

Ejercicios de Integrales

Cálculo

<http://synergy.vision/>

Contenido

Use la Parte 1 del Teorema Fundamental del Cálculo para encontrar la derivada de la función.

$$1. g(x) = \int_1^x \frac{1}{t^3 + 1} dt$$

$$2. g(x) = \int_1^x (2 + t^4)^5 dt$$

$$3. g(s) = \int_5^s (t - t^2)^8 dt$$

$$4. g(r) = \int_0^r \sqrt{x^2 + 4} dx$$

$$5. F(x) = \int_x^\pi \sqrt{1 + \sec t} dt$$

$$\left[\int_x^\pi \sqrt{1 + \sec t} dt = - \int_\pi^x \sqrt{1 + \sec t} dt \right]$$

$$6. G(x) = \int_1^x \cos \sqrt{t} dt$$

$$7. h(x) = \int_2^{1/x} \sin^4 t dt$$

$$8. h(x) = \int_1^{\sqrt{x}} \frac{z^2}{z^4 + 1} dz$$

$$9. y = \int_0^{\tan x} \sqrt{t + \sqrt{t}} dt$$

$$10. y = \int_0^{x^4} \cos^2 \theta d\theta$$

$$11. y = \int_{1-3x}^1 \frac{u^3}{1 + u^2} du$$

$$12. y = \int_{\sin x}^1 \sqrt{1 + t^2} dt$$

Evaluar la integral.

$$13. \int_{-1}^2 (x^3 - 2x) dx$$

$$14. \int_{-1}^1 x^{100} dx$$

$$15. \int_1^4 (5 - 2t + 3t^2) dt$$

$$16. \int_0^1 \left(1 + \frac{1}{2}u^4 - \frac{2}{5}u^9 \right) du$$

$$17. \int_1^9 \sqrt{x} dx$$

$$18. \int_1^8 x^{-2/3} dx$$

$$19. \int_{\pi/6}^{\pi} \sin \theta d\theta$$

$$20. \int_{-5}^5 \pi dx$$

$$21. \int_0^1 (u+2)(u-3) du$$

$$22. \int_0^4 (4-t)\sqrt{t} dt$$

$$23. \int_1^9 \frac{x-1}{\sqrt{x}} dx$$

$$24. \int_0^2 (y-1)(2y+1) dy$$

$$25. \int_0^{\pi/4} \sec^2 t dt$$

$$26. \int_0^{\pi/4} \sec \theta \tan \theta d\theta$$

$$27. \int_1^2 (1+2y)^2 dy$$

$$28. \int_1^2 \frac{s^4+1}{s^2} ds$$

$$29. \int_1^2 \frac{v^5+3v^6}{v^4} dv$$

$$30. \int_1^{18} \sqrt{\frac{3}{z}} dz$$

$$31. \int_0^{\pi} f(x) dx \quad \text{donde} \quad f(x) = \begin{cases} \sin x & \text{si } 0 \leq x < \pi/2 \\ \cos x & \text{si } \pi/2 \leq x \leq \pi \end{cases}$$

$$32. \int_{-2}^2 f(x) dx \quad \text{donde} \quad f(x) = \begin{cases} 2 & \text{si } -2 \leq x \leq 0 \\ 4-x^2 & \text{si } 0 < x \leq 2 \end{cases}$$

¿Qué está mal con la ecuación?

$$33. \int_{-2}^1 x^{-4} dx = \frac{x^{-3}}{-3} \Big|_{-2}^1 = -\frac{3}{8}$$

$$34. \int_{-1}^2 \frac{4}{x^3} dx = -\frac{2}{x^2} \Big|_{-1}^2 = \frac{3}{2}$$

$$35. \int_{\pi/3}^{\pi} \sec \theta \tan \theta d\theta = \sec \theta \Big|_{\pi/3}^{\pi} = -3$$

$$36. \int_0^{\pi} \sec^2 x dx = \tan x \Big|_0^{\pi} = 0$$

Use un gráfico para dar una estimación aproximada del área de la región que se encuentra debajo de la curva dada. Luego encuentra el área exacta.

$$37. y = \sqrt[3]{x}, \quad 0 \leq x \leq 27$$

$$38. y = x^{-4}, \quad 1 \leq x \leq 6$$

$$39. y = \sin x, \quad 0 \leq x \leq \pi$$

$$40. y = \sec^2 x, \quad 0 \leq x \leq \pi/3$$

Evaluar la integral.

$$41. \int_1^9 \frac{1}{2x} dx$$

$$42. \int_0^1 10^x dx$$

$$43. \int_{1/2}^{\sqrt{3/2}} \frac{6}{\sqrt{1-t^2}} dt$$

$$44. \int_0^1 \frac{4}{t^2 + 1} dt$$

$$45. \int_{-1}^1 e^{u+1} du$$

$$46. \int_1^2 \frac{4+u^2}{u^3} du$$

Verificar por diferenciación que la fórmula es correcta.

$$1. \int \frac{1}{x^2 \sqrt{1+x^2}} dx = -\frac{\sqrt{1+x^2}}{x} + C$$

$$2. \int \cos^2 x dx = \frac{1}{2}x + \frac{1}{4} \sin 2x + C$$

$$3. \int \cos^3 x dx = \sin x - \frac{1}{3} \sin^3 x + C$$

$$4. \int \frac{x}{\sqrt{a+bx}} dx = \frac{2}{3b^2} (bx - 2a) \sqrt{a+bx} + C$$

Encuentra la integral general indefinida.

$$5. \int (x^2 + x^{-2}) dx$$

$$6. \int (\sqrt{x^3} + \sqrt[3]{x^2}) dx$$

$$7. \int (x^4 - \frac{1}{2}x^3 + \frac{1}{4}x - 2) dx$$

$$8. \int (y^3 + 1.8y^2 - 2.4y) dy$$

$$9. \int (u + 4)(2u + 1) du$$

$$10. \int v(v^2 + 2)^2 dv$$

$$11. \int \frac{x^3 - 2\sqrt{x}}{x} dx$$

$$12. \int \left(u^2 + 1 + \frac{1}{u^2} \right) du$$

$$13. \int (\theta - \csc \theta \cot \theta) d\theta$$

$$14. \int \sec t (\sec t + \tan t) dt$$

$$15. \int (1 + \tan^2 \alpha) d\alpha$$

$$16. \int \frac{\sin 2x}{\sin x} dx$$

Evaluar la integral.

$$17. \int_{-2}^3 (x^2 - 3) dx$$

$$18. \int_1^2 (4x^3 - 3x^2 + 2x) dx$$

$$19. \int_{-2}^0 \left(\frac{1}{2}t^4 + \frac{1}{4}t^3 - t \right) dt$$

$$20. \int_0^3 (1 + 6w^2 - 10w^4) dw$$

$$21. \int_0^2 (2x - 3)(4x^2 + 1) dx$$

$$22. \int_{-1}^1 t(1 - t)^2 dt$$

$$23. \int_0^\pi (4 \sin \theta - 3 \cos \theta) d\theta$$

$$24. \int_1^2 \left(\frac{1}{x^2} - \frac{4}{x^3} \right) dx$$

$$25. \int_1^4 \left(\frac{4 + 6u}{\sqrt{u}} \right) du$$

$$26. \int_1^2 \left(x + \frac{1}{x} \right)^2 dx$$

$$27. \int_1^2 \sqrt{\frac{5}{x}} dx$$

$$28. \int_1^9 \frac{3x - 2}{\sqrt{x}} dx$$

$$29. \int_1^4 \sqrt{t}(1 + t) dt$$

$$30. \int_{\pi/4}^{\pi/3} \csc^2 \theta d\theta$$

$$31. \int_0^{\pi/4} \frac{1 + \cos^2 \theta}{\cos^2 \theta} d\theta$$

$$32. \int_0^{\pi/3} \frac{\sin \theta + \sin \theta \tan^2 \theta}{\sec^2 \theta} d\theta$$

$$33. \int_1^{64} \frac{1 + \sqrt[3]{x}}{\sqrt{x}} dx$$

$$34. \int_1^8 \frac{x - 1}{\sqrt[3]{x^2}} dx$$

$$35. \int_0^1 (\sqrt[4]{x^5} + \sqrt[5]{x^4}) dx$$

$$36. \int_0^1 (1 + x^2)^3 dx$$

$$37. \int_2^5 |x - 3| dx$$

$$38. \int_0^2 |2x - 1| dx$$

$$39. \int_{-1}^2 (x - 2|x|) dx$$

$$40. \int_0^{3\pi/2} |\sin x| dx$$

$$41. \int (\sin x + \sin hx) dx$$

$$42. \int_{-10}^{10} \frac{2e^x}{\sinh x + \cosh x} dx$$

$$43. \int \left(x^2 + 1 + \frac{1}{x^2 + 1} \right) dx$$

$$44. \int_1^2 \frac{(x-1)^3}{x^2} dx$$

$$45. \int_0^{1/\sqrt{3}} \frac{t^2 - 1}{t^4 - 1} dt$$

Evalúa la integral haciendo la sustitución dada.

$$1. \int \sin \pi x dx, \quad u = \pi x$$

$$2. \int x^3 (2 + x^4)^5 dx, \quad u = 2 + x^4$$

$$3. \int x^2 \sqrt{x^3 + 1} dx, \quad u = x^3 + 1$$

$$4. \int \frac{dt}{(1 - 6t)^4}, \quad u = 1 - 6t$$

$$5. \int \cos^3 \theta \sin \theta d\theta, \quad u = \cos \theta$$

$$6. \int \frac{\sec^2(1/x)}{x^2} dx, \quad u = 1/x$$

Evaluar la integral indefinida.

$$7. \int x \sin(x^2) dx$$

$$8. \int x^2 \cos(x^3) dx$$

9. $\int (1 - 2x)^9 dx$

10. $\int (3t + 2)^{2.4} dt$

11. $\int (x + 1)\sqrt{2x + x^2} dx$

12. $\int \sec^2 2\theta d\theta$

13. $\int \sec 3t \tan 3t dt$

14. $\int u\sqrt{1 - u^2} du$

15. $\int \frac{a + bx^2}{\sqrt{3ax + bx^3}} dx$

16. $\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$

17. $\int \sec^2 \theta \tan^3 \theta d\theta$

18. $\int \cos^4 \theta \sin \theta d\theta$

19. $\int (x^2 + 1)(x^3 + 3x)^4 dx$

20. $\int \sqrt{x} \sin(1 + x^{3/2}) dx$

21. $\int \frac{\cos x}{\sin^2 x} dx$

22. $\int \frac{\cos(\pi/x)}{x^2} dx$

23. $\int \frac{z^2}{\sqrt[3]{1 + z^3}} dz$

24. $\int \frac{dt}{\cos^2 t \sqrt{1 + \tan t}}$

25. $\int \sqrt{\cot x} \csc^2 x dx$

26. $\int \sin t \sec^2(\cos t) dt$

27. $\int \sec^3 x \tan x dx$

$$28. \int x^2 \sqrt{2+x} dx$$

$$29. \int x(2x+5)^8 dx$$

$$30. \int x^3 \sqrt{x^2+1} dx$$

Evalúa la integral indefinida. Ilustre y compruebe que su respuesta es razonable graficando tanto la función como su antiderivada (tomar $C = 0$).

$$31. \int x(x^2-1)^3 dx$$

$$32. \int \tan^2 \theta \sec^2 \theta d\theta$$

$$33. \int \sin^3 x \cos x dx$$

$$34. \int \sin x \cos^4 x dx$$

Evaluar la integral definida.

$$35. \int_0^1 \cos(\pi t/2) dt$$

$$36. \int_0^1 (3t-1)^{50} dt$$

$$37. \int_0^1 \sqrt[3]{1+7x} dx$$

$$38. \int_0^{\sqrt{\pi}} x \cos(x^2) dx$$

$$39. \int_0^{\pi} \sec^2(t/4) dt$$

$$40. \int_{1/6}^{1/2} \csc \pi t \cot \pi t dt$$

$$41. \int_{-\pi/4}^{\pi/4} (x^3 + x^4 \tan x) dx$$

$$42. \int_0^{\pi/2} \cos x \sin(\sin x) dx$$

$$43. \int_0^{13} \frac{dx}{\sqrt[3]{(1+2x)^2}}$$

$$44. \int_0^a x \sqrt{a^2 - x^2} dx$$

$$45. \int_0^a x\sqrt{x^2 + a^2}dx \quad (a > 0)$$

$$46. \int_{-\pi/3}^{\pi/3} x^4 \sin x dx$$

$$47. \int_1^2 x\sqrt{x-1}dx$$

$$48. \int_0^4 \frac{x}{\sqrt{1+2x}}dx$$

$$49. \int_{1/2}^1 \frac{\cos(x^{-2})}{x^3}dx$$

$$50. \int_0^{T/2} \sin(2\pi t/T - \alpha)dt$$

$$51. \int_0^1 \frac{dx}{(1 + \sqrt{x})^4}$$

Evaluar la integral.

$$52. \int \frac{dx}{5 - 3x}$$

$$53. \int e^x \sin(e^x)dx$$

$$54. \int \frac{(\ln x)^2}{x}dx$$

$$55. \int \frac{dx}{ax + b} (a \neq 0)$$

$$56. \int e^{\tan x} \sec^2 x dx$$

$$57. \int e^{\cos t} \sin t dt$$

$$58. \int e^{\tan x} \sec^2 x dx$$

$$59. \int \frac{\tan^{-1} x}{1 + x^2}dx$$

$$60. \int \frac{1+x}{1+x^2}dx$$

$$61. \int \frac{\sin(\ln x)}{x}dx$$

$$62. \int \frac{\sin 2x}{1 + \cos^2 x}dx$$

$$63. \int \frac{\sin 2x}{1 + \cos 2x} dx$$

$$64. \int \frac{\sin x}{1 + \cos^2 x} dx$$

$$65. \int \cot x dx$$

$$66. \int \frac{\sin x}{1 + \cos^2 x} dx$$

$$67. \int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$$

$$68. \int_0^1 x e^{-x^2} dx$$

$$69. \int_0^1 \frac{e^z + 1}{e^z + z} dz$$

$$70. \int_0^{1/2} \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$$

$$71. \int_1^2 (8x^3 + 3x^2) dx$$

$$72. \int_0^T (x^4 - 8x + 7) dx$$

$$73. \int_0^1 (1 - x^9) dx$$

$$74. \int_0^1 (\sqrt[4]{u} + 1)^2 du$$

$$75. \int_0^1 y(y^2 + 1)^5 dy$$

$$76. \int_0^2 y^2 \sqrt{1 + y^3} dy$$

$$77. \int_1^5 \frac{dt}{(t-4)^2}$$

$$78. \int_0^1 \sin(3\pi t) dt$$

$$79. \int_0^1 v^2 \cos(v^3) dv$$

$$80. \int_{-1}^1 \frac{\sin x}{1 + x^2} dx$$

$$81. \int_{-\pi/4}^{\pi/4} \frac{t^4 \tan t}{2 + \cos t} dt$$

$$82. \int \frac{x+2}{\sqrt{x^2+4x}} dx$$

$$83. \int \sin \pi t \cos \pi t dt$$

$$84. \int \sin x \cos(\cos x) dx$$

$$85. \int_0^{\pi/8} \sec 2\theta \tan 2\theta d\theta$$

$$86. \int_0^{\pi/4} (1 + \tan t)^3 \sec^2 t dt$$

$$87. \int_0^3 |x^2 - 4| dx$$

$$88. \int_0^4 |\sqrt{x} - 1| dx$$

Encuentra la derivada de la función.

$$89. F(x) = \int_0^x \frac{t^2}{1+t^3} dt$$

$$90. F(x) = \int_x^1 \sqrt{t + \sin t} dt$$

$$91. g(x) = \int_0^{x^4} \cos(t^2) dt$$

$$92. g(x) = \int_1^{\sin x} \frac{1-t^2}{1+t^4} dt$$

$$93. y = \int_{\sqrt{x}}^x \frac{\cos \theta}{\theta} d\theta$$

$$94. y = \int_{2x}^{3x+1} \sin(t^4) dt$$

$$1. \int \frac{\sqrt[5]{x^3} + \sqrt[6]{x}}{\sqrt{x}} dx.$$

$$2. \int \frac{dx}{\sqrt{x-1} + \sqrt{x+1}}.$$

$$3. \int \frac{e^x + e^{2x} + e^{3x}}{e^{4x}} dx.$$

$$4. \int \frac{a^x}{b^x} dx.$$

$$5. \int \operatorname{tg}^2 x dx.$$

$$6. \int \frac{dx}{a^2 + x^2}.$$

$$7. \int \frac{dx}{\sqrt{a^2 - x^2}}.$$

$$8. \int \frac{dx}{1 + \operatorname{sen} x}.$$

$$9. \int \frac{8x^2 + 6x + 4}{x + 1} dx.$$

$$10. \int \frac{1}{\sqrt{2x - x^2}} dx.$$

$$11. \int e^x \operatorname{sen} e^x dx.$$

$$12. \int x e^{-x^2} dx.$$

$$13. \int \frac{\log x}{x} dx.$$

$$14. \int \frac{e^x dx}{e^{2x} + 2e^x + 1}.$$

$$15. \int e^{x^2} e^x dx.$$

$$16. \int \frac{x dx}{\sqrt{1 - x^4}}.$$

$$17. \int \frac{e\sqrt{x}}{\sqrt{x}} dx.$$

$$18. \int x\sqrt{1 - x^2} dx.$$

$$19. \int \log(\cos x) \operatorname{tg} x dx.$$

$$20. \int \frac{\log(\log x)}{x \log x} dx.$$

$$21. \int x^2 e^x dx.$$

$$22. \int x^3 e^{x^2} dx.$$

$$23. \int e^{ax} \operatorname{sen} bx dx.$$

$$24. \int x^2 \operatorname{sen} x dx.$$

$$25. \int (\log x)^3 dx.$$

$$26. \int \frac{\log(\log x)}{x} dx.$$

$$27. \int \sec^3 x dx.$$

$$28. \int \cos(\log x) dx.$$

$$29. \int \sqrt{x} \log x dx.$$

$$30. \int x (\log x)^2 dx.$$

$$31. \int \frac{dx}{\sqrt{1-x^2}}.$$

$$32. \int \frac{dx}{\sqrt{1+x^2}}.$$

$$33. \int \frac{dx}{\sqrt{x^2-1}}.$$

$$34. \int \frac{dx}{x \sqrt{x^2-1}}.$$

$$35. \int \frac{dx}{x \sqrt{1-x^2}}.$$

$$36. \int \frac{dx}{x \sqrt{1+x^2}}.$$

$$37. \int x^2 \sqrt{1-x^2} dx.$$

$$38. \int \sqrt{1-x^2} dx.$$

$$39. \int \sqrt{1+x^2} dx.$$

$$40. \int \sqrt{x^2-1} dx.$$

$$41. \int \frac{dx}{1+\sqrt{x+1}}.$$

$$42. \int \frac{dx}{1+e^x}.$$

$$43. \int \frac{dx}{\sqrt{x} + \sqrt[3]{x}}.$$

$$44. \int \frac{dx}{\sqrt{1+e^x}}.$$

$$45. \int \frac{dx}{2+\operatorname{tg} x}.$$

$$46. \int \frac{dx}{\sqrt{\sqrt{x+1}}}.$$

$$47. \int \frac{4^x+1}{2^x+1} dx.$$

$$48. \int e\sqrt{x} dx.$$

$$49. \int \frac{\sqrt{1-x}}{1-\sqrt{x}} dx.$$

$$50. \int \sqrt{\frac{x-1}{x+1}} \cdot \frac{1}{x^2} dx.$$

$$51. \int \frac{2x^2+7x-1}{x^3+x^2-x-1} dx.$$

$$52. \int \frac{2x+1}{x^3-3x^2+3x-1} dx.$$

$$53. \int \frac{x^3+7x^2-5x+5}{(x-1)^2(x+1)^3} dx.$$

$$54. \int \frac{2x^2+x+1}{(x+3)(x-1)^2} dx.$$

$$55. \int \frac{x+4}{x^2+1} dx.$$

56. $\int \frac{x^3 + x + 2}{x^4 + 2x^2 + 1} dx.$
57. $\int \frac{3x^2 + 3x + 1}{x^3 + 2x^2 + 2x + 1} dx.$
58. $\int \frac{dx}{x^4 + 1}.$
59. $\int \frac{2x}{(x^2 + x + 1)^3} dx.$
60. $\int \frac{3x}{(x^2 + x + 1)^3} dx.$
61. $\int \frac{\arctg x}{1 + x^2} dx.$
62. $\int \frac{x \arctg x}{(1 + x^2)^3}.$
63. $\int \log \sqrt{1 + x^2} dx.$
64. $\int x \log \sqrt{1 + x^2} dx.$
65. $\int \frac{x^2 - 1}{x^2 + 1} \cdot \frac{1}{\sqrt{1 + x^4}} dx.$
66. $\int \arcsen \sqrt{x} dx.$
67. $\int \frac{x}{1 + \operatorname{sen} x} dx.$
68. $\int e^{\operatorname{sen} x} \cdot \frac{x \cos^3 x - \operatorname{sen} x}{\cos^2 x} dx.$
69. $\int \sqrt{tg x} dx.$
70. $\int \frac{dx}{x^6 + 1}.$
71. $\int \log(a^2 + x^2) dx.$
72. $\int \frac{1 + \cos x}{\operatorname{sen}^2 x} dx.$
73. $\int \frac{x + 1}{\sqrt{4 - x^2}} dx.$

$$74. \int x \arctg x dx.$$

$$75. \int \sen^3 x dx.$$

$$76. \int \frac{\sen^3 x}{\cos^2 x} dx.$$

$$77. \int x^2 \arctg x dx.$$

$$78. \int \frac{x dx}{\sqrt{x^2 - 2x + 2}}.$$

$$79. \int \sec^3 x \operatorname{tg} x dx.$$

$$80. \int x \operatorname{tg}^2 x dx.$$

$$81. \int \frac{dx}{(a^2 + x^2)^2}$$

$$82. \int \sqrt{1 - \sen x} dx.$$

$$83. \int \arctg \sqrt{x} dx.$$

$$84. \int \sen \sqrt{x+1} dx.$$

$$85. \int \frac{\sqrt{x^3 - 2}}{x} dx.$$

$$86. \int \log(x + \sqrt{x^2 - 1}) dx.$$

$$87. \int \log(x + \sqrt{x}) dx.$$

$$88. \int \frac{dx}{x - x^{3/5}}.$$

$$89. \int \frac{dx}{1 + \sen x}.$$

$$90. \int \frac{dx}{3 + 5 \sen x}.$$

Evaluar la integral utilizando la integración por partes con las elecciones indicadas de u y dv .

$$1. \int x \ln x dx; \quad u = \ln x, \quad dv = x dx$$

$$2. \int \theta \sec^2 \theta d\theta; \quad u = \theta, \quad dv = \sec^2 \theta d\theta$$

Evaluar la integral

$$3. \int x \cos 5x dx$$

$$4. \int x e^{-x} dx$$

$$5. \int r e^{r/2} dr$$

$$6. \int t \sin 2t dt$$

$$7. \int x^2 \sin \pi x dx$$

$$8. \int x^2 \cos mx dx$$

$$9. \int \ln(2x + 1) dx$$

$$10. \int \sin^{-1} x dx$$

$$11. \int \arctan 4t dt$$

$$12. \int p^5 \ln p dp$$

$$13. \int (\ln x^2) dx$$

$$14. \int t^3 e^t dt$$

$$15. \int e^{2\theta} \sin 3\theta d\theta$$

$$16. \int e^{\theta} \cos 2\theta d\theta$$

$$17. \int y \sinh y dy$$

$$18. \int y \cosh ay dy$$

$$19. \int_0^{\pi} t \sin 3t dt$$

$$20. \int_0^1 (x^2 + 1) e^{-x} dx$$

$$21. \int_1^2 \frac{\ln x}{x^2} dx$$

$$22. \int_1^4 \sqrt{t} \ln t dt$$

$$23. \int_0^1 \frac{y}{e^{2y}} dy$$

$$24. \int_{\pi/4}^{\pi/2} x \csc^2 x dx$$

$$25. \int_0^{1/2} \cos^{-1} x dx$$

$$26. \int_0^1 x 5^x dx$$

$$27. \int \cos x \ln(\sin x) dx$$

$$28. \int_1^{\sqrt{3}} \arctan(1/x) dx$$

$$29. \int \cos(\ln x) dx$$

$$30. \int_0^1 \frac{r^3}{\sqrt{4+r^2}} dr$$

$$31. \int_1^2 x^4 (\ln x)^2 dx$$

$$32. \int_0^t e^s \sin(t-s) ds$$

Primero realice una sustitución y luego use la integración por partes para evaluar la integral.

$$33. \int \sin \sqrt{x} dx$$

$$34. \int_1^4 e^{\sqrt{x}} dx$$

$$35. \int_{\sqrt{\pi/2}}^{\sqrt{\pi}} \theta^3 \cos(\theta^2) d\theta$$

$$36. \int x^5 e^{x^2} dx$$

$$37. \int x \cos \pi x dx$$

$$38. \int x^{3/2} \ln x dx$$

39. $\int (2x + 3)e^x dx$

40. $\int x^3 e^{x^2} dx$

41.

a. Demuestre que:

$$\int \sin^2 x dx = \frac{x}{2} - \frac{\sin 2x}{4} + C$$

b. Use la parte (a) y la fórmula de reducción para evaluar $\int \sin^4 x dx$.

42.

a. Demuestre la fórmula de reducción

$$\int \cos^n x dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x dx$$

b. Use la parte (a) para evaluar $\int \cos^2 x dx$.

c. Use las partes (a) y (b) para evaluar $\int \cos^4 x dx$.

43.

a. Demuestre que:

$$\int_0^{\pi/2} \sin^n x dx = \frac{n-1}{n} \int_0^{\pi/2} \sin^{n-2} x dx$$

Cuando $n \geq 2$ es un numero entero.

b. Use la parte (a) para evaluar, $\int_0^{\pi/2} \sin^3 x dx$ y $\int_0^{\pi/2} \sin^5 x dx$.

c. Use la parte (a) para mostrar que, para las potencias impares del seno,

$$\int_0^{\pi/2} \sin^{2n+1} x dx = \frac{2.4.6.....2n}{3.5.7.....(2n+1)}$$

44. Demuestre que, incluso para las potencias del seno:

$$\int_0^{\pi/2} \sin^{2n} x dx = \frac{1.3.5.....(2n-1)}{2.4.6.....2n} \frac{\pi}{2}$$

Use la integración por partes para probar la fórmula de reducción

45. $\int (\ln x)^n dx = x(\ln x)^n - n \int (\ln x)^{n-1} dx$

$$46. \int x^n e^x dx = x^n e^x - n \int x^{n-1} e^x dx$$

$$47. \int (x^2 + a^2)^n dx = \frac{x(x^2 + a^2)^n}{2n + 1} + \frac{2na^2}{2n + 1} \int (x^2 + a^2)^{n-1} dx \quad (n \neq -\frac{1}{2})$$

$$48. \int \sec^n x dx = \frac{\tan x \sec^{n-2} x}{n-1} + \frac{n-1}{n-1} \int \sec^{n-2} x dx \quad (n \neq 1)$$

$$49. \text{ Usa el ejercicio 45 para encontrar } \int (\ln x)^3 dx.$$

$$50. \text{ Usa el ejercicio 46 para encontrar } \int x^4 e^x dx.$$

Encuentra el área de la región delimitada por las curvas dadas.

$$51. y = xe^{-0,4x}, \quad y = 0, \quad x = 5$$

$$52. y = 5 \ln x, \quad y = x \ln x$$

Integrales trigonométricas

Evaluar la integral.

$$1. \int \sin^3 x \cos^2 x dx$$

$$2. \int \sin^6 x \cos^3 x dx$$

$$3. \int_{-\pi/2}^{3\pi/4} \sin^5 x \cos^3 x dx$$

$$4. \int_0^{\pi/2} \cos^5 x dx$$

$$5. \int \cos^5 x \sin^4 x dx$$

$$6. \int \sin^3(mx) dx$$

$$7. \int_0^{\pi/2} \cos^2 \theta d\theta$$

$$8. \int_0^{\pi/2} \sin^2(2\theta) d\theta$$

$$9. \int_0^{\pi} \sin^4(3t) dt$$

$$10. \int_0^{\pi} \cos^6 \theta d\theta$$

$$11. \int (1 + \cos \theta)^2 d\theta$$

$$12. \int x \cos^2 x dx$$

$$13. \int_0^{\pi/4} \sin^4 x \cos^2 x dx$$

$$14. \int_0^{\pi/2} \sin^2 x \cos^2 x dx$$

$$15. \int \sin^3 x \sqrt{\cos x} dx$$

$$16. \int \cos \theta \cos^5(\sin \theta) d\theta$$

$$17. \int \cos^2 x \tan^3 x dx$$

$$18. \int \cot^5 \theta \sin^4 \theta d\theta$$

$$19. \int \frac{1 - \sin x}{\cos x} dx$$

$$20. \int \cos^2 x \sin 2x dx$$

$$21. \int \sec^2 x \tan x dx$$

$$22. \int_0^{\pi/2} \sec^4(t/2) dt$$

$$23. \int \tan^2 x dx$$

$$24. \int \tan^4 x dx$$

$$25. \int \sec^6 t dt$$

$$26. \int_0^{\pi/4} \sec^4 \theta \tan^4 \theta d\theta$$

$$27. \int_0^{\pi/3} \tan^5 \sec^4 x dx$$

$$28. \int \tan^3(2x) \sec^5(2x) dx$$

$$29. \int \tan^3 \sec x dx$$

$$30. \int_0^{\pi/3} \tan^5 x \sec^6 x dx$$

$$31. \int \tan^5 x dx$$

$$32. \int \tan^6(ay) dy$$

$$33. \int \frac{\tan^3 \theta}{\cos^4 \theta} d\theta$$

$$34. \int \tan^2 x \sec x dx$$

$$35. \int_{\pi/6}^{\pi/2} \cot^2 x dx$$

$$36. \int_{\pi/4}^{\pi/2} \cot^3 x dx$$

$$37. \int \cot^3 \alpha \csc^3 \alpha d\alpha$$

$$38. \int \csc^4 x \cot^5 x dx$$

$$39. \int \csc x dx$$

$$40. \int_{\pi/6}^{\pi/3} \csc^3 x dx$$

$$41. \int \sin 5x \sin 2x dx$$

$$42. \int \sin 3x \cos x dx$$

$$43. \int \cos 7\theta \cos 5\theta d\theta$$

$$44. \int \frac{\cos x + \sin x}{\sin 2x} dx$$

$$45. \int \frac{1 - \tan^2 x}{\sec^2 x} dx$$

$$46. \int \frac{dx}{\cos x - 1}$$

$$47. \int t \sec^2(t^2) \tan^4(t^2) dt$$

$$48. \text{ Si } \int_0^{\pi/4} \tan^6 x \sec x dx = I \text{ exprese el valor de } \int_0^{\pi/4} \tan^8 x \sec x dx \text{ en terminos de } I.$$

Evalúa la integral indefinida. Ilustre y compruebe que su respuesta es razonable, graficando tanto el integrando como su antiderivada (tomando $C = 0$)

$$49. \int \sin^5 x dx$$

$$50. \int \sin^4 x \cos^4 x dx$$

$$51. \int \sin 3x \sin 6x dx$$

$$52. \int \sec^4 \frac{x}{2} dx$$

Evaluar la integral usando la sustitución trigonométrica indicada. Dibuje y etiquete el triángulo rectángulo asociado.

$$1. \int \frac{1}{x^2 \sqrt{x^2 - 9}} dx; \quad x = 3 \sec \theta$$

$$2. \int x^3 \sqrt{9 - x^2} dx; \quad x = 3 \sin \theta$$

$$3. \int \frac{x^3}{\sqrt{x^2 + 9}} dx; \quad x = 3 \tan \theta$$

$$4. \int_0^{2\sqrt{3}} \frac{x^3}{\sqrt{16 - x^2}} dx$$

$$5. \int_{\sqrt{2}}^2 \frac{1}{t^3 \sqrt{t^2 - 1}} dt$$

$$6. \int_0^2 x^3 \sqrt{x^2 + 4} dx$$

$$7. \int \frac{1}{x^2 \sqrt{25 - x^2}} dx$$

$$8. \int \frac{\sqrt{x^2 - a^2}}{x^4} dx$$

$$9. \int \frac{dx}{\sqrt{x^2 + 16}}$$

$$10. \int \frac{t^5}{\sqrt{t^2 + 2}} dt$$

$$11. \int \sqrt{1 - 4x^2} dx$$

$$12. \int_0^1 x \sqrt{x^2 + 4} dx$$

$$13. \int \frac{x^2 - 9}{x^3} dx$$

$$14. \int \frac{du}{u \sqrt{5 - u^2}}$$

$$15. \int \frac{x^2}{(a^2 - x^2)^{3/2}} dx$$

$$16. \int \frac{dx}{x^2 \sqrt{16x^2 - 9}}$$

$$17. \int \frac{x}{\sqrt{x^2 - 7}} dx$$

$$18. \int \frac{dx}{[(ax)^2 - b^2]^{3/2}}$$

$$19. \int \frac{\sqrt{1 + x^2}}{x} dx$$

20. $\int \frac{t}{\sqrt{25-t^2}} dt$

21. $\int_0^{2/3} x^3 \sqrt{4-9x^2} dx$

22. $\int_0^1 \sqrt{x^2+1} dx$

23. $\int \sqrt{5+4x-x^2} dx$

24. $\int \frac{dt}{\sqrt{t^2-6t+13}}$

25. $\int \frac{1}{\sqrt{9x^2+6x-8}} dx$

26. $\int \frac{x^2}{\sqrt{4x-x^2}} dx$

27. $\int \frac{dx}{(x^2+2x+2)^2}$

28. $\int \frac{dx}{(5-4x-x^2)^{5/2}}$

29. $\int x\sqrt{1-x^4} dx$

30. $\int_0^{\pi/2} \frac{\sqrt{\cos t}}{\sqrt{1+\sin^2 t}} dt$

31.

a) Use la sustitución trigonométrica para mostrar que

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln(x + \sqrt{x^2+a^2}) + C$$

b) Usa la sustitución hiperbólica $x = a \sinh t$ para mostrar que

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \sinh^{-1} \left(\frac{x}{a} \right) + C$$

Evaluar la integral

7. $\int \frac{x}{x-6} dx$

8. $\int \frac{r^2}{r+4} dr$

$$9. \int \frac{x-9}{(x+5)(x-2)} dx$$

$$10. \int \frac{1}{(t+4)(t-1)} dt$$

$$11. \int_2^3 \frac{1}{x^2-1} dx$$

$$12. \int_0^1 \frac{x-1}{x^2+3x+2} dx$$

$$13. \int \frac{ax}{x^2-bx} dx$$

$$14. \int \frac{1}{(x+a)(x+b)} dx$$

$$15. \int_0^1 \frac{2x+3}{(x+1)^2} dx$$

$$16. \int_0^1 \frac{x^3-4x-10}{x^2-x-6} dx$$

$$17. \int_1^2 \frac{4y^2-7y-12}{y(y+2)(y-3)} dy$$

$$18. \int \frac{x^2+2x-1}{x^3-x} dx$$

$$19. \int \frac{1}{(x+5)^2(x-1)} dx$$

$$20. \int \frac{x^2}{(x-3)(x+2)^2} dx$$

$$21. \int \frac{5x^2+3x-2}{x^3+2x^2} dx$$

$$22. \int \frac{ds}{s^2(s-1)^2}$$

$$23. \int \frac{x^2}{(x+1)^3} dx$$

$$24. \int \frac{x^3}{(x+1)^3} dx$$

$$25. \int \frac{10}{(x-1)(x^2+9)} dx$$

$$26. \int \frac{x^2 - x + 6}{x^3 + 3x} dx$$

$$27. \int \frac{x^3 + x^2 + 2x + 1}{(x^2 + 1)(x^2 + 2)} dx$$

$$28. \int \frac{x^2 - 2x - 1}{(x - 1)^2(x^2 + 1)} dx$$

$$29. \int \frac{x + 4}{x^2 + 2x + 5} dx$$

$$30. \int \frac{x^3 - 2x^2 + x + 1}{x^4 + 5x^2 + 4} dx$$

$$31. \int \frac{1}{x^3 - 1} dx$$

$$32. \int_0^1 \frac{x}{x^2 + 4x + 13} dx$$

$$33. \int_2^5 \frac{x^2 + 2x}{x^3 + 3x^2 + 4} dx$$

$$34. \int \frac{x^3}{x^3 + 1} dx$$

$$35. \int \frac{dx}{x^4 - x^2}$$

$$36. \int_0^1 \frac{2x^3 + 5x}{x^4 + 5x^2 + 4} dx$$

$$37. \int \frac{x - 3}{(x^2 + 2x + 4)^2} dx$$

$$38. \int \frac{x^4 + 1}{x(x^2 + 1)^2} dx$$

Haga una sustitución para expresar el integrando como una función racional y luego evalúe la integral.

$$39. \int \frac{1}{x\sqrt{x+1}} dx$$

$$40. \int \frac{1}{x - \sqrt{x+2}} dx$$

$$41. \int_9^{16} \frac{\sqrt{x}}{x - 4} dx$$

$$42. \int_0^1 \frac{1}{1 + \sqrt[3]{x}} dx$$

$$43. \int \frac{x^3}{\sqrt[3]{x^2 + 1}} dx$$

$$44. \int_{1/3}^3 \frac{\sqrt{x}}{x^2 + x} dx$$

$$45. \int \frac{1}{\sqrt{x} - \sqrt[3]{x}} dx \text{ Sugerencia: Sustituto } \mu = \sqrt[6]{x}.$$

$$46. \int \frac{1}{\sqrt[3]{x} + \sqrt[4]{x}} dx \text{ Sugerencia: Sustituto } \mu = \sqrt[12]{x}.$$

$$47. \int \frac{e^{2x}}{e^{2x} + 3e^x + 2} dx$$

$$48. \int \frac{\cos x}{\sin^2 x + \sin x} dx$$

Use la integración por partes, junto con las técnicas de esta sección, para evaluar la integral.

$$49. \int \ln(x^2 - x + 2) dx$$

$$50. \int x \tan^{-1} x dx$$