Role of Haptic Feedback in Rhythmic Dynamic Tasks: Paddle Juggling

M. Mert Ankaralı, H. Tutkun Şen, Allison M. Okamura and Noah J. Cowan

Abstract— **In this paper we investigate the role of haptic feedback in paddle juggling behavior, towards our ultimate goal of understanding the mechanisms by which the human nervous system controls rhythmic behaviors. Although there are studies that have investigated the role of haptic feedback in dynamic tasks, this paper is the first that analyzes the role of haptic feedback in a rhythmic dynamic behavior. We first developed an experimental haptic paddle device for our purpose. Using our haptic device we performed subject tests with eighteen people and asked them to stabilize the apex height of the ball within a goal region. We collected juggling data both with haptic feedback and without haptic feedback. We analyzed the performance of subjects based on two performance metrics. Results show that haptic feedback significantly improves the performance in juggling behavior. We also analyzed the open loop stability. As in two similar studies we characterized the open loop stability with the acceleration of the paddle at the impact instants. Results show that with haptic feedback humans select strictly open loop stable trajectories, whereas the trajectories without haptic feedback are distributed around the marginal stable point. From These results it is highly possible that open loop stability is a fundamental component in the control of human paddle juggling when haptic feedback exists. In addition, results show that open loop stability is not a part of the human behavior when haptic feedback is not available. We hypothesize that haptic feedback changes the structure of the motor control polices in the rhythmic dynamic behavior, and performance comes as a product of this policy change.**

I. Introduction

A. Motivation

The long-term objective of this study is understanding the mechanisms by which the human nervous systems controls rhythmic dynamic behaviors, which is one of the grand challenges in neuroscience. Examples of such behaviors are running, walking and juggling. Finding answers to this question will possibly lead us to develop advanced systems for HCI (Human Computer Interaction), neural prostheses, new effective rehabilitation strategies and building robust bio-inspired robotic systems (i.e. robots capable of dynamic locomotion).

Towards the ultimate goal, the first step is finding the factors that most influence the nervous system in the selection of control policies during those dynamic tasks. In some of these behaviors, it has been discovered that mechanical system dynamics plays a crucial role [6, 10].

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In this paper, we concentrated on discovering the importance of haptic feedback in rhythmic dynamic tasks in the context of paddle juggling behavior. Previously the role of haptic feedback in dynamic tasks was analyzed in several studies [9, 15] and the results showed that haptic feedback improves the performance in dynamic tasks. However, this paper is the first study that systematically analyzes the role of haptic feedback in the context of rhythmic dynamic tasks.

In this study, in order to analyze the role of haptic feedback in a rhythmic dynamic behavior, we have created a controlled environment using a haptic interface to simulate 1-D paddle juggling behavior both visually and haptically. We selected paddle juggling behavior due its experimental tractability. The reasons for choosing the 1-D paddle juggling in our rhythmic task are as follow:

- The mechanical system model of paddle juggling is much simpler compared to the mechanical system models in other rhythmic behaviors such as walking and running [2, 4, 20]. The mechanical system model of paddle juggling is formally written in Section III.
- It is easy to simulate the paddle juggling task in a virtual reality system that enables us to collect data easily and apply inputs to the system.

B. Background

Previously, the paddle juggling problem was analyzed by De in his master theses [7] in which the user paddles a ball in a simplified juggling setup with a haptic interface and computer-simulated physics for the ball, paddle, and their interaction. In the virtual reality he created, there is a goal region within which the user should stabilize the apex height (maximum height) of the ball. After stabilizing the ball, virtual perturbations were created on the ball dynamics (velocity perturbations) and he analyzed the neuromechanical responses to these perturbations. Using this interface, he performed only one subject experiment based on the result of which he found several interesting results.

For example, he found that in his experiment, open loop trajectories of the user was unstable from which one can deduce that neuromechanical system is open loop unstable. Explanation of open loop stability is given in Section V-B. However, this result contradicts with the study of Schaal et. al [22] in which they found that in real paddle juggling, the open loop trajectories of the users are stable since they tend to create a negative acceleration at the impact instant. This difference may occur due to lack of haptic feedback in De's study [7]. Also it is possible that, since in De's study there was only one sample trajectory, his finding can be a special result not common in all humans. Either way, it's sure that in terms of open loop stability a detailed comparison between with and without haptic feedback would be very helpful.

Another interesting result in De's study [7] was the biggest change in human response to the perturbations being observed just after the apex instant. It indicates that, the dominant data which human neural control system uses is the error between goal and actual apex height of the ball. Actually, this result is kind of intuitive because human visual perception is not very good at estimating the velocities of the objects. In such a task, since the only reliable information is the visual feedback, neural controller checks the apex height of the ball (where ball velocity is equal to zero) and gives decisions accordingly. Thus, again in the presence of haptic feedback this result would be questionable since haptic feedback can also provide useful information to the neural controllers.

In his study, he performed his analysis without knowing the role of haptic feedback in rhythmic paddle juggling behavior which is the main missing part of this research. In many haptic related studies, the role of haptic feedback found to be critical in enhancing human behavior and performance [8, 17, 23]. Thus, ignoring a significant factor can lead us to wrong answers. Besides, De used only one data to perform his analysis, but in human related studies more than one data is always needed to make inferences and to reach convincing conclusions.

In this paper we systematically investigated the role of haptic feedback in paddle juggling. In this context, we performed subject tests with eighteen people and analyzed their data.

II. Experimental 1-DOF Haptic Paddle

We developed an experimental 1-DOF haptic paddle device based on the design of "Stanford haptic paddle" interface [19] for measuring data and applying force feedback to the user. On the haptic paddle, there is a Maxon DC motor driven by a current amplifier. In order to control the torque of the DC motor we send the voltage commands to the amplifier via a PCI-DAS6014 DAQ Card. The motor shaft is connected to the haptic paddle via a Capstan drive mechanism and the position of the paddle is measured via a quadrature laser encoder on the DC motor. To read encoder inputs, a PCI-QUAD04 encoder card is used. A picture of the haptic paddle is shown in Fig. 1.

The virtual reality part of the system was developed in C# programming language and the loop rate of the whole system is 1 kHz. In order to perform subject tests, a user-friendly GUI was designed a snapshot of which is shown in Fig. 2. In the GUI prepared, in addition to the images of the paddle and the ball, there exists a plus sign image showing the last apex height (where the velocity is zero) of the ball during the experiment. Also there are two lines showing the limits of the goal region where the plus sign (apex height) should be stabilized.

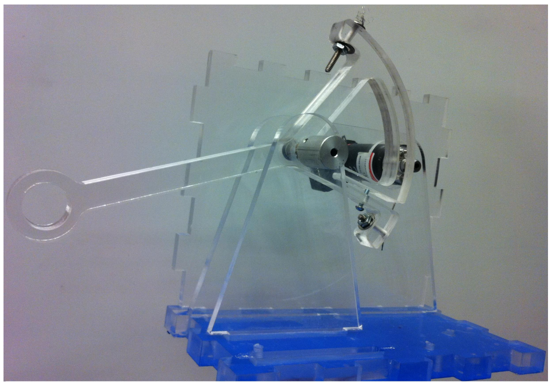

Fig. 1. The experimental 1-DOF Haptic Paddle

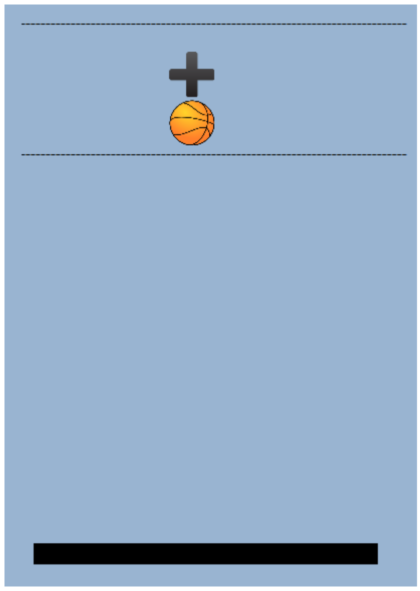

Fig. 2. Graphical User Interface (GUI) used in the experiments

III. Mechanical System Model

Paddle juggling, as in many other rhythmic dynamic tasks, is a hybrid dynamical system. A hybrid dynamic system is a collection of continuous flows and discrete transformations that switch between these flows with the events triggered by threshold functions [13]. In this context, we divide the ball dynamics into two parts: a continuous flight phase describing the dynamics in flight, and discrete transition phase describing the state transitions due to collision between the paddle and the ball. Table I provides the notation we used throughout the paper. When we ignore the drag friction force, flight dynamics of the ball takes the form

$$\begin{aligned} \dot{h}_b &= v_b, \\ \dot{v}_b &= -g. \end{aligned} \tag{1}$$

However, control loop of the haptic paddle juggling setup runs in the discrete domain, such that in our software we

TABLE I

NOTATION

Mechanical System States and Parameters	
h_b	Height of the ball
v_b	Velocity of the ball
v_b^+	Velocity of the ball just after the collision
h_p	Position of the paddle
v_p	Velocity of the paddle
a_p	Acceleration of the paddle
$a_{p,c}$	Acceleration of the paddle at the collision instant
g	Gravitational acceleration
α	Ball-paddle Coefficient of Restitution

discretized the continuos dynamics in (1) as

$$\begin{aligned} h_b[k+1] &= h_b[k] + v_b[k+1]\Delta t - \frac{1}{2} g \Delta t^2, \\ v_b[k+1] &= v_b[k+1] - g\Delta t, \end{aligned}$$

where Δt is the time between two sampling instants. In the real dynamics, collision event is triggered when the following conditions are satisfied :

$$\begin{aligned} h_b &= h_o, \\ v_b &< v_p. \end{aligned}$$

However, in the digital control loop it is impossible to detect an exact collision instant. So in the program the collision event is triggered when the following conditions are satisfied:

$$\begin{aligned} h_b &< h_p, \\ v_b &< v_p. \end{aligned}$$

In order to model the discrete transition due to collision between the paddle and the ball, we assume that the collision is purely elastic and the mass of the paddle is infinite such that the paddle velocity is not affected by the collision. Based on these assumptions, using the model in [7], discrete transition at the collision instant is defined as

As in [7], discrete transition due to collision between the paddle and the ball is modeled using coefficient of restitution rule. We assume that collision between the paddle and the ball is instantaneous and mass of paddle is much larger than the mass of the ball such that the paddle velocity is not affected by the impact. Based on these assumptions, discrete transition at the collision instant is defined as

$$\begin{aligned} h_b^+ &= h_b, \\ v_b^+ &= \alpha v_b + (1+\alpha) v_p. \end{aligned}$$

In the experiments with haptic feedback, we apply an impulsive force (short duration, constant magnitude force) proportional to the impact velocity to the haptic paddle, $v_{imp} = v_p - v_b$ right after the collision.

IV. Experimental Procedure

In order to figure out the significance and the role of haptic feedback in paddle juggling behavior, we performed subject tests with eighteen people using the experimental setup detailed in Section II. All of the experiments were conducted according to a fixed protocol.

Experimental procedure consists of a demo session having four training sessions and two actual sessions. At the beginning of the demo session, the experimenter explains how to use the haptic device to the participant and makes a short juggling demo. For the training sessions and experimental sessions we asked the participants to stabilize the apex height of the ball, showed with the plus sign, in the GUI within the goal region. In training sessions, the subjects perform four training blocks based on a fixed order:

1) Training without Haptic Feedback
2) Training with Haptic Feedback
3) Training without Haptic Feedback
4) Training with Haptic Feedback

In this paper we are only interested in the trained juggling behavior, thus the purpose of the training sessions is to make people get used to the environment and behavior. In the experimental sessions, one test is performed without haptic feedback and the other test is performed with haptic feedback. Half of the subjects started the actual sessions without haptic feedback, where the other half started with haptic feedback in order to eliminate bias that may occur due to a fixed order in the experiments. Duration of each training and experimental session is 2.5 min and in between each session the subjects are waited 30 sec to prevent the fatigue in the experiments. Since we are only interested in trained behavior for this paper, we only collected data from actual sessions and analyzed them in Section V. In a future work we may also collect data from training sessions and analyze the learning process in paddle juggling behavior.

V. Results and Discussion

This section presents our analysis of performed subject experiments. As explained in Section IV we performed subject tests with eighteen people and we analyzed the role of haptic feedback based on two performance criteria and then we examined the open loop stability of the paddle data. The goal, in the experiments, was to keep the apex height of the ball within the goal region which is selected as $48 \pm 4.3cm$.

We also asked the participants their opinions about the difficulty level of completing the task both for with and without haptic feedback cases. Fourteen of the subjects found that completing the task is easier with haptic feedback, whereas three of them told that there is no big difference in both cases. Only one subject stated that without haptic feedback is slightly easier. This is the first sign of the significance and improvement of haptic feedback in the juggling behavior because a majority of the subjects thinks that completing the task is easier with the presence of feedback.

Since we are only interested in the trained (learned) rhythmic juggling behavior for this paper, we decided to exclude the data that is unsuccessful. Two of the subjects couldn't catch a rhythm in both with and without haptic feedback experiments and their performance was very low in both cases for both metrics (PO values $> 80\%$ and PSD values $> 60\%$). Due to this reason we didn't include

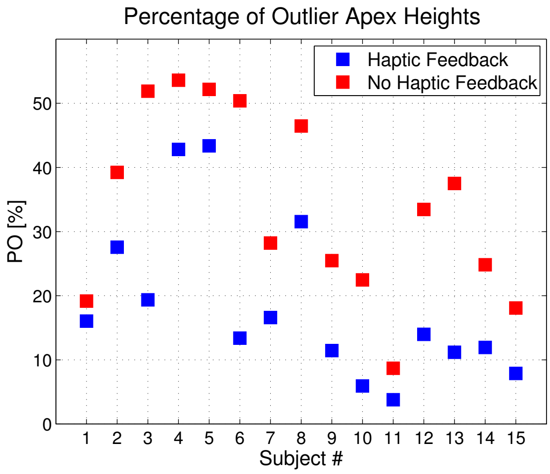

Fig. 3. Percentage of Outlier Apex heights. Blue markers belongs to the experiments with haptic feedback, whereas red markers belongs to the experiments without haptic feedback

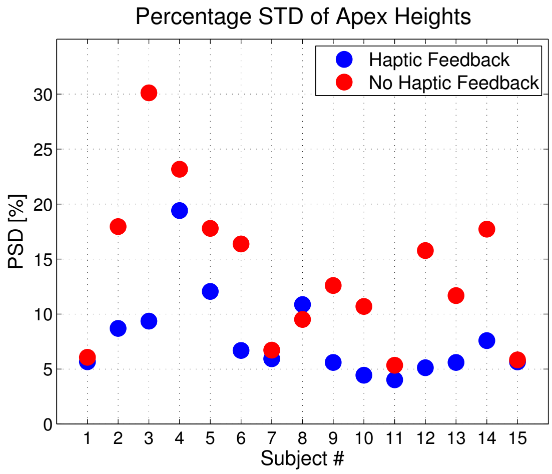

Fig. 4. Percentage of Standard Deviation of Apex Heights. Blue markers belongs to the experiments with haptic feedback, whereas red markers belongs to the experiments without haptic feedback

their data in our statistical analysis considering them as the outliers. Also, although one subject did a very good job in the experiment with haptic feedback (PO = 19.8% and PO = 6.2%), he/she couldn't catch a rhythm when haptic feedback is not available and performance on both metrics was very bad (PO = 70.2% and PO = 85.3%). Since we couldn't include the data without haptic feedback, we also excluded his/her data from our statistical analysis. As a result we analyzed the data of 15 subjects in the following sections.

A. *Performance Analysis*

In order to analyze the performance of the subjects in the two cases discussed, we developed two performance measures. Our first performance metric is the percentage of outliers, PO, which is calculated as:

$$PO = \frac{\text{\# Apex heights outside the goal region}}{\text{\# Apex heights}} \times 100. \quad (2)$$

Based on PO performance metric in (2) we plotted the performances of each subject (excluding the three outliers) for both cases in Fig. 3. From the results it can easily be seen that, each subject's performance is better with haptic feedback and the difference between with and without haptic feedback is significant. Even though the difference is obvious in Fig. 3, we also performed a t-test to emphasize this difference. Our statistical analysis showed that, performances with haptic feedback based on PO criteria is significantly better ($p < 0.00001$, Paired t-test) than the performance without haptic feedback.

Our second performance metric is the percentage standard deviation of apex heights, PSD, which is calculated as :

$$CV = \frac{std(\text{Apex heights values})}{mean(\text{Apex heights values})} \times 100, \quad (3)$$

which can be interpreted as a measure of overall stability of the experiment. Based on the PSD performance metric, the performances of each fifteen subjects for both cases can be seen in Fig. 4. Results show that, for only one subject, the PSD value for without haptic feedback case is slightly lower than the PSD value for the haptic feedback case. For all other participants, performances based on PSD value is better and the difference is generally significant. Similar to the PO case, in order to verify the significance of the difference between two cases, we performed another t-test to PSD values. The statistical analysis shows that haptic feedback increases the PSD performance significantly ($p < 0.001$, Paired t-test).

In addition to the individual performances illustrated in Fig. 3 and Fig. 4, we also analyzed the overall performances using both PO and PSD performance measures. Overall performance results are illustrated in Fig. 5, where the mean, the standard deviation and the maximum values of the data for both performance measures can be seen in these figures. The overall results also show that, there is a notable performance improvement with haptic feedback.

B. *Open Loop Stability*

General approach in analyzing the stability of rhythmic behaviors is modeling the behavior as a nonlinear dynamical system in which the periodic motion is a stable limit cycle [1, 3, 16, 22]. An example of such a stable limit cycle behavior obtained with our experimental setup is illustrated in Fig. 6. In the experiment user stabilizes the ball and reaches a limit cycle after 15 s.

Although we can accurately describe the ball dynamics, the human neuromechanical system is a "black box" for us, such that we can only observe the input-output relations hence, it is very difficult to analyze the dynamic behavior as a whole. One approach to this problem is the system identification methodology which is successfully applied in several studies [7, 18]. For this paper we left this part as a future work and for the stability point of view we only analyze the open loop stability using our data obtained in subject tests. Open loop stability is an important concept in

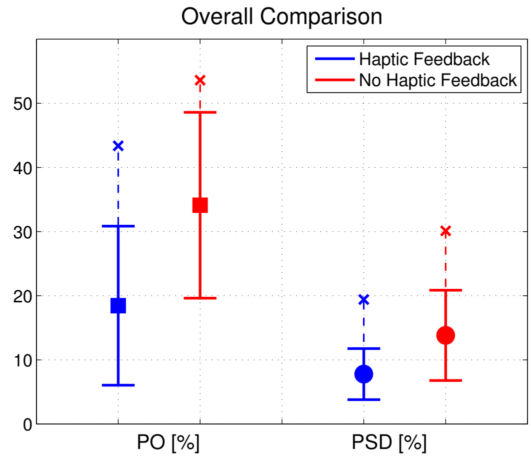

Fig. 5. Overall Performance Results. Blue color corresponds to the results with haptic feedback, whereas red color corresponds to the results without haptic feedback. Solid markers, cross markers and colored vertical bars represent mean, maximum and standard deviations of the associated cases and performance measures

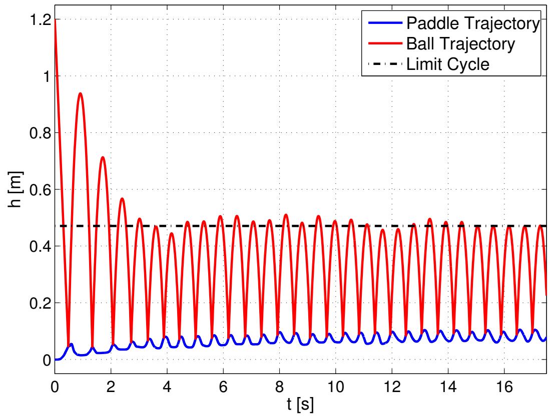

Fig. 6. Example stable limit cycle behavior obtained with the haptic paddle juggling setup

rhythmic dynamic systems and it is analyzed deeply in the literature especially for legged locomotion [2, 11].

Open loop stability criterion for our paddle juggling problem can be characterized as follows: if the behavior can be stabilized by applying the selected trajectories of the user in an open-loop fashion, we say that the system is open-loop or passive stable. However, this does not mean that actual motion is applied at that instant in an open-loop manner. Open-loop stability can be thought as a component that can increase the robustness of the behavior and it may eliminate some low level perturbations that high level feedback may not achieve. We can also hypothesis that if the selected open-loop trajectories are able to stabilize the system, then it is possible that control policies created by the human nervous system is based on creating passively stable trajectories to increase the robustness and closed-loop performance of the behavior. With this approach Open-loop or passive stability of a behavior can be interpreted as a fundamental component for the whole task.

Fortunately, we do not need to analyze the behavior deeply in order to characterize the open loop stability of the paddle juggling experiments because previously in [21, 22] open loop stability of paddle juggling is studied and characterized. In [21] it's found that, if the acceleration of the paddle at the impact instant is negative, then the open loop trajectories can stabilize the rhythmic juggling. On the other hand, in [22], Schall showed that if the acceleration of the paddle at the impact instant satisfies the following condition

$$-2g\frac{1+\alpha^2}{(1+\alpha)^2} < a_{p,c} < 0 \qquad (4)$$

then the eigenvalue of the linearized Poincaré map is in the unit circle which makes the open loop trajectories stable. Poincaré map is one of the most powerful and common tool being used in the analysis of rhythmic dynamic systems [2, 5, 12]. Poincaré return maps reduces the complex continuous rhythmic dynamical system into a lower dimensional discrete-time system that describes the motion as step-to-step transitions, i.e. a nonlinear discrete dynamical system. The stability of this reduced discrete dynamical system ensures the stability of the whole system [14]. Based on this approach, Schall characterizes the passive stability of Paddle Juggling behavior using the eigenvalue of the Poincaré return map.

The only thing that is left to us in analyzing the open loop stability of the experimental results is looking at the accelerations at the collision instants. In Fig. 7 we illustrated the averages and standard deviations of the impact paddle accelerations for each subject and for both with and without haptic feedback cases. Results shows that, selected impact accelerations with haptic feedback is significantly lower than the selected accelerations in experiments without haptic feedback ($p < 0.00000005$, Paired t-test for the average values of the impact accelerations). From this we can conclude that, with haptic feedback subjects tend to create significantly different trajectories in the paddle juggling behavior.

Additionally, we can also deduce that, with haptic feedback subjects create significantly open-loop stable trajectories since the paddle accelerations together with the standard deviations are within the stable green region which is determined according to (4) as defined by Schall in [22]. This verifies the result obtained with real paddle juggling experiments in [22]. However, the situation in data obtained without feedback is different. Average acceleration values of some of the subjects are positive and for the other people they are negative. If we also look at the standard deviations, we see that there are significant number of accelerations that are positive for subjects whose average impact acceleration is negative. From this we can conclude that, without haptic feedback open loop trajectories are distributed around marginal stable point which occurs when paddle acceleration is equal to zero.

In order to analyze the overall stability for both cases

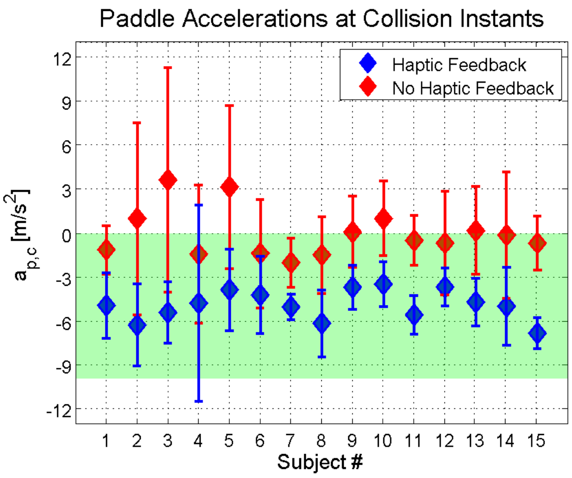

Fig. 7. Mean and standard deviations of paddle accelerations at collision instants for each subject. Blue color corresponds to experiments with haptic feedback, whereas red color corresponds to the experiments without haptic feedback. Shaded green region illustrates the open-loop stable trajectories according to (4) for $\alpha = 0.8$.

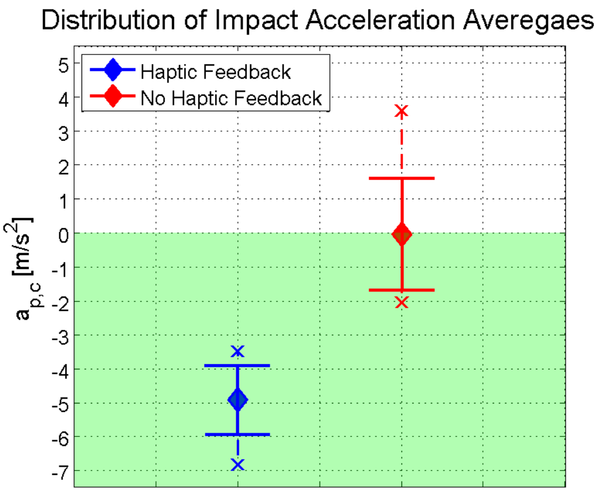

Fig. 8. Distributions of impact acceleration averages for cases. Solid markers, colored vertical bars and cross markers represent mean, standard deviation and extreme (maximum and minimum) values of acceleration averages. Shaded green region belongs to the open-loop stable trajectories for $\alpha = 0.8$.

we illustrated the mean, standard deviation, maximum and minimum values of the acceleration averages in Fig. 8. In this figure we observe that the trajectories with haptic feedback is strictly open loop stable and we can hypothesize that open loop stability is an essential component in the closed loop neuromechanical system when haptic feedback is available. This result agrees with the conclusions obtained in [22]. On the other hand, if we look at the impact acceleration distributions obtained without haptic feedback, we observe that the mean is almost zero. Thus, we can say that, human nervous system selects trajectories around metastable region thus open loop stability is definitely not a component of the control policies when haptic feedback is not available. Since the open loop stability is not a part of the behavior without haptic feedback, control loop most probably relies on the visual feedback correction. This result matches with the outcomes in De's study [7] with paddle juggling without haptic feedback.

Using our open loop stability analysis, we can see that, haptic feedback in paddle juggling behavior not only increases the performance of the task, but it also changes the motor control policy by helping the human nervous system creating negative impact accelerations, thus bringing open loop stable behavior. So, we can hypothesize that haptic feedback helps the nervous system to create passively stable trajectories to create a more robust behavior and to reach a better performance. Actually, this difference can be more important than the results that we obtained in performance analysis in Section V-A.

VI. Conclusions and Future Work

In this paper, we analyzed the role of haptic feedback in paddle juggling behavior to take a step towards our ultimate goal of discovering the mechanisms by which the human nervous systems controls rhythmic dynamic behaviors. Although, there are studies that investigated the role of haptic feedback in dynamic tasks [9, 15] , this paper is the first study that analyzed the significance and the role of haptic feedback in a rhythmic dynamic task.

We selected paddle juggling behavior due to its relatively simple dynamics compared to behaviors such as walking and running and due to its nice experimental tractability properties. In order to accomplish our aim we first developed an experimental haptic paddle interface as detailed in Section II. The main components of our GUI are the paddle simulating the motion of the user, the ball, the plus sign showing the last apex height of the ball and the goal lines indicating the region in which the users should stabilize the apex height of the ball.

Using our experimental haptic device we performed subject tests with eighteen people and collected paddle juggling data both with haptic feedback and without haptic feedback. In Section V-A we analyzed the performance of subjects in paddle juggling behavior based on two performance metrics, PO (Percentage of Outliers) and PSD (Percentage Standard Deviation of the Apex Height Values). Both individual results shown in Fig. 3 and Fig. 4 and combined overall results Fig. 5 show that the haptic feedback significantly improves the performance in juggling behavior.

Although, this result is enough to emphasize the significance and the importance of haptic feedback in rhythmic juggling problem, we analyzed the structural differences in both cases in terms of open loop stability. Based on the outcomes obtained in previous studies [21, 22] with paddle juggling problem, we characterized the open loop stability with the acceleration of the paddle at the impact. Both the individual results illustrated in Fig. 7 and the combined results illustrated in Fig. 8 show that with haptic feedback subjects select strictly open loop trajectories, whereas the impact accelerations for the trajectories without haptic feedback are distributed around the marginal stable point.

From this results it's highly possible that the open-loop stability is an essential mechanism in the human juggling behavior when haptic feedback is available which matches with the findings in the study of Schall [22] with physical paddle juggling experiments. On the other hand, it is sure that open loop stability is not a part of the closed loop behavior when haptic feedback is not available. Thus, most probably visual feedback highly is used for correcting the errors. These results verify the results obtained in [7].

From the open loop stability analysis we see that, the role of haptic feedback in rhythmic juggling is much more than just a significant performance increase because haptic feedback changes the structure of the motor control polices and the performance increase is most probably an outcome of this control policy change.

In the future, we will design new experimental scenarios to analyze different characteristics of paddle juggling behavior and we will try to find different answers relating to the control of rhythmic behavior in humans. We will also use advanced system identification techniques to test several hypothesis pertinent to the internal structure of the neural control mechanism.

VII. Acknowledgements

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