Tutorial 3

1. Show that these statements about the integer x are equivalent:
   a. $3x+2$ is even
   b. $x+5$ is odd
   c. x^2 is even
2. Prove that at least one of the real numbers $a_1, a_2, \cdots, a_n$ is greater than or equal to the average of these numbers. What kind of proof did you use? And show your proof.
3. Prove that there is no positive integer n such that $n^2 + n^3 = 100$.

4. Any dollar sum greater than 12 can be formed by the combination of 4 and 5 dollar coins (Using mathematical induction).
5. Prove that

$$3+3\times5+3\times5^2+\cdots+3\times5^n=3(5^{n+1}-1)/4$$

whenever n is a nonnegative integer.

6. Prove that if $A_1, A_2, \cdots, A_n$ and $B_1, B_2, \cdots, B_n$ are sets such that $A_j \subseteq B_j$ for $j = 1, 2, \cdots, n$, then

$$\bigcap_{j=1}^{n} A_j \subseteq \bigcap_{j=1}^{n} B_j.$$