Instructions

In the exam project for Advanced Financial Engineering, you are applying different methods from the course to real data.

Along with the report, you must submit a file/folder with code that documents the calculations. The report itself should not contain any code (with the exception of pseudo-code, if deemed necessary to explain methods).

The exam report has no page limit, but the student's ability to clearly and concisely present the relevant background, methods, and results and discuss these is an integrated part of the evaluation. For all parts of the report, you should provide the relevant background/methods/theory and not only the results. The calculations/analyses to be made are outlined on the following pages.

Disclaimer: The project uses real data; the results depend on the data and modeling choices. If it turns out that some of the below tasks are not possible to complete, I might rephrase the task if it cannot be solved by changing the data. Keep an eye on the announcements on Learn.

Market Risk

Choose four different stocks from the S&P500-index and download adjusted closing prices[1]. You are trying to assess the overall (portfolio) risk of investing $1 million dollar in each stock using the risk measures Value-at-Risk and Expected Shortfall. The ultimate goal is to make 1-day VaR and ES predictions at a 95% and a 99% level on September 30, 2023. The method(s) you recommend for calculating VaR and ES on that date must be carefully backtested over at least a year (using the most recent data) before including/excluding it. For all approaches, choose (and specify in the report) relevant parameters (e.g., choice of data points, parameters in methods). Remember that your calculations for the 1-day VaR calculated at the end of day t for day $t+1$, should only include information available up to and including day t. Also, give a bit of thought to how much data your VaR estimations should be based on.

VaR methods

The approaches to be implemented and tested are

1. Non-parametric approach with equal weights put on past observations
2. Non-parametric approach with *either* probability weighting *or* volatility adjustment
3. Parametric model using the *multivariate* normal distribution
4. Parametric model using either the multivariate t-distribution[2] *or* the EWMA method for the covariance matrix of returns along with a multivariate normal distribution.
5. Univariate GARCH model applied to portfolio returns. Remember to consider how often the GARCH parameters should be re-estimated.

Discussion

Along with your results of the various models, also include a discussion of the pros and cons of the different methods for determining VaR as well as a discussion of VaR and ES as methods for measuring risk.

[1] These are, among other things, adjusted for dividends which is also useful for option pricing later

[2] Fitting a multivariate student-t is not the easiest task. If you want to do it, you can find a description of how to do it in R in Rueppert-Matteson chapter 7.7. Basically, you fix the degrees of freedom and then estimate the rest. You then do this for multiple values of degrees of freedom and find out which one that gives you the highest likelihood. Then you use that degree of freedom. I have not found a Python-package that does an estimation of a multivariate t-distribution for fixed ν. But maybe you can find a workaround or have better Google skills than me.

Derivatives pricing

Choose one of the stocks from the Market Risk part and use the data to estimate model parameters and price derivatives

Model estimation

Fit a Geometric Brownian motion to the stock data. You may assume that the data is observed at equidistant times. Secondly, choose an appropriate value for the annual continuously compounded spot interest rate r. Include a discussion of the use of this particular model for financial data.

Pricing options

After estimating a GBM-model for your chosen stock, the task is now to compute and compare various option prices for options with three months to expiry, and assuming they are priced on September 30, 2023:

- Compute the price of an ATM call option using a binomial model. The stock price tree is set up such that $U = e^{\sigma\sqrt{\Delta t}} - 1$, $D = e^{-\sigma\sqrt{\Delta t}} - 1$ and $R = e^{r\Delta t} - 1$, where Δt is the size of the time step measured in years. Consequently, the number of time steps is $n = \frac{0.25}{\Delta t}$. Illustrate that the price computed by the binomial model converges to the Black-Scholes price when $\Delta t \to 0$ (or equivalently $n \to \infty$).
- Also, in the binomial model, compute the prices of both an ATM European put option and an ATM American put option.
- Discuss the pros and cons of using the binomial model and the Black-Scholes model for pricing options.

Delta hedging in the Black-Scholes model

Consider the European options that were priced earlier. Download data for your chosen stock from October 1, 2023, to December 31, 2023 (the expiry of the option). You want to delta hedge the options over the lifetime of the option.

- Calculate the initial Delta of the options and re-balance this hedge daily over the lifetime of the options. Illustrate your results.
- Compare the hedging costs with the theoretical option prices.
- Redo the Delta-hedge using less frequent rebalancing and discuss if you obtain the expected results.

Credit Value Adjustment

You have a portfolio of interest rate swaps (note that two of them are off-market swaps, meaning that they have a value different from zero at initiation):

- A 10-year interest swap on a notional of \$1 million, where you pay a fixed swap rate of 2% and receive the floating rate
- A 7-year interest swap on a notional of \$6 million, where you pay a fixed swap rate set such that the initial value of the contract is zero and receive the floating rate
- A 4-year interest swap on a notional of \$4 million dollars, where you pay the floating rate and receive a fixed rate of 2.5%

All swap payments are made on a quarterly basis, and you believe that the evolution of the interest rate can be described using a Vasicek[3] model estimated from data uploaded on Learn[4].

The interest rate swaps are entered with Credit Suisse, and their creditworthiness can be evaluated using the following CDS spreads along with an assumption that R = 32%.

Tenor (in bps)	**March 10**	**March 14**
1Y	362	836
2Y	404	713
3Y	412	654
4Y	403	599
5Y	416	555
7Y	418	530
10Y	423	522

Interest rate model

Estimate the parameters in the Vasicek model using the uploaded data. The data is simulated for the purpose of this project and can be seen as daily observed data for five years.

Calculation of exposure

For each of the swap contracts, use interest rate simulations to calculate the future values of the interest swaps. Use these simulated values to find the expected exposure.

Calculation of CVA charge

Calculate the CVA charge for each swap on the two dates in the table above. You may use the same interest simulations for the calculation for the purpose of this project.

Impact of netting and collateral

Calculate the expected exposure and the CVA charge in the following cases

- Netting is allowed, and no collateral is posted
- No netting, but Credit Suisse sets a collateral of \$50.000 per swap contract
- Netting is allowed, and Credit Suisse sets a collateral of \$50.000 in total
- Discuss which, if any, of the above is best suited to mitigate the counterparty risk associated with the portfolio of swaps

[3] The Vasicek model has the same form as the Ornstein-Uhlenbeck process.

[4] Assume that the parameters that you obtain from the estimation are obtained under the risk-neutral measure such that you can directly use them for simulating the interest rates and calculating future bond prices and swap values