Computer Simulation

Module 1: Intro + Course Tour

Dave Goldsman, Ph.D.
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Getting to Know You

Course Info

Course Objectives:

- Identify simulation models and recognize simulation studies
- Illustrate organization of simulation languages including Modeling with Arena, a comprehensive simulation package with animation capabilities
- Analyze statistical aspects of simulations including input analysis, random variate generation, output analysis, and variance reduction techniques

Prerequisites

- You *must* know probability and statistics at the level of ISyE 2027 and 2028, and maybe even a little stochastic processes.
- You should be familiar with some programming language and maybe even a spreadsheet package.
- Good News: Don’t panic! I’ll make the course as self-contained as possible!

Suggested Resources

- Law, A.M., *Simulation Modeling and Analysis*, 5th ed., McGraw-Hill Education, New York, 2015.
- Kelton, W.D., Sadowski, R.P., and Zupick, N.B., *Simulation with Arena*, 6th edition, McGraw-Hill, New York, 2015.
- **FREE** Arena software download: www.arenasimulation.com/academic/students

Grading

Test 1 (30%)

Test 2 (30%)

Test 3 (30%)

HW + Project + if I like you (10%)

HW will be assigned after every module.

Course Notes

We provide pretty extensive notes on the website. This doesn't mean that you can simply print out the notes and skip class!

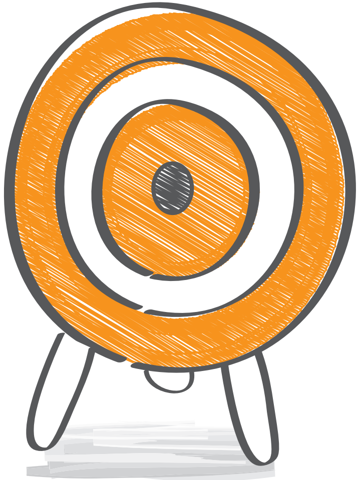
Programming

This course will involve extensive computer programming. You'll have some choice, but you can expect to use:

- A spreadsheet package, e.g., Excel.
- Some spreadsheet add-ons.
- A "real" language, e.g., Matlab or Python.
- A simulation language, e.g., Arena.

Summary

- Hi Everyone!
- Just gave some high-level course info.
- Next Time: Give some details on the course syllabus.



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Syllabus

GTx

Lesson Goals

Last Time: Gave some high-level course info.

This Time: Now some details on the course syllabus.

Generally speaking, the course will have lessons that emphasize math/stats issues, and lessons that are mostly modeling and programming of a variety of systems.

Syllabus – Let's Go!

- Introduction
- Calculus, Probability, and Statistics Boot Camp (Law, Chapter 4)
- Hand Simulations; Spreadsheet Simulations
- General Modeling Concepts (Law, 1&2)
- Verification+Validation (Law, 5)
  - Is the simulation doing what you think?

Syllabus – Arena Fun

- Arena Basics (KSZ, Chapter 4)
- A Generic Call Center in Arena (KSZ, 5)
- An Inventory Model (KSZ, 5)
- A Manufacturing Center (KSZ, 6)
- Entity Transfers in Arena (KSZ, 7)
- Advanced Arena Stuff (KSZ, 8)

Syllabus – Randomness

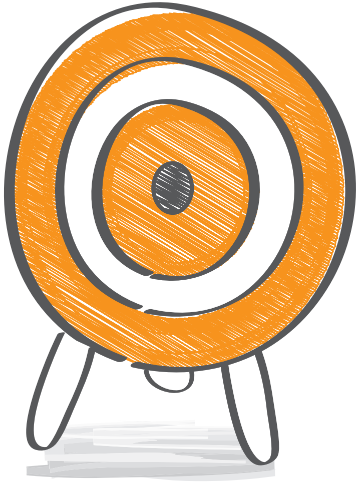
- Random Number Generation (Law, 7)
  - generate “randomness” on a computer
- Random Variate Generation (Law, 8)
  - single random variables
  - multivariate random variables
  - random processes
  - financial models

Syllabus – Stats Issues

- Input Analysis (Law, 6)
  - What should drive the simulation?
- Output Analysis (Law, 9)
  - Analyze what comes out of the simulation
- Comparing Systems (Law, 10)
  - Which system is better / best?
- Variance Reduction + Other Cool Stuff

Summary

- Chatted about the syllabus.
- Next Time: Let’s finally get into simulation with a Whirlwind Tour!



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Whirlwind Tour

Lesson Goals

Last Time: Chatted about the syllabus.

This Time: Let’s finally get into simulation!

We’ll first talk about general modeling issues, and why we would even consider using simulation.

Models

- Models are high-level representations of the operation of a real-world process or system.
- Our concern will be with models that are:
  - Discrete (vs. continuous)
  - Stochastic (vs. deterministic)
  - Dynamic (vs. static)
- How can you “solve” a model?
  - Analytic methods
  - Numerical methods
  - Simulation methods

Examples of Models

- Toss a stone off of a cliff. You can model its position via the usual physics equations – analytical models.
- Model the weather. Too tough for exact analytical models, so you might use numerical methods.
- Add a little randomness, and you may have to resort to a simulation model (plenty of examples coming up).

What is Simulation?

- Simulation is the imitation of a real-world process or system over time.
- Simulation involves the generation of an artificial history to draw inferences concerning the operating characteristics of the real system that is represented.

Simulation is…

- One of the top three industrial engineering / operations research / management science technologies.
- Used by academics and practitioners on a wide array of theoretical and applied problems.
- An indispensable problem-solving methodology.

What is It Good for?

- Describe / analyze real or conceptual system behavior.
- Ask “what if” questions.
- Aid in system design and optimization.
- Can simulate almost anything.
  - Customer-based systems like Manufacturing Processes, Supply Chains, Health Systems.
  - Systems with no “customers”, e.g., stock option prices.

Reasons to Simulate

- Will the system accomplish its goals?
- Current system won't accomplish its goals. Now what?
- Need incremental improvement.
- Create a specification or action plan.
- Solve a problem, like a bottleneck.
- Resolve disputes.
- Sell an idea.

Advantages ☺

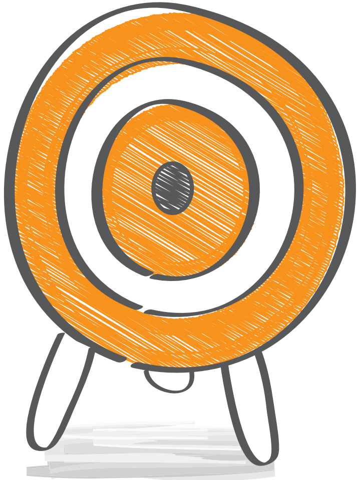
- Can study models too complicated for analytical / numerical treatment.
- Study detailed relations that might be lost in the analytical or numerical treatment.
- Use as a basis for experimental studies of systems.
- Use to check results and give credibility to conclusions obtained by other methods.
- Reduce design blunders.
- Really nice demo method.
- (Sometimes) very easy.

Disadvantages ☹

- Sometimes not so easy.
- Sometimes very time consuming / costly.
- Simulations give “random” output (and lots of misinterpretation of results is possible).
- To do a certain problem, better methods than simulation may exist.
- …

Summary

- Finally started our Whirlwind Tour with a discussion on the nature of simulation models.
- Next Time: A historical (hysterical?) presentation.



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Whirlwind Tour – History

Lesson Overview

Last Time: Talked about modeling and introduced simulation.

This Time: Let’s talk history. Talk history. (History repeating itself.)

Simulation has been around a long time, but has really come into its own with the rise of the machines.

History

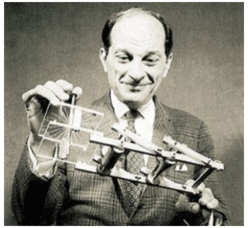
1777 – Buffon’s Needle Problem – a new spin on things

History

Early 1900’s – Beer and Student’s t distribution

More History

- 1946 – Ulam, Metropolis, von Neumann, and the H-Bomb



- 1960's – Industrial Applications
  - Manufacturing
  - Queueing Models

Recent History

- Development of Simulation Languages
  - Easy-to-use modeling tools
  - Graphics

← Harry Markowitz

- Rigorous Theoretical Work
  - Computational algorithms
  - Probabilistic and statistical methods

Origins: Manufacturing

Simulation is the technique of choice

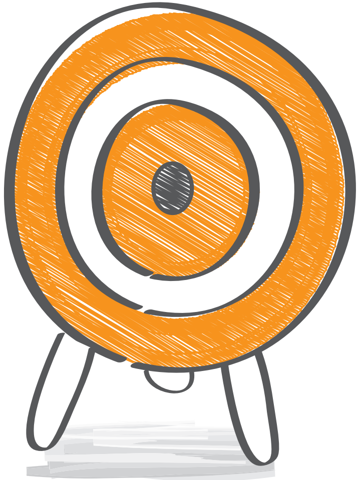
- Calculates movement of parts and interaction of system components.
- Evaluates part flow thru the system.
- Examines conflicting demand for resources.
- Studies contemplated changes before their introduction.
- Prevents design blunders.

Typical Questions

- What will be the throughput?
- How can we change it?
- Where are the bottlenecks?
- Which is the best design?
- What is the reliability of the system?
- What is the impact of breakdowns?

Summary

- You can take your seat belts off… the history lesson is over!
- Next Time: What kinds of stuff is simulation used for?



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What Can We Do For You?

Lesson Overview

Last Time: Discussed the history of simulation.

This Time: What kinds of stuff is simulation used for?

I’ll give away the punch line: Simulation can be used for an amazing array of different applications.

What I Used to Think Simulation Was

$$\sum_{i=nc_{k-1}+1}^{nc_k} \sum_{j=nc_k+1}^{nc_{k+1}} h_i h_j R_{j-i}$$

$$= \left\{ \sum_{i=nc_{k-1}+1}^{n(c_k-\epsilon_k)} \sum_{j=nc_k+1}^{nc_{k+1}} + \sum_{i=n(c_k-\epsilon_k)+1}^{nc_k} \sum_{j=nc_k+1}^{n(c_k+\epsilon_k)} + \sum_{i=n(c_k-\epsilon_k)+1}^{nc_k} \sum_{j=n(c_k+\epsilon_k)+1}^{nc_{k+1}} \right\} h_i h_j R_{j-i}$$

$$= \sum_{i=nc_k-n\epsilon_k+1}^{nc_k} \sum_{j=nc_k+1}^{nc_k+n\epsilon_k} h_i h_j R_{j-i} + O(n^4 \delta^{n\epsilon_k})$$

By complete enumeration as done in Appendix A.2, we get

$$\sum_{i=nc_k-n\epsilon_k+1}^{nc_k} \sum_{j=nc_k+1}^{nc_k+n\epsilon_k} h_i h_j R_{j-i} = \sum_{i=1}^{n\epsilon_k} \sum_{j=1}^{i} h_{nc_k-j+1} h_{nc_k-j+i+1} R_i + O(n^4 \delta^{n\epsilon_k})$$

Therefore,

$$\mathrm{E}[\mathcal{A}^2_{\mathcal{C}_m}(f;n)] = \mathrm{E}[\mathcal{A}^2(f;n)] - \frac{4}{n^3} \sum_{k=1}^{m} \sum_{i=1}^{n\epsilon_k} \sum_{j=1}^{i} h_{nc_k-j+1} h_{nc_k-j+i+1} R_i + O(n\delta^{n\epsilon}) \qquad (9)$$

Actual Applications

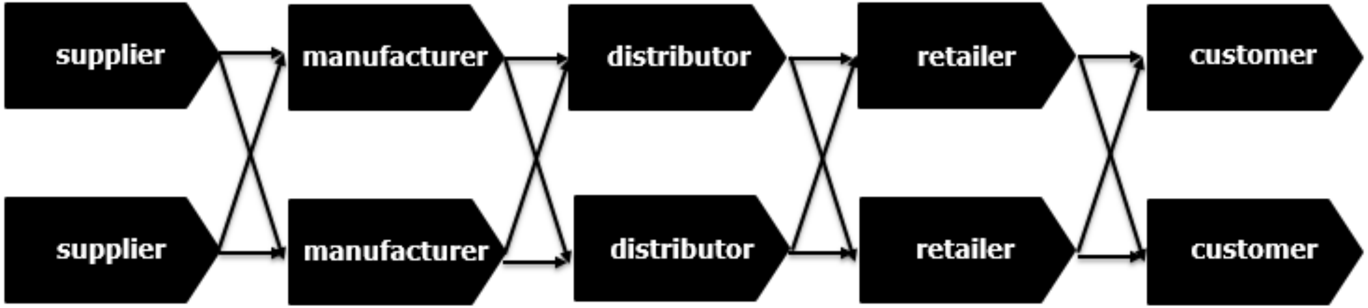
- Manufacturing
  - Automobile Production Facility
  - Carpet Production Facility
- Queueing Problems
  - Call Center Analysis
  - Fast Food Drive-Thru
  - Fast Food Drive-Thru Call Center
  - Airport Security Line

Some Queueing Systems

- Ice Cream Shoppe
- Combination meat processing center / fast-food restaurant / amusement park

← Denis (8) and Mina (6)

Generic Supply Chain



- Think of supply chain as a network where any node can potentially connect to any other node.
- In order to be lazy, let's have things flow from left to right. Looks a bit like a flowchart, eh?

Supply Chains (cont'd)

- 1985: Professor and CEO of SC company: "Why would anyone want to simulate a SC?"
- Now: Many SC tools with simulation capability.
  - Use simulation to determine how much value-added a forecasting application provides.
  - Use simulation to analyze how SC randomness or model errors affect a proposed solution. Is it robust? What's the best solution?

More Applications

- Inventory and Supply Chain Analysis
- Financial Analysis
  - Portfolio Analysis
  - Options Pricing
- Traffic Simulation
- Airspace Simulation
- Service Sector
- Health Systems

Health Systems

- Patient Flow in a Hospital
- Hospital Room Allocation
- Optimization of Doctor / Nurse Scheduling
- Procurement of Supplies
- Disease Surveillance
- Propagation of Disease Spread
- Humanitarian Logistics

Surveilance Application

- Use simulation to monitor certain time series.
- Predict issues as or before they happen.
- Is a disease in the process of becoming an outbreak?
- When is something out of the ordinary occurring?
- Take advantage of HUGE data sets.

Who’s Mr. Handsome?

Dr. Harold Shipman

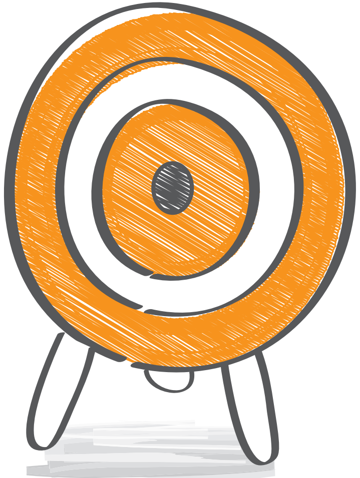
- Killed his patients using heroin overdoses (#2 UK serial killer).
- Caught after carelessly revising a patient’s will, leaving all her assets to himself (don’t use your own typewriter next time, dummy).
- Doctored his records to show his patients had needed morphine. But the software recorded the dates of these modifications.
- Hung himself in prison, never confessed.

Simulation Can Help!

- Surveillance uses sequential hypothesis tests, where the null hyp H_0 is “no disease” (“no murder”).
- The test statistics have very difficult distributions, even under H_0.
- Use simulation to approximate the probability distributions of the statistics under the null hyp.
- If sampling casts doubt on this distribution, then reject.

Summary

- Looked at a wide variety of practical simulation applications.
- Next Time: We'll give some very easy examples of the actual use of simulation in simple settings.



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Some Baby Examples

Lesson Overview

Last Time: Gave high-level examples of the kinds of stuff simulation is used for.

This Time: Some very easy examples of the use of simulation in simple settings.

The idea here is to start us off on our simulation journey via some really simple problems.

Happy Birthday

How many people do you need in a room in order to have a 50% chance that at least two will have the same birthday?

- 9
- 23
- 42
- 183

GTx

Birthday Paradox

Birthday Paradox

1. Enter Seed:

47475

2. Click Start Button:

START

New Birthday Date:

June 22

Total # Birthdays:

24

!!! MATCH !!!

BACK

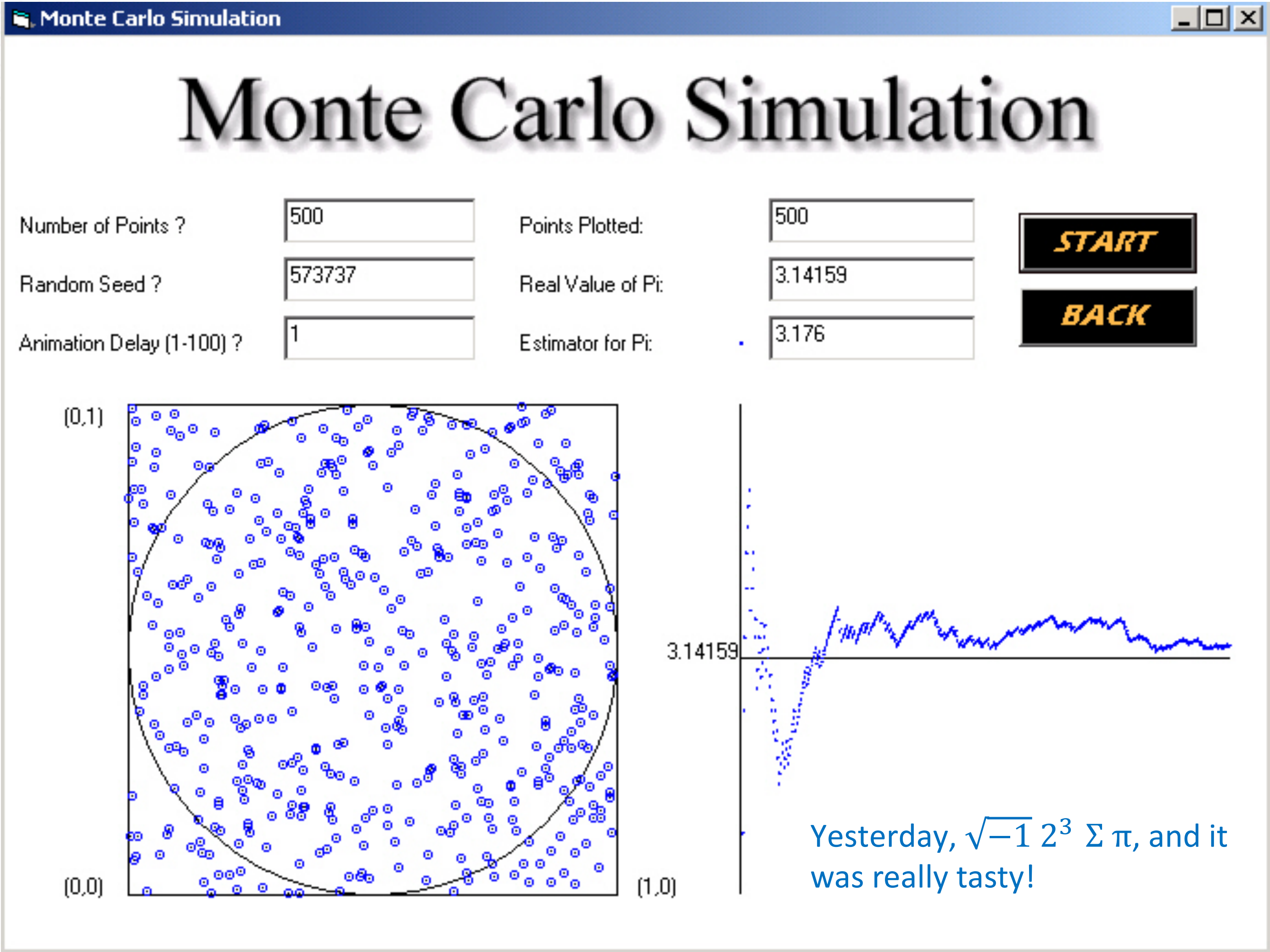
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1		happy				happy			happy			
2												
3		happy			happy							
4										happy		
5										happy		
6								happy				
7		happy										
8							happy					
9												
10												
11						happy						
12												
13					happy							
14												
15		happy					happy					
16					happy							
17				happy								
18												
19							happy					
20												
21												
22						happy						
23		happy										
24				happy								
25												
26										happy		
27												
28												
29		XXXX						happy				
30	happy	XXXX										
31		XXXX		XXXX		XXXX			XXXX		XXXX	

GTx

Let’s Make Some Pi

- Use simulation to estimate π.
- Idea:
  - Area of a unit square is 1.
  - Area of an inscribed circle is π/4.
  - Probability that a dart thrown at the square will hit the circle is π/4.
  - Throw lots of darts. Proportion that will land in circle should approach π/4.

Bonus Quiz: What is the volume of a pizza with radius z and height a?



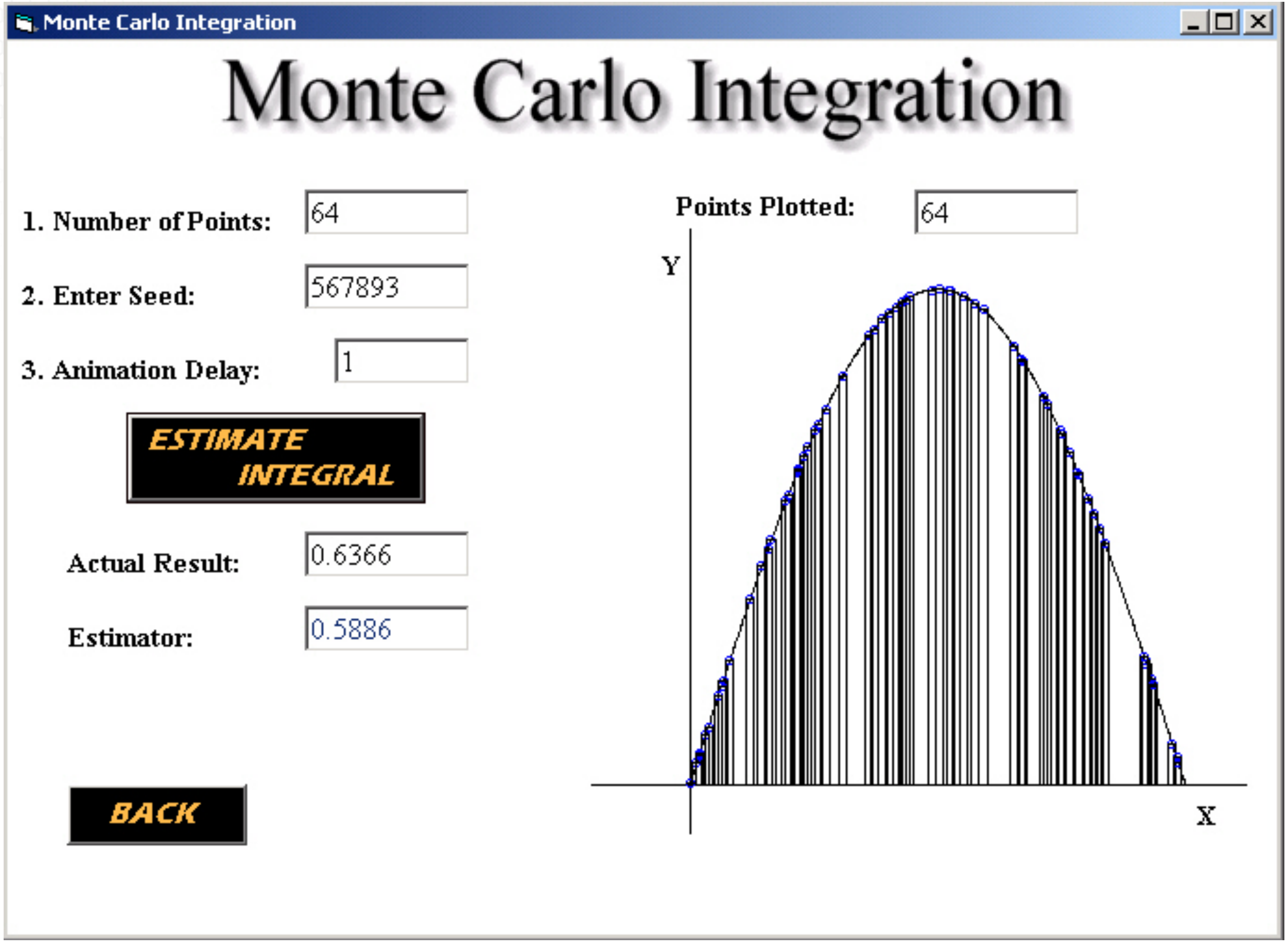
Yesterday, $\sqrt{-1}\ 2^3\ \Sigma\ \pi$, and it was really tasty!

Fun With Calculus

- Use simulation to integrate

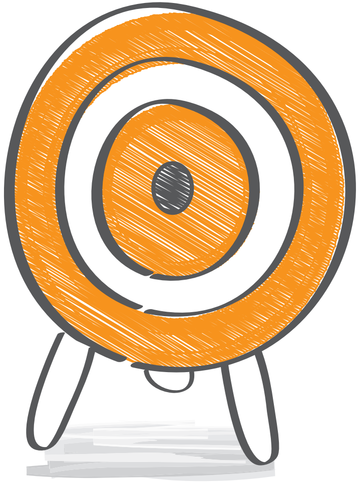
  $f(x) = \sin(\pi x)$ over [0,1].

- Idea:
  - Sample n rectangles.
  - Each is centered randomly on [0,1] and has width 1/n and height f(x).
  - Add up areas.
  - Make n really, really big.
  - Sum of areas approaches the integral of f(x).


Monte Carlo Integration
Monte Carlo Integration
1. Number of Points: 64
2. Enter Seed: 567893
3. Animation Delay: 1
ESTIMATE INTEGRAL
Actual Result: 0.6366
Estimator: 0.5886
BACK
Points Plotted: 64
Y
X

Summary

- Looked at some easy examples of the use of simulation applications in simple settings.
- Next Time: More more more! How do you like it? How do you like it?



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More Baby Examples

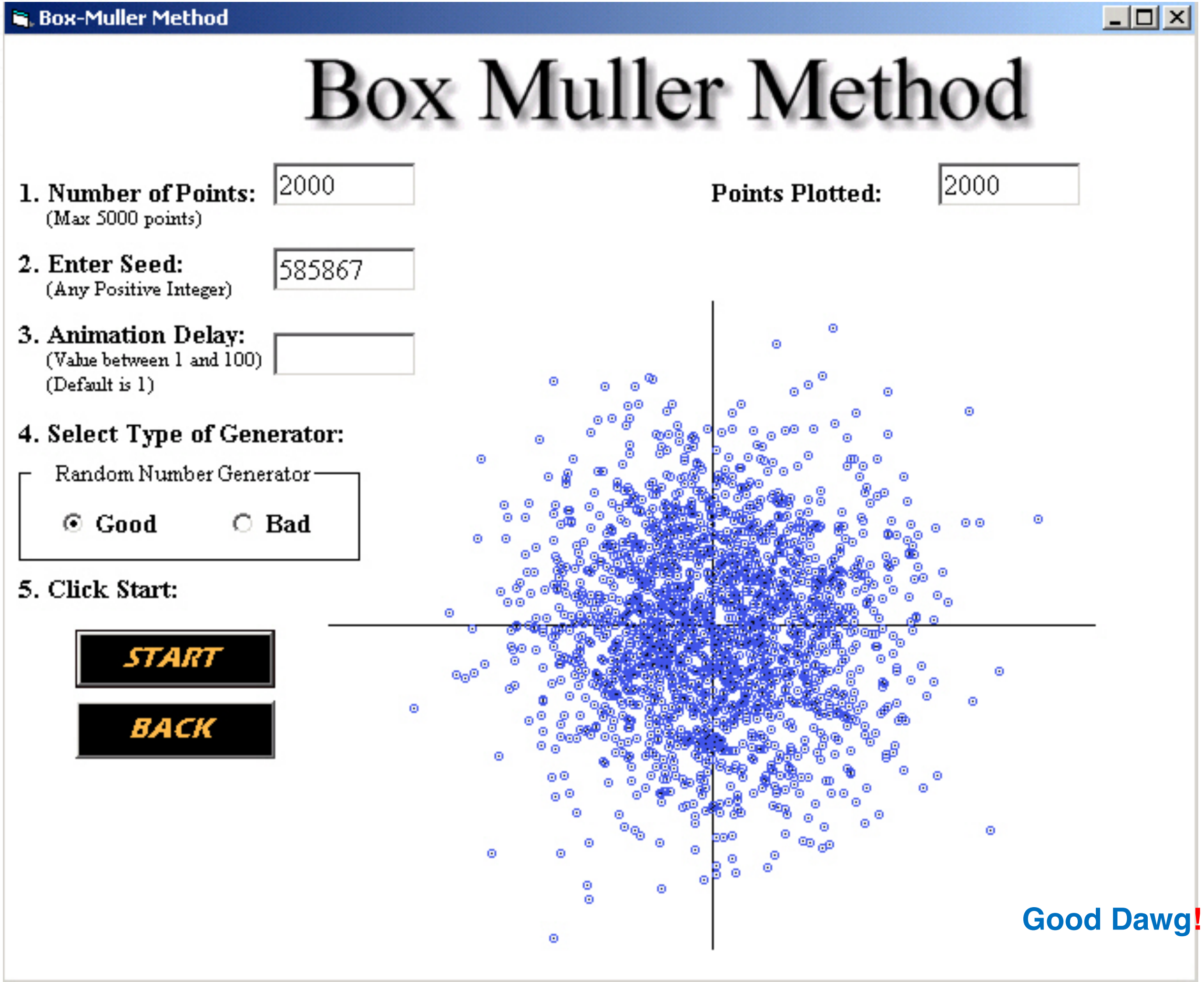
Lesson Overview

Last Time: Some baby examples.

This Time: More baby examples.

Evil Random Numbers

- See what happens when you use a bad random number generator.
- Idea:
  - Simulate heights vs weights.
  - Should be a 2-D bell curve (normal distribution) with most observations in the middle and some on the outside.
  - Do observations “look” random?


Box-Muller Method
Box Muller Method
1. Number of Points:
(Max 5000 points)
2000
Points Plotted:
2000
2. Enter Seed:
(Any Positive Integer)
585867
3. Animation Delay:
(Value between 1 and 100)
(Default is 1)
4. Select Type of Generator:
Random Number Generator
Good
Bad
5. Click Start:
START
BACK

Good Dawg!

Box-Muller Method

Box Muller Method

1. Number of Points: 2000
(Max 5000 points)

Points Plotted: 2000

2. Enter Seed: 64
(Any Positive Integer)

3. Animation Delay:
(Value between 1 and 100)
(Default is 1)

4. Select Type of Generator:

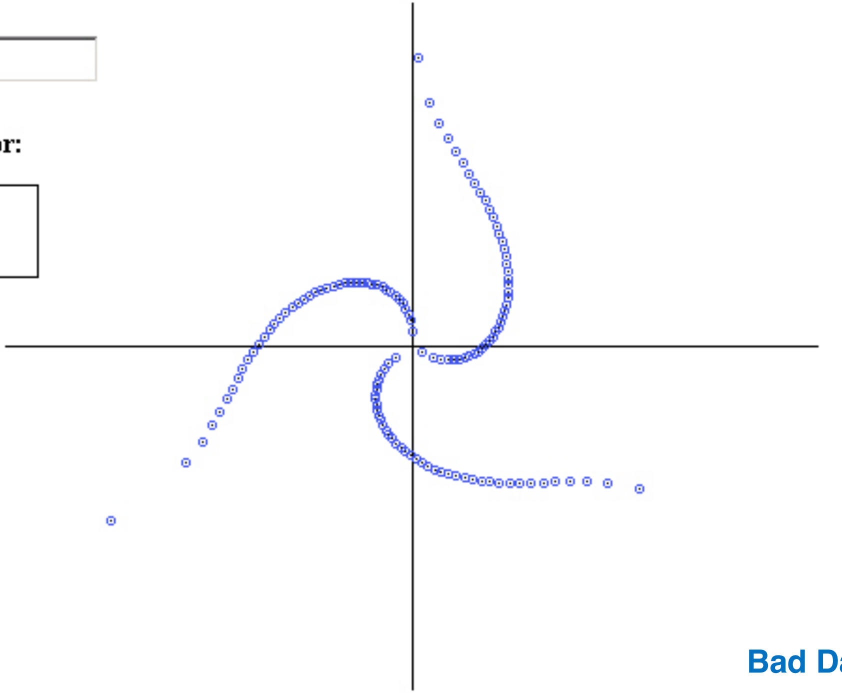
Random Number Generator

- ○ Good
- ◉ Bad

5. Click Start:

START

BACK



Bad Dawg!

Queues 'R Us

- Single-server queue at McWendy's.
- Customers show up, wait in line, get served first-in-first-out.
- What happens as arrival rate approaches service rate?
  - Nothing much?
  - Line gets pretty long?
  - Hamburgers start to taste better?

Queues ‘R Us (cont’d)

- Can analyze queues via simulation.
- Can analyze via numerical or exact methods.

- Fun fact: Notice anything interesting about the word “queueing”? How about “queueoid”?

M/M/1 Queue Simulation

MM1 Queue Simulation

Interarrival Mean? 4

Service Mean? 3

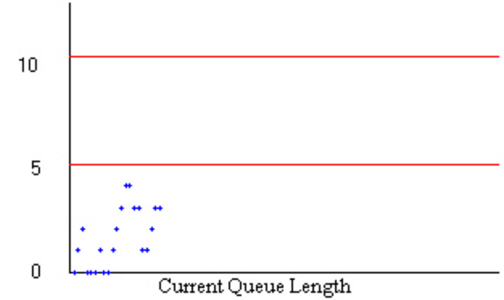
STOP

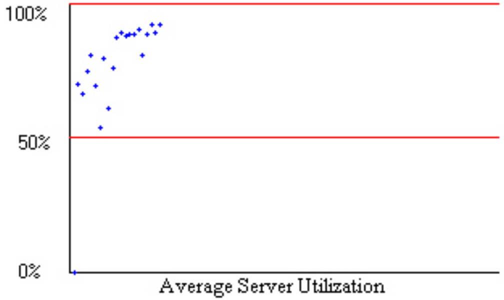
NEXT CUSTOMER

Interarrival time 6

Service Time 1

i	Arrive	Start	Service	Leave	Wait
1	0	0	7	7	0
2	2	7	3	10	5
3	5	10	5	15	5
4	18	18	2	20	0
5	20	20	1	21	0
6	24	24	2	26	0
7	25	26	11	37	1
8	37	37	2	39	0
9	39	39	15	54	0
10	43	54	9	63	11
11	47	63	2	65	16
12	50	65	1	66	15
13	51	66	2	68	15
14	61	68	2	70	7
15	65	70	2	72	5
16	66	72	1	73	6
17	72	73	10	83	1
18	74	83	4	87	9
19	76	87	1	88	11
20	77	88	4	92	11
21	83	92	1	93	9





M/M/1 Queue Simulation: Output Analysis

Output Analysis

DISPLAY DATA BACK

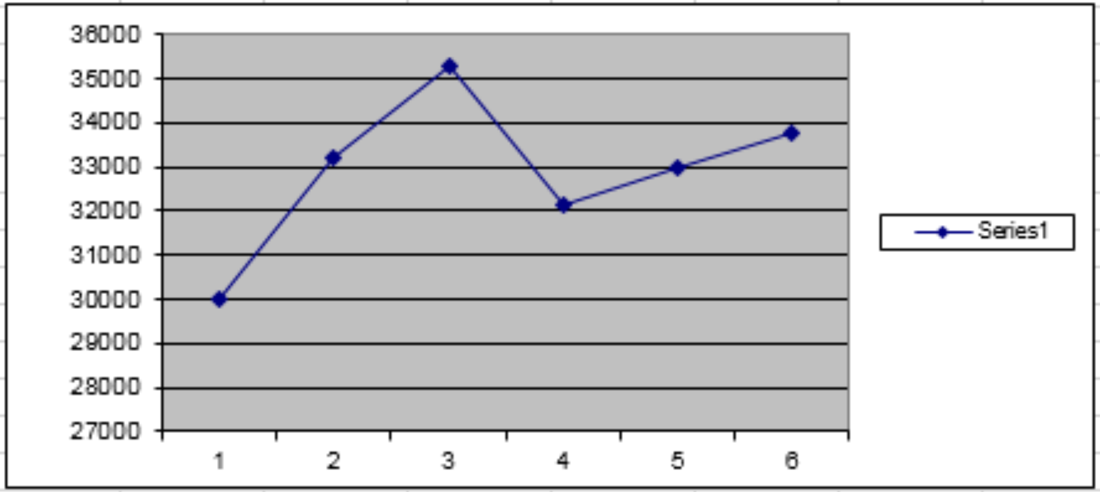
SIMTIME: 093 THROUGHPUT: 021

Average # of Customers in the System:	2.30
Average Time in the System:	10.19
Server Utilization:	93 %
Average Length of the Queue:	1.37
Average Waiting Time in the Queue:	6.05

Stock Market Follies

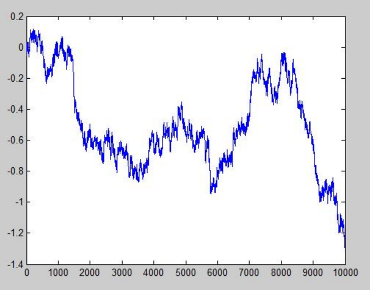
- Simulate a portfolio of various stocks.
- Stock prices change randomly from year to year, with various volatilities.
- Can consider different mixes for portfolio.
- Simple spreadsheet application.

	mean	sd	year0	year1	year2	year3	year4	year5
general year performance (so stocks are correlated)				1.27	1.05	0.77	0.90	0.87
energy	0.05	0.30	5000	2701	3889	3889	3355	4262
pharmaceuticals	0.06	0.20	5000	7730	6913	6522	8764	8136
entertainment	0.04	0.10	5000	6813	8078	6633	5900	5683
insurance	0.07	0.05	5000	6977	7820	6234	6616	6213
banking	0.06	0.30	5000	5238	5935	6803	5837	6240
computer technology	0.18	0.50	5000	3751	2654	2030	2491	3226
Totals			30000	33210.8	35290.2	32110.5	32963.9	33759.2



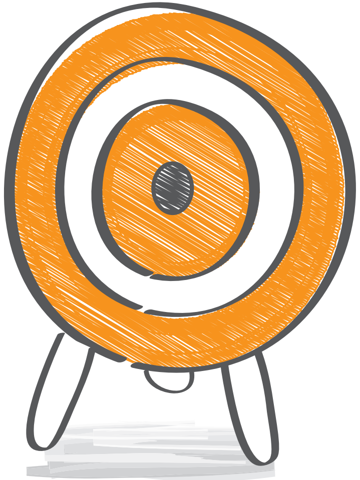
Taking a Random Walk

- Take a normal step up or down every time unit and plot where you are as time progresses.
  - This "random walk" converges to Brownian motion.
  - Einstein and Black+Scholes won Nobel Prizes for this research.



Summary

- We ran simulations on more easy examples, all of which involved randomness.
- Next Time: Let’s generate that randomness on a computer!



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Generating Randomness

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Lesson Overview

Last Time: Presented some really simple baby examples of simulations, all of which used some sort of randomness.

This Time: So how can we generate that randomness?

The big reveal: The algorithms that generate “randomness” aren’t random at all! They just appear to be to you and me.

Randomness

- Need random variables (RV's) to run the simulation, e.g., interarrival times, service times, etc.
- Generate Unif(0,1) pseudo-random numbers (PRN's)
  - Use a deterministic algorithm
  - Not really random, but seem to be
- Generate other RV's
  - Start with Unif(0,1)'s
  - Apply transformations to get any other type of random variable

Unif(0,1) PRN's

- Deterministic algorithm
- Example: Linear Congruential Generator
  - Choose an integer "seed," $X(0)$
  - Set $X(i) = a\,X(i-1) \bmod (m)$, where a and m are carefully chosen constants, and mod is the modulus function
  - Set the ith PRN as $U(i) = X(i)/m$

Unif(0,1) PRN's

Pretend Example:

- Start with $X(0) = 4$ (for no reason)
- Set $X(i) = 5\,X(i-1)$ mod(7

- Then $X(1) = 20 \bmod 7 = 6$
- $X(2) = 2$, $X(3) = 3$, $X(4) = 1$, $X(5) = 5$, etc.

- So $U(1) = X(1)/m = 6/7$
- $U(2) = 2/7$, $U(3) = 3/7$, etc.

- Numbers not so random.

Unif(0,1) PRN's

Real Example

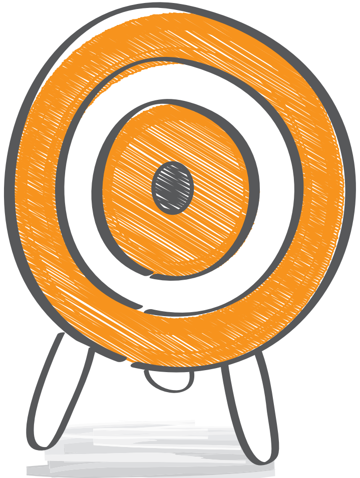
- $X(i) = 16807\ X(i-1) \bmod (2^{31} - 1)$
- $U(i) = X(i) / m$
  - This generator is used in a number of simulation languages
  - Has nice properties, including long "cycle times"
  - Better generators are out there

Generating Other RV's

- Start with U(i) ~ Unif(0,1)
- Apply some appropriate transformation
- Example: -(1/λ) ln(U(i)) ~ Exp(λ)
  - Inverse transform method – can use this for various important distributions
  - Many other more-sophisticated methods available, e.g., Box-Muller method for normals

Summary

- Showed how to generate *appears* to be randomness on a computer!
- Next Time: Random input means random output. Trouble with a Capital T. What to do?



Computer Simulation

Module 1: Introduction + Whirlwind Tour

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Simulation Output Analysis

GTx

Lesson Overview

Last Time: How can we generate that randomness so as to run the simulation?

This Time: Random input means random output, which requires careful analysis. What can we do about that?

Bottom Line: Everything that taught you in your Baby Stats class is a big fat lie – simulation output is never independent and normal. So we'll need new methods.

Analyzing Randomness

- Simulation output is nasty. Consider consecutive customer waiting times.
  - Not normally distributed – usually skewed
  - Not identically distributed – patterns change during the day
  - Not independent – usually correlated
- Can't analyze via "usual" statistics methods! Trouble!

Analyzing Randomness

Two general cases to consider

- Terminating Simulations
  - Interested in short-term behavior
  - Example: Avg customer waiting time in a bank over the course of a day
  - Example: Avg # of infected victims during a pandemic
- Steady-State Simulations
  - Interested in long-term behavior
  - Example: Long-running assembly line

Terminating Simulations

Usually analyzed via Independent Replications

- Make independent runs (replications) of the simulation, each under identical conditions
- Sample means from each replication are assumed to be approximately i.i.d. normal
- Use classical statistics techniques on the i.i.d. sample means (not on the original observations)

Steady-State Simulations

- First deal with initialization (start-up) bias.
  - Usually “warm up” simulation before collecting data
  - Failure to do so can ruin statistical analysis
- Many methods for dealing with steady-state data
  - Batch Means
  - Overlapping Batch Means / Spectral Analysis
  - Standardized Time Series
  - Regeneration

Steady-State Simulations

Method of Batch Means

- Make one long run (vs. many shorter reps)
- Warm up simulation before collecting data
- Chop remaining observations into contiguous batches
- Sample means from each batch are approximately i.i.d. normal
- Use classical statistics on the i.i.d. batch means

Summary

- Talked about analysis of troublesome simulation output.
- This completes Module 1, which went over introductory material to get us salivating for more!
- Coming up: Module 2 will present lots of calc/prob/stats review material from your distant past, albeit it with a little bit of simulation thrown in.