

# COMP 182: Algorithmic Thinking

## 6 February 2014

A biologist comes back from a field trip with  $n$  mice, and she believes that each of these mice belongs to one of two different species, which we will call  $A$  and  $B$ . The biologist would like to divide the  $n$  specimens into two groups—those that belong to  $A$  and those that belong to  $B$ —but it’s very hard for her to directly label any one specimen. So, she adopts the following approach.

For each pair of specimen  $i$  and  $j$ , she studies them carefully side by side and makes one of three decisions:

- *same*: she is certain that  $i$  and  $j$  belong to the same species;
- *different*: she is certain that  $i$  and  $j$  belong to two different species; or,
- *ambiguous*: she can’t decide with certainty whether  $i$  and  $j$  belong to the same species or not.

So, now the biologist has a collection of  $n$  specimens and a collection of  $m$  judgments (*same* or *different*) for the pairs that were not declared ambiguous. The biologist would like to know if this data is consistent with the idea that each mouse is from one of species  $A$  or  $B$ . We declare the  $m$  judgments to be *consistent* if it is possible to label each specimen  $A$  or  $B$  in such a way that for each pair  $i$  and  $j$ , the following hold:

- if  $i$  and  $j$  were declared “same,” then  $i$  and  $j$  have the same label, and
- if  $i$  and  $j$  were declared “different,” then  $i$  and  $j$  have different labels.

Help the biologist by devising an efficient algorithm for the problem. First, formulate the problem. Then, give the pseudo-code of an  $O(m+n)$  algorithm. Finally, analyze the algorithm’s running time to show it indeed takes  $O(m+n)$  time.