Karnatak Law Society's

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(Approved by AICTE, New Delhi)

Department of Civil Engineering

IV SEMESTER

Notes on

Unit 3: Simple, Compound and Reverse Curves

Advanced Surveying

(Course Code: 16CV42)

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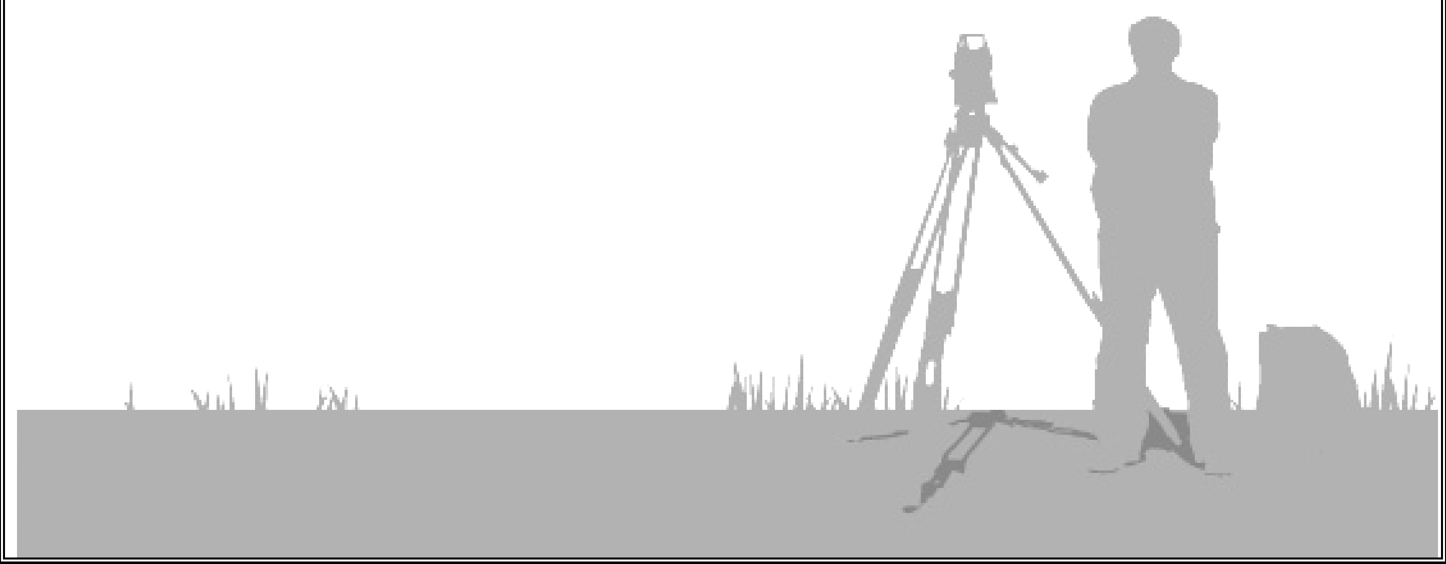
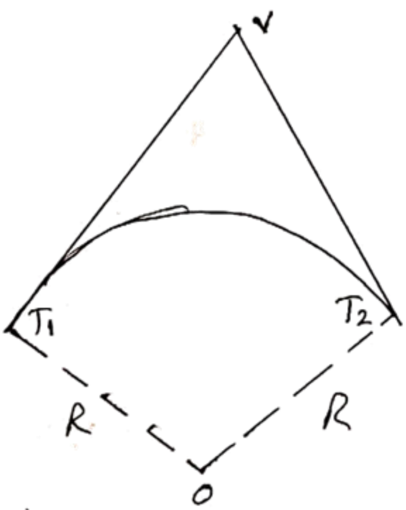


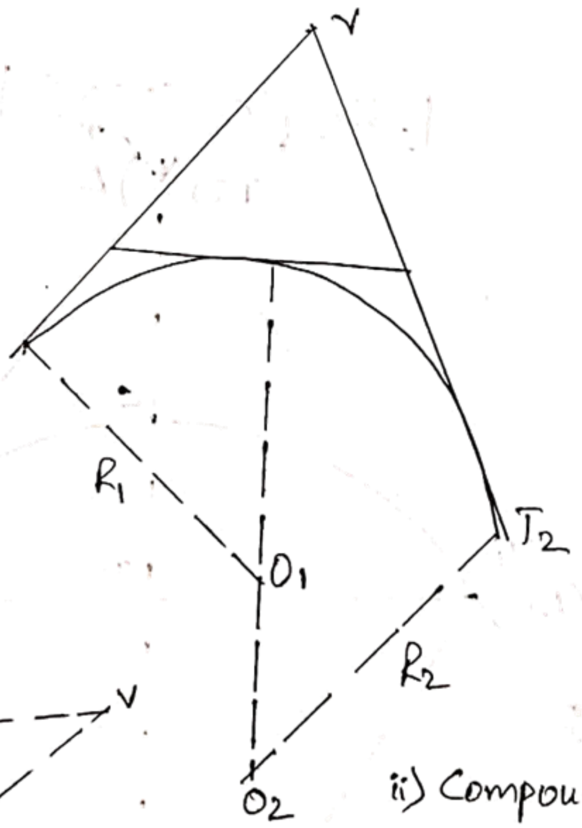
Circular curves *

Curves are generally used on highways and railways where it is necessary to change the direction of motion. A curve may be circular, parabolic or spiral and is always tangential to the two straight directions.

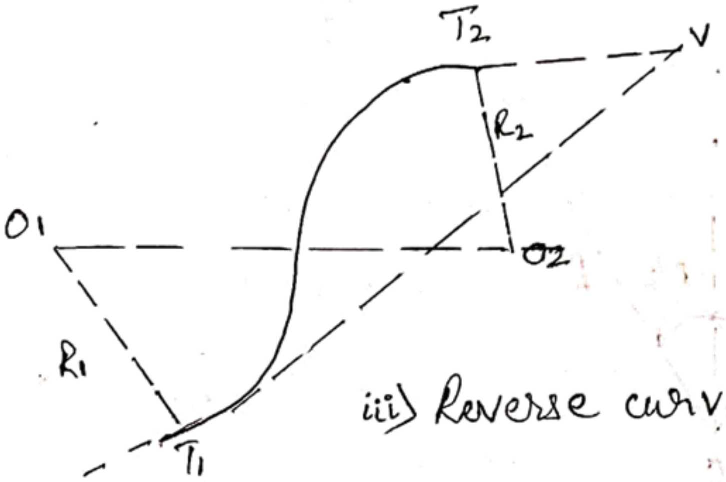
Circular curves are further devided in to three classes :
i) Simple curve
ii) Compound curve
iii) Reverse curve.



i) Simple curve



ii) Compound curve



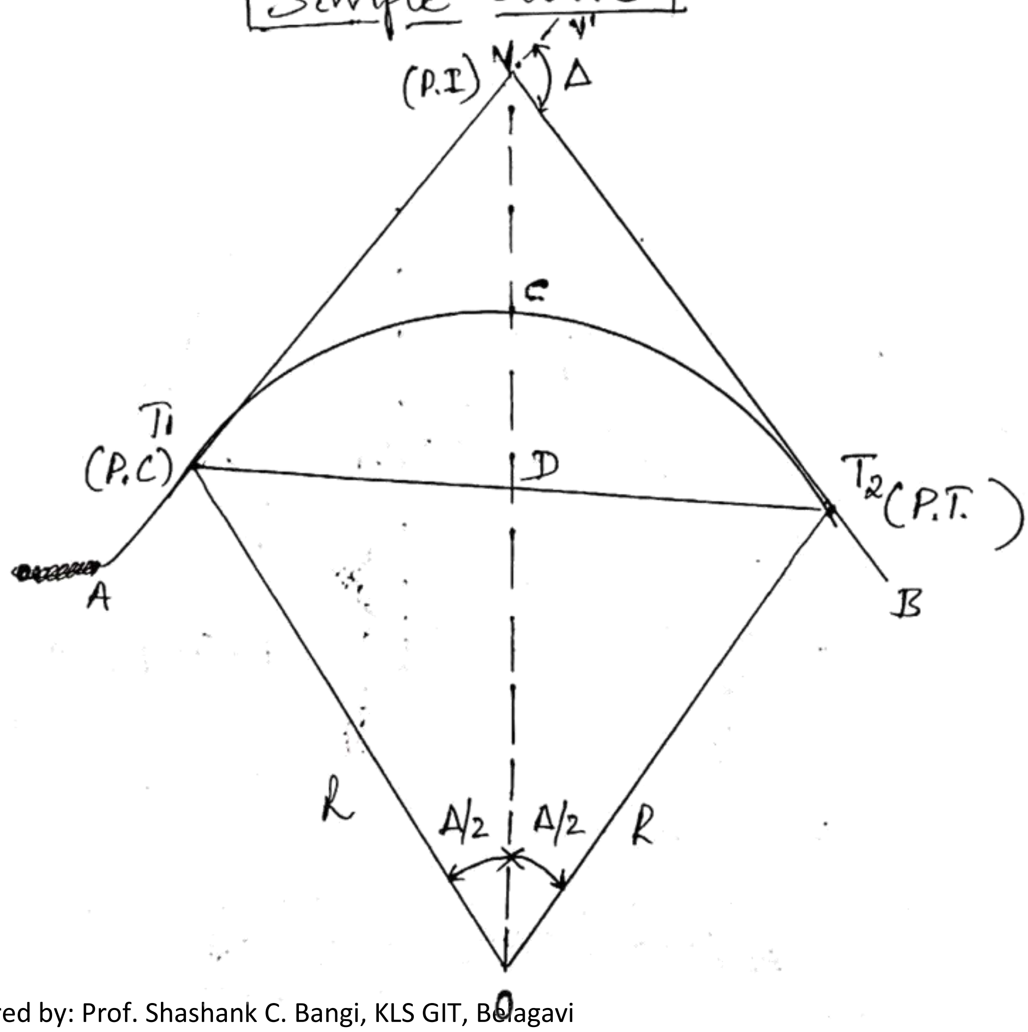
iii) Reverse curve

i) Simple curve is the one which consists of a single arc of a circle. It is tangential to both the straight lines.

ii) Compound curve consists of two or more simple arcs that turn in the same direction & join at common tangent points.

iii) Reverse curve is the one which consists of two circular arcs of same or different radii, having their centres to the different sides of the common tangent. Both the arcs ~~tend~~ thus bend in different direction with common tangent at their junction.

|Simple curve|



Definitions & Notations :

1) Back tangent : The tangent (AT_1) previous to the curve is called the back tangent or first tangent.

2) Forward tangent : The tangent (T_2B) following the curve is called the forward tangent or second tangent

3) Point of intersection : If two tangents AT_1 and BT_2 are produced, they will meet in a point, called the point of intersection (P.I) or Vertex (V.).

4) Point of curve (P.C) : It is beginning of the curve where the alignment changes from a tangent to a curve.

5) Point of tangency (P.T) : It is end of the curve where the alignment changes from a curve to tangent.

6) Intersection angle : The angle $V'VB$ between the tangent AV produced and VB is called the intersection angle (Δ) or the external deflection angle between the two tangents.

7) Deflection angle to any point: It is the angle at P.C between the back tangent and the chord from P.C to point on the curve.

8) Tangent distance (T): It is the distance between P.C to P.I & also the distance from P.I to P.T

9) External distance (E): It is distance from the mid-point of the curve to P.I

10) Length of curve (L): It is the total length of the curve from P.C to P.T

11) Long chord: It is chord joining P.C to P.T

12) Mid ordinate (M): It is the ordinate from the midpoint of the long chord to the midpoint of the curve

13) Normal chord (C): A chord between two successive regular stations on a curve.

14) Sub chord (c): Sub chord is any chord shorter than the normal chord.

15) Right hand curve: If the curve deflects to the right of the direction of the progress of survey. It is called the right-hand curve.

16). Left hand curve : If the curve deflects to the left of the direction of the progress of survey it is called the left hand curve,

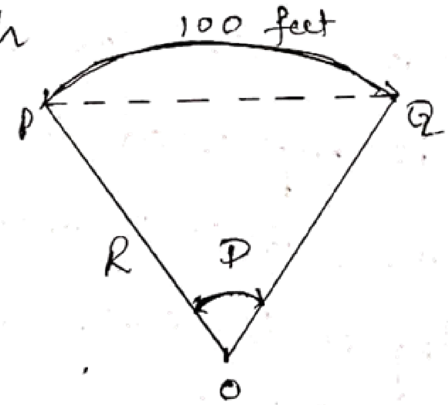
* **Designation of curve**

The sharpness of the curve is designated by its degree of curvature.

The degree of curvature has several slightly different definition. According to,

→ Arc Definition : (Generaly used in highway practice)

According to Arc definition, the degree of the curve is defined as the central angle of the curve that is subtended by an arc of 100 ft length



Relation between D & R

$$100 : 2\pi R = D^\circ : 360^\circ$$

$$R = \frac{360}{D} \times \frac{100}{2\pi}$$

$$R = \frac{5729.578}{D} \text{ ft.}$$

R → Radius

D → Degree of curve

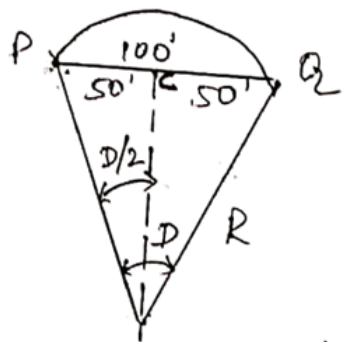
Thus Radius of 1° curve is 5729.578 ft.

To the first approximation, we have

$$R = 5730/D$$

→ Chord definition: (Generaly used in railway practice).

According to chord definition, the degree of the curve is defined as the central angle of the curve that is subtended by its chord of 100 ft length.



From Δ POC

$$\sin \frac{1}{2} D = \frac{50}{R}$$

$$R = \frac{50}{\sin \frac{1}{2} D}$$

when D is small $\sin \frac{1}{2} D \approx \frac{1}{2} D$ radians

$$R = \frac{50}{\frac{D}{2} \times \frac{\pi}{180}}$$, where D is in degrees.

$$= \frac{50 \times 360}{D \times \pi} = \frac{5729.578}{D} = \frac{5730}{D} \text{ (aprox)}$$

Every curve is chosen so that either its radius or its degree of curvature is expressed in round numbers. If radius is even, it is known as even radius curve. If degree is even, it is known as even degree curve.

Metric Degree of curve:

Two definitions for the degree of curve are in use

1) Angle at the centre subtended by an arc (or chord) of 20 metres.

2) Angle at the centre subtended by an arc (or chord) of 10 metres.

* If 20 metres arc (or chord) length is the basis for the degree of the curve, we get

$$D^\circ : 360^\circ = 20 : 2\pi R$$

From which, $R = \frac{1145.92}{D} \approx \frac{1146}{D}$ metres (approx)

* If the definition is based on 10 m arc length,

$$D^\circ : 360 = 10^\circ : 2\pi R$$

$$R = \frac{572.958}{D} \approx \frac{573}{D} \text{ metres}$$

Elements of Simple curve :

i) Length of the curve (l) :

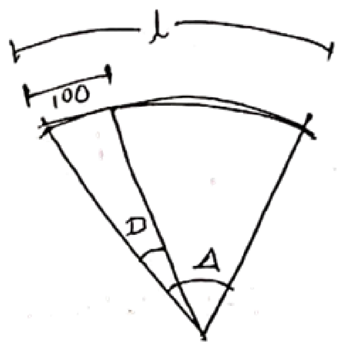
$$l = T_1 C T_2 = R\Delta$$ (where Δ is in ~~radians~~ degree)

$$= \frac{\pi R \Delta}{180^\circ}$$ (where Δ is in ~~degrees~~ radians)

* if curve is designed by its degree of curvature, the length of the curve will depend upon the criteria used for the definition of the degree of the curve

a) Arc definition :

if length of Arc = 100 ft.



$$\frac{\Delta}{D} = \frac{l}{100}$$

$$l = \frac{100\Delta}{D} \text{ ft.}$$

if length of arc = 20 m.

$$l = \frac{20\Delta}{D} \text{ metres}$$

2) Tangent length (T)

$$\text{Tangent length} = T = T_1V = VT_2 = OT_1 \tan \Delta/2$$
$$= R \tan \Delta/2$$

3) Length of the long chord (L)

$$L = T_1T_2 = 2OT_1 \sin\frac{\Delta}{2} = 2R\sin\frac{\Delta}{2}$$

4) Apex distance or external distance (E)

$$E = CV = VO - CO = R\sec\frac{\Delta}{2} - R$$

$$= R\left(\sec\frac{\Delta}{2} - 1\right) = R\,\text{exsec}\frac{\Delta}{2}$$

5) Mid-ordinate (M)

$$M = CD = CO - DO = R - R\cos\frac{\Delta}{2}$$

$$= R\left(1 - \cos\frac{\Delta}{2}\right) = R\,\text{versin}\frac{\Delta}{2}$$

Setting out Simple curves

The methods of setting out curves can be mainly devided into two methods depending upon instruments used:

1) **Linear Methods**: In linear method only chain or tape is used. Linear methods are used when.

a) a high degree of accuracy is not required

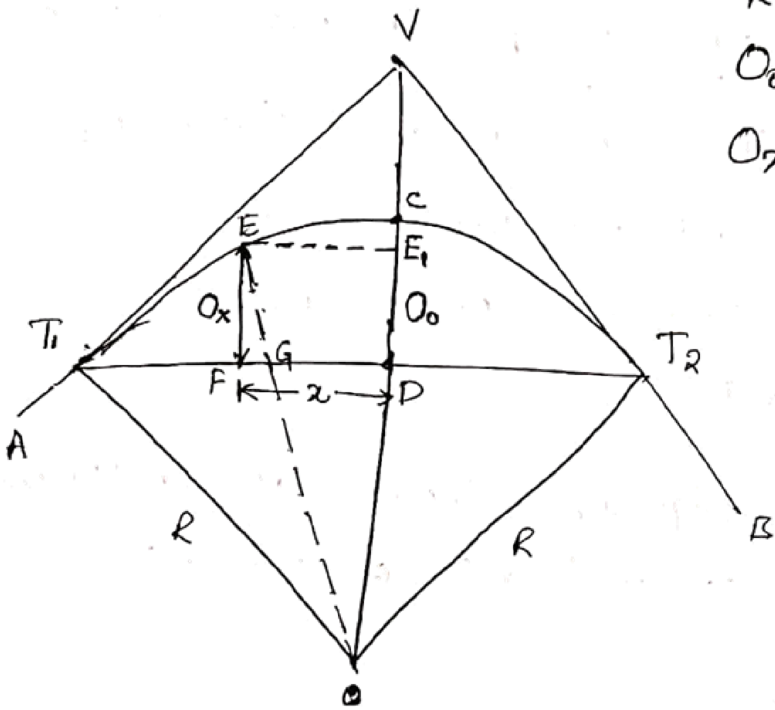
b) the curve is short

2) **Angular Methods**: In angular method, an instrument such as a theodolite is used with or without a chain (or tape).

★ **Linear Methods of Setting out**

1) By ordinates or offsets from the long chord
2) By successive bisection of arcs
3) By offsets from the tangents.
4) By offsets from chords produced (or by deflection distance)

* By ordinates from the long chord



$R \rightarrow$ Radius of the curve

$O_0 \rightarrow$ Mid-ordinate

$O_x \rightarrow$ Ordinate at distance x from the midpoint of the chord.

T_1 & $T_2 \rightarrow$ Tangent Points

$L \rightarrow$ Length of the chord actually measured on the ground.

Bisect the long chord at Point D.

From $\Delta\, OT_1D$

$$OT_1^2 = T_1D^2 + DO^2$$

$$R^2 = \left(\frac{L}{2}\right)^2 + [CO - CD]^2$$

$$R^2 = (L/2)^2 + (R - O_0)^2$$

$$(R - O_0) = \sqrt{R^2 - (L/2)^2}$$

$$O_0 = R - \sqrt{R^2 - (L/2)^2} \quad \text{--- (1)}$$

In order to calculate the ordinate O_x to any point E, draw the line EE_1 parallel to the long chord T_1T_2. Join EO to cut the long chord in G.

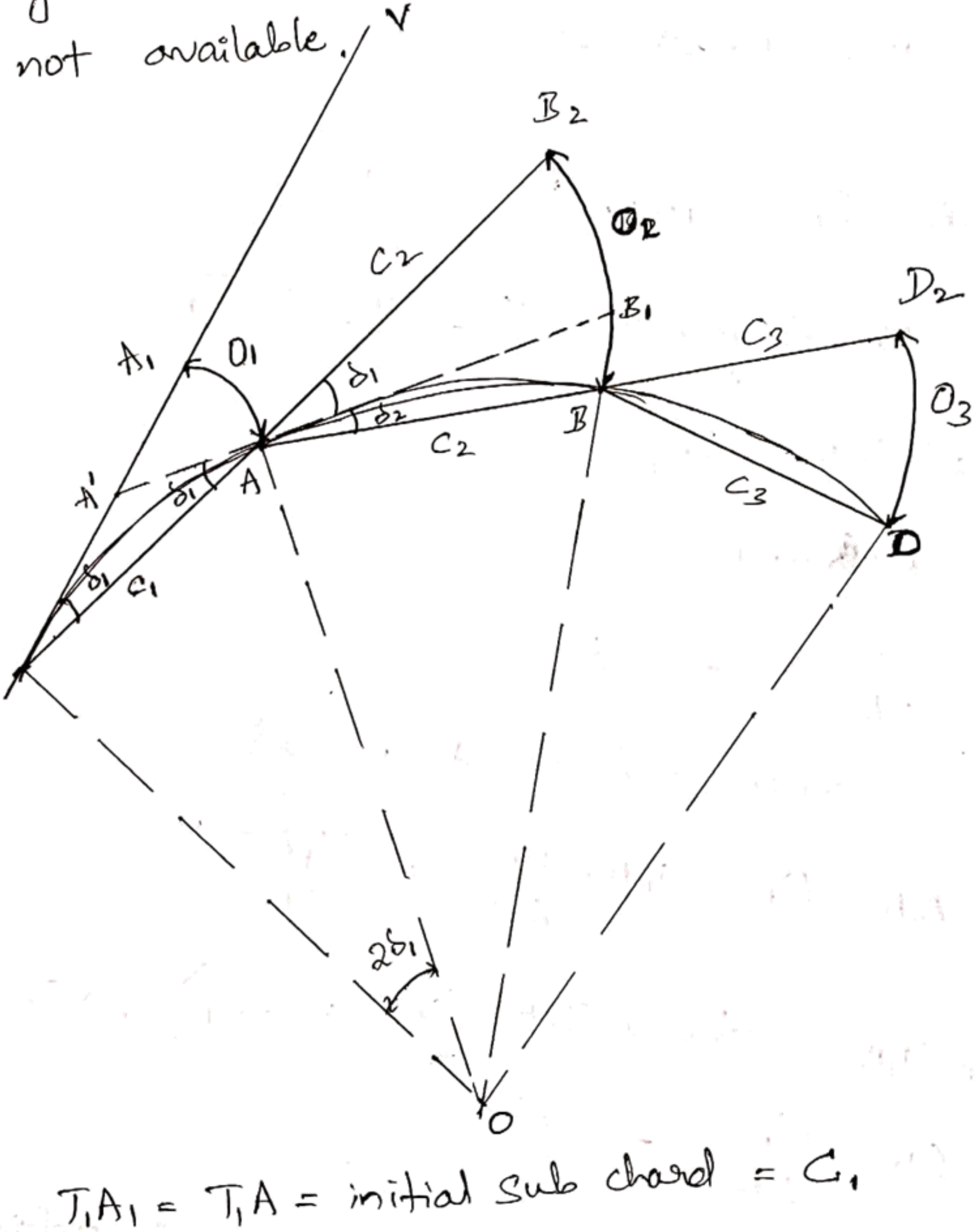
Then $O_x = EF = E_1D = E_1O - DO$

$$= \sqrt{(EO)^2 - (EE_1)^2} - (CO - CD)$$

$$= \sqrt{R^2 - x^2} - (R - O_0) \quad \text{--- (2)}$$

By Offsets from the chords produced

* The method is very much useful for long curves & is generally used on highway curves when a theodolite is not available.



Let $T_1A_1 = T_1A$ = initial sub chord = C_1

A, B, C etc = points on the curve

$AB = C_2$

$BD = C_3$ etc.

T_1V = Rear tangent

$\angle A_1T_1A = \delta$ = deflection angle of the first chord

$A_1A = O_1$ = first offset

$B_2B = O_2$ = Second offset

$D_2D = O_3$ = third offset, etc.

Now Arc $A_1A = O_1 = T_1A \cdot \delta$ —— (1)

Since T_1V is the tangent to the circle at T_1

$$\angle T_1OA = 2\angle A_1T_1A = 2\delta_1$$

$$T_1A = R \cdot 2\delta_1$$

$$\delta_1 = \frac{T_1A}{2R} \quad \text{—— (2)}$$

Substituting the value of δ in (1) we get

$$\text{Arc } A_1A = O_1 = T_1A \cdot \frac{T_1A}{2R} = \frac{T_1A^2}{2R}$$

Taking arc T_1A = chord T_1A (very nearly), we get

$$O_1 = \frac{C_1^2}{2R} \quad \text{—— (3)}$$

In order to obtain the value of the second offset O_2 for getting the point B on the curve, draw a tangent AB_1 to the curve at A to cut the rear tangent in A'. Join T_1A & Prolong it to a point B_2 such that $AB_2 = AB = C_2$ = length of the Second chord

then $O_2 = B_2B$.

As from the eqn (3)

$$B_1B = \frac{C_2^2}{2R}$$

$$B_1B = C_2 \times \delta_2$$

$$B_1B = C_2 \times \frac{C_2}{2R}$$

$$\boxed{B_1B = \frac{C_2^2}{2R}}$$

$\angle B_2AB_1 = \angle A'AT_1$ being opposite angles

Since T_1A' & $A'A$ are both tangents, they are equal in length.

$$\angle A'T_1A = \delta_1 = \angle A'AT_1$$

$$\angle B_2AB_1 = \angle A'AT_1 = \delta_1$$

$$\text{arc } B_2B_1 = AB_2 \cdot \delta_1 = C_2 \cdot \delta_1$$

Substituting the value of δ from eqn (2) we get

$$B_2B_1 = C_2 \cdot \frac{T_1A}{2R} = \frac{C_2 \cdot C_1}{2R}$$

$$\text{arc } B_2B = B_2B_1 + B_1B_2$$

$$O_2 = \frac{C_2C_1}{2R} + \frac{C_2^2}{2R}$$

$$O_2 = \frac{C_2}{2R}(C_1 + C_2) \quad \text{—— (4)}$$

Similarly the third offset $O_3 = D_2D$ is given by.

$$O_3 = \frac{C_3}{2R}(C_2 + C_3)$$

The last or n^{th} offset is given by

$$O_n = \frac{C_n}{2R}(C_{n-1} + C_n) \quad \text{--- (5)}$$

Generally the first chord is sub-chord, say of length c, & the intermediate chords are normal chords, say of length C. ~~...~~ and the last sub chord say of length c'. In that case, the above formula reduces to

$$O_1 = \frac{c^2}{R}$$

$$O_2 = \frac{C}{2R}(c + C)$$

$$O_3 = O_4 = \ldots O_{n-1} = \frac{C}{2R}(2C) = \frac{C^2}{R}.$$

$$O_n = \frac{c'}{2R}(C + c')$$

* Procedure for Setting out Simple curve by the offsets from the chords produced.

1) Locate the tangent points T_1 & T_2 & find out their chainages as explained earlier. Calculate the length (c) of the first sub-chord so that the first peg is the full station (regular station)

2) With zero mark at T_1, spread the chain (or tape) along the first tangent to point A_1 on it such that $T_1A_1 = C =$ length of the first sub-chord.

~~3) With zero mark at T_1, spread the chain (or tape)~~

3) With T_1 as centre and T_1A_1 as radius, swing the chain such that the arc $A_1A =$ calculated offset O. Fix the point A on the curve.

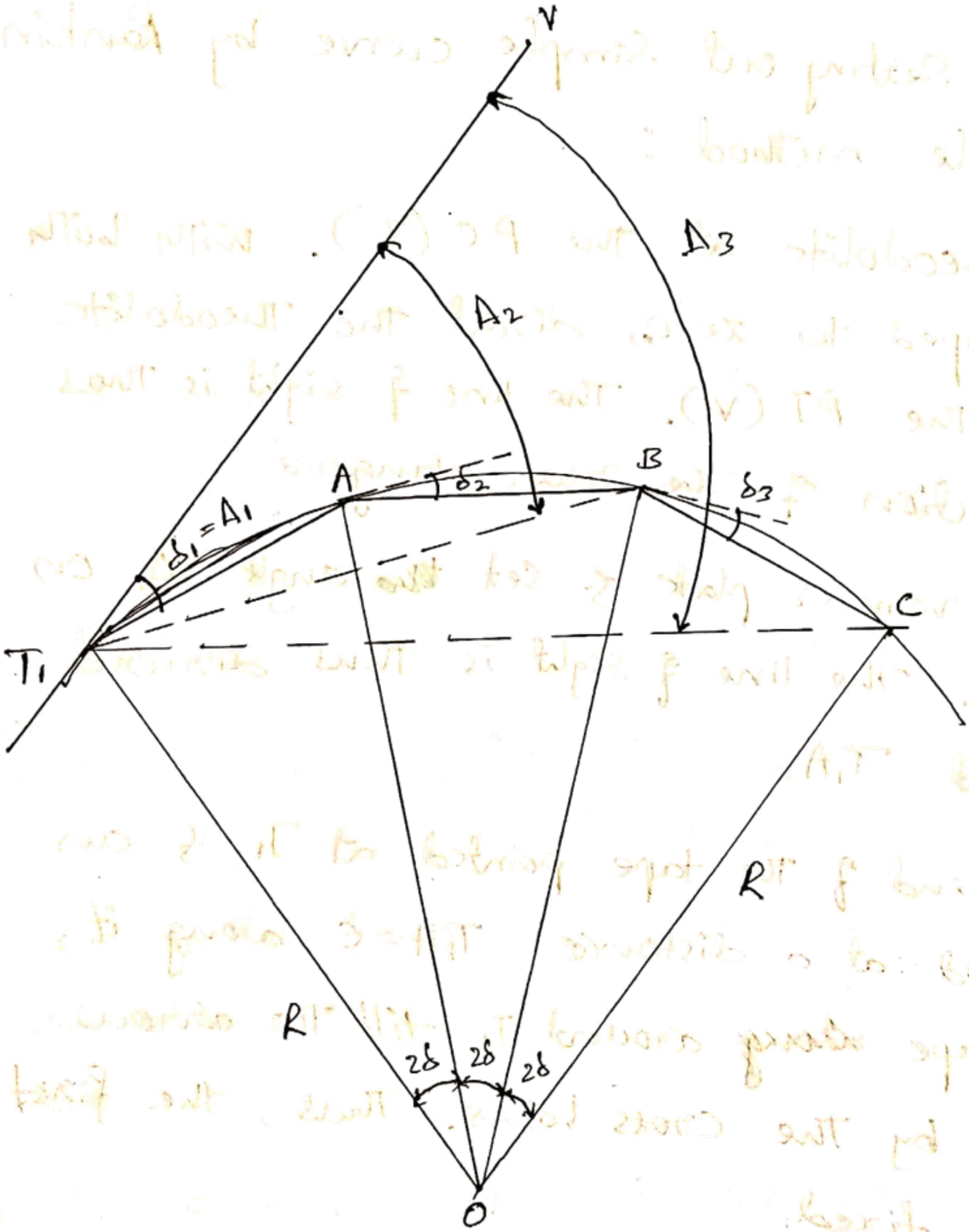
4) Spread the chain along T_1A & pull it straight in this direction to point B_2 such that the zero of the chain is at A & the distance $AB_2 = C =$ length of the normal chord.

5) With zero of the chain centred at A & AB_2 as radius, swing the chain to a point B such that $B_2B = O_2 =$ length of the second offset. Fix point B on the curve.

6) Spread the chain along AB & repeat the steps (4) & (5) till the point of tangency (T_2) is reached. All intermediate offsets will be equal to $\frac{C^2}{R}$, while the last offset will be equal to $\frac{C'}{2R}(C+C')$

* Rankine's method of tangential (or deflection) angles.

* A deflection angle to any point on the curve is the angle at P.C. between the back tangent & the chord from P.C to that point.



Here

$$\delta_n = 1718.9 \frac{C_n}{R} \text{ minutes}$$

$A_1 = \delta_1$

$A_2 = A_1 + \delta_2$

$A_3 = A_2 + \delta_3$

$\vdots$

$A_n = A_{n-1} + \delta_n$

Check :- $A_n = \frac{\Delta}{2}$

* Procedure for Setting out Simple curve by Rankine's deflection angle method :

1) Set the theodolite at the P.C. (T_1). With both plates clamped to zero, direct the theodolite to bisect the PI (V). The line of sight is thus in the direction of the rear tangent

2) Release the vernier plate & set ~~the~~ angle A_1 on the vernier. The line of sight is thus directed along chord T_1A.

3) With zero end of the tape pointed at T_1 & an arrow held at a distance $T_1A = C$ along it, Swing the tape ~~along~~ around T_1 till the arrow is bisected by the cross hairs. Thus, the first point A is fixed.

4) Set the second deflection angle A_2 on the vernier So that line of sight is directed along T_1B.

5) With the zero end of the pinned at A, & an arrow held at a distance AB = C_1 along it, Swing the tape around 'A' till the arrow is bisected by the cross hairs, thus fixing the point B

6) Repeat steps (4) & (5) till the last point T_2 is reached

Check :-

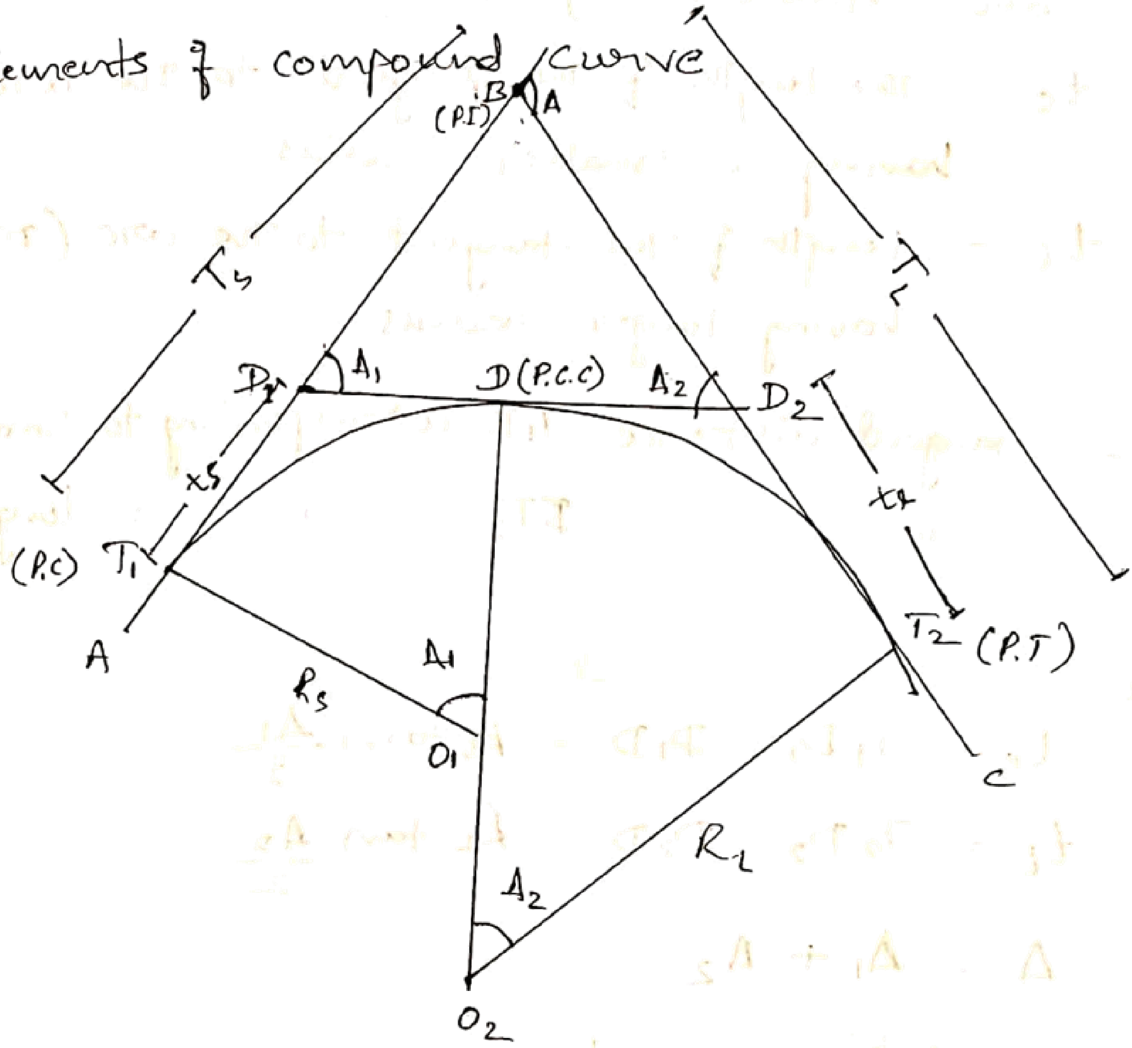
* The last point so located must coincide with the point of Tangency (P.T.) or (T_2) fixed independently by measurements from the P.I. If the discrepancy is small, last few pegs may be adjusted. If it is more, the whole curve should be reset.

Note:-
* In the case of left hand curve, each of the calculated angles of deflection angle (Δ_1, Δ_2 ... etc) should be subtracted from 360°. The angles so obtained are to be set on the vernier of theodolite for setting out the curve.

* In the above method 3 men are required :
① → The surveyor to operate theodolite
② → two chainmen to measure chord length with chain/Tape

Compound curve

★ Elements of compound curve



* T_1DT_2 is two centered compound curve having two circular arcs T_1D & DT_2 meeting at a common point D known as the point of compound curvature (P.C.C.)

→ T_1 → Point of curve (P.C.)
→ T_2 → Point of tangency (P.T.)
→ O_1 & O_2 → Centres of two arcs.
→ R_S = Smaller radius (O_1T_1)
→ R_L = Longer radius (O_2T_2)
→ D_1D_2 = Common tangent
→ Δ_1 = Deflection angle b/w rear and common tangent
→ Δ_2 = " " " Common & Forward tangent

Δ = Total deflection angle

t_s = The length of the tangent to the arc (T_1D) having a smaller radius

t_l = Length of the tangent to the arc (DT_2) having longer radius

T_s = tangent distance T_1B corresponding to smaller radius

T_L = " " BT_2 " " Longer "

We have.

$$t_s = T_1D_1 = D_1D = R_s \tan\frac{\Delta_1}{2}$$

$$t_L = T_2D_2 = D_2D = R_L \tan\frac{\Delta_2}{2}$$

$$\Delta = \Delta_1 + \Delta_2$$

From ΔBD_1D_2 we have

* $$\frac{D_1B}{\sin\Delta_2} = \frac{D_1D_2}{\sin\Delta} \Rightarrow D_1B = D_1D_2 \cdot \frac{\sin\Delta_2}{\sin\Delta}$$

$$= (t_s + t_l)\frac{\sin\Delta_2}{\sin\Delta}$$

* $$\frac{D_2B}{\sin\Delta_1} = \frac{D_1D_2}{\sin\Delta} \Rightarrow D_2B = D_1D_2 \cdot \frac{\sin\Delta_1}{\sin\Delta}$$

$$= (t_s + t_l)\frac{\sin\Delta_1}{\sin\Delta}$$

$$\therefore T_s = T_1D_1 + D_1B = t_s + (t_s + t_l)\frac{\sin\Delta_2}{\sin\Delta}$$

$$\therefore T_L = T_2D_2 + D_2B = t_l + (t_s + t_l)\frac{\sin\Delta_1}{\sin\Delta}$$

* Setting out of compound curve :-

The compound curve can be set by method of deflection angles. The first branch is set out by setting the Theodolite at T_1 (P.C.) & the second branch is set out by setting the Theodolite at the point D (P.C.C.).

The procedure is as follows:

1) After having known any four parts, calculate the rest of the three parts by the formulae.
2) Knowing T_s & T_L, Locate points T_1 & T_2 by linear measurement from P.I.
3) Calculate length of curve l_s & l_L. Calculate the chainage of T_1, D and T_2 as usual.
4) For 1st curve, calculate the tangential angles etc. for setting out the curve by Rankine's method.
5) Set the Theodolite at T_1 & set out the 1st branch of the curve as already explained in Rankine's method.
6) After having located the last point D (P.C.C) shift the theodolite to D & set it there. With the vernier set to $(360° - \Delta_1/2)$ reading, take back sight on T_1 & plunge/transit the telescope. The line of sight is thus oriented along T_1D produced and

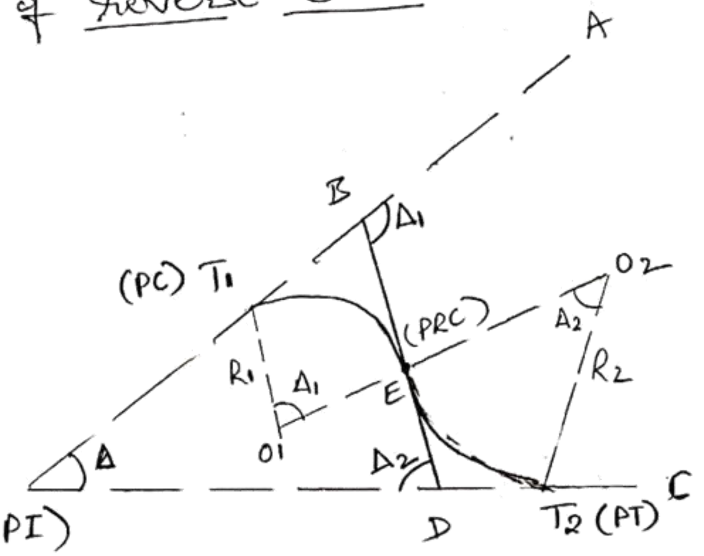
if the theodolite is now swing through $\Delta_1/2$;. The line of sight will be directed along the common tangent DD_2. Thus the theodolite is correctly oriented at D.

7) Calculate the tangential angles for the 2nd branch & set out the curve by observation from D, till T_2 is reached

8) Check the observations by measuring the angle $T_1 D T_2$; which should be equal to $\left(180^\circ - \frac{\Delta_1 + \Delta_2}{2}\right)$ or $\left(180^\circ - \Delta/2\right)$

REVERSE CURVES

★ Elements of reverse curve



VA & VC → tangents
T_1ET_2 → Reverse curve
T_1 → PC
E → PRC
T_2 → PT
BD → common tangent
O_1 & O_2 → center of two curves

A reverse curve consists of two simple curves of opposite direction that join at a common tangent point called the point of reverse curvature (PCC)

R_1 = Smaller Radius
R_2 = Greater radius
Δ_1 = central angle for the curve having smaller radius
Δ_2 = " " " " " " Greater "
Δ = total deviation between the tangents

Here.

$$EB = T_1B = t_1 = R_1 \tan \Delta_1/2.$$

$$T_2D = DE = t_2 = R_2 \tan \Delta_2/2.$$

$$BD = t_1 + t_2 = R_1 \tan \Delta_1/2 + R_2 \tan \Delta_2/2$$

$$R = \frac{BD}{\tan(\Delta_1/2) + \tan(\Delta_2/2)} \quad \text{if } (R_1 = R_2)$$

$$l_1 = \frac{R_1 \Delta_1 \pi}{180}$$

$$l_2 = \frac{R_2 \Delta_2 \pi}{180}.$$

From Δ VBD

$$\Delta = 180 - [\Delta_2 + (180 - \Delta_1)]$$

$$\Delta = \cancel{180} - \Delta_2 - \cancel{180} + \Delta_1$$

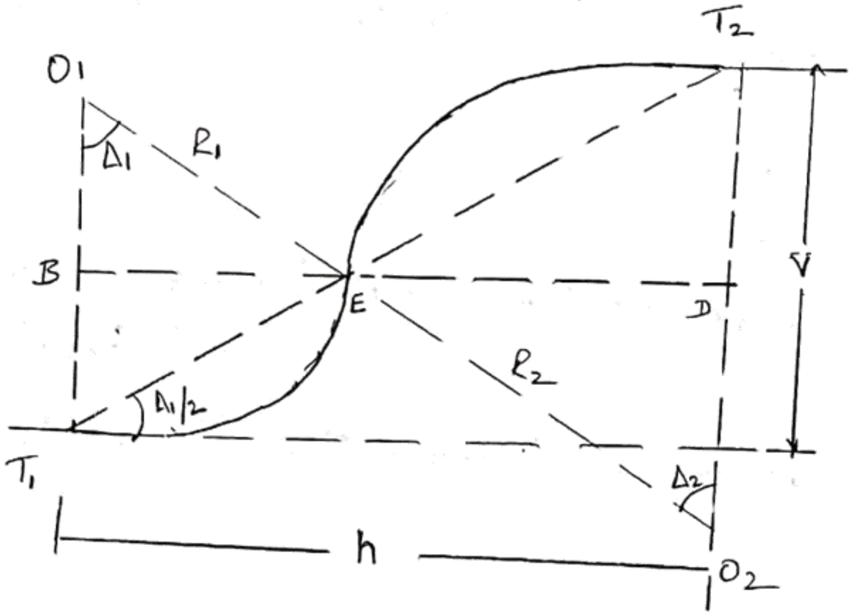
$$\Delta = \Delta_1 - \Delta_2 \quad (\text{Here } \Delta_1 > \Delta_2)$$

If $\Delta_1 < \Delta_2$.

$$\Delta = \Delta_2 - \Delta_1$$

$$\therefore \quad \Delta = \pm(\Delta_1 - \Delta_2)$$

★ Condition for Parallel straights ($\Delta_1 = \Delta_2$)



R_1 = Smaller radius

R_2 = Larger Radius

Δ_1 = Central angle corresponding to R_1

Δ_2 = " " " " R_2

V = Perpendicular distance between the two straights

h = distance between the perpendiculars at T_1 & T_2

E = Point of reverse curvature

Here $\Delta_1 = \Delta_2$

$$\therefore \quad V = (R_1 + R_2)(1 - \cos \Delta_1)$$

$$h = (R_1 + R_2) \sin \Delta_1$$

$$T_1T_2 = L = \sqrt{2V(R_1 + R_2)}$$

$$\tan \frac{\Delta_1}{2} = \frac{V}{h}$$

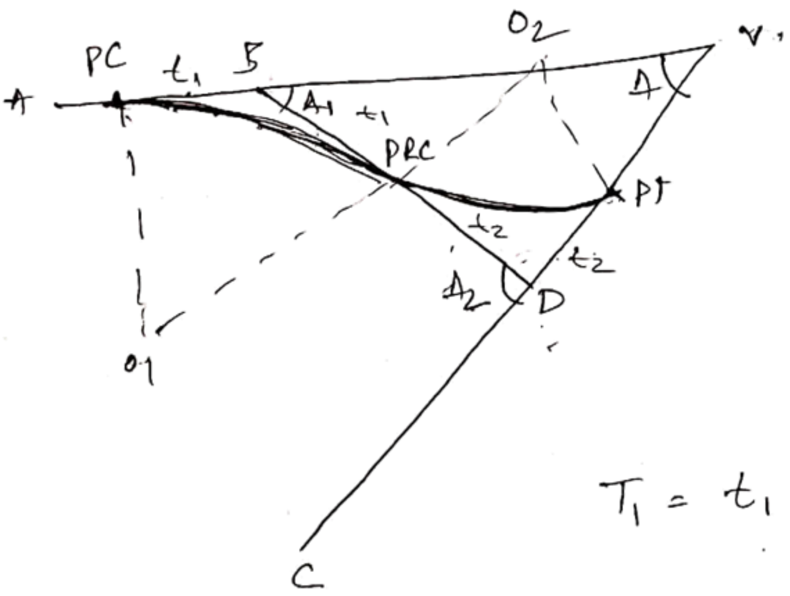
If $(R_1 = R_2) = R$

$$V = 2R(1 - \cos \Delta_1)$$

$$h = 2R \sin \Delta_1$$

$$L = \sqrt{4VR}$$

For non-parallel straight $(R_1 = R_2)$



From ΔBVD

$$\frac{BV}{\sin(180 - \Delta_2)} = \frac{BD}{\sin \Delta}$$

$$BV = \frac{BD}{\sin \Delta} \times \sin \Delta_2$$

$$T_1 = t_1 + (t_1 + t_2)\frac{\sin \Delta_2}{\sin \Delta} \quad (BD = t_1 + t_2)$$

~~Paraesh~~ |||ly. $T_2 = (t_1 + t_2)\frac{\sin \Delta_1}{\sin \Delta} - t_2$