RSM8423 Assignment 1: BioPharma Case Analysis

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March 12th, 2024

1 Problem Overview

BioPharma, Inc. is a global manufacturer of bulk chemicals and recently, the company has been considering restructuring its global production network due to its decline in profits and high costs at its German and Japanese plants. The company produces two patented chemicals, Highcal and Relax, and serves 6 different regions with its 6 plants located across the world. Tables 6-18, 6-19, 6-20, 6-21 and 6-22 in Chapter 6 of Supply Chain Management, provide data on the historical sales, production, costs and other information related to the company's current production network. This study uses historical data to construct production network models to try and find an optimal production network structure that can minimize the company's costs.

1.1 Capacitated Plant Location Models (Q1)

Originally, BioPharma had all 6 plants in operation, with all of them producing both chemicals, except for the plants in Japan and Germany, which only produced HighCal in 2013. The company is considering shutting down some plants or limiting the production of some plants to only one chemical. To determine which ones to change, we can use a capacitated plant location model which can determine the lowest-cost network. The model returns the number of plant locations and how much they should produce for each market by minimizing the cost of meeting demand given a set of demand and capacity constraints. Figure 1 in the Appendix shows the objective function, constraints and variables for our first model.

In the original model, where all plants are in operation and producing both chemicals, the total cost was 1,288.84 million USD including import duties. From the results of the first model, we can see that total cost dropped by about 22.66 million USD to 1,267.179 million USD including import duties after changing the production of some of the plants. Specifically, the optimal low-cost configuration had all plants producing both chemicals but the German and Japanese plant; the German plant is limited to only producing Relax and the Japanese plant is shut down (See Figure 2). This could be because, for one, the German plant has a much lower capacity per variable HighCal cost compared to the other plants that produce HighCal (See Figure 3); therefore, it might be more coefficient to source HighCal from other plants instead of the German one. We can also see that Japan has a much lower capacity across all different types of costs, which suggests that it may be more cost-efficient to close the Japan plant and produce the chemicals from the other plants.

1.2 Proposed Network Structures (Q2)

To further our network analysis, we can construct two more models. The second model adds more flexibility to the first model by incorporating multiple scenarios, each with a different set of exchange rates for a year. This allows the model to be more robust to the potential effects of different exchange rates. The third model builds on the second model by adding in the expected 10% increase in demand for the next 5 years for Asia without Japan. As the model accounts for the change in demand, the objective function minimizes the total cost accumulated over the five years instead. Figures 4 and 5 show the model details.

Figures 6 and 7 show the results of the second and third models. From the second model, we found that the minimum total cost was 1278.76 million USD, while for the third model, the minimum cost was at 6660.66 million USD over five years or 1332.13 million USD on average per year. We can see that the total cost for the second model is slightly higher than the first model, as it incorporates the possibility of different exchange rates, however, even while incorporating these scenarios, the total cost is still lower than the initial network. The average total cost per year for the third model is much higher than the original network as for this model, we assume that facility implementation decisions are made once at the very beginning and that we cannot make any changes to these decisions later on. As a result, the set of operating facilities and their chemical production choices may not be optimal for the years following the initial decision, which can result in an increase in the average total cost over the next 5 years. In 1.6, we will discuss a model that allows for decisions to be made over time using a multi-period model.

While the two models incorporate new aspects, the resulting facility operation choices remained the same as the first model. Both of the new models still recommend closing the plant in Japan and limiting the chemical production of the plant in Germany to only Relax.

1.3 The Effect of Increasing Plant Capacity (Q3)

To explore the effect of increasing plant capacity, we can increase the capacity of a single plant by one million units at an increased fixed cost of 3 million per year. Figure 8 shows the model's output costs after running the model 6 times, where for each time, we add additional capacity and costs to only one specific plant in the country column. The first table compares the resulting costs across the different plants, where more green values correspond to a lower cost in the column. From the final column, Total Cost, we can see that Japan benefits the most from increasing its capacity by 1 million units. This could be because as mentioned before, Japan has the least capacity per cost out of all the plants. However, while it may seem beneficial at first, the second table in Figure 8 shows that when we compare the results to the original second model results, adding capacity to any plant results still results in a higher total cost.

Figure 9 shows the results for when we tested the increase in capacity using the third model. Similarly, the results show that when comparing between plants, Japan would benefit the most from a one million unit increase in capacity, however, when compared to the original model without additional capacity, we can see that adding capacity will always increase the total cost.

Therefore, these results show that it would not be beneficial to increase the capacity of any plant. This increase in cost after increasing capacity can indicate that the additional capacity is unnecessary. If the plants do not need additional capacity, it can result in an increase in fixed costs without a similar increase in units produced and sold. As a result, it is recommended to not increase the capacity of the plants to reduce costs and inefficiencies in the network.

1.4 The Effect of Reduced Duties (Q4)

Another factor to consider for the models is the effect of reduced duties. To better understand this effect, we can run the second and third models multiple times while increasing the reduction in duties for every iteration. Figures 10 and 11 show the results of decreasing duties on the costs of the production network. From the tables, values that are more green indicate smaller costs for a column. For the Total Cost column, values that tend toward green are smaller, while values that tend toward red are larger.

From both model results, we can see that as we increase the reduction of duties, there is a stable decrease in the fixed costs. The first table in Figures 10 and 11 shows that higher decreases in duties result in lower total costs incurred from the production network. The second table shows that when we compare the results to the original model results without any reduction in duties, we find that the original models have higher costs than any scenario where duties are decreased. This shows that even a small reduction in duties will reduce costs for a given network. As a result, when making network production decisions, it is important to consider the duties associated with sending products to another country. If the duties associated with sending a product to another country are high, producing and supplying products locally may be more cost-effective, as it can avoid the high duties associated with exporting goods. On the other hand, suppose a region has much lower duties compared to some others. In that case, it may be beneficial to produce products in a region where the products have a lower production cost, and then import the products into the region for sale. As a result, we can see that the percentage of duties for a specific region plays an important role in finding the optimal solution for a given production network.

1.5 Multi-Period Models (Q5)

In addition to the three models from before, we can construct a multi-period model to model scenarios where plants can be shut down at any period, and the production of either chemical can also be altered for any period. The results can be seen in Figures 12, 13 and 14, as well as in the code. From these models, we found that the optimal plant locations remained the same, as we assumed that demand is constant. However, through these models, we could see that once a plant is implemented, the must continue to operate, however, chemical production can be stopped if needed to reduce costs. This can be seen when varying demands are tested against the network. As a result, these networks can be helpful for scenarios where decision-making can occur over time and is not limited to a here-and-now decision.

1.6 Factors Influencing Network Design (Q6)

Finally, in addition to the factors discussed above, there are also additional factors, that may not be quantifiable but should also be considered when designing a global supply chain network. For example, a company should also consider strategic factors when designing its production network. Some companies may want to focus on responsiveness, while others may rely more on a low-cost strategy. Depending on the usage of the chemicals that BioPharma uses, the company could decide to relocate plants to prioritize responsiveness, or they could relocate to reduce costs for both the company and its customers. Some other factors that the company should take into consideration are the political stability and infrastructure of a plant's region. Countries that are more politically unstable may have higher risks associated with them, which can potentially impact and disrupt the supply chain. Good infrastructure can reduce the cost of production in a given region

Conclusion

From this study, we compared and analyzed the optimal production network solutions. We found that the optimal low-cost network would shut down the plant in Japan and limit the production of chemicals in the German plant to only Relax. There are multiple factors to take into consideration when planning global supply chains. Factors such as plant capacity and duties can have a measurable effect on the total cost of a production network. Other factors such as strategy, political risk and infrastructure also play an important role in making supply chain network decisions. To optimize a global supply chain, it is important to not only understand the effect of these factors but also to build strategies that make the most out of the positive factors.

Appendix

Figure 1: Capacitated Plant Location Model 1

Parameters

- n = number of potential plant locations/capacity
- m = number of markets or demand points
- $k = 1$ refers to Highcal and $k = 2$ refers to Relax
- D_{jk} = annual demand from market j for product k
- L_i = potential capacity of plant i
- g_i = fixed cost of keeping plant i open in scenario s
- f_{ik} = fixed cost of keeping plant i open and producing product k in scenario s
- c_{ijk} = cost of producing and shipping one unit of product k from plant i to market j in scenario s

Decision Variables

- $y_i = 1$ to keep plant i, 0 to shut down plant i
- $h_i = 1$ to produce Highcal at plant i, 0 to not produce Highcal at plant i
- $r_i = 1$ to produce Relax at plant i, 0 to not produce Relax at plant i
- x_{ijk} = quantity of product k shipped from plant i to market j

Model

$$\begin{aligned}
\min_{z,y,x} \quad & 0.2\sum_{i=1}^{n}(g_i + \sum_{k=1}^{2} f_{ik}) + 0.8\sum_{i=1}^{n}(g_i y_i + f_{i1}h_i + f_{i2}r_i) + \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{2} c_{ijk}x_{ijk} & \\
\text{s.t.} \quad & \sum_{i=1}^{n} x_{ijk} = D_{jk} & \forall j = 1,\dots,m; k = 1,2 \\
& \sum_{j=1}^{m}\sum_{k=1}^{2} x_{ijk} \le L_i y_i & \forall i = 1,\dots,n \\
& \sum_{j=1}^{m} x_{ij1} \le L_i h_i & \forall i = 1,\dots,n \\
& \sum_{j=1}^{m} x_{ij2} \le L_i r_i & \forall i = 1,\dots,n \\
& x_{ijk} \ge 0 & \forall i = 1,\dots,n; j = 1,\dots,m; k = 1,2 \\
& y_i, h_i, r_i \in \{0,1\} & \forall i = 1,\dots,n \\
& h_i, r_i \le y_i & \forall i = 1,\dots,n
\end{aligned}$$

Figure 2: Model 1 Results

```
Fixed Cost =  189.8
Highcal Variable Cost: 494.71500000000003
Relax Variable Cost: 582.664
Total Variable Cost: 1077.379
Total Cost: 1267.179
```

	Plant	Open/Close	Produce HighCal	Produce Relax
0	Brazil	1.0	1.0	1.0
1	Germany	1.0	0.0	1.0
2	India	1.0	1.0	1.0
3	Japan	0.0	0.0	0.0
4	Mexico	1.0	1.0	1.0
5	US	1.0	1.0	1.0

```
Optimal Highcal Quantity (Million Kilograms):
```

	From\To	Latin America	Europe	Asia w/o Japan	Japan	Mexico	U.S.
0	Brazil	7.0	4.0	0.0	0.0	0.0	0.0
1	Germany	0.0	0.0	0.0	0.0	0.0	0.0
2	India	0.0	0.0	5.0	7.0	0.0	0.0
3	Japan	0.0	0.0	0.0	0.0	0.0	0.0
4	Mexico	0.0	11.0	0.0	0.0	3.0	13.0
5	US	0.0	0.0	0.0	0.0	0.0	5.0

```
Optimal Relax Quantity (Million Kilograms):
```

	From\To	Latin America	Europe	Asia w/o Japan	Japan	Mexico	U.S.
0	Brazil	7.0	0.0	0.0	0.0	0.0	0.0
1	Germany	0.0	12.0	0.0	5.0	0.0	0.0
2	India	0.0	0.0	3.0	3.0	0.0	0.0
3	Japan	0.0	0.0	0.0	0.0	0.0	0.0
4	Mexico	0.0	0.0	0.0	0.0	3.0	0.0
5	US	0.0	0.0	0.0	0.0	0.0	17.0

Figure 3: Capacity per USD

Plant	Capacity/FC	Capacity /HFC	Capacity/RFC	Capacity/HRM	Capacity/HP	CapacityRRM	Capacity/RP
Brazil	0.90	3.60	3.60	5.00	3.53	3.91	2.73
Germany	1.00	3.46	3.46	4.62	3.00	3.60	2.57
India	1.29	6.00	6.00	5.00	4.00	4.00	3.00
Japan	0.77	2.50	2.50	4.62	3.00	3.53	2.57
Mexico	1.00	5.00	5.00	5.00	3.60	3.91	2.77
U.S.	0.96	4.40	4.40	5.00	3.60	4.00	2.77

FC	Plant Fixed Cost (Million $)	Green	Higher capacity/cost
HFC	Highcal Fixed Cost (Million $)	Red	Lower capacity/cost
RFC	Relax Fixed Cost (Million $)		
HRM	Highcal Raw Material Cost ($/kg)		
HP	Highcal Production Cost ($/kg)		
RPM	Relax Raw Material Cost ($/kg)		
RP	Relax Production Cost ($/kg)		

Figure 4: Model 2

Parameters

- n = number of potential plant locations/capacity
- m = number of markets or demand points
- $k = 1$ refers to Highcal and $k = 2$ refers to Relax
- D_{jk} = annual demand from market j for product k
- L_i = potential capacity of plant i
- g_{is} = fixed cost of keeping plant i open in scenario s
- f_{iks} = fixed cost of keeping plant i open and producing product k in scenario s
- c_{ijks} = cost of producing and shipping one unit of product k from plant i to market j in scenario s
- s = scenario number given a set of exchange rates for a year
- p_s = 0.125 = probability of scenario s given equal weighting

Decision Variables

- y_i = 1 to keep plant i, 0 to shut down plant i
- h_i = 1 to produce Highcal at plant i, 0 to not produce Highcal at plant i
- r_i = 1 to produce Relax at plant i, 0 to not produce Relax at plant i
- x_{ijk} = quantity of product k shipped from plant i to market j

$$
\begin{aligned}
\min_{z,y,x} \quad & \sum_{s=1}^{S} p_s \left(0.2 \sum_{i=1}^{n} (g_{is} + \sum_{k=1}^{2} f_{iks}) + 0.8 \sum_{i=1}^{n} (g_{is} y_i + f_{i1s} h_i + f_{i2s} r_i) + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{2} c_{ijks} x_{ijk} \right) & \\
\text{s.t.} \quad & \sum_{i=1}^{n} x_{ijk} = D_{jk} & \forall j = 1, \dots, m; k = 1, 2 \\
& \sum_{j=1}^{m} \sum_{k=1}^{2} x_{ijk} \le L_i y_i & \forall i = 1, \dots, n \\
& \sum_{j=1}^{m} x_{ij1} \le L_i h_i & \forall i = 1, \dots, n \\
& \sum_{j=1}^{m} x_{ij2} \le L_i r_i & \forall i = 1, \dots, n \\
& x_{ijk} \ge 0 & \forall i = 1, \dots, n; j = 1, \dots, m; k = 1, 2 \\
& y_i, h_i, r_i \in \{0, 1\} & \forall i = 1, \dots, n \\
& h_i, r_i \le y_i & \forall i = 1, \dots, n
\end{aligned}
$$

Figure 5: Model 3

Parameters

- n = number of potential plant locations/capacity
- m = number of markets or demand points
- $k = 1$ refers to Highcal and $k = 2$ refers to Relax
- t = number of years after 2013
- D_{jkt} = annual demand from market j for product k at t years after 2013
- L_i = potential capacity of plant i
- g_{is} = fixed cost of keeping plant i open in scenario s
- f_{iks} = fixed cost of keeping plant i open and producing product k in scenario s
- c_{ijks} = cost of producing and shipping one unit of product k from plant i to market j in scenario s
- s = scenario number given a set of exchange rates for a year
- p_s = 0.125 = probability of scenario s given equal weighting

Decision Variables

- y_i = 1 to keep plant i, 0 to shut down plant i
- h_i = 1 to produce Highcal at plant i, 0 to not produce Highcal at plant i
- r_i = 1 to produce Relax at plant i, 0 to not produce Relax at plant i
- x_{ijkt} = quantity of product k shipped from plant i to market j at t years after 2013

$$
\begin{aligned}
\min_{z,y,x} \quad & \sum_{t=1}^{T}\sum_{s=1}^{S} p_s\left(0.2\sum_{i=1}^{n}(g_{is}+\sum_{k=1}^{2} f_{iks}) + 0.8\sum_{i=1}^{n}(g_{is}y_i + f_{i1s}h_i + f_{i2s}r_i) + \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{2} c_{ijks}x_{ijkt}\right) & \\
\text{s.t.} \quad & \sum_{i=1}^{n} x_{ijkt} = D_{jkt} & \forall j = 1,\dots,m; k = 1,2; t = 1,\dots,T \\
& \sum_{j=1}^{m}\sum_{k=1}^{2} x_{ijkt} \le L_i y_i & \forall i = 1,\dots,n; t = 1,\dots,T \\
& \sum_{j=1}^{m} x_{ij1t} \le L_i h_i & \forall i = 1,\dots,n; t = 1,\dots,T \\
& \sum_{j=1}^{m} x_{ij2t} \le L_i r_i & \forall i = 1,\dots,n; t = 1,\dots,T \\
& x_{ijkt} \ge 0 & \forall i = 1,\dots,n; j = 1,\dots,m; k = 1,2; t = 1,\dots,T \\
& y_i, h_i, r_i \in \{0,1\} & \forall i = 1,\dots,n \\
& h_i, r_i \le y_i & \forall i = 1,\dots,n
\end{aligned}
$$

Figure 6: Model 2 Results

```
Fixed Cost =  201.38123716900628
Highcal Variable Cost: 494.7150000000001
Relax Variable Cost: 582.6639999999999
Total Variable Cost: 1077.379
Total Cost: 1278.7602371690064
```

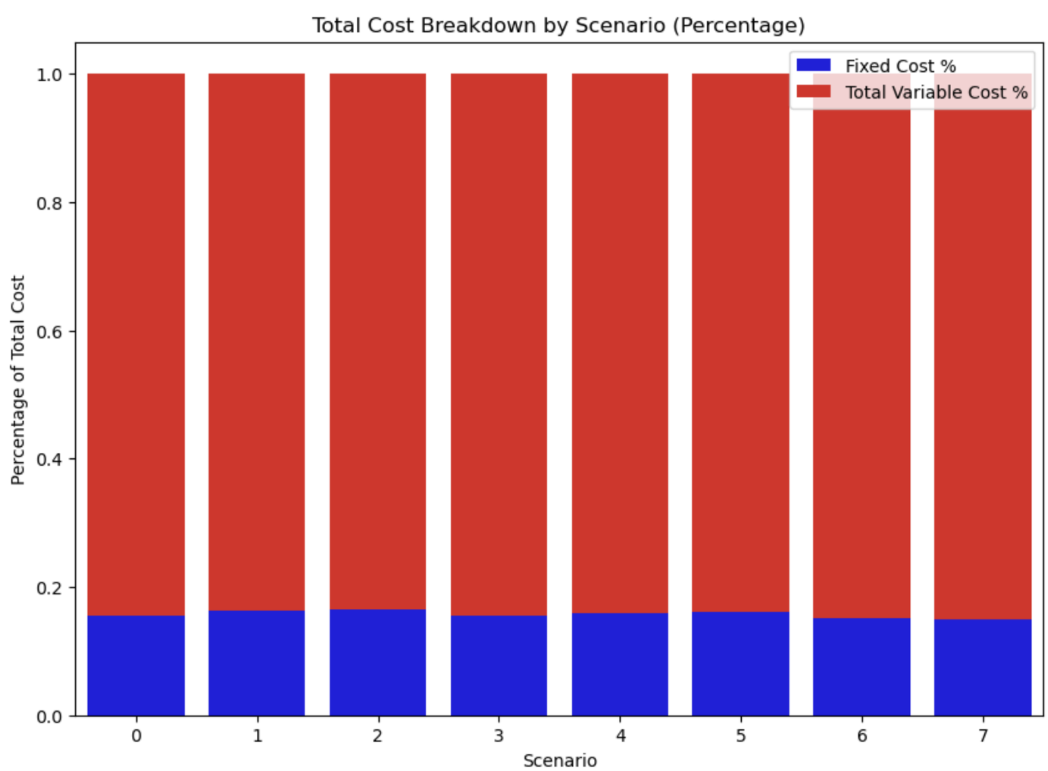
	Plant	Open/Close	Produce HighCal	Produce Relax
0	Brazil	1.0	1.0	1.0
1	Germany	1.0	0.0	1.0
2	India	1.0	1.0	1.0
3	Japan	0.0	0.0	0.0
4	Mexico	1.0	1.0	1.0
5	US	1.0	1.0	1.0

```
Optimal Highcal Quantity (Million Kilograms):
```

	From\To	Latin America	Europe	Asia w/o Japan	Japan	Mexico	U.S.
0	Brazil	7.0	4.0	0.0	0.0	0.0	0.0
1	Germany	0.0	0.0	0.0	0.0	0.0	0.0
2	India	0.0	0.0	5.0	7.0	0.0	0.0
3	Japan	0.0	0.0	0.0	0.0	0.0	0.0
4	Mexico	0.0	11.0	0.0	0.0	3.0	13.0
5	US	0.0	0.0	0.0	0.0	0.0	5.0

```
Optimal Relax Quantity (Million Kilograms):
```

	From\To	Latin America	Europe	Asia w/o Japan	Japan	Mexico	U.S.
0	Brazil	7.0	0.0	0.0	0.0	0.0	0.0
1	Germany	0.0	12.0	0.0	5.0	0.0	0.0
2	India	0.0	0.0	3.0	3.0	0.0	0.0
3	Japan	0.0	0.0	0.0	0.0	0.0	0.0
4	Mexico	0.0	0.0	0.0	0.0	3.0	0.0
5	US	0.0	0.0	0.0	0.0	0.0	17.0



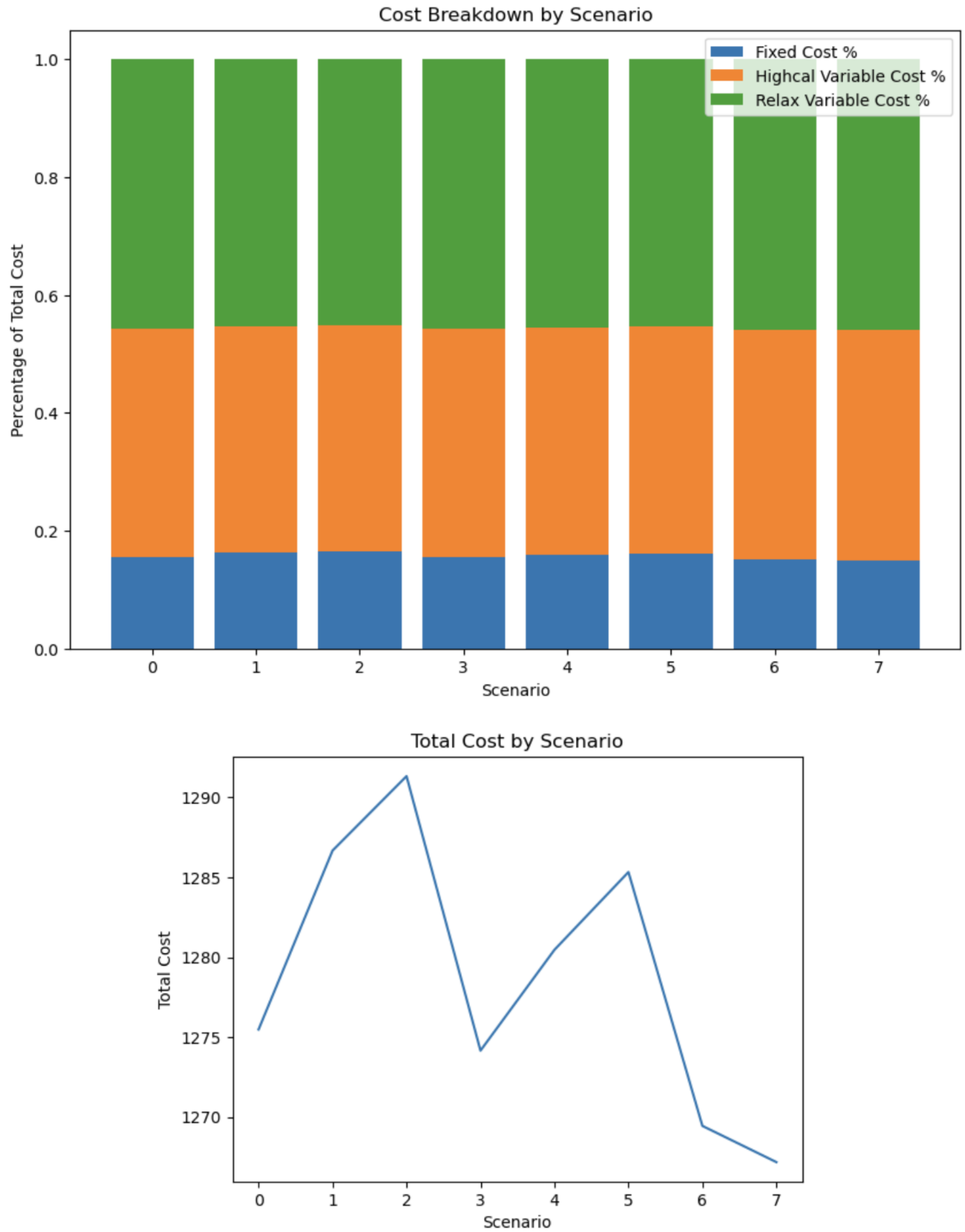


Figure 7: Model 3 Results

```
Fixed Cost =  1006.9061858450316
Highcal Variable Cost: 2602.0127295999987
Relax Variable Cost: 3051.744131800001
Total Variable Cost: 5653.756861399998
Total Cost: 6660.663047245032
```

	Plant	Open/Close	Produce HighCal	Produce Relax
0	Brazil	1.0	1.0	1.0
1	Germany	1.0	0.0	1.0
2	India	1.0	1.0	1.0
3	Japan	0.0	0.0	0.0
4	Mexico	1.0	1.0	1.0
5	US	1.0	1.0	1.0

Figure 8: Model 2 Costs After Increasing Capacity

Country	Fixed Cost	HVC	RVC	Total Variable Cost	Total Cost
Brazil	204.7461	495.563	580.968	1076.531	1281.277
Germany	204.4235	494.715	582.664	1077.379	1281.803
India	205.0835	494.715	580.968	1075.683	1280.766
Japan	201.9972	494.715	582.664	1077.379	1279.376
Mexico	204.5445	495.3055	580.968	1076.2735	1280.818
US	204.3812	494.9015	580.968	1075.8695	1280.251

Country	Fixed Cost	HVC	RVC	Total Variable	Total Cost
Brazil	204.7461	495.563	580.968	1076.531	1281.277
Germany	204.4235	494.715	582.664	1077.379	1281.803
India	205.0835	494.715	580.968	1075.683	1280.766
Japan	201.9972	494.715	582.664	1077.379	1279.376
Mexico	204.5445	495.3055	580.968	1076.2735	1280.818
US	204.3812	494.9015	580.968	1075.8695	1280.251
Original Model	201.3812	494.715	582.664	1077.379	1278.76

Figure 9: Model 3 Costs After Increasing Capacity

Country	Fixed Cost	HVC	RVC	Total Variab	Total Cost
Brazil	1023.73	2605.32	3043.784	5649.104	6672.835
Germany	1022.118	2602.013	3051.744	5653.757	6675.874
India	1025.417	2601.08	3043.784	5644.864	6670.282
Japan	1009.986	2602.013	3051.744	5653.757	6663.743
Mexico	1022.723	2604.033	3043.784	5647.817	6670.539
US	1021.906	2602.013	3043.784	5645.797	6667.703

Country	Fixed Cost	HVC	RVC	Total Variab	Total Cost
Brazil	1023.73	2605.32	3043.784	5649.104	6672.835
Germany	1022.118	2602.013	3051.744	5653.757	6675.874
India	1025.417	2601.08	3043.784	5644.864	6670.282
Japan	1009.986	2602.013	3051.744	5653.757	6663.743
Mexico	1022.723	2604.033	3043.784	5647.817	6670.539
US	1021.906	2602.013	3043.784	5645.797	6667.703
Original	1006.906	2602.013	3051.744	5653.757	6660.663

Figure 10: Model 2 Costs After Reducing Duties

Percent Reduction	Fixed Cost	HVC	RVC	Total Variable	Total Cost
10%	201.3812372	493.4985	582.0976	1075.5961	1276.977
30%	197.3812372	490.1848	585.4644	1075.6492	1273.03
50%	192.3199776	488.854	586.9272	1075.78125	1268.101
70%	192.3199776	486.0724	583.4164	1069.48875	1261.809
90%	165.4014069	490.8093	597.3368	1088.1461	1253.548

Percent Reduction	Fixed Cost	HVC	RVC	Total Variat	Total Cost
10%	201.3812	493.4985	582.0976	1075.596	1276.977
30%	197.3812	490.1848	585.4644	1075.649	1273.03
50%	192.32	488.854	586.9272	1075.781	1268.101
70%	192.32	486.0724	583.4164	1069.489	1261.809
90%	165.4014	490.8093	597.3368	1088.146	1253.548
Original Model	201.3812	494.715	582.664	1077.379	1278.76

Figure 11: Model 3 Costs After Reducing Duties

Percent Reduction	Fixed Cost	HVC	RVC	Total Variat	Total Cost
10%	1006.906	2596.226	3048.305	5644.53	6651.436
30%	986.9062	2584.378	3059.117	5643.494	6630.401
50%	986.9062	2576.536	3045.672	5622.207	6609.114
70%	946.791	2553.697	3083.805	5637.502	6584.293
90%	827.007	2580.727	3121.656	5702.383	6529.39

Percent Reduction	Fixed Cost	HVC	RVC	Total Variable	Total Cost
10%	1006.906	2596.226	3048.305	5644.53021	6651.436
30%	986.9062	2584.378	3059.117	5643.49437	6630.401
50%	986.9062	2576.536	3045.672	5622.2075	6609.114
70%	946.791	2553.697	3083.805	5637.50197	6584.293
90%	827.007	2580.727	3121.656	5702.38326	6529.39
Original	1006.906	2602.013	3051.744	5653.75686	6660.663

Figure 12: Model 4

Parameters

- n = number of potential plant locations/capacity
- m = number of markets or demand points
- $k = 1$ refers to Highcal and $k = 2$ refers to Relax
- D_{jk} = annual demand from market j
- L_i = potential capacity of plant i
- g_{is} = fixed cost of keeping plant i open in scenario s
- f_{iks} = fixed cost of keeping plant i open and producing product k in scenario s
- c_{ijks} = cost of producing and shipping one unit of product k from plant i to market j in scenario s
- s = scenario number given a set of exchange rates for a year
- p_s = 0.125 = probability of scenario s given equal weighting

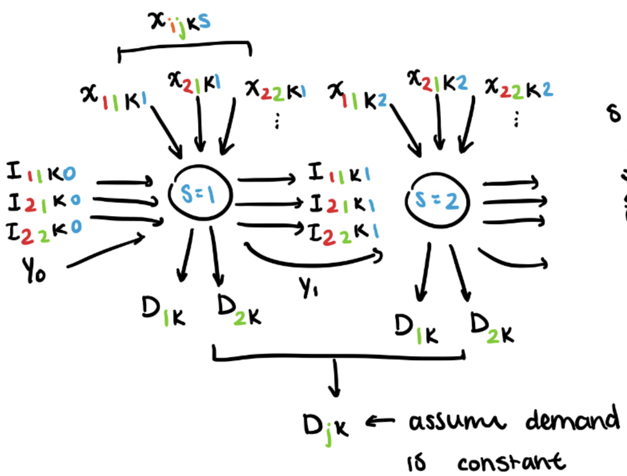
Decision Variables

- y_{is} = 1 to keep plant i, 0 to shut down plant i in scenario s
- h_{is} = 1 to produce Highcal at plant i, 0 to not produce Highcal at plant i in scenario s
- r_{is} = 1 to produce Relax at plant i, 0 to not produce Relax at plant i in scenario s
- x_{ijks} = quantity of product k shipped from plant i to market j in scenario s

$$\begin{aligned}
\min_{z,y,x} \quad & \sum_{s=1}^{S} p_s \left(0.2 \sum_{i=1}^{n} (g_{is} + \sum_{k=1}^{2} f_{iks}) + 0.8 \sum_{i=1}^{n} (g_{is} y_{is} + f_{i1s} h_{is} + f_{i2s} r_{is}) + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{2} c_{ijks} x_{ijks} \right) \\
\text{s.t.} \quad & \sum_{i=1}^{n} x_{ijks} = D_{jk} && \forall j = 1, \dots, m; k = 1, 2 \\
& \sum_{j=1}^{m} \sum_{k=1}^{2} x_{ijks} \le L_i y_{is} && \forall i = 1, \dots, n; s = 1, \dots, S \\
& \sum_{j=1}^{m} x_{ij1s} \le L_i h_{is} && \forall i = 1, \dots, n; s = 1, \dots, S \\
& \sum_{j=1}^{m} x_{ij2s} \le L_i r_{is} && \forall i = 1, \dots, n; s = 1, \dots, S \\
& x_{ijks} \ge 0 && \forall i = 1, \dots, n; j = 1, \dots, m; k = 1, 2; s = 1, \dots, S \\
& y_{is}, h_{is}, r_{is} \in \{0, 1\} && \forall i = 1, \dots, n; s = 1, \dots, S \\
& y_{is} \ge y_{i(s-1)} && \forall i = 1, \dots, n; s = 1, \dots, S \\
& h_{is}, r_{is} \le y_{is} && \forall i = 1, \dots, n; s = 1, \dots, S
\end{aligned}$$

```
Fixed Cost =  201.38123716900628
Highcal Variable Cost: 494.7150000000001
Relax Variable Cost: 582.6639999999999
Total Variable Cost: 1077.379
Total Cost: 1278.7602371690064
```

Figure 13: Model 5 diagram



s.t.

$$\sum_{i=1}^{n} I_{ijks} = \sum_{i=1}^{n} I_{ijk(s-1)} + \sum_{i=1}^{n} x_{ijks} - D_{jk}$$

$\forall i = 1, \ldots, n;\ j = 1, \ldots, m;\ k = 1, 2;\ s = 1, \ldots, S$ (1)

$y_{is} \geq y_{i(s-1)}$ $\forall i = 1, \ldots, n;\ s = 1, \ldots, S$ (2)

⋮

see all constraints in

Figure 14: Model 5

Parameters

- n = number of potential plant locations/capacity
- m = number of markets or demand points
- $k = 1$ refers to Highcal and $k = 2$ refers to Relax
- D_{jk} = annual demand from market j for product k
- L_i = potential capacity of plant i
- g_{is} = fixed cost of keeping plant i open in scenario s
- f_{iks} = fixed cost of keeping plant i open and producing product k in scenario s
- c_{ijks} = cost of producing and shipping one unit of product k from plant i to market j in scenario s
- s = scenario number given a set of exchange rates for a year
- p_s = 0.125 = probability of scenario s given equal weighting
- I_{ijks} = inventory of product k shipped from plant i to market j in scenario s

Decision Variables

- y_{is} = 1 to keep plant i, 0 to shut down plant i in scenario s
- h_{is} = 1 to produce Highcal at plant i, 0 to not produce Highcal at plant i in scenario s
- r_{is} = 1 to produce Relax at plant i, 0 to not produce Relax at plant i in scenario s
- x_{ijks} = quantity of product k shipped from plant i to market j in scenario s

$$
\begin{aligned}
\min_{z,y,x} \quad & \sum_{s=1}^{S} p_s\left(0.2\sum_{i=1}^{n}(g_{is}+\sum_{k=1}^{2} f_{iks}) + 0.8\sum_{i=1}^{n}(g_{is}y_{is}+f_{i1s}h_{is}+f_{i2s}r_{is}) + \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{2} c_{ijks}x_{ijks} + I_{ijks}\right) \\
\text{s.t.} \quad & \sum_{i=1}^{n} x_{ijks} = D_{jk} && \forall j=1,\dots,m; k=1,2; s=1,\dots,S \\
& \sum_{i=1}^{n} I_{ijks} = \sum_{i=1}^{n} I_{ijk(s-1)} + \sum_{i=1}^{n} x_{ijks} - D_{jk} && \forall i=1,\dots,n; j=1,\dots,m; k=1,2; s=1,\dots,S \\
& \sum_{j=1}^{m}\sum_{k=1}^{2} x_{ijks} \le L_i y_{is} && \forall i=1,\dots,n; s=1,\dots,S \\
& \sum_{j=1}^{m} x_{ij1s} \le L_i h_{is} && \forall i=1,\dots,n; s=1,\dots,S \\
& \sum_{j=1}^{m} x_{ij2s} \le L_i r_{is} && \forall i=1,\dots,n; s=1,\dots,S \\
& && \forall i=1,\dots,n; j=1,\dots,m; k=1,2; s=1,\dots,S \\
& x_{ijks}, I_{ijks} \ge 0 && \forall i=1,\dots,n; j=1,\dots,m; k=1,2 \\
& I_{ijk-1} = 0 && \forall i=1,\dots,n; s=1,\dots,S \\
& y_{is}, h_{is}, r_{is} \in \{0,1\} && \forall i=1,\dots,n; s=1,\dots,S \\
& y_{is} \ge y_{i(s-1)} && \forall i=1,\dots,n; s=1,\dots,S \\
& h_{is}, r_{is} \le y_{is}
\end{aligned}
$$

```
Fixed Cost =  201.38123716900628
Highcal Variable Cost: 494.7150000000001
Relax Variable Cost: 582.6639999999999
Total Variable Cost: 1077.379
Total Cost: 1278.7602371690064
```

References

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