Kinetic Modeling Equation for LIRA

Abbreviations

Abbreviation	Description
k_{di21}	miR21 diffusion rate constant
k_{di92}	miR92a diffusion rate constant
k_{de_aeB}	aeBlue degradation rate constant
k_{de21}	miR21 degradation rate constant
k_{de92}	miR92a degradation rate constant
k_{de_OLS}	Open LIRA Switch degradation rate constant
k_{de_CLS}	Close LIRA Switch degradation rate constant
k_{tr}	Trancription Rate Constant
k_{tl}	Translation Rate Constant
k_{21_CLS}	miR21-CLS Complex Formation Constant
k_{92_CLS}	miR92a-CLS Complex Formation Constant
k_{OLS192}	OLS1-miR92 Complex Formation Constant or OLSc Formation
k_{OLS221}	OLS2-miR21 Complex Formation Constant or OLSc Formation
$miR21_0$	miR21 extracellular
$miR92_0$	miR92a extracellular
$miR21_1$	miR21 intracellular
$miR92_1$	miR92a intracellular
Gene	Gene Coding for LIRA and Reporter
CLS	Close LIRA Switch
OLS	Open LIRA Switch
OLS_1	Open LIRA Switch formed by CLS-miR21 complex formation

OLS_2	Open LIRA Switch formed by CLS-miR92 complex formation
OLS_c	Open LIRA Switch formed by CLS-miR21-miR92a complex formation
aeB_1	aeBlue chromprotein translated from OLS_1
aeB_2	aeBlue chromprotein translated from OLS_2
aeB_c	aeBlue chromprotein translated from OLS_c
aeB_t	aeBlue chromprotein total = aeB_1 + aeB_2 + aeB_c
∅	Degradated substance

Chemical Equation for two miRNAs

$$[miR21_0] \rightarrow^{k_{di21}} [miR21_1] \rightarrow^{k_{de21}} \emptyset$$

$$[miR92_0] \rightarrow^{k_{di92}} [miR92_1] \rightarrow^{k_{de92}} \emptyset$$

$$[Gene] \rightarrow^{k_{tr}} [CLS] \rightarrow^{k_{de\_CLS}} \emptyset$$

$$[CLS] + [miR21_1] \rightarrow^{k_{21\_CLS}} [OLS_1] \rightarrow^{k_{de\_OLS_1}} \emptyset$$

$$[OLS_1] + [miR92_1] \rightarrow^{k_{OLS_1 92}} [OLS_c] \rightarrow^{k_{de\_OLS_c}} \emptyset$$

$$[CLS] + [miR92_1] \rightarrow^{k_{92\_CLS}} [OLS_2] \rightarrow^{k_{de\_OLS_2}} \emptyset$$

$$[OLS_2] + [miR21_1] \rightarrow^{k_{OLS_2 21}} [OLS_c] \rightarrow^{k_{de\_OLS_c}} \emptyset$$

$$[OLS_1] \rightarrow^{k_{tl}} [aeB_1] \rightarrow^{k_{de\_aeB}} \emptyset$$

$$[OLS_2] \rightarrow^{k_{tl}} [aeB_2] \rightarrow^{k_{de\_aeB}} \emptyset$$

$$[OLS_c] \rightarrow^{k_{tl}} [aeB_c] \rightarrow^{k_{de\_aeB}} \emptyset$$

ODE for two miRNAs

$$\frac{d[miR21_0]}{dt} = -k_{di21}[miR21_0]$$

$$\frac{d[miR92_0]}{dt} = -k_{di92}[miR92_0]$$

$$\frac{d[miR21_1]}{dt} = k_{di21}[miR21_0] - k_{de21}[miR21_1] - k_{21\_CLS}[CLS][miR21_1] - k_{OLS_221}[OLS_2][miR21_1]$$

$$\frac{d[miR92_1]}{dt} = k_{di92}[miR92_0] - k_{de92}[miR92_1] - k_{92\_CLS}[CLS][miR92_1] - k_{OLS_192}[OLS_1][miR92_1]$$

$$\frac{d[CLS]}{dt} = k_{tr} - k_{de\_CLS}[CLS] - k_{21\_CLS}[CLS][miR21_1] - k_{92\_CLS}[CLS][miR92_1]$$

$$\frac{d[OLS_1]}{dt} = k_{21\_CLS}[CLS][miR21_1] - k_{OLS_192}[OLS_1][miR92_1] - k_{de\_OLS_1}[OLS_1]$$

$$\frac{d[OLS_2]}{dt} = k_{92\_CLS}[CLS][miR92_1] - k_{OLS_221}[OLS_2][miR21_1] - k_{de\_OLS_2}[OLS_2]$$

$$\frac{d[OLS_c]}{dt} = k_{OLS_192}[OLS_1][miR92_1] + k_{OLS_221}[OLS_2][miR21_1] - k_{de\_OLS_c}[OLS_c]$$

$$\frac{d[aeB_1]}{dt} = k_{tl}[OLS_1] - k_{de\_aeB}[aeB_1]$$

$$\frac{d[aeB_2]}{dt} = k_{tl}[OLS_2] - k_{de\_aeB}[aeB_2]$$

$$\frac{d[aeB_c]}{dt} = k_{tl}[OLS_c] - k_{de\_aeB}[aeB_c]$$

$$\frac{d[aeB_t]}{dt} = \frac{d[aeB_1]}{dt} + \frac{d[aeB_2]}{dt} + \frac{d[aeB_c]}{dt}$$

Chemical Equation for one miRNA

To simulate 1 miRNA, we take it as miR21, because the parameter values we use in this simulation for both miR21 and miR92a are the same.

$$[miR21_0] \rightarrow^{k_{di21}} [miR21_1] \rightarrow^{k_{de21}} \emptyset$$
$$[Gene] \rightarrow^{k_{tr}} [CLS] \rightarrow^{k_{de\_CLS}} \emptyset$$
$$[CLS] + [miR21_1] \rightarrow^{k_{21\_CLS}} [OLS_1] \rightarrow^{k_{de\_OLS_1}} \emptyset$$
$$[OLS_1] \rightarrow^{k_{tl}} [aeB_1] \rightarrow^{k_{de\_aeB}} \emptyset$$

ODE for one miRNA

$$\frac{d[miR21_0]}{dt} = -k_{di21}[miR21_0]$$

$$\frac{d[miR21_1]}{dt} = k_{di21}[miR21_0] - k_{de21}[miR21_1] - k_{21\_CLS}[CLS][miR21_1]$$

$$\frac{d[CLS]}{dt} = k_{tr} - k_{de\_CLS}[CLS] - k_{21\_CLS}[CLS][miR21_1]$$

$$\frac{d[OLS_1]}{dt} = k_{21\_CLS}[CLS][miR21_1] - k_{de\_OLS_1}[OLS_1]$$

$$\frac{d[aeB]}{dt} = k_{tl}[OLS_1] - k_{de\_aeB}[aeB_1]$$

Modeling Parameters of LIRA

Symbol	Description	Value/Range	Unit	Reference(s)	Notes
k_{di21}	miR21 diffusion rate constant	1 x 10^{-2}	s^{-1}	(Levine *et al.,* 2007)	General miRNA rate diffusion with size ~20 nucleotide
k_{di92}	miR92a diffusion rate constant	1 x 10^{-2}	s^{-1}	(Levine *et al.,* 2007)	General miRNA rate diffusion with size ~20 nucleotide
k_{de_aeB}	aeBlue degradation rate constant	2.7 x 10^{-4}	s^{-1}	(Alon 2020)	General reporter degradation rate (1 hour)
k_{de21}	miR21 degradation rate constant	3 x 10^{-4}	s^{-1}	(Baabu *et al.,* 2022)	General RNA degradation rate
k_{de92}	miR92a degradation rate constant	3 x 10^{-4}	s^{-1}	(Baabu *et al.,* 2022)	General RNA degradation rate
k_{de_OLS}	Open LIRA Switch degradation rate constant	3 x 10^{-4}	s^{-1}	(Baabu *et al.,* 2022)	General RNA degradation rate
k_{de_CLS}	Close LIRA Switch degradation rate constant	3 x 10^{-4}	s^{-1}	(Baabu *et al.,* 2022)	General RNA degradation rate
k_{tr}	Trancription Rate Constant*	0.000779	$M.s^{-1}$	(Stögbauer *et al.,* 2012)	RNA Toehold and reporter transcription rate ~2.2 NtPs, this TU has around 1603 nucleotides
k_{tl}	Translation Rate Constant*	0.0344	s^{-1}	Average translation rate -	Average translation rate in *E coli* ~8 amino acid per second

				Bacteria Escherichia coli - BNID 111689 (harvard.edu)	and aeBlue chromoprotein has 231 amino acids
k_{21_CLS}	miR21-CLS Complex Formation Constant	1 x 10^5	$M^{-1}.s^{-1}$	(Baabu *et al.,* 2022)	miRNA-RNA toehold complex formation rate
k_{92_CLS}	miR92a-CLS Complex Formation Constant	1 x 10^5	$M^{-1}.s^{-1}$	(Baabu *et al.,* 2022)	miRNA-RNA toehold complex formation rate
k_{OLS192}	OLS1-miR92 Complex Formation Constant or OLSc Formation	1 x 10^5	$M^{-1}.s^{-1}$	(Baabu *et al.,* 2022)	miRNA-RNA toehold complex formation rate
k_{OLS221}	OLS2-miR21 Complex Formation Constant or OLSc Formation	1 x 10^5	$M^{-1}.s^{-1}$	(Baabu *et al.,* 2022)	miRNA-RNA toehold complex formation rate

*assumption: OLS_1, OLS_2, OLS_c have same degradation rate

Known that In E coli translation rate is 8 amino acids/second (Average translation rate - Bacteria Escherichia coli - BNID 111689 (harvard.edu)) and our aeBlue chromoprotein has 231 amino acids. So k_{tl} (**Translation Rate Constant**) = 1/average time for translation

Average time for translation = Number of amino acids × Time for addition of one amino acid

Time for addition of one amino acid = ⅛ = 0.125 second per one amino acid

Average time for translation = 231 × 0.125

Translation Rate Constant = 1/231 × 0.125 = ~0.0344 / second

Trancription Rate Constant

molar_sec = 1603.62 nucleotide/sec / (330 grams/mol * 6.022 x 10^{23} molecules/mol)

molar_sec = 0.000779 molar/sec

References

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