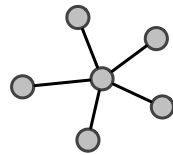


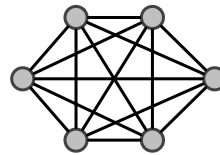
COMP 182: Algorithmic Thinking

16 January 2014

A network topology specifies how computers, printers, and other devices are connected over a network. The figure below illustrates two common topologies of networks: the star and the fully connected mesh.



star



fully connected mesh

Two notations that are useful in writing formal definitions are:

- \forall (pronounced “for all”). For example, the notation “ $\forall v \in V$ ” means “for every node v in the set V of nodes”, and “ $\forall e \in E$ ” means “for every edge e in the set E of edges”. More generally, “ $\forall a \in A$ ” means “for every element a in the set A .” The notation “ $\forall v \in V, \text{degree}(v) = 1$ ” means “every node v in V has degree 1.”
- \exists (pronounced “exists” or “there exists”). For example, the notation “ $\exists v \in V$ ” means “there exists a node v in V ”. The notation “ $\exists v \in V, \text{degree}(v) = 0$ ” means “there exists a node v in V that is isolated.”

1 Formal definitions

Let $g = (V, E)$ be a graph on n nodes ($n \geq 4$), where $V = \{0, 1, \dots, n-1\}$. Define formally what it means for g to be a star and to be a fully connected mesh (that is, give two definitions).

2 Problem formulations

Formulate two graph-theoretic, decision problems that correspond to checking whether a network topology is a star or a fully connected mesh (again, assume $n \geq 4$).

3 Brute-force algorithms

Write the pseudo-code of two algorithms **IsStar** and **IsFullyConnectedMesh** for solving the two problems you defined in the previous section (assume the graph is given by its adjacency matrix and that the number of nodes is ≥ 4).

4 Efficiency

For each of your two algorithms, which of the following terms most closely captures the number of steps that your algorithm takes as a function of the number of nodes n in the graph:

1000 $\log n$ n n^2 n^3 n^4 2^n $n!$