Tutorial 2

1. Translate these specifications into English where $F(p)$ is "Printer p is out of service," $B(p)$ is "Printer p is busy," $L(j)$ is "Print job j is lost," and $Q(j)$ is "Print job j is queued."
   a. $\exists p(F(p) \wedge B(p)) \rightarrow \exists j L(j)$;
   b. $\forall p B(p) \rightarrow \exists j Q(j)$;
   c. $\exists j(Q(j) \wedge L(j)) \rightarrow \exists p F(p)$;
   d. $(\forall p B(p) \wedge \forall j Q(j)) \rightarrow \exists j L(j)$.
2. Express the negations of each of these statements so that all negation symbols immediately precede predicates.
   a. $\forall x \exists y \forall z T(x,y,z)$;
   b. $\forall x \forall y P(x,y) \vee \forall x \forall y Q(x,y)$;
   c. $\forall x \exists y (P(x,y) \wedge \exists z R(x,y,x))$;
3. Let $P(x,y)$ be $2x+y=0$, where $x \in R$ and $y \in R$.
   a. $\forall x \exists y P(x,y) \equiv \forall y \exists x P(x,y)$?
   b. If $x \in R$ and $y \in Z$, is the statement in [a.] true?
   c. If $P(x,y)$ is not $2x+y=0$, is the statement in [a.] true? If it is true, please show your reason. Otherwise, please give a counterexample.

4. Let $L(x, y)$ be the statement "x loves y," where the domain for both x and y consists of all people in the world. Use quantifiers to express each of these statements.
   a. Everybody loves Jerry;
   b. Everybody loves somebody;
   c. There is somebody whom everybody loves;
   d. There is somebody whom Lydia does not love;
   e. There is somebody whom no one loves;
   f. There is someone who loves no one besides himself or herself.
5. Argue the following arguments:
   - $p \to r$ and $q \to r$ logically lead to the conclusion $(p \vee q) \to r$.
   - $p \to q$, $r \to s$ and $\neg q \vee \neg s$ logically lead to the conclusion $p \to \neg r$.
   - $p \to q$, $r \to s$ and $p \vee r$ logically lead to the conclusion $q \vee s$.
   - $(w \vee r) \to v$, $v \to (c \vee s)$, $s \to u$, $\neg c$, and $\neg u$ logically lead to the conclusion $\neg w$.
6. Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$, $\forall x(\neg Q(x) \vee S(x))$, $\forall x(R(x) \to \neg S(x))$, and $\exists x \neg P(x)$ are true, then $\exists x \neg R(x)$ is true.

7. For each of these arguments, explain which rules of inference are used for each step.
   a. "Doug, a student in this class, knows how to write programs in JAVA. Everyone who knows how to write programs in JAVA can get a high-paying job. Therefore, someone in this class can get a high-paying job."
   b. "Each of the 93 students in this class owns a personal computer. Everyone who owns a personal computer can use a word processing program. Therefore, Zeke, a student in this class, can use a word processing program."
8. Use resolution to show that the hypotheses "It is not raining or Yvette has her umbrella," "Yvette does not have her umbrella or she does not get wet," and "It is raining or Yvette does not get wet" imply that "Yvette does not get wet."