

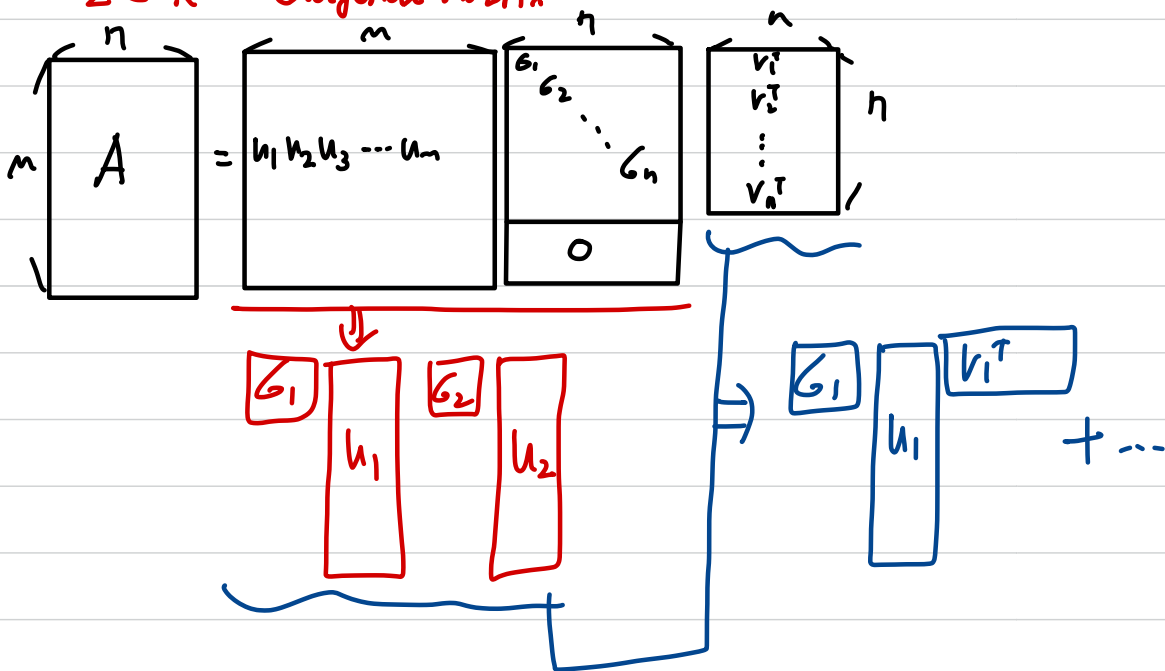
# \* SVD

rectangular matrix  $A \in \mathbb{R}^{m \times n}$

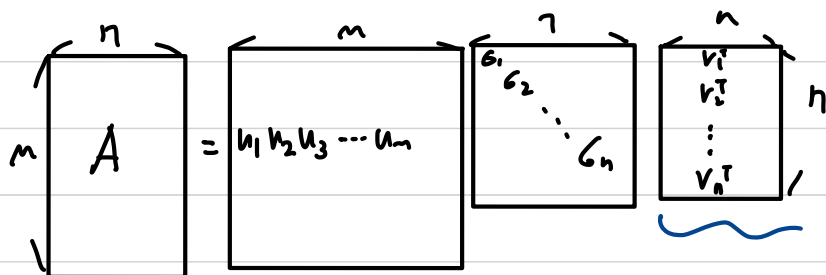
$$A = V \Sigma V^T \quad \text{eigen decomposition} \Rightarrow V D V^{-1}$$

$\hookrightarrow V \in \mathbb{R}^{m \times m}, V \in \mathbb{R}^{n \times n}$  orthonormal columns

$\Sigma \in \mathbb{R}^{m \times n}$  diagonal matrix



## • Reduced form of SVD



- $Av_i = \sigma_i u_i$

ex)

$$A \begin{matrix} \xrightarrow{2} \\ \xrightarrow{3} \end{matrix} \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} \sigma_i \\ \\ \end{bmatrix} \begin{bmatrix} \\ \\ \end{bmatrix}$$

- $AV = V\Sigma \Leftrightarrow [Av_1, Av_2, \dots, Av_n] = [\sigma_1 u_1, \sigma_2 u_2, \dots, \sigma_n u_n]$

$$V^{-1} = V^T \quad (V \in \mathbb{R}^{n \times n} \text{ orthonormal columns})$$

$$\therefore AV = V\Sigma \Leftrightarrow A = V\Sigma V^T$$