Karnatak Law Society's

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Belagavi – 590 008, Karnataka, India

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Department of Civil Engineering

IV SEMESTER

Notes on

Unit 1: Trigonometric Leveling

(Course Code: 16CV42)

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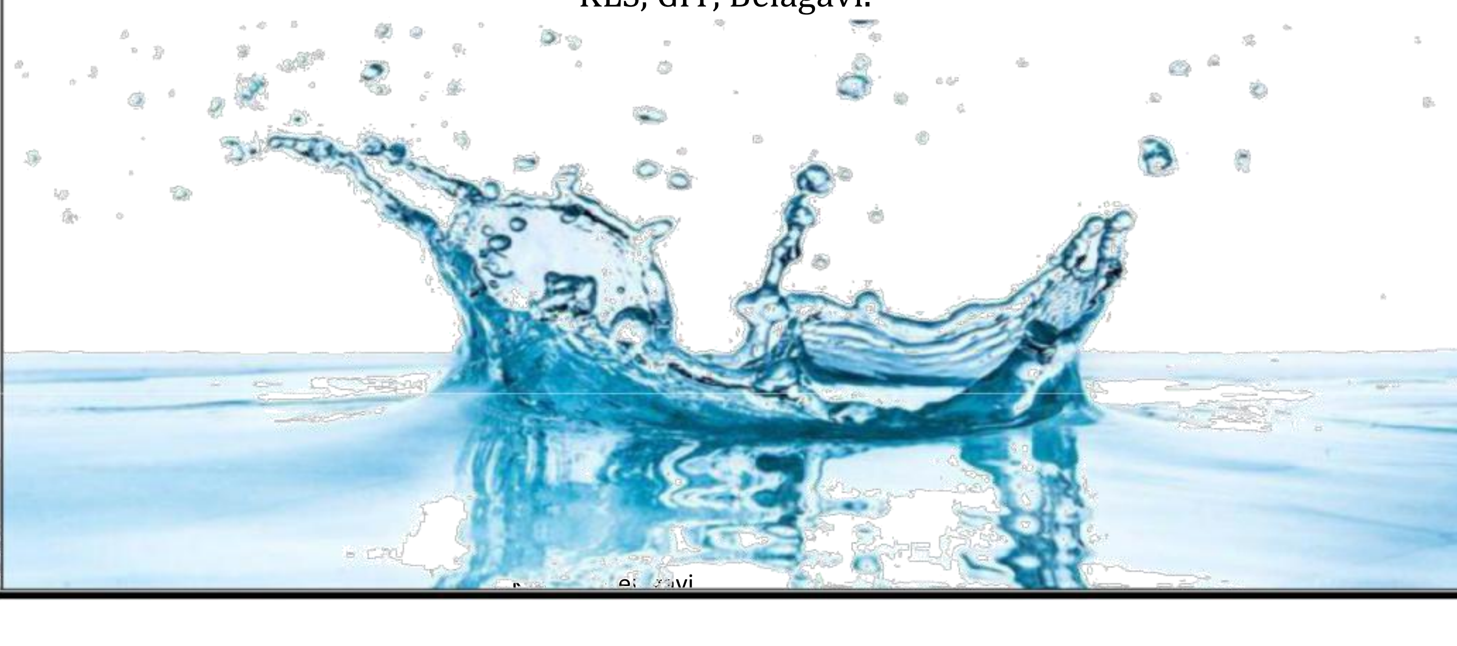


Trigonometric Levelling

Trigonometric levelling is the process of determining the difference of elevations of stations from observed vertical angles and known distance, which are assumed to be either horizontal or geodetic lengths at mean sea level

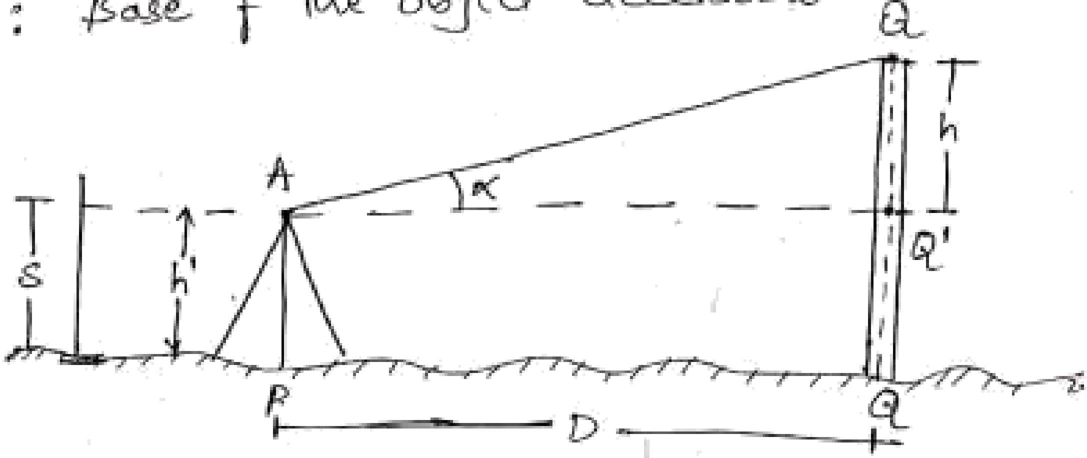
* In order to get the difference in elevation b/w the instrument station & the object under observation, we shall consider the following cases:

Case 1: Base of the object accessible

Case 2: Base of the object inaccessible: Instrument stations in the same vertical plane as the elevated object

Case 3: Base of the object inaccessible: Instrument stations ~~is~~ ~~are~~ not in the same vertical plane as the elevated object

Case 1: Base of the object accessible



P → Instrument station

Q → Point to be observed

A → Centre of the instrument

Q' → Projection of Q on horizontal plane through A

D = AQ' = horz dist b/w P & Q

h' = ht of instrument at P

h = QQ'

S = reading of staff kept at B.M. with line of sight horz.

α = angle of elevation from A to Q

From $\Delta AQQ'$

$$\tan\alpha = \frac{h}{D}$$

$$\therefore h = D\tan\alpha$$

RL of Q = R.L of instrument axis + $D\tan\alpha$

If the R.L. of P is known:

$$RL \text{ of } Q = RL \text{ of } P + h' + D\tan\alpha$$

If the reading on the staff kept at the B.M is S with the line of sight horizontal

$$RL \text{ of } Q = RL \text{ of } BM + S + D\tan\alpha.$$

Case 2: Base of the object inaccessible: instrument stations in the same vertical plane as the elevated object

If the horz dist b/w the inst and object can not be measured due to obstacles, two instrument stns are used so that they are in the same vertical plane as the elevated object.

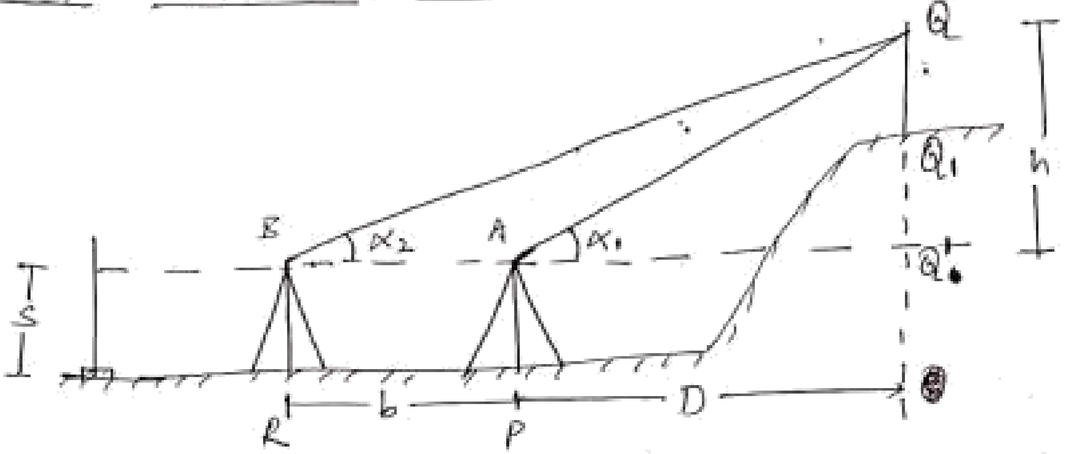
Procedure

1) Set up the theodolite at P. & level it accurately.
2) Bisect point Q accurately & read vertical angle α_1
3) Transit the telescope so that the line of sight is reversed. mark 2nd inst stn 'R' on the ground. Measure dist b/w P & R (i.e 'b')
4) With the vertical vernier set to zero & take the reading on the staff kept at the nearby B.M. (i.e S_1)
5) Shift instrument from P to 'R'. & measure vertical angle α_2 after bisecting point Q accurately from stn 'R'

6) With vertical vernier set to zero, take the reading on the staff kept at the nearby B.M (i.e S_2)

In order to calculate RL of Q, we will consider three cases

<u>Case i) Instrument axes at the same level</u> ($S_1 = S_2 = S$)



From $\Delta AQQ'$

$$\tan\alpha_1 = \frac{h}{D} \Rightarrow h = D\tan\alpha_1 \quad — ①$$

From $\Delta BQQ'$

$$\tan\alpha_2 = \frac{h}{(b+D)} \Rightarrow h = (b+D)\tan\alpha_2 \quad — ②$$

Equating ① & ②, we get

$$D\tan\alpha_1 = (b+D)\tan\alpha_2$$

$$D\tan\alpha_1 - D\tan\alpha_2 = b\tan\alpha_2$$

$$D(\tan\alpha_1 - \tan\alpha_2) = b\tan\alpha_2$$

$$D = \frac{b\tan\alpha_2}{\tan\alpha_1 - \tan\alpha_2}$$

$$h = D\tan\alpha_1 = \frac{b\tan\alpha_2\cdot\tan\alpha_1}{\tan\alpha_1 - \tan\alpha_2} \text{ or } \frac{b\tan\alpha_1\cdot\tan\alpha_2}{\tan\alpha_1 - \tan\alpha_2}$$

or

$$h = \frac{b\cdot\sin\alpha_1\cdot\sin\alpha_2}{\sin(\alpha_1 - \alpha_2)}$$

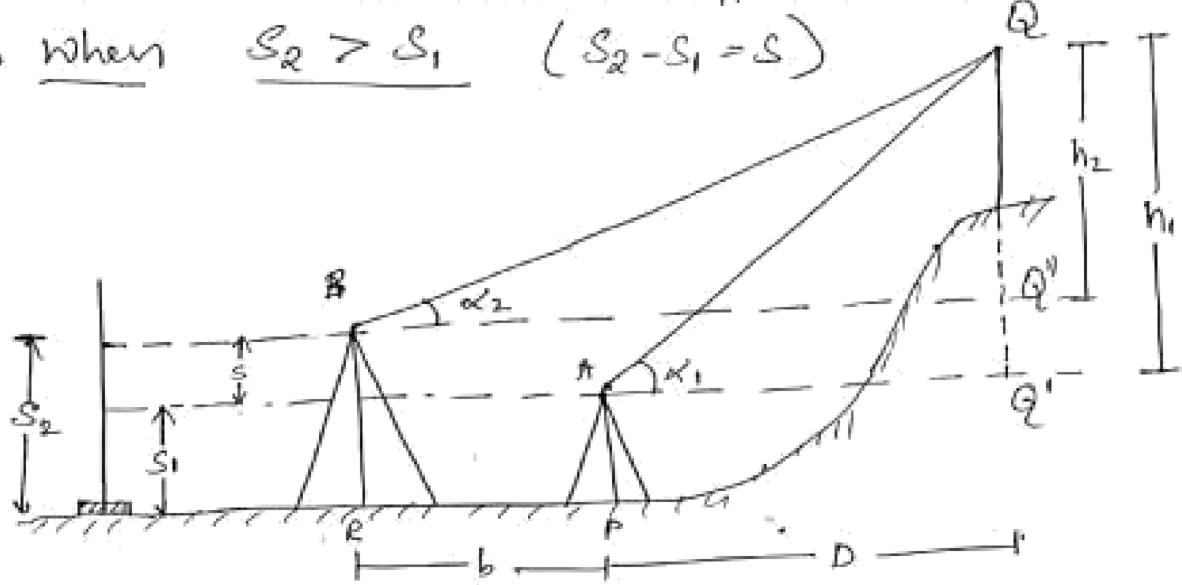
RL of Q = RL of BM + S + h

Check ⇒ $h = (b+D)\tan\alpha_2$

RL of Q = RL of BM + S + h.

Case ii) Instrument axes at different levels

A) When $S_2 > S_1$ $(S_2 - S_1 = S)$



From $\Delta QAQ'$ $\Rightarrow$ $h_1 = D \tan\alpha_1$ —— (1)

From $\Delta BQQ''$ $\Rightarrow$ $h_2 = (b+D)\tan\alpha_2$ —— (2)

(1) − (2)

$$h_1 - h_2 = D\tan\alpha_1 - (b+D)\tan\alpha_2$$

Here $h_1 - h_2 = S_2 - S_1 = S$

$$\therefore S = D\tan\alpha_1 - b\tan\alpha_2 - D\tan\alpha_2$$

$$D(\tan\alpha_1 - \tan\alpha_2) = S + b\tan\alpha_2$$

$$D = \frac{S + b\tan\alpha_2}{\tan\alpha_1 - \tan\alpha_2} = \frac{(b + S\cot\alpha_2)\tan\alpha_2}{\tan\alpha_1 - \tan\alpha_2}$$

$$h_1 = D\tan\alpha_1$$

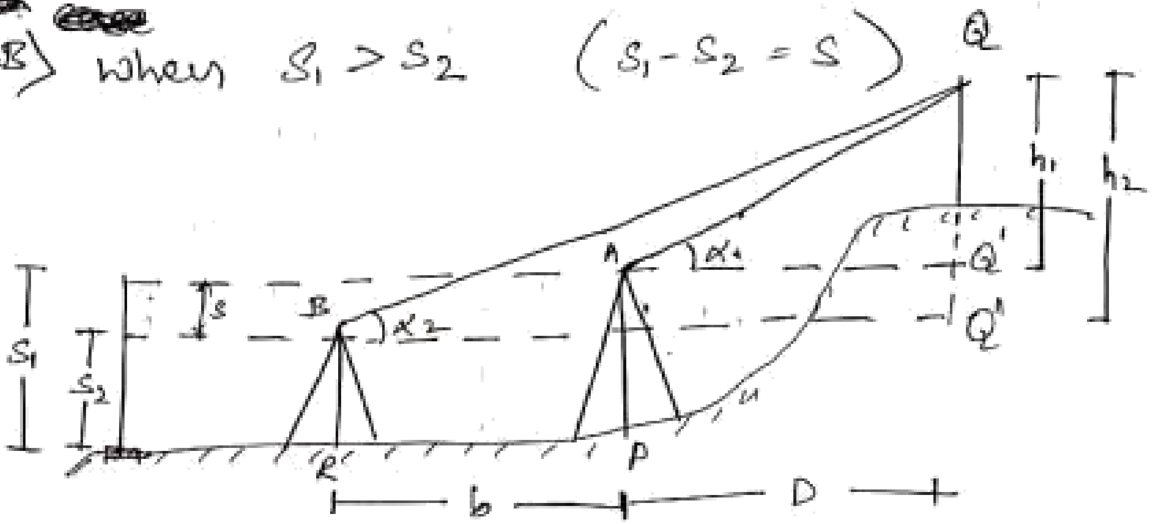
$$= \frac{(b + S\cot\alpha_2)\tan\alpha_1\tan\alpha_2}{\tan\alpha_1 - \tan\alpha_2} \quad \text{or} \quad \frac{(b + S\cot\alpha_2)\sin\alpha_1\sin\alpha_2}{\sin(\alpha_1 - \alpha_2)}$$

RL of Q = RL of BM + S_1 + h_1

Check :- $h_2 = (b+D)\tan\alpha_2$

RL of Q = RL of BM + $S_2 + h_2$

B) when $S_1 > S_2$ $(S_1 - S_2 = S)$



From $\Delta AQ'Q \Rightarrow h_1 = D\tan\alpha_1$ — ①

From $\Delta BQ'Q \Rightarrow h_2 = (b+D)\tan\alpha_2$ — ②

② − ①

$$h_2 - h_1 = (b+D)\tan\alpha_2 - D\tan\alpha_1$$

Here $h_2 - h_1 = S_1 - S_2 = S$

$$\therefore S = (b+D)\tan\alpha_2 - D\tan\alpha_1$$

$$S = b\tan\alpha_2 + D\tan\alpha_2 - D\tan\alpha_1$$

$$D(\tan\alpha_1 - \tan\alpha_2) = b\tan\alpha_2 - S$$

$$D = \frac{b\tan\alpha_2 - S}{\tan\alpha_1 - \tan\alpha_2} = \frac{(b - S\cot\alpha_2)\tan\alpha_2}{\tan\alpha_1 - \tan\alpha_2}$$

$$h_1 = D\tan\alpha_1$$

$$= \frac{(b - S\cot\alpha_2)\tan\alpha_1\tan\alpha_2}{\tan\alpha_1 - \tan\alpha_2} \text{ or } \frac{(b - S\cot\alpha_2)\sin\alpha_1\sin\alpha_2}{\sin(\alpha_1 - \alpha_2)}$$

RL of Q = RL of BM $+ S_1 + h_1$

Check! –

$h_2 = (b+D)\tan\alpha_2$

RL of Q = RL of BM $+ S_2 + h_2$

For Both case A & B.

$$D = \frac{(b \pm S\cot\alpha_2)\tan\alpha_2}{\tan\alpha_1 - \tan\alpha_2}$$

$$h_1 = \frac{(b \pm S\cot\alpha_2)\tan\alpha_1\tan\alpha_2}{\tan\alpha_1 - \tan\alpha_2}$$

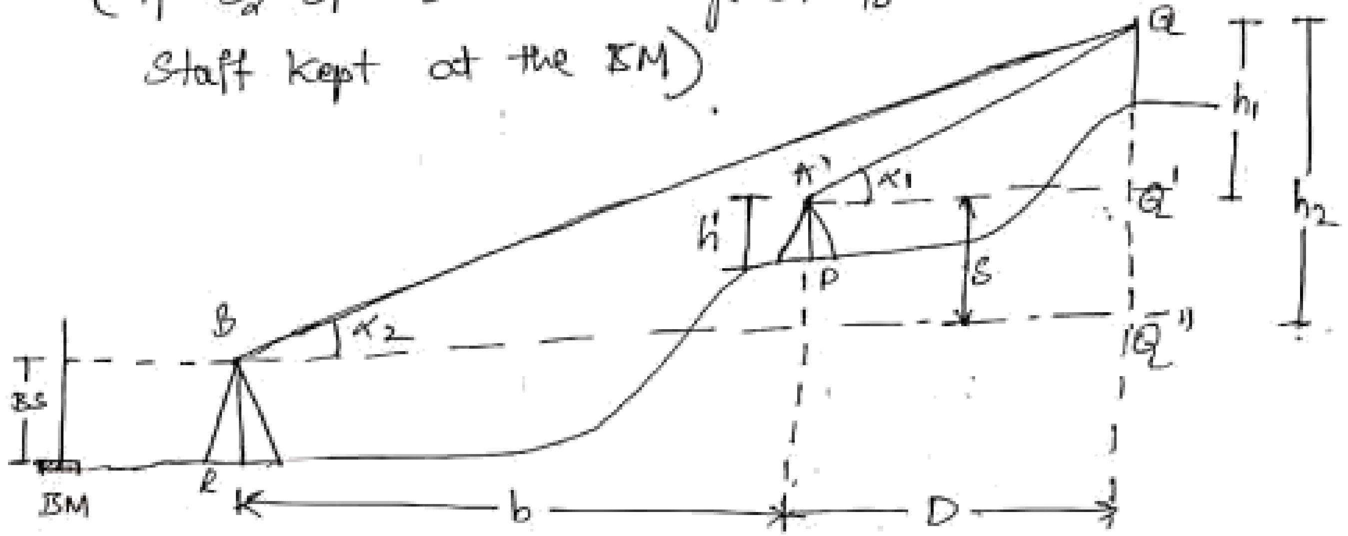
RL of Q = RL of BM $+ S_1 + h_1$

Use

'+' when $S_1 < S_2$

'−' when $S_1 > S_2$

Case iii) Instrument axes at very different Level
(if $S_2 - S_1$ or S is too great to be measured on a Staff kept at the BM)



$h_1 = D \tan \alpha_1$

$h_2 = (b + D) \tan \alpha_2$

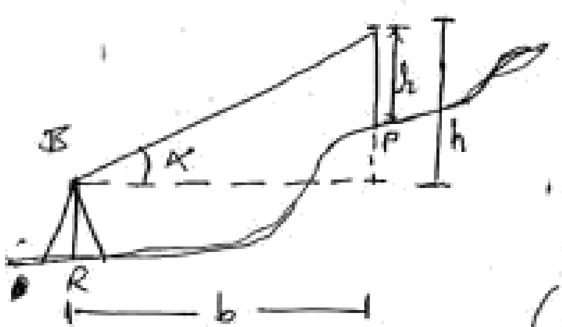
(2) − (1)

$(h_2 - h_1) = S = (b + D) \tan \alpha_2 - D \tan \alpha_1$

$D(\tan \alpha_1 - \tan \alpha_2) = b \tan \alpha_2 - S$

$$D = \frac{b \tan \alpha_2 - S}{\tan \alpha_1 - \tan \alpha_2} \quad \text{—(3)}$$

$$h_1 = D \tan \alpha_1 = \frac{(b \tan \alpha_2 - S) \tan \alpha_1}{\tan \alpha_1 - \tan \alpha_2} = \frac{(b - S \cot \alpha_2) \sin \alpha_1 \sin \alpha_2}{\sin(\alpha_1 - \alpha_2)} \quad \text{—(4)}$$



Ht of stn P above the axis at B

$= h - r = b \tan \alpha - r$

Ht of axis at A above the axis at B

$= S = b \tan \alpha - r + h'$

(h' = ht of inst at P)

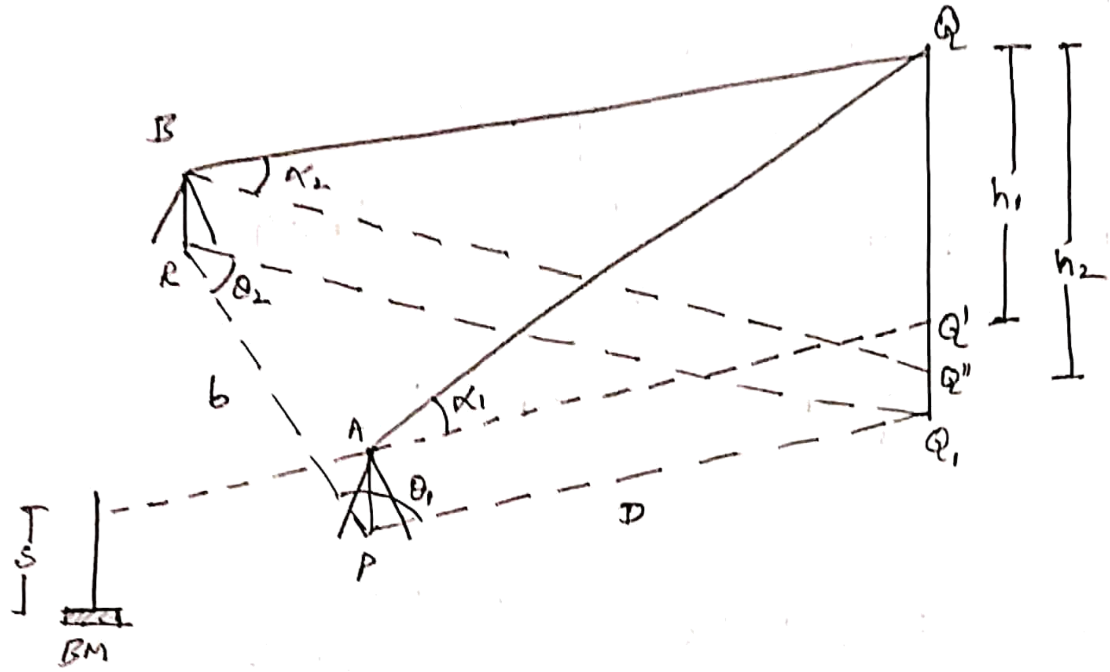
Equating the value of S in eqn (3) & (4) we can get D & h_1

RL of Q = RL of A + h_1 = RL of B + S + h_1

= (RL of BM + BS taken from B) + S + h_1

Here $S = b \tan \alpha - r + h'$

Case 3 : Base of the object inaccessible : Instrument stations not in the same vertical plane as the elevated object



Procedure

1) Set the inst at P, Measure angle of elevation α_1 to Q.
2) Sight the Point R on the ground with reading on horizontal circle as zero & measure the angle RPQ_1 (i.e hz angle θ_1 at P)
3) Take the backsight 'S' on the staff kept at BM.
4) Shift the instrument to R & Measure α_2 & θ_2 there.

Here $\Delta AQQ'$ & $\Delta BQQ''$ are in vertical plane.
ΔPQ_1R in horizontal plane.

From $\Delta AQQ'$ => $h_1 = D \tan\alpha_1$. — (1)

From ΔPRQ_1 => $\angle PQ_1R = 180° - (\theta_1+\theta_2) = \pi - (\theta_1+\theta_2)$

From the Sine rule,

$$\frac{PQ_1}{\sin\theta_2} = \frac{RQ_1}{\sin\theta_1} = \frac{RP}{\sin[\pi-(\theta_1+\theta_2)]}$$

$$\frac{PQ_1}{\sin\theta_2} = \frac{RQ_1}{\sin\theta_1} = \frac{b}{\sin(\theta_1+\theta_2)}$$

$$PQ_1 = D = \frac{b \sin \theta_2}{\sin(\theta_1 + \theta_2)} \quad \text{— (2)}$$

$$RQ_1 = \frac{b \sin \theta_1}{\sin(\theta_1 + \theta_2)}$$

Sub.. value of D in eq (1)

$$h_1 = D \tan \alpha_1 = \frac{b \sin\theta_2 \cdot \tan \alpha_1}{\sin(\theta_1 + \theta_2)}$$

$$RL \text{ of } Q = RL \text{ of } BM + S + h_1$$

As check.

$$h_2 = RQ_1 \tan \alpha_2 = \frac{b \sin\theta_1 \tan\alpha_2}{\sin(\theta_1 + \theta_2)}$$

$$RL \text{ of } Q = RL \text{ of } BM + S\left(\overset{B.S.}{\text{taken from B instrument}}\right) + h_2.$$