cos[\nu\cos^{-1}(1+t)^{-1}]$	$\pi^{\frac{1}{2}}e^pD_{\nu-\frac{1}{2}}(2^{\frac{1}{2}}p^{\frac{1}{2}})D_{-\nu-\frac{1}{2}}(2^{\frac{1}{2}}p^{\frac{1}{2}})$ $\mathrm{Re}\,p>0$
(15)	$\dfrac{\cos[\nu\cos^{-1}e^{-t})}{(1-e^{-2t})^{\frac{1}{2}}}$	$\dfrac{\pi 2^{-p}}{p\,\mathrm{B}(\frac{1}{2}p+\frac{1}{2}\nu+\frac{1}{2},\frac{1}{2}p-\frac{1}{2}\nu+\frac{1}{2})}$ $\mathrm{Re}\,p>0$

4.9. Hyperbolic functions

(1)	$\sinh(at)$	$a(p^2-a^2)^{-1} \quad \mathrm{Re}\,p>\lvert\mathrm{Re}\,a\rvert$
(2)	$\cosh(at)$	$p(p^2-a^2)^{-1} \quad \mathrm{Re}\,p>\lvert\mathrm{Re}\,a\rvert$
(3)	$\sinh^2(at)$	$2a^2(p^3-4a^2p)^{-1}$ $\mathrm{Re}\,p>2\lvert\mathrm{Re}\,a\rvert$

Hyperbolic functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(4)	$\cosh^2(at)$	$(p^2-2a^2)(p^3-4a^2p)^{-1}$ $\operatorname{Re} p>2\lvert\operatorname{Re} a\rvert$
(5)	$[\sinh(at)]^\nu$ $\operatorname{Re} a>0,\quad \operatorname{Re}\nu>-1$	$2^{-\nu-1}a^{-1}\,\mathrm{B}(\tfrac{1}{2}p/a-\tfrac{1}{2}\nu,\ \nu+1)$ $\operatorname{Re} p>\operatorname{Re}\nu a$
(6)	$[\cosh(at)-1]^\nu$ $\operatorname{Re} a>0,\quad \operatorname{Re}\nu>-\tfrac{1}{2}$	$2^{-\nu}a^{-1}\,\mathrm{B}(p/a-\nu,\ 2\nu+1)$ $\operatorname{Re} p>\operatorname{Re}\nu a$
(7)	$\operatorname{sech} t$	$\tfrac{1}{2}\psi(\tfrac{1}{4}p+\tfrac{3}{4})-\tfrac{1}{2}\psi(\tfrac{1}{4}p+\tfrac{1}{4})$ $\operatorname{Re} p>-1$
(8)	$\operatorname{sech}^2 t$	$\tfrac{1}{2}p[\psi(\tfrac{1}{4}p+\tfrac{1}{2})-\psi(\tfrac{1}{4}p)]-1$ $\operatorname{Re} p>-2$
(9)	$\tanh t$	$\tfrac{1}{2}\psi(\tfrac{1}{4}p+\tfrac{1}{2})-\tfrac{1}{2}\psi(\tfrac{1}{4}p)-p^{-1}$ $\operatorname{Re} p>0$
(10)	$t^{-1}-\operatorname{csch} t$	$\psi(\tfrac{1}{2}p+\tfrac{1}{2})-\log(\tfrac{1}{2}p)\qquad \operatorname{Re} p>0$
(11)	$t^{-1}-\operatorname{ctnh} t$	$\psi(\tfrac{1}{2}p)+p^{-1}-\log(\tfrac{1}{2}p)\qquad \operatorname{Re} p>0$
(12)	$\dfrac{2}{t}\sinh(at)$	$\log\dfrac{p+a}{p-a}\qquad \operatorname{Re} p>\lvert\operatorname{Re} a\rvert$
(13)	$0 \qquad 0<t<1$ $\dfrac{2}{t}\sinh(at) \qquad t>1$	$-\mathrm{Ei}(a-p)+\mathrm{Ei}(-a-p)$ $\operatorname{Re} p>\lvert\operatorname{Re} a\rvert$
(14)	$\dfrac{2}{t}\sinh(at) \qquad 0<t<1$ $0 \qquad t>1$	$\log\dfrac{p+a}{p-a}+\mathrm{Ei}(a-p)-\mathrm{Ei}(-a-p)$

Hyperbolic functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(15)	$0 \qquad 0<t<1$ $\frac{2}{t}\cosh(at) \qquad t>1$	$-\mathrm{Ei}(a-p)-\mathrm{Ei}(-a-p)$ $\mathrm{Re}\, p > \lvert\mathrm{Re}\, a\rvert$
(16)	$\frac{1}{t}\tanh t$	$\log(\frac{1}{4}p)+2\log\frac{\Gamma(\frac{1}{4}p)}{\Gamma(\frac{1}{4}p+\frac{1}{2})}$ $\mathrm{Re}\, p>0$
(17)	$\frac{1}{t}(1-\mathrm{sech}\, t)$	$2\log\frac{\Gamma(\frac{1}{4}p+\frac{3}{4})}{\Gamma(\frac{1}{4}p+\frac{1}{4})}-\log(\frac{1}{4}p)$ $\mathrm{Re}\, p>0$
(18)	$t^{\nu-1}\sinh(at) \qquad \mathrm{Re}\,\nu>-1$	$\frac{1}{2}\Gamma(\nu)[(p-a)^{-\nu}-(p+a)^{-\nu}]$ $\mathrm{Re}\, p > \lvert\mathrm{Re}\, a\rvert$
(19)	$t^{\nu-1}\cosh(at) \qquad \mathrm{Re}\,\nu>0$	$\frac{1}{2}\Gamma(\nu)[(p-a)^{-\nu}+(p+a)^{-\nu}]$ $\mathrm{Re}\, p > \lvert\mathrm{Re}\, a\rvert$
(20)	$t^{\nu-1}\,\mathrm{csch}\, t \qquad \mathrm{Re}\,\nu>1$	$2^{1-\nu}\Gamma(\nu)\,\zeta(\nu,\frac{1}{2}p+\frac{1}{2})$ $\mathrm{Re}\, p>-1$
(21)	$t^{\nu-1}\,\mathrm{ctnh}\, t \qquad \mathrm{Re}\,\nu>1$	$\Gamma(\nu)[2^{1-\nu}\zeta(\nu,\frac{1}{2}p)-p^{-\nu}]$ $\mathrm{Re}\, p>0$
(22)	$t^{\nu-1}(\mathrm{ctnh}\, t-1) \qquad \mathrm{Re}\,\nu>1$	$2^{1-\nu}\Gamma(\nu)\,\zeta(\nu,\frac{1}{2}p+1) \qquad \mathrm{Re}\, p>-2$
(23)	$0 \qquad 0<t<b$ $(\cosh t-\cosh b)^{\nu-1} \qquad t>b$ $\mathrm{Re}\,\nu>0$	$-i\,2^{\frac{1}{2}}\pi^{-\frac{1}{2}}e^{\nu\pi i}\Gamma(\nu)(\sinh b)^{\nu-\frac{1}{2}}$ $\times Q_{p-\frac{1}{2}}^{\frac{1}{2}-\nu}(\cosh b)$ $\mathrm{Re}\, p>\mathrm{Re}\,\nu-1$
(24)	$\sin(at)\sinh(at)$	$2a^2p(p^4+4a^4)^{-1}$ $\mathrm{Re}\, p > \lvert\mathrm{Re}\, a\rvert+\lvert\mathrm{Im}\, a\rvert$

Hyperbolic functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(25)	$\cos(at)\sinh(at)$	$(ap^2-2a^3)(p^4+4a^4)^{-1}$ $\operatorname{Re} p > \lvert\operatorname{Re} a\rvert + \lvert\operatorname{Im} a\rvert$
(26)	$\sin(at)\cosh(at)$	$(ap^2+2a^3)(p^4+4a^4)^{-1}$ $\operatorname{Re} p > \operatorname{Re}(\pm a \pm i a)$
(27)	$\cos(at)\cosh(at)$	$p^3(p^4+4a^4)^{-1}$ $\operatorname{Re} p > \operatorname{Re}(\pm a \pm i a)$
(28)	$e^{-a\sinh t}$ $\operatorname{Re} a>0$	$\pi\csc(\pi p)\,[\mathbf{J}_p(a)-J_p(a)]$
(29)	$e^{-a\sinh(t+i\psi)}$ $-\pi/2<\psi<\pi/2$ $\lvert\arg a\rvert<\pi/2-\psi$	$\csc(\pi p)\,[\int_0^\pi e^{ia\sin\psi\cos\theta}$ $\times\cos(p\theta-a\cos\psi\sin\theta)\,d\theta$ $-\pi e^{ip\psi}J_p(a)]$
(30)	$e^{-a\cosh t}$ $\operatorname{Re} a>0$	$\csc(\pi p)\,[\int_0^\pi e^{a\cos\theta}\cos(p\theta)\,d\theta$ $-\pi I_p(a)]$
(31)	$(\sinh \tfrac{1}{2}t)^{2\beta}e^{-2a\operatorname{ctnh}\frac{1}{2}t}$ $\operatorname{Re} a>0$	$\tfrac{1}{2}a^{\frac{1}{2}\beta-\frac{1}{2}}\Gamma(p-\beta)\,[W_{-p+\frac{1}{2},\beta}(4a)$ $-(p-\beta)W_{-p-\frac{1}{2},\beta}(4a)]$ $\operatorname{Re} p>\operatorname{Re}\beta$
(32)	$\log\cosh t$	$\tfrac{1}{2}p^{-1}[\psi(\tfrac{1}{4}p+\tfrac{1}{2})-\psi(\tfrac{1}{4}p)]-p^{-2}$ $\operatorname{Re} p>0$
(33)	$\log(\sinh t)-\log t$	$p^{-1}[\log(\tfrac{1}{2}p)-\tfrac{1}{2}p^{-1}-\psi(\tfrac{1}{2}p)]$ $\operatorname{Re} p>0$
(34)	$\sinh(2a^{\frac{1}{2}}t^{\frac{1}{2}})$	$\pi^{1/2}a^{1/2}p^{-3/2}e^{a/p}$ $\operatorname{Re} p>0$

Hyperbolic functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(35)	$\cosh(2a^{1/2}t^{1/2})$	$\pi^{1/2}a^{1/2}p^{-3/2}e^{a/p}\,\mathrm{Erf}(a^{1/2}p^{-1/2})+p^{-1}$ $\quad \mathrm{Re}\,p>0$
(36)	$t^{1/2}\sinh(2a^{1/2}t^{1/2})$	$\pi^{1/2}p^{-5/2}(\tfrac{1}{2}p+a)e^{a/p}$ $\times\,\mathrm{Erf}(a^{1/2}p^{-1/2})-a^{1/2}p^{-2}$ $\quad \mathrm{Re}\,p>0$
(37)	$t^{1/2}\cosh(2a^{1/2}t^{1/2})$	$\pi^{1/2}p^{-5/2}(\tfrac{1}{2}p+a)e^{a/p}$ $\quad \mathrm{Re}\,p>0$
(38)	$t^{-1/2}\sinh(2a^{1/2}t^{1/2})$	$\pi^{1/2}p^{-1/2}e^{a/p}\,\mathrm{Erf}(a^{1/2}p^{-1/2})$ $\quad \mathrm{Re}\,p>0$
(39)	$t^{-1/2}\cosh(2a^{1/2}t^{1/2})$	$\pi^{1/2}p^{-1/2}e^{a/p}$ $\quad \mathrm{Re}\,p>0$
(40)	$t^{-1/2}\sinh^2(a^{1/2}t^{1/2})$	$\tfrac{1}{2}\pi^{1/2}p^{-1/2}(e^{a/p}-1)$ $\quad \mathrm{Re}\,p>0$
(41)	$t^{-1/2}\cosh^2(a^{1/2}t^{1/2})$	$\tfrac{1}{2}\pi^{1/2}p^{-1/2}(e^{a/p}+1)$ $\quad \mathrm{Re}\,p>0$
(42)	$t^{-3/4}\sinh(2^{3/2}a^{1/2}t^{1/2})$	$\pi(2a)^{1/4}p^{-1/2}e^{a/p}I_{1/4}(a/p)$ $\quad \mathrm{Re}\,p>0$
(43)	$t^{-3/4}\cosh(2^{3/2}a^{1/2}t^{1/2})$	$\pi(2a)^{1/4}p^{-1/2}e^{a/p}I_{-1/4}(a/p)$ $\quad \mathrm{Re}\,p>0$
(44)	$t^{\nu-1}\sinh(2^{1/2}a^{1/2}t^{1/2})$ $\quad \mathrm{Re}\,\nu>-\tfrac{1}{2}$	$\Gamma(2\nu)(2p)^{-\nu}e^{\frac{1}{4}a/p}[D_{-2\nu}(-a^{1/2}p^{-1/2})$ $-D_{-2\nu}(a^{1/2}p^{-1/2})]$ $\quad \mathrm{Re}\,p>0$
(45)	$t^{\nu-1}\cosh(2^{1/2}a^{1/2}t^{1/2})$ $\quad \mathrm{Re}\,\nu>0$	$\Gamma(2\nu)(2p)^{-\nu}e^{\frac{1}{4}a/p}[D_{-2\nu}(-a^{1/2}p^{-1/2})$ $+D_{-2\nu}(a^{1/2}p^{-1/2})]$ $\quad \mathrm{Re}\,p>0$
(46)	$\begin{matrix}\sin\\ \cos\end{matrix}(bt)\begin{matrix}\tanh\\ \mathrm{ctnh}\end{matrix}(at^{1/2})$	see Mordell, L. J., 1920: *Mess. of Math.* 49, 65-72

Hyperbolic functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\, dt$
(47)	$\dfrac{\sinh[(2\nu+1)\pi^{1/2} t^{1/2}]}{\pi^{1/2} t^{1/2} \sinh(\pi^{1/2} t^{1/2})}$	For this and more generally for the Laplace transform of $\dfrac{a\cosh(At^{1/2})+b\cosh(Bt^{1/2})}{t^{1/2}\{a^2+2ab\cosh[(A+B)t^{1/2}]+b^2\}}$ see Mordell, L. J., 1933: *Acta Math.* 61, 323-360 and *Quart. J. Math.* 1920, 48, 329-342.
(48)	$(e^t-1)^{-1/2}\sinh[a(1-e^{-t})^{1/2}]$	$\pi^{1/2}\Gamma(p+\tfrac{1}{2})\, 2^p a^{-p} \mathbf{L}_p(a)$ $\quad \mathrm{Re}\, p > -\tfrac{1}{2}$
(49)	$(e^t-1)^{-1/2}\cosh[a(1-e^{-t})^{1/2}]$	$\pi^{1/2}\Gamma(p+\tfrac{1}{2})\, 2^p a^{-p} I_p(a)$ $\quad \mathrm{Re}\, p > -\tfrac{1}{2}$
(50)	$\tanh[\tfrac{1}{2}\pi(e^{2t}-1)^{1/2}]$	$2^{-p}\zeta(p-1)$ $\quad \mathrm{Re}\, p > 0$

4.10. Inverse hyperbolic functions

(1)	$\sinh^{-1} t$	$\tfrac{1}{2}\pi p^{-1}[\mathbf{H}_0(p)-Y_0(p)]$ $\quad \mathrm{Re}\, p > 0$
(2)	$0 \quad 0<t<b$ $\cosh^{-1}(t/b) \quad t>b$	$p^{-1} K_0(bp)$ $\quad \mathrm{Re}\, p > 0$
(3)	$\cosh^{-1}(1+t/a) \quad \lvert\arg a\rvert < \pi$	$p^{-1} e^{ap} K_0(ap)$ $\quad \mathrm{Re}\, p > 0$
(4)	$t \sinh^{-1} t$	$\pi\dfrac{\mathbf{H}_0(p)-Y_0(p)}{2p^2}+\pi\dfrac{\mathbf{H}_1(p)-Y_1(p)}{2p}-\dfrac{1}{p}$ $\quad \mathrm{Re}\, p > 0$
(5)	$\sinh[(2n+1)\sinh^{-1} t]$	$O_{2n+1}(p)$ $\quad \mathrm{Re}\, p > 0$

Inverse hyperbolic functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(6)	$\cosh(2n \sinh^{-1} t)$	$O_{2n}(p)$ $\quad$ Re $p>0$
(7)	$\sinh(\nu \sinh^{-1} t)$	$\nu p^{-1} S_{0,\nu}(p)$ $\quad$ Re $p>0$
(8)	$\cosh(\nu \sinh^{-1} t)$	$p^{-1} S_{1,\nu}(p)$ $\quad$ Re $p>0$
(9)	$0 \quad 0<t<b$ $\sinh[\nu \cosh^{-1}(t/b)] \quad t>b$	$\nu p^{-1} K_\nu(bp)$ $\quad$ Re $p>0$
(10)	$\sinh[\nu \cosh^{-1}(1+t/a)]$ $\lvert\arg a\rvert<\pi$	$\nu p^{-1} e^{ap} K_\nu(ap)$ $\quad$ Re $p>0$
(11)	$(1+t^2)^{-1/2} \exp(n \sinh^{-1} t)$	$\tfrac{1}{2}[S_n(p)-\pi \mathbf{E}_n(p)-\pi Y_n(p)]$ Re $p>0$
(12)	$(1+t^2)^{-1/2} \exp(-n \sinh^{-1} t)$	$\tfrac{1}{2}(-1)^{n+1}[S_n(p)+\pi \mathbf{E}_n(p)+\pi Y_n(p)]$ Re $p>0$
(13)	$(1+t^2)^{-1/2} \exp(-\nu \sinh^{-1} t)$	$\pi \csc(\nu\pi)[\mathbf{J}_\nu(p)-J_\nu(p)]$ Re $p>0$
(14)	$\dfrac{\sinh(\nu \sinh^{-1} t)}{(t^2+1)^{1/2}}$	$\nu S_{-1,\nu}(p)$ $\quad$ Re $p>0$
(15)	$0 \quad 0<t<b$ $\dfrac{\cosh(n \sinh^{-1} t)}{(t^2+1)^{1/2}} \quad t>b$	$S_n(\sinh^{-1} b,\, p)$ $\quad$ Re $p>0$
(16)	$\dfrac{\cosh(\nu \sinh^{-1} t)}{(t^2+1)^{1/2}}$	$S_{0,\nu}(p)$ $\quad$ Re $p>0$

Inverse hyperbolic functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(17)	$0 \qquad 0<t<b$ $\dfrac{\cosh(n \cosh^{-1} t)}{(t^2-1)^{1/2}} \qquad t>b>1$	$C_n(\cosh^{-1} b,\, p) \qquad \mathrm{Re}\, p>0$
(18)	$0 \qquad 0<t<b$ $\dfrac{\cosh[\nu \cosh^{-1}(t/b)]}{(t^2-b^2)^{1/2}} \qquad t>b$	$K_\nu(bp) \qquad \mathrm{Re}\, p>0$
(19)	$\dfrac{\cosh[\nu \cosh^{-1}(1+t/a)]}{(t^2+at)^{1/2}}$ $\lvert\arg a\rvert<\pi$	$e^{ap} K_\nu(ap) \qquad \mathrm{Re}\, p>0$
(20)	$(t^3+4a^2t)^{-1/2}$ $\times \exp(2\nu \sinh^{-1}(\tfrac{1}{2}t/a)]$ $\mathrm{Re}\, a>0$	$(\tfrac{1}{2}\pi)^{3/2} p^{1/2} [J_{\nu+1/4}(ap) J_{\nu-1/4}(ap)$ $+ Y_{\nu+1/4}(ap)\, Y_{\nu-1/4}(ap)]$ $\mathrm{Re}\, p>0$
(21)	$(t^3+4a^2t)^{-1/2}$ $\times \exp[-2\nu \sinh^{-1}(\tfrac{1}{2}t/a)]$ $\mathrm{Re}\, a>0$	$(\tfrac{1}{2}\pi)^{3/2} p^{1/2} [J_{\nu+1/4}(ap)\, Y_{\nu-1/4}(ap)$ $- J_{\nu-1/4}(ap)\, Y_{\nu+1/4}(ap)]$ $\mathrm{Re}\, p>0$
(22)	$(t^3+4a^2t)^{-1/2}\{\cos[(\nu+\tfrac{1}{4})\pi]$ $\times \exp[-2\nu \sinh^{-1}(\tfrac{1}{2}t/a)]$ $+\sin[(\nu+\tfrac{1}{4})\pi]$ $\times \exp[2\nu \sinh^{-1}(\tfrac{1}{2}t/a)]\}$ $\mathrm{Re}\, a>0$	$(\tfrac{1}{2}\pi)^{3/2} p^{1/2} [J_{1/4+\nu}(ap) J_{1/4-\nu}(ap)$ $+ Y_{1/4+\nu}(ap)\, Y_{1/4-\nu}(ap)]$ $\mathrm{Re}\, p>0$
(23)	$(t^3+4a^2t)^{-1/2}\{\sin[(\nu+\tfrac{1}{4})\pi]$ $\times \exp[-2\nu \sinh^{-1}(\tfrac{1}{2}t/a)]$ $-\cos[(\nu+\tfrac{1}{4})\pi]$ $\times \exp[2\nu \sinh^{-1}(\tfrac{1}{2}t/a)]\}$ $\mathrm{Re}\, a>0$	$(\tfrac{1}{2}\pi)^{3/2} p^{1/2} [J_{1/4+\nu}(ap)\, Y_{1/4-\nu}(ap)$ $- J_{1/4-\nu}(ap)\, Y_{1/4+\nu}(ap)] \qquad \mathrm{Re}\, p>0$

Inverse hyperbolic functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(24)	$\dfrac{\sinh[2\nu\sinh^{-1}(\frac{1}{2}t/a)]}{(t^3+4a^2t)^{\frac{1}{2}}}$ $\lvert\arg a\rvert<\pi$	$\dfrac{\pi^{3/2}p^{1/2}}{8i}[e^{\nu\pi i}H^{(1)}_{\frac{1}{2}+\nu}(ap)H^{(2)}_{\frac{1}{2}-\nu}(ap)$ $-e^{-\nu\pi i}H^{(1)}_{\frac{1}{2}-\nu}(ap)H^{(2)}_{\frac{1}{2}+\nu}(ap)]$ $\mathrm{Re}\,p>0$
(25)	$\dfrac{\cosh[2\nu\sinh^{-1}(\frac{1}{2}t/a)]}{(t^3+4a^2t)^{\frac{1}{2}}}$ $\lvert\arg a\rvert<\pi$	$\dfrac{\pi^{3/2}p^{1/2}}{8}[e^{\nu\pi i}H^{(1)}_{\frac{1}{2}+\nu}(ap)H^{(2)}_{\frac{1}{2}-\nu}(ap)$ $+e^{-\nu\pi i}H^{(1)}_{\frac{1}{2}-\nu}(ap)H^{(2)}_{\frac{1}{2}+\nu}(ap)]$ $\mathrm{Re}\,p>0$
(26)	$0 \qquad 0<t<2b$ $\dfrac{\cosh[2\nu\cosh^{-1}(\frac{1}{2}t/b)]}{(t^3-4b^2t)^{\frac{1}{2}}} \qquad t>2b$	$\dfrac{p^{\frac{1}{2}}}{(2\pi)^{\frac{1}{2}}}K_{\nu+\frac{1}{4}}(bp)K_{\nu-\frac{1}{4}}(bp)$ $\mathrm{Re}\,p>0$
(27)	$\dfrac{\cosh[2\nu\cosh^{-1}(1+\frac{1}{2}t/a)]}{[t(t+2a)(t+4a)]^{\frac{1}{2}}}$ $\lvert\arg a\rvert<\pi$	$\dfrac{p^{\frac{1}{2}}}{(2\pi)^{\frac{1}{2}}}e^{2ap}K_{\nu+\frac{1}{4}}(ap)K_{\nu-\frac{1}{4}}(ap)$ $\mathrm{Re}\,p>0$

4.11. Orthogonal polynomials

(1)	$P_n(t)$	Sum of powers with negative indices in the expansion, in ascending powers of p, of $(-1)^n(\frac{1}{2}\pi)^{\frac{1}{2}}p^{-\frac{1}{2}}I_{-n-\frac{1}{2}}(p)$ $\mathrm{Re}\,p>0$

Orthogonal polynomials (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(2)	$P_n(1-t)$	$e^{-p}p^n\left(\frac{1}{p}\frac{d}{dp}\right)^n\left(\frac{e^p}{p}\right)$ $=p^n\left(1+\frac{1}{2}\frac{d}{dp}\right)^n\left(\frac{1}{p^{n+1}}\right)$ $\mathrm{Re}\,p>0$
(3)	$P_n(e^{-t})$ $n\geq 2$	$\frac{(p-1)(p-2)(p-3)\cdots(p-n+1)}{(p+n)(p+n-2)\cdots(p-n+2)}$ $\mathrm{Re}\,p>0$
(4)	$P_{2n}(\cos t)$	$\frac{(p^2+1^2)(p^2+3^2)\cdots[p^2+(2n-1)^2]}{p(p^2+2^2)(p^2+4^2)\cdots[p^2+(2n)^2]}$ $\mathrm{Re}\,p>0$
(5)	$P_{2n+1}(\cos t)$	$\frac{p(p^2+2^2)(p^2+4^2)\cdots[p^2+(2n)^2]}{(p^2+1^2)(p^2+3^2)\cdots[p^2+(2n+1)^2]}$ $\mathrm{Re}\,p>0$
(6)	$P_{2n}(\cosh t)$	$\frac{(p^2-1^2)(p^2-3^2)\cdots[p^2-(2n-1)^2]}{p(p^2-2^2)(p^2-4^2)\cdots[p^2-(2n)^2]}$ $\mathrm{Re}\,p>2n$
(7)	$P_{2n+1}(\cosh t)$	$\frac{p(p^2-2^2)(p^2-4^2)\cdots[p^2-(2n)^2]}{(p^2-1^2)(p^2-3^2)\cdots[p^2-(2n+1)^2]}$ $\mathrm{Re}\,p>2n+1$
(8)	$2^\nu i^n(n+\nu)\Gamma(\nu)\,C_n^\nu(-it)$	$A_{n,\nu}(p)$ $\mathrm{Re}\,p>0$
(9)	$[t(2a-t)]^{\nu-1/2}\,C_n^\nu(t/a-1)$ $0<t<2a$ 0 $t>2a$ $\mathrm{Re}\,\nu>-1/2$	$(-1)^n\frac{\pi\Gamma(2\nu+n)}{n!\Gamma(\nu)}\left(\frac{a}{2p}\right)^\nu e^{-ap}$ $\times I_{\nu+n}(ap)$

Orthogonal polynomials (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(10)	$P_n^{(\alpha,\beta)}(t)$	$2(1-2\mu)_n\, A_{\kappa,\mu,n}(2p)$ Re $p>0$ $\kappa = \tfrac{1}{2}\alpha - \tfrac{1}{2}\beta$ $\mu = \tfrac{1}{2}\alpha + \tfrac{1}{2}\beta + \tfrac{1}{2} + n$
(11)	$t^{\alpha-1}\,\mathrm{He}_n(t)$ Re $\alpha > \begin{cases} 0 & \text{if } n \text{ is even} \\ -1 & \text{if } n \text{ is odd} \end{cases}$	$\sum_{m=0}^{[n/2]} \frac{n!\,\Gamma(\alpha+n-2m)}{m!\,(n-2m)!} (-\tfrac{1}{2})^m p^{2m-\alpha-n}$ Re $p>0$ $[n/2] = \begin{cases} \tfrac{1}{2}n & \text{if } n \text{ is even} \\ \tfrac{1}{2}n - \tfrac{1}{2} & \text{if } n \text{ is odd} \end{cases}$
(12)	$\mathrm{He}_{2n+1}(t^{1/2})$	$\pi^{1/2}\, 2^{-n-1} \frac{(2n+1)!}{n!} \frac{(\tfrac{1}{2}-p)^n}{p^{n+3/2}}$ Re $p>0$
(13)	$t^{-1/2}\,\mathrm{He}_{2n}(t^{1/2})$	$\pi^{1/2}\, 2^{-n} \frac{(2n)!}{n!} \frac{(\tfrac{1}{2}-p)^n}{p^{n+1/2}}$ Re $p>0$
(14)	$t^{\alpha-\frac{1}{2}n-1}\,\mathrm{He}_n(t^{1/2})$ Re $\alpha > \begin{cases} \tfrac{1}{2}n & \text{if } n \text{ is even} \\ \tfrac{1}{2}n - \tfrac{1}{2} & \text{if } n \text{ is odd} \end{cases}$	$\Gamma(\alpha)\, p^{-\alpha}\, {}_2F_1(-\tfrac{1}{2}n, \tfrac{1}{2}-\tfrac{1}{2}n; 1-\alpha; 2p)$ If α is an integer, take the first $1+[n/2]$ terms of the series. Re $p>0$
(15)	$e^{\beta t}\,\mathrm{He}_{2n+1}[2^{1/2}(\alpha-\beta)^{1/2} t^{1/2}]$	$(-2)^{-n}(\tfrac{1}{2}\pi)^{1/2}(\alpha-\beta)^{1/2}$ $\times \frac{(2n+1)!}{n!} \frac{(p-\alpha)^n}{(p-\beta)^{n+3/2}}$ Re $p >$ Re β
(16)	$e^{\beta t} t^{-1/2}\,\mathrm{He}_{2n}[2^{1/2}(\alpha-\beta)^{1/2} t^{1/2}]$	$(-2)^{-n}\pi^{1/2} \frac{(2n)!}{n!} \frac{(p-\alpha)^n}{(p-\beta)^{n+1/2}}$ Re $p >$ Re β

Orthogonal polynomials (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(17)	$t^{-1/2}\left[\mathrm{He}_n\left(\dfrac{x+t^{1/2}}{\lambda}\right)+\mathrm{He}_n\left(\dfrac{x-t^{1/2}}{\lambda}\right)\right]$	$(2\pi/p)^{1/2}(1-\tfrac{1}{2}\lambda^{-2}p^{-1})^{1/2 n} \times \mathrm{He}_n\left[\dfrac{x}{(\lambda^2-\frac{1}{2}p^{-1})^{1/2}}\right]$ $\mathrm{Re}\,p>0$
(18)	$t^{-1/2(n+1)}e^{-1/2\alpha/t}\mathrm{He}_n(\alpha^{1/2}t^{-1/2})$ $\mathrm{Re}\,\alpha>0$	$2^{1/2 n}\pi^{1/2}p^{1/2 n-1/2}e^{-(2\alpha p)^{1/2}}$ $\mathrm{Re}\,p>0$
(19)	$t^{-1/2}\mathrm{He}_{2n}(2^{1/2}\alpha^{1/2}t^{1/2}) \times \mathrm{He}_{2m}(2^{1/2}\beta^{1/2}t^{1/2})$	$\dfrac{\pi^{1/2}(2m+2n)!}{(-2)^{m+n}(m+n)!}\dfrac{(p-\alpha)^n(p-\beta)^m}{p^{m+n+1/2}} \times {}_2F_1\left[-m,-n;-m-n+\tfrac{1}{2};\dfrac{p(p-\alpha-\beta)}{(p-\alpha)(p-\beta)}\right]$ $\mathrm{Re}\,p>0$
(20)	$(\alpha\beta t)^{-1/2}\mathrm{He}_{2n+1}(2^{1/2}\alpha^{1/2}t^{1/2}) \times \mathrm{He}_{2m+1}(2^{1/2}\beta^{1/2}t^{1/2})$	$\dfrac{-\pi^{1/2}(2m+2n+2)!}{(-2)^{m+n+1}(m+n+1)!}\dfrac{(p-\alpha)^n(p-\beta)^m}{p^{m+n+3/2}} \times {}_2F_1\left[-m,-n;-m-n-\tfrac{1}{2};\dfrac{p(p-\alpha-\beta)}{(p-\alpha)(p-\beta)}\right]$ $\mathrm{Re}\,p>0$
(21)	$t^{-1/2}e^{-(\alpha+\beta)t}\mathrm{He}_n(2\alpha^{1/2}t^{1/2}) \times \mathrm{He}_n(2\beta^{1/2}t^{1/2})$	$\pi^{1/2}n!\,\dfrac{(\alpha+\beta-p)^{1/2 n}}{(\alpha+\beta+p)^{1/2 n+1/2}} \times P_n\left\{\dfrac{2\alpha^{1/2}\beta^{1/2}}{[(\alpha+\beta)^2-p^2]^{1/2}}\right\}$ $\mathrm{Re}(\alpha+\beta+p)>0$
(22)	$t^{-1/2}\left[\mathrm{He}_m\left(\dfrac{x+t^{1/2}}{\lambda}\right)\mathrm{He}_n\left(\dfrac{y+t^{1/2}}{\mu}\right)+\mathrm{He}_m\left(\dfrac{x-t^{1/2}}{\lambda}\right)\mathrm{He}_n\left(\dfrac{y-t^{1/2}}{\mu}\right)\right]$	$\dfrac{2\pi^{1/2}\lambda^{-m}\mu^{-n}}{(2p)^{1/2 m+1/2 n+1/2}}\sum_{k=0}^{\min(m,n)}\left\{\binom{m}{k}\binom{n}{k}k! \times(2\lambda^2p-1)^{1/2 m+1/2 k}(2\mu^2p-1)^{1/2 n+1/2 k} \times \mathrm{He}_{m-k}\left[\dfrac{x}{(\lambda^2-\frac{1}{2}p^{-1})^{1/2}}\right] \times \mathrm{He}_{n-k}\left[\dfrac{y}{(\mu^2-\frac{1}{2}p^{-1})^{1/2}}\right]\right\}$ $\mathrm{Re}\,p>0$

Orthogonal polynomials (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(23)	$(e^t-1)^{-\frac{1}{2}}\mathrm{He}_{2n}[x^{\frac{1}{2}}(1-e^{-t})^{\frac{1}{2}}]$	$\dfrac{(-2)^n \pi^{\frac{1}{2}}(2n)!\,\Gamma(p+\frac{1}{2})}{\Gamma(n+p+1)} L_n^p(\tfrac{1}{2}x)$ Re $p>-\frac{1}{2}$
(24)	$\mathrm{He}_{2n+1}[x^{\frac{1}{2}}(1-e^{-t})^{\frac{1}{2}}]$	$\dfrac{(-2)^n \pi^{\frac{1}{2}}(2n+1)!\,\Gamma(p)\,x^{\frac{1}{2}}}{\Gamma(n+p+3/2)} L_n^p(\tfrac{1}{2}x)$ Re $p>0$
(25)	$L_n(t)$	$(p-1)^n p^{-n-1}$ Re $p>0$
(26)	$t^n L_n(t)$	$n!\,p^{-n-1} P_n(1-2p^{-1})$ Re $p>0$
(27)	$L_n^\alpha(t)$	$\displaystyle\sum_{m=0}^{n} \binom{\alpha+m-1}{m} \frac{(p-1)^{n-m}}{p^{n-m+1}}$ Re $p>0$
(28)	$t^\alpha L_n^\alpha(t)$ Re $\alpha>-1$	$\dfrac{\Gamma(\alpha+n+1)}{n!}\dfrac{(p-1)^n}{p^{\alpha+n+1}}$ Re $p>0$
(29)	$t^\beta L_n^\alpha(t)$ Re $\beta>-1$	$\dfrac{\Gamma(\beta+n+1)}{n!}\dfrac{(p-1)^n}{p^{\beta+n+1}}$ $\times\,{}_2F_1[-n,\alpha-\beta;-\beta-n;p/(p-1)]$ Re $p>0$
(30)	$t^{2\alpha}[L_n^\alpha(t)]^2$ Re $\alpha>-\frac{1}{2}$	$\dfrac{2^{2\alpha}\Gamma(\alpha+\frac{1}{2})\Gamma(n+\frac{1}{2})}{\pi(n!)^2 p^{2\alpha+1}}$ $\times\,{}_2F_1[-n,\alpha+\frac{1}{2};\frac{1}{2}-n;(1-2/p)^2]$ Re $p>0$

Orthogonal polynomials (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(31)	$t^{\alpha} e^{\lambda t} L_n^{\alpha}(\kappa t)$ Re $\alpha>-1$	$\dfrac{\Gamma(\alpha+n+1)}{n!}\dfrac{(p-\kappa-\lambda)^n}{(p-\lambda)^{\alpha+n+1}}$ Re $(p-\lambda)>0$
(32)	$e^{-t}\sum_{m=0}^{n} a_{mn} L_m(2t)$ where a_{mn} is given by $P_n(z)=\sum_{m=0}^{n} a_{mn} z^m$	$\dfrac{1}{p+1} P_n\left(\dfrac{p-1}{p+1}\right)$ Re $p>-1$
(33)	$t^{-n} e^{-\lambda/t} L_n^{\alpha}(\lambda/t)$ Re $\lambda>0$	$(-1)^n(2/n!)\,\lambda^{-\frac{1}{2}\alpha} p^{\frac{1}{2}\alpha+n} K_{\alpha}(2\lambda^{\frac{1}{2}} p^{\frac{1}{2}})$ Re $p>0$
(34)	$L_n(\lambda t) L_n(\kappa t)$	$\dfrac{(p-\lambda-\kappa)^n}{p^{n+1}} P_n\left[\dfrac{p^2-(\lambda+\kappa)p+2\lambda\kappa}{p(p-\lambda-\kappa)}\right]$ Re $p>0$
(35)	$t^{\alpha} L_n^{\alpha}(\lambda t)\, L_m^{\alpha}(\kappa t)$ Re $\alpha>-1$	$\dfrac{\Gamma(m+n+\alpha+1)}{m!\,n!}\dfrac{(p-\lambda)^n(p-\kappa)^m}{p^{m+n+\alpha+1}} \times {}_2F_1\left[-m,-n;-m-n-\alpha;\dfrac{p(p-\lambda-\kappa)}{(p-\lambda)(p-\kappa)}\right]$ Re $p>0$
(36)	$t^{2\alpha} L_n^{\alpha}(\lambda t)\, L_n^{\alpha}(\kappa t)$ Re $\alpha>-\frac{1}{2}$	$\dfrac{\Gamma(2\alpha+1)\Gamma(n+\alpha+1)}{n!\,p^{2\alpha+1}} \times \sum_{r=0}^{\infty}\left\{\dfrac{(-1)^r[1-(\lambda+\kappa)/(2p)]^{n-r}}{r!\,\Gamma(\alpha-r+1)} \times C_{n+r}^{\alpha+\frac{1}{2}}\left[\dfrac{p^2-(\lambda+\kappa)p+2\lambda\kappa}{p(p-\lambda-\kappa)}\right]\right\}$ Re $p>0$

Orthogonal polynomials (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(37)	$t^m\, p_n(m,\,t)$	$m\,!\,(p-1)^n\, p^{-m-1}$ $\quad$ $\operatorname{Re} p>0$
(38)	$t^{\alpha-1}\, p_n(m,\,t)$ $\quad$ $\operatorname{Re}\alpha>\min(n,m)$	$\dfrac{m\,!\,\Gamma(\alpha-n)}{(m-n)\,!\,p^{\alpha-n}}$ $\times {}_2F_1(-n,\,\alpha-n;\,m-n+1;\,r/p)$ $\quad$ $\operatorname{Re} p>0$

4.12. Gamma function, error function, exponential integral and related functions

(1)	$\binom{t}{n} t^{\nu-1}$ $\quad$ $\operatorname{Re}\nu>0$	$\dfrac{\Gamma(\nu)}{p^\nu}\,\Phi_n\left(\nu,\,\dfrac{1}{p}\right)$ $\quad$ $\operatorname{Re} p>0$ where $\sum_0^\infty h^n\Phi_n(\nu,\,z)=[1-z\log(h+1)]^{-\nu}$
(2)	$\operatorname{Erf}(\tfrac{1}{2}t/a)$ $\quad$ $\lvert\arg a\rvert<\tfrac{1}{4}\pi$	$p^{-1}e^{a^2p^2}\operatorname{Erfc}(ap)$ $\quad$ $\operatorname{Re} p>0$
(3)	$e^{-a^2t^2}\operatorname{Erf}(iat)$ $\quad$ $\lvert\arg a\rvert<\tfrac{1}{4}\pi$	$(2ai\pi^{1/2})^{-1}\,e^{\frac{1}{4}a^{-2}p^2}\operatorname{Ei}(-\tfrac{1}{4}a^{-2}p^2)$ $\quad$ $\operatorname{Re} p>0$
(4)	$\operatorname{Erf}(a^{1/2}t^{1/2})$	$a^{1/2}p^{-1}(p+a)^{-1/2}$ $\quad$ $\operatorname{Re} p>0,\quad -\operatorname{Re} a$
(5)	$e^{at}\operatorname{Erf}(a^{1/2}t^{1/2})$	$a^{1/2}p^{-1/2}(p-a)^{-1}$ $\quad$ $\operatorname{Re} p>0,\quad \operatorname{Re} a$
(6)	$\operatorname{Erf}(\tfrac{1}{2}a^{1/2}t^{-1/2})$ $\quad$ $\operatorname{Re} a>0$	$p^{-1}(1-e^{-a^{1/2}p^{1/2}})$ $\quad$ $\operatorname{Re} p>0$

Gamma function etc. (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(7)	$\mathrm{Erfc}\,(\tfrac{1}{2}t/a)$ $\quad |\arg a| < \tfrac{1}{4}\pi$	$p^{-1}[1-e^{a^2p^2}\,\mathrm{Erfc}\,(ap)]$
(8)	$e^{-a^2t^2}\,\mathrm{Erfc}\,(i\,at)$	$\tfrac{1}{2}\pi^{1/2}a^{-1}e^{1/4\,a^{-2}p^2}[\mathrm{Erfc}\,(\tfrac{1}{2}a^{-1}p)$ $+i\pi^{-1}\,\mathrm{Ei}\,(-\tfrac{1}{4}a^{-2}p^2)]$ $\quad \mathrm{Re}\,p>0$
(9)	$\mathrm{Erfc}\,(a^{1/2}t^{1/2})$	$p^{-1}(p+a)^{-1/2}[(p+a)^{1/2}-a^{1/2}]$ $\quad \mathrm{Re}\,p>-\mathrm{Re}\,a$
(10)	$e^{at}\,\mathrm{Erfc}\,(a^{1/2}t^{1/2})$	$p^{-1/2}(p^{1/2}+a^{1/2})^{-1}$ $\quad \mathrm{Re}\,p>0$
(11)	$\mathrm{Erfc}\,(\tfrac{1}{2}a^{1/2}t^{-1/2})$ $\quad \mathrm{Re}\,a>0$	$p^{-1}e^{-a^{1/2}p^{1/2}}$ $\quad \mathrm{Re}\,p>0$
(12)	$e^{at}\,\mathrm{Erfc}\,(a^{1/2}t^{1/2}+\tfrac{1}{2}\beta^{1/2}t^{-1/2})$ $\quad \mathrm{Re}\,\beta>0$	$p^{-1/2}(p^{1/2}+a^{1/2})^{-1}$ $\times\exp(-a^{1/2}\beta^{1/2}-\beta^{1/2}p^{1/2})$ $\quad \mathrm{Re}\,p>0$
(13)	$S(t)$	$\dfrac{[(p^2+1)^{1/2}-p]^{1/2}}{2p(p^2+1)^{1/2}}$ $\quad \mathrm{Re}\,p>0$
(14)	$C(t)$	$\dfrac{[(p^2+1)^{1/2}-p]^{-1/2}}{2p(p^2+1)^{1/2}}$ $\quad \mathrm{Re}\,p>0$
(15)	$S(t^{1/2})$	$p^{-1}[\tfrac{1}{2}-\cos(\tfrac{1}{4}p^2)\,C(\tfrac{1}{4}p^2)$ $-\sin(\tfrac{1}{4}p^2)\,S(\tfrac{1}{4}p^2)]$
(16)	$C(t^{1/2})$	$p^{-1}[\tfrac{1}{2}\cos(\tfrac{1}{2}p^2)-\cos(\tfrac{1}{4}p^2)S(\tfrac{1}{4}p^2)$ $+\sin(\tfrac{1}{4}p^2)\,C(\tfrac{1}{4}p^2)]$
(17)	$\mathrm{Si}\,(t)$	$p^{-1}\,\mathrm{ctn}^{-1}\,p$ $\quad \mathrm{Re}\,p>0$
(18)	$\mathrm{si}\,(t)$	$-p^{-1}\tan^{-1}p$ $\quad \mathrm{Re}\,p>0$

Gamma function etc. (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(19)	$\mathrm{Ci}(t)=-\mathrm{ci}(t)$	$\frac{1}{2}p^{-1}\log(p^2+1)$ $\quad$ $\mathrm{Re}\,p>0$
(20)	$\cos t\,\mathrm{Si}(t)-\sin t\,\mathrm{Ci}(t)$	$(p^2+1)^{-1}\log p$ $\quad$ $\mathrm{Re}\,p>0$
(21)	$\sin t\,\mathrm{Si}(t)+\cos t\,\mathrm{Ci}(t)$	$-(p^2+1)^{-1}\,p\log p$ $\quad$ $\mathrm{Re}\,p>0$
(22)	$\mathrm{Si}(t^2)$	$\pi[\frac{1}{2}-C(\frac{1}{4}p^2)]^2+\pi[\frac{1}{2}-S(\frac{1}{4}p^2)]^2$ $\quad$ $\mathrm{Re}\,p>0$
(23)	$\overline{\mathrm{Ei}}(t)$	$-p^{-1}\log(p-1)$ $\quad$ $\mathrm{Re}\,p>1$
(24)	$\mathrm{Ei}(-t)$	$-p^{-1}\log(p+1)$ $\quad$ $\mathrm{Re}\,p>0$
(25)	$t^{-\frac{1}{2}}\,\mathrm{Ei}(-t)$	$-2\pi^{\frac{1}{2}}p^{-\frac{1}{2}}\log[p^{\frac{1}{2}}+(p+1)^{\frac{1}{2}}]$ $\quad$ $\mathrm{Re}\,p>0$
(26)	$\sin(at)\,\mathrm{Ei}(-t)$	$-(p^2+a^2)^{-1}\{\frac{1}{2}a\log[(p+1)^2+a^2]$ $-p\tan^{-1}[a/(p+1)]\}$ $\quad$ $\mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(27)	$\cos(at)\,\mathrm{Ei}(-t)$	$-(p^2+a^2)^{-1}\{\frac{1}{2}p\log[(p+1)^2+a^2]$ $+a\tan^{-1}[a/(p+1)]\}$ $\quad$ $\mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(28)	$\mathrm{li}(e^t)$	$-p^{-1}\log(p-1)$ $\quad$ $\mathrm{Re}\,p>1$
(29)	$\mathrm{li}(e^{-t})$	$-p^{-1}\log(p+1)$ $\quad$ $\mathrm{Re}\,p>0$
(30)	$\Gamma(\nu,at)$ $\quad$ $\mathrm{Re}\,\nu>-1$	$\Gamma(\nu)p^{-1}[1-(1+p/a)^{-\nu}]$ $\quad$ $\mathrm{Re}\,p>-\mathrm{Re}\,a$
(31)	$e^{at}\Gamma(\nu,at)$ $\quad$ $\mathrm{Re}\,\nu>-1$	$\Gamma(\nu)(p-a)^{-1}(1-a^{\nu}p^{-\nu})$ $\quad$ $\mathrm{Re}\,p>0$

Gamma function etc. (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(32)	$\Gamma(\nu, a/t)$ $\quad\lvert\arg a\rvert < \frac{1}{2}\pi$	$2a^{\frac{1}{2}\nu} p^{\frac{1}{2}\nu-1} K_\nu(2a^{\frac{1}{2}}p^{\frac{1}{2}})$ $\quad\mathrm{Re}\, p > 0$
(33)	$t^{\mu-1} e^{a/t}\Gamma(\nu, a/t)$ $\mathrm{Re}(\nu-\mu) < 1, \quad \lvert\arg a\rvert < \pi$	$2^{2+\mu-2\nu}\Gamma(1+\mu-\nu)(a/p)^{\frac{1}{2}\mu}$ $\times S_{2\nu-\mu-1,\,\mu}(2a^{\frac{1}{2}}p^{\frac{1}{2}})$ $\quad\mathrm{Re}\, p > 0$
(34)	$e^{\beta t}\gamma(\nu, at)$ $\quad\mathrm{Re}\,\nu > -1$	$\Gamma(\nu)\, a^\nu (p-\beta)^{-1}(p+a-\beta)^{-\nu}$ $\mathrm{Re}\, p > \mathrm{Re}\,\beta, \quad \mathrm{Re}(\beta-a)$
(35)	$\gamma(\frac{1}{4}, 2^{-3}a^{-2}t^2)$ $\quad\lvert\arg a\rvert < \frac{1}{4}\pi$	$2^{\frac{3}{4}} a^{\frac{1}{2}} p^{-\frac{1}{2}} e^{a^2p^2} K_{\frac{1}{4}}(a^2p^2)$ $\mathrm{Re}\, p > 0$
(36)	$\gamma(\nu, 2^{-3}a^{-2}t^2)$ $\lvert\arg a\rvert < \frac{1}{4}\pi, \quad \mathrm{Re}\,\nu > -\frac{1}{2}$	$2^{-\nu-1}\Gamma(2\nu)\, p^{-1} e^{a^2p^2} D_{-2\nu}(2ap)$ $\mathrm{Re}\, p > 0$
(37)	$e^{-\frac{1}{4}t^2/a}\gamma(\nu, \frac{1}{4}e^{i\pi}t^2/a)$ $\lvert\arg a\rvert < \frac{1}{2}\pi, \quad \mathrm{Re}\,\nu > -\frac{1}{2}$	$2^{1-2\nu}\Gamma(2\nu)\, a^{\frac{1}{2}} e^{ap^2+\nu\pi i}$ $\times\Gamma(\frac{1}{2}-\nu, ap^2)$ $\quad\mathrm{Re}\, p > 0$

4.13. Legendre functions

(1)	$[t(1+t)]^{-\frac{1}{2}\mu} P_\nu^\mu(1+2t)$ $\mathrm{Re}\,\mu < 1$	$\pi^{-\frac{1}{2}} p^{\mu-\frac{1}{2}} e^{\frac{1}{2}p} K_{\nu+\frac{1}{2}}(\frac{1}{2}p)$ $\mathrm{Re}\, p > 0$
(2)	$(1+t^{-1})^{\frac{1}{2}\mu} P_\nu^\mu(1+2t)$ $\mathrm{Re}\,\mu < 1$	$p^{-1} e^{\frac{1}{2}p} W_{\mu,\,\nu+\frac{1}{2}}(p)$ $\quad\mathrm{Re}\, p > 0$
(3)	$t^{\lambda+\frac{1}{2}\mu-1}(t+2)^{\frac{1}{2}\mu} P_\nu^{-\mu}(1+t)$ $\mathrm{Re}(\lambda+\mu) > 0$	$-\pi^{-1}\sin(\nu\pi)\, p^{-\lambda-\mu}$ $\times E(-\nu, \nu+1, \lambda+\mu : \mu+1 : 2p)$ $\mathrm{Re}\, p > 0$
(4)	$t^{\lambda-\frac{1}{2}\mu-1}(t+2)^{-\frac{1}{2}\mu} P_\nu^{-\mu}(1+t)$ $\mathrm{Re}\,\lambda > 0$	$\dfrac{E(\mu+\nu+1, \mu-\nu, \lambda : \mu+1 : 2p)}{2^\mu p^\lambda \Gamma(\mu+\nu+1)\Gamma(\mu-\nu)}$ $\mathrm{Re}\, p > 0$

Legendre functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(5)	$(\alpha+t)^{\frac12\nu}(\beta+t)^{\frac12\nu}$ $\times P_\nu[2\alpha^{-1}\beta^{-1}(\alpha+t)(\beta+t)-1]$ $\lvert\arg\alpha\rvert<\pi, \quad \lvert\arg\beta\rvert<\pi$	$\pi^{-1}(\alpha\beta)^{\frac12\nu+\frac12} e^{\frac12(\alpha+\beta)p}$ $\times K_{\nu+\frac12}(\tfrac12\alpha p)\, K_{\nu+\frac12}(\tfrac12\beta p)$ $\lvert\arg(\alpha p)\rvert<\pi, \quad \lvert\arg(\beta p)\rvert<\pi$
(6)	$(1+t)^{-1} P_\nu[2(1+t)^{-2}-1]$	$p^{-1} e^{p} W_{\nu+\frac12,0}(p)\, W_{-\nu-\frac12,0}(p)$ $\mathrm{Re}\, p>0$
(7)	$t^{-\frac12\mu} P_\nu^\mu[(1+t)^{\frac12}]$ $\mathrm{Re}\,\mu<1$	$2^\mu p^{\mu/2-5/4} e^{p/2} W_{\frac12\mu+\frac14,\frac12\nu+\frac14}(p)$ $\mathrm{Re}\, p>0$
(8)	$t^{-\frac12\mu}(1+t)^{-\frac12} P_\nu^\mu[(1+t)^{\frac12}]$ $\mathrm{Re}\,\mu<1$	$2^\mu p^{\frac12\mu-\frac34} p^{\frac12 p} W_{\frac12\mu-\frac14,\frac12\nu+\frac14}(p)$ $\mathrm{Re}\, p>0$
(9)	$t^{\frac12} P_\nu^{\frac14}[(1+t^2)^{\frac12}] P_\nu^{-\frac14}[(1+t^2)^{\frac12}]$	$\tfrac12(\tfrac12\pi/p)^{\frac12} H_{\nu+\frac12}^{(1)}(\tfrac12 p)\, H_{\nu+\frac12}^{(2)}(\tfrac12 p)$ $\mathrm{Re}\, p>0$
(10)	$(\alpha+t)^{-\frac12\nu-\frac12}(\beta+t)^{\frac12\nu}$ $\times[-1-(\alpha+\beta)/t]^{\frac12\mu}$ $\times P_\nu^\mu[\alpha^{\frac12}\beta^{\frac12}(\alpha+t)^{-\frac12}(\beta+t)^{-\frac12}]$ $\lvert\arg\alpha\rvert<\pi, \quad \lvert\arg\beta\rvert<\pi$ $\mathrm{Re}\,\mu<1$	$2^{\frac12} p^{-\frac12} e^{\frac12(\alpha+\beta)p} D_{\mu-\nu-1}(2^{\frac12}\alpha^{\frac12}p^{\frac12})$ $\times D_{\mu+\nu}(2^{\frac12}\beta^{\frac12}p^{\frac12})$ $\mathrm{Re}\, p>0$ $\lvert\arg(\alpha p)\rvert<\pi, \quad \lvert\arg(\beta p)\rvert<\pi$
(11)	$(1-e^{-2t})^{\frac12\mu} P_\nu^{-\mu}(e^t)$ $\mathrm{Re}\,\mu>-1$	$\dfrac{2^{p-1}\Gamma(\tfrac12 p+\tfrac12\nu+\tfrac12)\Gamma(\tfrac12 p-\tfrac12\nu)}{\pi^{\frac12}\Gamma(p+\mu+1)}$ $\mathrm{Re}\, p>\mathrm{Re}\,\nu, \quad -1-\mathrm{Re}\,\nu$
(12)	$\left[(e^t-1)\left(\dfrac{ae^t}{a-2}-1\right)\right]^{\frac12\mu}$ $\times P_\nu^{-\mu}(ae^t-a+1)$ $\mathrm{Re}\, a>0, \quad \mathrm{Re}\,\mu>-1$	$\dfrac{\Gamma(p-\mu+\nu+1)\Gamma(p-\nu-\mu)}{\Gamma(p+1)}\left(\dfrac{a}{a-2}\right)^{\frac12 p}$ $\times P_\nu^{\mu-p}(a-1)$ $\mathrm{Re}\, p>\mathrm{Re}(\mu-\nu)-1, \quad \mathrm{Re}(\mu+\nu)$

Legendre functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(13)	$(1-z^2+z^2e^{-t})^{\mu}$ $\times\{P_{2\nu}^{2\mu}[z(1-e^{-t})^{1/2}]$ $-P_{2\nu}^{2\mu}[-z(1-e^{-t})^{1/2}]\}\qquad \lvert z\rvert<1$	$\dfrac{-2^{2\mu+1}\pi z\,\Gamma(p)}{\Gamma(-\mu-\nu)\Gamma(\frac{1}{2}-\mu+\nu)\Gamma(p+3/2)}$ $\times{}_2F_1(1/2-\mu-\nu,\nu-\mu+1;p+3/2;z^2)$ $\mathrm{Re}\,p>0$
(14)	$(1-e^{-t})^{-1/2}(1-z^2+z^2e^{-t})^{\mu}$ $\times\{P_{2\nu}^{2\mu}[z(1-e^{-t})^{1/2}]$ $+P_{2\nu}^{2\mu}[-z(1-e^{-t})^{1/2}]\}\quad \lvert z\rvert<1$	$\dfrac{2^{2\mu+1}\pi\,\Gamma(p)}{\Gamma(\frac{1}{2}-\mu-\nu)\Gamma(1-\mu+\nu)\Gamma(p+\frac{1}{2})}$ $\times{}_2F_1(-\mu-\nu,\frac{1}{2}-\mu+\nu;p+\frac{1}{2};z^2)$ $\mathrm{Re}\,p>0$
(15)	$\sinh^{2\mu}(\frac{1}{2}t)P_{2n}^{-2\mu}[\cosh(\frac{1}{2}t)]$ $\mathrm{Re}\,\mu>-\frac{1}{4}$	$\dfrac{\Gamma(2\mu+\frac{1}{2})\Gamma(p-n-\mu)\Gamma(p+n-\mu+\frac{1}{2})}{4^{\mu}\pi^{1/2}\Gamma(p+n+\mu+1)\Gamma(p-n+\mu+\frac{1}{2})}$ $\mathrm{Re}\,p>n+\mathrm{Re}\,\mu$
(16)	$t^{\lambda+\frac{1}{2}\mu-1}(t+2)^{\frac{1}{2}\mu}Q_{\nu}^{\mu}(1+t)$ $\mathrm{Re}\,\lambda>0,\quad \mathrm{Re}(\lambda+\mu)>0$	$\dfrac{\Gamma(\nu+\mu+1)}{\Gamma(\nu-\mu+1)}\Big\{\dfrac{\sin(\nu\pi)}{2p^{\lambda+\mu}\sin(\mu\pi)}$ $\times E(-\nu,\nu+1,\lambda+\mu:\mu+1:2p)$ $-\dfrac{\sin[(\mu+\nu)\pi)}{2^{1-\mu}p^{\lambda}\sin(\mu\pi)}$ $\times E(\nu-\mu+1,-\nu-\mu,\lambda:1-\mu:2p)\Big\}$ $\mathrm{Re}\,p>0$
(17)	$t^{\lambda-\frac{1}{2}\mu-1}(t+2)^{\frac{1}{2}\mu}Q_{\nu}^{\mu}(1+t)$ $\mathrm{Re}\,\lambda>0,\quad \mathrm{Re}(\lambda-\mu)>0$	$-\dfrac{\sin(\nu\pi)}{2p^{\lambda-\mu}\sin(\mu\pi)}$ $\times E(-\nu,\nu+1,\lambda-\mu:1-\mu:2p)$ $-\dfrac{\sin[(\mu-\nu)\pi]}{2^{1+\mu}p^{\lambda}\sin(\mu\pi)}$ $\times E(\mu+\nu+1,\mu-\nu,\lambda:1+\mu:2p)$ $\mathrm{Re}\,p>0$

4.14. Bessel functions of arguments kt and $kt^{1/2}$

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\, dt$
(1)	$J_\nu(at)$ $\quad$ $\mathrm{Re}\,\nu > -1$	$r^{-1}(a/R)^\nu = r^{-1} e^{-\nu \sinh^{-1}(p/a)}$ $\quad$ $\mathrm{Re}\,p > \lvert \mathrm{Im}\,a \rvert$
(2)	$t J_\nu(at)$ $\quad$ $\mathrm{Re}\,\nu > -2$	$r^{-3}(p+\nu r)(a/R)^\nu$ $\quad$ $\mathrm{Re}\,p > \lvert \mathrm{Im}\,a \rvert$
(3)	$t^2 J_\nu(at)$ $\quad$ $\mathrm{Re}\,\nu > -3$	$\left(\dfrac{\nu^2-1}{r^3} + 3p\,\dfrac{p+\nu r}{r^5}\right)\left(\dfrac{a}{R}\right)^\nu$ $\quad$ $\mathrm{Re}\,p > \lvert \mathrm{Im}\,a \rvert$
(4)	$t^n J_n(at)$	$1\cdot 3\cdot 5\cdot \cdots \cdot (2n-1)\, a^n r^{-2n-1}$ $\quad$ $\mathrm{Re}\,p > \lvert \mathrm{Im}\,a \rvert$
(5)	$t^{-1} J_\nu(at)$ $\quad$ $\mathrm{Re}\,\nu > 0$	$\nu^{-1}(a/R)^\nu$ $\quad$ $\mathrm{Re}\,p \ge \lvert \mathrm{Im}\,a \rvert$
(6)	$t^{-2} J_\nu(at)$ $\quad$ $\mathrm{Re}\,\nu > 1$	$\dfrac{a}{2\nu}\left[\dfrac{1}{\nu-1}\left(\dfrac{a}{R}\right)^{\nu-1} + \dfrac{1}{\nu+1}\left(\dfrac{a}{R}\right)^{\nu+1}\right]$ $\quad$ $\mathrm{Re}\,p \ge \lvert \mathrm{Im}\,a \rvert$
(7)	$t^\nu J_\nu(at)$ $\quad$ $\mathrm{Re}\,\nu > -\tfrac{1}{2}$	$2^\nu \pi^{-1/2}\Gamma(\nu+\tfrac{1}{2})\, a^\nu r^{-2\nu-1}$ $\quad$ $\mathrm{Re}\,p > \lvert \mathrm{Im}\,a \rvert$
(8)	$t^{\nu+1} J_\nu(at)$ $\quad$ $\mathrm{Re}\,\nu > -1$	$2^{\nu+1}\pi^{-1/2}\Gamma(\nu+3/2)\, a^\nu r^{-2\nu-3} p$ $\quad$ $\mathrm{Re}\,p > \lvert \mathrm{Im}\,a \rvert$
(9)	$t^\mu J_\nu(at)$ $\quad$ $\mathrm{Re}(\mu+\nu) > -1$	$\Gamma(\mu+\nu+1)\, r^{-\mu-1} P_\mu^{-\nu}(p/r)$ $\quad$ $\mathrm{Re}\,p > \lvert \mathrm{Im}\,a \rvert$
(10)	$t^\mu \sin(at)\, J_\mu(at)$ $\quad$ $a > 0,\quad \mathrm{Re}\,\mu > -1$	$\dfrac{\Gamma(\mu+1)\, a^{\mu+1}}{2^{1/2}\pi} \times \displaystyle\int_0^{\frac{1}{2}\pi} \dfrac{(\cos\theta)^{\mu+\frac{1}{2}} \cos[(\mu-\tfrac{1}{2})\theta]}{(\tfrac{1}{4}p^2 + a^2\cos^2\theta)^{\mu+1}}\, d\theta$ $\quad$ $\mathrm{Re}\,p > 0$

$r = (p^2+a^2)^{1/2}, \qquad R = p + r$

Bessel functions of kt and $kt^{1/2}$ (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(11)	$t^{\mu-1}\cos(at)\,J_\mu(at)$ $a>0,\quad \mathrm{Re}\,\mu>0$	$\dfrac{\Gamma(\mu)\,a^\mu}{2^{1/2}\,\pi}$ $\times\displaystyle\int_0^{\frac{1}{2}\pi}\frac{(\cos\theta)^{\mu-\frac{1}{2}}\cos[(\mu+\frac{1}{2})\,\theta]}{(\frac{1}{4}p^2+a^2\cos^2\theta)^\mu}\,d\theta$ $\mathrm{Re}\,p>0$
(12)	$[J_0^2(\frac{1}{2}at)]^2$	$2\pi^{-1}r^{-1}K(a/r)\qquad \mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(13)	$[J_1^2(\frac{1}{2}at)]^2$	$2\pi^{-1}a^{-2}r^{-2}[(2p^2+a^2)K(a/r)$ $-2(p^2+a^2)E(a/r)]$ $\mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(14)	$J_\nu(at)\,J_\nu(\beta t)\qquad \mathrm{Re}\,\nu>-\frac{1}{2}$	$\dfrac{1}{\pi a^{1/2}\beta^{1/2}}\,Q_{\nu-\frac{1}{2}}\left(\dfrac{p^2+a^2+\beta^2}{2a\beta}\right)$ $\mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert+\lvert\mathrm{Im}\,\beta\rvert$
(15)	$t\,J_0(\frac{1}{2}at)\,J_1(\frac{1}{2}at)$	$2\pi^{-1}a^{-1}r^{-1}[K(a/r)-E(a/r)]$ $\mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(16)	$t\,J_\nu^2(at)\qquad \mathrm{Re}\,\nu>-1$	$2^{2\nu+1}(\nu+\frac{1}{2})\pi^{-1}a^{2\nu}p^{-2\nu-2}$ $\times\mathrm{B}(\nu+\frac{1}{2},\ \nu+\frac{1}{2})$ $\times\ {}_2F_1(\nu+1/2,\nu+3/2;2\nu+1;-4a^2/p^2)$ $\mathrm{Re}\,p>2\lvert\mathrm{Im}\,a\rvert$
(17)	$t^{-2}J_1^2(t)$	$\frac{1}{2}\pi^{-1}\int_0^\pi[(p^2+2-2\cos\phi)^{1/2}-p]$ $\times(1+\cos\phi)\,d\phi\qquad \mathrm{Re}\,p>0$
(18)	$t^{1/2}J_\nu^2(\frac{1}{2}at)\qquad \mathrm{Re}\,\nu>-\frac{3}{4}$	$\dfrac{a\,\Gamma(2\nu+3/2)}{2^{\nu+3/2}p^{1/2}r}\,P_{1/4}^{-\nu}\left(\dfrac{r}{p}\right)P_{-1/4}^{-\nu}\left(\dfrac{r}{p}\right)$ $\mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$

$r=(p^2+a^2)^{1/2},\quad R=p+r$

Bessel functions of kt and $kt^{1/2}$ (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(19)	$t^{-1/2} J_\nu^2(\tfrac{1}{2}at)$ Re $\nu > -\tfrac{1}{4}$	$2^{-\nu-1/2}\Gamma(2\nu+\tfrac{1}{2})\,p^{-1/2}[P_{-1/4}^{-\nu}(r/p)]^2$ Re $p > \lvert \mathrm{Im}\, a\rvert$
(20)	$t^{1/2} J_\nu(\tfrac{1}{2}at)\, J_{-\nu}(\tfrac{1}{2}at)$	$\dfrac{a\pi^{1/2}}{2p^{1/2} r}[(\nu+\tfrac{1}{4})P_{-1/4}^{\nu}(r/p)\,P_{1/4}^{-\nu}(r/p)$ $-(\nu-\tfrac{1}{4})P_{1/4}^{\nu}(r/p)\,P_{-1/4}^{-\nu}(r/p)]$ Re $p > \lvert \mathrm{Im}\, a\rvert$
(21)	$t^{1/2} J_\nu(\tfrac{1}{2}at)\, J_{\nu+1}(\tfrac{1}{2}at)$ Re $\nu > -5/4$	$\dfrac{a\,\Gamma(2\nu+5/2)}{2^{\nu+5/2}p^{1/2}r} P_{-1/4}^{-\nu}\left(\dfrac{r}{p}\right) P_{-1/4}^{-\nu-1}\left(\dfrac{r}{p}\right)$ Re $p > \lvert \mathrm{Im}\, a\rvert$
(22)	$t^{-1/2} J_\nu(\tfrac{1}{2}at)\, J_{-\nu}(\tfrac{1}{2}at)$	$2^{-1/2}\pi^{1/2}p^{-1/2} P_{-1/4}^{\nu}(r/p)\, P_{-1/4}^{-\nu}(r/p)$ Re $p > \lvert \mathrm{Im}\, a\rvert$
(23)	$t^{2\nu} J_\nu^2(at)$ Re $\nu > -\tfrac{1}{4}$	$\dfrac{2^{4\nu}a^{2\nu}\Gamma(\nu+\tfrac{1}{2})\Gamma(2\nu+\tfrac{1}{2})}{\pi\Gamma(\nu+1)\,p^{4\nu+1}}$ $\times {}_2F_1(\nu+\tfrac{1}{2}, 2\nu+\tfrac{1}{2}; \nu+1; -4a^2/p^2)$ Re $p > 2\lvert \mathrm{Im}\, a\rvert$
(24)	$t^{\mu-1} J_{\nu_1}(a_1 t)\cdots J_{\nu_n}(a_n t)$ Re$(\mu+N)>0$ $N=\nu_1+\cdots+\nu_n$ $a=a_1+\cdots+a_n$	$\dfrac{2^{-N}\Gamma(\mu+N)}{\Gamma(\nu_1+1)\cdots\Gamma(\nu_n+1)} a_1^{\nu_1}\cdots a_n^{\nu_n}$ $\times(p+ia)^{-\mu-N} F_A\left(\mu+N; \nu_1+\tfrac{1}{2},\ldots,\nu_n+\tfrac{1}{2}; 2\nu_1+1,\ldots,2\nu_n+1; \dfrac{2a_1 i}{p+ia},\ldots,\dfrac{2a_n i}{p+ia}\right)$ Re$(p\pm i a_1 \pm\cdots\pm i a_n)>0$ If $n=2$, F_A is to be replaced by F_2

$r=(p^2+a^2)$, $R=p+r$

Bessel functions of kt and $kt^{1/2}$ (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(25)	$J_0(2a^{1/2}t^{1/2})$	$p^{-1}e^{-a/p}$ Re $p>0$
(26)	$J_\nu(2a^{1/2}t^{1/2})$ Re $\nu>-2$	$\tfrac{1}{2}a^{1/2}\pi^{1/2}p^{-3/2}e^{-\frac{1}{2}a/p}[I_{\frac{1}{2}\nu-\frac{1}{2}}(\tfrac{1}{2}a/p) - I_{\frac{1}{2}\nu+\frac{1}{2}}(\tfrac{1}{2}a/p)]$ Re $p>0$
(27)	$t^{1/2}J_1(2a^{1/2}t^{1/2})$	$a^{1/2}p^{-2}e^{-a/p}$ Re $p>0$
(28)	$t^{n-\frac{1}{2}}J_1(2a^{1/2}t^{1/2})$	$(-1)^{n-1}a^{-\frac{1}{2}}n!\,p^{-n}e^{-\frac{1}{2}a/p}k_{2n}(\tfrac{1}{2}a/p)$ Re $p>0$
(29)	$t^{-\frac{1}{2}}J_\nu(2a^{1/2}t^{1/2})$ Re $\nu>-1$	$\pi^{1/2}p^{-\frac{1}{2}}e^{-\frac{1}{2}a/p}I_{\frac{1}{2}\nu}(\tfrac{1}{2}a/p)$ Re $p>0$
(30)	$t^{\frac{1}{2}\nu}J_\nu(2a^{1/2}t^{1/2})$ Re $\nu>-1$	$a^{\frac{1}{2}\nu}p^{-\nu-1}e^{-a/p}$ Re $p>0$
(31)	$t^{-\frac{1}{2}\nu}J_\nu(2a^{1/2}t^{1/2})$	$\dfrac{e^{i\nu\pi}p^{\nu-1}}{a^{\frac{1}{2}\nu}\Gamma(\nu)}e^{-a/p}\,\gamma\left(\nu,\dfrac{a}{p}e^{-i\pi}\right)$ Re $p>0$
(32)	$t^{\frac{1}{2}\nu-1}J_\nu(2a^{1/2}t^{1/2})$ Re $\nu>0$	$a^{-\frac{1}{2}\nu}\gamma(\nu,\ a/p)$ Re $p>0$
(33)	$t^{\frac{1}{2}\nu+n}J_\nu(2a^{1/2}t^{1/2})$ Re $\nu+n>-1$	$n!\,a^{\frac{1}{2}\nu}p^{-n-\nu-1}e^{-a/p}L_n^\nu(a/p)$ Re $p>0$
(34)	$t^{\mu-\frac{1}{2}}J_{2\nu}(2a^{1/2}t^{1/2})$ Re $(\mu+\nu)>-\tfrac{1}{2}$	$\dfrac{\Gamma(\mu+\nu+\frac{1}{2})}{a^{1/2}\Gamma(2\nu+1)}p^{-\mu}e^{-\frac{1}{2}a/p}M_{\mu,\nu}(a/p)$ Re $p>0$

Bessel functions of kt and $kt^{1/2}$ (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(35)	$t^{\mu-1}\,J_{2\nu}(2a^{1/2}t^{1/2})$ $\quad\mathrm{Re}\,(\mu+\nu)>0$	$\dfrac{\Gamma(\mu+\nu)\,a^{\nu}}{\Gamma(2\nu+1)\,p^{\mu+\nu}}$ $\times\,{}_1F_1(\mu+\nu;2\nu+1;-a/p)$ $\quad\mathrm{Re}\,p>0$
(36)	$t^{\nu-1/2}\{J_{2\mu}(2t^{1/2})\cos[(\nu+\mu)\pi]$ $-J_{-2\mu}(2t^{1/2})\cos[(\nu-\mu)\pi]\}$ $\quad\mathrm{Re}(\nu\pm\mu)>-1/2$	$-\sin(2\mu\pi)\,p^{-\nu}e^{-1/2p^{-1}}\,W_{\nu,\mu}(p^{-1})$ $\quad\mathrm{Re}\,p>0$
(37)	$t^{1/2\nu}L_n^{\nu}(t)\,J_{\nu}(2a^{1/2}t^{1/2})$	$a^{1/2\nu}e^{-a/p}\dfrac{(p-1)^n}{p^{\nu+n+1}}\,L_n^{\nu}\left[\dfrac{a}{p(1-p)}\right]$ $\quad\mathrm{Re}\,p>0$
(38)	$J_{\nu}(t)\,J_{2\nu}(2a^{1/2}t^{1/2})$ $\quad\mathrm{Re}\,\nu>-1/2$	$\dfrac{e^{-ap/(p^2+1)}}{(p^2+1)^{1/2}}\,J_{\nu}\left(\dfrac{a}{p^2+1}\right)$ $\quad\mathrm{Re}\,p>0$
(39)	$J_{\nu}(at^{1/2})\,J_{\nu}(\beta t^{1/2})$ $\quad\mathrm{Re}\,\nu>-1/2$	$p^{-1}\,e^{-1/4(a^2+\beta^2)/p}\,I_{\nu}(1/2\,a\beta/p)$ $\quad\mathrm{Re}\,p>0$
(40)	$t^{-1}\,J_{\nu}^2(2t^{1/2})$ $\quad\mathrm{Re}\,\nu>0$	$\nu^{-1}\,e^{-2/p}\,[I_{\nu}(2/p)+2\sum_{n=1}^{\infty} I_{\nu+n}(2/p)]$ $\quad\mathrm{Re}\,p>0$
(41)	$t^{-1/2}\,J_{\nu}(ae^{1/4\pi i}t^{1/2})$ $\times J_{\nu}(ae^{-1/4\pi i}t^{1/2})$ $\quad\mathrm{Re}\,\nu>-1/2$	$\dfrac{p^{1/2}\,2^{1-\nu}\,\Gamma(\nu+1/2)}{a^2[\Gamma(\nu+1)]^2}\,M_{1/4,1/2\nu}\left(\dfrac{a^2}{2p}\right)$ $\times M_{-1/4,1/2\nu}\left(\dfrac{a^2}{2p}\right)$ $\quad\mathrm{Re}\,p>0$

Bessel functions of kt and $kt^{1/2}$ (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(42)	$t^{\lambda-1} J_{2\mu}(2a^{1/2}t^{1/2}) J_{2\nu}(2a^{1/2}t^{1/2})$ $\mathrm{Re}(\lambda+\mu+\nu)>0$	$\dfrac{2\Gamma(\lambda+\mu+\nu)\,a^{\mu+\nu}}{\Gamma(2\mu+1)\,\Gamma(2\nu+1)\,p^{\lambda+\mu+\nu}}$ $\times {}_3F_3(\mu+\nu+\frac{1}{2},\, \mu+\nu+1,\, \lambda+\mu+\nu;$ $2\mu+1, 2\nu+1, 2\mu+2\nu+1; -4a/p)$ $\mathrm{Re}\, p>0$
(43)	$t^{\nu-1} J_{2\mu_1}(2a_1^{1/2}t^{1/2}) \cdots J_{2\mu_n}(2a_n^{1/2}t^{1/2})$ $M=\mu_1+\cdots+\mu_n,\quad \mathrm{Re}(\nu+M)>0$	$\dfrac{\Gamma(\nu+M)\,p^{-\nu-M}\,a_1^{\mu_1}\cdots a_n^{\mu_n}}{\Gamma(2\mu_1+1)\cdots\Gamma(2\mu_n+1)}$ $\times \Psi_2\left(\nu+M; 2\mu_1+1,\ldots,2\mu_n+1; \dfrac{a_1}{p},\ldots,\dfrac{a_n}{p}\right)$ $\mathrm{Re}\, p>0$
(44)	$Y_0(at)$	$-2\pi^{-1} r^{-1} \sinh^{-1}(p/a)$ $\mathrm{Re}\, p > \lvert \mathrm{Im}\, a \rvert$
(45)	$Y_\nu(at)$ $\quad \lvert \mathrm{Re}\,\nu \rvert < 1$	$a^\nu \operatorname{ctn}(\nu\pi) r^{-1} R^{-\nu} - a^{-\nu} \csc(\nu\pi) r^{-1} R^{\nu}$ $\mathrm{Re}\, p > \lvert \mathrm{Im}\, a \rvert$
(46)	$t\,Y_0(at)$	$2\pi^{-1} r^{-2}[1-pr^{-1}\log(R/a)]$ $\mathrm{Re}\, p > \lvert \mathrm{Im}\, a \rvert$
(47)	$t\,Y_1(at)$	$-2\pi^{-1} r^{-2}[p\,a^{-1}+ar^{-1}\log(R/a)]$ $\mathrm{Re}\, p > \lvert \mathrm{Im}\, a \rvert$
(48)	$t^\mu Y_\nu(at)$ $\quad \mathrm{Re}(\mu\pm\nu)>-1$	$r^{-\mu-1}[\Gamma(\mu+\nu+1)\operatorname{ctn}(\nu\pi) P_\mu^{-\nu}(p/r)$ $-\Gamma(\mu-\nu+1)\csc(\nu\pi) P_\mu^{\nu}(p/r)]$ $\mathrm{Re}\, p > \lvert \mathrm{Im}\, a \rvert$
(49)	$t^{-1/2} Y_{2\nu}(2a^{1/2}t^{1/2})$ $\quad \lvert \mathrm{Re}\,\nu \rvert < \frac{1}{2}$	$-\pi^{1/2} p^{-1/2} e^{-\frac{1}{2}a/p}[\tan(\nu\pi) I_\nu(\frac{1}{2}a/p)$ $+\pi^{-1}\sec(\nu\pi) K_\nu(\frac{1}{2}a/p)]$ $\mathrm{Re}\, p>0$

$r=(p^2+a^2)^{1/2}, \quad R=p+r$

Bessel functions of kt and $kt^{1/2}$ (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(50)	$t^{\mu-1/2}\, Y_{2\nu}(2a^{1/2}t^{1/2})$ $\qquad \mathrm{Re}(\mu\pm\nu)>-1/2$	$a^{-1/2}\,p^{-\mu}e^{-1/2\,a/p}$ $\times\left\{\dfrac{\tan[(\mu-\nu)\pi]\,\Gamma(\mu+\nu+1/2)}{\Gamma(2\nu+1)}M_{\mu,\nu}(a/p)\right.$ $\left.-\sec[(\mu-\nu)\pi]\,W_{\mu,\nu}(a/p)\right\}$ $\qquad \mathrm{Re}\,p>0$
(51)	$t^{-1}\{[at^{1/2}J_1(t^{1/2})+b\,J_0(t^{1/2})]^2$ $+[at^{1/2}Y_1(t^{1/2})+b\,Y_0(t^{1/2})]^2\}^{-1}$	$2\,I(a,b;p)$ $\qquad \mathrm{Re}\,p>0$ For expansions of $I(a,b;p)$ and a short numerical table of $I(0,1;p)$ cf. Jaeger, J. C., and Martha Clarke 1942: *Proc. Roy. Soc. Edinburgh Sect. A* 61, 229-230.
(52)	$H_0^{(1)}(at)$	$r^{-1}-2i\pi^{-1}r^{-1}\sinh^{-1}(p/a)$ $\qquad \mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(53)	$H_0^{(2)}(at)$	$r^{-1}+2i\pi^{-1}r^{-1}\sinh^{-1}(p/a)$ $\qquad \mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(54)	$H_\nu^{(1)}(at)$ $\qquad \lvert\mathrm{Re}\,\nu\rvert<1$	$ir^{-1}\csc(\nu\pi)(e^{-i\nu\pi}a^\nu R^{-\nu}-a^{-\nu}R^\nu)$ $\qquad \mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(55)	$H_\nu^{(2)}(at)$ $\qquad \lvert\mathrm{Re}\,\nu\rvert<1$	$ir^{-1}\csc(\nu\pi)(a^{-\nu}R^\nu-e^{i\nu\pi}a^\nu R^{-\nu})$ $\qquad \mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(56)	$t\,H_0^{(1)}(at)$	$\dfrac{p}{r^3}\left(1-\dfrac{2i}{\pi}\log\dfrac{R}{a}\right)+\dfrac{2i}{\pi r^2}$ $\qquad \mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(57)	$t\,H_0^{(2)}(at)$	$\dfrac{p}{r^3}\left(1+\dfrac{2i}{\pi}\log\dfrac{R}{a}\right)-\dfrac{2i}{\pi r^2}$ $\qquad \mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$

$r=(p^2+a^2)^{1/2}, \quad R=p+r$

Bessel functions of kt and $kt^{1/2}$ (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(58)	$t\,H_1^{(1)}(at)$	$\dfrac{a}{r^3}\left(1-\dfrac{2i}{\pi}\log\dfrac{R}{a}\right)-\dfrac{2ip}{\pi a r^2}$ $\quad \operatorname{Re} p > \lvert\operatorname{Im} a\rvert$
(59)	$t\,H_1^{(2)}(at)$	$\dfrac{a}{r^3}\left(1+\dfrac{2i}{\pi}\log\dfrac{R}{a}\right)+\dfrac{2ip}{\pi a r^2}$ $\quad \operatorname{Re} p > \lvert\operatorname{Im} a\rvert$
(60)	$t^{-1/2}\,H_{2\nu}^{(1)}(2a^{1/2}t^{1/2})\quad \lvert\operatorname{Re}\nu\rvert < \tfrac{1}{2}$	$\pi^{1/2}p^{-1/2}\sec(\nu\pi)\,e^{-\frac{1}{2}a/p}\times[e^{-i\nu\pi}I_\nu(\tfrac{1}{2}a/p)-i\pi^{-1}K_\nu(\tfrac{1}{2}a/p)]$ $\quad \operatorname{Re} p>0$
(61)	$t^{-1/2}\,H_{2\nu}^{(2)}(2a^{1/2}t^{1/2})\quad \lvert\operatorname{Re}\nu\rvert < \tfrac{1}{2}$	$\pi^{1/2}p^{-1/2}e^{-\frac{1}{2}a/p}\sec(\nu\pi)\times[e^{i\nu\pi}I_\nu(\tfrac{1}{2}a/p)+i\pi^{-1}K_\nu(\tfrac{1}{2}a/p)]$ $\quad \operatorname{Re} p>0$
(62)	$t^{-\frac{1}{2}\nu}\,H_\nu^{(1)}(2a^{1/2}t^{1/2})\quad \operatorname{Re}\nu<1$	$\dfrac{p^{\nu-1}e^{-a/p}}{\pi i\,a^{\frac{1}{2}\nu}}\Gamma(1-\nu)\,\Gamma(\nu, e^{-i\pi}a/p)$ $\quad \operatorname{Re} p>0$
(63)	$t^{-\frac{1}{2}\nu}\,H_\nu^{(2)}(2a^{1/2}t^{1/2})\quad \operatorname{Re}\nu<1$	$-\dfrac{p^{\nu-1}e^{-a/p}}{\pi i\,a^{\frac{1}{2}\nu}}\Gamma(1-\nu)\,\Gamma(\nu, e^{i\pi}a/p)$ $\quad \operatorname{Re} p>0$
(64)	$t^{\nu-1/2}\,H_1^{(1)}(2a^{1/2}t^{1/2})\quad \operatorname{Re}\nu>0$	$\dfrac{\Gamma(\nu+1)}{ip^\nu\sin(\nu\pi)}\,e^{-\frac{1}{2}a/p}\,k_{-2\nu}\!\left(\dfrac{ae^{-\pi i}}{2p}\right)$ $\quad \operatorname{Re} p>0$
(65)	$t^{\nu-1/2}\,H_1^{(2)}(2a^{1/2}t^{1/2})\quad \operatorname{Re}\nu>0$	$\dfrac{i\,\Gamma(\nu+1)}{p^\nu\sin(\nu\pi)}\,e^{-\frac{1}{2}a/p}\,k_{-2\nu}\!\left(\dfrac{ae^{\pi i}}{2p}\right)$ $\quad \operatorname{Re} p>0$

$r=(p^2+a^2)^{1/2},\quad R=p+r$

Bessel functions of kt and $kt^{1/2}$ (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(66)	$t^{\mu-1/2}\,H^{(1)}_{2\nu}(2a^{1/2}t^{1/2})$ $\mathrm{Re}\,(\mu\pm\nu)>-1/2$	$\dfrac{\Gamma(\mu+\nu+1/2)\Gamma(\mu-\nu+1/2)}{\pi a^{1/2}e^{\nu\pi i+1/2 a/p}\,p^{\mu}}$ $\times W_{-\mu,\nu}(e^{-i\pi}a/p)$ $\mathrm{Re}\,p>0$
(67)	$t^{\mu-1/2}\,H^{(2)}_{2\nu}(2a^{1/2}t^{1/2})$ $\mathrm{Re}(\mu\pm\nu)>-1/2$	$\dfrac{\Gamma(\mu+\nu+1/2)\Gamma(\mu-\nu+1/2)}{\pi a^{1/2}e^{-\nu\pi i+1/2 a/p}\,p^{\mu}}$ $\times W_{-\mu,\nu}(e^{i\pi}a/p)$ $\mathrm{Re}\,p>0$

4.15. Bessel functions of other arguments

(1)	$J_{\nu+1/2}(1/2\,t^2)$ $\mathrm{Re}\,\nu>-1$	$\pi^{-1/2}\,\Gamma(\nu+1)D_{-\nu-1}(pe^{1/4\pi i})$ $\times D_{-\nu-1}(pe^{-1/4\pi i})$ $\mathrm{Re}\,p>0$
(2)	$t^{1/2}\,J_{1/4}(at^2)$ $a>0$	$\dfrac{\pi^{1/2}p^{1/2}}{4a}\left[\mathbf{H}_{-1/4}\left(\dfrac{p^2}{4a}\right)-Y_{-1/4}\left(\dfrac{p^2}{4a}\right)\right]$ $\mathrm{Re}\,p>0$
(3)	$t^{1/2}\,J_{-1/4}(at^2)$ $a>0$	$\dfrac{\pi^{1/2}p^{1/2}}{4a}\left[\mathbf{H}_{1/4}\left(\dfrac{p^2}{4a}\right)-Y_{1/4}\left(\dfrac{p^2}{4a}\right)\right]$ $\mathrm{Re}\,p>0$
(4)	$t^{3/2}\,J_{-1/4}(at^2)$ $a>0$	$-\dfrac{\pi^{1/2}p^{3/2}}{8a^2}\left[\mathbf{H}_{-3/4}\left(\dfrac{p^2}{4a}\right)-Y_{-3/4}\left(\dfrac{p^2}{4a}\right)\right]$ $\mathrm{Re}\,p>0$
(5)	$t^{3/2}\,J_{-3/4}(at^2)$ $a>0$	$\dfrac{\pi^{1/2}p^{3/2}}{8a^2}\left[\mathbf{H}_{-1/4}\left(\dfrac{p^2}{4a}\right)-Y_{-1/4}\left(\dfrac{p^2}{4a}\right)\right]$ $\mathrm{Re}\,p>0$

Bessel functions of other arguments (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(6)	$t^{1/2}\,J_{1/8}(t^2/16)\,J_{-1/8}(t^2/16)$	$2^{-1/2}\pi^{1/2}p^{1/2}\sec(\pi/8)$ $\times H^{(1)}_{1/8}(p^2)\,H^{(2)}_{1/8}(p^2)$ $\quad \mathrm{Re}\,p>0$
(7)	$t^{1/2}\,J_{\nu+1/8}(t^2/16)J_{\nu-1/8}(t^2/16)$ $\quad \mathrm{Re}\,\nu>-3/8$	$2^{1/2}(\pi p)^{-3/2}\Gamma(\nu+3/8)\Gamma(\nu+5/8)$ $\times W_{-\nu,\,1/8}(2e^{\pi i/2}p^2)$ $\times W_{-\nu,\,1/8}(2e^{-\pi i/2}p^2)$ $\quad \mathrm{Re}\,p>0$
(8)	$t^{-1}\,J_\nu(t^{-1})$	$2\,J_\nu(2^{1/2}p^{1/2})\,K_\nu(2^{1/2}p^{1/2})$ $\quad \mathrm{Re}\,p>0$
(9)	$0 \quad 0<t<b$ $J_0(a\gamma) \quad t>b$	$r^{-1}e^{-br}$ $\quad \mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(10)	$0 \quad 0<t<b$ $t\,J_0(a\gamma) \quad t>b$	$pr^{-3}(br+1)e^{-br}$ $\quad \mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(11)	$0 \quad 0<t<b$ $\dfrac{J_0(a\gamma)}{t-\lambda} \quad t>b$ $\lvert\arg(b-\lambda)\rvert<\pi$	$-e^{-br}\int_0^\infty e^{-u}\,[u^2-2(\lambda p-br)u$ $+\lambda^2(r-p)^2]^{-1/2}\,du$ $\quad \mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(12)	$0 \quad 0<t<b$ $\gamma\,J_1(a\gamma) \quad t>b$	$ar^{-3}(br+1)e^{-br}$ $\quad \mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(13)	$0 \quad 0<t<b$ $\gamma^{-1}\,J_1(a\gamma) \quad t>b$	$a^{-1}b^{-1}(e^{-bp}-e^{-br})$ $\quad \mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(14)	$0 \quad 0<t<b$ $\gamma^{-1}\,J_\nu(a\gamma) \quad t>b$ $\mathrm{Re}\,\nu>-1$	$I_{\frac12\nu}[\tfrac12 b(r-p)]\,K_{\frac12\nu}[\tfrac12 b(r+p)]$ $\quad \mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$

$\gamma=(t^2-b^2)^{1/2}, \quad r=(p^2+a^2)^{1/2}, \quad R=p+r$

Bessel functions of other arguments (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\, dt$
(15)	$0 \qquad 0<t<b$ $t y^{-1} J_1(a y) \qquad t>b$	$a^{-1} e^{-bp} - a^{-1} p r^{-1} e^{-br}$ $\mathrm{Re}\, p > \lvert \mathrm{Im}\, a \rvert$
(16)	$0 \qquad 0<t<b$ $(t-b)^{\frac{1}{2}\nu} (t+b)^{-\frac{1}{2}\nu} J_\nu(a y)$ $t>b$ $\mathrm{Re}\, \nu > -1$	$a^\nu r^{-1} R^{-\nu} e^{-br} \qquad \mathrm{Re}\, p > \lvert \mathrm{Im}\, a \rvert$
(17)	$0 \qquad 0<t<b$ $y^\nu J_\nu(a y) \qquad t>b$ $\mathrm{Re}\, \nu > -1$	$2^{\frac{1}{2}} \pi^{-\frac{1}{2}} a^\nu b^{\nu+\frac{1}{2}} r^{-\nu-\frac{1}{2}} K_{\nu+\frac{1}{2}}(br)$ $\mathrm{Re}\, p > \lvert \mathrm{Im}\, a \rvert$
(18)	$0 \qquad 0<t<b$ $y^{2\mu} J_{2\nu}(a y) \qquad t>b$ $\mathrm{Re}(\mu+\nu) > -1$	$\sum_{n=0}^\infty \frac{(-1)^n (ab)^{2\nu+2n} (2b)^{2\mu+1} \Gamma(\mu+\nu+n+1)}{\pi^{\frac{1}{2}} n! \Gamma(2\nu+n+1) (2p)^{\mu+\nu+n+\frac{1}{2}}}$ $\times K_{\mu+\nu+n+\frac{1}{2}}(bp) \qquad \mathrm{Re}\, p > \lvert \mathrm{Im}\, a \rvert$
(19)	$J_0[a(t^2+\beta t)^{\frac{1}{2}}] \qquad \lvert \arg \beta \rvert < \pi$	$r^{-1} e^{\frac{1}{2}\beta(p-r)} \qquad \mathrm{Re}\, p > \lvert \mathrm{Im}\, a \rvert$
(20)	$(t^2+\beta t)^{\frac{1}{2}\nu} J_\nu[a(t^2+\beta t)^{\frac{1}{2}}]$ $\mathrm{Re}\, \nu > -1, \quad \lvert \arg \beta \rvert < \pi$	$\pi^{-\frac{1}{2}} (\tfrac{1}{2} a)^\nu (\beta/r)^{\nu+\frac{1}{2}} e^{\frac{1}{2}\beta p} K_{\nu+\frac{1}{2}}(\tfrac{1}{2}\beta r)$ $\mathrm{Re}\, p > \lvert \mathrm{Im}\, a \rvert$
(21)	$t^{\frac{1}{2}\nu} (t+\beta)^{-\frac{1}{2}\nu} J_\nu[a(t^2+\beta t)^{\frac{1}{2}}]$ $\mathrm{Re}\, \nu > -1, \quad \lvert \arg \beta \rvert < \pi$	$a^\nu r^{-1} R^{-\nu} e^{\frac{1}{2}\beta(p-r)} \qquad \mathrm{Re}\, p > \lvert \mathrm{Im}\, a \rvert$
(22)	$t^{\frac{1}{2}\nu-1} (t+1)^{-\frac{1}{2}\nu} J_\nu[a(t^2+t)^{\frac{1}{2}}]$ $\mathrm{Re}\, \nu > 0$	$2^\nu a^{-\nu} \gamma(\nu, \tfrac{1}{2} r - \tfrac{1}{2} p) \qquad \mathrm{Re}\, p > \lvert \mathrm{Im}\, a \rvert$

$y = (t^2 - b^2)^{\frac{1}{2}}, \quad r = (p^2 + a^2)^{\frac{1}{2}}, \quad R = p + r$

Bessel functions of other arguments (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(22)	$t^{\lambda-\frac{1}{2}\nu-1}(t+1)^{-\frac{1}{2}\nu} J_\nu[a(t^2+t)^{\frac{1}{2}}]$ $\operatorname{Re}\nu+1>\operatorname{Re}\lambda>0$	$\dfrac{2^\nu a^{-\nu}}{\Gamma(\nu-\lambda+1)}$ $\times\int_0^{\frac{1}{2}r-\frac{1}{2}p} e^{-u} u^{\lambda-1}(\frac{1}{4}a^2-pu-u^2)^{\nu-\lambda}\,du$ $\operatorname{Re}p>\lvert\operatorname{Im}a\rvert$
(23)	$(t^2+2it)^{\frac{1}{2}\nu} J_\nu[a(t^2+2it)^{\frac{1}{2}}]$ $\operatorname{Re}\nu>-1$	$-i\,2^{-\frac{1}{2}}\pi^{\frac{1}{2}} a^\nu r^{-\nu-\frac{1}{2}} e^{ip} H^{(2)}_{\nu+\frac{1}{2}}(r)$ $\operatorname{Re}p>\lvert\operatorname{Im}a\rvert$
(24)	$(t^2+2it)^{\lambda-\frac{1}{2}\nu} J_\nu[a(t^2+2it)^{\frac{1}{2}}]$ $\operatorname{Re}\lambda>-1$	$\dfrac{2^{\lambda-\nu-\frac{1}{2}}\pi^{\frac{1}{2}} e^{ip}\Gamma(\lambda+1)}{ir^{\lambda+\frac{1}{2}}\Gamma(\nu-\lambda)}$ $\times\sum_{n=0}^{\infty}\dfrac{\Gamma(\nu-\lambda+n)}{2^n n!\,\Gamma(\nu+n+1)\,r^n} H^{(2)}_{\lambda+n+\frac{1}{2}}(r)$ $\operatorname{Re}p>\lvert\operatorname{Im}a\rvert$
(25)	$\exp[i\,a(1-e^{-t})]\,J_\nu(ae^{-t})$	$\dfrac{J_\nu(a)}{\nu+p}+2\sum_{n=1}^{\infty} i^n\dfrac{(\nu-p+1)_{n-1}}{(\nu+p)_{n+1}}(\nu+n)$ $\times J_{\nu+n}(a)$ $\operatorname{Re}p>-\operatorname{Re}\nu$
(26)	$\sin[a(1-e^{-t})]\,J_\nu(ae^{-t})$	$2\sum_{n=0}^{\infty}\dfrac{(-1)^n(\nu-p+1)_{2n}}{(\nu+p)_{2n+2}}(\nu+2n-1)$ $\times J_{\nu+2n+1}(a)$ $\operatorname{Re}p>-\operatorname{Re}\nu$
(27)	$\cos[a(1-e^{-t})]J_\nu(ae^{-t})$	$\dfrac{J_\nu(a)}{\nu+p}+\sum_{n=0}^{\infty} 2(-1)^n\dfrac{(\nu-p+1)_{2n-1}}{(\nu+p)_{2n+1}}$ $\times(\nu+2n)\,J_{\nu+2n}(a)$ $\operatorname{Re}p>-\operatorname{Re}\nu$

$y=(t^2-b^2)^{\frac{1}{2}}$, $r=(p^2+a^2)^{\frac{1}{2}}$, $R=p+r$

Bessel functions of other arguments (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(28)	$J_\mu(ae^{-t})\, J_\nu[a(1-e^{-t})]$ $\operatorname{Re}\nu>-1$	$\left(\frac{2}{a}\right)^p \sum_{n=0}^{\infty} \frac{(-1)^n\,\Gamma(p+n)}{(\mu+n)\,n!\,\mathrm{B}(p,\mu+n)}$ $\times J_{\mu+\nu+p+2n}(a)$ $\quad\operatorname{Re}p>-\operatorname{Re}\mu$
(29)	$(1-e^{-t})^{\frac{1}{2}\nu}\, J_\nu[a(1-e^{-t})^{\frac{1}{2}}]$ $\operatorname{Re}\nu>-1$	$\Gamma(p)\,(2/a)^p\, J_{\nu+p}(a)$ $\quad\operatorname{Re}p>0$
(30)	$(1-e^{-t})^{-\frac{1}{2}\nu}\, J_\nu[a(1-e^{-t})^{\frac{1}{2}}]$	$\frac{s_{\nu+p-1,\,p-\nu}(a)}{2^\nu\, a^p\,\Gamma(\nu)}$ $\quad\operatorname{Re}p>0$
(31)	$(e^t-1)^{\frac{1}{2}\nu}\, J_\nu[2a(e^t-1)^{\frac{1}{2}}]$ $a>0,\quad \operatorname{Re}\nu>-1$	$\frac{2a^p}{\Gamma(p+1)} K_{\nu-p}(2a)$ $\operatorname{Re}p>\frac{1}{2}\operatorname{Re}\nu-\frac{3}{4}$
(32)	$(e^t-1)^{\mu}\quad J_{2\nu}[2a(e^t-1)^{\frac{1}{2}}]$ $a>0,\quad \operatorname{Re}(\mu+\nu)>-1$	$\frac{a^{2\nu}\,\mathrm{B}(\mu+\nu+1,\, p-\mu-\nu)}{\Gamma(2\nu+1)}$ $\times {}_1F_2(\mu+\nu+1;\, \mu+\nu+1-p;\, 2\nu+1;\, a^2)$ $+\frac{a^{2p-2\mu}\,\Gamma(\mu+\nu-p)}{\Gamma(\nu-\mu+p+1)}$ $\times {}_1F_2(p+1;\, p+1+\nu-\mu,\, p+1-\mu-\nu;\, a^2)$ $\operatorname{Re}p>\operatorname{Re}\mu-7/4$
(33)	$J_\nu(2a\sinh t)$ $\operatorname{Re}\nu>-1,\quad a>0$	$I_{\frac{1}{2}\nu+\frac{1}{2}p}(a)\, K_{\frac{1}{2}\nu-\frac{1}{2}p}(a)$ $\quad\operatorname{Re}p>-\frac{1}{2}$
(34)	$\operatorname{csch}(t)\, J_\nu(a\operatorname{csch} t)\qquad a>0$	$\frac{\Gamma(\frac{1}{2}p+\frac{1}{2}\nu+\frac{1}{2})}{a\,\Gamma(\nu+1)} W_{-\frac{1}{2}p,\,\frac{1}{2}\nu}(a)$ $\times M_{\frac{1}{2}p,\,\frac{1}{2}\nu}(a)$ $\quad\operatorname{Re}p>-\operatorname{Re}\nu-1$

Bessel functions of other arguments (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(35)	$\csc(\frac{1}{2}t)\exp\left(\frac{\alpha-\beta e^{t}}{e^{t}-1}\right)$ $\times J_{2\nu}\left[\frac{\alpha^{1/2}\beta^{1/2}}{\sinh(\frac{1}{2}t)}\right]$ $\operatorname{Re}\alpha>0, \quad \operatorname{Re}\beta>0$	$\frac{2\Gamma(p+\nu+\frac{1}{2})}{\alpha^{1/2}\beta^{1/2}\Gamma(2\nu+1)}e^{-\frac{1}{2}(\alpha+\beta)}$ $\times W_{-p,\nu}(\beta)\,M_{p,\nu}(\alpha)$ $\operatorname{Re}p>-\operatorname{Re}\nu-\frac{1}{2}$
(36)	$t^{-1}Y_{\nu}(t^{-1})$	$2Y_{\nu}(2^{1/2}p^{1/2})\,K_{\nu}(2^{1/2}p^{1/2}) \qquad \operatorname{Re}p>0$
(37)	$t^{-1}H_{\nu}^{(1)}(t^{-1})$	$2H_{\nu}^{(1)}(2^{1/2}p^{1/2})\,K_{\nu}(2^{1/2}p^{1/2}) \qquad \operatorname{Re}p>0$
(38)	$t^{-1}H_{\nu}^{(2)}(t^{-1})$	$2H_{\nu}^{(2)}(2^{1/2}p^{1/2})\,K_{\nu}(2^{1/2}p^{1/2}) \qquad \operatorname{Re}p>0$

4.16. Modified Bessel functions of arguments kt and $kt^{1/2}$

(1)	$I_{\nu}(at) \qquad \operatorname{Re}\nu>-1$	$a^{\nu}s^{-1}S^{-\nu} \qquad \operatorname{Re}p>\lvert\operatorname{Re}a\rvert$
(2)	$t\,I_{\nu}(at) \qquad \operatorname{Re}\nu>-2$	$a^{\nu}(p+\nu s)s^{-3}S^{-\nu} \qquad \operatorname{Re}p>\lvert\operatorname{Re}a\rvert$
(3)	$t^{-1}I_{1}(at)$	$\frac{(p+a)^{1/2}-(p-a)^{1/2}}{(p+a)^{1/2}+(p-a)^{1/2}} \qquad \operatorname{Re}p>\lvert\operatorname{Re}a\rvert$
(4)	$t^{-1}I_{\nu}(at) \qquad \operatorname{Re}\nu>0$	$\nu^{-1}a^{\nu}S^{-\nu} \qquad \operatorname{Re}p>\lvert\operatorname{Re}a\rvert$
(5)	$t^{-1/2}I_{\nu}(t) \qquad \operatorname{Re}\nu>-\frac{1}{2}$	$2^{1/2}\pi^{-1/2}Q_{\nu-1/2}(p) \qquad \operatorname{Re}p>1$
(6)	$t^{\nu}I_{\nu}(at) \qquad \operatorname{Re}\nu>-\frac{1}{2}$	$2^{\nu}\pi^{-1/2}\Gamma(\nu+\frac{1}{2})\,a^{\nu}s^{-2\nu-1}$ $\operatorname{Re}p>\lvert\operatorname{Re}a\rvert$

$s=(p^{2}-a^{2})^{1/2}, \qquad S=p+s$

Modified Bessel functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(7)	$t^{\nu+1} I_\nu(at)$ $\quad$ $\mathrm{Re}\,\nu>-1$	$2^{\nu+1}\pi^{-1/2}\Gamma(\nu+3/2)\,a^\nu p s^{-2\nu-3}$ $\quad$ $\mathrm{Re}\,p>\lvert\mathrm{Re}\,a\rvert$
(8)	$t^\mu I_\nu(at)$ $\quad$ $\mathrm{Re}(\mu+\nu)>-1$	$\Gamma(\mu+\nu+1)\,s^{-\mu-1}P_\mu^{-\nu}(p/s)$ $\quad$ $\mathrm{Re}\,p>\lvert\mathrm{Re}\,a\rvert$
(9)	$t^{\mu-1/2} I_{\nu+1/2}(at)$ $\quad$ $\mathrm{Re}(\mu+\nu)>-1$	$\dfrac{2^{1/2}\sin(\nu\pi)\,s^{-\mu}}{\pi^{1/2}a^{1/2}\sin[(\mu+\nu)\pi]}\,Q_\nu^\mu\left(\dfrac{p}{a}\right)$ $\quad$ $\mathrm{Re}\,p>\lvert\mathrm{Re}\,a\rvert$
(10)	$I_0^2(\tfrac12 at)$	$2\pi^{-1}p^{-1}E(a/p)$ $\quad$ $\mathrm{Re}\,p>\lvert\mathrm{Re}\,a\rvert$
(11)	$t\,I_0(\tfrac12 at)\,I_1(\tfrac12 at)$	$\dfrac{2p\,E(a/p)}{\pi a(p^2-a^2)}-\dfrac{2K(a/p)}{\pi a p}$ $\quad$ $\mathrm{Re}\,p>\lvert\mathrm{Re}\,a\rvert$
(12)	$t^{-1/2} I_\mu(at)\,I_\nu(bt)$ $\quad$ $\mathrm{Re}(\mu+\nu)>-\tfrac12$	$c^{1/2}\Gamma(\mu+\nu+\tfrac12)\,P_{\nu-1/2}^{-\mu}(\cosh\alpha)$ $\times P_{\mu-1/2}^{-\nu}(\cosh\beta)$ $\quad$ $\mathrm{Re}(p\pm a\pm b)>0$ where $\sinh\alpha=ac,\quad \sinh\beta=bc,$ $\cosh\alpha\cosh\beta=pc,$ $\lvert\mathrm{Im}\,\alpha\rvert,\quad \lvert\mathrm{Im}\,\beta\rvert<\tfrac12\pi$
(13)	$t^{2\lambda-1} I_{2\mu}(at)\,I_{2\nu}(\beta t)$ $\quad$ $\mathrm{Re}(\lambda+\mu+\nu)>0$	$\dfrac{2^{2\lambda-1}a^{2\mu}\beta^{2\nu}\,\Gamma(\lambda+\mu+\nu)\,\Gamma(\lambda+\mu+\nu+\tfrac12)}{\pi^{1/2}p^{2\lambda+2\mu+2\nu}\,\Gamma(2\mu+1)\,\Gamma(2\nu+1)}$ $\times F_4(\lambda+\mu+\nu,\lambda+\mu+\nu+\tfrac12;2\mu+1,$ $2\nu+1;a^2/p^2,\beta^2/p^2)$ $\quad$ $\mathrm{Re}\,p>\lvert\mathrm{Re}\,a\rvert+\lvert\mathrm{Re}\,\beta\rvert$

$s=(p^2-a^2)^{1/2},\quad S=p+s$

Modified Bessel functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(14)	$I_0(2a^{1/2}t^{1/2})$	$p^{-1}e^{a/p}$ $\quad$ $\operatorname{Re} p>0$
(15)	$t^{-1/2}I_0(2^{3/2}a^{1/2}t^{1/2})$	$\pi^{1/2}p^{-1/2}e^{a/p}I_0(a/p)$ $\quad$ $\operatorname{Re} p>0$
(16)	$t^{-1/2}I_1(2a^{1/2}t^{1/2})$	$a^{-1/2}(e^{a/p}-1)$ $\quad$ $\operatorname{Re} p>0$
(17)	$t^{-1/2}I_\nu(2^{3/2}a^{1/2}t^{1/2})$ $\quad$ $\operatorname{Re}\nu>-1$	$\pi^{1/2}p^{-1/2}e^{a/p}I_{\frac{1}{2}\nu}(a/p)$ $\quad$ $\operatorname{Re} p>0$
(18)	$t^{\frac{1}{2}\nu}I_\nu(2a^{1/2}t^{1/2})$ $\quad$ $\operatorname{Re}\nu>-1$	$a^{\frac{1}{2}\nu}p^{-\nu-1}e^{a/p}$ $\quad$ $\operatorname{Re} p>0$
(19)	$t^{-\frac{1}{2}\nu}I_\nu(2a^{1/2}t^{1/2})$	$a^{-\frac{1}{2}\nu}[\Gamma(\nu)]^{-1}p^{\nu-1}e^{a/p}\gamma(\nu,a/p)$ $\quad$ $\operatorname{Re} p>0$
(20)	$t^{\mu-\frac{1}{2}}I_{2\nu}(2a^{1/2}t^{1/2})$ $\quad$ $\operatorname{Re}(\mu+\nu)>-\frac{1}{2}$	$\dfrac{\Gamma(\mu+\nu+\frac{1}{2})\,e^{\frac{1}{2}a/p}}{a^{1/2}\Gamma(2\nu+1)\,p^{\mu}}M_{-\mu,\nu}\left(\dfrac{a}{p}\right)$ $\quad$ $\operatorname{Re} p>0$
(21)	$I_\nu^2(2^{1/2}a^{1/2}t^{1/2})$ $\quad$ $\operatorname{Re}\nu>-1$	$p^{-1}e^{a/p}I_\nu(a/p)$ $\quad$ $\operatorname{Re} p>0$
(22)	$I_\nu(2^{1/2}a^{1/2}t^{1/2})\,I_\nu(2^{1/2}\beta^{1/2}t^{1/2})$ $\quad$ $\operatorname{Re}\nu>-1$	$p^{-1}\exp[\frac{1}{2}(a+\beta)/p]\,I_\nu(a^{1/2}\beta^{1/2}/p)$ $\quad$ $\operatorname{Re} p>0$
(23)	$K_0(at)$	$s^{-1}\log(S/a)=s^{-1}\sinh^{-1}(s/a)$ $\quad$ $\operatorname{Re} p>-\operatorname{Re} a$
(24)	$K_\nu(at)$ $\quad$ $\lvert\operatorname{Re}\nu\rvert<1$	$\frac{1}{2}\pi\csc(\nu\pi)s^{-1}[a^{-\nu}S^{\nu}-a^{\nu}S^{-\nu}]$ $\quad$ $\operatorname{Re} p>-\operatorname{Re} a$

$s=(p^2-a^2)^{1/2}$, $\quad S=p+s$

Modified Bessel functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(25)	$tK_0(at)$	$ps^{-3}\log(S/a)-s^{-2}$ $\operatorname{Re} p>-\operatorname{Re} a$
(26)	$tK_1(at)$	$a^{-1}ps^{-2}-as^{-3}\log(S/a)$ $\operatorname{Re} p>-\operatorname{Re} a$
(27)	$t^{\mu-\frac{1}{2}}K_{\nu+\frac{1}{2}}(at)$ $\operatorname{Re}(\mu+\nu)>-1$ $\operatorname{Re}(\mu-\nu)>0$	$2^{-\frac{1}{2}}a^{-\frac{1}{2}}\pi^{\frac{1}{2}}\Gamma(\mu-\nu)\Gamma(\mu+\nu+1)$ $\times s^{-\mu}P_\nu^{-\mu}(p/a)$ $\operatorname{Re} p>-\operatorname{Re} a$
(28)	$t^{\mu}K_\nu(at)$ $\quad\operatorname{Re}(\mu\pm\nu)>-1$	$\dfrac{\sin(\mu\pi)\,\Gamma(\mu-\nu+1)}{\sin[(\mu+\nu)\pi]\,s^{\mu+1}}\,Q_\mu^\nu\left(\dfrac{p}{s}\right)$ $\operatorname{Re} p>-\operatorname{Re} a$
(29)	$\dfrac{1}{2t}\exp\left(-\dfrac{\lambda}{2at}\right)K_\nu(a\lambda t)$ $\operatorname{Re}(\lambda/a)>0$	$K_\nu(a^{-\frac{1}{2}}\lambda^{\frac{1}{2}}S^{\frac{1}{2}})\,K_\nu(a^{\frac{1}{2}}\lambda^{\frac{1}{2}}S^{-\frac{1}{2}})$ $\operatorname{Re} p>-\operatorname{Re}(a\lambda)$
(30)	$t^{-\frac{1}{2}}I_\mu(at)\,K_\nu(bt)$ $\operatorname{Re}(\mu\pm\nu)>-\frac{1}{2}$	$\dfrac{c^{\frac{1}{2}}\Gamma(\mu-\nu+\frac{1}{2})\cos(\mu\pi)}{\cos(\mu+\nu)\pi}\,P_{\nu-\frac{1}{2}}^{-\mu}(\cosh\alpha)$ $\times Q_{\mu-\frac{1}{2}}^{-\nu}(\cosh\beta)$ $\operatorname{Re}(p\pm a+b)>0$ for definition of α, β, and c see (12) of this section.
(31)	$t^{-\frac{1}{2}}K_\mu(at)\,K_\nu(bt)$ $\lvert\operatorname{Re}\mu\rvert+\lvert\operatorname{Re}\nu\rvert<\frac{1}{2}$	$\dfrac{c^{\frac{1}{2}}\Gamma(\frac{1}{2}-\mu-\nu)\cos(\mu\pi)\cos(\nu\pi)}{\cos(\mu+\nu)\pi\,\cos(\mu-\nu)\pi}$ $\times Q_{\nu+\frac{1}{2}}^{-\mu}(\cosh\alpha)\,Q_{\mu-\frac{1}{2}}^{-\nu}(\cosh\beta)$ $\operatorname{Re}(p+a+b)>0$ α, β, c defined in (12).

$s=(p^2-a^2)^{\frac{1}{2}},\qquad S=p+s$

Modified Bessel functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(32)	$K_0(2a^{1/2}t^{1/2})$	$-\frac{1}{2}p^{-1}e^{a/p}\,\mathrm{Ei}(-a/p) \qquad \mathrm{Re}\,p>0$
(33)	$K_1(2^{3/2}a^{1/2}t^{1/2})$	$2^{-3/2}a^{1/2}\pi^{1/2}p^{-3/2}e^{a/p}$ $\times[K_1(a/p)-K_0(a/p)] \qquad \mathrm{Re}\,p>0$
(34)	$t^{-1/2}K_0(2^{3/2}a^{1/2}t^{1/2})$	$\frac{1}{2}\pi^{1/2}p^{-1/2}e^{a/p}K_0(a/p) \qquad \mathrm{Re}\,p>0$
(35)	$t^{-1/2}K_\nu(2^{3/2}a^{1/2}t^{1/2})$ $\lvert\mathrm{Re}\,\nu\rvert<1$	$\frac{1}{2}\pi^{1/2}p^{-1/2}\sec(\frac{1}{2}\nu\pi)\,e^{a/p}K_{\frac{1}{2}\nu}(a/p)$ $\mathrm{Re}\,p>0$
(36)	$t^{\frac{1}{2}\nu}K_\nu(2a^{1/2}t^{1/2}) \qquad \mathrm{Re}\,\nu>-1$	$\frac{1}{2}a^{\frac{1}{2}\nu}\Gamma(\nu+1)p^{-\nu-1}e^{a/p}$ $\times\Gamma(-\nu,\,a/p) \qquad \mathrm{Re}\,p>0$
(37)	$t^{\mu-\frac{1}{2}}K_{2\nu}(2a^{1/2}t^{1/2})$ $\mathrm{Re}(\mu\pm\nu)>-\frac{1}{2}$	$\dfrac{\Gamma(\mu+\nu+\frac{1}{2})\Gamma(\mu-\nu+\frac{1}{2})}{2a^{1/2}p^{\mu}}e^{\frac{1}{2}a/p}$ $\times W_{-\mu,\nu}(a/p) \qquad \mathrm{Re}\,p>0$
(38)	$t^{-1/2}K_{2\nu}(2^{1/2}a^{1/2}t^{1/2})$ $\times\{\sin[(\nu-\frac{1}{4})\pi]J_{2\nu}(2^{1/2}a^{1/2}t^{1/2})$ $+\cos[(\nu-\frac{1}{4})\pi]Y_{2\nu}(2^{1/2}a^{1/2}t^{1/2})\}$ $\lvert\mathrm{Re}\,\nu\rvert<\frac{1}{4}$	$-2^{-3/2}\pi^{-1/2}p^{1/2}a^{-1}\Gamma(\frac{1}{4}+\nu)$ $\times\Gamma(\frac{1}{4}-\nu)\,W_{\frac{1}{4},\nu}(e^{\frac{1}{2}\pi i}a/p)$ $\times W_{\frac{1}{4},\nu}(e^{-\frac{1}{2}\pi i}a/p)$ $\mathrm{Re}\,p>0$
(39)	$t^{2\nu}K_{2\nu}(t^{1/2})\,I_{2\nu}(t^{1/2})$ $\mathrm{Re}\,\nu>-\frac{1}{4}$	$\frac{1}{2}\Gamma(2\nu+\frac{1}{2})p^{-3\nu-\frac{1}{2}}e^{\frac{1}{2}p^{-1}}$ $\times W_{-\nu,\nu}(p^{-1}) \qquad \mathrm{Re}\,p>0$

4.17. Modified Bessel functions of other arguments

(1)	$\exp\left(-\dfrac{t^2}{16a}\right)I_0\left(\dfrac{t^2}{16a}\right)$ $\mathrm{Re}\,a\geq 0$	$\dfrac{2^{1/2}a^{1/2}}{\pi^{1/2}}e^{ap^2}K_0(ap^2) \qquad \mathrm{Re}\,p>0$

Modified Bessel functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(2)	$t^{1/2} \exp\left(-\frac{t^2}{8a}\right) I_{1/4}\left(\frac{t^2}{8a}\right)$ $\quad \operatorname{Re} a \geq 0$	$\frac{2}{\Gamma(1/4)} \frac{a^{1/2}}{p^{1/2}} e^{ap^2} \Gamma(1/4, ap^2)$ $\quad \operatorname{Re} p > 0$
(3)	$t^{2\nu} \exp\left(-\frac{t^2}{8a}\right) I_\nu\left(\frac{t^2}{8a}\right)$ $\quad \operatorname{Re} a \geq 0, \quad \operatorname{Re} \nu > -1/4$	$\frac{a^{1/2\nu}\Gamma(4\nu+1)}{2^{4\nu}\Gamma(\nu+1)} p^{-\nu-1} e^{1/2 ap^2} \times W_{-3\nu/2,\nu/2}(ap^2)$ $\quad \operatorname{Re} p > 0$
(4)	$\frac{1}{t} \exp\left(-\frac{\alpha+\beta}{2t}\right) I_\nu\left(\frac{\alpha-\beta}{2t}\right)$ $\quad \operatorname{Re} \alpha \geq \operatorname{Re} \beta > 0$	$2K_\nu[(\alpha^{1/2}+\beta^{1/2})p^{1/2}]\, I_\nu[(\alpha^{1/2}-\beta^{1/2})p^{1/2}]$ $\quad \operatorname{Re} p > 0$
(5)	$0 \quad 0<t<b$ $I_0(a\gamma) \quad t>b$	$s^{-1} e^{-bs}$ $\quad \operatorname{Re} p > \lvert\operatorname{Re} a\rvert$
(6)	$0 \quad 0<t<b$ $t\, I_0(a\gamma) \quad t>b$	$p(bs^{-2}+s^{-3})e^{-bs}$ $\quad \operatorname{Re} p > \lvert\operatorname{Re} a\rvert$
(7)	$0 \quad 0<t<b$ $\gamma\, I_1(a\gamma) \quad t>b$	$a(bs^{-2}-s^{-3})e^{-bs}$ $\quad \operatorname{Re} p > \lvert\operatorname{Re} a\rvert$
(8)	$0 \quad 0<t<b$ $\gamma^{-1} I_1(a\gamma) \quad t>b$	$a^{-1} b^{-1}(e^{-bs}-e^{-bp})$ $\quad \operatorname{Re} p > \lvert\operatorname{Re} a\rvert$
(9)	$0 \quad 0<t<b$ $\gamma^{-1} t\, I_1(a\gamma) \quad t>b$	$a^{-1} ps^{-1}e^{-bs} - a^{-1}e^{-bp}$ $\quad \operatorname{Re} p > \lvert\operatorname{Re} a\rvert$

$\gamma = (t^2 - b^2)^{1/2} \qquad s = (p^2 - a^2)^{1/2}, \quad S = p + s$

Modified Bessel functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(10)	$0 \qquad 0<t<b$ $y^{\nu} I_{\nu}(ay) \qquad t>b$ $\mathrm{Re}\,\nu>-1$	$2^{1/2}\pi^{-1/2}a^{\nu}b^{\nu+1/2}s^{-\nu-1/2}K_{\nu+1/2}(bs)$ $\mathrm{Re}\,p>\lvert\mathrm{Re}\,a\rvert$
(11)	$0 \qquad 0<t<b$ $(t-b)^{1/2\nu}(t+b)^{-1/2\nu}I_{\nu}(ay)$ $t>b$ $\mathrm{Re}\,\nu>-1$	$a^{\nu}s^{-1}S^{-\nu}e^{-bs} \qquad \mathrm{Re}\,p>\lvert\mathrm{Re}\,a\rvert$
(12)	$I_0[a(t^2+\beta t)^{1/2}] \qquad \lvert\arg\beta\rvert<\pi$	$s^{-1}e^{1/2\beta(p-s)} \qquad \mathrm{Re}\,p>\lvert\mathrm{Re}\,a\rvert$
(13)	$(t^2+\beta t)^{1/2\nu}I_{\nu}[a(t^2+\beta t)^{1/2}]$ $\mathrm{Re}\,\nu>-1, \quad \lvert\arg\beta\rvert<\pi$	$\pi^{-1/2}(\tfrac{1}{2}a)^{\nu}(\beta/s)^{\nu+1/2}e^{1/2\beta p}$ $\times K_{\nu+1/2}(\tfrac{1}{2}\beta s) \qquad \mathrm{Re}\,p>\lvert\mathrm{Re}\,a\rvert$
(14)	$t^{1/2\nu}(t+\beta)^{-1/2\nu}I_{\nu}[a(t^2+\beta t)^{1/2}]$ $\mathrm{Re}\,\nu>-1, \quad \lvert\arg\beta\rvert<\pi$	$a^{\nu}s^{-1}S^{-\nu}e^{1/2\beta(p-s)} \qquad \mathrm{Re}\,p>\lvert\mathrm{Re}\,a\rvert$
(15)	$t^{\mu-1}(t+\beta)^{-\mu}I_{2\nu}[a(t^2+\beta t)^{1/2}]$ $\mathrm{Re}(\mu+\nu)>0, \quad \lvert\arg\beta\rvert<\pi$	$\dfrac{2\Gamma(\mu+\nu)e^{1/2\beta p}}{a\beta\Gamma(2\nu+1)}M_{1/2-\mu,\nu}\left(\dfrac{a^2\beta}{2S}\right)$ $\times W_{1/2-\mu,\nu}\left(\dfrac{\beta S}{2}\right) \qquad \mathrm{Re}\,p>\lvert\mathrm{Re}\,a\rvert$
	For more general formulas see MacRobert, T. M., 1948: *Philos. Mag.* (7) 39, pp. 466-471.	
(16)	$(2t-t^2)^{1/2\nu-1/4}C_n^{\nu}(t-1)$ $\times I_{\nu-1/2}[a(2t-t^2)^{1/2}]$ $0<t<2$ $0 \qquad t>2$ $\mathrm{Re}\,\nu>-\tfrac{1}{2}$	$(-1)^n\dfrac{2^{1/2}\pi^{1/2}a^{\nu-1/2}}{r^{\nu}e^{p}}C_n^{\nu}\left(\dfrac{p}{r}\right)I_{\nu+n}(r)$ $r=(p^2+a^2)^{1/2}$

$y=(t^2-b^2)^{1/2}, \quad s=(p^2-a^2)^{1/2}, \quad S=p+s$

Modified Bessel functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(17)	$\exp[a(1-e^{-t})]\, I_\nu(ae^{-t})$	$\frac{I_\nu(a)}{\nu+p}+\sum_{n=1}^{\infty}\frac{(\nu-p+1)_{n-1}}{(\nu+p)_{n+1}}(\nu+n)$ $\times I_{\nu+p}(a)$ $\quad$ Re $p>-$ Re ν
(18)	$t^{-1/2} e^{-a/t} K_\nu(a/t)$ $\quad$ Re $a>0$	$2\pi^{1/2}p^{-1/2}K_{2\nu}(2^{3/2}a^{1/2}p^{1/2})$ $\quad$ Re $p>0$
(19)	$\frac{1}{t}\exp\left(-\frac{a+\beta}{2t}\right)K_\nu\left(\frac{a-\beta}{2t}\right)$ $\quad$ Re $a>$ Re $\beta>0$	$2K_\nu[(a^{1/2}+\beta^{1/2})p^{1/2}]K_\nu[(a^{1/2}-\beta^{1/2})p^{1/2}]$ $\quad$ Re $p>0$
(20)	$t^{\mu-1}(t+\beta)^{-\mu}K_{2\nu}[a(t^2+\beta t)^{1/2}]$ $\quad$ Re$(\mu\pm\nu)>0$, $\quad$ $\lvert\arg\beta\rvert<\pi$	$a^{-1}\beta^{-1}\Gamma(\mu+\nu)\Gamma(\mu-\nu)e^{1/2\beta p}$ $\times W_{1/2-\mu,\nu}(\tfrac12 a^2\beta S^{-1})$ $\times W_{1/2-\mu,\nu}(\tfrac12\beta S)$ $\quad$ Re $p>\lvert$Re $a\rvert$
(21)	$-2\pi^{-1}K_0[2a\sinh(\tfrac12 t)]$ $\quad$ Re $a>0$	$J_p(a)\,\frac{\partial Y_p(a)}{\partial p}-Y_p(a)\,\frac{\partial J_p(a)}{\partial p}$
(22)	$-2\pi^{-1}\cosh t\; K_0[2a\sinh(\tfrac12 t)]$ $\quad$ Re $a>0$	$J_p'(a)\,\frac{\partial Y_p'(a)}{\partial p}-Y_p'(a)\,\frac{\partial J_p'(a)}{\partial p}$ $+\frac{p^2}{a^2}\Big[J_p(a)\,\frac{\partial Y_p(a)}{\partial p}$ $-Y_p(a)\,\frac{\partial J_p(a)}{\partial p}\Big]$ $\quad$ $\left[J_p'=\frac{dJ_p}{da}\right]$
(23)	$2\pi^{-2}\sin(2\nu\pi)$ $\times K_{2\nu}[2a\sinh(\tfrac12 t)]$ $\quad$ Re $a>0$	$J_{\nu-p}(a)\,Y_{-\nu-p}(a)-J_{-\nu-p}(a)\,Y_{\nu-p}(a)$

$\gamma=(t^2-b^2)^{1/2}, \quad s=(p^2-a^2)^{1/2}, \quad S=p+s$

Modified Bessel functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(24)	$\operatorname{csch}(\tfrac{1}{2}t)\,K_{2\nu}[a\operatorname{csch}(\tfrac{1}{2}t)]$ $\quad \operatorname{Re} a>0$	$a^{-1}\Gamma(p+\nu+\tfrac{1}{2})\Gamma(p-\nu+\tfrac{1}{2})$ $\times W_{-p,\nu}(ia)\,W_{-p,\nu}(-ia)$ $\quad \operatorname{Re}(p\pm\nu)>-1$
(25)	$\dfrac{1}{\sinh(\tfrac{1}{2}t)}\exp\left(-\dfrac{ae^{t}+\beta}{e^{t}-1}\right)$ $\times K_{2\nu}\left[\dfrac{a^{1/2}\beta^{1/2}}{\sinh(\tfrac{1}{2}t)}\right]$ $\quad \operatorname{Re} a>0,\quad \operatorname{Re}\beta>0$	$\dfrac{1}{a^{1/2}\beta^{1/2}}\Gamma(p+\nu+\tfrac{1}{2})\Gamma(p-\nu+\tfrac{1}{2})$ $\times e^{-\frac{1}{2}a+\frac{1}{2}\beta}W_{-p,\nu}(a)\,W_{-p,\nu}(\beta)$ $\quad \operatorname{Re}(p\pm\nu)>-\tfrac{1}{2}$

4.18. Kelvin's functions and related functions

(1)	$\operatorname{ber} t$	$[\tfrac{1}{2}(p^4+1)^{-1/2}+\tfrac{1}{2}p^2(p^4+1)^{-1}]^{1/2}$ $\quad \operatorname{Re} p>2^{-1/2}$
(2)	$\operatorname{bei} t$	$[\tfrac{1}{2}(p^4+1)^{-1/2}-\tfrac{1}{2}p^2(p^4+1)^{-1}]^{1/2}$ $\quad \operatorname{Re} p>2^{-1/2}$
(3)	$\operatorname{ber}(2t^{1/2})$	$p^{-1}\cos p^{-1}$ $\quad \operatorname{Re} p>0$
(4)	$\operatorname{bei}(2t^{1/2})$	$p^{-1}\sin p^{-1}$ $\quad \operatorname{Re} p>0$
(5)	$t^{\frac{1}{2}\nu}\operatorname{ber}_{\nu}(t^{1/2})$ $\quad \operatorname{Re}\nu>-1$	$2^{-\nu}p^{-\nu-1}\cos[\tfrac{1}{4}(1+3\nu\pi p)/p]$ $\quad \operatorname{Re} p>0$
(6)	$t^{\frac{1}{2}\nu}\operatorname{bei}_{\nu}(t^{1/2})$ $\quad \operatorname{Re}\nu>-1$	$2^{-\nu}p^{-\nu-1}\sin[\tfrac{1}{4}(1+3\nu\pi p)/p]$ $\quad \operatorname{Re} p>0$
(7)	$0 \quad 0<t<b$ $\operatorname{ber}(ay)+i\operatorname{bei}(ay) \quad t>b$	$v^{-1}e^{-bv}$ $\quad \operatorname{Re}(p\pm ai^{1/2})>0$

$y=(t^2-b^2)^{1/2},\quad v=(p^2-ia^2)^{1/2},\quad V=p+v$

Kelvin's functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(8)	$0 \qquad 0<t<b$ $t[\mathrm{ber}(ay)+i\,\mathrm{bei}(ay)] \qquad t>b$	$pv^{-3}(bv+1)e^{-bv}$ $\mathrm{Re}\,(p \pm ai^{1/2})>0$
(9)	$0 \qquad 0<t<b$ $y[\mathrm{ber}_1(ay)+i\,\mathrm{bei}_1(ay)] \qquad t>b$	$av^{-3}(bv+1)e^{-bv+\frac{3}{4}\pi i}$ $\mathrm{Re}\,(p \pm ai^{1/2})>0$
(10)	$0 \qquad 0<t<b$ $y^{-1}[\mathrm{ber}_1(ay)+i\,\mathrm{bei}_1(ay)]$ $t>b$	$a^{-1}b^{-1}e^{-\frac{3}{4}\pi i}(e^{-bp}-e^{-bv})$ $\mathrm{Re}\,(p \pm ai^{1/2})>0$
(11)	$0 \qquad 0<t<b$ $ty^{-1}[\mathrm{ber}_1(ay)+i\,\mathrm{bei}_1(ay)]$ $t>b$	$a^{-1}e^{-\frac{3}{4}\pi i}(e^{-bp}-pv^{-1}e^{-bv})$ $\mathrm{Re}\,(p \pm ai^{1/2})>0$
(12)	$0 \qquad 0<t<b$ $\left(\dfrac{t-b}{t+b}\right)^{\frac{1}{2}\nu}[\mathrm{ber}_\nu(ay)$ $+\,i\,\mathrm{bei}_\nu(ay)] \qquad t>b$ $\mathrm{Re}\,\nu>-1$	$a^{\nu}v^{-1}V^{-\nu}e^{\frac{3}{4}\nu\pi i-bv}$ $\mathrm{Re}\,(p \pm ai^{1/2})>0$
(13)	$t[\mathrm{ker}(at)+i\,\mathrm{kei}(at)]$	$pv^{-3}\log(i^{-1/2}V/a)-v^{-2}$ $\mathrm{Re}\,(p \pm ai^{1/2})>0$
(14)	$t[\mathrm{ker}_1(at)+i\,\mathrm{kei}_1(at)]$	$i^{1/2}p\,a^{-1}v^{-2}+ai^{3/2}v^{-3}\log(i^{-1/2}V/a)$ $\mathrm{Re}\,(p \pm ai^{1/2})>0$
(15)	$0 \qquad 0<t<b$ $\mathrm{ker}(ay)+i\,\mathrm{kei}(ay) \qquad t>b$	$v^{-1}e^{-bv}\log(i^{-1/2}V/a)$ $\mathrm{Re}\,(p+ai^{1/2})>0$

$y=(t^2-b^2)^{1/2}, \quad v=(p^2-ia^2)^{1/2}, \quad V=p+v$

Kelvin's functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(16)	$0 \qquad 0<t<b$ $t[\operatorname{ker}(a\gamma)+i\operatorname{kei}(a\gamma)] \qquad t>b$	$-v^{-2}e^{-bv}[1+(bp-pv^{-1})\log(i^{-1/2}V/a)]$ $\operatorname{Re}(p+ai^{1/2})>0$
(17)	$0 \qquad 0<t<b$ $\gamma[\operatorname{ker}_1(a\gamma)+i\operatorname{kei}_1(a\gamma)] \qquad t>b$	$v^{-1}e^{-bv}[i^{1/2}p/a+i^{3/2}a(b+1/v)$ $\times\log(i^{-1/2}V/a)] \quad \operatorname{Re}(p+ai^{1/2})>0$
(18)	$0 \qquad 0<t<b$ $\left(\dfrac{t-b}{t+b}\right)^{\frac{1}{2}\nu}[\operatorname{ker}_\nu(a\gamma)$ $+i\operatorname{kei}_\nu(a\gamma)] \qquad t>b$ $\lvert\operatorname{Re}\nu\rvert<1$	$\dfrac{\pi e^{-bp-\frac{1}{2}i\nu\pi}}{2v\sin(\nu\pi)}\left[\left(\dfrac{V}{ai^{1/2}}\right)^\nu-\left(\dfrac{ai^{1/2}}{V}\right)^\nu\right]$ $\operatorname{Re}(p+ai^{1/2})>0$
(19)	$V_\nu^{(b)}(2t^{1/2}) \qquad \operatorname{Re}\nu>0$	$p\,I_\nu(2p^{-1}) \qquad \operatorname{Re}p>0$
(20)	$t^{1/2}W_\nu^{(b)}(2t^{1/2}) \qquad \operatorname{Re}\nu>-2$	$p^{-2}I_\nu(2p^{-1}) \qquad \operatorname{Re}p>0$
(21)	$X_\nu^{(b)}(2t^{1/2}) \qquad \operatorname{Re}\nu>-1$	$p^{-1}I_\nu(2p^{-1}) \qquad \operatorname{Re}p>0$
(22)	$t^{-1/2}Z_\nu^{(b)}(2t^{1/2}) \qquad \operatorname{Re}\nu>0$	$I_\nu(2p^{-1}) \qquad \operatorname{Re}p>0$

4.19. Functions related to Bessel functions, Struve, Lommel, and Bessel integral functions

(1)	$\mathbf{H}_0(at)$	$2\pi^{-1}r^{-1}\log(r/p+a/p)$ $\operatorname{Re}p>\lvert\operatorname{Im}a\rvert$
(2)	$\mathbf{H}_1(at)$	$\dfrac{2}{\pi p}-\dfrac{2p}{\pi a r}\log\dfrac{r+a}{p} \qquad \operatorname{Re}p>\lvert\operatorname{Im}a\rvert$

$\gamma=(t^2-b^2)^{1/2}, \quad v=(p^2-ia^2)^{1/2}, \quad V=p+v, \quad r=(p^2+a^2)^{1/2}$

Related functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(3)	$\mathbf{H}_2(at)$	$\frac{2}{\pi}\left(-\frac{2}{a}+\frac{a}{3p^2}+\frac{a^2+2p^2}{a^2 r}\log\frac{r+a}{p}\right)$ $\operatorname{Re} p > \lvert\operatorname{Im} a\rvert$
(4)	$\mathbf{H}_3(at)$	$2\pi^{-1}p^{-1}(1/3+4a^{-2}p^2+2a^2p^{-2}/15)$ $-\frac{6a^2p+8p^3}{\pi a^3 r}\log\frac{r+a}{p}$ $\operatorname{Re} p > \lvert\operatorname{Im} a\rvert$
(5)	$\mathbf{H}_{\frac{1}{2}}(at)$ $\operatorname{Re} a>0$	$(\tfrac{1}{2}ap)^{-\frac{1}{2}}-a^{-\frac{1}{2}}R^{\frac{1}{2}}/r$ $\operatorname{Re} p > \lvert\operatorname{Im} a\rvert$
(6)	$\mathbf{H}_{-n-\frac{1}{2}}(at)$	$(-1)^n a^{n+\frac{1}{2}}R^{-n-\frac{1}{2}}/r$ $\operatorname{Re} p > \lvert\operatorname{Im} a\rvert$
(7)	$t^{-1}\mathbf{H}_1(at)$	$\frac{2}{\pi}\left(-1+\frac{r}{a}\log\frac{r+a}{p}\right)$ $\operatorname{Re} p > \lvert\operatorname{Im} a\rvert$
(8)	$t^{-1}\mathbf{H}_2(at)$	$\frac{2}{\pi}\left(\frac{p}{a}+\frac{a}{3p}-\frac{r}{a}\log\frac{a+r}{p}\right)$ $\operatorname{Re} p > \lvert\operatorname{Im} a\rvert$
(9)	$t^{-1}\mathbf{H}_3(at)$	$\frac{2}{\pi}\left(\frac{a^2}{15p^2}-\frac{4p^2}{3a^2}-\frac{7}{9}+\frac{4p^2r+a^2r}{3a^3}\log\frac{r+a}{p}\right)$ $\operatorname{Re} p > \lvert\operatorname{Im} a\rvert$
(10)	$t^{\frac{1}{2}}\mathbf{H}_{\frac{1}{2}}(at)$	$2^{1/2}\pi^{-1/2}a^{3/2}p^{-1}r^{-2}$ $\operatorname{Re} p > \lvert\operatorname{Im} a\rvert$
(11)	$t^{\frac{1}{2}}\mathbf{H}_{-\frac{1}{2}}(at)$	$2^{\frac{1}{2}}\pi^{-\frac{1}{2}}a^{\frac{1}{2}}r^{-2}$ $\operatorname{Re} p > \lvert\operatorname{Im} a\rvert$

$r=(p^2+a^2)^{\frac{1}{2}}, \quad R=p+r$

Related functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(12)	$t^{1/2}\,\mathbf{H}_{3/2}(at)$	$2^{1/2}\pi^{-1/2}a^{1/2}[\tfrac{1}{2}p^{-2}-r^{-2}+a^{-2}\log(r/p)]$ $\quad \operatorname{Re} p > \lvert\operatorname{Im} a\rvert$
(13)	$t^{1/2}\,\mathbf{H}_{-3/2}(at)$	$2^{1/2}\pi^{-1/2}[a^{-1/2}pr^{-2} - a^{-3/2}\tan^{-1}(a/p)]$ $\quad \operatorname{Re} p > \lvert\operatorname{Im} a\rvert$
(14)	$t^{-1/2}\,\mathbf{H}_{1/2}(at)$	$(\tfrac{1}{2}\pi a)^{-1/2}\log(r/p)$ $\quad \operatorname{Re} p > \lvert\operatorname{Im} a\rvert$
(15)	$t^{-1/2}\,\mathbf{H}_{-1/2}(at)$	$(\tfrac{1}{2}\pi a)^{-1/2}\tan^{-1}(a/p)$ $\quad \operatorname{Re} p > \lvert\operatorname{Im} a\rvert$
(16)	$t^{-1/2}\,\mathbf{H}_{3/2}(at)$	$(\tfrac{1}{2}\pi a)^{-1/2}[\tfrac{1}{2}ap^{-1}-a^{-1}p\log(r/p)]$ $\quad \operatorname{Re} p > \lvert\operatorname{Im} a\rvert$
(17)	$t^{3/2}\,\mathbf{H}_{3/2}(at)$	$2^{1/2}\pi^{-1/2}a^{5/2}(3p^2+a^2)p^{-3}r^{-4}$ $\quad \operatorname{Re} p > \lvert\operatorname{Im} a\rvert$
(18)	$\mathbf{L}_0(at)$	$2\pi^{-1}s^{-1}\sin^{-1}(a/p)$ $\quad \operatorname{Re} p > \lvert\operatorname{Re} a\rvert$
(19)	$\mathbf{L}_1(at)$	$2\pi^{-1}p^{-1}[-1+a^{-1}p^2s^{-1}\sin^{-1}(a/p)]$ $\quad \operatorname{Re} p > \lvert\operatorname{Re} a\rvert$
(20)	$\mathbf{L}_2(at)$	$\dfrac{2}{a\pi}\left(-2-\dfrac{a^2}{3p^2}+\dfrac{2p^2-a^2}{as}\sin^{-1}\dfrac{a}{p}\right)$ $\quad \operatorname{Re} p > \lvert\operatorname{Re} a\rvert$
(21)	$\mathbf{L}_3(at)$	$\dfrac{2}{\pi p}\left(\dfrac{1}{3}-\dfrac{4p^2}{a^2}-\dfrac{2a^2}{15p^2}+\dfrac{4p^4-3ap^2}{a^3s}\sin^{-1}\dfrac{a}{p}\right)$ $\quad \operatorname{Re} p > \lvert\operatorname{Re} a\rvert$

$r=(p^2+a^2)^{1/2}, \quad R=p+r, \quad s=(p^2-a^2)^{1/2}, \quad S=p+s$

Related functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(22)	$t^{-1}\,\mathbf{L}_1(at)$	$2\pi^{-1}[1-a^{-1}s\sin^{-1}(a/p)]$ $\quad \mathrm{Re}\,p > \lvert\mathrm{Re}\,a\rvert$
(23)	$t^{-2}\,\mathbf{L}_2(at)$	$2\pi^{-1}[p/a-ap^{-1}/3$ $-a^{-2}ps\sin^{-1}(a/p)]$ $\quad \mathrm{Re}\,p > \lvert\mathrm{Re}\,a\rvert$
(24)	$t^{-1}\,\mathbf{L}_3(at)$	$\dfrac{2}{\pi}\left(\dfrac{4p^2}{3a^2}-\dfrac{7}{9}-\dfrac{a^2}{15p^2} - \dfrac{4p^2s-a^2s}{3a^3}\sin^{-1}\dfrac{a}{p}\right)$ $\quad \mathrm{Re}\,p > \lvert\mathrm{Re}\,a\rvert$
(25)	$\mathbf{L}_{1/2}(at)$	$a^{-1/2}S^{1/2}/s-(\tfrac{1}{2}ap)^{-1/2}$ $\quad \mathrm{Re}\,p > \lvert\mathrm{Re}\,a\rvert$
(26)	$\mathbf{L}_{-n-1/2}(at)$	$a^{n+1/2}s^{-1}S^{-n-1/2}$ $\quad \mathrm{Re}\,p > \lvert\mathrm{Re}\,a\rvert$
(27)	$t^{1/2}\,\mathbf{L}_{1/2}(at)$	$2^{1/2}\pi^{-1/2}a^{3/2}p^{-1}s^{-2}$ $\quad \mathrm{Re}\,p > \lvert\mathrm{Re}\,a\rvert$
(28)	$t^{1/2}\,\mathbf{L}_{-1/2}(at)$	$2^{1/2}\pi^{-1/2}a^{1/2}s^{-2}$ $\quad \mathrm{Re}\,p > \lvert\mathrm{Re}\,a\rvert$
(29)	$t^{1/2}\,\mathbf{L}_{3/2}(at)$	$2^{1/2}a^{1/2}\pi^{-1/2}[s^{-2}-\tfrac{1}{2}p^{-2}$ $-a^{-2}\log(s/p)]$ $\quad \mathrm{Re}\,p > \lvert\mathrm{Re}\,a\rvert$
(30)	$t^{1/2}\,\mathbf{L}_{-3/2}(at)$	$(\tfrac{1}{2}\pi a)^{-1/2}[ps^{-2}-a^{-1}\operatorname{ctnh}^{-1}(p/a)]$ $\quad \mathrm{Re}\,p > \lvert\mathrm{Re}\,a\rvert$
(31)	$t^{-1/2}\,\mathbf{L}_{1/2}(at)$	$-(\tfrac{1}{2}\pi a)^{-1/2}\log(s/p)$ $\quad \mathrm{Re}\,p > \lvert\mathrm{Re}\,a\rvert$

$s=(p^2-a^2)^{1/2}, \quad S=p+s$

Related functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(32)	$t^{-1/2}\,\mathbf{L}_{-1/2}(at)$	$(\tfrac{1}{2}a\pi)^{-1/2}\,\mathrm{ctnh}^{-1}(p/a)$ $\mathrm{Re}\,p > \lvert\mathrm{Re}\,a\rvert$
(33)	$t^{-1/2}\,\mathbf{L}_{3/2}(at)$	$(\tfrac{1}{2}a\pi)^{-1/2}[a^{-1}p\log(s/p)-\tfrac{1}{2}ap^{-1}]$ $\mathrm{Re}\,p > \lvert\mathrm{Re}\,a\rvert$
(34)	$t^{3/2}\,\mathbf{L}_{3/2}(at)$	$2^{1/2}\pi^{-1/2}a^{5/2}(3p^2-a^2)p^{-3}s^{-4}$ $\mathrm{Re}\,p > \lvert\mathrm{Re}\,a\rvert$
(35)	$t^{\nu}\,\mathbf{L}_{\nu}(at)$ $\quad \mathrm{Re}\,\nu > -\tfrac{1}{2}$	$\dfrac{(2a)^{\nu}\,\Gamma(\nu+\tfrac{1}{2})}{\pi^{1/2}\,s^{2\nu+1}} - \dfrac{\Gamma(2\nu+1)\,(a/p)^{\nu}}{(\tfrac{1}{2}\pi p)^{1/2}(a^2-p^2)^{-\frac{1}{2}\nu-\frac{1}{4}}}\,P^{-\nu-\frac{1}{2}}_{-\nu-\frac{1}{2}}\left(\dfrac{a}{p}\right)$ $\mathrm{Re}\,p > \lvert\mathrm{Re}\,a\rvert$
(36)	$t^{\frac{1}{2}\nu}\,\mathbf{L}_{\nu}(t^{1/2})$ $\quad \mathrm{Re}\,\nu > -3/2$	$2^{-\nu}p^{-\nu-1}e^{\frac{1}{4}p^{-1}}\,\mathrm{Erf}(\tfrac{1}{2}p^{-1/2})$ $\mathrm{Re}\,p > 0$
(37)	$t^{\frac{1}{2}\nu}\,\mathbf{L}_{-\nu}(t^{1/2})$	$\dfrac{2^{-\nu}p^{-\nu-1}}{\Gamma(\tfrac{1}{2}-\nu)}\,e^{\frac{1}{4}p^{-1}}\,\gamma(\tfrac{1}{2}-\nu,\tfrac{1}{4}p^{-1})$ $\mathrm{Re}\,p > 0$
(38)	$t^{1/2}\,s_{\mu,\frac{1}{4}}(\tfrac{1}{2}t^2)$ $\quad \mathrm{Re}\,\mu > -\tfrac{3}{4}$	$2^{-2\mu-1}p^{1/2}\,\Gamma(2\mu+3/2)$ $\quad \mathrm{Re}\,p > 0$ $\times\, S_{-\mu-1,\frac{1}{4}}(\tfrac{1}{2}p^2)$
	For further similar formulas see Meijer, C. S., 1935: *Nederl. Akad. Wetensch., Proc.* 38, 628-634.	
(39)	$Ji_0(t)$	$-p^{-1}\sinh^{-1}p$ $\quad \mathrm{Re}\,p>0$
(40)	$Ji_{\nu}(t)$ $\quad \mathrm{Re}\,\nu > 0$	$\nu^{-1}p^{-1}[(p^2+1)^{1/2}-p]^{\nu}-\nu^{-1}p^{-1}$ $\mathrm{Re}\,p>0$
(41)	$Ji_0(2t^{1/2})$	$\tfrac{1}{2}p^{-1}\,\mathrm{Ei}(-p^{-1})$ $\quad \mathrm{Re}\,p>0$

$s=(p^2-a^2)^{1/2}, \quad S=p+s$

4.20. Parabolic cylinder functions

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\, dt$
(1)	$t^{\nu} e^{\frac{1}{4}t^2} D_{-\mu}(t) \qquad \operatorname{Re}\nu>-1$	$\dfrac{\Gamma(\nu+1)}{\Gamma(\mu)} \int_0^\infty x^{\mu-1}(p+x)^{-\nu-1} \times e^{-\frac{1}{2}x^2}\, dx$ $= p^{\mu-\nu-1} \sum_{r=0}^{\infty} (r!)^{-1} (\mu)_{2r} \times \Gamma(\nu-2r-\mu+1)(-\tfrac{1}{2}p^2)^r$ $\operatorname{Re} p>0$
(2)	$\exp\left(-\dfrac{t^2}{4a}\right)\left[D_{-2\nu}\left(-\dfrac{t}{a}\right) - D_{-2\nu}\left(\dfrac{t}{a}\right)\right]$	$2^{\frac{1}{2}} \pi^{\frac{1}{2}} a^{1-2\nu} p^{-2\nu} e^{\frac{1}{2}a^2p^2} \times \dfrac{\Gamma(\nu, \frac{1}{2}a^2p^2)}{\Gamma(\nu)} \qquad \operatorname{Re} p>0$
(3)	$D_{2n+1}(2^{\frac{1}{2}} t^{\frac{1}{2}})$	$(-2)^n \Gamma(n+3/2)(p-1/2)^n \times (p+1/2)^{-n-3/2} \qquad \operatorname{Re} p>-\tfrac{1}{2}$
(4)	$D_{2\nu}(-2a^{\frac{1}{2}}t^{\frac{1}{2}}) - D_{2\nu}(2a^{\frac{1}{2}}t^{\frac{1}{2}})$	$\dfrac{2^{\nu+3/2} \pi a^{1/2} (p-a)^{\nu-1/2}}{\Gamma(-\nu)(p+a)^{\nu+1}}$ $\operatorname{Re} p>\lvert\operatorname{Re} a\rvert$
(5)	$t^{-\frac{1}{2}} D_{2n}(2^{\frac{1}{2}}t^{\frac{1}{2}})$	$(-2)^n \Gamma(n+\tfrac{1}{2})(p-\tfrac{1}{2})^n (p+\tfrac{1}{2})^{-n-\frac{1}{2}}$ $\operatorname{Re} p>-\tfrac{1}{2}$
(6)	$t^{-\frac{1}{2}}[D_{2\nu}(2a^{\frac{1}{2}}t^{\frac{1}{2}}) + D_{2\nu}(-2a^{\frac{1}{2}}t^{\frac{1}{2}})]$	$2^{\nu+1}\pi(p-a)^{\nu}(p+a)^{-\nu-\frac{1}{2}}/\Gamma(\tfrac{1}{2}-\nu)$ $\operatorname{Re} p>\lvert\operatorname{Re} a\rvert$
(7)	$t^{-\frac{1}{2}\nu-\frac{1}{2}} e^{\frac{1}{4}t} D_{\nu}(t^{\frac{1}{2}}) \qquad \operatorname{Re}\nu<1$	$\pi^{\frac{1}{2}} p^{-\frac{1}{2}}(1+2^{\frac{1}{2}}p^{\frac{1}{2}})^{\nu} \qquad \operatorname{Re} p>0$
(8)	$t^{-\nu/2-3/2} e^{t/4} D_{\nu}(t^{1/2})$ $\operatorname{Re}\nu<-1$	$-2^{\frac{1}{2}}\pi^{\frac{1}{2}}(\nu+1)^{-1}(1+2^{\frac{1}{2}}p^{\frac{1}{2}})^{\nu+1}$ $\operatorname{Re} p>0$

Parabolic cylinder functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(9)	$t^{\nu-1}e^{\frac{1}{4}t}D_{2\nu+2n-1}(t^{\frac{1}{2}})$ $\operatorname{Re}\nu>0$	$\dfrac{\pi^{\frac{1}{2}}\Gamma(2n+2\nu)(1-2p)^n}{2^{2n-\frac{1}{2}+\nu}n!\,p^{n+\nu}}$ $\times {}_2F_1(n+\nu,\tfrac{1}{2}-\nu;n+1;1-\tfrac{1}{2}p^{-1})$ $\operatorname{Re}p>0$
(10)	$t^{\nu-1}e^{\frac{1}{4}t}D_{2\mu-1}(t^{\frac{1}{2}})$ $\operatorname{Re}\nu>0,\quad \operatorname{Re}(\nu-\mu)>-1$	$2^{\frac{1}{2}}\pi^{\frac{1}{2}}\Gamma(2\nu)(2p)^{-\frac{1}{2}\mu-\frac{1}{2}\nu}$ $\times(2p-1)^{\frac{1}{2}\mu-\frac{1}{2}\nu}P^{\mu-\nu}_{\mu+\nu-1}(2^{-\frac{1}{2}}p^{-\frac{1}{2}})$ $\operatorname{Re}p>0$
(11)	$[D_{-n-1}(-i2^{\frac{1}{2}}t^{\frac{1}{2}})]^2$ $-[D_{-n-1}(i2^{\frac{1}{2}}t^{\frac{1}{2}})]^2$	$\dfrac{2\pi i}{n!\,p^{\frac{1}{2}}}\dfrac{(p-1)^n}{(p+1)^{n+1}}$ $\qquad\operatorname{Re}p>0$
(12)	$t^{-\nu}e^{-a/(8t)}$ $\times D_{2\nu-1}(2^{-\frac{1}{2}}a^{\frac{1}{2}}t^{-\frac{1}{2}})$ $\operatorname{Re}a>0$	$2^{\nu-\frac{1}{2}}\pi^{\frac{1}{2}}p^{\nu-1}e^{-a^{\frac{1}{2}}p^{\frac{1}{2}}}$ $\operatorname{Re}p>0$
(13)	$\dfrac{e^{\frac{1}{2}t}}{(e^t-1)^{\mu+\frac{1}{2}}}\exp\left(-\dfrac{a}{1-e^{-t}}\right)$ $\times D_{2\mu}\left[\dfrac{2a^{\frac{1}{2}}}{(1-e^{-t})^{\frac{1}{2}}}\right]$ $\operatorname{Re}a>0$	$e^{-a}2^{p+\mu}\Gamma(p+\mu)D_{-2p}(2a^{\frac{1}{2}})$ $\operatorname{Re}p>-\operatorname{Re}\mu$

4.21. Gauss' hypergeometric function

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(1)	$t^{a-1}F(\tfrac{1}{2}+\nu, \tfrac{1}{2}-\nu;\ a; -\tfrac{1}{2}t)$ $\mathrm{Re}\ a > 0$	$\pi^{-1/2}\Gamma(a)(2p)^{1/2-a}K_\nu(p)$ $\mathrm{Re}\ p > 0$
(2)	$t^{\gamma-1}F(a, \beta; \delta; -t) \qquad \mathrm{Re}\ \gamma > 0$	$\dfrac{\Gamma(\delta)\,p^{-\gamma}}{\Gamma(a)\,\Gamma(\beta)} E(a, \beta, \gamma: \delta: p)$ $\mathrm{Re}\ p > 0$
(3)	$t^{\gamma-1}(1+t)^{a+\beta-\delta}F(a, \beta; \delta; -t)$ $\mathrm{Re}\ \gamma > 0$	$\dfrac{\Gamma(\delta)\,p^{-\gamma}}{\Gamma(\delta-a)\Gamma(\delta-\beta)}$ $\times E(\delta-a, \delta-\beta, \gamma: \delta: p) \quad \mathrm{Re}\ p > 0$
(4)	$t^{\gamma-1}F(2a, 2\beta; \gamma; -\lambda t)$ $\mathrm{Re}\ \gamma > 0, \quad \lvert\arg\lambda\rvert < \pi$	$\Gamma(\gamma)\,p^{-\gamma}(p/\lambda)^{a+\beta-1/2}e^{1/2 p/\lambda}$ $\times W_{1/2-a-\beta,\, a-\beta}(\tfrac{1}{2}p/\lambda) \quad \mathrm{Re}\ p > 0$
(5)	$0 \qquad 0 < t < 1$ $(t^2-1)^{2a-1/2}$ $\times F(a-\tfrac{1}{2}\nu, a+\tfrac{1}{2}\nu; 2a+\tfrac{1}{2}; 1-t^2)$ $t > 1$ $\mathrm{Re}\ a > -\tfrac{1}{4}$	$2^{2a}\pi^{-1/2}p^{-2a}\Gamma(2a+\tfrac{1}{2})K_\nu(p)$ $\mathrm{Re}\ p > 0$
(6)	$[(a+t)(\beta+t)]^{-1/2-\nu}$ $\times F\left[\dfrac{1}{2}+\nu, \dfrac{1}{2}+\nu; 1; \dfrac{t(a+\beta+t)}{(a+t)(\beta+t)}\right]$ $\lvert\arg a\rvert < \pi, \quad \lvert\arg\beta\rvert < \pi$	$\pi^{-1}(a\beta)^{-\nu}e^{1/2(a+\beta)p}K_\nu(\tfrac{1}{2}ap)K_\nu(\tfrac{1}{2}\beta p)$ $\lvert\arg ap\rvert < \pi, \quad \lvert\arg\beta\pi\rvert < \pi$ $\mathrm{Re}\ p > 0$
(7)	$t^{-1/2}(1+a/t)^{\mu}(1+\beta/t)^{\nu}$ $\times F\left[-\mu, -\nu; \tfrac{1}{2}-\mu-\nu; \dfrac{t(a+\beta+t)}{(a+t)(\beta+t)}\right]$ $\lvert\arg a\rvert < \pi, \quad \lvert\arg\beta\rvert < \pi$ $\mathrm{Re}(\mu+\nu) < 1$	$2^{-\mu-\nu}\Gamma(\tfrac{1}{2}-\mu-\nu)\,p^{-1/2}e^{1/2(a+\beta)p}$ $\times D_{2\mu}(2^{1/2}a^{1/2}p^{1/2})D_{2\nu}(2^{1/2}\beta^{1/2}p^{1/2})$ $\lvert\arg ap\rvert < \pi, \quad \lvert\arg\beta p\rvert < \pi$ $\mathrm{Re}\ p > 0$

Gauss' hypergeometric functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\, dt$
(8)	$t^{-\kappa-\lambda}(a+t)^{\kappa-\mu-\frac{1}{2}}(\beta+t)^{\lambda-\mu-\frac{1}{2}}$ $\times F\left[\frac{1}{2}-\kappa+\mu, \frac{1}{2}-\lambda+\mu; 1-\kappa-\lambda; \frac{t(a+\beta+t)}{(a+t)(\beta+t)}\right]$ $\lvert\arg a\rvert<\pi, \quad \lvert\arg\beta\rvert<\pi$ $\mathrm{Re}(\kappa+\lambda)<1$	$\Gamma(1-\kappa-\lambda)(a\beta)^{-\mu-\frac{1}{2}} p^{-1} e^{\frac{1}{2}(a+\beta)p}$ $\times W_{\kappa,\mu}(ap) W_{\lambda,\mu}(\beta p) \quad \mathrm{Re}\, p>0$ $\lvert\arg ap\rvert<\pi, \quad \lvert\arg\beta p\rvert<\pi$
(9)	$(1-e^{-t})^{\lambda-1} F(a,\beta;\gamma;\delta e^{-t})$ $\mathrm{Re}\,\lambda>0, \quad \lvert\arg(1-\delta)\rvert<\pi$	$\mathrm{B}(p,\lambda)\, {}_3F_2(a,\beta,p;\gamma,p+\lambda;\delta)$ $\mathrm{Re}\, p>0$
(10)	$(1-e^{-t})^{\mu}$ $\times F(-n,\mu+\beta+n;\beta;e^{-t})$ $\mathrm{Re}\,\mu>-1$	$\mathrm{B}(p,\mu+n+1)\mathrm{B}(p,\beta+n-p)/\mathrm{B}(p,\beta-p)$ $\mathrm{Re}\, p>0$
(11)	$(1-e^{-t})^{\gamma-1} F(a,\beta;\gamma;1-e^{-t})$ $\mathrm{Re}\,\gamma>0$	$\dfrac{\Gamma(p)\Gamma(\gamma-a-\beta+p)\Gamma(\gamma)}{\Gamma(\gamma-a+p)\Gamma(\gamma-\beta+p)}$ $\mathrm{Re}\, p>0, \quad \mathrm{Re}\, p>\mathrm{Re}(a+\beta-\gamma)$
(12)	$(1-e^{-t})^{\gamma-1}$ $\times F[a,\beta;\gamma;\delta(1-e^{-t})]$ $\mathrm{Re}\,\gamma>0, \quad \lvert\arg(1-\delta)\rvert<\pi$	$\mathrm{B}(p,\gamma)F(a,\beta;p+\gamma;\delta) \quad \mathrm{Re}\, p>0$
(13)	$(1-e^{-t})^{\lambda-1}$ $\times F[a,\beta;\gamma;\delta(1-e^{-t})]$ $\mathrm{Re}\,\lambda>0, \quad \lvert\arg(1-\delta)\rvert<\pi$	$\mathrm{B}(p,\lambda)\, {}_3F_2(a,\beta,\lambda;\gamma,p+\lambda;\delta)$ $\mathrm{Re}\, p>0$

4.22. Confluent hypergeometric functions

Particular confluent hypergeometric functions occur in sections 4.11, 4.12, 4.14 - 4.18, 4.20

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(1)	$k_0(t)$	$(p+1)^{-1}$ $\quad$ Re $p>-1$
(2)	$k_{2n+2}(t)$	$2(1-p)^n(1+p)^{-n-2}$ $\quad$ Re $p>-1$
(3)	$k_{2\nu}(t)$	$[2\pi\nu(1-\nu)]^{-1}\sin(\nu\pi)$ $\times\, {}_2F_1(1,2;2-\nu;\tfrac{1}{2}-\tfrac{1}{2}p)$ $\quad$ Re $p>0$
(4)	$t^{n-\frac{1}{2}}k_{2n+2}(t)$	$(-1)^{n-1}\dfrac{(2n)!\,\pi^{\frac{1}{2}}}{(n+1)!\,2^{2n+\frac{1}{2}}}(p+1)^{-n-1}$ $\times P^{1}_{2n+1}(p-1)^{\frac{1}{2}}(p+1)^{-\frac{1}{2}}$ $\quad$ Re $p>0$
(5)	$e^{-t^2}k_{2n}(t^2)$	$(-1)^{n-1}2^{-1/4-3n/2}p^{n-3/2}e^{p^2/16}$ $\times W_{-\frac{3}{4}-\frac{1}{2}n,\,\frac{1}{4}-\frac{1}{2}n}(p^2/8)$
(6)	$t^{-\frac{1}{2}}e^{-t^{\frac{1}{2}}}k_{2n}(t^{\frac{1}{2}})$	$\sum_{m=0}^{n-1}(-1)^r\binom{n-1}{r}\left(\dfrac{2}{p}\right)^{\frac{1}{2}(n+1-r)}$ $\times\, e^{1/(2p)}D_{-n+r-1}(2^{\frac{1}{2}}p^{-\frac{1}{2}})$ $\quad$ Re $p>0$
(7)	$t^{-1}k_{2m+2}(\tfrac{1}{2}t)k_{2n+2}(\tfrac{1}{2}t)$	$(-p)^{m+n}(p+1)^{-m-n-2}$ $\times\, {}_2F_1(-m,-n;2;p^{-2})$ $\quad$ Re $p>-1$

Confluent hypergeometric functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(8)	$\dfrac{e^{\frac{1}{2}(\alpha+\beta)t}}{\alpha\beta t}\, k_{2m+2}(\tfrac{1}{2}\alpha t)$ $\times k_{2n+2}(\tfrac{1}{2}\beta t)$	$\dfrac{(-1)^{m+n}(m+n+1)!}{(m+1)!(n+1)!}\,\dfrac{(p-\alpha)^m(p-\beta)^n}{p^{m+n+2}}$ $\times {}_2F_1\left[-m,-n;-m-n-1;\dfrac{p(p-\alpha-\beta)}{(p-\alpha)(p-\beta)}\right]$ Re $p>0$
(9)	$t^{\lambda-1} k_{2m_1+2}(\alpha_1 t)\cdots k_{2m_n+2}(\alpha_n t)$ Re $\lambda+n>0$	$(-1)^M 2^n \alpha_1\cdots\alpha_n (p+A)^{-\lambda-n}\Gamma(\lambda+n)$ $\times F_A\left(\lambda+n;-m_1,\dots,-m_n;2,\dots,2;\dfrac{2\alpha_1}{p+A},\dots,\dfrac{2\alpha_n}{p+A}\right)$ Re $p>0$ $M=m_1+\cdots+m_n$ $A=\alpha_1+\cdots+\alpha_n$
(10)	$t^{\mu-\frac{1}{2}} M_{\kappa,\mu}(\alpha t)$ Re $\mu>-\tfrac{1}{2}$	$\alpha^{\mu+\frac{1}{2}}\Gamma(2\mu+1)\,\dfrac{(p-\frac{1}{2}\alpha)^{\kappa-\mu-\frac{1}{2}}}{(p+\frac{1}{2}\alpha)^{\kappa+\mu+\frac{1}{2}}}$ Re $p>\tfrac{1}{2}\lvert\text{Re }\alpha\rvert$
(11)	$t^{\nu-1} M_{\kappa,\mu}(\alpha t)$ Re $(\mu+\nu)>-\tfrac{1}{2}$	$\alpha^{\mu+\frac{1}{2}}\Gamma(\mu+\nu+\tfrac{1}{2})(p+\tfrac{1}{2}\alpha)^{-\mu-\nu-\frac{1}{2}}$ $\times {}_2F_1[\mu+\nu+\tfrac{1}{2},\mu-\kappa+\tfrac{1}{2};2\mu+1;\alpha/(p+\tfrac{1}{2}\alpha)]$ Re $p>\tfrac{1}{2}\lvert\text{Re }\alpha\rvert$
(12)	$t^{2\nu-1} e^{-\frac{1}{2}t^2/\alpha} M_{-3\nu,\nu}(t^2/\alpha)$ Re $\alpha>0$, Re $\nu>-\tfrac{1}{4}$	$\tfrac{1}{2}\pi^{-\frac{1}{2}}\Gamma(4\nu+1)\alpha^{-\nu}p^{-4\nu}e^{\alpha p^2/8}$ $\times K_{2\nu}(\alpha p^2/8)$ Re $p>0$
(13)	$t^{2\mu-1} e^{-\frac{1}{2}t^2/\alpha} M_{\kappa,\mu}(t^2/\alpha)$ Re $\alpha>0$, Re $\mu>-\tfrac{1}{4}$	$2^{-3\mu-\kappa}\Gamma(4\mu+1)\alpha^{\frac{1}{2}(\kappa+\mu-1)}p^{\kappa-\mu-1}$ $\times e^{\alpha p^2/8} W_{-\frac{1}{2}(\kappa+3\mu),\frac{1}{2}(\kappa-\mu)}(\tfrac{1}{4}\alpha p^2)$ Re $p>0$

Confluent hypergeometric functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(14)	$t^{\nu-1}M_{\kappa_1,\mu_1-\frac{1}{2}}(a_1t)\cdots M_{\kappa_n,\mu_n-\frac{1}{2}}(a_nt)$ $M=\mu_1+\cdots+\mu_n$ $\operatorname{Re}(\nu+M)>0$	$a_1^{\mu_1}\cdots a_n^{\mu_n}(p+A)^{-\nu-M}\Gamma(\nu+M)$ $\times F_A\left(\nu+M;\mu_1-\kappa_1,\dots,\mu_n-\kappa_n;\; 2\mu_1,\dots,2\mu_n;\frac{a_1}{p+A},\dots,\frac{a_n}{p+A}\right)$ $A=\frac{1}{2}(a_1+\cdots+a_n)$ $\operatorname{Re}(p\pm\frac{1}{2}a_1\pm\cdots\pm\frac{1}{2}a_n)>0$
(15)	$(e^t-1)^{\mu-\frac{1}{2}}\exp(-\frac{1}{2}\lambda e^t)$ $\times M_{\kappa,\mu}(\lambda e^t-\lambda)\quad \operatorname{Re}\mu>-\frac{1}{2}$	$\dfrac{\Gamma(2\mu+1)\Gamma(\frac{1}{2}+\kappa-\mu+p)}{\Gamma(p+1)}$ $\times W_{-\kappa-\frac{1}{2}p,\,\mu-\frac{1}{2}p}(\lambda)$ $\operatorname{Re}p>\operatorname{Re}(\mu-\kappa)-\frac{1}{2}$
(16)	$t^{\nu-1}W_{\kappa,\mu}(at)$ $\operatorname{Re}(\nu\pm\mu)>-\frac{1}{2}$	$\dfrac{\Gamma(\mu+\nu+\frac{1}{2})\Gamma(\nu-\mu+\frac{1}{2})a^{\mu+\frac{1}{2}}}{\Gamma(\nu-\kappa+1)(p+\frac{1}{2}a)^{\mu+\nu+\frac{1}{2}}}$ $\times {}_2F_1\left(\mu+\nu+\frac{1}{2},\mu-\kappa+\frac{1}{2};\nu-\kappa+1;\frac{p-\frac{1}{2}a}{p+\frac{1}{2}a}\right)$ $\operatorname{Re}(p+\frac{1}{2}a)>0$
(17)	$t^{-1}\exp(-\frac{1}{2}a/t)W_{\frac{1}{2},\mu}(a/t)$ $\operatorname{Re}a>0$	$2\pi^{-\frac{1}{2}}(2ap)^{\frac{1}{2}}K_{\mu+\frac{1}{2}}(a^{\frac{1}{2}}p^{\frac{1}{2}})$ $\times K_{\mu-\frac{1}{2}}(a^{\frac{1}{2}}p^{\frac{1}{2}})\qquad \operatorname{Re}p>0$
(18)	$t^{-1}\exp(\frac{1}{2}a/t)W_{-\frac{1}{2},\mu}(a/t)$ $\lvert\arg a\rvert<\pi$	$\frac{1}{4}\mu^{-1}(a\pi^3p)^{\frac{1}{2}}[H^{(1)}_{\mu+\frac{1}{2}}(a^{\frac{1}{2}}p^{\frac{1}{2}})$ $\times H^{(2)}_{\mu-\frac{1}{2}}(a^{\frac{1}{2}}p^{\frac{1}{2}})$ $+H^{(1)}_{\mu-\frac{1}{2}}(a^{\frac{1}{2}}p^{\frac{1}{2}})H^{(2)}_{\mu+\frac{1}{2}}(a^{\frac{1}{2}}p^{\frac{1}{2}})]$ $\operatorname{Re}p>0$
(19)	$t^{3\nu-\frac{1}{2}}\exp(\frac{1}{2}a/t)W_{\nu,\nu}(a/t)$ $\lvert\arg a\rvert<\pi,\quad \operatorname{Re}\nu>-\frac{1}{4}$	$\frac{1}{2}\Gamma(2\nu+\frac{1}{2})a^{\nu+\frac{1}{2}}p^{-2\nu}H^{(1)}_{2\nu}(a^{\frac{1}{2}}p^{\frac{1}{2}})$ $\times H^{(2)}_{2\nu}(a^{\frac{1}{2}}p^{\frac{1}{2}})\qquad \operatorname{Re}p>0$

Confluent hypergeometric functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(20)	$t^{-3\nu-\frac{1}{2}}\exp(-\tfrac{1}{2}a/t)\,W_{\nu,\nu}(a/t)$ $\mathrm{Re}\,a>0$	$2\pi^{-\frac{1}{2}}a^{\frac{1}{2}-\nu}p^{2\nu}[K_{2\nu}(a^{\frac{1}{2}}p^{\frac{1}{2}})]^2$ $\mathrm{Re}\,p>0$
(21)	$t^{\kappa}\exp(\tfrac{1}{2}a/t)\,W_{\kappa,\mu}(a/t)$ $\lvert\arg a\rvert<\pi,\quad \mathrm{Re}(\kappa\pm\mu)>-\tfrac{1}{2}$	$2^{1-2\kappa}a^{\frac{1}{2}}p^{-\kappa-\frac{1}{2}}S_{2\kappa,2\mu}(2a^{\frac{1}{2}}p^{\frac{1}{2}})$ $\mathrm{Re}\,p>0$
(22)	$t^{-\kappa}\exp(-\tfrac{1}{2}a/t)\,W_{\kappa,\mu}(a/t)$ $\mathrm{Re}\,a>0$	$2a^{\frac{1}{2}}p^{\kappa-\frac{1}{2}}K_{2\mu}(2a^{\frac{1}{2}}p^{\frac{1}{2}})\qquad \mathrm{Re}\,p>0$
(23)	$(1-e^{-t})^{-\kappa}\exp\left[-\dfrac{\lambda}{2(e^{t}-1)}\right]$ $\times W_{\kappa,\mu}\left[\dfrac{\lambda}{(e^{t}-1)}\right]\quad \mathrm{Re}\,\lambda>0$	$\dfrac{\Gamma(\frac{1}{2}+\mu+p)\Gamma(\frac{1}{2}-\mu+p)}{\Gamma(1-\kappa+p)}e^{\frac{1}{2}\lambda}W_{-p,\mu}(\lambda)$ $\mathrm{Re}(\tfrac{1}{2}\pm\mu+p)>0$
(24)	$\lambda e^{t}(e^{t}-1)^{-\kappa-1}\exp\left[-\dfrac{\lambda}{2(e^{t}-1)}\right]$ $\times W_{\kappa,\mu}\left(\dfrac{\lambda}{1-e^{-t}}\right)\quad \mathrm{Re}\,\lambda>0$	$\Gamma(\kappa+p)\,W_{-p,\mu}(\lambda)\qquad \mathrm{Re}\,p>-\mathrm{Re}\,\kappa$

4.23. Generalized hypergeometric series

(1)	$t^{\gamma-1}\,{}_1F_1(\alpha;\gamma;\lambda t)\qquad \mathrm{Re}\,\gamma>0$	$\Gamma(\gamma)\,p^{\alpha-\gamma}(p-\lambda)^{-\alpha}$ $\mathrm{Re}\,p>0,\quad \mathrm{Re}\,\lambda>0$
(2)	$\dfrac{t^{\gamma-1}e^{-t}}{(1-\lambda)^{2\alpha}}\,{}_1F_1\left[\alpha;\gamma;\dfrac{-4\lambda t}{(1-\lambda)^2}\right]$ $\mathrm{Re}\,\gamma>0$	$\dfrac{\Gamma(\gamma)}{(p+1)^{\gamma}}\left(1-2\,\dfrac{p-1}{p+1}\lambda+\lambda^2\right)^{-\alpha}$ $\mathrm{Re}\,p>-1$ $\mathrm{Re}\,p>-\mathrm{Re}[(1+\lambda)^2(1-\lambda)^{-2}]>0$
(3)	$t^{\alpha+\nu-\frac{1}{2}}$ $\times{}_1F_2(\tfrac{1}{2}+\nu;1+2\nu,\tfrac{1}{2}+\nu+\alpha;-2t)$ $\mathrm{Re}(\alpha+\nu+\tfrac{1}{2})>0$	$2^{\nu}\Gamma(\nu+1)\Gamma(\alpha+\nu+\tfrac{1}{2})\,p^{-\alpha-\frac{1}{2}}$ $\times e^{-1/p}I_{\nu}(p^{-1})\qquad \mathrm{Re}\,p>0$

Generalized hypergeometric series (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(4)	$t^{\beta-1}\,{}_1F_2(-n;a+1,\beta;\lambda t)$ Re $\beta>0$	$n![\Gamma(\beta)/(a+1)_n]p^{-\beta}L_n^{a}(\lambda/p)$ Re $p>0$
(5)	${}_2F_2(-n,n+1;1,1;t)$	$p^{-1}P_n(1-2/p)$ Re $p>0$
(6)	$t^{\gamma-1}\,{}_2F_2(-n,n+1;1,\gamma;t)$ Re $\gamma>0$	$\Gamma(\gamma)p^{-\gamma}P_n(1-2/p)$ Re $p>0$
(7)	$t^{\gamma-1}\,{}_2F_2(-n,n;\gamma,\tfrac{1}{2};t)$ Re $\gamma>0$	$\Gamma(\gamma)p^{-\gamma}\cos[2n\sin^{-1}(p^{-\frac{1}{2}})]$ Re $p>0$
(8)	$t^{\gamma-1}\,{}_2F_2(-n,n+1;\gamma,3/2;t)$ Re $\gamma>0$	$\dfrac{\Gamma(\gamma)}{(2n+1)p^{\gamma}}\sin[(2n+1)\sin^{-1}(p^{-\frac{1}{2}})]$ Re $p>0$
(9)	$t^{\gamma-1}\,{}_2F_2(-n,n+2\nu;\nu+\tfrac{1}{2},\gamma;t)$ Re $\gamma>0$	$n\,\mathrm{B}(n,2\nu)\Gamma(\gamma)p^{-\gamma}C_n^{\nu}(1-2/p)$ Re $p>0$
(10)	$t^{\gamma-1}\,{}_2F_2(-n,a+n;\beta,\gamma;t)$ Re $\gamma>0$	$\Gamma(\gamma)p^{-\gamma}F(-n,a+n;\beta;p^{-1})$ Re $p>0$
(11)	$t^{\mu+\nu-1}e^{-\frac{1}{2}t^2}$ $\times\,{}_2F_2\left(\mu,\nu;\dfrac{\mu+\nu}{2},\dfrac{1+\mu+\nu}{2};\dfrac{t^2}{4}\right)$ Re$(\mu+\nu)>0$	$\Gamma(\mu+\nu)e^{\frac{1}{4}p^2}D_{-\mu}(p)D_{-\nu}(p)$
(12)	t^{2a-1} $\times\,{}_3F_2(1,\tfrac{1}{2}-\mu+\nu,\tfrac{1}{2}-\mu-\nu;$ $a,a+\tfrac{1}{2};-\lambda^2t^2)$ Re $\lambda>0$, Re $a>0$	$\Gamma(2a)\lambda^{2\mu-1}p^{1-2a-2\mu}S_{2\mu,2\nu}(p/\lambda)$ Re $p>0$

Generalized hypergeometric series (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(13)	$t^{2\alpha-1} \times {}_4F_3(\tfrac12+\mu+\nu, \tfrac12-\mu+\nu, \tfrac12+\mu-\nu, \tfrac12-\mu-\nu; \tfrac12, \alpha, \alpha+\tfrac12; -\lambda^2 t^2/4)$ $\operatorname{Re}\alpha>0, \quad \operatorname{Re}\lambda>0$	$\dfrac{\pi\Gamma(2\alpha)}{4\lambda p^{2\alpha-1}}[e^{(\mu-\nu)\pi i} H_{2\mu}^{(1)}(p/\lambda) H_{2\nu}^{(2)}(p/\lambda) + e^{(\nu-\mu)\pi i} H_{2\mu}^{(2)}(p/\lambda) H_{2\nu}^{(1)}(p/\lambda)]$ $\lvert\arg p\rvert<\tfrac12\pi$
(14)	$t^{2\alpha-1} \times {}_4F_3(1+\mu+\nu, 1-\mu+\nu, 1+\mu-\nu, 1-\mu-\nu; 3/2, \alpha, \alpha+\tfrac12; -\lambda^2 t^2/4)$ $\operatorname{Re}\lambda>0, \quad \operatorname{Re}\alpha>0$	$\dfrac{\pi\Gamma(2\alpha)p^{2-2\alpha}}{8i\lambda^2(\mu^2-\nu^2)} \times [e^{(\mu-\nu)\pi i} H_{2\mu}^{(1)}(p/\lambda) H_{2\nu}^{(2)}(p/\lambda) - e^{(\nu-\mu)\pi i} H_{2\mu}^{(2)}(p/\lambda) H_{2\nu}^{(1)}(p/\lambda)]$ $\operatorname{Re}p>0$
(15)	$t^{2\alpha-1} \times {}_4F_3(\tfrac12+\mu-\kappa, \tfrac12-\mu-\kappa, \tfrac12-\kappa, 1-\kappa; 1-2\kappa, \alpha, \alpha+\tfrac12; -\lambda^2 t^2)$ $\operatorname{Re}\lambda>0, \quad \operatorname{Re}\alpha>0$	$\Gamma(2\alpha)\lambda^{2\kappa} p^{-2\alpha-2\kappa} W_{\kappa,\mu}(ip/\lambda) \times W_{\kappa,\mu}(-ip/\lambda) \qquad \operatorname{Re}p>0$
(16)	$t^{\rho_n-1} \times {}_mF_n(\alpha_1,\dots,\alpha_m; \rho_1,\dots,\rho_n; \lambda t)$ $m\le n, \quad \operatorname{Re}\rho_n>0$	$\Gamma(\rho_n)\, p^{-\rho_n} \times {}_mF_{n-1}(\alpha_1,\dots,\alpha_m; \rho_1,\dots,\rho_{n-1}; \lambda/p)$ $\operatorname{Re}p>0$ if $m<n$ $\operatorname{Re}p>\operatorname{Re}\lambda$ if $m=n$
(17)	$t^{\sigma-1} \times {}_mF_n(\alpha_1,\dots,\alpha_m; \rho_1,\dots,\rho_n; \lambda t)$ $m\le n, \quad \operatorname{Re}\sigma>0$	$\Gamma(\sigma)p^{-\sigma} \times {}_{m+1}F_n(\alpha_1,\dots,\alpha_m,\sigma; \rho_1,\dots,\rho_n; \lambda/p)$ $\operatorname{Re}p>0$ if $m<n$ $\operatorname{Re}p>\operatorname{Re}\lambda$ if $m=n$
(18)	$t^{2\sigma-1} \times {}_mF_n(\alpha_1,\dots,\alpha_m; \rho_1,\dots,\rho_n; \lambda^2 t^2)$ $m<n, \quad \operatorname{Re}\sigma>0$	$\Gamma(2\sigma)p^{-2\sigma} \times {}_{m+2}F_n(\alpha_1,\dots,\alpha_m, \tfrac12\sigma, \tfrac12\sigma+\tfrac12; \rho_1,\dots,\rho_n; 4\lambda^2 p^{-2})$ $\operatorname{Re}p>0$ if $m<n-1$ $\operatorname{Re}p>\lvert\operatorname{Re}\lambda\rvert$ if $m=n-1$

Generalized hypergeometric series (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(19)	$t^{\sigma-1}$ $\times {}_mF_n[\alpha_1,\dots,\alpha_m;\rho_1,\dots,\rho_n;(\lambda t)^k]$ $m+k\le n+1,\quad \mathrm{Re}\,\sigma>0$	$\Gamma(\sigma)p^{-\sigma}$ $\times {}_{m+k}F_n\left[\alpha_1,\dots\alpha_m,\frac{\sigma}{k},\frac{\sigma+1}{k},\dots,\frac{\sigma+k-1}{k};\ \rho_1,\dots,\rho_n;\left(\frac{k\lambda}{p}\right)^k\right]$ $\mathrm{Re}\,p>0$ if $m+k\le n$ $\mathrm{Re}\,(p+k\lambda e^{2\pi ir/k})>0$ for $r=0,1,\dots,k-1$ if $m+k=n+1$
(20)	$t^{-1/2}$ $\times {}_{2m}F_{2n}\left(\frac{\alpha_1}{2},\frac{\alpha_1+1}{2},\dots,\frac{\alpha_m}{2},\frac{\alpha_m+1}{2};\ \frac{\rho_1}{2},\frac{\rho_1+1}{2},\dots,\frac{\rho_n}{2},\frac{\rho_n+1}{2};\ -2^{m-n-2}\frac{k^2}{t}\right)$ $k>0,\quad m\le n$	$\pi^{1/2}p^{-1/2}\,{}_mF_n(\alpha_1,\dots,\alpha_m;\rho_1,\dots,\rho_n;-kp^{1/2})$ $\mathrm{Re}\,p>0$
(21)	$(1-e^{-t})^{\lambda-1}$ $\times {}_mF_n(\alpha_1,\dots,\alpha_m;\rho_1,\dots,\rho_n;\gamma e^{-t})$ $\mathrm{Re}\,\lambda>0,\quad m\le n$ Valid for $m=n+1$ if $\lvert\gamma\rvert<1$	$\mathrm{B}(\lambda,p)$ $\times {}_{m+1}F_{n+1}(\alpha_1,\dots,\alpha_m,p;\rho_1,\dots,\rho_n,p+\lambda;\gamma)$ $\mathrm{Re}\,p>0$
(22)	$(1-e^{-t})^{\lambda-1}$ $\times {}_mF_n[\alpha_1,\dots,\alpha_m;\rho_1,\dots,\rho_n;\ \gamma(1-e^{-t})]$ $\mathrm{Re}\,\lambda>0,\quad m\le n$ Valid for $m=n+1$ if $\lvert\gamma\rvert<1$	$\mathrm{B}(\lambda,p)$ $\times {}_{m+1}F_{n+1}(\alpha_1,\dots,\alpha_m,\lambda;\rho_1,\dots,\rho_n,p+\lambda;\gamma)$ $\mathrm{Re}\,p>0$
(23)	$t^{\alpha_{m+1}-1}E(m;\alpha_r:n;\beta_s:t^{-1})$ $\mathrm{Re}\,\alpha_{m+1}>0$	$p^{-\alpha_{m+1}}E(m+1;\alpha_r:n;\beta_s:p)\quad \mathrm{Re}\,p>0$

Generalized hypergeometric series (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(24)	$(e^t-1)^{\alpha_{m+1}}$ $\times E\left(m;\alpha_r:n;\beta_s:\dfrac{\lambda}{1-e^{-t}}\right)$ $\mathrm{Re}\,\alpha_{m+1}>-1$	$\Gamma(p-\alpha_{m+1})E(m+1;\alpha_r:n;\beta_s,p:\lambda)$ $\mathrm{Re}\,p>\mathrm{Re}\,\alpha_{m+1}$
(25)	$t^{-2\nu}S_1(\nu,\nu-\tfrac12,-\nu-\tfrac12,\nu-\tfrac12;at)$	$2^{-2\nu-\frac12}\pi^{-\frac12}p^{2\nu-1}\mathbf{H}_{2\nu}(4a/p)$ $\mathrm{Re}\,p>0$
(26)	$t^{-2\nu-1}$ $\times S_1(\nu,\nu-\tfrac12,-\nu-\tfrac12,\nu+\tfrac12;at)$	$2^{-2\nu-3/2}\pi^{-1/2}p^{2\nu}\mathbf{H}_{2\nu}(4a/p)$ $\mathrm{Re}\,p>0$
(27)	$t^{-2\lambda-1}$ $\times S_1(\nu-\tfrac12,-\nu-\tfrac12,\lambda,\lambda+\tfrac12;at)$ $\mathrm{Re}(\nu-\lambda)>0$	$2^{-2\lambda-1}\pi^{-\frac12}p^{2\lambda}J_{2\nu}(4a/p)$ $\mathrm{Re}\,p>0$
(28)	$t^{-2\nu}S_2(\nu,\nu-\tfrac12,-\nu-\tfrac12,\nu-\tfrac12;at)$	$2^{-2\nu}\pi^{\frac12}p^{2\nu-1}[I_{2\nu}(4a/p)$ $-\mathbf{L}_{2\nu}(4a/p)]$ $\mathrm{Re}\,p>0$
(29)	$t^{-2\nu-1}$ $\times S_2(\nu,-\nu-\tfrac12,\nu-\tfrac12,\nu+\tfrac12;at)$ $\mathrm{Re}\,\nu<0$	$2^{-2\nu-1}\pi^{\frac12}\sec(2\nu\pi)p^{2\nu}$ $\times[I_{-2\nu}(4a/p)-\mathbf{L}_{2\nu}(4a/p)]$ $\mathrm{Re}\,p>0$
(30)	$t^{-2\nu}S_2(\nu,-\nu-\tfrac12,\nu-\tfrac12,\nu-\tfrac12;at)$ $\mathrm{Re}\,\nu<\tfrac12$	$2^{-2\nu}\pi^{\frac12}\sec(2\nu\pi)p^{2\nu-1}$ $\times[I_{-2\nu}(4a/p)-\mathbf{L}_{2\nu}(4a/p)]$ $\mathrm{Re}\,p>0$
(31)	$t^{-2\lambda-1}$ $\times S_2(\nu-\tfrac12,-\nu-\tfrac12,\lambda+\tfrac12,\lambda;at)$ $\mathrm{Re}(\lambda\pm\nu)<0$	$2^{-2\lambda}\pi^{-\frac12}p^{2\lambda}K_{2\nu}(4a/p)$ $\mathrm{Re}\,p>0$
(32)	$t^{-2\nu}S_3(\nu,\nu-\tfrac12,-\nu-\tfrac12,\nu-\tfrac12;at)$ $\mathrm{Re}\,\nu<\tfrac12$	$2^{-2\nu}\pi^{3/2}\sec(2\nu\pi)p^{2\nu-1}$ $\times[\mathbf{H}_{2\nu}(4a/p)-Y_{2\nu}(4a/p)]$ $\mathrm{Re}\,p>0$

Generalized hypergeometric series (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(33)	$t^{\alpha-1}$ $\times E(a_1,\dots,a_h:\rho_1,\dots,\rho_k:t^{-1})$ $\mathrm{Re}\,\alpha>0$	$p^{-\alpha}E(a_1,\dots,a_h,\alpha:\rho_1,\dots,\rho_k:p)$
(34)	$t^{-\alpha}G_{h,k}^{m,n}\left(t\,\middle\vert\,\begin{matrix}a_1,\dots,a_h\\ b_1,\dots,b_k\end{matrix}\right)$ $h+k<2(m+n)$ $\mathrm{Re}\,\alpha>\mathrm{Re}\,b_j+1,\quad j=1,\dots,m$	$p^{\alpha-1}\,G_{h+1,k}^{m,n+1}\left(p^{-1}\,\middle\vert\,\begin{matrix}\alpha,a_1,\dots,a_h\\ b_1,\dots,b_k\end{matrix}\right)$ $\lvert\arg p\rvert<(m+n-\tfrac{1}{2}h-\tfrac{1}{2}k)\pi$
	The same formula is valid if $h<k$ (or $h=k$ and $\mathrm{Re}\,p>1$) and $\mathrm{Re}\,\alpha<\mathrm{Re}\,b_j+1,\quad j=1,\dots,m.$	

4.24. Hypergeometric functions of several variables

(1)	$t^{\beta'-1}\Phi_1(\alpha,\beta,\gamma;x,yt)$ $\mathrm{Re}\,\beta'>0$	$\Gamma(\beta')p^{-\beta'}F_1(\alpha,\beta,\beta',\gamma;x,y/p)$ $\mathrm{Re}\,p>0,\quad \mathrm{Re}\,p>\mathrm{Re}\,y$
(2)	$t^{\beta-1}\Phi_2(\alpha,\alpha',\gamma;xt,y)$ $\mathrm{Re}\,\beta>0$	$\Gamma(\beta)p^{-\beta}\Xi_1(\alpha,\alpha',\beta,\gamma;x/p,y)$ $\mathrm{Re}\,p>0,\quad \mathrm{Re}\,p>\mathrm{Re}\,x$
(3)	$t^{\gamma-1}\Phi_2(\beta,\beta',\gamma;xt,yt)$ $\mathrm{Re}\,\gamma>0$	$\Gamma(\gamma)p^{-\gamma}(1-x/p)^{-\beta}(1-y/p)^{-\beta'}$ $\mathrm{Re}\,p>0,\quad \mathrm{Re}\,x,\quad \mathrm{Re}\,y$
(4)	$t^{\alpha-1}\Phi_2(\beta,\beta',\gamma;xt,yt)$ $\mathrm{Re}\,\alpha>0$	$\Gamma(\alpha)p^{-\alpha}F_1(\alpha,\beta,\beta',\gamma;x/p,y/p)$ $\mathrm{Re}\,p>0,\quad \mathrm{Re}\,x,\quad \mathrm{Re}\,y$
(5)	$t^{\gamma-1}\Phi_2(\beta_1,\dots,\beta_n;\gamma;\lambda_1t,\dots,\lambda_nt)$ $\mathrm{Re}\,\gamma>0$	$\dfrac{\Gamma(\gamma)}{p^{\gamma}}\left(1-\dfrac{\lambda_1}{p}\right)^{-\beta_1}\cdots\left(1-\dfrac{\lambda_n}{p}\right)^{-\beta_n}$ $\mathrm{Re}\,p>0,\quad \mathrm{Re}\,\lambda,\quad m=1,\dots,n$
(6)	$t^{\alpha-1}\Phi_3(\beta,\gamma;xt,y)\quad \mathrm{Re}\,\alpha>0$	$\Gamma(\alpha)p^{-\alpha}\Xi_2(\alpha,\beta,\gamma;x/p,y)$ $\mathrm{Re}\,p>0,\quad \mathrm{Re}\,x$

Hypergeometric functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(7)	$t^{\beta'-1}\Phi_3(\beta,\gamma;x,yt)$ Re $\beta'>0$	$\Gamma(\beta')p^{-\beta'}\Phi_2(\beta,\beta',\gamma;x,y/p)$ Re $p>0$, Re γ
(8)	$t^{2\alpha-1}\Phi_3(\beta,\gamma;x,yt^2)$ Re $\alpha>0$	$\Gamma(2\alpha)p^{-2\alpha}\Xi_1(\alpha,\beta,\alpha+\tfrac{1}{2},\gamma;4yp^{-2},x)$ Re $p>2\lvert\text{Re}\,\gamma^{1/2}\rvert$
(9)	$t^{\gamma-1}\Phi_3(\beta,\gamma;xt,yt)$ Re $\gamma>0$	$\Gamma(\gamma)p^{-\gamma}(1-x/p)^{-\beta}e^{y/p}$ Re $p>0$, Re x
(10)	$t^{\alpha-1}\Phi_3(\beta,\gamma;xt,yt)$ Re $\alpha>0$	$\Gamma(\alpha)p^{-\alpha}\Phi_1(\alpha,\beta,\gamma;x/p,y/p)$ Re $p>0$, Re x
(11)	$t^{\beta'-1}\Psi_1(\alpha,\beta,\gamma,\gamma';x,yt)$ Re $\beta'>0$	$\Gamma(\beta')p^{-\beta'}F_2(\alpha,\beta,\beta',\gamma,\gamma';x,y/p)$ Re $p>0$, Re γ
(12)	$t^{\beta-1}\Psi_2(\alpha,\gamma,\gamma';xt,y)$ Re $\beta>0$	$\Gamma(\beta)p^{-\beta}\Psi_1(\alpha,\beta,\gamma,\gamma';x/p,y)$ Re $p>0$, Re x
(13)	$t^{\alpha-1}\Psi_2(\beta,\gamma,\gamma';xt,yt)$ Re $\alpha>0$	$\Gamma(\alpha)p^{-\alpha}F_4(\alpha,\beta,\gamma,\gamma';x/p,y/p)$ Re $p>0$, Re x, Re γ
(14)	$t^{\beta'-1}\Xi_1(\alpha,\alpha',\beta,\gamma;x,yt)$ Re $\beta'>0$	$\Gamma(\beta')p^{-\beta'}F_3(\alpha,\alpha',\beta,\beta';\gamma;x,y/p)$ Re $p>0$, Re γ
(15)	$t^{\alpha'-1}\Xi_2(\alpha,\beta,\gamma;x,yt)$ Re $\alpha'>0$	$\Gamma(\alpha')p^{-\alpha'}\Xi_1(\alpha,\alpha',\beta,\gamma;x,y/p)$ Re $p>0$, Re γ
(16)	$t^{2\alpha'-1}\Xi_2(\alpha,\beta,\gamma;x,yt^2)$ Re $\alpha'>0$	$\Gamma(2\alpha')p^{-2\alpha'}$ $\times F_3(\alpha,\alpha',\beta,\alpha'+\tfrac{1}{2},\gamma;x,4yp^{-2})$ Re $p>2\lvert\text{Re}\,\gamma^{1/2}\rvert$

4.25. Elliptic functions

	$f(t)$		$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(1)	$\theta_4(\tfrac{1}{2}x/l \mid i\pi t/l^2)$	$-l \leq x \leq l$	$lp^{-1/2}\cosh(xp^{1/2})\operatorname{csch}(lp^{1/2})$ $\quad \operatorname{Re} p > 0$
(2)	$\theta_1(\tfrac{1}{2}x/l \mid i\pi t/l^2)$	$-l \leq x \leq l$	$-lp^{-1/2}\sinh(xp^{1/2})\operatorname{sech}(lp^{1/2})$ $\quad \operatorname{Re} p > 0$
(3)	$\theta_2(\tfrac{1}{2}+\tfrac{1}{2}x/l \mid i\pi t/l^2)$	$-l \leq x \leq l$	$-lp^{-1/2}\sinh(xp^{1/2})\operatorname{sech}(lp^{1/2})$ $\quad \operatorname{Re} p > 0$
(4)	$\theta_3(\tfrac{1}{2}+\tfrac{1}{2}x/l \mid i\pi t/l^2)$	$-l \leq x \leq l$	$lp^{-1/2}\cosh(xp^{1/2})\operatorname{csch}(lp^{1/2})$ $\quad \operatorname{Re} p > 0$
(5)	$\hat{\theta}_4(\tfrac{1}{2}x/l \mid i\pi t/l^2)$	$-l \leq x \leq l$	$-lp^{-1/2}\sinh(xp^{1/2})\operatorname{csch}(lp^{1/2})$ $\quad \operatorname{Re} p > 0$
(6)	$\hat{\theta}_3(\tfrac{1}{2}+\tfrac{1}{2}x/l \mid i\pi t/l^2)$	$-l \leq x \leq l$	$-lp^{-1/2}\sinh(xp^{1/2})\operatorname{csch}(lp^{1/2})$ $\quad \operatorname{Re} p > 0$
(7)	$e^{\alpha t}\,\theta_3(\alpha^{1/2} t \mid i\pi t)$		$\tfrac{1}{2}p^{-1/2}[\tanh(p^{1/2}+\alpha^{1/2}) + \tanh(p^{1/2}-\alpha^{1/2})]$ $\quad \operatorname{Re} p > 0$
(8)	$e^{\alpha t}\,\hat{\theta}_3(\alpha^{1/2} t \mid i\pi t)$		$\tfrac{1}{2}p^{-1/2}[\tanh(p^{1/2}+\alpha^{1/2}) - \tanh(p^{1/2}-\alpha^{1/2})+2]$ $\quad \operatorname{Re} p > 0$
(9)	$\dfrac{\partial}{\partial x}\,\theta_4\left(\dfrac{x}{2l}\middle\vert\dfrac{i\pi t}{l^2}\right)$	$-l < x < l$	$\dfrac{l\sinh(xp^{1/2})}{\sinh(lp^{1/2})}$ $\quad \operatorname{Re} p > 0$
(10)	$\dfrac{\partial}{\partial x}\,\theta_1\left(\dfrac{x}{2l}\middle\vert\dfrac{i\pi t}{l^2}\right)$	$-l < x < l$	$-\dfrac{l\cosh(xp^{1/2})}{\sinh(lp^{1/2})}$ $\quad \operatorname{Re} p > 0$
(11)	$\dfrac{\partial}{\partial x}\,\theta_2\left(\dfrac{x+l}{2l}\middle\vert\dfrac{i\pi t}{l^2}\right)$	$-l < x < l$	$-\dfrac{l\cosh(xp^{1/2})}{\cosh(lp^{1/2})}$ $\quad \operatorname{Re} p > 0$

Elliptic functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	
(12)	$\dfrac{\partial}{\partial x}\theta_3\left(\dfrac{x+l}{2l}\,\middle\vert\,\dfrac{i\pi t}{l^2}\right)$	$\dfrac{l\sinh(xp^{1/2})}{\sinh(lp^{1/2})}$	$\operatorname{Re} p>0$

4.26. Miscellaneous functions

(1)	$\nu(t)$		$(p\log p)^{-1}$	$\operatorname{Re} p>0$
(2)	$(1-e^{-t})^{-1}\nu(t)$		$\int_0^\infty \zeta(u+1,\,p)\,du$	$\operatorname{Re} p>1$
(3)	$t^{-1/2}\nu(2t^{1/2})$		$2\pi^{1/2}p^{-1/2}\nu(p^{-1})$	$\operatorname{Re} p>0$
(4)	$\nu(e^{-t})$		$\displaystyle\int_0^\infty \frac{du}{(p+u)\Gamma(u+1)}$	$\operatorname{Re} p>0$
(5)	$\nu(1-e^{-t})$		$\Gamma(p)\,\nu(1,p)$	$\operatorname{Re} p>0$
(6)	$\nu(t,\,a)$	$\operatorname{Re} a>-1$	$p^{-a-1}/\log p$	$\operatorname{Re} p>1$
(7)	$\nu(2t^{1/2},\,2a)$	$\operatorname{Re} a>-1$	$\tfrac{1}{2}\pi^{1/2}p^{-3/2}\nu(p^{-1},\,a-\tfrac{1}{2})$	$\operatorname{Re} p>0$
(8)	$t^{-1/2}\nu(2t^{1/2},2a)$	$\operatorname{Re} a>-\tfrac{1}{2}$	$2\pi^{1/2}p^{-1/2}\nu(p^{-1},\,a)$	$\operatorname{Re} p>0$
(9)	$\mu(t,\,a-1)$	$\operatorname{Re} a>0$	$\Gamma(a)p^{-1}(\log p)^{-a}$	$\operatorname{Re} p>1$
(10)	$t^{-1/2}\mu(2t^{1/2},\,a)$		$2^{a+1}\pi^{1/2}p^{-1/2}\mu(p^{-1};a)$	$\operatorname{Re} p>0$

Miscellaneous functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t)\, dt$
(11)	$V_n(t)$ $V_n(t)$ is defined by the generating function $\frac{1}{1-z} \exp\left(-\frac{1+z}{1-z}\, t\right)$ $= \sum_{n=0}^{\infty} (n+\tfrac{1}{2})\, V_n(t) P_n(z)$	$\frac{2}{p-1}\, Q_n\left(\frac{p+1}{p-1}\right)$ $\quad$ $\mathrm{Re}\, p > 0$
(12)	$U^{m,n}(t)$ $U^{m,n}(t)$ is defined by the generating function $\frac{e^{-at} I_0(bt)}{(1-x)(1-y)} = \sum_{m,n=0}^{\infty} x^m y^n U^{m,n}(t)$ where $a+b = \left(\frac{1+x}{1-x}\right)^2, \quad a-b = \left(\frac{1+y}{1-y}\right)^2$	$\frac{1}{p+1}\, P_m\left(\frac{p-1}{p+1}\right)\, P_n\left(\frac{p-1}{p+1}\right)$ $\mathrm{Re}\, p > -1$

CHAPTER V

INVERSE LAPLACE TRANSFORMS

5.1. General formulas

Most general formulas are in section 4.1. The present section contains a few additions.

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(1)	$g(p)$	$f(t)=\dfrac{1}{2\pi i}\displaystyle\int_{c-i\infty}^{c+i\infty} e^{pt}g(p)\,dp$
(2)	$g(p+p^{1/2})$	$\tfrac{1}{2}\pi^{-1/2}\int_0^t u(t-u)^{-3/2}e^{-\frac{1}{4}u^2/(t-u)}$ $\times f(u)\,du$
(3)	$p^{-1/2}g(p+p^{1/2})$	$\pi^{-1/2}\int_0^t (t-u)^{-1/2}e^{-\frac{1}{4}u^2/(t-u)}f(u)du$
(4)	$(p+a)^{-\nu}g[cp+(p+a)^{1/2}]$ $c>0$	$2^{\nu-1/2}\pi^{-1/2}\int_0^{t/c}(t-cu)^{\nu-1}\exp\left[-a(t-cu)-\dfrac{u^2}{8(t-cu)}\right]D_{1-2\nu}\left[\dfrac{u}{2^{1/2}(t-cu)^{1/2}}\right]$ $\times f(u)\,du$
(5)	$g(r)$	$f(t)-a\int_0^t f[(t^2-u^2)^{1/2}]\,J_1(au)\,du$
(6)	$r^{-1}g(r)$	$\int_0^t J_0[a(t^2-u^2)^{1/2}]\,f(u)\,du$

$r=(p^2+a^2)^{1/2}, \quad R=p+r$

General formulas (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(7)	$pr^{-1} g(r)$	$f(t)-at\int_0^t (t^2-u^2)^{-1/2} \times J_1[a(t^2-u^2)^{1/2}]\,f(u)\,du$
(8)	$r^{-1}R^{-\nu}g(r)$ $\quad$ Re $\nu>-1$	$a^{-\nu}\int_0^t \left(\frac{t-u}{t+u}\right)^{\frac{1}{2}\nu} \times J_\nu[a(t^2-u^2)^{1/2}]\,f(u)\,du$
(9)	$g(\beta+r-p)-g(\beta)$	$-at^{-1/2}\int_0^t t^{-1/2}(t+2u)^{-1/2} \times J_1[at^{1/2}(t+2u)^{1/2}]\,e^{-\beta u}\,uf(u)\,du$
(10)	$r^{-1}R^{-\nu}g(r-p)$ $\quad$ Re $\nu>-1$	$a^{-\nu}t^{\frac{1}{2}\nu}\int_0^\infty (t+2u)^{-\frac{1}{2}\nu} \times I_\nu[at^{1/2}(t+2u)^{1/2}]\,f(u)\,du$
(11)	$r^{-1}R^{-\nu}g(p-r)$ $\quad$ Re $\nu>-1$	$a^{-\nu}t^{\frac{1}{2}\nu}\int_0^\infty (t-2u)^{-\frac{1}{2}\nu} \times I_\nu[at^{1/2}(t-2u)^{1/2}]\,f(u)\,du$
(12)	$g(s)$	$f(t)+a\int_0^t f[(t^2-u^2)^{1/2}]\;I_1(u)\,du$
(13)	$s^{-1}g(s)$	$\int_0^t I_0[a(t^2-u^2)^{1/2}]\,f(u)\,du$
(14)	$ps^{-1}g(s)$	$f(t)+at\int_0^t (t^2-u^2)^{-1/2} \times I_1[a(t^2-u^2)^{1/2}]\,f(u)\,du$
(15)	$s^{-1}S^{-\nu}g(s)$ $\quad$ Re $\nu>-1$	$a^{-\nu}\int_0^t \left(\frac{t-u}{t+u}\right)^{\frac{1}{2}\nu} \times I_\nu[a(t^2-u^2)^{1/2}]\,f(u)\,du$

$r=(p^2+a^2)^{1/2}$, $\quad R=p+r$, $\quad s=(p^2-a^2)^{1/2}$, $\quad S=p+s$

General formulas (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(16)	$g(\beta+s-p)-g(\beta)$	$at^{-\frac{1}{2}} \int_0^\infty e^{-\beta u}(t+2u)^{-\frac{1}{2}}$ $\times I_1[at^{\frac{1}{2}}(t+2u)^{\frac{1}{2}}]\, uf(u)\,du$
(17)	$s^{-1} S^{-\nu} g(s-p)$ Re $\nu>-1$	$a^{-\nu} t^{\frac{1}{2}\nu} \int_0^\infty (t+2u)^{-\frac{1}{2}\nu}$ $\times I_\nu[at^{\frac{1}{2}}(t+2u)^{\frac{1}{2}}]\, f(u)\,du$
(18)	$s^{-1} S^{-\nu} g(p-s)$ Re $\nu>-1$	$a^{-\nu} t^{\frac{1}{2}\nu} \int_0^\infty (t-2u)^{-\frac{1}{2}\nu}$ $\times I_\nu[at^{\frac{1}{2}}(t-2u)^{\frac{1}{2}}]\, f(u)\,du$

5.2. Rational functions

(1)	$(p+\alpha)^{-1}$	$e^{-\alpha t}$
(2)	$(\lambda p+\mu)(p+\alpha)^{-2}$	$[\lambda+(\mu-\alpha\lambda)t]\, e^{-\alpha t}$
(3)	$(\lambda p+\mu)[(p+\alpha)^2-\beta^2]^{-1}$	$\lambda e^{-\alpha t}\cosh(\beta t)+\beta^{-1}(\mu-\alpha\lambda)$ $\times e^{-\alpha t}\sinh(\beta t)$
(4)	$(\lambda p+\mu)[(p+\alpha)^2+\beta^2]^{-1}$	$\lambda e^{-\alpha t}\cos(\beta t)+\beta^{-1}(\mu-\alpha\lambda)$ $\times e^{-\alpha t}\sin(\beta t)$
(5)	$\dfrac{\lambda p+\mu}{(p+\alpha)(p+\beta)}$	$\dfrac{\alpha\lambda-\mu}{\alpha-\beta}\, e^{-\alpha t}+\dfrac{\beta\lambda-\mu}{\beta-\alpha}\, e^{-\beta t}$
(6)	$(\lambda p^2+\mu p+\nu)(p+\alpha)^{-3}$	$[\lambda+(\mu-2\alpha\lambda)t+\frac{1}{2}(\alpha^2\lambda-\alpha\mu+\nu)t^2]$ $\times e^{-\alpha t}$

$s=(p^2-a^2)^{\frac{1}{2}}$, $\quad S=p+s$

Rational functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(7)	$3\alpha^2 \dfrac{\lambda p^2+\mu p+\nu}{p^3+\alpha^3}$	$(\lambda\alpha^2-\mu\alpha+\nu)\,e^{-\alpha t}$ $-(2\lambda\alpha^2-\mu\alpha-\nu)\,e^{\frac{1}{2}\alpha t}\cos(\tfrac{1}{2}3^{\frac{1}{2}}\alpha t)$ $+3^{\frac{1}{2}}(\mu\alpha-\nu)\,e^{\frac{1}{2}\alpha t}\sin(\tfrac{1}{2}3^{\frac{1}{2}}\alpha t)$
(8)	$\dfrac{\lambda p^2+\mu p+\nu}{(p+\alpha)^2(p+\beta)}$	$\left[\dfrac{\alpha(\alpha-2\beta)\lambda+\mu\beta-\nu}{(\alpha-\beta)^2} - \dfrac{\alpha^2\lambda-\alpha\mu+\nu}{\alpha-\beta}\,t\right] e^{-\alpha t}$ $+\dfrac{\beta^2\lambda-\beta\mu+\nu}{(\alpha-\beta)^2}\,e^{-\beta t}$
(9)	$\dfrac{\lambda p^2+\mu p+\nu}{[(p+\alpha)^2+\beta^2](p+\gamma)}$	$\left[\lambda-\dfrac{\lambda\gamma^2-\mu\gamma+\nu}{(\alpha-\gamma)^2+\beta^2}\right] e^{-\alpha t}\cos(\beta t)$ $+\dfrac{1}{\beta}\left[\mu-(\alpha+\gamma)\lambda - (\alpha-\gamma)\dfrac{\lambda\gamma^2-\mu\gamma+\nu}{(\alpha-\gamma)^2+\beta^2}\right] e^{-\alpha t}\sin(\beta t)$ $+\dfrac{\lambda\gamma^2-\mu\gamma+\nu}{(\alpha-\gamma)^2+\beta^2}\,e^{-\gamma t}$
(10)	$\dfrac{\lambda^2 p+\mu p+\nu}{(p+\alpha)(p+\beta)(p+\gamma)}$	$\dfrac{\lambda\alpha^2+\mu\alpha+\nu}{(\alpha-\beta)(\alpha-\gamma)}\,e^{-\alpha t}$ $+\dfrac{\lambda\beta^2-\mu\beta+\nu}{(\beta-\alpha)(\beta-\gamma)}\,e^{-\beta t}$ $+\dfrac{\lambda\gamma^2-\mu\gamma+\nu}{(\gamma-\alpha)(\gamma-\beta)}\,e^{-\gamma t}$
(11)	$4\alpha^3 \dfrac{\lambda p^3+\mu p^2+\nu p+\rho}{p^4+4\alpha^4}$	$4\alpha^3\lambda\cos(\alpha t)\cosh(\alpha t)$ $+(2\alpha^2\mu-\rho)\cos(\alpha t)\sinh(\alpha t)$ $+(2\alpha^2\mu+\rho)\sin(\alpha t)\cosh(\alpha t)$ $+2\alpha\nu\sin(\alpha t)\sinh(\alpha t)$

Rational functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(12)	$2a^3 \dfrac{\lambda p^3+\mu p^2+\nu p+\rho}{p^4-a^4}$	$(a^3\lambda-a\nu)\cos(at)+(a^2\mu-\rho)\sin(at)$ $+(a^3\lambda+a\nu)\cosh(at)$ $+(a^2\mu+\rho)\sinh(at)$
(13)	$\dfrac{\lambda p^3+\mu p^2+\nu p+\rho}{(p^2+a^2)^2}$	$\lambda\cos(at)+\dfrac{\rho+a^2\mu}{2a^3}\sin(at)$ $+\dfrac{\nu-a^2\lambda}{2a}t\sin(at)-\dfrac{\rho-a^2\mu}{2a^2}t\cos(at)$
(14)	$(\beta^2-a^2)\dfrac{\lambda p^3+\mu p^2+\nu p+\rho}{(p^2+a^2)(p^2+\beta^2)}$	$(\nu-a^2\lambda)\cos(at)+(\rho/a-a\mu)\sin(at)$ $-(\nu-\beta^2\lambda)\cos(\beta t)-(\rho/\beta-\beta\mu)\sin(\beta t)$
For further similar formulas see Gardner, M. F. and J. L. Barnes, 1942: *Transients in linear systems*, I, Wiley.		
(15)	$p^{-1}(p^{-1}-1)(p^{-1}-\tfrac{1}{2})\cdots(p^{-1}-1/n)$	$A_n(t)$
(16)	$2(1-p)^n(1+p)^{-n-2}$	$k_{2n+2}(t)$
(17)	$\dfrac{\lambda_1 p^{n-1}+\lambda_2 p^{n-2}+\cdots+\lambda_n}{(p+a)^n}$	$\Big\{\lambda_1+\left[\lambda_2-\binom{n-1}{1}\lambda_1 a\right]t$ $+\left[\lambda_3-\binom{n-2}{1}\lambda_2 a+\binom{n-1}{2}\lambda_1 a^2\right]\dfrac{t^2}{2!}$ $+\left[\lambda_4-\binom{n-3}{1}\lambda_3 a+\binom{n-2}{2}\lambda_2 a^2\right.$ $\left.-\binom{n-1}{3}\lambda_1 a^3\right]\dfrac{t^3}{3!}$ $+\cdots+[\lambda_n-\lambda_{n-1}a+\cdots+\lambda_1(-a)^{n-1}]$ $\times\dfrac{t^{n-1}}{(n-1)!}\Big\}e^{-at}$

Rational functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(18)	$p^{-1}(p+a)^{-n}$	$a^{-n}[1-e^{-at}e_{n-1}(at)]$ $e_n(z)=1+\frac{z}{1!}+\cdots+\frac{z^n}{n!}$
(19)	$\frac{\lambda_1 p^{n-1}+\lambda_2 p^{n-2}+\cdots+\lambda_n}{(p+a_1)(p+a_2)\cdots(p+a_n)}$	$\frac{\lambda_1(-a_1)^{n-1}+\cdots+\lambda_n}{(a_2-a_1)(a_3-a_1)\cdots(a_n-a_1)} e^{-a_1 t}$ + $n-1$ similar terms obtained by cyclic permutation of $a_1, \ldots, a_n$
(20)	$\frac{Q(p)}{P(p)}$ $P(p)=(p-a_1)(p-a_2)\cdots(p-a_n)$ $Q(p)$ = polynomial of degree $\le n-1$ $a_i \neq a_k$, for $i \neq k$	$\sum_{m=1}^{n} \frac{Q(a_m)}{P_m(a_m)} e^{a_m t}$ $P_m(p)=\frac{P(p)}{p-a_m}$
(21)	$\frac{Q(p)}{P(p)}$ $P(p)=(p-a_1)^{m_1}\cdots(p-a_n)^{m_n}$ $Q(p)$ = polynomial of degree $< m_1+\cdots+m_n-1$ $a_i \neq a_k$, for $i \neq k$	$\sum_{k=1}^{n}\sum_{l=1}^{m_k} \frac{\Phi_{kl}(a_k)}{(m_k-l)!(l-1)!} t^{m_k-l} e^{a_k t}$ $\Phi_{kl}(p)=\frac{d^{l-1}}{dp^{l-1}}\left[\frac{Q(p)}{P_k(p)}\right]$ $P_k(p)=\frac{P(p)}{(p-a_k)^{m_k}}$
(22)	$\frac{(2n+1)!\,a^{2n+1}}{(p^2+a^2)(p^2+3^2a^2)\cdots[p^2+(2n+1)^2a^2]}$	$\sin^{2n+1}(at)$
(23)	$\frac{(2n)!\,a^{2n}}{p(p^2+2^2a^2)(p^2+4^2a^2)\cdots(p^2+4n^2a^2)}$	$\sin^{2n}(at)$

Rational functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(24)	$\dfrac{p(p^2+2^2\alpha^2)(p^2+4^2\alpha^2)\cdots[p^2+(2n)^2\alpha^2]}{(p^2+\alpha^2)(p^2+3^2\alpha^2)\cdots[p^2+(2n+1)^2\alpha^2]}$	$P_{2n+1}[\cos(\alpha t)]$
(25)	$\dfrac{(p^2+\alpha^2)(p^2+\ {}^2\alpha^2)\cdots[p^2+(2n-1)^2\alpha^2]}{p(p^2+2^2\alpha^2)(p^2+4^2\alpha^2)\cdots[p^2+(2n)^2\alpha^2]}$	$P_{2n}[\cos(\alpha t)]$
(26)	$\dfrac{Q(p)+p\,\eta(p)}{P(p)}$ $P(p)=(p^2+\alpha_1^2)\cdots(p^2+\alpha_n^2)$ $Q(p)$, $\eta(p)$ even polynomials of degree $\leq 2n-2$ $\alpha_i \neq \alpha_k$, for $i \neq k$	$\sum_{m=1}^{n} \dfrac{1}{P_m(i\alpha_m)}[\eta(i\alpha_m)\cos(\alpha_m t)$ $+(\alpha_m)^{-1}Q(i\alpha_m)\sin(\alpha_m t)]$ $P_m(p)=\dfrac{P(p)}{p^2+\alpha_m^2}$

5.3. Irrational algebraic functions

(1)	$(p-\alpha)^{-1}p^{-1/2}$	$\alpha^{-1/2}e^{\alpha t}\operatorname{Erf}(\alpha^{1/2}t^{1/2})$
(2)	$p^{-1}(p-\alpha)^{-1}p^{-1/2}$	$\alpha^{-3/2}e^{\alpha t}\operatorname{Erf}(\alpha^{1/2}t^{1/2})$ $-2\alpha^{-1}\pi^{-1/2}t^{1/2}$
(3)	$2\pi i(p-1)^n(p+1)^{-n-1}p^{-1/2}$	$n![D^2_{-n-1}(-i\,2^{1/2}t^{1/2})-D^2_{-n-1}(i2^{1/2}t^{1/2})]$
(4)	$(p^{1/2}+\alpha)^{-1}$	$\pi^{-1/2}t^{-1/2}-\alpha e^{\alpha^2 t}\operatorname{Erfc}(\alpha t^{1/2})$
(5)	$\alpha p^{-1}(p^{1/2}+\alpha)^{-1}$	$1-e^{\alpha^2 t}\operatorname{Erfc}(\alpha t^{1/2})$
(6)	$(\alpha-\beta)p^{1/2}(p^{1/2}+\alpha^{1/2})^{-1}(p-\beta)^{-1}$	$\alpha e^{\alpha t}\operatorname{Erfc}(\alpha^{1/2}t^{1/2})$ $+\alpha^{1/2}\beta^{1/2}e^{\beta t}\operatorname{Erfc}(\beta^{1/2}t^{1/2})-\beta e^{\beta t}$

Irrational algebraic functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(7)	$p^{-1/2}(p^{1/2}+a)^{-1}$	$e^{a^2t}\,\mathrm{Erfc}(at^{1/2})$
(8)	$a^2p^{-3/2}(p^{1/2}+a)$	$2\pi^{-1/2}at^{1/2}+e^{a^2t}\,\mathrm{Erfc}(at^{1/2})-1$
(9)	$(a-\beta)\beta^{1/2}(p-\beta)^{-1}$ $\times p^{-1/2}(p^{1/2}+a^{1/2})^{-1}$	$\beta^{1/2}e^{at}\,\mathrm{Erfc}(a^{1/2}t^{1/2})$ $+a^{1/2}e^{\beta t}\,\mathrm{Erf}(\beta^{1/2}t^{1/2})-\beta^{1/2}e^{\beta t}$
(10)	$(p^{1/2}+a^{1/2})^{-2}$	$1-2\pi^{-1/2}a^{1/2}t^{1/2}$ $+(1-2at)e^{at}[\mathrm{Erf}(a^{1/2}t^{1/2})-1]$
(11)	$p^{-1}(p^{1/2}+a^{1/2})^{-2}$	$a^{-1}+(2t-a^{-1})e^{at}\,\mathrm{Erfc}(a^{1/2}t^{1/2})$ $-2\pi^{-1/2}a^{-1/2}t^{1/2}$
(12)	$p^{-1/2}(p^{1/2}+a)^{-2}$	$2\pi^{-1/2}t^{1/2}-2ate^{a^2t}\,\mathrm{Erfc}(at^{1/2})$
(13)	$(p^{1/2}+a)^{-3}$	$2\pi^{-1/2}(a^2t+1)t^{1/2}$ $-ate^{a^2t}(2a^2t+3)\,\mathrm{Erfc}(at^{1/2})$
(14)	$p^{1/2}(p^{1/2}+a)^{-3}$	$(2a^4t^2+5a^2t+1)e^{a^2t}\,\mathrm{Erfc}(at^{1/2})$ $-2\pi^{-1/2}a(a^2t+2)t^{1/2}$
(15)	$p^{-1/2}(p^{1/2}+a)^{-3}$	$(2at^2+1)te^{a^2t}\mathrm{Erfc}(at^{1/2})-2\pi^{-1/2}at^{3/2}$
(16)	$3(p^{1/2}+a)^{-4}$	$-2\pi^{-1/2}a^3t^{5/2}(2a^2t+5)$ $+t(4a^4t^2+12a^2t+3)e^{a^2t}\,\mathrm{Erfc}(at^{1/2})$
(17)	$p^{-1}(p^{1/2}-a)(p^{1/2}+a)^{-1}$	$2e^{a^2t}\,\mathrm{Erfc}(at^{1/2})-1$
(18)	$p^{-1}(p^{1/2}-a)^2(p^{1/2}+a)^{-2}$	$1+8a^2te^{a^2t}\,\mathrm{Erfc}(at^{1/2})-8\pi^{-1/2}at^{1/2}$

Irrational algebraic functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(19)	$p^{-1}(p^{1/2}-\alpha)^3(p^{1/2}+\alpha)^{-3}$	$2(8\alpha^4t^2+8\alpha^2t+1)e^{\alpha^2t}\operatorname{Erfc}(\alpha t^{1/2})$ $-8\pi^{-1/2}\alpha t^{1/2}(2\alpha^2t+1)-1$
(20)	$(p-\alpha)^{1/2}-(p-\beta)^{1/2}$	$\frac{1}{2}\pi^{-1/2}t^{-3/2}(e^{\beta t}-e^{\alpha t})$
(21)	$(p+\alpha)^{-1/2}$	$\pi^{-1/2}t^{-1/2}e^{-\alpha t}$
(22)	$(p+\beta)^{-1}(p+\alpha)^{1/2}$	$\pi^{-1/2}t^{-1/2}e^{-\alpha t}$ $+(\alpha-\beta)^{1/2}e^{-\beta t}\operatorname{Erf}[(\alpha-\beta)^{1/2}t^{1/2}]$
(23)	$(p+\alpha)^{-1}(p+\beta)^{-1/2}$	$(\beta-\alpha)^{-1/2}e^{-\alpha t}\operatorname{Erf}[(\beta-\alpha)^{1/2}t^{1/2}]$
(24)	$(p+\alpha)^{1/2}(p-\alpha)^{-1/2}-1$	$\alpha[I_0(\alpha t)+I_1(\alpha t)]$
(25)	$(p+\alpha+\beta)^{1/2}(p+\alpha-\beta)^{-3/2}$	$e^{-\alpha t}[(1+2\beta t)I_0(\beta t)+2\beta t\,I_1(\beta t)]$
(26)	$(p+\alpha+\beta)^{-1/2}(p+\alpha-\beta)^{-3/2}$	$te^{-\alpha t}[I_0(\beta t)+I_1(\beta t)]$
(27)	$p^{-1}(p+\alpha-\beta)^{1/2}(p+\alpha+\beta)^{-1/2}$	$e^{-\alpha t}I_0(\beta t)$ $+(\alpha-\beta)\int_0^t e^{-\alpha u}I_0(\beta u)\,du$
(28)	$\dfrac{(p+\alpha)^{1/2}-(p-\alpha)^{1/2}}{(p+\alpha)^{1/2}+(p-\alpha)^{1/2}}$	$t^{-1}I_1(\alpha t)$
	Campbell, G. A., and R. M. Foster, 1931: *Fourier integrals for practical applications*, Bell Telephone Laboratories, New York, contains other similar integrals.	
(29)	$(p+\alpha)^{-n-1/2}$	$\dfrac{\pi^{-1/2}t^{n-1/2}e^{-\alpha t}}{\frac{1}{2}\cdot\frac{3}{2}\cdots(n-\frac{1}{2})}$

Irrational algebraic functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(30)	$(p-\alpha)^n(p-\beta)^{-n-1/2}$	$\frac{(-2)^n n!\,e^{\beta t}}{(2n)!\,\pi^{1/2}t^{1/2}}\ \mathrm{He}_{2n}[2^{1/2}(\alpha-\beta)^{1/2}t^{1/2}]$
(31)	$(p-\alpha)^n(p-\beta)^{-n-3/2}$	$\frac{(-2)^n 2^{1/2} n!}{(2n+1)!\,\pi^{1/2}}e^{\beta t}\,\mathrm{He}_{2n+1}[2^{1/2}(\alpha-\beta)^{1/2}t^{1/2}]$
(32)	$(p-\alpha)^n(p-\beta)^{-m-n-3/2}$	$\frac{(-1)^n e^{\beta t}2^{n+k+1/2}}{(\alpha-\beta)^m\,\pi^{1/2}}$ $\times\sum_{k=1}^{m}\binom{m}{k}\frac{(n+k)!}{(2n+2k+1)!}$ $\times\,\mathrm{He}_{2n+2k+1}[2^{1/2}(\alpha-\beta)^{1/2}t^{1/2}]$
(33)	$(p-\alpha)^n(p-\beta)^{-m-n-1/2}$	$\frac{(-1)^n e^{\beta t}2^{n+k}}{(\alpha-\beta)^m\,\pi^{1/2}t^{1/2}}\sum_{k=1}^{m}\binom{m}{k}\frac{(n+k)!}{(2n+2k)!}$ $\times\,\mathrm{He}_{2n+2k}[2^{1/2}(\alpha-\beta)^{1/2}t^{1/2}]$
(34)	$(p^2+\alpha p+\beta)^{-1/2}$	$e^{-1/2\,\alpha t}J_0[(\beta-1/4\,\alpha^2)^{1/2}t]$
(35)	r^{-1}	$J_0(at)$
(36)	$p^{-1}r^{-3}$	$1/2\,\pi a^{-2}t[J_1(at)\,\mathbf{H}_0(at)$ $-J_0(at)\,\mathbf{H}_1(at)]$
(37)	$r^{-1}R^{1/2}$	$2^{1/2}\pi^{-1/2}t^{-1/2}\cos(at)$
(38)	$(r-p)^{1/2}=aR^{-1/2}$	$2^{-1/2}\pi^{-1/2}t^{-3/2}\sin(at)$
(39)	$ar^{-1}R^{-1/2}$	$2^{1/2}\pi^{-1/2}t^{-1/2}\sin(at)$

$r=(p^2+a^2)^{1/2},\quad R=p+r$

Irrational algebraic functions (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(40)	$p^{-1} r^{-1} R^{\frac{1}{2}}$	$2a^{-\frac{1}{2}} C(at)$
(41)	$p^{-1} r^{-1} R^{-\frac{1}{2}}$	$2a^{-3/2} S(at)$
(42)	r^{-2n-1}	$\dfrac{a^{-n} t^n J_n(at)}{1\cdot3\cdot5\cdots(2n-1)}$
(43)	R^{-n}	$n a^{-n} t^{-1} J_n(at)$
(44)	s^{-1}	$I_0(at)$
(45)	$p^{-1} s^{-3}$	$\frac{1}{2}\pi a^{-2} t[I_1(at)\mathbf{L}_0(at) - I_0(at)\mathbf{L}_1(at)]$
(46)	$s^{-1} S^{\frac{1}{2}}$	$2^{\frac{1}{2}} \pi^{-\frac{1}{2}} t^{-\frac{1}{2}} \cosh(at)$
(47)	$as^{-1} S^{-\frac{1}{2}}$	$2^{\frac{1}{2}} \pi^{-\frac{1}{2}} t^{-\frac{1}{2}} \sinh(at)$
(48)	s^{-2n-1}	$\dfrac{t^n I_n(at)}{1\cdot3\cdot5\cdots(2n-1)a^n}$
(49)	S^{-n}	$n a^{-n} t^{-1} I_n(at)$
(50)	$[(p^4+a^4)^{\frac{1}{2}}+p^2]^{\frac{1}{2}} (p^4+a^4)^{-\frac{1}{2}}$	$2^{\frac{1}{2}} \operatorname{ber}(at)$
(51)	$[(p^4+a^4)^{\frac{1}{2}}-p^2]^{\frac{1}{2}} (p^4+a^4)^{-\frac{1}{2}}$	$2^{\frac{1}{2}} \operatorname{bei}(at)$

$r = (p^2 + a^2)^{\frac{1}{2}}$, $R = p + r$, $s = (p^2 - a^2)^{\frac{1}{2}}$, $S = p + s$

5.4. Powers with an arbitrary index

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(1)	$\Gamma(\nu)(p+\alpha)^{-\nu}$ $\quad \operatorname{Re}\nu>0$	$t^{\nu-1}e^{-\alpha t}$
(2)	$p^{-m-1}(p-1)^n$	$t^m p_n(m,t)/m!$
(3)	$\Gamma(\nu)(p+\alpha)^{-\nu}(p-\beta)^{-1}$ $\quad \operatorname{Re}\nu>0$	$(\alpha+\beta)^{-\nu}e^{\beta t}\gamma[\nu,(\alpha+\beta)t]$
(4)	$\Gamma(\nu+1)(p-\lambda)^n(p-\mu)^{-\nu-1}$ $\quad \operatorname{Re}\nu>n-1$	$n!\,t^{\nu-n}e^{\mu t}L_n^{\nu-n}[(\mu-\lambda)t]$
(5)	$\Gamma(\nu)(p+\alpha)^{-\nu}(p+\beta)^{-\nu}$ $\quad \operatorname{Re}\nu>0$	$\pi^{1/2}(\alpha-\beta)^{1/2-\nu}t^{\nu-1/2}e^{-1/2(\alpha+\beta)t}$ $\times I_{\nu-1/2}[1/2(\alpha-\beta)t]$
(6)	$(p-\alpha)^{\lambda}(p-\beta)^{-\lambda-1/2}$	$2^{-\lambda-1}\pi^{-1}\Gamma(1/2-\lambda)t^{-1/2}e^{1/2(\alpha+\beta)t}$ $\times\{D_{2\lambda}[-2^{1/2}(\alpha-\beta)^{1/2}t^{1/2}]$ $+D_{2\lambda}[2^{1/2}(\alpha-\beta)^{1/2}t^{1/2}]\}$
(7)	$(p-\alpha)^{\lambda}(p-\beta)^{-\lambda-3/2}$	$2^{-\lambda-3/2}\pi^{-1}(\alpha-\beta)^{-1/2}\Gamma(-1/2-\lambda)$ $\times e^{1/2(\alpha+\beta)t}\{D_{2\lambda+1}[-2^{1/2}(\alpha-\beta)^{1/2}t^{1/2}]$ $-D_{2\lambda+1}[2^{1/2}(\alpha-\beta)^{1/2}t^{1/2}]\}$
(8)	$\Gamma(2\nu-2\lambda)(p-\alpha)^{2\lambda}(p-\beta)^{-2\nu}$ $\quad \operatorname{Re}(\nu-\lambda)>0$	$t^{2\nu-2\lambda-1}e^{\alpha t}{}_1F_1[2\nu,2\nu-2\lambda;(\beta-\alpha)t]$ $=(\alpha-\beta)^{\lambda-\nu}t^{\nu-\lambda-1}e^{1/2(\alpha+\beta)t}$ $\times M_{\lambda+\nu,\,\nu-\lambda-1/2}[(\alpha-\beta)t]$
(9)	$\Gamma(\gamma)p^{-\gamma}$ $\times(1-\lambda_1/p)^{-\beta_1}\cdots(1-\lambda_n/p)^{-\beta_n}$ $\quad \operatorname{Re}\gamma>0$	$t^{\gamma-1}\Phi_2(\beta_1,\ldots,\beta_n;\gamma;\lambda_1 t,\ldots,\lambda_n t)$
(10)	$p^{-2\lambda}(p^2+\alpha^2)^{-\nu}$ $\quad \operatorname{Re}(\lambda+\nu)>0$	$[\Gamma(2\lambda+2\nu)]^{-1}t^{2\lambda+2\nu-1}$ $\times{}_1F_2(\nu;\lambda+\nu,\lambda+\nu+1/2;-1/4\alpha^2t^2)$

Arbitrary powers (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(11)	$p^{-3\lambda}(p^3+a^3)^{-\nu}$ $\quad \operatorname{Re}(\lambda+\nu)>0$	$\dfrac{t^{3\lambda+3\nu-1}}{\Gamma(3\lambda+3\nu)} \times {}_1F_3\left(\nu;\lambda+\nu,\lambda+\nu+\dfrac{1}{3},\lambda+\nu+\dfrac{2}{3};-\dfrac{a^3t^3}{27}\right)$
(12)	$\dfrac{2\Gamma(\nu)(\lambda p+\mu)}{(p^2-\beta^2)(p+a)^\nu}$ $\quad \operatorname{Re}\nu>0$	$\left(\lambda+\dfrac{\mu}{\beta}\right)e^{\beta t}\dfrac{\gamma[\nu,(a+\beta)t]}{(a+\beta)^\nu} + \left(\lambda-\dfrac{\mu}{\beta}\right)e^{-\beta t}\dfrac{\gamma[\nu,(a-\beta)t]}{(a-\beta)^\nu}$
(13)	$[(p+a)^{1/2}+\beta^{1/2}]^\nu$ $\quad \operatorname{Re}\nu<0$	$-2^{1/2}\pi^{-1/2}\nu(2t)^{-1/2\nu-1}e^{(1/2\beta-a)t} \times D_{\nu-1}(2^{1/2}\beta^{1/2}t^{1/2})$
(14)	$(p+a)^{-1/2}[(p+a)^{1/2}+\beta^{1/2}]^\nu$ $\quad \operatorname{Re}\nu<1$	$2^{1/2}\pi^{-1/2}(2t)^{-1/2\nu-1/2}e^{(1/2\beta-a)t} \times D_\nu(2^{1/2}\beta^{1/2}t^{1/2})$
(15)	$[(p+a)^{1/2}+(p+\beta)^{1/2}]^{-2\nu}$ $\quad \operatorname{Re}\nu>0$	$\nu(a-\beta)^{-\nu}t^{-1}e^{-1/2(a+\beta)t}I_\nu[1/2(a-\beta)t]$
(16)	$[(p+a)^{1/2}+(p-a)^{1/2}]^{-2\nu} \times(p+a)^{1/2}(p-a)^{-1/2}$ $\quad \operatorname{Re}\nu>0$	$1/4(2a)^{1-\nu}[I_{\nu-1}(at) + 2I_\nu(at)+I_{\nu+1}(at)]$
(17)	$[(p+a)^{1/2}+(p+\beta)^{1/2}]^{-2\nu} \times(p+a)^{-1/2}(p+\beta)^{-1/2}$ $\quad \operatorname{Re}\nu>-1$	$(a-\beta)^{-\nu}e^{-1/2(a+\beta)t}I_\nu[1/2(a-\beta)t]$
(18)	$\Gamma(\nu+1/2)r^{-2\nu-1}$ $\quad \operatorname{Re}\nu>-1/2$	$\pi^{1/2}(2a)^{-\nu}t^\nu J_\nu(at)$
(19)	$\Gamma(\nu+1/2)s^{-2\nu-1}$ $\quad \operatorname{Re}\nu>-1/2$	$\pi^{1/2}(2a)^{-\nu}t^\nu I_\nu(at)$

$r=(p^2+a^2)^{1/2},\quad R=p+r,\qquad s=(p^2-a^2)^{1/2},\quad S=p+s$

Arbitrary powers (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(20)	$\Gamma(\nu+\tfrac{1}{2})\,pr^{-2\nu-1}$ $\quad$ $\mathrm{Re}\,\nu>0$	$\pi^{\frac{1}{2}}a(2a)^{-\nu}t^{\nu}J_{\nu-1}(at)$
(21)	$a^{\nu}R^{-\nu}$ $\quad$ $\mathrm{Re}\,\nu>0$	$\nu t^{-1}J_{\nu}(at)$
(22)	$a^{\nu}pR^{-\nu}$ $\quad$ $\mathrm{Re}\,\nu>1$	$a\nu t^{-1}J_{\nu-1}(at)$ $-\nu(\nu+1)t^{-2}J_{\nu}(at)$
(23)	$r^{-1}R^{-\nu}$ $\quad$ $\mathrm{Re}\,\nu>-1$	$a^{-\nu}J_{\nu}(at)$
(24)	$a^{\nu-1}pr^{-1}R^{-\nu}$ $\quad$ $\mathrm{Re}\,\nu>0$	$\tfrac{1}{2}J_{\nu-1}(at)-\tfrac{1}{2}J_{\nu+1}(at)$
(25)	$p^{-1}[a^{-\nu}R^{\nu}+a^{\nu}R^{-\nu}\cos(\nu\pi)]$ $\lvert\mathrm{Re}\,\nu\rvert<1$	$1+\cos(\nu\pi)-\nu\sin(\nu\pi)$ $\times\int_t^\infty x^{-1}\,Y_{\nu}(ax)\,dx$
(26)	$(p+\nu r)r^{-3}R^{-\nu}$ $\quad$ $\mathrm{Re}\,\nu>-2$	$a^{-\nu}t\,J_{\nu}(at)$
(27)	$\Gamma(\nu+\tfrac{1}{2})\,ps^{-2\nu-1}$ $\quad$ $\mathrm{Re}\,\nu>0$	$\pi^{\frac{1}{2}}a(2a)^{-\nu}t^{\nu}I_{\nu-1}(at)$
(28)	$a^{\nu}S^{-\nu}$ $\quad$ $\mathrm{Re}\,\nu>0$	$\nu t^{-1}I_{\nu}(at)$
(29)	$a^{\nu}pS^{-\nu}$ $\quad$ $\mathrm{Re}\,\nu>1$	$a\nu t^{-1}I_{\nu-1}(at)$ $-\nu(\nu+1)t^{-2}I_{\nu}(at)$
(30)	$a^{\nu}s^{-1}S^{-\nu}$ $\quad$ $\mathrm{Re}\,\nu>-1$	$I_{\nu}(at)$
(31)	$a^{\nu-1}ps^{-1}S^{-\nu}$ $\quad$ $\mathrm{Re}\,\nu>0$	$\tfrac{1}{2}I_{\nu-1}(at)+\tfrac{1}{2}I_{\nu+1}(at)$
(32)	$s^{-1}(a^{-\nu}S^{\nu}-a^{\nu}S^{-\nu})$ $\quad$ $\lvert\mathrm{Re}\,\nu\rvert<1$	$2\pi^{-1}\sin(\nu\pi)K_{\nu}(at)$

$r=(p^2+a^2)^{\frac{1}{2}},\quad R=p+r,\qquad s=(p^2-a^2)^{\frac{1}{2}},\quad S=p+s$

Arbitrary powers (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(33)	$(p+\nu s)s^{-3}S^{-\nu}$ $\quad \mathrm{Re}\,\nu>-2$	$a^{-\nu}t\, I_\nu(at)$
(34)	$a^\nu v^{-1}V^{-\nu}$ $\quad \mathrm{Re}\,\nu>-1$	$e^{-\frac{1}{4}i\pi\nu}[\mathrm{ber}_\nu(at)+i\,\mathrm{bei}_\nu(at)]$
(35)	$v^{-1}(a^{-\nu}V^\nu-e^{i\pi\nu}a^\nu V^{-\nu})$ $\quad \lvert\mathrm{Re}\,\nu\rvert<1$	$2\pi^{-1}e^{3/2\,i\pi\nu}\sin(\nu\pi)[\mathrm{ker}_\nu(at)$ $+i\,\mathrm{kei}_\nu(at)]$

5.5. Exponential functions of arguments p and $1/p$

(1)	$p^{-1}e^{-ap}$	$a>0$	0 1		$0<t<a$ $t>a$
(2)	$p^{-1}(1-e^{-ap})$	$a>0$	1 0		$0<t<a$ $t>a$
(3)	$p^{-1}(e^{-ap}-e^{-bp})$	$0\le a<b$	0 1 0		$0<t<a$ $a<t<b$ $t>b$
(4)	$p^{-2}(e^{-ap}-e^{-bp})$	$0\le a<b$	0 $t-a$ $b-a$		$0<t<a$ $a<t<b$ $t>b$
(5)	$p^{-3}(e^{-ap}-e^{-bp})$	$0\le a<b$	0 $\frac{1}{2}(t-a)^2$ $t(b-a)+\frac{1}{2}(a^2-b^2)$		$0<t<a$ $a<t<b$ $t>b$

$s=(p^2-a^2)^{\frac{1}{2}},\quad S=p+s,\qquad v=(p^2-ia^2)^{\frac{1}{2}},\quad V=p+v$

Exponential functions of p and 1/p (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(6)	$p^{-2}(e^{-ap}-e^{-bp})^2 \qquad 0 \leq a < b$	$0 \qquad t<2a$ $t-2a \qquad 2a<t<a+b$ $2b-t \qquad a+b<t<2b$ $0 \qquad t>2b$
(7)	$p^{-3}(e^{-ap}-e^{-bp})^2 \qquad 0 \leq a < b$	$0 \qquad t<2a$ $\frac{1}{2}(t-2a)^2 \qquad 2a<t<a+b$ $(b-a)^2-\frac{1}{2}(t-2b)^2$ $a+b<t<2b$ $(b-a)^2 \qquad t>2b$
(8)	$p^{-3}(e^{-ap}-e^{-bp})^3 \qquad 0 \leq a < b$	$0 \qquad t<3a$ $\frac{1}{2}(t-3a)^2 \qquad 3a<t<2a+b$ $[(3/4)(b-a)^2]-[t-(3/2)(a+b)]^2$ $2a+b<t<a+2b$ $\frac{1}{2}(3b-t)^2 \qquad a+2b<t<3b$ $0 \qquad t>3b$
(9)	$(p+\beta)^{-1}e^{-ap} \qquad a>0$	$0 \qquad 0<t<a$ $e^{-\beta(t-a)} \qquad t>a$
(10)	$(\lambda p+\mu)(p^2-\beta^2)^{-1}e^{-ap}$ $a>0$	$0 \qquad 0<t<a$ $\lambda\cosh[\beta(t-a)]+\mu\beta^{-1}\sinh[\beta(t-a)]$ $t>a$
(11)	$(\lambda p+\mu)(p^2+\beta^2)^{-1}e^{-ap}$ $a>0$	$0 \qquad 0<t<a$ $\lambda\cos[\beta(t-a)]+\mu\beta^{-1}\sin[\beta(t-a)]$ $t>a$
(12)	$(p^2+a^2)^{-1}(1-e^{-2m\pi p/a})$ $a>0, \quad m=1,2,\ldots$	$a^{-1}\sin(at) \qquad 0<t<2m\pi/a$ $0 \qquad t>2m\pi/a$

Exponential functions of p and $1/p$ (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(13)	$p(p^2+a^2)^{-1}(1-e^{-2m\pi p/a})$ $a>0, \quad m=1,2,3,\ldots$	$\cos(at) \qquad 0<t<2m\pi/a$ $0 \qquad t>2m\pi/a$
(14)	$a^2p^{-1}(p^2+a^2)^{-1}$ $\times(1-e^{-2m\pi p/a})$	$2\sin^2(\tfrac{1}{2}at) \qquad 0<t<2m\pi/a$ $0 \qquad t>2m\pi/a$
(15)	$p^{-1}(e^{ap}-1)^{-1} \qquad a>0$	$n \qquad na<t<(n+1)a$
(16)	$p^{-2}(e^{ap}-1)^{-1} \qquad a>0$	$nt-\tfrac{1}{2}an(n+1) \qquad na<t<(n+1)a$
(17)	$p^{-1}(e^{ap}+1)^{-1} \qquad a>0$	$0 \qquad 2na<t<(2n+1)a$ $1 \qquad (2n+1)a<t<(2n+2)a$
(18)	$p^{-2}(e^{ap}+1)^{-1} \qquad a>0$	$\tfrac{1}{4}[1-(-1)^n](2t-a)+\tfrac{1}{2}(-1)^n na$ $na<t<(n+1)a$
(19)	$p^{-1}(e^{ap}-\beta)^{-1} \qquad a>0$	$(1-\beta^n)/(1-\beta) \qquad na<t<(n+1)a$
(20)	$p^{-2}(e^{ap}-\beta)^{-1} \qquad a>0$	$(1-\beta^n)(1-\beta)^{-1}t-a(1-\beta)^{-2}$ $\times[1-(n+1)\beta^n+n\beta^{n+1}]$ $na<t<(n+1)a$
(21)	$\dfrac{1}{(p^2+c^2)(e^{-ap}+1)} \qquad a>0$ $c>0, \quad ac\neq(2n+1)\pi$	$\dfrac{\sin(ct+\tfrac{1}{2}ac)}{2c\cos(\tfrac{1}{2}ac)}$ $+2a\sum_{n=0}^{\infty}\dfrac{\cos[(2n+1)\pi t/a]}{a^2c^2-(2n+1)^2\pi^2}$
(22)	$g(p)(e^{ap}+\beta)^{-c} \qquad a,\ c>0$	$\sum_{0\le n<t/a-c}\binom{-c}{n}\beta^n f(t-ac-an)$

Exponential functions of p and $1/p$ (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(23)	$g(p)(1+\beta e^{-ap})^{\nu}$ $\quad a>0$	$\sum_{0\le n\le t/a}\binom{\nu}{n}\beta^n f(t-an)$
(24)	$a(p^2+a^2)^{-1}(1+e^{-2m\pi p/a})^{-1}$ $a>0,\quad m=1,2,\ldots$	$\sin(at)\quad 2n<at/(2m\pi)<2n+1$ $0\quad 2n+1<at/(2m\pi)<2n+2$ $n=0,1,2,\ldots$
(25)	$p(p^2+a^2)^{-1}(1+e^{-2m\pi p/a})^{-1}$ $a>0,\quad m=1,2,\ldots$	$\cos(at)\quad 2n<at/(2m\pi)<2n+1$ $0\quad 2n+1<at/(2m\pi)<2n+2$ $n=0,1,2,\ldots$
(26)	$a^2p^{-1}(p^2+a^2)^{-1}$ $\times(1+e^{-2m\pi p/a})^{-1}$ $a>0,\quad m=1,2,\ldots$	$2\sin^2(\tfrac{1}{2}at)$ $2n<at/(2m\pi)<2n+1$ $0\quad 2n+1<at/(2m\pi)<2n+2$ $n=0,1,2,\ldots$
(27)	$(p^2+a^2)^{-1}(1+e^{-\pi p/a})$ $\times(1-e^{-\pi p/a})^{-1}$ $\quad a>0$	$a^{-1}\lvert\sin(at)\rvert$
(28)	$p(p^2+a^2)^{-1}(1+e^{-\pi p/a})$ $\times(1-e^{-\pi p/a})^{-1}$	$\cos(at)\quad 2n\pi<at<(2n+1)\pi$ $-\cos(at)$ $(2n+1)\pi<at<(2n+2)\pi$ $n=0,1,2,\ldots$
(29)	$g(p)(1+e^{-ap})(1-e^{-ap})^{-1}$ $a>0$	$f(t)+2\sum_{1\le n<t/a}f(t-an)$
(30)	$p^{-1}e^{-ap}(e^{ap}-1)^n$ $\quad a>0$	$\binom{n}{m}\quad na<t<(n+1)a$ $n=0,1,2,\ldots$
(31)	$e^{a/p}-1$	$a^{1/2}t^{-1/2}I_1(2a^{1/2}t^{1/2})$

Exponential functions of p and $1/p$ (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$		$f(t)$
(32)	$p^{-1/2} e^{a/p}$		$\pi^{-1/2} t^{-1/2} \cosh(2a^{1/2} t^{1/2})$
(33)	$p^{-3/2} e^{a/p}$		$\pi^{-1/2} a^{-1/2} \sinh(2a^{1/2} t^{1/2})$
(34)	$p^{-5/2} e^{a/p}$		$\pi^{-1/2} a^{-1} t^{1/2} \cosh(2a^{1/2} t^{1/2})$ $-\tfrac{1}{2}\pi^{-1/2} a^{-3/2} \sinh(2a^{1/2} t^{1/2})$
(35)	$p^{-\nu-1} e^{a/p}$	$\operatorname{Re} \nu > -1$	$a^{-\frac{1}{2}\nu} t^{\frac{1}{2}\nu} I_\nu(2a^{1/2} t^{1/2})$
(36)	$1-e^{-a/p}$		$a^{1/2} t^{-1/2} J_1(2a^{1/2} t^{1/2})$
(37)	$p^{-1/2} e^{-a/p}$		$\pi^{-1/2} t^{-1/2} \cos(2a^{1/2} t^{1/2})$
(38)	$p^{-3/2} e^{-a/p}$		$\pi^{-1/2} a^{-1/2} \sin(2a^{1/2} t^{1/2})$
(39)	$p^{-5/2} e^{-a/p}$		$\tfrac{1}{2}\pi^{-1/2} a^{-3/2} \sin(2a^{1/2} t^{1/2})$ $-a^{-1}\pi^{-1/2} t^{1/2} \cos(2a^{1/2} t^{1/2})$
(40)	$p^{-\nu-1} e^{-a/p}$	$\operatorname{Re} \nu > -1$	$a^{-\frac{1}{2}\nu} t^{\frac{1}{2}\nu} J_\nu(2a^{1/2} t^{1/2})$

5.6. Exponential functions of other arguments

(1)	$e^{-a^{1/2} p^{1/2}}$	$\operatorname{Re} a > 0$	$\tfrac{1}{2}\pi^{-1/2} a^{1/2} t^{-3/2} e^{-\frac{1}{4}a/t}$
(2)	$p e^{-a^{1/2} p^{1/2}}$	$\operatorname{Re} a > 0$	$\tfrac{1}{4}\pi^{-1/2} a^{1/2} (\tfrac{1}{2} a t^{-1} - 3) t^{-5/2} e^{-\frac{1}{4}a/t}$
(3)	$p^{-1} e^{-a^{1/2} p^{1/2}}$	$\operatorname{Re} a \geq 0$	$\operatorname{Erfc}(\tfrac{1}{2} a^{1/2} t^{-1/2})$
(4)	$p^{-2} e^{-a^{1/2} p^{1/2}}$	$\operatorname{Re} a \geq 0$	$(t+\tfrac{1}{2}a) \operatorname{Erfc}(\tfrac{1}{2} a^{1/2} t^{-1/2})$ $-\pi^{-1/2} a^{1/2} t^{1/2} e^{-\frac{1}{4}a/t}$

Exponential functions of other arguments (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(5)	$p^{1/2}e^{-\alpha^{1/2}p^{1/2}}$ Re $\alpha>0$	$\pi^{-1/2}t^{-5/2}(\frac{1}{4}\alpha-\frac{1}{2}t)e^{-\frac{1}{4}\alpha/t}$
(6)	$p^{-1/2}e^{-\alpha^{1/2}p^{1/2}}$ Re $\alpha\geq 0$	$\pi^{-1/2}t^{-1/2}e^{-\frac{1}{4}\alpha/t}$
(7)	$p^{-3/2}e^{-\alpha^{1/2}p^{1/2}}$ Re $\alpha\geq 0$	$2\pi^{-1/2}t^{1/2}e^{-\frac{1}{4}\alpha/t}$ $-\alpha^{1/2}\operatorname{Erfc}(\frac{1}{2}\alpha^{1/2}t^{-1/2})$
(8)	$p^{\frac{1}{2}n-\frac{1}{2}}e^{-\alpha^{1/2}p^{1/2}}$ Re $\alpha>0$	$2^{-\frac{1}{2}n}\pi^{-1/2}t^{-\frac{1}{2}n-\frac{1}{2}}e^{-\frac{1}{4}\alpha/t}$ $\times \operatorname{He}_n(2^{-1/2}\alpha^{1/2}t^{-1/2})$
(9)	$p^{\nu-1/2}e^{-\alpha^{1/2}p^{1/2}}$ Re $\alpha>0$	$2^{-\nu}\pi^{-1/2}t^{-\nu-1/2}e^{-\alpha t^{-1}/8}$ $\times D_{2\nu}(2^{-1/2}\alpha^{1/2}t^{-1/2})$
(10)	$2(p+\beta)^{-1}e^{-\alpha^{1/2}p^{1/2}}$ Re $\alpha\geq 0$	$e^{-\beta t}[e^{-i\alpha^{1/2}\beta^{1/2}}\operatorname{Erfc}(\frac{1}{2}\alpha^{1/2}t^{-1/2}-i\beta^{1/2}t^{1/2})$ $+e^{i\alpha^{1/2}\beta^{1/2}}\operatorname{Erfc}(\frac{1}{2}\alpha^{1/2}t^{-1/2}+i\beta^{1/2}t^{1/2})]$
(11)	$p(p^2+\beta^2)^{-1}e^{-\alpha^{1/2}p^{1/2}}$ Re $\alpha\geq 0$, Re $\beta\geq 0$	$\exp[-(\frac{1}{2}\alpha\beta)^{1/2}]\cos[\beta t-(\frac{1}{2}\alpha\beta)^{1/2}]$ $-\dfrac{1}{\pi}\displaystyle\int_0^\infty e^{-ut}\sin(\alpha^{1/2}u^{1/2})\frac{u}{u^2+\beta^2}\,du$
(12)	$(p^{1/2}+\beta)^{-1}e^{-\alpha p^{1/2}}$ Re $\alpha^2\geq 0$	$\pi^{-1/2}t^{-1/2}e^{-\frac{1}{4}\alpha^2/t}$ $-\beta e^{\alpha\beta+\beta^2 t}\operatorname{Erfc}(\frac{1}{2}\alpha t^{-1/2}+\beta t^{1/2})$
(13)	$p(p^{1/2}+\beta)^{-1}e^{-\alpha p^{1/2}}$ Re $\alpha^2>0$	$\pi^{-1/2}t^{-3/2}(\frac{1}{4}\alpha^2 t^{-1}-\frac{1}{2}$ $-\frac{1}{2}\alpha\beta+\beta^2 t)e^{-\frac{1}{4}\alpha^2/t}$ $-\beta^3 e^{\alpha\beta+\beta^2 t}\operatorname{Erfc}(\frac{1}{2}\alpha t^{-1/2}+\beta t^{1/2})$
(14)	$\beta p^{-1}(p^{1/2}+\beta)^{-1}e^{-\alpha p^{1/2}}$ Re $\alpha^2\geq 0$	$\operatorname{Erfc}(\frac{1}{2}\alpha t^{-1/2})$ $-e^{\alpha\beta+\beta^2 t}\operatorname{Erfc}(\frac{1}{2}\alpha t^{-1/2}+\beta t^{1/2})$

Exponential functions of other arguments (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(15)	$p^{1/2}(p^{1/2}+\beta)^{-1}e^{-ap^{1/2}}$ Re $a^2 \geq 0$	$\pi^{-1/2}t^{-1}(\tfrac{1}{2}at^{-1/2}-\beta t^{1/2})e^{-\frac{1}{4}a^2/t}$ $+\beta^2 e^{a\beta+\beta^2 t}\,\mathrm{Erfc}(\tfrac{1}{2}at^{-1/2}+\beta t^{1/2})$
(16)	$p^{-1/2}(p^{1/2}+\beta)^{-1}e^{-ap^{1/2}}$ Re $a^2 \geq 0$	$e^{a\beta+\beta^2 t}\,\mathrm{Erfc}(\tfrac{1}{2}at^{-1/2}+\beta t^{1/2})$
(17)	$p^{-3/2}(p^{1/2}+\beta)^{-1}e^{-ap^{1/2}}$ Re $a^2 \geq 0$	$2\pi^{-1/2}\beta^{-1}t^{1/2}e^{-\frac{1}{4}a^2/t}$ $-(\beta^{-2}+a\beta^{-1})\,\mathrm{Erfc}(\tfrac{1}{2}at^{-1/2})$ $+\beta^{-2}e^{a\beta+\beta^2 t}\,\mathrm{Erfc}(\tfrac{1}{2}at^{-1/2}+\beta t^{1/2})$
(18)	$p^{-1}(p^{1/2}+\beta)^{-2}e^{-ap^{1/2}}$ Re $a^2 > 0$	$\beta^{-2}\,\mathrm{Erfc}(\tfrac{1}{2}at^{-1/2})$ $-2\pi^{-1/2}\beta^{-1}t^{1/2}e^{-\frac{1}{4}a^2/t}$ $+(2t+a\beta^{-1}-\beta^{-2})e^{a\beta+\beta^2 t}$ $\times\,\mathrm{Erfc}(\tfrac{1}{2}at^{-1/2}+\beta t^{1/2})$
(19)	$p^{-1/2}(p^{1/2}+\beta)^{-2}e^{-ap^{1/2}}$ Re $a^2 > 0$	$2\pi^{-1/2}t^{1/2}e^{-\frac{1}{4}a^2/t}-(2\beta t+a)e^{a\beta+\beta^2 t}$ $\times\,\mathrm{Erfc}(\tfrac{1}{2}at^{-1/2}+\beta t^{1/2})$
	See also Campbell, G. A. and R. M. Foster, 1931: *Fourier integrals for practical applications*, Bell Telephone Laboratories, New York	
(20)	$(p-1)^{-1/2}e^{-ap^{1/2}}$ Re $a^2 > 0$	$\pi^{-1/2}t^{-1/2}e^{t}\{\exp(-\tfrac{1}{4}a^2/t)$ $-a\int_0^\infty e^{-\frac{1}{4}u^2/t}(u^2-a^2)^{-1/2}$ $\times J_1[(u^2-a^2)^{1/2}]\,du\}$
(21)	$p^{-\frac{1}{2}\nu-\frac{1}{2}}e^{-ap^{1/2}}$ Re $a>0$, Re $\nu>-1$	$\tfrac{1}{2}\pi^{-1/2}a^{-\nu/2}t^{-3/2}$ $\times\int_0^\infty u^{1+\nu/2}e^{-\frac{1}{4}u^2/t}$ $\times J_\nu(2a^{1/2}u^{1/2})\,du$

Exponential functions of other arguments (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(22)	$p^{-\nu-1}\exp[-a^{-1}p^{-1}(p^2+1)^{1/2}]$ $\quad \mathrm{Re}\,\nu>-1$	$a^{-\frac{1}{2}\nu}t^{\frac{1}{2}\nu}\left\{J_\nu(2a^{\frac{1}{2}}t^{\frac{1}{2}}) - a\int_0^\infty \frac{J_\nu(2a^{\frac{1}{2}}t^{\frac{1}{2}})J_1[(u^2-a^2)^{\frac{1}{2}}]}{(u^2-a^2)^{\frac{1}{2}}}du\right\}$
(23)	$e^{-bp}-e^{-br}$ $\quad b>0$	$0 \quad 0<t<b$ $aby^{-1}J_1(ay) \quad t>b$
(24)	$r^{-1}e^{-br}$ $\quad b>0$	$0 \quad 0<t<b$ $J_0(ay) \quad t>b$
(25)	$(1-pr^{-1})e^{-br}$ $\quad b>0$	$0 \quad 0<t<b$ $a\left(\frac{t-b}{t+b}\right)^{\frac{1}{2}}J_1(ay) \quad t>b$
(26)	$r^{-2}(b+r^{-1})e^{-br}$ $\quad b>0$	$0 \quad 0<t<b$ $a^{-1}y\,J_1(ay) \quad t>b$
(27)	$e^{-bp}-pr^{-1}e^{-br}$ $\quad b>0$	$0 \quad 0<t<b$ $aty^{-1}J_1(ay) \quad t>b$
(28)	$r^{-1}R^{\frac{1}{2}}e^{-br}$ $\quad b>0$	$0 \quad 0<t<b$ $2^{\frac{1}{2}}\pi^{-\frac{1}{2}}(t+b)^{-\frac{1}{2}}\cos(ay) \quad t>b$
(29)	$ar^{-1}R^{-\frac{1}{2}}e^{-br}$ $\quad b>0$	$0 \quad 0<t<b$ $2^{\frac{1}{2}}\pi^{-\frac{1}{2}}(t+b)^{-\frac{1}{2}}\sin(ay) \quad t>b$

$y=(t^2-b^2)^{\frac{1}{2}}, \quad r=(p^2+a^2)^{\frac{1}{2}}, \quad R=p+r$

Exponential functions of other arguments (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(30)	$a^\nu r^{-1} R^{-\nu} e^{-br}$ $\quad b>0$, $\mathrm{Re}\,\nu>-1$	$0 \quad 0<t<b$ $(t-b)^{\frac{1}{2}\nu}(t+b)^{-\frac{1}{2}\nu} J_\nu(a y) \quad t>b$
(31)	$1-e^{-\beta(r-p)}$	$a\beta(t^2+2\beta t)^{-\frac{1}{2}} J_1[a(t^2+2\beta t)^{\frac{1}{2}}]$
(32)	$r^{-1} R^{\frac{1}{2}} e^{\beta(p-r)}$	$2^{\frac{1}{2}}\pi^{-\frac{1}{2}}(t+2\beta)^{-\frac{1}{2}}\cos[a(t^2+2\beta t)^{\frac{1}{2}}]$
(33)	$ar^{-1} R^{-\frac{1}{2}} e^{\beta(p-r)}$	$2^{\frac{1}{2}}\pi^{-\frac{1}{2}}(t+2\beta)^{-\frac{1}{2}}\sin[a(t^2+2\beta t)^{\frac{1}{2}}]$
(34)	$a^\nu r^{-1} R^{-\nu} e^{-\beta(r-p)}$ $\quad \mathrm{Re}\,\nu>-1$	$t^{\frac{1}{2}\nu}(t+2\beta)^{-\frac{1}{2}\nu} J_\nu[a(t^2+2\beta t)^{\frac{1}{2}}]$
(35)	$e^{-bs}-e^{-bp}$ $\quad b>0$	$0 \quad 0<t<b$ $aby^{-1} I_1(ay) \quad t>b$
(36)	$s^{-1} e^{-bs}$ $\quad b>0$	$0 \quad 0<t<b$ $I_0(ay) \quad t>b$
(37)	$ps^{-1} e^{-bs}-e^{-bp}$ $\quad b>0$	$0 \quad 0<t<b$ $aty^{-1} I_1(ay) \quad t>b$
(38)	$(ps^{-1}-1)e^{-bs}$ $\quad b>0$	$0 \quad 0<t<b$ $a\left(\frac{t-b}{t+b}\right)^{\frac{1}{2}} I_1(ay) \quad t>b$
(39)	$s^{-2}(b+s^{-1})e^{-bs}$ $\quad b>0$	$0 \quad 0<t<b$ $a^{-1} y\, I_1(ay) \quad t>b$

$y=(t^2-b^2)^{\frac{1}{2}}$, $r=(p^2+a^2)^{\frac{1}{2}}$, $R=p+r$,
$s=(p^2-a^2)^{\frac{1}{2}}$, $S=p+s$

Exponential functions of other arguments (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(40)	$a^\nu s^{-1}S^{-\nu}e^{-bs}$ $\quad b>0$, $\operatorname{Re}\nu>-1$	$0 \quad 0<t<b$ $(t-b)^{\frac12\nu}(t+b)^{-\frac12\nu}I_\nu(a\gamma) \quad t>b$
(41)	$e^{\beta(p-s)}-1$	$a\beta(t^2+2\beta t)^{-\frac12}I_1[a(t^2+2\beta t)^{\frac12}]$
(42)	$s^{-1}e^{\beta(p-s)}$	$I_0[a(t^2+2\beta t)]$
(43)	$1-ps^{-1}e^{\beta(p-s)}$	$-a(t+\beta)(t^2+2\beta t)^{-\frac12}I_1[a(t^2+2\beta t)^{\frac12}]$
(44)	$a^\nu s^{-1}S^{-\nu}e^{\beta(p-s)}$ $\quad \operatorname{Re}\nu>-1$	$t^{\frac12\nu}(t+2\beta)^{-\frac12\nu}I_\nu[a(t^2+2\beta t)^{\frac12}]$
(45)	$(p+a)^{-\frac12}(p+\beta)^{-\frac12}[p+\tfrac12(a+\beta)$ $+(p+a)^{\frac12}(p+\beta)^{\frac12}]^{-\nu}$ $\times\exp[-b(p+a)^{\frac12}(p+\beta)^{\frac12}]$ $\operatorname{Re}\nu>-1, \quad b>0$	$0 \quad 0<t<b$ $[\tfrac12(a-\beta)]^{-\nu}(t-b)^{\frac12\nu}(t+b)^{-\frac12\nu}$ $\times e^{-\frac12(a+\beta)t}I_\nu[\tfrac12(a-\beta)\gamma] \quad t>b$
(46)	$(p+a)^{-\frac12}(p+\beta)^{-\frac12}[p+\tfrac12(a+\beta)$ $+(p+a)^{\frac12}(p+\beta)^{\frac12}]^{-\nu}$ $\times\exp[\gamma p-\gamma(p+a)^{\frac12}(p+\beta)^{\frac12}]$ $\operatorname{Re}\nu>-1$	$[\tfrac12(a-\beta)]^{-\nu}t^{\frac12\nu}(t+2\gamma)^{-\frac12\nu}$ $\times\exp[-\tfrac12(a+\beta)(t+\gamma)]$ $\times I_\nu[\tfrac12(a-\beta)(t^2+2\gamma t)^{\frac12}]$

5.7. Logarithmic functions

(1)	$p^{-1}\log p$	$-\log(\gamma t)$
(2)	$p^{-n-1}\log p$	$[1+1/2+1/3+\cdots+1/n-\log(\gamma t)]t^n/n!$
(3)	$p^{-n-\frac12}\log p$	$\dfrac{2^n t^{n-\frac12}}{1\cdot3\cdot5\cdots(2n-1)\pi^{\frac12}}\left[2\left(1+\dfrac13+\cdots+\dfrac{1}{2n-1}\right)-\log(4\gamma t)\right]$

$\gamma=(t^2-b^2)^{\frac12}; \quad s=(p^2-a^2)^{\frac12}, \quad S=p+s$

Logarithmic functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(4)	$\Gamma(\nu)p^{-\nu}\log p$ $\qquad$ $\operatorname{Re}\nu>0$	$t^{\nu-1}[\psi(\nu)-\log t]$
(5)	$\dfrac{\log(p+\beta)}{p+a}$	$e^{-at}\{\log(\beta-a)-\operatorname{Ei}[(a-\beta)t]\}$
(6)	$a(p^2+a^2)^{-1}\log p$	$\cos(at)\operatorname{Si}(at)$ $+\sin(at)[\log a-\operatorname{Ci}(at)]$
(7)	$p(p^2+a^2)^{-1}\log p$	$\cos(at)[\log a-\operatorname{Ci}(at)]$ $-\sin(at)\operatorname{Si}(at)$
(8)	$p^{-1}(\log p)^2$	$[\log(\gamma t)]^2-\pi^2/6$
(9)	$p^{-1}[\log(\gamma p)]^2$	$(\log t)^2-\pi^2/6$
(10)	$p^{-2}(\log p)^2$	$t\{[1-\log(\gamma t)]^2+1-\pi^2/6\}$
(11)	$p^{-a}(\log p)^{-1}$ $\qquad$ $a\geq 0$	$\nu(t,a-1)$
(12)	$\log\dfrac{p+\beta}{p+a}$	$\dfrac{e^{-at}-e^{-\beta t}}{t}$
(13)	$p\log\dfrac{p+a}{p+\beta}+\beta-a$	$(at^{-1}+t^{-2})e^{-at}-(\beta t^{-1}+t^{-2})e^{-\beta t}$
(14)	$p^{-1}\log(p^2+a^2)$	$2\operatorname{ci}(at)+2\log a$
(15)	$\log\dfrac{p^2+\beta^2}{p^2+a^2}$	$\dfrac{2}{t}[\cos(at)-\cos(\beta t)]$
(16)	$p\log\dfrac{p^2+\beta^2}{p^2+a^2}$	$\dfrac{2}{t^2}[\cos(\beta t)+\beta t\sin(\beta t)$ $-\cos(at)-at\sin(at)]$

Logarithmic functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(17)	$\log\dfrac{(p+\alpha)^2+\lambda^2}{(p+\beta)^2+\lambda^2}$	$2t^{-1}\cos(\lambda t)(e^{-\beta t}-e^{-\alpha t})$
(18)	$\dfrac{\log[(p+\alpha)^{1/2}+(p+\beta)^{1/2}]}{(p+\beta)^{1/2}}$	$\dfrac{e^{-\beta t}}{2\pi^{1/2}t^{1/2}}\{\log(\alpha-\beta)-\mathrm{Ei}[(\beta-\alpha)t]\}$
(19)	$p^{-1}\log r$	$\log a+\mathrm{ci}(at)$
(20)	$p^{-2}\log r$ Re $a>0$	$t[\log a+a^{-1}t^{-1}\sin(at)+\mathrm{ci}(at)]$
(21)	$ar^{-2}\log r$	$\tfrac{1}{2}\sin(at)\left[\log\left(\dfrac{2a}{\gamma t}\right)-\mathrm{Ci}(2at)\right]$ $+\tfrac{1}{2}\cos(at)\,\mathrm{Si}(2at)$
(22)	$pr^{-2}\log r$	$\tfrac{1}{2}\cos(at)\left[\log\dfrac{2a}{\gamma t}-\mathrm{Ci}(2at)\right]$ $-\tfrac{1}{2}\sin(at)\,\mathrm{Si}(2at)$
(23)	$p\log(r/p)$	$t^{-2}[\cos(at)-1]+at^{-1}\sin(at)$
(24)	$r\log\dfrac{a+r}{p}-a$	$\tfrac{1}{2}\pi a t^{-1}\mathbf{H}_1(at)$
(25)	$pr^{-1}\log\dfrac{a+r}{p}$	$a-\tfrac{1}{2}\pi a\,\mathbf{H}_1(at)$
(26)	$r^{-1}\log R$	$\log a\,J_0(at)-\tfrac{1}{2}\pi Y_0(at)$
(27)	$ar^{-3}\log R$	$t\,[J_1(at)\log a-\tfrac{1}{2}\pi Y_1(at)]$ $-a^{-1}\cos(at)$

Logarithmic functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(28)	$pr^{-3}\log R$	$t\{J_0(at)\log a-\frac{1}{2}\pi Y_0(at)]$ $+a^{-1}\sin(at)$
(29)	$p\log(s/p)$	$t^{-2}[\cosh(at)-1]-at^{-1}\sinh(at)$
(30)	$s^{-1}\log S$	$I_0(at)\log a+K_0(at)$
(31)	$as^{-3}\log S$	$t[I_1(at)\log a-K_1(at)]$ $+a^{-1}\cosh(at)$
(32)	$ps^{-3}\log S$	$t[I_0(at)\log a+K_0(at)]$ $+a^{-1}\sinh(at)$
(33)	$2(A^2-1)^{-\frac{1}{2}}(B^2-1)^{-\frac{1}{2}}$ $\times\log\dfrac{(A+1)(B+1)+(A^2-1)^{\frac{1}{2}}(B^2-1)^{\frac{1}{2}}}{(A+1)(B+1)-(A^2-1)^{\frac{1}{2}}(B^2-1)^{\frac{1}{2}}}$ where $A^2=p+\beta,\quad B^2=p-\beta$	$-e^{t}I_0(\beta t)\,\mathrm{Ei}(-t)$

5.8. Trigonometric functions

(1)	$p^{-1}\sin(ap^{-1})$	$\mathrm{bei}(2a^{\frac{1}{2}}t^{\frac{1}{2}})$
(2)	$p^{-1}\cos(ap^{-1})$	$\mathrm{ber}(2a^{\frac{1}{2}}t^{\frac{1}{2}})$
(3)	$p^{-\frac{1}{2}}\sin(ap^{-1})$	$\pi^{-\frac{1}{2}}t^{-\frac{1}{2}}\sinh(2^{\frac{1}{2}}a^{\frac{1}{2}}t^{\frac{1}{2}})\sin(2^{\frac{1}{2}}a^{\frac{1}{2}}t^{\frac{1}{2}})$
(4)	$p^{-\frac{1}{2}}\cos(ap^{-1})$	$\pi^{-\frac{1}{2}}t^{-\frac{1}{2}}\cosh(2^{\frac{1}{2}}a^{\frac{1}{2}}t^{\frac{1}{2}})\cos(2^{\frac{1}{2}}a^{\frac{1}{2}}t^{\frac{1}{2}})$
(5)	$p^{-3/2}\sin(ap^{-1})$	$\pi^{-\frac{1}{2}}a^{-\frac{1}{2}}\cosh(2^{\frac{1}{2}}a^{\frac{1}{2}}t^{\frac{1}{2}})\sin(2^{\frac{1}{2}}a^{\frac{1}{2}}t^{\frac{1}{2}})$

Trigonometric functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(6)	$p^{-3/2}\cos(ap^{-1})$	$\pi^{-1/2} a^{-1/2}\sinh(2^{1/2}a^{1/2}t^{1/2})\cos(2^{1/2}a^{1/2}t^{1/2})$
(7)	$p^{-\nu-1}\sin(ap^{-1})$ Re $\nu>-2$	$(t/a)^{1/2\nu}[\cos(\tfrac{3}{4}\pi\nu)\,\mathrm{bei}_\nu(2a^{1/2}t^{1/2})$ $-\sin(\tfrac{3}{4}\pi\nu)\,\mathrm{ber}_\nu(2a^{1/2}t^{1/2})]$
(8)	$p^{-\nu-1}\cos(ap^{-1})$ Re $\nu>-1$	$(t/a)^{1/2\nu}[\cos(\tfrac{3}{4}\pi\nu)\,\mathrm{ber}_\nu(2a^{1/2}t^{1/2})$ $+\sin(\tfrac{3}{4}\pi\nu)\,\mathrm{bei}_\nu(2a^{1/2}t^{1/2})]$
(9)	$p^{-1/2}e^{-a^{1/2}p^{1/2}}\sin(a^{1/2}p^{1/2})$	$\pi^{-1/2}t^{-1/2}\sin(\tfrac{1}{2}at^{-1})$
(10)	$p^{-1/2}e^{-a^{1/2}p^{1/2}}\cos(a^{1/2}p^{1/2})$	$\pi^{-1/2}t^{-1/2}\cos(\tfrac{1}{2}at^{-1})$
(11)	$p^{-\mu-1/2}\sin(a^{-1/2}p^{-1/2})$ Re $\mu>-1$	$a^{-1/2}[\Gamma(\mu+1)]^{-1}t^{\mu}$ $\times\,{}_0F_2(\mu+1,3/2;-\tfrac{1}{4}t/a)$
(12)	$p^{-\mu-1}\cos(a^{-1/2}p^{-1/2})$ Re $\mu>-1$	$[\Gamma(\mu+1)]^{-1}t^{\mu}\,{}_0F_2(\mu+1,\tfrac{1}{2};-\tfrac{1}{4}t/a)$
(13)	$\Gamma(\nu+\tfrac{1}{2})p^{-\nu}\sin[(2n+1)\sin^{-1}(p^{-1/2})]$ Re $\nu>-\tfrac{1}{2}$	$t^{\nu-1/2}(2n+1)$ $\times\,{}_2F_2(-n,n;\nu+1/2,3/2;t)$
(14)	$\Gamma(\nu)p^{-\nu}\cos[2n\sin^{-1}(p^{-1/2})]$ Re $\nu>0$	$t^{\nu-1}\,{}_2F_2(-n,n;\nu,\tfrac{1}{2};t)$
(15)	$p^{-1}\tan^{-1}p$	$-\mathrm{si}(t)$
(16)	$p^{-1}\mathrm{ctn}^{-1}p$	$\mathrm{Si}(t)$
(17)	$\tan^{-1}(ap^{-1})$	$t^{-1}\sin(at)$

Trigonometric functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(18)	$p\tan^{-1}(ap^{-1})-a$	$t^{-2}[at\cos(at)-\sin(at)]$
(19)	$\log(p^2+a^2)^{\frac{1}{2}}\tan^{-1}(ap^{-1})$	$-t^{-1}\log(\gamma t)\sin(at)$
(20)	$\dfrac{\sin[\beta+\tan^{-1}(ap^{-1})]}{(p^2+a^2)^{\frac{1}{2}}}$	$\sin(at+\beta)$
(21)	$\dfrac{\cos[\beta+\tan^{-1}(ap^{-1})]}{(p^2+a^2)^{\frac{1}{2}}}$	$\cos(at+\beta)$
(22)	$\tan^{-1}[2ap(p^2+\beta^2)^{-1}]$	$2t^{-1}\sin(at)\cos[(a^2+\beta^2)^{\frac{1}{2}}t]$

5.9. Hyperbolic functions

(1)	$p^{-1}\operatorname{sech}(ap)$ $\quad a>0$	$0 \quad 4n-1<t/a<4n+1$ $2 \quad 4n+1<t/a<4n+3$
(2)	$p^{-2}\operatorname{sech}(ap)$ $\quad a>0$	$t-(-1)^n(t-2an)$ $2n-1<t/a<2n+1$
(3)	$p^{-1}\operatorname{csch}(ap)$ $\quad a>0$	$2n \quad 2n-1<t/a<2n+1$
(4)	$p^{-2}\operatorname{csch}(ap)$ $\quad a>0$	$2n(t-an) \quad 2n-1<t/a<2n+1$
(5)	$a(p^2+a^2)^{-1}\operatorname{csch}(\frac{1}{2}\pi p/a)$ $a>0$	$\lvert\cos(at)\rvert-\cos(at)$
(6)	$p^{-1}\tanh(pa)$ $\quad a>0$	$(-1)^{n-1} \quad n-1<\frac{1}{2}t/a<n$
(7)	$p^{-2}\tanh(ap)$ $\quad a>0$	$a+(-1)^n(2an-a-t)$ $n-1<\frac{1}{2}t/a<n$

Hyperbolic functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(8)	$p^{-1}\operatorname{ctnh}(ap)$ $\quad a>0$	$2n-1$ $\quad n-1<\tfrac{1}{2}t/a<n$
(9)	$p^{-2}\operatorname{ctnh}(ap)$ $\quad a>0$	$(2n-1)t-2an(n-1)$ $\quad n-1<\tfrac{1}{2}t/a<n$
(10)	$a(p^2+a^2)^{-1}\operatorname{ctnh}(\tfrac{1}{2}\pi p/a)$ $\quad a>0$	$\lvert\sin(at)\rvert$
(11)	$p^{-\frac{1}{2}}\sinh(ap^{-1})$	$\tfrac{1}{2}\pi^{-\frac{1}{2}}t^{-\frac{1}{2}}[\cosh(2a^{\frac{1}{2}}t^{\frac{1}{2}}) - \cos(2a^{\frac{1}{2}}t^{\frac{1}{2}})]$
(12)	$p^{-3/2}\sinh(ap^{-1})$	$\tfrac{1}{2}a^{-\frac{1}{2}}\pi^{-\frac{1}{2}}[\sinh(2a^{\frac{1}{2}}t^{\frac{1}{2}}) - \sin(2a^{\frac{1}{2}}t^{\frac{1}{2}})]$
(13)	$p^{-5/2}\sinh(ap^{-1})$	$\tfrac{1}{2}a^{-1}\pi^{-1/2}t^{1/2}[\cosh(2a^{1/2}t^{1/2}) + \cos(2a^{1/2}t^{1/2})] - \tfrac{1}{4}a^{-3/2}\pi^{-1/2} \times [\sinh(2a^{1/2}t^{1/2}) + \sin(2a^{1/2}t^{1/2})]$
(14)	$p^{-\nu-1}\sinh(ap^{-1})$ $\quad \operatorname{Re}\nu>-2$	$\tfrac{1}{2}a^{-\frac{1}{2}\nu}t^{\frac{1}{2}\nu}[I_\nu(2a^{\frac{1}{2}}t^{\frac{1}{2}}) - J_\nu(2a^{\frac{1}{2}}t^{\frac{1}{2}})]$
(15)	$p^{-\frac{1}{2}}\cosh(ap^{-1})$	$\tfrac{1}{2}\pi^{-\frac{1}{2}}t^{-\frac{1}{2}}[\cos(2a^{\frac{1}{2}}t^{\frac{1}{2}}) + \cosh(2a^{\frac{1}{2}}t^{\frac{1}{2}})]$
(16)	$p^{-3/2}\cosh(ap^{-1})$	$\tfrac{1}{2}a^{-\frac{1}{2}}\pi^{-\frac{1}{2}}[\sinh(2a^{\frac{1}{2}}t^{\frac{1}{2}}) + \sin(2a^{\frac{1}{2}}t^{\frac{1}{2}})]$
(17)	$p^{-5/2}\cosh(ap^{-1})$	$\tfrac{1}{2}a^{-1}\pi^{-1/2}t^{1/2}[\cosh(2a^{1/2}t^{1/2}) - \cos(2a^{1/2}t^{1/2})] - \tfrac{1}{4}a^{-3/2}\pi^{-1/2} \times [\sinh(2a^{1/2}t^{1/2}) - \sin(2a^{1/2}t^{1/2})]$
(18)	$p^{-\nu-1}\cosh(ap^{-1})$ $\quad \operatorname{Re}\nu>-1$	$\tfrac{1}{2}a^{-\frac{1}{2}\nu}t^{\frac{1}{2}\nu}[I_\nu(2a^{\frac{1}{2}}t^{\frac{1}{2}}) + J_\nu(2a^{\frac{1}{2}}t^{\frac{1}{2}})]$

Hyperbolic functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(19)	$\operatorname{sech}(p^{1/2})$	$-\left[\frac{\partial}{\partial v}\theta_1(\tfrac{1}{2}v\|i\pi t)\right]_{v=0}$
(20)	$p^{-1/2}\operatorname{sech}(p^{1/2})$	$\hat{\theta}_2(\tfrac{1}{2}\|i\pi t)$
(21)	$\operatorname{csch}(p^{1/2})$	$-\left[\frac{\partial}{\partial v}\hat{\theta}_4(\tfrac{1}{2}v\|i\pi t)\right]_{v=0}$
(22)	$p^{-1/2}\operatorname{csch}(p^{1/2})$	$\theta_4(0\|i\pi t)$
(23)	$p^{-1}\tanh(p^{1/2})$	$\int_0^1 \hat{\theta}_2(\tfrac{1}{2}v\|i\pi t)\,dv$
(24)	$p^{-1/2}\tanh(p^{1/2})$	$\theta_2(0\|i\pi t)$
(25)	$p^{-1/2}\tanh(p^{1/2}+a)$	$e^{a^2t}[\theta_3(at\|i\pi t)+\hat{\theta}_3(at\|i\pi t)]-\pi^{-1/2}t^{-1/2}$
(26)	$p^{-1/2}\operatorname{ctnh}(p^{1/2})$	$\theta_3(0\|i\pi t)$
(27)	$\dfrac{\sinh(xp)}{p\cosh(ap)}$ $\quad 0\le x\le a$	$\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n-\frac{1}{2}}\sin[(n-\tfrac{1}{2})\pi x/a]$ $\times\sin[(n-\tfrac{1}{2})\pi t/a]$
(28)	$\dfrac{\cosh(xp)}{p\cosh(ap)}$ $\quad -a\le x\le a$	$1+\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n}}{n-\frac{1}{2}}\cos[(n-\tfrac{1}{2})\pi x/a]$ $\times\cos[(n-\tfrac{1}{2})\pi t/a]$

Hyperbolic functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$		$f(t)$
(29)	$\dfrac{\sinh(xp)}{p^2\cosh(ap)}$	$0 \le x \le a$	$x+\dfrac{2a}{\pi^2}\sum_{n=1}^{\infty}\dfrac{(-1)^n}{(n-\frac{1}{2})^2}\sin[(n-\frac{1}{2})\pi x/a]$ $\times\cos[(n-\frac{1}{2})\pi t/a]$
(30)	$\dfrac{\cosh(xp)}{p^2\cosh(ap)}$	$-a \le x \le a$	$t+\dfrac{2a}{\pi^2}\sum_{n=1}^{\infty}\dfrac{(-1)^n}{(n-\frac{1}{2})^2}\cos[(n-\frac{1}{2})\pi x/a]$ $\times\sin[(n-\frac{1}{2})\pi t/a]$
(31)	$\dfrac{\sinh(xp^{1/2})}{\sinh(lp^{1/2})}$	$-l < x < l$	$\dfrac{1}{l}\dfrac{\partial}{\partial x}\ \theta_4\left(\dfrac{x}{2l}\middle\vert\dfrac{i\pi t}{l^2}\right)$
(32)	$\dfrac{\sinh(xp^{1/2})}{p^{1/2}\sinh(lp^{1/2})}$	$-l \le x \le l$	$-\dfrac{1}{l}\ \hat{\theta}_4\left(\dfrac{x}{2l}\middle\vert\dfrac{i\pi t}{l^2}\right)$
(33)	$\dfrac{\sinh(xp^{1/2})}{\cosh(lp^{1/2})}$	$-l < x < l$	$-\dfrac{1}{l}\dfrac{\partial}{\partial x}\ \hat{\theta}_1\left(\dfrac{x}{2l}\middle\vert\dfrac{i\pi t}{l^2}\right)$
(34)	$\dfrac{\sinh(xp^{1/2})}{p^{1/2}\cosh(lp^{1/2})}$	$-l \le x \le l$	$-\dfrac{1}{l}\ \theta_1\left(\dfrac{x}{2l}\middle\vert\dfrac{i\pi t}{l^2}\right)$
(35)	$\dfrac{\cosh(xp^{1/2})}{\sinh(lp^{1/2})}$	$-l \le x \le l$	$-\dfrac{1}{l}\dfrac{\partial}{\partial x}\ \hat{\theta}_4\left(\dfrac{x}{2l}\middle\vert\dfrac{i\pi t}{l^2}\right)$
(36)	$\dfrac{\cosh(xp^{1/2})}{p^{1/2}\sinh(lp^{1/2})}$	$-l \le x \le l$	$\dfrac{1}{l}\ \theta_4\left(\dfrac{x}{2l}\middle\vert\dfrac{i\pi t}{l^2}\right)$
(37)	$\dfrac{\cosh(xp^{1/2})}{\cosh(lp^{1/2})}$	$-l \le x \le l$	$-\dfrac{1}{l}\dfrac{\partial}{\partial x}\ \theta_1\left(\dfrac{x}{2l}\middle\vert\dfrac{i\pi t}{l^2}\right)$

Hyperbolic functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(38)	$\dfrac{\cosh(xp^{1/2})}{p^{1/2}\cosh(lp^{1/2})}$ $\quad -l \le x \le l$	$-\dfrac{1}{l}\,\hat{\theta}_1\left(\dfrac{x}{2l},\dfrac{t}{l^2}\right)$
(39)	$\dfrac{1}{p-i\omega}\,\dfrac{\sinh(xp^{1/2})}{\sinh(lp^{1/2})}$ $\quad l \ge x > 0$	$\dfrac{\sinh(xi^{1/2}\omega^{1/2})}{\sinh(li^{1/2}\omega^{1/2})}\,e^{i\omega t}$ $+2\pi\sum_{n=1}^{\infty}\dfrac{n(-1)^n\sin(n\pi x/l)}{n^2\pi^2+i\omega l^2}$ $\times e^{-n^2\pi^2 t/l^2}$
(40)	$\dfrac{1}{p-i\omega}\,\dfrac{\cosh(xp^{1/2})}{\cosh(lp^{1/2})}$	$\dfrac{\cosh(xi^{1/2}\omega^{1/2})}{\cosh(xi^{1/2}\omega^{1/2})}\,e^{i\omega t}$ $-2p\sum_{n=0}^{\infty}\dfrac{(n+\frac{1}{2})(-1)^n\cos[(n+\frac{1}{2})\pi x/l]}{(n+\frac{1}{2})^2\pi^2+i\omega l^2}$ $\times e^{-(n+\frac{1}{2})^2\pi^2 t/l^2}$
(41)	$p^{-1}\sinh^{-1}(p/a)$	$-Ji_0(at)$
(42)	$(p^2+a^2)^{-1/2}\sinh^{-1}(p/a)$	$-\frac{1}{2}\pi Y_0(at)$
(43)	$(p^2-a^2)^{-1/2}\cosh^{-1}(p/a)$	$K_0(at)$
(44)	$\mathrm{ctnh}^{-1}(p/a)$	$t^{-1}\sin(at)$
(45)	$p^{-1}(\sinh^{-1}p)^2$	$\int_t^\infty \tau^{-1} Y_0(\tau)\,d\tau$

5.10. Orthogonal polynomials

(1)	$(p+\beta)^{-n-1}P_n\left(\dfrac{p+a}{p+\beta}\right)$	$\dfrac{t^n}{n!}\,e^{-\beta t}L_n[\frac{1}{2}(\beta-a)t]$

Orthogonal polynomials (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(2)	$(p+\beta)^{-\nu}P_n\left(\dfrac{p+\alpha}{p+\beta}\right)$ Re $\nu>0$	$\dfrac{t^{\nu-1}}{\Gamma(\nu)}e^{-\beta t}{}_2F_2[-n,n+1;1,\nu;\tfrac{1}{2}(\beta-\alpha)t]$
(3)	$\dfrac{(p-\alpha-\beta)^n}{p^{n+1}}\times P_n\left[\dfrac{p^2-(\alpha+\beta)p+2\alpha\beta}{p(p-\alpha-\beta)}\right]$	$L_n(\alpha t)\,L_n(\beta t)$
(4)	$n!p^{-\frac{1}{2}}P_n(p^{-1})$	$i^{-n}\pi^{-\frac{1}{2}}t^{-\frac{1}{2}}\mathrm{He}_n(2^{\frac{1}{2}}t^{\frac{1}{2}})\,\mathrm{He}_n(i\,2^{\frac{1}{2}}t^{\frac{1}{2}})$
(5)	$\dfrac{(\alpha+\beta-p)^{\frac{1}{2}n}}{(\alpha+\beta+p)^{\frac{1}{2}n+\frac{1}{2}}}\times P_n\left\{\dfrac{2\alpha^{\frac{1}{2}}\beta^{\frac{1}{2}}}{[(\alpha+\beta)^2-p^2]^{\frac{1}{2}}}\right\}$	$\dfrac{e^{-2\alpha t}\mathrm{He}_n(2\alpha^{\frac{1}{2}}t^{\frac{1}{2}})\mathrm{He}_n(2\beta^{\frac{1}{2}}t^{\frac{1}{2}})}{n!\pi^{\frac{1}{2}}t^{\frac{1}{2}}}$
(6)	$(p+\beta)^{-\mu}C_n^{\nu}\left(\dfrac{p+\alpha}{p+\beta}\right)$ Re $\mu>0$, Re $\nu>0$	$\dfrac{t^{\mu-1}e^{-\beta t}}{n\,\mathrm{B}(n,2\nu)\Gamma(\mu)}\times{}_2F_2[-n,n+2\nu;\mu,\nu+\tfrac{1}{2};\tfrac{1}{2}(\beta-\alpha)t]$
(7)	$p^{-n-\frac{1}{2}}e^{-\alpha/p}\mathrm{He}_{2n}(2^{\frac{1}{2}}\alpha^{\frac{1}{2}}p^{-\frac{1}{2}})$	$(-2)^n\pi^{-\frac{1}{2}}t^{n-\frac{1}{2}}\cos(2\alpha^{\frac{1}{2}}t^{\frac{1}{2}})$
(8)	$p^{-n-1}e^{-\alpha/p}\mathrm{He}_{2n+1}(2^{\frac{1}{2}}\alpha^{\frac{1}{2}}p^{-\frac{1}{2}})$	$(-1)^n2^{n+\frac{1}{2}}\pi^{-\frac{1}{2}}t^n\sin(2\alpha^{\frac{1}{2}}t^{\frac{1}{2}})$
(9)	$p^{-\beta}L_n^{\alpha}\left(\dfrac{\lambda}{p}\right)$ Re $\beta>0$	$\dfrac{t^{\beta-1}{}_1F_2(-n;\alpha+1,\beta;\lambda t)}{n\Gamma(\beta)\mathrm{B}(n,\alpha+1)}$
(10)	$n!p^{-n-\alpha-1}e^{-\lambda/p}L_n^{\alpha}(\lambda p^{-1})$ Re $\alpha>-n-1$	$\lambda^{-\frac{1}{2}\alpha}t^{\frac{1}{2}\alpha+n}J_\alpha(2\lambda^{\frac{1}{2}}t^{\frac{1}{2}})$

Orthogonal polynomials (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(11)	$\dfrac{(p-1)^n e^{-\lambda/p}}{p^{n+\alpha+1}} L_n^{\alpha}\left[\dfrac{\lambda}{p(1-p)}\right]$ $\quad \mathrm{Re}\,\alpha>-1$	$\lambda^{-\frac{1}{2}\alpha} t^{\frac{1}{2}\alpha} L_n^{\alpha}(t)\, J_\alpha(2\lambda^{\frac{1}{2}} t^{\frac{1}{2}})$
(12)	$n!\,\mathrm{B}(n+\frac{1}{2}, p+\frac{1}{2})\, L_n^{p}(\lambda)$	$(-2)^{-n}(e^t-1)^{-\frac{1}{2}}\,\mathrm{He}_{2n}\{[2\lambda(1-e^{-t})]^{\frac{1}{2}}\}$
(13)	$n!\,\mathrm{B}(n+3/2, p)\, L_n^{p}(\lambda)$	$(-1)^n 2^{-n-\frac{1}{2}} \lambda^{-\frac{1}{2}} \pi^{-\frac{1}{2}}$ $\times \mathrm{He}_{2n+1}\{[2\lambda(1-e^{-t})]^{\frac{1}{2}}\}$

5.11. Gamma function, incomplete gamma functions, zeta function and related functions

(1)	$\Gamma(\nu)\,\Gamma(ap)/\Gamma(ap+\nu)$ $\quad \mathrm{Re}\,a>0, \quad \mathrm{Re}\,\nu>0$	$a^{-1}(1-e^{-t/a})^{\nu-1}$
(2)	$2^{1-2p}\Gamma(2p)[\Gamma(p+\lambda+\frac{1}{2})$ $\times\Gamma(p-\lambda+\frac{1}{2})]^{-1}$	$\pi^{-1}(1-e^{-t})^{-\frac{1}{2}}\cos[2\lambda\cos^{-1}(e^{-\frac{1}{2}t})]$
(3)	$\dfrac{2^{p-1}\Gamma(\frac{1}{2}p+\frac{1}{2}\nu+\frac{1}{2})\,\Gamma(\frac{1}{2}p-\frac{1}{2}\nu)}{\pi^{\frac{1}{2}}\Gamma(p+\mu+1)}$ $\quad \mathrm{Re}\,\mu>-\frac{1}{2}$	$(1-e^{-2t})^{\frac{1}{2}\mu} P_\nu^{-\mu}(e^t)$
(4)	$2^{-p}\Gamma(p)[\Gamma(\frac{1}{2}p+\frac{1}{2}n+\frac{1}{2})$ $\times\Gamma(\frac{1}{2}p-\frac{1}{2}n+\frac{1}{2})]^{-1}$	$\pi^{-1}(1-e^{-2t})^{-\frac{1}{2}} T_n(e^{-t})$
(5)	$\dfrac{\Gamma(p+a)}{\Gamma(p+\beta)}(\gamma+p)_n \quad \mathrm{Re}(\beta-a)>n$	$\dfrac{e^{-at}}{\Gamma(\beta-a-n)}(1-e^{-t})^{\beta-a-n-1}$ $\times {}_2F_1(-n, \beta-\gamma-n; \beta-a-n; 1-e^{-t})$

Gamma functions etc. (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(6)	$\dfrac{\Gamma[\frac12(p-n-\mu)]\,\Gamma[\frac12(p+n-\mu+1)]}{\Gamma[\frac12(p+n+\mu)+1]\,\Gamma[\frac12(p-n+\mu+1)]}$ $\quad \operatorname{Re}\mu>-\frac12$	$\dfrac{2^{\mu+1}\pi^{\frac12}}{\Gamma(\mu+\frac12)}\sinh^{\mu}t\,P_n^{-\mu}(\cosh t)$
(7)	$\dfrac{\Gamma(p+\alpha)\,\Gamma(p+\beta)}{\Gamma(p+\gamma)\,\Gamma(p+\delta)}$ $\quad \operatorname{Re}(\gamma+\delta-\alpha-\beta)>0$	$\dfrac{e^{-\alpha t}(1-e^{-t})^{\gamma+\delta-\alpha-\beta-1}}{\Gamma(\gamma+\delta-\alpha-\beta)}$ $\times {}_2F_1(\delta-\beta,\gamma-\beta;\gamma+\delta-\alpha-\beta;1-e^{-t})$
(8)	$\log\dfrac{e^{p}\Gamma(p)}{2^{\frac12}\pi^{\frac12}p^{p-\frac12}}$	$\dfrac{1}{t}\left(\dfrac{1}{1-e^{-t}}-\dfrac{1}{t}-\dfrac{1}{2}\right)$
(9)	$\log\dfrac{(p+\alpha)^{\frac12}\Gamma(p+\alpha)}{\Gamma(p+\alpha+\frac12)}$	$\frac12 t^{-1}e^{-\alpha t}\tanh(\frac14 t)$
(10)	$\log\dfrac{\Gamma(p+\alpha+\frac34)}{(p+\alpha)^{\frac12}\Gamma(p+\alpha+\frac14)}$	$\frac12 t^{-1}e^{-\alpha t}[1-\operatorname{sech}(\frac14 t)]$
(11)	$\log\dfrac{\Gamma(p+\alpha)\,\Gamma(p+\beta+\frac12)}{\Gamma(p+\alpha+\frac12)\,\Gamma(p+\beta)}$	$\dfrac{e^{-\alpha t}-e^{-\beta t}}{t(1+e^{-\frac12 t})}$
(12)	$\log\dfrac{\Gamma(p+\alpha)\,\Gamma(p+\beta+\gamma)}{\Gamma(p+\alpha+\gamma)\,\Gamma(p+\beta)}$	$\dfrac{(e^{-\alpha t}-e^{-\beta t})(1-e^{-\gamma t})}{t(1-e^{-t})}$
(13)	$p^{-1}\psi(ap)$ $\quad \operatorname{Re}a>0$	$-\log[\gamma(e^{t/a}-1)]$
(14)	$\psi(\frac12 p+\frac12)-\psi(\frac12 p)$	$2(1+e^{-t})^{-1}$
(15)	$p^{-1}[\psi(\frac12 p+\frac12)-\psi(\frac12 p)]$	$2\log(\frac12+\frac12 e^{t})$

Gamma functions etc. (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(16)	$\psi(ap+\beta)-\psi(ap+\gamma)$ $\quad$ Re $a>0$	$a^{-1}(e^{-\gamma t/a}-e^{-\beta t/a})(1-e^{-t/a})^{-1}$
(17)	$\psi(p+a)+\psi(p+\beta)-\psi(p)$ $-\psi(p+a+\beta)$	$(1-e^{-at})(1-e^{-\beta t})(1-e^{-t})^{-1}$
(18)	$p^{-1}[\psi(p)-\log p]$	$\log[t(e^t-1)^{-1}]$
(19)	$\psi(p)-\log p$	$t^{-1}-(1-e^{-t})^{-1}$
(20)	$\dfrac{\Gamma(p)\Gamma(a)}{\Gamma(p+a)}[\psi(p+a)-\psi(p)]$ $\quad$ Re $a>0$	$t(1-e^{-t})^{a-1}$
(21)	$\log\dfrac{\Gamma(ap+\beta)}{\Gamma(ap+\lambda)}+(\lambda-\beta)\psi(ap+\delta)$ $\quad$ Re $a>0$	$(1-e^{-t/a})^{-1}[t^{-1}(e^{-\beta t/a}-e^{-\lambda t/a})$ $+a^{-1}(\beta-\lambda)e^{-\delta t/a}]$
(22)	$\psi^{(n)}(ap)$	$(-a)^{-n-1}t^n(1-e^{-t/a})^{-1}$
(23)	$p^{-\nu}\gamma(\nu, bp)$ $\quad$ Re $\nu>0, \quad b>0$	$t^{\nu-1}$ $\quad 0<t<b$ 0 $\quad t>b$
(24)	$p^{-\nu}e^{-bp}\gamma(\nu,-bp)$ $\quad$ Re $\nu>0, \quad b>0$	$(b-t)^{\nu-1}$ $\quad 0<t<b$ 0 $\quad t>b$
(25)	$\gamma(\nu, a/p)$ $\quad$ Re $\nu>0$	$a^{\frac{1}{2}\nu}t^{\frac{1}{2}\nu-1}J_\nu(2a^{\frac{1}{2}}t^{\frac{1}{2}})$
(26)	$p^{\nu-1}e^{a/p}\gamma(\nu, a/p)$ $\quad$ Re $\nu>0$	$\Gamma(\nu)a^{\frac{1}{2}\nu}t^{-\frac{1}{2}\nu}I_\nu(2a^{\frac{1}{2}}t^{\frac{1}{2}})$
(27)	$p^{\nu-3/2}e^{a/p}\gamma(\nu, a/p)$	$\Gamma(\nu)(t/a)^{\frac{1}{4}-\frac{1}{2}\nu}\mathbf{L}_{\nu-\frac{1}{2}}(2a^{\frac{1}{2}}t^{\frac{1}{2}})$

Gamma functions etc. (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(28)	$p^{\mu}\gamma(\nu, a/p)$ $\mathrm{Re}\,\nu>0,\quad \mathrm{Re}(\nu-\mu)>0$	$t^{-\mu-1}\int_0^{at} u^{\frac{1}{2}\nu+\frac{1}{2}\mu-\frac{1}{2}} J_{\nu-\mu-1}(2u^{\frac{1}{2}})\,du$
(29)	$\nu\Gamma(\nu-\mu)\,p^{\mu}e^{a/p}\gamma(\nu, a/p)$ $\mathrm{Re}\,\nu>0,\quad \mathrm{Re}\,\mu>0$	$a^{\nu}t^{\nu-\mu-1}\,{}_1F_2(1;\ \nu+1, \nu-\mu; at)$
(30)	$\gamma[\nu, \frac{1}{2}(p^2+a^2)^{\frac{1}{2}}-\frac{1}{2}p]\quad \mathrm{Re}\,\nu>0$	$t^{\frac{1}{2}\nu-1}(t+1)^{-\frac{1}{2}\nu}J_{\nu}[at^{\frac{1}{2}}(t+1)^{\frac{1}{2}}]$
(31)	$a^{-p}\gamma(p, a)$	$\exp(-ae^{-t})$
(32)	$\Gamma(\nu, bp)\qquad b>0,\quad \mathrm{Re}\,\nu<1$	$0\qquad 0<t<b$ $\dfrac{b^{\nu}}{\Gamma(1-\nu)t(t-b)^{\nu}}\qquad t>b$
(33)	$\Gamma(1-\nu)e^{ap}\Gamma(\nu, ap)$ $\mathrm{Re}\,\nu<1$	$a^{\nu}(t+a)^{-1}t^{-\nu}$
(34)	$p^{-\nu}\Gamma(\nu, bp)\qquad b>0$	$0\qquad 0<t<b$ $t^{\nu-1}\qquad t>b$
(35)	$p^{-\nu}e^{ap}\Gamma(\nu, ap)$	$(t+a)^{\nu-1}$
(36)	$a^{\frac{1}{2}}\Gamma(1-2\nu)e^{ap^2}\Gamma(\nu, ap^2)$ $\mathrm{Re}\,a>0,\quad \mathrm{Re}\,\nu<1$	$4^{\nu}e^{i\pi(\nu-\frac{1}{2})-\frac{1}{4}t^2/a}$ $\times\gamma(\frac{1}{2}-\nu, \frac{1}{4}e^{i\pi}t^2/a)$
(37)	$p^{-\frac{1}{2}}e^{\frac{1}{4}ap^2}\Gamma(\frac{1}{4}, \frac{1}{4}ap^2)$ $\mathrm{Re}\,a>0$	$\Gamma(\frac{1}{4})a^{-\frac{1}{2}}t^{\frac{1}{2}}e^{-\frac{1}{2}t^2/a}I_{\frac{1}{4}}(\frac{1}{2}t^2/a)$
(38)	$p^{-2\nu}e^{\frac{1}{2}a^2p^2}\Gamma(\nu, \frac{1}{2}a^2p^2)$	$2^{-\frac{1}{2}}\pi^{-\frac{1}{2}}\Gamma(\nu)a^{2\nu-1}e^{-\frac{1}{4}t^2/a^2}$ $\times[D_{-2\nu}(-t/a)-D_{-2\nu}(t/a)]$

Gamma functions etc. (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(39)	$\Gamma(1-\nu)p^{\nu-1}e^{a/p}\Gamma(\nu,a/p)$ $\quad\mathrm{Re}\,\nu<1$	$2a^{\frac{1}{2}\nu}t^{-\frac{1}{2}\nu}K_\nu(2a^{\frac{1}{2}}t^{\frac{1}{2}})$
(40)	$p^{\nu-3/2}e^{a/p}\Gamma(\nu,a/p)$ $\quad\mathrm{Re}\,\nu<3/2$	$\Gamma(\nu)(a/t)^{\frac{1}{2}\nu-\frac{1}{4}}[I_{\frac{1}{2}-\nu}(2a^{\frac{1}{2}}t^{\frac{1}{2}}) - \mathbf{L}_{\nu-\frac{1}{2}}(2a^{\frac{1}{2}}t^{\frac{1}{2}})]$
(41)	$p^\mu\Gamma(\nu,a/p)$ $\quad\mathrm{Re}(\mu+\nu)<-\frac{1}{2},\quad\mathrm{Re}\,\mu>0$	$t^{-\mu-1}\int_{at}^\infty u^{\frac{1}{2}\mu+\frac{1}{2}\nu-\frac{1}{2}}J_{\nu-\mu-1}(2u^{\frac{1}{2}})\,du$
(42)	$a^p\Gamma(-p,a)$ $\quad\mathrm{Re}\,a>0$	e^{-ae^t}
(43)	$p^{-1}\zeta(p)$	$n \qquad \log n<t\le\log(n+1)$
(44)	$p^{-1}\zeta(p+a)$	$\sum_{1\le n\le\exp t} n^{-a}$
(45)	$\Gamma(a)p^{-a}\zeta(p)$ $\quad\mathrm{Re}\,a>0$	$\sum_{1\le n\le\exp t}(t-\log n)^{a-1}$
(46)	$p^{-1}\zeta'(p)/\zeta(p)$	$-\psi(e^t)$
(47)	$\Gamma(a)\,\zeta(a,\beta p)$ $\quad\mathrm{Re}\,a>1,\quad\mathrm{Re}\,\beta>0$	$\beta^{-a}t^{a-1}(1-e^{-t/\beta})^{-1}$

5.12. Error function, exponential integral and related functions

(1)	$e^{ap^2}\,\mathrm{Erfc}(a^{\frac{1}{2}}p)$ $\quad\mathrm{Re}\,a>0$	$\pi^{-\frac{1}{2}}a^{-\frac{1}{2}}e^{-\frac{1}{4}t^2/a}$

Error function etc. (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)dt$	$f(t)$
(2)	$e^{ap^2} \operatorname{Erfc}(a^{1/2} p+\frac{1}{2}a^{-1/2}b)$ $\quad \operatorname{Re} a>0, \quad b>0$	$0 \qquad 0<t<b$ $\pi^{-1/2} a^{-1/2} e^{-\frac{1}{4}t^2/a} \qquad t>b$
(3)	$p^{-1} e^{ap^2} \operatorname{Erfc}(a^{1/2} p)$ $\quad \operatorname{Re} a>0$	$\operatorname{Erf}(\frac{1}{2} a^{-1/2} t)$
(4)	$(p-a)^{-1} e^{p^2} \operatorname{Erfc}(p)$	$e^{a(t+a)} [\operatorname{Erf}(\frac{1}{2}t+a)-\operatorname{Erf}(a)]$
(5)	$1-a^{1/2}\pi^{1/2} p e^{ap^2} \operatorname{Erfc}(a^{1/2} p)$ $\quad \operatorname{Re} a>0$	$\frac{1}{2}a^{-1} t e^{-\frac{1}{4}t^2/a}$
(6)	$p^{-1}(p+1)^{-1} e^{\frac{1}{4}p^2} \operatorname{Erfc}(\frac{1}{2}p)$	$e^{t+\frac{1}{4}} [\operatorname{Erf}(t+\frac{1}{2})-\operatorname{Erf}(\frac{1}{2})]$
(7)	$p^{-1} e^{p^2} [\operatorname{Erf}(p)-\operatorname{Erf}(p+b)]$ $\quad b>0$	$\operatorname{Erf}(\frac{1}{2}t) \qquad 0<t<2b$ $\operatorname{Erf}(b) \qquad t>2b$
(8)	$\operatorname{Erfc}(b^{1/2} p^{1/2}) \quad b>0$	$0 \qquad 0<t<b$ $\pi^{-1} b^{1/2} t^{-1} (t-b)^{-1/2} \qquad t>b$
(9)	$e^{-bp}-\pi^{1/2} b^{1/2} p^{1/2} \operatorname{Erfc}(b^{1/2} p^{1/2})$ $\quad b>0$	$0 \qquad 0<t<b$ $\frac{1}{2} b^{1/2} t^{-3/2} \qquad t>b$
(10)	$p^{-1/2} \operatorname{Erf}(b^{1/2} p^{1/2}) \quad b>0$	$\pi^{-1/2} t^{-1/2} \qquad 0<t<b$ $0 \qquad t>b$
(11)	$p^{-1/2} \operatorname{Erfc}(b^{1/2} p^{1/2}) \quad b>0$	$0 \qquad 0<t<b$ $\pi^{-1/2} t^{-1/2} \qquad t>b$
(12)	$e^{ap} \operatorname{Erfc}(a^{1/2} p^{1/2}) \quad \lvert\arg a\rvert<\pi$	$\pi^{-1} a^{1/2} (t+a)^{-1} t^{-1/2}$

Error function etc. (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(13)	$1-(\pi a p)^{1/2} e^{ap}\,\mathrm{Erfc}\,(a^{1/2} p^{1/2})$ $\qquad \lvert\arg a\rvert < \pi$	$\frac{1}{2} a^{1/2}(t+a)^{-3/2}$
(14)	$p^{-1/2} e^{ap}\,\mathrm{Erfc}\,(a^{1/2} p^{1/2})$ $\qquad \lvert\arg a\rvert < \pi$	$\pi^{-1/2}(t+a)^{-1/2}$
(15)	$\Gamma(\nu+\frac{1}{2})\, p^{-\nu} e^{ap}\,\mathrm{Erfc}\,(a^{1/2} p^{1/2})$ $\qquad \mathrm{Re}\,\nu > -\frac{1}{2},\quad \lvert\arg a\rvert < \pi$	$\pi^{-1/2} a^{-1/2} t^{\nu-1/2}\ {}_2F_1(1,\frac{1}{2};\nu+\frac{1}{2};-t/a)$
(16)	$\mathrm{Erf}\,(a^{1/2} p^{-1/2})$	$\pi^{-1} t^{-1} \sin(2a^{1/2} t^{1/2})$
(17)	$p^{-1/2} e^{a/p}\,\mathrm{Erf}\,(a^{1/2} p^{-1/2})$	$\pi^{-1/2} t^{-1/2} \sinh(2a^{1/2} t^{1/2})$
(18)	$p^{-3/2} e^{a/p}\,\mathrm{Erf}\,(a^{1/2} p^{-1/2})$	$\pi^{-1/2} a^{-1/2}[\cosh(2a^{1/2} t^{1/2})-1]$
(19)	$\pi^{1/2} p^{-5/2} e^{a/p}\,\mathrm{Erf}\,(a^{1/2} p^{-1/2})$	$a^{-1} t^{1/2} \sinh(2a^{1/2} t^{1/2}) - a^{-1/2} t$ $\quad -\frac{1}{2} a^{-3/2}[\cosh(2a^{1/2} t^{1/2})-1]$
(20)	$p^{-1/2} e^{a/p}\,\mathrm{Erfc}\,(a^{1/2} p^{-1/2})$	$\pi^{-1/2} t^{-1/2} e^{-2a^{1/2} t^{1/2}}$
(21)	$p^{-3/2} e^{a/p}\,\mathrm{Erfc}\,(a^{1/2} p^{-1/2})$	$a^{-1/2}\pi^{-1/2}(1-e^{-2a^{1/2} t^{1/2}})$
(22)	$p^{-\nu-1} e^{a/p}\,\mathrm{Erfc}\,(a^{1/2} p^{-1/2})$ $\qquad \mathrm{Re}\,\nu > -1$	$(t/a)^{\frac{1}{2}\nu}[I_\nu(2a^{1/2} t^{1/2}) - \mathbf{L}_\nu(2a^{1/2} t^{1/2})]$
(23)	$p^{-\nu-1} e^{a/p}\,\mathrm{Erf}\,(a^{1/2} p^{-1/2})$ $\qquad \mathrm{Re}\,\nu > -1$	$(t/a)^{\frac{1}{2}\nu}\,\mathbf{L}_\nu(2a^{1/2} t^{1/2})$
(24)	$\mathrm{Ei}\,(-bp)$ $\qquad b > 0$	$0 \qquad 0 < t < b$ $-1/t \qquad t > b$
(25)	$p^{-1}\,\mathrm{Ei}\,(-bp)$ $\qquad b > 0$	$0 \qquad 0 < t < b$ $\log(b/t) \qquad t > 0$

Error function etc. (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(26)	$p^{-1}\,\mathrm{Ei}[-b(p+a)]$ $b>0,\quad a\neq 0$	$0 \qquad 0<t<b$ $\mathrm{Ei}(-ba)-\mathrm{Ei}(-at) \qquad t>b$
(27)	$p^{-1}[\mathrm{Ei}(-bp)-\log(\gamma p)] \qquad b>0$	$\log t \qquad 0<t<b$ $\log b \qquad t>b$
(28)	$e^{ap}\,\mathrm{Ei}(-ap) \qquad \lvert\arg a\rvert<\pi$	$-(t+a)^{-1}$
(29)	$p^{-1}e^{ap}\,\mathrm{Ei}(-ap) \qquad \lvert\arg a\rvert<\pi$	$-\log(1+t/a)$
(30)	$-p^{-1}e^{-bp}\,\overline{\mathrm{Ei}}(bp) \qquad b>0$	$\log\lvert 1-t/b\rvert$
(31)	$pe^{ap}\,\mathrm{Ei}(-ap)+a^{-1} \qquad \lvert\arg a\rvert<\pi$	$(t+a)^{-2}$
(32)	$e^{-bp}+bp\,\mathrm{Ei}(-bp) \qquad b>0$	$0 \qquad 0<t<b$ $bt^{-2} \qquad t>b$
(33)	$[\mathrm{Ei}(-\tfrac{1}{2}p)]^2$	$0 \qquad 0<t<1$ $2t^{-1}\log(2t-1) \qquad t>1$
(34)	$\mathrm{Ei}(-ap)\,\mathrm{Ei}(-bp)$ $a,\ b>0$	$0 \qquad 0<t<a+b$ $t^{-1}\log[a^{-1}b^{-1}(t-a)(t-b)]$ $t>a+b$
(35)	$\overline{\mathrm{Ei}}(p)\,\mathrm{Ei}(-p)$	$t^{-1}\log\lvert 1-t^2\rvert$
(36)	$e^{bp}[\mathrm{Ei}(-bp)]^2 \qquad b>0$	$0 \qquad 0<t<b$ $2(t+b)^{-1}\log(t/b) \qquad t>b$
(37)	$e^{bp}\{[\mathrm{Ei}(-bp)]^2$ $-2\log b\,\mathrm{Ei}(-2bp)\} \qquad b>0$	$0 \qquad 0<t<b$ $2(t+b)^{-1}\log t \qquad t>b$

Error function etc. (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(38)	$e^{(\alpha+\beta)p}\,\mathrm{Ei}(-\alpha p)\,\mathrm{Ei}(-\beta p)$ $\lvert\arg(\alpha+\beta)\rvert<\pi$	$(t+\alpha+\beta)^{-1}$ $\times\log[\alpha^{-1}\beta^{-1}(t+\alpha)(t+\beta)]$
(39)	$e^{(\alpha+\beta)p}[\mathrm{Ei}(-\alpha p)\,\mathrm{Ei}(-\beta p)$ $-\log(\alpha\beta)\,\mathrm{Ei}(-\alpha p-\beta p)]$ $\lvert\arg(\alpha+\beta)\rvert<\pi$	$(t+\alpha+\beta)^{-1}\log[(t+\alpha)(t+\beta)]$
(40)	$\exp(\tfrac{1}{4}\alpha^{-2}p^{2})\,\mathrm{Ei}(-\tfrac{1}{4}\alpha^{-2}p^{2})$ $\lvert\arg\alpha\rvert<\tfrac{1}{4}\pi$	$2i\pi^{-1/2}\,\alpha e^{-\alpha^2t^2}\,\mathrm{Erf}(i\alpha t)$
(41)	$p^{-1}\,\mathrm{Ei}(-p^{-1})$	$2\,\mathcal{J}i_0(2t^{1/2})$
(42)	$p^{-\nu-1}\,\mathrm{Ei}(-\alpha p^{-1})$ $\mathrm{Re}\,\nu>-1,\quad \mathrm{Re}\,\alpha>0$	$2t^{\nu}\int_\infty^{\alpha^{1/2}t^{1/2}}u^{-\nu-1}\,J_\nu(2u)\,du$
(43)	$-p^{-1}e^{\alpha/p}\,\mathrm{Ei}(-\alpha p^{-1})$	$2K_0(2\alpha^{1/2}t^{1/2})$
(44)	$p^{-\nu-1}e^{\alpha/p}\,\mathrm{Ei}(-\alpha p^{-1})$ $\mathrm{Re}\,\nu>-1$	$t^{\nu}\int_\infty^{\alpha t}u^{-\frac{1}{2}\nu-1}\,J_\nu[2(u-\alpha t)^{1/2}]\,du$
(45)	$\mathrm{ci}(\alpha p)\cos(\alpha p)-\mathrm{si}(\alpha p)\sin(\alpha p)$ $\mathrm{Re}\,\alpha>0$	$t(t^2+\alpha^2)^{-1}$
(46)	$\mathrm{ci}(\alpha p)\sin(\alpha p)+\mathrm{si}(\alpha p)\cos(\alpha p)$ $\mathrm{Re}\,\alpha>0$	$-\alpha(t^2+\alpha^2)^{-1}$
(47)	$p^{-1}[\mathrm{ci}(\alpha p)\cos(\alpha p)$ $-\,\mathrm{si}(\alpha p)\sin(\alpha p)]\quad \mathrm{Re}\,\alpha>0$	$\tfrac{1}{2}\log(1+t^2/\alpha^2)$
(48)	$p^{-1}[\mathrm{ci}(\alpha p)\sin(\alpha p)$ $+\,\mathrm{si}(\alpha p)\cos(\alpha p)]\quad \mathrm{Re}\,\alpha>0$	$-\tan^{-1}(t/\alpha)$

Error function etc. (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(49)	$[\mathrm{ci}\,(ap)]^2 + [\mathrm{si}\,(ap)]^2$	$t^{-1} \log(1 + t^2/a^2)$
(50)	$\tfrac{1}{2} - \cos(\tfrac{1}{4}p^2)\,C(\tfrac{1}{4}p^2)$ $- \sin(\tfrac{1}{4}p^2)\,S(\tfrac{1}{4}p^2)$	$2^{1/2}\pi^{-1/2} \sin(t^2)$
(51)	$\tfrac{1}{2}\cos(\tfrac{1}{2}p^2) - \cos(\tfrac{1}{4}p^2)S(\tfrac{1}{4}p^2)$ $+ \sin(\tfrac{1}{4}p^2)\,C(\tfrac{1}{4}p^2)$	$2^{1/2}\pi^{-1/2} \cos(t^2)$
(52)	$[\tfrac{1}{2} - C(\tfrac{1}{4}p^2)]^2 + [\tfrac{1}{2} - S(\tfrac{1}{4}p^2)]^2$	$2\pi^{-1} t^{-1} \sin(t^2)$
(53)	$C_n(a, p)$ $a > 0$	0 $0 < t < \cosh a$ $(t^2 - 1)^{-1/2} \cosh(n \cosh^{-1} t)$ $t > \cosh a$
(54)	$S_n(a, p)$ $a > 0$	0 $0 < t < \sinh a$ $(t^2 - 1)^{-1/2} \cosh(n \sinh^{-1} t)$ $t > \sinh a$

5.13. Legendre functions

(1)	$\pi p^{-1} P_\nu(p)$ $0 < \mathrm{Re}\,\nu < 1$	$-t^{-1} \sin(\nu\pi)\, W_{0,\,\nu+1/2}(2t)$
(2)	$s^\mu P_\nu^\mu\left(\dfrac{p}{a}\right)$ $\mathrm{Re}\,\mu - 1 < \mathrm{Re}\,\nu < -\mathrm{Re}\,\mu$	$\dfrac{2^{1/2} a^{1/2} t^{-\mu-1/2} K_{\nu+1/2}(at)}{\pi^{1/2}\,\Gamma(-\mu+\nu+1)\Gamma(-\mu-\nu)}$
(3)	$s^{-\mu} Q_\nu^\mu\left(\dfrac{p}{a}\right)$ $\mathrm{Re}(\mu + \nu) > -1$	$\dfrac{\pi^{1/2} a^{1/2} \sin[(\mu+\nu)\pi]}{2^{1/2}\,\sin(\nu\pi)}\, t^{\mu-1/2} I_{\nu+1/2}(at)$
(4)	$Q_\nu\left(\dfrac{p^2 + a^2 + \beta^2}{2a\beta}\right)$ $\mathrm{Re}\,\nu > -1$	$\pi a^{1/2} \beta^{1/2} J_{\nu+1/2}(at)\, J_{\nu+1/2}(\beta t)$

$s = (p^2 - a^2)^{1/2}$

Legendre functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(5)	$\Gamma(-2\nu)p^{\nu}(a^2-p^2)^{\frac{1}{2}\nu}P_\nu^\nu(a/p)$ $\operatorname{Re}\nu<0$	$2^{-\frac{1}{2}}\pi^{\frac{1}{2}}(t/a)^{-\nu-\frac{1}{2}}[I_{-\nu-\frac{1}{2}}(at)$ $-\mathbf{L}_{-\nu-\frac{1}{2}}(at)]$
(6)	$\Gamma(-2\nu)p^{\nu+1}(a^2-p^2)^{\frac{1}{2}\nu}P_\nu^\nu(a/p)$ $\operatorname{Re}\nu<-\frac{1}{2}$	$2^{-\frac{1}{2}}\pi^{\frac{1}{2}}a(t/a)^{-\nu-\frac{1}{2}}[I_{-\nu-3/2}(at)$ $-\mathbf{L}_{-\nu-3/2}(at)]$
(7)	$p^{-\frac{1}{2}\nu-\frac{1}{2}}(p-a)^{\frac{1}{2}\mu}P_\nu^\mu(a^{\frac{1}{2}}p^{-\frac{1}{2}})$ $\operatorname{Re}\mu<1,\quad \operatorname{Re}(\nu-\mu)>-1$	$\dfrac{t^{\frac{1}{2}(\nu-\mu-1)}e^{\frac{1}{2}at}D_{\mu+\nu}(2^{\frac{1}{2}}a^{\frac{1}{2}}t^{\frac{1}{2}})}{\pi^{\frac{1}{2}}2^{\frac{1}{2}(\mu-\nu-1)}\Gamma(\nu-\mu+1)}$
(8)	$s^{-\nu-1}P_\nu^\mu(p/s)\quad \operatorname{Re}(\nu-\mu)>-1$	$[\Gamma(\nu-\mu+1)]^{-1}t^\nu I_{-\mu}(at)$
(9)	$s^{-\nu-1}Q_\nu^\mu(p/s)$ $\operatorname{Re}(\nu\pm\mu)>-1$	$\dfrac{\sin(\mu+\nu)\pi}{\sin(\nu\pi)}\;\dfrac{t^\nu K_\mu(at)}{\Gamma(\nu-\mu+1)}$
(10)	$p^{-\lambda}Q_{2\nu}(p^{\frac{1}{2}})\quad \operatorname{Re}(\lambda+\nu)>-\frac{1}{2}$	$\dfrac{\pi^{\frac{1}{2}}\Gamma(2\nu+1)t^{\lambda+\nu-\frac{1}{2}}}{2^{2\nu+1}\Gamma(2\nu+3/2)\Gamma(\lambda+\nu+1/2)}$ $\times{}_2F_2\left(\nu+\dfrac{1}{2},\nu+1;2\nu+\dfrac{3}{2},\lambda+\nu+\dfrac{1}{2};t\right)$
(11)	$p^{-\frac{1}{2}}[P_{-\frac{1}{4}}^\mu(r/p)]^2\quad \operatorname{Re}\mu<\frac{1}{4}$	$2^{\frac{1}{2}-\mu}[\Gamma(\frac{1}{2}-2\mu)]^{-1}t^{-\frac{1}{2}}$ $\times[J_{-\mu}(\frac{1}{2}at)]^2$
(12)	$2^{-\frac{1}{2}}\pi^{\frac{1}{2}}p^{-\frac{1}{2}}P_{-\frac{1}{4}}^\mu(r/p)P_{-\frac{1}{4}}^{-\mu}(r/p)$	$t^{-\frac{1}{2}}J_\mu(\frac{1}{2}at)J_{-\mu}(\frac{1}{2}at)$
(13)	$ar^{-1}p^{-\frac{1}{2}}P_{\frac{1}{4}}^\mu(r/p)P_{-\frac{1}{4}}^\mu(r/p)$ $\operatorname{Re}\mu<\frac{3}{4}$	$2^{3/2-\mu}[\Gamma(3/2-2\mu)]^{-1}t^{1/2}$ $\times[J_{-\mu}(\frac{1}{2}at)]^2$

$s=(p^2-a^2)^{\frac{1}{2}},\quad r=(p^2+a^2)^{\frac{1}{2}}$

Legendre functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(14)	$\dfrac{\Gamma(p-\mu+\nu+1)\Gamma(p-\mu-\nu)}{\Gamma(p+1)}$ $\times\left(\dfrac{a}{a-2}\right)^{\frac{1}{2}p} P_\nu^{\mu-p}(a-1)$ $\mathrm{Re}\, a>0,\quad \mathrm{Re}\,\mu>-1$	$\left[(e^t-1)\left(\dfrac{ae^t}{a-2}-1\right)\right]^{\frac{1}{2}\mu}$ $\times P_\nu^{-\mu}(ae^t+1-a)$
(15)	$\Gamma(\frac{1}{2}-\mu)\, Q^\mu_{p-\frac{1}{2}}(\cosh a)$ $\mathrm{Re}\,\mu<\frac{1}{2},\quad a>0$	$0 \qquad 0<t<a$ $2^{-\frac{1}{2}}\pi^{\frac{1}{2}}e^{\mu\pi i}(\sinh a)^\mu$ $\times(\cosh t-\cosh a)^{-\mu-\frac{1}{2}} \qquad t>a$
(16)	$\Gamma(\frac{1}{2}-\mu)e^{ap}\, Q^\mu_{p-\frac{1}{2}}(\cosh a)$ $\mathrm{Re}\,\mu<\frac{1}{2},\quad \lvert\arg a\rvert<\pi$	$\pi^{\frac{1}{2}}2^{-\mu-1}e^{\mu\pi i}\sinh^\mu a$ $\times[\sinh(\frac{1}{2}t)\sinh(a+\frac{1}{2}t)]^{-\mu-\frac{1}{2}}$
(17)	$2^{p+1}e^{(p-\alpha)\pi i}(\mu^2-1)^{\frac{1}{2}(p-\alpha)}$ $\times\Gamma(p)\, Q^{\alpha-p}_{p-1}(\mu)$	$\Gamma(\alpha)(1-e^{-t})^{-\frac{1}{2}}\{[\mu+(1-e^{-t})^{\frac{1}{2}}]^{-\alpha}$ $+[\mu-(1-e^{-t})^{\frac{1}{2}}]^{-\alpha}\}$
(18)	$\pi^{\frac{1}{2}}2^{p+\frac{1}{2}}\Gamma(p)(\mu^2-1)^{\frac{1}{4}-\frac{1}{2}p}$ $\times P^{\frac{1}{2}-p}_{\alpha+p-\frac{1}{2}}(\mu)$	$(1-e^{-t})^{-\frac{1}{2}}\{[\mu+(\mu^2-1)^{\frac{1}{2}}(1-e^{-t})^{\frac{1}{2}}]^\alpha$ $+[\mu-(\mu^2-1)^{\frac{1}{2}}(1-e^{-t})^{\frac{1}{2}}]^\alpha\}$
(19)	$\dfrac{\pi^{\frac{1}{2}}\Gamma(2p)\Gamma(2\nu+1)}{2^{p+\nu-1}\Gamma(p+\nu+\frac{1}{2})}e^{-pa}$ $\times P^{-\nu-p}_{\nu-p}[(1-e^{-2a})^{\frac{1}{2}}]$ $\mathrm{Re}\,\nu>-\frac{1}{2},\quad a>0$	$0 \qquad 0<t<2a$ $e^{\nu a}(1-e^{-t})^{-\frac{1}{2}}[e^{-a}(1-e^{-t})^{\frac{1}{2}}$ $-e^{-\frac{1}{2}t}(1-e^{-2a})^{\frac{1}{2}}]^{2\nu} \qquad t>2a$

5.14. Bessel functions

(1)	$\pi e^{-ap}[\frac{1}{2}\pi\, Y_0(iap)$ $-J_0(iap)\log(\frac{1}{2}\gamma)] \qquad a>0$	$\dfrac{\log[4t(2a-t)/a^2]}{t^{\frac{1}{2}}(2a-t)^{\frac{1}{2}}} \qquad 0<t<2a$ $0 \qquad t>2a$

Bessel functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(2)	$\cos(ap)\,J_0(ap)+\sin(ap)\,Y_0(ap)$ $\operatorname{Re} a>0$	$\dfrac{2^{3/2}a}{\pi}\,\dfrac{[t+(t^2+4a^2)^{1/2}]^{-1/2}}{t^{1/2}(t^2+4a^2)^{1/2}}$
(3)	$\sin(ap)\,J_0(ap)-\cos(ap)\,Y_0(ap)$ $\operatorname{Re} a>0$	$\dfrac{2^{1/2}}{\pi}\,\dfrac{[t+(t^2+4a^2)^{1/2}]^{1/2}}{t^{1/2}(t^2+4a^2)^{1/2}}$
(4)	$\cos(ap)\,J_1(ap)+\sin(ap)\,Y_1(ap)$ $\operatorname{Re} a>0$	$-\dfrac{2^{5/2}a^2}{\pi}\,\dfrac{[t+(t^2+4a^2)^{1/2}]^{-3/2}}{t^{1/2}(t^2+4a^2)^{1/2}}$
(5)	$\sin(ap)\,J_1(ap)-\cos(ap)\,Y_1(ap)$ $\operatorname{Re} a>0$	$\dfrac{1}{2^{1/2}\pi a}\,\dfrac{[t+(t^2+4a^2)^{1/2}]^{3/2}}{t^{1/2}(t^2+4a^2)^{1/2}}$
(6)	$p^{-\nu}[\cos(ap)\,J_\nu(ap)$ $+\sin(ap)\,Y_\nu(ap)]$ $\operatorname{Re}\nu>-\tfrac{1}{2},\quad \operatorname{Re} a>0$	$-\dfrac{2t^{\nu-1/2}(t^2+4a^2)^{\frac{1}{2}\nu-\frac{1}{4}}}{\pi^{1/2}(2a)^\nu\Gamma(\nu+\frac{1}{2})}$ $\times\sin[(\nu-\tfrac{1}{2})\operatorname{ctn}^{-1}(\tfrac{1}{2}t/a)]$
(7)	$p^{-\nu}[\sin(ap)\,J_\nu(ap)$ $-\cos(ap)\,Y_\nu(ap)]$ $\operatorname{Re}\nu>-\tfrac{1}{2},\quad \operatorname{Re} a>0$	$\dfrac{2t^{\nu-1/2}(t^2+4a^2)^{\frac{1}{2}\nu-\frac{1}{4}}}{\pi^{1/2}(2a)^\nu\Gamma(\nu+\frac{1}{2})}$ $\times\cos[(\nu-\tfrac{1}{2})\operatorname{ctn}^{-1}(\tfrac{1}{2}t/a)]$
(8)	$p^{-\nu}[\cos(ap-\beta)\,J_\nu(ap)$ $+\sin(ap-\beta)\,Y_\nu(ap)]$ $\operatorname{Re}\nu>-\tfrac{1}{2},\quad \operatorname{Re} a>0$	$\dfrac{2t^{\nu-1/2}(t^2+4a^2]^{\frac{1}{2}\nu-\frac{1}{4}}}{\pi^{1/2}(2a)^\nu\Gamma(\nu+\frac{1}{2})}$ $\times\sin[(\tfrac{1}{2}-\nu)\operatorname{ctn}^{-1}(\tfrac{1}{2}t/a)+\beta]$
(9)	$p^{-\nu}e^{-iap}H_\nu^{(1)}(ap)\quad \operatorname{Re}\nu>-\tfrac{1}{2}$ $-\pi/2<\arg a<3\pi/2$	$-i\,\dfrac{2(t^2-2ait)^{\nu-1/2}}{\pi^{1/2}(2a)^\nu\Gamma(\nu+\frac{1}{2})}$

Bessel functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(10)	$\Gamma(\nu+\tfrac{1}{2})\,p^{-\nu} e^{iap} H_\nu^{(2)}(ap)$ $\operatorname{Re}\nu>-\tfrac{1}{2}, \quad -3\pi/2<\arg a<\pi/2$	$i\pi^{-1/2}\,2^{1-\nu} a^{-\nu}(t^2+2ait)^{\nu-1/2}$
(11)	$p^{-1} J_\nu(2a/p) \qquad \operatorname{Re}\nu>-1$	$J_\nu(2a^{1/2}t^{1/2})\,I_\nu(2a^{1/2}t^{1/2})$
(12)	$\Gamma(\nu+1)\Gamma(\lambda)\,p^{\nu-\lambda} J_\nu(4a/p)$ $\operatorname{Re}\lambda>0$	$(2a)^\nu t^{\lambda-1}\,{}_0F_3(\nu+1, \tfrac{1}{2}\lambda, \tfrac{1}{2}\lambda+\tfrac{1}{2}; -a^2t^2)$
(13)	$p^{-1} e^{(a^2-\beta^2)/p}\, J_\nu(2a\beta/p)$ $\operatorname{Re}\nu>-1$	$J_\nu(2\beta t^{1/2})\; I_\nu(2at^{1/2})$
(14)	$(p^2+1)^{-1/2} e^{-ap/(p^2+1)}$ $\times J_\nu\left(\dfrac{a}{p^2+1}\right) \qquad \operatorname{Re}\nu>-\tfrac{1}{2}$	$J_\nu(t)\; J_{2\nu}(2a^{1/2}t^{1/2})$
(15)	$p^{-\mu} J_\nu(p^{-1/2}) \qquad \operatorname{Re}(\mu+\tfrac{1}{2}\nu)>0$	$\dfrac{t^{\mu+\frac{1}{2}\nu-1}\,{}_0F_2(\mu+\tfrac{1}{2}\nu, \nu+1; -\tfrac{1}{4}t)}{2^\nu\Gamma(\mu+\tfrac{1}{2}\nu)\,\Gamma(\nu+1)}$
(16)	$(p^2+a^2)^{-\frac{1}{2}\nu} e^{ip} H_\nu^{(2)}[(p^2+a^2)^{1/2}]$ $\operatorname{Re}\nu>-\tfrac{1}{2}$	$i\,2^{1/2}\pi^{-1/2} a^{1/2-\nu}(t^2+2it)^{\frac{1}{2}\nu-\frac{1}{4}}$ $\times J_{\nu-1/2}[a(t^2+2it)^{1/2}]$
(17)	$\Gamma(p+\tfrac{1}{2})(\tfrac{1}{2}a)^{-p} J_p(a)$	$\pi^{-1/2}(e^t-1)^{-1/2}\cos[a(1-e^{-t})^{1/2}]$
(18)	$\Gamma(p)(\tfrac{1}{2}a)^{-p} J_{p+\mu}(a)$ $\operatorname{Re}\mu>-1$	$(1-e^{-t})^{\frac{1}{2}\mu} J_\mu[a(1-e^{-t})^{1/2}]$
(19)	$p^{1/2}[J_{\nu+1/4}(ap)\,J_{\nu-1/4}(ap)$ $+\,Y_{\nu+1/4}(ap)\,Y_{\nu-1/4}(ap)]$ $\operatorname{Re}a>0$	$(\tfrac{1}{2}\pi)^{-3/2}(t^3+4a^2t)^{-1/2}$ $\times e^{2\nu\sinh^{-1}(\frac{1}{2}t/a)}$

Bessel functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(20)	$p^{1/2}[J_{1/4+\nu}(ap)\,J_{1/4-\nu}(ap)$ $+Y_{1/4+\nu}(ap)\,Y_{1/4-\nu}(ap)]$ $\mathrm{Re}\,a>0$	$(\tfrac{1}{2}\pi)^{-3/2}(t^3+4a^2t)^{-1/2}$ $\times\{\cos[(\nu+\tfrac{1}{4})\pi]\,e^{-2\nu\sinh^{-1}(\frac{1}{2}t/a)}$ $+\sin[(\nu+\tfrac{1}{4})\pi]\,e^{2\nu\sinh^{-1}(\frac{1}{2}t/a)}\}$
(21)	$p^{1/2}[J_{\nu+1/4}(ap)\,Y_{\nu-1/4}(ap)$ $-J_{\nu-1/4}(ap)\,Y_{\nu+1/4}(ap)]$ $\mathrm{Re}\,a>0$	$(\tfrac{1}{2}\pi)^{-3/2}(t^3+4a^2t)^{-1/2}$ $\times e^{-2\nu\sinh^{-1}(\frac{1}{2}t/a)}$
(22)	$p^{1/2}[J_{1/4+\nu}(ap)\,Y_{1/4-\nu}(ap)$ $-J_{1/4-\nu}(ap)\,Y_{1/4+\nu}(ap)]$ $\mathrm{Re}\,a>0$	$(\tfrac{1}{2}\pi)^{-3/2}(t^3+4a^2t)^{-1/2}$ $\times\{\sinh[(\nu+\tfrac{1}{4})\pi]\,e^{-2\nu\sinh^{-1}(\frac{1}{2}t/a)}$ $-\cos[(\nu+\tfrac{1}{4})\pi]\,e^{2\nu\sinh^{-1}(\frac{1}{2}t/a)}\}$
(23)	$J_{\nu-p}(a)\,Y_{-\nu-p}(a)$ $-J_{-\nu-p}(a)\,Y_{\nu-p}(a)$ $\mathrm{Re}\,a>0,\quad \lvert\mathrm{Re}\,\nu\rvert<\tfrac{1}{2}$	$2\pi^{-2}\sin(2\nu\pi)K_{2\nu}[2a\sinh(\tfrac{1}{2}t)]$
(24)	$J_p(a)\dfrac{\partial Y_p(a)}{\partial p}-Y_p(a)\dfrac{\partial J_p(a)}{\partial p}$ $\mathrm{Re}\,a>0$	$-\dfrac{2}{\pi}K_0[2a\sinh(\tfrac{1}{2}t)]$
(25)	$p^{1/2}H^{(1)}_{1/8}\left(\dfrac{p^2}{a}\right)H^{(2)}_{1/8}\left(\dfrac{p^2}{a}\right)$ $a>0$	$a\cos\left(\dfrac{\pi}{8}\right)\dfrac{(2t)^{1/2}}{\pi^{1/2}}J_{1/8}\left(\dfrac{at^2}{16}\right)$ $\times J_{-1/8}\left(\dfrac{at^2}{16}\right)$
(26)	$p^{-1/2}H^{(1)}_{\nu}(\tfrac{1}{2}p/a)\,H^{(2)}_{\nu}(\tfrac{1}{2}p/a)$	$2a(2t/\pi)^{1/2}P^{1/4}_{\nu-1/4}[(1+a^2t^2)^{1/2}]$ $\times P^{-1/4}_{\nu-1/4}[(1+a^2t^2)^{1/2}]$

Bessel functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(27)	$p^{1/2} H^{(1)}_{1/2+\nu}(ap)\, H^{(2)}_{1/2-\nu}(ap)$ $\mathrm{Re}\ a>0$	$4[\pi^3 t(t^2+4a^2)]^{-1/2} e^{-\nu\pi i}$ $\times\{\cosh[2\nu \sinh^{-1}(\tfrac{1}{2}t/a)]$ $+i\sinh[2\nu\sinh^{-1}(\tfrac{1}{2}t/a)]\}$
(28)	$\pi\Gamma(2\lambda+2)e^{(\mu-\nu)\pi i} p^{-2\lambda}$ $\times H^{(1)}_{2\mu}(p/a)\, H^{(2)}_{2\nu}(p/a)$ $\mathrm{Re}\ \lambda>-\tfrac{1}{2}$	$2(2\lambda+1)\,a t^{2\lambda}$ $\times {}_4F_3(\tfrac{1}{2}+\mu+\nu, \tfrac{1}{2}-\mu+\nu, \tfrac{1}{2}+\mu-\nu, \tfrac{1}{2}-\mu-\nu;$ $\tfrac{1}{2}, \lambda+\tfrac{1}{2}, \lambda+1; -\tfrac{1}{4}a^2t^2)$ $+i\,4a^2(\mu^2-\nu^2)t^{2\lambda+1}$ $\times {}_4F_3(1+\mu+\nu, 1+\nu-\mu, 1-\mu-\nu, 1+\mu-\nu;$ $3/2, \lambda+1, \lambda+3/2; -\tfrac{1}{4}a^2t^2)$
(29)	$\Gamma(2\nu+\tfrac{1}{2})p^{-2\nu} H^{(1)}_{2\nu}(a^{1/2}p^{1/2})$ $\times H^{(2)}_{2\nu}(a^{1/2}p^{1/2})$ $\quad\mathrm{Re}\ \nu>-\tfrac{1}{4}$	$2a^{-\nu-1/2} t^{3\nu-1/2} e^{\frac{1}{2}a/t} W_{\nu,\nu}(a/t)$
(30)	$p^{1/2}[H^{(1)}_{\nu}(a^{1/2}p^{1/2})\, H^{(2)}_{\nu+1}(a^{1/2}p^{1/2})$ $+H^{(1)}_{\nu+1}(a^{1/2}p^{1/2})\, H^{(2)}_{\nu}(a^{1/2}p^{1/2})]$	$a^{-1/2}\pi^{-3/2}(4\nu+2)e^{\frac{1}{2}a/t} W_{-1/2,\nu+1/2}(a/t)$

5.15. Modified Bessel functions of arguments kp and kp^2

(1)	$e^{-bp} I_0(bp)$ $\quad b>0$	$\pi^{-1}(2bt-t^2)^{-1/2}$ $\quad 0<t<2b$ 0 $\quad t>2b$
(2)	$\pi b e^{-bp} I_1(bp)$ $\quad b>0$	$(b-t)(2bt-t^2)^{-1/2}$ $\quad 0<t<2b$ 0 $\quad t>2b$
(3)	$e^{-\frac{1}{2}(a+b)p} I_n[\tfrac{1}{2}(b-a)p]$ $b>a\geq 0$	0 $\quad 0<t<a$ $\dfrac{\cos\left(n\cos^{-1}\frac{2t-a-b}{b-a}\right)}{\pi(t-a)^{1/2}(b-t)^{1/2}}$ $\quad a<t<b$ 0 $\quad t>b$

Modified Bessel functions of kp and kp^2 (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(4)	$\dfrac{\pi^{3/2}e^{-bp/2}p^{\nu}}{\Gamma(\nu+\frac{1}{2})\,b^{\nu}}I_\nu(\frac{1}{2}bp)$ $\mathrm{Re}\,\nu<\frac{1}{2},\quad b>0$	$\cos(2\pi\nu)(bt-t^2)^{-\nu-\frac{1}{2}}\qquad 0<t<b$ $-\sin(2\nu\pi)(t^2-bt)^{-\nu-\frac{1}{2}}\qquad t>b$
(5)	$\pi^{\frac{1}{2}}\Gamma(\nu+\frac{1}{2})e^{-\frac{1}{2}bp}b^{\nu}p^{-\nu}I_\nu(\frac{1}{2}bp)$ $\mathrm{Re}\,\nu>-\frac{1}{2},\quad b>0$	$(bt-t^2)^{\nu-\frac{1}{2}}\qquad 0<t<b$ $0\qquad t>b$
(6)	$\Gamma(2\nu+n)e^{-\frac{1}{2}bp}b^{\nu}p^{-\nu}I_{\nu+n}(\frac{1}{2}bp)$ $\mathrm{Re}\,\nu>-\frac{1}{2},\quad b>0$	$\dfrac{(-1)^n n!\,\Gamma(\nu)\,2^{2\nu}}{\pi(bt-t^2)^{\frac{1}{2}-\nu}}C_n^{\nu}(2t/b-1)$ $0<t<b$ $0\qquad t>b$
(7)	$\Gamma(2\nu)\,p^{-\nu}\operatorname{csch}(ap)\,I_\nu(ap)$ $a>0,\quad \mathrm{Re}\,\nu>-\frac{1}{2}$	$\pi^{-1}2^{\nu}a^{-\nu}\Gamma(\nu)[2a(t-2ak)$ $-(t-2ak)^2]^{\nu-\frac{1}{2}}\qquad k=0,1,2,\ldots$ $2ak<t<2a(k+1)$
(8)	$K_0(bp)\qquad b>0$	$0\qquad 0<t<b$ $y^{-1}\qquad t>b$
(9)	$p^{-1}K_0(bp)\qquad b>0$	$0\qquad 0<t<b$ $\cosh^{-1}(t/b)\qquad t>b$
(10)	$K_1(bp)\qquad b>0$	$0\qquad 0<t<b$ $b^{-1}ty^{-1}\qquad t>b$
(11)	$p^{-1}K_1(bp)\qquad b>0$	$0\qquad 0<t<b$ $b^{-1}y\qquad t>b$
(12)	$K_\nu(bp)\qquad b>0$	$0\qquad 0<t<b$ $y^{-1}\cosh[\nu\cosh^{-1}(t/b)]\qquad t>b$

$y=(t^2-b^2)^{\frac{1}{2}}$

Modified Bessel functions of kp and kp^2 (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(13)	$p^{-1} K_\nu(bp)$ $\quad b>0$	$0 \quad 0<t<b$ $\nu^{-1}\sinh[\nu\cosh^{-1}(t/b)] \quad t>b$
(14)	$\Gamma(\nu+\tfrac{1}{2})p^{-\nu}K_\nu(bp)$ $b>0, \quad \mathrm{Re}\,\nu>-\tfrac{1}{2}$	$0 \quad 0<t<b$ $2^{-\nu}\pi^{\frac{1}{2}} b^{-\nu} y^{2\nu-1} \quad t>b$
(15)	$2^{2\mu}\Gamma(2\mu+\tfrac{1}{2})(p/b)^{-2\mu}K_{2\nu}(bp)$ $\mathrm{Re}\,\mu>-\tfrac{1}{4}, \quad b>0$	$0 \quad 0<t<b$ $\pi^{\frac{1}{2}} y^{4\mu-1}$ $\times\, {}_2F_1(\mu-\nu,\mu+\nu;2\mu+\tfrac{1}{2};1-t^2/b^2)$ $t>b$
(16)	$e^{ap}K_0(ap)$ $\quad \lvert\arg a\rvert<\pi$	$(t^2+2at)^{-\frac{1}{2}}$
(17)	$e^{ap}K_1(ap)$ $\quad \lvert\arg a\rvert<\pi$	$a^{-1}(t+a)(t^2+2at)^{-\frac{1}{2}}$
(18)	$e^{ap}K_\nu(ap)$ $\quad \lvert\arg a\rvert<\pi$	$(t^2+2at)^{-\frac{1}{2}}\cosh[\nu\cosh^{-1}(1+t/a)]$
(19)	$p^{-1}e^{ap}K_0(ap)$ $\quad \lvert\arg a\rvert<\pi$	$\cosh^{-1}(1+t/a)$
(20)	$p^{-1}e^{ap}K_1(ap)$ $\quad \lvert\arg a\rvert<\pi$	$a^{-1}(t^2+2a)^{-\frac{1}{2}}$
(21)	$p^{-1}e^{ap}K_\nu(ap)$ $\quad \lvert\arg a\rvert<\pi$	$\nu^{-1}\sinh[\nu\cosh^{-1}(1+t/a)]$
(22)	$p^{-\nu}e^{ap}K_\nu(ap)$ $\mathrm{Re}\,\nu>-\tfrac{1}{2}, \quad \lvert\arg a\rvert<\pi$	$\pi^{\frac{1}{2}}[\Gamma(\nu+\tfrac{1}{2})]^{-1}(2a)^{-\nu}(t^2+2at)^{\nu-\frac{1}{2}}$
(23)	$p^{\mu}e^{ap}K_\nu(ap)$ $\mathrm{Re}\,\mu<\tfrac{1}{2}, \quad \lvert\arg a\rvert<\pi$	$2^{-\frac{1}{2}}\pi^{\frac{1}{2}}a^{-\frac{1}{2}}(t^2+2at)^{-\frac{1}{2}\mu-\frac{1}{4}}$ $\times\, P^{\mu+\frac{1}{2}}_{\nu-\frac{1}{2}}(1+t/a)$

$y=(t^2-b^2)^{\frac{1}{2}}$

Modified Bessel functions of kp and kp^2 (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(24)	$e^{ap^2}K_0(ap^2)$ Re $a>0$	$2^{-1/2}a^{-1/2}\pi^{1/2}\exp\left(-\frac{t^2}{16a}\right)I_0\left(\frac{t^2}{16a}\right)$
(25)	$p^{1/2}e^{ap^2}K_{1/4}(ap^2)$ Re $a>0$	$(2at)^{-1/2}\exp\left(-\frac{t^2}{8a}\right)$
(26)	$p^{-1/2}e^{ap^2}K_{1/4}(ap^2)$ Re $a>0$	$(8a)^{-1/4}\gamma\left(\frac{1}{4},\frac{t^2}{8a}\right)$
(27)	$\Gamma(4\nu+1)p^{-4\nu}e^{ap^2}K_{2\nu}(ap^2)$ Re $\nu>-1/4$, Re $a>0$	$2^{3\nu+1}\pi^{1/2}a^{\nu}t^{2\nu-1}\exp\left(-\frac{t^2}{16a}\right)\times M_{-3\nu,\nu}\left(\frac{t^2}{8a}\right)$

5.16. Modified Bessel functions of other arguments

(1)	$I_\nu(2a/p)$ Re $\nu>0$	$a^{1/2}t^{-1/2}Z_\nu^{(b)}(2a^{1/2}t^{1/2})$
(2)	$p\,I_\nu(2a/p)$ Re $\nu>1$	$a^2V_\nu^{(b)}(2a^{1/2}t^{1/2})$
(3)	$p^{-1}I_\nu(2a/p)$ Re $\nu>-1$	$X_\nu^{(b)}(2a^{1/2}t^{1/2})$
(4)	$p^{-2}I_\nu(2a/p)$ Re $\nu>-2$	$a^{-1/2}t^{1/2}W_\nu^{(b)}(2a^{1/2}t^{1/2})$
(5)	$p^{-\lambda}\,I_\nu\left(\frac{2a}{p}\right)$ Re$(\lambda+\nu)>0$	$\frac{a^\nu t^{\lambda+\nu-1}}{\Gamma(\nu+1)\Gamma(\lambda+\nu)}\times{}_0F_3\left(\nu+1,\frac{\lambda+\nu}{2},\frac{\lambda+\nu+1}{2};\frac{1}{4}a^2t^2\right)$

Modified Bessel functions of other arguments (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(6)	$p^{-1/2}\,e^{a/p}\,I_{1/4}(a/p)$	$\pi^{-1}(2a)^{-1/4}\,t^{-3/4}\sinh[(8at)^{1/2}]$
(7)	$p^{-1/2}\,e^{-a/p}\,I_{1/4}(a/p)$	$\pi^{-1}(2a)^{-1/4}\,t^{-3/4}\sin[(8at)^{1/2}]$
(8)	$p^{-1/2}\,e^{a/p}\,I_{-1/4}(a/p)$	$\pi^{-1}(2a)^{-1/4}\,t^{-3/4}\cosh[(8at)^{1/2}]$
(9)	$p^{-1/2}\,e^{-a/p}\,I_{-1/4}(a/p)$	$\pi^{-1}(2a)^{-1/4}\,t^{-3/4}\cos[(8at)^{1/2}]$
(10)	$\pi p^{-1/2}\,e^{a/p}\,I_{3/4}(a/p)$	$2^{-1/4}a^{-1/4}t^{-3/4}\cosh[(8at)^{1/2}]$ $-2^{-7/4}a^{-3/4}t^{-5/4}\sinh[(8at)^{1/2}]$
(11)	$p^{-1/2}\,e^{-a/p}\,I_{3/4}(a/p)$	$2^{-7/4}a^{-3/4}t^{-5/4}\sin[(8at)^{1/2}]$ $-2^{-1/4}a^{-1/4}t^{-3/4}\cos[(8at)^{1/2}]$
(12)	$\pi p^{-1/2}\,e^{a/p}\,I_{-3/4}(a/p)$	$2^{-1/4}a^{-1/4}t^{-3/4}\sinh[(8at)^{1/2}]$ $-2^{-7/4}a^{-3/4}t^{-5/4}\cosh[(8at)^{1/2}]$
(13)	$\pi p^{-1/2}\,e^{-a/p}\,I_{-3/4}(a/p)$	$-2^{-1/4}a^{-1/4}t^{-3/4}\sin[(8at)]^{1/2}]$ $-2^{-7/4}a^{-3/4}t^{-5/4}\cos[(8at)^{1/2}]$
(14)	$p^{-1}\,e^{a/p}\,I_\nu(a/p)$ $\quad \mathrm{Re}\,\nu>-1$	$\{I_\nu[(2at)^{1/2}]\}^2$
(15)	$p^{-1}\,e^{-a/p}\,I_\nu(a/p)$ $\quad \mathrm{Re}\,\nu>-1$	$\{J_\nu[(2at)^{1/2}]\}^2$
(16)	$p^{-1/2}\,e^{a/p}\,I_\nu(a/p)$ $\quad \mathrm{Re}\,\nu>-1/2$	$(\pi t)^{-1/2}\,I_{2\nu}[(8at)^{1/2}]$
(17)	$p^{-1/2}\,e^{-a/p}\,I_\nu(a/p)$ $\quad \mathrm{Re}\,\nu>-1/2$	$(\pi t)^{-1/2}\,J_{2\nu}[(8at)^{1/2}]$

Modified Bessel functions of other arguments (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(18)	$p^{-\lambda}e^{a/p}\,I_\nu(a/p)$ $\mathrm{Re}(\lambda+\nu)>0$	$\dfrac{2^{-\nu}a^\nu t^{\lambda+\nu-1}}{\Gamma(\nu+1)\Gamma(\lambda+\nu)}$ $\times\,{}_1F_2(\nu+\tfrac{1}{2};2\nu+1,\lambda+\nu;2at)$
(19)	$p^{-\lambda}e^{-a/p}\,I_\nu(a/p)$ $\mathrm{Re}(\lambda+\nu)>0$	$\dfrac{2^{-\nu}a^\nu t^{\lambda+\nu-1}}{\Gamma(\nu+1)\Gamma(\lambda+\nu)}$ $\times\,{}_1F_2(\nu+\tfrac{1}{2};2\nu+1,\lambda+\nu;-2at)$
(20)	$p^{-1/2}\sinh(a/p)\,I_\nu(a/p)$ $\mathrm{Re}\,\nu>-\tfrac{1}{2}$	$\tfrac{1}{2}\pi^{-1/2}t^{-1/2}\{I_{2\nu}[(8at)^{1/2}]$ $-J_{2\nu}[(8at)^{1/2}]\}$
(21)	$p^{-1/2}\cosh(a/p)\,I_\nu(a/p)$ $\mathrm{Re}\,\nu>-\tfrac{1}{2}$	$\tfrac{1}{2}\pi^{-1/2}t^{-1/2}\{I_{2\nu}[(8at)^{1/2}]$ $+J_{2\nu}[(8at)^{1/2}]\}$
(22)	$p^{-1}e^{(\alpha^2+\beta^2)/p}\,I_\nu(2\alpha\beta/p)$ $\mathrm{Re}\,\nu>-1$	$I_\nu(2\alpha t^{1/2})\,I_\nu(2\beta t^{1/2})$
(23)	$p^{-1}e^{-(\alpha^2+\beta^2)/p}\,I_\nu(2\alpha\beta/p)$ $\mathrm{Re}\,\nu>-1$	$J_\nu(2\alpha t^{1/2})\,J_\nu(2\beta t^{1/2})$
(24)	$2p^{-1}\sinh[(\alpha^2+\beta^2)/p]$ $\times I_\nu(2\alpha\beta/p)$ $\mathrm{Re}\,\nu>-1$	$I_\nu(2\alpha t^{1/2})\,I_\nu(2\beta t^{1/2})$ $-J_\nu(2\alpha t^{1/2})\,J_\nu(2\beta t^{1/2})$
(25)	$2p^{-1}\cosh[(\alpha^2+\beta^2)/p]$ $\times I_\nu(2\alpha\beta/p)$ $\mathrm{Re}\,\nu>-1$	$I_\nu(2\alpha t^{1/2})\,I_\nu(2\beta t^{1/2})$ $+J_\nu(2\alpha t^{1/2})\,J_\nu(2\beta t^{1/2})$

Modified Bessel functions of other arguments (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(26)	$2^{1/2}\pi^{1/2}r^{-\nu}e^{-p}C_n^{\nu}(p/r)$ $\times I_{\nu+n}(r) \qquad r=(p^2+a^2)^{1/2}$	$(-1)^n a^{1/2-\nu}(2t-t^2)^{1/2\nu-1/4}C_n^{\nu}(t-1)$ $\times I_{\nu-1/2}[a(2t-t^2)^{1/2}] \qquad 0<t<2$ $0 \qquad t>2$
(27)	$\pi e^{-p}I_0[(p^2-a^2)^{1/2}]$	$(2t-t^2)^{-1/2}\cos[a(2t-t^2)^{1/2}]$ $\qquad 0<t<2$ $0 \qquad t>2$
(28)	$p^{1/2}[I_{\nu-1/4}(bp)I_{-\nu-1/4}(bp)$ $-I_{\nu+1/4}(bp)I_{-\nu+1/4}(bp)]$	$\dfrac{2^{3/2}\cos[2\nu\cos^{-1}(\frac{1}{2}t/b)]}{\pi^{3/2}(4b^2t-t^3)^{1/2}} \qquad 0<t<2b$ $0 \qquad t>2b$
(29)	$p^{-1/2}\sinh(a/p)K_0(a/p)$	$\pi^{-1/2}t^{-1/2}K_0[(8at)^{1/2}]$ $+\frac{1}{2}\pi^{1/2}t^{-1/2}Y_0[(8at)^{1/2}]$
(30)	$p^{-1/2}\cosh(a/p)K_0(a/p)$	$\pi^{-1/2}t^{-1/2}K_0[(8at)^{1/2}]$ $-\frac{1}{2}\pi^{1/2}t^{-1/2}Y_0[(8at)^{1/2}]$
(31)	$a^{1/2}p^{-1/2}e^{a/p}K_{1/4}(a/p)$	$(2t)^{-3/4}\,e^{-(8at)^{1/2}}$
(32)	$\pi^{-1/2}p^{2\lambda}K_{2\nu}(2a/p)$ $\qquad \mathrm{Re}(\lambda\pm\nu)<0$	$2^{2\lambda}t^{-2\lambda-1}$ $\times S_2(\nu-\frac{1}{2},-\nu-\frac{1}{2},\lambda+\frac{1}{2},\lambda;\frac{1}{2}at)$
(33)	$p^{-1/2}e^{a/p}K_{\nu}(a/p) \qquad \lvert\mathrm{Re}\,\nu\rvert<\frac{1}{2}$	$2\pi^{-1/2}t^{-1/2}\cos(\nu\pi)K_{2\nu}[(8at)^{1/2}]$
(34)	$\pi^{-1/2}p^{-1/2}e^{-a/p}K_{\nu}(a/p)$ $\qquad \lvert\mathrm{Re}\,\nu\rvert<\frac{1}{2}$	$-t^{-1/2}\sin(\nu\pi)J_{2\nu}[(8at)^{1/2}]$ $-t^{-1/2}\cos(\nu\pi)Y_{2\nu}[(8at)^{1/2}]$

Modified Bessel functions of other arguments (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(35)	$K_0(a^{1/2}p^{1/2})$ Re $a\geq 0$, $a\neq 0$	$\frac{1}{2}t^{-1}e^{-\frac{1}{4}a/t}$
(36)	$a^{1/2}p^{-1/2}K_1(a^{1/2}p^{1/2})$ Re $a\geq 0$	$e^{-\frac{1}{4}a/t}$
(37)	$a^{-1/2}p^{1/2}K_1(a^{1/2}p^{1/2})$ Re $a>0$	$\frac{1}{4}t^{-2}e^{-\frac{1}{4}a/t}$
(38)	$p^{-1/2}K_\nu(2a^{1/2}p^{1/2})$ Re $a>0$	$\frac{1}{2}\pi^{-1/2}t^{-1/2}e^{-\frac{1}{2}a/t}K_{\frac{1}{2}\nu}(\frac{1}{2}a/t)$
(39)	$a^{\frac{1}{2}\nu}p^{-\frac{1}{2}\nu}K_\nu(2a^{1/2}p^{1/2})$ Re $a>0$	$\frac{1}{2}t^{\nu-1}e^{-a/t}$
(40)	$a^{-\frac{1}{2}\nu}p^{\frac{1}{2}\nu}K_\nu(2a^{1/2}p^{1/2})$ Re $a>0$	$\frac{1}{2}t^{-\nu-1}e^{-a/t}$
(41)	$p^{\frac{1}{2}\nu-1}K_\nu(2a^{1/2}p^{1/2})$ Re $a>0$	$\frac{1}{2}a^{-\frac{1}{2}\nu}\Gamma(\nu, a/t)$
(42)	$p^{\frac{1}{2}\nu+n}K_\nu(2a^{1/2}p^{1/2})$ Re $a>0$	$\frac{1}{2}(-1)^n n!\,a^{\frac{1}{2}\nu}t^{-n}e^{-a/t}L_n^\nu(a/t)$
(43)	$2a^{1/2}p^{\mu-1}K_{2\nu}(2a^{1/2}p^{1/2})$ Re $a>0$	$t^{\frac{1}{2}-\mu}e^{-\frac{1}{2}a/t}W_{\mu-\frac{1}{2},\nu}(a/t)$
(44)	$r^{-1}K_1(br)$ $b>0$	0 $0<t<b$ $a^{-1}b^{-1}\sin a\gamma$ $t>b$
(45)	$2^{1/2}\pi^{-1/2}r^{-\nu}K_\nu(br)$ Re $\nu>-\frac{1}{2}$	0 $0<t<b$ $a^{\frac{1}{2}-\nu}b^{-\nu}\gamma^{\nu-\frac{1}{2}}J_{\nu-\frac{1}{2}}(a\gamma)$ $t>b$
(46)	$2^{1/2}\pi^{-1/2}r^{-\nu}e^{\beta p}K_\nu(\beta r)$ Re $\nu>-\frac{1}{2}$, $\lvert\arg\beta\rvert<\pi$	$a^{\frac{1}{2}-\nu}\beta^{-\nu}(t^2+2\beta t)^{\frac{1}{2}\nu-\frac{1}{4}}$ $\times J_{\nu-\frac{1}{2}}[a(t^2+2\beta t)^{1/2}]$

$r=(p^2+a^2)^{1/2}$

Modified Bessel functions of other arguments (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(47)	$2^{1/2}\pi^{-1/2}s^{-\nu}K_\nu(bs)$ $\quad \mathrm{Re}\,\nu>-\frac{1}{2}$	$0 \qquad 0<t<b$ $a^{1/2-\nu}b^{-\nu}y^{\nu-1/2}I_{\nu-1/2}(ay) \qquad t>b$
(48)	$2^{1/2}\pi^{-1/2}s^{-\nu}e^{\beta p}K_\nu(\beta s)$ $\mathrm{Re}\,\nu>-\frac{1}{2}, \quad \lvert\arg\beta\rvert<\pi$	$a^{1/2-\nu}\beta^{-\nu}(t^2+2\beta t)^{\frac{1}{2}\nu-\frac{1}{4}}$ $\times I_{\nu-1/2}[a(t^2+2\beta t)^{1/2}]$
(49)	$\dfrac{(\frac{1}{2}c)^p K_p(c)}{\Gamma(p+\frac{1}{2})} \qquad c>0$	$\dfrac{\cos[c(e^t-1)^{1/2}]}{2\pi^{1/2}(1-e^{-t})^{1/2}}$
(50)	$\dfrac{a^p K_{\nu-p}(a)}{\Gamma(p+1)}$ $\mathrm{Re}\,\nu>-1, \quad a>0$	$\frac{1}{2}(e^t-1)^{\frac{1}{2}\nu}J_\nu[2a(e^t-1)^{1/2}]$
(51)	$J_\nu(a^{1/2}p^{1/2})K_\nu(a^{1/2}p^{1/2}) \qquad a>0$	$\frac{1}{2}t^{-1}J_\nu(\frac{1}{2}a/t)$
(52)	$Y_\nu(a^{1/2}p^{1/2})K_\nu(a^{1/2}p^{1/2}) \qquad a>0$	$\frac{1}{2}t^{-1}Y_\nu(\frac{1}{2}a/t)$
(53)	$H_\nu^{(1)}(a^{1/2}p^{1/2})K_\nu(a^{1/2}p^{1/2}) \qquad a>0$	$\frac{1}{2}t\,H_\nu^{(1)}(\frac{1}{2}a/t)$
(54)	$H_\nu^{(2)}(a^{1/2}p^{1/2})K_\nu(a^{1/2}p^{1/2}) \qquad a>0$	$\frac{1}{2}t^{-1}H_\nu^{(2)}(\frac{1}{2}a/t)$
(55)	$p^{1/2}I_n(bp)K_{n+1/2}(bp) \qquad b>0$	$\dfrac{(-1)^n\cos[(2n+\frac{1}{2})\cos^{-1}(\frac{1}{2}t/b)]}{[\frac{1}{2}\pi(4b^2t-t^3)]^{1/2}} \qquad 0<t<2b$ $0 \qquad t>2b$
(56)	$K_\nu(a^{1/2}p^{1/2}+\beta^{1/2}p^{1/2})$ $\times I_\nu(a^{1/2}p^{1/2}-\beta^{1/2}p^{1/2})$ $\mathrm{Re}\,a>0, \quad \mathrm{Re}\,\beta>0$	$\frac{1}{2}t^{-1}e^{-\frac{1}{2}(a+\beta)/t}I_\nu[\frac{1}{2}(a-\beta)/t]$

$s=(p^2-a^2)^{1/2}, \quad y=(t^2-b^2)^{1/2}$

Modified Bessel functions of other arguments (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(57)	$I_\nu[\tfrac12 b(r-p)]K_\nu[\tfrac12 b(r+p)]$ $r=(p^2+a^2)^{1/2}$, $\mathrm{Re}\,\nu>-\tfrac12$	$0 \qquad 0<t<b$ $(t^2-b^2)^{-1/2}J_{2\nu}[a(t^2-b^2)^{1/2}] \qquad t>b$
(58)	$I_{\nu+p}(c)K_{\nu-p}(c)$ $c>0$, $\mathrm{Re}\,\nu>-\tfrac12$	$\tfrac12 J_{2\nu}(2c\sinh\tfrac12 t)$
(59)	$p^{2\nu}[K_{2\nu}(a^{1/2}p^{1/2})]^2 \qquad \mathrm{Re}\,a>0$	$\tfrac12\pi^{1/2}a^{\nu-1/2}t^{-3\nu-1/2}e^{-\frac12 a/t}W_{\nu,\nu}(a/t)$
(60)	$e^{\frac12(\alpha+\beta)p}K_{2\nu}(\tfrac12\alpha p)K_{2\nu}(\tfrac12\beta p)$ $\lvert\arg\alpha\rvert<\pi$, $\lvert\arg\beta\rvert<\pi$	$\pi(\alpha\beta)^{\nu-1/4}(\alpha+t)^{-\nu-1/4}(\beta+t)^{-\nu-1/4}$ $\times P_{2\nu-1/2}[2\alpha^{-1}\beta^{-1}(\alpha+t)(\beta+t)-1]$
(61)	$p^{1/2}K_{\nu+1/2}(a^{1/2}p^{1/2})K_{\nu-1/2}(a^{1/2}p^{1/2})$ $\mathrm{Re}\,a>0$	$\tfrac12(2a)^{-1/2}\pi^{1/2}t^{-1}e^{-\frac12 a/t}W_{1/2,\nu}(a/t)$
(62)	$p^{1/2}K_{\nu+1/4}(bp)K_{\nu-1/4}(bp) \qquad b>0$	$0 \qquad 0<t<2b$ $\dfrac{2^{1/2}\pi^{1/2}\cosh[2\nu\cosh^{-1}(\tfrac12 t/b)]}{(t^3-4b^2t)^{1/2}}$ $t>2b$
(63)	$p^{1/2}e^{2ap}K_{\nu+1/4}(ap)K_{\nu-1/4}(ap)$ $\lvert\arg a\rvert<\pi$	$(2\pi)^{1/2}[t(t+2a)(t+4a)]^{-1/2}$ $\times\cosh[2\nu\cosh^{-1}(1+\tfrac12 t/a)]$
(64)	$K_\nu(\alpha^{1/2}p^{1/2}+\beta^{1/2}p^{1/2})$ $\times K_\nu(\alpha^{1/2}p^{1/2}-\beta^{1/2}p^{1/2})$ $\mathrm{Re}\,\alpha>0$, $\mathrm{Re}\,\beta>0$	$\tfrac12 t^{-1}e^{-\frac12(\alpha+\beta)/t}K_\nu[\tfrac12(\alpha-\beta)/t]$
(65)	$K_\nu[p^{1/2}+(p-1)^{1/2}]K_\nu[p^{1/2}-(p-1)^{1/2}]$	$\tfrac12 t^{-1}e^{\frac12 t-1/t}K_\nu(\tfrac12 t)$
(66)	$K_\nu[(\lambda S/a)^{1/2}]K_\nu[(\lambda a/S)^{1/2}]$ $S=(p^2-a^2)^{1/2}+p$ $\mathrm{Re}(\lambda/a)>0$	$\tfrac12 t^{-1}e^{-\frac12\lambda a^{-1}t^{-1}}K_\nu(a\lambda t)$

5.17. Functions related to Bessel functions

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(1)	$\mathbf{J}_\nu(p)-J_\nu(p)$	$\pi^{-1}\sin(\nu\pi)(t^2+1)^{-1/2}[(t^2+1)^{1/2}-t]^\nu$
(2)	$\csc(\pi p)[\mathbf{J}_p(a)-J_p(a)]$ $\quad\mathrm{Re}\,a\geq 0$	$\pi^{-1}e^{-a\sinh t}$
(3)	$\mathbf{E}_\nu(p)+Y_\nu(p)$	$-(t^2+1)^{-1/2}\{[(t^2+1)^{1/2}+t]^\nu$ $+\cos(\nu\pi)[(t^2+1)^{1/2}-t]^\nu\}$
(4)	$p^{-1}[\mathbf{H}_0(ap)-Y_0(ap)]$	$2\pi^{-1}\sinh^{-1}(t/a)$
(5)	$\tfrac{1}{2}\pi[\mathbf{H}_1(ap)-Y_1(ap)]-1$ $\quad\lvert\arg a\rvert<\tfrac{1}{2}\pi$	$a^{-1}t(t^2+a^2)^{-1/2}$
(6)	$p^{-\nu}[\mathbf{H}_\nu(ap)-Y_\nu(ap)]$ $\quad\mathrm{Re}\,a>0$	$\dfrac{2^{1-\nu}a^{-\nu}}{\pi^{1/2}\Gamma(\nu+\frac{1}{2})}(t^2+a^2)^{\nu-1/2}$
(7)	$p^{1/2}[\mathbf{H}_{1/4}(p^2/a)-Y_{1/4}(p^2/a)]$ $\quad a>0$	$a\pi^{-1/2}t^{1/2}J_{-1/4}(\tfrac{1}{4}at^2)$
(8)	$p^{1/2}[\mathbf{H}_{-1/4}(p^2/a)-Y_{-1/4}(p^2/a)]$ $\quad a>0$	$a\pi^{-1/2}t^{1/2}J_{1/4}(\tfrac{1}{4}at^2)$
(9)	$p^{3/2}[\mathbf{H}_{-3/4}(p^2/a)-Y_{-3/4}(p^2/a)]$ $\quad a>0$	$-\tfrac{1}{2}a^2\pi^{-1/2}t^{3/2}J_{-1/4}(\tfrac{1}{4}at^2)$
(10)	$p^{3/2}[\mathbf{H}_{-1/4}(p^2/a)-Y_{-1/4}(p^2/a)]$ $\quad a>0$	$\tfrac{1}{2}a^2\pi^{-1/2}t^{3/2}J_{-3/4}(\tfrac{1}{4}at^2)$
(11)	$p^{-\lambda}\mathbf{H}_\nu\left(\dfrac{2a}{p}\right)$ $\quad\mathrm{Re}(\lambda+\nu)>-1$	$\dfrac{2\pi^{-1/2}a^{\nu+1}t^{\lambda+\nu}}{\Gamma(\nu+3/2)\Gamma(\lambda+\nu+1)}$ $\times{}_1F_4\left(1;\dfrac{3}{2},\nu+\dfrac{3}{2},\dfrac{\lambda+\nu+1}{2},\dfrac{\lambda+\nu}{2}+1;-\dfrac{a^2t^2}{4}\right)$

Functions related to Bessel functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(12)	$p^{-1/2}[\mathbf{H}_0(2ap^{1/2})-Y_0(2ap^{1/2})]$	$2\pi^{-3/2}t^{-1/2}e^{1/2 a^2/t}K_0(\tfrac{1}{2}a^2/t)$
(13)	$p^{-1/2\nu}[\mathbf{H}_{-\nu}(ap^{1/2})-Y_{-\nu}(ap^{1/2})]$ $\operatorname{Re}\nu>-\tfrac{1}{2}$	$2^{\nu}\pi^{-1}a^{-\nu}\cos(\nu\pi)t^{\nu-1}\exp(\tfrac{1}{4}a^2t^{-1})$ $\times\operatorname{Erfc}(\tfrac{1}{2}at^{-1/2})$
(14)	$p^{-1/2\nu-1/2}[\mathbf{H}_\nu(ap^{1/2})-Y_\nu(ap^{1/2})]$	$2\pi^{-1/2}a^{-1}[\Gamma(\tfrac{1}{2}+\nu)]^{-1}t^{-1/2\nu}$ $\times\exp\left(\frac{a^2}{8t}\right)W_{1/2\nu,1/2\nu}\left(\frac{a^2}{4t}\right)$
(15)	$\Gamma(p+\tfrac{1}{2})\,2^p a^{-p}\,\mathbf{H}_p(a)$	$\pi^{-1/2}(e^t-1)^{-1/2}\sin[a(1-e^{-t})^{1/2}]$
(16)	$p^{-1}[I_0(bp)-\mathbf{L}_0(bp)]$ $\quad b>0$	$2\pi^{-1}\sin^{-1}(t/b)$ $\quad 0<t<b$ 0 $\quad t>b$
(17)	$\tfrac{1}{2}\pi[\mathbf{L}_1(bp)-I_1(bp)]+1$ $\quad b>0$	$b^{-1}t(b^2-t^2)^{-1/2}$ $\quad 0<t<b$ 0 $\quad t>b$
(18)	$\pi^{1/2}\Gamma(\nu+\tfrac{1}{2})p^{-\nu}[I_\nu(bp)-\mathbf{L}_\nu(bp)]$ $\operatorname{Re}\nu>-\tfrac{1}{2},\quad b>0$	$2^{1-\nu}b^{-\nu}(b^2-t^2)^{\nu-1/2}$ $\quad 0<t<b$ 0 $\quad t>b$
(19)	$\pi^{1/2}(2b)^{\nu}\Gamma(\nu+\tfrac{1}{2})p^{-\nu}e^{-bp}\mathbf{L}_\nu(bp)$ $\operatorname{Re}\nu>-\tfrac{1}{2},\quad b>0$	$(2bt-t^2)^{\nu-1/2}$ $\quad 0<t<b$ $-(2bt-t^2)^{\nu-1/2}$ $\quad b<t<2b$ 0 $\quad t>2b$
(20)	$\tfrac{1}{2}\pi^{1/2}\Gamma(\nu+\tfrac{1}{2})p^{-\nu}\operatorname{csch}p\,\mathbf{L}_\nu(p)$ $\operatorname{Re}\nu>-\tfrac{1}{2}$	$[2(t-2k)-(t-2k)^2]^{\nu-1/2}$ $2k<t<2k+1$ $-[2(t-2k)-(t-2k)^2]^{\nu-1/2}$ $2k+1<t<2k+2$

Functions related to Bessel functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(21)	$\Gamma(\nu+\frac{1}{2})p^{-\nu}\operatorname{csch}(\frac{1}{2}p)[I_\nu(p)-\mathbf{L}_\nu(p)]$ $\quad \operatorname{Re}\nu>-\frac{1}{2}$	$0 \qquad 0<t<1/2$ $4\pi^{-\frac{1}{2}}[\frac{3}{4}+t-k-(t-k)^2]^{\nu-\frac{1}{2}}$ $\quad k+1/2<t<k+3/2$ $k=0,1,2,\ldots$
(22)	$p^{-\lambda}\mathbf{L}_\nu(2a/p)$ $\quad \operatorname{Re}(\lambda+\nu)>-1$	$\dfrac{2\pi^{-\frac{1}{2}}a^{\nu+1}t^{\lambda+\nu}}{\Gamma(\nu+3/2)\Gamma(\lambda+\nu+1)}$ $\times {}_1F_4\left(1;\dfrac{3}{2},\nu+\dfrac{3}{2},\dfrac{\lambda+\nu+1}{2},\dfrac{\lambda+\nu}{2}+1;\dfrac{a^2t^2}{4}\right)$
(23)	$p^{-\frac{1}{2}}[I_0(2ap^{\frac{1}{2}})-\mathbf{L}_0(2ap^{\frac{1}{2}})]$ $\quad \operatorname{Re}a>0$	$(\pi t)^{-\frac{1}{2}}e^{-\frac{1}{2}a^2/t}I_0(\frac{1}{2}a^2/t)$
(24)	$p^{-\frac{1}{2}\nu}[\mathbf{L}_{-\nu}(ap^{\frac{1}{2}})-I_\nu(ap^{\frac{1}{2}})]$ $\quad \operatorname{Re}\nu>-\frac{1}{2}$	$i\pi^{-1}2^\nu a^{-\nu}\cos(\nu\pi)t^{\nu-1}$ $\times\exp(\frac{1}{4}a^2t^{-1})\operatorname{Erf}(\frac{1}{2}iat^{-\frac{1}{2}})$
(25)	$\Gamma(\frac{1}{2}-p)(\frac{1}{2}b)^p[I_p(b)-\mathbf{L}_{-p}(b)]$ $\quad b>0$	$\pi^{-\frac{1}{2}}(1-e^{-t})^{-\frac{1}{2}}\sin[b(e^t-1)^{\frac{1}{2}}]$
(26)	$S_{0,\nu}(p)$	$(1+t^2)^{-\frac{1}{2}}\cosh(\nu\sinh^{-1}t)$
(27)	$S_{-1,\nu}(p)$	$\nu^{-1}(1+t^2)^{-\frac{1}{2}}\sinh(\nu\sinh^{-1}t)$
(28)	$p^{-1}S_{0,\nu}(p)$	$\nu^{-1}\sinh(\nu\sinh^{-1}t)$
(29)	$p^{-1}S_{1,\nu}(p)$	$\cosh(\nu\sinh^{-1}t)$
(30)	$p^{1-2\lambda-\mu}S_{\mu,\nu}\left(\dfrac{p}{a}\right)$ $\quad \operatorname{Re}\lambda>0,\quad \operatorname{Re}a>0$	$\dfrac{a^{1-\mu}t^{2\lambda-1}}{\Gamma(2\lambda)}$ $\times {}_3F_2\left(1,\dfrac{1-\mu+\nu}{2},\dfrac{1-\mu-\nu}{2};\lambda,\lambda+\dfrac{1}{2};-a^2t^2\right)$

Functions related to Bessel functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(31)	$p^{1/2} S_{-\mu-1,1/4}(\tfrac{1}{2}p^2) \qquad \operatorname{Re}\mu > -\tfrac{3}{4}$	$2^{2\mu+1}[\Gamma(2\mu+3/2)]^{-1} t^{1/2}$ $\times s_{\mu,1/4}(\tfrac{1}{2}t^2)$
	Further similar formulas may be found in *Nederl. Akad. Wetensch. Proc.*, 1935: 38, Part II, p. 629.	
(32)	$p^{-\mu-1/2} S_{2\mu,2\nu}(2a^{1/2}p^{1/2})$ $\operatorname{Re}(\mu \pm \nu) > -\tfrac{1}{2}, \quad \lvert\arg a\rvert < \pi$	$2^{2\mu-1} a^{-1/2} t^{\mu} e^{1/2 a/t} W_{\mu,\nu}(a/t)$
(33)	$p^{-1/2\nu} S_{\mu,\nu}(2a^{1/2}p^{1/2})$ $\operatorname{Re}(\mu-\nu) < 1, \quad \lvert\arg a\rvert < \pi$	$2^{\mu-1} a^{-1/2\nu} t^{\nu-1} e^{a/t}$ $\times \Gamma(\tfrac{1}{2}\mu+\tfrac{1}{2}\nu+\tfrac{1}{2},\ a/t)$
(34)	$p^{-1} S_{2,\nu}(ap) - a$	$(\nu - 1/\nu)\sinh[\nu \sinh^{-1}(t/a)]$
(35)	$2\,O_n(p)$	$[t+(1+t^2)^{1/2}]^n + [t-(1+t^2)^{1/2}]^n$
(36)	$S_n(p)$	$\dfrac{[t+(1+t^2)^{1/2}]^n - [t-(1+t^2)^{1/2}]^n}{(1+t^2)^{1/2}}$

5.18. Parabolic cylinder functions

(1)	$\Gamma(\nu)\, e^{1/4 a^2 p^2} D_{-\nu}(ap)$ $\operatorname{Re}\nu > 0, \quad \lvert\arg a\rvert < \tfrac{1}{4}\pi$	$a^{-\nu} t^{\nu-1} e^{-1/2 t^2/a^2}$
(2)	$\Gamma(2\nu)\, p^{-1} e^{1/4 a^2 p^2} D_{-2\nu}(ap)$ $\operatorname{Re}\nu > 0, \quad \lvert\arg a\rvert < \tfrac{1}{4}\pi$	$2^{\nu+1}\gamma(\nu, \tfrac{1}{2}t^2/a^2)$
(3)	$D_{-2\nu}(2b^{1/2}p^{1/2})$ $\operatorname{Re}\nu > 0, \quad b > 0$	$0 \qquad 0 < t < b$ $\dfrac{2^{1/2-\nu} b^{1/2}}{\Gamma(\nu)} \dfrac{(t-b)^{\nu-1}}{(t+b)^{\nu+1/2}} \qquad t > b$

Parabolic cylinder functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(4)	$p^{-1/2}D_{1-2\nu}(2b^{1/2}p^{1/2})$ $\mathrm{Re}\,\nu>0,\quad b>0$	$0 \qquad 0<t<b$ $\dfrac{2^{1/2-\nu}(t-b)^{\nu-1}}{\Gamma(\nu)(t+b)^{\nu-1/2}} \qquad t>b$
(5)	$\Gamma(\nu)e^{1/2 ap}D_{-2\nu}(2^{1/2}a^{1/2}p^{1/2})$ $\mathrm{Re}\,\nu>0,\quad \lvert\arg a\rvert<\pi$	$2^{-\nu}a^{1/2}t^{\nu-1}(t+a)^{-\nu-1/2}$
(6)	$\Gamma(\nu)p^{-1/2}e^{1/2 ap}D_{1-2\nu}(2^{1/2}a^{1/2}p^{1/2})$ $\mathrm{Re}\,\nu>0,\quad \lvert\arg a\rvert<\pi$	$2^{1/2-\nu}t^{\nu-1}(t+a)^{1/2-\nu}$
(7)	$\Gamma(2\nu)p^{-\nu}e^{1/4 a/p}D_{-2\nu}(a^{1/2}p^{-1/2})$ $\mathrm{Re}\,\nu>0$	$(2t)^{\nu-1}e^{-2^{1/2}a^{1/2}t^{1/2}}$
(8)	$p^{-\nu}e^{-1/4 a/p}D_{2\nu-1}(a^{1/2}p^{-1/2})$ $\mathrm{Re}\,\nu>0$	$2^{\nu+1/2}\pi^{-1/2}t^{\nu-1}\sin(\nu\pi-2^{1/2}a^{1/2}t^{1/2})$
(9)	$2^{p+\nu}\Gamma(p+\nu)D_{-2p}(a)$ $\lvert\arg a\rvert<\tfrac{1}{4}\pi$	$e^{1/2 t}(e^t-1)^{-\nu-1/2}\exp\left[-\dfrac{a^2e^{-t}}{4(1-e^{-t})}\right]$ $\times D_{2\nu}\left[\dfrac{a}{(1-e^{-t})^{1/2}}\right]$
(10)	$\Gamma(\nu+1)D_{-\nu-1}(pe^{1/4\pi i})$ $\times D_{-\nu-1}(pe^{-1/4\pi i})\quad \mathrm{Re}\,\nu>-1$	$\pi^{1/2}J_{\nu+1/2}(\tfrac{1}{2}t^2)$
(11)	$2^{1/2}\Gamma(\nu)D_{-\nu}(2^{1/2}p^{1/2}e^{1/4\pi i})$ $\times D_{-\nu}(2^{1/2}p^{1/2}e^{-1/4\pi i})\quad \mathrm{Re}\,\nu>0$	$t^{\nu-1}(1+t^2)^{-1/2}[1+(1+t^2)^{1/2}]^{1/2-\nu}$
(12)	$e^pD_{\nu-1/2}(2^{1/2}p^{1/2})D_{-\nu-1/2}(2^{1/2}p^{1/2})$	$\dfrac{\cos\{\nu\cos^{-1}[(1+t)^{-1}]\}}{[\pi t(t+1)(t+2)]^{1/2}}$

Parabolic cylinder functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(13)	$p^{-\frac{1}{2}}e^{\frac{1}{2}(\alpha+\beta)p}D_{4\mu}(2^{\frac{1}{2}}\alpha^{\frac{1}{2}}p^{\frac{1}{2}})$ $\times D_{4\nu}(2^{\frac{1}{2}}\beta^{\frac{1}{2}}p^{\frac{1}{2}})$ $\operatorname{Re}(\mu+\nu)<\frac{1}{4}$ $\lvert\arg\alpha\rvert<\pi,\quad \lvert\arg\beta\rvert<\pi$	$2^{-\frac{1}{2}}t^{-\mu-\nu-\frac{1}{4}}(t+\alpha)^{\mu-\nu-\frac{1}{4}}$ $\times(t+\beta)^{\nu-\mu-\frac{1}{4}}(-t-\alpha-\beta)^{\mu+\nu+\frac{1}{4}}$ $\times P^{\nu+\mu+\frac{1}{4}}_{2\nu-2\mu-\frac{1}{2}}[\alpha^{\frac{1}{2}}\beta^{\frac{1}{2}}(t+\alpha)^{-\frac{1}{2}}(t+\beta)^{-\frac{1}{2}}]$

5.19. Gauss' hypergeometric function

(1)	$F(\alpha,\beta;\gamma;\frac{1}{2}-p/\lambda)$ $\operatorname{Re}\alpha>0,\quad \operatorname{Re}\beta>0$	$\dfrac{\lambda\Gamma(\gamma)}{\Gamma(\alpha)\Gamma(\beta)}(\lambda t)^{\frac{1}{2}(\alpha+\beta-3)}$ $\times W_{\frac{1}{2}(\alpha+\beta+1)-\gamma,\,\frac{1}{2}(\alpha-\beta)}(\lambda t)$
(2)	$\Gamma(\alpha)p^{-\alpha}F(\alpha,\beta;\gamma;\lambda/p)$ $\operatorname{Re}\alpha>0$	$\lambda^{-\frac{1}{2}\gamma}t^{\alpha-\frac{1}{2}\gamma-1}M_{\frac{1}{2}\gamma-\beta,\,\frac{1}{2}\gamma-\frac{1}{2}}(\lambda t)$
(3)	$p^{\gamma-1}(p-1)^nF[-n,\alpha;\gamma;p/(p-1)]$ $\operatorname{Re}\gamma<1-n$ $\operatorname{Re}(\alpha-\gamma)>n-1$	$n!\,[\Gamma(1-\gamma)]^{-1}t^{-\gamma-n}L_n^{\alpha-\gamma-n}(t)$
(4)	$p^{m+n}(1+p)^{-m-n-2}$ $\times F(-m,-n;2;p^{-2})$	$(-1)^{m+n}t^{-1}k_{2m+2}(\frac{1}{2}t)k_{2n+2}(\frac{1}{2}t)$
(5)	$(p+1)^{-2\alpha}F\left[-n,\alpha;\dfrac{1}{2}-\nu;\left(\dfrac{p-1}{p+1}\right)^2\right]$ $\operatorname{Re}\alpha>0$	$\dfrac{(n!)^2\pi 2^{1-\alpha}}{\Gamma(\alpha)\Gamma(\frac{1}{2}+n)}t^{2\alpha-1}[L_n^{\alpha-\frac{1}{2}}(t)]^2$
(6)	$(p^2+a^2)^{-\lambda}$ $\times F\left(\lambda,\mu;\lambda+\mu+\dfrac{1}{2};\dfrac{a^2}{p^2+a^2}\right)$ $\operatorname{Re}\lambda>0$	$\dfrac{\Gamma(\lambda+\mu+\frac{1}{2})}{\Gamma(2\lambda)}\left(\dfrac{a}{2}\right)^{\frac{1}{2}-\lambda-\mu}$ $\times t^{\lambda-\mu-\frac{1}{2}}J_{\lambda+\mu-\frac{1}{2}}(at)$

Gauss' hypergeometric function (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(7)	$(p-\alpha)^n(p-\beta)^m p^{-m-n-2} \times F\left[-m,-n;-m-n-1;\ \frac{p(p-\alpha-\beta)}{(p-\alpha)(p-\beta)}\right]$	$\frac{(m+1)!(n+1)!}{(m+n+1)!}\frac{(-1)^{m+n}}{\alpha\beta t}e^{\frac{1}{2}(\alpha+\beta)t} \times k_{2n+2}(\tfrac{1}{2}\alpha t)\,k_{2m+2}(\tfrac{1}{2}\beta t)$
(8)	$(p-\alpha)^n(p-\beta)^m p^{-m-n-\frac{1}{2}} \times F\left[-m,-n;-m-n+\tfrac{1}{2};\ \frac{p(p-\alpha-\beta)}{(p-\alpha)(p-\beta)}\right]$	$\frac{(-2)^{m+n}(m+n)!}{(2m+2n)!\,\pi^{\frac{1}{2}}t^{\frac{1}{2}}}e^{\frac{1}{2}(\alpha+\beta)t} \times D_{2n}(2^{\frac{1}{2}}\alpha^{\frac{1}{2}}t^{\frac{1}{2}})\,D_{2m}(2^{\frac{1}{2}}\alpha^{\frac{1}{2}}t^{\frac{1}{2}})$
(9)	$(p-\alpha)^n(p-\beta)^m p^{-m-n-3/2} \times F\left[-m,-n;-m-n-\tfrac{1}{2};\ \frac{p(p-\alpha-\beta)}{(p-\alpha)(p-\beta)}\right]$	$-\frac{(-1)^{m+n}(-2)^{m+n+1}(m+n+1)!}{(2m+2n+2)!\,(\pi\alpha\beta t)^{\frac{1}{2}}} \times e^{\frac{1}{2}(\alpha+\beta)t}D_{2n+1}(2^{\frac{1}{2}}\alpha^{\frac{1}{2}}t^{\frac{1}{2}}) \times D_{2m+1}(2^{\frac{1}{2}}\alpha^{\frac{1}{2}}t^{\frac{1}{2}})$
(10)	$(p-\alpha)^n(p-\beta)^m p^{-m-n-\lambda-1} \times F\left[-m,-n;-m-n-\lambda;\ \frac{p(p-\alpha-\beta)}{(p-\alpha)(p-\beta)}\right]$ Re $\lambda>-1$	$\frac{m!\,n!\,t^{\lambda}}{\Gamma(m+n+\lambda+1)}L_n^{\lambda}(\alpha t)\,L_m^{\lambda}(\beta t)$
(11)	$B(p,\gamma)F(\alpha,\beta;\gamma+p;z)$ Re $\gamma>0$, $\lvert\arg(z-1)\rvert<\pi$	$(1-e^{-t})^{\gamma-1}F[\alpha,\beta;\gamma;z(1-e^{-t})]$
(12)	$\frac{\Gamma(p)}{\Gamma(p+\frac{1}{2})} \times F(-\mu-\nu,\tfrac{1}{2}-\mu+\nu;p+\tfrac{1}{2};z^2)$ $\lvert z\rvert<1$	$\frac{\Gamma(\frac{1}{2}-\mu-\nu)\Gamma(\frac{1}{2}-\mu+\nu)}{2^{2\mu+1}\pi(1-e^{-t})^{\frac{1}{2}}} \times(1-z^2+z^2e^{-t})^{\mu} \times\{P_{2\nu}^{2\mu}[z(1-e^{-t})^{\frac{1}{2}}] +P_{2\nu}^{2\mu}[-z(1-e^{-t})^{\frac{1}{2}}]\}$

Gauss' hypergeometric function (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(13)	$\dfrac{\Gamma(p)}{\Gamma(p+3/2)}$ $\times F(\tfrac{1}{2}-\mu-\nu, 1-\mu+\nu; p+3/2; z^2)$ $\quad \lvert z\rvert < 1$	$-\dfrac{\Gamma(-\mu-\nu)\Gamma(\tfrac{1}{2}-\mu+\nu)}{4^{\mu+\frac{1}{2}}\pi z}$ $\times(1-z^2+z^2e^{-t})^{\mu}$ $\times\{P^{2\mu}_{2\nu}[z(1-e^{-t})^{\frac{1}{2}}]$ $-P^{2\mu}_{2\nu}[-z(1-e^{-t})^{\frac{1}{2}}]\}$
(14)	$B(p,\nu)F(a,p;p+\nu;z)$ $\quad \operatorname{Re}\nu>0, \quad \lvert\arg(z-1)\rvert<\pi$	$(1-e^{-t})^{\nu-1}(1-ze^{-t})^{-a}$

5.20. Confluent hypergeometric functions

(1)	$p^{-\mu-\frac{1}{2}} e^{-\frac{1}{2}(a+b)p} M_{\kappa,\mu}[(b-a)p]$ $\quad \operatorname{Re}(\mu\pm\kappa)>-\tfrac{1}{2}, \quad b>a\ge 0$	$0 \qquad 0<t<a$ $\dfrac{(b-a)^{\frac{1}{2}-\mu}}{B(\tfrac{1}{2}+\kappa+\mu, \tfrac{1}{2}-\kappa+\mu)} \dfrac{(t-a)^{\kappa+\mu-\frac{1}{2}}}{(b-t)^{\kappa-\mu+\frac{1}{2}}} \qquad a<t<b$ $0 \qquad t>b$
(2)	$p^{\kappa} e^{\frac{1}{2}a/p} M_{\kappa,\mu}(a/p)$ $\quad \operatorname{Re}(\kappa-\mu)<\tfrac{1}{2}$	$\dfrac{a^{\frac{1}{2}}\Gamma(2\mu+1)}{\Gamma(\mu-\kappa+\frac{1}{2})} t^{-\kappa-\frac{1}{2}} I_{2\mu}(2a^{\frac{1}{2}}t^{\frac{1}{2}})$
(3)	$p^{\frac{1}{2}} M_{\frac{1}{4},\nu}(a/p)\, M_{-\frac{1}{4},\nu}(a/p)$ $\quad \operatorname{Re}\nu>-\tfrac{1}{4}$	$2^{2\nu} a t^{-\frac{1}{2}} \dfrac{[\Gamma(2\nu+1)]^2}{\Gamma(2\nu+\frac{1}{2})}$ $\times J_{2\nu}[e^{\frac{1}{4}\pi i}(2at)^{\frac{1}{2}}]$ $\times J_{2\nu}[e^{-\frac{1}{4}\pi i}(2at)^{\frac{1}{2}}]$
(4)	$p^{-\mu-\frac{1}{2}} W_{\kappa,\mu}(p)$ $\quad \operatorname{Re}(\mu-\kappa)>\tfrac{1}{2}$	$0 \qquad 0<t<\tfrac{1}{2}$ $\dfrac{1}{\Gamma(\mu+\frac{1}{2}-\kappa)} \dfrac{(t-\frac{1}{2})^{\mu-\kappa-\frac{1}{2}}}{(t+\frac{1}{2})^{-\mu-\kappa+\frac{1}{2}}} \qquad t>\tfrac{1}{2}$

Confluent hypergeometric functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(5)	$p^{-1}e^{\frac12 ap}W_{\kappa,\mu}(ap)$ $\lvert\arg a\rvert<\pi,\quad \mathrm{Re}\,\kappa<1$	$(1+at^{-1})^{\frac12\kappa}P^{\kappa}_{\mu-\frac12}(1+2t/a)$
(6)	$p^{-\mu-\frac12}e^{\frac12 ap}W_{\kappa,\mu}(ap)$ $\lvert\arg\lvert a\rvert<\pi$ $\mathrm{Re}\,(\frac12-\kappa+\mu)>0$	$\dfrac{a^{\frac12-\mu}t^{\mu-\kappa-\frac12}(a+t)^{\mu+\kappa-\frac12}}{\Gamma(\frac12-\kappa+\mu)}$
(7)	$p^{\kappa-\frac12}e^{\frac12 p}W_{\kappa,\mu}(p)$ $\mathrm{Re}\,\kappa<\frac14$	$2^{-2\kappa-\frac12}t^{-\kappa-\frac14}(1+t)^{-\frac12}$ $\times P^{2\kappa+\frac12}_{2\mu-\frac12}[(1+t)^{\frac12}]$
(8)	$p^{\kappa-1}e^{\frac12 p}W_{\kappa,\mu}(p)\qquad \mathrm{Re}\,\kappa<\frac34$	$2^{-2\kappa+\frac12}t^{-\kappa+\frac14}P^{2\kappa-\frac12}_{2\mu-\frac12}[(1+t)^{\frac12}]$
(9)	$p^{-\sigma}e^{\frac12 p/a}W_{\kappa,\mu}(p/a)$ $\lvert\arg a\rvert<\pi,\quad \mathrm{Re}\,(\sigma-\kappa)>0$	$a^{-\kappa}[\Gamma(\sigma-\kappa)]^{-1}t^{\sigma-\kappa-1}$ $\times\,{}_2F_1(\frac12-\kappa+\mu,\frac12-\kappa-\mu;\sigma-\kappa;-at)$
(10)	$p^{-2\mu-1}e^{\frac12 ap^2}W_{-3\mu,\mu}(ap^2)$ $\mathrm{Re}\,a>0,\quad \mathrm{Re}\,\mu>-1/8$	$2^{8\mu}a^{-\mu}\dfrac{\Gamma(2\mu+1)}{\Gamma(8\mu+1)}t^{4\mu}$ $\times\exp\left(-\dfrac{t^2}{8a}\right)I_{2\mu}\left(\dfrac{t^2}{8a}\right)$
(11)	$p^{-2\mu-1}e^{\frac12 ap^2}W_{\kappa,\mu}(ap^2)$ $\mathrm{Re}\,a>0,\quad \mathrm{Re}(\kappa-\mu)<\frac12$	$\dfrac{2^{1-\kappa+\mu}a^{\frac12(\mu+\kappa+1)}}{\Gamma(1-2\kappa+2\mu)}t^{\mu-\kappa-1}\exp\left(-\dfrac{t^2}{8a}\right)$ $\times M_{-\frac12(\kappa+3\mu),\frac12(\mu-\kappa)}\left(\dfrac{t^2}{4a}\right)$
(12)	$p^{\kappa}W_{\kappa,\mu}(a/p)\qquad \mathrm{Re}(\kappa\pm\mu)<\frac12$	$\dfrac{2a^{\frac12}t^{-\kappa-\frac12}K_{2\mu}(2a^{\frac12}t^{\frac12})}{\Gamma(\frac12-\kappa+\mu)\Gamma(\frac12-\kappa-\mu)}$

Confluent hypergeometric functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(20)	$p^{1/2}W_{1/4,\nu}(i\,a/p)W_{1/4,\nu}(-i\,a/p)$ $\lvert\mathrm{Re}\,\nu\rvert<1/4$	$-\dfrac{4a(\frac{1}{2}\pi/t)^{1/2}K_{2\nu}[(2at)^{1/2}]}{\Gamma(\frac{1}{4}+\nu)\Gamma(\frac{1}{4}-\nu)}$ $\times\{J_{2\nu}[(2at)^{1/2}]\sin[(\nu-\frac{1}{4})\pi]$ $+Y_{2\nu}[(2at)^{1/2}]\cos[(\nu-\frac{1}{4})\pi]\}$
(21)	$p^{-3/2}W_{\kappa,1/8}(\frac{1}{4}ip^2/a)$ $\times W_{\kappa,1/8}(-\frac{1}{4}ip^2/a)$ $\mathrm{Re}\,\kappa<3/8,\quad a>0$	$(\frac{1}{2}\pi^3t)^{1/2}\dfrac{J_{-\kappa+1/8}(\frac{1}{2}at^2)J_{-\kappa-1/8}(\frac{1}{2}at^2)}{\Gamma(3/8-\kappa)\Gamma(5/8-\kappa)}$
(22)	$\dfrac{\Gamma(\frac{1}{2}-\kappa+\mu+p)}{\Gamma(1+2\mu+p)}a^{-\frac{1}{2}(1+2\mu+p)}$ $\times M_{\kappa-\frac{1}{2}p,\mu+\frac{1}{2}p}(a)$ $\mathrm{Re}(\frac{1}{2}+\kappa+\mu)>0$	$\dfrac{e^{-(\frac{1}{2}+\kappa+\mu)t}}{\Gamma(\frac{1}{2}+\kappa+\mu)}(1-e^{-t})^{\kappa+\mu-\frac{1}{2}}$ $\times\exp[-a(\frac{1}{2}-e^{-t})]$
(23)	$\Gamma(\frac{1}{2}-\kappa-\mu+p)W_{\kappa-p,\mu}(a)$ $\mathrm{Re}\,a>0$	$a^{\frac{1}{2}-\mu}(e^t-1)^{2\mu-1}\exp[-\frac{1}{2}a$ $+(\frac{1}{2}-\kappa-\mu)t-a/(e^t-1)]$
(24)	$\dfrac{\Gamma(\frac{1}{2}+\mu+p)\Gamma(\frac{1}{2}-\mu+p)}{\Gamma(1-\kappa+p)}W_{-p,\mu}(a)$ $\lvert\arg a\rvert<\pi$	$(1-e^{-t})^{-\kappa}\exp\left[-\dfrac{\frac{1}{2}a}{(1-e^{-t})}\right]$ $\times W_{\kappa,\mu}\left[\dfrac{a}{(e^t-1)}\right]$
(25)	$\dfrac{\Gamma(\frac{1}{2}-\kappa-\mu+p)}{\Gamma(1+p)}W_{\kappa-\frac{1}{2}p,\mu-\frac{1}{2}p}(a)$ $\mathrm{Re}\,\mu>-\frac{1}{2}$	$\dfrac{1}{\Gamma(2\mu+1)}(e^t-1)^{\mu-\frac{1}{2}}\exp(-\frac{1}{2}ae^t)$ $\times M_{-\kappa,\mu}[a(e^t-1)]$
(26)	$a^{\mu-\frac{1}{2}+\frac{1}{2}p}W_{\kappa-\frac{1}{2}p,\mu+\frac{1}{2}p}(a)$ $\mathrm{Re}(\mu+\kappa)<\frac{1}{2},\quad \mathrm{Re}\,a>0$	$[\Gamma(\frac{1}{2}-\mu-\kappa)]^{-1}(e^t-1)^{-\frac{1}{2}-\mu-\kappa}$ $\times\exp[-(\frac{1}{2}-\mu+\kappa)t-a(e^t-\frac{1}{2})]$

Confluent hypergeometric functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(27)	$\Gamma(\frac{1}{2}+\mu+p)\,M_{p,\mu}(\alpha)\,W_{-p,\mu}(\beta)$ $\mathrm{Re}\,\alpha>0,\quad \mathrm{Re}\,\beta>0$	$\frac{1}{2}\Gamma(2\mu+1)\,\alpha^{\frac{1}{2}}\beta^{\frac{1}{2}}\,\mathrm{csch}\,(\frac{1}{2}t)$ $\times e^{\frac{1}{2}(\alpha-\beta)\mathrm{ctnh}\,(\frac{1}{2}t)}$ $\times J_{2\mu}[\alpha^{\frac{1}{2}}\beta^{\frac{1}{2}}\,\mathrm{csch}\,(\frac{1}{2}t)]$
(28)	$\Gamma(\frac{1}{2}+\mu+p)\,\Gamma(\frac{1}{2}-\mu+p)$ $\times W_{-p,\mu}(\alpha)\,W_{+p,\mu}(\beta)$ $\mathrm{Re}\,\alpha>0,\quad \mathrm{Re}\,\beta>0$	$\frac{1}{2}\alpha^{\frac{1}{2}}\beta^{\frac{1}{2}}\,\mathrm{csch}\,(\frac{1}{2}t)\,e^{-\frac{1}{2}(\alpha+\beta)\mathrm{ctnh}\,(\frac{1}{2}t)}$ $\times K_{2\mu}[\alpha^{\frac{1}{2}}\beta^{\frac{1}{2}}\,\mathrm{csch}\,(\frac{1}{2}t)]$
(29)	$p^{-\nu}e^{-1/(8p)}\,k_{2n}(2^{-3}p^{-1})$	$\frac{1}{2}(-1)^{n-1}(t^{n-\frac{1}{2}}/n!)\,J_1(t^{\frac{1}{2}})$
(30)	$\Gamma(\nu+1)\,p^{-\nu}e^{-\frac{1}{2}\alpha/p}$ $\times k_{-2\nu}(\frac{1}{2}e^{\pi i}\alpha/p)\quad \mathrm{Re}\,\nu>0$	$-i\,\sin(\nu\pi)\,t^{\nu-\frac{1}{2}}H_1^{(2)}(2\alpha^{\frac{1}{2}}t^{\frac{1}{2}})$
(31)	$\Gamma(\nu+1)\,p^{-\nu}e^{-\frac{1}{2}\alpha/p}$ $\times k_{-2\nu}(\frac{1}{2}e^{-\pi i}\alpha/p)\quad \mathrm{Re}\,\nu>0$	$i\,\sin(\nu\pi)\,t^{\nu-\frac{1}{2}}H_1^{(1)}(2\alpha^{\frac{1}{2}}t^{\frac{1}{2}})$

5.21. Generalized hypergeometric functions

(1)	$\Gamma(\sigma)\,p^{-\sigma}$ $\times {}_mF_n(\alpha_1,\ldots,\alpha_m;\rho_1,\ldots,\rho_n;\lambda/p)$ $m\le n+1,\quad \mathrm{Re}\,\sigma>0$	$t^{\sigma-1}\,{}_mF_{n+1}(\alpha_1,\ldots,\alpha_m;\rho_1,\ldots,\rho_n,\sigma;\lambda t)$
(2)	$\Gamma(2\sigma)\,p^{-2\sigma}$ $\times {}_mF_n(\alpha_1,\ldots,\alpha_m;\rho_1,\ldots,\rho_n;\lambda^2/p^2)$ $m\le n+1)\quad \mathrm{Re}\,\sigma>0$	$t^{2\sigma-1}$ $\times {}_mF_{n+2}(\alpha_1,\ldots,\alpha_m;\rho_1,\ldots,\rho_n,\sigma,\sigma+\frac{1}{2};$ $\frac{1}{4}\lambda^2t^2)$
(3)	$\Gamma(k\sigma)\,p^{-k\sigma}$ $\times {}_mF_n(\alpha_1,\ldots,\alpha_m;\rho_1,\ldots,\rho_n;\lambda^k/p^k)$ $m\le n+1,\quad \mathrm{Re}\,\sigma>0$	$t^{k\sigma-1}\,{}_mF_{n+k}(\alpha_1,\ldots,\alpha_m;$ $\rho_1,\ldots,\rho_n,\sigma,\sigma+1/k,\ldots,\sigma+k-1/k;$ $\lambda^kt^k/k^k)$

Generalized hypergeometric functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(4)	$p^{-1/2}$ $\times {}_mF_n(a_1,\ldots,a_m;\rho_1,\ldots,\rho_n;-\lambda p^{1/2})$ $m \le n$	$\pi^{-1/2}t^{-1/2}\ {}_{2m}F_{2n}(\tfrac12 a_1,\tfrac12 a_1+\tfrac12,\ldots,\tfrac12 a_m,$ $\tfrac12 a_m+\tfrac12;\tfrac12\rho_1,\tfrac12\rho_1+\tfrac12,\ldots,\tfrac12\rho_n,$ $\tfrac12\rho_n+\tfrac12;-2^{m-n-2}\lambda^2/t)$
(5)	$\mathrm{B}(p,\lambda)$ $\times {}_3F_2(\alpha,\beta,p;\gamma,p+\lambda;z)$ $\mathrm{Re}\,\lambda>0,\quad \lvert\arg(z-1)\rvert<\pi$	$(1-e^{-t})^{\lambda-1}F(\alpha,\beta;\gamma;ze^{-t})$
(6)	$\mathrm{B}(p,\lambda)$ $\times {}_3F_2(\alpha,\beta,\lambda;\gamma,p+\lambda;z)$ $\mathrm{Re}\,\lambda>0,\quad \lvert\arg(z-1)\rvert<\pi$	$(1-e^{-t})^{\lambda-1}F(\alpha,\beta;\gamma;z(1-e^{-t})]$
(7)	$2^{2p+\alpha}\mathrm{B}(p,p+\alpha)$ $\times {}_4F_3(-n,n+1,p+\alpha;1,2p+\alpha;1)$	$\theta^{-1}[(1-\theta)^{\alpha}+(-1)^n(1+\theta)^{\alpha}]\,P_n(\theta)$ $\theta=(1-e^{-t})^{1/2}$
(8)	$\mathrm{B}(p,\sigma)$ $\times {}_{m+1}F_{n+1}(a_1,\ldots,a_m,p;$ $\rho_1,\ldots,\rho_n,p+\sigma;z)$ $\mathrm{Re}\,\sigma>0,\quad m\le n+1$ $\lvert z\rvert<1$ if $m=n+1$	$(1-e^{-t})^{\sigma-1}$ $\times {}_mF_n(a_1,\ldots,a_m;\rho_1,\ldots,\rho_n;ze^{-t})$
(9)	$\mathrm{B}(p,\sigma)$ $\times {}_{m+1}F_{n+1}(a_1,\ldots,a_m,\sigma;$ $\rho_1,\ldots,\rho_n,p+\sigma;z)$ $\mathrm{Re}\,\sigma>0,\quad m\le n+1$ $\lvert z\rvert<1$ if $m=n+1$	$(1-e^{-t})^{\sigma-1}$ $\times {}_mF_n(a_1,\ldots,a_m;\rho_1,\ldots,\rho_n;z(1-e^{-t})]$
(10)	$p^{-\lambda-\mu}$ $\times E(-\nu,\nu+1,\lambda+\mu:\mu+1:2p)$ $\mathrm{Re}(\lambda+\mu)>0$	$-\pi\csc(\nu\pi)\,t^{\lambda+\frac12\mu-1}(t+2)^{\frac12\mu}$ $\times P_\nu^{-\mu}(t+1)$

Generalized hypergeometric functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(11)	$p^{-\lambda}E(\mu+\nu+1,\mu-\nu,\lambda:\mu+1:2p)$ $\operatorname{Re}\lambda>0$	$2^{\mu}\Gamma(\mu+\nu+1)\Gamma(\mu-\nu)t^{\lambda-\frac12\mu-1}$ $\times(t+2)^{-\frac12\mu}P_\nu^{-\mu}(t+1)$
(12)	$p^{-\gamma}E(\alpha,\beta,\gamma:\delta:p)$ $\operatorname{Re}\gamma>0$	$\dfrac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\delta)}t^{\gamma-1}F(\alpha,\beta;\delta;-t)$
(13)	$p^{-\alpha_m}E(m;\alpha_r:n;\beta_s:p)$ $\operatorname{Re}\alpha_m>0$	$t^{\alpha_m-1}E(m-1;\alpha_r:n;\beta_s:1/t)$
(14)	$\Gamma(p-\alpha_m)$ $\times E(m;\alpha_r:n+1;\beta_1,\dots,\beta_n,p:z)$ $\operatorname{Re}\alpha_m>0$	$(e^t-1)^{\alpha_m}$ $\times E[m-1;\alpha_1,\dots,\alpha_{m-1}:n;$ $\beta_1,\dots,\beta_n:z/(1-e^{-t})$

5.22. Elliptic functions and theta functions

(1)	$p^{-1}K(a/p)$	$\tfrac12\pi\, I_0^2(\tfrac12 at)$
(2)	$K(a/p)-\tfrac12\pi$	$\tfrac12\pi a\, I_0(\tfrac12\pi t)\, I_1(\tfrac12 at)$
(3)	$pK(a/p)-\tfrac12\pi p$	$\tfrac14\pi a^2\{\tfrac12[I_0(\tfrac12 at)]^2+[I_1(\tfrac12 at)]^2$ $+\tfrac12 I_0[\tfrac12 at)\, I_2(\tfrac12 at)\}$
(4)	$\tfrac12\pi p-pE(a/p)$	$\tfrac12\pi a t^{-1}\, I_0(\tfrac12 at)\, I_1(\tfrac12 at)$
(5)	$p[K(a/p)-E(a/p)]$	$\tfrac14\pi a^2[I_0^2(\tfrac12 at)+I_1^2(\tfrac12 at)]$
(6)	$p(p^2-a^2)^{-1}E(a/p)$	$\tfrac12\pi\, I_0(\tfrac12 at)[I_0(\tfrac12 at)$ $+at\, I_1(\tfrac12 at)]$

Elliptic functions and theta functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(7)	$r^{-1} E(a/r)$	$\tfrac{1}{2}\pi J_0(\tfrac{1}{2}at)[J_0(\tfrac{1}{2}at) - at\, J_1(\tfrac{1}{2}at)]$
(8)	$r^{-1} K(a/r)$	$\tfrac{1}{2}\pi J_0^2(\tfrac{1}{2}at)$
(9)	$(1-\tfrac{1}{2}a^2 r^{-2})K(a/r) - E(a/r)$	$\tfrac{1}{4}\pi a^2 J_1^2(\tfrac{1}{2}at)$
(10)	$r^{-1}[K(a/r) - E(a/r)]$	$\tfrac{1}{2}\pi a t\, J_0(\tfrac{1}{2}at)\, J_1(\tfrac{1}{2}at)$
(11)	$p^{-\frac{1}{2}}\, \theta_1(a, i\pi p)$	$\pi^{-\frac{1}{2}} \sum_{n=-\infty}^{\infty} (-1)^n J_0[2(a+n-\tfrac{1}{2})t^{\frac{1}{2}}]$
(12)	$p^{-\frac{1}{2}}\, \theta_2(a, i\pi p)$	$\pi^{-\frac{1}{2}} \sum_{n=-\infty}^{\infty} (-1)^n J_0[2(a+n)t^{\frac{1}{2}}]$
(13)	$p^{-\frac{1}{2}}\, \theta_3(a, i\pi p)$	$\pi^{-\frac{1}{2}} \sum_{n=-\infty}^{\infty} J_0[2(a+n)t^{\frac{1}{2}}]$
(14)	$p^{-\frac{1}{2}}\, \theta_4(a, i\pi p)$	$\pi^{-\frac{1}{2}} \sum_{n=-\infty}^{\infty} J_0[2(a+n+\tfrac{1}{2})t^{\frac{1}{2}}]$
(15)	$p^{-1}\, \theta_1(a, i\pi p)$ $\quad \lvert \mathrm{Re}\, a \rvert < \tfrac{1}{2}$	$\pi^{-1} \sum_{n=-\infty}^{\infty} \dfrac{(-1)^n \sin[2(a+n-\tfrac{1}{2})t^{\frac{1}{2}}]}{a+n-\tfrac{1}{2}}$
(16)	$p^{-1}\, \theta_2(a, i\pi p)$ $\quad 0 < \mathrm{Re}\, a < 1$	$\pi^{-1} \sum_{n=-\infty}^{\infty} \dfrac{(-1)^n \sin[2(a+n)t^{\frac{1}{2}}]}{a+n}$
(17)	$p^{-1}\, \theta_3(a, i\pi p)$ $\quad 0 < \mathrm{Re}\, a < 1$	$\pi^{-1} \sum_{n=-\infty}^{\infty} \dfrac{\sin[2(a+n)t^{\frac{1}{2}}]}{a+n}$

$r = (p^2 + a^2)^{\frac{1}{2}}$

Elliptic functions and theta functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(18)	$p^{-1}\,\theta_4(a, i\pi p)$ $\quad |\mathrm{Re}\, a| < \tfrac{1}{2}$	$\pi^{-1} \sum_{n=-\infty}^{\infty} \dfrac{\sin[2(a+n+\frac{1}{2})t^{1/2}]}{a+n+\frac{1}{2}}$
(19)	$p^{-1}\,\theta_2(0, i\pi p)$	$0 \qquad 0 < t < \tfrac{1}{4}\pi^2$ $2n+2 \qquad \pi^2(n+1/2)^2 < t < \pi^2(n+3/2)^2$
(20)	$p^{-1}\,\theta_3(0, i\pi p)$	$2n+1 \qquad \pi^2 n^2 < t < \pi^2(n+1)^2$
(21)	$p^{-1}\,\theta_4(0, i\pi p)$	$1 \qquad (2k)^2\pi^2 < t < (2k+1)^2\pi^2$ $-1 \qquad (2k+1)^2\pi^2 < t < (2k+2)^2\pi^2$
(22)	$p^{-\nu}\,\theta_1(a, i\pi p)$ $\mathrm{Re}\,\nu \geq \tfrac{1}{2}, \quad |\mathrm{Re}\, a| < \tfrac{1}{2}$	$\pi^{-1/2} t^{\frac{1}{2}\nu-\frac{1}{4}}$ $\times \sum_{n=-\infty}^{\infty} \dfrac{(-1)^n J_{\nu-1/2}[2(a+n-\frac{1}{2})t^{1/2}]}{(a+n-\frac{1}{2})^{\nu-1/2}}$
(23)	$p^{-\nu}\,\theta_2(a, i\pi p)$ $\mathrm{Re}\,\nu \geq \tfrac{1}{2}, \quad 0 < \mathrm{Re}\, a < 1$	$\pi^{-1/2} t^{\frac{1}{2}\nu-\frac{1}{4}} \sum_{n=-\infty}^{\infty} \dfrac{(-1)^n J_{\nu-1/2}[2(a+n)t^{1/2}]}{(a+n)^{\nu-1/2}}$
(24)	$p^{-\nu}\,\theta_3(a, i\pi p)$ $\mathrm{Re}\,\nu \geq \tfrac{1}{2}, \quad 0 < \mathrm{Re}\, a < 1$	$\pi^{-1/2} t^{\frac{1}{2}\nu-\frac{1}{4}} \sum_{n=-\infty}^{\infty} \dfrac{J_{\nu-1/2}[2(a+n)t^{1/2}]}{(a+n)^{\nu-1/2}}$
(25)	$p^{-\nu}\,\theta_4(a, i\pi p)$ $\mathrm{Re}\,\nu > \tfrac{1}{2}, \quad |\mathrm{Re}\, a| < \tfrac{1}{2}$	$\pi^{-1/2} t^{\frac{1}{2}\nu-\frac{1}{4}} \sum_{n=-\infty}^{\infty} \dfrac{J_{\nu-1/2}[2(a+n+\frac{1}{2})t^{1/2}]}{(a+n+\frac{1}{2})^{\nu-1/2}}$