

$$\liminf A_n = \{x \in X : N \setminus \mathcal{J}_x \text{ is finite}\};$$