COMP 182: Algorithmic Thinking
13 February 2014

In 2000, Broder *et al.* set out to build a global map of the Web, using *strongly connected components* (SCCs) as the basic building blocks. For their raw data, they used the index of pages and links from one of the largest commercial search engines a the time, Alta Vista. They found that the Web has a central "core" containing many of its most prominent pages, with many other nodes that lie upstream, downstream, or "off to the side" relative to this core (see Fig. 1). Obviously, to conduct this study, the authors had to deal with the algorithmic question of how to compute the SCCs of a directed graph.

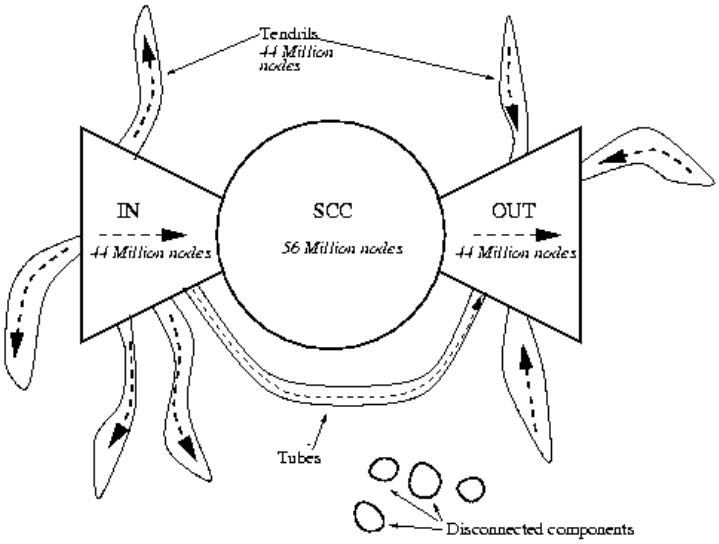

Figure 1: A schematic picture of the bow-tie structure of the Web (Broder *et al.*, WWW 2000). Although the numbers are now outdated, the structure has persisted.

A directed graph is called *strongly connected* if for any pair of distinct nodes u and v, there exists a directed path from u to v and a directed path from v to u. A directed graph's nodes can be partitioned into disjoint maximal subsets of nodes that are mutually accessible via directed paths; these subsets are called *strongly connected components* (SCCs) of the directed graph.

1. What are the SCCs of the graph $g = (V, E)$ with $V = \{a, b, c, d, e, f, g, h\}$ and $E = \{(a, b), (b, g), (c, d), (c, e), (d, b), (d, g), (e, h), (f, a), (f, b), (g, f), (h, c), (h, d)\}$.
2. How many SCCs does a DAG with n nodes and m edges have?
3. Give the pseudo-code of a polynomial-time algorithm that uses **DFS** to compute the SCCs of a directed graph.
4. What is the running time of your algorithm?