$$E = 9K - \frac{3KE}{G}$$

$$E = \left(1 + \frac{3K}{G}\right) = 9K$$

or

$$E = \left(\frac{G+3K}{G}\right) = 9K$$

or

$$E = \frac{9KG}{G+3K} \quad \text{—— (6)}$$

Eqn (6) may be expressed as

$$\frac{9}{E} = \frac{G+3K}{KG}$$

$$\boxed{\frac{9}{E} = \frac{3}{G} + \frac{1}{K}}$$

Numericals :-

1. A bar of 20mm diameter is tested in tension. It is observed that when a load of 37.7 KN is applied, the extension measured over a guage length of 200mm is 0.12mm & Contraction in diameter is 0.0036MM find poissons ratio & elastic Constants E, G, K.

Solution Given Data

1. $P = 37.7\,KN = 37.7 \times 10^3 N$
2. Gauge length $L = 200mm$
3. $\Delta = 0.12mm$
4. $\Delta d = 0.0036mm.$

Step 1 → Area $A = \frac{\pi}{4} \times (20)^2$

$A = 314.15\,mm^2$

Step 2 → Linear Strain $= \frac{\Delta}{L} = \frac{0.12}{200} = 0.0006$

Step 3 → Lateral Strain $\frac{\Delta d}{d} = \frac{0.0036}{20}$

$= 0.00018$

Step 4 → Poissons ratio

$$\mu = \frac{\text{Lateral Strain}}{\text{Linear Strain}}$$

$$\mu = \frac{0.00018}{0.0006}$$

$$\mu = 0.3$$

Step 5 → $\Delta = \frac{PL}{AE}$

$$0.12 = \frac{37.7 \times 10^3 \times 200}{314.15 \times E}$$

$$E = 200004.8 \ N/mm^2$$

Step 6 → $E = 2G(1+\mu)$

We get $G = \frac{E}{2(1+\mu)}$

$$G = \frac{200004.8}{2(1+0.3)}$$

$$G = 76924.92 \ N/mm^2$$

Step 7 → from the relation

$E = 3K(1-2\mu)$ We get

$$K = \frac{E}{3(1-2\mu)} = \frac{200004.8}{3(1-2\times 0.3)}$$

$$K = 1{,}66{,}670.66 \ N/mm^2$$

2] Determine the change in length, breadth & thickness of a Steel bar which is 4m long 30mm wide & 20mm thick & is subjected to an axial pull of 30KN in the direction of its length Take $E = 2\times10^5\ N/mm^2$ and poisson's ratio $\mu = 0.3$.

Solve Given Data

1. length of the bar $L = 4m = 4000mm$
2. Breadth of the bar $b = 30mm$
3. Thickness of the bar $t = 20mm$
4. Area of Cross Section $A = b\times t$

$A = 30\times20$

$A = 600mm^2$

5. Axial pull $P = 30kN = 30\times10^3 N$.
6. Young's Modulus $E = 2\times10^5\ N/mm^2$
7. Poisson's ratio $\mu = 0.3$.

Step 1 → Now Strain in the direction of Load (or Longitudinal Strain)

$$\text{Strain} = \frac{\text{Stress}}{E} = \frac{\text{Load}}{A\times E}$$

$\text{Stress} = \frac{\text{Load}}{\text{Area}}$

$\sigma = E.e$ — Hooke's Law

$$\frac{P}{A\times E} = \frac{30\times10^3}{600\times2\times10^5}$$

$$= 0.00025$$

Step 2 → But longitudinal Strain $= \frac{\delta L}{L}$

$$\frac{\delta L}{L} = 0.00025$$

Step 3 → δL (or change in length)

$$= 0.00025\times L$$

$$= 0.00025\times4000$$

$$= 1mm.$$

Step 4 → Poisson's ratio $= \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}}$

$$0.3 = \frac{\text{Latural Strain}}{0.00025}$$

$$\text{Lateral Strain} = 0.3 \times 0.00025$$
$$= 0.000075$$

Step 5 → Lateral Strain $= \frac{\delta b}{b}$ or $\frac{\delta d}{d}$ (or $\frac{\delta t}{t}$)

$$\delta b = b \times \text{Lateral Strain}$$
$$= 30 \times 0.000075$$
$$\delta b = 0.00225\text{mm}$$

Similarly $\delta t = t \times \text{Lateral Strain}$

$$\delta t = 20 \times 0.000075$$
$$\delta t = 0.0015\text{mm}.$$

3] Determine the Value of young's modulus & poisson's ratio of a metallic bar of length 30cm, breadth 4cm & depth 4cm. when the bar is Subjected to an axial Compressive Load of 400kN. The decrease in length is given as 0.075cm & increase in breadth is 0.003cm.

Solution Given Data

1. Length $L = 30\text{cm}$
2. Breadth $b = 4\text{cm}$
3. Depth $d = 4\text{cm}$.

Step 1 → Area of Cross Section

$$A = b \times d$$
$$A = 4 \times 4$$
$$A = 16\text{cm}^2$$
$$A = 16 \times 100 = 1600\text{mm}^2$$

Step 2 → Axial Compressive Load

$P = 400 KN = 400 \times 10^3 N$

Step 3 → Decrease in length $\delta L = 0.075 cm$

Step 4 → Increase in breadth $\delta b = 0.003 cm$.

Step 5 → Longitudinal Strain

$$\frac{\delta L}{L} = \frac{0.075}{30} = 0.0025$$

Step 6 → Lateral Strain $\frac{\delta b}{b} = \frac{0.003}{4} = 0.00075$.

Step 7 → poissons ratio $\mu = \frac{\text{Lateral Strain}}{\text{longitudinal strain}}$

$$\mu = \frac{0.00075}{0.0025} = 0.3$$

Step 8 → Longitudinal Strain

$$e = \frac{\text{Stress}}{E} = \frac{P}{A.E}$$

$$0.0025 = \frac{400 \times 10^3}{1600 \times E}$$

$$E = \frac{400 \times 10^3}{1600 \times 0.0025} = 1 \times 10^5 N/mm^2$$

4] A Circular rod of 100mm diameter & 500mm long is Subjected to a tensile force of 1000KN. Determine the modulus of rigidity, bulk modulus & change in Volume if poissons ratio $\mu = 0.3$ & Youngs modulus $E = 2 \times 10^5 N/mm^2$.

Solve from the relationship.

$$E = 2G(1+\mu)$$

$$E = 3K(1-2\mu)$$

Step 1 → We get $G = \frac{E}{2(1+\mu)} = \frac{2\times10^5}{2(1+0.3)}$

$G = 0.7692\times10^5 \ N/mm^2$

Step 2 → $K = \frac{E}{3(1-2\mu)} = \frac{2\times10^5}{3(1-2\times0.3)}$

$K = 1.667\times10^5 \ N/mm^2$

Step 3 → Longitudinal Stress $= \frac{P}{A}$

$= \frac{1000\times10^3}{\frac{\pi}{4}\times(100)^2}$

$= 127.324 \ N/mm^2$

Step 4 → Linear Strain $= \frac{\text{Stress}}{E}$

$= \frac{127.324}{2\times10^5}$

$= 63.662\times10^{-5}$

Step 5 → Lateral Strain

$e_y = -\mu e_x$

& $e_z = -\mu e_x$

Step 6 → Volumetric Strain

$e_v = e_x + e_y + e_z$

$e_v = e_x(1-2\mu)$

$e_v = 63.662\times10^{-5} \times (1-2\times0.3)$

$e_v = 25.4648\times10^{-5}$

Step 7 → $\frac{\text{change in Volume}}{V} = e_v$

Change in Volume $= e_v \times V$

$= 25.468\times10^{-5} \times \frac{\pi}{4}\times(100)^2\times500$

$= 1000 \ mm^3$

5] In a Laboratory tensile test is Conducted & young's modulus of the material is found to be $2.1 \times 10^5 \, N/mm^2$, on the Same material torsion test is Conducted and modulus of rigidity is found to be $0.78 \times 10^5 \, N/mm^2$. Determine poissons ratio & bulk modulus of the material

Solution Given Data

1. $E = 2.1 \times 10^5 \, N/mm^2$
2. $G = 0.78 \times 10^5 \, N/mm^2$

Step 1 → $E = 2G(1+\mu)$

$$2.1 \times 10^5 = 2 \times 0.78 \times 10^5 (1+\mu)$$

$$1.346 = 1+\mu$$

$$\mu = 0.346$$

Step 2 → $E = 3K(1-2\mu)$

$$2.1 \times 10^5 = 3K(1-2 \times 0.346)$$

$$K = 2.275 \times 10^5 \, N/mm^2$$

6] A material has modulus of rigidity equal to $0.4 \times 10^5 \, N/mm^2$ & bulk modulus equal to $0.75 \times 10^5 \, N/mm^2$. find its young's modulus & poisson's ratio

Solution

$$G = 0.4 \times 10^5 \, N/mm^2$$

$$K = 0.75 \times 10^5 \, N/mm^2$$

Using the relation $E = \dfrac{9GK}{3K+G}$

$$E = \frac{9 \times 0.4 \times 10^5 \times 0.75 \times 10^5}{3 \times 0.75 \times 10^5 + 0.4 \times 10^5}$$

$$E = 1.019 \times 10^5 \, N/mm^2$$

from the relation

$$E = 2G(1+\mu)$$

$$1.019 \times 10^5 = 2 \times 0.4 \times 10^5 (1+\mu)$$

$$1.2736 = 1+\mu$$

$$\mu = 0.2736.$$

7] Determine the poissons ratio & bulk modulus of a material, for which youngs modulus is 1.2×10^5 N/mm^2 & modulus of rigidity is 4.8×10^4 N/mm^2.

Solution Given Data

1. Youngs Modulus $E = 1.2 \times 10^5$ N/mm^2
2. Modulus of rigidity $G = 4.8 \times 10^4$ N/mm^2

Let poissons ratio 'μ'

Step 1 → $E = 2G(1+\mu)$

$$1.2 \times 10^5 = 2 \times 4.8 \times 10^4 (1+\mu)$$

$$(1+\mu) = \frac{1.2 \times 10^5}{2 \times 4.8 \times 10^4}$$

$$\mu = 1.25 - 1$$

$$\mu = 0.25$$

Step 2 → Bulk Modulus

$$K = \frac{E}{3(1-2\mu)} = \frac{1.2 \times 10^5}{3(1-0.25 \times 2)}$$

$$K = 8 \times 10^4 \text{ N/mm}^2$$

8] A bar of 30mm diameter is Subjected to a pull of 60KN. The measured extension on gauge length of 200mm is 0.1mm & change in diameter is 0.004mm.
Calculate
1. Young's Modulus
2. poisson's ratio
3. Bulk Modulus.

Solution Given Data
1. Diameter of bar $d = 30mm$
2. pull $P = 60KN = 60\times10^3 N$
3. Gauge length $L = 200mm$
4. Extension $\delta L = 0.1mm$
5. Change in diameter $= 0.004mm = \delta d$

Step 1 → Area of bar
$$A = \frac{\pi}{4}(30)^2 = 225\pi\ mm^2$$

Step 2 → Tensile Stress $\sigma = \frac{P}{A} = \frac{60\times10^3}{225\pi} = 84.87\ N/mm^2$

Step 3 → Longitudinal Strain
$$\frac{\delta L}{L} = \frac{0.1}{200} = 0.0005$$

Step 4 → Young's Modulus
$$E = \frac{\text{Tensile Stress}}{\text{Longitudinal Strain}} = \frac{84.87}{0.0005} = 16.975\times10^4\ N/mm^2 = 1.6975\times10^5\ N/mm^2$$

Step 5 → Poisson's ratio (μ)
$$\text{poisson's ratio } \mu = \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}}$$
$$\mu = \frac{\left(\frac{\delta d}{d}\right)}{0.0005} = \frac{\left(\frac{0.004}{30}\right)}{0.0005} = \frac{0.000133}{0.0005}$$
$$\mu = 0.266$$