ST2004 Applied Probability I

Group assignment

See instructions at the end of this document.

Question 1 (30 marks)

- A 12-sided die is rolled continuously until all the possible outcomes have occurred at least once.

  – Estimate the expected number of dice rolls needed using a simulation study.

  – Compute the expected number of dice rolls needed analytically.

- A pair of 6-sided fair dice are rolled continuously until all the possible outcomes (i.e. all possible sums of two dice, $2, 3, \ldots, 12$) have occurred at least once. Estimate the expected number of dice rolls needed using a simulation study.

Question 2 (20 marks)

A deck of 100 cards - numbered $1, 2, \ldots, 100$ - is shuffled and then turned over one card at a time. We say that a "hit" occurs whenever card i is the ith card to be turned over, $i = 1, \ldots, 100$. Simulate 10 000 repetitions of the game to estimate the expectation and variance of the total number of hits.

Question 3 (20 marks)

Consider the following game: you begin with $20. You flip a coin, winning $10 if the coin lands heads and losing $10 if the coin lands tails. The game is played continuously until you either go broke or have $100 (i.e. a net profit of $80). Estimate using simulation studies

1. The probability you win the game.

2. The probability the game ends within ten coin flips.

Question 4 (30 marks)

Consider a coin tossing game between two players. The first player picks a combination of heads (H) and tails (T) of length 3 (e.g. HHH, HTH) that they think will come up. The second player subsequently picks a different combination of heads and tails of the same length. The coin is then flipped continuously until a player completes their combination of heads and tails. For each of the 8 combinations player 1 picks (i.e. HHH, HTH, HHT, etc.), which combinations should player 2 pick to maximize the chance of winning? For each of the 8 combinations player 1 picks, estimate the chance of winning for player 2 using the optimal strategy.

Instructions

Submission

Submission date: **Friday 2nd of December 2022**. Late submissions will not be accepted. You can submit your project by uploading to Blackboard.

Assessment

The project will carry 20% of the course marks. 80% of the marks will be awarded for doing a competent job; an extra 20% will be awarded for flair, imagination, thoroughness (good projects will go well beyond the lectures).

Project and report

To complete this project you can use Excel or any software or resource that you think might help. The report should comprise no more than 10 pages[1], including tables, plots and diagrams, a good use of which is encouraged and will be rewarded, as long as they provide useful information. Details on the computations should be provided concisely. The report should be prepared **professionally** using any appropriate software (e.g. Word or Latex.

Teamwork

A group project is proposed mainly because group learning can be valuable, but also because team-working is a useful skill. **All members of each team must make significant contribution** to the group project. Clearly the project work does naturally break into sections which can be conducted in parallel before being brought together. The team will receive a mark and all members will receive by default the same mark. In case particular situations arise I might decide to allocate the group mark to the individual team members in a different way.

Plagiarism and sources

While learning from each other and from other groups is encouraged, plagiarism is remarkably easy to spot, especially with online resources. This includes projects that were submitted for the same course in the last years. The same work submitted by two teams will receive a single mark which will be then divided between the teams involved, following discussion. Sources must be cited.

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[1]this page limit is only indicative but reports that are too long will be penalised