COMP 182: Algorithmic Thinking
20 February 2014*

You are given a set of cities, along with the pattern of highways between them, in the form of an undirected graph $g = (V, E)$. Each stretch of highway $e \in E$ connects two of the cities, and you know its length in miles, ℓ_e. You want to get from city s to city t. There's one problem: your car can only hold enough gas to cover L miles. There are gas stations in each city, but not between cities. Therefore, you can only take a route if every one of its edges has length $\ell_e \leq L$.

1. Given the limitation on your car's fuel tank capacity, show how to determine in linear time whether there is a feasible route from s to t.

2. You are now planning to buy a new car, and you want to know the minimum fuel tank capacity that is needed to travel from s to t. Give an $O((|V| + |E|) \log |V|)$ algorithm to determine this.

*This problem is from the book Algorithms, by Dasgupta *et al.*