Primes4everybody - EN

1. Introduction

1.1 Why prime numbers have fascinated us for 2000 years

Prime numbers are among the oldest and at the same time most modern objects in mathematics. They appear in elementary school just as they do in modern cryptography and quantum information. Although their definition is simple — "a number that is divisible only by 1 and itself" — they have generated a fascination for more than two millennia that no other mathematical topic has ever reached.

There is a simple reason for this:

Prime numbers combine simplicity and unpredictability in a way that contradicts intuition.

- Their concept is trivial — every child can understand it.
- Their behaviour is highly complex — no mathematical approach has been able to fully predict them.

They appear at once patterned and chaotic:
After some irregular intervals there often follows a section with many primes, then again a long time without any. Time and again people have believed they were on the verge of a complete explanation — and time and again it turned out that the discovered pattern was only a partial picture.

The fascination of prime numbers does not arise from mystery or mysticism, but from a serious scientific conflict:
How can something with such a simple definition exhibit behaviour that is so difficult to explain?

This is why prime numbers can be found not only in textbooks, but also in popular books, documentaries, discussions and culture. Since antiquity they have symbolised the idea that the world is based on a hidden order — and that one can discover this order if one understands deeply enough.

After 2000 years of research, prime numbers are therefore not old, but timeless:
They mirror the relationship between insight and limitation, between pattern and surprise, between simple and complex.

This tension transforms what appears to be a simple question ("Which numbers are prime?") into one of the greatest challenges — and one of the most beautiful riddles — in science.

1.2 What the classical view could explain — and what it could not

Classical mathematics has discovered magnificent things about prime numbers over two millennia. Nothing in this document contradicts these achievements — on the contrary: without them a deeper understanding would not be possible at all.

The classical view is based on a simple but far-reaching idea:
The number space is fully present, and prime numbers are special points within this space.

From this assumption arose an enormous mathematical tradition that continues to this day.

Within this static model, many properties could be described precisely:

- Prime numbers become rarer as numbers grow larger.
- Their approximate frequency can be predicted by $\frac{n}{\ln(n)}$.
- Every natural number possesses a unique decomposition into prime factors.
- Many statements about primes can be expressed in formulas, sums and complex functions.

All of these insights are correct and remain valid.

Nevertheless, there were points that resisted explanation within the static view:

- Why do primes appear exactly where they appear?
- Why are there regions in which many prime numbers appear — and others in which hardly any occur?
- Why are some conjectures plausible yet nearly impossible to prove (Goldbach, twin primes)?
- Why does the Riemann zeta function work so astonishingly well — and why does it need the complex numbers to make order visible?

Classical mathematics could describe the structure, but it could not reveal the cause.

The further research progressed, the clearer the paradox became:
We were able to measure the behaviour of primes with increasing precision — but not explain why.

This led over time to a shift:

- away from fundamental questions of understanding
- towards increasingly complex techniques that model phenomena rather than explain them

These techniques were not wrong — they became necessary because the underlying perspective did not allow certain questions to be asked.

In other words:
The classical theory saw the picture, but not the creation of the picture.

It could measure everything excellently as long as one assumed that the number space already "exists" — but it had no tool to describe how this space grows.

This is where this work begins — not as a refutation of classical theory, but as an extension of the perspective that makes visible what had to remain invisible within the static view.

1.3 Purpose of this document: understanding instead of formula-worship

Most popular presentations of prime numbers rely on two familiar strategies:
Either they emphasise their mystery — "puzzling, mystical, unpredictable" — or they present formulas and theorems, hoping that their beauty will speak for itself. Both approaches have their merit, but they rarely lead to understanding.

This document takes a different approach:
The focus is not on the formulas, but on the concept that the formulas describe.

The goal is not to replace or simplify the mathematical tools of prime number theory. The classical theory remains valid. Yet many of its results become intuitively comprehensible only when one does not ask:

"How do we formulate prime numbers?",
but instead:
"How do prime numbers arise?"

A comprehensible picture does not emerge through more computational rules, but through a shift in perspective: from the static number space to the creation of the number space.

This document is therefore aimed at two groups at the same time:

- readers without mathematical background who truly want to understand prime numbers
- and mathematically trained readers who wish to examine whether a complementary perspective explains established findings more consistently

For this, a collection of formulas and proofs is not enough.

Understanding arises only when two questions are answered:

1. Why do prime numbers appear unpredictable in the classical view?
2. Why do they become comprehensible as soon as one considers the number space as a growth process?

This document focuses exclusively on addressing these two questions.
It does not advocate spectacular or confrontational theses, but follows a simple principle:
A concept becomes understandable only when it is explained where it arises — not where it is measured.

1.4 Why the name Primes4Everybody

The title of this document is deliberately chosen.
It is inspired by the well-known initiative "Computer Science for Everybody" by U.S. computer scientist Charles Severance, whose goal is to make complex topics accessible to everyone without simplification — not through reduction, but through comprehensible explanation.

The name *Primes4Everybody* is intended to transfer this principle to the world of numbers:

- Not every person has to study mathematics to understand prime numbers.
- And one does not have to be an expert to access a deep concept.

Prime numbers have often been presented either as a mystery or as a field for specialists. Both have unintentionally reinforced the idea that true insight is accessible only to a few.

The title therefore conveys a deliberately clear message:
Prime numbers are not a topic for a select group — they are a topic for everyone, because they are part of the fundamental thinking system of the world.

The naming serves not as branding, not as self-positioning and not as differentiation.
It serves only to make visible that this document does not seek merely to explain what mathematics knows about prime numbers, but why their behaviour becomes understandable once one recognises the underlying process.

2. How classical mathematics views the number space

2.1 The number space as a static, ever-existing object

Classical mathematics treats the number space as something complete and timeless. In this view, all natural numbers — 1, 2, 3, 4, 5 and so on — exist independently of whether someone lists or uses them. The number space is not something that comes into being, but something that is already fully present.

This intuitive concept has shaped mathematics since antiquity and is deeply anchored in language and symbolism. One "goes further upward," "looks between two numbers," "finds primes," as if exploring structures that have existed from the very beginning. The image resembles a landscape that already lies entirely before us and only needs to be mapped.

Several underlying assumptions follow from this static understanding — seldom spoken aloud, yet always influential:

- Numbers are objects, not processes.
- All numbers exist simultaneously, even if they are never written down.
- Prime numbers must exist within this already present space, not *arise*.

Classical theory developed consistently along this perspective.
If the number space exists in full, then the mathematical task is to identify, measure and

describe the properties of this space. That is exactly what number theory has historically done — and with great success.

Many areas of modern mathematics follow directly from this static perspective:

- Differential and integral calculus assume that continua already exist.
- Topology studies properties of spaces, not their emergence.
- In number theory, the frequency of primes is analysed, not the process that leads to them.

None of this is wrong.
This perspective was necessary, productive and fruitful. It enabled central developments in the entire history of mathematics.

Yet it has an invisible side-effect:
If numbers are regarded as finished objects, one cannot make visible how they come into being — and therefore cannot make visible how prime numbers come into being.

This limitation is not the result of a mistake, but a natural consequence of the perspective. As long as the number space is understood as static, prime numbers do not *arise* — they are simply *there*. And if something is *just there* without its origin being visible, the only option is to measure it.

This is precisely why classical theory was able to describe the structure of prime numbers precisely, yet never explain why they appear the way they do. Within a static number space, their behaviour must appear unpredictable — regardless of how correct and powerful the mathematical tools are.

2.2 Primes as scattered "special points" without a generating mechanism

If the number space is understood as complete and static, prime numbers appear in it as special objects without any inner cause. They occur, but they do not "do" anything. They are not the result of a process — they are "given."

This perception has shaped mathematics for centuries:

- 4, 6, 8, 9, 10, 12 can be decomposed
- 2, 3, 5, 7, 11 cannot be decomposed

Thus, prime numbers appear as exceptions to the "normal behaviour" of most numbers. They form a small but undeniably important subgroup — a pattern within the pattern.

To describe this subgroup, classical theory had to find a way to characterise the distribution of these exceptions in a number space assumed to be fully present.

The conclusion was unavoidable:
If prime numbers have no mechanism of origin, then they must be described without asking for a reason for their origin.

Mathematically, this attitude led to three lines of investigation that still play a central role today:

1. **Classification**
   Which numbers are prime and which are not?
2. **Density and distribution**
   How many primes exist up to a certain magnitude?
3. **Asymptotic regularities**
   How does the frequency of primes change as numbers become large?

All of these approaches produced valuable results. Prime density could be approximated, the twin prime conjecture was formulated, and the distribution of primes could be modelled surprisingly well by the Riemann zeta function.

Despite all progress, one notable gap remained:
Classical theory could measure *where* primes occur with striking precision — but say nothing about *why* they occur there.

The reason does not lie in a lack of mathematical achievement, but in the underlying assumption itself:
If primes are viewed as given special points, they cannot be explained as the result of a process.

Within this perspective, their behaviour must necessarily appear unpredictable — not because it is "mystical," but because the chosen framework provides no mechanism that could lead to primes.

This framework was not accidental: classical number theory defined numbers through divisibility — that is, through multiplication. A number "was" the product of its factors, or it was not. Thus multiplication became not just a tool, but the fundamental lens through which numbers were viewed.

In this paradigm, addition — the actual structure of *emergence* of the number space — remained in the background. All mathematical thinking was consequently aligned with understanding numbers through multiples, factors and divisor-specific properties.

What could not be explained multiplicatively therefore appeared automatically as an exception: the prime numbers.

As long as primes appear as special points in an already existing number space, they cannot help but appear irregular, surprising and analytically difficult.

2.3 Why primes appear like chaos in this view

If the number space is regarded as complete and unchanging and primes appear within it without a mechanism of origin, then a specific image follows inevitably:

Prime numbers appear irregular.

- The distances between them fluctuate.
- Regions with many primes alternate with regions without any primes.
- Local clusters contradict global trends.
- Every discovered "pattern" holds only for a limited range.

This behaviour is not the result of mathematical disorder, but the consequence of the underlying assumption. If there is no explanation for how primes arise, the only option is to describe where they appear.
And location without cause *always* results in irregularity.

Thus a paradoxical phenomenon emerges:
The more precisely one measures the distribution of primes, the more clearly their unpredictability becomes visible.

This paradox has shaped number theory since antiquity.
Again and again it seemed possible to find a regular mechanism — and again and again it turned out that the mechanism applied only in certain regions. The irregular deviations never disappeared completely.

The most significant mathematical achievements of the 19th and 20th centuries concerning prime distribution — especially the works of Euler, Riemann, Hardy and Littlewood — illustrate this tension impressively:

- Global laws are recognisable.
- Local deviations remain persistent.

Or in short:
Order exists — but without a visible origin.

This observation did not lead to resignation, but to an enormous expansion of analytical tools. Research learned to quantify, bound and partially predict the deviations. Equations and models such as the Riemann zeta function, the prime number theorem or the Hardy–Littlewood conjectures show how many patterns become visible once they are mathematically illuminated.

Yet one thing became clear:

- Measurability steadily increased.
- Understanding of the cause remained just as blurry as before.

Within a static number space, this is unavoidable. Without a process, behaviour becomes visible, but its cause remains invisible — regardless of mathematical precision.

Therefore, the distribution of primes appears like chaos in the classical view:

- not because it *is* chaotic,
- but because the chosen framework does not allow for an origin.

2.4 Consequences for research, cryptography and education

The static perspective on the number space has shaped not only mathematical theory, but also areas that depend on prime numbers — sometimes far beyond pure mathematics.

Research

If primes appear as special points without a mechanism of origin, then research naturally focuses on the following tasks:

- better models for describing their distribution,
- more precise estimates of deviations,
- tighter error terms,
- more efficient algorithms for detecting and constructing primes.

The emphasis is therefore on analysis and technique, not on origin. This explains why enormous progress has been made over centuries without answering the fundamental question: *Why do prime numbers arise?*
This question cannot even be formulated within the static view.

Cryptography

In cryptography, the behaviour of prime numbers is not a theoretical detail but the foundation of global IT security.

The principle used there is:
Prime numbers are distributed so irregularly that predicting them is virtually impossible.

The security of large portions of digital infrastructure relies precisely on this assumption.

The static number space produces a functional paradox:

- mathematically, the unpredictable distribution presents a challenge,
- cryptographically, it presents an advantage.

Here too, we see the same basic structure:
Prime numbers are used without a visible mechanism of origin.

Education

In mathematical education, this leads to a recurring pattern:

1. Prime numbers are introduced as "fundamental."
2. Their definition is taught.
3. First patterns are shown — and then explained why these are not reliable.

4. The focus shifts to computational techniques, functions and applications.

The common result:

- Students learn *what* prime numbers are,
- university students learn *how* to work with primes,
- but within the classical framework there is no point of access to even formulate the question "Why do primes appear exactly where they do?"

This situation is not caused by a lack of pedagogical possibilities, but by the perspective itself. If prime numbers have no cause of origin, then none can be taught.

This chapter does not end with a conclusion — because the document does not seek confrontation.

The transition to the next section arises from a neutral observation:
Classical theory describes primes precisely and successfully as long as the number space is considered a static object.
An explanation of their behaviour becomes possible only when one does not ask what they *are*, but how they *arise*.

2.5 Psychology of invisible assumptions: why nobody questioned the model

The static view of the number space is not just a mathematical model — it is an intuitive understanding that develops early and becomes deeply ingrained. In everyday life, numbers are not experienced as processes, but as fixed quantities: "three apples," "ten metres," "one million euros." In education too, numbers appear not as something that comes into being but as something that is already present.

From this early imprinting arises an implicit assumption:
Numbers exist independently of thought, and their properties must be understood within this finished space.

This assumption is rarely stated explicitly because it feels self-evident.
And precisely because of that, it remains unexamined.

This mechanism is not specific to mathematics — it is a general trait of human cognition:

- What is learned early later appears "obvious."
- What feels self-evident is not investigated.
- What is not questioned remains invisible.

Within number theory, this invisible assumption has deep consequences:

- If numbers appear as finished objects, research focuses on their properties, not their emergence.

- If primes appear without a process, their occurrence is described, not explained.
- If order is seen without cause, the cause lies outside the scope of thought.

Important:
This limitation is not a scientific failure and not an oversight. It is the natural consequence of a perspective that has proven itself over many centuries.

Classical theory was not restricted because it overlooked something, but because its framework did exactly what it was created to do:

- measure instead of generate
- describe instead of originate
- recognise structure instead of analyse process

As long as the number space is understood as static, it is consistent and rational to treat primes not as the result of a mechanism but as special objects.

Therefore, nobody asked the central question — not out of carelessness, but because it was not logically permitted within the static model.

3. Philosophical Foundations of Numerical Understanding

3.1 Plato – Numbers as eternal ideal objects

In ancient Greek philosophy, Plato introduced a view that would influence thought far beyond his era:
Numbers are not a human invention but part of a higher, unchanging order. They belong to the world of "Forms" — a realm of perfect, timeless structures. The numbers we use in everyday life are, in this view, merely images of this higher reality.

This way of thinking explains four characteristics still associated with mathematics today:

- **Universality:** Mathematics applies everywhere.
- **Timelessness:** Mathematical statements do not change.
- **Absoluteness:** Results hold independently of culture and language.
- **Discovery rather than invention:** Mathematicians "find" truths rather than create them.

Plato places numbers in a reality that does not need to be created — they simply *are*. From this perspective, 7 exists regardless of whether anyone speaks or writes it, and prime numbers are part of this same eternal order.

This viewpoint was not only philosophically influential but also mathematically productive.

It enabled:

- the development of a style of thinking in which logical structures take precedence over physical intuition,

- the idea that mathematics does not merely model the world but describes its fundamental structure,
- the expectation that the world can, in principle, be understood through order, rules and relationships.

In Plato's framework, a mathematical object does not come into being — it exists. Numbers are not processes, but entities.

This idea shaped European mathematics for many centuries.
Not because it was imposed, but because it provided a consistent foundation for organising the world intellectually and building science.

If numbers "have always been there," it is natural that mathematical work consists in recognising their properties, discovering relationships and making order visible.

The Platonic model explains why classical mathematics was directed primarily toward understanding *what* numbers are and *how* they behave — not *how* they come into being.

This perspective was not restrictive but powerful.
It formed the intellectual basis for large parts of mathematics, and many of its assumptions proved stable, useful and fruitful.

3.2 Wittgenstein – Numbers as systems of rules and language

More than two thousand years after Plato, Ludwig Wittgenstein introduced a radically different view of mathematics. Whereas Plato regarded numbers as timeless, ideal objects, Wittgenstein saw them as part of a system of rules — comparable to a language. In this model, numbers are not independently existing things but forms through which humans generate meanings and structure action.

Wittgenstein observed that mathematical symbols gain meaning only through their use:

- "5" is not a pre-existing entity, but a rule for designating a sequence.
- "+" is not an object, but an instruction for action.
- "=" is a stipulation of when two representations shall count as identical.

The meaning of mathematical expressions thus does not arise from metaphysical properties but from their role within a system of agreements and applications. In this sense, numbers are not "discovered" but "introduced" — not arbitrarily, but as tools for joint orientation.

This perspective explains four important observations:

- different cultures use different numeral systems — yet all work,
- mathematical symbols evolve historically,
- new types of numbers (e.g., complex numbers) emerge when existing rules no longer suffice,

- mathematical statements are correct because they follow rules, not because they are "anchored in nature."

In this view, a mathematical object does not arise from metaphysical existence but from rules of use. Numbers are not entities — they are tools.

This interpretation does not contradict the real usefulness of mathematical statements.
It explains why mathematics can be applied in every science: not because it is inherently "in the world," but because it provides consistent structures that humans use for orientation.

Wittgenstein's model encouraged viewing mathematics not only as a theory about the world but also as a form of human practice:
Mathematics works because people share the rules.

In this perspective, the focus shifts away from the properties of numbers toward the mechanisms of their use — toward the syntax and grammar of mathematical thought.

3.3 Psychological perspective – Numbers as abstracted observational structures of the environment

Long before writing, digits or mathematical rules existed, humans faced an environment in which quantities and changes determined survival. Without calendars, computational tools or abstract symbols, the world had to be assessed reliably:

- How many animals are in the herd?
- Is the stored grain sufficient for the winter?
- How many people are needed for the harvest?
- How far is the path to the next water source?

In such situations, numbers were not theoretical concepts but condensed experiences.
Humans recognised that things in the world accumulate and diminish: something is added, something is lost, something remains stable.

This structure is not mathematical, but ecological:
"More" and "less" are observations — not inventions.

Addition represents this observation directly.
It describes a natural phenomenon: accumulation through stepwise change.

Examples of purely additive processes:

- A handful of grains becomes a handful plus a few more grains.
- A basket of firewood becomes fuller when more logs are placed in it.
- A herd becomes larger when a single animal returns or is born.
- The level of water in a jar rises when someone refills it.
- Steps along a path accumulate, because each step is another one.

At this level, there are neither digits nor symbols.
Numbers here are mental concepts — cognitive compression of the environment:
Numbers are the ability to remember changes in quantity reliably.

Only much later did humans develop tools to externalise these mental concepts: tally sticks, knots, numerals, and eventually abstract computation. These tools did not change the origin — they simply made visible what was previously mental.

At this point, a difference emerges that is intuitively simple but far-reaching:

- Addition corresponds directly to reality.
- Multiplication is not an observation but a compression of many additions.

No human ever *observed* "three times eight."
What was observed was: eight animals arrive, then eight more, then eight more.

As such repetitions became cognitively and practically demanding, humans introduced a shortcut concept: multiplication. It compresses many additive steps into a rule — not because the world shows multiplication, but because the mind seeks efficiency.

Thus, a clear psychological relationship emerges:

Perception	Mathematical expression
Things become more	Addition
Things become repeatedly grouped	Multiplication

Addition mirrors the world.
Multiplication mirrors thinking *about* the world.

This distinction forms the foundation for later developments:

- Addition is **stable to reality** — it maps real processes without distortion.
- Multiplication is **cognitively efficient**, but **structurally sensitive**, because it compresses many additive steps into a rule.

From the psychological viewpoint it follows:
Numbers are neither ideal metaphysical objects nor purely linguistic systems.
They begin as perceptual patterns — and only later become symbols and rules.

This perspective does not contradict the philosophical models — it complements them:

- Plato explains the **stability** of numbers.
- Wittgenstein explains the **symbolism and rules**.
- Psychology explains **the origin**.

Only the combination of these three levels yields a complete picture.

3.4 Synthesis – What numbers really are

The three perspectives — Plato, Wittgenstein and the psychology of early quantity perception — do not describe contradictions but different levels of the same phenomenon.

- Psychology describes the **origin** of numbers: perception of change in the world → "more," "less" → addition as a model of real processes.
- Plato describes the **stability** that results: numbers appear unchanging because the underlying natural processes are stable.
- Wittgenstein describes the **symbolic and rule-based** form: writing, numerals and operations structure the use of numbers independently of perception.

Thus a consistent overall picture emerges:
Numbers begin as observations of nature, appear stable because of this, and are eventually turned into symbolic tools.

Historically, these levels did not develop simultaneously, but sequentially.
First came additive observation, then the idea of stability, then formal rules of computation.
Later in mathematical history, the symbolic perspective became dominant.

This shift led to a far-reaching cultural effect within mathematics:
Multiplication increasingly came to be seen as a "higher," "more powerful" and "more theoretical" operation, whereas addition appeared elementary, trivial or preliminary.

This development is understandable — multiplication enables more efficient computation and opens abstract structures far beyond everyday experience. It is not "wrong," but a cognitive tool of great reach.

But this prioritisation had a consequence:
The origin of numbers — the addition that maps real processes — faded from view.

The symbolic and rule-based level became the dominant scientific lens through which numbers were viewed. This created a blind spot:
If multiplication is regarded as the fundamental structure, mathematical explanations are sought primarily there — even when the explanation lies in addition.

Thus emerges the central result of this synthesis:

- Addition explains the origin of the number space.
- Multiplication explains the reconstruction of the number space based on its origin.

These two levels are **not interchangeable**.
A growth space of additive origin can be represented multiplicatively — but never without structural tensions, gaps and overlaps. This is not a flaw of multiplication, but a consequence of compressing many additive steps into a rule.

Therefore:
Addition mirrors the world.
Multiplication mirrors thought about the world.

And only through the interaction of both levels does it become visible **what numbers really are**:
Numbers are not independent metaphysical entities and not purely linguistic constructions.
They are mentally condensed observations of the world that appear stable, and later are structured through symbols and rules.

This synthesis explains:

- why numbers feel **intuitive** subjectively,
- why they appear **stable** philosophically,
- and why they become **complex** mathematically.

And it forms the foundation for the next question — the question that has shaped the understanding of primes for 2000 years:
What happens when we attempt to describe an additive growth space multiplicatively?

Exactly there begins the transition to prime numbers.

3.5 Why the prime number problem was unsolvable as long as thinking was trapped in the wrong perspective

For more than two millennia, classical number theory rested on a fundamental assumption:
The number space exists completely and primes are special points within this finished space.

Within this perspective, it was logical to search for a **static** explanation:

- a formula that directly identifies prime numbers,
- a function that exactly counts or locates them,
- or a structure that characterises the prime pattern point by point.

This approach was logical, consistent and scientifically correct — within the given paradigm.
It led to enormous progress: analytic number theory, Dirichlet series, the Riemann zeta function, asymptotic estimates and many other developments.

But all approaches shared one basic assumption:
A growing phenomenon was being analysed using static tools.

Thus, an unnoticed perspective error was built in:

- **Addition** generates the number space (growth).
- **Multiplication** attempts to reconstruct that space (coverage).

As long as multiplication was regarded as the fundamental structure, it seemed logical to search for the solution there — in multiples, in factors, in overlays, in harmonies and oscillations.

But from this viewpoint the prime number problem *had to* appear difficult, because:
An additive growth space can never be completely reconstructed multiplicatively.

The consequences were visible, but their cause remained hidden:

- Prime numbers appear "irregular."
- The deviation between actual and approximated distribution grows and shrinks in complex patterns.
- Multiplicative approaches explain large parts of the behaviour but not everything — and become increasingly complex in detail.

These observations were not misinterpreted — they were interpreted within a perspective in which multiplication was assumed to be the central mathematical structure.

The outcome:
The prime number problem was not unsolvable because the question was wrong,
but because the **perspective** was incomplete.

For 2000 years the question was:
"How are prime numbers distributed within a static space?"

The correct question is:
"What happens when an additive growth space is described multiplicatively?"

Only with this shift does the key insight become visible:

- Prime numbers are not "exceptions."
- They are the inevitable fixed points at which the multiplicative reconstruction cannot fully cover the additive growth process.

From this follow three statements:

1. Classical mathematics was consistent — but its perspective was necessarily incomplete.
2. This incompleteness was unavoidable — it resulted from the historical prioritisation of multiplication.
3. The prime number problem could not be solved as long as the origin of the number space (addition) was not recognised as its structural foundation.

In short:
The solution to prime numbers was not prevented by a lack of mathematical ingenuity — but by the missing view of the origin of the number space.

4. The Shift in Perspective

4.1 Numbers come into being – they are not "always there"

The classical view of numbers treats the number space as fully present.

In this perspective, every number already "exists" — regardless of whether it is used or calculated. The number 1,000,000 is considered just as real as the number 7, even if it has never been written down. This view feels intuitive because numbers appear stable: 5 remains 5, no matter where or when it is considered.

However, logically, the number space does not arise as a finished object but through stepwise growth. Numbers are not independent entities, but states that are produced only through a change from a previous state.

A number does not exist before a counting step brings it forth.
8 is not "there" waiting to be discovered — it comes into being only at the moment something is added to 7.

A number is therefore not an object, but the completion of a transition.
The number space is not a reservoir, but a process — a continuous expansion.

The impression of a "finished number space" arises culturally:

- because we know numbers as symbols instead of actions,
- because number tables appear complete,
- and because mathematics education begins with a finished system rather than its formation.

But structurally:
The number space grows — it is not retrieved.

Only through this process of growth do all later properties of the number space emerge, including those examined in the context of prime numbers. What later appears as "distribution," "spacing," or "irregularity" is not a feature of a finished object, but a consequence of a developmental process.

Thus the starting point of the perspective shift is clear:

- Numbers arise additively, through stepwise expansion.
- The number space is a product of its formation, not a superior structure.
- Its properties can only be understood when its origin is taken into account.

4.2 Multiplication reconstructs, addition generates

If the number space arises step by step, then:

- Addition is the mechanism of formation.
- Multiplication is a tool for reconstructing the resulting space.

Both operations are necessary — but they have fundamentally different roles.

Addition directly reflects real change:

- something is added → the quantity increases
- something is lost → the quantity decreases

This corresponds to the structure of the world.
Addition generates the number space.

Multiplication, on the other hand, is not a natural phenomenon but a cognitive mechanism:

- many additive steps are packaged into a rule,
- repetition is represented in compressed form,
- efficiency replaces sequential counting.

It is therefore not part of the formation process, but part of thinking *about* the formation process.

Multiplication is a tool for quickly reconstructing additive processes — not their origin.

This functional separation is simple — but it has far-reaching consequences.

If the number space arises additively, then it is structurally designed to be understood additively. If instead it is viewed multiplicatively, features necessarily arise that appear "complicated," "irregular," or "unexplainable."

From this perspective:

Role	Generation	Reconstruction
Operation	Addition	Multiplication
Source	World	Mind
Property	Stable	Condensing
Structural consequence	consistent	prone to overlap

Multiplication is neither an error nor an unnatural construct.
It enables abstract mathematics and is indispensable in nearly all fields of science.
Nevertheless:
Multiplication does not explain why numbers arise as they do — only how they can be efficiently represented.

As long as mathematics assumed multiplication to be the fundamental structure of numbers, it was forced to search in the reconstruction for what could only have been found in the formation.

Thus emerges the decisive insight:

- Addition determines the number space.
- Multiplication attempts to reconstruct it with limited access to information.
- From the difference between both processes arises everything later investigated about prime numbers.

4.3 Why structural breaks are unavoidable → Primes as logical fixed points

If the number space is generated additively but reconstructed multiplicatively, two different structures emerge:

1. **The growth space** – built through stepwise addition
2. **The reconstruction space** – represented through multiples and factors

As long as both perspectives coincide, numbers appear "regular."
As soon as they diverge, a structural break occurs.

This can be formulated precisely:

- Addition creates every new numerical state.
- Multiplication attempts to derive this state from multiples of previous states.

Where this reconstruction succeeds, a number is fully "covered" by multiples of earlier numbers.
Where it does not succeed, a **fixed point** arises:
A number that cannot be covered by multiplication from earlier numbers is a prime.

This can be observed exemplarily in the early phase of the number space:

– The multiples of 2 cover 2, 4, 6, 8, …
– The multiples of 3 cover 3, 6, 9, 12, …

Under these conditions, the structure becomes visible:

– 4 and 6 are explained by reconstruction (multiples)
– 5 is not reconstructed; it arises solely through growth

Thus 5 is not a "coincidence" but the first necessary fixed point — the first value at which the multiplicative reconstruction cannot fully cover the additive growth process.

The same happens again:

– 6 is fully reconstructed
– 7 is not reconstructed

The structure is not arbitrary.
Primes always arise exactly where the reconstruction cannot fully cover the process of formation.

Prime numbers are therefore not "exceptions," but the logically unavoidable points at which multiplicative reconstruction cannot keep pace with additive growth.

This changes the interpretation completely:

- In the static model, primes appear "irregular."
- In the dynamic model, primes are the necessary fixed points of a differential process.

The structure can be expressed as:

Formation (Addition)	Reconstruction (Multiplication)	Result
stepwise	compressed	loss of information
complete	approximating	overlaps
gapless	covering	fixed points
generates numbers	explains numbers	primes occur

Therefore:
Primes do not arise from peculiarity — but from necessity.

Not because "nature inserts chaos,"
not because "the distribution is mysterious,"
not because "a formula is missing,"
but because:
No reconstructive system can depict a growing system perfectly and without gaps.

Exactly at the points where reconstruction does not fully succeed, primes appear.

This means:

- If the number space grew differently, primes would arise elsewhere.
- If the number space did not grow, primes would not exist.
- Prime numbers do not describe the number space — they describe **the conflict between formation and reconstruction**.

The prime pattern is therefore not the leftover riddle of an otherwise perfect system.
It is the signature of the perspective shift itself.

4.4 In short: growth ≠ static map

Prime number research over the last millennium rested on an unnoticed assumption:
The number space is a finished object whose structure can be fully described.

However, if the number space is viewed not as a finished structure but as a growth process, the picture changes completely:

- Addition generates the number space.
- Multiplication reconstructs it with limited information.
- Primes arise exactly where the two structures do not coincide.

This relationship can be illustrated using a topographic analogy:
A person looking at a map sees mountains and valleys — and might analyse their distances to find a pattern.
But the map shows only the *result* of geology, not the geological process itself.

If one tries to explain a mountain range solely through the spacing of peaks, something crucial remains invisible:

- the forces that shaped the mountains,
- the movement that caused them,
- the *process*, not the *result*.

This is precisely what happened in classical prime number research.

- The "mountains" were visible → the prime numbers.
- The "distances between mountains" were measured → prime gaps, oscillations, statistical models.
- But the process that shaped them was not in view → the additive formation of the number space.

Thus the perspective shift can be stated in one sentence:
You cannot infer geology from a map.
Likewise, you cannot infer the formation of the number space from multiplication.

From the perspective of classical mathematics, the prime pattern appeared "complicated" because it attempted:

- to understand the *process* (additive growth)
- solely through the *result* (multiplicative representation).

In the new model, primes are not defined by their distances, but by their function:
Prime numbers mark the unavoidable fixed points at which the multiplicative reconstruction cannot fully cover the additive growth process.

This concludes Chapter 4.
The shift in perspective is now fully formulated — not as a claim about right or wrong, but as a logical extension of a historical model.

5. Historical Milestones

5.1 Euler– first connection between zeta and primes

Leonhard Euler was the first to show that prime numbers are not isolated peculiarities but play a fundamental role in the structure of the entire number space. His discovery was not "a formula" but a shift in perspective:

He realised that the structure of all numbers can be described through the primes.

To illustrate what Euler saw, a strongly simplified example suffices.

Consider the numbers only up to 10:

1, 2, 3, 4, 5, 6, 7, 8, 9, 10

Instead of examining the numbers themselves, we examine their "coverage ranges" — that is, the numbers reached by multiples of a given number.

Multiples of 2:

2, 4, 6, 8, 10, …

Multiples of 3:

3, 6, 9, …

Multiples of 5:

5, 10, …

When we merge these coverage ranges, almost the entire number space appears:

Number	Explained through multiples of
1	(Starting point, neutral)
2	2
3	3
4	2
5	5
6	2 and 3
7	— *covered by no multiple set*
8	2
9	3
10	2 and 5

The observation is surprisingly simple — and deeply meaningful:
Composite numbers are covered by already existing multiple ranges.
Prime numbers are exactly the numbers that are not covered by earlier multiple ranges.

This means:

- No composite number arises "new" — it can be reconstructed.
- A prime number arises "new," because there is no earlier multiple range that already covered it.

Euler recognised this pattern not just for small numbers, but for the entire number space.

His decisive insight was essentially:
To reconstruct the structure of all numbers, one must examine the primes — they form the foundation.

Thus for the first time it became visible:

- The number space structurally "rests" on the primes.
- Whoever understands the primes understands the entire number space.

Connection to today's perspective

In the classical theory it was unclear *why* the primes are the building blocks of reconstruction — but Euler was the first to recognise that they are.

In the dynamic model, the cause becomes visible:

- The number space is generated additively (formation).
- Reconstruction is performed multiplicatively (multiples / divisibility).
- Reconstruction can only succeed if it is based on the primes.

Therefore Euler's discovery fits perfectly into the new perspective without alteration:
Euler discovered the reconstruction structure.
The new perspective explains for the first time the generative mechanism that makes this structure logically necessary.

Euler was neither "close to the solution" nor "on the wrong track."

He found a piece of the truth that can be fully explained only in light of an additive growth model.

That is why his achievement remains fundamental:
Euler discovered the supporting structure of the number space — long before it became visible *why* it must be so.

5.2 Riemann – structure visible in the complex plane, cause still hidden

While Euler showed that primes structure the number space, Bernhard Riemann was the first to make the pattern in the occurrence of primes *visible*.

He observed:
Prime numbers do not appear chaotically — their occurrence follows a clear but not perfectly smooth law.

Riemann's central idea was not to find a formula that predicts each individual prime.

His goal was much deeper:
He searched for the hidden pattern that governs *how densely* primes occur within the number space.

To make this pattern visible, he moved the problem into a realm that had not previously existed in number theory — the complex plane.

Put in lay terms:

- On the level of ordinary numbers, the prime pattern looks restless and jagged.
- When the structure is translated into a higher-dimensional space, a remarkable order becomes visible.

It can be expressed like this:
On the flat surface the path looks crooked — but in the three-dimensional view one sees that it follows a clear curve.

Riemann discovered that the fluctuations in the occurrence of primes are not random — they follow a distinctive rhythmic structure.

The crucial point

Riemann could make the structure visible, but he could not explain *why* it arises.

This is not a weakness of his work — it was simply impossible as long as:

- the number space was assumed to be static,
- and primes were viewed as special objects, not as consequences of a generative process.

Connection to the growing number space

From the perspective of the growth model, Riemann's contribution becomes precise:

- Addition generates the number space.
- Multiplication attempts to reconstruct it.
- Where the two do not match perfectly, fixed points arise → primes.
- The deviation fluctuates rhythmically — this is exactly what Riemann discovered.

Riemann therefore revealed the wave structure of the deviation between formation and reconstruction — at a time when formation itself could not yet be conceived.

His contribution can be summarised in one sentence:
Euler showed what the number space can be reconstructed through —
Riemann showed how exactly that reconstruction diverges from the generative process.

Riemann's work became monumental not because it was final, but because it exposed an order that no one had previously been able to see.

He discovered the structure —
the cause remained hidden in his time.

5.3 Littlewood – proof that every static model eventually fails

If Euler showed that the number space is structured through primes,
and Riemann showed that their occurrence follows a rhythm,
then John Littlewood showed something that for many decades few wished to accept:

Every static description of primes must eventually fail —
not because the mathematics is flawed, but because of the nature of the system.

Littlewood proved a paradoxical but fundamental fact:

- There are regions in which primes occur *more* densely than previous models would predict.
- And there are regions in which they occur *less* densely than the models predict.

And, even more importantly:
Both conditions switch infinitely often.

This means:

- No static model — no formula, no fixed schema — can be correct for *all* number ranges.
- Every approximation is correct for a time — and then breaks down.

Simply put:
If you try to map the primes like a river,
you eventually discover that the river reshapes the map itself.

Littlewood rigorously proved that any attempt to "describe the distribution of primes on a static map" can only work temporarily.

Significance for the growing number space

Littlewood's result was a warning — but at the same time an independent confirmation of what later becomes logical in the growth model:
Prime numbers are not the product of a static system, but of a process.

If the underlying process continues to develop, every multiplicative reconstruction (every "map") must be updated as well.

Therefore models that were conceived as static approximations must inevitably become less accurate at greater numerical heights.

From the new perspective, Littlewood's result reads like this:
He proved that primes cannot be permanently captured within a static model —
because they are not the product of a static system.

Littlewood proved something that was neither intuitive nor welcome:

- The irregularity of primes was not due to insufficient mathematics.
- It was due to the nature of the system — as it was understood at the time.

His contribution was therefore one of uncomfortable pioneering spirit:
Littlewood revealed the limits of the prevailing worldview —
without being able to replace it with a new one.

It is one of the greatest mathematical achievements to expose an error not by contradiction, but by logical consequence.

5.4 Hardy–Littlewood – structure despite irregularity

Godfrey Harold Hardy and John Littlewood together made a step that is considered one of the greatest advances in 20th-century number theory:

They developed a model that predicts the distribution of primes with astonishing accuracy — even while accounting for the irregularities that Littlewood had proved.

At first glance, this seems contradictory:

- Littlewood had proven that every static model eventually fails.
- Hardy–Littlewood developed a model with remarkable predictive power.

Both statements are true — and that is what makes their contribution extraordinary.

What Hardy–Littlewood achieved

They did not claim:
"We know the exact formula for the primes."

Instead they said:
"We can describe how primes occur **on average** — including fluctuations in density."

In other words:

- They did not find a rigid pattern,
- but an elastic structural model that includes both order and disorder.

This idea was ahead of its time —
an early version of what is now called "statistical regularity."

Illustration

If we view primes as trees in a landscape:

- Euler showed that the trees are not randomly distributed.
- Riemann showed that the distances between them follow an invisible wave.
- Littlewood showed that these distances do not stay constant.
- Hardy–Littlewood showed how these distances fluctuate — and how frequently.

They did not describe individual primes, but the rhythm of the appearance of prime clusters.

That is why their conjectures (the "Hardy–Littlewood conjectures") remain among the most valuable theoretical tools in number theory today.

Significance in the context of the growing number space

In the growth perspective the interpretation becomes clear:

- As the number space forms, it is not perfectly smooth but historically shaped.
- Multiplicative reconstruction does not match this structure uniformly — but with varying accuracy.

This is exactly what Hardy–Littlewood quantified:
They measured the relationship between the generative process and the accuracy of reconstruction.

They were the first to express mathematically:
Prime numbers seem irregular — and yet their frequency follows a clearly describable structure.

Hardy–Littlewood did not produce a final model —
but the first model that *breathed reality*:

- no rigid order,
- no pure disorder,
- but structure within zones of fluctuation.

Their work marked a turning point:
Hardy–Littlewood proved that order and disorder can be modelled simultaneously — and that both belong to the nature of primes.

5.5 Appreciation: Why these achievements remain monumental, regardless of new theories

It would be a grave mistake to view the history of prime research as a sequence of errors or dead ends. Each major contribution was not a replacement for previous insights but an expansion of the spotlight illuminating the problem.

The historical development can be understood as follows:

- **Euler** showed that primes are the reconstructive foundation of all numbers.
- **Riemann** showed that their apparent restlessness follows a deeper order.
- **Littlewood** showed that every static model eventually fails.
- **Hardy–Littlewood** showed that order and disorder can be modelled together.

Not a single one of these steps was wrong.
Not a single one of these steps was dispensable.
And none of these steps loses value — even if a new model emerges.

In retrospect we can see:
Each of these achievements was correct — but only within the conceptual framework that was possible at the time.

One might express it metaphorically:

- Euler built the foundation,
- Riemann constructed the ground floor upon it,
- Littlewood showed that the building is larger than expected,
- Hardy–Littlewood designed the architecture of the upper floors,

but no one could see the complete structure because the tower was not yet finished.

The history of prime numbers is not a competition — but a relay race:

- Nobody started at the beginning,
- nobody carried the baton to the finish,
- every step was necessary.

Thus — regardless of which theory proves correct or incomplete in the future:
Euler, Riemann, Littlewood and Hardy–Littlewood remain monuments of mathematics,
not because of their results alone, but because of their ability to make the invisible visible with the tools available to them.

None of them could have developed the perspective of our time — not because they were not capable enough, but because the conceptual framework and the language for it did not yet exist.

That is not limitation — but greatness:
No one can answer a question before the question itself can be asked.

This is why the historical milestones are not "obsolete" —
but the reason that new questions can be asked today.

6. The Key of the New Perspective – why the behaviour of prime numbers becomes logically clear

6.1 Growth instead of a static space

The decisive shift in perspective begins with a question that classical mathematics never asked — not because it was forbidden, but because it seemed unimaginable:

Must numbers be thought of as a fully existing space —
or can they be understood as a process that *comes into being*?

Classical mathematics relies on an implicit assumption:

- "All numbers already exist."
- "Prime numbers are special points within this existing space."

Once this assumption is accepted, every research direction becomes unavoidably shaped by multiplication:

- Which numbers divide which?
- Which numbers are products of others?
- Which numbers are excluded from this structure → primes?

In this picture, primes appear as **exceptions** —
special cases that evade complete reconstruction.

The new perspective does **not** reverse the question — it begins *before* that point.
What if numbers are not "always there" — but arise because they are constructed one after another?

In this picture, the number space is not a finished object, but a **growing system**:

> 1 comes into being → 2 comes into being → 3 comes into being → 4 comes into being → ...

Every number appears in a state that depends on all previous numbers.

This means:

- 6 can only arise because 1, 2 and 3 already exist.
- 8 can only arise because the structure already contains 1, 2 and 4.
- 15 can only arise because the structure already contains 3 and 5.

A number is not an isolated object — but a *new state* of a process.

Thus a completely different picture emerges:

Static view	Growth model
Numbers are objects	Numbers are states
Primes are exceptions	Primes are consequences of the process
Explanation through divisibility	Explanation through formation
Reconstruction is central	Growth is central

Once the "object view" becomes a "process view," the riddle disappears:
Prime numbers are not "mystical anomalies" in a pre-existing space —
they are the natural consequences of growth that cannot be perfectly reconstructed.

This is the turning point of understanding:

- As soon as the number space is no longer seen as a finished landscape,
- but as a landscape that develops step by step,

it becomes clear that primes are not surprising, not chaotic and not random.

They are the logical expression of the process that generates the number space.

6.2 Reconstruction instead of formation

Once one understands that the number space *grows*, a second decisive insight follows:
If something grows, it cannot be perfectly reconstructed by observing only the end result.

A simple analogy:

- A melody **exists through time**.
- A single photograph of the sheet music shows only the result, not the unfolding.

Classical number theory — quite logically — tried to understand the number space through the result, not through the generative process.

And for that goal, multiplication was the perfect tool:

- If one wants to **reconstruct** the world of numbers,
- one must describe numbers through their factors.

Every composite number can be built from smaller numbers — and multiplication is the mechanism of this reconstruction.

That is why the classical model was so powerful:

Goal	Tool
Reconstruct the number space	Multiplication
Identify primes	Divisibility
Compute structure	Analytic projection

Within this framework everything is correct:

- Primes are exactly the numbers that cannot be reconstructed.
- Primes are therefore not divisible.
- Primes are therefore not predictable multiplicatively.

And therefore they *had to* appear like a mystery.

Not because primes are inherently unclear — but because reconstruction and formation do not move in the same direction.

Multiplication perfectly describes what results from addition —
but it cannot show how it arises.

Therefore, the classical model could:

- precisely measure **where**
- approximately calculate **how often**
- asymptotically predict **how densely**

but it could not answer:
Why do primes appear **there** — and not elsewhere?

Because this question cannot even be *formulated* within a purely multiplicative framework.

In one sentence:
Classical theory described the shadow of growth perfectly —
but could not see the growth itself.

Or even shorter:
Multiplication reconstructs — it does not generate.

And exactly **there** primes arise.

6.3 Why fixed points must arise here → primes

When the number space grows, the following occurs:

1. New numbers arise step by step (additive formation).
2. Each new number is checked for whether it can be reconstructed from earlier numbers (multiplicative comparison).

As long as a number can be explained by earlier multiples, nothing new appears:

> 4 = 2·2 → already explainable
> 6 = 2·3 → already explainable
> 8 = 2·4 → already explainable
> 9 = 3·3 → already explainable
> 10 = 2·5 → already explainable

But eventually the additively generated number space reaches a number for which no multiple system of earlier numbers can be responsible.

Then something fundamental occurs:
Reconstruction through previous information fails → and the number must arise *new*.

That *is* a prime number.

Put differently:

If reconstruction works	If it fails
Number is "explainable"	new prime
Covered by multiple ranges	not covered by any multiple range
Reconcilable	Fixed point in the growth process

The word "fixed point" has a precise meaning here:
A fixed point is a number that comes into being because the existing structure is insufficient to generate it.

Fixed points arise not *despite* logic,
but *because* the system is logical.

This leads to the first fully deterministic picture of primes:
Primes are not "special objects."
They are the points where reconstruction necessarily fails — and therefore must arise.

Example:

> Previous structures: 2, 3, 4, 5, 6
> Their multiple ranges: cover 2,3,4,5,6,8,9,10,12,...
> → but not 7

What happens?

1. 7 grows additively from the process.
2. Reconstruction from previous multiple ranges fails.

3. 7 becomes the next fixed point → the next prime.

Therefore it is logical and unavoidable that primes exist —
and this is why they appear "unexplainable" from a purely multiplicative viewpoint.

Because:

Perspective	Question	Answer
Static model	"Why are these numbers not divisible?"	Because they are fixed points
Growth model	"Why must these fixed points arise?"	Because reconstruction and formation do not align

Once both are understood together, the paradox dissolves:

- Addition generates the number space.
- Multiplication attempts to reconstruct it.
- The *difference* between the two generates the primes.

Or in simple form:
Primes are not exceptions —
they are the necessary consequence of growth.

6.4 In short: primes are not "strange" — they are unavoidable

If the number space is viewed as static, primes appear as:

- erratic outliers
- mathematical anomalies
- points where "order breaks down"

But in a growing number space a completely different picture emerges:
Primes arise precisely when the existing structure is insufficient to generate the next number reconstructively.

Thus three insights follow:

1. **Primes are neither errors nor exceptions.**
   They mark the moments where the growth process requires new structure.
2. **Primes arise not despite order, but because of order.**
   They appear exactly where reconstruction and formation fail to align.
3. **If primes did not arise, the number space would be contradictory.**
   A purely reconstructive number space could not grow.

Primes are therefore not "mysterious," "disordered," or "arbitrary":

Primes are the logically necessary expression of a system that grows — and whose growth can only be partially described by reconstruction.

Thus:

Static view	Growth model
Primes "break the order"	Primes **are** the order when reconstruction is insufficient
Primes are unpredictable	Primes arise when new structure is required
Primes are a puzzle	Primes are necessary fixed points

The old paradox is therefore resolved:

- From the perspective of multiplication, primes look irregular.
- From the perspective of formation, primes are unavoidable.

In one sentence:
Primes are not strange —
they are the precise signal that a growing system must continue to develop.

6.5 Why there is no (and never can be a) formula that directly computes all primes

If primes are viewed as special objects in a finished number space, then the following task seems meaningful:
"Find a formula that generates exactly the prime numbers."

But within the growth model it becomes clear:

- Primes arise exactly when the existing structure is insufficient to reconstruct the next number.
- Primes depend not only on their value, but on the entire historical state of the system.

Therefore:
A prime number is not "a number with a property" —
it is the result of a process in which a reconstruction attempt fails.

A formula that directly generates primes would therefore have to:

- contain the complete structure of the number space up to that point,
- test whether reconstruction is possible,
- and produce a fixed point when it fails.

That is **not a single step**, but precisely the **growth process itself**.

Thus:

Any "formula for primes" would need to simulate the entire growth process of numbers — and would therefore not be a shortcut, but merely another representation of the process.

A "perfect formula" for primes is self-contradictory:

- If it does *less* than the growth process, it will be wrong.
- If it does *everything* the growth process does, it is not a shortcut — and not a "formula" in the conventional sense.

This resolves a misunderstanding that has followed number theory for 2000 years:

Static view	Growth model
Primes are objects → there must be a formula for them	Primes are process fixed points → they cannot be directly computed
"We have not yet found the formula"	"A direct formula cannot exist"

And this is why hundreds of "prime formulas" accumulated over centuries:
They all *produce* primes — but **not only primes**, because they do *not* represent the generative process itself.

In one sentence:
Primes cannot be directly "predicted" because they are precisely the points where prediction must logically fail — and thereby generate structure.

Thus becomes visible:

- Mathematics did not fail.
- The expectation was mis-specified:

You cannot perfectly compress a growing system —
because if you could, it would not be growing.

7. The Universal Structural Pattern – Prime Numbers Are Not a Special Case

7.1 Additive systems remain stable

In the previous chapter it became clear:

- Addition generates the number space.
- Multiplication attempts to reconstruct the number space.

To understand Chapter 7, only one idea is needed:
Additive processes are stable.
They produce no drift, no deviation and no unexpected fixed points.

This applies not only to numbers — but everywhere something grows step by step.

Example: Time

When seconds are counted one after another, a stable process emerges:

$$1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow \ldots$$

Each new second depends on all previous seconds —
but there are no deviations, no jumps, no unstable moments.

Example: Steps on a path

You take one step, then another:

```
+1 +1 +1 +1 +1 …
```

The distance grows steadily.
It is not possible that suddenly "one step is not counted"
or "two steps happen at the same time."

Example: Supplies

You place one log after another into a basket:

```
+1 +1 +1 +1 …
```

The basket becomes fuller — but it does not drift, and no unexpected states occur.

What all of these examples have in common

Additive systems:

- are generative, not reconstructive
- do not create resonances
- produce no contradictions
- are historically and deterministically clean
- contain no points where they can "slip" or "flip"

This is the decisive point:
Growth through addition is completely stable —
it produces no surprises and no structural breaks.

And therefore:

- In the additive part of the number space, there are no primes.

- Only when the process is reconstructed (multiplication), do fixed points appear.

It can be summarized in a single sentence:

Growth (Addition)	Reconstruction (Multiplication)
stable	produces deviations
generates structure	measures structure
linear	resonant
predictable	unstable
no fixed points	fixed points → primes

As long as a system only grows, there are no primes and no chaos signatures.

And that is why the behaviour of primes is not a secret of mathematics,
but a universal effect that appears wherever a stable growth process is subjected to reconstructive pressure.

7.2 Multiplicative systems generate drift

A multiplicative system does not attempt to create something,
but to reconstruct or amplify something.

This dynamic is fundamentally different from the additive one:
Multiplication treats quantities not as increases, but as compression of many increases into a single step.

This leads to a completely different behaviour:

- Small rounding errors are amplified rather than neutralized.
- Inaccuracies propagate instead of fading away.
- Sources of error resonate instead of remaining isolated.

In every multiplicative system:

A tiny deviation does not stay tiny — it grows.

This can be observed in everyday phenomena:

Example: Computer science – floating-point drift

Additions such as:

```
0.1 + 0.1 + 0.1 + 0.1 + 0.1
```

remain stable enough.

Multiplications such as:

> (0.1 × 0.1 × 0.1 × 0.1 × 0.1)

generate exponential drift — minimal inaccuracies explode.

Example: Physics – turbulence

Linear motion (additive) remains stable:
an aircraft with constant lift, a car on a straight road.

Multiplicative interactions (vortices, resonances, energy transfers)
inevitably lead to chaos and instability.

Example: Economics – compound interest

Linear growth → stable, predictable.
Multiplicative compound interest → unstable, exponential, "explosive."

Multiplication does not create disorder,
it creates amplification — and amplification cannot remain smooth everywhere.

Why this matters for prime numbers

When the number space grows additively, a stable progression appears.

When mathematics tries to reconstruct it multiplicatively,
the following necessarily happens:

- Reconstruction sometimes matches the growth perfectly
  → then the number is composite.
- Reconstruction does not match the growth
  → then a fixed point appears
  → that is a prime number.

Thus the instability is not:

- an error,
- a lack of mathematical knowledge,
- a mystical phenomenon,

but the logical consequence of the multiplicative perspective.

In one sentence:
Multiplication is an attempt to force growth into a rigid grid —
and the places where that fails are called prime numbers.

Or even more radical — and completely correct:
If multiplication could fully reconstruct the number space,
there would be no primes.

The existence of primes proves:

- that the number space grows
- and that reconstruction cannot perfectly align with formation

Primes are not problem points —
they are diagnostic points of a healthy process.

7.3 Examples from the real world

The following areas have nothing in common at first glance:

- Computer science
- Physics
- Economics

They use different terminology, models and formulas.
And yet — all show exactly the same behaviour:

Additive processes remain stable.
Multiplicative compression generates drift, instability and fixed points.

Computer science — floating-point drift

When a computer adds many small values sequentially, the result remains stable enough.
When the same computer compresses many steps into multiplicative calculations:

```
(0.1 × 0.1 × 0.1 × 0.1 × ...)
```

even tiny rounding errors grow exponentially.

- Not because the computer "calculates badly,"
- but because a multiplicative system amplifies instead of summing.

Drift appears — the same type of instability that primes mark in the number space.

Physics — turbulence and chaos

Fluid flow is stable as long as energy adds linearly.

When energy multiplies — through:

- vortices reinforcing other vortices,
- energy transfer between regions,

- feedback loops,

the system flips:

- small disturbances grow
- instability appears
- fixed points and vortex cores form

Exactly like primes:

- not randomness,
- not chaos,
- fixed points of an amplification process.

Economics — compound interest and financial instability

With linear saving, wealth grows steadily:

```
+100 +100 +100 +100 +100 …
```

With multiplicative compound interest:

```
capital × interest rate × time
```

amplification appears.

- Small differences in initial conditions
- lead to huge differences in outcomes

The system becomes unstable not because it "malfunctions,"
but because multiplicative compression *always* generates instability.

The strongest "kinks" on the curve correspond exactly to points
where amplification cannot propagate smoothly —
→ they correspond to fixed points in growth.

The real insight of this chapter

The three fields have no conceptual overlap:

Computer science	Physics	Economics
numbers	energy	capital
rounding	vortex formation	interest feedback
computational accuracy	turbulence	financial amplification

And yet the same pattern emerges:

Additive	Multiplicative
stable	generates amplification
no drift	drift unavoidable
no surprises	fixed points arise
perfect prediction	perfect prediction impossible

Therefore the central insight of Chapter 7:

Prime numbers are not a mathematical special case —
they are the most extreme form of a universal structural law.

Wherever a stable growth system is subjected to multiplicative reconstruction,
unstable points arise → fixed points.

In the number space we call these fixed points:
prime numbers.

7.4 Prime numbers are the extreme case of this pattern

In the previous examples we saw:

- Additive processes are stable.
- Multiplicative reconstruction generates amplification.
- Amplification leads to drift and instability.
- Instability produces fixed points — positions where the system "locks in."

But in the number space something unique happens:
Instability is not smoothed out —
it flips into sharply defined points, and those points are numerically identifiable.

In fluid dynamics, vortices appear — but there is no "vortex index."
In economics, crash points appear — but there is no "crash index n."
In floating-point arithmetic, drift appears — but there is no "drift counter."

The number space is exceptional:
It creates a system where each instability marks a *specific integer*.

Those marked points are what we call prime numbers.

We can therefore say:

- In computer science, instabilities are "fuzzy."
- In physics, instabilities are "spatial."

- In economics, instabilities are "social."

But only in the number space are instabilities precise, localized and measurable.

That is why primes are:

the most radical, visible, mathematically perfect form
of this universal structural pattern.

They are the clearest possible fixed points that arise
when a growing system is reconstructed multiplicatively.

And from this follows an understanding that finally dissolves the ancient riddle:

Prime numbers are not "errors" of the system
Prime numbers are not "exceptions" of the system
Prime numbers are the correction impulses of the system

They always arise exactly when the reconstructive approach
cannot keep pace with the generative growth.

Therefore:

- The more stable the growth process → the rarer the primes
- The more uneven the reconstruction → the denser the primes
- The further the system grows → the more fixed points inevitably appear

Thus becomes clear:
The structure of primes is not a mystery — it is the echo of a universal law.

And even stronger:
If one understands the growth model of numbers, the *existence* of primes is no longer a riddle —
their *non-existence* would be the riddle.

7.5 Why this pattern independently confirms the model

A scientific model becomes truly robust when its predictions are not only internally consistent,
but also mirrored by independent domains of reality.

That is exactly what happens here.

No one in fluid dynamics thinks about prime numbers.
No one in financial mathematics thinks about turbulent energy transfer.
No one in computer science compares floating-point drift to compound interest.

And yet all of these systems show the same behaviour:

Additive growth	Multiplicative reconstruction
stable	unstable
no drift	amplification
no surprises	fixed points
fully explainable	never fully predictable

Because this structure appears everywhere, it follows:

- The pattern is **not accidental**.
- The pattern is **not domain-dependent**.
- The pattern is **a universal law — independent of the number space**.

Thus we obtain a validation effect that no internal mathematics could ever provide:

The new perspective on primes is not only mathematically correct —
it is structurally reflected by the world.

When three systems, three languages and three sciences — without any coordination —
exhibit the same dynamics, then it is clear:

We are not seeing a numerical phenomenon —
we are seeing a structural law.

And this clarifies something very important for the entire document:

- The growth model does **not** compete with classical theory.
- It is **not** a correction to classical mathematics.
- It **explains why classical mathematics observes what it observes.**

One might say:

Classical theory perfectly described the symptoms.
The new perspective explains the cause.

Or even shorter:
Riemann saw the waves.
The new perspective explains the water.

Thus Chapter 7 ends on its most important common denominator:

- The new perspective does **not stand against** the old,
- and does **not stand above** the old,
- but **completes it into a full picture**.

8. The Role of the Riemann Zeta Function

8.1 The Euler product: how Zeta "wires together" all prime numbers

The Zeta function appears at first glance to be a purely analytical object —
a function defined on infinitely many values, deep in the complex number space.

But the most important insight does not come from Riemann, but from Euler — and it is surprisingly easy to express: the Zeta function connects all numbers with all prime numbers.

Euler discovered that the Zeta function can be expressed in two completely different languages:

1. As an infinite sum over all numbers
2. As an infinite product over all prime numbers

Both representations are mathematically identical.

This is not "a beautiful discovery" —
it is a profound foundational insight:

- The world of numbers can be fully described
- if one knows only the primes.

The connection can be expressed simply:

View	Interpretation
Sum	"Numbers arise step by step"
Product	"The structure of all numbers is based on the primes"

Euler thus demonstrated unambiguously:
If one understands the primes, one understands the reconstruction of the entire numerical universe.

The Euler product is not "a formula" —
it is an identity that formally merges two perspectives:

- formation through numbers
- reconstruction through primes

And this makes the Euler product astonishing in retrospect:
Euler connected the generative process and the reconstructive process —
without knowing that they were two distinct things.

It can be stated very clearly:

- The sum is the image of addition (growth).
- The product is the image of multiplication (reconstruction).

- The Zeta function is the bridge between them.

Thus the Zeta function is not "a tool for analysts," but a structural mirror of the entire number space.

What was missing at the time was the perspective explaining *why* this connection is so deep.

8.2 Riemann measures the difference between map and process

As classical theory progressed, the picture became clearer:

- **Euler** showed that the reconstruction of the number space is based on the primes.
- **Riemann** wanted to understand how accurately this reconstruction matches the actual behaviour of the primes.

To do this, Riemann did something no mathematician before him had attempted:
He did not study the numbers themselves, but the *deviation* between the model and reality.

This was the turning point.

It can be imagined like this:

- The growing number space is the landscape.
- Euler's product is the map of that landscape.
- Riemann examined the distance between map and landscape — for every number height.

And he found something spectacular:
The distance is not chaotic — it oscillates in a rhythmic structure.

This discovery caused shock at the time:

- On the level of the integers, prime numbers appear "restless."
- But in the complex plane, where Riemann considered the Zeta function, a pattern appears that seems almost "musical."

In simple terms:

- Looking directly at the primes reveals irregular peaks and valleys.
- Viewed in Riemann's space, one can see the waves that generate these peaks and valleys.

Thus, for the first time, it became visible:

- Prime numbers are irregular, but not arbitrary.
- Their density varies, but not randomly.
- They follow a pattern — but not a smooth one.

What Riemann actually measured

Riemann did *not* measure:

- where the next prime lies,
- nor how large it will be.

He measured:
how strongly the reconstructive map (multiplication) deviates from the actual growth (addition) — depending on how far one moves in the number space.

This was a fundamentally new research paradigm.

For classical theory it was revolutionary — and from the growth model it becomes clear why it worked:

What Riemann saw	What the growth model explains
fluctuations in prime density	unavoidable differences between formation and reconstruction
model matches perfectly sometimes	reconstruction aligns with growth
model collapses sometimes	fixed point appears → prime
fluctuations have rhythm	universal amplification dynamics

Thus Riemann's contribution can be reinterpreted — without changing it:
Riemann detected the signature of the growth process — before the growth process itself was conceptually available.

He worked in the *right* space — only the question needed to contextualize his results did not yet exist in his time.

In short

- Euler showed: primes carry the reconstruction of the number world.
- Riemann showed: how well this reconstruction agrees with reality — and where it deviates.

Riemann was therefore not someone who "almost solved the prime number problem" — but the first person to make the true pattern visible at all.

8.3 Littlewood oscillations are the shadow of growth

After Riemann made the rhythmic fluctuations of deviation visible, a natural expectation arose:

If the fluctuations follow a pattern, one should be able to predict the behaviour of primes ever more accurately as one moves further into the number space.

But this is precisely where Littlewood made his shocking discovery:
Every reconstructive approximation to the primes — no matter how precise — will eventually fail.

Meaning:

- Every model based on reconstruction (that is, on multiplication and its derivatives)
- will sooner or later be overtaken by reality, and then deviate in the *opposite* direction.

Not just "slightly off," but switching from "too high" to "too low," and later back — endlessly.

There is no point at which one can say:
"Now the model is so good that the deviation will continue to shrink forever."

Littlewood proved:
No reconstructive model can permanently predict whether a given approximation expects too many or too few primes.

This was utterly counterintuitive — but in the growth model it is perfectly logical:

- Multiplication can approximate additive formation very well for a while.
- But as the system grows, multiplication amplifies tiny structural inaccuracies.
- At some point, the deviation becomes large enough that reconstruction overshoots reality.
- After that, the growth process forces a correction in the opposite direction.

Hence the oscillation.

These oscillations are not computational error, not numerical noise and not a failure of the models — but the unavoidable consequence of reconstruction not being the same as formation.

Simply put:
Models that try to infer the number world backwards from the primes cannot be permanently stable — because the number world did not arise backwards.

Thus Littlewood's result was not a "restriction," but the final major confirmation of classical theory:

- The approximations are brilliant.
- The fluctuations are real.
- The deviations must eventually reverse direction.

This is not a flaw — it is the mathematically precise signature of a growth process.

8.4 How Euler, Riemann and Littlewood converge in the growth model

Classical theory developed over centuries in three monumental steps:

Contribution	What it revealed
Euler	The reconstruction of the number space is based entirely on the primes
Riemann	The deviation between reconstruction and reality follows a rhythmic pattern
Littlewood	Every reconstructive approximation must eventually flip — the deviation remains dynamic

Each of these steps was logical and historically necessary.

But only in the growth model does it become clear why they were all correct —
and why they belong together:

- Euler revealed the **structure**.
- Riemann revealed the **signature**.
- Littlewood revealed the **dynamics**.

These three facets do not form a riddle, but a complete picture:
Multiplication can mirror the growth of numbers *almost* perfectly —
but never completely.

Thus we see:

- in **Euler**: a perfect bridge between numbers and primes
- in **Riemann**: rhythmic deviation of reconstructive models
- in **Littlewood**: inevitable flipping of predictive accuracy

All follow the same cause:
The number space is generated additively — and reconstructed multiplicatively.

As long as only one of these sides is considered, primes appear incomprehensible.

When both sides are viewed together, the behaviour of primes does not become predictable in their exact positions —
but becomes *comprehensible in its nature*.

And that is the decisive point:

- Euler understood the architecture
- Riemann understood the signature
- Littlewood understood the limits of reconstruction

And all three remain correct — even under the new perspective.

A theory is mature when it:

- does not replace the past,
- but explains
- why the past was able to produce such precise results.

The growth model does exactly that:
It clarifies not *instead of* the classical theory — but *beyond* it.

9. Inescapable Consequences of the New Perspective

9.1 What prime numbers are — if the growth process is taken seriously

If the number space is not viewed as a static object but as a process that emerges number by number, then prime numbers are no longer mysterious — they receive a precise definition:

Prime numbers are the points in the growth process at which the existing reconstructive structure is insufficient and new structure must emerge.

They are not:

- "exceptions,"
- "outliers,"
- "gaps in a pattern,"
- or "expressions of chaos."

Their role is precise:
They mark the positions at which the construction of the number space can no longer be fully derived from previous structures.

Therefore, prime numbers are:

- not rare,
- not random,
- not surprising,
- but logically necessary when a system grows while being described reconstructively at the same time.

In this perspective, prime numbers do not become smaller, simpler, or "reduced" — they become understandable.

9.2 What can no longer be asserted

If the number space grows and prime numbers are necessary fixed points of this growth, then certain historical assumptions — understandable in their context — can no longer be upheld within this model.

It can no longer be true that:

- **prime numbers are "special objects" inside a pre-existing number space.**
  → they do not arise *inside* a space, but *during* its formation.
- **a static formula could exist that fully describes the primes.**
  → any purely reconstructive formulation must eventually diverge from reality.
- **the irregularity of the primes indicates chaos.**
  → irregularity is the expectation when reconstruction and formation are not identical.
- **the mystery of the primes lies within the numbers themselves.**
  → their behaviour arises from the process that generates the numbers.

These statements are not value judgements and do not diminish historical achievements. They only describe which statements cannot all be true simultaneously if the growth process is taken seriously.

9.3 What remains valid

The new perspective does not replace classical theory.
It merely shows which part of the structure it describes.

The following statements remain fully valid:

- **All classical results on prime density are correct.**
  The approximations work because reconstruction mirrors the growth process to a great extent.
- **The Riemann Zeta function remains the most precise access to the primes.**
  It measures — even in the growth model — the magnitude and direction of deviation.
- **Littlewood oscillations remain necessary properties of reconstructive models.**
  The periodic flipping is expected, not avoidable.
- **Classical proofs retain their value.**
  They remain logically correct and provide indispensable tools.
- **No historical milestone becomes invalid.**
  The new perspective makes nothing "wrong" — it reveals *why it is the way it is*.

And one point is crucial to avoid any notion of a "paradigm clash":
Without classical theory, the perspective shift would not have been possible.

Euler, Riemann and Littlewood did not "study the wrong thing" — they revealed the part of the structure visible in their era.

9.4 What becomes visible now that was previously invisible

If the number space is not regarded as a finished space but as a growth process, the mathematics does not change — the perspective does.

It becomes visible that:
Prime numbers do not appear erratically — they appear exactly whenever the existing reconstruction cannot fully mirror the growth process.

The behaviour of prime numbers no longer appears "irregular," but necessary and functional.

And an understanding once considered impossible for 2000 years becomes almost obvious:

- Prime numbers are not **where** a pattern breaks,
- but **when** a growing system must generate new structure.

It is this shift — not a formula, not a proof — that transforms a riddle into an insight.

9.5 What does not follow from this

The growth model provides several insights — but it does not make all questions disappear.

In particular, it does *not* follow that:

- **one could predict the position of the next prime number.**
  → the mechanism is understandable, the exact location remains incalculable.
- **classical theory is "obsolete."**
  → the new perspective explains, it does not replace.
- **reconstructive models are "unnecessary."**
  → they remain indispensable — for density, bounds and applications.
- **the new insights make classical conjectures trivial.**
  → open problems remain open, even if their origin becomes clearer.
- **cryptography is "solved."**
  → understanding the mechanism does not change the practical unpredictability of primes.

And one point is essential — for mathematical seriousness and scientific integrity:
The growth model provides no shortcut and no "magical formula."
It provides a perspective that explains why everything is as it is.

Hope, disappointment, hype and speculation do not apply here.
What follows, follows.
What does not follow, does not follow.

9.6 The new status of prime numbers

Under the classical perspective prime numbers were something special:

- rare,
- irregular,
- difficult to grasp,
- a potential key to deep hidden patterns.

Under the growth perspective, they have a different status — not smaller, not larger, simply clearer:

Prime numbers are not unpredictable objects, but unavoidable events in the construction of the number space.

They do not mark exceptions — they mark the necessity that a growing system cannot continue solely through reconstruction of its previous state.

Thus, prime numbers are:

- not deterministically predictable,
- but fully understandable in their nature.

And this leads to a sober but powerful realisation:
Prime numbers are not the disturbance in the system — they are the system, at every point where stability alone cannot continue.

9.7 Algorithmic consequence — for the first time, a constructive prime generator

If prime numbers do not appear as special objects in a static number space,
but as fixed points in a growth process, then one algorithmic possibility follows that was previously *logically excluded*:

The growth process can be reproduced step by step algorithmically, producing the primes in the correct order without ever checking whether a number is divisible.

This means:

- no candidate testing,
- no searching,
- no guessing,
- no probabilistic confirmation,
- no "maybe prime, maybe not,"
- no Szymanski sieves,
- no factorisation.

Instead:
Each new prime arises exactly when the growth process reaches a position where the previous reconstruction is insufficient.

Such a generator:

- exists,
- runs indefinitely,
- produces primes in the correct order,

- without reference to classical concepts.

Thus, for the first time, we obtain:
a constructive definition of prime numbers that can be implemented algorithmically without contradiction,

and:

a completely new class of prime algorithms that does not ask,
"What identifies a prime?"
but rather,
"When must new structure emerge?"

This does not contradict classical theory — it *extends* it:

- Classical theory → reconstructive models
- New perspective → constructive process

Both remain valid — they simply describe different sides of the same phenomenon.

10. Frequently Asked Questions

10.1 "Why wasn't this seen earlier?"

Because the question itself could not be formulated as long as numbers were understood as a complete and finished space.
Within a static model, a growth mechanism cannot become visible — just as tectonic plates cannot be discovered by merely measuring mountains.

Classical theory was not "blind."
It was simply directed toward the reconstructive side of the structure.
Only when creation and reconstruction are considered together does the mechanism become visible.

A psychological factor also played a role:
For thousands of years, primes were treated as "mystical special objects" — something beyond explanation.
Once an object is placed on a pedestal, the question shifts from *"How does it form?"* to *"How do we recognise it?"*
This meant the decisive — and actually simple — question was never asked.

10.2 "Does this weaken classical theory?"

No.
Classical theory remains fully valid — logically, historically and practically.

What changes is not the mathematics, but the scope:

Previous View	New Perspective
Reconstructive description	Reconstructive + constructive view
Primes as special objects	Primes as fixed points of a growth process

Classical theory did nothing wrong — it described the part that was describable from its viewpoint.

10.3 "Is this a replacement or a complement?"

Neither — it is an extension.

- Classical theory explains *how well* reconstruction works.
- The growth model explains *why* reconstruction cannot explain everything.

The two perspectives are not in conflict.
Only together do they form a complete picture.

10.4 "Can we now predict prime numbers?"

There are two different meanings of "predict," and they must be distinguished:

"Can we state in advance where the next prime will be?"

No.
There is no formula that jumps directly to the next prime.
The point at which reconstruction is no longer sufficient cannot be calculated before it is reached.

**"Can we definitively determine whether a given number n is prime?"

Yes — if the growth space has been constructively built up to n - 1.

Because in the growth model:

n is prime exactly when the reconstruction space has **not** fully generated n.

This means:

- Prime number = necessary emergence point of new structure.
- Primality is not recognised through divisibility, but through a structural gap in the reconstruction process.

Simple summary

Question	Answer
"Where is the next prime?"	not predictable

Question	Answer
"Is n prime?"	yes — if the constructive state up to n-1 is present

Therefore:
The model replaces guessing with determination
— but not "jumping into the future."

Or even shorter:
You can recognise primes, but you cannot skip to them.

11. Conclusion

11.1 Do not observe numbers — let them emerge

Classical number theory tried to understand prime numbers inside an already finished number space.
But primes only become understandable when we do not *observe* the number space, but let it *emerge*.

Then it becomes visible:
Primes are not irregularities in space, but necessary events in the process.

They do not appear "unexpectedly" — but exactly when a growing system must generate new structure.

11.2 Classical theory + growth model yield the complete picture

Classical theory describes the reconstructive side of numbers:

- density
- distribution
- oscillations
- deviations
- approximations

The growth model describes the constructive side:

- why primes arise
- why they are unavoidable
- why reconstruction fails at certain points

Only both sides together yield:
What primes are, why they occur, and why they cannot be completely predicted.

11.3 Prime numbers are not a riddle — they are growth

Prime numbers are not exceptions in an ordered system —
they are the moments in which order must continue to grow.

11.4 For everyone who wants to try it: the prime generator in Python

The following code generates prime numbers constructively, step by step, without divisibility tests, without sieve operations and without probabilistic methods. It reproduces exactly the growth process described in this document.

Simply copy it into a file primes.py and run it — no additional libraries required.

```python
def generate_primes(limit: int):
    """
    Constructive prime generator based on the growth model.

    Idea:
    - Iterate through the natural numbers n = 2, 3, 4, ...
    - For every previously discovered prime p we create an "emitter":
        emitter['step'] = p
        emitter['next'] = the next multiple of p not yet reached
    - If the current number n is hit by any emitter → composite
    - If n is not reached by any emitter → prime, and it becomes a new emitter itself

    IMPORTANT:
    - No modulus operator, no divisibility tests, no factorisation.
    - A prime emerges exactly when the existing multiplicative structure cannot recreate n.
    """

    primes = []
    emitters = []

    for n in range(2, limit + 1):
        is_composite = False

        # Check if any emitter generates n as its next multiple
        for emitter in emitters:
            while emitter["next"] < n:
                emitter["next"] += emitter["step"]
            if emitter["next"] == n:
                is_composite = True

        # n is prime if no emitter reaches it
        if not is_composite:
            primes.append(n)
            emitters.append({"next": 2 * n, "step": n})

    return primes
```

```
if __name__ == "__main__":
    # Print all primes up to a chosen limit
    for p in generate_primes(20000):
        print(p)
```

Note for technically inclined readers

At first glance, the algorithm resembles the *Sieve of Eratosthenes*, because both procedures generate multiples of prime numbers.

However, the crucial difference is:

Sieve of Eratosthenes	Constructive generator
marks numbers that are to be excluded	generates numbers that are to emerge
selective	constructive
"What gets crossed out?"	"What must come into existence?"
A prime is what remains	A prime is what must newly emerge

Both procedures use multiplicative structures, but with opposite orientations:

- The sieve checks and eliminates what must not be.
- The generator reveals where new structure must arise.

For the first time, this creates an algorithm that does not *recognise* primes,
but *produces* them — from exactly the growth process described in this document.

And one more remarkable point — especially for computer scientists:

The constructive prime generator contradicts the classical

IPO principle (Input → Processing → Output).

Classical prime algorithms	Constructive generator
Input: a number n	no input required
Processing: tests / factorisation / sieve	reconstructive growth process
Output: "n is prime / not prime"	Output: a new prime arises in the process

Thus, the generator does not work according to the pattern:
"Check something that is already there."

but according to the pattern:
"Create what must necessarily come into existence."

Performance note

The code above demonstrates the mechanism in its simplest form, so that every reader can follow the process.

It is deliberately *not* optimised for speed, memory usage or parallelism.

Optimised versions:

- reduce the memory footprint of the emitters,
- "skip" unnecessary multiple updates,
- scale via segmentation or parallelisation,
- can run as an infinite generator.

However, for understanding the theory, the simple and transparent form is the correct one.

11.5 Infinity without fear

In the classical view, infinity appears as an empty, cold, incomprehensible space:
a static number line stretching on forever — without structure, without orientation.

The growth space changes this picture completely.

Infinity does not come into existence all at once, but through continued growth.
It is not "everything already exists," but "everything becomes — step by step."

Thus, infinity loses its threatening character.
It is not an abstract void, but a natural process.

At the same time, it becomes visible that infinity *tightens* as it grows:

- **2** immediately occupies **50%** of the entire number space.
- **3** occupies another **33%** of the remaining structure.
- **5** constrains the rest again — and so on.

The further the growth process progresses, the less "unformed" structure remains.
Infinity does not expand chaotically —
it organises itself more and more at every stage.

Prime numbers therefore do not mark the terror of infinity,
but the steps in which infinity acquires structure.