Discrete Math — Homework 1 Solutions

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Q1

(a) $\exists p(F(p) \land B(p)) \to \exists j L(j)$: If there exists a printer being out of service and busy, then then there exists a print job being lost.

(b) $\forall p B(p) \to \exists j Q(j)$: If all printers are busy, then there must be a print job being queued.

(c) $\exists j(Q(j) \land L(j)) \to \exists p F(p)$: If there exists a print job being lost and queued, then there exists a printer being out of service.

(d) $(\forall p B(p) \land \forall j Q(j)) \to \exists j L(j)$: If all printers are busy and all print jobs are queued, then there exists a print job being lost.

Q2

(a) $\neg\forall x\exists y\forall z T(x, y, z) \iff \exists x\forall y\exists z(\neg T(x, y, z))$

(b) $\neg(\forall x\forall y P(x, y) \lor \forall x\forall y Q(x, y)) \iff (\exists x\exists y\neg P(x, y)) \land (\exists x\exists y\neg Q(x, y))$

(c) $\neg(\forall x\exists y(P(x, y) \land \exists z R(x, y, z))) \iff \exists x\forall y\neg(P(x, y) \land \exists z R(x, y, z))$
$\iff \exists x\forall y(\neg P(x, y) \lor \forall z\neg R(x, y, z))$

Q3

$P(x, y) : 2x + y = 0$ where $x, y \in \mathbb{R}$

(a) $\forall x\exists y P(x, y)$ means for every x, there's a solution for y, which is a tautology.
$\forall y\exists x P(x, y)$ means for every y, there's a solution for x, which is a tautology as well.
Two tautologys has the same truth value all the time. Thus, they're logically equivalent.

(b) $2x + y = 0 \implies y = -2x$, let $x := 0.1$, then $y = -0.2 \notin \mathbb{Z}$, so the LHS(left hand side) is not a tautology.
$2x + y = 0 \implies x = -\frac{y}{2} \in \mathbb{R}$, so the RHS is always true.
So, the statement is not true.

(c) No. Let $P(x, y)$ be $x^2 = y$, where $x, y \in \mathbb{R}$.

Q4

Let $L(x, y)$ be "x loves y", where the domain for both x and y consists of all people in the world.

(a) Everybody loves Jerry: $\forall x L(x, \text{Jerry})$.

(b) Everybody loves somebody: $\forall x \exists y L(x, y)$.

(c) There is somebody whom everybody loves: $\exists y \forall x L(x, y)$.

(d) There is somebody whom Lydia does not love: $\exists y \neg L(\text{Lydia}, y)$.

(e) There is somebody whom no one loves: $\exists y \forall x \neg L(x, y)$.

(f) There is someone who loves no one besides himself or herself: $\exists x(L(x, x) \wedge (\forall p(x \neq p \rightarrow \neg L(x, p)))$.

Q5

Q6

Q7

Q8