World Coordinate Systems Representations Within the FITS Format[1]

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1 Introduction

When the original FITS format definition was published (Wells, Greisen, and Harten, 1981, *Astron. Astrophys. Suppl.* **44**, 363), a number of keywords were provided for the association of a *world coordinate system* (WCS) with the data array. Unfortunately, the explicit definitions of these keywords were not given, so that thus far it has not generally been possible to convey WCS information from one data analysis system to another in a concise and unambiguous manner. In other words, even though the syntax of the WCS information was defined, the semantics were not. The purpose of this document is to define the syntax *and* semantics for the expression of world coordinate systems within the FITS format.

The AIPS Group of the National Radio Astronomy Observatory adopted a number of conventions for use within the AIPS system for the expression of world coordinates. These conventions are fully expressed within the FITS format for sky images in several standard projection geometries. It is these basic conventions that are to be defined as the standard for use within FITS. The AIPS conventions, however, do not extend easily to data arrays of higher dimensionality, or to data arrays with skew world coordinate axes. Moreover, in order to have the WCS be accurate to of order 0.1 arcsec, the coordinate reference frame (FK4, FK5), equinox, and epoch must all be specified. This paper presents the complete set of standards to be used within the FITS format for the specification of the most common sky geometries.

2 The Basic Problem

The location of a pixel in a data array can be easily identified by giving its *pixel coordinate*, given that a pixel counting convention (0-based, 1-based) is defined. However, the vast amount of astronomical data requires an association between the pixel coordinate system and the *world* or *physical* coordinate system. For example, an array of spectral flux measurements needs to be associated with wavelengths through some sort of dispersion solution, or an array of time-resolved flux measurements needs to be associated with actual times in UTC. In the simplest cases, these associations can be expressed in linear terms:

$$\textit{world coordinate} = \textit{pixel coordinate} \cdot \textit{scale} + \textit{offset}$$

where *scale* is the pixel size (in world coordinate units) and *offset* is a zero-point correction. In more common cases, however, the relation between pixel coordinates and world coordinates is non-linear.

[1] This document is a DRAFT only and has not been reviewed or approved by any of the standing FITS committees or the IAU Commission 5 Working Group on FITS.

Perhaps the single most important example from astronomy is images of the sky. Any image of the sky is intrinsically a projection of the celestial sphere onto a plane. Thus, the objective in giving a description of the coordinate system in a FITS header is to provide an unambiguous and accurate mapping of pixel coordinates in the image to coordinates on the celestial sphere (equatorial, ecliptic, galactic).

There are three standard projection geometries which apply to the majority of optical, X-ray, and radio telescopes: a tangent-plane projection (also known as a gnomonic projection), in which the linear displacement from the point of tangency is proportional to the tangent of the angular displacement, a sine projection (used by most radio interferometers), and a Schmidt camera projection. A few east-west radio interferometers (such as the Westerbork Synthesis Radio Telescope) utilize a projection geometry based on the north celestial pole. These geometries are illustrated in Figure 1.

The problem of converting from pixel coordinates to world coordinates can be viewed in three major steps:

1. Convert the raw pixel coordinates into an idealized set of pixel coordinates, including any corrections for the rotation of the data array from the natural sky coordinates. *This is a linear operation.*

2. Convert the idealized pixel coordinates into sky coordinates by applying the proper geometric corrections for the type of projection.

3. Correct the sky coordinates (if necessary) to include the effects of equinox, epoch, and coordinate reference frame.

In practice, the first two steps above are generally combined mathematically into one step. AIPS Memos 27 and 46 describe in detail the conversions corresponding to the second step[2]. The third step may be required for comparing image data from different telescopes or taken at different times. The standard coordinate reference frames are equinox B1950.0 within the FK4 reference frame (e.g., the SAO Catalog), and J2000.0 within the FK5 reference frame (e.g., the Space Telescope Guide Star Catalog).

3 FITS WCS Definitions for Standard Sky Geometries

For a two-dimensional sky image (`NAXIS` = 2, with two spatial coordinates), a standard FITS header should use the following keywords to define the WCS:

`CRVAL1, CRVAL2` – the values of the world coordinates *at the point of tangency.* Values are given in decimal degrees. It is suggested that the values of `CRVAL1` and `CRVAL2` be given to double precision accuracies. The values of `CRVAL1` and `CRVAL2` *do not change* with image subsetting operations. The only way to change the values of `CRVAL1` and `CRVAL2` is to resample the data array onto a grid with a different point of tangency, or to transform the data into a different coordinate system (such as galactic longitude and latitute instead of right ascension and declination). In the latter case, however, `CRVAL1` and `CRVAL2` still correspond to the same point on the celestial sphere.

[2]Copies of these memos, written by Eric Greisen, are available from the National Radio Astronomy Observatory, AIPS Group, Edgemont Road, Charlottesville, VA 22903.

`CRPIX1, CRPIX2` – the pixel values corresponding to the point of tangency. Note that these values need not be integral values, and they need not correspond to a pixel that is physically within the array. For example, if a subset has been extracted from a larger image, the reference pixel/point of tangency may well lie outside the data array.

`CDELT1, CDELT2` – the pixel size (in physical units, decimal degrees) along each coordinate axis *at the point of tangency.*

`CROTA2` – the rotation angle (decimal degrees) between the 'declination-like' coordinate axis and the 2nd axis of the pixel array. `CROTA2` is measured positive in a counterclockwise direction. See Figure 2. `CROTA1` is not used. If there is no rotation, the keyword `CROTA2` need not appear. FITS reading programs should assume a value for `CROTA2` of zero if a value is not explicitly given in the FITS header.

`CTYPE1, CTYPE2` – the type of world coordinate axes. The `CTYPE` value strings are eight characters in length, and are constructed from two four-character segments. The first four characters give the type of world coordinate, and the second four characters give the type of projection geometry. The first four-character string is left-justified and filled out to a length of four characters by appending hyphens; the second four-character string is right-justified and filled out to a length of four characters by prepending hyphens. The following coordinate pairs and geometries are recognized:

Coordinates	Description
`RA--, DEC-`	equatorial coordinates (α, δ)
`GLON, GLAT`	galactic coordinates (ℓ^{II}, b^{II})
`ELON, ELAT`	ecliptic coordinates (λ, β)

Projection	Description
`-TAN`	tangent plane (gnomonic)
`-SIN`	sine
`-ARC`	arc (Schmidt cameras)
`-NCP`	north celestial pole (east-west interferometers)
`-STG`	stereographic
`-MER`	Mercator
`-AIT`	Aitoff
`-GLS`	global sinusoidal

An example of a valid `CTYPE` is '`DEC--TAN`'. Rigorous interpretation is not possible where unrelated coordinate types or geometries are mixed – a combination of '`RA---TAN`' with '`GLON-AIT`', for example, is not meaningful. Spatial coordinates must be supplied in logically consistent pairs.

The fact that these `CTYPE`s are defined does not imply that all systems supporting FITS input and output must understand them. However, data arrays which map into one of the above coordinate systems and geometries should use this notation. Extensions of the notation to other types of projections and other types of coordinates are certainly possible.

Equatorial coordinates '`RA--/DEC-`' require further qualification; see the discussion of coordinate reference frames below (section **??**).

Galactic coordinates '`GLON/GLAT`' conform to the IAU definitions of 1958 known as (ℓ^{II}, b^{II}).

Ecliptic coordinates '`ELON/ELAT`' refer to the mean ecliptic and equinox of date, in the post-IAU 1976 system. The date is determined via the `MJD-OBS` or `DATE-OBS` keyword (section **??**).

For the tangent-plane, sine, and Schmidt projection geometries, transformations between the three coordinate systems (α, δ), (ℓ^{II}, b^{II}), and (λ, β), can be expressed via simple header modifications (to CRVAL1, CRVAL2, and CROTA2).

The conventions described above do not accommodate coordinate systems with three or more data axes (three rotation angles are needed for NAXIS = 3, six for NAXIS = 4, and ten for NAXIS = 5, and the semantic definitions for the rotation angles become unwieldly) or with skew data axes. Rather than define a CSKEW keyword, which would only have a straightforward interpretation for two-dimensional arrays, a generic coordinate description matrix can be used. The coordinate description (CD) matrix gives the partial derivatives of each physical coordinate axis with respect to each pixel coordinate axis at the point of tangency. That is, we have new FITS keywords of the form CDiiijjj[3]:

$$\mathtt{CDiiijjj} = \left(\frac{\partial \text{world coordinate iii}}{\partial \text{pixel coordinate jjj}} \right) \Bigg|_{\text{tangent point}}$$

The indices iii and jjj run from 1 to NAXIS, so that a NAXIS $\times$ NAXIS matrix is formed. For simple, non-rotated sky images, CD001001 = CDELT1, CD002002 = CDELT2, and the cross-terms in the matrix are both zero and can be omitted from the FITS header.

One transforms from raw pixel coordinates (x, y) to linear sky coordinates (l, m) (with rotation and skew removed), using the following relations:

$$\begin{bmatrix} \Delta l \\ \Delta m \end{bmatrix} = \begin{bmatrix} \mathtt{CD001001} & \mathtt{CD001002} \\ \mathtt{CD002001} & \mathtt{CD002002} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

where $(\Delta x, \Delta y)$ is the displacement in pixels from the reference pixel (CRPIX1, CRPIX2), and $(\Delta l, \Delta m)$ is the displacement in degrees from the reference world coordinate (CRVAL1, CRVAL2).

In the case where the sky image is rotated with respect to the pixel array, it is straightforward to convert from the CDELT/CROTA notation to the CD-matrix notation:

$$\begin{aligned}
\mathtt{CD001001} &= \mathtt{CDELT1} \cos\theta \\
\mathtt{CD001002} &= |\mathtt{CDELT2}| \operatorname{sign}(\mathtt{CDELT1}) \sin\theta \\
\mathtt{CD002001} &= -|\mathtt{CDELT1}| \operatorname{sign}(\mathtt{CDELT2}) \sin\theta \\
\mathtt{CD002002} &= \mathtt{CDELT2} \cos\theta
\end{aligned}$$

(The *sign* function represents the sign of its argument, and has the value $+1$ or -1.)

The reverse transformation, from CD-matrix to CDELT/CROTA notation, is a bit more complicated. The magnitudes of the CDELTs are found from

$$\begin{aligned}
|\mathtt{CDELT1}| &= \sqrt{\mathtt{CD001001}^2 + \mathtt{CD002001}^2} \\
|\mathtt{CDELT2}| &= \sqrt{\mathtt{CD001002}^2 + \mathtt{CD002002}^2}
\end{aligned}$$

[3]The STSDAS software uses the notation CDi_j rather than CDiiijjj. The advantage is that for typical WCS notations, the keywords are simpler and easier to read (CD1_1, CD1_2, CD2_1, CD2_2 instead of CD001001, CD001002, etc.). The notation CDiiijjj was originally proposed in order to assure compatibility with the most general FITS image, i.e., a 999 $\times$ 999 dimensional image with all possible cross-terms in the CD matrix. As this is a rather unlikely occurrence, and since STSDAS routinely handles only 2 (and occasionally 3) dimensional images, it uses the simpler notation. [Note added 30 October 1992 by RJH.]

The signs of the CDELTs are determined from the determinant of the CD-matrix. That is, the sign of the determinant is equal to the sign of the product of the CDELTs:

$$\text{sign}(\texttt{CDELT1} \cdot \texttt{CDELT2}) = \text{sign}(\texttt{CD001001} \cdot \texttt{CD002002} - \texttt{CD001002} \cdot \texttt{CD002001})$$

This equation does not determine the signs of CDELT1 and CDELT2 uniquely, but as long as the rest of the transformation is done consistently, it does not really matter. As a matter of convention for astronomical images, if the determinant of the CD-matrix is negative, then CDELT1 should be negative (consistent with a data matrix more or less aligned with right ascension and declination).

Finally, the rotation angle is given by

$$\begin{aligned}
\texttt{CROTA2} &= \arctan\left(\frac{\text{sign}(\texttt{CDELT1} \cdot \texttt{CDELT2})\texttt{CD001002}}{\texttt{CD002002}}\right) \\
&= \arctan\left(\frac{-\text{sign}(\texttt{CDELT1} \cdot \texttt{CDELT2})\texttt{CD002001}}{\texttt{CD001001}}\right)
\end{aligned}$$

In these expressions it is strongly recommended that implementors use the Fortran ATAN2 function or its equivalent in order to avoid the 180 degree ambiguity of the ATAN function. Since both expressions are equally valid, it is recommended that the average values of the arguments to the *arctan* function be used in order to minimize computational round-off errors. Similarly, if the values differ beyond the expected computational precision, users should be warned that the coordinate axes are not orthogonal. If the sky image is skew or rotated and skew, then the CD matrix notation must be used.

The CD matrix should be incorporated into FITS headers according to the following plan:

- Any image with skew coordinate axes must use the CD matrix notation if the coordinate system is being represented.
- FITS images may be written using the CDELT/CROTA notation for non-skew coordinate axes, but are strongly encouraged to add the CD matrix as well.
- All FITS reading programs should be able to interpret the CDELT/CROTA notation if the CD matrix is not present. If both the CD matrix and CDELT/CROTA notations are given, the CD matrix takes precedence. Discrepancies between the two notations should be reported to the user.
- Since the CD matrix notation is directly extensible to data arrays of three or more dimensions, it is the preferred notation for such cases.

The developers of FITS I/O programs are urged to make the transition from the CDELT/CROTA notation to the CDiiijjj notation.

4 Coordinate Reference Frames

In order to rigorously define the coordinate systems being used, several additional keywords are required. For example, equatorial coordinates (α, δ) are relative to a given equinox and epoch, and relative to a fundamental coordinate system.

RADECSYS is a new keyword which specifies the frame of reference for the equatorial coordinate system. RADECSYS is a character string with one of four possible values:

RADECSYS	Definition
FK4	mean place, old (pre-IAU 1976) system
FK4-NO-E	mean place, old system but without e-terms
FK5	mean place, new (post-IAU 1976) system
GAPPT	geocentric apparent place, post-IAU 1976 system

If the CTYPEs are an RA--/DEC- pair, RADECSYS should also be specified. If RADECSYS is not given, the reference frame is assumed to be FK4.

All three mean place types require knowing the epoch of the mean equator and equinox. The term *equinox* is generally used to refer to this time, and this should be given using the new floating point keyword EQUINOX. The default equinox for FK4 and FK4-NO-E is 1950.0, and for FK5 it is 2000.0. For RADECSYS = 'FK4' or RADECSYS = 'FK4-NO-E' the number is interpreted as a Besselian epoch, whereas for RADECSYS = 'FK5' the number is interpreted as a Julian epoch.

The previous FITS definition included the keyword EPOCH, however, the use of this keyword was inconsistent with standard astrometric conventions. The use of the keyword EPOCH should be discontinued in FITS headers. FITS readers encountering EPOCH should interpret it as equivalent to EQUINOX.

The FK4 reference frame is not inertial – there is a small but significant rotation relative to distant objects. For this reason an epoch (specifying when the mean place was correct) is required in addition to an equinox (which specifies the reference frame). The epoch should be given in keywords MJD-OBS or DATE-OBS. MJD-OBS is a new keyword which specifies the time of observation in the TAI timescale in the form of a Modified Julian Date (JD - 2400000.5). The time is assumed to be representative of the observation for the purposes of velocity corrections, etc., and may, for example, be midway between the start and end of an exposure. For many applications the distinction between International Atomic Time (TAI) and the more familiar UTC may be ignored. If MJD-OBS is absent, DATE-OBS (in the format DD/MM/YY) may be used instead. However, because the latter is given only to one day resolution and without the timescale being specified, the Modified Julian Date specification is preferable.

In the absence of MJD-OBS or DATE-OBS it may be assumed that the date of observation is the same as the equinox, with a corresponding error introduced into coordinate calculations.

Coordinates given as geocentric apparent positions GAPPT require only the additional specification of the epoch of the observation, supplied via the MJD-OBS or DATE-OBS keywords.

5 Extensions/Enhancements of the FITS Definitions

The conventions described here allow for the expression of standard sky projection geometries within FITS format data files. Problems such as non-linear spectral dispersion solutions (polynomials, splines, piecewise linear), irregularly spaced data, and distortions to otherwise standard geometries, are not considered. Rather than issue a recommended standard in these areas, where general solutions have yet to be designed and implemented, discussions and trial implementations will continue before a standard is defined.

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