

## SHEAR FORCE AND BENDING MOMENT DIAGRAMS

### IN STATICALLY DETERMINATE BEAMS :- ①

BEAM :- A Beam is a structural member subjected to a system of external forces at right angles to its longitudinal axis.

### STATICALLY DETERMINATE BEAM :

A Beam is said to be statically determinate, if its reaction components can be determined by using equations of static equilibrium conditions only ( $\sum F_x = 0$ ,  $\sum F_y = 0$  and  $\sum M = 0$ ).

i.e No of unknown reactions = No of equm conditions.

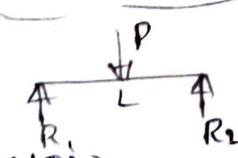
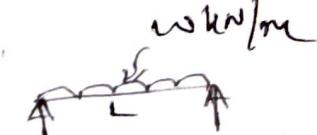
### STATICALLY INDETERMINATE BEAM :

A Beam is said to be statically indeterminate, if its reaction components cannot be determined by using equilibrium equation.

i.e No of unknowns reactions > Available equm equations

- Simply supported beam
  - Overhanging beam
  - Cantilever beam
  - Fixed beam
  - Continuous beam
  - Propped cantilever beam
- } statically determinate beam
- } statically indeterminate beam.

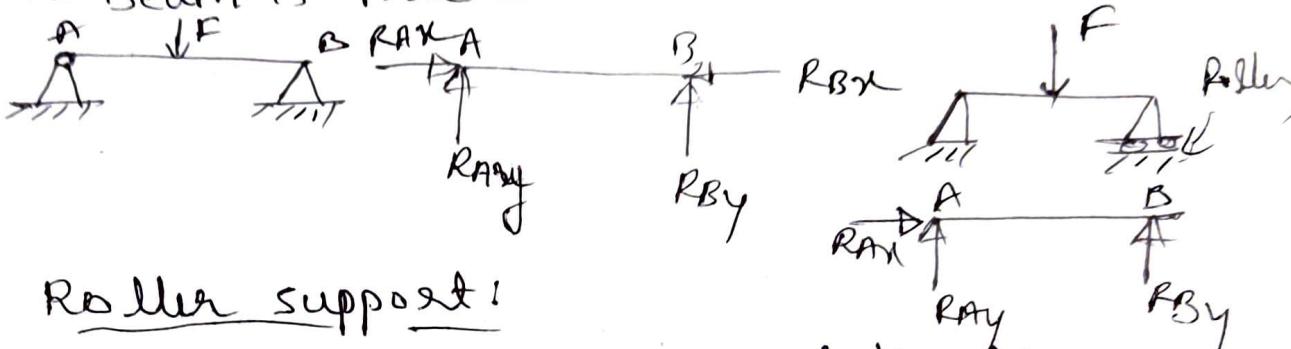
### Types of loads acting on the beam :

- Concentrated load - 
- Uniformly distributed load (UDL) - 
- " - Varying load (VVL) - 
- Externally applied moment.

## Types of supports:

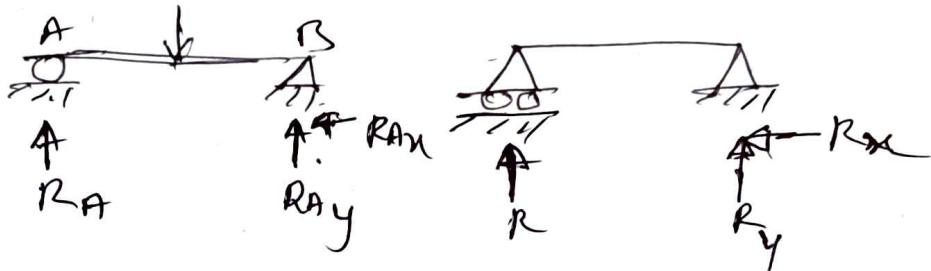
- Simple support --- One reaction along vertical direction  
→ beam is free to move along the direction & rotate about support
- Pinned or Hinged support:

Two reactions, one along Horizontal & other along vertical direction.  
→ Beam is free to rotate about the support.



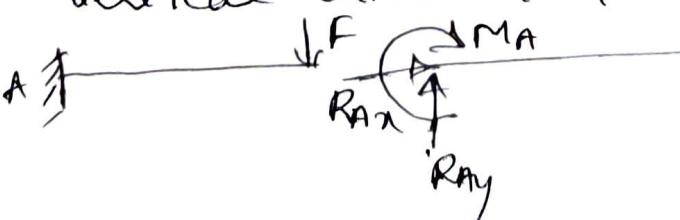
## Roller support:

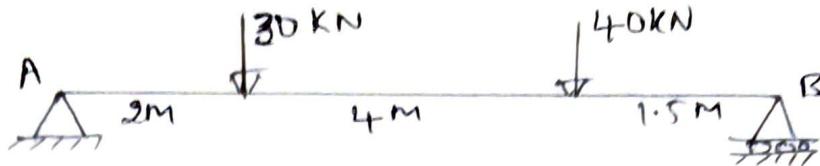
- One reaction along vertical direction
- beam is free to move along the direction & free to rotate about the support.



## Fixed support:

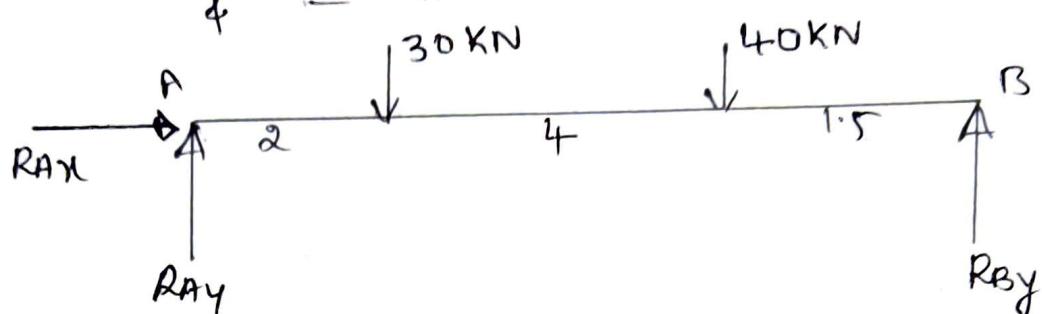
Two reactions, along the two directions and a moment.  
beam is prevented from moving along the two directions & also rotation





Find the support reactions @ A & B

Soln: First draw FBD  
since the support @ A is hinged - 2 reactions  
& " - @ B is roller - one "



$$\sum M_A = 0, \text{ to get } R_{By}$$

$$\text{i.e. } R_{Ax} \times 0 + R_{By} \times 7.5 + 30 \times 2 + 40 \times 6 - R_{By} \times 7.5 = 0$$

$$\Rightarrow 300 - 7.5 R_{By} = 0 \Rightarrow 7.5 R_{By} = 300$$

$$\Rightarrow R_{By} = 40 \text{ kN}$$

$$\sum F_y = 0 \text{ or } \sum M_B = 0, \text{ to get } R_{Ay}$$

$$R_{Ay} - 30 - 40 + R_{By} = 0$$

$$R_{Ay} = 30 + 40 - 40 = 30 \text{ kN. } R_{Ay} = 30 \text{ kN}$$

$$\sum F_x = 0, \text{ to get } R_{Ax} = 0$$

$$\boxed{R_{Ax} = 0} \quad (\because \text{there is no any free force other than reaction } R_{Ax})$$

FBD cross check

$$30 + 40 = 40 + 30$$

$$\cancel{70} = \cancel{70}$$

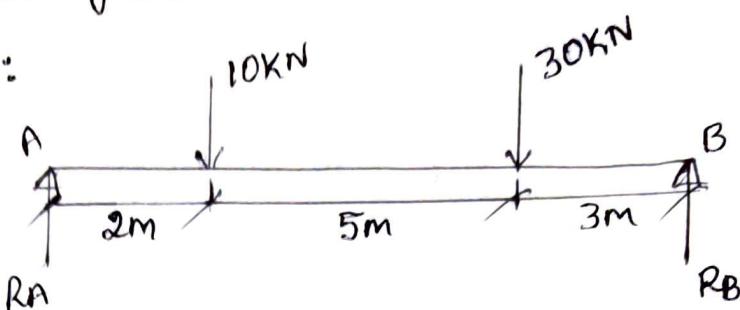
Then the beam is under equilibrium condition.

# 30

Calculation of support reactions for  
statically determinate beams

Example:

(1)



Find  $R_A = ?$  &  $R_B = ?$

Procedure:

- ①  $R_A$  &  $R_B$  can be calculated by using 3 static equilibrium conditions.  
if  $\sum F_x = 0$ ,  $\sum F_y = 0$  &  $\sum M = 0$
- ② First consider moment about A is equal to zero if  $\sum M_A = 0$  or  $\sum M_B = 0$ , whichever is easy for calculation
- ③ Sign conventions  $\uparrow(+), \downarrow(-), \leftarrow(+), \rightarrow(-)$
- ④ Consider  $\sum M_A = 0$   
if  $R_A \times 0 + 10 \times 2 + 30 \times 7 - R_B \times 10 = 0$   
LHS  $\qquad\qquad\qquad$  RHS  
 $290 - 10R_B = 0$

$$290 - 10R_B = 0 \\ \Rightarrow 10R_B = 290 \Rightarrow R_B = 29 \text{ KN}$$

$\sum F_y = 0$  or  $\sum M_B = 0$  to get  $R_A$

$$R_A - 10 - 30 + R_B = 0$$

$$R_A = 10 + 30 - 23 = 17 \text{ KN}$$

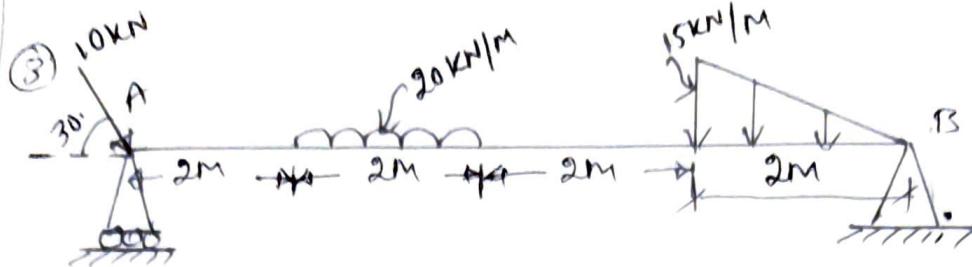
For cross check:

summation of applied force = summation of  $R_A$  &  $R_B$

$$\Rightarrow 10 + 30 = 23 + 17$$

$$40 = 40$$

$$\therefore R_A = 17 \text{ KN} \\ R_B = 23 \text{ KN}$$

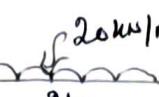


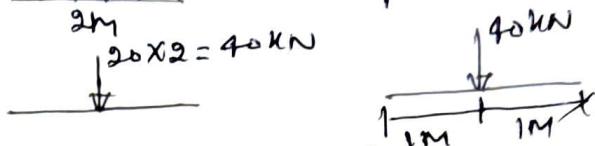
soln.

support A is roller - one reaction -  $R_{Ay}$

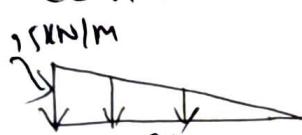
support B is hinged - two reaction -  $R_{Ay} + R_{Bx}$

- Resolve 10kN force along x + y direction

- Convert UDL  into point load

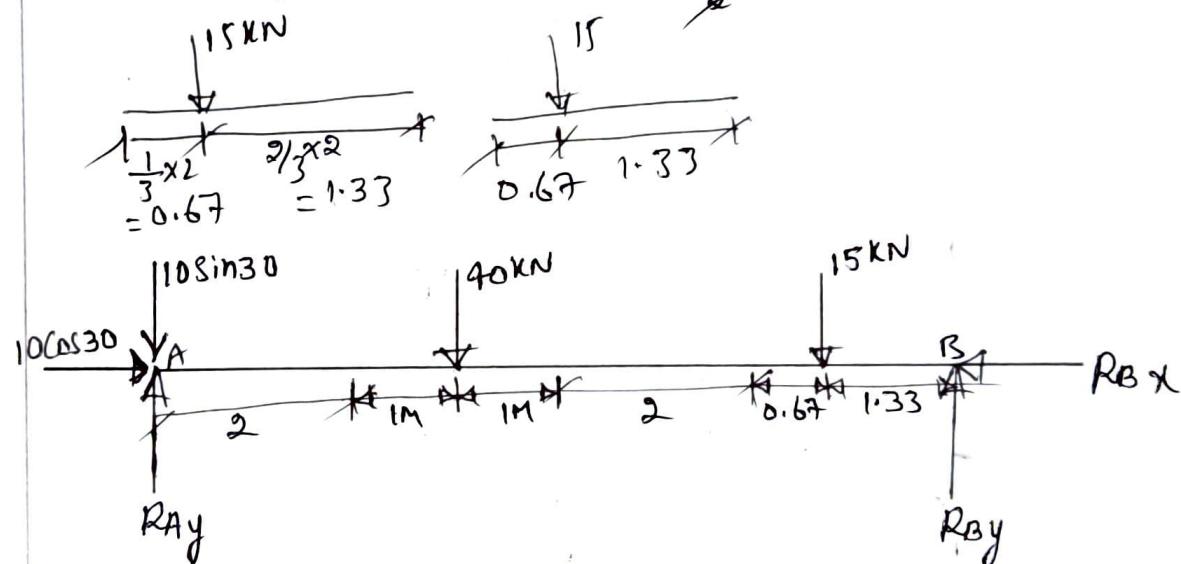


- Convert UVL into pt load



$$\text{pt load} = \text{Area of } \Delta$$

$$= \frac{1}{2} \times 2 \times 15 = 15 \text{ kN}$$



$$\sum M_A = 0, \text{ to get } R_{By}$$

$$40 \times 3 + 15 \times \frac{2+2+2+0.67}{6.67} - R_{By} \times 8 + R_{Bx} \times 0 = 0$$

$$220.05 - 8R_{By} = 0 \Rightarrow R_{By} = 27.50 \text{ kN}$$

$$\sum F_y = 0 \Rightarrow R_{Ay} - 10\sin 30 - 40 - 15 + R_{By} = 0$$

$$\Rightarrow R_{Ay} = +10\sin 30 + 40 + 15 - 27.5$$

$$\Rightarrow R_{Ay} = 32.5$$

$$\sum F_x = 0 \Rightarrow 10\cos 30 - R_{Bx} = 0$$

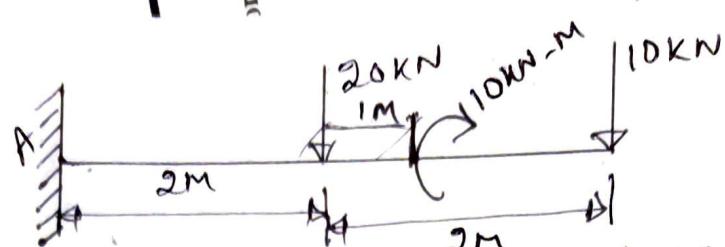
$$\Rightarrow R_{Bx} = 8.66 \text{ kN}$$

For cross check. Vertically up force = vertically upward reaction

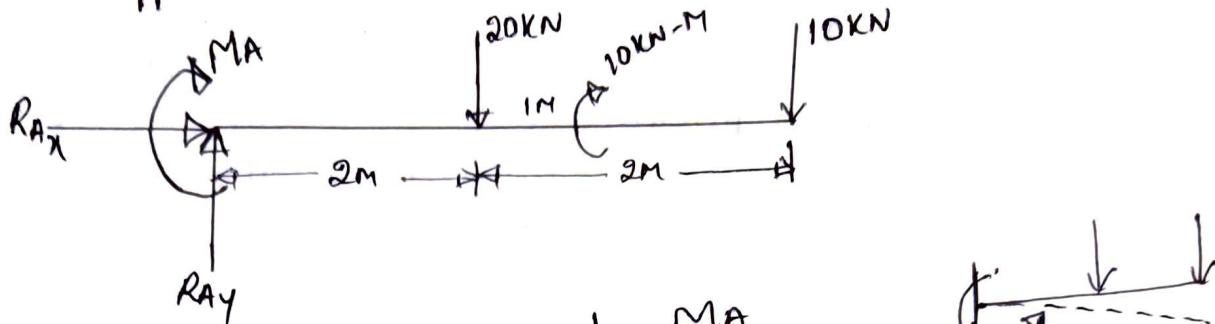
$$\Rightarrow 10\sin 30 + 40 + 15 = 27.5 + 32.5$$

$$\Rightarrow 60 = 60$$

(4)



support A is fixed - 2 reactions & moment



$$\sum M_A = 0, \text{ to get } M_A$$

$$\Rightarrow M_A + 20 \times 2 + 10 + 10 \times 4 = 0$$

$$M_A = -90 \text{ kN-m} = 90 \text{ kN-m} \quad (5)$$

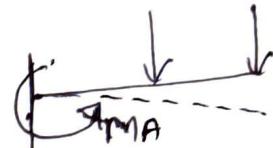
$$\sum F_x = 0 \Rightarrow \underline{R_{Ax} = 0}$$

$$\sum F_y = 0 \Rightarrow R_{Ay} - 20 - 10 = 0$$

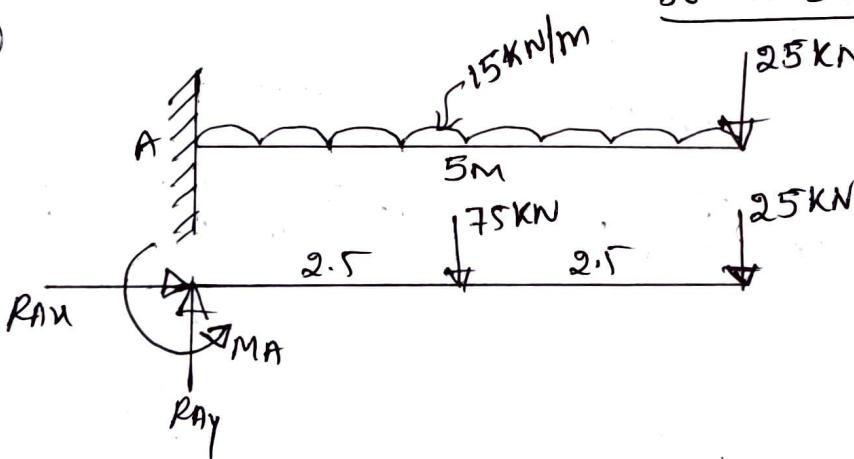
$$\Rightarrow R_{Ay} = 30 \text{ kN}$$

$$\text{For cross check, } 20 + 30 = R_{Ay}$$

$$\underline{30 = 30}$$



(5)



$$\sum M_A = 0 \Rightarrow -M_A + 75 \times 2.5 + 25 \times 5 = 0$$

$$\Rightarrow -M_A = -312.5 \text{ kN-m}$$

$$\Rightarrow \underline{M_A = 312.5 \text{ kN-m}}$$

$$\sum F_y = 0$$

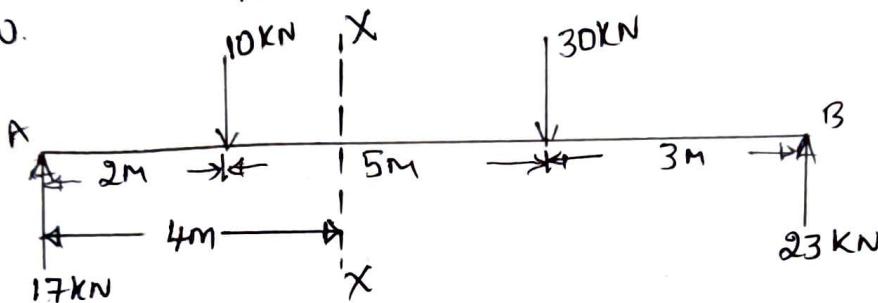
$$R_{Ay} - 75 - 25 = 0 \Rightarrow R_{Ay} = 100 \text{ kN}$$

$$\sum F_x = 0 \Rightarrow R_{Ax} = 0$$

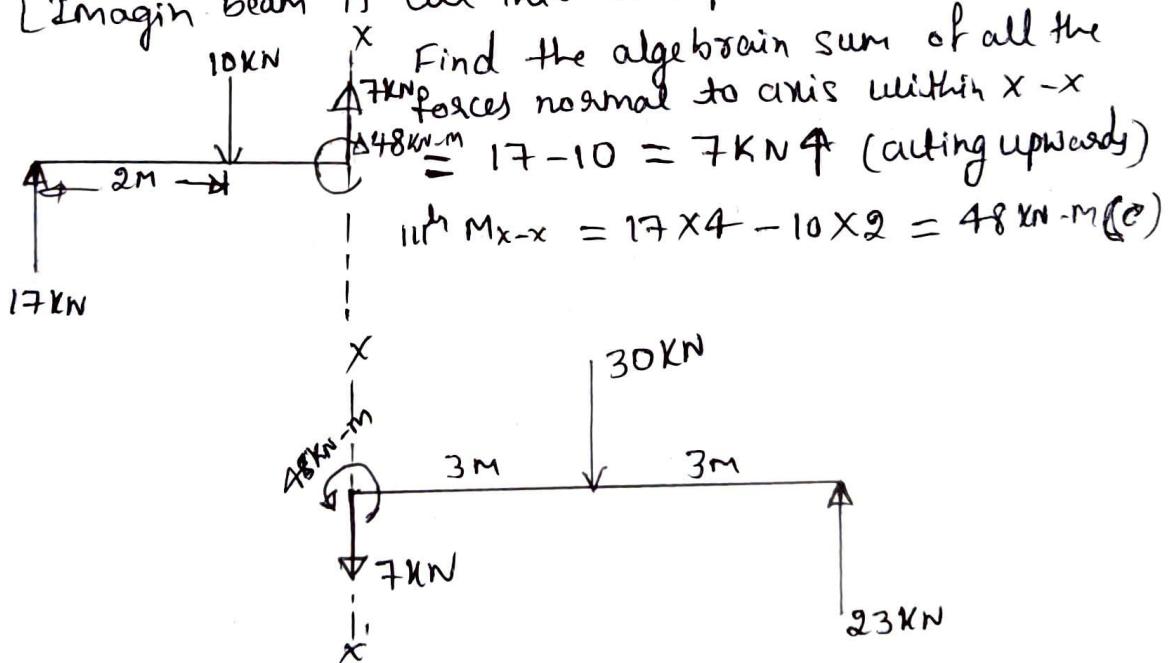
Understanding the meaning of shear force (SF) and Bending moment in beam (BM) :-

The beam transfer the applied loads to the support. In the process of transferring the applied loads to the supports, the beam develop resistance to moment and transverse shear forces at all its cross sections. The moment of SF developed at all sections of beam are determined and will be shown diagrammatically.

To understand the meaning of SF & BM, consider a section X-X at a distance 4m from left hand support A in the beam shown in Fig below.



Consider the left hand portion of section X-X  
[Imagin beam is cut into two pieces]



If we consider right portion

$$\text{Algebraic sum of forces} = 23 - 30 = -7 \text{ kN} = 7 \text{ kN } \downarrow$$

$$\text{f } \Sigma M_{X-X} = -23 \times 6 + 30 \times 3 = -48 \text{ kN-m} = 48 \text{ kN-m } (S)$$

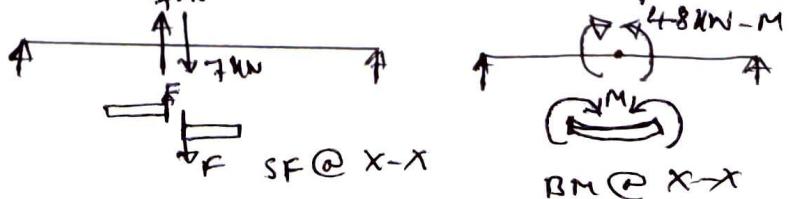
$\therefore$  The force 7 kN is called as SF & moment 48 kN-m is called as BM

Considering left portion,  
 vertical forces acting are reaction 17 kN acting  
 upwards and the external load 10 kN acting  
 vertically downwards. ∴ the resultant force acting  
 on the left portion is upward ( $F_{7\text{ kN}}$ ), to  
 maintain the equilibrium of the left portion  
 there must be an induced force of 7 kN acting  
 vertically downwards at the cut section.  
 This 7 kN force is known as shear force.  
 Therefore the resultant force acting  
 on any one of the portions normal to the  
 axis of the beam is called shear force  
 at the section.

⇒ Thus the section x-x is subjected to force  
 7 kN and moment 48 kN-m as shown above.  
 The force is trying to shear off the  
 section is called shear force. The moment  
 bends the section is called bending moment.

⇒ SHEAR FORCE: "Shear force at a section  
 in a beam is the force that is trying to shear  
 off the section and is obtained as the algebraic  
 sum of all the forces including the reactions acting  
 normal to the axis of the beam either to the left  
 or to the right of the section."

⇒ BENDING MOMENT: Bending moment at a section  
 in a beam is the moment that is trying to bend  
 bend it and is obtained as the algebraic sum of all  
 the forces including the moment about the section  
 of all the forces, either to the left or to the right  
 position).

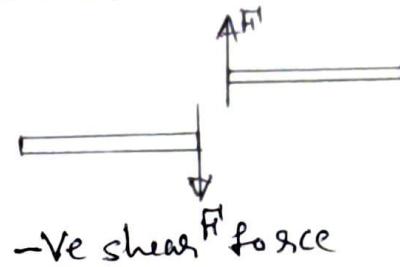
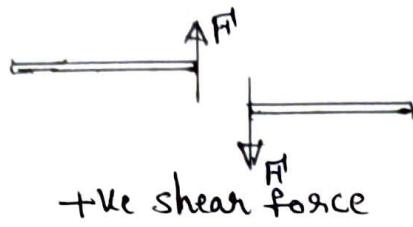


## SIGN CONVENTIONS FOR SFF & BM :- (5)

SF: → Shear force at a section is considered to be positive when the resultant force to the left of the section is upwards or to the right of the section downwards.

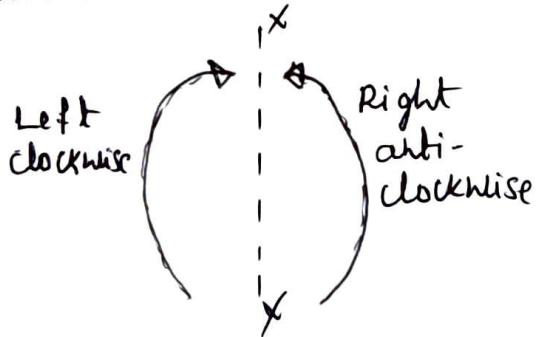
OR

→ The SF is +ve if it tends to move left portion upward relative to the right portion.



BM: → If the algebraic sum of the moments to the left of the section is clockwise or to the right of the section anticlockwise, then the moment M is taken as positive. Its effect is to create sagging of the beam as shown below.

OR  
→ The BM is +ve if it tries to sag the beam. If left portion of the beam is considered the moment works out to be clockwise.



+ve Bending moment  
[Sagging Bending moment]

→ If the  $\Sigma M$  to the left of the section is anticlockwise or to the right of the section clockwise, the BM is taken as negative. Its effect is to cause hogging of the beam.

OR  
→ The BM is -ve if it tries to hog the beam.



-ve BM  
[Hogging BM]

# SHEAR FORCE & BENDING MOMENT DIAGRAMS

(6) shear force and bending moment values vary from section to section. A designer needs not only the value's of shear force and bending moment at salient points, but also needs information about the nature of variation. Hence, the diagrams showing these at all the sections of the beam is necessary.

## SFD [Shear Force Diagram]:

A diagram in which ordinate represents shear force and abscissa represents the position of the section is called shear force diagram.

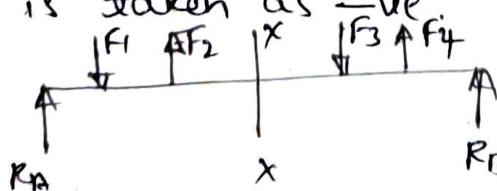
## BMD [Bending Moment Diagram]:

A diagram in which ordinate represents bending moment and abscissa represents the position of the section is called BMD.

⇒ The following sign conventions are used for drawing B.M & SF diagram.

## Important points for drawing shear force and bending moment diagram:

- ① Consider the left or right portion of the section
- ② Add the forces (including the reactions) normal to the beam on one side of the portion. If the left portion of the section is considered, a force on the left portion acting upwards is +ve ( $F_1, F_2$ ), while a force on the left portion acting downwards is taken as -ve.



In left position  $R_A$  &  $F_2$  are +ve  
if  $F_1$  is -ve  
In right position  $F_3$  is +ve  
if  $R_B$  &  $F_4$  are -ve

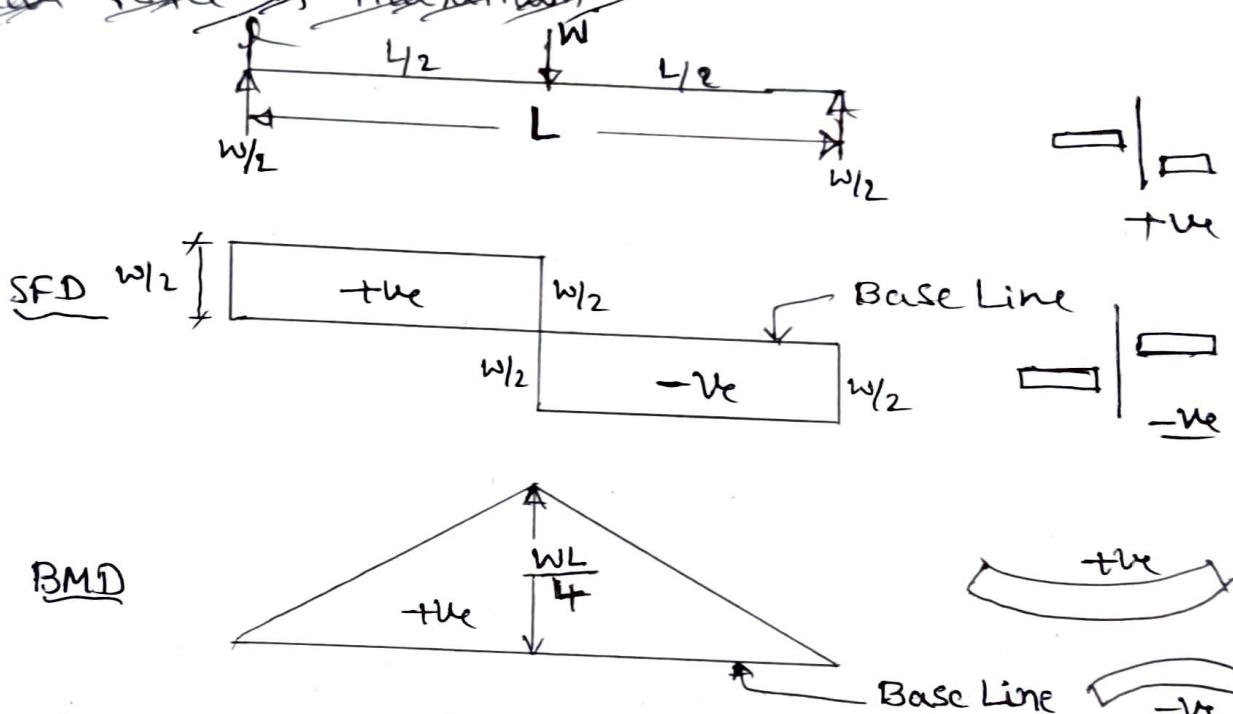
⇒ Left portion

- upward forces are +ve
- downward are -ve

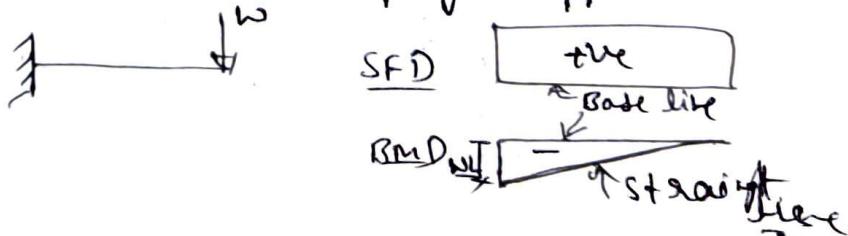
Right portion

- downward are +ve
- upward are -ve

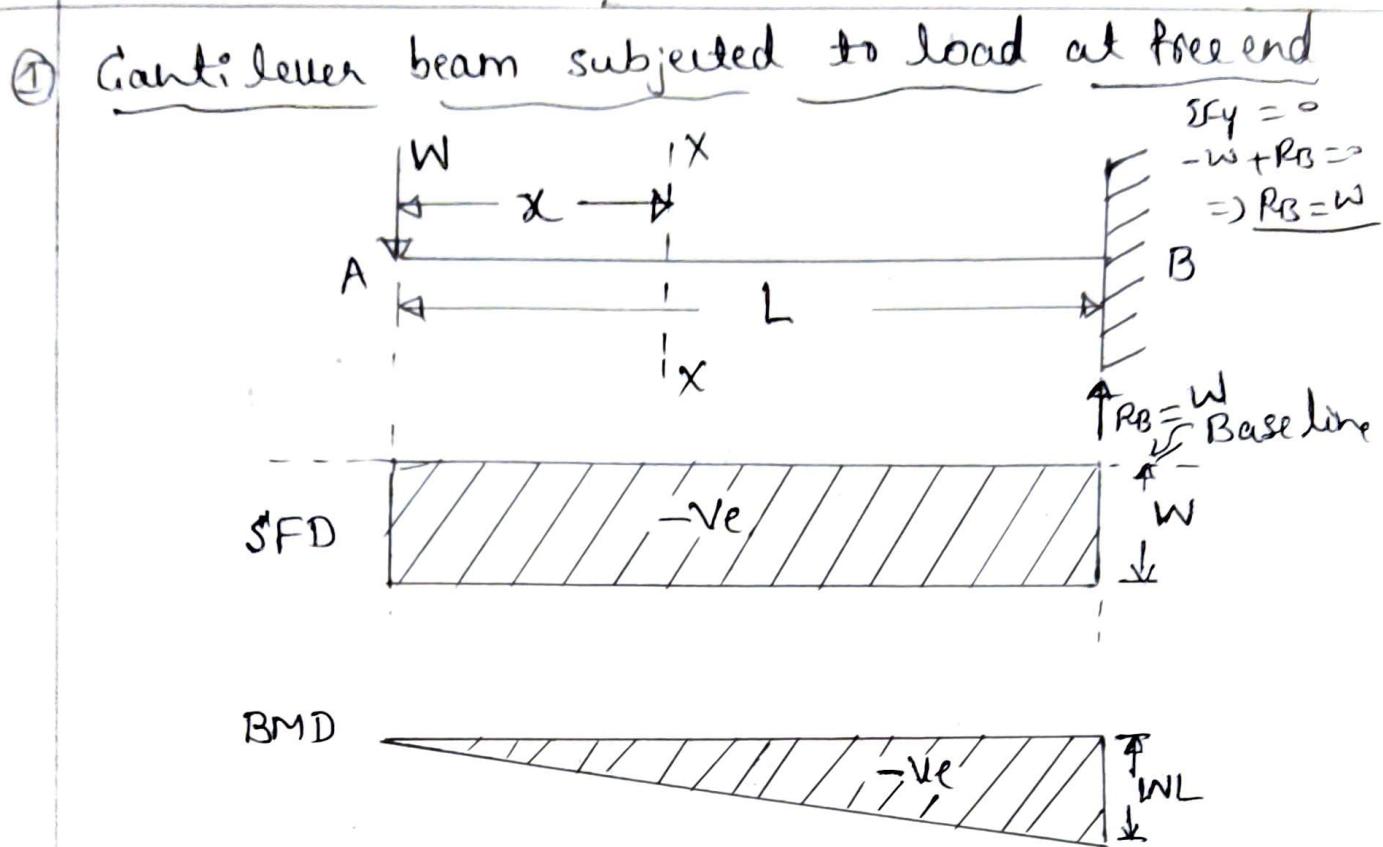
- ③ The +ve values of SF and BM are plotted above the base line.  
 The -ve values of SF and BM are plotted below the base line.
- ④ The shear force will increase or decrease suddenly by a vertical straight line at a section where there is a vertical point load.
- ⑤ SF is maximum at support, SF is zero at a section where BM is maximum. And BM is zero at a point or section where shear force is maximum.



- ⑥ If there is no loading at a section, the shear force will not change at the section or in other words, the SF between any two vertical load will remain unchanged if SFD will be horizontal.
- ⑦ The BM at the free end of a Cantilever and the two supports of a simply supported beam will be zero.



# SFD AND BMD FOR STANDARD CASES ; (7)



Consider the beam shown in fig above,  
 Take a section  $x-x$  @ a distance ' $x$ ' from end A.  
 Considering left hand position of section  $x-x$   
 for calculating SF and BM.

SF - Adding all the forces within  $x-x$   
 BM - Taking moment about  $x-x$  due to  
 all the forces which are in left position.

i.e. shear force if SF at section  $x-x$  is,  
 if  $F_1 = -W$  [ $\because$  Force  $W$  is acting downward]

Hence SF @ before B =  $-W$  ( $\because$  there is no other force than  $W$ )

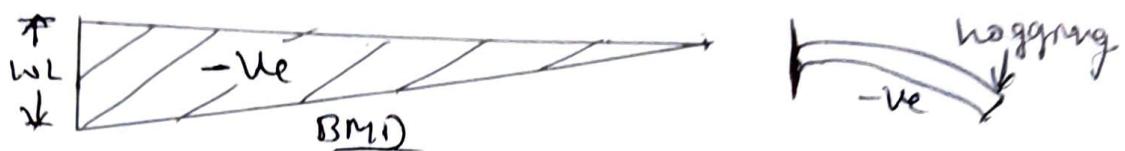
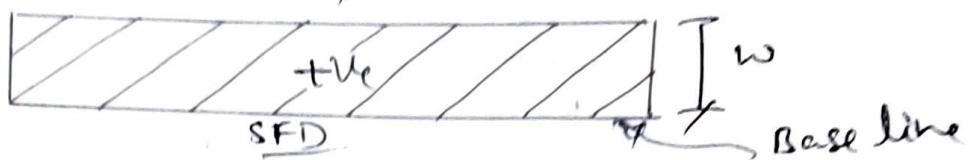
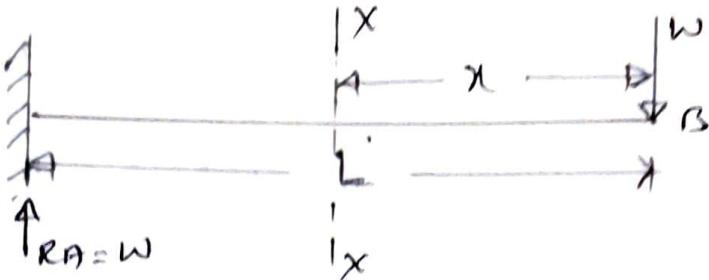
$$\therefore SF @ B = -W + W = 0$$

Therefore SFD and BMD are shown in Fig  
 above. BM @  $x-x \Rightarrow M = -Wx$

$$\begin{aligned} \text{At } x=0 &\Rightarrow M = 0 \\ \text{At } x=L &\Rightarrow M = -WL \end{aligned}$$

HW

$$\begin{aligned}\Sigma F_y &= 0 \\ -W + RA &= 0 \\ \Rightarrow RA &= W\end{aligned}$$



Base line

Hogging  
-ve

considering right portion of x-x section

$$SF @ x-x \Rightarrow F = w$$

$$SF @ \text{before A} = w \quad [\text{section@ A}]$$

$$SF @ A = w - w = 0$$

$$BM @ x = -wx$$

[In left portion  $\uparrow$  +ve f  $\rightarrow$  -ve  
In right "  $\uparrow$  -ve f  $\rightarrow$  +ve]

$$@ x = 0 M = 0$$

$$@ x = L M = -WL$$

OR considering, calculation of SF from left hand side

$$SF @ A = RA = w \quad (\uparrow +ve)$$

$$SF @ x-x = w$$

$$SF @ \text{before B} = w$$

$$SF @ B = RA - w = w - w = 0 \quad [(\uparrow +ve, \downarrow -ve)]$$

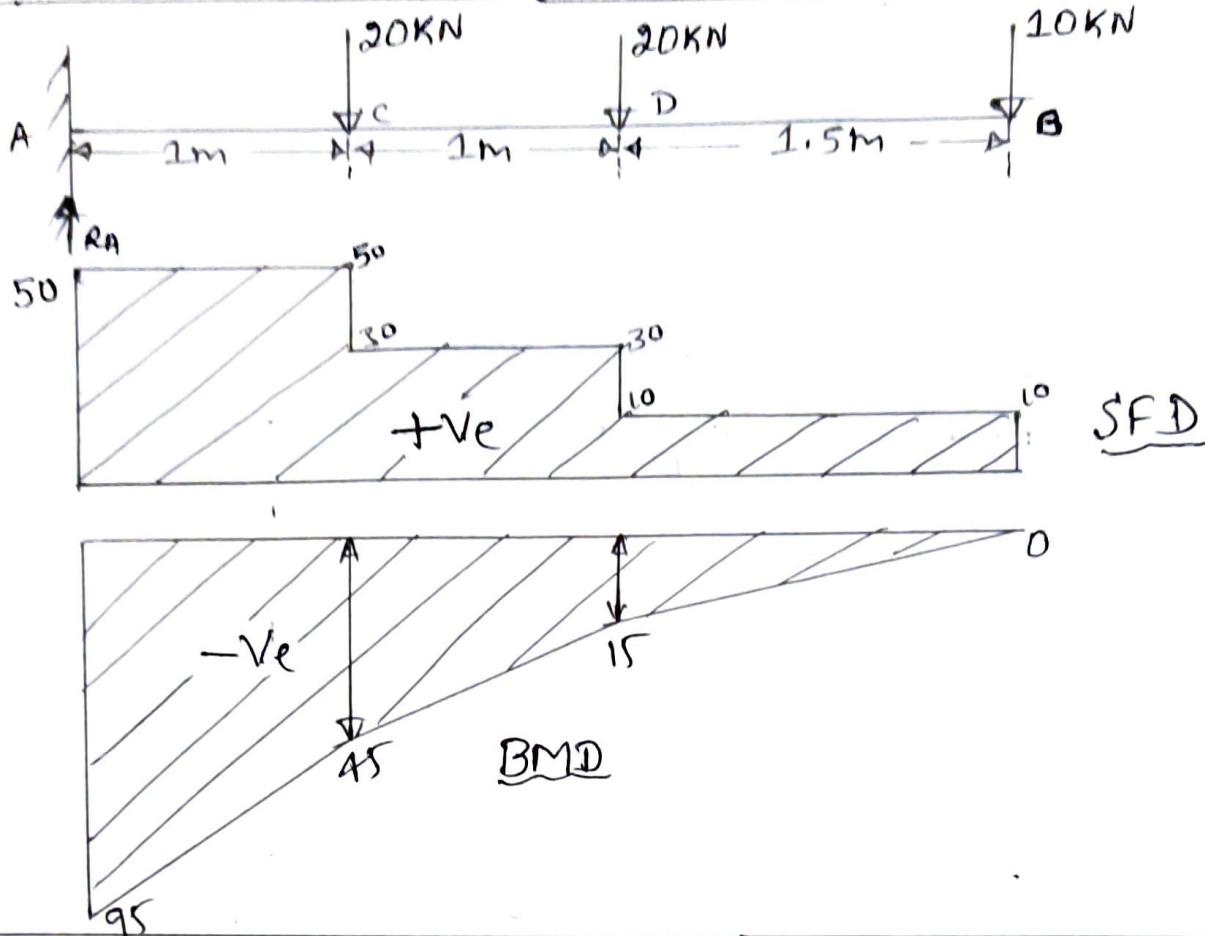
$$BM @ B = 0$$

$$BM @ A = wXL = WL$$

since the load w is trying to hog the beam, so that moment about A is taken as -ve.

$$\therefore MA = -WL$$

Q) Cantilever beam subjected to several pt loads. (8)



Calculation of Reaction @ A

$$\sum F_y = 0 \Rightarrow R_A - 20 - 20 - 10 = 0 \Rightarrow R_A = 50 \text{ kN}$$

Calculation of SF @ A, C, D & B:

$$SF @ A = R_A = 50 \text{ kN} \quad (+ve)$$

$$SF @ \text{before } C = R_A = 50 \text{ kN}$$

$$SF @ C = R_A - 20 \text{ (down)} = 50 - 20 = 30 \text{ kN}$$

$$SF @ \text{before } D = 30 \text{ kN}$$

$$SF @ D = 30 - 20 \text{ (down)} = 10 \text{ kN}$$

$$SF @ \text{before } B = 10 \text{ kN}$$

$$SF @ B = 10 - 10 = 0$$

Calculation of BM @ A, C, D & B:

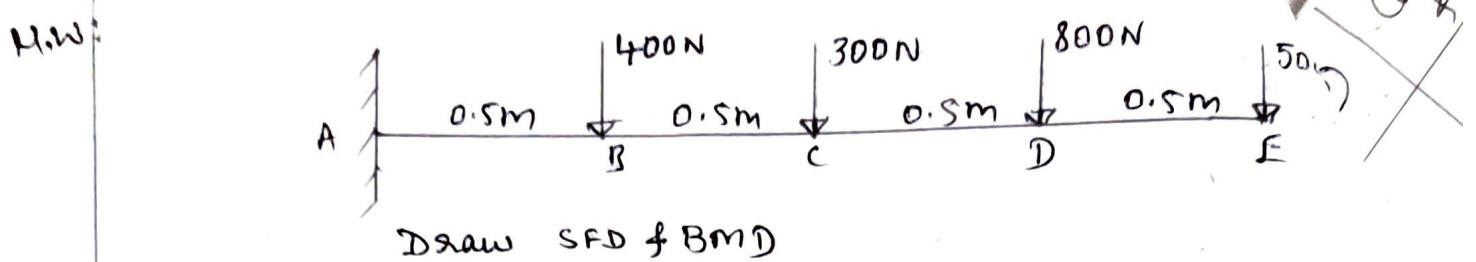
$$BM @ A \text{ re } M_A = -10 \times 3.5 - 20 \times 2 - 20 \times 1$$

$$M_A = -95 \text{ kN-m}$$

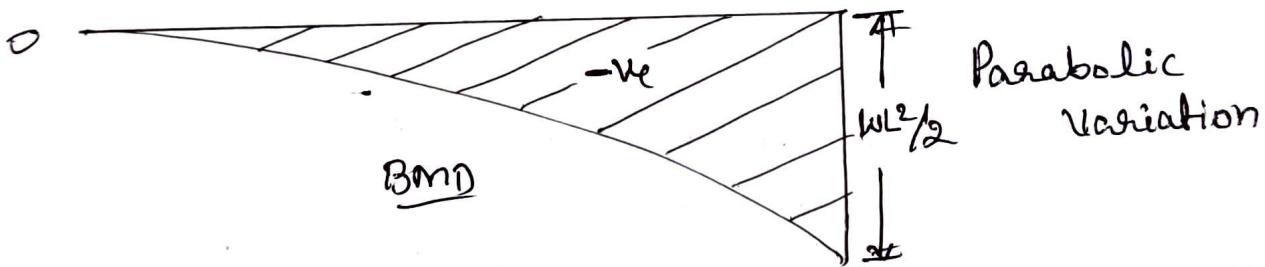
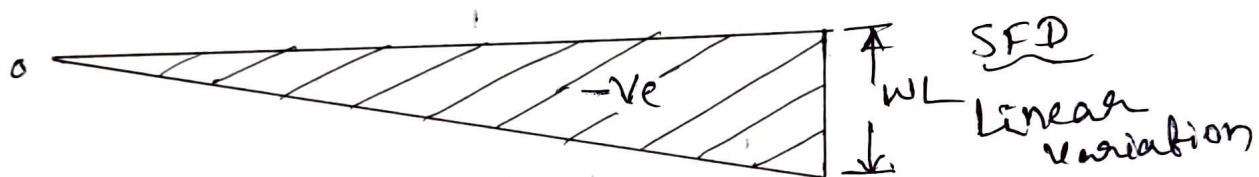
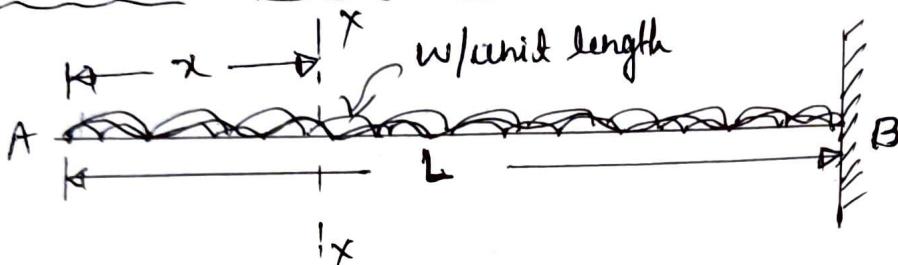
$$BM @ C \text{ re } M_C = -10 \times 2.5 - 20 \times 1 = -45 \text{ kN-m}$$

$$\text{Hence } M_D = -10 \times 1.5 = -15 \text{ kN-m}$$

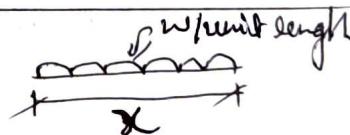
$$\text{at } M_B = 0$$



③ Cantilever beam subjected to UDL over entire span:



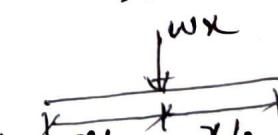
Consider left hand portion



$$SF @ x = -wx$$

$$\text{at } x=0 \text{ (at A)} \quad SF = 0$$

$$@ x=L, SF = -WL \text{ (linear variation)}$$

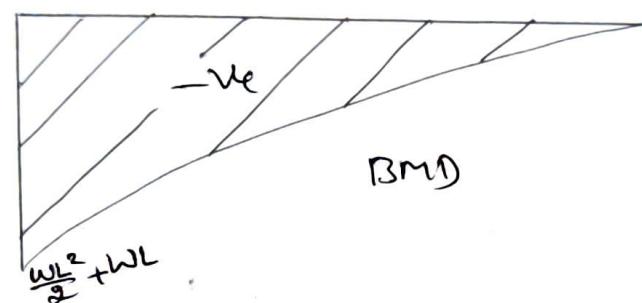
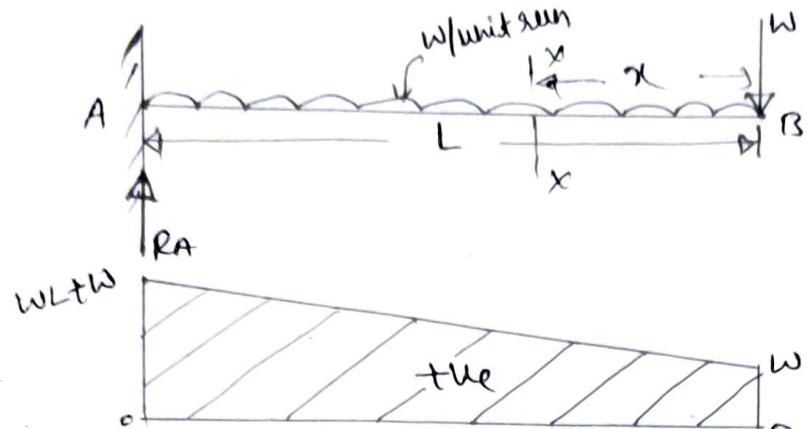


$$BM @ x = -wx \times \frac{x}{2} = -\frac{wx^2}{2}$$

$$@ x=0, BM = 0$$

$$@ x=L, BM = -\frac{WL^2}{2} \quad (\text{Parabolic variation})$$

Cantilever beam subjected to UDL over entire span + point load @ free end (9)



calculation of RA,  $\sum F_y = 0$ ,  $RA - WL - w = 0$   
 $\Rightarrow RA = WL + w$ .

$$SF @ A = RA(\uparrow) = WL + w$$

$$\begin{aligned} SF @ B &= \cancel{(WL + w)} - \cancel{w(L)} - \cancel{WL} - \cancel{wL} \\ &= RA(\uparrow) - WL(\downarrow) - w(\downarrow) \\ &= (WL + w) - WL - w \\ &= WL + w - WL - w \end{aligned}$$

$$SF @ \text{before } B = RA - WL = WL + w - WL = w$$

$$SF @ B = w - w = 0$$

OR considering Right portion

$$SF @ x = w + wx$$

$$@ x = 0, SF = w \quad (SF @ B)$$

$$@ x = L \quad SF = \underline{w + WL}$$

$$BM @ x = -wx - w \cdot x \cdot \frac{x}{2} = -wx - \frac{wx^2}{2}$$

$$@ x = 0, BM = 0$$

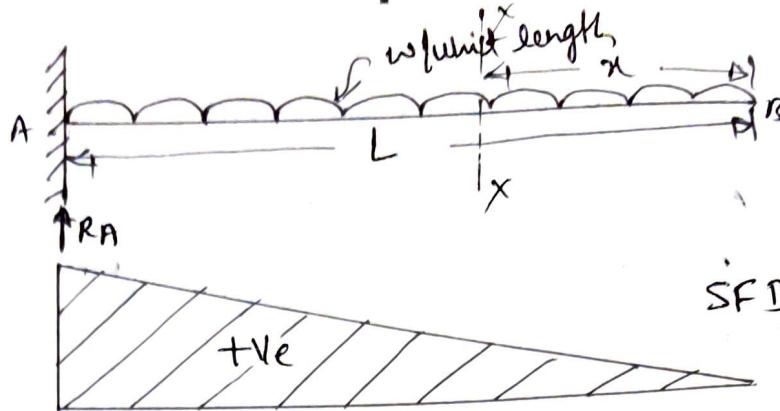
$$@ x = L, BM = -(WL + \frac{WL^2}{2}) = -WL \left[ 1 + \frac{WL}{2} \right] = -WL \left[ \frac{2+WL}{2} \right]$$

OR

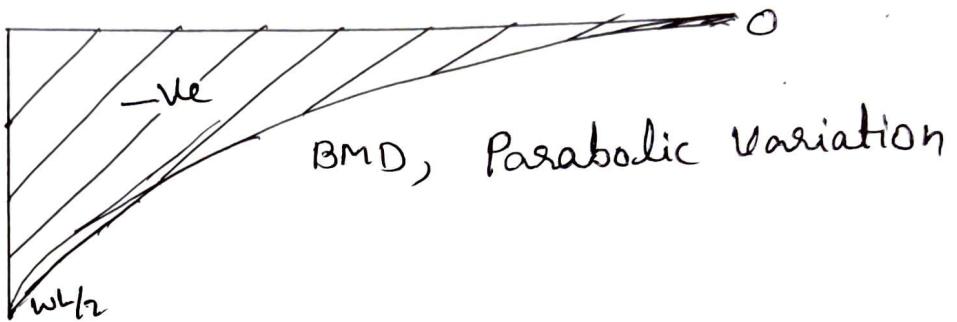
$$BM @ A = - \left[ WL + WL \cdot \frac{L}{2} \right] = - \left[ WL + \frac{WL^2}{2} \right]$$

$$BM @ B = 0$$

H.W.



SFD, Linear Variation



Consider right portion

$$SF @ x-x = wx \quad (+ve)$$

$$@ x=0, SF = 0$$

$$@ x=L, SF = WL$$

OR

$$\text{Calculate } RA, \text{ so } y=0 \Rightarrow RA - WL = 0 \Rightarrow RA = WL$$

$$SF @ A = RA(\uparrow) = WL, SF @ \text{before } B = RA = WL$$

$$SF @ B = RA(\uparrow) - WL(\downarrow) = WL - WL = 0$$

BM @ x-x (from right side)

$$BM = -wx \times \frac{x}{2} = -\frac{wx^2}{2}$$

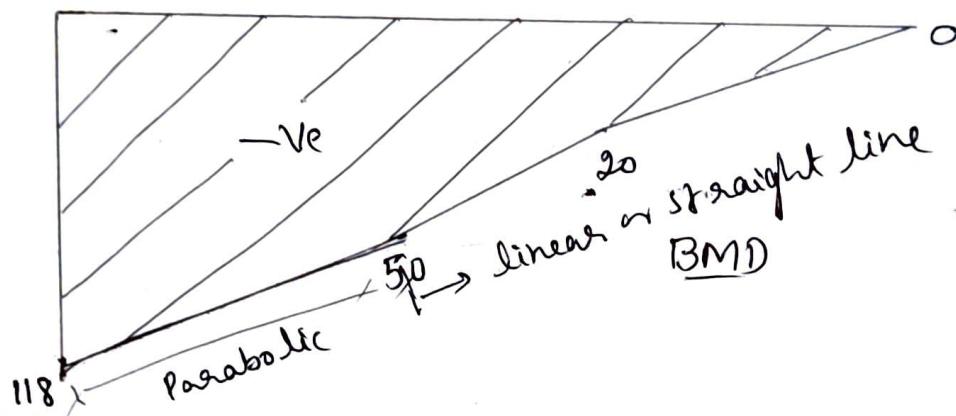
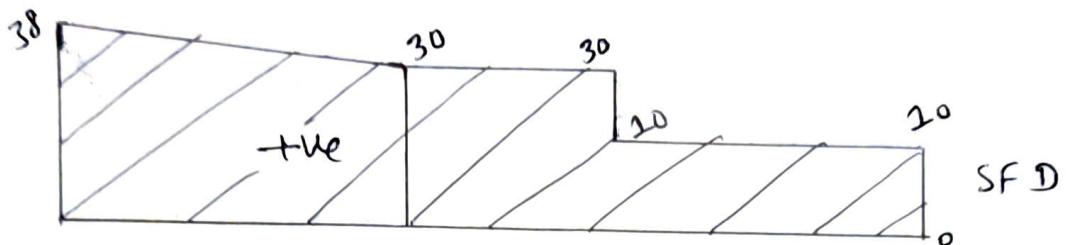
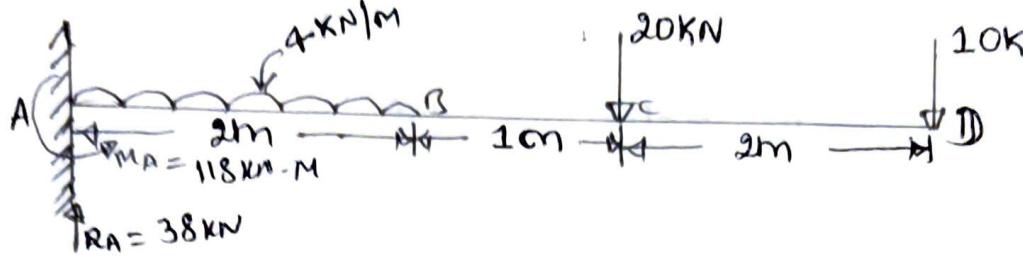
$$@ x=0, BM = 0, @ x=L, BM = -\frac{WL^2}{2}$$

OR

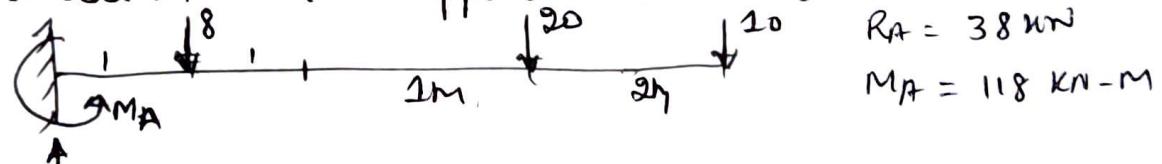
$$BM @ A = -WL \times \frac{L}{2} = \frac{WL^2}{2}$$

$$\underline{BM @ B = 0}$$

Cantilever beam subjected to UDL & Point load (10)



Calculation of Support reactions



$$RA = 38 \text{ kN} \quad \sum M_A = 0 \Rightarrow -MA + 8 \times 1 + 20 \times 3 + 10 \times 5 = 0 \\ \Rightarrow MA = 118 \text{ kN-m} \quad (3)$$

$$\sum y = 0 \Rightarrow RA - 8 - 20 - 10 = 0 \Rightarrow RA = 38 \text{ kN}$$

Calculation of SF

Consider section @ A, B, C & D, left hand sides

$$\therefore SF @ A = RA \uparrow = 38 \text{ kN}$$

$$SF @ \text{before } B = RA - 8(4 \times 2) \uparrow = 38 - 8 = 30 \text{ kN}$$

$$SF @ B = 30 \text{ kN}$$

$$SF @ \text{before } C = 30 \text{ kN}$$

$$SF @ C = 30 - 20 = 10 \text{ kN}$$

$$SF @ \text{before } D = 10$$

$$SF @ D = 10 - 10 = 0$$

means SF @ D = 10 kN

# Calculation of BM at A, B, C, D

Consider sections @ A, B, C + D, Left hand pos.

$$\therefore \text{BM} @ A = -M_A = -118 \text{ KN-M}$$

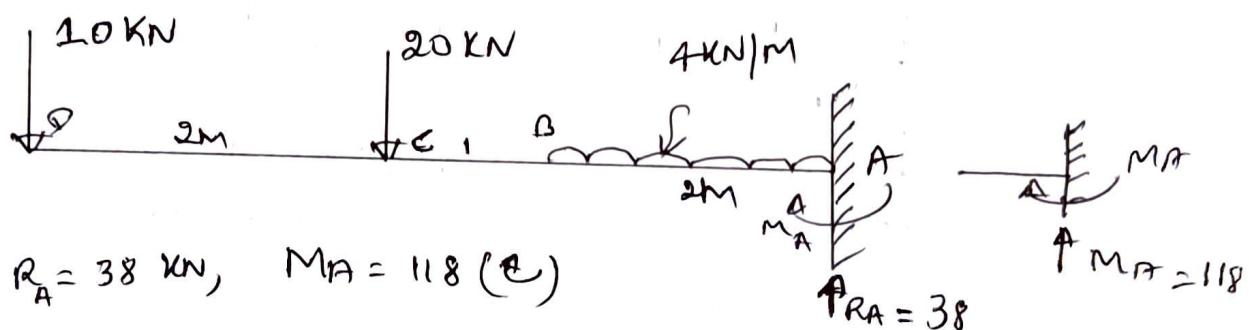
$$\text{BM} @ B = -M_A - 8 \times 1 + 38 \times 2 = -118 - 8 + 38 \times 2 = -50$$

$$\text{MB} @ C = -M_A + 38 \times 3 - 8 \times 2 = -118 + 38 \times 3 - 8 \times 2 = -20 \text{ KN-M}$$

$$\text{BM} @ D = -M_A + 38 \times 5 - 8 \times 4 - 20 \times 2 = 0$$

## Nature of Force & BM [SF+BM] variation

Load	SF	BM
No Load	Constant	Linear
LIDL	Linear	Parabolic
LVL	Parabolic	Cubic
.	.	.



$$\text{SF} @ D = -10$$

$$\text{SF} @ \text{before } C = -10$$

$$\text{SF} @ C = -10 - 20 = -30$$

$$\text{SF} @ \text{before } B = -30$$

$$\text{SF} @ B = -30$$

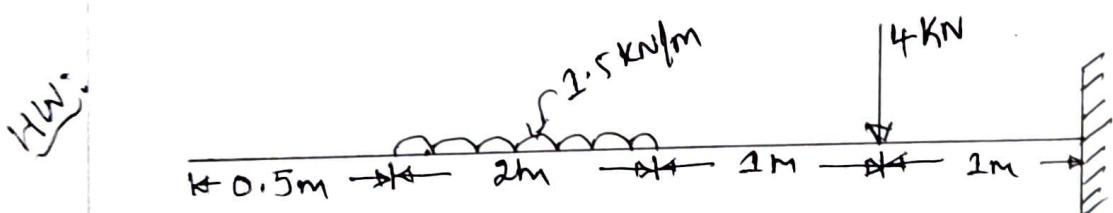
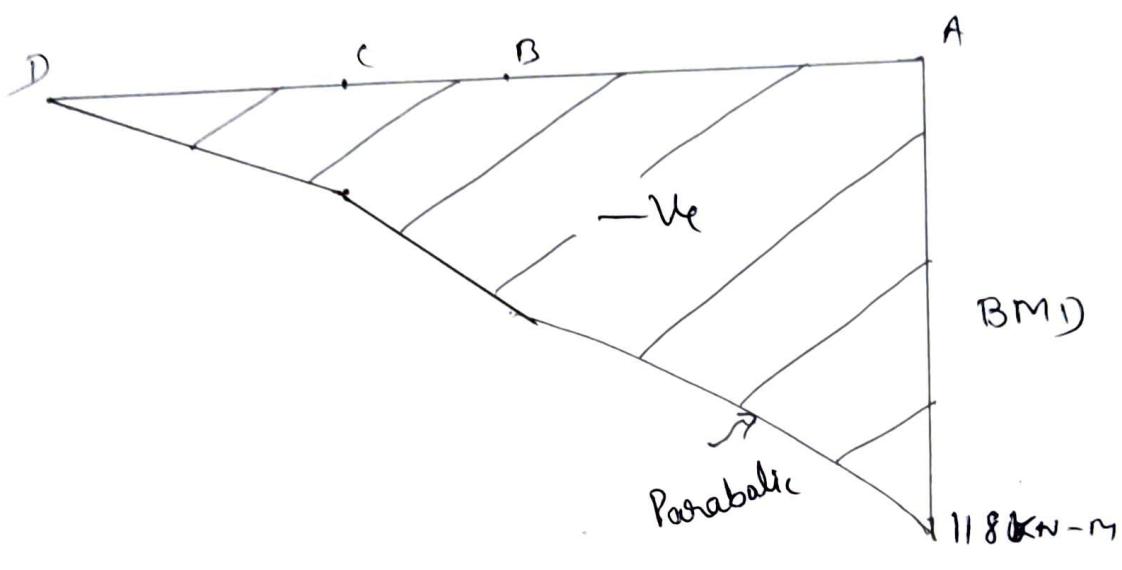
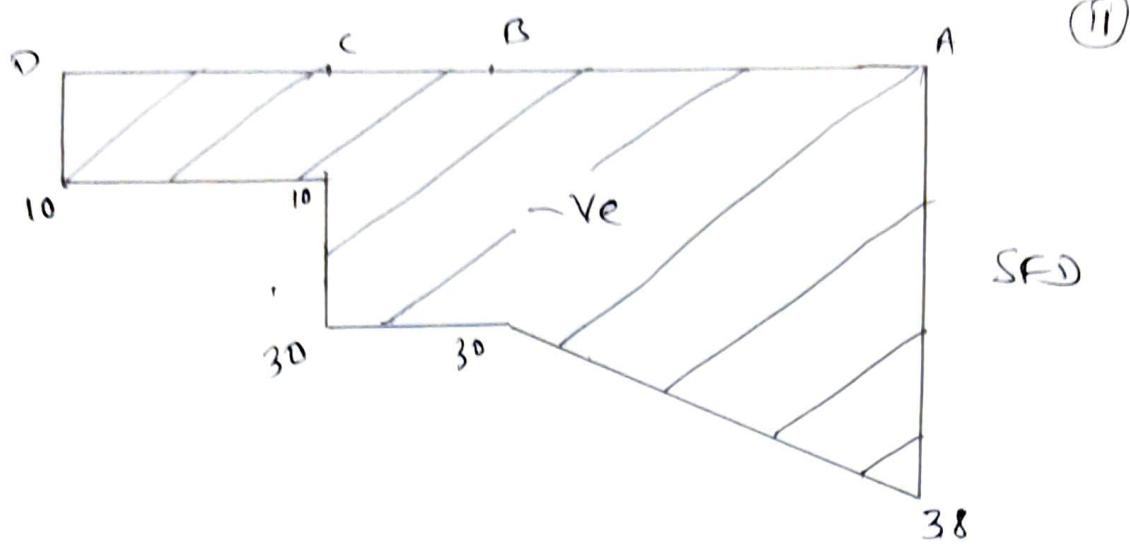
$$\text{SF} @ \text{before } A = -30 - 8 = -38$$

$$\text{SF} @ A = -38 + 38 = 0$$

$$\text{BM} @ D = 0, \text{BM} @ C = -20, \text{BM} @ B = -50$$

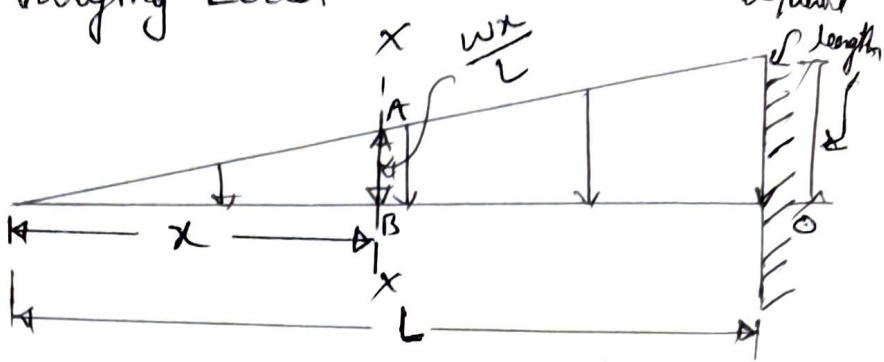
$$\text{BM} @ \text{before } A = -10 \times 5 - 20 \times 3 - 8 \times 1 = -118 \text{ KN-M}$$

$$\text{BM} @ A = -118 + 118 = 0$$



② Cantilever beam subjected to UVL over entire length (12)

UVL - Uniformly Varying Load



Magnitude of the UVL @  $x = 0$  =  $w/\text{unit length}$ .

$\therefore$  magnitude @  $x = L$  is calculated as follows

$$\text{At } x = L, \frac{w}{L} = \frac{AB}{x} \Rightarrow AB = \frac{wL}{2}$$

$$SF @ x = L = -\frac{1}{2} x \times x \times \frac{wx}{L} = -\frac{wx^2}{2L}$$

$$\text{At } x = 0, SF = 0$$

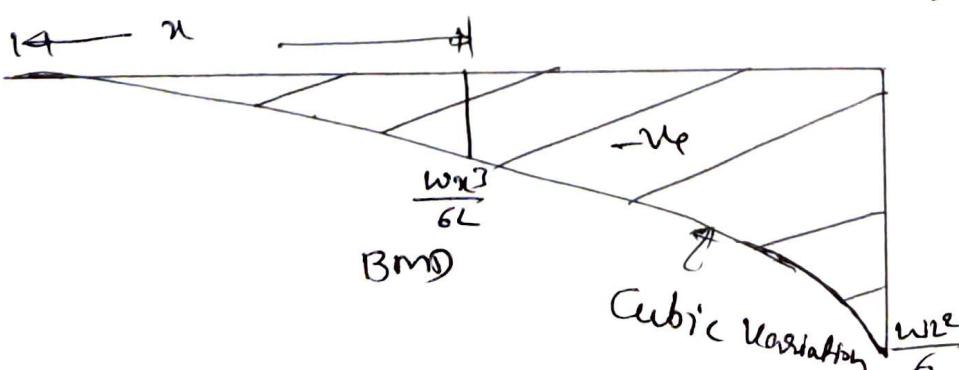
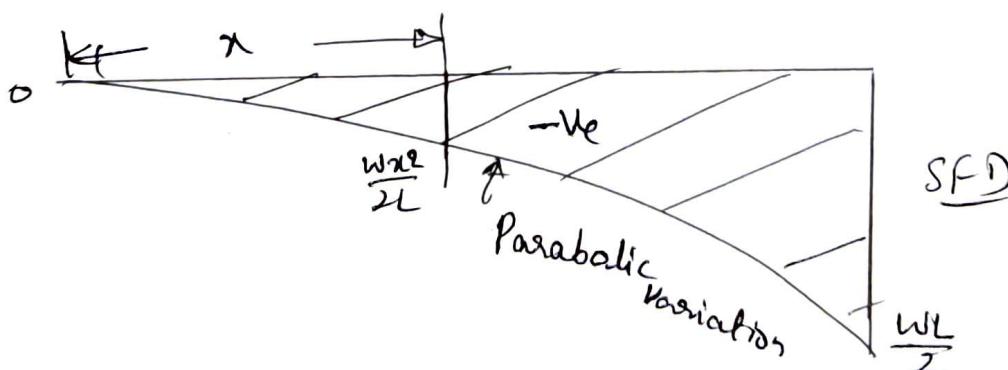
$$\text{At } x = L, SF = -\frac{wL^2}{2L} = -\frac{wL}{2}$$

$$BM @ x = L = -\frac{wx^2}{2L} \times \frac{x}{3} = -\frac{wx^3}{6L}$$

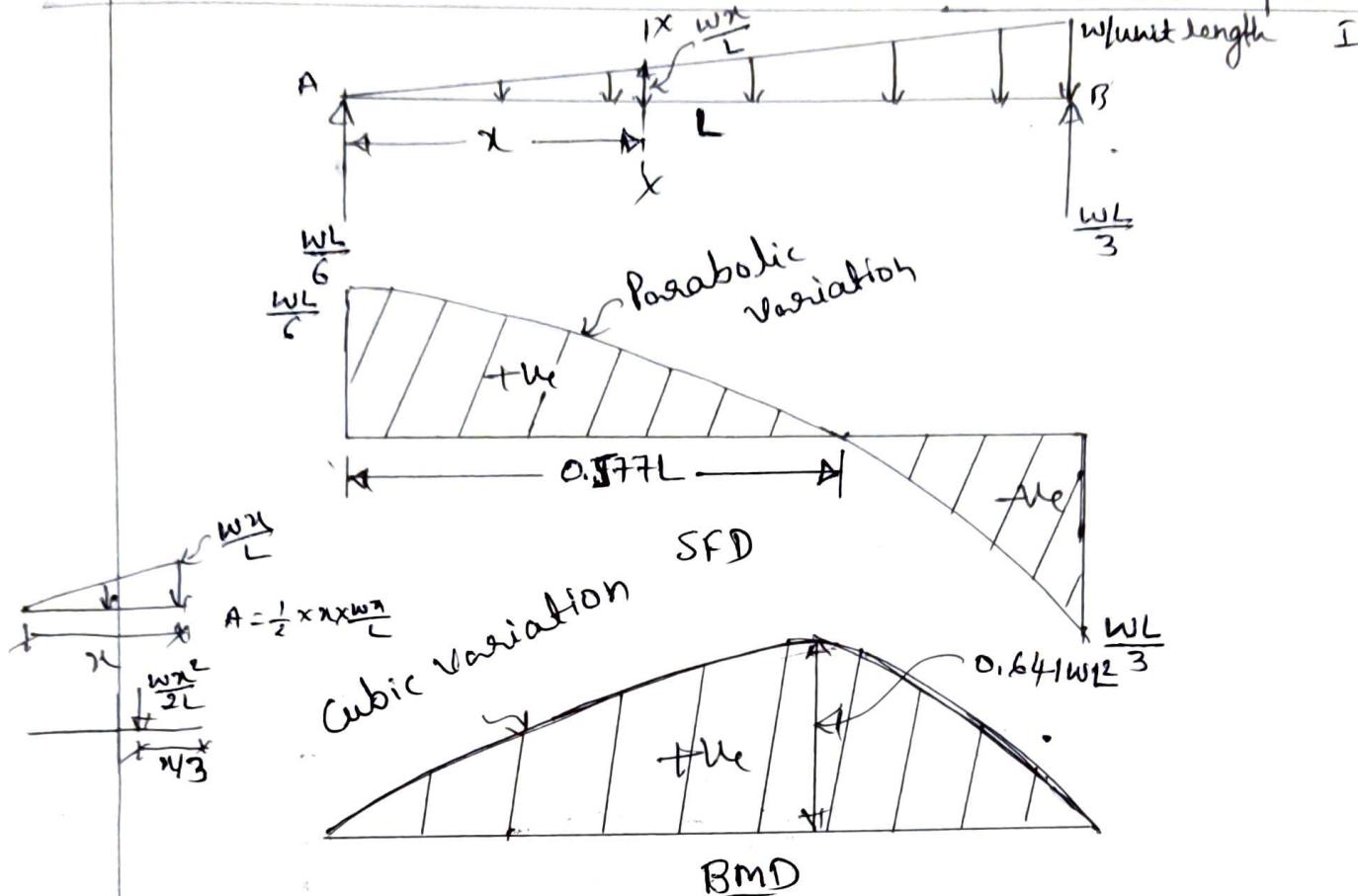
(Cubic Variation)

$$\text{At } x = 0, BM = 0$$

$$\text{At } x = L, BM = -\frac{wL^3}{6L} = -\frac{wL^2}{6}$$



: Simply Supported beam subjected to UVL over entire span:



$$SF @ x = \frac{WL}{6} - \frac{wx^2}{2L}$$

$$\text{at } x=0, SF @ A = \frac{WL}{6}, \quad x=L, SF @ B = \frac{WL}{6} - \frac{WL}{2L} \\ = \frac{WL}{6} - \frac{WL}{2} = \frac{WL-3WL}{6} \\ = -\frac{2WL}{6} = -\frac{WL}{3}$$

since in case of WDL, SF is zero at  $x = \frac{L}{2}$

But in UVL SF is zero at a distance  $x$  from end A.

$$SF @ x = 0$$

$$\Rightarrow \frac{WL}{6} - \frac{wx^2}{2L} = 0$$

$$\Rightarrow \frac{wx^2}{2L} = \frac{WL}{6} \Rightarrow x^2 = \frac{WL}{6} \times \frac{2L}{w} = \frac{2L^2}{6}$$

$$\Rightarrow x^2 = \frac{2L^2}{6}$$

Taking square root on both sides

$$\sqrt{x^2} = \sqrt{\frac{2L^2}{6}} \Rightarrow x = \frac{L}{\sqrt{3}} = 0.577L$$

∴ SF is zero at a distance  $0.577L$  from end A

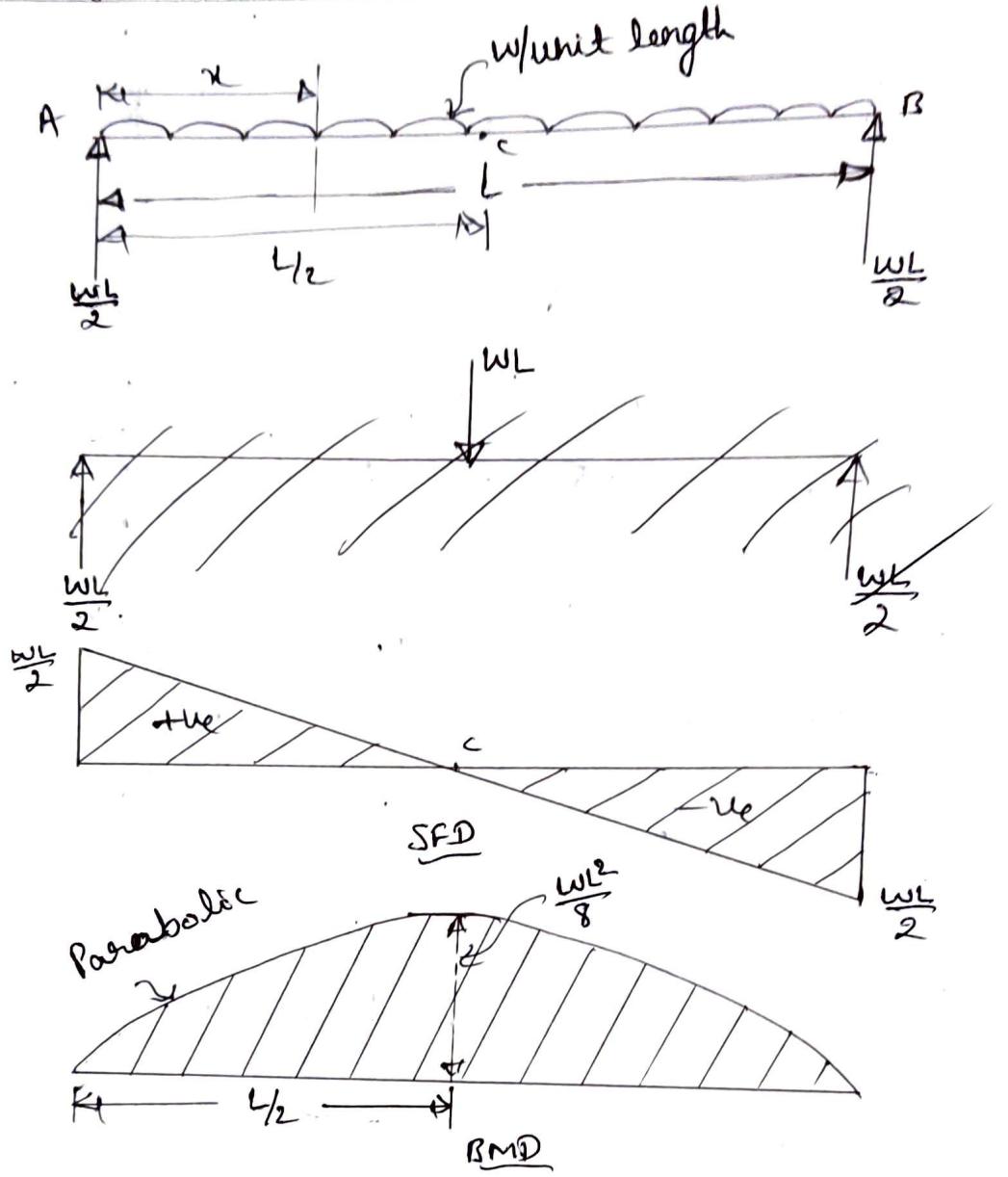
$$BM @ x = \frac{WLx}{6} - \frac{wx^3}{2L} \times \frac{x}{3} = \frac{WLx}{6} - \frac{wx^3}{6L}$$

$$BM @ x = 0.577L = \frac{WL \times 0.577L}{6} - \frac{w(0.577L)^3}{6L}$$

$$\Rightarrow BM @ x = 0.0961WL^2 - 0.032WL^2 = 0.064WL^2$$

$$= 0.064WL^2$$

\* Simply supported Beam subjected to UDL entire



\* BM is max at a pt where SF is zero

$$\text{SFD: } SF @ x = \frac{WL}{2} - wx$$

$$@ x=0, SF @ A = \frac{WL}{2} - 0 = \frac{WL}{2}$$

$$@ x=L, SF @ B = \frac{WL}{2} - WL = -\frac{WL}{2}$$

$$@ x = \frac{L}{2}, SF @ C = \frac{WL}{2} - \frac{WL}{2} = 0$$

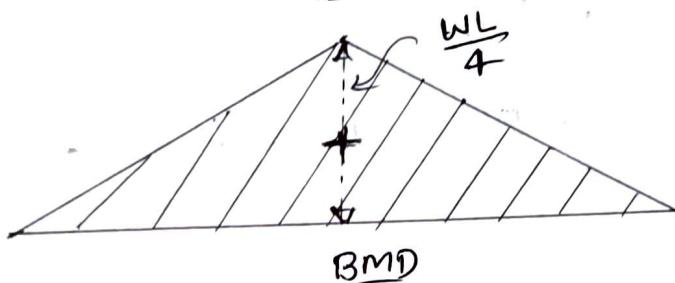
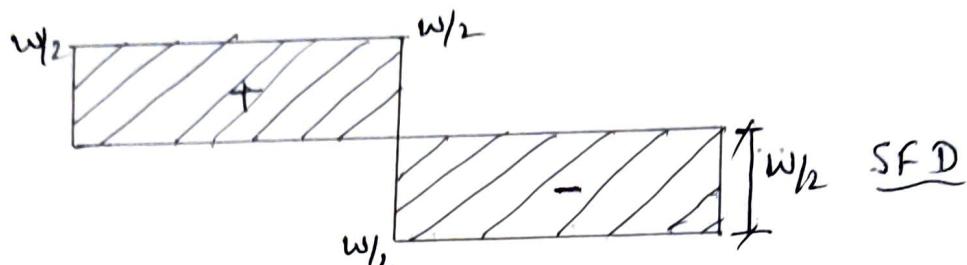
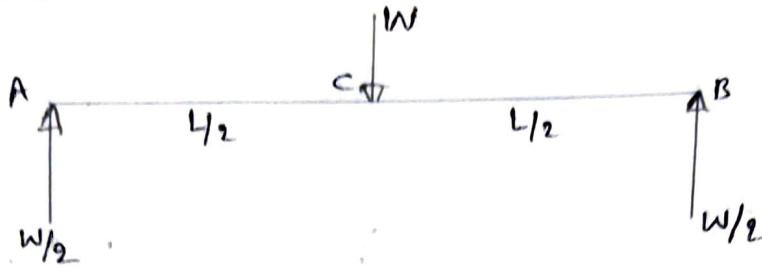
$$\text{BMD: } BM @ x = \frac{WLx}{2} - \frac{wx^2}{2}$$

$$@ x=0, BM @ A = 0$$

$$@ x=L, BM @ B = \frac{WL^2}{2} - \frac{WL^2}{2} = 0$$

$$@ x = \frac{L}{2}, BM @ C = \frac{WLxL}{2 \times 2} - \frac{wxL^2}{2} \\ = \frac{WL^2}{4} - \frac{WL^2}{8} \\ = \frac{2WL^2 - WL^2}{8} = \frac{WL^2}{8} //$$

: Simply supported beam with point load @ centre



SFD:  $SF @ A = \frac{w}{2}, SF @ \text{before } C = \frac{w}{2}$

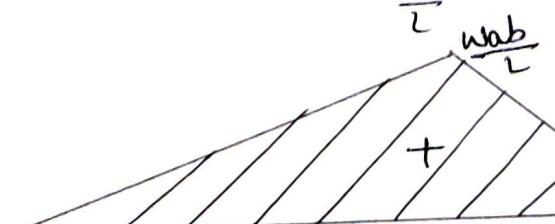
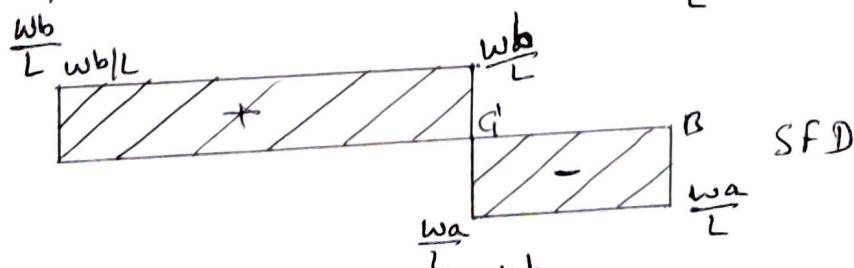
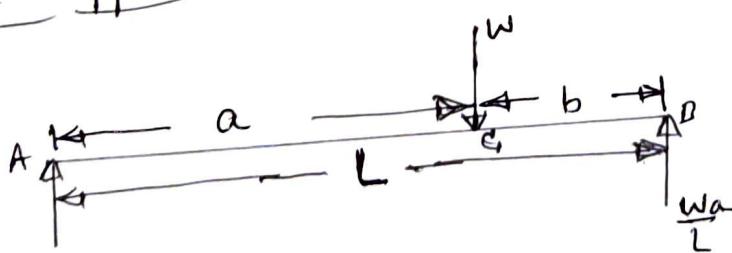
$$SF @ C = \frac{w}{2} - w = \frac{w - 2w}{2} = -\frac{w}{2}$$

$$SF @ \text{before } B = -\frac{w}{2}, SF @ B = -\frac{w}{2} + \frac{w}{2} = 0$$

BMD:  $BM @ A \neq @ B = 0$

$$BM @ C = \frac{w}{2} \times \frac{L}{2} = \frac{WL}{4}$$

+ simply supported beam, subjected to eccentric load



From SFD & BMD max BM occurs @ II where the SF changes sign

Simply supported beam subjected to semi-point loads (Q5)

