Numericals

1) A Circular rod of diameter 20mm and 500mm Long is Subjected to a tensile force 45KN. The modulus of elasticity for Steel may be taken as 200 KN/mm^2 find Stress, Strain & elongation of the bar due to applied Load.

Solution Given Data

Load $P = 45KN = 45 \times 10^3 N$

$E = 200 KN/mm^2 = 200 \times 10^3 N/mm^2$

$L = 500mm$

Diameter of the rod $d = 20mm$

Solution

Step 1 → Cross Sectional area

$$\text{Area } A = \frac{\pi d^2}{4} = \frac{\pi}{4} \times (20)^2 = \boxed{314.159\ mm^2}$$

Step 2 → Stress $\sigma = \frac{P}{A} = \frac{45 \times 10^3}{314.159} = \boxed{143.24\ N/mm^2}$

Step 3 → Strain $e = \frac{\sigma}{E} = \frac{143.24}{200 \times 10^3} = \boxed{0.0007162}$

$$E = \frac{\sigma}{e}$$

$$e = \frac{\sigma}{E}$$

Step 4 → Elongation $\Delta = \frac{PL}{AE} = \frac{45 \times 10^3 \times 500}{314.159 \times 200 \times 10^3}$

$$\boxed{\Delta = 0.358\ mm.}$$

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2) A rod 150cm long & of diameter 2cm is Subjected to an axial pull of 20kN. If the modulus of elasticity of the material of the rod is $2 \times 10^5\ N/mm^2$. Determine

i) Stress
ii) Strain
iii) Elongation of rod.

Solution Given Data

* Length of the rod $L = 150cm$
* Diameter of the rod $D = 2cm = 20mm$
* Axial pull $P = 20kN = 20 \times 10^3 N$
* Modulus of Elasticity $E = 2 \times 10^5\ N/mm^2$

Step 1 → Area $A = \frac{\pi}{4}(D)^2 = \frac{\pi}{4}(20)^2 = \boxed{314.15\,mm^2}$

Step 2 → Stress $\sigma = \frac{P}{A} = \frac{20\times10^3}{314.15} = \boxed{63.66\,N/mm^2}$

Step 3 → Strain $e = \frac{\sigma}{E} = \frac{63.66}{2\times10^5} = \boxed{0.000318}$

Step 4 → Elongation $e = \frac{\Delta}{L}$ or $\frac{dL}{L}$

$$\Delta = e \times L$$
$$= 0.000318 \times 150$$
$$\Delta = 0.0477\text{ cm.}$$

or

$$\Delta = \frac{PL}{AE} = \frac{20\times10^3 \times 150}{314.15 \times 2\times10^5} = \boxed{0.0477\,cm.}$$

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3) A Steel rod of 30mm diameter and 400mm length was tested in a tension testing machine. At a Load of 135 KN, the extension in a gauge length of 50mm was measured to be 0.045 mm & the reduction in diameter was 0.008mm. Determine the poissons ratio & Young's modulus for the material.

Solution Given Data

* Diameter $D = 30mm$
* Length $L' = 50mm$
* Load $P = 135KN = 135\times10^3 N$
* Gauge length $L = 50mm$
* Extension $\Delta L = 0.045mm$
* Reduction in diameter $\Delta D = 0.008\,mm$.

Requirement → poissons ratio & Young's modulus.

Solution

Step 1 → Cross Sectional area of the Steel rod.

$$A = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times (30)^2 = \boxed{706.86 mm^2}$$

Step 2 → Stress in the rod $\sigma = \frac{P}{A} = \frac{135 \times 10^3}{706.86}$

$$\boxed{\sigma = 190.99 \, N/mm^2}$$

Step 3 → Longitudinal Strain $\varepsilon = \frac{\Delta L}{L} = \frac{0.045}{50}$

$$\boxed{\varepsilon = 0.9 \times 10^{-3}}$$

Step 4 → Lateral Strain $\frac{\Delta D}{D} = \frac{0.008}{30} = \boxed{0.267 \times 10^{-3}}$

Step 5 → Young's Modulus $E = \frac{\sigma}{\varepsilon} = \frac{190.99}{0.9 \times 10^{-3}}$

$$\boxed{E = 212.21 \times 10^3 \, N/mm^2}$$

Step 6 → poisson's ratio ν or μ or $\frac{1}{m}$

$$\mu = \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}}$$

$$\mu = \frac{0.267 \times 10^{-3}}{0.9 \times 10^{-3}} = \boxed{0.2967}$$

4] Find the minimum diameter of a steel wire, which is used to raise a load of 4000N if the stress in the rod is not to exceed $95 \, MN/m^2$.

Solution

Given Data

* Load $P = 4000 N$
* Stress $\sigma = 95 MN/m^2$

$= 95 \times 10^6 \, N/m^2$

$\sigma = 95 \, N/mm^2$

* Diameter of wire in 'mm'

Mega $= 10^6$

$10^6 N/m^2 = 1 N/mm^2$

Step 1 → Area $A = \frac{\pi}{4} D^2$

Now Stress $\sigma = \frac{Load}{Area}$

$\sigma = \frac{P}{A}$

$95 = \frac{4000}{\frac{\pi}{4} D^2}$ or $D^2 = \frac{4000 \times 4}{\pi \times 95}$

$D^2 = 53.61$

$D = \sqrt{53.61}$

$\boxed{D = 7.32mm}$

5] Find the Young's modulus of a brass rod of diameter 25mm & of Length 250mm which is subjected to a tensile Load of 50KN. When the extension of the rod is equal to 0.3mm.

Solution Given Data

* Diameter of rod $D = 25mm$
* Area of rod $A = \frac{\pi}{4}(25)^2 = 490.87 mm^2$
* Tensile Load $P = 50 KN = 50 \times 10^3 N$
* Extension of rod $dL = 0.3mm$
* Length of rod $L = 250mm$

Step 1 → Area $A = \frac{\pi}{4}(D)^2 = \frac{\pi}{4}(25)^2 = \boxed{490.87 mm^2}$

Step 2 → Stress $\sigma = \frac{P}{A} = \frac{50 \times 10^3}{490.87} = \boxed{101.86 N/mm^2}$

Step 3 → Strain $e = \frac{\Delta}{L}$ or $\frac{dL}{L} = \frac{0.3}{250} = \boxed{0.0012}$

or $e = \frac{\sigma}{E}$ if Young's modulus is given.

Step 4 → Young's Modulus

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{e} = \frac{101.86}{0.0012} = 84883.33 \ N/mm^2$$

$G = 10^9$

$1 \ N/mm^2 = 10^6 \ N/m^2$

7] A Specimen of Steel 25mm diameter with a gauge length of 200mm is tested to destruction. It has an extension of 0.16mm under a load of 80KN and the Load at elastic limit is 160KN. The maximum Load is 180KN.

The total extension at fracture is 56mm & diameter at neck is 18mm find

1] The Stress at elastic limit
2] Youngs modulus
3] percentage Elongation
4] percentage reduction in area.
5] Ultimate tensile Stress.

Solution Given Data

* Diameter of Steel d = 25mm
* Gauge Length = 200mm
* Extension = 0.16mm
* Load = 80KN
* Loat at Elastic Limit = 160KN
* Maximum Load = 180KN

Step 1 → Area $A = \frac{\pi}{4} d^2$

$$A = \frac{\pi}{4}(25)^2$$

$$\boxed{A = 490.874 mm^2}$$

Step 2 → Stress at Elastic limit

$$\sigma = \frac{\text{Load at Elastic limit}}{\text{Area}}$$

$$\sigma = \frac{160 \times 10^3}{490.874}$$

$$\boxed{\sigma = 325.949 N/mm^2}$$

Step 3 → Youngs Modulus

$E = \frac{\text{Stress}}{\text{Strain}}$ Within Elastic Limit

$$E = \frac{P/A}{\Delta/L} = \frac{80\times10^3/490.874}{0.16/200} = \boxed{203718\ N/mm^2}$$

Step 4 → percentage Elongation

$$= \frac{\text{final Extension}}{\text{Original Length}} = \frac{56}{200}\times100$$

$$\boxed{= 28\ \%}$$

Step 5 → percentage reduction in area

$$\frac{\text{Initial area} - \text{final area}}{\text{Initial area}}\times 100$$

$$= \frac{\frac{\pi}{4}(25)^2 - \frac{\pi}{4}\times(18)^2}{\frac{\pi}{4}\times(25)^2}\times100$$

$$\boxed{= 48.16\ \%}$$

Step 6 → Ultimate tensile Stress

$$\frac{\text{Ultimate Load}}{\text{Area}}$$

$$= \frac{180\times10^3}{490.874}$$

$$\boxed{= 366.693\ N/mm^2}$$

8] A metal bar 50mm x 50mm is subjected to an axial Compressive Load of 500KN. The Contraction of a 200mm gauge length is found to be 0.5mm & the increase in thickness 0.04mm. Find the values of Young's modulus and poisson's ratio. 16

Solution Given Data

- ★ Cross Section – Square 50mm x 50mm
- ★ Load $P = 500 kN = 500 \times 10^3 N$ (Compressive)
- ★ Gauge Length $L = 200mm$
- ★ Contraction $\Delta L = 0.5mm$
- ★ Increase in thickness $\Delta a = 0.04mm$

Step 1 → Cross Sectional area of bar

$$A = 50 \times 50 = 2500 mm^2$$

Step 2 → Stress in the bar

$$\sigma = \frac{P}{A} = \frac{500 \times 10^3}{2500} = \boxed{200 N/mm^2}$$

Step 3 → Longitudinal Strain

$$\epsilon = \frac{\Delta L}{L} = \frac{0.5}{200} = \boxed{2.5 \times 10^{-3}}$$

Step 4 → Lateral Strain

$$\frac{\Delta a}{a} = \frac{0.04}{50} = \boxed{0.8 \times 10^{-3}}$$

Step 5 → Young's Modulus

$$E = \frac{\sigma}{\epsilon} = \frac{200}{2.5 \times 10^{-3}} = \boxed{80 \times 10^3 N/mm^2}$$

Step 6 → Poisson's ratio

$$\mu \text{ or } \nu = \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}} = \frac{0.8 \times 10^{-3}}{2.5 \times 10^{-3}} = \boxed{0.32}$$

9) A tensile test was conducted on a mild steel bar. The following data was obtained from the test.

(a) Diameter of the steel bar = 3cm.
(b) Gauge length of the bar = 20cm.
(c) Load at Elastic limit = 250KN
(d) Extension at a load of 150KN = 0.21mm
(e) Maximum Load = 380KN
(f) Total Extension = 60mm
(g) Diameter of the rod at the failure = 2.25cm

Determine
(a) Young's modulus
(b) The stress at Elastic limit
(c) The percentage Elongation
(d) The percentage decrease in area.

Solution

Step 1 → Area of the rod

$$\text{Area } A = \frac{\pi}{4} d^2 \qquad d = 3\text{cm} = 3\times10,\ d = 30\text{mm}$$

$$= \frac{\pi}{4}(30)^2$$

$$\boxed{A = 706.85\text{mm}^2}$$

Step 2 → $\text{Stress} = \frac{\text{Load}}{\text{Area}} = \frac{150\times10^3}{706.85}$

$$\boxed{\sigma = 212.20\,\text{N/mm}^2}$$

Step 3 → $\text{Strain} = \frac{\text{Increase in length or Extension}}{\text{Original length or Gauge length}}$

Gauge length 20cm = 200mm

$$= \frac{0.21}{200}$$

$$\boxed{\varepsilon \text{ or } e = 0.00105}$$

Step 4 → Young's Modulus.

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{212.20}{0.00105} = \boxed{202.09\times10^3\,\text{N/mm}^2}$$

II] The Stress at the Elastic limit is given by.

Step 5 → Stress = $\frac{\text{Load at elastic Limit}}{\text{Area}}$

$$\sigma = \frac{250 \times 10^3}{706.85}$$

$$\boxed{\sigma = 353.68 \ N/mm^2}$$

Step 6 → percentage Elongation

$$= \frac{\text{Total increase in length}}{\text{Original length or Gauge length}} \times 100$$

$$= \frac{60}{200} \times 100$$

$$\boxed{= 30\%.}$$

Step 7 → Percentage decrease in area

percentage decrease in area

$$= \frac{\left(\text{Original area} - \text{Area at the failure}\right)}{(\text{Original area})} \times 100$$

$$= \left(\frac{\frac{\pi}{4} \times (3)^2 - \frac{\pi}{4} \times (2.25)^2}{\frac{\pi}{4} \times (3)^2} \right) \times 100$$

$$= \left(\frac{(3)^2 - (2.25)^2}{(3)^2} \right) \times 100$$

$$= \left(\frac{9 - 5.0625}{9} \right) \times 100$$

$$\boxed{= 43.75\%.}$$

10] A Steel tube 25mm outer diameter and 12mm inner diameter carries an axial tensile load of 40 KN. What will be the stress in the tube if E = 200 GPa? What further increase in load is possible if the stress is limited to 225 MN/m^2?

Solution Given Data

* Outer diameter D = 25mm
* Inner diameter d = 12mm
* Load P = 40 KN = 40×10^3 N
* Modulus of Elasticity E = 200 GPa = 200×10^3 N/mm^2
* Upper limit of stress = 225 MN/M^2
  = 225 N/mm^2

Step 1 → Cross Sectional area of tube

$$A = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4} \times (25^2 - 12^2)$$

$$= 377.78 \ mm^2$$

Step 2 → Stress in the tube

$$\sigma = \frac{P}{A} = \frac{40 \times 10^3}{377.78} = 105.88 \ N/mm^2$$

Step 3 → Increase in Stress

$$\Delta\sigma = 225 - 105.88 = 119.12 \ N/mm^2$$

Step 4 → Increase in Load

$$\Delta P = \Delta\sigma \times A = 119.12 \times 377.78$$

$$= 45001.15 N$$

$$= 45 KN$$

11] The Safe Stress for a hollow steel Column which Carries an axial Load of 2.1×10^3 KN is $125\ MN/m^2$. If the external diameter of the Column is 30cm. Determine the internal diameter.

Solution Given Data

* Safe Stress $\sigma = 125\ MN/m^2$

$\sigma = 125 \times 10^6\ N/m^2$

* Axial Load $P = 2.1 \times 10^3$ KN

$P = 2.1 \times 10^6$ N

* External diameter $D = 30cm = 0.3m$
* Let d = Internal diameter

Step 1 → Area of Cross Section of the Column

$$A = \frac{\pi}{4}(D^2 - d^2)$$

$$A = \frac{\pi}{4}\left[(0.30)^2 - d^2\right] m^2 \quad \text{——— (1)}$$

Step 2 → Stress $\sigma = \frac{P}{A}$

$$125 \times 10^6 = \frac{2.1 \times 10^6}{\frac{\pi}{4}\left[(0.30)^2 - d^2\right]}$$

or

$$\left[(0.30)^2 - d^2\right] = \frac{4 \times 2.1 \times 10^6}{\pi \times 125 \times 10^6}$$

$$0.09 - d^2 = 213.9$$

or

$$0.09 - 0.02139 = d^2$$

$$d = \sqrt{0.09 - 0.02139}$$

$$d = 0.2619\ m$$

$$\boxed{d = 26.19\ cm.}$$

12) The ultimate Stress for a hollow steel Column which Carries an axial Load of 1.9 MN is 480 N/mm^2. If the external diameter of the Column is 200mm. Determine the internal diameter. Take the factor of Safety as 4.

Solution Given Data

* Ultimate Stress = 480 N/mm^2 — M = Mega
* Axial Load $P = 1.9\,MN$
  $= 1.9 \times 10^6\,N$
* External diameter $D = 200mm$
* factor of Safety = 4
* Let d = Internal diameter in mm.

Step 1 → Area of Cross Section of the Column

$$A = \frac{\pi}{4}(D^2 - d^2)$$

$$A = \frac{\pi}{4}\left[(200)^2 - d^2\right] mm^2$$

Step 2 → We have

$$\text{factor of Safety} = \frac{\text{Ultimate Stress}}{\text{Working Stress or permissible Stress}}$$

$$4 = \frac{480}{\text{Working Stress}}$$

$$\text{Working Stress} = \frac{480}{4}$$

$$\text{Working Stress } \boxed{\sigma = 120\,N/mm^2}$$

Step 3 →

$$\sigma = \frac{P}{A}$$

$$120 = \frac{1.9 \times 10^6}{\frac{\pi}{4}\left[(200)^2 - d^2\right]}$$

$$120 = \frac{1.9 \times 10^6 \times 4}{\pi(40{,}000 - d^2)}$$

$$40{,}000 - d^2 = \frac{1.9 \times 10^6 \times 4}{\pi \times 120} = 20159.6$$

$$d^2 = 40{,}000 - 20159.6$$

$$d^2 = 19840.4$$

$$d = \sqrt{19840.4}$$

$$d = 140.85\,mm$$

13] The following data refer to a mild Steel Specimen tested in a Laboratory. Diameter of Specimen = 25mm, Gauge Length of Specimen = 300mm, Length of Specimen after failure = 360mm, Extension observed under a Load of 20KN = 0.06mm. Load at yield point = 150KN & Load at failure = 252KN. Neck diameter at failure point = 18.25mm. Determine

1) Young's modulus
2) Yield Stress
3) Ultimate Stress
4) percentage Elongation
5) Percentage reduction of Cross Sectional area
6) Safe Stress adopting a factor of Safety as 2

Solution Given Data

- ★ Diameter $D = D_i = 25mm$
- ★ Gauge Length $L = L_i = 300mm$
- ★ Length of Specimen at failure $L_f = 360mm$
- ★ Extension under a Load of $P = 20KN$, $\Delta L = 0.06mm$
- ★ Load at yield point $P_y = 150KN$
- ★ Load at failure $P_u = 252KN$
- ★ Neck diameter at failure point $D_f = 18.25mm$
- ★ factor of Safety (FOS) = 2

Step 1 → Cross Sectional area of the Steel rod

$$A = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times 25^2 = 490.87 \, mm^2$$

Step 2 → Stress in the rod $\sigma = \frac{P}{A} = \frac{20 \times 10^3}{490.87} = \boxed{40.74 \, N/mm^2}$

Step 3 → Longitudinal Strain $\varepsilon = \frac{\Delta L}{L} = \frac{0.06}{300}$

$$\boxed{\varepsilon = 2 \times 10^{-4}}$$

Step 4 → Young's Modulus $E = \frac{\sigma}{\varepsilon} = \frac{40.74}{2 \times 10^{-4}}$

$$\boxed{E = 203.7 \times 10^{3} \, N/mm^2}$$

Step 5 → Yield Stress $\sigma_y = \frac{P_y}{A} = \frac{150 \times 10^3}{490.87}$

$\sigma_y = 305.58 \ N/mm^2$

Step 6 → Ultimate Stress $\sigma_u = \frac{P_u}{A} = \frac{252 \times 10^3}{490.87}$

$\sigma_u = 513.37 \ N/mm^2$

Step 7 → percentage Elongation

$$= \frac{L_f - L_i}{L_i} \times 100$$

$$= \frac{360 - 300}{300} \times 100$$

$$= 20\%$$

Step 8 → percentage reduction in area

$$\frac{A_i - A_f}{A_i} \times 100 = \frac{D_i^2 - D_f^2}{D_i^2} \times 100$$

$$= \frac{25^2 - 18.25^2}{25^2} \times 100$$

$$= 46.71\%$$

Step 9 → Safe Stress $= \frac{\text{Yield stress}}{\text{factor of safety}}$

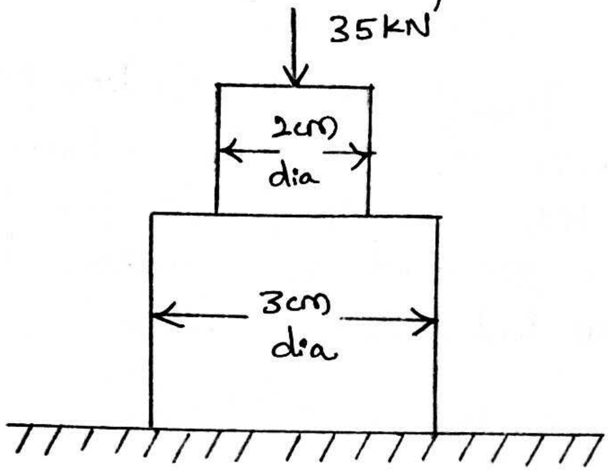
$$= \frac{\sigma_y}{FOS} = \frac{305.58}{2}$$

$$= 152.79 \ N/mm^2$$

14] A stepped bar shown in figure, is subjected to an axially applied Compressive Load of 35 KN. find the maximum & minimum stress produced.

Solution Given Data

* Axial Load $P = 35\,KN = 35 \times 10^3\,N$
* Diameter of upper part $D_1 = 2\,cm = 20\,mm$



Step 1 → Area of upper part $A = \frac{\pi}{4}(20)^2$

$A_1 = 314.15\,mm^2$

Step 2 → Area of Lower part $A_2 = \frac{\pi}{4}(30)^2$

$A_2 = 706.85\,mm^2$

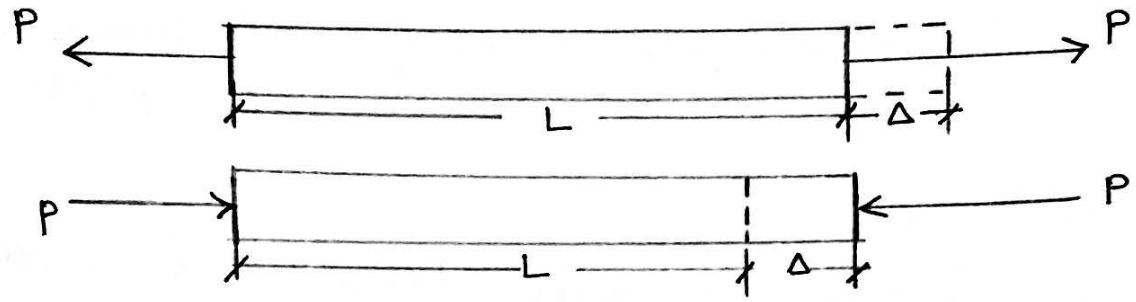
The Stress is equal to Load divided by area. Hence Stress will be maximum where area is minimum. Hence Stress will be maximum in upper part & minimum in Lower part.

Maximum Stress $\sigma = \frac{Load}{A_1} = \frac{35 \times 10^3}{314.15} = 111.408\,N/mm^2$

Minimum Stress $\sigma = \frac{Load}{A_2} = \frac{35 \times 10^3}{706.85} = 49.514\,N/mm^2$

Extension / shortening of a bar :

Consider the bars shown in fig ①



Stress $\sigma = \frac{P}{A}$ Strain $\varepsilon = \frac{\Delta}{L}$

From Hooke's law we have

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\varepsilon} = \frac{P/A}{\Delta/L} = \frac{PL}{A\Delta}$$

$$\boxed{\Delta = \frac{PL}{AE}}$$