

# **COMP3308/COMP3608, Lecture 3b**

# **ARTIFICIAL INTELLIGENCE**

## **Local Search Algorithms**

**Reference: Russell and Norvig, ch. 4**

# Outline

- **Optimisation problems**
- **Local search algorithms**
  - **Hill-climbing**
  - **Beam search**
  - **Simulated annealing**
  - **Genetic algorithms**

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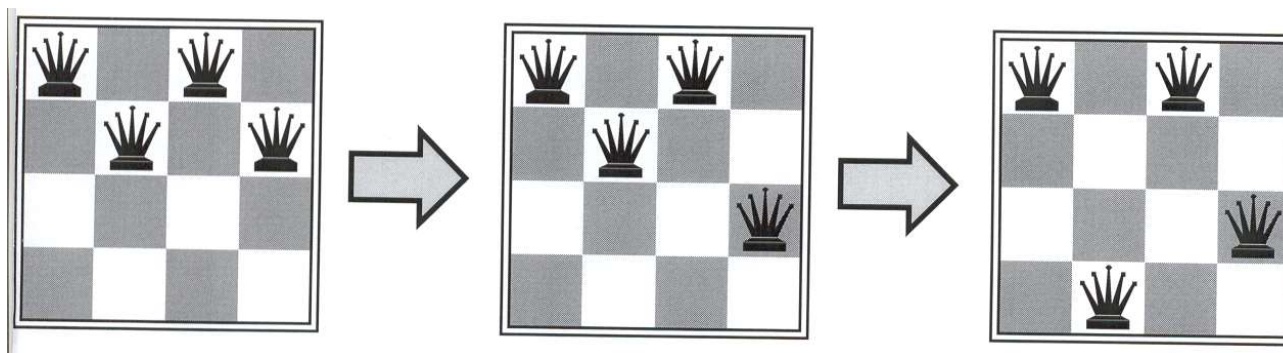
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# Optimisation Problems

- **Problem setting so far: path finding**
  - **Goal:** find a path from S to G
  - **Solution:** the path
  - **Optimal solution:** least cost path
  - **Search algorithms:**
    - **Uninformed:** BFS, UCS, DFS, IDS
    - **Informed:** greedy, A\*
- **Now a new problem setting: optimisation problem**
  - **Each state has a value  $v$**
  - **Goal:** find the **optimal state**
    - = the state with the highest or lowest  $v$  score (depending on what is desirable, maximum or minimum)
  - **Solution:** the state; the path is not important
- **A large number of states => can't be enumerated**
  - => **We can't apply the previous algorithms – too expensive**

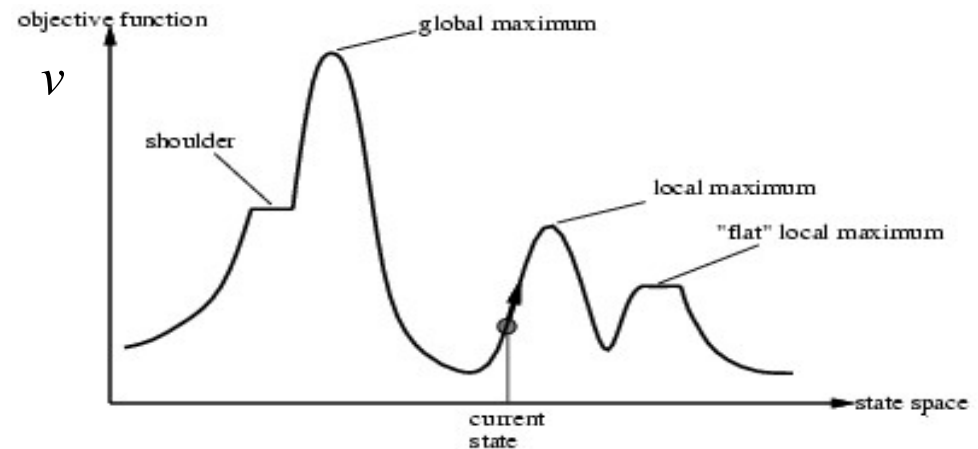
# Optimisation Problems - Example

- **n-queens problem**
  - The solution is the goal configuration, not the path to it
- **Non-incremental formulation**
  - **Initial state:** n-queens on the board (given or randomly chosen)
  - **States:** any configuration with n-queens on the board
  - **Goal:** no queen is attacking each other
  - **Operators:** “move a queen” or “move a queen to reduce the number of attacks”



# V-value Landscape

- Each state has a value  $v$  that we can compute
- This value is defined by a heuristic *evaluation function* (also called *objective function*)
- Goal - 2 variations depending on the task:
  - find the state with the highest value (global maximum) or
  - find the state with the lowest value (global minimum)
- Complete local search – finds a goal state if one exists
- Optimal local search – finds the best goal state – the state associated with the global maximum/minimum



# Hill-Climbing Algorithm - Idea

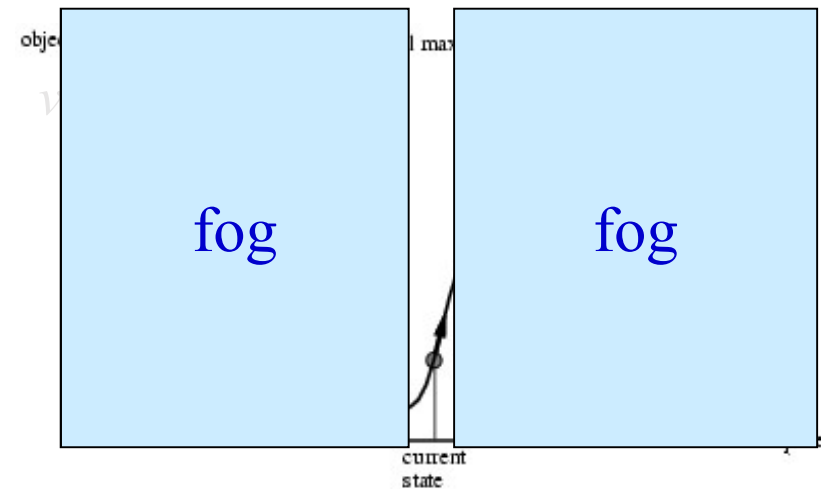
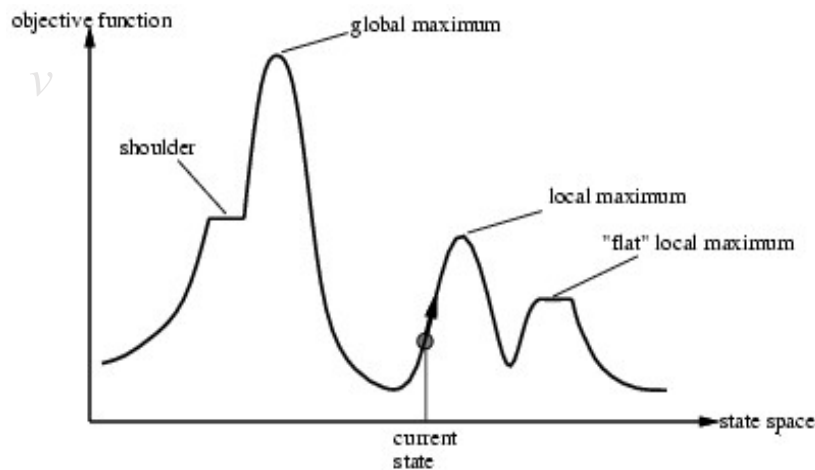
- Also called *iterative improvement* algorithm
- Idea: Keep only a single state in memory, try to improve it
- Two variations:
  - Steepest *ascent* – the goal is the *maximum* value
  - Steepest *descent* – the goal is the *minimum* value

# Hill-climbing Search

- Assume that we are looking for a maximum value (i.e. hill-climbing ascend)
- Idea: move around trying to find the highest peak
  - Store only the current state
  - Do not look ahead beyond the immediate neighbors of the current state
  - If a neighboring state is better, move to it & continue, otherwise stop
  - “Like climbing Everest in thick fog with amnesia”

we can't see the whole landscape,  
only the neighboring states

keeps only 1 state in memory, no  
backtracking to previous states



# Hill-climbing Algorithm

## Hill-climbing descent

**1) Set current node  $n$  to the initial state  $s$**

(The initial state can be given or can be randomly selected)

**2) Generate the successors of  $n$ . Select the best successor  $n_{best}$ ; *it is the successor with the best  $v$  score,  $v(best)$  (i.e. the lowest score for descent)***

**3) If  $v(best) > v(n)$ , return  $n$  //compare the child with the parent; if child not  
//better – stop; local or global minimum found**

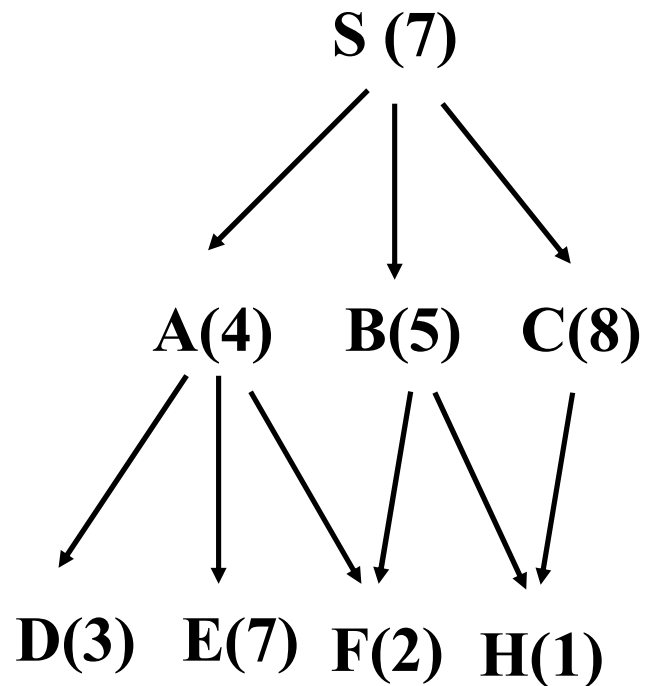
**Else set  $n$  to  $n_{best}$  . Go to step 2 //if better - accept the child and keep  
// searching**

- **Summary: Always expands the best successor, no backtracking**



# Hill-climbing – Example 1

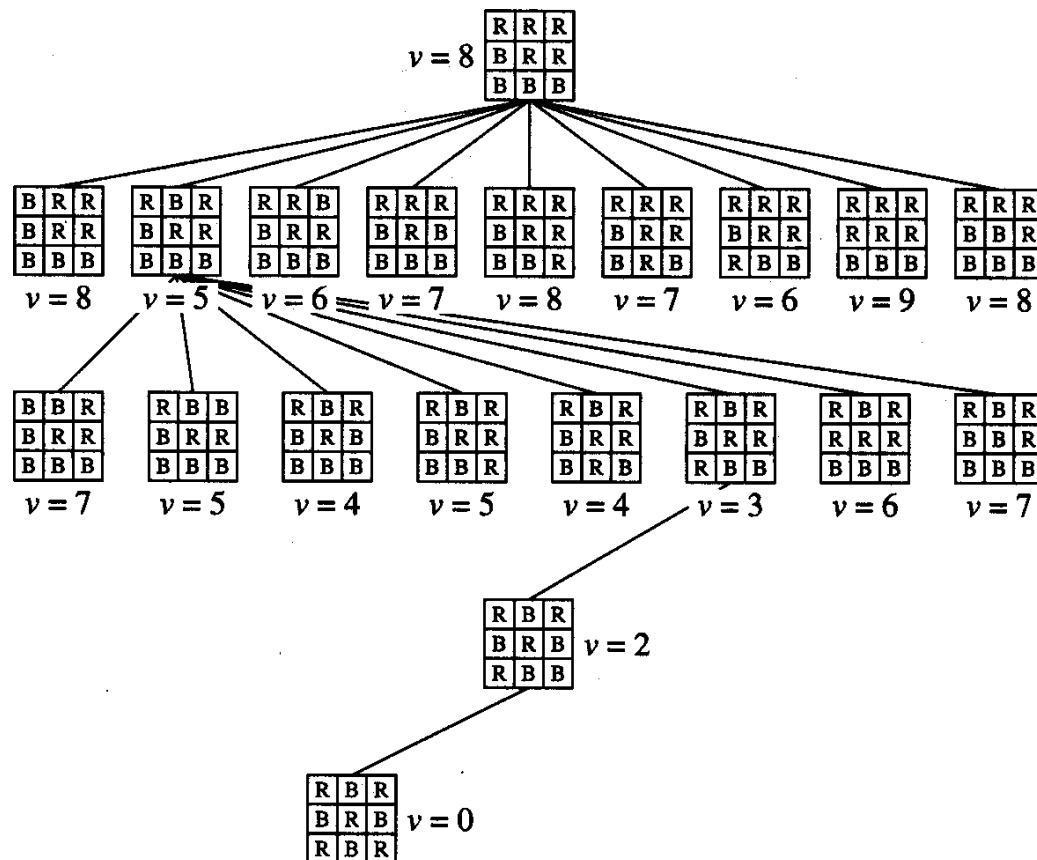
- **$v$  - value is in brackets; the lower, the better (i.e. descent)**
- **Expanded nodes: SAF**



## Hill-climbing – Example 2

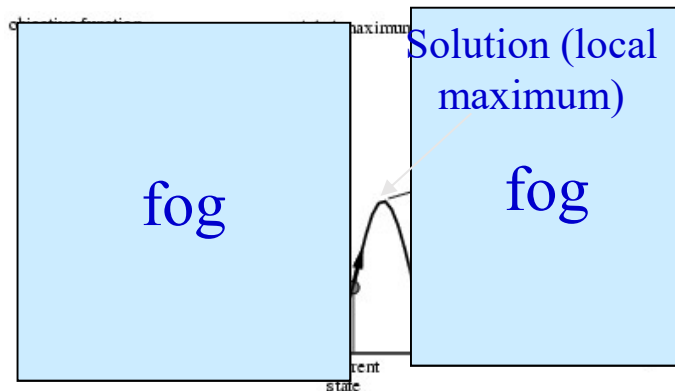
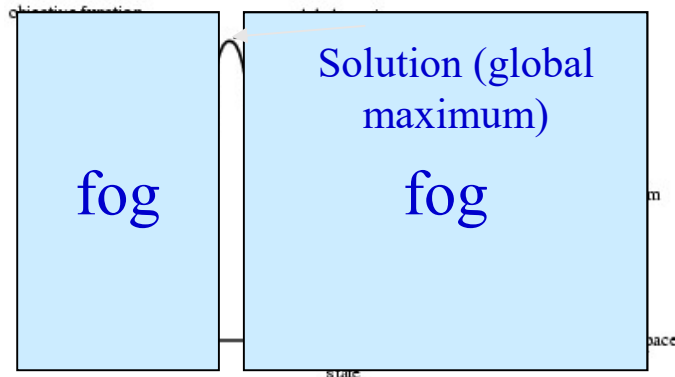
- **Given:** 3x3 grid, each cell is colored either red (R) or blue (B)
- **Aim:** find the coloring with minimum number of pairs of adjacent cells with the same color
- **Ascending or descending?**

$v$  – # pairs of adjacent cells with the same color



Picture from N. Nielsen, AI, 1998

# Hill-climbing Search



## Weaknesses:

- Not a very clever algorithm – can easily get stuck in a local optimum (maximum/minimum)
- However, not all local maxima/minima are bad – some may be reasonably good even though not optimal

## Advantages: good choice for hard, practical problems

- Uses very little memory
- Finds reasonable solutions in large spaces where systematic algorithms are not useful

**Not complete, not optimal but keeps just one node in memory!**

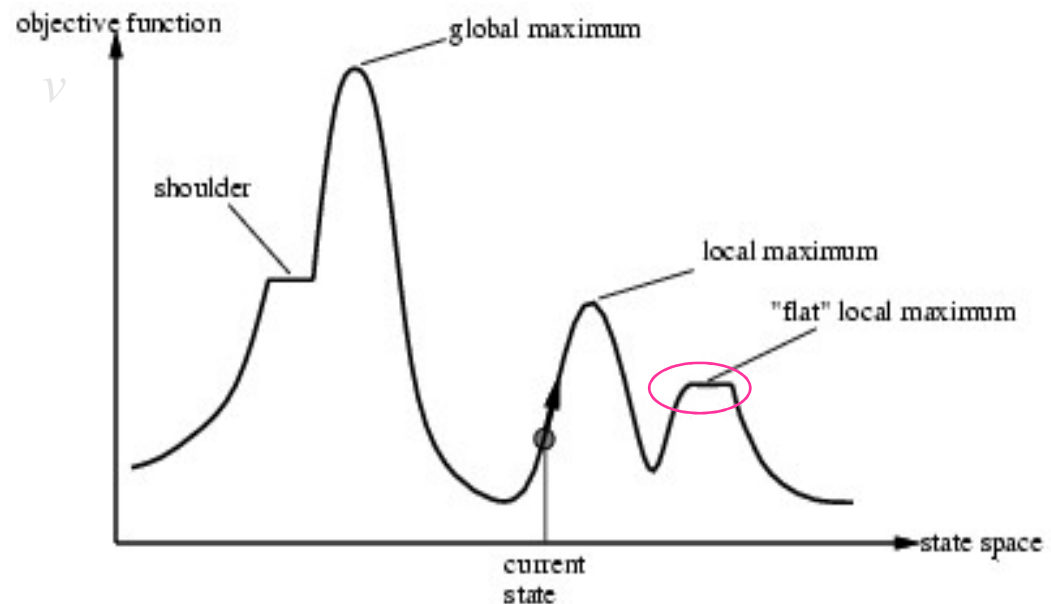
# Hill-climbing – Escaping Bad Local Optima

- Hill climbing finds the closest local optimum (minimum or maximum, depending on the version – descent or ascent)
  - Which may or may not be the global optimum
- The solution that is found depends on the initial state
- When the solution found is not good enough - random restart:
  - run the algorithm several times starting from different points; select the best solution found (i.e. the best local optimum)
  - *If at first you don't succeed, try, try again!*
  - This is applicable for tasks without a fixed initial state

# Hill-climbing – Escaping Bad Local Optima (2)

- **Plateaus (flat areas):** no change or very small change in  $v$
- **Our version of hill-climbing does not allow visiting states with the same  $v$  as it terminates if the best child's  $v$  is the same as the parent's**
- **But other versions keep searching if the values are the same and this may result in visiting the same state more than once and walking endlessly**

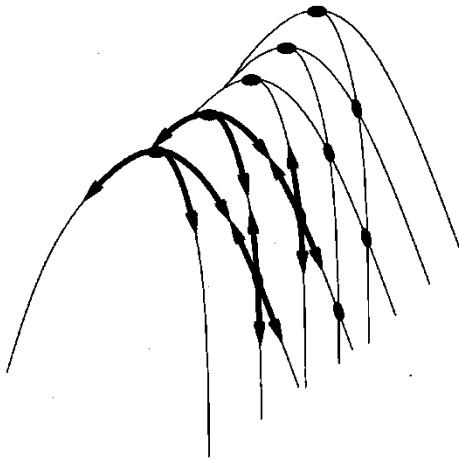
- **Solution: keep track of the number of times  $v$  is the same and do not allow revisiting of nodes with the same  $v$**



# Hill-climbing – Escaping Bad Local Optima (3)

- **Ridges** – the current local maximum is not good enough; we would like to move up but all moves go down

- **Example:**



- Dark circles = states
- A sequence of local maxima that are not connected with each other
- From each of them all available actions point downhill.

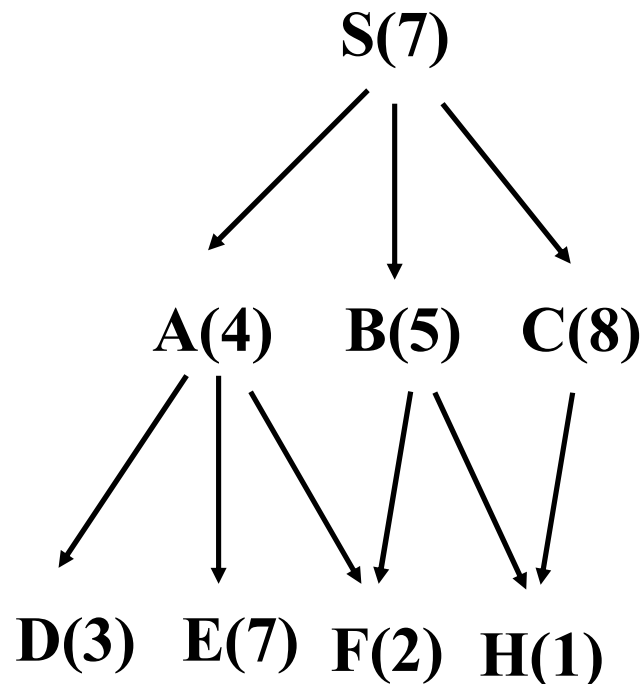
- **Possible solutions:** combine 2 or more moves in a macro move that can increase the height or allow a limited number of look-ahead search

# Beam Search

- It keeps track of  $k$  states rather than just 1
- **Version 1: Starts with 1 given state**
  - At each level: generate all successors of the given state
  - If any one is a goal state, stop; else select the  $k$  best successors from the list and go to the next level
- **Version 2: Starts with  $k$  randomly generated states**
  - At each level: generate all successors of all  $k$  states
  - If any one is a goal state, stop; else select the  $k$  best successors from the list and go to the next level
- In nutshell: keeps only  $k$  best states

## Beam Search - Example

- Consider the version that starts with 1 given state
- Starting from S, run beam search with  $k=2$  using the values in brackets as evaluation function (the smaller, the better)
- Expanded nodes = ?



- S
- generate ABC
- select AB (the best 2 children)
- generate DEFH
- select FH (the best 2 children)
- expanded nodes: SABFH



# Beam Search and Hill-Climbing Search

- **Compare beam search with 1 initial state and hill climbing with 1 initial state**
  - **Beam – 1 start node, at each step keeps  $k$  best nodes**
  - **Hill climbing – 1 start node, at each step keeps 1 best node**
- **Compare beam search with  $k$  random initial states and hill-climbing with  $k$  random initial states**
  - **Beam –  $k$  starting positions,  $k$  threads run in parallel, passing of useful information among them as at each step the best  $k$  children are selected**
  - **Hill climbing –  $k$  starting positions,  $k$  threads run individually, no passing of information among them**

## Beam Search with A\*

- **Recall that memory was a big problem for A\***
- **Idea: keep only the best  $k$  nodes in the fringe, i.e. use a priority queue of size  $k$**
- **Advantage: memory efficient**
- **Disadvantage: neither complete, nor optimal**

# Simulated Annealing

- **What is annealing in metallurgy?**
    - a material's temperature is gradually decreased (very slowly) allowing its crystal structure to reach a minimum energy state
  - **Similar to hill-climbing but selects a random successor instead of the best successor (step 2 below)**
- 1) **Set current node  $n$  to the initial state  $s$ .  
Randomly select  $m$ , one of  $n$ 's successors**
  - 2) **If  $v(m)$  is better than  $v(n)$ ,  $n=m$  //accept the child  $m$   
Else  $n=m$  with a probability  $p$  //accept the child  $m$  with probability  $p$**
  - 3) **Go to step 2 until a predefined number of iterations is reached or the state reached (i.e. the solution found) is good enough**

# The Probability $p$

parent  $n$



child  $m$

- Assume that we are looking for a minimum
- There are different ways to define  $p$ , e.g. 
$$p = e^{\frac{v(n)-v(m)}{T}}$$
- Main ideas:
  - 1)  $p$  decreases exponentially with the badness of the child (move) and
  - 2) bad children (moves) are more likely to be allowed at the beginning than at the end
- nominator — shows how good the child  $m$  is
  - Bad move (the child is worse than the parent):  $v(n) < v(m)$ , e.g.
  - case1:  $v(n)=10, v(m)=20, p1=e^{-10/T}$
  - case2:  $v(n)=10, v(m)=11, p2=e^{-1/T}$
  - $m$  (the child) in case1 is worse than in case2
  - $p1 < p2$  as  $T$  is positive  $\Rightarrow p$  exponentially decreases with the badness of the move

## The Probability $p$ (2)

$$p = e^{\frac{v(n)-v(m)}{T}}$$

- **denominator: parameter  $T$  that decreases (anneals) over time based on a *schedule*, e.g.  $T=T*0.8$** 
  - **high  $T$  – bad moves are more likely to be allowed**
  - **low  $T$  – more unlikely; becomes more like hill-climbing**
  - **$T$  decreases with time and depends on the number of iterations completed, i.e. until “bored”**
- **Some versions have an additional step (Metropolis criterion for accepting the child):**
  - **$p$  is compared with  $p'$ , a randomly generated number  $[0,1]$**
  - **If  $p > p'$ , accept the child, otherwise reject it**
- **In summary, simulated annealing combines a **hill-climbing step** (accepting the best child) with a **random walk step** (accepting bad children with some probability). The random walk step can help escape bad local minima.**

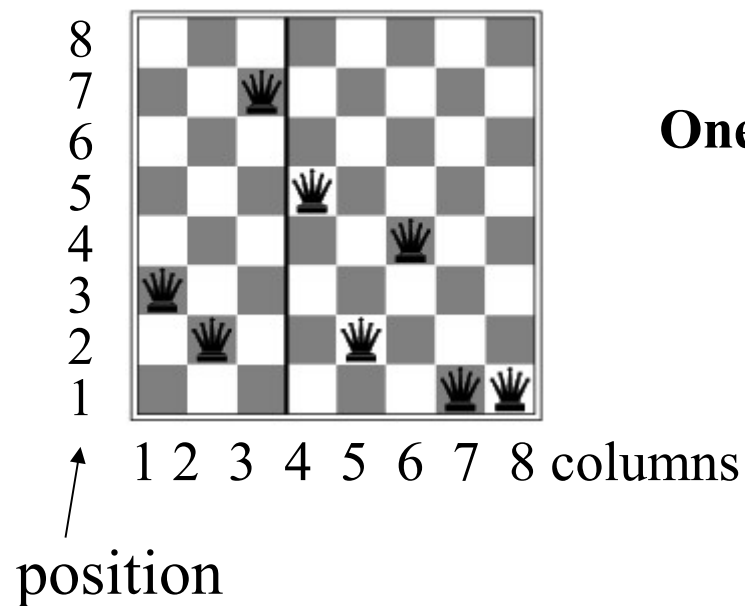
# Simulated Annealing - Theorem

- **What is the correspondence?**
  - $v$  – total energy of the atoms in the material
  - $T$  – temperature
  - *schedule* – the rate at which  $T$  is lowered
- **Theorem: If the schedule lowers  $T$  slowly enough, the algorithm will find global optimum**
  - i.e. is complete and optimal given a long enough cooling schedule => annealing schedule is very important
- **Simulated annealing has been widely used to solve VLSI layout problems, factory scheduling and other large-scale optimizations**
- **It is easy to implement but a “slow enough” schedule may be difficult to set**

# Genetic Algorithms

- Inspired by mechanisms used in evolutionary biology, e.g. selection, crossover, mutation
- Similar to beam search, in fact a variant of stochastic beam search
- Each state is called an *individual*. It is coded as a string.
- Each state  $n$  has a fitness score  $f(n)$  (evaluation function). The higher the value, the better the state.
- Goal: starting with  $k$  randomly generated individuals, find the optimal state
- Successors are produced by **selection, crossover and mutation**
- At any time keep a fixed number of states (the population)

## Example – 8-queens Problem



One possible encoding is (3 2 7 5 2 4 1 1)

column 1: a queen  
at position 3

column 2: a queen  
at position 2

...



## Example – 8-queens Problem (2)

- Suppose that we are given 4 individuals (initial population) with their fitness values
  - **Fitness values = number of non-attacking pairs of queens (given 8 queens, how many different pairs of queens are there?  $28 \Rightarrow$  max value is 28)**
  - **Let the probability for reproduction is proportional to the fitness (expressed as %)**

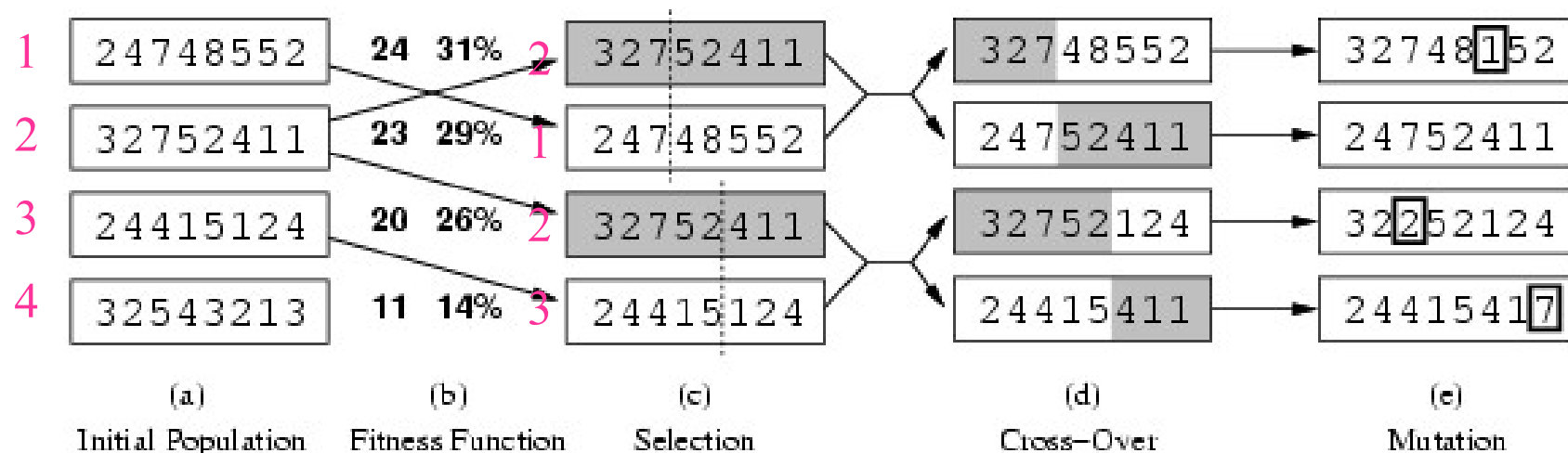
24748552	24	31%
32752411	23	29%
24415124	20	26%
32543213	11	14%

(a)

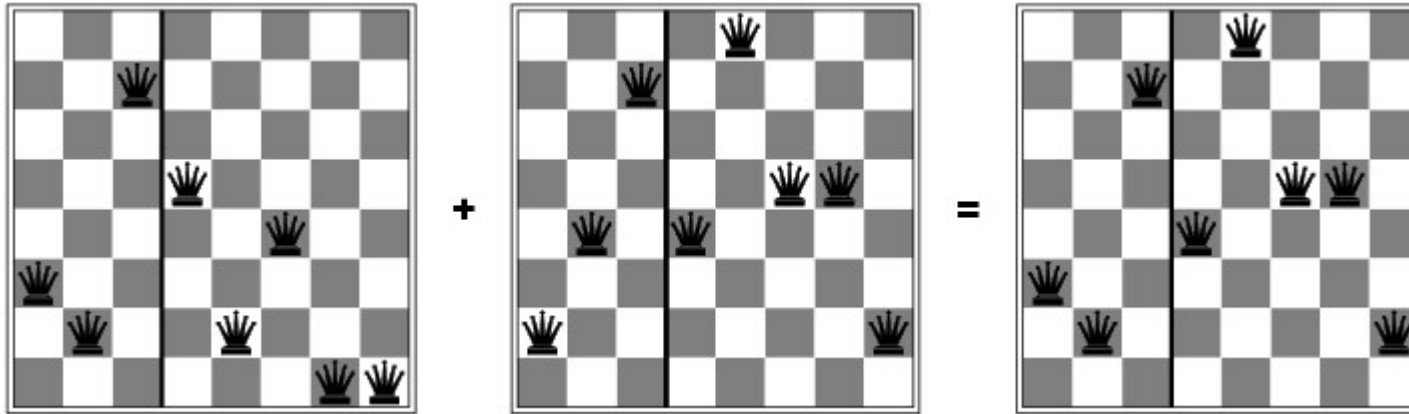
Initial Population

## Example – 8-queens Problem (3)

- **Select** 4 individuals for reproduction based on the fitness function
  - The higher the fitness function, the higher the probability to be selected
  - Let individuals 2, 1, 2 and 3 be selected, i.e. individual 2 is selected twice while 4 is not selected
- **Crossover** – random selection of crossover point; crossing over the parents strings
- **Mutation** – random change of bits (in this case 1 bit was changed in each individual)



## A Closer Look at the Crossover



$$(3\ 2\ 7\ 5\ 2\ 4\ 1\ 1) + (2\ 4\ 7\ 4\ 8\ 5\ 5\ 2) = (3\ 2\ 7\ 4\ 8\ 5\ 5\ 2)$$

- When the 2 states are different, crossover produces a state which is a long way from either parents
- Given that the population is diverse at the beginning of the search, crossover takes **big** steps in the state space early in the process and **smaller** later, when more individuals are similar

# Genetic Algorithm – Pseudo Code (1 variant)

from <http://pages.cs.wisc.edu/~jerryzhu/cs540.html>

1. Let  $s_1, \dots, s_N$  be the current population
  2. Let  $p_i = f(s_i) / \sum_j f(s_j)$  be the reproduction probs
  3. FOR  $k = 1; k < N; k += 2$ 
    - parent1 = randomly pick  $s$  with probs  $p$
    - parent2 = randomly pick another  $s$  with probs  $p$
    - randomly select a crossover point, swap strings of parents 1, 2 to generate children  $t[k], t[k+1]$
  4. FOR  $k = 1; k \leq N; k++$ 
    - Randomly mutate each position in  $t[k]$  with a small probability
  5. The new generation replaces the old:  $\{s\} \leftarrow \{t\}$ .
- Repeat until some individual is fit enough or a predefined maximum number of iterations has been reached**

# Genetic Algorithms - Discussion

- **Combine:**
  - **uphill tendency (based on the fitness function)**
  - **random exploration (based on crossover and mutation)**
- **Exchange information among parallel threads - the population consists of several individuals**
- **The main advantage comes from crossover**
- **Success depends on the representation (encoding)**
- **Easy to implement**
- **Not complete, not optimal**

# Links

- **Simulated annealing as a training algorithm for backpropagation neural networks:**
  - **R.S. Sexton, R.E. Dorsey, J.D. Johnson, *Beyond backpropagation: using simulated annealing for training neural networks*, [people.missouristate.edu/randallsexton/sabp.pdf](http://people.missouristate.edu/randallsexton/sabp.pdf)**