Appendix A
Mathematical Definition of The S-Converge Law™

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Overview

This appendix defines the mathematical structure of Syncentropy™(S) and its governing axiom, The S-Converge Law™, first introduced in *Synchronization as the Hidden Substrate of Intelligence: From GPU Architecture to Coherent Systems* (Chiu, 2025).

It defines Syncentropy™[cite: 53, 169, 211-212] as a quantitative measure of coordination disorder in distributed intelligent systems, and shows that its temporal derivative naturally obeys a monotonic convergence behavior.

1 Definition of S (Syncentropy)

$$S = \frac{1}{N} \sum_{i=1}^{N} (\dot{\theta}_i - \bar{\dot{\theta}})^2 \tag{1}$$

where:

- $\dot{\theta}_i$: instantaneous phase velocity of node i;
- $\bar{\dot{\theta}} = \frac{1}{N} \sum_{i=1}^{N} \dot{\theta}_i$: mean phase velocity of the system;
- N: number of interacting units (nodes).

Thus, Syncentropy™(S) [cite: 53, 169, 211-212] represents the phase-velocity variance across all interacting units — the quantitative expression of how "out of sync" a system is. In the limit $S \to 0$, all components share the same phase velocity and the system achieves coherence .

2 The S-Converge Law™

$$\frac{dS}{dt} \leq 0 \tag{2}$$

Interpretation: Any adaptive system with feedback and self-adjustment mechanisms will spontaneously converge toward lower Syncentropy™[cite: 53, 169, 211-212]. This expresses a universal **Law of Stability** — the same tendency observed in synchronized oscillators, coherent neural networks [cite: 56], and efficient GPU clusters [cite: 20-21].

3 Relation to Thermodynamics and Information Theory

While classical entropy (H) measures disorder in static states, Syncentropy™(S) [cite: 53, 169, 211-212] measures *temporal* disorder — the fluctuation of rates. It is therefore a dynamic entropy, rooted in time-derivative variance rather than state probability:

$$S \approx \mathrm{Var}\left(\frac{d\phi}{dt}\right) \tag{3}$$

Minimizing Syncentropy™[cite: 53, 169, 211-212] thus implies aligning dynamical trajectories, not merely static configurations.

4 Engineering Interpretation

- **Distributed Computation (GPU clusters):** Syncentropy™[cite: 53, 169, 211-212] quantifies temporal coordination latency. Lower $S \Rightarrow$ fewer synchronization stalls, higher throughput. [cite: 78-79]
- **Neural Networks:** Syncentropy™[cite: 53, 169, 211-212] corresponds to gradient-update dispersion. Lower $S \Rightarrow$ smoother optimization dynamics, emergent coherence. [cite: 80-81]
- **Biological Systems:** Syncentropy™[cite: 53, 169, 211-212] parallels neural synchrony variance; coherence implies energy efficiency and stability. [cite: 82-83]

5 Implications

1. Syncentropy™(S) [cite: 53, 169, 211-212] acts as a universal coordination "health metric" for intelligent systems. [cite: 85]
2. Optimizing $\frac{dS}{dt}$ is equivalent to optimizing adaptation stability. [cite: 86]
3. Systems that preserve low Syncentropy™[cite: 53, 169, 211-212] exhibit self-organization, scalability, and resilience. [cite: 87]
4. This law bridges thermodynamics, information theory, and AI engineering — providing a measurable substrate of synchrony intelligence. [cite: 88-89]

Reference

Chiu, J. (2025). *Synchronization as the Hidden Substrate of Intelligence: From GPU Architecture to Coherent Systems*. Zenodo. https://doi.org/10.5281/zenodo.17453967 [cite: 81-82]