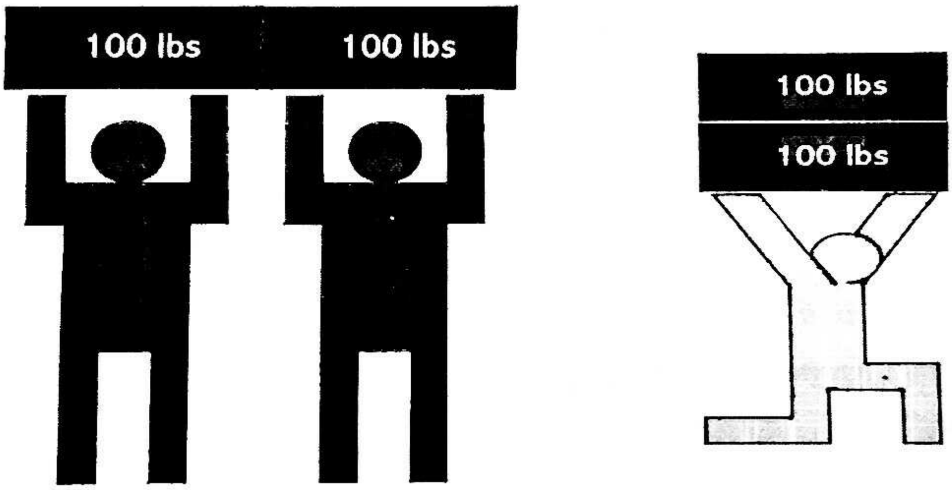
Simple Stresses and Strains

Introduction:

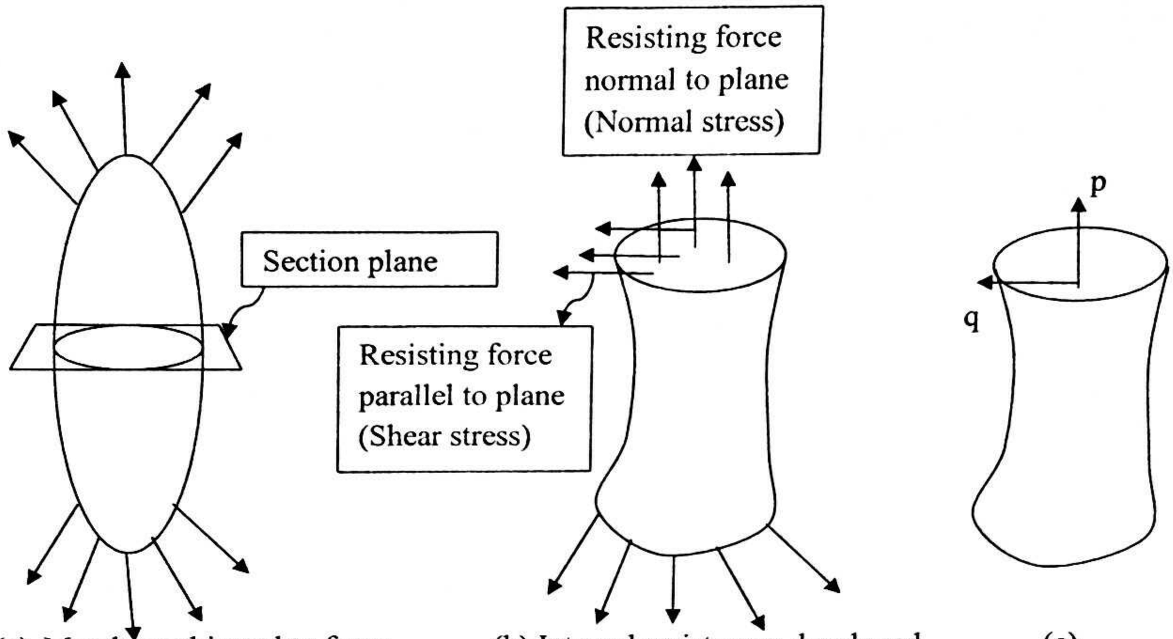
Strength of material is a basic course for almost all the engineering branches. Many structural elements like bars, tubes, beams, columns, trusses, cylinders, spheres, shafts are used for the benefit of mankind. They may be made up of timber, steel, copper, aluminium, concrete or any other materials. The application of laws of mechanics to find the support reactions due to applied loads is covered under the subject engineering mechanics. In transferring, these forces from their point of application to supports the material of the structure develops resisting forces and undergoes deformation. The effect of these resisting forces on the structural element is treated under the subject Strength of materials / mechanics of solids.

General meaning of stress: **Stress = Force /Area**



* (When a material or member is subjected to external load, it undergoes deformations (*Deformation* of the material is the change in geometry created when load is applied). While undergoing deformations, the particles of the material exert a resisting or internal force. When this resisting force is equal to the applied load, an equilibrium condition takes place and deformation stops. This internal resistance is called '**stress**'. The internal resistance per unit area is called intensity of stress.)

To find the resisting force developed by the particles of material or member a section plane is passed (Fig. a) through the member and then consider the equilibrium of any one part. Each part is in equilibrium under the action of applied forces and internal resisting forces. The resisting forces may be split into normal and parallel to the section plane. The resisting force normal or perpendicular to the sectional plane is called '**intensity of normal stresses.** And the resisting force parallel to the sectional plane is called '**shearing resistance or intensity of shear stress' (ref. Fig b).**



(a) Member subjected to force (b) Internal resistances developed (c)

In practice intensity of stress is called as "**stress**" only. Mathematically

$$\textbf{Normal stress} = \mathbf{p} = \lim_{\Delta A-0} \Delta R/\Delta A = \mathrm{d}R/\mathrm{d}A$$

Where R is the normal resisting force.

$$\textbf{Shearing stress} = \mathbf{q} = \lim_{\Delta A-0} \Delta Q/\Delta A = \mathrm{d}Q/\mathrm{d}A$$

Where Q is the shearing resistance force.

- **Thus stress at any point may be defined as internal resistance developed per unit area.**

At any cross section, stress developed may or may not be uniform. In a bar of uniform cross-section subject to axial concentrated loads as shown in **Fig.2 a** below, the stress is uniform at a section away from the applied loads **Fig.2 b.** But there is a variation of stress at the section near the applied loads **Fig. 2 c.**

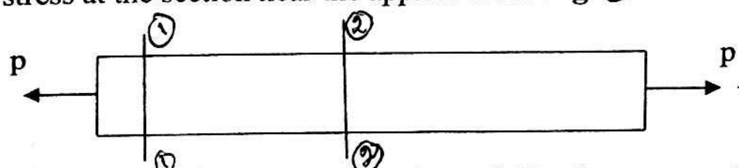


Fig. 2 a: bar subjected to axial load

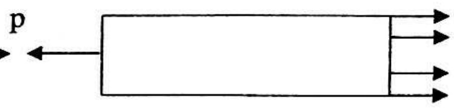
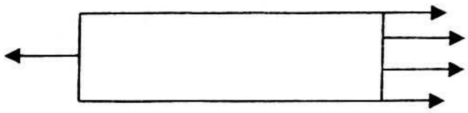


Fig. 2 b: variation of stresses
(Section at 2-2)

away from ends



(Section @ ①-①)

Fig. [illegible] c: variation of stresses near ends

Unit of stress: N/mm^2, KN/mm^2, N/m^2, KN/m^2

- A stress of **1 N/m^2** is known as **Pascal** and is represented by **'Pa'**
- Hence **1 MPa** = $1MN/m^2 = 1 \times 10^6\ N/(1000mm)^2 = 1\ N/mm^2$
- Thus 1 MPa = 1 N/mm^2

Types of stresses:

There are basically two types of stresses namely,

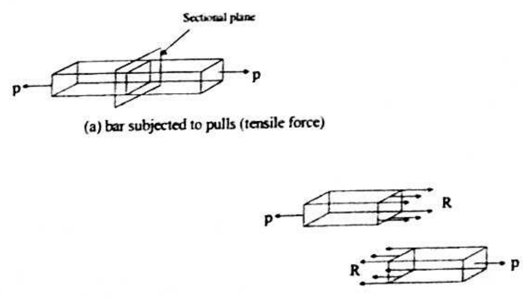
1. Normal stress
   - Tensile stress
   - Compressive stress
2. Shear stress or tangential stress

Normal stress: It is a stress acting normal or perpendicular to the sectional plane of the member. These are of two types, tensile and compressive stress.

Tensile stress: The stress which is causing extension or elongation of the bar in the direction of applied load is called as tensile stress.

Following figure shows the bar subjected to tensile force 'P'. To maintain the equilibrium the end forces applied must be the same say P.

Tensile stresses



(a) bar subjected to pulls (tensile force)

(b) Resisting force developed

Resisting forces acting on a section are shown in Fig. (b). now since the stresses are uniform,

$$R = \int p\,dA = p\int dA = pA$$

Where A is the cross sectional area.

Considering the equilibrium of a cut piece of the bar, we get

$$P(\text{applied force}) = R(\text{internal resistance})$$

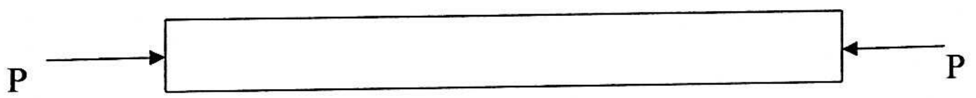
But R = pA

$$P = pA$$

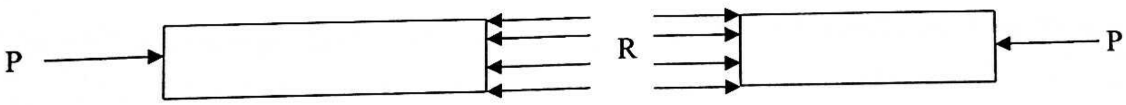
$$p\ (\text{tensile stress}) = P/A$$

Where p is tensile stress, P is applied tensile force and A is cross sectional area.

Compressive stresses: The force which is causing shortening of the bar is called compressive force and stresses caused by these forces are called compressive stresses.



(a) bar subjected to compressive forces



(b) Resisting force deeloped

Now since stresses are uniform, for equilibrium

$$P = R$$

$$P = pA$$

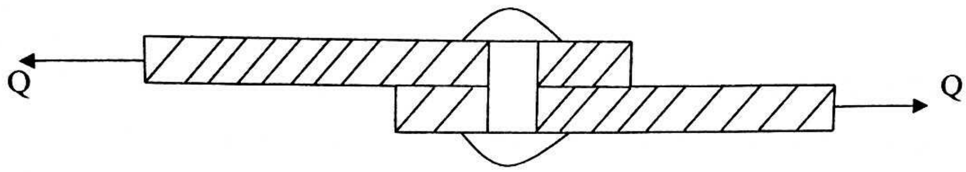
$$p\ (\text{compressive stress}) = P/A$$

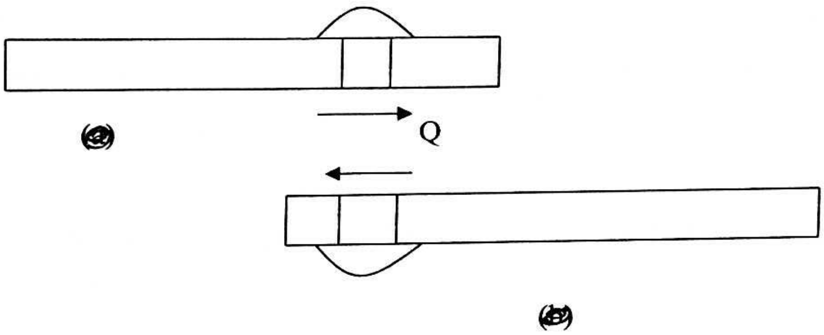
where P is compressive force and p is compressive stress.

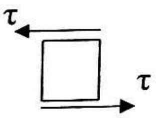
Thus whether it is tensile or compressive force, the stress developed in a bar subjected to axial forces, is equal to force per unit area.

Therefore tensile stress $= \dfrac{\textit{tensile force}}{\textit{cross sectional area}}$, **Compressive stress** $= \dfrac{\textit{compressive force}}{\textit{cross sectional area}}$

Shear stress: When two parts of a body exert an equal and opposite force on each other in a direction tangential to their surface of contact, shear stress is said to exist between the two parts.







(c)

In the figure the section of the rivet or bolt is subjected to equal and opposite forces Q (shearing forces, forces parallel to the section of rivet).

Let Q – shearing force and τ be the shearing stress.

Then the direct shear stress is equal to shearing force per unit area.

$$\therefore \tau = \frac{\text{shearing force}}{\text{c/s area}} = \frac{Q}{A}$$

Materials may be classified into

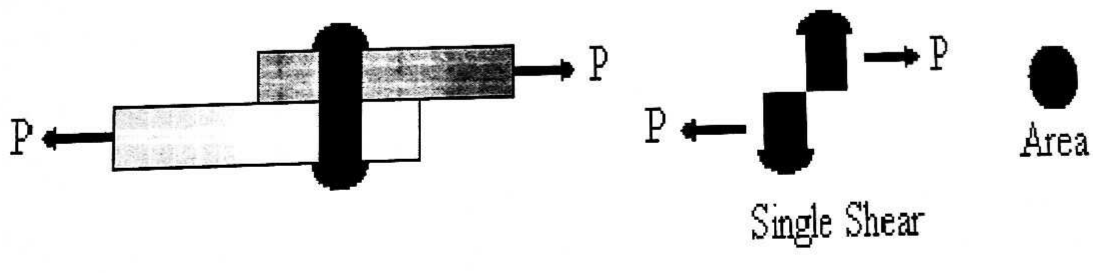
1. **Elastic material,** which undergoes a deformation when subjected to an external loading such that, the deformation disappears on the removed of the loading, (Rubber).
2. A plastic material undergoes a continuous deformation during the period of loading and the deformation is permanent and the material does not regain its original dimensions on the removal of the loading, (Aluminum).
3. A rigid material does not undergo any deformation when subjected to an external loading,

(Glass and Cast iron).

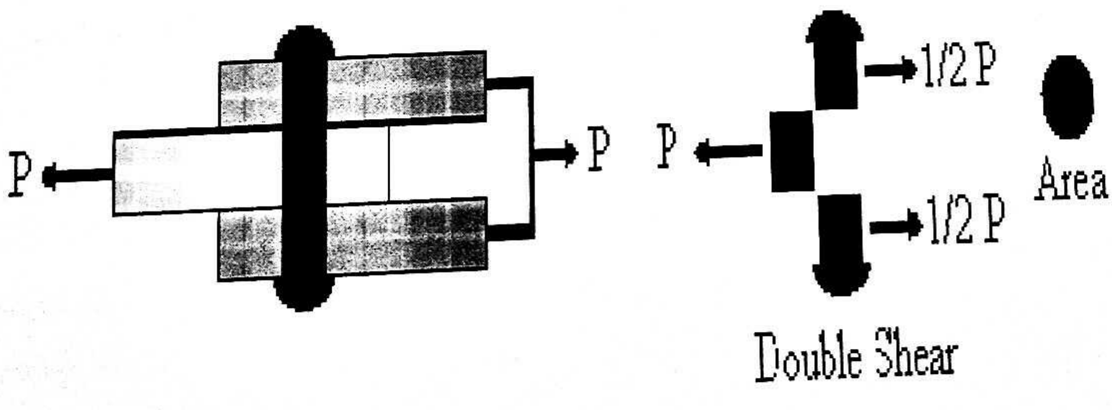
Loads

1. Dead loads: static in nature such as the self weight of the roof.
2. Live loads: fluctuating in nature, does not remain constant such as a weight of a vehicle moving on a bridge.
3. Tensile loads.
4. Compressive loads.
5. Shearing loads.

Single Shear: One section of rivet is subjected to shearing stress.



Double Shear: Two sections are subjected to shearing stress.



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PROPERTIES OF MATERIALS:

Hardness: Resistance of a material to deformation, indentation, or penetration by means such as abrasion, drilling, impact, scratching, and wear, measured by hardness tests such as Brinell, Rockwell,Vickers.
Hardwood does not mark as easily as softwood.

Strength : Amount of force needed to break a material usually by pushing or pulling down.

Toughness : toughness is the ability of a material to absorb energy and plastically deform without fracturing. Or toughness is the amount of energy per unit volume that a material can absorb before rupturing. Or It is also defined as a material's resistance to fracture when stressed

Stiffness : Amount of force needed to change the shape of a material. or The ability to *resist bending*

Elasticity: Ability to return its original shape when a force is removed eg rubber band.

Or The ability of a material to return to its original form after a load has been applied and removed. Good examples include rubber, mild steel and some plastics such as nylon.

Plasticity: Ability to retain the new shape when a force is removed eg Steel.

The materials which deform permanently when small forces are applied, show plasticity

Ductility : Ability of a material (such as a metal) to undergo permanent deformation through elongation (reduction in cross-sectional area) or bending at room temperature without fracturing. Opposite of brittleness.

Ductile materials can be stretched without breaking and drawn into thin wires.

Example - **Aluminum, copper, mild steel, platinum, etc.**

Malleability: is the ability of a metal to be hammered into thin sheets. Gold and silver are highly malleable. When a piece of hot iron is hammered it takes the shape of a sheet. The property is not seen in non-metals. Non-malleable metals may break apart when struck by a hammer.

Brittleness: Tendency of a material to fracture or fail upon the application of a relatively small amount of force, impact.Oppositeof toughness.
Example- Glass
Compressive strength: The ability to withstand *pushing* or squeezing forces (compression).

Tensile strength: The ability to withstand *pulling* or stretching forces (tension).

Density: Density is mass per unit volume

<u>Strain</u>: when a load acts on a material, it will undergo deformation. strain is a measure of the deformation produced by the application of external forces.

No material is perfectly rigid. under the action of forces a rubber undergoes changes in shape & size. Even for small forces deformations are quite large. Actually all materials including steel, Cast iron, brass, concrete, etc, undergo similar deformation when loaded But the deformations are very small & hence we cannot see them with naked eye. There are instruments like extensometer, electric strain gauges which can measure extension of magnitude $\frac{1}{100^{th}}$, $\frac{1}{1000^{th}}$ of a millimeter.

There are machines like universal testing machines in which bars of different materials can be subjected to accurately known forces of magnitude as high as 1000 KN. The studies have shown that the bars extend under tensile forces & shorten under compressive forces as shown in fig below.

strain is measured as the ratio of change in length divided by the original length. It is usually denoted by e or ϵ.

If 'l' is the Original length & Δl is the change in length, then

$$\text{strain} = e = \frac{\Delta l}{l}$$

The change in length per unit is known as linear strain.

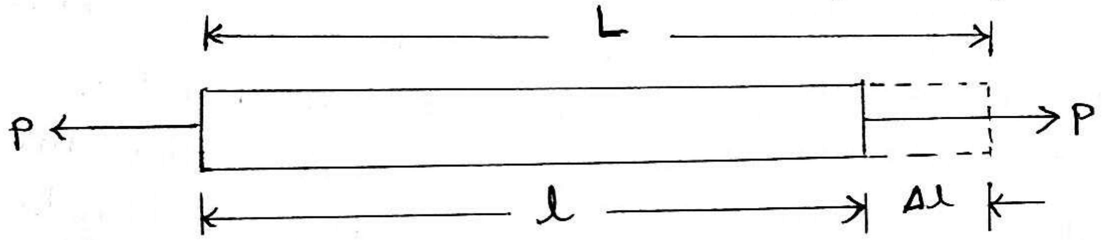
Tensile strain:

Let 'l' be the initial length of the bar before the application of the load. After a tensile load 'P' is applied, let its length be 'L'

Then change in length (extension) $= L - l = \Delta l$

Therefore extension or elongation per unit length is defined as tensile strain

$$\Rightarrow \text{Tensile strain} = \frac{\text{Extension}}{\text{Original length}} = \frac{\Delta l}{l}$$



Compressive strain:

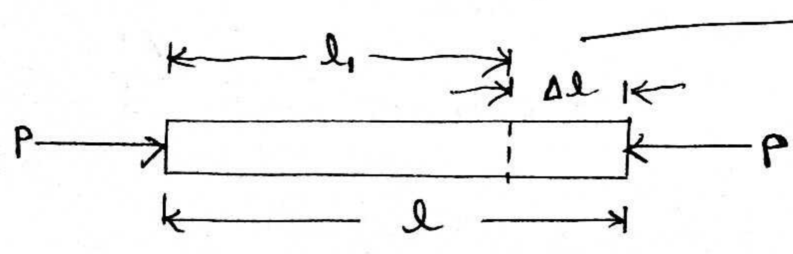
Consider a uniform bar of initial length l, which is subjected to compressive force 'P', after the application of load 'P' the length reduces to l_1 as shown in fig below. Then

shortening or change in length $= \Delta l = l - l_1$

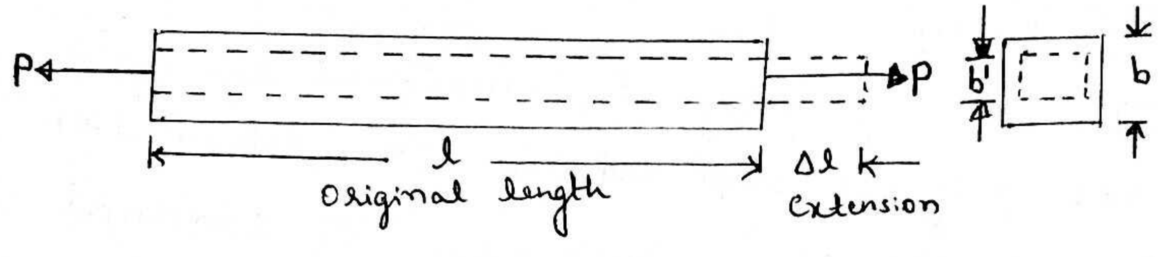
Therefore compressive strain is defined as shortening per unit length.

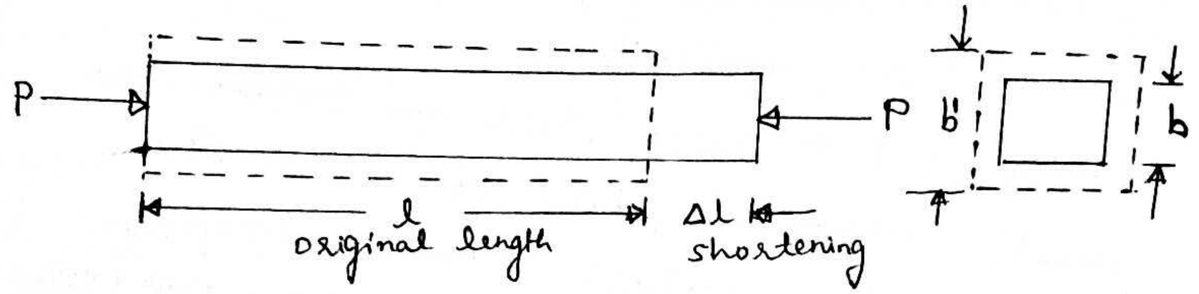
$$\therefore \text{compressive strain} = e = \frac{\text{shortening of bar}}{\text{original length}}$$

$$\Rightarrow e = \frac{\Delta l}{l}$$



Lateral strain :





when changes in longitudinal direction is taking place, changes in lateral direction also takes place. The nature of these changes in lateral direction are exactly opposite to that of changes in longitudinal direction ie, if extension is taking place in longitudinal direction, the shortening of lateral dimension takes place & if the shorte-ning is taking place in longitudinal direction as extension takes place in lateeal direction as shown in fig above.

Therefore lateral strain may be defined as changes in the lateral dimension per unit lateral dimensi-ons.

Thus

$$\text{lateral strain} = \frac{\text{change in lateral directions}}{\text{Original lateral dimension}}$$

$$= \frac{b' - b}{b} = \frac{\Delta b}{b}$$

Unit of strain : strain has no unit. It is expressed as a percentage or in microstrain (μS).

$1 \mu S$ is an extension of one part/million.

A strain of 0.2% is equal to $2000 \mu S$.

Difference between Stress and Pressure	
Stress	**Pressure**
Stress can be defined as the internal resistive force to the deformation per unit area.	Pressure can be defined as the amount of force applied per unit area.
Stress can be represented as (strain) / (Young's modulus)	Pressure can be mathematically represented as (force) / (area)
Due to stress, the pressure will not be developed.	Due to pressure, stress will be developed.
Stress can be either a positive or a negative force	The pressure is always a positive force
Pressure is exerted externally	Stress is developed internally
Stress may be tensile, compressive and shear	Pressure is always compressive

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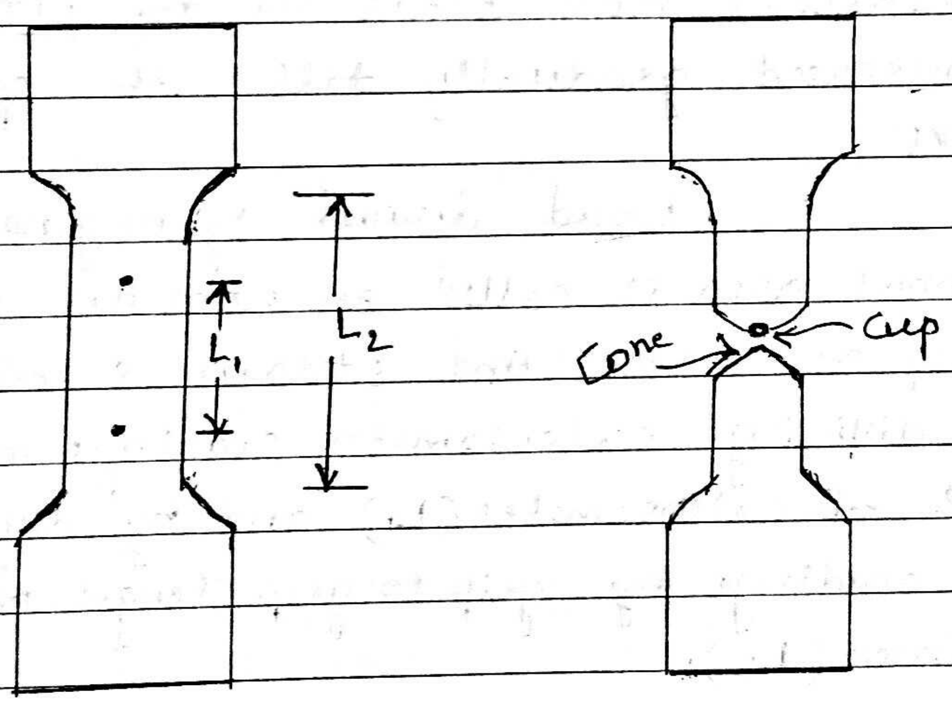
Stress, strain relation

The stress-strain relation of any material is obtained by conducting tension test in the laboratories on standard specimen. Different materials behaves differently and their behavior in tension & in compression differ slightly.

: Behaviour in Tension :

Stress-strain Curve for Mild steel
[Ductile Material]



Tension Test Specimen — Tension Test specimen after Breaking

Fig. above shows a typical tensile test specimen of mild steel. It's ends are gripped into Universal testing Machine.

Extensometer is fitted to test specimen which measures the extension over the length L_1, shown in fig. The length over which exte-nsion is measured is called gauge length. The load is applied gradually and at regular intervals of loads extension is measured. After certain load, extension increases at faster rate and the capacity of extensometer to measure the extension comes to an end & hence it is removed before this stage is reached and extension is measured from scale on the UTM. Load is increased gradually till the specimen breaks.

Load divided by original cross-sectional area is called an nominal stress or simply as stress. And strain is obtained by dividing extensometer readings by gauge length of extensometer (L_1) and by dividing scale readings by grip to grip length of the specimen (L_2)

The following fig. shows the stress-strain diagram for the mild steel specimen.

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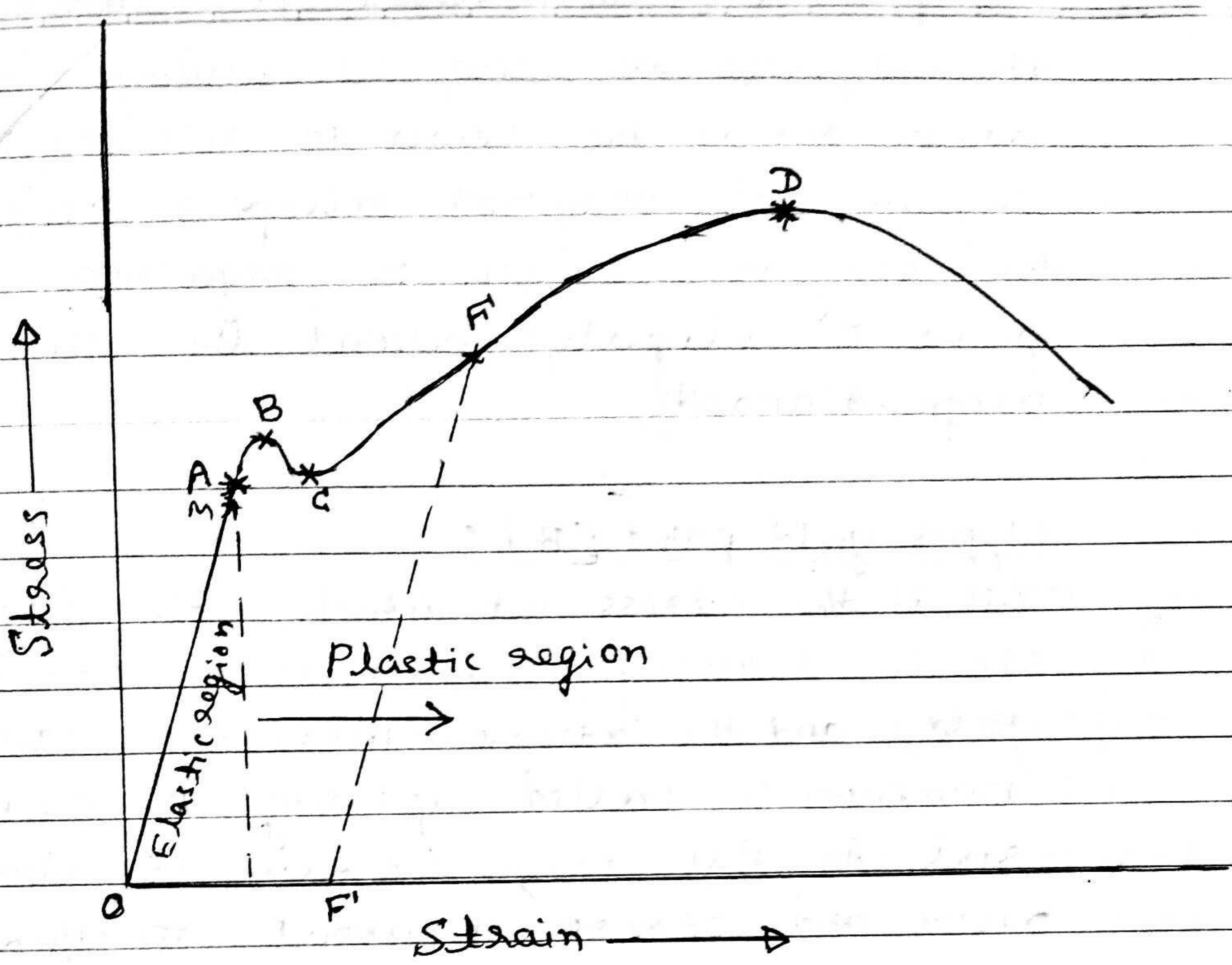
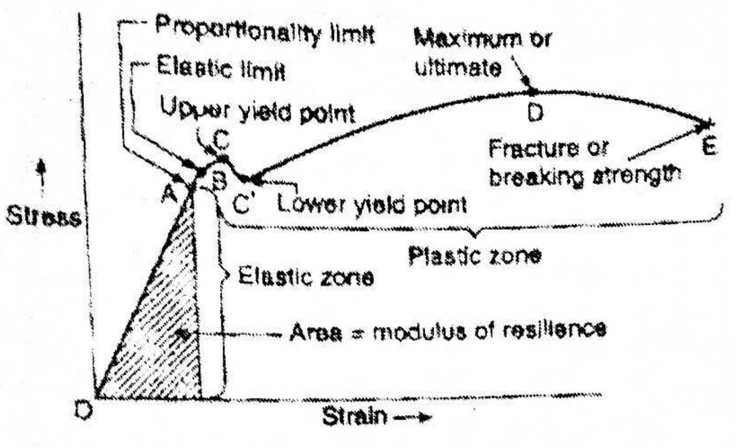


The following important points are observed from stress-strain curve

Limit of proportionality [A]:
It is the limiting value of the stress up to which stress is proportional to strain.

Elastic limit: (A)
This is the limiting value of the stress up to which if the material is stressed (loaded) and then released (unloaded) strain disappears


Proportionality limit
Elastic limit
Maximum or
ultimate
Upper yield point
C
D
A
B
Fracture or
breaking strength
E
C'
Lower yield point
Stress
Plastic zone
Elastic zone
Area = modulus of resilience
O
Strain

completely [means when the material is subjected to load, it undergo deformation and if the load is released, it regains it's original shape & size] and the original length is regained. This point is slightly beyond the limit of proportionality.

Upper yield point [B]:
This is the stress at which, the load starts reducing (means material starts to yield) and the extension increases. This phenomenon is called yielding of the material. At this stage strain is about 0.125% and stress is about 250 N/mm²

Lower yield point [C]:
At this stage the stress remains same but strain increases for some time.

Ultimate stress [D]:
This is the maximum stress the material can resist. This stress is about 370-400 N/mm². At this stage the cross sectional area at a particular section starts reducing very fast, this is called neck formation

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Breaking stress [E] :

This is the stress at which finally the specimen fails. At this stage strain is about 20 to 25%.

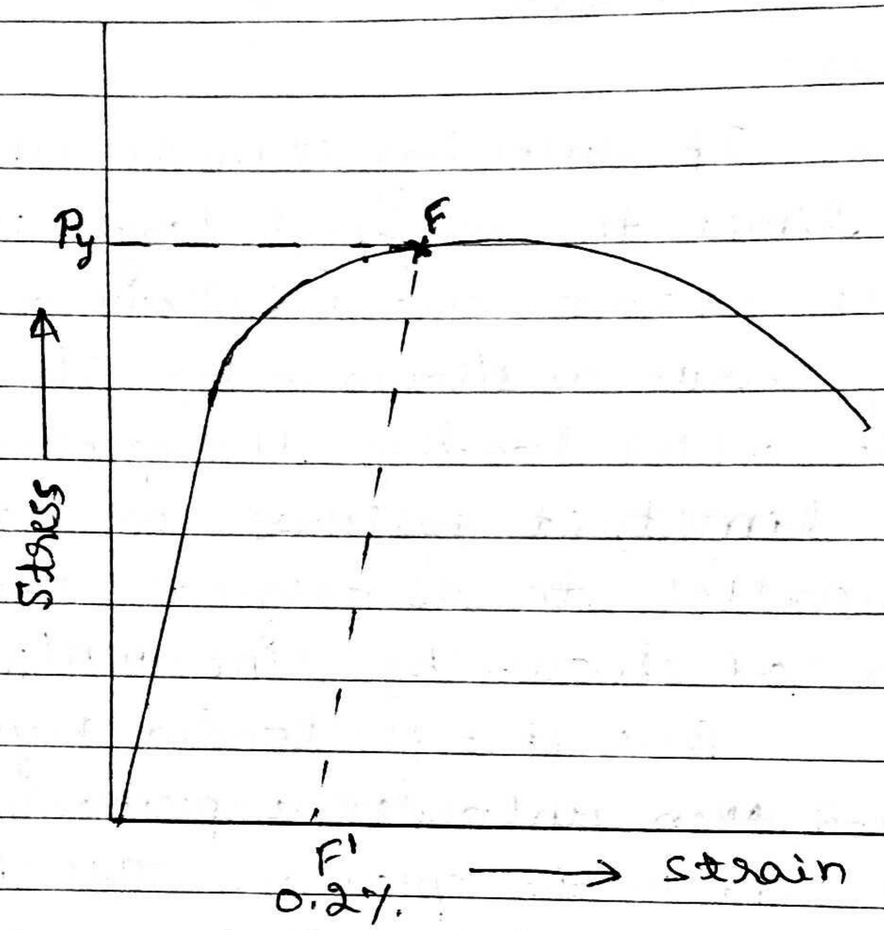
If unloading is made within elastic limit the original length is regained i.e. stress - strain curve follows down the loading curve as shown in fig. If unloading is made after loading the specimen beyond elastic limit, it follows a straight line parallel to the original straight portion as shown by line FF' in fig.

Thus if it is loaded beyond elastic limit and then unloaded a permanent strain (OF') is left in the specimen. This is called permanent set

Stress - strain relation in aluminium and high strength steel :

In these elastic materials there is no clear cut yield point. The necking takes place at ultimate stress and eventually the breaking point is lower than the ultimate point.

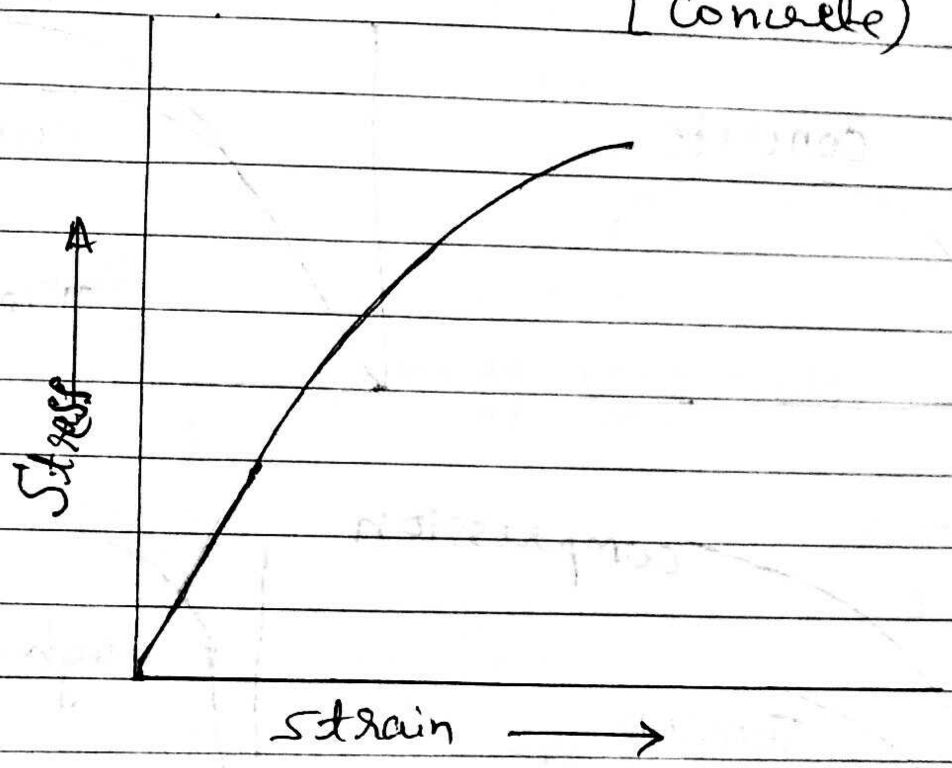
The typical stress- strain diagram is shown in fig. below.



stress- strain relation in aluminum & high strength steel

The stress P_y at which if unloading is made there will be 0.2% permanent set is known as 0.2% proof stress and this point is treated as yield point for all practical purposes.

Stress - strain relation in brittle material
(Concrete)



The typical stress- strain relation in a brittle material like Cast iron is shown in fig above.

In these materials

→ There is no appreaciable change in rate of strain

→ There is no yield point

→ & No. necking takes place

→ Ultimat point & the breaking point are one and the same.

→ The strain at failure is very small

Grade of Steel & their minimum % elongation

Grade	Minimum % elongation
Fe250	23%
Fe415	14.5%
Fe500	12%
Fe550	8%

Engg Materials

- Metals
  - Ferrous
    - Iron
    - Pig-Iron
    - Cast-Iron
    - Steel
  - Non-Ferrous
    - Copper & alloys
    - Aluminium
    - Zinc
    - Tin, Lead, gold
- Non-Metals
  - Rubber
  - Plastic
  - Resin.

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Percentage elongation :

It is used to measure the ductility of material.

It is defined as the ratio of the final extension at rupture to original length, expressed as percentage.

Thus Percentage elongation $= \frac{L_1' - L}{L} \times 100$

where $L \rightarrow$ original length

L_1' – Length at rupture

The code specify that the original length (gauge length) is to be five times the diameter and the portion considered must include neck (whenever it occurs). Usually marking are made on tension rod at every '2.5d' distance and after failure the portion in which necking takes place is considered.

In case of ductile material % elongation is 20 to 25.

Percentage reduction in Area :

It is defined as the ratio of maximum changes in the cross-sectional area to original cross-sectional area, expressed as percentage,

Thus % reduction in area = $\frac{A - A'}{A} \times 100$
where A - original c/s area
A' - Minimum c/s area at failure
A' is calculated after measuring the diameter at the neck. For this, the two broken pieces of the specimen are to be kept joining each other properly.

For steel the % reduction in area is 60 to 70.

Behaviour of materials under compression.
For compression tests short specimens are used. Because in case of long specimen there is a chance of buckling or lateral bending of the specimen.

Some experiments (compression tests on materials) have shown the following results.
→ In case of ductile material stress-strain curve is same as that of in tension test up to ~~&~~ even slightly beyond yield point.
→ There is no necking in case of compression tests

→ For most brittle materials ultimate compressive stress in compression is much larger than in tension

Nominal stress and True stress :-

As we know that direct stress (Normal stress) is based on the value obtained by dividing the load by original cross-sectional area. For this reason the value of stress started dropping after neck is formed in mild steel as shown in fig ie stress- strain curve for mild steel.

But actually as material is stressed it's cross-sectional area changes. ~~We~~ should divide load by the actual cross-sectional area to get real stress in the material.

To distinguish between the two value's we introduce the terms nominal stress and true stress & are given by,

$$\text{Nominal stress} = \frac{\text{Load}}{\text{Original cross-sectional area}}$$

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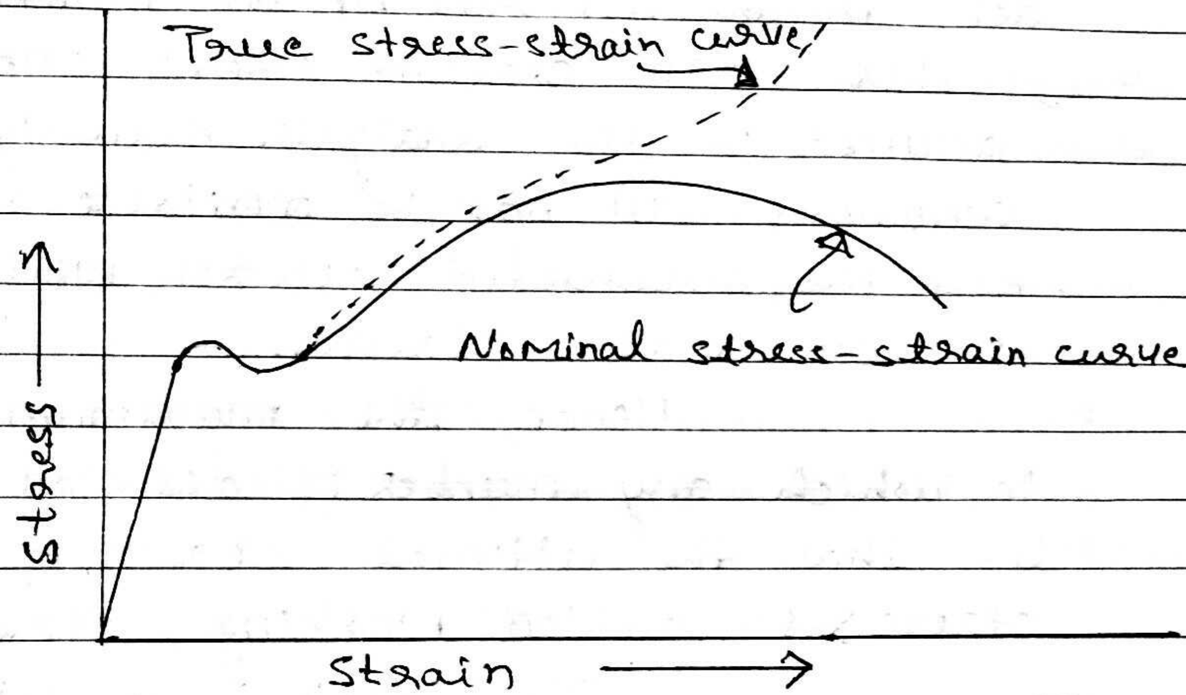
$$\text{True stress} = \frac{\text{Load}}{\text{Actual cross sectional area}}$$



Nominal & true stress-strain curve for mild steel

Factor OF Safety:

In practice it is not possible to design a mechanical component or structural component permitting stre-ssing up to ultimate stress for the following reasons

→ Reliability of material may not be 100%

→ The resulting deformation may obstruct the functional performance of the compo-nent

→ The loads taken by the designer are only estimated loads, occationally

Hook's Law :

An important relationship between stress and strain is given by Hook's law. Which states that stress is proportio-nal to strain so long as the mate-rial behaves elastically.

Statement ; Stress is propostional to strain within the elastic limit.

In stress-strain curve of mild steel or any ductile material, the linear portion (straight line) follows or obeys the hook's law.

Thus stress $\propto$ strain

$\Rightarrow$ stress = a constant $\times$ strain

$$p = E \times e$$

where p is stress, e - strain

& E - constant of proportionality called as (of material) modulus of elasticity or young's modulus of elasticity

$$\text{i.e. } E = \frac{\text{stress}}{\text{strain}} = \frac{p}{e}$$

E can be expressed in N/mm^2, Pa, MPa, GPa etc. $\left[E = \frac{\text{stress}}{\text{strain}} = \frac{N/mm^2}{\text{no unit}} = N/mm^2 \right.$

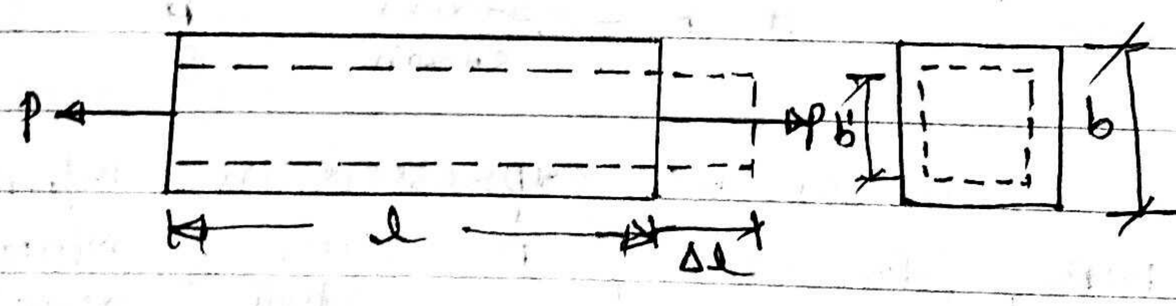
E for concrete = $5000\sqrt{fck}$

fck - 28 days compressive strength

E for steel = 2×10^5 N/mm^2 [For all steel]

POISSON'S RATIO:

Let us consider an member (bar) shown in fig subjected to tensile load 'P' it undergoes changes in length, also undergoes changes of opposite nature in lateral directions.



As we know linear strain is defined as the ratio of change in axial direction (length i.e Δl) to original length. And lateral strain is the ratio of change in lateral directions to the original lateral dimension.

Thus Poisson's ratio is a constant ratio between the lateral strain and linear strain, exists within the elastic limit.

$$\text{Thus Poisson's ratio} = \frac{\text{Lateral strain}}{\text{Linear strain}}$$

It is denoted by $\frac{1}{m}$ or μ.

For most of the metals this value is between 0.25 to 0.33.

→ For steel — 0.3 in elastic range
— 0.5 in Plastic range

→ For concrete — 0.15.

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Volumetric strain :

when a member is subjected to stresses (loads), it undergoes deformation in all the directions. Hence, there will be change in volume.

Thus the ratio of change in volume to original volume is called volumetric strain.

$$\Rightarrow e_v = \frac{\Delta V}{V}$$

e_v = volumetric strain

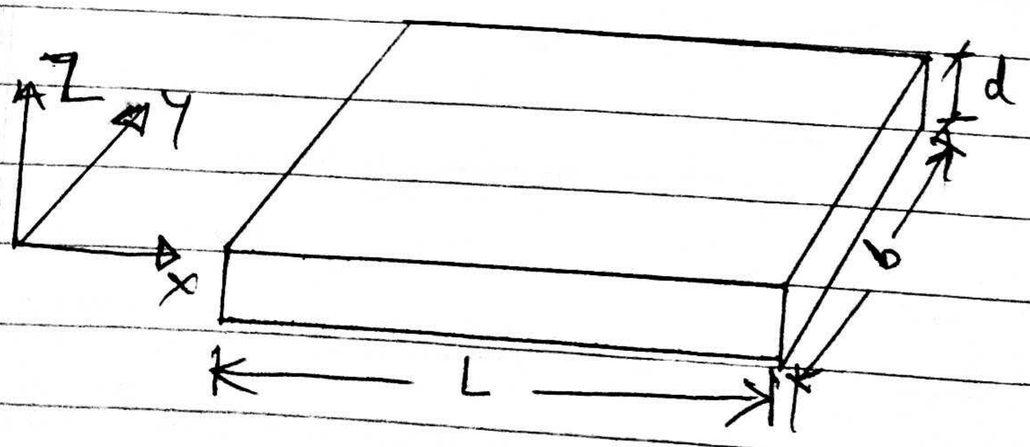
ΔV = change in volume

V = original volume

It can be shown that 'e_v' is sum of strains in three mutually $\perp$r directions

ie $e_v = e_x + e_y + e_z$

For example consider a bar of length 'L', breadth 'b' and depth 'd' as shown in fig. below.



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Elastic Constants :-

→ Modulus of elasticity (E)
→ Modulus of rigidity (shear modulus)
→ Bulk Modulus.

Modulus of elasticity or Young's Modulus of elasticity (E):

It is defined as the ratio of linear stress to linear strain within the elastic limit.

ie $E = \frac{\text{stress}}{\text{strain}} = \frac{p}{e}$

Modulus of rigidity or shear modulus (G)

It is defined as the ratio of shearing stress to shearing strain within the elastic limit & is denoted by G or N

Thus $G = \frac{q}{\phi}$

where G - Modulus of rigidity
q - shearing stress
ϕ = shearing strain

Bulk Modulus : (K)

when a body is subjected to identical stresses 'p' in three mutually ⊥r directions, the body undergoes uniform changes in three Mutually ⊥r directions without distortion of shape.

The ratio of change in volume to original volume has been defined as Volumetric strain (e_v). Then the Bulk Modulus K is given by

$$K = \frac{P}{e_v}$$

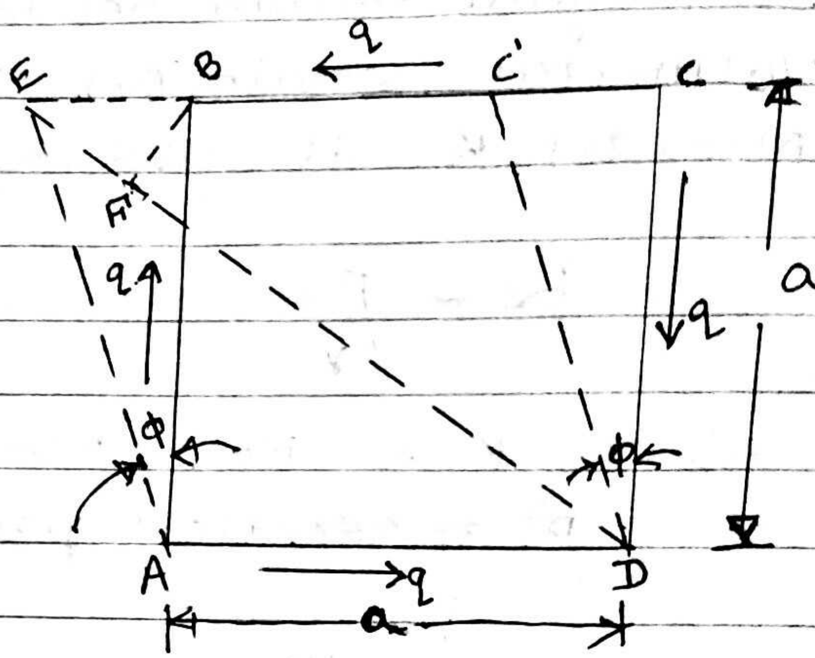
where K = Bulk Modulus

P = Identical pressure (Volumetric stress)

$e_v = \frac{\Delta V}{V}$ – Volumetric strain

"Thus the ratio of Volumetric stress (P) to Volumetric strain (e_v) is called Bulk Modulus."

Relationship between Modulus of elasticity (E) and Modulus of rigidity (G):



Consider a square element ABCD of sides 'a' subjected to pure shear 'q' as shown in fig above. And AEC'D is the deformed shape due to shear (q).

Let ϕ be the shear strain

G be the Modulus of rigidity

ie $G = \dfrac{q}{\phi}$

Draw a line BF ⊥r to diagonal DE.

Now strain in diagonal BD is given by

ie $BD = \dfrac{DE - DF}{DF}$

$[\because DF \simeq DB]$

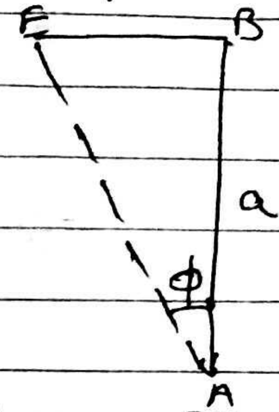
$= \dfrac{EF}{DB}$ $\quad DB = \sqrt{AB^2 + AD^2}$

$= \dfrac{EF}{a\sqrt{2}}$ $\quad DB = \sqrt{a^2 + a^2} = a\sqrt{2}$

Since angle of deformation is very small, we can assume $\angle BEF = 45°$

Hence $EF = BE \cos 45°$ $\left[\cos 45 = \frac{EF}{BE} \Rightarrow EF = BE \cos 45°\right]$

$\therefore$ strain in diagonal $BD = \frac{EF}{BD} = \frac{BE \cos 45}{a\sqrt{2}}$



$\Rightarrow$ 'e' in $BD = \frac{(a \tan\phi) \times \cos 45}{a\sqrt{2}}$

$\Rightarrow$ 'e' in $BD = \frac{1/\sqrt{2}}{\sqrt{2}} \times \tan\phi$

$= \frac{1}{(\sqrt{2})^2} \times \tan\phi$

$= \frac{1}{2} \tan\phi$

$\tan\phi = \frac{BE}{a}$

$\Rightarrow BE = a \tan\phi$

Since ϕ is very small

$\Rightarrow \frac{1}{2}\phi$

But $\phi = \frac{q}{G}$ $\left[G = \frac{q}{\phi}\right]$

$\therefore$ strain in $BD = \frac{1}{2} \times \frac{q}{G}$ $\longrightarrow$ (1)

we know that the pure shear 'q' gives rise to axial tensile stress 'q' in the diagonal direction of DB & axial compression q at right angles to it. These are two stresses cause tensile strain along the diagonal DB.

$\hookrightarrow$

$\therefore$ Tensile strain along the diagonal

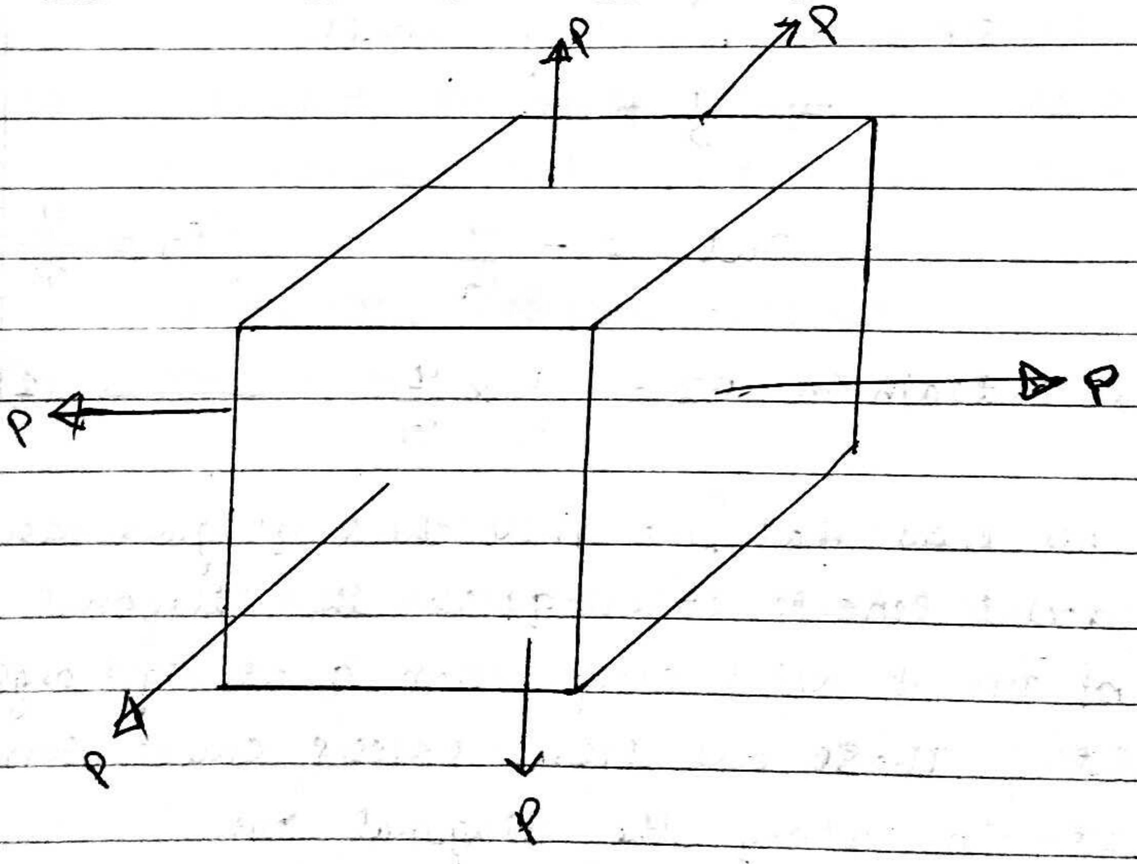
$$DB = \frac{q}{E} + \mu\frac{q}{E} = \frac{q}{E}[1+\mu] \rightarrow (2)$$

From equations (1) & (2), we get

$$\frac{1}{2} \times \frac{\cancel{q}}{G} = \frac{\cancel{q}}{E}[1+\mu]$$

$$\Rightarrow \boxed{E = 2G[1+\mu]}$$

Relationship between Modulus of elasticity and Bulk modulus [E & K]



Consider a cubic element subjected to stresses 'p' in the three mutually perpendicular

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direction x, y, z ~~direction~~ as shown in fig.
Now the stress 'p' in 'x' direction causes tensile strain $\frac{p}{E}$ in x direction & stress 'p' in y & z direction causes compressive strains $\frac{\mu p}{E}$ in x direction.

The expression for strains along each direction can be written as

Hence $e_x = \frac{p}{E} - \frac{\mu p}{E} - \frac{\mu p}{E}$

$$= \frac{p}{E}[1-2\mu]$$

$|||^{ly}$ $e_y = \frac{p}{E}[1-2\mu]$

& $e_z = \frac{p}{E}[1-2\mu]$

$\Rightarrow$ Volumetric strain $= e_v = e_x + e_y + e_z$

$$\Rightarrow e_v = \frac{3p}{E}[1-2\mu]$$

But, Bulk modulus $K = \frac{p}{e_v} = \frac{\cancel{p}}{\frac{3\cancel{p}[1-2\mu]}{E}}$

$$\Rightarrow \boxed{E = 3K[1-2\mu]}$$

Relationship between E G K :

We know, $E = 2G[1+\mu] \rightarrow (1)$

& $E = 3K[1-2\mu] \rightarrow (2)$

By eliminating μ between the above two equation we can get the relationship between E, G, K, free from the term 'μ'.

From eqn (1) $\mu = \frac{E}{2G} - 1$

sub this in eqn (2)

ie $E = 3K\left[1 - 2\left(\frac{E}{2G} - 1\right)\right]$

$= 3K\left(1 - \frac{E}{G} + 2\right) = 3K\left[3 - \frac{E}{G}\right]$

$E = 9K - \frac{3KE}{G}$

$E + \frac{3KE}{G} = 9K$

$E\left[1 + \frac{3K}{G}\right] = 9K$ or $\frac{9}{E} = \frac{G+3K}{KG}$

$E\left[\frac{G+3K}{G}\right] = 9K$ $\quad \frac{9}{E} = \frac{3}{G} + \frac{1}{K}$

$\Rightarrow \boxed{E = \frac{9KG}{G+3K}}$