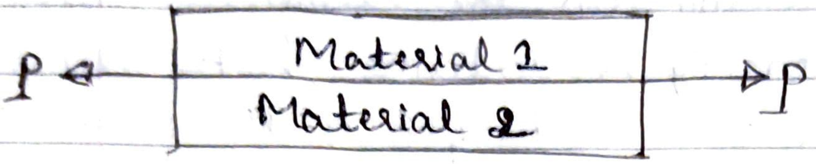
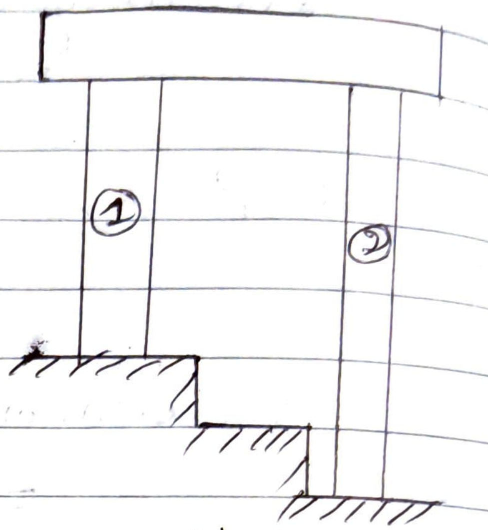
Compound bars :



{a}



[b]

Bars made with two or more materials is called compound bars.

compound bars may have same length or different lengths as shown in fig (a) & (b) above

Consider a member with two material. Let the forces developed due to applied loads in material ① & ② be P_1 & P_2 respectively. Then according to static equilibrium conditions along the axis of the member

$$P_1 + P_2 = P$$

Compatibility condition shows,

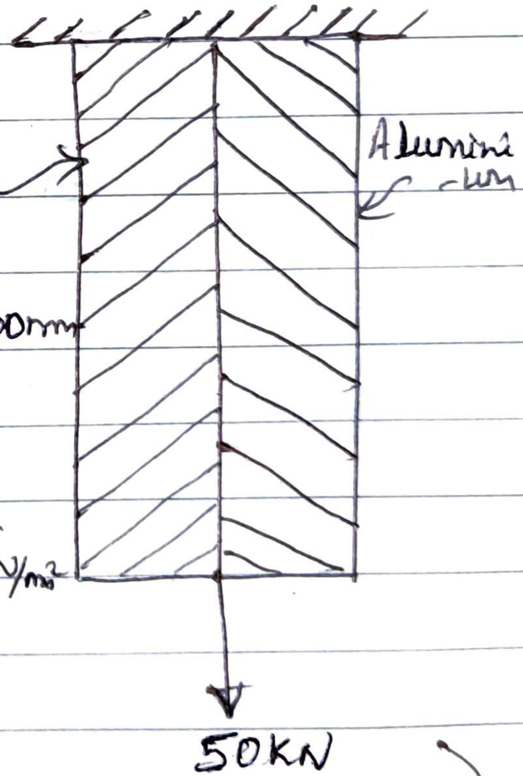
$$\Delta_1 = \Delta_2$$

$$\Rightarrow \frac{P_1 L_1}{A_1 E_1} = \frac{P_2 L_2}{A_2 E_2}$$

From the above two equations the two unknowns P_1 and P_2 can be found uniquely.

Problems on compound bars

① A compound bar of length 500 mm consists of a strip of aluminium 50 mm wide × 20 mm thick and a strip of steel 50 mm wide × 15 mm thick rigidly joined at ends. If the bar is subjected to a load of 50 KN, find the stresses developed in each material and the extension of the bar. Take E for aluminium = 1×10^5 N/mm² & for steel as 2×10^5 N/mm².



Soln: Given data

$L_{alu} = L_{steel} = 500 mm$

$P_{alu} = P_{steel} = 50 \times 10^3 N$

$A_{alu} = 50 \times 20 = \cancel{10,000}$ 1,000

$A_{steel} = 50 \times 15 = 750 mm^2$

$E_{alu} = 1 \times 10^5$ $E_{steel} = 2 \times 10^5 N/mm^2$

⇒ stress in Aluminium

$= \frac{50 \times 10^3}{1000} = 50 N/mm^2$

wrong method

stress in steel $= \frac{50 \times 10^3}{750} = 66.67$

Since the 50KN force is shared by ~~the~~ aluminium & steel.

∴ From equilibrium condition

$$P_{alu} + P_{steel} = 50 \times 10^3 \longrightarrow (1)$$

From the compatibility condition

$$\Delta l_{alu} = \Delta l_{steel} \qquad L_a = L_s = 500mm$$

$$\frac{P_{alu} \times L_{alu}}{A_{alu} \times E_{alu}} = \frac{P_{steel} \times L_{steel}}{A_{steel} \times E_{steel}}$$

$$\Rightarrow \frac{P_{alu} \times 1}{1000 \times 1 \times 10^5} = \frac{P_s \times 1}{750 \times 2 \times 10^5}$$

$$\Rightarrow P_a = 0.67 P_s \longrightarrow (2)$$

From eqn (1)

$$P_a + P_s = 50 \times 10^3$$

$$0.67 P_s + P_s = 50 \times 10^3$$

$$\Rightarrow 1.67 P_s = 50 \times 10^3$$

$$\Rightarrow P_s = 29.940 \text{ KN} \approx 30 \text{KN}$$

$$\& \quad P_a = 20 \text{ KN}$$

∴ stress in alumn strip $= \dfrac{P_a}{A_{alu}}$

$$= \frac{20 \times 10^3}{1000}$$

$$= 20 \text{ N/mm}^2$$

$|||^{ly}$ in steel $= \frac{30\times10^3}{750} = 40\ N/mm^2$

$\therefore$ extension of the bar,

$\Delta_a = \Delta_s = \frac{P_s L_s}{A_s \times E_s} = \frac{30\times10^3\times500}{750\times2\times10^5}$

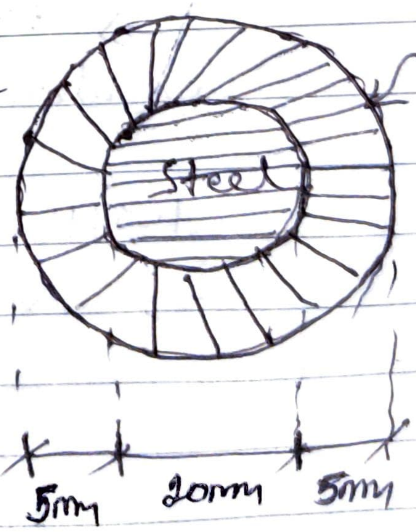
$\boxed{\Delta = 0.1mm}$

② A compound bar consists of a circular rod of steel of diameter 20 mm rigidly fitted into a copper tube of internal diameter 20mm and thickness 5mm as shown in fig below. If the bar is subjected to a load of 100KN, find the stresses developed in the two materials.

Take $E_s = 2\times10^5\ N/mm^2$, $E_c = 1.2\times10^5\ N/mm^2$

Soln: Area of steel rod

ie $A_s = \frac{\pi}{4}\times20^2 = 100\pi\ mm^2$



$|||^{ly}$ Area of copper rod

ie $A_c = \frac{\pi}{4}[D^2 - d^2]$

$\Rightarrow A_c = \frac{\pi}{4}[30^2 - 20^2]$

$A_c = 125\pi\ mm^2$

Since the compound bar is subjected to a load of 100 KN.
This 100 KN load is shared by steel & copper strips.

Let P_s be the load shared by steel & P_c be the " " — copper
∴ From equilibrium condition,

$$P_s + P_c = 100 \longrightarrow (1)$$

& From compatibility condition,

$$\Delta_s = \Delta_c$$

$$\frac{P_s L}{A_s E_s} = \frac{P_c L}{A_c E_c}$$

$$\frac{P_s}{100\pi \times 2\times10^5} = \frac{P_c}{125\pi \times 1.2\times10^5}$$

$$\Rightarrow P_s = 1.33 P_c \longrightarrow (2)$$

considering eqns (1) & (2)

$$1.33P_c + P_c = 100$$

$$\Rightarrow 2.33 P_c = 100$$

$$\Rightarrow P_c = 42.91 \text{ KN}$$

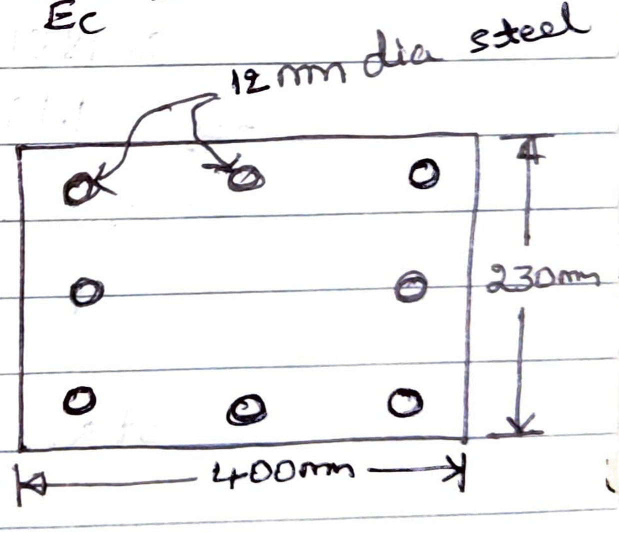
$$\& \quad P_s = 57.08 \text{ KN}$$

$\therefore$ stress in steel $= \frac{57.08 \times 10^3}{100\pi} = 181.7 \, N/mm^2$

$|||^{ly}$ stress in copper $= \frac{42.91 \times 10^3}{125\pi} = 109.27 \, N/mm^2$

(3) A reinforced concrete column of size 230 × 400mm has 8 steel bars of 12mm diameter as shown in fig. If the column is subjected to an axial compression of 600 KN, find the stresses developed in steel and concrete.

Take modular ratio $= \frac{E_s}{E_c} = 18.67$



soln: Total area of steel

i.e $A_s = \frac{\pi}{4} \times 12^2 \times 8 = 288\pi \, mm^2$

Area of concrete

$A_c = (400 \times 230) - A_s$

$A_c = 91.095 \times 10^3 \, mm^2$

600 KN axial compressive load is shared by steel and the concrete

Let P_s be the load shared by steel

& P_c be the — " — concrete

FROM equilibrium condition

$P_s + P_c = 600 \rightarrow (1)$

& From compatibility condition

$$\Delta_s = \Delta_c$$

$$\frac{P_s L}{A_s E_s} = \frac{P_c L}{A_c E_c}$$

$$P_s = P_c \times \frac{A_s}{A_c} \times \frac{E_s}{E_c}$$

$$P_s = P_c \times \frac{288\pi}{91.095 \times 10^3} \times 18.67$$

$$P_s = 0.185 P_c \longrightarrow (2)$$

From eqns (1) & (2)

$$0.185 P_c + P_c = 600$$

$$1.185 P_c = 600$$

$$\Rightarrow \quad P_c = 506.329 \text{ kN}$$

$$\& \quad P_s = 93.67 \text{ kN}$$

$\therefore$ stress in concrete $= \dfrac{506.329 \times 10^3}{91.095 \times 10^3}$

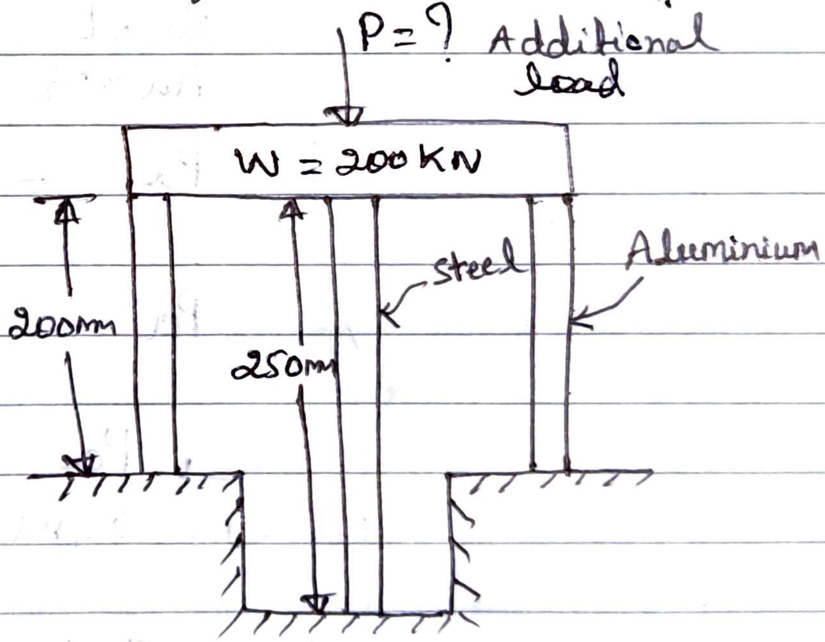
$= \underline{5.556 \text{ N/mm}^2}$

III^{ly} stress in steel $= \dfrac{93.67 \times 10^3}{288\pi}$

$= \underline{103.52 \text{ N/mm}^2}$

(4) Three pillars, two of aluminium and one of steel support a rigid platform of 20KN as shown in the fig. If area of each aluminium pillar is 1000mm² and that of steel pillar is 800mm², find the stresses developed in each pillar.

Take $E_a = 1 \times 10^5\ N/mm^2$ and $E_s = 2 \times 10^5\ N/mm^2$. What additional load 'P' can it take if working stresses are $65\ N/mm^2$ in aluminium and $150\ N/mm^2$ in steel?



$A_a = 1000\ mm^2$

$A_s = 800\ mm^2$

$E_a = 1 \times 10^5\ N/mm^2$ & $E_s = 2 \times 10^5\ N/mm^2$

$P_a = 65\ N/mm^2$ & $P_s = 150\ N/mm^2$ [working stresses]

200 KN load is shared by two aluminium pillars & one steel pillars

Let P_a be the load shared by each aluminium pillar

& P_s be the load shared by steel pillar

Therefore from equilibrium condition

$$P_a + P_s + P_a = 200$$

$$2P_a + P_s = 200 \text{ kN} \rightarrow (1)$$

$$\Delta a = \Delta s$$

$$\frac{P_a \times L_a}{A_a \times E_a} = \frac{P_s \times L_s}{A_s \times E_s}$$

$$\frac{P_a \times 200}{1000 \times 1 \times 10^5} = \frac{P_s \times 250}{800 \times 2 \times 10^5}$$

$$2 \times 10^{-6}\, P_a = 1.562 \times 10^{-6}\, P_s$$

$$\Rightarrow P_a = 0.781\, P_s \rightarrow ②$$

From eqns ① & ②

$$2 \times 0.781 \times P_s + P_s = 200$$

$$2.562\, P_s = 200$$

$$\Rightarrow P_s = 78.06 \text{ KN}$$

$$\& \quad P_a = 60.97 \text{ KN}$$

∴ stress developed in aluminium pillar

$$= \frac{60.97 \times 10^3}{1000} = 60.97 \text{ N/mm}^2$$

$||^{ly}$ in steel pillar $= \frac{78.06 \times 10^3}{800} = 97.57 \text{ N/mm}^2$

Calculation of additional load capacity of steel & aluminium pillar.

i) Additional load carrying capacity of aluminium. [If Pa governs load carrying capacity]

Since Permissible stress of alum

or $P_a = 65 \text{ N/mm}^2$

W.K.T $= \rho_a = \dfrac{\text{Additional load}}{A_a}$

$$\rho_a = \frac{P_a}{A_a}$$

$\Rightarrow$ Additional load carrying capacity of aluminium pillar

$\therefore P_a = \rho_a \times A_a = 65 \times 1000 = 65000 N$

$P_s = 1.28 P_a$

$III^{ly}.\ P_s = 1.28 \times 65000 = 83200 N$

$\therefore$ Total load carrying capacity

$= 2P_a + P_s = 213200 N$

$= \underline{213.2 \text{ KN}}$

If ρ_s governs load carrying capacity

$\rho_s = 150 N/mm^2$

$\Rightarrow P_s = \rho_s \times A_s = 150 \times 800 = 120 KN$

$P_a = 0.781 P_s = 93.72 KN$

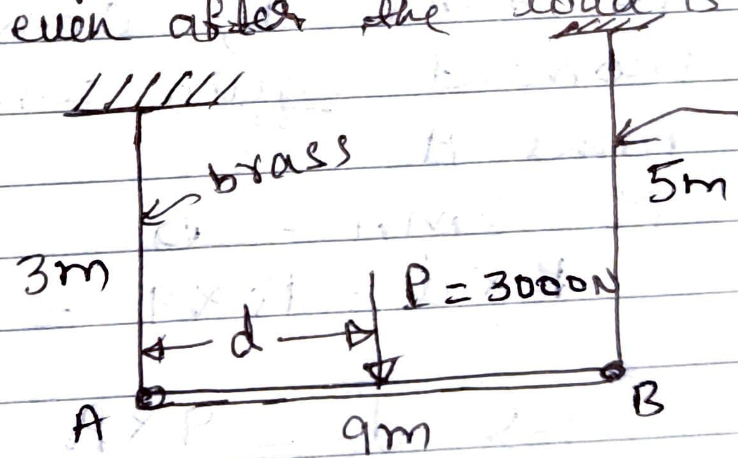
$\therefore$ Total load carrying capacity

$= 2P_a + P_s = \underline{307.44 \text{ KN}}$

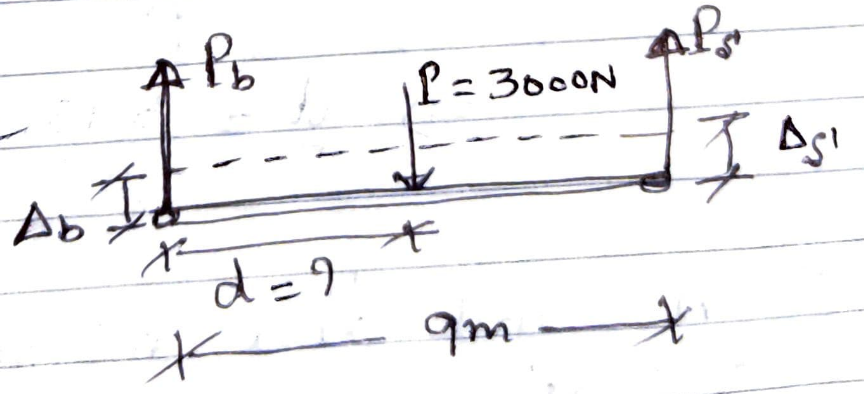
$\therefore$ Actual load carrying capacity is 213.2 KN, which means additional load $P = 213.2 - 200 = \underline{13.2 \text{ kN}}$

⑤ A rigid bar AB is 9m long is suspended by two vertical rods at it's ends A & B and hangs in a horizontal position by it's own weight. The rod at A is brass, length 3m, cross-sectional area is 1000 mm² and $E_b = 1 \times 10^5\ N/mm^2$. The rod at B is steel, length 5m, cross-sectional area is 445 mm² & $E_s = 2 \times 10^5\ N/mm^2$. At what distance 'd' from A may vertical load P = 3000N be applied if the bar is to remain horizontal even after the load is applied.

$L_b = 3000\ mm$
$E_b = 1 \times 10^5$
$A_b = 1000\ mm^2$



$L_s = 5000\ mm$
$E_s = 2 \times 10^5$
$A_s = 445\ mm^2$



From static equm condition

$P_b + P_s = 3000 \longrightarrow (i)$

To bar remains in H^{l} position

$\Delta_b = \Delta_s$

$$\frac{P_b L_b}{A_b E_b} = \frac{P_s \times L_s}{A_s \times E_s}$$

After simplyfying

$P_b = 1.87\ P_{s'} \rightarrow (ii)$

considering (i) & (ii)

$P_{s'} = 1045.296\ N$

$P_b = 1954.70\ N$

To find 'd', considering moment equm condition about Point A.

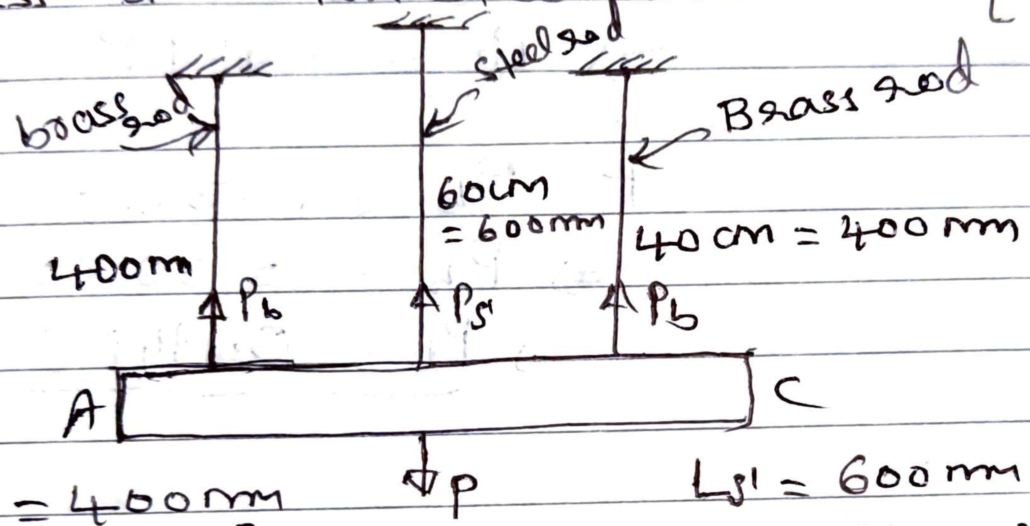
ie $\Sigma M_A = 0$

$P \times d - P_s \times 9 = 0$

$$\Rightarrow d = \frac{9 \times P_{s'}}{P} = \frac{9 \times 1045.296}{3000}$$

$$\Rightarrow d = 3.135\ m$$

⑥ A rigid ~~rod~~ bar AC supported by 3 rods in the same vertical plane and equidistant. The outer rods are of the brass and of length 40 cms & diameter 20mm. The central rod is of steel & of 60cm length & 25mm diameter. Calculat the forces in the bars due to an applied force 'P.' If the bar AC remains H^le after the load has been applied. Take E for brass to E for steel ratio = 2. $\left[\frac{E_b}{E_s} = 2\right]$



$L_b = 400mm$ $\qquad$ $L_{s'} = 600mm$

$A_b = \frac{\pi}{4} \times 20^2 = 314.2$ $\qquad$ $A_s = \frac{\pi}{4} \times 25^2 = 490.93$

$\frac{E_b}{E_s} = 2$

From static equm condition

$P_b + P_b + P_{s'} = P$

$\Rightarrow 2P_b + P_{s'} = P \rightarrow (i)$

The bar AC to be remains in H^le position, $\quad \Delta_b = \Delta_c$

$$\frac{P_b L_b}{A_b E_b} = \frac{P_s L_{s'}}{A_s E_{s'}}$$

$$\Rightarrow P_b = P_{s'} \times \frac{A_b}{A_s} \times \frac{E_b}{E_s} \times \frac{L_s}{L_b}$$

$$= P_{s'} \times \frac{314.2}{490.93} \times 2 \times \frac{600}{400}$$

$$\Rightarrow P_b = \frac{628.4}{1963.72} \times P_{s'}$$

$$\Rightarrow P_b = 0.32 P_{s'} \rightarrow (ii)$$

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From (i) & (ii)

$$2 \times 0.32 P_{S'} + P_{S'} = P$$

$$0.64 P_{S'} + P_{S'} = P$$

$$1.64 P_{S'} = P$$

$$\Rightarrow \underline{P_{S'} = 0.609 P}$$

$$\& \quad P_b = 0.32 P_{S'}$$

$$\Rightarrow P_b = 0.32 \times 0.609 P$$

$$\Rightarrow \underline{P_b = 0.1948 P}$$