CS 325 HW 6 SP18

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Homework #6

You may solve the problems using your choice of software, state which software package/language(s) you used and provide the code or spreadsheet. There is no submission to TEACH this week.

1. Shortest Paths using LP: **(7 points)**

Shortest paths can be cast as an LP using distances dv from the source s to a particular vertex v as variables.

- We can compute the shortest path from s to t in a weighted directed graph by solving.

  max dt
  subject to
  $$ds = 0$$
  $$dv - du \leq w(u,v) \quad \text{for all } (u,v) \in E$$

- We can compute the single-source by changing the objective function to

  $$\max \sum_{v \in V} dv$$

Use linear programming to answer the questions below. State the objective function and constraints for each problem and include a copy of the LP code and output.

a) Find the distance of the shortest path from G to C in the graph below.

Objective Function: maximize dc

Constraints: s.t dg = 0, dg = 0, da-df <= 5, da-dh <= 4, df-dd <= 18, df-da <= 10, dh-dg <= 3, dg-de <= 7, db-da <= 8, db-dh <= 9, db-df <= 7, dc-df <= 3, dc-db <= 4, dd-dc <= 3, dd-dg <= 2, dd-de <= 9, de-dd <= 25, de-df <= 2 and de-db <= 10

Solution: The shortest path is 16

```
LP OPTIMUM FOUND AT STEP      6

       OBJECTIVE FUNCTION VALUE

       1)      16.00000

 VARIABLE        VALUE          REDUCED COST
       DC        16.000000          0.000000
       DG         0.000000          0.000000
       DA         4.000000          0.000000
       DF        13.000000          0.000000
       DH         3.000000          0.000000
       DD         0.000000          0.000000
       DE         0.000000          0.000000
       DB        12.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          1.000000
       3)        14.000000          0.000000
       4)         3.000000          0.000000
       5)         5.000000          0.000000
       6)         1.000000          0.000000
       7)         0.000000          1.000000
       8)         7.000000          0.000000
       9)         0.000000          0.000000
      10)         0.000000          1.000000
      11)         8.000000          0.000000
      12)         0.000000          0.000000
      13)         0.000000          1.000000
      14)        19.000000          0.000000
      15)         2.000000          0.000000
      16)         9.000000          0.000000
      17)        25.000000          0.000000
      18)        15.000000          0.000000
      19)        22.000000          0.000000

NO. ITERATIONS=       6
```

```
max dc
ST
      dg = 0
      da-df <= 5
      da-dh <= 4
      df-dd <= 18
      df-da <= 10
      dh-dg <= 3
      dg-de <= 7
      db-da <= 8
      db-dh <= 9
      db-df <= 7
      dc-df <= 3
      dc-db <= 4
      dd-dc <= 3
      dd-dg <= 2
      dd-de <= 9
      de-dd <= 25
      de-df <= 2
      de-db <= 10
END
```

b) Find the distances of the shortest paths from G to all other vertices.

Objective Function: maximize da+db+dc+dd+de+df

Constraints: s.t dg = 0, dg = 0, da-df <= 5, da-dh <= 4, df-dd <= 18, df-da <= 10, dh-dg <= 3, dg-de <= 7, db-da <= 8, db-dh <= 9, db-df <= 7, dc-df <= 3, dc-db <= 4, dd-dc <= 3, dd-dg <= 2, dd-de <= 9, de-dd <= 25, de-df <= 2 and de-db <= 10

Solution:

Shortest path from G to A is 7

Shortest path from G to B is 12

Shortest path from G to C is 16

Shortest path from G to D is 2

Shortest path from G to E is 19

Shortest path from G to F is 17

Shortest path from G to H is 3

```
max da+db+dc+dd+de+df+dh
ST
        dg  =  0
        da-df  <=  5
        da-dh  <=  4
        df-dd  <=  18
        df-da  <=  10
        dh-dg  <=  3
        dg-de  <=  7
        db-da  <=  8
        db-dh  <=  9
        db-df  <=  7
        dc-df  <=  3
        dc-db  <=  4
        dd-dc  <=  3
        dd-dg  <=  2
        dd-de  <=  9
        de-dd  <=  25
        de-df  <=  2
        de-db  <=  10
END
```

```
LP OPTIMUM FOUND AT STEP      0

      OBJECTIVE FUNCTION VALUE

      1)      76.00000

 VARIABLE        VALUE          REDUCED COST
       DA         7.000000          0.000000
       DB        12.000000          0.000000
       DC        16.000000          0.000000
       DD         2.000000          0.000000
       DE        19.000000          0.000000
       DF        17.000000          0.000000
       DH         3.000000          0.000000
       DG         0.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          7.000000
       3)        15.000000          0.000000
       4)         0.000000          3.000000
       5)         3.000000          0.000000
       6)         0.000000          2.000000
       7)         0.000000          6.000000
       8)        26.000000          0.000000
       9)         3.000000          0.000000
      10)         0.000000          2.000000
      11)        12.000000          0.000000
      12)         4.000000          0.000000
      13)         0.000000          1.000000
      14)        17.000000          0.000000
      15)         0.000000          1.000000
      16)        26.000000          0.000000
      17)         8.000000          0.000000
      18)         0.000000          1.000000
      19)         3.000000          0.000000

NO. ITERATIONS=      0
```

2. <u>Product Mix</u>: **(7 points)**
Acme Industries produces four types of men's ties using three types of material. Your job is to determine how many of each type of tie to make each month. The goal is to maximize profit, profit per tie = selling price - labor cost – material cost. Labor cost is $0.75 per tie for all four types of ties. The material requirements and costs are given below.

Formulate the problem as a linear program with an objective function and all constraints. Determine the optimal solution for the linear program using any software you want. Include a copy of the code and output. What are the optimal numbers of ties of each type to maximize profit?

Product	Labor	Material	Profit	Min Unit	Max Unit
Silk	0.75	0.125s	6.7	6000	7000
Polyester	0.75	0.08p	3/55	10000	14000
Blend1	0.75	0.5C/0.5P	4.31	13000	16000
Blend2	0.75	0.03C/0.07P	4.81	6000	8500

Material	Cost Per Yard	Yards available per month
Silk	20	1000
Polyester	6	2000
Cotton	9	1250

Profit of Silk – 6.7 – 0.125 * 20 – 0.75 = 3.45

Profit of Polyester – 3.55 – 0.08 * 6 – 0.75 = 2.32

Profit of Blend 1- 4.31 – 0.05 * 6 – 0.05 * 9 – 0.75 = 2.81

Profit of Blend 2 – 4.81 – 0.03 * 6 – 0/07 * 9 – 0.75 = 3.25

Objective – Maximize Z such that it is the profit for the day. Z = 3.45s + 2.32p + 2.81b1 + 3.25b2

Constraint – S.t

0.125s <= 1000

0.08p + 0.05b1 + 0.03b2 <= 2000

0.05b1 + 0.07b2 <= 1250

s >= 6000

s <= 7000

p >= 10000

p <= 14000

b1 >= 13000

b1 <= 16000

b2 >= 6000

b2 <= 8500

The final profit (maximized) is 120196. This is concluded by making 7000 silk ties, 13625 polyster ties, 13100 blend1 ties and 8500 blend 2 tie

```
max 3.45s + 2.32p + 2.81b + 3.25c
ST
        0.125s <= 1000
        0.08p + 0.05b + 0.03c <= 2000
        0.05b + 0.07c <= 1250
        s >= 6000
        s <= 7000
        p >= 10000
        p <= 14000
        b >= 13000
        b <= 16000
        c >= 6000
        c <= 8500
END
```

```
      OBJECTIVE FUNCTION VALUE

      1)      120196.0

VARIABLE        VALUE          REDUCED COST
       S      7000.000000          0.000000
       P     13625.000000          0.000000
       B     13100.000000          0.000000
       C      8500.000000          0.000000

     ROW   SLACK OR SURPLUS     DUAL PRICES
      2)       125.000000          0.000000
      3)         0.000000         29.000000
      4)         0.000000         27.200001
      5)      1000.000000          0.000000
      6)         0.000000          3.450000
      7)      3625.000000          0.000000
      8)       375.000000          0.000000
      9)       100.000000          0.000000
     10)      2900.000000          0.000000
     11)      2500.000000          0.000000
     12)         0.000000          0.476000
```

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3. Transshipment Model **(10 points)**

This is an extension of the transportation model. There are now intermediate transshipment points added between the sources (plants) and destinations (retailers). Items being shipped from a Plant (p_i) must be shipped to a Warehouse (w_j) before being shipped to the Retailer (r_k). Each Plant will have an associated supply (s_i) and each Retailer will have a demand (d_k). The number of plants is n, number of warehouses is q and the number of retailers is m. The edges (i,j) from plant (p_i)to warehouse (w_j) have costs associated denoted cp(i,j). The edges (j,k) from a warehouse (w_j)to a retailer (r_k) have costs associated denoted cw(j,k).

The graph below shows the transshipment map for a manufacturer of refrigerators. Refrigerators are produced at four plants and then shipped to a warehouse (weekly) before going to the retailer.

Part A: Determine the number of refrigerators to be shipped from the plants to the warehouses and then warehouses to retailers to minimize the cost. Formulate the problem as a linear program with an objective function and all constraints. Determine the optimal solution for the linear program using any software you want. What are the optimal shipping routes and minimum cost?

HW3

- Plant → Warehouse → Retailer
- Goal Minimalize total transportation cost
- Constiaints

Supply

$P_{1w1} + P_{1w2} \leq 150$

$P_{2w1} + P_{2w2} \leq 450$

$P_{3w1} + P_{3w2} + P_{3w3} \leq 250$

$P_{4w2} + P_{4w3} \leq 150$

Demand

$R_1 W_1 \geq 100$

$R_{2w1} \geq 150$

$R_{3w1} + R_{3w2} \geq 100$

$R_{4w1} + R_{4w2} + R_{4w3} \geq 200$

$R_{5w2} + R_{5w3} \geq 200$

$R_{6w2} + R_{6w3} \geq 150$

$R_{7w3} \geq 100$

Send

$P_1 W_1 + P_2 W_1 + P_3 W_1 = W_1 R_1 + W_1 R_2 + W_1 R_3 + W_1 R_4$

$P_1 W_2 + P_2 W_2 + P_3 W_2 + P_4 W_2 = W_2 R_3 + W_2 R_4 + W_2 R_5 + W_2 R_6$

$P_3 W_3 + P_4 W_4 = W_3 R_4 + W_3 R_5 + W_3 R_6 + W_3 R_7$

^ Formulation of the data provided, now added appropriated transportation cost to their functions and flip all variables to the left with numbers to fit LANDO model.

```
min 10cp11 + 15cp12 + 11cp21 + 8cp22 + 13cp31 + 8cp32 + 9cp33 +
14cp42 + 8cp43 + 5cw11 + 6cw12 + 7cw13 + 10cw14 + 12cw23 + 8cw24 +
10cw25 + 14cw26 + 14cw34 + 12cw35 + 12cw36 + 6cw37
ST
        cp11 + cp12 <= 150
        cp21 + cp22 <= 450
        cp31 + cp32 + cp33 <= 250
        cp42 + cp43 <= 150
        cw11 >= 100
        cw12 >= 150
        cw13 + cw23 >= 100
        cw14 + cw24 + cw34 >= 200
        cw25 + cw35 >= 200
        cw26 + cw36 >= 150
        cw37 >= 100
        cp11 + cp21 + cp31 - cw11 - cw12 - cw13 - cw14 = 0
        cp12 + cp22 + cp32 + cp42 - cw23 - cw24 - cw25 - cw26 = 0
        cp33 + cp43 - cw34 - cw35 - cw36 - cw37 = 0
        cp11 >= 0
        cp12 >= 0
        cp21 >= 0
        cp22 >= 0
        cp31 >= 0
        cp32 >= 0
        cp33 >= 0
        cp42 >= 0
        cp43 >= 0
        cw11 >= 0
        cw12 >= 0
        cw13 >= 0
        cw14 >= 0
        cw23 >= 0
        cw24 >= 0
        cw25 >= 0
        cw26 >= 0
        cw34 >= 0
        cw35 >= 0
        cw36 >= 0
        cw37 >= 0
END
```

```
LP OPTIMUM FOUND AT STEP      13

        OBJECTIVE FUNCTION VALUE

        1)      17100.00

 VARIABLE        VALUE          REDUCED COST
     CP11       150.000000          0.000000
     CP12         0.000000          8.000000
     CP21       200.000000          0.000000
     CP22       250.000000          0.000000
     CP31         0.000000          2.000000
     CP32       150.000000          0.000000
     CP33       100.000000          0.000000
     CP42         0.000000          7.000000
     CP43       150.000000          0.000000
     CW11       100.000000          0.000000
     CW12       150.000000          0.000000
     CW13       100.000000          0.000000
     CW14         0.000000          5.000000
     CW23         0.000000          2.000000
     CW24       200.000000          0.000000
     CW25       200.000000          0.000000
     CW26         0.000000          1.000000
     CW34         0.000000          7.000000
     CW35         0.000000          3.000000
     CW36       150.000000          0.000000
     CW37       100.000000          0.000000
```

Therefore the optimal solution/minimum cost is 17100 and the optimal route is as follow,

P1W1/150, P2W1/200, P2W2/250, P3W2/150, P3W3/100, P4W3/150

W1R1/100, W1R2/150, W1R3/100, W2R4/200, W2R5/200, W3R6/150, W3R7/100

Part B: Due to old infrastructure Warehouse 2 is going to close eliminating all of the associated routes. What is the optimal solution for this modified model? Is it feasible to ship all the refrigerators to either warehouse 1 or 3 and then to the retailers without using warehouse 2? Why or why not?

It is not feasible because it now violates constraints. One example of this is that with the removal of Warehouse 2, R5 will no longer be reachable from P1. No LINDO programming is necessary as an examination of the table will provide the answer/explanation needed.

```
NO FEASIBLE SOLUTION AT STEP          10.
SUM OF INFEASIBILITIES=       50.000000000000000
VIOLATED ROWS HAVE  NEGATIVE SLACK,  OR
(EQUALITY ROWS) NONZERO  SLACKS.   ROWS
CONTRIBUTING TO  INFEASIBILITY  HAVE  A
NONZERO  DUAL  PRICE.   USE THE "DEBUG"
COMMAND FOR MORE INFORMATION.
```

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Part C: Instead of closing Warehouse 2 management has decide to keep a portion of it open but limit shipments to 100 refrigerators per week. Is this feasible? If so what is the optimal solution when warehouse 2 is limited to 100 refrigerators?

This is feasible. After adding the constraint of limiting the amount of refrigerators that can go into Warehouse 2, we see that the optimal cost (minimum cost) is now 18300. And the optimal route is as follow.

P1W1/150, P2W1/350, P2W2/100, P3W3/250, P4W3/150

W1R1/100, W1R2/150, W1R3/100, W1R3/100, W1R4/150, W2R4/50, W2R5/50, W3R5/150, W3R6/150, W3R7/100

```
min 10cp11 + 15cp12 + 11cp21 + 8cp22 + 13cp31 + 8cp32 + 9cp33 +
14cp42 + 8cp43 + 5cw11 + 6cw12 + 7cw13 + 10cw14 + 12cw23 + 8cw24 +
10cw25 + 14cw26 + 14cw34 + 12cw35 + 12cw36 + 6cw37
ST
        cp11 + cp12 <= 150
        cp21 + cp22 <= 450
        cp31 + cp32 + cp33 <= 250
        cp42 + cp43 <= 150
        cw11 >= 100
        cw12 >= 150
        cw13 + cw23 >= 100
        cw14 + cw24 + cw34 >= 200
        cw25 + cw35 >= 200
        cw26 + cw36 >= 150
        cw37 >= 100
        cp11 + cp21 + cp31 - cw11 - cw12 - cw13 - cw14 = 0
        cp12 + cp22 + cp32 + cp42 - cw23 - cw24 - cw25 - cw26 = 0
        cp33 + cp43 - cw34 - cw35 - cw36 - cw37 = 0
        cp12 + cp22 + cp32 + cp42 <= 100
        cp11 >= 0
        cp21 >= 0
        cp31 >= 0
        cp33 >= 0
        cp43 >= 0
        cw11 >= 0
        cw12 >= 0
        cw13 >= 0
        cw14 >= 0
        cw34 >= 0
        cw35 >= 0
        cw36 >= 0
        cw37 >= 0
END
```

```
LP OPTIMUM FOUND AT STEP     15

      OBJECTIVE FUNCTION VALUE

      1)      18300.00

 VARIABLE        VALUE          REDUCED COST
     CP11       150.000000          0.000000
     CP12         0.000000          8.000000
     CP21       350.000000          0.000000
     CP22       100.000000          0.000000
     CP31         0.000000          4.000000
     CP32         0.000000          2.000000
     CP33       250.000000          0.000000
     CP42         0.000000          9.000000
     CP43       150.000000          0.000000
     CW11       100.000000          0.000000
     CW12       150.000000          0.000000
     CW13       100.000000          0.000000
     CW14       150.000000          0.000000
     CW23         0.000000          7.000000
     CW24        50.000000          0.000000
     CW25        50.000000          0.000000
     CW26         0.000000          4.000000
     CW34         0.000000          4.000000
     CW35       150.000000          0.000000
     CW36       150.000000          0.000000
     CW37       100.000000          0.000000
```

Note: *Include a copy of the code for all parts of the problem.*

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4. Making Change Revisited **(6 points)**

Given coins of denominations (value) $1 = v_1 < v_2 < \ldots < v_n$, we wish to make change for an amount A using as few coins as possible. Assume that v_i's and A are integers. Since $v_1 = 1$ there will always be a solution. Solve the coin change problem from HW 3 using integer programming. For each the following denomination sets and amounts formulate the problem as an integer program with an objective function and constraints, determine the optimal solution. What is the minimum number of coins used in each case and how many of each coin is used? Include a copy of your code.

Since 1 = V1 < V2 < V3 where V1 and Vn and amount A are integers. If we want to make a change based on the provided lists of coins and since our objective is to use the least amount of coins possible, we should start from the largest coin. We would then need to focus on Vn where n is largest and work our way down.

a) V = [1, 5, 10, 25] and A = 202.

Objective Function – Minimize V1 + V2 + V3 + V4 Where V1 = number of $1, V2 = number of $5, V3 = number of $10 and V4 = number of $25

Constraints - 1.00V1 + 5.00V2 + 10.0V3 + 25.0V4 = A, where A equal to 202,

V1,V2,V3,V4 = integers and larger than 0

```
min v1 + v2 + v3 + v4
ST
        v1 + 5v2 + 10v3 + 25v4 = 202
        v1 >= 0
        v2 >= 0
        v3 >= 0
        v4 >= 0
END
GIN v1
GIN v2
GIN v3
GIN v4
```

```
        OBJECTIVE FUNCTION VALUE

        1)      10.00000

  VARIABLE        VALUE          REDUCED COST
        V1         2.000000          1.000000
        V2         0.000000          1.000000
        V3         0.000000          1.000000
        V4         8.000000          1.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000          0.000000
        3)         2.000000          0.000000
        4)         0.000000          0.000000
        5)         0.000000          0.000000
        6)         8.000000          0.000000

NO. ITERATIONS=      64
BRANCHES=   12 DETERM.=  1.000E    0
```

Optimal Solution

a) Minimum # of coins needed? The minimum # of coins needed is 10
b) How many coins of each type? We need 2 $1 and 8 $25

c) V = [1, 3, 7, 12, 27] and A = 293

Objective Function – Minimize V1 + V2 + V3 + V4 + V5 Where V1 = number of $1, V2 = number of $3, V3 = number of $7, V4 = number of $12 and V5 = number of $27

Constraints - 1V1 + 3V2 + 7V3 + 12V4 + 27V5 = A, where A equal to 293

V1,V2,V3,V4 and V5 = integers and larger than 0

```
min v1 + v2 + v3 + v4 + v5
ST
        v1 + 3v2 + 7v3 + 12v4 + 27v5 = 293
        v1 >= 0
        v2 >= 0
        v3 >= 0
        v4 >= 0
END
GIN v1
GIN v2
GIN v3
GIN v4
GIN v5
```

```
        OBJECTIVE FUNCTION VALUE

        1)      14.00000

  VARIABLE        VALUE          REDUCED COST
        V1         0.000000          1.000000
        V2         0.000000          1.000000
        V3         2.000000          1.000000
        V4         3.000000          1.000000
        V5         9.000000          1.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000          0.000000
        3)         0.000000          0.000000
        4)         0.000000          0.000000
        5)         2.000000          0.000000
        6)         3.000000          0.000000

 NO. ITERATIONS=      97
 BRANCHES=   34 DETERM.=  1.000E    0
```

Optimal Solution

a) Minimum # of coins needed? We need 14 coins minimum
b) How many coins of each type? We need 2 \$7, 3 \$12, 9 \$27.