Tutorial 13

1. What is the probability of these events when we randomly select a permutation of $\{1,2,3,4\}$?
   a. 1 precedes 4;
   b. 4 precedes 1 and 4 precedes 2;
   c. 4 precedes 3 and 2 precedes 1.
2. What is the probability that a positive integer not exceeding 100 selected at random is divisible by 5 or 7?
3. For each of the following pairs of events, which are subsets of the set of all possible outcomes when a coin is tossed three times, determine whether or not they are independent.
   a. E_1: tails comes up with the coin is tossed the first time; E_2: heads comes up when the coin is tossed the second time.
   b. E_1: the first coin comes up tails; E_2: two, and not three, heads come up in a row.
   c. E_1: the second coin comes up tails; E_2: two, and not three, heads come up in a row.
4. Let E_1 and E_2 be events in sample space Ω. Then we have

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2).$$

Tutorial 17

1. What is the conditional probability that a randomly generated bit string of length four contains at least two consecutive 0s, given that the first bit is a 1? (Assume the probabilities of a 0 and a 1 are the same.)
2. What is the conditional probability that exactly four heads appear when a fair coin is flipped five times, given that the first flip came up heads?
3. Find each of the following probabilities when a coin is flipped n times, and head appears with probability p.
   a. the probability of no failures
   b. the probability of at least one failure
   c. the probability of at most one failure
   d. the probability of at least two failures

4. Suppose that E, F_1, F_2, and F_3 are events from a sample space Ω and that F_1, F_2, and F_3 are pairwise disjoint and their union is Ω. Find $P(E)$ if $P(E|F_1) = 1/8$, $P(E|F_2) = 1/4$, $P(E|F_3) = 1/6$, $P(F_1) = 1/4, P(F_2) = 1/4$, and $P(F_3) = 1/2$.
5. When a test for steroids is given to soccer players, 98% of the players taking steroids test positive and 12% of the players not taking steroids test positive. Suppose that 5% of soccer players take steroids. What is the probability that a soccer player who tests positive takes steroids?