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## 1 aclfoundation Theory

Built: 25 February 2018

Parent Theories: indexedLists, patternMatches

### 1.1 Datatypes

```
Form =
    TT
  | FF
  | prop 'aavar
  | notf (('aavar, 'apn, 'il, 'sl) Form)
  | (andf) (('aavar, 'apn, 'il, 'sl) Form)
           (('aavar, 'apn, 'il, 'sl) Form)
  | (orf) (('aavar, 'apn, 'il, 'sl) Form)
          (('aavar, 'apn, 'il, 'sl) Form)
  | (impf) (('aavar, 'apn, 'il, 'sl) Form)
           (('aavar, 'apn, 'il, 'sl) Form)
  | (eqf) (('aavar, 'apn, 'il, 'sl) Form)
          (('aavar, 'apn, 'il, 'sl) Form)
  | (says) ('apn Princ) (('aavar, 'apn, 'il, 'sl) Form)
  | (speaks_for) ('apn Princ) ('apn Princ)
  | (controls) ('apn Princ) (('aavar, 'apn, 'il, 'sl) Form)
  | reps ('apn Princ) ('apn Princ)
         (('aavar, 'apn, 'il, 'sl) Form)
  | (domi) (('apn, 'il) IntLevel) (('apn, 'il) IntLevel)
  | (eqi) (('apn, 'il) IntLevel) (('apn, 'il) IntLevel)
  | (doms) (('apn, 'sl) SecLevel) (('apn, 'sl) SecLevel)
  | (eqs) (('apn, 'sl) SecLevel) (('apn, 'sl) SecLevel)
  | (eqn) num num
  | (lte) num num
  | (lt) num num
Kripke =
    KS ('aavar -> 'aaworld -> bool)
       ('apn -> 'aaworld -> 'aaworld -> bool) ('apn -> 'il)
       ('apn -> 'sl)
Princ =
   Name 'apn
  | (meet) ('apn Princ) ('apn Princ)
  | (quoting) ('apn Princ) ('apn Princ);
IntLevel = iLab 'il | il 'apn ;
SecLevel = sLab 'sl | sl 'apn
```

#### 1.2 Definitions

```
[imapKS_def]
  \vdash \forall \mathit{Intp} \ \mathit{Jfn} \ \mathit{ilmap} \ \mathit{slmap}.
        imapKS (KS Intp Jfn ilmap slmap) = ilmap
[intpKS_def]
  \vdash \ \forall \mathit{Intp} \ \mathit{Jfn} \ \mathit{ilmap} \ \mathit{slmap}.
        intpKS (KS Intp Jfn ilmap slmap) = Intp
[jKS_def]
  \vdash \forall Intp \ Jfn \ ilmap \ slmap. jKS (KS Intp \ Jfn \ ilmap \ slmap) = Jfn
[01_def]
 ⊢ 01 = PO one_weakorder
[one_weakorder_def]
 \vdash \forall x \ y. \ \text{one\_weakorder} \ x \ y \iff \mathtt{T}
[po_TY_DEF]
 \vdash \exists \mathit{rep}. TYPE_DEFINITION WeakOrder \mathit{rep}
[po_tybij]
 \vdash (\forall\,a. PO (repPO a) = a) \land
     \forall r. WeakOrder r \iff (repPO (PO r) = r)
[prod_PO_def]
  \vdash \forall PO_1 \ PO_2.
        prod_PO PO_1 PO_2 = PO (RPROD (repPO PO_1) (repPO PO_2))
[smapKS_def]
  \vdash \forall Intp \ Jfn \ ilmap \ slmap.
        smapKS (KS Intp Jfn ilmap slmap) = slmap
[Subset_PO_def]
 \vdash Subset_P0 = P0 (\subseteq)
1.3 Theorems
[abs_po11]
  \vdash \forall r \ r'.
        \texttt{WeakOrder} \ r \ \Rightarrow \ \texttt{WeakOrder} \ r' \ \Rightarrow \ \texttt{((PO} \ r \ \texttt{=} \ \texttt{PO} \ r') \ \Longleftrightarrow \ (r \ \texttt{=} \ r'))
[absPO_fn_onto]
 \vdash \ \forall \, a . \ \exists \, r . \ (a = \texttt{PO} \ r) \ \land \ \texttt{WeakOrder} \ r
```

```
[antisym_prod_antisym]
 \vdash \forall r \ s.
       antisymmetric r \wedge \text{antisymmetric } s \Rightarrow
       antisymmetric (RPROD r s)
[EQ_WeakOrder]
 ⊢ WeakOrder (=)
[KS_bij]
 \vdash \forall M. M = KS \text{ (intpKS } M) \text{ (jKS } M) \text{ (imapKS } M) \text{ (smapKS } M)
[one_weakorder_WO]
 ⊢ WeakOrder one_weakorder
[onto_po]
 \vdash \forall r. WeakOrder r \iff \exists a. r = repPO a
[po_bij]
 \vdash (\forall a. PO (repPO a) = a) \land
    \forall r. WeakOrder r \iff (repPO (PO r) = r)
[PO_repPO]
 \vdash \forall a. \ PO \ (repPO \ a) = a
[refl_prod_refl]
 \vdash \ \forall \, r \ s. reflexive r \ \land reflexive s \ \Rightarrow reflexive (RPROD r \ s)
[repPO_iPO_partial_order]
 \vdash (\forall x. repPO iPO x x) \land
     (\forall x \ y. \ \texttt{repPO} \ iPO \ x \ y \ \land \ \texttt{repPO} \ iPO \ y \ x \ \Rightarrow \ (x = y)) \ \land
    \forall x \ y \ z. repPO iPO \ x \ y \ \land repPO iPO \ y \ z \Rightarrow repPO iPO \ x \ z
[repP0_01]
 ⊢ repPO 01 = one_weakorder
[repPO_prod_PO]
 \vdash \forall po_1 po_2.
       repPO (prod_PO po_1 po_2) = RPROD (repPO po_1) (repPO po_2)
[repPO_Subset_PO]
 \vdash repPO Subset_PO = (\subseteq)
[RPROD_THM]
 \vdash \forall r \ s \ a \ b.
       RPROD r s a b \iff r (FST a) (FST b) \wedge s (SND a) (SND b)
```

```
[SUBSET\_WO] \\ \vdash WeakOrder (\subseteq) \\ [trans\_prod\_trans] \\ \vdash \forall r \ s. \ transitive \ r \land \ transitive \ s \Rightarrow \ transitive \ (RPROD \ r \ s) \\ [WeakOrder\_Exists] \\ \vdash \exists R. \ WeakOrder \ R \\ [WO\_prod\_WO] \\ \vdash \forall r \ s. \ WeakOrder \ r \land \ WeakOrder \ s \Rightarrow \ WeakOrder \ (RPROD \ r \ s) \\ [WO\_repPO] \\ \vdash \forall r. \ WeakOrder \ r \iff (repPO \ (PO \ r) = r) \\
```

### 2 aclsemantics Theory

Built: 25 February 2018

Parent Theories: aclfoundation

#### 2.1 Definitions

```
[Efn_def]
 \vdash (\forall Oi \ Os \ M. Efn Oi \ Os \ M TT = \mathcal{U}(:,v)) \land
     (\forall Oi \ Os \ M. \ Efn \ Oi \ Os \ M \ FF = \{\}) \land
     (\forall~Oi~Os~M~p. Efn Oi~Os~M (prop p) = intpKS M~p) \land
     (\forall Oi \ Os \ M \ f.
         Efn Oi\ Os\ M (notf f) = \mathcal{U}(:'v) DIFF Efn Oi\ Os\ M f) \land
     (\forall Oi \ Os \ M \ f_1 \ f_2.
         Efn Oi Os M (f_1 and f_2) =
         Efn Oi Os M f_1 \cap Efn Oi Os M f_2) \wedge
     (\forall Oi \ Os \ M \ f_1 \ f_2.
         Efn Oi Os M (f_1 orf f_2) =
         Efn Oi Os M f_1 \cup Efn Oi Os M f_2) \wedge
     (\forall Oi \ Os \ M \ f_1 \ f_2.
         Efn Oi Os M (f_1 \text{ impf } f_2) =
         \mathcal{U}(:, v) DIFF Efn Oi\ Os\ M\ f_1\ \cup Efn Oi\ Os\ M\ f_2)\ \wedge
     (\forall Oi \ Os \ M \ f_1 \ f_2.
         Efn Oi Os M (f_1 eqf f_2) =
         (\mathcal{U}(:\,\,{}^{\backprime}\mathtt{v}) DIFF Efn Oi\ Os\ M\ f_1\ \cup Efn Oi\ Os\ M\ f_2) \cap
         (\mathcal{U}(:, v) DIFF Efn Oi\ Os\ M\ f_2\ \cup Efn Oi\ Os\ M\ f_1)) \wedge
     (\forall Oi \ Os \ M \ P \ f.
         Efn Oi\ Os\ M\ (P\ says\ f) =
         \{w \mid \text{Jext (jKS } M) \mid P \mid w \subseteq \text{Efn } Oi \mid Os \mid M \mid f\} \}) \land 
     (\forall Oi \ Os \ M \ P \ Q.
         Efn Oi \ Os \ M (P speaks_for Q) =
```

```
if Jext (jKS M) Q RSUBSET Jext (jKS M) P then U(:'v)
         else { }) \ \
     (\forall Oi \ Os \ M \ P \ f.
         Efn Oi\ Os\ M (P controls f) =
         \mathcal{U}(: `v) DIFF \{w \mid \text{Jext (jKS } M) \mid P \mid w \subseteq \text{Efn } Oi \mid Os \mid M \mid f\} \cup \mathcal{U}(: `v)
         Efn Oi Os M f) \land
     (\forall Oi \ Os \ M \ P \ Q \ f.
         Efn Oi\ Os\ M (reps P\ Q\ f) =
         \mathcal{U}(:,v) DIFF
         \{w \mid \text{Jext (jKS } M) \mid (P \text{ quoting } Q) \mid w \subseteq \text{Efn } Oi \mid Os \mid M \mid f\} \cup G
         \{w \mid \text{Jext (jKS } M) \mid Q \mid w \subseteq \text{Efn } Oi \mid Os \mid M \mid f\} \}
     (\forall Oi \ Os \ M \ intl_1 \ intl_2.
         Efn Oi \ Os \ M \ (intl_1 \ domi \ intl_2) =
         if repPO Oi (Lifn M intl_2) (Lifn M intl_1) then \mathcal{U}(:,v)
         else { }) \ \
     (\forall Oi \ Os \ M \ intl_2 \ intl_1.
         Efn Oi \ Os \ M \ (intl_2 \ eqi \ intl_1) =
         (if repPO Oi (Lifn M intl_2) (Lifn M intl_1) then \mathcal{U}(:,v)
          else { }) ∩
         if repPO Oi (Lifn M intl_1) (Lifn M intl_2) then \mathcal{U}(:,v)
         else { }) \
     (\forall Oi \ Os \ M \ secl_1 \ secl_2.
         Efn Oi \ Os \ M \ (secl_1 \ doms \ secl_2) =
         if repPO Os (Lsfn M secl_2) (Lsfn M secl_1) then \mathcal{U}(:,v)
         else { }) \ \
     (\forall Oi \ Os \ M \ secl_2 \ secl_1.
         Efn Oi\ Os\ M\ (secl_2\ eqs\ secl_1) =
         (if repPO Os (Lsfn M secl_2) (Lsfn M secl_1) then \mathcal{U}(:,v)
          else { }) ∩
         if repPO Os (Lsfn M secl_1) (Lsfn M secl_2) then \mathcal{U}(:,v)
         else { }) \ \
     (\forall Oi \ Os \ M \ numExp_1 \ numExp_2.
         Efn Oi\ Os\ M\ (numExp_1\ eqn\ numExp_2) =
         if numExp_1 = numExp_2 then \mathcal{U}(:,v) else \{\}) \land
     (\forall Oi \ Os \ M \ numExp_1 \ numExp_2.
         Efn Oi\ Os\ M\ (numExp_1\ lte\ numExp_2) =
         if numExp_1 \leq numExp_2 then \mathcal{U}(:'v) else \{\}) \land
     \forall Oi \ Os \ M \ numExp_1 \ numExp_2.
       Efn Oi\ Os\ M (numExp_1 lt numExp_2) =
       if numExp_1 < numExp_2 then \mathcal{U}(:,v) else \{\}
[Jext_def]
 \vdash (\forall J \ s. Jext J (Name s) = J \ s) \land
     (\forall J P_1 P_2.
         Jext J (P_1 meet P_2) = Jext J P_1 RUNION Jext J P_2) \wedge
     \forall J \ P_1 \ P_2. Jext J \ (P_1 \ \text{quoting} \ P_2) = Jext J \ P_2 O Jext J \ P_1
[Lifn_def]
 \vdash (\forall M \ l. Lifn M (iLab l) = l) \land
    \forall M \ name. Lifn M (il name) = imapKS M name
```

[Lsfn\_def]

```
\vdash (\forall M \ l. Lsfn M (sLab l) = l) \land
    \forall\,M name. Lsfn M (sl name) = smapKS M name
2.2
        Theorems
[andf_def]
 \vdash \ \forall \ Oi \ Os \ M \ f_1 \ f_2.
       Efn Oi\ Os\ M (f_1\ {\rm andf}\ f_2) = Efn Oi\ Os\ M\ f_1 \cap Efn Oi\ Os\ M\ f_2
[controls_def]
 \vdash \forall Oi \ Os \ M \ P \ f.
       Efn Oi\ Os\ M (P controls f) =
       \mathcal{U}(:"v) DIFF \{w \mid \text{Jext (jKS } M) \mid P \mid w \subseteq \text{Efn } Oi \mid Os \mid M \mid f\} \cup \mathcal{U}(:"v)
       Efn Oi Os M f
[controls_says]
 \vdash \forall M \ P \ f.
       Efn Oi\ Os\ M (P controls f) = Efn Oi\ Os\ M (P says f impf f)
[domi_def]
 \vdash \ \forall \ Oi \ Os \ M \ intl_1 \ intl_2.
       Efn Oi Os M (intl_1 domi intl_2) =
       if repPO Oi (Lifn M intl_2) (Lifn M intl_1) then \mathcal{U}(:'v)
       else { }
[doms_def]
 \vdash \ \forall \ Oi \ Os \ M \ secl_1 \ secl_2.
       Efn Oi \ Os \ M \ (secl_1 \ doms \ secl_2) =
       if repPO Os (Lsfn M secl_2) (Lsfn M secl_1) then \mathcal{U}(:,v)
       else { }
[eqf_def]
 \vdash \forall Oi \ Os \ M \ f_1 \ f_2.
       Efn Oi \ Os \ M \ (f_1 \ eqf \ f_2) =
        (\mathcal{U}(:\mbox{`v}) DIFF Efn Oi Os M f_1 \cup Efn Oi Os M f_2) \cap
        (\mathcal{U}(:, v) DIFF Efn Oi\ Os\ M\ f_2\ \cup Efn Oi\ Os\ M\ f_1)
[eqf_impf]
 \vdash \ \forall M \ f_1 \ f_2.
       Efn Oi Os M (f_1 eqf f_2) =
       Efn Oi Os M ((f_1 impf f_2) andf (f_2 impf f_1))
```

```
[eqi_def]
 \vdash \ \forall \ \mathit{Oi} \ \ \mathit{Os} \ \ \mathit{M} \ \ \mathit{intl}_2 \ \ \mathit{intl}_1 \, .
       Efn Oi\ Os\ M\ (intl_2\ eqi\ intl_1) =
        (if repPO Oi (Lifn M intl_2) (Lifn M intl_1) then \mathcal{U}(:,v)
         else { }) ∩
       if repPO Oi (Lifn M intl_1) (Lifn M intl_2) then \mathcal{U}(:,v)
       else { }
[eqi_domi]
 \vdash \ \forall M \ intL_1 \ intL_2.
       Efn Oi \ Os \ M \ (intL_1 \ eqi \ intL_2) =
       Efn Oi Os M (intL_2 domi intL_1 and intL_1 domi intL_2)
[eqn_def]
 \vdash \ \forall \ Oi \ Os \ M \ numExp_1 \ numExp_2.
       Efn Oi\ Os\ M (numExp_1 eqn numExp_2) =
       if numExp_1 = numExp_2 then \mathcal{U}(:,v) else \{\}
[eqs_def]
 \vdash \forall Oi \ Os \ M \ secl_2 \ secl_1.
       Efn Oi \ Os \ M \ (secl_2 \ eqs \ secl_1) =
        (if repPO Os (Lsfn M secl_2) (Lsfn M secl_1) then \mathcal{U}(:,v)
         else { }) ∩
       if repPO Os (Lsfn M secl_1) (Lsfn M secl_2) then \mathcal{U}(:,v)
       else { }
[eqs_doms]
 \vdash \forall M \ secL_1 \ secL_2.
       Efn Oi\ Os\ M\ (secL_1\ eqs\ secL_2) =
       Efn Oi Os M (secL_2 doms secL_1 and secL_1 doms secL_2)
[FF_def]
 \vdash \forall Oi \ Os \ M. Efn Oi \ Os \ M FF = {}
[impf_def]
 \vdash \ \forall \ Oi \ Os \ M \ f_1 \ f_2.
       Efn Oi \ Os \ M \ (f_1 \ \text{impf} \ f_2) =
       \mathcal{U}(:, v) DIFF Efn Oi Os M f_1 \cup Efn Oi Os M f_2
[lt_def]
 \vdash \ \forall \ Oi \ Os \ M \ numExp_1 \ numExp_2.
       Efn Oi\ Os\ M\ (numExp_1\ lt\ numExp_2) =
       if numExp_1 < numExp_2 then \mathcal{U}(:,v) else \{\}
[lte_def]
 \vdash \forall Oi \ Os \ M \ numExp_1 \ numExp_2.
       Efn Oi\ Os\ M (numExp_1 lte numExp_2) =
       if numExp_1 \leq numExp_2 then \mathcal{U}(:,v) else \{\}
```

```
[meet_def]
 \vdash \forall J \ P_1 \ P_2. Jext J \ (P_1 \ \text{meet} \ P_2) = Jext J \ P_1 RUNION Jext J \ P_2
[name_def]
 \vdash \forall J \ s. \ \texttt{Jext} \ J \ (\texttt{Name} \ s) = J \ s
[notf_def]
 \vdash \forall Oi \ Os \ M \ f. \ \texttt{Efn} \ Oi \ Os \ M \ (\texttt{notf} \ f) = \mathcal{U}(:'\texttt{v}) \ \texttt{DIFF} \ \texttt{Efn} \ Oi \ Os \ M \ f
[orf_def]
 \vdash \forall Oi \ Os \ M \ f_1 \ f_2.
        Efn Oi Os M (f_1 orf f_2) = Efn Oi Os M f_1 \cup Efn Oi Os M f_2
[prop_def]
 \vdash \ \forall \ Oi \ Os \ M \ p. Efn Oi \ Os \ M (prop p) = intpKS M p
[quoting_def]
 \vdash \forall J \ P_1 \ P_2. Jext J (P_1 quoting P_2) = Jext J P_2 O Jext J P_1
[reps_def]
  \vdash \ \forall \ Oi \ Os \ M \ P \ Q \ f.
        Efn Oi\ Os\ M (reps P\ Q\ f) =
        \mathcal{U}(:,v) DIFF
        \{w \mid \mathsf{Jext} \ (\mathsf{jKS} \ M) \ (P \ \mathsf{quoting} \ Q) \ w \subseteq \mathsf{Efn} \ \mathit{Oi} \ \mathit{Os} \ M \ f\} \ \cup
        \{w \mid \text{Jext (jKS } M) \mid Q \mid w \subseteq \text{Efn } Oi \mid Os \mid M \mid f\}
[says_def]
 \vdash \forall Oi \ Os \ M \ P \ f.
        Efn Oi \ Os \ M \ (P \ \text{says} \ f) =
        \{w \mid \text{Jext (jKS } M) \mid P \mid w \subseteq \text{Efn } Oi \mid Os \mid M \mid f\}
[speaks_for_def]
  \vdash \forall Oi \ Os \ M \ P \ Q.
        Efn Oi\ Os\ M (P speaks_for Q) =
        if Jext (jKS M) Q RSUBSET Jext (jKS M) P then \mathcal{U}(:'v)
[TT_def]
 \vdash \ \forall \ Oi \ Os \ M . Efn Oi \ Os \ M TT = \mathcal{U}(: `v)
```

## 3 aclrules Theory

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Parent Theories: aclsemantics

#### 3.1 Definitions

```
[sat_def]
 \vdash \forall M \ Oi \ Os \ f. \ (M,Oi,Os) \ \text{sat} \ f \iff (\text{Efn} \ Oi \ Os \ M \ f = \mathcal{U}(:'world))
3.2
       Theorems
[And_Says]
 \vdash \forall M \ Oi \ Os \ P \ Q \ f.
       (M,Oi,Os) sat P meet Q says f eqf P says f and Q says f
[And_Says_Eq]
 \vdash (M,Oi,Os) sat P meet Q says f \iff
     (M,Oi,Os) sat P says f and Q says f
[and_says_lemma]
 \vdash \forall M \ Oi \ Os \ P \ Q \ f.
       (M,Oi,Os) sat P meet Q says f impf P says f and f says f
[Controls_Eq]
 \vdash \forall M \ Oi \ Os \ P \ f.
       (M,Oi,Os) sat P controls f\iff (M,Oi,Os) sat P says f impf f
[DIFF_UNIV_SUBSET]
 \vdash (\mathcal{U}(:'a) DIFF s \cup t = \mathcal{U}(:'a)) \iff s \subseteq t
[domi_antisymmetric]
 \vdash \ \forall M \ Oi \ Os \ l_1 \ l_2.
       (M, Oi, Os) sat l_1 domi l_2 \Rightarrow
       (M, Oi, Os) sat l_2 domi l_1 \Rightarrow
       (M,Oi,Os) sat l_1 eqi l_2
[domi_reflexive]
 \vdash \ \forall \, M \ Oi \ Os \ l. \ (M,Oi,Os) \ {\it sat} \ l \ {\it domi} \ l
[domi_transitive]
 \vdash \ \forall M \ Oi \ Os \ l_1 \ l_2 \ l_3.
       (M, Oi, Os) sat l_1 domi l_2 \Rightarrow
       (M, Oi, Os) sat l_2 domi l_3 \Rightarrow
       (M,Oi,Os) sat l_1 domi l_3
[doms_antisymmetric]
 \vdash \ \forall M \ Oi \ Os \ l_1 \ l_2.
       (M,Oi,Os) sat l_1 doms l_2 \Rightarrow
       (M,Oi,Os) sat l_2 doms l_1 \Rightarrow
       (M,Oi,Os) sat l_1 eqs l_2
```

```
[doms_reflexive]
 \vdash \ \forall \, M \ Oi \ Os \ l. \ (M,Oi,Os) \ {\it sat} \ l \ {\it doms} \ l
[doms_transitive]
 \vdash \ \forall M \ Oi \ Os \ l_1 \ l_2 \ l_3.
        (M, Oi, Os) sat l_1 doms l_2 \Rightarrow
        (M,Oi,Os) sat l_2 doms l_3 \Rightarrow
        (M, Oi, Os) sat l_1 doms l_3
[eqf_and_impf]
 \vdash \ \forall M \ Oi \ Os \ f_1 \ f_2.
        (M,Oi,Os) sat f_1 eqf f_2 \iff
        (M, Oi, Os) sat (f_1 \text{ impf } f_2) and (f_2 \text{ impf } f_1)
[eqf_andf1]
 \vdash \ \forall \, M \ \ Oi \ \ Os \ f \ \ f' \ \ g \, .
        (M, Oi, Os) sat f \neq f' \Rightarrow
        (M,Oi,Os) sat f and g \Rightarrow
        (M, Oi, Os) sat f' and g
[eqf_andf2]
 \vdash \ \forall \, M \ \ Oi \ \ Os \ f \ \ f' \ \ g \, .
        (M,Oi,Os) sat f eqf f' \Rightarrow
        (M,Oi,Os) sat g and f \Rightarrow
        (M,Oi,Os) sat g and f'
[eqf_controls]
 \vdash \forall M \ Oi \ Os \ P \ f \ f'.
        (M,Oi,Os) sat f \neq f' \Rightarrow
        (M,Oi,Os) sat P controls f \Rightarrow
        (M, Oi, Os) sat P controls f'
[eqf_eq]
 \vdash (Efn Oi\ Os\ M\ (f_1\ \mathsf{eqf}\ f_2) = \mathcal{U}(:\ \mathsf{'b})) \iff
     (Efn Oi Os M f_1 = Efn Oi Os M f_2)
[eqf_eqf1]
 \vdash \ \forall M \ Oi \ Os \ f \ f' \ g.
        (M,Oi,Os) sat f eqf f' \Rightarrow
        (M, Oi, Os) sat f \neq g \Rightarrow
        (M,Oi,Os) sat f' eqf g
[eqf_eqf2]
 \vdash \ \forall \, M \ \ Oi \ \ Os \ f \ f' \ g \, .
        (M,Oi,Os) sat f \neq f' \Rightarrow
        (M,Oi,Os) sat g eqf f \Rightarrow
        (M,Oi,Os) sat g eqf f'
```

```
[eqf_impf1]
 \vdash \forall M \ Oi \ Os \ f \ f' \ g.
       (M,Oi,Os) sat f eqf f' \Rightarrow
       (M,Oi,Os) sat f impf g \Rightarrow
       (M,Oi,Os) sat f' impf g
[eqf_impf2]
 \vdash \forall M \ Oi \ Os \ f \ f' \ g.
       (M,Oi,Os) sat f \neq f' \Rightarrow
       (M,Oi,Os) sat g impf f \Rightarrow
       (M,Oi,Os) sat g impf f'
[eqf_notf]
 \vdash \forall M \ Oi \ Os \ f \ f'.
       (M,Oi,Os) sat f \neq f' \Rightarrow
       (M,Oi,Os) sat notf f \Rightarrow
       (M,Oi,Os) sat notf f'
[eqf_orf1]
 \vdash \ \forall M \ Oi \ Os \ f \ f' \ g.
       (M,Oi,Os) sat f eqf f' \Rightarrow
       (M,Oi,Os) sat f orf g \Rightarrow
       (M,Oi,Os) sat f' orf g
[eqf_orf2]
 \vdash \forall M \ Oi \ Os \ f \ f' \ g.
       (M,Oi,Os) sat f eqf f' \Rightarrow
       (M,Oi,Os) sat g orf f \Rightarrow
       (M,Oi,Os) sat g orf f'
[eqf_reps]
 \vdash \forall M \ Oi \ Os \ P \ Q \ f \ f'.
       (M,Oi,Os) sat f eqf f' \Rightarrow
       (M,Oi,Os) sat reps P Q f \Rightarrow
       (M,Oi,Os) sat reps P Q f'
[eqf_sat]
 \vdash \forall M \ Oi \ Os \ f_1 \ f_2.
       (M,Oi,Os) sat f_1 eqf f_2 \Rightarrow
       ((M,Oi,Os) \text{ sat } f_1 \iff (M,Oi,Os) \text{ sat } f_2)
[eqf_says]
 \vdash \forall M \ Oi \ Os \ P \ f \ f'.
       (M,Oi,Os) sat f eqf f' \Rightarrow
       (M,Oi,Os) sat P says f \Rightarrow
       (M,Oi,Os) sat P says f'
```

```
[eqi_Eq]
 \vdash \forall M \ Oi \ Os \ l_1 \ l_2.
        (M,Oi,Os) sat l_1 eqi l_2 \iff
        (M,Oi,Os) sat l_2 domi l_1 andf l_1 domi l_2
[eqs_Eq]
 \vdash \ \forall M \ Oi \ Os \ l_1 \ l_2.
        (M,Oi,Os) sat l_1 eqs l_2 \iff
        (M,Oi,Os) sat l_2 doms l_1 and l_1 doms l_2
[Idemp_Speaks_For]
 \vdash \ \forall M \ Oi \ Os \ P. (M,Oi,Os) sat P speaks_for P
[Image_cmp]
 \vdash \forall R_1 \ R_2 \ R_3 \ u. (R_1 \ \mathsf{O} \ R_2) u \subseteq R_3 \iff R_2 \ u \subseteq \{y \mid R_1 \ y \subseteq R_3\}
[Image_SUBSET]
 \vdash \ \forall \, R_1 \ R_2 \,. \ R_2 \ \text{RSUBSET} \ R_1 \ \Rightarrow \ \forall \, w \,. \ R_2 \ w \ \subseteq \ R_1 \ w
[Image_UNION]
 \vdash \forall R_1 R_2 w. (R_1 RUNION R_2) w = R_1 w \cup R_2 w
[INTER_EQ_UNIV]
 \vdash (s \cap t = \mathcal{U}(:'a)) \iff (s = \mathcal{U}(:'a)) \land (t = \mathcal{U}(:'a))
[Modus_Ponens]
 \vdash \ \forall M \ Oi \ Os \ f_1 \ f_2.
        (M, Oi, Os) sat f_1 \Rightarrow
        (M,Oi,Os) sat f_1 impf f_2 \Rightarrow
        (M,Oi,Os) sat f_2
[Mono_speaks_for]
 \vdash \ \forall M \ Oi \ Os \ P \ P' \ Q \ Q'.
        (M,Oi,Os) sat P speaks_for P' \Rightarrow (M,Oi,Os) sat Q speaks_for Q' \Rightarrow
        (M,Oi,Os) sat P quoting Q speaks_for P' quoting Q'
[MP_Says]
 \vdash \forall M \ Oi \ Os \ P \ f_1 \ f_2.
        (M,Oi,Os) sat
        P says (f_1 \text{ impf } f_2) impf P says f_1 impf P says f_2
Quoting
 \vdash \forall M \ Oi \ Os \ P \ Q \ f.
        (M,Oi,Os) sat P quoting Q says f eqf P says Q says f
```

```
[Quoting_Eq]
 \vdash \ \forall M \ Oi \ Os \ P \ Q \ f.
        (M, Oi, Os) sat P quoting Q says f \iff
       (M,Oi,Os) sat P says Q says f
[reps_def_lemma]
 \vdash \ \forall M \ Oi \ Os \ P \ Q \ f.
       Efn Oi\ Os\ M (reps P\ Q\ f) =
       Efn Oi Os M (P quoting Q says f impf Q says f)
[Reps_Eq]
 \vdash \forall M \ Oi \ Os \ P \ Q \ f.
        (M,Oi,Os) sat reps P Q f \iff
       (M,Oi,Os) sat P quoting Q says f impf Q says f
[sat_allworld]
 \vdash \ \forall \ M \ f. \ (M,Oi,Os) \ {\sf sat} \ f \iff \forall \ w. \ w \in {\sf Efn} \ Oi \ Os \ M \ f
[sat_andf_eq_and_sat]
 \vdash (M, Oi, Os) sat f_1 and f_2 \iff
     (M,Oi,Os) sat f_1 \wedge (M,Oi,Os) sat f_2
sat_TT
 \vdash (M, Oi, Os) sat TT
[Says]
 \vdash \ \forall M \ Oi \ Os \ P \ f. \ (M,Oi,Os) \ {\tt sat} \ f \ \Rightarrow \ (M,Oi,Os) \ {\tt sat} \ P \ {\tt says} \ f
[says_and_lemma]
 \vdash \ \forall \, M \ Oi \ Os \ P \ Q \ f \, .
       (M,Oi,Os) sat P says f and f says f impf P meet f says f
[Speaks_For]
 \vdash \forall M \ Oi \ Os \ P \ Q \ f.
        (M,Oi,Os) sat P speaks_for Q impf P says f impf Q says f
[speaks_for_SUBSET]
 \vdash \forall R_3 \ R_2 \ R_1.
       R_2 RSUBSET R_1 \Rightarrow \forall w. \{w \mid R_1 \mid w \subseteq R_3\} \subseteq \{w \mid R_2 \mid w \subseteq R_3\}
[SUBSET_Image_SUBSET]
 \vdash \ \forall R_1 \ R_2 \ R_3.
       (\forall w_1. R_2 w_1 \subseteq R_1 w_1) \Rightarrow
       \forall w. \{w \mid R_1 \ w \subseteq R_3\} \subseteq \{w \mid R_2 \ w \subseteq R_3\}
```

```
Trans_Speaks_For
  \vdash \forall M \ Oi \ Os \ P \ Q \ R.
           (M,Oi,Os) sat P speaks_for Q \Rightarrow
           (M, Oi, Os) sat Q speaks_for R \Rightarrow
           (M,Oi,Os) sat P speaks_for R
[UNIV_DIFF_SUBSET]
  \vdash \forall R_1 \ R_2. \ R_1 \subseteq R_2 \Rightarrow (\mathcal{U}(:\ 'a)\ \mathtt{DIFF}\ R_1 \cup R_2 = \mathcal{U}(:\ 'a))
[world_and]
  \vdash \forall M \ Oi \ Os \ f_1 \ f_2 \ w.
           w \in \mathsf{Efn}\ \mathit{Oi}\ \mathit{Os}\ \mathit{M}\ (\mathit{f}_1\ \mathsf{andf}\ \mathit{f}_2) \iff
          w \in \text{Efn } Oi \ Os \ M \ f_1 \ \land \ w \in \text{Efn } Oi \ Os \ M \ f_2
[world_eq]
  \vdash \ \forall M \ Oi \ Os \ f_1 \ f_2 \ w.
           w \in \text{Efn } Oi \ Os \ M \ (f_1 \ \text{eqf} \ f_2) \iff
           (w \in \mathsf{Efn}\ \mathit{Oi}\ \mathit{Os}\ \mathit{M}\ \mathit{f}_1 \iff w \in \mathsf{Efn}\ \mathit{Oi}\ \mathit{Os}\ \mathit{M}\ \mathit{f}_2)
world_eqn
  \vdash \forall M \ Oi \ Os \ n_1 \ n_2 \ w. \ w \in \texttt{Efn} \ Oi \ Os \ m \ (n_1 \ \texttt{eqn} \ n_2) \iff (n_1 \ \texttt{=} \ n_2)
[world_F]
  \vdash \ \forall \, M \ Oi \ Os \ w \, . \ w \, \notin \, \mathtt{Efn} \ Oi \ Os \ M \ \mathtt{FF}
world_imp
  \vdash \forall M \ Oi \ Os \ f_1 \ f_2 \ w.
          w \in \text{Efn } Oi \ Os \ M \ (f_1 \ \text{impf} \ f_2) \iff
           w \in \texttt{Efn} \ Oi \ Os \ M \ f_1 \ \Rightarrow \ w \in \texttt{Efn} \ Oi \ Os \ M \ f_2
[world_lt]
  \vdash \ orall \ \mathit{M} \ \mathit{Oi} \ \mathit{Os} \ \mathit{n}_1 \ \mathit{n}_2 \ \mathit{w} . \ \mathit{w} \in \mathsf{Efn} \ \mathit{Oi} \ \mathit{Os} \ \mathit{m} \ (\mathit{n}_1 \ \mathsf{lt} \ \mathit{n}_2) \iff \mathit{n}_1 < \mathit{n}_2
[world_lte]
  \vdash \ orall \ \mathit{M} \ \mathit{Oi} \ \mathit{Os} \ \mathit{n}_1 \ \mathit{n}_2 \ \mathit{w} . \ \mathit{w} \ \in \ \mathsf{Efn} \ \mathit{Oi} \ \mathit{Os} \ \mathit{m} \ (\mathit{n}_1 \ \mathsf{lte} \ \mathit{n}_2) \ \Longleftrightarrow \ \mathit{n}_1 \ \leq \ \mathit{n}_2
[world_not]
  [world_or]
  \vdash \forall M \ f_1 \ f_2 \ w.
          w \in \text{Efn } Oi \ Os \ M \ (f_1 \ \text{orf} \ f_2) \iff
          w \in \mathtt{Efn}\ Oi\ Os\ M\ f_1\ \lor\ w \in \mathtt{Efn}\ Oi\ Os\ M\ f_2
[world_says]
  \vdash \ \forall M \ Oi \ Os \ P \ f \ w.
          w \in \text{Efn } Oi \ Os \ M \ (P \ \text{says} \ f) \iff
          \forall v. v \in \text{Jext (jKS } M) \ P \ w \Rightarrow v \in \text{Efn } Oi \ Os \ M \ f
world_T
  \vdash \forall M \ Oi \ Os \ w. \ w \in \texttt{Efn} \ Oi \ Os \ M \ \texttt{TT}
```

## 4 aclDrules Theory

Built: 25 February 2018

Parent Theories: aclrules

#### 4.1 Theorems

```
[Conjunction]
 \vdash \ \forall M \ Oi \ Os \ f_1 \ f_2.
       (M,Oi,Os) sat f_1 \Rightarrow
       (M,Oi,Os) sat f_2 \Rightarrow
       (M, Oi, Os) sat f_1 and f_2
[Controls]
 \vdash \ \forall M \ Oi \ Os \ P \ f.
       (M,Oi,Os) sat P says f \Rightarrow
       (M, Oi, Os) sat P controls f \Rightarrow
       (M,Oi,Os) sat f
[Derived_Controls]
 \vdash \ \forall M \ Oi \ Os \ P \ Q \ f.
       (M,Oi,Os) sat P speaks_for Q \Rightarrow
       (M,Oi,Os) sat Q controls f \Rightarrow
       (M,Oi,Os) sat P controls f
[Derived_Speaks_For]
 \vdash \ \forall M \ Oi \ Os \ P \ Q \ f.
       (M,Oi,Os) sat P speaks_for Q \Rightarrow
       (M,Oi,Os) sat P says f \Rightarrow
       (M,Oi,Os) sat Q says f
[Disjunction1]
 \vdash \forall M \ Oi \ Os \ f_1 \ f_2. \ (M,Oi,Os) \ sat \ f_1 \Rightarrow (M,Oi,Os) \ sat \ f_1 \ orf \ f_2
[Disjunction2]
 \vdash \forall M \ Oi \ Os \ f_1 \ f_2. (M,Oi,Os) sat f_2 \Rightarrow (M,Oi,Os) sat f_1 orf f_2
[Disjunctive_Syllogism]
 \vdash \ \forall M \ Oi \ Os \ f_1 \ f_2.
       (M,Oi,Os) sat f_1 orf f_2 \Rightarrow
       (M,Oi,Os) sat notf f_1 \Rightarrow
       (M,Oi,Os) sat f_2
[Double_Negation]
 \vdash \forall M \ Oi \ Os \ f. \ (M,Oi,Os) \ \text{sat notf (notf } f) \Rightarrow (M,Oi,Os) \ \text{sat } f
```

```
[eqn_eqn]
 \vdash (M, Oi, Os) sat c_1 eqn n_1 \Rightarrow
     (M,Oi,Os) sat c_2 eqn n_2 \Rightarrow
     (M,Oi,Os) sat n_1 eqn n_2 \Rightarrow
     (M,Oi,Os) sat c_1 eqn c_2
[eqn_lt]
 \vdash (M, Oi, Os) sat c_1 eqn n_1 \Rightarrow
     (M,Oi,Os) sat c_2 eqn n_2 \Rightarrow
     (M, Oi, Os) sat n_1 lt n_2 \Rightarrow
     (M, Oi, Os) sat c_1 lt c_2
[eqn_lte]
 \vdash (M,Oi,Os) sat c_1 eqn n_1 \Rightarrow
     (M,Oi,Os) sat c_2 eqn n_2 \Rightarrow
     (M,Oi,Os) sat n_1 lte n_2 \Rightarrow
     (M,Oi,Os) sat c_1 lte c_2
[Hypothetical_Syllogism]
 \vdash \ \forall M \ Oi \ Os \ f_1 \ f_2 \ f_3.
        (M, Oi, Os) sat f_1 impf f_2 \Rightarrow
        (M,Oi,Os) sat f_2 impf f_3 \Rightarrow
        (M, Oi, Os) sat f_1 impf f_3
[il_domi]
 \vdash \ \forall \, M \ \ Oi \ \ Os \ \ P \ \ Q \ \ l_1 \ \ l_2 \, .
        (M, Oi, Os) sat il P eqi l_1 \Rightarrow
        (M, Oi, Os) sat il Q eqi l_2 \Rightarrow
        (M,Oi,Os) sat l_2 domi l_1 \Rightarrow
        (M,Oi,Os) sat il Q domi il P
[INTER_EQ_UNIV]
 \vdash \forall s_1 \ s_2. \ (s_1 \cap s_2 = \mathcal{U}(:'a)) \iff (s_1 = \mathcal{U}(:'a)) \land (s_2 = \mathcal{U}(:'a))
[Modus_Tollens]
 \vdash \ \forall M \ Oi \ Os \ f_1 \ f_2.
        (M,Oi,Os) sat f_1 impf f_2 \Rightarrow
        (M,Oi,Os) sat notf f_2 \Rightarrow
        (M, Oi, Os) sat notf f_1
[Rep_Controls_Eq]
 \vdash \forall M \ Oi \ Os \ A \ B \ f.
        (M,Oi,Os) sat reps A B f \iff
        (M, Oi, Os) sat A controls B says f
```

```
[Rep_Says]
 \vdash \ \forall M \ Oi \ Os \ P \ Q \ f.
       (M,Oi,Os) sat reps P Q f \Rightarrow
       (M,Oi,Os) sat P quoting Q says f \Rightarrow
       (M,Oi,Os) sat Q says f
[Reps]
 \vdash \ \forall \, M \ Oi \ Os \ P \ Q \ f \, .
       (M,Oi,Os) sat reps P Q f \Rightarrow
       (M,Oi,Os) sat P quoting Q says f \Rightarrow
       (M,Oi,Os) sat Q controls f \Rightarrow
       (M, Oi, Os) sat f
[Says_Simplification1]
 \vdash \ \forall M \ Oi \ Os \ P \ f_1 \ f_2.
       (M,Oi,Os) sat P says (f_1 \text{ andf } f_2) \Rightarrow (M,Oi,Os) sat P says f_1
[Says_Simplification2]
 \vdash \forall M \ Oi \ Os \ P \ f_1 \ f_2.
       (M,Oi,Os) sat P says (f_1 \text{ andf } f_2) \Rightarrow (M,Oi,Os) sat P says f_2
[Simplification1]
 \vdash \ \forall \ M \ Oi \ Os \ f_1 \ f_2. (M,Oi,Os) sat f_1 andf f_2 \Rightarrow (M,Oi,Os) sat f_1
[Simplification2]
 \vdash \forall M \ Oi \ Os \ f_1 \ f_2. (M,Oi,Os) sat f_1 and f_2 \Rightarrow (M,Oi,Os) sat f_2
[sl_doms]
 \vdash \ \forall \, M \ \ Oi \ \ Os \ \ P \ \ Q \ \ l_1 \ \ l_2 \, .
       (M, Oi, Os) sat sl P eqs l_1 \Rightarrow
       (M,Oi,Os) sat sl Q eqs l_2 \Rightarrow
       (M,Oi,Os) sat l_2 doms l_1 \Rightarrow
       (M,Oi,Os) sat sl Q doms sl P
```

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