Tutorial 18

1. A shipment of 20 similar laptop computers to a retail outlet contains 3 that are defective. If a school makes a random purchase of 2 of these computers, find the probability distribution for the number of defectives.
2. An experiment consists of tossing 4 fair coins. Compute the probability and distribution functions for the following r.v.s.
   a. The number of heads before the first tail;
   b. The number of heads after the first tail;
   c. The number of heads less the number of tails.
3. A coin is tossed three times. If X is a r.v. giving the number of heads that arise, construct a table showing the probability distribution of X.
4. An urn holds 5 white and 3 black marbles. If 2 marbles are to be drawn at random without replacement and X denotes the number of white marbles, find the probability distribution for X.

5. Suppose that Frida selects a ball by first picking one of two boxes at random and then selecting a ball from this box at random. The first box contains two white balls and three blue balls, and the second box contains four white balls and one blue ball. What is the probability that Frida picked a ball from the first box if she has selected a blue ball?
6. The joint probability function of two r.v.s X and Y is given by $f(x,y) = c(2x+y)$, where x and y can assume all integers such that $0 \leq x \leq 2, 0 \leq y \leq 3$, and $f(x,y) = 0$ otherwise.
   a. Find the value of the constant c;
   b. Find $P(X = 2, Y = 1)$;
   c. Find $P(X \geq 1, Y \leq 2)$;
   d. Find the marginal probability functions of X and Y;
   e. Show that the r.v.s X and Y are dependent;
   f. Compute $f(y = 1|X = 2)$;

Discrete Mathematics and Its Applications
Welcome Tutorial :-)

Tutorial 19

Tutorial 19

1. Suppose that a Bayesian spam filter is trained on a set of 1000 spam messages and 400 messages that are not spam. The word "opportunity" appears in 175 spam messages and 20 messages that are not spam. Would an incoming message be rejected as spam if it contains the word "opportunity" and the threshold for rejecting a message is 0.9?
2. Suppose that 8% of the patients tested in a clinic are infected with HIV. Furthermore, suppose that when a blood test for HIV is given, 98% of the patients infected with HIV test positive and that 3% of the patients not infected with HIV test positive. What is the probability that
   - a patient testing positive for HIV is infected with it?
   - a patient testing positive for HIV is not infected with it?
   - a patient testing negative for HIV is infected with it?
   - a patient testing negative for HIV is not infected with it?

3. Let X be the number appearing on the first die when two fair dice are rolled and let Y be the sum of the numbers appearing on the two dice. Show that $E(X)E(Y) \neq E(XY)$.
4. The **law of total expectation** states that if sample space Ω is the disjoint union of events $S_1, S_2, \cdots, S_n$ and X is a r.v., then $E(X) = \sum_{j=1}^{n} E(X|S_j)P(S_j)$, where $E(X|S_j)$ is the **conditional expectation** of r.v. given event S_j from sample space Ω, and can be computed as $E(X|S_j) = \sum_{r \in X(\Omega)} r \cdot P(X = r|S_j)$.
   a. Prove the law of total expectation;
   b. Use the law of total expectation to find the average weight of a breeding elephant seal, given that 12% of the breeding elephant seals are male and the rest are female, and the expected weights of a breeding elephant seal is 4,200 pounds for a male and 1,100 pounds for a female.