1. How many 6-element RNA sequences
   a. do not contain U?
   b. end with GU?
   c. start with C?
   d. contain only A or U?

2. Find the value of each of these quantities.
   a. $P(6,4), P(7,5)$;
   b. $C(6,4), C(7,5)$;
   c. $C(6,2), C(7,2)$.

3. How many permutations of the letters ABCDEFG contain
   a. string BCD?
   b. strings BA and GF?
   c. strings ABC and CDE?
   d. strings CBA and BED?

4. A multiple-choice test contains 10 questions. There are four possible answers for each question.
   a. In how many ways can a student answer the questions on the test if the student answers every question?
   b. In how many ways can a student answer the questions on the test if the student can leave answers blank?

5. How many positive integers less than 1000
   a. have distinct digits?
   b. have distinct digits and are even?

6. How many bit strings of length 10 contain
   a. exactly four 1s?
   b. at most four 1s?
   c. at least four 1s?
   d. an equal number of 0s and 1s?

Tutorial 8

1. Find the number of elements in $A_1 \cup A_2 \cup A_3$ if there are 100 elements in each set and if
   a. the sets are pairwise disjoint;
   b. there are 50 common elements in each pair of sets and no elements in all three sets;
   c. there are 50 common elements in each pair of sets and 25 elements in all three sets.
   d. the sets are equal.

2. How many derangements are there of a set with seven elements?

3. How many positive integers less than 200 are
   a. either odd or the square of an integer;
   b. second or higher powers of integers?
   c. either primes or second or higher powers of integers?
   d. not divisible by the square of an integer greater than 1?

4. How many ways are there to choose eight coins from a piggy
   bank containing 100 identical pennies and 80 identical nickels?
5. How many solutions are there to the equation
   $x_1 + x_2 + x_3 + x_4 = 17$
   a. if $x_1, x_2, x_3,$ and x_4 are nonnegative integers?
   b. if $x_1, x_2, x_3,$ and x_4 are positive integers?
   c. if $x_1 \geq 2, x_2 \geq 3,$ $x_3,$ and x_4 are positive integers?
6. How many solutions are there to the equation
   $x_1 + x_2 + x_3 + x_4 \leq 17$
   a. if $x_1, x_2, x_3,$ and x_4 are nonnegative integers?
   b. if $x_1, x_2, x_3,$ and x_4 are positive integers?

1. Find the next larger permutation in lexicographic order after each of these permutations.
   a. 1432;
   b. 54123;
   c. 12453;
   d. 31528764.
2. Find the next larger 5-combination of the set $\{1, 2, 3, 4, 5, 6, 7\}$ after each of these 4-combinations
   a. $\{1, 2, 4, 5, 7\}$;
   b. $\{1, 4, 5, 6, 7\}$;
3. Write the pseudo-code for generating the next permutation in a reverse lexicographic order.
4. Given set $\{n, n-1, \cdots, 1\}$, write the pseudo-code for generating the next r-combination in a reverse lexicographic order.

5. Show that among any group of five (not necessarily consecutive) integers, there are two with the same remainder when divided by 4.

6. Let n be a positive integer. Show that in any set of n consecutive integers there is exactly one divisible by n.

7. Show that whenever 25 girls and 25 boys are seated around a circular table there is always a person both of whose neighbors are boys.