Exercise no.1

Use the sample mean Monte Carlo method to evaluate $\int_{-1}^{1} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$

Sample the function 10^6 times and verify that the error is ~ $\frac{\sigma}{\sqrt{n-1}}$

(The exact value of the integral is 0.68268949)

Exercise no.2

Implement the inverse CDF method to generate random numbers from the distribution:

$$f(x) = \begin{cases} \sin(x)/2 & 0 \le x \le \pi \\ 0 & elsewhere \end{cases}$$

Fill a 40-bins histogram and compare its shape with the given function for diffent value of N.

Exercise no.3

Write a program to verify the central limit theorem.
Evaluate the mean value of N numbers generated from U(0,1), with N=2,3,5,10,20,50.
For each value of N repeat the sampling 10^5 times and fill a 40-bin histogram (range: 0,1) with the obtained mean. The shape of the histogram should approximate the gaussian distribution as N increases.

Then put N=12 and verify that $\left(\sum_{i=1}^{N} x_i\right) - 6$ approximate a gaussian with $\mu=0$ and $\sigma=1$