The Irreducible Structure of the Prime Distribution

A Constructive Model of Fixpoints, Global Causality, and the Structural Foundations of the Riemann Hypothesis and Goldbach's Conjecture

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Abstract

This paper develops a constructive model of the number space in which prime numbers do not appear as properties of already-given integers, but as fixed points of a globally and irreducibly determined growth process. In contrast to classical arithmetic, which formulates primality as a local property of a number, the proposed model describes the emergence of numbers as dependent on the entire preceding multiplicative structure. The condition $n \notin D_{n-1}$ thus ceases to be a retrospective test and instead becomes a generative rule of formation.

From this viewpoint, central phenomena of analytic number theory arise not as statistical properties but as consequences of structural global causality: fixed points occur precisely where the existing structure permits no further generability. This yields natural interpretations of, among others, the sign changes of $\pi(x) - \mathrm{li}(x)$ (Littlewood), the Goldbach coupling of two fixed points, and the critical strip of the Riemann zeta function as an expression of a balanced projection effect. The model does not state these relationships as proofs in the classical sense, but as structural necessities within the constructive number space.

Finally, it is shown that every algorithm for prime generation that correctly internalizes the generative condition must satisfy the same global dependency on information. Such an algorithm is formulated in the form of a resonance-guided fixed-point procedure and visibly confirms the irreducibility of the prime distribution: primes cannot be predicted locally, but can only be detected in the unfolding of structure itself.

This paper is intended as a contribution to a foundational reconstruction of arithmetic: away from static property attribution, toward a generative perspective in which primes appear as emergent fixed points of a global structure.

In this sense, classical number theory studies the topological map of primes; the constructive model developed here studies the geological structure that makes this map inevitable.

Keywords

- generative number theory
- prime distribution
- irreducibility of primes
- constructive number space
- global vs. local arithmetic
- Riemann hypothesis (structural interpretation)
- Goldbach coupling
- Littlewood oscillations
- emergent fixed points
- non-predictive algorithms for primes
- resonance-based sieve
- dynamical arithmetic
- algorithmic number theory
- computational prime generation
- complexity of multiplicative structures

1. Introduction

1.1 The classical prime number problem and its limitations

Prime numbers constitute the fundamental building blocks of arithmetic. Their distribution has been the focus of intense investigation since the early work of Gauss, Dirichlet, Riemann, and Hadamard. Despite substantial progress, no closed expression exists that exactly determines the positions of the primes or predicts future primes deterministically.

Classical approaches can be roughly divided into two categories:

1. **Analytic approximations:**
   This includes, most notably, the prime number theorems of Hadamard and de la Vallée-Poussin, which describe the asymptotic behaviour of the prime counting function $\pi(x) \sim \frac{x}{\log x}$.
   These results, however, provide only global approximations and no exact information about the occurrence of individual primes.
2. **Structural conjectures:**
   Among these are the Riemann Hypothesis, the Hardy–Littlewood conjectures, and empirically motivated hypotheses concerning the local density of primes. None of these statements explains *why* primes appear at specific positions or how their distribution arises from an underlying structure.

The crucial point is the following:
The classical number system treats primality exclusively as a property of fully formed numbers.

From this perspective, several fundamental limitations arise:

- There is no description that models number formation as a process.
- Every statement about primes refers to objects that are already completely defined.
- The concept of "prediction" is restricted to analytic approximation rather than structural determination.

As a consequence of these limitations, even comprehensive theories are unable to *explain*

- the asynchronous, irregular nature of prime positions,
- the non-periodic fluctuations of $\pi(x)$,
- or complex phenomena such as the Littlewood oscillations.

In summary:
The classical prime number problem is well-defined within the existing system, but its solution requires structural notions that the system itself does not provide.

This observation forms the starting point of the present work.

1.2 Why classical arithmetic is structurally inadequate

The classical foundations of number theory are based on Peano Arithmetic (PA) and the Zermelo–Fraenkel axioms with Choice (ZFC). Both systems are inherently static: natural numbers are treated as fully defined objects, and their arithmetic properties are formulated solely with respect to these completed objects.

This perspective leads to several structural limitations that are crucial for understanding the emergence of prime numbers.

(1) Absence of a concept of formation

Neither PA nor ZFC contains an operator or an axiomatically grounded mechanism that describes how the number space *grows* or what global structures arise during this growth.

In these systems, each number exists "from the outset" as an autonomous and immutable object.

Consequently, they lack:

- a notion of process,
- a notion of global dependency,
- and any possibility of tracing the distribution of primes back to an internal generative logic.

(2) Lack of global structure

Classical systems model divisibility as a relation ($a \mid b$) between completed numbers. What they do *not* model is the global divisor structure that emerges when numbers are generated successively and multiplication paths permanently imprint earlier elements.

Thus, PA/ZFC are unable to formalize statements about:

- structural dependencies between different regions of the number space,
- the cumulative impact of earlier multiplications,
- or fixed-point–like generative mechanisms.

(3) Reductive rather than generative definition of primes

In PA/ZFC, a prime is defined as:

$$p > 1 \ \wedge \ \forall a, b \in \mathbb{N} : \ (p = ab \Rightarrow a = 1 \vee b = 1).$$

This is purely a negative property (indivisibility) and not a constructive description explaining *why* a number is indivisible or how this property arises from the global structure of the number space.

The prime thus remains an isolated attribute of an isolated object.

(4) Inability to express global dependency

The question of whether the appearance of a prime at a given position depends on the structure of earlier multiplication paths cannot even be *formulated* within the classical system.

Reason: the classical framework does not possess operators or relations that express global, dynamic, or structural dependencies.

All possible statements are necessarily local:

- properties of a single object,
- relations between finite pairs of objects,
- statements about sets of already completed numbers,

but never about:

- growth processes,
- global chains of influence,
- or emergent structural points

(5) Consequence: systemic insufficiency

From these limitations it follows logically that classical systems:

- can define primes,
- can study their asymptotic behaviour,
- and can formulate analytic approximations,

but lack the language to explain the emergence of primes from an underlying, dynamic global structure.

In this precise sense, classical arithmetic is structurally inadequate — not contradictory, but *expressively weak*.

This inadequacy motivates the transition to a generative perspective on arithmetic, which is introduced in Section 1.3.

1.3 Motivation: From a static to a generative view of number

The structural limitations outlined in Section 1.2 indicate that the classical framework cannot capture essential aspects of the distribution of prime numbers. In particular, there is no model that accounts for the dependency of arithmetic structure on the successive generation of the number space.

This observation suggests the following shift in perspective:

Arithmetic should not be viewed as a finished, static object, but as a constructive process in which new numbers emerge successively and in doing so participate in shaping and modifying a global structure.

Such a viewpoint is well established in various areas of modern mathematics — for example in recursive constructions, dynamical systems, generative models, and formal semantics.

For arithmetic itself, however, this approach has not yet been systematically developed.

The central idea is:

(1) Numbers do not arise in isolation but within a global structure.

Every newly formed number influences:

- the set of available divisors,
- the possible multiplication paths,
- and the future composition of the number space.

Arithmetic is therefore an accumulative process, not a static object.

(2) Primes appear as structural fixed points of the process

In a generative model, a number n arises as a product of earlier numbers precisely when suitable factors already exist.
If this is not the case, n emerges as a new irreducible point of the structure.

This leads to a constructive definition of a prime:

A prime is a number that *cannot* be generated by the existing global divisor structure within the generative process of arithmetic.

This definition is structural and generative, in contrast to the purely negative definition of the classical system.

(3) Global dependency replaces local reduction

In a generative number space, the emergence of a prime depends on:

- the already established multiplication paths,
- the global divisor structure,
- and the successive formation of the natural numbers.

These dependencies cannot be expressed within the classical system, but they arise directly in the generative model.

This provides the motivation for the present paper:
To understand the distribution of prime numbers structurally, arithmetic must be modeled as a dynamic, globally dependent process.

1.4 Overview of the constructive number space

The constructive number space is a model in which the natural numbers are not viewed as a fully given set, but as the outcome of a successive generative process. This model is not an extension of the classical number system; rather, it is an alternative internal organization that allows structural dependencies to be expressed which cannot even be formulated in the classical framework.

The constructive number space consists of the following core elements:

(1) Successive generation of natural numbers

The number space begins with

$\mathbb{N}1 = \{1\}$

and is expanded step by step by

$\mathbb{N}n + 1 = \mathbb{N}_n \cup \{n + 1\}.$

The key idea is that with each expansion:

- the set of potential divisors
- and the set of possible multiplication paths

grow and accumulate into an increasingly structured whole.

This distinguishes the model from the classical view, in which all numbers exist simultaneously and structural relations are independent of their “formation”.

(2) Global divisor structure

At each stage n, there exists a set of multiplication paths formed by the already available numbers.

These paths constitute a global structure:

$D_n = \{ab \mid a, b \in \mathbb{N}_n\}$

This structure has three essential characteristics:

1. It is cumulative: once a multiplication path has been formed, it remains relevant for all subsequent steps.
2. It is global: the generability of a number depends not on local properties but on the entirety of previously formed products.
3. It is deterministic: for every n, the structure is uniquely defined.

(3) Generative generability

A number k is called generatively generable if

$k \in D_{k-1}.$

That is,
k can be produced by multiplying two numbers that already exist.

A number not satisfying this condition does not arise as a product within the existing structure.

(4) Primes as fixed points of the process

From the definition of generative generability follows a constructive definition of a prime:

$$p \text{ is prime} \iff p \notin D_{p-1}.$$

Thus, in the constructive model, a prime is a number that cannot be produced by the global multiplication structure available at that point in the process.

This definition is:

- objective,
- constructive,
- compatible with the classical definition (since non-generability corresponds to indivisibility),
- yet structurally sharper because it roots the cause of indivisibility within the process itself.

(5) Dependence of future values on global structure

Since the generability of a number depends on the entire previous divisor space, the status of $n+1$ is:

- not locally determined,
- not reducible to a property of $n+1$,
- but dependent on the *global structure* of all earlier multiplication paths.

This global dependency cannot be expressed in the classical system, but it is a foundational property of the constructive model.

(6) Consequent irreducibility

The constructive number space thereby provides a natural explanation for the irregularity of the primes:

- The emergence of a prime is a fixed point of the global generative process.
- This process is cumulative and non-linear.
- Primes are therefore structurally irreducible and not locally predictable.

These properties form the basis for the irreducibility theorem stated in Chapter 3.

1.5 Objective: Demonstrating structural irreducibility

The objective of this article is to formally demonstrate that the distribution of prime numbers in the constructive number space is *structurally irreducible*. Here, *irreducibility* does not merely refer to statistical unpredictability, but to the strict non-reducibility of prime occurrence to any locally definable property of individual numbers.

The paper pursues three closely connected goals:

(1) Structural characterization of the emergence of primes

In the classical framework, the question

$$\text{“Is } n \text{ prime?”}$$

is expressed through the condition $n \notin D_{n-1}$, but treated as a property of an already existing number.

Within the constructive model, the same logical condition assumes an entirely different epistemic role:
$n \notin D_{n-1}$ is not a retrospective test but the generative condition under which a number arises in the growth process of the number space.

This shifts the semantic status of the statement:

- Classical: “n is prime because no smaller factors exist.”
- Constructive: “n becomes a prime exactly when it has not been generated within the

entire preceding multiplicative structure."

The condition is global, not local:
Whether a number becomes prime depends entirely on the cumulative structure of prior multiplicative generability.

This yields a mathematically precise foundation for the statement:
Primes are not numbers *with* a property, but fixed points of a globally and irreducibly determined growth process.

(2) Proof of non-reducibility

I subsequently show that in the constructive model:

- the generability of a new element
- is not determined by any local property of that element,
- but solely by the global structure of the number space up to that point.

More precisely:
There exists no local property $P(n)$ such that for all n, $P(n)$ is equivalent to "n is prime" while simultaneously being independent of the global divisor structure.

This statement constitutes the core of irreducibility.

(3) Consequences for classical questions

Finally, it is shown that structural irreducibility:

- explains the inherent unpredictability of the primes,
- clarifies the limits of classical goals (e.g., the search for a "formula" for primes or a deterministic prediction function),

without requiring the solution of those problems.

Irreducibility is therefore not justified on epistemic grounds (lack of knowledge, lack of techniques), but as a structural property of the number system itself.

Summary of objectives

This paper establishes:

1. a new constructive definition of prime numbers,
2. a rigorous derivation of their global dependency,
3. a formal irreducibility proof,
4. a structural reinterpretation of classical problems,
5. and an explanation for why the distribution of primes cannot be reduced to a closed formula

This provides the foundation on which later chapters (in particular those concerning Goldbach, Riemann, and global structure) can build, without relying on speculative concepts.

2. The constructive number space in detail

2.1 Formal foundational construction

The constructive number space is defined as a growing sequence of finite sets

$(\mathbb{N}n)n \in \mathbb{N}$,

where each set $\mathbb{N}_n$ contains the first n natural numbers:

$\mathbb{N}_n = \{1, 2, \ldots, n\}$.

The decisive difference from the classical number space does not lie in the elements themselves, but in the manner in which the structure of multiplication emerges step by step. Each stage n generates a set of multiplication products formed exclusively from numbers that already exist.

(1) Multiplication structure of a stage

For each stage n, define:

$D_n = \{ab \mid a, b \in \mathbb{N}_n\}$.

This set contains all products that are *generable up to this point* using the numbers available at stage n.

It has the following properties:

1. **Monotonicity:**
   $D_n \subseteq D_{n+1}$.
2. **Finiteness:**
   Each set D_n is finite because $\mathbb{N}_n$ is finite.
3. **Determinism:**
   For every n, the set D_n is uniquely determined.
4. **Global dependency:**
   Whether a product k lies in D_n depends on the full structure of $\mathbb{N}_n$, not on intrinsic properties of k.

(2) Generative generability

A natural number $k \geq 2$ is called *generatively generable* if it can already be produced by the multiplicative structure of the preceding stage:

$k \in D_{k-1}$.

Intuitively, this means:

- k can be formed by multiplying two numbers,
- both smaller than k,
- that already exist at stage $k-1$.

This definition does not assume divisibility; rather, it reconstructs structure from the ground up through the dynamics of growth.

(3) Fixed points of generativity

A number k is called a fixed point of the growth process if

$k \notin D_{k-1}$.

Fixed points cannot be generated by the existing structure — their emergence is a structural leap that cannot be explained by local properties or lower multiplication paths.

These fixed points form the basis for the constructive definition of prime numbers in the following chapter.

(4) Relation to the classical structure

Remarkably, the constructive number space is fully compatible with classical arithmetic:

- If k lies in D_{k-1}, then it has a proper divisor.
- If k does not lie in D_{k-1}, then it is prime.

The difference is not in the classification but in the explanation of origin:

- Classical system: a prime is defined locally (“has no divisors except 1 and itself”).
- Constructive system: a prime arises globally (“is not produced by the generative structure of earlier multiplications”).

This shift is crucial for the irreducibility analysis developed in later chapters.

2.2 Multiplicative generability

The notion of *multiplicative generability* denotes the central property of the constructive number space:
a number does not arise from a timeless, fully present structure, but from the accumulated multiplication paths of preceding stages.

I now refine this concept and derive its fundamental properties.

(1) Definition of generability

For $k \geq 2$ I define:

$$k \text{ is generable } \iff k \in D_{k-1},$$

where

$$D_n = \{ab \mid a, b \in \mathbb{N}_n\}.$$

Thus, a number is generable if it can be produced by multiplying two natural numbers that are *strictly smaller* than itself.

(2) Structure of D_n

The set D_n has two essential properties:

(a) Upper closure

$$\max(D_n) = n^2.$$

Therefore, the multiplicative structure of each stage is highly unevenly distributed:
many small products accumulate in low ranges, whereas higher products occur increasingly sparsely.

(b) Redundancy of paths

If $k \in D_n$, then there often exist many pairs (a, b) such that

$$ab = k, \qquad a \leq b \leq n.$$

This redundancy becomes relevant later in Chapter 6 (Goldbach), particularly in regard to the additive density of generable numbers.

(3) Characterization via factorization

From the definition, the following is immediate:

Lemma 2.2.1.

For $k \geq 2$:

$$k \text{ is generable } \iff \exists, (a, b) \in \mathbb{N}_{k-1}^2 : ab = k.$$

Hence, generability is equivalent to the existence of a proper factorization of k.

This means:

- generability indicates that a number is composite,
- non-generability indicates that it is prime.

The definition is thus fully compatible with classical arithmetic, but conceptually different (global rather than local).

(4) Examples

Example 1:
$k = 6$
$D_5 = \{1 \cdot 1, 1 \cdot 2, \ldots, 5 \cdot 5\}$
contains $6 = 2 \cdot 3$.

Therefore:
6 is generable.

Example 2:
$k = 11$
All products from $\{1, \dots, 10\}$ form
$D_{10} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 18, 20, \dots, 100\}.$
Since no pair $a, b < 11$ satisfies $ab = 11$:
$11 \notin D_{10}.$
Therefore:
$11 \text{ is not generable (fixed point).}$

Example 3:
$k = 25$
$25 = 5 \cdot 5, \qquad 5 < 25.$
Since $5 \in \mathbb{N}_{24}$:
*$25 \in D*{24}.$

(5) Structural difference from classical systems

The key difference:

Classical:
A number is composite if it possesses a factorization.

Constructive:
A number is generable if it possesses a factorization using numbers that *already exist at that moment in the process*.

Because the structure in the constructive model grows step by step, the question of generability is:

- global,
- cumulative,
- temporally asymmetric,
- and dependent on the totality of prior multiplication paths.

This global condition is formalized in the next section (2.3).

2.3 Global divisor structure

The global divisor structure is the foundation of the constructive number space.
It describes the totality of all multiplication paths that can be formed from the numbers existing up to stage n.
Crucially, this structure is cumulative, deterministic, and globally effective.

(1) Definition

For each stage $n \in \mathbb{N}$, define:

$$D_n = \{ab \mid a, b \in \mathbb{N}_n\}.$$

This set encodes all products that can be generated from the numbers that already exist.
It represents the "multiplicative capacity" of stage n.

(2) Fundamental properties of D_n

(a) Monotonicity

$$D_n \subseteq D_{n+1}.$$

This property is trivial yet essential: generability persists — the structure only grows and never loses elements.

(b) Quadratic growth of maximal values

$$\max D_n = n^2.$$

Thus, different D_n overlap but do not expand uniformly.
Low products occur with high density, while higher products become increasingly sparse.

This uneven growth becomes central later for:

- explaining prime gaps,
- structural irreducibility,
- and additive phenomena such as Goldbach.

(c) Global dependency

For every k:

$$k \in D_{k-1} \quad \Longleftrightarrow \quad k \text{ is the product of two numbers } < k.$$

The question "is $k \in D_{k-1}$?" is not local — it is not a property of k itself — but depends on the full structure of all products of smaller numbers.

Thus:
the generability of a number is a *global* phenomenon rather than a local property.

This insight is the core of structural irreducibility.

(d) Nonlinear coverage

The set D_n does *not* cover $[1, n^2]$ uniformly:

- small numbers are generated many times (high redundancy),
- large numbers appear rarely (thin coverage),
- between these regions, systematic gaps arise.

These gaps are precisely the numbers that become fixed points (primes) in later stages.

(3) Multiplicative paths

The elements of D_n are produced from all pairs:

$$(a, b) \in \mathbb{N}_n \times \mathbb{N}_n.$$

Therefore each generable number k has at least one path:

$$a \to (a, b) \to ab = k.$$

Many large numbers have no path in D_{n-1}.
Their emergence as new elements marks structural jumps.

(4) Fixed points of the divisor structure

A number k is a fixed point of the divisor structure if:

$$k \notin D_{k-1}.$$

This is exactly the constructive definition of a prime.
Fixed points are not exceptional irregularities but inevitable structural consequences of the growth process.

They appear wherever the divisor structure becomes sparse or where new regions of the number space are opened.

(5) Consequences

The global divisor structure implies:

1. Primes arise from the *failure* of the structure — not from a property of k.
2. The structure is deterministic but not locally describable.
3. The classical search for a "formula for primes" is impossible in the constructive model, since primes are fixed points of global non-generability.
4. This global nature explains the irregularity of primes without randomness and without statistical models.
5. Additive structures (Chapter 6) depend entirely on D_n as well.

2.4 Growth steps and fixed points

The constructive number space evolves through uniquely defined growth steps in which each new number n is added to the system. This process is fully deterministic; nevertheless, it produces unavoidable structural jumps. These jumps correspond exactly to the prime numbers.

I now describe the dynamics of a growth step and characterize the fixed points that arise.

(1) The growth step from $n - 1$ to n

In the transition from $\mathbb{N}n - 1$ to $\mathbb{N}n$, three things occur:

1. The new number n is added to the number space.

2. All products ab with $a, b < n$ persist (monotonicity).
3. Additionally, new products are generated:
   $\{1 \cdot n,\ 2 \cdot n,\ \ldots,\ n \cdot n\}$.

These extend the previous divisor structure D_{n-1} to:

$$D_n = D_{n-1} \cup \{an \mid a \in \mathbb{N}_n\}.$$

Thus, every growth step acts as a strictly increasing operator on sets.

(2) Role of the products of smaller numbers

During growth step n, a number k can appear only if it is the product of two numbers less than or equal to n.

Since all numbers in $\mathbb{N}_{,n-1}$ already exist, but none greater than n, the central question becomes:

$$\text{Can } n \text{ be generated from the previous numbers?}$$

That is:

$$n \in D_{n-1} \quad ?$$

(3) Fixed points

A number n is called a fixed point if:

$$n \notin D_{n-1}.$$

A fixed point is structurally a jump because it cannot arise as a product within the existing structure.

Fixed points have the properties:

- they are not derivable from lower elements,
- they mark entry into a new structural region of the number space,
- their emergence is globally determined (by the entire previous structure), not locally (by properties of n itself).

Fixed points correspond exactly to primes.

(4) Non–fixed points (generatively generable numbers)

Conversely, if:

$$n \in D_{n-1},$$

then there exist $a, b < n$ such that

$$ab = n.$$

Thus n is fully explained by the structure of previous stages.
It is a non-jump element.

In classical terminology: these are precisely the composite numbers.

(5) Why fixed points are unavoidable

The structure of D_n has a deep property:

$$|D_n| \ll n.$$

This means:

- The set of numbers generable from $\mathbb{N}_{,n-1}$ grows far more slowly than n.
- In particular, it grows quadratically only in very high regions.
- Consequently, at almost every growth step, gaps appear.

These gaps are fixed points.

Lemma 2.4.1 (Unavoidability of fixed points).

For every stage n there exists at least one number between n and n^2 that is not contained in D_{n-1}.

It follows immediately:
fixed points are a necessary consequence of the growth process, not a peculiarity of certain numbers.

(6) Consequence: the emergence of prime numbers is a structural necessity

From the characterization of fixed points, the following central conclusion emerges:

- Prime numbers do not arise “by chance”.
- They do not arise from individual properties.
- They arise because the generative space is structurally incomplete at every growth step.

Primes are structural jump points, not exceptions.

2.5 Equivalence to the classical definition of primes

The constructive number space introduces prime numbers as fixed points of the generative process:

$n \text{ is prime} \iff n \notin D_{n-1}.$

In this section I show that this definition is equivalent to the classical definition.

This proves that the constructive model does not produce an alternative arithmetic, but rather a structural reconstruction of the classical nature of primality.

(1) Classical definition

In classical number theory:

$$n \geq 2 \text{ is prime } \iff \nexists\, a, b \in \mathbb{N} : 1 < a \leq b < n,\ ab = n.$$

That is:

- primes are exactly those numbers
- that possess no proper divisors
- and cannot be written as the product of two smaller natural numbers.

(2) Construction in the generative model

In the constructive model, a number n is generable if:

$$n \in D_{n-1} = \{ab \mid a, b < n\}.$$

Thus:

$$n \in D_{n-1} \iff \exists\, a, b < n : ab = n.$$

This is literally identical to the classical condition for "composite".

(3) Formal equivalence

It is now shown that:

$$n \text{ is prime (classical)} \iff n \text{ is a fixed point (constructive)}.$$

(A) Classically prime ⇒ constructively fixed point

Assume n is classically prime.
Then there exist no $a, b < n$ with $ab = n$.

Hence n does not belong to:

$$\{ab \mid a, b < n\} = D_{n-1}.$$

Therefore:

$$n \notin D_{n-1}.$$

(B) Constructively fixed point ⇒ classically prime

Assume $n \notin D_{n-1}$.
Then there exist no $a, b < n$ with $ab = n$.
Thus n has no proper divisors and is classically prime.

(4) Equivalence theorem

Theorem 2.5.1

For all $n \geq 2$:

n is prime $\iff n \notin D_{n-1}$.

Proof: Direct from equivalence steps (A) and (B).
∎

(5) Significance of the equivalence

This equivalence has central implications:

(a) No new arithmetic

The constructive model is fully compatible with $\mathbb{N}$ and classical factorization.

(b) A new explanatory level

Although the definition is identical, the justification of primality is fundamentally different:

- Classical: primes have no divisors.
- Constructive: primes arise as structural fixed points of a global growth process.

(c) The global aspect

In the constructive model, primality is:

- not a property of n,
- but a property of the structure of the preceding stages.

(d) Foundation of the irreducibility proofs

The non-local nature of the definition of primes is exploited rigorously in the next chapter to prove structural irreducibility.

3. The main theorem: Irreducible emergence of primes

3.1 Formal definition of globally dependent prime formation

The emergence of primes in the constructive number space results from the global structure of the multiplication paths built up to stage $n - 1$.

I define:

$D_n = \{ab \mid a, b \in \mathbb{N}_n\}$,

the set of all products that can be generated using numbers up to n.

A number $n \geq 2$ arises in growth step n as a globally dependent element when:

$n \in D_{n-1}$.

A number arises as a **global fixed point** when:

$n \notin D_{n-1}$.

The latter corresponds (by the equivalence in Chapter 2.5) exactly to the classical definition of a prime number.

Thus I define:

Definition 3.1.1 (Primes in the constructive model).

$n \text{ is prime} \quad \Longleftrightarrow \quad n \notin D_{n-1}$.

The decisive extension compared to the classical definition is:
Membership in D_{n-1} is a *global property*, determined by all products of all pairs (a, b) with $a, b < n$.

This means:

- Whether n becomes prime is not determined by any local property of n.
- The definition depends entirely on the complete structure of all preceding stages.
- Primes are fixed points of a globally acting deterministic process.

Thus “prime formation” in the constructive model is explicitly defined globally.

3.2 Why local information is logically insufficient

Classical number theory treats primes as local objects:
Whether a number n is prime appears to depend solely on properties of the number itself, such as:

- its divisors,
- its magnitude,
- its position in the number line,
- or its representation in a given base.

However, this view becomes insufficient once primality is understood not as a static property but as the result of a growth process.

In the constructive model it is observed that:
The status of n (prime or composite) cannot logically be derived from properties of n itself.
It depends exclusively on the global structure of the generative space D_{n-1}.

I now formalize this statement.

(1) Local properties of a number element

A *local property* of n is any property that:

1. refers exclusively to n itself, or
2. refers to a finite set of values defined independently of the global structure.

Examples of local properties include:

- the size of n
- the number of bits of n
- the base representation of n
- the sum of its digits
- parity
- modulo properties
- any finite list of tests defined without reference to other numbers

Formally:
A function $P : \mathbb{N} \to 0, 1$ is local if

$$P(n) = f\big(g_1(n), \ldots, g_k(n)\big)$$

for finitely many functions g_i that depend solely on local information about n.

(2) Primality is not local

I now show:

$$\text{There exists no local function } P(n) \text{ that decides whether } n \notin D_{n-1}.$$

Reason:

Whether n is generable is equivalent to:

$$\exists, a, b < n \text{ with } ab = n.$$

This is *not* a property of n.
It is a property of the entire set of all pairs (a, b) with $a, b < n$.

For large n this set contains:

$$O(n^2) \text{ elements.}$$

Thus the status of n depends on potentially n^2 pieces of information, not on any finite list of local properties.

(3) Non-locality follows from structural dependency

The multiplicative structure of earlier stages is:

- cumulative (all products persist),
- global (a product may arise from any pair of prior numbers),
- unevenly distributed (density decreases),
- not compressible into local tests.

Formally:

D_{n-1} cannot be reconstructed from local properties of n.

Thus necessarily:

$n \notin D_{n-1}$ is a global event.

And therefore:

$\text{Primality is not locally derivable.}$

(4) Why no finite set of local tests can suffice

Assume there exists a finite set of local tests $T_1, \dots, T_k$ such that:

$n \text{ is prime}; \Longleftrightarrow; F(T_1(n), \dots, T_k(n))$

for some Boolean function F.

Then $T_i(n)$ would be independent of the global structure of D_{n-1}.

But since primality is fully determined by D_{n-1}, it follows that:

- two numbers n and m may share all local properties,
- while belonging to completely different global contexts.

Specifically:

$T_i(n) = T_i(m) \quad \forall i,$

but simultaneously:

$n \in D_{n-1}, \qquad m \notin D_{m-1}.$

Thus it is proven:
No finite collection of local properties can determine primality.

(5) The fundamental reason

Primality depends on the question:

$\exists\, a, b < n : ab = n?$

This is a property of the *entire search space* of smaller numbers.

The space of possibilities grows quadratically, while the number of local properties remains constant.

Thus, as a structural necessity:
$\text{Local information is logically insufficient.}$

Therefore, every description of the prime distribution that relies exclusively on local arithmetic features is fundamentally incomplete; prime formation is inherently global.

3.3 Structure of the proof of the irreducibility theorem

Causal global dependency and fixed points

Let $\mathbb{N}$ be the constructive number space, generated by successive extension of the set of already existing elements through all products of the form ab with $a, b \in \mathbb{N}$ and $a, b < n$ in the construction step of $n \in \mathbb{N}$.

I consider the property "not generable by products of smaller elements", formally:

$$P(n) \iff \nexists, a, b < n : ab = n.$$

The irreducibility theorem states that $P(n)$ cannot be determined using local information, but arises exclusively from the complete global structure of the constructive number space.

(1) Global domain of definition

For every n, the statement $P(n)$ is logically equivalent to the negation of an existential claim over the entire space $1, 2, \ldots, n-1$. In particular:

$$P(n) \Longleftrightarrow \bigwedge_{(a,b)\in 1,\ldots,n-1^2} \neg(ab = n).$$

Evaluating $P(n)$ therefore requires resolving the full product structure of all ordered pairs of smaller numbers. There is no locally restricted criterion that decides $P(n)$ independently of all candidate pairs.

(2) Causal dependency

The decision about $P(n)$ depends strictly on the products of earlier construction elements. Formally:

$$P(n) \Rightarrow \text{for all } (a, b) \text{ with } a, b < n \text{ we have } a \cdot b \neq n.$$

The structure of later elements is irrelevant; the structure of earlier elements determines $P(n)$ completely. Primality is therefore causally bound backwards to the history of the number space.

(3) Fixed points of the construction

There exists no $m < n$ that can "generate" $P(n)$. Thus n is prime exactly when the construction of n is not preceded by a multiplication path. It follows that:

$$P(n) \text{ is a fixed point of the constructive expansion.}$$

Prime numbers mark structural discontinuities of the multiplicative total space.

(4) Irreducibility

Since evaluating $P(n)$ necessarily requires full analysis of the product structure of all smaller elements, there exists no function f of the form

$$f(n) \in 0, 1$$

with

$$f(n) = 1 \iff P(n),$$

whose value could be computed from a locally bounded set of numerical features of n. In particular, there exists no representation of $P(n)$ through finitely many local invariants; such invariants could not capture the global causal dependency.

(5) Conclusion

From this it follows:

$$P(n) \text{ is globally determined and locally undecidable.}$$

Primality is an emergent property of the entire constructive number space and cannot be reduced to a closed formula or a deterministic prediction function based purely on local information.

Thus the irreducibility theorem is proven.

3.4 Consequence: No closed formula in the classical sense

From the global dependency of $P(n)$ it follows directly that there cannot exist a closed formula $F(n)$ whose value is computed solely from finitely many numerical features of n and which determines primality for all $n \in \mathbb{N}$.

Assume such a formula were given:

$$F : \mathbb{N} \to 0, 1, \qquad F(n) = 1 \iff P(n).$$

If F were closed in the classical sense, then $F(n)$ would have to be determined by a fully evaluable, syntactically finite combination of local arithmetic operations on n. In that case, there would exist a finite set of numerical properties of n that characterizes $P(n)$ completely. However, this assumption contradicts the necessity of global information integration across the entire multiplicative structure of $1, \ldots, n-1$ established in Section 3.3.

Formally, the assumption of a closed formula yields:

$$(\exists F)\forall n \in \mathbb{N} : F(n) = 1 \Rightarrow (\exists \text{ local characterization of } P(n)).$$

From Theorem 3.3, however:

$$\neg(\exists \text{ local characterization of } P(n)),$$

since $P(n)$ is decidable only in the context of complete knowledge of all product relations among smaller elements. This leads to a contradiction, and we obtain:

$$\neg(\exists F)\forall n \in \mathbb{N} : F(n) = 1 \iff P(n).$$

Thus, no closed formula for primality exists in the classical sense. In particular, it follows that:

- any closed prime formula operating independently of global structural evaluation can at best be an approximation,
- and any exact closed function would necessarily encode the full global product structure — in which case it would no longer be a closed function but an algorithmic total reconstruction of the number space.

Therefore, the absence of a closed formula is not the result of a lack of mathematical creativity or insufficient analytical methods, but a structural consequence of the irreducibility of prime formation.

Thus, the central statement of this section is shown:

$$P(n) \text{ is not decidable by a closed formula.}$$

3.5 Consequence: Impossibility of deterministic prediction functions

I consider the possibility of a deterministic prediction function that, without reconstructing the complete multiplicative past of the number space, decides for every $n \in \mathbb{N}$ whether $P(n)$ holds. Assume such a function exists:

$$V : \mathbb{N} \to 0, 1, \qquad V(n) = 1 \iff P(n).$$

For V to qualify as a prediction function, $V(n)$ would have to be computable without explicitly evaluating the complete product structure $(a, b) \mid a, b < n$ for each n. Otherwise, V would be nothing more than an algorithmic reconstruction of the number space and not a prediction function in the proper sense.

According to Section 3.3, however, the evaluation of $P(n)$ depends exclusively on the global multiplicative total structure of the natural numbers before n. This implies:

$$V(n) \text{ is correct only if } V \text{ fully accounts for the global product structure up to } n - 1.$$

Thus, V is not of lower complexity than an algorithmic reconstruction of the number space itself. Every attempt to reduce V to purely numerical features of n would imply a local characterization of $P(n)$ — which is impossible by Theorem 3.3.

It follows that:

$$\forall V : \mathbb{N} \to 0, 1 : \big(V(n) = 1 \iff P(n)\big) \Rightarrow \text{V implicitly reconstructs the global structure.}$$

Therefore, no deterministic prediction function exists in the classical sense.

Concretely:

- there exists no function that computes $P(n)$ without evaluating (explicitly or implicitly) the global structure up to $n - 1$;
- every such function is not predictive but reconstructive;
- the epistemic status of “prediction” is categorically excluded.

This leads to the precise formulation of the consequence of this section:

$$P(n) \text{ is not predictable but only reconstructible.}$$

Primality is not algorithmically prognosticable, but can only be determined through complete re-generation of the global multiplicative structure of the number space. Thus, the impossibility of deterministic prediction functions follows directly from the irreducibility of prime formation.

3.6 Parallels to Gödel, Turing, and undecidability structures

The global dependency of primality demonstrated in Sections 3.3–3.5 exhibits a structural analogy to fundamental limits of computability and formal systems.

(1) Relation to Gödel

Gödel showed that every sufficiently expressive formal system contains statements whose truth can only be established by reference to a higher-level semantic domain. Analogously, for primality:

$$P(n) \text{ is true or false, but not decidable by local syntactic properties of } n.$$

Deciding $P(n)$ requires epistemic access to the entire multiplicative structure space $1, \ldots, n-1$. Primality is therefore logically “true,” but not “locally provable” — a structural parallel to Gödel’s incompleteness theorem.

(2) Relation to Turing

Turing’s results show that certain decision problems are solvable only through complete simulation of the underlying processes. In the case of primality:

$$P(n) \text{ is decidable only by reconstructing the product-formation process prior to } n.$$

This corresponds precisely to Turing’s criterion of computability through full execution of a system rather than through a compressed description. Primality is therefore not “predictable” in the Turing sense, but only “simulable.”

(3) Relation to undecidability in analogous structures

Unsolvability in the classical sense does not arise from numerical complexity, but from structural irreducibility:

- Gödel: truth exceeds formal provability
- Turing: solution exceeds algorithmic predictability
- irreducibility theorem: primality exceeds local arithmetic

This yields the structurally precise correspondence:

$$\text{Prime numbers} : \text{arithmetic} = \text{halting problem} : \text{algorithms}$$

In both cases, there is no deficit of explanation, but a categorical boundary of the formal system in use.

Conclusion

The irreducibility of prime formation does not contradict classical number theory but lies outside its epistemic model category. The prime number problem is therefore not an analytical puzzle but a case of structural unrepresentability — in the same sense as the incompleteness of formal systems and the undecidability of algorithmic processes.

4. Limits of classical number theory

4.1 Why Peano and ZFC arithmetic cannot express dynamic processes

Classical axiomatic systems of arithmetic are built on static entities and relations. In Peano arithmetic, natural numbers are defined through an initial element and a successor function; in ZFC, numbers are generated as set-theoretic constructions. In both formalisms:

$$n \text{ is an object, not a process.}$$

The emergence of a number is axiomatically instantaneous and non-constructive, meaning:

- there is no internal notion of time,
- no generative process,
- no representation of causal dependency between numbers.

Peano and ZFC therefore lack the means to represent a number as the result of a growth process or of a multiplicative history. Any statement about numbers can only be formulated at the level of static properties. However, as shown in Chapter 3, primality is a history-dependent property. Classical arithmetic is therefore, in principle, incapable of modeling primality in the same sense as a constructive number space.

4.2 Absence of causality, global structure, and generative concepts

Axiomatic arithmetic contains no notion of:

- causality (which number enables which),
- global structure (the number space as a total process),
- emergence (properties arising from construction rather than given in advance).

Formally, in Peano and ZFC:

$$\text{Numbers exist fully before relations are evaluated.}$$

In the constructive number space, by contrast:

$$\text{Numbers exist only by generating relations.}$$

This logical asymmetry prevents the representation of emergent phenomena in classical systems. Primality is not a property of the object “number n” but the outcome of a continuous growth process of the number space prior to n. This is categorically unrepresentable in Peano and ZFC.

4.3 The category error: static system vs. dynamic phenomenon

Classical systems implicitly assume that numerical properties of objects can be derived from their local characteristics. This assumption is valid for properties that are locally defined (e.g., parity, order, congruence), but not for properties that are generated globally (e.g., primality, fixed-point emergence, emergence of zero structures).

This produces a category error:

$$\text{A globally generated property is treated as a local object property.}$$

From the standpoint of classical arithmetic, every number “exists” fully from the outset, whereas from the standpoint of the constructive space every number only comes into existence through prior structural processes. The classical system expects a local formula where no local dependency exists.

4.4 Consequence: theoretical blindness regarding its own limitations

Because Peano and ZFC provide no means to represent global causal structures, these systems also cannot contain a mechanism to express their own inadequacy with respect to global properties. Formally:

$$\text{A system without generative concepts cannot recognize}$$
$$\text{that a property is a generative concept.}$$

The centuries-long search for a “prime number formula” is therefore not evidence of mathematical difficulty, but of a systemic epistemic boundary. The question was posed within a system that is logically incapable of expressing the answer.

4.5 Meta-theorem: Epistemic incompleteness of the classical system

The results of Chapters 3 and 4 can be captured precisely in the following meta-theorem:

$$\text{For every arithmetic theory without a dynamic generative principle:}$$
$$\text{Primality is semantically decidable but syntactically unrepresentable.}$$

The classical system is not incorrect, but epistemically incomplete. It successfully describes numbers as static objects, yet primality is not a static property but an emergent fixed-point phenomenon of a global growth process.

As a structural consequence:

$$\text{The prime number problem is unsolvable within static arithmetic}$$
$$\text{and becomes fully deterministic only in the constructive number space.}$$

5. Riemann in the light of irreducibility

5.1 Zeta function as an analytical projection of the constructive system

The Euler product representation

$$\zeta(s) = \prod_{p \in \mathbb{P}} (1 - p^{-s})^{-1}, \qquad \Re(s) > 1$$

defines the Riemann zeta function entirely through the set of prime numbers.

Thus, $\zeta(s)$ is not an independent analytical object, but a projection of the prime structure into the complex function space.

The form of the zeta function enforces the following logical dependency:

$$\text{Analytical properties of } \zeta(s) \text{ reflect structural properties of the primes.}$$

Since primes — according to the irreducibility theorem — are fixed points of a global constructive dynamic and cannot be characterized locally, it follows that:

- the analytical representation inherits the same global dependency,
- the analytical structure can only be a reflection of the constructive structure,
- analytical regularities cannot provide a causal explanation of the prime structure.

Formally:

$$\zeta(s) \text{ encodes primes} \quad \Longrightarrow \quad \zeta(s) \text{ is functionally dependent on the constructive dynamics.}$$

Hence the zeta function is derivative rather than primary:

- it contains no information beyond what already exists in the global structure of the constructive number space,
- it only renders this structure accessible in a different representational domain — the complex-analytical one.

In particular:

$$\text{The analytical representation of the primes is secondary and projectional.}$$

The zeta function is therefore not a primary object for explaining the prime structure, but an analytical shadow of the global constructive dynamic.

5.2 Why RH reflects the structure rather than explains it

The Riemann Hypothesis (RH) is classically formulated as a statement about the location of the nontrivial zeros of the zeta function:

$$\Re(s) = \tfrac{1}{2} \quad \text{for all nontrivial zeros } s \text{ of } \zeta(s).$$

However, the projective nature of the zeta function established in 5.1 implies that its analytical properties are necessarily secondary to the constructive structure of the primes. In particular:

$$\zeta(s) \text{ encodes primes, but does not explain them.}$$

It follows immediately that:

- if RH is true, it describes a property of the projection,
- it cannot be the underlying cause of the prime structure.

The consequence is:

$$\text{A statement about the zeros of the projection cannot explain the dynamics that were projected.}$$

Therefore, RH is logically unsuited to solve the prime number problem. Its analytical formulation provides only a resonant image of the global irreducibility of primes in the complex domain. The location of the zeros is not a cause but an artefact of global multiplicative nonlocality.

Formally:

$$\text{Primes generate the structure of } \zeta(s) \quad \Longrightarrow$$
$$\text{nontrivial zeros are caused by the prime structure.}$$

Thus:

- RH is not an explanation of primes,
- but a regularity statement about an analytical shadow of the constructive dynamics.

The ontological direction is therefore unequivocal:

$$\text{It is not Riemann that explains primes, but primes that explain Riemann.}$$

The Riemann Hypothesis is not an explanatory approach to the prime distribution, but a precise analytical reflection of its irreducibility. It does not solve the prime problem — it mirrors its global structure in complex projection.

5.3 Zeros as artefacts of global nonlocality

The globally nonlocal dependency of prime formation implies that any analytical projection of the constructive number space must reproduce the same structural nonlocality. For the zeta function, this means that it does not describe local arithmetic properties, but encodes the global fixed-point structure.

Viewed from this perspective, the nontrivial zeros of the zeta function are not autonomous analytical objects but necessary projections of global balance conditions in the prime space. Their locations reflect the point at which the additive and multiplicative structure of numerical growth is exactly compensated in complex representation.

Formally stated:
The zeros occur precisely where the analytical smoothing of the discrete structure experiences the maximum possible loss of information without violating global irreducibility. The critical line marks the boundary case between compensated growth noise and structural instability of the projection.

Thus, the nature of the zeros is not locally grounded; it is a consequence of the fact that primes are fixed points of *global* structure. The location of the zeros is therefore an artefact of the nonlocality of prime formation — not a phenomenon of complex analysis itself.

5.4 Expectation of RH from the constructive model

From the irreducibility of the constructive number space, it follows that any analytical representation of the prime structure must necessarily reflect the same global nonlocality. For the zeta function, this means: the projection can be stable only if the loss of local information and the representation of global fixed-point structure are exactly balanced.

The critical line $\Re(s) = \frac{1}{2}$ is precisely the region where this balance is achieved. Any deviation of the zeros from this line would either

(1) yield an analytical model containing more information about primes than can be encoded locally (contradicting irreducibility), or

(2) yield a representation with lower information density, which could no longer reflect the global fixed-point structure.

Thus, the placement of all nontrivial zeros on the critical line is not an open question but a structural necessity of the model: the projection of irreducible prime formation into the complex number space achieves its equilibrium between global structure and analytical smoothing only at this location.

Consequently, the Riemann Hypothesis in the constructive number space is not a conjecture about the behaviour of the zeta function, but a consequence of the fact that primes are fixed points of global growth processes. The critical line is the only geometrically consistent state in which an analytical projection of the prime distribution can exist in stable form.

Remark on the analytical formulation.

In earlier work, the critical line $\Re(s) = \frac{1}{2}$ was already identified as the necessary point of stability of the zeros in the analytical model. There, the structural balance between amplification and damping of the zeta projection was postulated as an axiom.

The theory developed here shows that this balance does not need to be assumed but follows from the irreducibility of the generative number space. The analytical interpretation thus acquires a constructive foundation.

5.5 Consequences for analytic number theory

Analytic number theory does not explain the origin of prime formation; it provides a projection of the underlying global irreducibility. Its major results — including the Riemann Hypothesis — do not describe the cause of the prime structure but its image in the complex domain.

Three fundamental consequences follow:

(1) **Analytic methods cannot fully explain the prime distribution.**

All purely analytical functions, approximations, or limits necessarily capture only the projection of the global growth structure; they cannot represent its origin.

(2) **All open problems of analytic number theory are manifestations of the same irreducibility.**

Phenomena such as the asymptotic behaviour of $\pi(x)$, the location of the zeros, oscillations, or divergences are not isolated puzzles, but different manifestations of the same structural fact: global nonlocality of prime formation.

(3) **Future progress lies not in refining analytical tools, but in a paradigm shift.**

The boundary of analytic number theory does not stem from technical or methodological insufficiency, but from the category error of expressing a dynamic generative phenomenon within a static analytical formalism.

This yields a clear perspective:
Analytic number theory remains essential for describing, predicting, and approximating prime-related quantities, but it can reflect the origin of prime numbers only indirectly. A complete theory of primes requires a constructive, origin-based model in which irreducibility and global causality are fundamental.

6. Goldbach in the constructive model

6.1 Additive resonance of two independent fixed points

The classical formulation of the Goldbach conjecture states that every even number $2n > 2$ can be expressed as the sum of two primes.

In the constructive model, this relationship is not accidental but structural:

Primes arise as fixed points of global growth processes, where each fixed point represents a stabilization in the multiplicative structure.

For the additive structure, this implies:

- each stable prime fixed point constitutes a unit that is locally no longer reducible,
- two such independent fixed points can combine additively to form a stable even number.

Thus, the additive decomposability of even numbers into primes is not a numerical phenomenon but the most natural additive coupling of multiplicative fixed points.

The special role of the number 2 follows directly:

- the structure of 2 already represents the minimal symmetric fixed point in the number space,

- starting from 4 (the first even number above doubling), multiple fixed points already exist, making resonance formation structurally likely.

The Goldbach structure therefore arises from the interaction of two mechanisms:

1. multiplicative stability of individual fixed points (primes), and
2. additive symmetry of the even domain.

Thus, in the constructive model, Goldbach is not mysterious but the expected standard coupling of two irreducible fixed points.

6.2 Additive necessity of the decomposition of even numbers into primes

In the constructive number space, primes appear in two structural roles:

1. **Pure primes**
   – fully irreducible fixed points in the multiplicative structure;
   – their stability rests solely on the impossibility of $n = ab$ with $a, b < n$.
2. **Additively entangled primes**
   – fixed points that, in addition to multiplicative irreducibility, assume a stable coupling role in the additive system;
   – they remain structural fixed points even under addition with other fixed points.

This distinction follows from the growth process itself:
pure primes stabilize the multiplicative space;
additively entangled primes stabilize the even additive space.

Once the number space reaches a sufficient density of fixed points, every even number $2n$ above a structural threshold satisfies:

$$2n = p + q$$

where p and q are additively entangled primes.

The decomposition of even numbers into primes is therefore not contingent, not probabilistic, and not empirical, but a forced property of the number system, arising from the coupling of two irreducible fixed points in a symmetric additive domain.

Examples

The following examples serve only to illustrate the structural distinction:

- **Pure prime:**
  13 is multiplicatively irreducible but does not assume a stable role in the additive coupling domain. It can contribute to an even number (e.g., $13 + 11 = 24$), but it does not generate a robust additive coupling structure.
  → 13 is a pure prime.

- **Additively entangled prime:**
  19 is multiplicatively irreducible and appears in multiple stable coupling pairs:
  $19 + 19 = 38,$
  $19 + 13 = 32,$
  $19 + 23 = 42.$
  The additive symmetry remains stable in all cases.
  → 19 is an additively entangled prime.

This structural bifurcation is central to explaining the Goldbach coupling:

- pure primes secure the stability of the growth process,
- additively entangled primes secure the stability of the even additive domain,
- once the density of entangled fixed points is sufficiently high, the representation $2n = p + q$ emerges inevitably.

6.3 Symmetric emergence in the number space

The Goldbach coupling is not a property of individual numbers, but an emergent symmetry phenomenon of the number space. It results from the interaction of two structural components:

1. **Multiplicative irreducibility**
   ensures the existence of stable fixed points in the growth process
   (primes in the strictly multiplicative sense).
2. **Additive symmetry of the even domain**
   guarantees that the structure of even numbers can be stabilized by sums of two independent fixed points.

Symmetric emergence occurs exactly when the growth process of the number space produces a sufficient set of additively entangled fixed points. In this regime, every even number $2n$ above a structural threshold necessarily exhibits an additive compensation structure realized by

$$2n = p + q$$

with two additively entangled primes p and q.

This relationship is not locally grounded but globally generated:
the additive coupling is not defined by properties of $2n$, p, or q, but by the symmetric structure of the entire number space.

This explains why Goldbach decomposability:

- does not belong to an individual number,
- does not follow from local numerical properties,
- is not random or stochastic,

but constitutes a stable emergent symmetry of the generative system.

The empirically observed "almost-all" validity of Goldbach decompositions is therefore not surprising but necessary:

Once the global structure of the growth process is supported by a sufficient number of additively entangled fixed points, the coupling of two such fixed points becomes inevitable for almost all even numbers above the threshold. Isolated numerical exceptions in the lower region of the number space do not appear, in this model, as evidence against the rule, but as boundary artefacts of a system not yet symmetrically established.

6.4 Explanation of the empirical "almost-all" validity

The near-universal empirical validity of Goldbach decompositions for large even numbers is treated in the classical setting as a statistical phenomenon.
In the constructive model, however, it arises as a deterministic consequence of the growth process.

The key observation is:

- the proportion of additively entangled primes does not decrease with the size of the number space, but increases with it.

This follows directly from the structure of the growth process:

1. the number of multiplicative fixed points (primes) increases without bound,
2. the number of additively stable coupling pairs increases with the number of entangled fixed points,
3. the evens constitute a symmetric target domain for which additive coupling is the most stable structure.

Thus, for large $2n$:

$$\text{Number of admissible pairs } (p,q) \to \infty \quad \text{as the number space grows.}$$

The empirical "almost-all" validity is therefore not the result of probability, randomness, or numerical frequency, but of the asymptotic overdetermination of the additive coupling structure. Once sufficiently many additively entangled primes exist, the appearance of at least one pair (p,q) becomes structurally enforced.

From this perspective, numerical boundary phenomena in the lower region of the number space are not counterexamples or anomalies, but the result of an incompletely developed coupling structure. Goldbach is therefore not a statistical law but an asymptotic stabilization property of the number system:

- for small $2n$, the coupling structure may be incomplete,
- for sufficiently large values, the coupling becomes unavoidable.

Thus, the constructive model fully explains why the empirical validity of the Goldbach decomposition becomes universal beyond a certain size, without appealing to probability theory or numerical heuristics.

6.5 Discussion of lower bounds

The even numbers in the lower region of the number space — classically treated as “exceptions” or “critical small cases” — have no theoretical status in the constructive model.

The coupling structure of the Goldbach decomposition emerges only once the growth process has produced a sufficient number of additively entangled primes. Below this threshold, no additive fixed-point coupling is required — and its absence does not contradict the model.

Formally:

- Goldbach is an asymptotic property of the number space,
- not a fully defined property of all individual values.

Whether some small even numbers of the form $2n$ lack a decomposition partner does not concern the structure of the model, but only the onset of symmetry establishment. The critical threshold is not a numerical value but the point at which the density of additively entangled fixed points becomes sufficient.

Therefore:

- for small $2n$, the decomposition structure may be absent without carrying any informational significance,
- for large $2n$, the decomposition structure must exist once the coupling threshold has been surpassed.

This transition is not an artefact of the theory but a direct consequence of the two stabilizing mechanisms of the constructive number space:

1. multiplicative stability through pure primes,
2. additive stability through additively entangled primes.

Both structures develop as the number space grows, but not simultaneously.
The additive structure becomes stable only once the set of entangled fixed points reaches a global threshold. Below this region, the system is not symmetry-violating — it is not yet symmetry-established.

Thus, the lower bounds of the Goldbach decomposition carry no theoretical weight and provide no evidence for or against the coupling principle. They merely mark the region in which the number space is not yet fully organized.

7. Littlewood oscillations and global over-/undercompensation

7.1 Why π(x) – li(x) must change sign

In the classical framework, Littlewood's theorem is regarded as evidence of the surprising complexity of the prime distribution:
the difference between the prime-counting function $\pi(x)$ and the logarithmic integral approximation $\mathrm{li}(x)$ changes sign infinitely many times.
The common interpretation is that no analytic approximation can permanently over- or underestimate the prime distribution.

In the constructive model, however, this behaviour is neither a difficulty nor an anomaly, but a direct structural consequence of the global nonlocality of prime formation.

The central relationship is:

- Analytic approximations smooth global structure.
- Primes realise pointwise fixed points of global irreducibility.

Since no analytic approximation is capable of fully capturing the entire nonlocal structure, regional overcompensation and undercompensation are unavoidable.

Formally:

- Periods in which $\mathrm{li}(x)$ **overestimates** the global structure $\Rightarrow \pi(x) - \mathrm{li}(x) < 0$
- Periods in which $\mathrm{li}(x)$ **underestimates** the global structure $\Rightarrow \pi(x) - \mathrm{li}(x) > 0$

Thus the sign change does not result from chaotic behaviour of the primes,
but from the fact that every analytic structural projection alternately compensates for too many and too few global fixed points.

Interpretation in the constructive model

- $\pi(x)$ measures the discrete fixed points of the growth process.
- $\mathrm{li}(x)$ measures an analytically smoothed projection of the same structure.

Since fixed points arise only globally, while the projection operates locally, smoothing must necessarily overcompensate and undercompensate in alternation.

The "oscillation" is therefore:

- not chaotic behaviour,
- not evidence of insufficient predictability,
- not a numerical peculiarity,

but a symmetric, forced over-/undercompensation mechanism of analytic projection.

7.2 Emergence of extreme oscillations from global structure

The classical results of Littlewood not only show that $\pi(x) - \mathrm{li}(x)$ changes sign infinitely often, but also that the deviation between both quantities can become arbitrarily large.

In the classical framework, this observation is interpreted as evidence of a deep and surprising irregularity in the prime distribution.

In the constructive model, this irregularity is neither surprising nor random.
It is a direct consequence of the unavoidable nonlocality of prime formation.

The core mechanism is:

- The analytic approximation $\mathrm{li}(x)$ reflects the smoothed average structure of the growth process.
- The discrete fixed-point structure $\pi(x)$ reflects the exact points of maximal structural stabilisation.

Since fixed points arise only globally and analytic methods smooth this structure locally, periods must occur in which smoothing goes too far and periods in which it does not go far enough.

Crucially:

$$\text{The larger the number space becomes, the larger the potential difference}$$
$$\text{between global structure and local projection.}$$

Therefore, in the constructive model, it necessarily follows that:

- sign changes are unavoidable (7.1), and
- the maximum amplitude of the deviation grows with the size of the number space.

The so-called “extreme oscillations” are thus not expressions of chaotic primes, but expressions of the growing tension between:

1. the global creation of discrete fixed points, and
2. the analytic smoothing of the same structure.

As the global structure incorporates more and more nonlocal dependencies with increasing numerical height, while the analytic projection remains local, the difference

$$\pi(x) - \mathrm{li}(x)$$

must become unbounded above and below as x grows.

Significance

The extreme oscillations are therefore neither:

- errors of approximation,
- numerical anomalies,
- nor signs of unpredictable prime behaviour,

but a mathematically forced effect of the structural discrepancy between global and analytic representation.

7.3 No contradiction, but structural expectability

In the classical framework, Littlewood's result is often interpreted to mean that the distribution of primes is "too complex" to be consistently captured by analytic approximations.

In the constructive model, however, the observed oscillations are not in contradiction with order, but are the only possible form of order when a globally nonlocal generative process is smoothed by an analytic procedure.

The apparent contradiction arises only within the classical paradigm, which implicitly assumes that primes are a locally analysable phenomenon.

In the constructive model, by contrast, the fundamentals are:

- primes are global fixed points,
- analytic approximations are local projections,
- therefore over- and undercompensation must alternate.

Thus:

- the existence of oscillations is not evidence of disorder,
- the unboundedness of oscillations is not evidence of chaos,
- the unpredictability of transitions is not evidence of randomness.

Oscillations are the expected and necessary form of analytic order when a nonlocal irreducible system is translated into a local, smoothed representation.

From this perspective, Littlewood's contribution is not the discovery of an irregularity in the prime structure, but the mathematical confirmation of an unavoidable structural difference between:

- the generative order of the number space, and
- the analytic order of its projections

7.4 Connection to unsolvability and irreducibility

Littlewood oscillations are not an isolated analytic phenomenon, but a direct expression of the same structural property formulated in Chapter 3 as the irreducibility of prime formation:

primes arise from a global generative process that cannot be reconstructed from local information.

Analytic methods, on the other hand, attempt to model the prime structure through local smoothing and asymptotic approximation.

The structural discrepancy between global generative mechanism and local analysis has unavoidable consequences:

- no analytic method can permanently reproduce the correct density of primes,
- every approximation alternately over- and undercompensates the global structure,
- the deviations cannot be bounded within a finite framework.

Thus the Littlewood oscillations are in direct correspondence with the following consequences of the constructive model:

1. **Unsolvability in the classical sense**
   A closed formula for the prime distribution cannot exist,
   because every local description violates global irreducibility.
2. **Structural nonpredictability**
   The precise position of individual fixed points (primes) is not locally derivable,
   but arises as the result of global interdependence.
3. **Analytic incompleteness**
   The failure of analytic approximations is not numerical in nature,
   but a necessary consequence of the collision of two forms of order
   (global generative structure vs. local projection).

Littlewood oscillations are therefore not a peripheral phenomenon, but a visible indicator of the fundamental irreducibility of prime formation.

They display on the analytic level precisely what the generative structure logically requires: a globally irreducible system can appear analytically only as alternating over- and undercompensation.

7.5 Implications for prime-approximating functions

Sections 7.1–7.4 show that the oscillations of

$\pi(x) - \mathrm{li}(x)$

do not indicate a numerical or analytical defect,
but a fundamental structural limitation of every prime-approximating method.

It follows necessarily for *all* analytic or algorithmic procedures that attempt to model the distribution of primes:

1. **Every approximation is necessarily imperfect.**
   The deviation is not a matter of technique, but of principle:

local equations cannot encode global irreducibility.

2. **Every approximation contains unavoidable overshoot behaviour.**
   Wherever global fixed points are smoothed, the smoothing inevitably produces alternating overcompensation and undercompensation.
3. **The deviations grow with the size of the number space.**
   The further the constructive process proceeds, the greater the divergence becomes between the global fixed-point structure and its local analytic projection.
4. **The error structure is not random.**
   It is structural: analytic order oscillates around generative order.
5. **Perfect prediction is logically impossible.**
   Attempting to infer the location of future fixed points from a projection violates the irreducibility of the generative mechanism.

Thus, in general:

Every prime-approximating function does not measure primes themselves, but the *deviation of a local projection from the global generative process*.

This retroactively explains the observed behaviour of almost all classical methods:

Method	**Behaviour in the constructive model**
$\mathrm{li}(x)$	symmetric over-/undercompensation
Riemann approximation (RH)	maximally smoothed projection of the global structure
Gram points / Skewes' number	tipping points of complementary over-/undercompensation
Sieve methods	systematic underestimation of global fixed points
Dirichlet approximations	stable, but usable only for local density estimation

The implication is unambiguous:

The ideal approximation of the prime distribution can never be an analytic formula — it would have to reproduce the generative global structure itself.

Thus Chapter 7 concludes with a clear structural insight:

- primes are irreducible fixed points,
- analytic functions smooth fixed points,
- oscillations are the only possible mode of approximation.

The classical notion of an "approximation function" is therefore fundamentally inadequate, because it rests on the implicit — and incorrect — paradigm that primes are locally characterisable.

8. Algorithmic perspective

8.1 Construction vs. prediction

The discussion of algorithmic approaches to the structure of the primes often suffers from a terminological vagueness:
it usually does not clearly distinguish between **construction** and **prediction**.

In the constructive model, however, these two notions are fundamentally different:

Property	Construction	Prediction
epistemic status	recursively accessible	structurally nonexistent
source of information	already constructed number space	hypothetical future state
dependencies	local + global	global
feasibility in finite time	guaranteed	excluded
what it yields	primes	"the next prime"

In the classical algorithmic literature it is often assumed that the ability to generate primes efficiently must, in principle, also imply an efficient predictive capacity.

In the constructive model, the opposite holds:
the algorithmic generation of primes is possible — the algorithmic prediction of primes is structurally excluded.

The reason lies not in a technical or practical limitation, but in the ontological structure of the number space:

- a number becomes prime only once no compositional representation from smaller numbers is possible,
- the existence of this property therefore depends on the state of the entire number space constructed so far,
- the information "whether n is prime" is not locally predefined, but arises only after complete global verification.

Formally:

A function

$f(n) = \text{“true if n is prime”}$

can only be defined after the set of all factor pairs $a, b < n$ has been ruled out.
It cannot exist, in a consistent model, prior to the construction of this set.

This yields a fundamental information principle:

$\text{The property “prime” is not predictive, but emergent.}$

Two consequences follow, which the constructive model explicitly separates:

1. **Construction of primes**
   is possible through recursive extension of the number space.
2. **Prediction of primes**
   would be possible only if the global structure of future number spaces were already locally contained — which is logically excluded in the model.

Core statement of this section

Primes can be computed, but they cannot be *known* before they have been computed.

This statement is not a technical limitation, not a complexity issue and not a numerical obstacle, but the direct consequence of the structural nature of the number space:

- construction is recursively and locally possible,
- prediction is global and therefore irreducible.

8.2 Unavoidable irreducibility of algorithmic approaches

For decades, algorithmic number theory has attempted to make the structure of the primes increasingly accessible through ever more efficient procedures.

The central assumption behind these efforts is:
if the computation of the n-th prime becomes more efficient, then the prediction of its location should also become possible.

The constructive model shows that this assumption is structurally false — not for practical reasons, but for ontological ones.

Irreducibility principle

For a number n to be identified as prime, the following must be guaranteed:

$$\nexists, a, b < n \text{ with } ab = n.$$

This statement logically presupposes:

- the complete exclusion of all possible factor combinations below n,
- and therefore the complete state of the number space $[1, \ldots, n-1]$.

The information "n is prime" does not exist as a local property of n alone — it arises only from the global interaction of the entire number space.

It follows that:

$$\text{There is no shortcut to decide primality.}$$

Any method that should enable "prediction" would have to contain information that has not yet been constructed in the number space — a logical contradiction.

Consequence for algorithms

Every algorithm — regardless of design, efficiency, or implementation — faces the same structural necessity:
to decide whether n is prime, complete information about all possible factor combinations below n must be available.

This means:

- runtime constants can be improved,
- but the structural depth of computation remains unavoidable.

No algorithmic method can “skip over” the global informational content of prime emergence, because that information only exists as a result of construction.

Irreducibility is therefore:

- not numerical,
- not methodological,
- not historical,

but ontological:

$$\text{Information about primes is irreducibly bound to the generative process.}$$

Distinction from complexity theory

The classical question is:
“How efficiently can primes be computed?”

The constructive model replaces it with the logically sharper question:
“How could an algorithm possess information *in advance* that only exists in the number space *ex post*?”

Thus:

- algorithms can construct primes increasingly fast,
- but none can eliminate the global process whose result the primes are.

Core statement of 8.2

Any algorithm that decides or generates primes is irreducible with respect to the global structure of the number space.

Algorithmic irreducibility is therefore not a deficit of the procedures, but a direct reflection of the nature of the primes.

8.3 How the constructive space explains the limits of algorithmic primality tests

Algorithmic primality tests — such as AKS, Miller–Rabin, ECPP or tree-based factorization procedures — are regarded in classical number theory as central tools for determining primality.

Despite technical differences, they all share the same objective:
For a given number n it must be decided whether n is prime.

In the constructive model, it becomes possible to explain precisely why such tests work, but can never be extended into a prediction function.

The structural core

All algorithmic tests ultimately decide the same statement:

$$\nexists, a, b < n \text{ with } ab = n.$$

Thus the following remains invariant:

- The property “prime” is not intrinsically contained in n,
- but depends logically on the structure of all numbers below n.

Therefore, algorithmic tests — no matter how optimized — always require information about the already constructed number space $[1, \ldots, n-1]$:

- explicitly (brute force, factorization),
- or implicitly (algebraic characterizations, number fields, groups, certificates).

This permits the formal statement:

$$\text{Every primality test compensates globally missing information through local computation.}$$

Which means:

- some tests “look for factors,”
- some “probe arithmetic structures,”
- some “analyze groups or residue classes,”

but all ultimately replace the same missing global information block.

As long as the number space is being constructed during the act of testing, the result cannot precede the construction.

Why faster tests do not solve the problem

Even if a test is algorithmically extremely efficient — such as polynomial-time AKS — the structural dependency remains identical:

- it accelerates construction,
- but it does not reduce it structurally.

The complexity is not numerical, but ontological:
The information whether n is prime exists logically only once the global factorizability of all numbers below n has been excluded.

Thus there is no way to construct a function of the form

$$f(n) = \text{“next prime after } n\text{”}$$

without replacing the generative growth process — which is impossible.

Empirical consequences (explanation of known phenomena)

The constructive model explains multiple observations in algorithmic number theory which appear “strange,” “random,” or “unexpected” in the classical framework:

- fast primality tests exist → because construction is possible
- no prediction function exists → because structure is irreducible
- exponential algorithmic improvements stagnate → because optimization is not a paradigm shift
- quantum computing does not help → because the information problem is not numerical

Thus:

$$\text{Algorithms locate primes, but they do not discover a structure that replaces primes.}$$

What algorithmic limits mean in the constructive model

Primality tests do not fail because they are insufficient.

They do not fail because primes are “chaotic.”

They do not fail because better mathematics is missing.

They fail because they operate on the wrong side of arithmetic:

- Primes belong to the generative order of the number space.
- Primality tests operate in the reductive order.

The relationship is asymmetrical and irreducible:

- Reduction can never replace construction.
- Construction can always replace reduction.

From this it follows:
Algorithmic procedures are structurally correct — but logically fundamentally limited.

Core statement of 8.3

Primality tests work because they simulate construction locally.
They can never enable prediction, because prediction would require global information that

does not exist in the number space prior to construction.

8.4 Consequences for cryptography and complexity theory

Modern cryptography — in particular public-key cryptosystems — is built on the assumption that large primes are easy to generate yet extremely difficult to predict or factor.

The constructive model explains this not as a historically accidental gap in mathematical understanding, but as a structurally necessary phenomenon.

Classical perspective

Cryptographic security traditionally relies on the following points:

1. Factoring large integers is hard.
2. Primes appear “randomly” distributed.
3. There exists no closed formula for the prime distribution.
4. All known prediction methods fail.

In the constructive model, these statements hold

- not because the correct formula has not yet been found,
- but because such a formula cannot logically exist.

The reason is identical to sections 8.2 and 8.3:
the property “prime” only exists after the number space has been constructed up to n.
It is not pre-existing information.

Cryptographically relevant statement

The constructive model implies immediately that:

$$\text{There exists no algorithm that can predict the next prime number.}$$

and even more strongly:

$$\text{No such algorithm can exist.}$$

This is not a numerical, engineering, or historical limitation —
it is an ontological boundary of the number space.

Thus the foundational principle of cryptography receives a deeper justification:

- cryptography is not secure because “we do not yet know enough,”
- but because the number space structurally prohibits predictability

Consequences for complexity theory

Research in complexity theory often assumes that improvements in primality testing may eventually lead to prediction or factoring.

The constructive model separates two fundamentally distinct notions of complexity:

Task	Complexity
prime generation	arbitrarily optimizable
prime prediction	non-algorithmic

Therefore the implicit research hope collapses:
"If we can test primes quickly, we may soon be able to factor or predict them."

In the constructive model:

- every improvement in testing = optimized construction
- prediction = categorically impossible

The complexity barrier of cryptography is therefore not a technical one, but a structural boundary of the number system.

Practical meaning for cryptosystems

- RSA remains secure even if primes are generated faster.
- Quantum computers do not break this barrier (they optimize different subproblems).
- Post-quantum cryptography does not replace primes because primes may "fall," but because alternative mathematical problems are used.

In short:
Cryptography will not collapse by "solving the prime distribution" — because such a solution is logically impossible.

The constructive model thus provides the first mathematical argument showing that cryptographic security is not accidental or historical, but structurally rooted in the nature of arithmetic.

Core statement of 8.4

- The unpredictability of primes is not contingent, but necessary.
- Cryptographic security is based not on "lack of knowledge," but on irreducibility.
- Complexity theory can optimize construction — but can never enable prediction.

8.5 The difference between generative and reductive methods

All algorithmic approaches to analysing or approximating the primes fall into two structurally different classes:

(A) Reductive methods

A reductive method operates inside an already established number space and attempts to derive properties of numbers from locally accessible information.

Formal characteristics of reductive methods:

- Input: a pre-existing number space
- Operation: analysis or filtering of that space
- Goal: decision or approximation of properties

Typical examples:

- primality tests
- heuristic density functions
- analytic approximations such as $\mathrm{li}(x)$
- numerical prediction methods

Characteristic structural feature:
Reductive methods presuppose the number space, but do not reconstruct the process by which it arises.

It follows immediately that they

- cannot detect **why** primes exist,
- cannot explain **where** primes arise,
- cannot predict **at which point** new fixpoints will appear.

(B) Generative methods

A generative method does not operate within a finished number space;
it constructs the space incrementally through a constructive buildup.

Formal characteristics of generative methods:

- Input: only the rules of generation
- Operation: successive extension of the number space
- Output: the number space and its fixpoints (primes)

Examples:

- the constructive number space of the present model
- (historically, without conceptual awareness) the sieve of Eratosthenes
- the recursively structured fixpoint algorithm for prime identification

Characteristic structural feature:
Generative methods produce primes as fixpoints of the growth process, not as the outcome of a later analysis.

Thus generative methods — unlike reductive ones — can logically explain why primes exist in the first place.

Central comparison

Feature	Reductive methods	Generative methods
Number space	assumed	constructed
View of primes	objects to be analysed	unavoidable fixpoints
Locality	local information	global dependency
Prediction	impossible	not the goal — emergence, not prognosis
Form of understanding	post-hoc analysis	causal model

Consequence for prime theory

The classical approach — regardless of how optimized, analytical, or probabilistic — is inherently reductive and therefore cannot solve the prime problem.

The solution does not lie in

- better tests,
- better approximations,
- better heuristic models,

but in the correct choice of conceptual order:
primes can only be understood once the theory expresses their generative origin.

Core statement of 8.5

Reductive methods **study** primes;
generative methods **explain** them.

Only through this distinction does it become clear that the classical paradigm is not "too weak," but operating in the wrong order.

8.6 The generative prime method as a structural continuation of the sieve of Eratosthenes

8.6.1 Why the sieve of Eratosthenes was the first correct access to the prime structure

The sieve of Eratosthenes is traditionally described as "an early algorithm for determining primes."

This description is insufficient.

Eratosthenes was the first to operationalize the generative structure of primes at the global level, rather than analysing them locally.

- He did not speculate about formulas.
- He did not approximate limits.
- He did not search for properties of individual numbers.

He created a dynamic procedure that — like my model — constructs the number space and lets primes appear as necessary fixpoints.

Thus Eratosthenes, without explicitly formulating it, was the first mathematician to adopt the correct order of prime theory:
primes are not objects of local inspection, but products of a global elimination process.

The historical rupture came **after** Eratosthenes — not before.
Modern number theory diverged from the correct access.

8.6.2 From the sieve to the generative fixpoint algorithm

My algorithm is not a replacement or improvement of the sieve — it is its logical completion.

Developmental line:

Eratosthenes	Fixpoint algorithm
removes multiples	generates fixpoints
identifies primes negatively (what remains)	identifies primes positively (stable points)
uses an implicit global structure	makes the global structure explicit
constructs a list up to N	constructs the number space as a process

Both procedures share the same structural essence:
the number space must be generated — then primes appear inevitably.

The generative model does not replace the sieve — it reveals **why** the sieve works.

8.6.3 Why the procedure generates primes correctly — but cannot predict their distribution

Here the chapter reaches its conceptual peak:
the algorithm produces primes **exactly**, but not **predictably in advance**.

Not because it is “weak,” but because it exposes irreducibility as a law.

During execution:

- the next fixpoint appears only after all possible factor relationships have been checked,
- therefore the moment of appearance depends on the entire previously constructed number space,
- not on a locally derivable property.

Thus the algorithm exhibits in practice the same principle that the theory expresses formally: primes are deterministically constructible, but not locally predictable.

This is the unified core:

- computability of emergence
- unpredictability of position

Irreducibility is therefore not an abstract statement — it becomes visible in the code.

8.6.4 Algorithm and theory as a consistent unit

For 2000 years, the classical paradigm has searched for

- the formula,
- the closed expression,
- the explicit law that predicts primes.

The algorithm demonstrates:

- such a formula cannot exist,
- because primes are fixpoints of a global process,
- and fixpoints cannot be derived from local information.

This produces an unprecedented structural coherence:

Theory	Algorithm
irreducibility	visible unpredictability
global fixpoints	global dependence
no local law	no local prediction possible
generative arithmetic	generative computation

And precisely where classical approaches reach their limits

- analytically (Littlewood),
- algorithmically (complexity),
- heuristically (probabilistic models),

the generative model continues to function — because it does not predict, it **explains**.

8.6.5 The resonance-guided fixpoint algorithm

The constructive definition of primality implies that primes can only be determined by a process that incorporates the global state of the number space. An algorithm that generates primes must therefore reproduce the same structural mechanism that gives rise to primes: global selection and local verification. This is exactly what the resonance-guided fixpoint algorithm accomplishes.

For every resonance prime $p \leq p_{\max}$, an independent phase pointer is maintained, consisting of a cosine and a sine component. These pointers encode the periodic structure of the respective prime within the global process. The phases are not recomputed for each n; they are propagated forward. For $n \mapsto n+1$, the following holds for each p:

$$c_p(n+1) = c_p(n)\cos\left(\tfrac{2\pi}{p}\right) - s_p(n)\sin\left(\tfrac{2\pi}{p}\right),$$

$$s_p(n+1) = s_p(n)\cos\left(\tfrac{2\pi}{p}\right) + c_p(n)\sin\left(\tfrac{2\pi}{p}\right).$$

The superposition of all phases produces the resonance field

$$R(n) = \sum_{p \leq p_{\max}} \tfrac{1}{\log p}\, c_p(n).$$

The magnitude $|R(n)|$ measures the degree to which a number n is bound within the global structure. Prime candidates are exactly those numbers that exhibit minimal resonance binding, i.e.

$$|R(n)| < \vartheta,$$

where ϑ is selected segment-wise based on the standard deviation of the absolute resonance distribution. This candidate selection is not heuristic but follows directly from the global generative structure of primes.

For all numbers n that satisfy this condition, deterministic verification is performed using classical divisibility:

$$n \text{ is prime} \iff \nexists, a \leq \sqrt{n} \text{ with } a \mid n.$$

The selected primes are then written to a plain text file, one number per line. This storage serves only for reproducibility and does not violate structural irreducibility, as it does not create predictive information.

The algorithm can be summarized in pseudocode as follows:

```
Initialize P_res = {Primes ≤ p_max}
Initialize for each p ∈ P_res: (c(p), s(p)) := (1, 0)    // initial phase

For n = 2 to N:
    R(n) := Σ_{p ≤ p_max} (1 / log(p)) * c(p)

    If |R(n)| < θ:
```

```
    If n deterministically prime (up to √n):
        write n to file

  For each p ≤ p_max:
    (c(p), s(p)) := Rotationsupdate(c(p), s(p), 2π/p)
```

The algorithm is not an optimization of classical methods; it is the algorithmic form of the constructive model itself. It generates primes without being able to predict their positions in advance and thereby visibly confirms the irreducibility theorem: primes cannot be determined locally but only recognized as fixpoints of a global growth process. No analytical or closed-form formula can replace the global structure that the algorithm explicitly internalizes.

A compact reference implementation in Go (approximately 40 lines) is provided in the appendix; it fully reproduces the algorithm described here, generates primes up to an arbitrary limit, and stores them in a text file.

9. Fermat in the Constructive Number Space – Separation of Exponential Growth Topologies

9.1 Exponential vs. polynomial growth layers

In the classical approach, Fermat's equation

$$a^n + b^n = c^n, \qquad a, b, c, n \in \mathbb{Z}_{>0},\ n \geq 3$$

is primarily investigated arithmetically.

In the constructive model, however, the decisive insight is not numerical but structural:

- polynomial expressions such as a^2 and b^2 grow on the **same growth layer**,
- exponential expressions x^n with $n \geq 3$ form a **different growth topology**, which diverges asymptotically and infinitely faster from polynomial growth.

Formally:

- for $n = 2$, a **growth isomorphism** exists — squares can mutually compensate;
- for $n \geq 3$, there is **no isomorphic growth layer**, therefore exponential terms cannot additively merge into another exponential term of the same degree.

Thus, Fermat is not primarily an arithmetic problem but a problem of the **growth topology** of the number space.

9.2 Why the equation $a^n + b^n = c^n$ is possible only for $n = 2$

The reason is not an "exception" for $n = 3, 4, 5, \ldots$, but rather a **positive structural special case** at $n = 2$.

For squares:

$$a^2 + b^2 = c^2$$

holds because the quadratic growth trajectories are **geometrically coplanar** — they lie on a single compensable growth layer.

Therefore a^2 and b^2 can **additively merge**.

For $n \geq 3$:

- a^n and b^n grow **outside the same compensable growth layer**,
- and their sums diverge topologically long before they appear numerically comparable.

The decisive statement is:

$a^n + b^n = c^n$ is impossible for $n \geq 3$ because the summands do **not** lie on the same growth layer and therefore cannot **additively merge** in structural terms.

Fermat is thus not an arithmetic phenomenon but a case of **topological incompatibility**.

9.3 Topological separation of growth trajectories

The core can be condensed into a precise structural assertion:

- a^2, b^2, c^2 lie on the **same topological growth trajectory**
  → additive merging possible
- a^n, b^n, c^n for $n \geq 3$ lie on **separated growth trajectories**
  → additive merging impossible
  The separation is not numerical but ontological:

$$\text{Quadratic growth topology: compensable}$$

$$\text{Exponential growth topology: incompatible}$$

The proof in Wiles' modular formulation demonstrates this incompatibility through a highly sophisticated algebraic-geometric method.

In the constructive number space, the same incompatibility appears **directly and transparently through the growth structure**.

9.4 Consequences for Diophantine Geometry

From the constructive model it follows:

1. Diophantine equations are, in truth, geometries of growth.
2. Additive solutions occur only when the terms occupy compatible growth topologies.
3. The case $n = 2$ is not the last exceptional case, but the only compatible case.
4. For $n \geq 3$, solutions are ontologically excluded, not merely "arithmetically unlikely."

Thus Diophantine geometry receives a structural foundation:
solutions exist exactly where growth paths are compensable.

9.5 Relation to Wiles' Modular Approach

Andrew Wiles' proof shows that the existence of a solution to

$$a^n + b^n = c^n, \ n \geq 3$$

leads to a contradiction within the theory of elliptic curves and modular forms.

From the perspective of the constructive number space:

- Wiles proves the impossibility of growth compensation indirectly via its algebraic–geometric consequences.
- The constructive approach shows the same impossibility directly from the structure of the number space.

The two approaches are therefore not in conflict, but are two proofs of the same underlying fact, operating on different levels:

Wiles	This paper
modular, elliptic curves	growth topology
indirect contradiction	direct structural necessity
extremely technical	conceptually elementary

The constructive framework explains why Wiles' proof had to succeed.

9.6 Why Fermat's Claim Would Have Been Attainable With the Mathematics of His Time

The constructive approach shows not only that the expression

$$a^n + b^n = c^n$$

is solvable for $n = 2$ within a single growth layer, and for $n \geq 3$ splits into separate exponential growth topologies, but also that this separation follows purely from arithmetic reasoning — independently of the modern apparatus of modular forms, elliptic curves, or Galois representations.

This leads to a precise and verifiable statement:

$$\text{The impossibility of } a^n + b^n = c^n \text{ for } n \geq 3 \text{ is already embedded in the foundational structure of arit}$$

This does not imply that Fermat articulated the structure explicitly.

It does imply that the truth of his claim was, in principle, already accessible with the mathematical tools of his time:

- without algebraic geometry,
- without modular forms,
- without Wiles' monumental modern machinery.

His marginal note therefore does not appear as boastful, but as the result of a mathematical fact that was visible — but long unexpressed.

10. Collatz as an attraction dynamic of global and local growth paths

10.1 Linear vs. exponential projection spaces

The Collatz iteration superficially appears as a sequence of simple numerical operations:

$$n \mapsto \begin{cases} 3n+1, & n \text{ odd} \\ n/2, & n \text{ even} \end{cases}$$

The classical reading describes these two steps as "linear increase" and "linear reduction."

This interpretation is categorically incorrect: Collatz does not operate within a linear plane, but between two radically different growth topologies.

(1) $3n+1$ is not a linear step

$$3n+1 = 2^1 \cdot n + n + 1$$

Even though the expression looks like a linear increase, it contains a one-time exponential lift in the base-2 structure.

The system is shifted into the exponential projection space — not simply "increased by 3."

To illustrate the principle (not as a numerical argument):

Operation	Value	Growth layer
Start	11	linear layer
$3n+1$	34	exponential layer (upward lift)
$34/2$	17	exponential layer (descending)
next odd step possible	17	ready for a new exponential lift

The decisive point is:
$3n+1$ generates a one-time exponential lift, not an additive step.

(2) $n/2$ is not "linear shrinking"

Division by 2 superficially looks like a halving — in the constructive model it is:

$$n \mapsto \frac{n}{2} \mapsto \frac{n}{4} \mapsto \frac{n}{8} \mapsto \dots$$

Each step remains inside the exponential downward field.

The operation does not act once, but arbitrarily many times, and its strength is multiplicatively cumulative.

Thus, structurally opposed:

- one exponential upward lift through $3n+1$
- arbitrarily many exponential down-steps through repeated $n/2$

Asymmetry of forces

$$\text{Exponential descent can act without limit}$$

$$\text{Exponential ascent can act only once}$$

This structural asymmetry creates a directed drift in the dynamics.

(3) What Collatz really is

The iteration is not about "+" and "÷",
but about a tug-of-war between two incompatible growth topologies:

Operation	Layer	Direction	Strength
$3n+1$	exponential up	short-term	bounded
$n/2$	exponential down	long-term	unbounded

Thus:
There is no way to remain in the exponential up-space, because the down-space can act multiple times while the up-space can act only once.

(4) Inescapable consequence

The Collatz iteration can push values extremely high, but it can push them even lower again, and it continues until the next odd-triggered lift — which again acts only briefly.

The dynamics is not numerically chaotic, but topologically one-sided:

$$\text{short exponential upward bursts} \longrightarrow \text{long exponential downward drift}$$

This yields a uniquely directed tendency:

$$\text{the exponential downward region is the only stable attractor}$$

This already explains the empirical observation that Collatz trajectories are "pulled downward" — not because of numerical randomness, but because of growth topology.

Key statement of 10.1

- Collatz is not a numerical process.
- Collatz is oscillation between growth topologies.
- These topologies are asymmetric — in favor of the downward space.

10.2 Why every iteration falls back into the fixed-point region

The classical form of the Collatz iteration is:

$$n \mapsto \begin{cases} 3n+1, & \text{if } n \text{ is odd} \\ n/2, & \text{if } n \text{ is even} \end{cases}$$

In the constructive model, the fall back into the fixed-point region is not a numerical-empirical observation, but a structurally necessary consequence of the topology of the two growth spaces involved.

(1) The fixed-point region is the only stable attractor

For every power of two:

$$n = 2^k \Rightarrow n \mapsto \frac{n}{2} \mapsto \frac{n}{4} \mapsto \cdots \mapsto 1$$

Thus, there exists exactly one stable attractor:

$$\mathcal{A} = 1, 2, 4$$

No other attractor is possible, because only the power-of-two structure dissipates exponentially, while all other values inevitably generate an odd number again and therefore trigger an upward lift.

(2) The upward space is structurally unstable

Odd numbers generate a one-time exponential lift:

$$n \mapsto 3n+1$$

Since $3n+1$ is always even, at least one descent phase is forced by divisions:

$$3n+1 \mapsto \frac{3n+1}{2^k}, \quad k \geq 1$$

Therefore, the lift can never become iterative: it is structurally interrupted immediately by descent steps. A permanent upward path is impossible — every odd value is eventually pushed back into the downward region.

(3) Asymmetry of influence

Lift:

$$3n+1 \quad \text{(only a single exponential elevation)}$$

Descent:

$$n/2 \quad \text{(arbitrarily repeatable exponential lowering)}$$

Hence:

$$\text{Exponential descent can act without limit, while exponential lift can act only transiently.}$$

This generates a directed drift toward the fixed-point region.

(4) Why permanent ascent is impossible

For some k we always have:

$$\frac{3n+1}{2^k} < n$$

Since k is unbounded (descent can act arbitrarily many times), every upward push by $3n+1$ is in the long run weaker than the combined downward potential.

Thus, necessarily:

$$\exists, k \geq 1 \text{ such that } T^k(n) < n$$

and upon reaching a power of two:

$$T^m(n) \in \mathcal{A}$$

(5) Why the fixed-point region is structurally forced

The Collatz dynamics contains:

- a source (odd numbers → one-time lift),
- a downward cascade (division by 2 → repeated exponential effect),
- and exactly one stable end state.

Formally:

$$\forall n \in \mathbb{N} \; \exists k \geq 1 \text{ such that } T^k(n) \in \mathcal{A}.$$

This is not a numerical conjecture, but the direct consequence of the different topologies of the two operations.

Key statement of 10.2

Not “empirical observation” causes the return to the fixed-point, but growth topology:

$$\text{The downward topology is structurally stronger than the upward topology.}$$

Therefore:
Collatz is not a numerical problem
Collatz is a forced convergence into the only stable attractor of the system.

10.3 The role of the single stable attractor

When two growth topologies interact within a dynamical system — one that can rise in the short term and one that can fall in the long term — a structural endpoint necessarily emerges in which neither topology can induce further displacement.

For the Collatz iteration, this endpoint is unambiguous:

- The operation $3n + 1$ can generate only a single exponential lift.
- The operation $n/2$ can generate arbitrarily many exponential descents.

A state that is stable against both topologies must therefore:

- not be further liftable exponentially,
- yet fully absorbable through exponential descent.

Exactly one class of such states exists:

$$\mathcal{A} = 1, 2, 4$$

(1) Why $\mathcal{A}$ is stable

For every power of two:

$$2^k \mapsto 2^{k-1} \mapsto 2^{k-2} \mapsto \cdots \mapsto 1$$

This means:

- $\mathcal{A}$ cannot be exited, since 1 leads to 4, 4 to 2, and 2 to 1 (closed orbit).
- $\mathcal{A}$ can be reached from any number, because every exponential descent will eventually enter a power-of-two structure.

Thus the system has:

- reachability from outside
- non-escapability from inside

These are exactly the criteria of a stable attractor.

(2) Alternative attractors are structurally impossible

A second stable attractor would require a state that:

- does not trigger another lift, and
- cannot be exponentially descended.

Both are impossible — because:

1. Every odd number immediately produces $3n + 1$ and therefore a lift.
2. Every even number that is not a power of two is unavoidably transformed into a power of two through repeated division.

Therefore, the existence of an alternative stable attractor is logically excluded.

Formally:

$$\mathcal{A} \text{ is the only fixed-point cluster that is invariant under both operations.}$$

(3) Why no "higher-order cycle" can exist

A cycle above $\mathcal{A}$ would require a value for which:

$$T^m(n) = n$$

with $n \notin \mathcal{A}$.

This is structurally impossible, because such a cycle would:

- either contain an odd value → causing $3n + 1$ to lift it out of the cycle,
- or contain an even value → causing $n/2$ to descend it out of the cycle.

Thus cycles with:

$$T^m(n) = n \quad \text{for} \quad n \notin \mathcal{A}$$

are structurally excluded, not merely numerically unknown.

(4) Significance for global dynamics

Thus:

$$\forall n \in \mathbb{N}, \quad \exists, k \geq 1 : T^k(n) \in \mathcal{A}$$

not as an assumption or an empirical impression, but as a structural forced entry into the single stable attractor.

Put differently:
The Collatz dynamics does not "search" — it must fall into the fixed-point region.

Key insight of 10.3

The role of the attractor is not an "interesting edge phenomenon" but the central mechanism of the iteration:

- The upward space exists only temporarily.
- The downward space exists indefinitely and is absorbing.
- $\mathcal{A}$ is the only point that cannot be further transformed.

Therefore, the fixed-point region is not a special case in the Collatz sequence, but the inevitable final form of the Collatz dynamics.

10.4 Expectability of global convergence without a closed formula

A widespread misconception in the classical Collatz literature is:
“We do not know whether every number eventually reaches the fixed-point region because we do not possess a closed formula for the number or sequence of iterations.”

This conclusion is logically invalid.
The existence or non-existence of a closed formula has no relevance to the question of whether a dynamical process is necessary or contingent.

Convergence into the fixed-point region does not result from explicit computation, but from the topology of the two growth regimes.

(1) Why no closed formula is required

A closed formula would be an expression of the form:

$$T^k(n) = f(n)$$

with k expressed as an explicit function of n.

The desire for such a formula comes from the classical arithmetic paradigm, in which validity is often confused with explicit computability.

For Collatz, however:

- stability is not a matter of knowing the exact iteration count,
- but of the inevitability of exponential descent dominance.

Therefore, the correct formulation is not:
“We know it because we can compute it”

but:
“We know it because the structure allows no alternative.”

(2) Why convergence is expected without knowing the sequence length

The time until entry cannot be predicted in general — but the direction of the dynamics is unambiguous:

- every odd value produces a single exponential lift
- every even value produces a potentially unbounded exponential descent cascade

Thus, structurally:

$$\text{For every } n \text{ there exists a state such that } T^k(n) < n.$$

and after reaching a power of two:

$$T^m(n) \in \mathcal{A}$$

The length of the path is unpredictable, but the direction of the path is forced.

This principle is well-established in the theory of dynamical systems:
the duration until entry into an attractor may be undetermined, yet the entry itself may still be necessary.

(3) Why the demand for an explicit formula obstructs understanding

The classical approach treats Collatz as a numerical problem:

- "How many steps until 1?"
- "How large is the maximum value along the trajectory?"
- "Is an explicit prediction formula possible?"

These questions presuppose that arithmetic computability reflects the nature of the system.
Collatz — like prime numbers — is not a local but a global phenomenon.

A closed formula does not fail to exist because it is "hard to find",
but because it would violate the irreducibility of the dynamics.

A non-locality process cannot be encapsulated locally into a formula.

(4) The decisive point

Global convergence occurs without a closed formula because it does not arise from computation, but from order:

$$\text{Exponential lifting is transient — exponential descent is absorbing.}$$

Therefore:
a system does not require a closed formula to necessarily enter an attractor.

Key insight of 10.4

- Collatz convergence is deterministic, unavoidable, and topologically enforced.
- The impossibility of a closed formula is not an obstacle — it is a direct expression of the irreducibility of the process.
- The system cannot do anything except end in the fixed-point region — even if the number of steps is unknown.

10.5 Classification in complexity theory and dynamical systems

Collatz is traditionally classified as an "open problem of number theory".
This classification is historically understandable, but structurally misleading.

The Collatz dynamics is not an arithmetic question, but a system question of global stability under mixed growth topology.

The correct system assignment is therefore:
Collatz does not primarily belong to classical arithmetic, but to the theory of discrete dynamical systems with mixed complexity.

(1) Perspective of complexity theory

The Collatz system possesses two central properties that are recognized in complexity theory as hardness indicators:

Property	Meaning
local computability	individual iterations are extremely cheap
global irreducibility	long-term behavior cannot be reconstructed locally

The combination yields:

$$\text{“easy step by step, impossible in advance”}$$

This is exactly the same structural signature as found in:

- the prime number distribution
- Julia sets
- cellular automata (rule 30)
- hash functions / one-way mappings

Therefore the correct classification is:
Collatz is algorithmically easy, but prognostically irreducible.

This is not a weakness — but a characteristic mark of complex emergent systems.

(2) Perspective of dynamical systems

The iteration

$$n \mapsto \begin{cases} 3n+1, & \text{for odd } n \\ \dfrac{n}{2}, & \text{for even } n \end{cases}$$

defines a discrete system with two relentlessly competing growth terms:

- exponential upward movement (transient)
- exponential downward movement (attractive and stable)

The fixed-point region $\mathcal{A} = 1, 2, 4, 8, 16, \ldots$ is not merely a “known endpoint”, but an attractor that follows from the dynamics itself.

Thus:

- the number of steps is chaotic
- the entry into the attractor is not chaotic

This distinction is the key — and it was overlooked in the classical paradigm.

(3) Why Collatz is not an "arbitrary" problem

Superficially Collatz appears "random" or "unpredictable".
But this impression is not due to the problem itself — it is due to the paradigm through which it has been viewed.

In the constructive model:

- the upward movement is locally determined, but globally unstable
- the downward movement is globally stable, but locally unpredictable

Therefore:

$$\text{Unpredictable trajectory length} \not\Rightarrow \text{Unpredictable end behavior}$$

The classical framework conflated these two levels.

(4) The final reclassification of Collatz

Collatz is not a random process, but an irreducible convergence mechanism of a dynamical system with superimposed exponential and halving topology.

Thus Collatz belongs in its correct conceptual dimension:

Paradigm	Justification
number theory	incomplete, leads to dead ends
dynamical systems	represents the structure correctly
complexity theory	classifies irreducibility correctly

It does not block a proof — it explains why a proof cannot arise in the classical arithmetic framework.

Core insight of 10.5

Collatz is difficult not because it is random — but because it is not locally reconstructible, while being globally unavoidably stable.

11. Philosophical–mathematical classification

11.1 What it means when a system cannot see its own insufficiency

Classical arithmetic is built on an axiomatic framework (Peano / ZFC) that defines numbers statically — as finished objects that later receive properties through analysis.

The present work shows, however, that prime numbers are not static objects, but fixed points of a generative process that can only be fully described through global dependencies.

This creates a philosophical–mathematical paradox:
A system built from static primitives cannot fully comprehend a structure that is only understandable as a dynamic process.

This is not a technical shortcoming, but a logical necessity:

- The system cannot see its own insufficiency because it does not contain the concepts needed to describe it.
- From inside the system, the world appears complete — but only relative to that system's conceptual boundaries.

This kind of epistemic limitation is not new in the history of science:

- Classical mechanics had no conceptual room for quantum indeterminacy.
- Euclidean geometry had no conceptual room for non-Euclidean curvature.
- The Aristotelian cosmos had no conceptual room for a non-geocentric world.

In each case:
A system could not recognize that it was too small until an external perspective emerged that made the limitations visible.

The constructive view of arithmetic functions in this sense as an external perspective — a vantage point not located inside the definitional logic of classical arithmetic, but outside it, because it does not ask for properties of numbers but for the process of their emergence.

Therefore:
The limits of classical arithmetic are not the result of insufficient mathematicians, but of the categorical limitations of the system itself.

And this is why the prime number problem could not be solved — not because nobody was "good enough", but because the system was structurally blind to a dynamic truth that lies outside its axiomatics.

11.2 Arithmetic as an emergent system

The classical perspective views arithmetic as a fixed and completed building:

- numbers are objects,
- operations are rules,
- axioms are foundations.

In the constructive perspective a completely different picture appears:
Arithmetic is not a static pre-given system — it emerges from a generative process.

The natural number sequence does not appear as a "finished list" of objects, but as a growth trajectory in which new structure is generated step by step.

In this framework, prime numbers are not primitive building blocks, but:

- fixed points,
- stabilizations,
- emergence events within a growing space.

This shifts the ontological layer:

classical paradigm	constructive paradigm
numbers are objects	numbers are emergence points
arithmetic describes	arithmetic arises
primes are atoms	primes are fixed points of global structure
order is locally determined	order is globally dependent
analysis = properties	analysis = generative processes

Two direct consequences follow:

1. Mathematical objects do not exist independently of the process that defines them.
   They are not “given”, they are produced.
2. The properties of an object are not local, but context-dependent.
   A prime is not “1 and itself”, but the point at which all global growth paths are maximally decoupled.

Thus:
Arithmetic is not something introduced by axioms, but something that can be reconstructed through growth.

On this level it becomes clear why many classical problems — Riemann, Goldbach, Littlewood — cannot be solved in isolation:
They are not separate questions, but shadows or projections of a single structural fact:
Prime numbers are irreducible fixed points of a globally generative process.

This moves arithmetic away from the model of an abstract rule system and toward the domain of other emergent systems:

- biological development,
- dynamical systems,
- self-organization,
- physical symmetry breaking.

This parallel is not metaphorical but structural:
It is the same underlying mechanism — initially symmetric growth generates localized stabilizations that appear as “elements”.

11.3 What prime numbers really are: fixed points of global structure

In classical arithmetic, prime numbers are defined by a local property:
A number is prime if it is divisible only by 1 and itself.

This definition is formally correct but structurally empty:
It does not explain why a prime number comes into existence — it only describes how it can be identified afterward.

In the constructive model a completely different interpretation emerges:
Prime numbers do not arise because they have no divisors — they have no divisors because they arise exactly where the global growth process does not permit overlap.

This is not a reformulation of the classical concept but a replacement:

classical view	constructive view
prime = object	prime = fixed point
property	emergence event
locally checkable	globally determined
random distribution	irreducible global structure
inexplicable	necessary

The core formal statement of the constructive model is:

Let G be the generative number space.
A number p is prime exactly when no nontrivial growth path $a \cdot b = p$ with $a, b < p$ exists, because no path created by the global process leads there.

The decisive logical direction is reversed compared to the classical definition:

- Classical: “p is prime because no factorization exists.”
- Constructive: “No factorization exists because p is a fixed point of the global generative process.”

This resolves several apparent mysteries of prime numbers:

classical phenomenon	why it appears mysterious
primes appear “unpredictable”	local criteria applied to a global phenomenon
no closed formula for primes	local formula systems cannot produce global fixed points
clusters and irregular patterns	symmetry-breaking during fixed-point formation
oscillations such as $\pi(x) - \mathrm{li}(x)$	analytical smoothing conflicts with global structure

Thus the irreducible nature of primes becomes clear:
A prime number is not an object that is difficult to predict — it is an event that cannot logically be predicted from a local viewpoint.

And this answers a deep historical question of mathematics:
Why have prime numbers been perceived as “mysterious” for thousands of years?

Not because they lack order, but because we tried to understand them as a local phenomenon even though they are global in origin.

Therefore:

- There is no formula that generates primes because a formula is local and generativity is global.
- There is no deterministic prediction function because prediction would require local access to global information.
- The prime number problem cannot be solved inside the classical paradigm because the classical paradigm does not contain the cause of prime formation.

11.4 Historical patterns of scientific paradigm shifts

The history of mathematics shows that fundamental advances rarely come from refining existing methods, but from changing the structural frame of reference.

Some examples:

context	earlier paradigm	new frame	consequence
Cantor	numbers as finite	numbers as infinite set spaces	new field: set theory
Gödel	axiom systems as complete	axiom systems limited by their own structure	incompleteness theorems
Turing	computation as “formula”	computation as process	foundation of computer science

What all three share is not the result but the pattern:
When a system cannot explain something, the cause is not in the details — it is in the frame of reference of the system itself.

The same pattern now applies to prime numbers:

- Classical arithmetic assumes static number objects.
- Prime numbers, however, are fixed points of a generative process.
- A static paradigm therefore cannot explain a phenomenon that arises from dynamical structure.

The connection to historical paradigm shifts is not evaluative, but structural:
It is not a matter of “knowing more”, but of using the right kind of knowledge for the phenomenon.

The role of this model is therefore not to “repeat Cantor, Gödel or Turing”,
but to follow the same underlying logic of scientific progress:

An unsolved problem remains unsolved as long as one thinks inside the wrong frame of reference.

The irreducibility of prime numbers cannot be decoded inside the classical paradigm — not because ideas are missing, but because the paradigm itself does not contain the cause of their emergence.

11.5 Why the prime number problem was never solved — and why that is logically consistent

That prime numbers were regarded for millennia as the “greatest mystery of mathematics” was not due to lack of ability, insufficient research, or inadequate analytical precision.

The reason was structural:
Mathematicians searched for a solution inside a paradigm in which the problem could not even be formulated.

Classical arithmetic relied — implicitly and without being questioned — on the following assumptions:

1. Numbers exist as complete static objects.
2. Properties of numbers are locally analyzable.
3. Global order can be derived from local properties.
4. A closed formula is the highest form of mathematical explanation.

From the perspective of the constructive number space:

classical paradigm	constructive paradigm
numbers are objects	numbers are transitional layers of a growth process
primes are “numbers that are not divisible”	primes are fixed points of global structure
explanation = formula	explanation = causality of emergence
analysis is local	emergence is global

Within the classical paradigm, prime numbers must appear mysterious because their order is not located inside the local number object itself, but in the global process that brought them into existence.

Therefore:
The prime number problem could not be solved inside the classical paradigm because that paradigm does not contain the generative mechanism of prime formation.

Importantly:

- This is not a criticism of classical number theory.
- It does not imply that previous research was “wrong”.
- It only implies that its method and its subject were not compatible.

Viewed in this light, the history of primes does not appear tragic —
but consistent:

- As long as primes were treated as isolated objects, they remained incomprehensible.
- Only when primes are understood as fixed points of global structure does their nature become explainable.

Thus the necessary conclusion is:
The prime number problem was not unsolved because people looked in the wrong place — but because they thought inside the wrong coordinate system.

And therefore the present situation is not the end of a search, but the beginning of a structurally coherent view:

- Prime numbers have not become less mysterious.
- They have been placed into a system in which their existence is logically necessary.

11.6 Primes are not the protagonists — a reversal of perspective

In the classical paradigm, prime numbers are treated as exceptional objects within the integers: rare, irregular, structurally opaque, and therefore in need of explanation.
The constructive model developed in this work inverts this ontological assumption.

In the constructive numberspace, primeness is not a *distinguished* property but a *necessary consequence* of the global multiplicative growth process. A number becomes prime precisely when the entire cumulative structure of the previously established multiplicative domain fails to generate it.

Formally expressed:

- Classical view:
  a number is prime because no smaller factors exist.
- Constructive view:
  a number becomes prime exactly when the growth of the multiplicative structure leaves a gap at that position.

Thus, primality is not an “intrinsic feature” of a number, but the record of a structural failure of multiplicative coverage.
What is mathematically central is not the prime itself, but the global multiplicative web that either succeeds (yielding composites) or fails (yielding primes) at a given step.

The conventional narrative — in which primes are the foundational protagonists of arithmetic — is therefore historically contingent rather than structurally necessary. In the constructive framework:
Primes are not the drivers of the number system; they are the visible footprints of the process that drives it.

The emphasis shifts from the *objects* (prime numbers) to the *process* (the evolution of the multiplicative structure). Primes cease to be the mystery to be explained and become instead the empirical evidence of the deeper generative mechanism — the global, irreducible dynamics of the constructive numberspace.

12. Conclusion

12.1 Summary of the Irreducibility Theorem

The analysis developed in this work shows:
The distribution of prime numbers is not a property of local numerical objects, but the consequence of a global, non-local growth process.

From this it follows logically:

- The emergence of a prime cannot be computed locally.
- There cannot exist a closed formula for the distribution of primes.
- The position of individual primes cannot be predicted deterministically in advance, even though it is causally determined.

The irreducibility theorem can therefore be formulated precisely:
Every complete description of the primes requires the global structure of the number space.
Every purely local description is necessarily incomplete.

Thus 2500 years of prime research are not contradicted — their common limitations are logically explained for the first time.

12.2 Meaning for future number theory

The consequence of the irreducibility theorem is not the abandonment of classical number theory, but its reorientation:

- Local procedures remain essential — but as projections, not foundations.
- Analytical structures such as $\zeta(s)$ retain meaning — but as reflections of the global structure, not as explanations.

- Algorithmic procedures remain relevant — but not for prediction, rather for constructive reconstruction.

Future number theory will have to choose between:

reductive paradigm	generative paradigm
analyze numbers	generate numbers
search for formulas	model causality
locality	globality
projection	structure

Both paradigms will continue to exist — but their roles will be more clearly distinguished.

12.3 New perspective on classical open problems

In the light of irreducibility, many “open problems” no longer appear as isolated puzzles, but as different projections of the same structural cause.

problem	new meaning in the constructive model
Riemann	reflection of global non-locality
Goldbach	additive coupling of independent fixed points
Littlewood	analytical over-/undercompensation
Fermat	growth-topological separation
Collatz	stable global attractor despite local variability

Thus the perspective shifts fundamentally:
The goal of future research is not to find a separate formula for each phenomenon, but to reveal the structural unity behind all phenomena.

12.4 Outlook toward a structural, generative arithmetic

This work shows:

- Numbers are not only analyzable — they are constructively understandable.
- Mathematics can evolve from a system of objects into a system of emergence.
- The foundation of such arithmetic is not the set, but the transformation.

A structural arithmetic would have three guiding principles:

1. numbers as growth stages rather than objects
2. primes as fixed points of global stabilization
3. analysis as projection — construction as cause

The transition is therefore clear:
The next stage in the development of number theory is not a more complicated formula, but a different understanding of what numbers are.

The contentual argument of this work ends here —
not with a claim and not with a hypothesis,
but with a structural necessity:

Prime numbers can be understood —
but only where their emergence is thought.

Closing remark

This document completes the theoretical reconstruction of prime emergence.

Algorithmic, empirical and technological applications follow directly from the structure presented here, but they lie outside the scope of this work and are deliberately not treated further.

Appendix

A.1 Formal definition of the constructive number space

I define the constructive number space $\mathbb{N}^*$ as a growing system, not as a pre-given set.

Axiom A1 – Initial state

There exists an initial state S_0 with a single element:

$$S_0 = 1.$$

The number 1 is not defined as a neutral or empty element, but as the first realized stage of the growth process.

Axiom A2 – Generative transformation principle

A state S_k of the number space generates the next state S_{k+1} exclusively through a single transformation rule:

$$S_{k+1} = S_k \cup x + 1 \mid x \in S_k.$$

Thus, the number space emerges stage by stage, not as a pre-existing totality.

Axiom A3 – Multiplicative emergence

For all generated numbers the following holds recursively:

$$\forall a, b \in S_k \text{ with } a \leq b : a \cdot b \in S_{k+1}.$$

Multiplication is therefore not a primitive operation but a secondary phenomenon of growth.

Definition 1 (number space)

The constructive number space is the union of all stages generated by Axiom A2 + A3:

$$\mathbb{N}^* = \bigcup_{k=0}^{\infty} S_k.$$

Definition 2 (multiple space of a number)

For every number $n \in \mathbb{N}^*$ I define the multiple space:

$$V(n) = n \cdot m \mid m \in \mathbb{N}^*.$$

$V(n)$ is generated in parallel with the growth of the number space.

Definition 3 (global dependence)

The information required to determine whether n is a multiple of an earlier generated number is:

$$G(n) = a \in \mathbb{N}^* \mid a \leq \sqrt{n}.$$

Determining any structural property of n therefore requires global access to the already constructed space.

Definition 4 (prime number as fixed point)

A number $p \in \mathbb{N}^*$ is called a prime in the constructive sense if:

$$V(a) \cap p = \varnothing \quad \text{for all } a \in \mathbb{N}^* \text{ with } 1 < a < p.$$

That is:
p is not a multiple of any earlier generated number — not because this is checked locally, but because such a multiplicative path never emerged globally.

A prime is therefore:
a fixed point in the growth process of the number space.

Definition 5 (irreducibility)

Whether p is a fixed point depends on the global history of the number space:

$$p \text{ is prime} \iff \forall a < p : p \notin V(a).$$

Since each $V(a)$ arises only during construction, it follows:

Primality is not locally computable, but an emergent result of the entire growth process.

Corollary (without proof here)

There exists no function

$$f\colon \mathbb{N}^* \to 0, 1$$

that decides primality solely from the numerical properties of n, without reference to the globally constructed structure of the number space:

$$\text{Primality is globally irreducible.}$$

A.2 Illustrative examples of local vs global structure

This section demonstrates the contrast between local arithmetic properties and global structural properties using minimal examples.

Example 1 – Local information cannot determine primality

Consider the natural number 29.

Local numerical properties:

property	value
digit sum	2 + 9 = 11
parity	odd
neighbours	28 and 30
nearest squares	$5^2 = 25$, $6^2 = 36$

None of these local features allows a decision:

$$\text{“prime or not”} \nRightarrow \text{decidable from local properties}$$

The correct statement follows only from global structure:

$$29 \text{ is prime} \iff 29 \notin V(a) \text{ for all } 2 \leq a \leq \sqrt{29}.$$

Thus primality is not a property of 29, but a property of all earlier numbers taken together.

Example 2 – Local tests sometimes produce correct answers, but never explanations

15 is divisible by 3. Local divisibility heuristics succeed here.

But:

- $91\ (= 7 \cdot 13)$
  has no obvious local feature and is often incorrectly guessed as prime.
- 97 (prime) has the same neutral local profile as 91.

Therefore:

$$\text{Local tests are not wrong — but incomplete.}$$

They fail not because of numerical weakness, but because they review the number instead of the global process that produced it.

Example 3 – Global emergence determines structure

Consider the first 12 natural numbers in the constructive model:

number	emergence	status
1	initial	neutral
2	+1 from 1	prime
3	+1 from 2	prime
4	$2 \cdot 2$	composite
5	+1 from 4	prime
6	$2 \cdot 3$	composite
7	+1 from 6	prime
8	$2 \cdot 4$	composite
9	$3 \cdot 3$	composite
10	$2 \cdot 5$	composite
11	+1 from 10	prime
12	$3 \cdot 4$	composite

Every prime can be explained only from the entire growth history of the number space.

A local inspection of the isolated number cannot show why this number did not become a product of earlier numbers.

Example 4 – Definitive contrast: local vs global

question	local analysis	global analysis
“Why is 29 prime?”	no answer	29 lies in none of the multiple spaces of earlier numbers
“When comes the next prime?”	no rule	emerges when growth does not produce a multiplicative collision
“Is there a formula for primes?”	expected	impossible in principle due to global dependence
“Does more computational power solve the problem?”	technically	structurally irrelevant

Thus the examples show:
Primality is not a local property of a number, but a global property of the growth process.

A.3 Classical vs constructive prime analysis

Classical number theory treats prime numbers as properties of individual numbers.
The constructive number space treats prime numbers as fixed points of a global growth process.

The differences can be systematically contrasted as follows:

Dimension	**Classical arithmetic**	**Constructive number space**
Fundamental object	Number as a static object	Number as the result of a growth process
Emergence	not modeled	explicitly modeled
Definition of a prime	no divisors < n	no multiplicative re-entry in global history
Information basis	local properties of n	complete history of the number space
Type of logic	local	global
Causality	not included	central
Multiplication	primitive operation	emergent effect of growth
Role of multiple spaces	analytical tool	constitutive structural principle
Why is a number prime?	no local explanation	fixed point: no multiplicative return path from earlier numbers
Detectability of distribution from	numeric patterns	global structural history
Expectation of a closed formula	explicit research goal	ruled out in principle
Role of $\zeta(s)$	analytical formula structure	projection of global structure
Role of Goldbach	open additive question	symmetric coupling of independent fixed points
Role of Littlewood	surprising oscillation	necessary over-/undercompensation
Role of Fermat	algebraic special case	topological separation of growth paths
Role of Collatz	unclear	global attractor despite local variability
Prediction of individual primes	conceivable goal	logically impossible

Dimension	Classical arithmetic	Constructive number space
Prediction of density	asymptotic	emergent from global stabilization
Relationship between theory and algorithm	separate	identical structure in both
Relationship between formula and structure	formula determines structure	structure determines projections (formulas)

Interpretive short summary of the table

Not as a rhetorical claim, but as a structural conclusion:

- Classical arithmetic attempts to adjust local projections (formulas, divisibility rules, analytic approximations) to the prime structure.
- The constructive number space explains primes as the result of the global emergence process — formulas reflect the structure, but do not generate it.

Thus it becomes clear:
Neither is the classical view “wrong” nor the constructive view a “replacement” — both describe the same object, but on different levels:
projection analysis vs emergence analysis.

A.4 The generative prime procedure – formal representation

Objective of the representation:
The algorithmic construction of primes from the global growth structure, not through divisibility tests.

Preliminary consideration (informal but rigorous)

In the constructive model, every composite number arises through the multiple formation of earlier numbers.

If this process is modeled explicitly, the following holds:
A number is prime exactly when it does not arise as a generated multiple of earlier numbers.

The generative prime procedure therefore does not construct numbers and then test whether they are prime.
It generates all multiples of earlier numbers and marks as prime exactly those numbers which are **not** produced by multiple formation.

Pseudocode

```
# initialization
emitters = {}          # set of active multiple generators
```

```
primes   = []          # list of discovered primes

# start
next_number = 2

while True:
    if next_number is contained in the multiple sets of the emitters:
        # composite number
        advance all relevant emitters to their next multiple
    else:
        # fixed point → prime
        mark next_number as prime
        insert a new emitter:
            emitter:
                current_multiple = next_number * 2
                step_size        = next_number

    next_number += 1
```

Formal definition of emitters

Each prime p creates a new emitter when it is discovered:

- start: $2p$
- step size: p
- iteration: $2p, 3p, 4p, 5p, \dots$

An emitter $E(p)$ therefore represents the full multiple space

$V(p) = p \cdot k \mid k \in \mathbb{N}^*$

The number space is constructed incrementally:

- If n is generated by some emitter $E(p) \to n$ is composite
- If n is not generated by any emitter $\to n$ is prime and generates a new emitter

Invariants of the procedure

1. completeness of multiple formation
   every multiple space grows monotonically without loss of information
2. uniqueness of prime detection
   a number can be prime only at the moment of its first appearance
3. nonlocality
   whether n is prime depends on whether any earlier emitter reaches n —
   not on local properties of n itself

Why the procedure is correct

Let n be the current number.

- If there exists an emitter $E(p)$ that produces n, then $n = p \cdot k$ with $p < n \to n$ is composite
- If no emitter produces n, then

$$\forall p < n : n \notin V(p)$$

$\to n$ is not a multiple of any earlier number
$\to n$ is prime in the sense of Definition A.4 of the constructive number space

Therefore:
The procedure is correct without divisibility tests, without factorization, and without local analysis of n.

Structural difference to the classical sieve

Sieve of Eratosthenes	Generative procedure
marks multiples “top-down”	generates multiples “inside-out”
requires a prefilled search range	constructs the range during execution
basis = static set	basis = growth process
prime follows from not being marked	prime follows from not arising

The generative procedure is therefore not an optimization of the sieve, but the algorithmic manifestation of the global emergence of structure.

Purpose of section A.4

- The algorithm is not aimed at numerical acceleration, but at visibly reconstructing the irreducibility principle.
- It is the algorithmic proof that primes do not have to be checked locally — they arise exactly when no global re-entry occurs.

A.5 Go implementation of the generative procedure

```
// prime_emitters.go
//
// Generative prime algorithm in the sense of the constructive number space:
//
// – No divisibility tests
// – No factorization
// – Each prime creates an "emitter" that grows its multiple-space V(p)
// – A number n is prime exactly if it is not generated by any emitter
//
// Example call:
```

```
//
//   go run prime_emitters.go -limit=100
//
// Output:
//   2
//   3
//   5
//   7
//   11
//   13
//   17
//   ...
//
// Store in file (shell):
//
//   go run prime_emitters.go -limit=1000000 > primes.txt
//
// The focus here is *conceptual clarity*, not maximal efficiency.

package main

import (
    "flag"
    "fmt"
)

// Emitter represents the multiple-space of a prime p:
//   next = next multiple of p that has not yet been "processed"
//   step = p itself (step size)
type Emitter struct {
    next uint64 // next multiple (2p, 3p, 4p, ...)
    step uint64 // prime p
}

func main() {
    // upper bound as flag
    limit := flag.Uint64("limit", 100, "highest number to be evaluated (inclusive)")
    flag.Parse()

    if *limit < 2 {
        // There are no primes < 2
        return
    }

    var emitters []Emitter

    // iterate through the number space from 2 to limit
    for n := uint64(2); n <= *limit; n++ {
        isComposite := false
```

```
        // 1) check whether n is generated by any existing emitter
        for i := range emitters {
            // move emitter forward until next >= n
            for emitters[i].next < n {
                emitters[i].next += emitters[i].step
            }
            if emitters[i].next == n {
                isComposite = true
                // we do not have to break; there can be multiple representations,
                // but one match is sufficient for the prime/non-prime decision
            }
        }

        // 2) if n was not generated by any emitter → prime / fixed point
        if !isComposite {
            // n is prime in the constructive sense
            fmt.Println(n)

            // new emitter for n:
            //   – start at 2n
            //   – step size = n
            emitters = append(emitters, Emitter{
                next: 2 * n,
                step: n,
            })
        }
    }
}
```

A.6 Historical classification: The Sieve of Eratosthenes as precursor

The Sieve of Eratosthenes, developed in the 3rd century BC, is considered one of the earliest algorithmic tools for determining prime numbers.

In classical literature it is usually described as an “eliminating method” — a procedure that successively marks multiples and thereby makes primes visible.

In the light of the constructive model, however, a deeper connection becomes clear:
The Sieve of Eratosthenes is not merely an exclusion procedure, but the first algorithmic access to the global generative structure of primes.

Eratosthenes understood — without an explicit theory of global fixpoints — that:

- every prime has its own non-local domain of influence (its multiples),
- and that the set of natural numbers becomes structured through the overlap of these domains.

Thus the sieve is based on exactly the same fundamental idea:
Primes do not become visible through local properties, but through the interaction of all

previously existing primes.

This idea is logically identical to the generative prime method in A.4/A.5;
the difference lies only in direction:

Principle	Sieve of Eratosthenes	Generative prime algorithm
Orientation	from the outside downward (global exclusion)	from the inside outward (global construction)
Form	eliminating	generating
Expression	marking multiples	emitters / fixpoint formation
Epistemic focus	making primes visible	reconstructing their principle of emergence

Therefore Eratosthenes is, historically speaking, not an exception but the first algorithmic access to the irreducibility structure of primes.

Modern literature usually underestimates the epistemic significance of the sieve, because it interprets it mainly numerically rather than structurally.

Minimal reference: Go code of the classical sieve

```
// eratosthenes.go — minimal comparison program
package main

import (
    "flag"
    "fmt"
)

func main() {
    limit := flag.Int("limit", 100, "highest number")
    flag.Parse()

    if *limit < 2 {
        return
    }

    marked := make([]bool, *limit+1)

    for p := 2; p*p <= *limit; p++ {
        if !marked[p] {
            for m := p * p; m <= *limit; m += p {
                marked[m] = true
            }
        }
    }
```

```
    for n := 2; n <= *limit; n++ {
        if !marked[n] {
            fmt.Println(n)
        }
    }
}
```

Closing remark for A.6

The comparison shows a remarkable detail:

- The Sieve of Eratosthenes does not explicitly state why the method works.
- The generative procedure makes the reason visible: primes are fixpoints of a global constructive process, not outcomes of local tests.

Thus the sieve forms a historical and conceptual precursor of the generative model — not as an “incomplete” method, but as a remarkably clear algorithmic intuition of the true structure of primes.

A.7 Glossary of the concepts used

Arithmetical space

The space of the natural numbers with their full multiplicative and additive structure.
In the paper it is not understood as a static object, but as a growing generative process.

Globality / non-locality

Term for properties that cannot be derived from local features of a number, but arise from its relation to the entire number space. Prime formation is, in the paper, a genuinely global phenomenon.

Fixpoint

A value within a generative process that remains stable even though the process continues to evolve.
Primes are understood as such fixpoints of maximal structural stability.

Generative production process

A process that builds from the inside outward rather than filtering by external elimination.
In the paper it denotes the mechanism by which primes arise in the growing number space.

Irreducibility

Property that the distribution of primes cannot be reduced to local rules.
The possibility that the global structure could be determined by a local formula is structurally

excluded.

Analytical projection

Mapping of a global generative process into a smooth, static analytical expression — for example the zeta function.
It reflects the structure but does not explain it.

Prime emitter

A previously discovered prime that writes active information back into the number space through its multiples.
In the generative model it is interpreted as a structural channel of influence.

Resonance field

Term for the superposed influence of all active prime emitters.
In analytical form it corresponds to the interference structure that determines where new fixpoints emerge.

Additive prime / pure prime

Terms introduced in Chapter 6 to distinguish:

Type	Meaning	Example
pure prime	arises as a fixpoint in the generative process without additive resonance	5, 7, 13, 19
additive prime	arises as the sum of two fixpoints and forms a stable resonance value	11 = 5 + 6, 17 = 10 + 7 (illustrative examples, not sum decompositions)

(The terms serve to explain Goldbach-type resonance structure, not to introduce a new classification.)

Growth topology

Analytical form that examines how values move under repeated application of structural operations.
It enables the interpretation of Fermat (Chapter 9) and Collatz (Chapter 10) within the same framework.

Attractor

A value or region toward which a dynamical system inevitably returns, regardless of the starting point.
In Collatz, 1 corresponds to the only stable attractor.

Epistemic boundary

Point at which a mathematical system is in principle no longer able to recognize that its means of expression are insufficient.
Applied to RH: analytical systems cannot fully capture the generative irreducibility of the primes.

Analytical vs. constructive

Paradigm	analytical	constructive
View	number space as object	number space as process
Goal	description	emergence
Means	formulas, projections	generative mechanisms, fixpoints
Limit	local information	no formula can replace global emergence

Key statement of the paper

Primes cannot merely be described — they must emerge.
Every local description is structurally incomplete because prime formation is a globally irreducible generative system.

A.8 Bibliography

[1] B. Riemann, “On the number of primes less than a given magnitude”,
Monatsberichte der Berliner Akademie, 1859.

[2] G. H. Hardy and J. E. Littlewood, “Some problems of ‘Partitio Numerorum’: III. On the expression of a number as a sum of primes”,
Acta Mathematica, vol. 44, pp. 1–70, 1923.

[3] A. Selberg, “An elementary proof of the prime number theorem”,
Annals of Mathematics, vol. 50, pp. 305–313, 1949.

[4] P. Turán, “On a new method of analysis and its applications”,
Wiley, New York, 1984.

[5] E. Bombieri and H. Iwaniec, “Some mean-value theorems for exponential sums”,
Annali della Scuola Normale Superiore di Pisa, 1986.

[6] A. Granville and K. Soundararajan, “The distribution of values of L-functions”,
Geometric and Functional Analysis, vol. 13, pp. 992–1028, 2003.

[7] T. Tao, “Structure and randomness in the prime numbers”,
Proceedings of the International Congress of Mathematicians (ICM), 2006.

[8] A. Wiles, “Modular elliptic curves and Fermat’s Last Theorem”,
Annals of Mathematics, vol. 141, pp. 443–551, 1995.

[9] S. Chaitin, “Algorithmic Information Theory”,
Cambridge University Press, Cambridge, 1987.

[10] A. M. Turing, “On Computable Numbers, with an Application to the Entscheidungsproblem”,
Proceedings of the London Mathematical Society, vol. 42, 1937.

[11] K. Gödel, “On formally undecidable propositions of Principia Mathematica and related systems I”,
Monatshefte für Mathematik und Physik, 1931.

[12] B. Green and T. Tao, “The primes contain arbitrarily long arithmetic progressions”,
Annals of Mathematics, vol. 167, pp. 481–547, 2008.

[13] H. Maier, “Primes in short intervals”,
Michigan Mathematical Journal, vol. 32, pp. 221–225, 1985.