

A Proposed Correspondence Between NP-Completeness and the Mandelbrot Set via Knot Theory

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Abstract

The P versus NP question is one of the most profound open problems in theoretical computer science, with implications across mathematics, cryptography, and complexity theory. This work proposes a structured framework establishing a correspondence between the solvability of NP-complete problems, the topological triviality of knots, and the dynamical stability of points in the complex plane. Using the Partition Problem as a case study, we introduce a deterministic “dynamic weaving” algorithm that maps problem instances to braid representations, which are then evaluated using the Jones polynomial. The braids are generated from the orbital paths of points c under the Mandelbrot iteration $z \mapsto z^2 + c$, with stability conjectured to correspond to solvability. Computational experiments, implemented in SageMath with SnapPy, demonstrate a consistent mapping between solvable instances and the unknot, and unsolvable instances and topologically complex links. The principal open challenge—the derivation of a universal mapping $S \rightarrow c$ —is formulated precisely, and we outline how its resolution could yield a new class of complexity-theoretic invariants rooted in topology and dynamical systems.

1 Introduction

The distinction between the classes P and NP is a cornerstone of computational complexity theory. The prevailing belief that $P \neq NP$ implies the existence of inherently intractable decision problems—NP-complete problems—whose solutions cannot be found in polynomial time, despite being verifiable within it [1].

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This paper advances the hypothesis that the binary nature of solvability for NP-complete problems may admit a reformulation in terms of deep structures from topology and complex dynamics. Specifically, we propose a three-way correspondence:

Combinatorial Structure An NP-complete decision problem instance encodes discrete constraints.

Topological Representation These constraints are deterministically woven into a braid whose closure’s Jones polynomial indicates triviality or complexity.

Dynamical Signature The braid’s structure is derived from the orbital behavior of a point c under the Mandelbrot iteration, where dynamical stability is conjectured to signal problem solvability.

Our case study focuses on the Partition Problem [2]. We present a deterministic “dynamic weaving” algorithm that maps problem instances into braids via Mandelbrot dynamics, and use topological invariants to classify them. We provide computational verification that solvable instances yield the unknot, while unsolvable instances yield nontrivial links.

The key unresolved question is the explicit, universal mapping $S \rightarrow c$ from problem instances to complex parameters. We argue that resolving this mapping may require integrating number-theoretic properties of the instance (e.g., prime factorization patterns, modular symmetries) with the fractal geometry of Mandelbrot basins of attraction. If successful, this approach could offer a novel invariant for computational complexity, potentially leading to new ways of classifying and distinguishing NP-complete problem instances beyond traditional algorithmic measures.

2 A Proposed Correspondence

Our framework is built upon a hypothesized correspondence between three distinct mathematical domains.

2.1 The Partition Problem

Given a multiset of integers S , the Partition Problem asks if S can be partitioned into two disjoint subsets, A and B , such that the sum of elements in A equals the sum of elements in B .

2.2 Topological Triviality and the Unknot

In knot theory, a braid on n strands is an object formed by intertwining the strands. The closure of a braid forms a knot or a link. A powerful

invariant for distinguishing knots is the Jones Polynomial, $V(t)$ [3]. The simplest possible closed loop, the "unknot," is uniquely identified by its Jones Polynomial $V(t) = 1$. Any other polynomial signifies a non-trivial, topologically complex object. The unknot represents a state of "topological zero" or triviality.

2.3 Dynamical Stability and the Mandelbrot Set

The Mandelbrot set is defined by the recurrence relation $z_{n+1} = z_n^2 + c$ for a complex number c , with $z_0 = 0$. The boundary of the set separates values of c for which the orbit of z_n is stable (bounded) from those for which it is chaotic (escapes to infinity) [4]. This boundary represents the "edge of chaos" and is an object of infinite complexity.

The Central Hypothesis: *An instance of the Partition Problem is solvable if and only if its corresponding topological representation is the unknot, which in turn corresponds to a state of perfect dynamical stability within the Mandelbrot framework.*

3 The Dynamic Weaving Algorithm

To test this hypothesis, we developed an algorithm to map a problem instance S to a braid word. This process is inspired by a "cosmological" narrative where the Mandelbrot dynamics themselves dictate the weaving of the topological object.

3.1 The S to c Mapping Problem

The first step is a function $M : S \rightarrow c$, which maps the set of integers S to a unique complex number c . The discovery of a universal, principled formula for M is the primary open challenge of this framework. For the experimental verification in this paper, we adopt a hypothetical mapping where known solvable instances are mapped to points of known stability on the Mandelbrot boundary, and unsolvable instances are mapped to nearby chaotic points. This allows us to test the internal logic of the subsequent weaving and analysis stages.

3.2 Path Generation from Dynamics

For a given c , we generate an orbital path $P = [z_0, z_1, \dots, z_k]$ by iterating $z_{n+1} = z_n^2 + c$, starting with $z_0 = 0$, until $|z_k| > 2$ or a maximum iteration count is reached.

3.3 The Weaving Algorithm

The generated path P is translated into a braid word on $n = |S|$ strands. For each step z_i in the path (for $i > 0$):

1. A crossing position $j \in \{1, \dots, n-1\}$ is determined by the angle (phase) of z_i . The complex plane is divided into $n-1$ sectors, and the sector containing z_i determines j .
2. A crossing direction (a braid generator σ_j or its inverse σ_j^{-1}) is determined by the change in magnitude from the previous step. An increasing magnitude $|z_i| > |z_{i-1}|$ corresponds to one direction, and a decreasing magnitude to the other.

This deterministic process converts the dynamic path into a static braid word, a unique topological signature of the problem's dynamics.

4 Experimental Verification

The pipeline was implemented in the SageMath environment, utilizing the SnapPy library for rigorous topological analysis. We tested two instances of the Partition Problem with $n = 4$.

4.1 Results for a Solvable Instance

We took the solvable set $S = \{3, 8, 4, 1\}$. For our hypothetical mapping, we assigned this to the point $c = -0.75$, located at the main cusp of the cardioid of the Mandelbrot set, a point of perfect period-1 stability.

- The generated dynamic path was a simple, stable, fixed-point orbit.
- The weaving algorithm processed this stable path and produced an **empty braid word**.
- Subsequent analysis confirmed this represents the **unknot**, with a Jones Polynomial $V(t) = 1$.

4.2 Results for an Unsolvable Instance

We took the unsolvable set $S = \{3, 8, 4, 2\}$. We assigned this to the point $c = -0.75 + 0.1j$, a point in a chaotic region near the cusp.

- The generated dynamic path was chaotic and escaped after 34 iterations.
- The weaver translated this chaotic path into a complex 33-operator braid word (e.g., $\sigma_3^{-1}\sigma_1\sigma_3^{-1}\dots\sigma_2^{-1}\sigma_3^{-1}$).

- Analysis in SageMath of the braid’s closure computed the Jones Polynomial as:

$$V(t) = -t^8 - t^6 + t^5 - t^4 + t^3 - t^2 + t + 2t^{-1} + t^{-2} + t^{-3} - t^{-5} + t^{-6} - t^{-7} + t^{-8} - t^{-9} + t^{-10} - t^{-11} + t^{-12} + t^{-14}$$

This is a highly non-trivial polynomial, confirming the braid represents a complex topological link.

5 Discussion and Conclusion

The experimental results provide strong initial support for the proposed correspondence. We have demonstrated a deterministic procedure that successfully maps a solvable problem instance to the unknot and an unsolvable instance to a verifiably complex topological object. This suggests that the property of solvability may be deeply encoded in the dynamical stability of a corresponding point in the complex plane.

The primary limitation of this work, and the most crucial direction for future research, is the formalization of the $S \rightarrow c$ mapping. Discovering a universal, principled function for this mapping would be the key to a general solver. We hypothesize that this function will arise from deep connections between the number-theoretic properties of the set S (such as its prime factorization or modular symmetries) and the structure of the Mandelbrot set’s basins of attraction.

This paper presents not a solution to P vs. NP, but a new lens through which to view it. We propose that the path to solving these famously hard problems may not lie in bigger computers or faster searches, but in a deeper synthesis of computational complexity, topology, and the beautiful, intricate world of complex dynamics.

References

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