Models of dynamic connectome properties

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Log-normal distributions of synaptic weights in noise driven networks

In cortical circuits the distribution of synaptic weights has been repeatedly reported to be log-normal [1]. A number of computational models address how such a weight distribution might arise from network dynamics. Here we test a hypothesis that log-normal weight distributions emerge from network dynamics in which LTP dominates and in which synaptic scaling takes the role of keeping weights from growing too large.

Network model

The network considered here consists of N randomly connected ($p = 0.1$) conductance based leaky integrate-and-fire neurons. The differential equation for a neuron's membrane voltage V is

$$\tau_m \frac{dV}{dt} = E_l - V + g_e (E_e - V) + \xi_{\text{ext}}(t). \tag{1}$$

When synaptic transmission is active ($\Theta_{\text{trans}} = 1$), the conductance g_e is increased by w_i at times t_i^k of spikes from connected neurons with index i,

$$\frac{dg_e}{dt} = -\frac{g_e}{\tau_e} + \Theta_{\text{trans}} \sum_i w_i \sum_k \delta(t - t_i^k) \tag{2}$$

Spike-timing dependent plasticity is implemented in the form of long-term potentiation (LTP). The weight w changes for $\Delta t = t_{\text{post}} - t_{\text{pre}} > 0$ as

$$\Delta w = A_{\text{LTP}} \exp\left(\frac{\Delta t}{\tau_{\text{LTP}}}\right). \tag{3}$$

Multiplicative synaptic scaling acts as a homeostatic mechanism and prevents weights from unbounded growth. The incoming weights w_j to a given neuron are scaled as

$$w_j \to w_j \left(1 + \eta_{\text{SN}} \left(\frac{W_{\text{target}}}{\sum_k w_k} - 1\right)\right), \tag{4}$$

where $\sum_k w_k$ sums over all incoming weights and W_{target} is a set target for the sum.

Results

At first we disabled synaptic transmission $\Theta_{\text{trans}} = 0$ resulting in network dynamics that are exclusively driven by the membrane noise $\xi_{\text{ext}}(t)$. We found that log-normal like distributions in synaptic weights naturally emerge from the interplay of additive changes through LTP and the scaling of weights through synaptic normalization, when roughly

$$A_{\text{LTP}} = \frac{W_{\text{target}}}{pN}, \tag{5}$$

that is when the sum of A_{LTP} for all incoming connections roughly matches the target weight (Fig. 1).

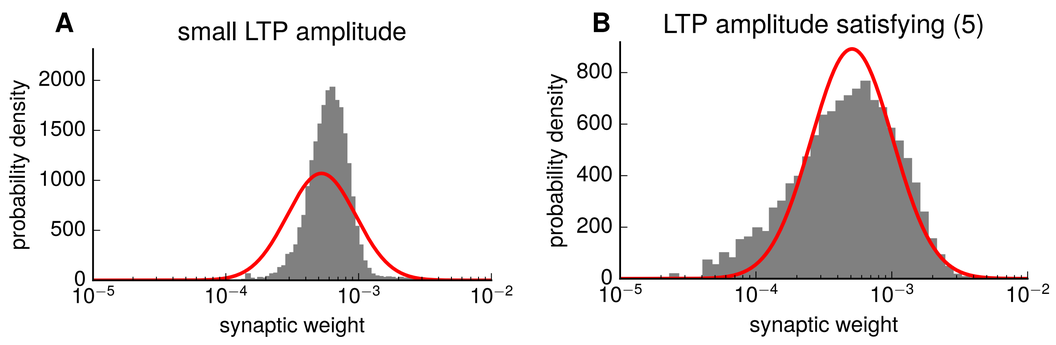

Figure 1: In a completely noise-driven network ($N = 400$, $p = 0.1$, $\Theta_{\text{trans}} = 0$), log-normal like distribution emerge when the relation (5) roughly holds. Shown is synaptic weight distribution after simulation of $T = 200$ s (grey) and log-normal fit to data (red).

We tested relation (5) by, for example, increasing p. In this case, to obtain good fits for a log-normal distribution in synaptic weights, A_{LTP} had to be decreased in accordance with the equation. We found that the rate of synaptic scaling η_{SN} did not significantly affect the shape of the weight distribution.

In the next step, we analyzed a network with an excitatory an inhibitory population, supplementing equations (1)-(4) in the typical way and additionally adding long-term depression (LTD) for $\Delta t < 0$. First results here indicate that when LTP dominates over LTD ($A_{\text{LTP}} = 10\,A_{\text{LTD}}$) a reduction of membrane noise lets log-normal like distribution in synaptic weights emerge more clearly (Fig. 2). Correlated activity might thus play a crucial in forming the long tailed distributions of weights observed in cortical networks.

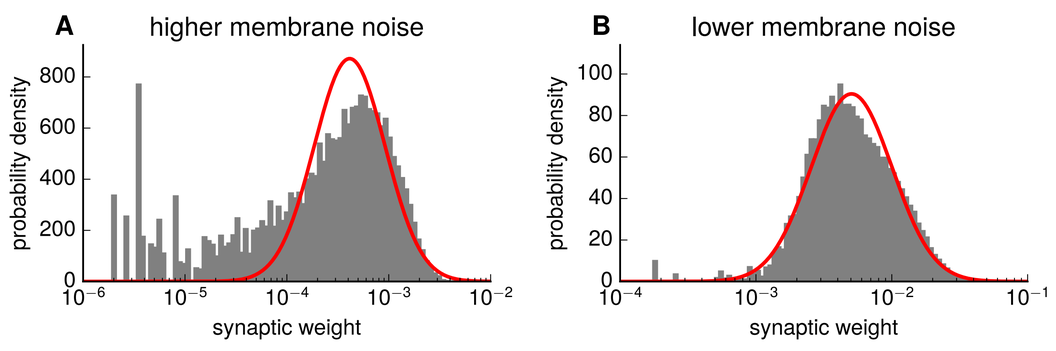

Figure 2: In a network with recurrent dynamics, reduction in membrane noise favours log-normal like weight distributions

Modelling synaptic lifetime distributions with Kesten processes

In an experimental study by Loewenstein, Yanover, and Rumpel [2] chronic in-vivo two-photon imaging suggested that the lifetime dynamics of spines in the neocortex follow a power law. Motivated by results from detailed network simulations, we here consider a simple stochastic model based on the Kesten process in order to analyze how different properties of a cortical network might affect the lifetime distributions of synaptic spines.

Model

Kesten processes have been used before [3] to describe spine size dynamics. In this model, a given spine size X_n at time step n is updated as

$$X_{n+1} = a_n X_n + b_n. \tag{6}$$

In this further simplified form we take a_n to be a fixed value of $a_n < 1$, while the additive change is in each time step drawn from a normal distribution, $b_n \sim \mathcal{N}(\mu_b, \sigma_b^2)$. Then, to model synapse growth and pruning processes, we consider a population of N synapses. Each synapse has a random time T_{init}, uniformly distributed in $[0, T_{\text{max}}]$, at which it is initialized with size X_0. The synapse size X_t then evolves according to (6).

The lifetime of the synapse is the number of time steps from T_{init} until for the first time $X_t < X_{\text{prune}}$. In this case the synapse gets pruned and a new synapse with size X_0 is inserted in the network (Fig. 3). In the case that X_t doesn't fall below X_{prune} until T_{max}, the lifetime is recorded as $T = T_{\text{max}} - T_{\text{init}}$. At the end of the simulation the sizes $X_{T_{\text{max}}}$ are recorded for all N synapses.

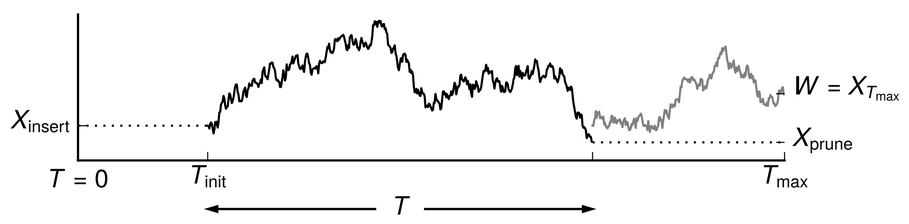

Figure 3: Adapted Kesten simulation model allows the tracking of lifetimes and size distributions

Results

We simulated 5×10^5 synapses evolving as Kesten processes and recorded lifetime and weight distributions. For unbiased additive change ($\mu_b = 0$), a power law like distribution of synaptic lifetimes emerges (Fig. 4A). The distribution of spine sizes $X_{T_{\text{max}}}$ resembles a log-normal distribution, as one might expect from findings on the synaptic weight distributions in cortical circuits [1].

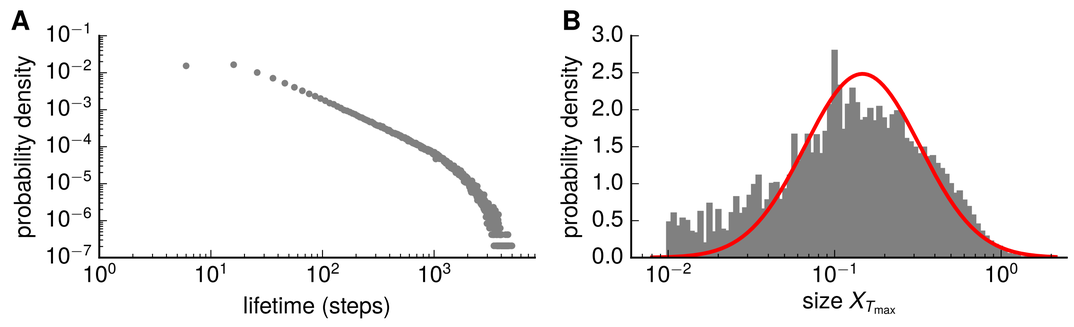

Figure 4: Dynamical properties of network connectivity modelled by Kesten processes. **A** Lifetime distribution of synapses created at time step T_{init} uniformly distributed in $[0, T_{\text{max}}]$. **B** Distribution of spine sizes $X_{T_{\text{max}}}$ at time step $X_{T_{\text{max}}}$ (grey) and log-normal fit (red). Parameters for both: $a = 0.9987$, $\mu_b = 0$, $\sigma_b^2 = 0.22$, $X_{\text{insert}} = 0.1$, $X_{\text{prune}} = 0.01$.

Next, we explored how different parameters in the model affect the lifetime and spine size distributions. We found that varying σ_b^2 has little effect on the distributions. Interestingly however, the bias in the additive change affects both distributions significantly. As one might expect, a bias towards increases in size moves the tail of the lifetime distribution towards higher lifetimes (Fig. 5A) while shifting the mean of the spine size towards higher values (Fig. 5B). This observation matches qualitatively with preliminary results from detailed network simulations in which a higher bias towards LTP resulted in similarly extended lifetimes.

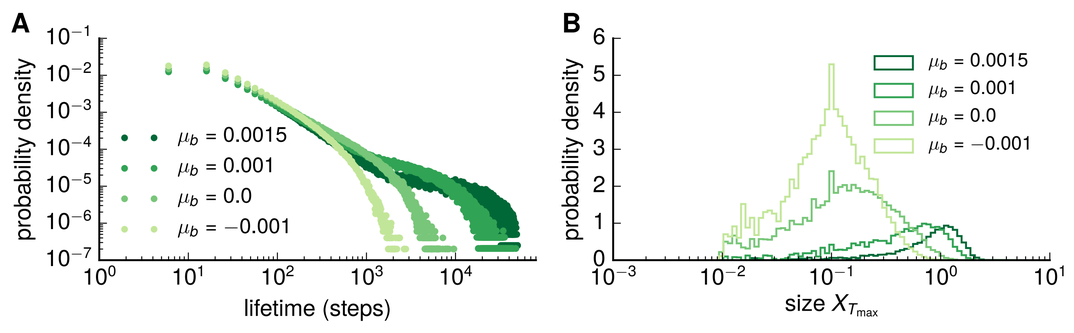

Figure 5: The bias of additive change in size strongly affects both lifetime and weight distributions.

References

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