

Discrete Math — Homework 2 Solutions

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Q1

- (a) $\exists p(F(p) \wedge B(p)) \rightarrow \exists jL(j)$: If there exists a printer being out of service and busy, then there exists a print job being lost.
- (b) $\forall pB(p) \rightarrow \exists jQ(j)$: If all printers are busy, then there must be a print job being queued.
- (c) $\exists j(Q(j) \wedge L(j)) \rightarrow \exists pF(p)$: If there exists a print job being lost and queued, then there exists a printer being out of service.
- (d) $(\forall pB(p) \wedge \forall jQ(j)) \rightarrow \exists jL(j)$: If all printers are busy and all print jobs are queued, then there exists a print job being lost.

Q2

- (a) $\neg\forall x\exists y\forall zT(x, y, z) \iff \exists x\forall y\exists z(\neg T(x, y, z))$
- (b) $\neg(\forall x\forall yP(x, y) \vee \forall x\forall yQ(x, y)) \iff (\exists x\exists y\neg P(x, y)) \wedge (\exists x\exists y\neg Q(x, y))$
- (c) $\neg(\forall x\exists y(P(x, y) \wedge \exists zR(x, y, z))) \iff \exists x\forall y\neg(P(x, y) \wedge \exists zR(x, y, z))$
 $\iff \exists x\forall y(\neg P(x, y) \vee \forall z\neg R(x, y, z))$

Q3

$P(x, y) : 2x + y = 0$ where $x, y \in \mathbb{R}$

- (a) $\forall x\exists yP(x, y)$ means for every x , there's a solution for y , which is a tautology.
 $\forall y\exists xP(x, y)$ means for every y , there's a solution for x , which is a tautology as well.
Two tautologies has the same truth value all the time. Thus, they're logically equivalent.
- (b) $2x + y = 0 \implies y = -2x$, let $x := 0.1$, then $y = -0.2 \notin \mathbb{Z}$, so the LHS(left hand side) is not a tautology.
 $2x + y = 0 \implies x = -\frac{y}{2} \in \mathbb{R}$, so the RHS is always true.
So, the statement is not true.
- (c) No. Let $P(x, y)$ be $x^2 = y$, where $x, y \in \mathbb{R}$.

Q4

Let $L(x, y)$ be “ x loves y ”, where the domain for both x and y consists of all people in the world.

- (a) Everybody loves Jerry: $\forall x L(x, \text{Jerry})$.
- (b) Everybody loves somebody: $\forall x \exists y L(x, y)$.
- (c) There is somebody whom everybody loves: $\exists y \forall x L(x, y)$.
- (d) There is somebody whom Lydia does not love: $\exists y \neg L(\text{Lydia}, y)$.
- (e) There is somebody whom no one loves: $\exists y \forall x \neg L(x, y)$.
- (f) There is someone who loves no one besides himself or herself: $\exists x (L(x, x) \wedge (\forall p (x \neq p \rightarrow \neg L(x, p))))$.

Q5

- (a) $(p \rightarrow r) \wedge (q \rightarrow r) \rightarrow ((p \vee q) \rightarrow r)$

Proof:

Hypotheses:

- $p \rightarrow r \iff (\neg p \vee r)$ is true.
- $q \rightarrow r \iff (\neg q \vee r)$ is true.

Thus,

$$\begin{aligned} (p \vee q) \rightarrow r &\iff \neg(p \vee q) \vee r \iff (\neg p \wedge \neg q) \vee r \\ &\iff (\neg p \vee r) \wedge (\neg q \vee r) \end{aligned}$$

the conclusion is drawn. \square

- (b) $(p \rightarrow q) \wedge (r \rightarrow s) \wedge (\neg q \vee \neg s) \rightarrow (p \rightarrow \neg r)$

Proof:

Hypotheses:

- $p \rightarrow q$ is true.
- $r \rightarrow s \iff \neg s \rightarrow \neg r$ is true.
- $\neg q \vee \neg s \iff q \rightarrow \neg s$ is true.

According to hypothetical syllogism, the conclusion is drawn. \square

- (c) $(p \rightarrow q) \wedge (r \rightarrow s) \wedge (p \vee r) \rightarrow (q \vee s)$

Proof:

Hypotheses:

- $p \rightarrow q \iff \neg q \rightarrow \neg p$ is true.
- $r \rightarrow s$ is true.

- $p \vee r \iff \neg p \rightarrow r$ is true.

According to hypothetical syllogism, $\neg q \rightarrow s \iff q \vee s$ is true. \square

(d) $((w \vee r) \rightarrow v) \wedge (v \rightarrow (c \vee s)) \wedge (s \rightarrow u) \wedge \neg c \wedge \neg u \rightarrow \neg w$

Proof:

Hypotheses:

- $(w \vee r) \rightarrow v$ is true.
- $v \rightarrow (c \vee s)$ is true.
- $s \rightarrow u$ is true.
- $\neg c$ is true, which means c is false.
- $\neg u$ is true, which means u is false.

Since u is false and $s \rightarrow u$ is true, s should be false.

Then, $c \vee s$ should be false, and given $v \rightarrow (c \vee s)$ is true, we can deduce that v is false.

Since $(w \vee r) \rightarrow v$ is true, $w \vee r$ is false. Hence, w is false, which means $\neg w$ is true. \square

Q6

If $\forall x(P(x) \vee Q(x))$, $\forall x(\neg Q(x) \vee S(x))$, $\forall x(R(x) \rightarrow \neg S(x))$, and $\exists x\neg P(x)$ are true, then $\exists x\neg R(x)$ is true.

Proof:

\square

Q7

Q8