

# Ejercicios de Integrales

Cálculo

*<http://synergy.vision/>*

## **Contenido**

Use la Parte 1 del Teorema Fundamental del Cálculo para encontrar la derivada de la función.

$$1. g(x) = \int_1^x \frac{1}{t^3 + 1} dt$$

$$2. g(x) = \int_1^x (2 + t^4)^5 dt$$

$$3. g(s) = \int_5^s (t - t^2)^8 dt$$

$$4. g(r) = \int_0^r \sqrt{x^2 + 4} dx$$

$$5. F(x) = \int_x^\pi \sqrt{1 + \sec t} dt$$

$$\left[ \int_x^\pi \sqrt{1 + \sec t} dt = - \int_\pi^x \sqrt{1 + \sec t} dt \right]$$

$$6. G(x) = \int_1^x \cos \sqrt{t} dt$$

$$7. h(x) = \int_2^{1/x} \sin^4 t dt$$

$$8. h(x) = \int_1^{\sqrt{x}} \frac{z^2}{z^4 + 1} dz$$

$$9. y = \int_0^{\tan x} \sqrt{t + \sqrt{t}} dt$$

$$10. y = \int_0^{x^4} \cos^2 \theta d\theta$$

$$11. y = \int_{1-3x}^1 \frac{u^3}{1 + u^2} du$$

$$12. y = \int_{\sin x}^1 \sqrt{1 + t^2} dt$$

Evaluar la integral.

$$13. \int_{-1}^2 (x^3 - 2x) dx$$

$$14. \int_{-1}^1 x^{100} dx$$

$$15. \int_1^4 (5 - 2t + 3t^2) dt$$

$$16. \int_0^1 \left( 1 + \frac{1}{2}u^4 - \frac{2}{5}u^9 \right) du$$

$$17. \int_1^9 \sqrt{x} dx$$

$$18. \int_1^8 x^{-2/3} dx$$

$$19. \int_{\pi/6}^{\pi} \sin \theta d\theta$$

$$20. \int_{-5}^5 \pi dx$$

$$21. \int_0^1 (u+2)(u-3) du$$

$$22. \int_0^4 (4-t)\sqrt{t} dt$$

$$23. \int_1^9 \frac{x-1}{\sqrt{x}} dx$$

$$24. \int_0^2 (y-1)(2y+1) dy$$

$$25. \int_0^{\pi/4} \sec^2 t dt$$

$$26. \int_0^{\pi/4} \sec \theta \tan \theta d\theta$$

$$27. \int_1^2 (1+2y)^2 dy$$

$$28. \int_1^2 \frac{s^4+1}{s^2} ds$$

$$29. \int_1^2 \frac{v^5+3v^6}{v^4} dv$$

$$30. \int_1^{18} \sqrt{\frac{3}{z}} dz$$

$$31. \int_0^{\pi} f(x) dx \quad \text{donde} \quad f(x) = \begin{cases} \sin x & \text{si } 0 \leq x < \pi/2 \\ \cos x & \text{si } \pi/2 \leq x \leq \pi \end{cases}$$

$$32. \int_{-2}^2 f(x) dx \quad \text{donde} \quad f(x) = \begin{cases} 2 & \text{si } -2 \leq x \leq 0 \\ 4-x^2 & \text{si } 0 < x \leq 2 \end{cases}$$

¿Qué está mal con la ecuación?

$$33. \int_{-2}^1 x^{-4} dx = \frac{x^{-3}}{-3} \Big|_{-2}^1 = -\frac{3}{8}$$

$$34. \int_{-1}^2 \frac{4}{x^3} dx = -\frac{2}{x^2} \Big|_{-1}^2 = \frac{3}{2}$$

$$35. \int_{\pi/3}^{\pi} \sec \theta \tan \theta d\theta = \sec \theta \Big|_{\pi/3}^{\pi} = -3$$

$$36. \int_0^{\pi} \sec^2 x dx = \tan x \Big|_0^{\pi} = 0$$

Use un gráfico para dar una estimación aproximada del área de la región que se encuentra debajo de la curva dada. Luego encuentra el área exacta.

$$37. y = \sqrt[3]{x}, \quad 0 \leq x \leq 27$$

$$38. y = x^{-4}, \quad 1 \leq x \leq 6$$

$$39. y = \sin x, \quad 0 \leq x \leq \pi$$

$$40. y = \sec^2 x, \quad 0 \leq x \leq \pi/3$$

Evaluar la integral.

$$41. \int_1^9 \frac{1}{2x} dx$$

$$42. \int_0^1 10^x dx$$

$$43. \int_{1/2}^{\sqrt{3/2}} \frac{6}{\sqrt{1-t^2}} dt$$

$$44. \int_0^1 \frac{4}{t^2 + 1} dt$$

$$45. \int_{-1}^1 e^{u+1} du$$

$$46. \int_1^2 \frac{4+u^2}{u^3} du$$

Verificar por diferenciación que la fórmula es correcta.

$$1. \int \frac{1}{x^2 \sqrt{1+x^2}} dx = -\frac{\sqrt{1+x^2}}{x} + C$$

$$2. \int \cos^2 x dx = \frac{1}{2}x + \frac{1}{4} \sin 2x + C$$

$$3. \int \cos^3 x dx = \sin x - \frac{1}{3} \sin^3 x + C$$

$$4. \int \frac{x}{\sqrt{a+bx}} dx = \frac{2}{3b^2} (bx - 2a) \sqrt{a+bx} + C$$

Encuentra la integral general indefinida.

$$5. \int (x^2 + x^{-2}) dx$$

$$6. \int (\sqrt{x^3} + \sqrt[3]{x^2}) dx$$

$$7. \int (x^4 - \frac{1}{2}x^3 + \frac{1}{4}x - 2) dx$$

$$8. \int (y^3 + 1.8y^2 - 2.4y) dy$$

$$9. \int (u + 4)(2u + 1) du$$

$$10. \int v(v^2 + 2)^2 dv$$

$$11. \int \frac{x^3 - 2\sqrt{x}}{x} dx$$

$$12. \int \left( u^2 + 1 + \frac{1}{u^2} \right) du$$

$$13. \int (\theta - \csc \theta \cot \theta) d\theta$$

$$14. \int \sec t (\sec t + \tan t) dt$$

$$15. \int (1 + \tan^2 \alpha) d\alpha$$

$$16. \int \frac{\sin 2x}{\sin x} dx$$

Evaluar la integral.

$$17. \int_{-2}^3 (x^2 - 3) dx$$

$$18. \int_1^2 (4x^3 - 3x^2 + 2x) dx$$

$$19. \int_{-2}^0 \left( \frac{1}{2}t^4 + \frac{1}{4}t^3 - t \right) dt$$

$$20. \int_0^3 (1 + 6w^2 - 10w^4) dw$$

$$21. \int_0^2 (2x - 3)(4x^2 + 1) dx$$

$$22. \int_{-1}^1 t(1 - t)^2 dt$$

$$23. \int_0^\pi (4 \sin \theta - 3 \cos \theta) d\theta$$

$$24. \int_1^2 \left( \frac{1}{x^2} - \frac{4}{x^3} \right) dx$$

$$25. \int_1^4 \left( \frac{4 + 6u}{\sqrt{u}} \right) du$$

$$26. \int_1^2 \left( x + \frac{1}{x} \right)^2 dx$$

$$27. \int_1^2 \sqrt{\frac{5}{x}} dx$$

$$28. \int_1^9 \frac{3x - 2}{\sqrt{x}} dx$$

$$29. \int_1^4 \sqrt{t}(1 + t) dt$$

$$30. \int_{\pi/4}^{\pi/3} \csc^2 \theta d\theta$$

$$31. \int_0^{\pi/4} \frac{1 + \cos^2 \theta}{\cos^2 \theta} d\theta$$

$$32. \int_0^{\pi/3} \frac{\sin \theta + \sin \theta \tan^2 \theta}{\sec^2 \theta} d\theta$$

$$33. \int_1^{64} \frac{1 + \sqrt[3]{x}}{\sqrt{x}} dx$$

$$34. \int_1^8 \frac{x - 1}{\sqrt[3]{x^2}} dx$$

$$35. \int_0^1 (\sqrt[4]{x^5} + \sqrt[5]{x^4}) dx$$

$$36. \int_0^1 (1 + x^2)^3 dx$$

$$37. \int_2^5 |x - 3| dx$$

$$38. \int_0^2 |2x - 1| dx$$

$$39. \int_{-1}^2 (x - 2|x|) dx$$

$$40. \int_0^{3\pi/2} |\sin x| dx$$

$$41. \int (\sin x + \sin hx) dx$$

$$42. \int_{-10}^{10} \frac{2e^x}{\sinh x + \cosh x} dx$$

$$43. \int \left( x^2 + 1 + \frac{1}{x^2 + 1} \right) dx$$

$$44. \int_1^2 \frac{(x-1)^3}{x^2} dx$$

$$45. \int_0^{1/\sqrt{3}} \frac{t^2 - 1}{t^4 - 1} dt$$

Evalúa la integral haciendo la sustitución dada.

$$1. \int \sin \pi x dx, \quad u = \pi x$$

$$2. \int x^3 (2 + x^4)^5 dx, \quad u = 2 + x^4$$

$$3. \int x^2 \sqrt{x^3 + 1} dx, \quad u = x^3 + 1$$

$$4. \int \frac{dt}{(1 - 6t)^4}, \quad u = 1 - 6t$$

$$5. \int \cos^3 \theta \sin \theta d\theta, \quad u = \cos \theta$$

$$6. \int \frac{\sec^2(1/x)}{x^2} dx, \quad u = 1/x$$

Evaluar la integral indefinida.

$$7. \int x \sin(x^2) dx$$

$$8. \int x^2 \cos(x^3) dx$$

9.  $\int (1 - 2x)^9 dx$

10.  $\int (3t + 2)^{2.4} dt$

11.  $\int (x + 1)\sqrt{2x + x^2} dx$

12.  $\int \sec^2 2\theta d\theta$

13.  $\int \sec 3t \tan 3t dt$

14.  $\int u\sqrt{1 - u^2} du$

15.  $\int \frac{a + bx^2}{\sqrt{3ax + bx^3}} dx$

16.  $\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$

17.  $\int \sec^2 \theta \tan^3 \theta d\theta$

18.  $\int \cos^4 \theta \sin \theta d\theta$

19.  $\int (x^2 + 1)(x^3 + 3x)^4 dx$

20.  $\int \sqrt{x} \sin(1 + x^{3/2}) dx$

21.  $\int \frac{\cos x}{\sin^2 x} dx$

22.  $\int \frac{\cos(\pi/x)}{x^2} dx$

23.  $\int \frac{z^2}{\sqrt[3]{1 + z^3}} dz$

24.  $\int \frac{dt}{\cos^2 t \sqrt{1 + \tan t}}$

25.  $\int \sqrt{\cot x} \csc^2 x dx$

26.  $\int \sin t \sec^2(\cos t) dt$

27.  $\int \sec^3 x \tan x dx$



$$28. \int x^2 \sqrt{2+x} dx$$

$$29. \int x(2x+5)^8 dx$$

$$30. \int x^3 \sqrt{x^2+1} dx$$

Evalúa la integral indefinida. Ilustre y compruebe que su respuesta es razonable graficando tanto la función como su antiderivada (tomar  $C = 0$ ).

$$31. \int x(x^2-1)^3 dx$$

$$32. \int \tan^2 \theta \sec^2 \theta d\theta$$

$$33. \int \sin^3 x \cos x dx$$

$$34. \int \sin x \cos^4 x dx$$

Evaluar la integral definida.

$$35. \int_0^1 \cos(\pi t/2) dt$$

$$36. \int_0^1 (3t-1)^{50} dt$$

$$37. \int_0^1 \sqrt[3]{1+7x} dx$$

$$38. \int_0^{\sqrt{\pi}} x \cos(x^2) dx$$

$$39. \int_0^{\pi} \sec^2(t/4) dt$$

$$40. \int_{1/6}^{1/2} \csc \pi t \cot \pi t dt$$

$$41. \int_{-\pi/4}^{\pi/4} (x^3 + x^4 \tan x) dx$$

$$42. \int_0^{\pi/2} \cos x \sin(\sin x) dx$$

$$43. \int_0^{13} \frac{dx}{\sqrt[3]{(1+2x)^2}}$$

$$44. \int_0^a x \sqrt{a^2 - x^2} dx$$

$$45. \int_0^a x\sqrt{x^2 + a^2}dx \quad (a > 0)$$

$$46. \int_{-\pi/3}^{\pi/3} x^4 \sin x dx$$

$$47. \int_1^2 x\sqrt{x-1}dx$$

$$48. \int_0^4 \frac{x}{\sqrt{1+2x}}dx$$

$$49. \int_{1/2}^1 \frac{\cos(x^{-2})}{x^3}dx$$

$$50. \int_0^{T/2} \sin(2\pi t/T - \alpha)dt$$

$$51. \int_0^1 \frac{dx}{(1 + \sqrt{x})^4}$$

Evaluar la integral.

$$52. \int \frac{dx}{5 - 3x}$$

$$53. \int e^x \sin(e^x)dx$$

$$54. \int \frac{(\ln x)^2}{x}dx$$

$$55. \int \frac{dx}{ax + b} (a \neq 0)$$

$$56. \int e^{\tan x} \sec^2 x dx$$

$$57. \int e^{\cos t} \sin t dt$$

$$58. \int e^{\tan x} \sec^2 x dx$$

$$59. \int \frac{\tan^{-1} x}{1 + x^2}dx$$

$$60. \int \frac{1 + x}{1 + x^2}dx$$

$$61. \int \frac{\sin(\ln x)}{x}dx$$

$$62. \int \frac{\sin 2x}{1 + \cos^2 x}dx$$

$$63. \int \frac{\sin 2x}{1 + \cos 2x} dx$$

$$64. \int \frac{\sin x}{1 + \cos^2 x} dx$$

$$65. \int \cot x dx$$

$$66. \int \frac{\sin x}{1 + \cos^2 x} dx$$

$$67. \int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$$

$$68. \int_0^1 x e^{-x^2} dx$$

$$69. \int_0^1 \frac{e^z + 1}{e^z + z} dz$$

$$70. \int_0^{1/2} \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$$

$$71. \int_1^2 (8x^3 + 3x^2) dx$$

$$72. \int_0^T (x^4 - 8x + 7) dx$$

$$73. \int_0^1 (1 - x^9) dx$$

$$74. \int_0^1 (\sqrt[4]{u} + 1)^2 du$$

$$75. \int_0^1 y(y^2 + 1)^5 dy$$

$$76. \int_0^2 y^2 \sqrt{1 + y^3} dy$$

$$77. \int_1^5 \frac{dt}{(t-4)^2}$$

$$78. \int_0^1 \sin(3\pi t) dt$$

$$79. \int_0^1 v^2 \cos(v^3) dv$$

$$80. \int_{-1}^1 \frac{\sin x}{1 + x^2} dx$$

$$81. \int_{-\pi/4}^{\pi/4} \frac{t^4 \tan t}{2 + \cos t} dt$$

$$82. \int \frac{x+2}{\sqrt{x^2+4x}} dx$$

$$83. \int \sin \pi t \cos \pi t dt$$

$$84. \int \sin x \cos(\cos x) dx$$

$$85. \int_0^{\pi/8} \sec 2\theta \tan 2\theta d\theta$$

$$86. \int_0^{\pi/4} (1 + \tan t)^3 \sec^2 t dt$$

$$87. \int_0^3 |x^2 - 4| dx$$

$$88. \int_0^4 |\sqrt{x} - 1| dx$$

Encuentra la derivada de la función.

$$89. F(x) = \int_0^x \frac{t^2}{1+t^3} dt$$

$$90. F(x) = \int_x^1 \sqrt{t + \sin t} dt$$

$$91. g(x) = \int_0^{x^4} \cos(t^2) dt$$

$$92. g(x) = \int_1^{\sin x} \frac{1-t^2}{1+t^4} dt$$

$$93. y = \int_{\sqrt{x}}^x \frac{\cos \theta}{\theta} d\theta$$

$$94. y = \int_{2x}^{3x+1} \sin(t^4) dt$$

$$1. \int \frac{\sqrt[5]{x^3} + \sqrt[6]{x}}{\sqrt{x}} dx.$$

$$2. \int \frac{dx}{\sqrt{x-1} + \sqrt{x+1}}.$$

$$3. \int \frac{e^x + e^{2x} + e^{3x}}{e^{4x}} dx.$$

$$4. \int \frac{a^x}{b^x} dx.$$

$$5. \int \operatorname{tg}^2 x dx.$$

$$6. \int \frac{dx}{a^2 + x^2}.$$

$$7. \int \frac{dx}{\sqrt{a^2 - x^2}}.$$

$$8. \int \frac{dx}{1 + \operatorname{sen} x}.$$

$$9. \int \frac{8x^2 + 6x + 4}{x + 1} dx.$$

$$10. \int \frac{1}{\sqrt{2x - x^2}} dx.$$

$$11. \int e^x \operatorname{sen} e^x dx.$$

$$12. \int x e^{-x^2} dx.$$

$$13. \int \frac{\log x}{x} dx.$$

$$14. \int \frac{e^x dx}{e^{2x} + 2e^x + 1}.$$

$$15. \int e^{x^2} e^x dx.$$

$$16. \int \frac{x dx}{\sqrt{1 - x^4}}.$$

$$17. \int \frac{e\sqrt{x}}{\sqrt{x}} dx.$$

$$18. \int x\sqrt{1 - x^2} dx.$$

$$19. \int \log(\cos x) \operatorname{tg} x dx.$$

$$20. \int \frac{\log(\log x)}{x \log x} dx.$$

$$21. \int x^2 e^x dx.$$

$$22. \int x^3 e^{x^2} dx.$$

$$23. \int e^{ax} \operatorname{sen} bx dx.$$

$$24. \int x^2 \operatorname{sen} x dx.$$

$$25. \int (\log x)^3 dx.$$

$$26. \int \frac{\log(\log x)}{x} dx.$$

$$27. \int \sec^3 x dx.$$

$$28. \int \cos(\log x) dx.$$

$$29. \int \sqrt{x} \log x dx.$$

$$30. \int x (\log x)^2 dx.$$

$$31. \int \frac{dx}{\sqrt{1-x^2}}.$$

$$32. \int \frac{dx}{\sqrt{1+x^2}}.$$

$$33. \int \frac{dx}{\sqrt{x^2-1}}.$$

$$34. \int \frac{dx}{x \sqrt{x^2-1}}.$$

$$35. \int \frac{dx}{x \sqrt{1-x^2}}.$$

$$36. \int \frac{dx}{x \sqrt{1+x^2}}.$$

$$37. \int x^2 \sqrt{1-x^2} dx.$$

$$38. \int \sqrt{1-x^2} dx.$$

$$39. \int \sqrt{1+x^2} dx.$$

$$40. \int \sqrt{x^2-1} dx.$$

$$41. \int \frac{dx}{1+\sqrt{x+1}}.$$

$$42. \int \frac{dx}{1+e^x}.$$

$$43. \int \frac{dx}{\sqrt{x} + \sqrt[3]{x}}.$$

$$44. \int \frac{dx}{\sqrt{1+e^x}}.$$

$$45. \int \frac{dx}{2+tgx}.$$

$$46. \int \frac{dx}{\sqrt{\sqrt{x+1}}}.$$

$$47. \int \frac{4^x+1}{2^x+1} dx.$$

$$48. \int e\sqrt{x} dx.$$

$$49. \int \frac{\sqrt{1-x}}{1-\sqrt{x}} dx.$$

$$50. \int \sqrt{\frac{x-1}{x+1}} \cdot \frac{1}{x^2} dx.$$

$$51. \int \frac{2x^2+7x-1}{x^3+x^2-x-1} dx.$$

$$52. \int \frac{2x+1}{x^3-3x^2+3x-1} dx.$$

$$53. \int \frac{x^3+7x^2-5x+5}{(x-1)^2(x+1)^3} dx.$$

$$54. \int \frac{2x^2+x+1}{(x+3)(x-1)^2} dx.$$

$$55. \int \frac{x+4}{x^2+1} dx.$$

56.  $\int \frac{x^3 + x + 2}{x^4 + 2x^2 + 1} dx.$
57.  $\int \frac{3x^2 + 3x + 1}{x^3 + 2x^2 + 2x + 1} dx.$
58.  $\int \frac{dx}{x^4 + 1}.$
59.  $\int \frac{2x}{(x^2 + x + 1)^3} dx.$
60.  $\int \frac{3x}{(x^2 + x + 1)^3} dx.$
61.  $\int \frac{\arctg x}{1 + x^2} dx.$
62.  $\int \frac{x \arctg x}{(1 + x^2)^3}.$
63.  $\int \log \sqrt{1 + x^2} dx.$
64.  $\int x \log \sqrt{1 + x^2} dx.$
65.  $\int \frac{x^2 - 1}{x^2 + 1} \cdot \frac{1}{\sqrt{1 + x^4}} dx.$
66.  $\int \arcsen \sqrt{x} dx.$
67.  $\int \frac{x}{1 + \operatorname{sen} x} dx.$
68.  $\int e^{\operatorname{sen} x} \cdot \frac{x \cos^3 x - \operatorname{sen} x}{\cos^2 x} dx.$
69.  $\int \sqrt{tg x} dx.$
70.  $\int \frac{dx}{x^6 + 1}.$
71.  $\int \log(a^2 + x^2) dx.$
72.  $\int \frac{1 + \cos x}{\operatorname{sen}^2 x} dx.$
73.  $\int \frac{x + 1}{\sqrt{4 - x^2}} dx.$



$$74. \int x \operatorname{arctg} x dx.$$

$$75. \int \operatorname{sen}^3 x dx.$$

$$76. \int \frac{\operatorname{sen}^3 x}{\cos^2 x} dx.$$

$$77. \int x^2 \operatorname{arctg} x dx.$$

$$78. \int \frac{x dx}{\sqrt{x^2 - 2x + 2}}.$$

$$79. \int \sec^3 x \operatorname{tg} x dx.$$

$$80. \int x \operatorname{tg}^2 x dx.$$

$$81. \int \frac{dx}{(a^2 + x^2)^2}$$

$$82. \int \sqrt{1 - \operatorname{sen} x} dx.$$

$$83. \int \operatorname{arctg} \sqrt{x} dx.$$

$$84. \int \operatorname{sen} \sqrt{x+1} dx.$$

$$85. \int \frac{\sqrt{x^3 - 2}}{x} dx.$$

$$86. \int \log(x + \sqrt{x^2 - 1}) dx.$$

$$87. \int \log(x + \sqrt{x}) dx.$$

$$88. \int \frac{dx}{x - x^{3/5}}.$$

$$89. \int \frac{dx}{1 + \operatorname{sen} x}.$$

$$90. \int \frac{dx}{3 + 5 \operatorname{sen} x}.$$