Useful Results in Econ

Synferlo

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1 SSOC for utility/profit maximization

1.1 Utility maximization

Given $U = U(X_1, X_2)$, quasi-concave, utility maximization requires

$$du = u_1 dx_1 + u_2 dx_2 = 0 \tag{1}$$

$$d^2u = (u_{11}dx_1 + u_{12}dx_2)dx_1 + (u_{21}dx_1 + u_{22}dx_2)dx_2 < 0. \tag{2}$$

Therefore, we can rewrite equation (2)

$$u_{11}dx_1^2 + u_{12}dx_1dx_2 + u_{21}dx_1dx_2 + u_{22}dx_2^2 < 0 \tag{3}$$

$$u_{11}(dx_1^2 + \frac{2u_{12}}{u_{11}}dx_1dx_2) + u_{22}dx_2^2 < 0 \tag{4}$$

$$u_{11}\left(dx_1 + \frac{2u_{12}}{u_{11}}dx_1dx_2 + (\frac{u_{12}}{u_{11}}dx_2)^2 - (\frac{u_{12}}{u_{11}}dx_2)^2\right) + u_{22}dx_2^2 < 0 \tag{5}$$

$$u_{11}(dx_1 + \frac{u_{12}}{u_{11}}dx_2)^2 + dx_2^2(u_{22} - \frac{u_{12}^2}{u_{11}}) < 0 \tag{6}$$

where $u_{11} < 0$, $(dx_1 + \frac{u_{12}}{u_{11}}dx_2)$ and dx_2^2 are positive. Hence,

$$u_{22} - \frac{u_{12}^2}{u_{11}} < 0 \tag{7}$$

$$u_{11}u_{22} - u_{12}^2 > 0. \tag{8}$$

1.2 Profit maximization

Given $\pi = pf - w_1x_1 - w_2x_2$, where $f(\cdot)$ stands for the production function, $f = f(x_1, x_2)$.

$$d\pi = (pf_1 - w_1)dx_1 + (pf_2 - w_2)dx_2 = 0 \tag{9}$$

$$d^2\pi = p(f_{11}dx_1 + f_{12}dx_2)dx_1 + p(f_{21}dx_1 + p_{22}dx_2)dx_2 < 0. \tag{10}$$

Therefore, we can rewrite equation (10) as the following,

$$p(f_{11}dx_1 + f_{12}dx_2)dx_1 + p(f_{21}dx_1 + f_{22}dx_2)dx_2 < 0 \tag{11}$$

$$f_{11}dx_1^2 + f_{12}dx_1dx_2 + f_{12}dx_1dx_2 + f_{22}dx_2^2 < 0 \tag{12}$$

$$f_{11}\left(dx_1 + \frac{f_{12}}{f_{11}}dx_2\right)^2 + \left(f_{22} - \frac{f_{12}^2}{f_{11}}\right)dx_2^2 < 0. \tag{13}$$

Hence, we have similar results as in utility maximization problem,

$$f_{22} - \frac{f_{12}^2}{f_{11}} < 0 \tag{14}$$

$$f_{11}f_{22} - f_{12}^2 > 0. \tag{15}$$