Discrete Mathematics and Its Applications
Welcome Tutorial :-)
Tutorial 1

DaSE @ ECNU

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Tutorial 1

1. Which of these sentences are propositions? What are the truth values of those that are propositions?
   a. $2+3=5$
   b. $5+7=10$
   c. $x+3=11$
   d. Answer this question.
   e. What time is it?
   f. $2^n \geq 100$
2. Let p, q, and r be the propositions

   p : You have the flu

   q : You miss the final examination

   r : You pass the course

   Express each of these propositions as a sentence
   a. $p \rightarrow q$
   b. $\neg p \leftrightarrow r$
   c. $q \rightarrow \neg r$
   d. $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$
   e. $(p \wedge q) \vee (\neg q \wedge r)$

3. Construct a truth table for each of these compound propositions
   a. $p \rightarrow (\neg q \vee r)$
   b. $(\neg p \vee q) \wedge r$
   c. $(p \oplus q) \vee (p \oplus \neg q)$
   d. $(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$
   e. $(p \vee q) \rightarrow (p \wedge q)$
   f. $(\neg p \oplus q) \rightarrow (\neg p \wedge q)$
4. Evaluate each of these expressions
   a. $11000 \wedge (01101 \vee 11011)$
   b. $(01111 \wedge 10101) \vee 01000$
   c. $(01010 \oplus 11011) \oplus 01000$
   d. $(11011 \vee 01010) \wedge (10001 \vee 11011)$

Tutorial 1 Cont'd

5. Express these system specifications using the propositions p "The message is scanned for viruses" and q "The message was sent from an unknown system" together with logical connectives.
   a. The message is scanned for viruses whenever the message was sent from an unknown system.
   b. The message was sent from an unknown system but it was not scanned for viruses.
   c. It is necessary to scan the message for viruses whenever it was sent from an unknown system.
6. An explorer is captured by a group of cannibals. There are two types of cannibals-those who always tell the truth and those who always lie. The cannibals will barbecue the explorer unless he can determine whether a particular cannibal always lies or always tells the truth. He is allowed to ask the cannibal exactly one question.
   a. Explain why the question "Are you a liar?" does not work.
   b. Find a question that the explorer can use to determine whether the cannibal always lies or always tells me truth.

7. Show that each of these conditional statements is a tautology.
   a. $(p \wedge q) \rightarrow p$;
   b. $p \rightarrow (p \vee q)$;
   c. $\neg p \rightarrow (p \rightarrow q)$;
   d. $(p \wedge q) \rightarrow (p \rightarrow q)$;
   e. $\neg(p \rightarrow q) \rightarrow p$;
   f. $\neg(p \rightarrow q) \rightarrow \neg q$.
8. Determine whether each of these compound propositions is satisfiable
   a. $(p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$;
   b. $(p \rightarrow q) \wedge (p \rightarrow \neg q) \wedge (\neg p \rightarrow q) \wedge (\neg p \rightarrow \neg q)$;
   c. $(p \leftrightarrow q) \wedge (\neg p \leftrightarrow q)$.