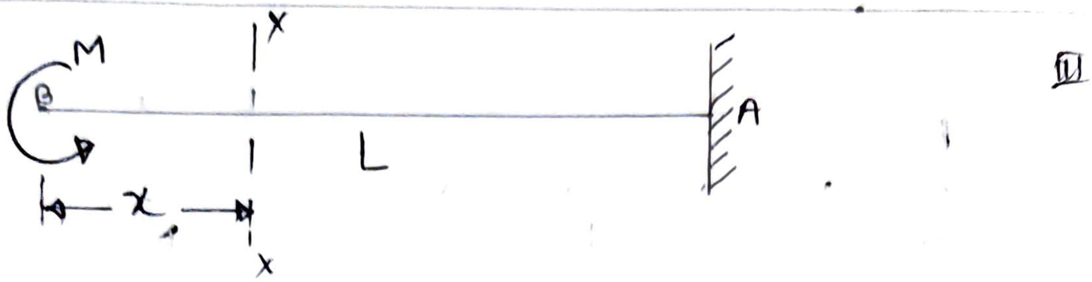
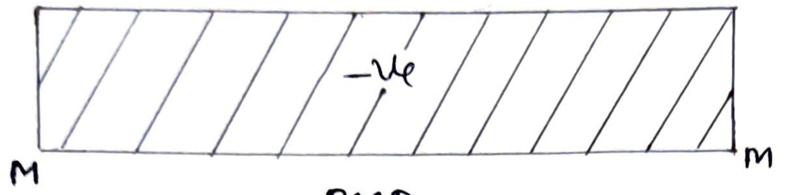
Cantilever beam with a couple acting at the free end





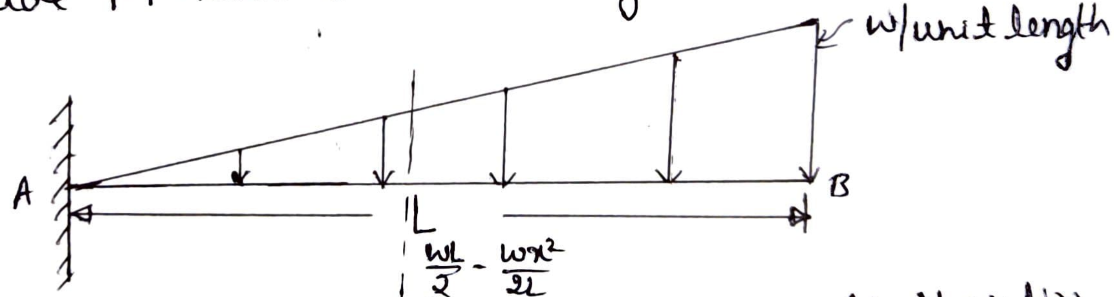
BMD

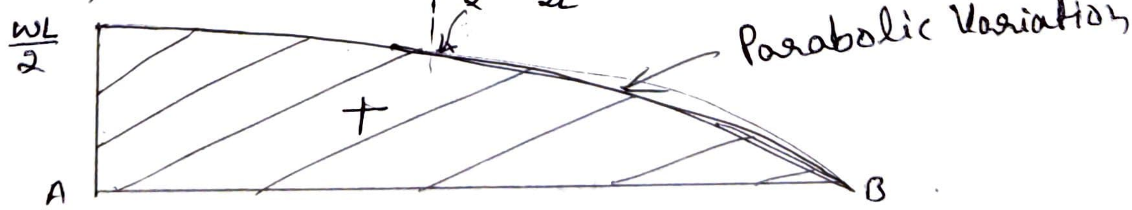
Fig. shows a cantilever beam subjected to a couple M at the free end B.

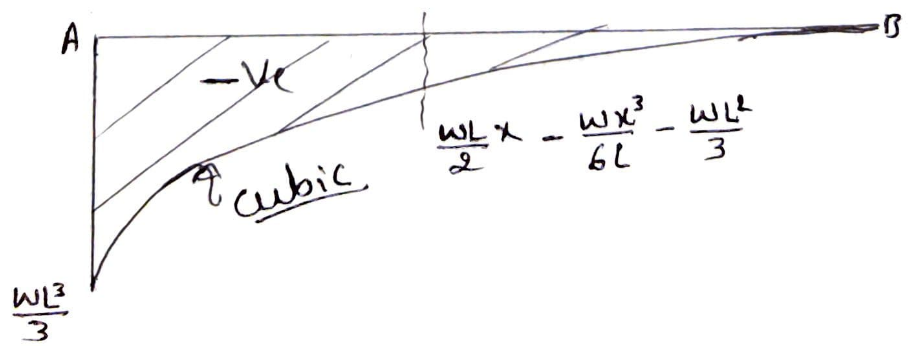
SFD :- Since there is no force acting on the beam, shearing force is zero throughout the length of the beam.

BMD:
$$BM @ x-x = -M$$
$$\text{at } x = 0,\ BM @ B = -M$$
$$\text{at } x = L,\ BM @ A = -M$$

Hence the bending moment is of constant magnitude M for the entire length of the beam.

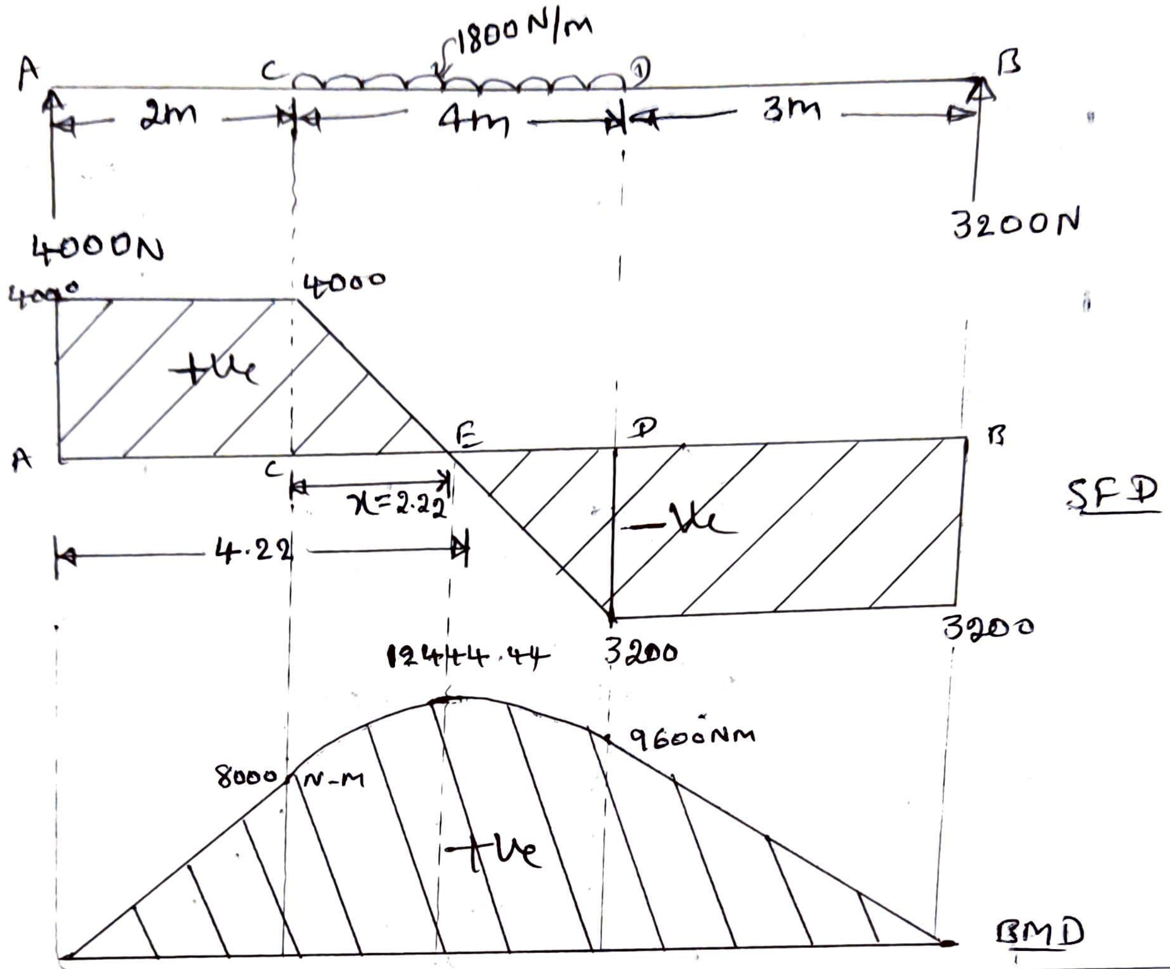






: SFD & BMD for beams Subjected to Various loads:

(1)



SFD:

SF @ A = 4000

SF @ before C = 4000

SF @ C = 4000

SF @ before D = 4000 − 7200

⇒ SF @ before D = − 3200

SF @ D = − 3200

SF @ before B = − 3200

SF @ B = 0 ie SF @ B = 3200

BMD

BM @ A & @ B = 0

BM @ C = 4000 × 2 = 8000 N-M

BM @ before D = 4000 × 6 − 7200 × 2

= 9600 N-M

BM @ D = 9600 N-M

Since SF is zero at a distance x from C.

ie SF @ E = 0

∴ shear force, SF @ E = 0

⇒ 4000 − 1800x = 0

⇒ 1800x = 4000

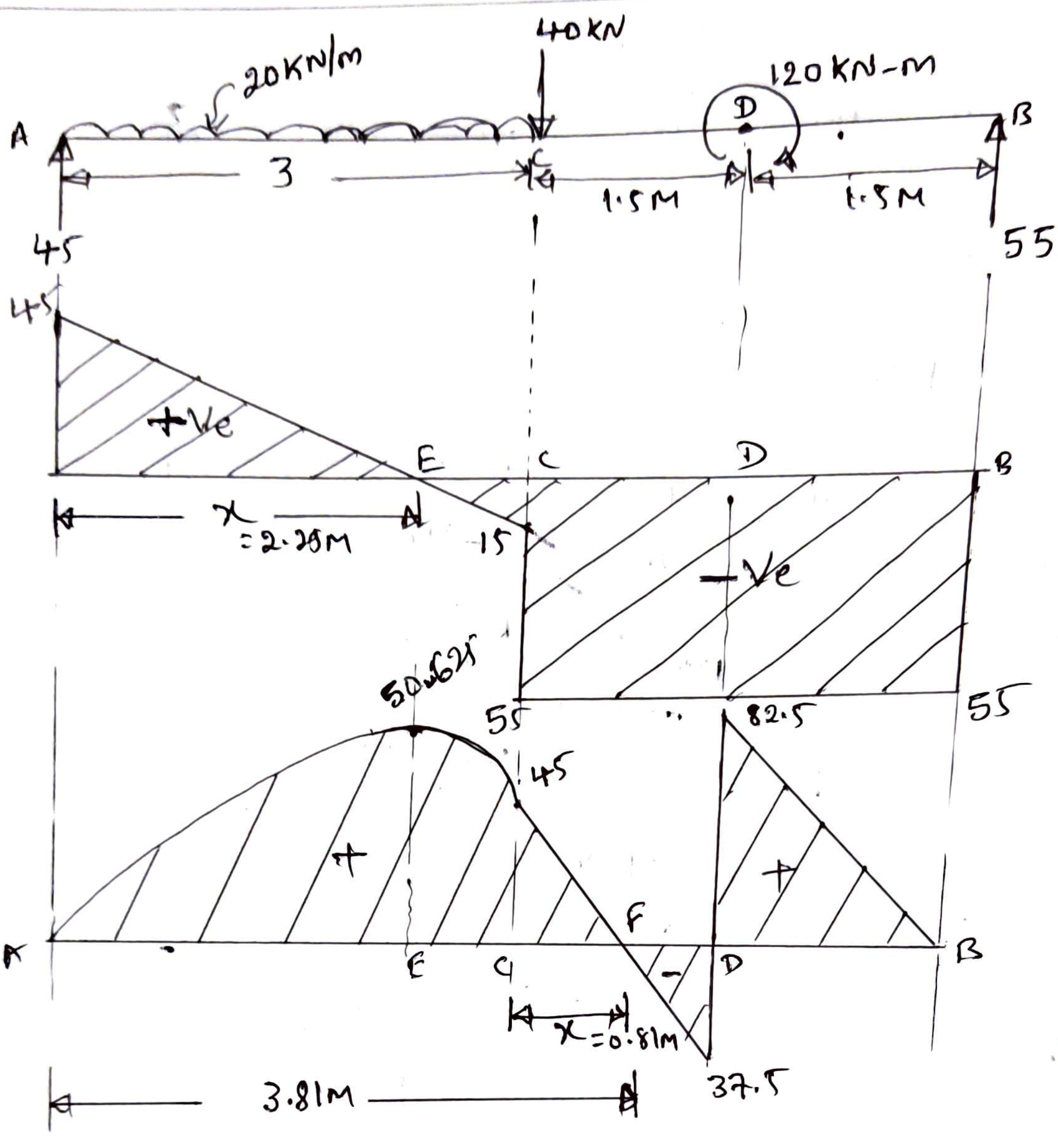
⇒ x = 2.22 M

BM @ E is given by

$= 4000 \times 4.22 - 1800 \times 2.22 \times \frac{2.22}{2}$

= 12444.44 N-M Max BM

(2)



SFD:

SF @ A = 45

SF @ before C = 45 − 60 = −15

SF @ C = −15 − 40 = −55

SF @ D = −55

SF @ before B = −55

SF @ B = 0 re SF @ B = −55

SF @ E (at a distance x from A) = 0

$\Rightarrow 45 - 20x = 0$

$\Rightarrow 20x = 45$

$\Rightarrow x = \frac{45}{20} = 2.25M$

BMD:

BM @ A = 0

BM @ before C = 45×3 − 60×1.5 = 45

BM @ C = 45

BM @ before D = 45×4.5 − 60×3 − 40×1.5 = −37.5

BM @ D = −37.5 + 120 = 82.5

BM @ B = 0

∴ BM @ E

ie $M_E = 45 \times 2.25 - 20 \times 2.25 \times \frac{2.25}{2}$

$= 50.625$ KN-M

NOTE: In BMD, the point F is called point of contraflexure where BM is zero and changes it's sign from +ve to −ve

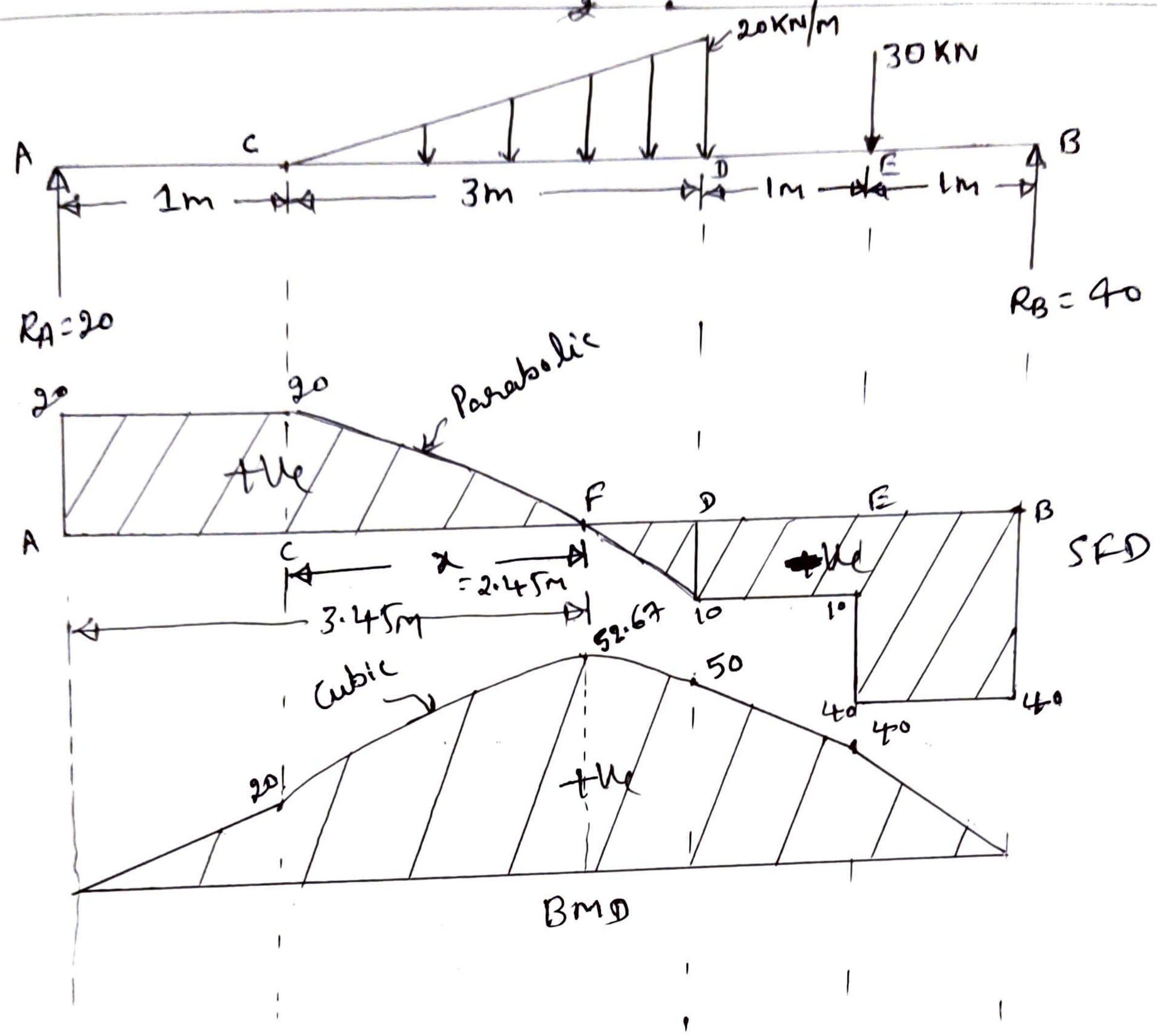
ie BM @ F = 0 $\Rightarrow 45(3+x) - 60 \times (1.5+x) - 40 \times x = 0$

$\Rightarrow 135 + 45x - 90 - 60x - 40x = 0 \Rightarrow -55x + 45 = 0$

$\Rightarrow x = 0.81m$

3

$A = \frac{1}{2} \times 3 \times 20 = 30$



SFD:

SF @ A = 20

SF @ C = 20

SF @ before D = 20 − 30 = −10

SF @ D = −10

SF @ before E = −10

SF @ E = −10 − 30 = −40

SF @ before B = −40

SF @ B = 0, i.e. SF @ B = −40

SF @ F = 0

$\Rightarrow 20 - \left[\frac{1}{2} \times x \times \frac{20x}{3}\right] = 0 \Rightarrow 20 - \frac{10x^2}{3} = 0$

$\Rightarrow \frac{10x^2}{3} = 20 \Rightarrow x^2 = \frac{20 \times 3}{10} = 6$

$\Rightarrow x = \sqrt{6} = 2.45\text{m}$

$\therefore$ BM @ F $= 20 \times 3.45 - 20 \times \frac{2.45}{3} = \underline{52.67}$

BMD:

BM @ C = 20 × 1 = 20

BM @ before D = 20 × 4 − 30 × 1 = 50

BM @ D = 50

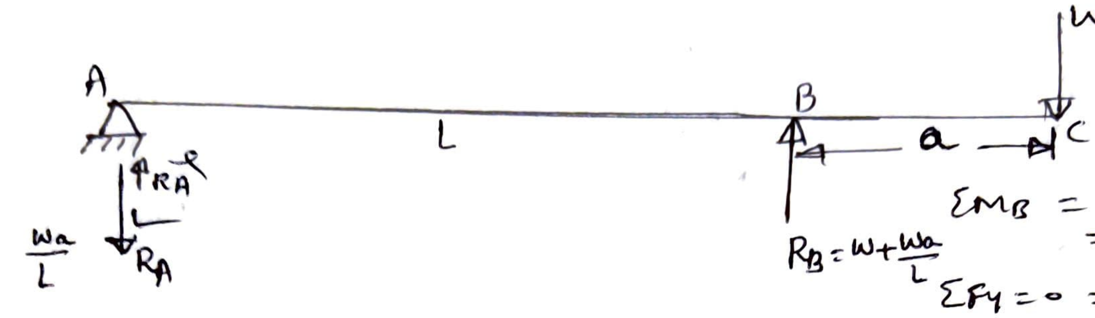
BM @ before E = 20 × 5 − 30 × 2 = 40

BM @ B = 0

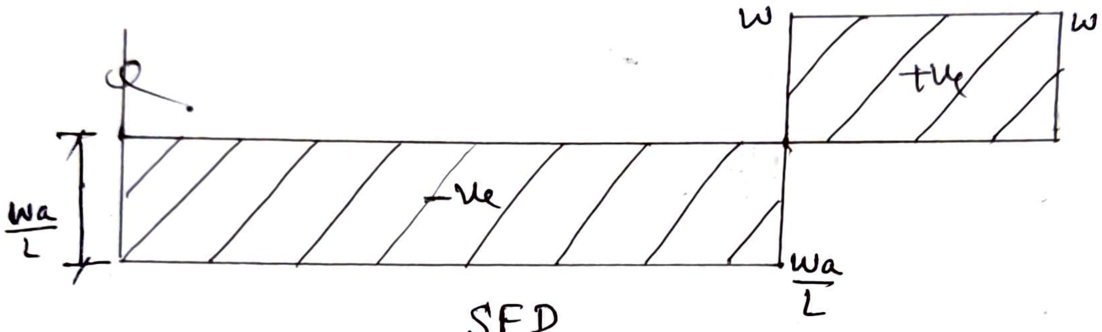
$\frac{20}{3} = \frac{h}{x} \Rightarrow h = \frac{20x}{3}$

$\frac{1}{2} \times 2.45 \times \frac{20}{3} \times 2.45 = 20$

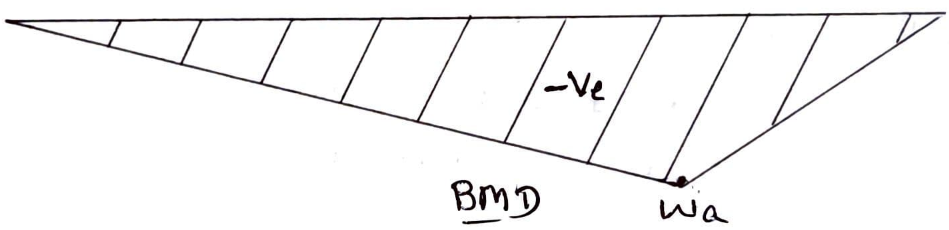
: Overhanging Beam subjected to concentrated load at free end. (14)



$\Sigma M_B = 0 \Rightarrow R_A L + Wa = 0$
$\Rightarrow R_A = -\frac{Wa}{L}$
$\Sigma F_y = 0 \Rightarrow -\frac{Wa}{L} - W + R_B = 0$
$\Rightarrow R_B = W + \frac{Wa}{L}$



SFD



BMD

SFD: SF @ A $= -\frac{Wa}{L}$

SF @ before B $= -\frac{Wa}{L}$

SF @ B $= -\cancel{\frac{Wa}{L}} + W + \cancel{\frac{Wa}{L}} = W$

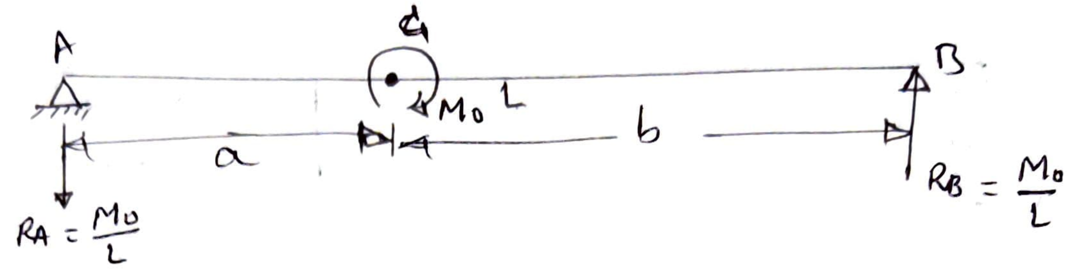
SF @ before C $= W$

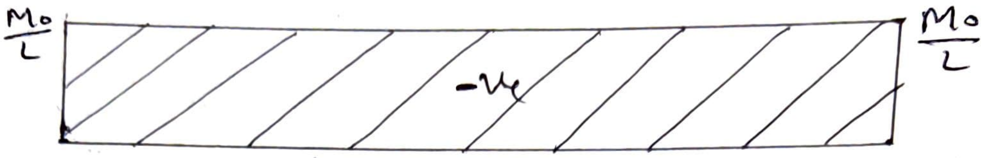
SF @ C $= W - W = 0$ * SF @ C = W

BMD: BM @ A & @ C $= 0$

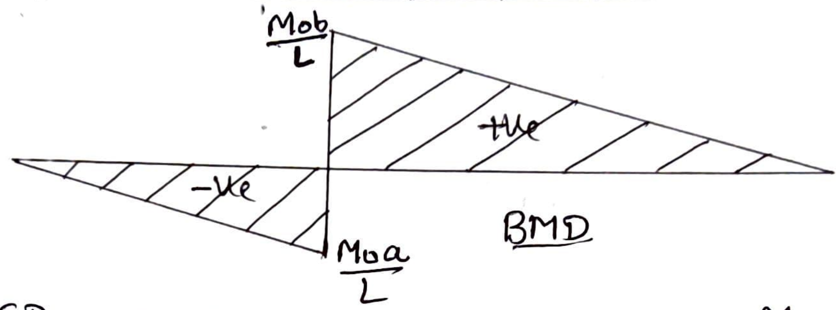
BM @ B $= -\frac{Wa}{\cancel{L}} \times \cancel{L} = -Wa$

* Simply supported beam subjected to External Mo M_0 @ $x = a$ from L





SFD:



BMD

SFD: SF @ A $= -\frac{M_0}{L}$, SF @ C $= -\frac{M_0}{L}$

SF @ before B $= -\frac{M_0}{L}$

SF @ B $= -\frac{M_0}{L} + \frac{M_0}{L} = 0$

BMD: LHS: BM @ A $= 0$

BM @ before C $= -\frac{M_0 a}{L} = -M_0 \times \frac{a}{L}$

RHS: BM @ B $= 0$

BM @ before C $= \frac{M_0}{L} \times b = \frac{M_0 b}{L}$

If $M_0 = 10$ KN-m, $L = 5$m, $a = 2$m, $b = 3$m

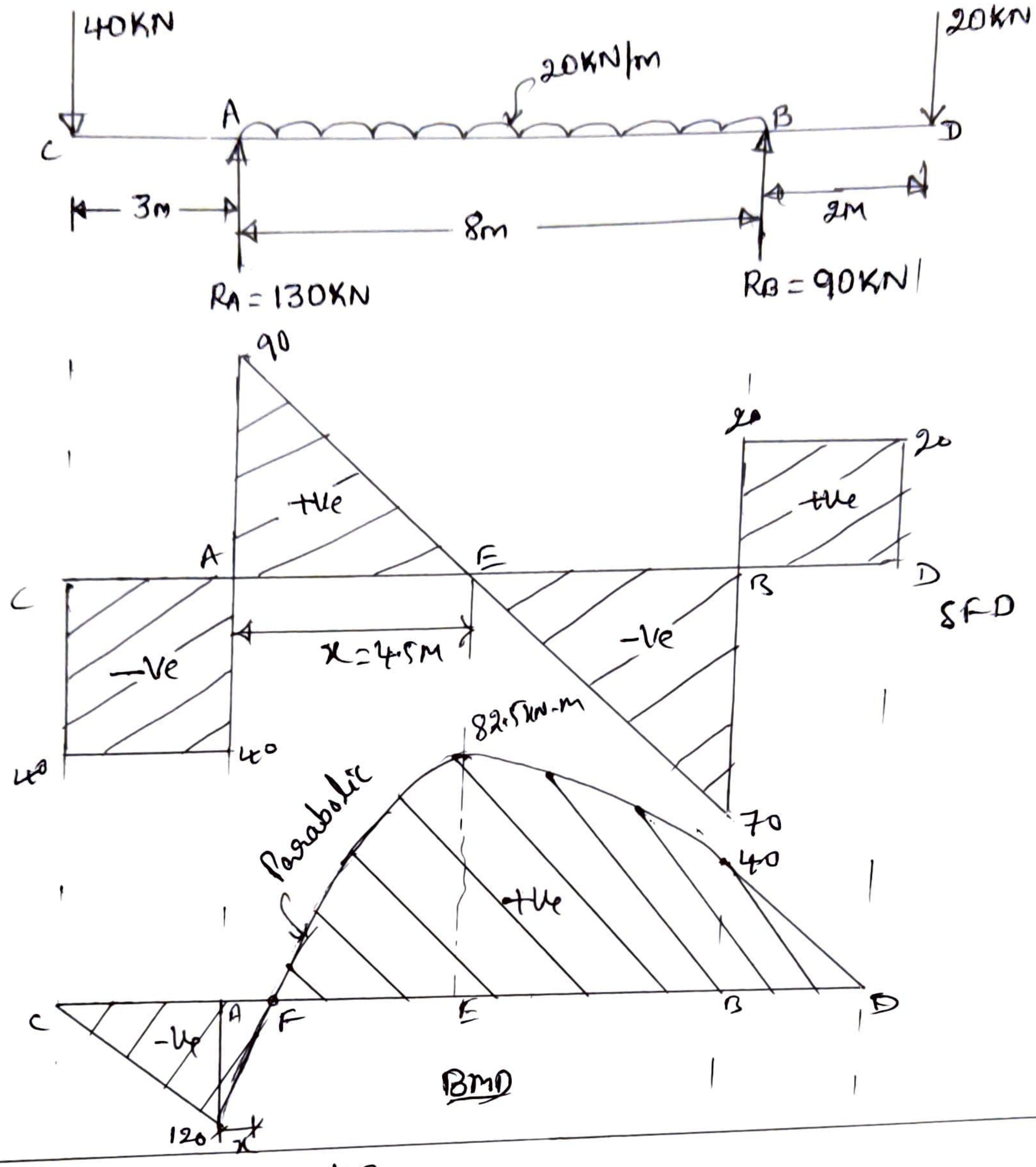
$R_A = 2$, $R_B = 2$ $\quad -R_A + R_B = 0$

SFD: SF @ A $= -2$, SF @ C $= -2$, SF @ before B $= -2$, SF @ B $= 0$

BMD: BM @ A $= 0$, BM @ before C $= -2 \times 2 = -4$ kN-m

BM @ C $= -4 + 10 = 6$ kNm,

BM @ B $= -2 \times 5 + 10 = 0$



SFD: SF @ C = -40

SF @ before A = -40

SF @ A = -40 + 130 = 90

SF @ before B = 90 - 160 = -70

SF @ B = -70 + 90 = 20

SF @ before D = 20

SF @ D = 20 - 20 = 0

SF @ x = 0

$\Rightarrow -40 + 130 - 20x = 0$

$\Rightarrow 20x = 90 \Rightarrow x = 4.5M$

BMD: BM @ C = 0

BM @ A = $-40 \times 3 = -120$

BM @ before B = $-40 \times 11 + 130 \times 8 - 160 \times 4 = -40$

BM @ B = -40 KN-M

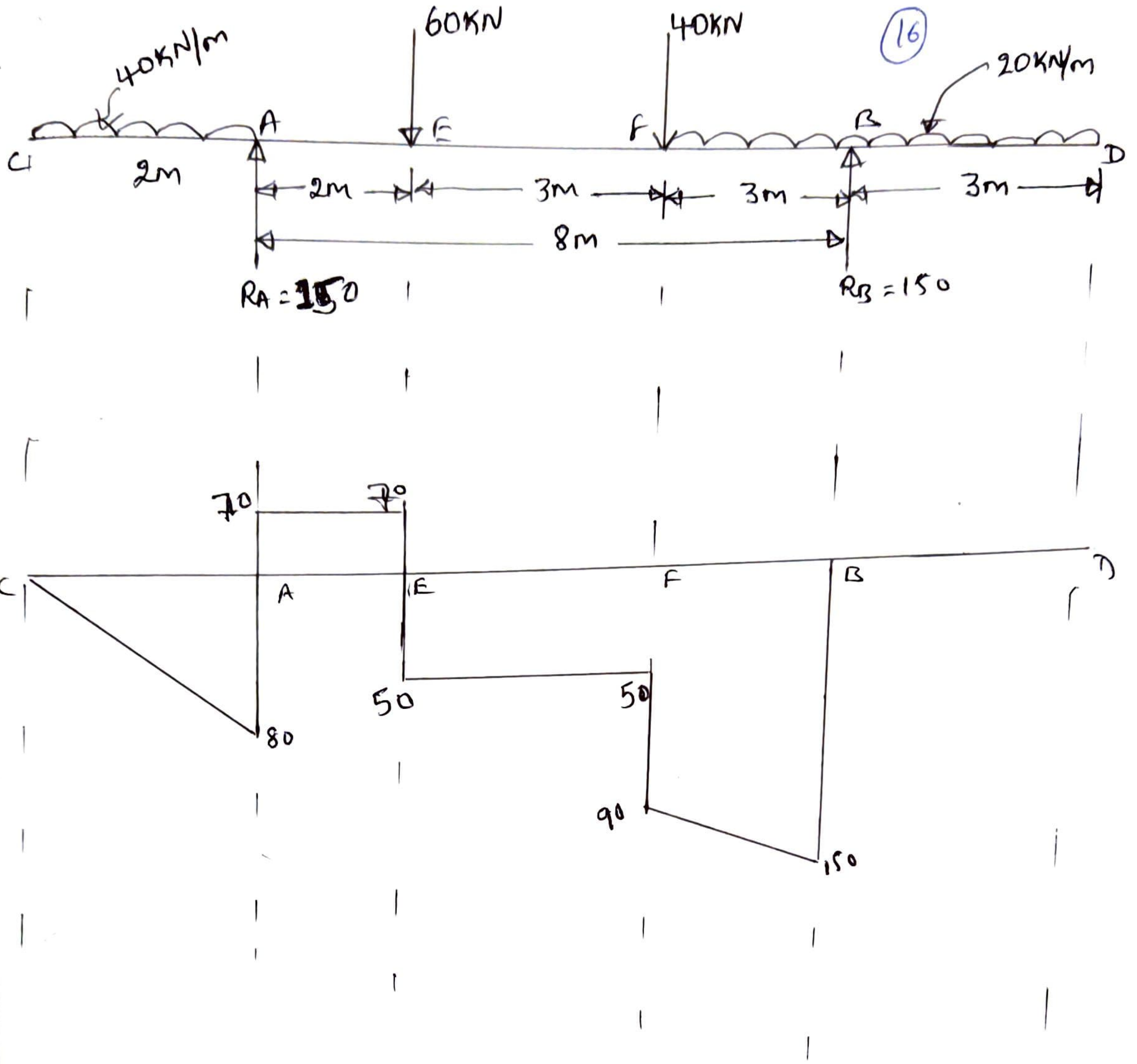
BM @ D = 0

BM @ E = $-40 \times 7.5 + 130 \times 4.5 - 20 \times 4.5 \times \frac{4.5}{2}$

BM @ E = 82.5, pt F is called pt of contraflexure

BM @ F = 0 $\Rightarrow -40 \times (3+x) + 130 \times x - 20 \frac{x^2}{2} = 0$

$\Rightarrow -120 - 40x + 130x - 10x^2 = 0 \Rightarrow x = 1.62M$ from A



Calculation of RA & RB :

$\Sigma M_A = 0 \Rightarrow -80\times 1 + 60\times 2 + 40\times 5 + 60\times 6.5 - R_B\times 8 + 60\times 9.5 = 0$

$\Rightarrow 8R_B = 1200 \Rightarrow R_B = 150$

$\Sigma F_y = 0 \Rightarrow -80 + R_A - 60 - 40 + \overset{150}{R_B} - 60 - 60 = 0$

$\Rightarrow R_A = 80 + 60 + 40 - 150 + 60 + 60 = 150$

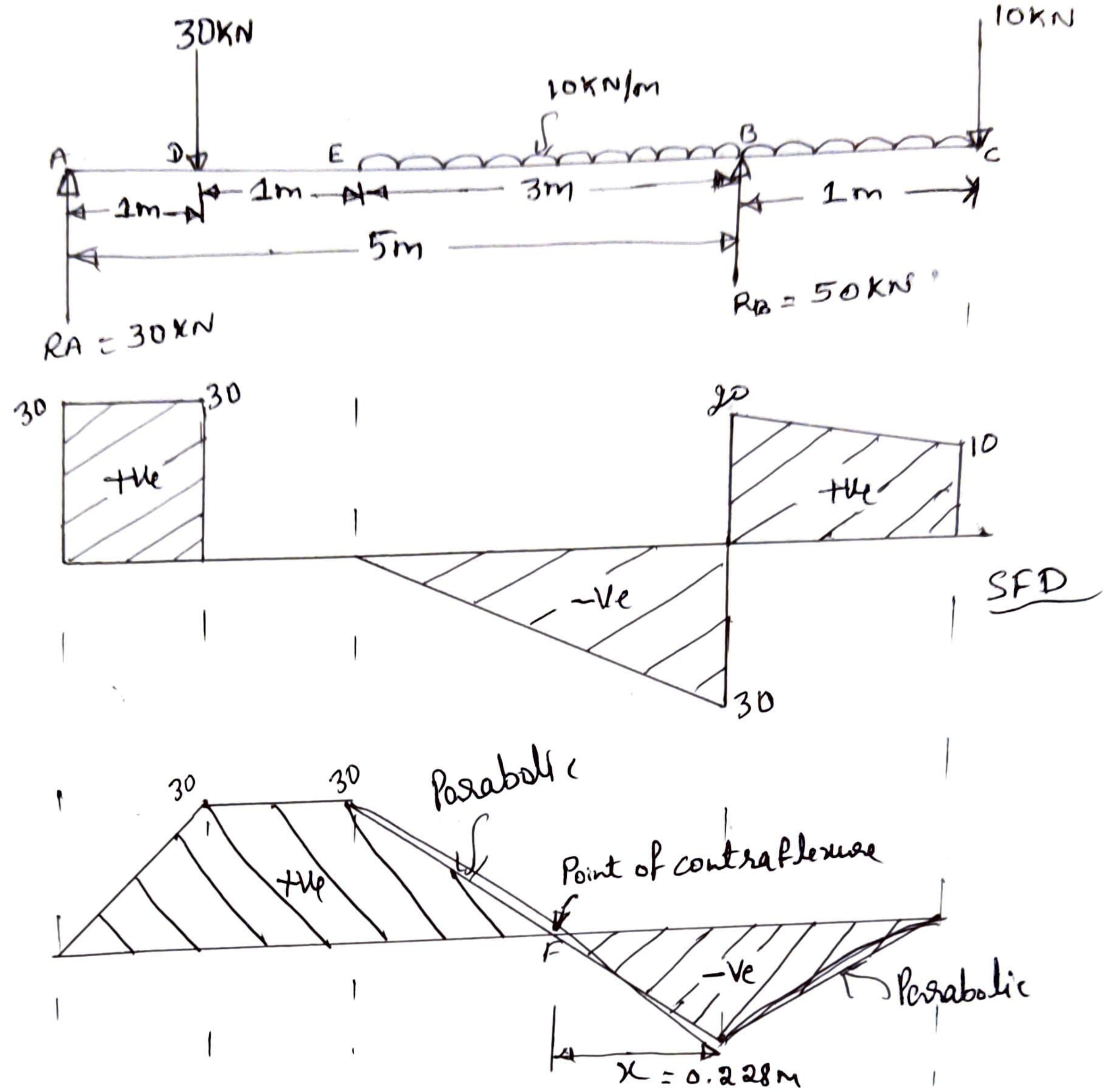
SFD: SF @ C = 0, SF @ before A = −80

SF@A = −80 + 150 = +70, SF @ before E = 70

SF@E = 70 − 60 = +10, SF@ before F = +10

SF@F = 10 − 40 = −30, SF @ before B = −30 − 60 = −90

SF @ B = −90 + 150 = 60, ~~SF @ before D = −60~~, SF@D = −60 + 60 = 0



Calculation of support reactions RA & RB:

$\Sigma M_A = 0 \Rightarrow 30 \times 1 + 30 \times 3.5 - R_B \times 5 + 10 \times 5.5 + 10 \times 6 = 0$

$\Rightarrow R_B = 50KN$

$\Sigma F_y = 0 \Rightarrow R_A - 30 - 30 + R_B - 10 - 10 = 0$

$\Rightarrow R_A = 30 + 30 + 10 + 10 - 50$

$\Rightarrow R_A = 30KN$

SFD: SF@ A = 30, SF @ before D = 30

SF@ D = 30 - 30 = 0, SF@ E = 0

SF@ before B = -30

SF@ B = -30 + 50 = 20

SF@ before C = 20 - 10 = 10

SF@ C = 10 - 10 = 0

BMD: BM @ A = 0

BM @ D = 30×1 = 30 KN-M

BM @ E = 30×2 − 30×1 = 60 − 30 = 30 KN-M

BM @ before B = 30×5 − 30×4 − 30×1.5 = −15 KN-M

BM @ B = −15 KN-M

BM @ before C = 30×6 − 30×5 − 30×2.5 + 50×1 − 10×0.5

= 0, BM @ C = 0

In BMD, pt F is called point of contraflexure

BM @ F = 0

$$-10\times(1+x) + 50x - \frac{10x^2}{2} = 0$$

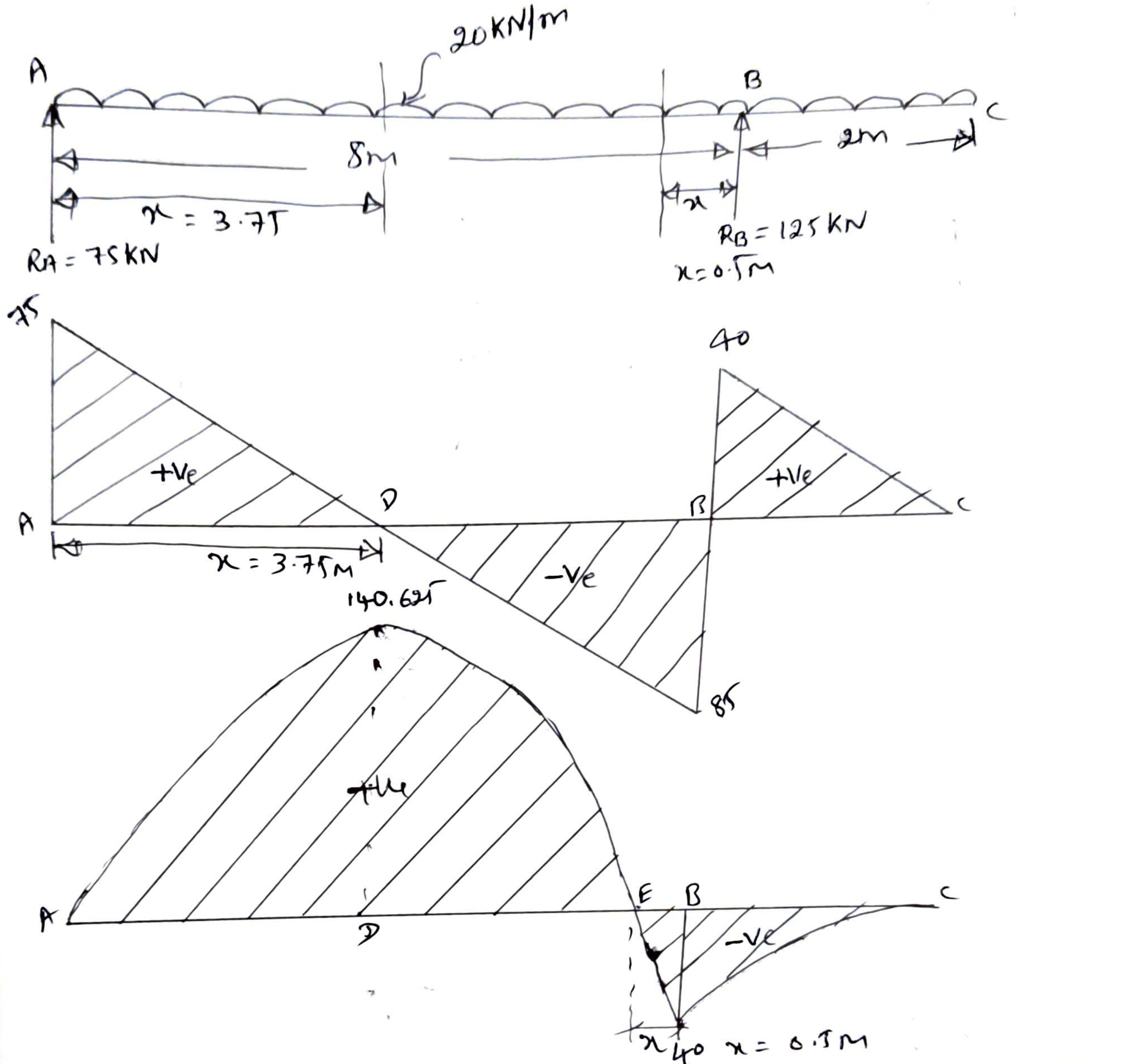
$$-10 - 10x + 50x - 5x^2 = 0$$

$$-5x^2 + 45x - 10 = 0$$

$$ax^2 + bx + c = 0$$

ie $a = -5$, $b = 45$, $c = -10$

$\Rightarrow x = 0.228$ M from B end



SFD: SF @ A = 75 KN, SF @ before B = 75 - 160 = -85 KN

SF @ B = - 85 + 125 = 40 KN

SF @ C = 40 - 40 = 0

From SFD, SF @ D = 0, => 75 - 20x = 0 => x = 3.75

BMD: BM @ A = 0, BM @ before B = 75 × 8 - 160 × 4 = - 40

BM @ B = - 40,

BM @ C = 75 × 10 - 160 × 6 + 125 × 2 - 40 × 1 = 0

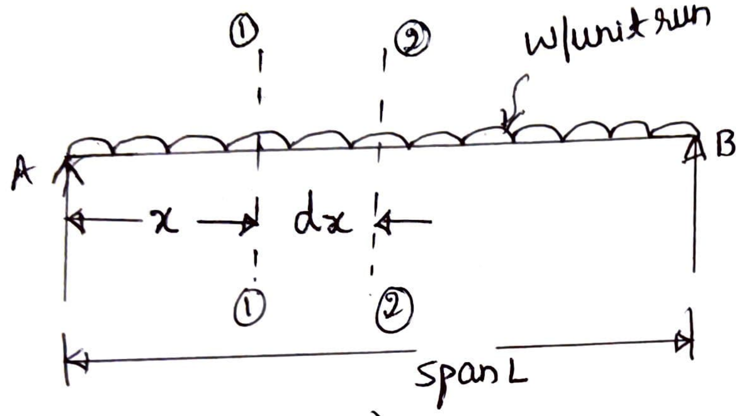
BM @ D = $75 \times 3.75 - 20 \times 3.75 \times \frac{3.75}{2}$ = 140.625 KN-m

In BMD, Pt E is called point of contraflexure

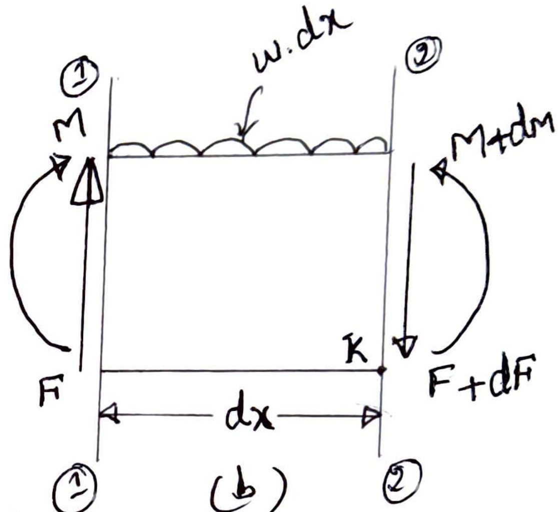
BM @ E = 0 => $125 \times x - \frac{20x^2}{2} - 40(1+x) = 0$

=> $125x - 10x^2 - 40 - 40x = 0$ => $-10x^2 + 85x - 40 = 0$
=> x = 0.5

Relationship between Load, shear force and Bending Moment :-



(a)



(b)

Consider a simply supported beam loaded as shown in fig. above (a) and consider an elemental length dx at a distance x from the left end A.

The free body diagram of the elemental length of the beam along with the forces and moments acting is shown in fig.(b) above

Let F = shear force at section 1-1

F+dF = shear force at section 2-2

M = Bending moment at section 1-1

M+dM = BM at section 2-2

Total load on a beam of length dx = w.dx

↳

For equilibrium of the elemental strip of beam

$\Sigma F_y = 0$

$\Rightarrow F - w.dx - (F+dF) = 0$

$\Rightarrow \cancel{F} - wdx - \cancel{F} - dF = 0$

$-wdx = dF \Rightarrow \boxed{\frac{dF}{dx} = -w}$

* $\Rightarrow$ ∴ the above relationship shows that the rate of change of shearforce is equal to the load.

Now taking moments about the point K or about 2-2

ie $M + F.dx - w.dx \times \left(\frac{dx}{2}\right) - (M+dM) = 0$

$\cancel{M} + Fdx - \cancel{\frac{wdx^2}{2}} - \cancel{M} - dM = 0$

Neglet

Neglecting higher powers of small quantities

∴ $Fdx - dM = 0$

$\Rightarrow Fdx = dM \Rightarrow \boxed{\frac{dM}{dx} = F}$

* $\Rightarrow$ ∴ the above equation shows that the rate of changes of BM is equal to shear force

From the above, it can be seen that in case of BM remaining constant over a length (i.e. M being constant), the shear force will be zero in that portion of the beam.