Analysis of Matrix Index Transformation

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We can represent a 2-dimensional index to 1-dimensional index and vice versa. When using a set to mark some elements in a given matrix, it is convenient to covert the index from 2-dimensional to 1-dimensional, because the conversion is unique so that the key of the hash will be unique as well. The conversion is known as

$$f : R^2 \to R^1 \tag{1}$$

By (1), an element in (r, c) in a $m \times n$ matrix can be converted to

$$f(r, c) = nr + c \tag{2}$$

By (2),

$$\frac{\partial f}{\partial r} = n > 0 \tag{3}$$

$$\frac{\partial f}{\partial c} = 1 > 0 \tag{4}$$

By (3) and (4), for fixed r or c, f is increasing function, so the reverse function exist, this implies that we can covert the 1-dimensional index (says i) back to the 2-dimensional index by reverse function as following

$$x(i) = i/n \tag{5}$$

$$y(i) = i\%n \tag{6}$$

With the discussion above, we can now generalize the transformation a bit. The reason why we can transform the index is because

(I) For each of the transformation, the mapping is unique so that the reverse image exists

(II) And the boundary m and n play a key role to ensure the mapping is unique

Here is the generalized form of the mapping: Let real function f_s defined on the set $X = \{(x_1, x_2)|x_1 \in \mathbb{N}, 0 \leq x_2 \leq n\}$ where $n \in \mathbb{N}$ and s is a real number greater than n, $f_s(x_1, x_2) = sx_1 + x_2$.

Theorem 1: For any $(x_1, x_2), (x_1^{'}, x_2^{'}) \in X$, if $f_s(x_1, x_2) = f_s(x_1^{'}, x_2^{'})$, then $x_1 = x_1^{'}$ and $x_2 = x_2^{'}$.

Proof. Suppose $x_i \neq x_i^{'}$ holds for all i. Then we have

$$s|x_1 - x_1^{'}| = |x_2 - x_2^{'}| \leq n < s \tag{7}$$

By (7), we have

$$|x_1 - x_1^{'}| < 1 \tag{8}$$

since $x_1, x_1^{'} \in \mathbb{N}$ and $x_1 \neq x_1^{'}$, (8) says precisely $x_1 = x_1^{'}$, which is contradict to our assumption, so $x_1 = x_1^{'}$. For fixed x_1, $f_s(x_1, x_2)$ is increasing, so $x_2 = x_2^{'}$, we thus complete the proof. $\square$