# 经典电磁学 基本理论 构架

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#### 理解经典电磁学理论

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摘 臺 本文先回顾和评述传统对经典表克斯韦方程组及洛伦兹力公式通过实验定律和作用 量的两种理解方式,再给出一种以四度电磁势 A、及规范对称为基础的另类理解方式。 美體調 麦克斯韦方程组;洛伦兹力;西度电磁势

#### UNDERSTANDING CLASSICAL ELECTROMAGNETIC THEORY

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Abstract Based on the review and comment on two kinds traditional understandings by experiment laws and actions for the classical Maxwell equations and Lorentz force formula we present an alternative understanding in terms of four-vector electromagnetic potential A, and gauge invariance. Key words Maxwell equations; Lorentz force; four-vector electromagnetic potential

经典电磁学理论的核心是表京斯韦方程组和 洛伦兹力公式。前者确定了电场强度和磁感应强 度对給定的电荷电流源密度分布的依赖方程组 (或者说已知电荷电流源的分布决定了电磁场)。 后耆则对给定的电场强度和磁感应强度给出了电 荷电流源所受的电磁力(已知电磁场的分布决定了 源的受力),若再辅以牛顿第二定律并加上源所受 的其他可能的非电磁力。则可以写出电荷电流源的 运动方程(或者说已知电磁场决定了电荷电流源的 运动),场方程和运动方程两者结合起来就确定了 一个电磁系统的完整运动规律。本文具限于讨论 真空中的电荷电流源及电磁场,如若考虑介质材 料。须要在上面所讨论的电荷电流源中加入由于介 面的极化和磁化导致的极 化磁化电荷电流的贡献。

在学习、研究和应用电磁学理论时我们总希 望能尽量深入地理解它,特别是能有直现图像的 理解,麦克斯韦方程组和洛伦兹力公式为什么会 是这个样子? 一定是需要目前电磁学理论所给出 的这种结构而不可能有所变化吗? 这个问题在一 般的教科书和研究论文中进行深入论述的比较 少,本文就此作一讨论。

谈论理解一个事物总要依据一些出发点,理 解麦克斯韦方程组和洛伦兹力公式同样需要有出 发点, 裁科书里一般有两类理解方式, 一是从实验 定律出发,另一是从作用量出发。对这两种做法 本文轉分別在第1和第2节作一系统回顾和评 述,在第3节给出一种自认为更优的新的理解方 法,并进行讨论。著名物理学家费曼普经说过"从 一个新角度看待旧问题是很有意思的"。如果觉得 本文前面的基本内容过于基本和简单,读者可以 直接从 2.3 节的对作用量做法的评述开始阅读。

#### 1 从实验定律推导经典电磁学理论

通常的静态和稳恒情形的麦克斯韦方程组和 洛伦兹 力公式可以从实验给出的电荷 源之间相互 作用力的摩仑定律,电流源之间相互作用力的事 奥-萨伐尔定律出发,将电场强度和磁感应强度分 别定义为单位电量的电荷电流源所受的电力和磁 力得到。具体地,若一团由电荷密度 a, (r)电流密

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简介 这是物理学科本科理论物理的四大力学中唯一的一门涉及基本相互作用力的课程,也是物理本科阶段最难(没有之一)的课...

章节 ....力学 5.6 相对论电动力学 5.7 磁单极-规范不变性-Witten效应 5.8 第五章作业 5.6 相对论电动力学...









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• 从实验定律出发构建经典电磁学理论\*\*\*

● 从作用量出发构建协变的经典电磁学理论……

• 经典电磁学理论的另类理解及扩展……



### 一. 从实验定律出发

构建经典电磁学理论







#### 电磁程 互作用的源: 电荷,电荷密度

电荷是电相互作用之源.物质的电相互作用强度与其携带的电荷成正比.电荷可正可负,也可为零(即不带电). 电荷的起源 摩擦生电

- ▲ 直电荷: 无体积,带电量,是基本电量单位的整数倍
   (1.6021917733(49) × 10<sup>-19</sup>库伦) 分立性在宏观物理中可忽略。
- 电荷密度: 电荷分布在空间体积中.

$$ho = \lim_{\Delta au o 0} rac{\Delta ext{q}}{\Delta au} =$$
场点处单位体积的电量 $\sigma = \lim_{\Delta ext{S} o 0} rac{\Delta ext{q}}{\Delta ext{S}} =$ 场点处单位面积的电量 $\eta = \lim_{\Delta ext{I} o 0} rac{\Delta ext{q}}{\Delta ext{I}} =$ 场点处单位长度的电量

• <u>点电荷的电荷密度</u>: 位于 $\vec{\mathbf{r}}_0$ 处电量为q 的点电荷在空间的电荷密度分布为 $\rho(\vec{\mathbf{r}},\vec{\mathbf{r}}_0)$ 

$$\int_{\tau} \mathbf{d}\tau \ \rho(\vec{\mathbf{r}}, \vec{\mathbf{r_0}}) = \mathbf{q} \ \rightarrow \rho(\vec{\mathbf{r}}, \vec{\mathbf{r_0}}) = \begin{cases} 0 & \vec{r} \neq \vec{r_0} \\ \infty & \vec{r} = \vec{r_0} \end{cases} = \mathbf{q}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r_0}})$$

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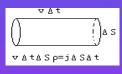
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运动电荷形成电流,有方向和电量大小。

电流密度矢量式的大小为单位时间流过垂直于电流方向单位面积的电量,方向为电流方向。

对任意一个不一定垂直于电流方向的面元 $\Delta \vec{S}$ ,单位时间通过的电量为 $j\Delta S_{\perp} = \vec{j} \cdot \Delta \vec{S}$ . 对有限大小面元,单位时间流过的电量叫电流强度  $\Delta S_{\perp} + \vec{j} = J_{S} d\vec{S} \cdot \vec{j}$ 

以速度 $\vec{v}$ 运动的电荷具有电流密度  $\vec{j} = \rho \vec{v}$ 



电流密度用图画出为电流线,它方向为式方向,密度为式大小。在电流线图上沿式方向作一小电流管:

$$\mathbf{J}\mathbf{dl} = \mathbf{j}\boldsymbol{\Delta}\mathbf{S}\mathbf{d}\vec{\mathbf{l}} = \vec{\mathbf{j}}\mathbf{d}\boldsymbol{\tau} = \rho\vec{\mathbf{v}}\mathbf{d}\boldsymbol{\tau} \stackrel{\$\text{phire}}{=} \sum_{i} \rho_{i}\vec{\mathbf{v}}_{i}\mathbf{d}\boldsymbol{\tau} = \sum_{i} \mathbf{q}_{i}\vec{\mathbf{v}}_{i}$$

$$\rho_{-} = -\rho_{+} \to \rho = \rho_{-} + \rho_{+} = 0$$
 $\vec{j} = \rho_{+}\vec{v}_{+} + \rho_{-}\vec{v}_{-} = \rho_{+}(\vec{v}_{+} - \vec{v}_{-})$ 

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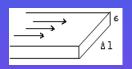
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#### 电磁相互作用的源: 电流,电流密度; 磁单极

电流分布在表面上用面电流密度 $\vec{i}$ ,方向为电流方向,大小为 单位时间流过垂直于电流方向单位长度的电量。 对任一线元 $\Delta \vec{i}$ ,单位 时间通过的电量 $\vec{i}$ .  $\Delta \vec{i}$ 。

对以速度 $\vec{\mathbf{v}}_i$ 运动的面电荷  $\sigma_i$ ,  $\vec{\mathbf{i}} = \sum_{\mathbf{v}} \sigma_i \vec{\mathbf{v}}_i$  面电流密度 与体电流密度有关,  $\vec{\mathbf{j}} = \frac{\vec{\mathbf{i}}}{\delta}$ ,

$$\mathbf{J}\mathbf{d}\vec{l} = \mathbf{i}\Delta\mathbf{l}\mathbf{d}\vec{l} = \vec{\mathbf{i}}\mathbf{d}\mathbf{S} = \sum_{i} \sigma_{i}\vec{\mathbf{v}}_{i}\mathbf{d}\mathbf{S} = \sum_{i} \mathbf{q}_{i}\vec{\mathbf{v}}_{i}$$



#### 磁单极

磁相互作用产生源的认识历史:

- ullet 磁铁  $\Rightarrow$  磁荷(磁单极)  $\mathbf{Q}_{\mathbf{m}}$ ; 磁荷密度 $ho_{\mathbf{m}}$ ; 磁流密度 $ec{j}_m$
- 磁荷之间遵从类似于库仑定律的平方反比率
- 电流也可以产生磁作用
- ●安培提出分子电流■電假说⇒磁相互作用由电流
  ■型地#产生
- 还有没有磁单极? Dirac Strings and Magnetic Monopoles in Spin Ice Dy<sub>2</sub>Ti<sub>2</sub>O<sub>7</sub>, Science.1178868,2009.9.3

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#### 电荷守恒定律:

电荷不会自动产生或湮灭,除非碰上其它电荷

$$\oint_{\tau} \mathbf{d\vec{S}} \cdot \vec{\mathbf{j}} = -\frac{\mathbf{d}}{\mathbf{dt}} \int_{\tau} \mathbf{d}\tau \rho$$

对一个任意体积<sub>7</sub>,若在某一段时间内其电量减少,则减少的电量一定是 从此体积的表面S流出(或流进等量的负电荷)。

#### 叠加原理:

电磁相互作用力具有可迭加性质

$$ec{\mathbf{f}} = \sum_{\mathrm{i}} ec{\mathbf{f}}_{\mathrm{i}}$$





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#### 库仑定律:

对静止点电荷,同号相斥,异号相吸,相互作用力沿电荷 连线方向,大小正比于电荷 的电量,反比于电荷之间距离平方

$$ec{\mathbf{f}_{21}} = \mathbf{k}_1 rac{\mathbf{q}_1 \mathbf{q}_2}{\mathbf{R}^3} ec{\mathrm{R}} \hspace{0.5cm} \mathbf{k}_1 = rac{1}{4\pi\epsilon_0} \hspace{0.5cm} \epsilon_0 = 8.854 imes 10^{-12}$$
库伦 $^2/$ 牛顿:米 $^2$ 

 $\vec{\mathbf{f}}_{21}$ 是 $\mathbf{q}_2$ 对 $\mathbf{q}_1$ 的作用力,  $\mathbf{q}_1$ 对 $\mathbf{q}_2$ 的作用力为 $\vec{f}_{12} = -\vec{f}_{21}$ ,  $\vec{\mathbf{R}}$ 是从 $\mathbf{q}_2$ 指向 $\mathbf{q}_1$ 的矢量。

$$egin{aligned} ec{\mathbf{F}}_{\mathbf{21}} &= \int \mathbf{d}ec{\mathbf{f}}_{\mathbf{21}} = \int_{ au_1} \int_{ au_2} rac{\mathbf{k}_1 
ho_1(ec{\mathbf{r}}') \mathbf{d} au' 
ho_2(ec{\mathbf{r}}'') \mathbf{d} au''}{\mathbf{R}^3} ec{\mathbf{R}} \end{aligned} \qquad ec{\mathbf{R}} &= ec{\mathbf{r}}' - ec{\mathbf{r}}'' \end{aligned}$$

$$\vec{f}_{21} = \int_{\tau_1} \!\! d\tau' \! \int_{\tau_2} \!\! d\tau'' \frac{k_1 q_1 \delta(\vec{r}'\!\!-\!\vec{r}_1) q_2 \delta(\vec{r}''\!\!-\!\vec{r}_2)}{|\vec{r}'\!\!-\!\vec{r}''|^3} (\vec{r}'\!\!-\!\vec{r}'') = \frac{k_1 q_1 q_2}{|\vec{r}_1\!\!-\!\vec{r}_2|^3} (\vec{r}_1\!\!-\!\vec{r}_2)$$



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#### 电磁相互作用的场与真空中的基本实验定律: 电流元的相互作用

电流元的相互作用:

恒定电流元 $J_2d\vec{l}_2$ 对 $J_1d\vec{l}_1$ 的作用力为



$$\mathbf{d^2\vec{F}_{21}} = k_2J_1\mathbf{d\vec{l}_1} \times (\frac{J_2\mathbf{dl_2} \times R}{R^3}) = \frac{k_2J_1J_2}{R^3}[(\vec{R} \cdot \vec{dl_1})\vec{dl_2} - (\vec{dl_1} \cdot \vec{dl_2})\vec{R}]$$

电流元 $J_1d\vec{l}_1$ 对  $J_2d\vec{l}_2$ 的作用力为

$$\mathbf{d^2} \vec{\mathbf{F}}_{12} = rac{\mathbf{k_2J_1J_2}}{\mathbf{R^3}} [-(\vec{\mathbf{R}}\cdot \mathbf{d}\vec{\mathbf{l_2}}) \mathbf{d}\vec{\mathbf{l_1}} + (\mathbf{d}\vec{\mathbf{l_1}}\cdot \mathbf{d}\vec{\mathbf{l_2}}) \vec{\mathbf{R}}]$$
 where

其中: 安培最早的工作测量的不是电流元,而是电流圈之间的相互作用力!

$$ec{\mathbf{R}}=ec{\mathbf{r_1}}-ec{\mathbf{r_2}}$$
  $\mathbf{k_2}=rac{\mu_0}{4\pi}$   $\mu_0=4\pi imes 10^{-7}$ 公斤・米/库伦 $^2$ 

两电流元之间的作用力与反作用力不一样, $\vec{F}_1 + \vec{F}_2 \neq 0$  (例:两运动点电荷 $\vec{v}_2 \parallel \vec{R} \perp \vec{v}_1$ )  $\rightarrow \frac{d\vec{P}_1}{dt} + \frac{d\vec{P}_2}{dt} \neq 0$ 或  $\vec{P}_1 + \vec{P}_2 \neq 0$ ,动量守恒不再成立。



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<u>→→</u>



<mark>律:</mark> 场

一 两个电流元的相互作用动量守恒不成立的现象启示两个电流元构成的 体系并不是一个封闭体系,还有其它携带动量的物质实在。

场的概念最早是法拉第提出的: 场是局域的; 弥漫于整个空间。源产生场, 场反作用于源。场是传递源之间相互作用的载体。

$$\begin{split} \vec{\mathbf{F}}_{\mathbf{21}} &= \int_{\tau_{\mathbf{1}}} \!\! \mathbf{d}\tau' \; \left[ \; \rho_{\mathbf{1}}(\vec{\mathbf{r}}') \int_{\tau_{\mathbf{2}}} \!\! \mathbf{d}\tau'' \frac{\mathbf{k}_{\mathbf{1}} \rho_{\mathbf{2}}(\vec{\mathbf{r}}'')}{\mathbf{R}^{3}} \vec{\mathbf{R}} \; + \; \vec{\mathbf{j}}_{\mathbf{1}}(\vec{\mathbf{r}}') \times \int_{\tau_{\mathbf{2}}} \!\! \mathbf{d}\tau'' \frac{\mathbf{k}_{\mathbf{2}} \vec{\mathbf{j}}_{\mathbf{2}}(\vec{\mathbf{r}}'') \times \vec{\mathbf{R}}}{\mathbf{R}^{3}} \; \right] \\ &= \int_{-} \!\! \mathbf{d}\tau' \; \left[ \; \rho_{\mathbf{1}}(\vec{\mathbf{r}}') \vec{\mathbf{E}}_{\mathbf{1}}(\vec{\mathbf{r}}') \; + \; \vec{\mathbf{j}}_{\mathbf{1}}(\vec{\mathbf{r}}') \times \vec{\mathbf{B}}_{\mathbf{1}}(\vec{\mathbf{r}}') \; \right] \qquad \vec{\mathbf{R}} = \vec{\mathbf{r}}' - \vec{\mathbf{r}}'' \end{split}$$

$$ec{\mathbf{E}}_{1}(ec{\mathbf{r}}') = \mathbf{k}_{1} \int \mathbf{d} au'' rac{
ho_{2}(ec{\mathbf{r}}'')}{\mathbf{R}^{3}} ec{\mathbf{R}} \qquad \qquad ec{\mathbf{B}}_{1}(ec{\mathbf{r}}') = \mathbf{k}_{2} \int \mathbf{d} au'' rac{ec{\mathbf{j}}_{2}(ec{\mathbf{r}}'')}{\mathbf{R}^{3}} imes ec{\mathbf{R}} \,.$$

库伦定律和安培比萨定律中源受力的公式分解为源产生场公式和源在场中受力公式的结合!

电磁力之间的迭加原理即转化为场的迭加原理: • 场方程是线性的!

场:

如此引进的场在一开始没有场受力的概念,只有源产生场,场作用源的概念

$$\vec{\mathbf{F}}_{\mathbf{11}} \equiv \int \!\! \mathbf{d}\tau [\rho_{\mathbf{1}}(\vec{\mathbf{r}})\vec{\mathbf{E}}_{\mathbf{s}}(\vec{\mathbf{r}}) + \vec{\mathbf{j}}_{\mathbf{1}}(\vec{\mathbf{r}}) \times \vec{\mathbf{B}}_{\mathbf{s}}(\vec{\mathbf{r}})] = \int_{\tau} \int_{\tau} \!\! \mathbf{d}\tau' \mathbf{d}\tau'' [\mathbf{k}_{\mathbf{1}}\rho_{\mathbf{1}}(\vec{\mathbf{r}}')\rho_{\mathbf{1}}(\vec{\mathbf{r}}'') + \mathbf{k}_{\mathbf{2}}\vec{\mathbf{j}}_{\mathbf{1}}(\vec{\mathbf{r}}'') \times \vec{\mathbf{j}}_{\mathbf{1}}(\vec{\mathbf{r}}'') \times ] \frac{(\vec{\mathbf{r}}' - \vec{\mathbf{r}}'')}{|\vec{\mathbf{r}}' - \vec{\mathbf{r}}''|^3} = 0$$

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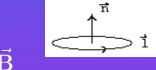
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电场由电荷产生,磁场由电流即运动的电荷产生,因而电场与磁场的关系一定与运动即时间的变化有关.

一根闭合导线所包围的磁通的改变,将在此闭合导线上产生感应电动势,其数值等于单位时间内磁通的变化率,由此产生的感应电动势所决定的感应电流方向阻止导线内磁通的变化.

$$arepsilon = -rac{\mathbf{d} \mathbf{\Phi}}{\mathbf{d} \mathbf{t}} \qquad arepsilon = \oint_{\mathbf{S}} \mathbf{d} ec{\mathbf{I}} \cdot ec{\mathbf{E}} \qquad \mathbf{\Phi} = \int_{\mathbf{S}} \mathbf{d} ec{\mathbf{S}} \cdot ec{\mathbf{B}}$$



<u>S</u>方向与Ī方向成右手螺旋关系。

此定律早期在很低频电磁场中建立,后来发现居然对很高频的电磁场仍然成立!

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麦克斯韦方程组的积分形式 电荷守恒:  $\oint_{\tau} \mathbf{d\vec{S}} \cdot \vec{\mathbf{j}} = -\int_{\tau} \mathbf{d} \tau \frac{\partial \rho}{\partial \mathbf{t}}$ 



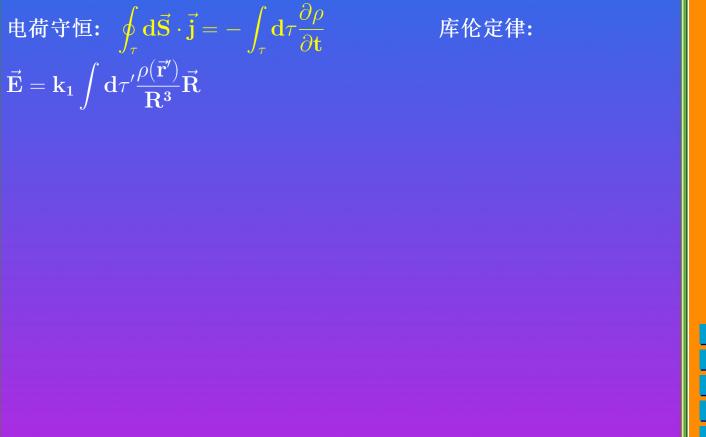






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## 真空中电磁相互作用的场方程: 麦克斯韦方程组的积分形式

电荷守恒:  $\oint_{\tau} \mathbf{d\vec{S}} \cdot \vec{\mathbf{j}} = -\int_{\tau} \mathbf{d}\tau \frac{\partial \rho}{\partial \mathbf{t}}$ 

库伦定律:

 $ec{\mathbf{E}} = \mathbf{k_1} \int d\tau' \frac{\rho(\vec{\mathbf{r}}')}{\mathbf{R}^3} \vec{\mathbf{R}} = -\mathbf{k_1} \int d\tau' \rho(\vec{\mathbf{r}}') \nabla \frac{1}{\mathbf{R}}$ 





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### 真空中电磁相互作用的场方程: 麦克斯韦方程组的积分形式

电荷守恒:  $\oint_{\tau} \mathbf{d\vec{S}} \cdot \vec{\mathbf{j}} = -\int_{\tau} \mathbf{d}\tau \frac{\partial \rho}{\partial \mathbf{t}}$ 库伦定律:

$$ec{\mathbf{E}} = \mathbf{k_1} \int \mathbf{d} au' rac{
ho(ec{\mathbf{r}'})}{\mathbf{R}^3} ec{\mathbf{R}} = -\mathbf{k_1} \int \mathbf{d} au' 
ho(ec{\mathbf{r}'}) 
abla rac{1}{\mathbf{R}} = -
abla ig[\int \mathbf{d} au' rac{
ho(ec{\mathbf{r}'})}{\mathbf{R}}ig]$$









# <u>真空中电磁相互作用的场方程</u>。麦克斯韦方程组的积分形式 $\int d\rho$

电荷守恒:  $\oint_{\tau} \mathbf{d\vec{S}} \cdot \vec{\mathbf{j}} = -\int_{\tau} \mathbf{d\tau} \frac{\partial \rho}{\partial t}$  库伦定律:  $\vec{\mathbf{E}} = \mathbf{k}_1 \int \mathbf{d\tau}' \frac{\rho(\vec{\mathbf{r}}')}{\mathbf{R}^3} \vec{\mathbf{R}} = -\mathbf{k}_1 \int \mathbf{d\tau}' \rho(\vec{\mathbf{r}}') \nabla \frac{1}{\mathbf{R}} = -\nabla \left[ \int \mathbf{d\tau}' \frac{\rho(\vec{\mathbf{r}}')}{\mathbf{R}} \right]$   $\phi(\vec{\mathbf{r}}) = \mathbf{k}_1 \int \mathbf{d\tau}' \frac{\rho(\vec{\mathbf{r}}')}{\mathbf{R}} + \mathring{\mathbf{r}} \mathfrak{B}$ 



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## 真空中电磁相互作用的场方程: 麦克斯韦方程组的积分形式

电荷守恒:  $\oint d\vec{\mathbf{S}} \cdot \vec{\mathbf{j}} = - \int_{\tau} d\tau \frac{\partial \rho}{\partial \mathbf{t}}$ 

库伦定律:

$$ec{\mathbf{E}} = \mathbf{k_1} \int \mathbf{d} au' rac{
ho(ec{\mathbf{r}'})}{\mathbf{R}^3} ec{\mathbf{R}} = -\mathbf{k_1} \int \mathbf{d} au' 
ho(ec{\mathbf{r}'}) 
abla rac{\mathbf{1}}{\mathbf{R}} = -
abla ig[\int \mathbf{d} au' rac{
ho(ec{\mathbf{r}'})}{\mathbf{R}}ig]$$

$$rac{
ho(\mathbf{r}')}{\mathbf{R^3}}ec{\mathbf{R}} = -\mathbf{k_1}\int \, \mathbf{d} au' 
ho$$

$$\mathrm{d} au'
ho(\vec{\mathbf{r}}')
abla rac{1}{\mathrm{R}} = -
abla \left[\int \mathrm{d} au' rac{
ho(\vec{\mathbf{r}}')}{\mathrm{R}}
ight]$$

 $\phi(\vec{\mathbf{r}}) = \mathbf{k_1} \int \mathbf{d} \tau' \frac{\rho(\vec{\mathbf{r}}')}{\mathbf{R}} + 常数$  $ec{\mathbf{E}}(ec{\mathbf{r}}) = -\nabla \phi(ec{\mathbf{r}})$ 





直空中电磁相互作用的场方程: 麦克斯韦方程组的积分形式 电荷守恒:  $\oint d\vec{\mathbf{S}} \cdot \vec{\mathbf{j}} = -\int_{\tau} d\tau \frac{\partial \rho}{\partial \mathbf{t}}$ 库伦定律:

电荷守恒: 
$$\int_{\tau} \mathbf{dS} \cdot \mathbf{j} = -\int_{\tau} \mathbf{d\tau} \frac{\mathbf{r}}{\partial \mathbf{t}}$$
 库伦定律: 
$$\vec{\mathbf{E}} = \mathbf{k}_1 \int \mathbf{d\tau}' \frac{\rho(\vec{\mathbf{r}}')}{\mathbf{R}^3} \vec{\mathbf{R}} = -\mathbf{k}_1 \int \mathbf{d\tau}' \rho(\vec{\mathbf{r}}') \nabla \frac{1}{\mathbf{R}} = -\nabla \left[ \int \mathbf{d\tau}' \frac{\rho(\vec{\mathbf{r}}')}{\mathbf{R}} \right]$$

$$\phi(\vec{\mathbf{r}}) = \mathbf{k}_1 \int \mathbf{d\tau}' \frac{\rho(\vec{\mathbf{r}}')}{\mathbf{R}} + 常数 \qquad \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -\nabla \phi(\vec{\mathbf{r}})$$

$$\oint_{\mathbf{S}} \mathbf{d\vec{l}} \cdot \vec{\mathbf{E}} = \int_{\mathbf{S}} \mathbf{d\vec{S}} \cdot (\nabla \times \vec{\mathbf{E}})$$



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直空中电磁相互作用的场方程: 麦克斯韦方程组的积分形式 电荷守恒:  $\oint_{\tau} d\vec{S} \cdot \vec{j} = - \int_{\tau} d\tau \frac{\partial \rho}{\partial t}$  库伦定律:

电荷守恒: 
$$\oint_{\tau} \mathbf{dS} \cdot \mathbf{j} = -\int_{\tau} \mathbf{d}\tau \frac{\mathbf{i}}{\partial \mathbf{t}}$$
 库伦定律: 
$$\vec{\mathbf{E}} = \mathbf{k}_1 \int \mathbf{d}\tau' \frac{\rho(\vec{\mathbf{r}}')}{\mathbf{R}^3} \vec{\mathbf{R}} = -\mathbf{k}_1 \int \mathbf{d}\tau' \rho(\vec{\mathbf{r}}') \nabla \frac{1}{\mathbf{R}} = -\nabla \left[ \int \mathbf{d}\tau' \frac{\rho(\vec{\mathbf{r}}')}{\mathbf{R}} \right]$$
 
$$\phi(\vec{\mathbf{r}}) = \mathbf{k}_1 \int \mathbf{d}\tau' \frac{\rho(\vec{\mathbf{r}}')}{\mathbf{R}} + \mathbf{\mathring{r}} \mathbf{\mathring{m}}$$
 
$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = -\nabla \phi(\vec{\mathbf{r}})$$

$$\oint_{\mathbf{S}} \mathbf{d}\vec{\mathbf{l}} \cdot \vec{\mathbf{E}} = \int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot (\nabla \times \vec{\mathbf{E}}) = -\int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot [\nabla \times (\nabla \phi)]$$



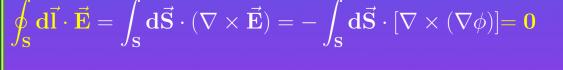






直空中电磁相互作用的场方程: 麦克斯韦方程组的积分形式 电荷守恒:  $\oint d\vec{\mathbf{S}} \cdot \vec{\mathbf{j}} = -\int_{\tau} d\tau \frac{\partial \rho}{\partial \mathbf{t}}$ 库伦定律: 15/96

电荷守恒: 
$$\oint_{\tau} \mathbf{dS} \cdot \mathbf{j} = -\int_{\tau} \mathbf{d}\tau \frac{\partial \rho}{\partial t}$$
 库伦定律: 
$$\vec{\mathbf{E}} = \mathbf{k}_1 \int \mathbf{d}\tau' \frac{\rho(\vec{\mathbf{r}}')}{\mathbf{R}^3} \vec{\mathbf{R}} = -\mathbf{k}_1 \int \mathbf{d}\tau' \rho(\vec{\mathbf{r}}') \nabla \frac{1}{\mathbf{R}} = -\nabla \left[ \int \mathbf{d}\tau' \frac{\rho(\vec{\mathbf{r}}')}{\mathbf{R}} \right]$$
 
$$\phi(\vec{\mathbf{r}}) = \mathbf{k}_1 \int \mathbf{d}\tau' \frac{\rho(\vec{\mathbf{r}}')}{\mathbf{R}} + \mathbf{\hat{r}} \mathbf{\hat{m}}$$
 
$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = -\nabla \phi(\vec{\mathbf{r}})$$









直空中电磁相互作用的场方程: 麦克斯韦方程组的积分形式 电荷守恒:  $\oint d\vec{\mathbf{S}} \cdot \vec{\mathbf{j}} = -\int_{\tau} d\tau \frac{\partial \rho}{\partial \mathbf{t}}$ 库伦定律:

电询引证: 
$$\int_{\tau} \mathbf{d}\mathbf{s} \cdot \mathbf{j} = -\int_{\tau} \mathbf{d}\tau \frac{\partial}{\partial t}$$
 库尼定律: 
$$\vec{\mathbf{E}} = \mathbf{k}_1 \int \mathbf{d}\tau' \frac{\rho(\vec{\mathbf{r}}')}{\mathbf{R}^3} \vec{\mathbf{R}} = -\mathbf{k}_1 \int \mathbf{d}\tau' \rho(\vec{\mathbf{r}}') \nabla \frac{1}{\mathbf{R}} = -\nabla \left[ \int \mathbf{d}\tau' \frac{\rho(\vec{\mathbf{r}}')}{\mathbf{R}} \right]$$

$$\phi(\vec{\mathbf{r}}) = \mathbf{k}_1 \int \mathbf{d}\tau' \frac{\rho(\vec{\mathbf{r}}')}{\mathbf{R}} + 常数 \qquad \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -\nabla \phi(\vec{\mathbf{r}})$$

$$\oint_{\mathbf{S}} \mathbf{d}\vec{\mathbf{I}} \cdot \vec{\mathbf{E}} = \int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot (\nabla \times \vec{\mathbf{E}}) = -\int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot [\nabla \times (\nabla \phi)] = \mathbf{0}$$

$$\oint_{\tau} \mathbf{d}\vec{\mathbf{S}} \cdot \vec{\mathbf{E}} = \int_{\tau} \mathbf{d}\tau \nabla \cdot \vec{\mathbf{E}}$$



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直空中电磁相互作用的场方程: 麦克斯韦方程组的积分形式 电荷守恒:  $\int d\vec{\mathbf{S}} \cdot \vec{\mathbf{j}} = -\int d\tau \frac{\partial p}{\partial t}$  库伦定律:

E = 
$$\mathbf{k}_1 \int \mathbf{d} \tau' \frac{\rho(\vec{\mathbf{r}}')}{\mathbf{R}^3} \vec{\mathbf{R}} = -\mathbf{k}_1 \int \mathbf{d} \tau' \rho(\vec{\mathbf{r}}') \nabla \frac{\mathbf{1}}{\mathbf{R}} = -\nabla \left[ \int \mathbf{d} \tau' \frac{\rho(\vec{\mathbf{r}}')}{\mathbf{R}} \right]$$

$$\phi(\vec{\mathbf{r}}) = \mathbf{k}_1 \int \mathbf{d}\tau' \frac{\rho(\vec{\mathbf{r}}')}{\mathbf{R}} + 常数$$
  $\vec{\mathbf{E}}(\vec{\mathbf{r}}) = -\nabla\phi(\vec{\mathbf{r}})$ 

$$\oint_{\mathbf{S}} \mathbf{d}\vec{\mathbf{l}} \cdot \vec{\mathbf{E}} = \int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot (\nabla \times \vec{\mathbf{E}}) = -\int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot [\nabla \times (\nabla \phi)] = \mathbf{0}$$

$$\oint_{\tau} \mathbf{d}\vec{\mathbf{S}} \cdot \vec{\mathbf{E}} = \int_{\tau} \mathbf{d}\tau \nabla \cdot \vec{\mathbf{E}} = -\int_{\tau} \mathbf{d}\tau \nabla \cdot (\nabla \phi)$$









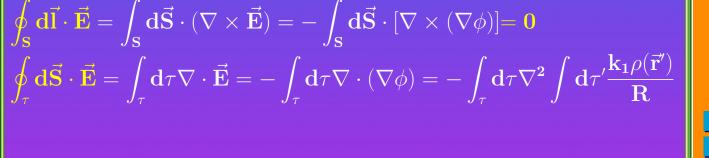
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直空中由磁准 互作目的场方程: 麦克斯韦方程组的积分形式 电荷守恒:  $\oint d\vec{\mathbf{S}} \cdot \vec{\mathbf{j}} = -\int_{\tau} d\tau \frac{\partial \rho}{\partial \mathbf{t}}$ 库伦定律:

$$\vec{\mathbf{E}} = \mathbf{k}_1 \int \mathbf{d}\tau' \frac{\rho(\vec{\mathbf{r}}')}{\mathbf{R}^3} \vec{\mathbf{R}} = -\mathbf{k}_1 \int \mathbf{d}\tau' \rho(\vec{\mathbf{r}}') \nabla \frac{1}{\mathbf{R}} = -\nabla \left[ \int \mathbf{d}\tau' \frac{\rho(\vec{\mathbf{r}}')}{\mathbf{R}} \right]$$

$$\phi(\vec{\mathbf{r}}) = \mathbf{k}_1 \int \mathbf{d}\tau' \frac{\rho(\vec{\mathbf{r}}')}{\mathbf{R}} + 常数$$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = -\nabla \phi(\vec{\mathbf{r}})$$





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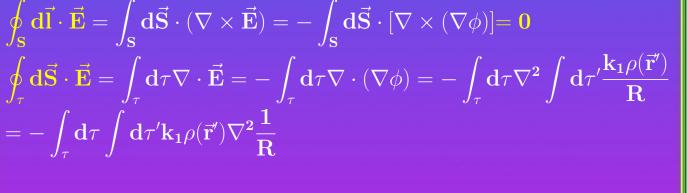


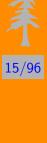


真空中电磁相互作用的场方程: 麦克斯韦方程组的积分形式 电荷守恒:  $\oint d\vec{S} \cdot \vec{j} = -\int d au \frac{\partial \rho}{\partial t}$  库伦定律:

$$ec{\mathbf{E}} = \mathbf{k}_1 \int \mathbf{d} au' \frac{
ho(ec{\mathbf{r}}')}{\mathbf{R}^3} ec{\mathbf{R}} = -\mathbf{k}_1 \int \mathbf{d} au' 
ho(ec{\mathbf{r}}') 
abla \delta \frac{1}{\mathbf{R}} = -\nabla [ \int \delta \tau' \rho(ec{\mathbf{r}}') \nabla \frac{1}{\mathbf{R}} = -\nabla [ \int \delta \tau' \frac{\rho(ec{\mathbf{r}}')}{\mathbf{R}} ]$$

$$\phi(ec{\mathbf{r}}) = \mathbf{k}_1 \int \mathbf{d} au' \frac{
ho(ec{\mathbf{r}}')}{\mathbf{R}} + \ddot{\mathbf{r}} \ddot{\mathbf{g}} \qquad \qquad \ddot{\mathbf{E}}(ec{\mathbf{r}}) = -
abla \phi(ec{\mathbf{r}}') = -
abla \phi(ec{\mathbf{r}}')$$









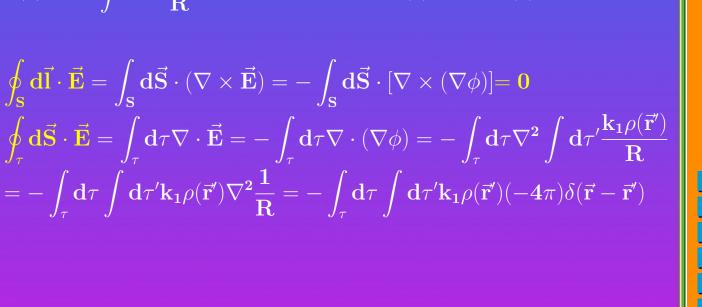




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# 直空中电磁相互作用的场方程: 麦克斯韦方程组的积分形式 电荷守恒: $\oint d\vec{S} \cdot \vec{j} = -\int d au \frac{\partial \rho}{\partial t}$ 库伦定律:

$$ec{\mathbf{E}} = \mathbf{k}_1 \int \mathbf{d} au' \frac{
ho(ec{\mathbf{r}}')}{\mathbf{R}^3} ec{\mathbf{R}} = -\mathbf{k}_1 \int \mathbf{d} au' 
ho(ec{\mathbf{r}}') 
abla \frac{1}{\mathbf{R}} = -\nabla \left[ \int \mathbf{d} au' \frac{
ho(ec{\mathbf{r}}')}{\mathbf{R}} \right]$$
 $\phi(ec{\mathbf{r}}) = \mathbf{k}_1 \int \mathbf{d} au' \frac{
ho(ec{\mathbf{r}}')}{\mathbf{R}} + 常数$ 
 $ec{\mathbf{E}}(ec{\mathbf{r}}) = -\nabla \phi(ec{\mathbf{r}})$ 















#### 直空中电磁相互作用的场方程: 麦克斯韦方程组的积分形式 电荷守恒: $\oint d\vec{\mathbf{S}} \cdot \vec{\mathbf{j}} = -\int_{\mathbf{c}} d\tau \frac{\partial \rho}{\partial \mathbf{t}}$ 库伦定律:

$$ec{\mathbf{E}} = \mathbf{k}_1 \int \mathbf{d} au' \frac{
ho(ec{\mathbf{r}'})}{\mathbf{R}^3} ec{\mathbf{R}} = -\mathbf{k}_1 \int \mathbf{d} au' 
ho(ec{\mathbf{r}'}) 
abla \vec{\mathbf{I}}{\mathbf{R}} = -\nabla \left[ \int \mathbf{d} au' \frac{
ho(ec{\mathbf{r}'})}{\mathbf{R}} 
ight]$$
 $\phi(ec{\mathbf{r}}) = \mathbf{k}_1 \int \mathbf{d} au' \frac{
ho(ec{\mathbf{r}'})}{\mathbf{R}} +$ 常数  $\vec{\mathbf{E}}(ec{\mathbf{r}}) = -\nabla \phi(ec{\mathbf{r}})$ 

$$\begin{split} &\oint_{\mathbf{S}} \mathbf{d}\vec{\mathbf{l}} \cdot \vec{\mathbf{E}} = \int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot (\nabla \times \vec{\mathbf{E}}) = -\int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot [\nabla \times (\nabla \phi)] = \mathbf{0} \\ &\oint_{\tau} \mathbf{d}\vec{\mathbf{S}} \cdot \vec{\mathbf{E}} = \int_{\tau} \mathbf{d}\tau \nabla \cdot \vec{\mathbf{E}} = -\int_{\tau} \mathbf{d}\tau \nabla \cdot (\nabla \phi) = -\int_{\tau} \mathbf{d}\tau \nabla^{2} \int \mathbf{d}\tau' \frac{\mathbf{k}_{1}\rho(\vec{\mathbf{r}}')}{\mathbf{R}} \\ &= -\int_{\tau} \mathbf{d}\tau \int \mathbf{d}\tau' \mathbf{k}_{1}\rho(\vec{\mathbf{r}}') \nabla^{2} \frac{1}{\mathbf{R}} = -\int_{\tau} \mathbf{d}\tau \int \mathbf{d}\tau' \mathbf{k}_{1}\rho(\vec{\mathbf{r}}')(-4\pi)\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \\ &= \frac{1}{\epsilon_{0}} \int_{\tau} \mathbf{d}\tau \rho(\vec{\mathbf{r}}) \end{split}$$



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直空中自磁相互作用的场方程: 麦克斯韦方程组的积分形式 电荷守恒:  $\oint d\vec{\mathbf{S}} \cdot \vec{\mathbf{j}} = -\int_{\mathbf{c}} d\tau \frac{\partial \rho}{\partial \mathbf{t}}$ 库伦定律:

$$\vec{\mathbf{E}} = \mathbf{k}_1 \int \mathbf{d}\tau' \frac{\rho(\vec{\mathbf{r}}')}{\mathbf{R}^3} \vec{\mathbf{R}} = -\mathbf{k}_1 \int \mathbf{d}\tau' \rho(\vec{\mathbf{r}}') \nabla \frac{1}{\mathbf{R}} = -\nabla \left[ \int \mathbf{d}\tau' \frac{\rho(\vec{\mathbf{r}}')}{\mathbf{R}} \right]$$

$$\phi(\vec{\mathbf{r}}) = \mathbf{k}_1 \int \mathbf{d}\tau' \frac{\rho(\vec{\mathbf{r}}')}{\mathbf{R}} + 常数 \qquad \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -\nabla \phi(\vec{\mathbf{r}})$$

 $=\frac{1}{\epsilon_0}\int_{\tau} d\tau \rho(\vec{\mathbf{r}}) = \frac{1}{\epsilon_0} \mathbf{Q}_{\mathbf{p}}$ 

$$\oint_{\mathbf{S}} \mathbf{d}\vec{\mathbf{I}} \cdot \vec{\mathbf{E}} = \int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot (\nabla \times \vec{\mathbf{E}}) = -\int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot [\nabla \times (\nabla \phi)] = \mathbf{0}$$

$$\oint_{\mathbf{T}} \mathbf{d}\vec{\mathbf{S}} \cdot \vec{\mathbf{E}} = \int_{\tau} \mathbf{d}\tau \nabla \cdot \vec{\mathbf{E}} = -\int_{\tau} \mathbf{d}\tau \nabla \cdot (\nabla \phi) = -\int_{\tau} \mathbf{d}\tau \nabla^{2} \int \mathbf{d}\tau' \frac{\mathbf{k}_{1}\rho(\vec{\mathbf{r}}')}{\mathbf{R}}$$

$$= -\int_{\tau} \mathbf{d}\tau \int \mathbf{d}\tau' \mathbf{k}_{1}\rho(\vec{\mathbf{r}}') \nabla^{2} \frac{1}{\mathbf{R}} = -\int_{\tau} \mathbf{d}\tau \int \mathbf{d}\tau' \mathbf{k}_{1}\rho(\vec{\mathbf{r}}')(-4\pi)\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}')$$













### 麦克斯韦方程组的积分形式

比萨定律:  $\vec{B} = \int \frac{k_2 J(\vec{r}') d\vec{l}' \times \vec{R}}{R^3}$ 









### 真空中电磁相互作用的场方程:麦克斯韦方程组的积分形式

比萨定律: 
$$\vec{B} = \int \frac{k_2 J(\vec{r}') d\vec{l}' \times \vec{R}}{R^3} = -\int k_2 J(\vec{r}') d\vec{l}' \times \nabla \frac{1}{R}$$









### 麦克斯韦方程组的积分形式

比萨定律: 
$$\vec{B} = \int \frac{k_2 J(\vec{r}') d\vec{l}' \times \vec{R}}{R^3} = -\int k_2 J(\vec{r}') d\vec{l}' \times \nabla \frac{1}{R}$$
$$= \nabla \times \int \frac{k_2 J(\vec{r}') d\vec{l}'}{R}$$









### <u>真空中电磁相互作用的场方程</u>: 麦克斯韦方程组的积分形式

比萨定律: 
$$\vec{B} = \int \frac{k_2 J(\vec{r}') d\vec{l}' \times \vec{R}}{R^3} = -\int k_2 J(\vec{r}') d\vec{l}' \times \nabla \frac{1}{R}$$
$$= \nabla \times \int \frac{k_2 J(\vec{r}') d\vec{l}'}{R}$$

$$ec{\mathbf{A}}(ec{\mathbf{r}}) = \mathbf{k_2} \int \mathbf{d}ec{\mathbf{l}} rac{\mathbf{J}(ec{\mathbf{r}}')}{\mathbf{R}}$$



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#### <u>真空中电磁相互作用的场方程</u>: 麦克斯韦方程组的积分形式

比萨定律: 
$$\vec{B} = \int \frac{k_2 J(\vec{r}') d\vec{l}' \times \vec{R}}{R^3} = -\int k_2 J(\vec{r}') d\vec{l}' \times \nabla \frac{1}{R}$$

$$= \nabla \times \int \frac{k_2 J(\vec{r}') d\vec{l}'}{R}$$

$$\vec{\mathbf{A}}(\vec{\mathbf{r}}) = \mathbf{k_2} \int d\vec{\mathbf{l}} \frac{\mathbf{J}(\vec{\mathbf{r}})}{\mathbf{R}} + \nabla \chi$$



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#### <u>真空中电磁相互作用的场方程</u>: 麦克斯韦方程组的积分形式

比萨定律: 
$$\vec{B} = \int \frac{k_2 J(\vec{r}') d\vec{l}' \times \vec{R}}{R^3} = -\int k_2 J(\vec{r}') d\vec{l}' \times \nabla \frac{1}{R}$$
$$= \nabla \times \int \frac{k_2 J(\vec{r}') d\vec{l}'}{R}$$

$$ec{f A}(ec{f r}) = {f k_2} \int dec{f l} rac{{f J}(ec{f r}')}{{f R}} + 
abla \chi = \int d au' rac{{f j}(ec{f r}')}{{f R}} + 
abla \chi$$
 where  ${f k_2}$ 



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#### 真空中电磁相互作用的场方程: 麦克斯韦方程组的积分形式

比萨定律: 
$$\vec{B} = \int \frac{k_2 J(\vec{r}') d\vec{l}' \times \vec{R}}{R^3} = -\int k_2 J(\vec{r}') d\vec{l}' \times \nabla \frac{1}{R}$$
$$= \nabla \times \int \frac{k_2 J(\vec{r}') d\vec{l}'}{R}$$

$$ec{\mathbf{A}}(ec{\mathbf{r}}) = \mathbf{k_2} \int \mathbf{d}ec{\mathbf{l}} rac{\mathbf{J}(ec{\mathbf{r}}')}{\mathbf{R}} + 
abla \chi = \int \mathbf{d} au' rac{\dot{\mathbf{j}}(ec{\mathbf{r}}')}{\mathbf{R}} + 
abla \chi$$
 where  $ec{\mathbf{j}}$ 

$$\vec{\mathbf{B}}(\vec{\mathbf{r}}) = 
abla imes \vec{\mathbf{A}}(\vec{\mathbf{r}})$$



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#### 真空中电磁相互作用的场方程: 麦克斯韦方程组的积分形式

比萨定律: 
$$\vec{\mathbf{B}} = \int \frac{\mathbf{k_2J}(\vec{\mathbf{r}}')d\vec{\mathbf{l}}' \times \vec{\mathbf{R}}}{\mathbf{R}^3} = -\int \mathbf{k_2J}(\vec{\mathbf{r}}')d\vec{\mathbf{l}}' \times \nabla \frac{1}{\mathbf{R}}$$
$$= \nabla \times \int \frac{\mathbf{k_2J}(\vec{\mathbf{r}}')d\vec{\mathbf{l}}'}{\mathbf{R}}$$

$$ec{f A}(ec{f r}) = {f k_2} \int dec{f l} rac{{f J}(ec{f r}')}{{f R}} + 
abla \chi = \int d au' rac{{f j}(ec{f r}')}{{f R}} + 
abla \chi$$
 where  ${f B}(ec{f r}) = 
abla imes {f A}(ec{f r})$ 

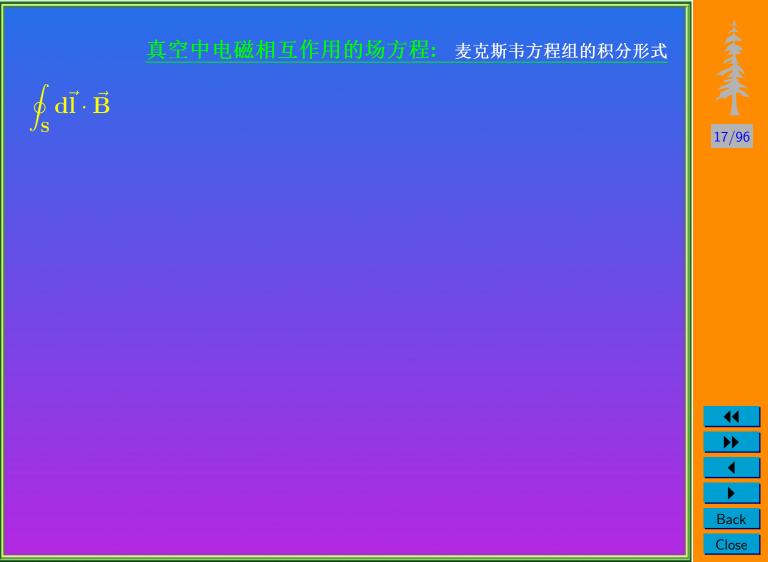
$$\oint_{\tau} \mathbf{d\vec{S}} \cdot \vec{\mathbf{B}} = \int_{\tau} \mathbf{d}\tau \nabla \cdot \vec{\mathbf{B}} = \int_{\tau} \mathbf{d}\tau \nabla \cdot (\nabla \times \vec{\mathbf{A}}) = \mathbf{0}$$



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# 17/96 真空中电磁相互作用的场方程:麦克斯韦方程组的积分形式 $\left| \oint_{\mathbf{S}} \mathbf{d} \vec{\mathbf{l}} \cdot \vec{\mathbf{B}} \right| = \left| \int_{\mathbf{S}} \mathbf{d} \vec{\mathbf{S}} \cdot (\nabla \times \vec{\mathbf{B}}) \right|$









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真空中电磁相互作用的场方程:麦克斯韦方程组的积分形式  $\overline{\int_{\mathbf{S}} \mathbf{d} ec{\mathbf{l}} \cdot ec{\mathbf{B}}} \ = \ \overline{\int_{\mathbf{S}} \mathbf{d} ec{\mathbf{S}} \cdot (
abla imes ec{\mathbf{B}})} = - \overline{\int_{\mathbf{S}} \mathbf{d} ec{\mathbf{S}} \cdot 
abla} abla imes [ \int \mathbf{d} ec{\mathbf{l}}' imes \mathbf{k_2} \mathbf{J} (ec{\mathbf{r}}') 
abla \overline{\mathbf{R}} ]$ 



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### 真空中电磁相互作用的场方程。麦克斯韦方程组的积分形式 $\oint_{\mathbf{S}} \mathbf{d} \vec{\mathbf{l}} \cdot \vec{\mathbf{B}} = \int_{\mathbf{S}} \mathbf{d} \vec{\mathbf{S}} \cdot (\nabla \times \vec{\mathbf{B}}) = -\int_{\mathbf{S}} \mathbf{d} \vec{\mathbf{S}} \cdot \nabla \times [\int \mathbf{d} \vec{\mathbf{l}}' \times \mathbf{k_2} \mathbf{J}(\vec{\mathbf{r}}') \nabla \frac{\mathbf{1}}{\mathbf{B}}]$ $oxed{\mathbf{S}} = -\int_{\mathbf{S}} \mathbf{d} ec{\mathbf{S}} \cdot \int_{\mathbf{S}} \mathbf{d} au' \mathbf{k_2} abla imes [ec{\mathbf{j}}(ec{\mathbf{r}}') imes abla rac{1}{\mathbf{B}}]$



# 直空中 电磁准 互作用 的场方程: 麦克斯韦方程组的积分形式 $\oint_{\mathbf{S}} \mathbf{d} \vec{\mathbf{I}} \cdot \vec{\mathbf{B}} = \int_{\mathbf{S}} \mathbf{d} \vec{\mathbf{S}} \cdot (\nabla \times \vec{\mathbf{B}}) = -\int_{\mathbf{S}} \mathbf{d} \vec{\mathbf{S}} \cdot \nabla \times [\int \mathbf{d} \vec{\mathbf{l}}' \times \mathbf{k_2} \mathbf{J}(\vec{\mathbf{r}}') \nabla \frac{\mathbf{I}}{\mathbf{B}}]$ $\overline{\mathbf{J}} = -\int_{\mathbf{S}} \mathbf{d} \vec{\mathbf{S}} \cdot \int_{\mathbf{S}} \mathbf{d} au' \mathbf{k_2} abla imes [\vec{\mathbf{j}}(\vec{\mathbf{r}}') imes abla rac{1}{\mathrm{R}}]$ $\mathbf{k_2} = \mathbf{k_2} \int_{\mathbf{S}} \mathbf{d} \vec{\mathbf{S}} \cdot \int_{\mathbf{S}} \mathbf{d} au' [\vec{\mathbf{j}}(\vec{\mathbf{r}}') \cdot abla abla - \vec{\mathbf{j}}(\vec{\mathbf{r}}') abla^2] rac{1}{\mathbf{R}} \mathbf{c}$







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<u>真空中电磁相互作用的场方程</u>: 麦克斯韦方程组的积分形式  $\oint d\vec{\mathbf{l}} \cdot \vec{\mathbf{B}} = \int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot (\nabla \times \vec{\mathbf{B}}) = - \int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot \nabla \times [\int d\vec{\mathbf{l}}' \times \mathbf{k_2} \mathbf{J}(\vec{\mathbf{r}}') \nabla \frac{\mathbf{I}}{\mathbf{B}}]$  ${f r} = -\int_{f S} {f d} ec{f S} \cdot \int_{f G} {f d} au' {f k_2} 
abla imes [ec{f j}(ec{f r}') imes 
abla rac{1}{{f R}}]$  $\mathbf{k}_{\mathbf{2}} = \mathbf{k}_{\mathbf{2}} \int_{\mathbf{S}} \mathbf{d} ec{\mathbf{S}} \cdot \int_{\mathbf{S}} \mathbf{d} au' [ec{\mathbf{j}} (ec{\mathbf{r}}') \cdot 
abla 
abla - ec{\mathbf{j}} (ec{\mathbf{r}}') 
abla^2] rac{1}{\mathbf{R}} \mathbf{k}_{\mathbf{2}}$  $\int_{\mathbf{R}} \mathbf{d} au' \vec{\mathbf{j}}(\vec{\mathbf{r}}') \cdot 
abla 
abla rac{1}{\mathbf{R}}$ 



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真空中电磁相互作用的场方程: 麦克斯韦方程组的积分形式  $\oint_{\mathbf{S}} \mathbf{d\vec{l}} \cdot \vec{\mathbf{B}} = \int_{\mathbf{S}} \mathbf{d\vec{S}} \cdot (\nabla \times \vec{\mathbf{B}}) = -\int_{\mathbf{S}} \mathbf{d\vec{S}} \cdot \nabla \times [\int \mathbf{d\vec{l}'} \times \mathbf{k_2} \mathbf{J}(\vec{\mathbf{r}'}) \nabla \frac{1}{\mathbf{R}}]$   $= -\int_{\mathbf{S}} \mathbf{d\vec{S}} \cdot \int_{\mathbf{C}} \mathbf{d\tau'} \mathbf{k_2} \nabla \times [\vec{\mathbf{j}}(\vec{\mathbf{r}'}) \times \nabla \frac{1}{\mathbf{R}}]$ 

$$\mathbf{k_2} \int_{\mathbf{S}} \mathbf{d} ec{\mathbf{S}} \cdot \int_{\infty} \mathbf{d} au' [ec{\mathbf{j}} (ec{\mathbf{r}}') \cdot 
abla 
abla - ec{\mathbf{j}} (ec{\mathbf{r}}') 
abla^2] rac{1}{\mathbf{R}} = \mathbf{k_2} \int_{\mathbf{S}} \mathbf{d} ec{\mathbf{r}}' [ec{\mathbf{j}} (ec{\mathbf{r}}') \cdot 
abla \nabla \nabla \cdot ec{\mathbf{j}} (ec{\mathbf{r}}') 
abla^2$$

$$\int_{\infty} \mathbf{d} au' ec{\mathbf{j}}(ec{\mathbf{r}}') \cdot 
abla 
abla rac{1}{R} = -\int_{\infty} \mathbf{d} au' ec{\mathbf{j}}(ec{\mathbf{r}}') \cdot 
abla' 
abla rac{1}{R}$$









真空中电磁相互作用的场方程。 麦克斯韦方程组的积分形式  $\oint_{\mathbf{S}} \mathbf{d}\vec{\mathbf{l}} \cdot \vec{\mathbf{B}} = \int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot (\nabla \times \vec{\mathbf{B}}) = -\int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot \nabla \times [\int \mathbf{d}\vec{\mathbf{l}}' \times \mathbf{k_2} \mathbf{J}(\vec{\mathbf{r}}') \nabla \frac{1}{\mathbf{R}}]$ 

$$egin{aligned} J_{\mathbf{S}} & J_{\mathbf{$$

$$\begin{split} \int_{\infty} \mathbf{d}\tau' \vec{\mathbf{j}}(\vec{\mathbf{r}}') \cdot \nabla \nabla \frac{1}{R} &= -\int_{\infty} \mathbf{d}\tau' \vec{\mathbf{j}}(\vec{\mathbf{r}}') \cdot \nabla' \nabla \frac{1}{R} \\ &= \int_{\infty} \mathbf{d}\tau' \big[ \nabla' \cdot [-\vec{\mathbf{j}}(\vec{\mathbf{r}}') \nabla \frac{1}{R}] + [\nabla' \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}}')] \nabla \frac{1}{R} \big] \end{split}$$









<u>真空中电磁相互作用的场方程</u>: 麦克斯韦方程组的积分形式  $-\int d\vec{S} (\nabla \times \vec{P}) - \int d\vec{S} \nabla \times [\int d\vec{l}' \times l_r I(\vec{s}') \nabla^{-1}]$ 

$$\begin{split} \oint_{\mathbf{S}} \mathbf{d}\vec{\mathbf{l}} \cdot \vec{\mathbf{B}} &= \int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot (\nabla \times \vec{\mathbf{B}}) = -\int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot \nabla \times [\int \mathbf{d}\vec{\mathbf{l}}' \times \mathbf{k}_2 \mathbf{J}(\vec{\mathbf{r}}') \nabla \frac{1}{\mathbf{R}}] \\ &= -\int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot \int_{\infty} \mathbf{d}\tau' \mathbf{k}_2 \nabla \times [\vec{\mathbf{j}}(\vec{\mathbf{r}}') \times \nabla \frac{1}{\mathbf{R}}] \\ &= \mathbf{k}_2 \int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot \int_{\infty} \mathbf{d}\tau' [\vec{\mathbf{j}}(\vec{\mathbf{r}}') \cdot \nabla \nabla - \vec{\mathbf{j}}(\vec{\mathbf{r}}') \nabla^2] \frac{1}{\mathbf{R}} \end{split}$$

$$\begin{split} \int_{\infty} d\tau' \vec{\mathbf{j}}(\vec{\mathbf{r}}') \cdot \nabla \nabla \frac{1}{R} &= -\int_{\infty} d\tau' \vec{\mathbf{j}}(\vec{\mathbf{r}}') \cdot \nabla' \nabla \frac{1}{R} \\ &= \int_{\infty} d\tau' \big[ \nabla' \cdot [-\vec{\mathbf{j}}(\vec{\mathbf{r}}') \nabla \frac{1}{R}] + [\nabla' \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}}')] \nabla \frac{1}{R} \big] \\ &= -\int_{\infty} d\vec{\mathbf{S}}' \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}}') \nabla \frac{1}{R} + \int_{\infty} d\tau' [\nabla' \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}}')] \nabla \frac{1}{R} \end{split}$$









# 真空中<u>电磁相互作用的场方程</u>。麦克斯韦方程组的积分形式

$$\begin{split} \oint_{\mathbf{S}} d\vec{\mathbf{I}} \cdot \vec{\mathbf{B}} &= \int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot (\nabla \times \vec{\mathbf{B}}) = -\int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot \nabla \times [\int d\vec{\mathbf{I}}' \times \mathbf{k}_2 \mathbf{J}(\vec{\mathbf{r}}') \nabla \frac{1}{R}] \\ &= -\int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot \int_{\infty} d\tau' \mathbf{k}_2 \nabla \times [\vec{\mathbf{j}}(\vec{\mathbf{r}}') \times \nabla \frac{1}{R}] \\ &= \mathbf{k}_2 \int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot \int_{\infty} d\tau' [\vec{\mathbf{j}}(\vec{\mathbf{r}}') \cdot \nabla \nabla - \vec{\mathbf{j}}(\vec{\mathbf{r}}') \nabla^2] \frac{1}{R} \end{split}$$

$$\int_{\infty} d\tau' \vec{\mathbf{j}}(\vec{\mathbf{r}}') \cdot \nabla \nabla \frac{1}{R} = -\int_{\infty} d\tau' \vec{\mathbf{j}}(\vec{\mathbf{r}}') \cdot \nabla' \nabla \frac{1}{R}$$

$$= \int_{\infty} d\tau' \left[ \nabla' \cdot \left[ -\vec{\mathbf{j}}(\vec{\mathbf{r}}') \nabla \frac{1}{R} \right] + \left[ \nabla' \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}}') \right] \nabla \frac{1}{R} \right]$$

$$= -\int_{\infty} d\vec{\mathbf{S}}' \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}}') \nabla \frac{1}{R} + \int_{\infty} d\tau' \left[ \nabla' \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}}') \right] \nabla \frac{1}{R}$$

$$= 0$$









<u>真空中电磁相互作用的场方程</u>: 麦克斯韦方程组的积分形式  $-\int d\vec{S} \cdot (\nabla \times \vec{B}) = -\int d\vec{S} \cdot \nabla \times [\int d\vec{l}' \times k_s \mathbf{I}(\vec{r}') \nabla - \mathbf{I}]$ 

$$\begin{split} \oint_{\mathbf{S}} \mathbf{d}\vec{\mathbf{l}} \cdot \vec{\mathbf{B}} &= \int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot (\nabla \times \vec{\mathbf{B}}) = -\int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot \nabla \times [\int \mathbf{d}\vec{\mathbf{l}}' \times \mathbf{k}_2 \mathbf{J}(\vec{\mathbf{r}}') \nabla \frac{1}{\mathbf{R}}] \\ &= -\int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot \int_{\infty} \mathbf{d}\tau' \mathbf{k}_2 \nabla \times [\vec{\mathbf{j}}(\vec{\mathbf{r}}') \times \nabla \frac{1}{\mathbf{R}}] \\ &= \mathbf{k}_2 \int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot \int_{\infty} \mathbf{d}\tau' [\vec{\mathbf{j}}(\vec{\mathbf{r}}') \cdot \nabla \nabla - \vec{\mathbf{j}}(\vec{\mathbf{r}}') \nabla^2] \frac{1}{\mathbf{R}} \end{split}$$

$$\int_{\infty} d\tau' \vec{\mathbf{j}}(\vec{\mathbf{r}}') \cdot \nabla \nabla \frac{1}{R} = -\int_{\infty} d\tau' \vec{\mathbf{j}}(\vec{\mathbf{r}}') \cdot \nabla' \nabla \frac{1}{R} 
= \int_{\infty} d\tau' \left[ \nabla' \cdot \left[ -\vec{\mathbf{j}}(\vec{\mathbf{r}}') \nabla \frac{1}{R} \right] + \left[ \nabla' \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}}') \right] \nabla \frac{1}{R} \right] 
= -\int_{\infty} d\vec{\mathbf{S}}' \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}}') \nabla \frac{1}{R} + \int_{\infty} d\tau' \left[ \nabla' \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}}') \right] \nabla \frac{1}{R} 
= 0 
0 = \oint d\vec{\mathbf{S}} \cdot \vec{\mathbf{j}} = \int d\tau \nabla \cdot \vec{\mathbf{j}}$$









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<u>真空中电磁相互作用的多方程</u>: 麦克斯韦方程组的积分形式  $= \int d\vec{S} \cdot (\nabla \times \vec{B}) = - \int d\vec{S} \cdot \nabla \times [\int d\vec{l}' \times k_s J(\vec{r}') \nabla \frac{1}{z}]$ 

$$\begin{split} \oint_{\mathbf{S}} \mathbf{d}\vec{\mathbf{l}} \cdot \vec{\mathbf{B}} &= \int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot (\nabla \times \vec{\mathbf{B}}) = -\int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot \nabla \times [\int \mathbf{d}\vec{\mathbf{l}}' \times \mathbf{k}_2 \mathbf{J}(\vec{\mathbf{r}}') \nabla \frac{\mathbf{l}}{\mathbf{R}}] \\ &= -\int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot \int_{\infty} \mathbf{d}\tau' \mathbf{k}_2 \nabla \times [\vec{\mathbf{j}}(\vec{\mathbf{r}}') \times \nabla \frac{\mathbf{l}}{\mathbf{R}}] \\ &= \mathbf{k}_2 \int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot \int_{\infty} \mathbf{d}\tau' [\vec{\mathbf{j}}(\vec{\mathbf{r}}') \cdot \nabla \nabla - \vec{\mathbf{j}}(\vec{\mathbf{r}}') \nabla^2] \frac{\mathbf{l}}{\mathbf{R}} \end{split}$$

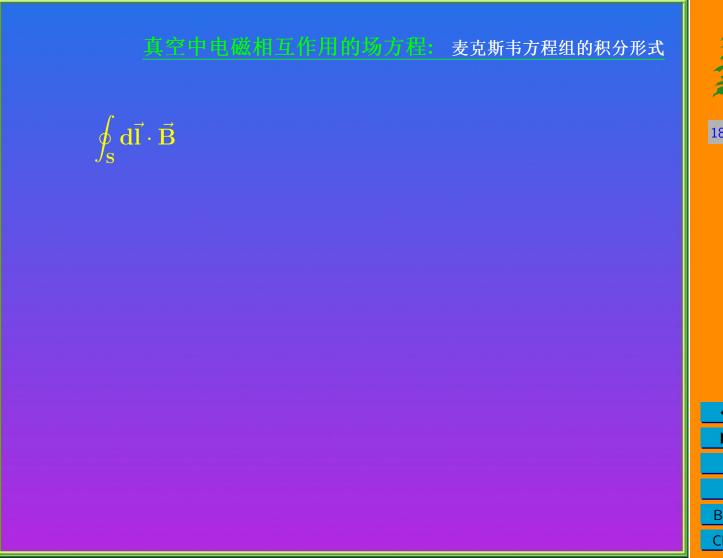
$$\begin{split} \int_{\infty} d\tau' \vec{\mathbf{j}}(\vec{\mathbf{r}}') \cdot \nabla \nabla \frac{1}{R} &= -\int_{\infty} d\tau' \vec{\mathbf{j}}(\vec{\mathbf{r}}') \cdot \nabla' \nabla \frac{1}{R} \\ &= \int_{\infty} d\tau' \big[ \nabla' \cdot [-\vec{\mathbf{j}}(\vec{\mathbf{r}}') \nabla \frac{1}{R}] + [\nabla' \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}}')] \nabla \frac{1}{R} \big] \\ &= -\int_{\infty} d\vec{\mathbf{S}}' \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}}') \nabla \frac{1}{R} + \int_{\infty} d\tau' [\nabla' \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}}')] \nabla \frac{1}{R} \\ &= 0 \\ 0 &= \oint d\vec{\mathbf{S}} \cdot \vec{\mathbf{j}} = \int d\tau \nabla \cdot \vec{\mathbf{j}} \rightarrow \qquad \nabla \cdot \vec{\mathbf{j}} = \mathbf{0} \end{split}$$







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## 真空中电磁相互作用的场方程: 麦克斯韦方程组的积分形式

$$\oint_{\mathbf{S}} \mathbf{d}\vec{\mathbf{l}} \cdot \vec{\mathbf{B}} = \mathbf{k_2} \int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot \int \mathbf{d}\tau' [\vec{\mathbf{j}}(\vec{\mathbf{r}}') \cdot \nabla \nabla - \vec{\mathbf{j}}(\vec{\mathbf{r}}') \nabla^2] \frac{1}{\mathbf{R}}$$









#### 真空中电磁相互作用的场方程:麦克斯韦方程组的积分形式

$$\begin{split} \oint_{\mathbf{S}} \mathbf{d} \vec{\mathbf{l}} \cdot \vec{\mathbf{B}} &= k_2 \int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot \int d\tau' [\vec{\mathbf{j}}(\vec{\mathbf{r}}') \cdot \nabla \nabla - \vec{\mathbf{j}}(\vec{\mathbf{r}}') \nabla^2] \frac{1}{R} \\ &= -k_2 \int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot \int d\tau' \vec{\mathbf{j}}(\vec{\mathbf{r}}') \nabla^2 \frac{1}{R} \end{split}$$









#### 真空中电磁相互作用的场方程: 麦克斯韦方程组的积分形式

$$\begin{split} \oint_{\mathbf{S}} \mathbf{d}\vec{\mathbf{l}} \cdot \vec{\mathbf{B}} &= k_2 \int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot \int d\tau' [\vec{\mathbf{j}}(\vec{\mathbf{r}}') \cdot \nabla \nabla - \vec{\mathbf{j}}(\vec{\mathbf{r}}') \nabla^2] \frac{1}{R} \\ &= -k_2 \int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot \int d\tau' \vec{\mathbf{j}}(\vec{\mathbf{r}}') \nabla^2 \frac{1}{R} \\ &= 4\pi k_2 \int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot \int d\tau' \vec{\mathbf{j}}(\vec{\mathbf{r}}') \delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \end{split}$$









#### 真空中电磁相互作用的场方程。麦克斯韦方程组的积分形式

$$\begin{split} \oint_{\mathbf{S}} \mathbf{d}\vec{\mathbf{l}} \cdot \vec{\mathbf{B}} &= k_2 \int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot \int \mathbf{d}\tau' [\vec{\mathbf{j}}(\vec{\mathbf{r}}') \cdot \nabla \nabla - \vec{\mathbf{j}}(\vec{\mathbf{r}}') \nabla^2] \frac{1}{R} \\ &= -k_2 \int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot \int \mathbf{d}\tau' \vec{\mathbf{j}}(\vec{\mathbf{r}}') \nabla^2 \frac{1}{R} \\ &= 4\pi k_2 \int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot \int \mathbf{d}\tau' \vec{\mathbf{j}}(\vec{\mathbf{r}}') \delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \\ &= 4\pi k_2 \int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}}) \end{split}$$



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#### 真空中电磁相互作用的场方程:麦克斯韦方程组的积分形式

$$\begin{split} \oint_{\mathbf{S}} \mathbf{d}\vec{\mathbf{l}} \cdot \vec{\mathbf{B}} &= k_2 \int_{S} \mathbf{d}\vec{S} \cdot \int \mathbf{d}\tau' [\vec{\mathbf{j}}(\vec{\mathbf{r}}') \cdot \nabla \nabla - \vec{\mathbf{j}}(\vec{\mathbf{r}}') \nabla^2] \frac{1}{R} \\ &= -k_2 \int_{S} \mathbf{d}\vec{S} \cdot \int \mathbf{d}\tau' \vec{\mathbf{j}}(\vec{\mathbf{r}}') \nabla^2 \frac{1}{R} \\ &= 4\pi k_2 \int_{S} \mathbf{d}\vec{S} \cdot \int \mathbf{d}\tau' \vec{\mathbf{j}}(\vec{\mathbf{r}}') \delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \\ &= 4\pi k_2 \int_{S} \mathbf{d}\vec{S} \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}}) = \mu_0 \mathbf{J} \end{split}$$



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#### 真空中电磁相互作用的场方程: 麦克斯韦方程组的积分形式

$$\begin{split} \oint_{\mathbf{S}} \mathbf{d}\vec{\mathbf{l}} \cdot \vec{\mathbf{B}} &= k_2 \int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot \int \mathbf{d}\tau' [\vec{\mathbf{j}}(\vec{\mathbf{r}}') \cdot \nabla \nabla - \vec{\mathbf{j}}(\vec{\mathbf{r}}') \nabla^2] \frac{1}{R} \\ &= -k_2 \int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot \int \mathbf{d}\tau' \vec{\mathbf{j}}(\vec{\mathbf{r}}') \nabla^2 \frac{1}{R} \\ &= 4\pi k_2 \int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot \int \mathbf{d}\tau' \vec{\mathbf{j}}(\vec{\mathbf{r}}') \delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \\ &= 4\pi k_2 \int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}}) = \mu_0 \mathbf{J} \end{split}$$

法拉第电磁感应定律:  $\oint_{\mathbf{S}} \mathbf{d\vec{l}} \cdot \vec{\mathbf{E}} = - \int_{\mathbf{S}} \mathbf{d\vec{S}} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}}$ 







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#### 真空中电磁相互作用的场方程: 麦克斯韦方程组的积分形式

$$\begin{split} \oint_{\mathbf{S}} \mathbf{d}\vec{\mathbf{l}} \cdot \vec{\mathbf{B}} &= k_2 \int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot \int \mathbf{d}\tau' [\vec{\mathbf{j}}(\vec{\mathbf{r}}') \cdot \nabla \nabla - \vec{\mathbf{j}}(\vec{\mathbf{r}}') \nabla^2] \frac{1}{R} \\ &= -k_2 \int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot \int \mathbf{d}\tau' \vec{\mathbf{j}}(\vec{\mathbf{r}}') \nabla^2 \frac{1}{R} \\ &= 4\pi k_2 \int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot \int \mathbf{d}\tau' \vec{\mathbf{j}}(\vec{\mathbf{r}}') \delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \\ &= 4\pi k_2 \int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}}) = \mu_0 \mathbf{J} \end{split}$$

库伦定律  $\oint_{\mathbf{S}} \mathbf{d} \vec{\mathbf{l}} \cdot \vec{\mathbf{E}} = \mathbf{0}$ 的推广!



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# 19/96 麦克斯韦方程组的微分形式 $\oint_{ au} \mathbf{d} ec{\mathbf{S}} \cdot ec{\mathbf{j}} = - \int_{ au} \mathbf{d} au rac{\partial ho}{\partial \mathbf{t}}$

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# 真空中电磁相互作用的场方程:麦克斯韦方程组的微分形式 $\overline{\int_{ au} \mathbf{d} ec{\mathbf{S}} \cdot ec{\mathbf{j}}} = - \int_{ au} \mathbf{d} au rac{\partial ho}{\partial \mathbf{t}} \; ightarrow \; \int_{ au} \mathbf{d} au ( abla \cdot ec{\mathbf{j}} + rac{\partial ho}{\partial \mathbf{t}}) = \mathbf{0}$



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## 真空中 电磁相 互作用 的场方程: 麦克斯韦方程组的微分形式 $\oint_{ au} d\vec{\mathbf{S}} \cdot \vec{\mathbf{j}} = -\int_{ au} d au rac{\partial ho}{\partial \mathbf{t}} \ ightarrow \ \int_{ au} d au ( abla \cdot \vec{\mathbf{j}} + rac{\partial ho}{\partial \mathbf{t}}) = \mathbf{0} \ ightarrow \ rac{\partial ho}{\partial \mathbf{t}} + abla \cdot \vec{\mathbf{j}} = \mathbf{0}$











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## 真空中电磁相互作用的场方程:麦克斯韦方程组的微分形式 $\int_{ au} \mathbf{d} \vec{\mathbf{S}} \cdot \vec{\mathbf{j}} = -\int_{ au} \mathbf{d} au rac{\partial ho}{\partial \mathbf{t}} \; o \; \int_{ au} \mathbf{d} au ( abla \cdot \vec{\mathbf{j}} + rac{\partial ho}{\partial \mathbf{t}}) = \mathbf{0} \; o \; rac{\partial ho}{\partial \mathbf{t}} + abla \cdot \vec{\mathbf{j}} = \mathbf{0}$

 $\oint_{ au} \mathbf{d} ec{\mathbf{S}} \cdot ec{\mathbf{E}} = rac{1}{\epsilon_{\mathbf{0}}} \int_{ au} \mathbf{d} au 
ho$ 













## 真空中电磁相互作用的场方程: 麦克斯韦方程组的微分形式 $\int_{ au} d ec{\mathbf{S}} \cdot ec{\mathbf{j}} = - \int_{ au} d au rac{\partial ho}{\partial \mathbf{t}} \; o \; \int_{ au} d au ( abla \cdot ec{\mathbf{j}} + rac{\partial ho}{\partial \mathbf{t}}) = \mathbf{0} \; o \; rac{\partial ho}{\partial \mathbf{t}} + abla \cdot ec{\mathbf{j}} = \mathbf{0}$

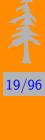
 $\oint_{ au} d\vec{\mathbf{S}} \cdot \vec{\mathbf{E}} = rac{1}{\epsilon_0} \int_{ au} d au 
ho \; 
ightarrow \; \int_{ au} d au (
abla \cdot \vec{\mathbf{E}} - rac{1}{\epsilon_0} 
ho) = \mathbf{0}$ 



# 真空中电磁相互作用的场方程: 麦克斯韦方程组的微分形式

$$\oint_{\tau} d\vec{S} \cdot \vec{j} = -\int_{\tau} d\tau \frac{\partial \rho}{\partial t} \rightarrow \int_{\tau} d\tau (\nabla \cdot \vec{j} + \frac{\partial \rho}{\partial t}) = 0 \rightarrow \frac{\partial \rho}{\partial t} + \nabla \cdot \vec{j} = 0$$

$$\oint_{\tau} d\vec{S} \cdot \vec{E} = \frac{1}{\epsilon_0} \int_{\tau} d\tau \rho \rightarrow \int_{\tau} d\tau (\nabla \cdot \vec{E} - \frac{1}{\epsilon_0} \rho) = 0 \rightarrow \nabla \cdot \vec{E} = \frac{1}{\epsilon_0} \rho$$









# 真空中电磁相互作用的场方程: 麦克斯韦方程组的微分形式 $-\int \mathbf{d}\tau \frac{\partial \rho}{\partial \tau} \rightarrow \int \mathbf{d}\tau (\nabla \cdot \vec{\mathbf{i}} + \frac{\partial \rho}{\partial \tau}) = \mathbf{0} \rightarrow \frac{\partial \rho}{\partial \tau} + \nabla \cdot \vec{\mathbf{i}} = \mathbf{0}$

$$\begin{split} \oint_{\tau} d\vec{S} \cdot \vec{j} &= -\int_{\tau} d\tau \frac{\partial \rho}{\partial t} \ \rightarrow \ \int_{\tau} d\tau (\nabla \cdot \vec{j} + \frac{\partial \rho}{\partial t}) = 0 \ \rightarrow \ \frac{\partial \rho}{\partial t} + \nabla \cdot \vec{j} = 0 \\ \oint_{\tau} d\vec{S} \cdot \vec{E} &= \frac{1}{\epsilon_0} \int_{\tau} d\tau \rho \ \rightarrow \ \int_{\tau} d\tau (\nabla \cdot \vec{E} - \frac{1}{\epsilon_0} \rho) = 0 \ \rightarrow \ \nabla \cdot \vec{E} = \frac{1}{\epsilon_0} \rho \\ \oint_{S} d\vec{l} \cdot \vec{B} &= \mu_0 \int_{S} d\vec{S} \cdot \vec{j} \end{split}$$









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# <u>有空中电磁和工作片的场方是</u> 麦克斯韦方程组的微分形式

$$\oint_{\tau} d\vec{S} \cdot \vec{j} = -\int_{\tau} d\tau \frac{\partial \rho}{\partial t} \rightarrow \int_{\tau} d\tau (\nabla \cdot \vec{j} + \frac{\partial \rho}{\partial t}) = 0 \rightarrow \frac{\partial \rho}{\partial t} + \nabla \cdot \vec{j} = 0$$

$$\oint_{\tau} d\vec{S} \cdot \vec{E} = \frac{1}{\epsilon_0} \int_{\tau} d\tau \rho \rightarrow \int_{\tau} d\tau (\nabla \cdot \vec{E} - \frac{1}{\epsilon_0} \rho) = 0 \rightarrow \nabla \cdot \vec{E} = \frac{1}{\epsilon_0} \rho$$

$$\oint_{S} d\vec{l} \cdot \vec{B} = \mu_0 \int_{S} d\vec{S} \cdot \vec{j} \rightarrow \int_{S} d\vec{S} \cdot (\nabla \times \vec{B} - \mu_0 \vec{j}) = 0$$









# <u>直卒中由版相下作用的场方程</u>。麦克斯韦方程组的微分形式

$$\oint_{\tau} d\vec{S} \cdot \vec{j} = -\int_{\tau} d\tau \frac{\partial \rho}{\partial t} \rightarrow \int_{\tau} d\tau (\nabla \cdot \vec{j} + \frac{\partial \rho}{\partial t}) = 0 \rightarrow \frac{\partial \rho}{\partial t} + \nabla \cdot \vec{j} = 0$$

$$\oint_{\tau} d\vec{S} \cdot \vec{E} = \frac{1}{\epsilon_{0}} \int_{\tau} d\tau \rho \rightarrow \int_{\tau} d\tau (\nabla \cdot \vec{E} - \frac{1}{\epsilon_{0}} \rho) = 0 \rightarrow \nabla \cdot \vec{E} = \frac{1}{\epsilon_{0}} \rho$$

$$\oint_{S} d\vec{I} \cdot \vec{B} = \mu_{0} \int_{S} d\vec{S} \cdot \vec{j} \rightarrow \int_{S} d\vec{S} \cdot (\nabla \times \vec{B} - \mu_{0} \vec{j}) = 0 \rightarrow \nabla \times \vec{B} = \mu_{0} \vec{j}$$







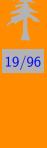


# <u>有空中电磁和工作片的场方是</u> 麦克斯韦方程组的微分形式 $\int_{ar{f c}} {f d} ec{f S} \cdot ec{f j} = - \int_{ar{f c}} {f d} au rac{\partial ho}{\partial {f t}} \; ightarrow \; \int_{ar{f c}} {f d} au ( abla \cdot ec{f j} + rac{\partial ho}{\partial {f t}}) = {f 0} \; ightarrow \; rac{\partial ho}{\partial {f t}} + abla \cdot ec{f j} = {f 0} \; {f 0}$

$$\oint_{\tau} d\vec{\mathbf{S}} \cdot \vec{\mathbf{E}} = \frac{1}{\epsilon_0} \int_{\tau} d\tau \rho \rightarrow \int_{\tau} d\tau (\nabla \cdot \vec{\mathbf{E}} - \frac{1}{\epsilon_0} \rho) = 0 \rightarrow \nabla \cdot \vec{\mathbf{E}} = \frac{1}{\epsilon_0} \rho$$

$$\oint_{\mathbf{S}} d\vec{\mathbf{I}} \cdot \vec{\mathbf{B}} = \mu_0 \int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot \vec{\mathbf{j}} \rightarrow \int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot (\nabla \times \vec{\mathbf{B}} - \mu_0 \vec{\mathbf{j}}) = 0 \rightarrow \nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}}$$

$$\oint_{\tau} d\vec{\mathbf{S}} \cdot \vec{\mathbf{B}} = 0$$









# 直空中自成相互作用的场方程: 麦克斯韦方程组的微分形式 $\int \mathbf{d} ec{\mathbf{S}} \cdot ec{\mathbf{j}} = - \int \mathbf{d} au rac{\partial ho}{\partial \mathbf{t}} \; ightarrow \; \int \mathbf{d} au ( abla \cdot ec{\mathbf{j}} + rac{\partial ho}{\partial \mathbf{t}}) = \mathbf{0} \; ightarrow \; rac{\partial ho}{\partial \mathbf{t}} + abla \cdot ec{\mathbf{j}} = \mathbf{0} \; ightarrow \;$

 $\oint_{ au} \mathbf{d} \vec{\mathbf{S}} \cdot \vec{\mathbf{E}} = rac{1}{\epsilon_{\mathbf{0}}} \int_{ au} \mathbf{d} au 
ho \; 
ightarrow \; \int_{ au} \mathbf{d} au (
abla \cdot \vec{\mathbf{E}} - rac{1}{\epsilon_{\mathbf{0}}} 
ho) = \mathbf{0} \; \overline{
ightarrow \; \mathbf{E}} = rac{1}{\epsilon_{\mathbf{0}}} 
ho$  $\oint_{\mathbf{S}} \mathbf{d\vec{l}} \cdot \vec{\mathbf{B}} = \mu_{\mathbf{0}} \int_{\mathbf{S}} \mathbf{d\vec{S}} \cdot \vec{\mathbf{j}} \to \int_{\mathbf{S}} \mathbf{d\vec{S}} \cdot (\nabla \times \vec{\mathbf{B}} - \mu_{\mathbf{0}} \vec{\mathbf{j}}) = \mathbf{0} \to \nabla \times \vec{\mathbf{B}} = \mu_{\mathbf{0}} \vec{\mathbf{j}}$  $\oint ext{d} ec{ ext{S}} \cdot ec{ ext{B}} = \mathbf{0} \; 
ightarrow \; \int ext{d} au 
abla \cdot ec{ ext{B}} = \mathbf{0}$ 









## <u>直卒中自成相互作具的场方是</u> 麦克斯韦方程组的微分形式 $\oint_{ar{f c}} {f d} ec{f S} \cdot ec{f j} = - \int_{ar{f c}} {f d} au rac{\partial ho}{\partial {f t}} \; ightarrow \; \int_{ar{f c}} {f d} au ( abla \cdot ec{f j} + rac{\partial ho}{\partial {f t}}) = {f 0} \; ightarrow \; rac{\partial ho}{\partial {f t}} + abla \cdot ec{f j} = {f 0} \; .$

$$\oint_{\tau} d\vec{\mathbf{S}} \cdot \vec{\mathbf{E}} = \frac{1}{\epsilon_{0}} \int_{\tau} d\tau \rho \rightarrow \int_{\tau} d\tau (\nabla \cdot \vec{\mathbf{E}} - \frac{1}{\epsilon_{0}} \rho) = \mathbf{0} \rightarrow \nabla \cdot \vec{\mathbf{E}} = \frac{1}{\epsilon_{0}} \rho$$

$$\oint_{\mathbf{S}} d\vec{\mathbf{I}} \cdot \vec{\mathbf{B}} = \mu_{0} \int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot \vec{\mathbf{j}} \rightarrow \int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot (\nabla \times \vec{\mathbf{B}} - \mu_{0} \vec{\mathbf{j}}) = \mathbf{0} \rightarrow \nabla \times \vec{\mathbf{B}} = \mu_{0} \vec{\mathbf{j}}$$

$$\oint_{\mathbf{S}} d\vec{\mathbf{S}} \cdot \vec{\mathbf{B}} = \mathbf{0} \rightarrow \int d\tau \nabla \cdot \vec{\mathbf{B}} = \mathbf{0} \rightarrow \nabla \cdot \vec{\mathbf{B}} = \mathbf{0}$$









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 $\oint_{\tau} d\vec{\mathbf{S}} \cdot \vec{\mathbf{E}} = \frac{1}{\epsilon_{0}} \int_{\tau} d\tau \rho \rightarrow \int_{\tau} d\tau (\nabla \cdot \vec{\mathbf{E}} - \frac{1}{\epsilon_{0}} \rho) = \mathbf{0} \rightarrow \nabla \cdot \vec{\mathbf{E}} = \frac{1}{\epsilon_{0}} \rho$   $\oint_{\mathbf{S}} d\vec{\mathbf{I}} \cdot \vec{\mathbf{B}} = \mu_{0} \int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot \vec{\mathbf{j}} \rightarrow \int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot (\nabla \times \vec{\mathbf{B}} - \mu_{0} \vec{\mathbf{j}}) = \mathbf{0} \rightarrow \nabla \times \vec{\mathbf{B}} = \mu_{0} \vec{\mathbf{j}}$   $\oint_{\mathbf{S}} d\vec{\mathbf{S}} \cdot \vec{\mathbf{B}} = \mathbf{0} \rightarrow \int_{\tau} d\tau \nabla \cdot \vec{\mathbf{B}} = \mathbf{0}$   $\oint_{\mathbf{S}} d\vec{\mathbf{I}} \cdot \vec{\mathbf{E}} = -\int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}}$ 









## $\int_{ar{f c}} {f d} ec{f S} \cdot ec{f j} = - \int_{ar{f c}} {f d} au rac{\partial ho}{\partial {f t}} \; ightarrow \; \int_{ar{f c}} {f d} au ( abla \cdot ec{f j} + rac{\partial ho}{\partial {f t}}) = {f 0} \; ightarrow \; rac{\partial ho}{\partial {f t}} + abla \cdot ec{f j} = {f 0} \; ightarrow \; {f j}$

$$\oint_{\tau} d\vec{\mathbf{S}} \cdot \vec{\mathbf{E}} = \frac{1}{\epsilon_{0}} \int_{\tau} d\tau \rho \rightarrow \int_{\tau} d\tau (\nabla \cdot \vec{\mathbf{E}} - \frac{1}{\epsilon_{0}} \rho) = \mathbf{0} \rightarrow \nabla \cdot \vec{\mathbf{E}} = \frac{1}{\epsilon_{0}} \rho$$

$$\oint_{\mathbf{S}} d\vec{\mathbf{I}} \cdot \vec{\mathbf{B}} = \mu_{0} \int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot \vec{\mathbf{j}} \rightarrow \int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot (\nabla \times \vec{\mathbf{B}} - \mu_{0} \vec{\mathbf{j}}) = \mathbf{0} \rightarrow \nabla \times \vec{\mathbf{B}} = \mu_{0} \vec{\mathbf{j}}$$

$$\oint_{\mathbf{S}} d\vec{\mathbf{S}} \cdot \vec{\mathbf{B}} = \mathbf{0} \rightarrow \int_{\tau} d\tau \nabla \cdot \vec{\mathbf{B}} = \mathbf{0} \rightarrow \nabla \cdot \vec{\mathbf{B}} = \mathbf{0}$$

$$\oint_{\mathbf{S}} d\vec{\mathbf{I}} \cdot \vec{\mathbf{E}} = -\int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}} \rightarrow \int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot (\nabla \times \vec{\mathbf{E}} + \frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}}) = \mathbf{0}$$



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$$\begin{split} \oint_{\tau} d\vec{\mathbf{S}} \cdot \vec{\mathbf{j}} &= -\int_{\tau} d\tau \frac{\partial \rho}{\partial t} \, \to \, \int_{\tau} d\tau (\nabla \cdot \vec{\mathbf{j}} + \frac{\partial \rho}{\partial t}) = 0 \, \to \, \frac{\partial \rho}{\partial t} + \nabla \cdot \vec{\mathbf{j}} = 0 \\ \oint_{\tau} d\vec{\mathbf{S}} \cdot \vec{\mathbf{E}} &= \frac{1}{\epsilon_0} \int_{\tau} d\tau \rho \, \to \, \int_{\tau} d\tau (\nabla \cdot \vec{\mathbf{E}} - \frac{1}{\epsilon_0} \rho) = 0 \, \to \, \nabla \cdot \vec{\mathbf{E}} = \frac{1}{\epsilon_0} \rho \\ \oint_{\mathbf{S}} d\vec{\mathbf{I}} \cdot \vec{\mathbf{B}} &= \mu_0 \int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot \vec{\mathbf{j}} \, \to \, \int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot (\nabla \times \vec{\mathbf{B}} - \mu_0 \vec{\mathbf{j}}) = 0 \, \to \nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} \\ \oint_{\mathbf{S}} d\vec{\mathbf{S}} \cdot \vec{\mathbf{B}} &= 0 \, \to \, \int_{\tau} d\tau \nabla \cdot \vec{\mathbf{B}} = 0 \, \to \, \nabla \cdot \vec{\mathbf{B}} = 0 \\ \oint_{\mathbf{S}} d\vec{\mathbf{I}} \cdot \vec{\mathbf{E}} &= -\int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial t} \, \to \, \int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot (\nabla \times \vec{\mathbf{E}} + \frac{\partial \vec{\mathbf{B}}}{\partial t}) = 0 \\ \to \, \nabla \times \vec{\mathbf{E}} &= -\frac{\partial \vec{\mathbf{B}}}{\partial t} \end{split}$$



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# <u>安中由磁和工作目的场方是</u> 麦克斯韦方程组的微分形式

$$\begin{split} &\oint_{\tau} d\vec{\mathbf{S}} \cdot \vec{\mathbf{j}} = -\int_{\tau} d\tau \frac{\partial \rho}{\partial t} \, \to \, \int_{\tau} d\tau (\nabla \cdot \vec{\mathbf{j}} + \frac{\partial \rho}{\partial t}) = 0 \, \to \, \frac{\partial \rho}{\partial t} + \nabla \cdot \vec{\mathbf{j}} = 0 \\ &\oint_{\tau} d\vec{\mathbf{S}} \cdot \vec{\mathbf{E}} = \frac{1}{\epsilon_0} \int_{\tau} d\tau \rho \, \to \, \int_{\tau} d\tau (\nabla \cdot \vec{\mathbf{E}} - \frac{1}{\epsilon_0} \rho) = 0 \, \to \, \nabla \cdot \vec{\mathbf{E}} = \frac{1}{\epsilon_0} \rho \\ &\oint_{\mathbf{S}} d\vec{\mathbf{I}} \cdot \vec{\mathbf{B}} = \mu_0 \int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot \vec{\mathbf{j}} \to \int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot (\nabla \times \vec{\mathbf{B}} - \mu_0 \vec{\mathbf{j}}) = 0 \to \nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} \\ &\oint_{\mathbf{S}} d\vec{\mathbf{S}} \cdot \vec{\mathbf{B}} = 0 \, \to \, \int_{\tau} d\tau \nabla \cdot \vec{\mathbf{B}} = 0 \, \to \, \nabla \cdot \vec{\mathbf{B}} = 0 \\ &\oint_{\mathbf{S}} d\vec{\mathbf{I}} \cdot \vec{\mathbf{E}} = -\int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}} \, \to \, \int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot (\nabla \times \vec{\mathbf{E}} + \frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}}) = 0 \\ &\to \, \nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}} \end{split}$$

$$\oint_{\mathbf{S}} \mathbf{d}\vec{\mathbf{l}} \cdot \vec{\mathbf{E}} = -\int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot \frac{\partial \mathbf{B}}{\partial \mathbf{t}} \rightarrow \int_{\mathbf{S}} \mathbf{d}\vec{\mathbf{S}} \cdot (\nabla \times \vec{\mathbf{E}} + \frac{\partial \mathbf{B}}{\partial \mathbf{t}}) = \mathbf{0}$$

$$\rightarrow \nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}}$$

$$\vec{\mathbf{F}} = \int \mathbf{d}\tau [\rho(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}}) + \vec{\mathbf{j}} \times \vec{\mathbf{B}}(\vec{\mathbf{r}})]$$









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$$\begin{split} &\oint_{\tau} d\vec{\mathbf{S}} \cdot \vec{\mathbf{j}} = -\int_{\tau} d\tau \frac{\partial \rho}{\partial t} \, \to \, \int_{\tau} d\tau (\nabla \cdot \vec{\mathbf{j}} + \frac{\partial \rho}{\partial t}) = 0 \, \to \, \frac{\partial \rho}{\partial t} + \nabla \cdot \vec{\mathbf{j}} = 0 \\ &\oint_{\tau} d\vec{\mathbf{S}} \cdot \vec{\mathbf{E}} = \frac{1}{\epsilon_0} \int_{\tau} d\tau \rho \, \to \, \int_{\tau} d\tau (\nabla \cdot \vec{\mathbf{E}} - \frac{1}{\epsilon_0} \rho) = 0 \, \to \, \nabla \cdot \vec{\mathbf{E}} = \frac{1}{\epsilon_0} \rho \\ &\oint_{\mathbf{S}} d\vec{\mathbf{I}} \cdot \vec{\mathbf{B}} = \mu_0 \int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot \vec{\mathbf{j}} \to \int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot (\nabla \times \vec{\mathbf{B}} - \mu_0 \vec{\mathbf{j}}) = 0 \to \nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} \\ &\oint_{\mathbf{S}} d\vec{\mathbf{S}} \cdot \vec{\mathbf{B}} = 0 \, \to \, \int_{\tau} d\tau \nabla \cdot \vec{\mathbf{B}} = 0 \, \to \, \nabla \cdot \vec{\mathbf{B}} = 0 \\ &\oint_{\mathbf{S}} d\vec{\mathbf{I}} \cdot \vec{\mathbf{E}} = -\int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial t} \, \to \, \int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot (\nabla \times \vec{\mathbf{E}} + \frac{\partial \vec{\mathbf{B}}}{\partial t}) = 0 \\ &\to \, \nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t} \end{split}$$

$$\vec{\mathbf{F}} = \int_{\tau} \mathbf{d}\tau [\rho(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}}) + \vec{\mathbf{j}} \times \vec{\mathbf{B}}(\vec{\mathbf{r}})] \rightarrow \int_{\tau} \mathbf{d}\tau (\vec{\mathbf{f}} - \rho\vec{\mathbf{E}} - \vec{\mathbf{j}} \times \vec{\mathbf{B}}) = \mathbf{0}$$









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# <u>真空中电磁相互作用的场方程</u>:麦克斯韦方程组的微分形式 $\int d\tau \frac{\partial \rho}{\partial \tau} \rightarrow \int d\tau (\nabla \cdot \vec{i} + \frac{\partial \rho}{\partial \tau}) = 0 \rightarrow \frac{\partial \rho}{\partial \tau} + \nabla \cdot \vec{i} = 0$

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$$\begin{split} &\oint_{\tau} d\vec{\mathbf{S}} \cdot \vec{\mathbf{j}} = -\int_{\tau} d\tau \frac{\partial \rho}{\partial t} \, \to \, \int_{\tau} d\tau (\nabla \cdot \vec{\mathbf{j}} + \frac{\partial \rho}{\partial t}) = 0 \, \to \, \frac{\partial \rho}{\partial t} + \nabla \cdot \vec{\mathbf{j}} = 0 \\ &\oint_{\tau} d\vec{\mathbf{S}} \cdot \vec{\mathbf{E}} = \frac{1}{\epsilon_{0}} \int_{\tau} d\tau \rho \, \to \, \int_{\tau} d\tau (\nabla \cdot \vec{\mathbf{E}} - \frac{1}{\epsilon_{0}} \rho) = 0 \, \to \, \nabla \cdot \vec{\mathbf{E}} = \frac{1}{\epsilon_{0}} \rho \\ &\oint_{\mathbf{S}} d\vec{\mathbf{I}} \cdot \vec{\mathbf{B}} = \mu_{0} \int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot \vec{\mathbf{j}} \to \int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot (\nabla \times \vec{\mathbf{B}} - \mu_{0} \vec{\mathbf{j}}) = 0 \to \nabla \times \vec{\mathbf{B}} = \mu_{0} \vec{\mathbf{j}} \\ &\oint_{\mathbf{S}} d\vec{\mathbf{S}} \cdot \vec{\mathbf{B}} = 0 \, \to \, \int_{\tau} d\tau \nabla \cdot \vec{\mathbf{B}} = 0 \, \to \, \nabla \cdot \vec{\mathbf{B}} = 0 \\ &\oint_{\mathbf{S}} d\vec{\mathbf{I}} \cdot \vec{\mathbf{E}} = -\int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial t} \, \to \, \int_{\mathbf{S}} d\vec{\mathbf{S}} \cdot (\nabla \times \vec{\mathbf{E}} + \frac{\partial \vec{\mathbf{B}}}{\partial t}) = 0 \\ &\to \, \nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t} \end{split}$$

$$\oint_{\mathbf{S}} \mathbf{d} \mathbf{l} \cdot \mathbf{E} = -\int_{\mathbf{S}} \mathbf{d} \mathbf{S} \cdot \frac{1}{\partial \mathbf{t}} \rightarrow \int_{\mathbf{S}} \mathbf{d} \mathbf{S} \cdot (\mathbf{V} \times \mathbf{E} + \frac{1}{\partial \mathbf{t}}) = \mathbf{0}$$

$$\rightarrow \nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}}$$

$$\vec{\mathbf{F}} = \int_{\tau} \mathbf{d} \tau [\rho(\vec{\mathbf{r}}) \vec{\mathbf{E}} (\vec{\mathbf{r}}) + \vec{\mathbf{j}} \times \vec{\mathbf{B}} (\vec{\mathbf{r}})] \rightarrow \int_{\tau} \mathbf{d} \tau (\vec{\mathbf{f}} - \rho \vec{\mathbf{E}} - \vec{\mathbf{j}} \times \vec{\mathbf{B}}) = \mathbf{0}$$

$$\rightarrow \vec{\mathbf{f}} = \rho \vec{\mathbf{E}} + \vec{\mathbf{j}} \times \vec{\mathbf{B}}$$

#### 真空中电磁相互作用的场方程: 麦克斯韦方程组的微分形式

$$rac{\partial 
ho}{\partial \mathbf{t}} + 
abla \cdot \vec{\mathbf{j}} = \mathbf{0} \qquad 
abla \cdot \vec{\mathbf{E}} = rac{1}{\epsilon_0} 
ho \qquad 
abla imes \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}}$$

$$\frac{\partial \rho}{\partial \mathbf{t}} = \epsilon_0 \frac{\partial}{\partial \mathbf{t}} \nabla \cdot \vec{\mathbf{E}} = \epsilon_0 \nabla \cdot \frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}}$$

$$\vec{\mathbf{E}} = \vec{\mathbf{E}} \cdot \vec{\mathbf{E}} = \vec{\mathbf{E}} \cdot \vec{\mathbf{E$$

$$\nabla \cdot \vec{\mathbf{j}} = \frac{1}{\mu_0} \nabla \cdot (\nabla \times \vec{\mathbf{B}}) = \frac{1}{\mu_0} (\nabla \times \nabla) \cdot \vec{\mathbf{B}} = \mathbf{0}$$

$$abla imes ec{\mathbf{B}} 
ightarrow 
abla imes ec{\mathbf{B}} = \mu_0 ec{\mathbf{j}} + \mu_0 \epsilon_0 \frac{\partial ec{\mathbf{E}}}{\partial \mathbf{t}} \leftarrow$$
位移电流项(电生磁)



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#### <u>真空中电磁指互作用的场方程</u>: 麦克斯韦方程组的微分形式

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 $abla \cdot \vec{\mathbf{E}} = \frac{1}{\epsilon_0} 
ho$   $abla_{\mathbf{i}=1} \partial_{\mathbf{i}} \mathbf{E}_{\mathbf{i}} = \frac{1}{\epsilon_0} 
ho$  电荷是电场发散聚敛的源

$$abla imes ec{\mathbf{E}} = -rac{\partial ec{\mathbf{B}}}{\partial t}$$

$$\sum_{\mathbf{i},\mathbf{j},\mathbf{k}} \delta_{\mathbf{i}} \mathbf{E_{j}} = -rac{\partial \mathbf{B_{k}}}{\partial t}$$
 磁场的时间变化率是电场旋转的源

$$abla \cdot ec{B} = 0$$
  $\sum_{i=1} \partial_i B_i = 0$  磁场不发散和聚敛  $\partial ec{E}$   $\partial ec{E}$ 

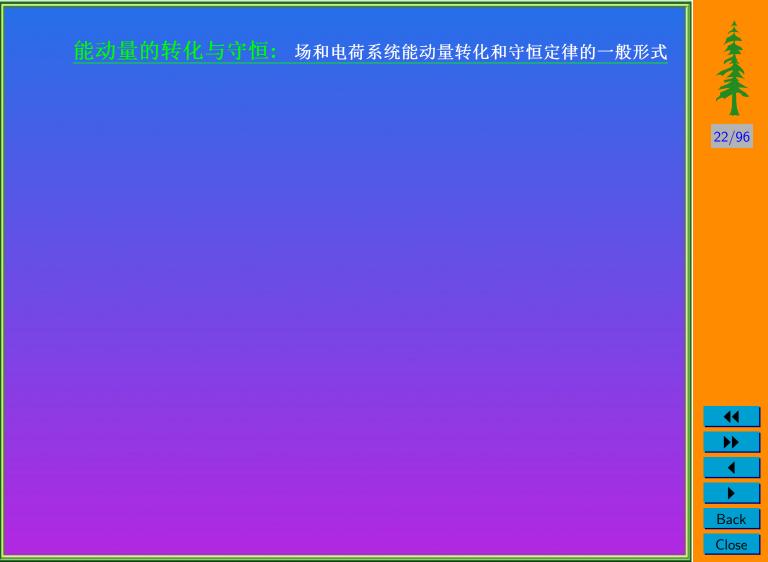
$$\nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t} \qquad \sum_{\mathbf{i}, \mathbf{j} = 1}^{3} \epsilon_{\mathbf{i} \mathbf{j} \mathbf{k}} \partial_{\mathbf{i}} \mathbf{B}_{\mathbf{j}} = \mu_0 \mathbf{j}_{\mathbf{k}} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}_{\mathbf{k}}}{\partial t}$$
 电流密度和电场的时间变化率是磁场旋转的源

$$ec{\mathbf{f}} = 
ho ec{\mathbf{E}} + ec{\mathbf{j}} imes ec{\mathbf{B}}$$
  $\mathbf{f_k} = 
ho \mathbf{E_k} + \sum^{\mathbf{3}} \epsilon_{ijk} \mathbf{j_i} \mathbf{B_j}$ 

i.i=1

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• 场的能量密度w: 单位体积的场所带的能量,是时空坐标的 函数  $\mathbf{w} = \mathbf{w}(\vec{\mathbf{r}}, \mathbf{t})$ 



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#### 能动量的装化与守恒: 场和电荷系统能动量转化和守恒定律的一般形式

- 场的能量密度w: 单位体积的场所带的能量,是时空坐标的 函数  $\mathbf{w} = \mathbf{w}(\vec{\mathbf{r}}, \mathbf{t})$
- 场的动量密度 $\vec{g}$ : 单位体积的场所带的动量,是时空坐标的函数  $\vec{g} = \vec{g}(\vec{r}, t)$

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- <mark>场的能量密度w:</mark> 单位体积的场所带的能量,是时空坐标的 函数  $\mathbf{w} = \mathbf{w}(\vec{\mathbf{r}}, \mathbf{t})$
- 场的动量密度 $\vec{g}$ : 单位体积的场所带的动量,是时空坐标的函数  $\vec{g} = \vec{g}(\vec{r}, t)$
- 场的能流密度 $\vec{S}$ : 描述能量的传播,大小等于单位时间垂直流过单位横 截面的能量,方向指向能量传播的方向,是时空坐标的函数  $\vec{S} = \vec{S}(\vec{r},t)$



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- 场的能量密度w: 单位体积的场所带的能量,是时空坐标的函数  $\mathbf{w} = \mathbf{w}(\vec{\mathbf{r}}, \mathbf{t})$
- 场的动量密度 $\vec{g}$ : 单位体积的场所带的动量,是时空坐标的函数  $\vec{g} = \vec{g}(\vec{r}, t)$
- 场的能流密度 $\vec{S}$ : 描述能量的传播,大小等于单位时间垂直流过单位横 截面的能量,方向指向能量传播的方向,是时空坐标的函数  $\vec{S} = \vec{S}(\vec{r},t)$
- 场的动量流密度  $\mathcal{J}$ : 它描述动量的传播, $\Delta \vec{S} \cdot \vec{J} = \Delta \vec{p}$ 定义为单位时间通过面元  $\Delta \vec{S}$ 流出的动量,是时空坐标的函数  $\mathcal{J} = \mathcal{J}$   $(\vec{r}, t)$





#### 能动量的技化与宁恒:场和电荷系统能动量转化和守恒定律的一般形式

● 能量转化与守恒定律:



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● 能量转化与守恒定律: 单位时间通过表面流入V的能量等于 单位时间场对 V内电荷所做的功(即功率)与V内单位时间 电磁场能量的增加之和。



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● 能量转化与守恒定律: 单位时间通过表面流入V的能量等于单位时间场对 V内电荷所做的功(即功率)与V内单位时间电磁场能量的增加之和。

$$-\oint_{\mathbf{V}} \mathbf{d}\vec{\sigma} \cdot \vec{\mathbf{S}} = \int_{\mathbf{V}} \mathbf{dV} \vec{\mathbf{f}} \cdot \vec{\mathbf{v}} + \frac{\mathbf{d}}{\mathbf{dt}} \int_{\mathbf{V}} \mathbf{dV} \mathbf{w}$$
  $\vec{f}$ :场对电荷作用力密度  $\vec{v}$ :电荷运动速度

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#### 能动量的转化与守恒: 场和电荷系统能动量转化和守恒定律的一般形式

● 能量转化与守恒定律: 单位时间通过表面流入V的能量等于单位时间场对 V内电荷所做的功(即功率)与V内单位时间电磁场能量的增加之和。

$$-\oint_{V} d\vec{\sigma} \cdot \vec{S} = \int_{V} dV \vec{f} \cdot \vec{v} + \frac{d}{dt} \int_{V} dV w$$

$$\vec{f} : 场对电荷作用力密度 \qquad \vec{v} : 电荷运动速度$$

$$-\nabla \cdot \vec{S} = \vec{f} \cdot \vec{v} + \frac{\partial w}{\partial t}$$







● 能量转化与守恒定律: 单位时间通过表面流入V的能量等于单位时间场对 V内电荷所做的功(即功率)与V内单位时间电磁场能量的增加之和。

$$\begin{split} &-\oint_{\mathbf{V}}\mathbf{d}\vec{\sigma}\cdot\vec{\mathbf{S}} = \int_{\mathbf{V}}\mathbf{d}\mathbf{V}\vec{\mathbf{f}}\cdot\vec{\mathbf{v}} + \frac{\mathbf{d}}{\mathbf{d}\mathbf{t}}\int_{\mathbf{V}}\mathbf{d}\mathbf{V}\mathbf{w} \\ &\vec{f}:$$
场对电荷作用力密度  $\vec{v}:$ 电荷运动速度 
$$&-\nabla\cdot\vec{\mathbf{S}} = \vec{\mathbf{f}}\cdot\vec{\mathbf{v}} + \frac{\partial\mathbf{w}}{\partial\mathbf{t}} \end{split}$$

● 动量转化与守恒定律: 单位时间通过表面流入V的动量等于V 内电荷所受的力与V内单位时间电磁场动量的增加之和。

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#### 能动量的转化与守恒: 场和电荷系统能动量转化和守恒定律的一般形式

● 能量转化与守恒定律: 单位时间通过表面流入V的能量等于单位时间场对 V内电荷所做的功(即功率)与V内单位时间电磁场能量的增加之和。

$$\begin{split} &-\oint_{V} d\vec{\sigma} \cdot \vec{S} = \int_{V} dV \vec{f} \cdot \vec{v} + \frac{d}{dt} \int_{V} dV w \\ &\vec{f} :$$
场对电荷作用力密度  $\vec{v} :$ 电荷运动速度 
$$-\nabla \cdot \vec{S} = \vec{f} \cdot \vec{v} + \frac{\partial w}{\partial t} \end{split}$$

● 动量转化与守恒定律: 单位时间通过表面流入V的动量等于V 内电荷所受的力与V内单位时间电磁场动量的增加之和。

$$-\oint_{\mathbf{V}}\mathbf{d}ec{\sigma}\cdot\stackrel{
ightharpoonup}{\mathcal{J}}=\int_{\mathbf{V}}\mathbf{d}\mathbf{V}ec{\mathbf{f}}+rac{\mathbf{d}}{\mathbf{dt}}\int_{\mathbf{V}}\mathbf{d}\mathbf{V}ec{\mathbf{g}}$$

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#### 能动量的转化与守恒:场和电荷系统能动量转化和守恒定律的一般形式

● 能量转化与守恒定律: 单位时间通过表面流入V的能量等于单位时间场对 V内电荷所做的功(即功率)与V内单位时间电磁场能量的增加之和。

$$\begin{split} &-\oint_{V} d\vec{\sigma} \cdot \vec{S} = \int_{V} dV \vec{f} \cdot \vec{v} + \frac{d}{dt} \int_{V} dV w \\ &\vec{f} :$$
场对电荷作用力密度  $\vec{v} :$ 电荷运动速度 
$$-\nabla \cdot \vec{S} = \vec{f} \cdot \vec{v} + \frac{\partial w}{\partial t} \end{split}$$

● 动量转化与守恒定律: 单位时间通过表面流入V的动量等于V 内电荷所受的力与V内单位时间电磁场动量的增加之和。

$$\begin{split} &-\oint_{V} d\vec{\sigma} \cdot \stackrel{\rightharpoonup}{\mathcal{J}} = \int_{V} dV \vec{f} + \frac{d}{dt} \int_{V} dV \vec{g} \\ &-\nabla \cdot \stackrel{\rightharpoonup}{\mathcal{J}} = \vec{f} + \frac{\partial \vec{g}}{\partial t} \end{split}$$

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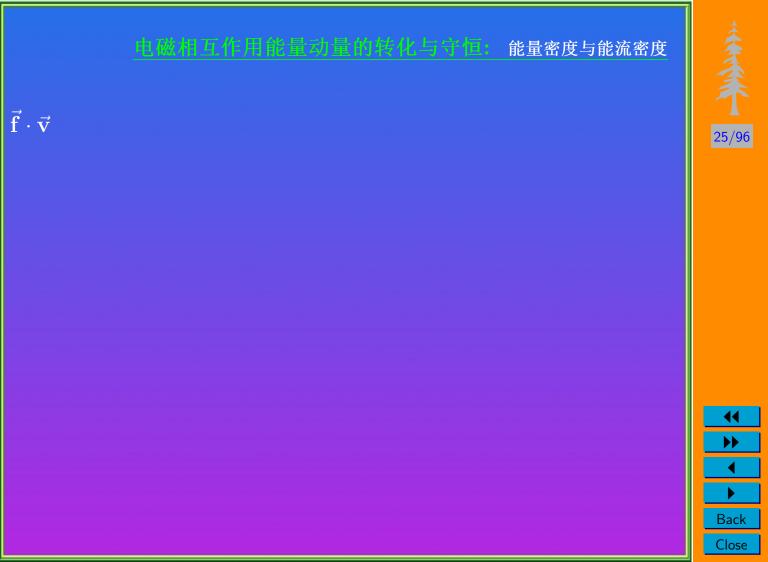


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物理量	密度	流密度	守恒定律	守恒荷
电荷	ρ	$ec{\mathbf{j}}$	$ abla \cdot ec{\mathbf{j}} + rac{\partial  ho}{\partial \mathbf{t}} = 0$	$\mathbf{Q}=\int\mathbf{dV} ho$
能量	W	$ec{\mathbf{S}}$	$ abla \cdot ec{\mathbf{S}} + rac{\partial \mathbf{w}}{\partial \mathbf{t}} + ec{\mathbf{f}} \cdot ec{\mathbf{v}} = 0$	${f E}_{f k}$ 量 $=\int { m d} {f V} {f w}$
动量	ďος	$ec{\mathcal{J}}$	$ abla \cdot \stackrel{ ightharpoonup}{\mathcal{J}} + rac{\partial ec{\mathbf{g}}}{\partial \mathbf{t}} + ec{\mathbf{f}} = 0$	$ec{ ext{P}}_{ec{ ext{d}}} = \int  ext{d} ext{V}$ g



### 能量密度与能流密度 $ec{\mathbf{f}} \cdot \vec{\mathbf{v}} = (\rho_{\mathbf{f}} \vec{\mathbf{E}} + \rho_{\mathbf{f}} \vec{\mathbf{v}} \times \vec{\mathbf{B}}) \cdot \vec{\mathbf{v}}$









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## 能量密度与能流密度

 $ec{\mathbf{f}} \cdot \vec{\mathbf{v}} = (\rho_{\mathbf{f}} \vec{\mathbf{E}} + \rho_{\mathbf{f}} \vec{\mathbf{v}} \times \vec{\mathbf{B}}) \cdot \vec{\mathbf{v}} = \rho_{\mathbf{f}} \vec{\mathbf{v}} \cdot \vec{\mathbf{E}}$ 





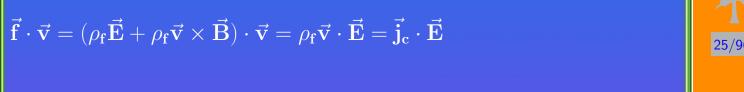






## 能量密度与能流密度











#### 电磁相互作用能量动量的转化与宁恒: 能量密度与能流密度



 $\vec{\mathbf{f}} \cdot \vec{\mathbf{v}} = (\rho_{\mathbf{f}} \vec{\mathbf{E}} + \rho_{\mathbf{f}} \vec{\mathbf{v}} \times \vec{\mathbf{B}}) \cdot \vec{\mathbf{v}} = \rho_{\mathbf{f}} \vec{\mathbf{v}} \cdot \vec{\mathbf{E}} = \vec{\mathbf{j}}_{\mathbf{c}} \cdot \vec{\mathbf{E}} = \vec{\mathbf{E}} \cdot (\nabla \times \frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}} - \frac{\partial \epsilon_{\mathbf{0}} \vec{\mathbf{E}}}{\partial \mathbf{t}})$ 







## 电磁相互作用能量动量的转化与守恒: 能量密度与能流密度 $\vec{\mathbf{f}} \cdot \vec{\mathbf{v}} = (\rho_{\mathbf{f}} \vec{\mathbf{E}} + \rho_{\mathbf{f}} \vec{\mathbf{v}} \times \vec{\mathbf{B}}) \cdot \vec{\mathbf{v}} = \rho_{\mathbf{f}} \vec{\mathbf{v}} \cdot \vec{\mathbf{E}} = \vec{\mathbf{j}}_{\mathbf{c}} \cdot \vec{\mathbf{E}} = \vec{\mathbf{E}} \cdot (\nabla \times \frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}} - \frac{\partial \epsilon_{\mathbf{0}} \vec{\mathbf{E}}}{\partial \mathbf{t}})$



$$\left(\frac{\vec{\Sigma}}{25/9}\right)$$

$$= \vec{\mathbf{E}} \cdot (\nabla \times \frac{\vec{\mathbf{B}}}{\mu_0}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_0 \vec{\mathbf{E}}}{\partial \mathbf{t}}$$









## 自, 磁相互作用能量动量的转化与守恒: 能量密度与能流密度

 $\vec{\mathbf{f}} \cdot \vec{\mathbf{v}} = (\rho_{\mathbf{f}} \vec{\mathbf{E}} + \rho_{\mathbf{f}} \vec{\mathbf{v}} \times \vec{\mathbf{B}}) \cdot \vec{\mathbf{v}} = \rho_{\mathbf{f}} \vec{\mathbf{v}} \cdot \vec{\mathbf{E}} = \vec{\mathbf{j}}_{\mathbf{c}} \cdot \vec{\mathbf{E}} = \vec{\mathbf{E}} \cdot (\nabla \times \frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}} - \frac{\partial \epsilon_{\mathbf{0}} \vec{\mathbf{E}}}{\partial \mathbf{t}})$   $= \vec{\mathbf{E}} \cdot (\nabla \times \frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_{\mathbf{0}} \vec{\mathbf{E}}}{\partial \mathbf{t}} = -\nabla \cdot (\vec{\mathbf{E}} \times \frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}}) + \frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}} \cdot (\nabla \times \vec{\mathbf{E}}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_{\mathbf{0}} \vec{\mathbf{E}}}{\partial \mathbf{t}}$ 





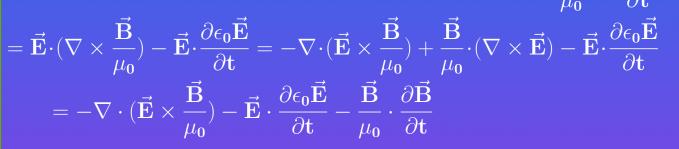




## 电磁相互作用能量动量的转化与守恒: 能量密度与能流密度

$$\vec{\mathbf{f}} \cdot \vec{\mathbf{v}} = (\rho_{\mathbf{f}} \vec{\mathbf{E}} + \rho_{\mathbf{f}} \vec{\mathbf{v}} \times \vec{\mathbf{B}}) \cdot \vec{\mathbf{v}} = \rho_{\mathbf{f}} \vec{\mathbf{v}} \cdot \vec{\mathbf{E}} = \vec{\mathbf{j}}_{\mathbf{c}} \cdot \vec{\mathbf{E}} = \vec{\mathbf{E}} \cdot (\nabla \times \frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}} - \frac{\partial \epsilon_{\mathbf{0}} \vec{\mathbf{E}}}{\partial \mathbf{t}})$$

$$= \vec{\mathbf{E}} \cdot (\nabla \times \frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_{\mathbf{0}} \vec{\mathbf{E}}}{\partial \mathbf{t}} = -\nabla \cdot (\vec{\mathbf{E}} \times \frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}}) + \frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}} \cdot (\nabla \times \vec{\mathbf{E}}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_{\mathbf{0}} \vec{\mathbf{E}}}{\partial \mathbf{t}}$$













## 电磁相互作用能量动量的转化与守恒: 能量密度与能流密度

$$\vec{\mathbf{f}} \cdot \vec{\mathbf{v}} = (\rho_{\mathbf{f}}\vec{\mathbf{E}} + \rho_{\mathbf{f}}\vec{\mathbf{v}} \times \vec{\mathbf{B}}) \cdot \vec{\mathbf{v}} = \rho_{\mathbf{f}}\vec{\mathbf{v}} \cdot \vec{\mathbf{E}} = \vec{\mathbf{j}}_{\mathbf{c}} \cdot \vec{\mathbf{E}} = \vec{\mathbf{E}} \cdot (\nabla \times \frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}} - \frac{\partial \epsilon_{\mathbf{0}}\vec{\mathbf{E}}}{\partial \mathbf{t}})$$

$$= \vec{\mathbf{E}} \cdot (\nabla \times \frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_{\mathbf{0}}\vec{\mathbf{E}}}{\partial \mathbf{t}} = -\nabla \cdot (\vec{\mathbf{E}} \times \frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}}) + \frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}} \cdot (\nabla \times \vec{\mathbf{E}}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_{\mathbf{0}}\vec{\mathbf{E}}}{\partial \mathbf{t}}$$

 $= \vec{\mathbf{E}} \cdot (\nabla \times \frac{\vec{\mathbf{B}}}{\mu_0}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_0 \vec{\mathbf{E}}}{\partial \mathbf{t}} = -\nabla \cdot (\vec{\mathbf{E}} \times \frac{\vec{\mathbf{B}}}{\mu_0}) + \frac{\vec{\mathbf{B}}}{\mu_0} \cdot (\nabla \times \vec{\mathbf{E}}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_0 \vec{\mathbf{E}}}{\partial \mathbf{t}}$   $= -\nabla \cdot (\vec{\mathbf{E}} \times \frac{\vec{\mathbf{B}}}{\mu_0}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_0 \vec{\mathbf{E}}}{\partial \mathbf{t}} - \frac{\vec{\mathbf{B}}}{\mu_0} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}} = -\nabla \cdot \vec{\mathbf{S}} - \frac{\partial \mathbf{w}}{\partial \mathbf{t}}$ 









#### 直磁相互作用能量动量的转化与守恒: 能量密度与能流密度

$$\vec{\mathbf{f}} \cdot \vec{\mathbf{v}} = (\rho_{\mathbf{f}} \vec{\mathbf{E}} + \rho_{\mathbf{f}} \vec{\mathbf{v}} \times \vec{\mathbf{B}}) \cdot \vec{\mathbf{v}} = \rho_{\mathbf{f}} \vec{\mathbf{v}} \cdot \vec{\mathbf{E}} = \vec{\mathbf{j}}_{\mathbf{c}} \cdot \vec{\mathbf{E}} = \vec{\mathbf{E}} \cdot (\nabla \times \frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}} - \frac{\partial \epsilon_{\mathbf{0}} \vec{\mathbf{E}}}{\partial \mathbf{t}})$$

$$\vec{\mathbf{E}} \cdot (\nabla \times \vec{\mathbf{B}}) = \vec{\mathbf{E}} \cdot (\nabla \times \vec{\mathbf{E}}) = \vec{\mathbf{E}} \cdot (\nabla \times \vec{$$

$$= \vec{\mathbf{E}} \cdot (\nabla \times \frac{\mathbf{B}}{\mu_0}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_0 \mathbf{E}}{\partial \mathbf{t}} = -\nabla \cdot (\vec{\mathbf{E}} \times \frac{\mathbf{B}}{\mu_0}) + \frac{\mathbf{B}}{\mu_0} \cdot (\nabla \times \vec{\mathbf{E}}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_0 \mathbf{E}}{\partial \mathbf{t}}$$

$$= -\nabla \cdot (\vec{\mathbf{E}} \times \frac{\vec{\mathbf{B}}}{\partial t}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_0 \vec{\mathbf{E}}}{\partial t} - \frac{\vec{\mathbf{B}}}{\partial t} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial t} = -\nabla \cdot \vec{\mathbf{S}} - \frac{\partial \mathbf{w}}{\partial t}$$

$$= \vec{\mathbf{E}} \cdot (\nabla \times \frac{\vec{\mathbf{B}}}{\mu_0}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_0 \vec{\mathbf{E}}}{\partial \mathbf{t}} = -\nabla \cdot (\vec{\mathbf{E}} \times \frac{\vec{\mathbf{B}}}{\mu_0}) + \frac{\vec{\mathbf{B}}}{\mu_0} \cdot (\nabla \times \vec{\mathbf{E}}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_0 \vec{\mathbf{E}}}{\partial \mathbf{t}}$$

$$= -\nabla \cdot (\vec{\mathbf{E}} \times \frac{\vec{\mathbf{B}}}{\mu_0}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_0 \vec{\mathbf{E}}}{\partial \mathbf{t}} - \frac{\vec{\mathbf{B}}}{\mu_0} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}} = -\nabla \cdot \vec{\mathbf{S}} - \frac{\partial \mathbf{w}}{\partial \mathbf{t}}$$

$$= -\nabla \cdot (\vec{\mathbf{E}} \times \frac{\vec{\mathbf{B}}}{\mu_0}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_0 \vec{\mathbf{E}}}{\partial \mathbf{t}} - \frac{\vec{\mathbf{B}}}{\mu_0} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}} = -\nabla \cdot \vec{\mathbf{S}} - \frac{\partial \mathbf{w}}{\partial \mathbf{t}}$$

$$\vec{\mathbf{S}} = \vec{\mathbf{E}} \times \frac{\vec{\mathbf{B}}}{\mu_0} \qquad \qquad \frac{\partial \mathbf{w}}{\partial \mathbf{t}} = \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_0 \vec{\mathbf{E}}}{\partial \mathbf{t}} + \frac{\vec{\mathbf{B}}}{\mu_0} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}}$$









# 直磁相互作用能量动量的转化与守恒: 能量密度与能流密度

 $\vec{\mathbf{f}} \cdot \vec{\mathbf{v}} = (\rho_{\mathbf{f}} \vec{\mathbf{E}} + \rho_{\mathbf{f}} \vec{\mathbf{v}} \times \vec{\mathbf{B}}) \cdot \vec{\mathbf{v}} = \rho_{\mathbf{f}} \vec{\mathbf{v}} \cdot \vec{\mathbf{E}} = \vec{\mathbf{j}}_{\mathbf{c}} \cdot \vec{\mathbf{E}} = \vec{\mathbf{E}} \cdot (\nabla \times \frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}} - \frac{\partial \epsilon_{\mathbf{0}} \vec{\mathbf{E}}}{\partial \mathbf{t}})$ 

$$\begin{split} & = \vec{\mathbf{E}} \cdot (\nabla \times \frac{\vec{\mathbf{B}}}{\mu_0}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_0 \vec{\mathbf{E}}}{\partial \mathbf{t}} = -\nabla \cdot (\vec{\mathbf{E}} \times \frac{\vec{\mathbf{B}}}{\mu_0}) + \frac{\vec{\mathbf{B}}}{\mu_0} \cdot (\nabla \times \vec{\mathbf{E}}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_0 \vec{\mathbf{E}}}{\partial \mathbf{t}} \\ & = -\nabla \cdot (\vec{\mathbf{E}} \times \frac{\vec{\mathbf{B}}}{\mu_0}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_0 \vec{\mathbf{E}}}{\partial \mathbf{t}} - \frac{\vec{\mathbf{B}}}{\mu_0} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}} = -\nabla \cdot \vec{\mathbf{S}} - \frac{\partial \mathbf{w}}{\partial \mathbf{t}} \end{split}$$

 $\vec{\mathbf{S}} = \vec{\mathbf{E}} \times \frac{\vec{\mathbf{B}}}{\mu_0} \qquad \qquad \frac{\partial \mathbf{w}}{\partial \mathbf{t}} = \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_0 \vec{\mathbf{E}}}{\partial \mathbf{t}} + \frac{\vec{\mathbf{B}}}{\mu_0} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}}$ 

 $\partial \mathbf{w}$  $\overline{\partial \mathbf{t}}$ 

# 自磁相 互作用能量动量的转化与守恒: 能量密度与能流密度

$$\vec{\mathbf{f}} \cdot \vec{\mathbf{v}} = (\rho_{\mathbf{f}} \vec{\mathbf{E}} + \rho_{\mathbf{f}} \vec{\mathbf{v}} \times \vec{\mathbf{B}}) \cdot \vec{\mathbf{v}} = \rho_{\mathbf{f}} \vec{\mathbf{v}} \cdot \vec{\mathbf{E}} = \vec{\mathbf{j}}_{\mathbf{c}} \cdot \vec{\mathbf{E}} = \vec{\mathbf{E}} \cdot (\nabla \times \frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}} - \frac{\partial \epsilon_{\mathbf{0}} \vec{\mathbf{E}}}{\partial \mathbf{t}})$$

$$= \vec{\mathbf{E}} \cdot (\nabla \times \frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_{\mathbf{0}} \vec{\mathbf{E}}}{\partial \mathbf{t}} = -\nabla \cdot (\vec{\mathbf{E}} \times \frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}}) + \frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}} \cdot (\nabla \times \vec{\mathbf{E}}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_{\mathbf{0}} \vec{\mathbf{E}}}{\partial \mathbf{t}}$$

$$= \vec{\mathbf{E}} \cdot (\nabla \times \frac{\vec{\mathbf{B}}}{\mu_0}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_0 \vec{\mathbf{E}}}{\partial \mathbf{t}} = -\nabla \cdot (\vec{\mathbf{E}} \times \frac{\vec{\mathbf{B}}}{\mu_0}) + \frac{\vec{\mathbf{B}}}{\mu_0} \cdot (\nabla \times \vec{\mathbf{E}}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_0 \vec{\mathbf{E}}}{\partial \mathbf{t}}$$

$$= -\nabla \cdot (\vec{\mathbf{E}} \times \frac{\vec{\mathbf{B}}}{\mu_0}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_0 \vec{\mathbf{E}}}{\partial \mathbf{t}} - \frac{\vec{\mathbf{B}}}{\mu_0} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}} = -\nabla \cdot \vec{\mathbf{S}} - \frac{\partial \mathbf{w}}{\partial \mathbf{t}}$$

$$\vec{\mathbf{S}} = \vec{\mathbf{E}} \times \frac{\vec{\mathbf{B}}}{\mu_0} \qquad \qquad \frac{\partial \mathbf{w}}{\partial \mathbf{t}} = \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_0 \vec{\mathbf{E}}}{\partial \mathbf{t}} + \frac{\vec{\mathbf{B}}}{\mu_0} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}}$$

$$\frac{\partial \mathbf{w}}{\partial \mathbf{t}} = \epsilon_0 \vec{\mathbf{E}} \cdot \frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}} + \frac{1}{\mu_0} \vec{\mathbf{B}} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}}$$

$$\frac{\partial \mathbf{w}}{\partial \mathbf{t}} = \epsilon_0 \vec{\mathbf{E}} \cdot \frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}} + \frac{1}{\mu_0} \vec{\mathbf{B}} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}}$$











# 白磁相互作用能量动量的转化与守恒: 能量密度与能流密度

$$\vec{\mathbf{f}} \cdot \vec{\mathbf{v}} = (\rho_{\mathbf{f}} \vec{\mathbf{E}} + \rho_{\mathbf{f}} \vec{\mathbf{v}} \times \vec{\mathbf{B}}) \cdot \vec{\mathbf{v}} = \rho_{\mathbf{f}} \vec{\mathbf{v}} \cdot \vec{\mathbf{E}} = \vec{\mathbf{j}}_{\mathbf{c}} \cdot \vec{\mathbf{E}} = \vec{\mathbf{E}} \cdot (\nabla \times \frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}} - \frac{\partial \epsilon_{\mathbf{0}} \vec{\mathbf{E}}}{\partial \mathbf{t}})$$

$$= \vec{\mathbf{E}} \cdot (\nabla \times \frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_{\mathbf{0}} \vec{\mathbf{E}}}{\partial \mathbf{t}} = -\nabla \cdot (\vec{\mathbf{E}} \times \frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}}) + \frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}} \cdot (\nabla \times \vec{\mathbf{E}}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_{\mathbf{0}} \vec{\mathbf{E}}}{\partial \mathbf{t}}$$

$$= -\nabla \cdot (\vec{\mathbf{E}} \times \frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_{\mathbf{0}} \vec{\mathbf{E}}}{\partial \mathbf{t}} - \frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}} = -\nabla \cdot \vec{\mathbf{S}} - \frac{\partial \mathbf{w}}{\partial \mathbf{t}}$$

$$\vec{\mathbf{S}} = \vec{\mathbf{E}} \times \frac{\vec{\mathbf{B}}}{\mu_0} - \vec{\mathbf{E}} \cdot \frac{\vec{\mathbf{D}}}{\partial \mathbf{t}} - \frac{\vec{\mathbf{D}}}{\mu_0} \cdot \frac{\vec{\mathbf{D}}}{\partial \mathbf{t}} = -\mathbf{V} \cdot \mathbf{S} - \frac{\vec{\mathbf{D}}}{\partial \mathbf{t}}$$

$$\vec{\mathbf{S}} = \vec{\mathbf{E}} \times \frac{\vec{\mathbf{B}}}{\mu_0} \qquad \frac{\partial \mathbf{w}}{\partial \mathbf{t}} = \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_0 \vec{\mathbf{E}}}{\partial \mathbf{t}} + \frac{\vec{\mathbf{B}}}{\mu_0} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}}$$

$$\frac{\partial \mathbf{w}}{\partial \mathbf{t}} = \epsilon_0 \vec{\mathbf{E}} \cdot \frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}} + \frac{1}{\mu_0} \vec{\mathbf{B}} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}} = \frac{\partial}{\partial \mathbf{t}} \frac{1}{2} (\epsilon_0 \mathbf{E}^2 + \frac{1}{\mu_0} \mathbf{B}^2)$$











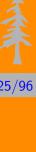


### $\vec{\mathbf{f}} \cdot \vec{\mathbf{v}} = (\rho_{\mathbf{f}} \vec{\mathbf{E}} + \rho_{\mathbf{f}} \vec{\mathbf{v}} \times \vec{\mathbf{B}}) \cdot \vec{\mathbf{v}} = \rho_{\mathbf{f}} \vec{\mathbf{v}} \cdot \vec{\mathbf{E}} = \vec{\mathbf{j}}_{\mathbf{c}} \cdot \vec{\mathbf{E}} = \vec{\mathbf{E}} \cdot (\nabla \times \frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}} - \frac{\partial \epsilon_{\mathbf{0}} \vec{\mathbf{E}}}{\partial \mathbf{t}})$

$$\begin{aligned} \mathbf{F} \cdot \mathbf{v} &= (\rho_{\mathbf{f}} \mathbf{E} + \rho_{\mathbf{f}} \mathbf{v} \times \mathbf{B}) \cdot \mathbf{v} = \rho_{\mathbf{f}} \mathbf{v} \cdot \mathbf{E} = \mathbf{J_{c}} \cdot \mathbf{E} = \mathbf{E} \cdot (\mathbf{v} \times \frac{\mathbf{J}}{\mu_{0}} - \frac{\mathbf{J}}{\partial \mathbf{t}}) \\ &= \vec{\mathbf{E}} \cdot (\nabla \times \frac{\vec{\mathbf{B}}}{\mu_{0}}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_{0} \vec{\mathbf{E}}}{\partial \mathbf{t}} = -\nabla \cdot (\vec{\mathbf{E}} \times \frac{\vec{\mathbf{B}}}{\mu_{0}}) + \frac{\vec{\mathbf{B}}}{\mu_{0}} \cdot (\nabla \times \vec{\mathbf{E}}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_{0} \vec{\mathbf{E}}}{\partial \mathbf{t}} \\ &= -\nabla \cdot (\vec{\mathbf{E}} \times \frac{\vec{\mathbf{B}}}{\mu_{0}}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_{0} \vec{\mathbf{E}}}{\partial \mathbf{t}} - \frac{\vec{\mathbf{B}}}{\mu_{0}} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}} = -\nabla \cdot \vec{\mathbf{S}} - \frac{\partial \mathbf{w}}{\partial \mathbf{t}} \end{aligned}$$

$$\mathbf{S} = \mathbf{E} \times \frac{1}{\mu_0} \qquad \overline{\partial \mathbf{t}} = \mathbf{E} \cdot \frac{\mathbf{v}}{\partial \mathbf{t}} + \frac{1}{\mu_0} \cdot \overline{\partial \mathbf{t}}$$

$$\frac{\partial \mathbf{w}}{\partial \mathbf{t}} = \epsilon_0 \vec{\mathbf{E}} \cdot \frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}} + \frac{1}{\mu_0} \vec{\mathbf{B}} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}} = \frac{\partial}{\partial \mathbf{t}} \frac{1}{2} (\epsilon_0 \mathbf{E}^2 + \frac{1}{\mu_0} \mathbf{B}^2) = \frac{\partial}{\partial \mathbf{t}} \frac{1}{2} (\vec{\mathbf{E}} \cdot \vec{\mathbf{D}} + \vec{\mathbf{H}} \cdot \vec{\mathbf{B}})$$













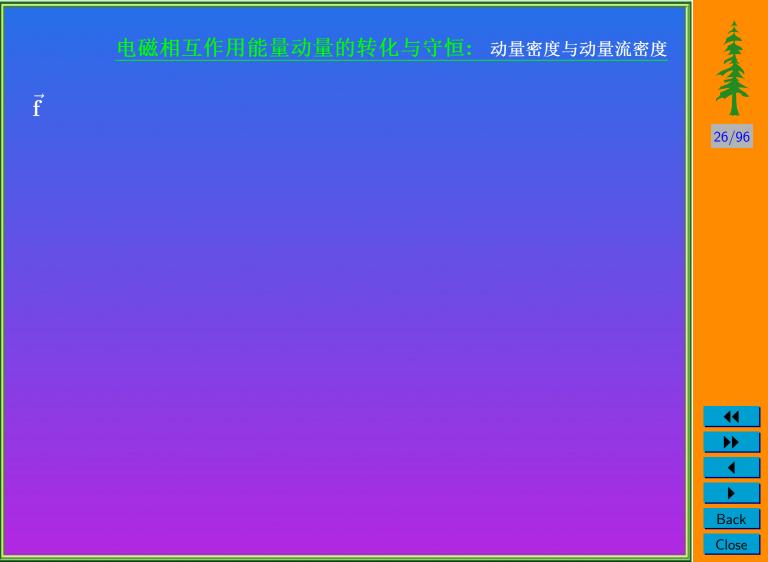
$$\vec{\mathbf{f}} \cdot \vec{\mathbf{v}} = (\rho_{\mathbf{f}} \vec{\mathbf{E}} + \rho_{\mathbf{f}} \vec{\mathbf{v}} \times \vec{\mathbf{B}}) \cdot \vec{\mathbf{v}} = \rho_{\mathbf{f}} \vec{\mathbf{v}} \cdot \vec{\mathbf{E}} = \vec{\mathbf{j}}_{\mathbf{c}} \cdot \vec{\mathbf{E}} = \vec{\mathbf{E}} \cdot (\nabla \times \frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}} - \frac{\partial \epsilon_{\mathbf{0}} \vec{\mathbf{E}}}{\partial \mathbf{t}})$$

$$= \vec{\mathbf{E}} \cdot (\nabla \times \frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_{\mathbf{0}} \vec{\mathbf{E}}}{\partial \mathbf{t}} = -\nabla \cdot (\vec{\mathbf{E}} \times \frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}}) + \frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}} \cdot (\nabla \times \vec{\mathbf{E}}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_{\mathbf{0}} \vec{\mathbf{E}}}{\partial \mathbf{t}}$$

$$= -\nabla \cdot (\vec{\mathbf{E}} \times \frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_{\mathbf{0}} \vec{\mathbf{E}}}{\partial \mathbf{t}} - \frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}} = -\nabla \cdot \vec{\mathbf{S}} - \frac{\partial \mathbf{w}}{\partial \mathbf{t}}$$

$$\mathbf{S} = \mathbf{E} imes \frac{\mathbf{E}}{\mu_0} imes \frac{\mathbf{E}}{\partial \mathbf{t}} = \mathbf{E} \cdot \frac{\mathbf{E}}{\partial \mathbf{t}} + \frac{\mathbf{E}}{\mu_0} \cdot \frac{\mathbf{E}}{\partial \mathbf{t}} + \frac{\mathbf{E}}{\mu_0} \cdot \frac{\mathbf{E}}{\partial \mathbf{t}}$$
 $\frac{\partial \mathbf{W}}{\partial \mathbf{t}} = \epsilon_0 \vec{\mathbf{E}} \cdot \frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}} + \frac{\mathbf{E}}{\mu_0} \cdot \frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}} = \frac{\partial}{\partial t} \frac{\mathbf{E}}{\partial t} \cdot \frac{\partial}{\partial t} = \frac{\partial}{\partial t} \frac{\mathbf{E}}{\partial t} \cdot \frac{\partial}{\partial t} = \frac{\partial}{\partial t} \frac{\mathbf{E}}{\partial t} \cdot \vec{\mathbf{E}} \cdot \vec{\mathbf{$ 

$$\begin{split} \frac{\partial \mathbf{w}}{\partial \mathbf{t}} &= \epsilon_0 \vec{\mathbf{E}} \cdot \frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}} + \frac{1}{\mu_0} \vec{\mathbf{B}} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}} = \frac{\partial}{\partial \mathbf{t}} \frac{1}{2} (\epsilon_0 \mathbf{E^2} + \frac{1}{\mu_0} \mathbf{B^2}) = \frac{\partial}{\partial \mathbf{t}} \frac{1}{2} (\vec{\mathbf{E}} \cdot \vec{\mathbf{D}} + \vec{\mathbf{H}} \cdot \vec{\mathbf{B}}) \\ \mathbf{w} &= \frac{1}{2} (\vec{\mathbf{E}} \cdot \vec{\mathbf{D}} + \vec{\mathbf{H}} \cdot \vec{\mathbf{B}}) & \underline{\mathbf{B}} \underline{\mathbf{A}} \underline{\mathbf{T}} \underline{\mathbf{R}} \underline{\mathbf{B}} \underline{\mathbf{W}} \underline{\mathbf{W}} \underline{\mathbf{B}} \underline{\mathbf{B}} \underline{\mathbf{W}} \underline{\mathbf{W}} \underline{\mathbf{W}} \underline{\mathbf{B}} \underline{\mathbf{W}} \underline{\mathbf{$$



# 动量密度与动量流密度 $ec{\mathbf{f}} = ho_{\mathbf{f}} ec{\mathbf{E}} + ec{\mathbf{j}}_{\mathbf{c}} imes ec{\mathbf{B}}$



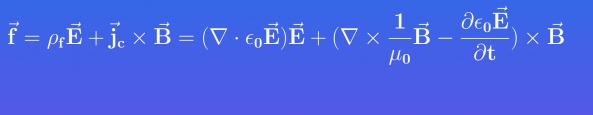






## 动量密度与动量流密度











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### 直磁相互作用能量动量的转化与守恒: 动量密度与动量流密度



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 $|\vec{\mathbf{f}} = \rho_{\mathbf{f}}\vec{\mathbf{E}} + \vec{\mathbf{j}}_{\mathbf{c}} \times \vec{\mathbf{B}} = (\nabla \cdot \epsilon_{\mathbf{0}}\vec{\mathbf{E}})\vec{\mathbf{E}} + (\nabla \times \frac{1}{\mu_{\mathbf{0}}}\vec{\mathbf{B}} - \frac{\partial \epsilon_{\mathbf{0}}\vec{\mathbf{E}}}{\partial \mathbf{t}}) \times \vec{\mathbf{B}}|$  $=\epsilon_{\mathbf{0}}(\nabla\cdot\vec{\mathbf{E}})\vec{\mathbf{E}}+rac{1}{\mu_{\mathbf{0}}}(
abla imesec{\mathbf{B}}) imesec{\mathbf{B}}-\epsilon_{\mathbf{0}}rac{\partialec{ec{\mathbf{E}}}}{\partial\mathbf{t}} imesec{\mathbf{B}}$ 







## 自磁相互作用能量动量的转化与守恒: 动量密度与动量流密度



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$$\vec{\mathbf{f}} = \rho_{\mathbf{f}} \vec{\mathbf{E}} + \vec{\mathbf{j}}_{\mathbf{c}} \times \vec{\mathbf{B}} = (\nabla \cdot \epsilon_{\mathbf{0}} \vec{\mathbf{E}}) \vec{\mathbf{E}} + (\nabla \times \frac{1}{\mu_{\mathbf{0}}} \vec{\mathbf{B}} - \frac{\partial \epsilon_{\mathbf{0}} \vec{\mathbf{E}}}{\partial \mathbf{t}}) \times \vec{\mathbf{B}}$$
$$= \epsilon_{\mathbf{0}} (\nabla \cdot \vec{\mathbf{E}}) \vec{\mathbf{E}} + \frac{1}{\mu_{\mathbf{0}}} (\nabla \times \vec{\mathbf{B}}) \times \vec{\mathbf{B}} - \epsilon_{\mathbf{0}} \frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}} \times \vec{\mathbf{B}}$$

$$= \epsilon_{0}(\nabla \cdot \vec{\mathbf{E}})\vec{\mathbf{E}} + \frac{1}{\mu_{0}}[(\nabla \cdot \vec{\mathbf{B}})\vec{\mathbf{B}} + (\nabla \times \vec{\mathbf{B}}) \times \vec{\mathbf{B}}]$$
$$-\epsilon_{0}[\frac{\partial \vec{\mathbf{E}}}{\partial t} \times \vec{\mathbf{B}} + \vec{\mathbf{E}} \times \frac{\partial \vec{\mathbf{B}}}{\partial t} + \vec{\mathbf{E}} \times (\nabla \times \vec{\mathbf{E}})]$$







# 自磁相互作用能量动量的转化与守恒: 动量密度与动量流密度 $|\vec{\mathbf{f}} = \rho_{\mathbf{f}}\vec{\mathbf{E}} + \vec{\mathbf{j}}_{\mathbf{c}} \times \vec{\mathbf{B}} = (\nabla \cdot \epsilon_{\mathbf{0}}\vec{\mathbf{E}})\vec{\mathbf{E}} + (\nabla \times \frac{1}{\mu_{\mathbf{0}}}\vec{\mathbf{B}} - \frac{\partial \epsilon_{\mathbf{0}}\vec{\mathbf{E}}}{\partial \mathbf{t}}) \times \vec{\mathbf{B}}|$

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$$= \epsilon_{0}(\nabla \cdot \vec{\mathbf{E}})\vec{\mathbf{E}} + \frac{1}{\mu_{0}}(\nabla \times \vec{\mathbf{B}}) \times \vec{\mathbf{B}} - \epsilon_{0}\frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}} \times \vec{\mathbf{B}}$$

$$= \epsilon_0 (\nabla \cdot \vec{\mathbf{E}}) \vec{\mathbf{E}} + \frac{1}{\mu_0} [(\nabla \cdot \vec{\mathbf{B}}) \vec{\mathbf{B}} + (\nabla \times \vec{\mathbf{B}}) \times \vec{\mathbf{B}}]$$

$$-\epsilon_{0}\left[\frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}} \times \vec{\mathbf{B}} + \vec{\mathbf{E}} \times \frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}} + \vec{\mathbf{E}} \times (\nabla \times \vec{\mathbf{E}})\right]$$

$$= \epsilon_{\mathbf{0}}[(\nabla \cdot \vec{\mathbf{E}})\vec{\mathbf{E}} + (\nabla \times \vec{\mathbf{E}}) \times \vec{\mathbf{E}}] + \frac{1}{\mu_{\mathbf{0}}}[(\nabla \cdot \vec{\mathbf{B}})\vec{\mathbf{B}} + (\nabla \times \vec{\mathbf{B}}) \times \vec{\mathbf{B}}]$$
$$-\epsilon_{\mathbf{0}}\frac{\partial}{\partial \mathbf{t}}(\vec{\mathbf{E}} \times \vec{\mathbf{B}})$$





# 中磁相互作用能量动量的转化与守恒: 动量密度与动量流密度 $ec{\mathbf{f}} = ho_{\mathbf{f}} \vec{\mathbf{E}} + \vec{\mathbf{j}}_{\mathbf{c}} \times \vec{\mathbf{B}} = (\nabla \cdot \epsilon_{\mathbf{0}} \vec{\mathbf{E}}) \vec{\mathbf{E}} + (\nabla \times \frac{1}{\mu_{\mathbf{0}}} \vec{\mathbf{B}} - \frac{\partial \epsilon_{\mathbf{0}} \vec{\mathbf{E}}}{\partial \mathbf{t}}) \times \vec{\mathbf{B}}$

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$$= \epsilon_0 (\nabla \cdot \vec{\mathbf{E}}) \vec{\mathbf{E}} + \frac{1}{\mu_0} (\nabla \times \vec{\mathbf{B}}) \times \vec{\mathbf{B}} - \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}} \times \vec{\mathbf{B}}$$

$$= \epsilon_0 (\nabla \cdot \vec{\mathbf{E}}) \vec{\mathbf{E}} + \frac{1}{\mu_0} [(\nabla \cdot \vec{\mathbf{B}}) \vec{\mathbf{B}} + (\nabla \times \vec{\mathbf{B}}) \times \vec{\mathbf{B}}]$$

$$-\epsilon_{\mathbf{0}}\left[\frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}} \times \vec{\mathbf{B}} + \vec{\mathbf{E}} \times \frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}} + \vec{\mathbf{E}} \times (\nabla \times \vec{\mathbf{E}})\right]$$

$$= \epsilon_0 [(\nabla \cdot \vec{\mathbf{E}}) \vec{\mathbf{E}} + (\nabla \times \vec{\mathbf{E}}) \times \vec{\mathbf{E}}] + \frac{1}{\mu_0} [(\nabla \cdot \vec{\mathbf{B}}) \vec{\mathbf{B}} + (\nabla \times \vec{\mathbf{B}}) \times \vec{\mathbf{B}}]$$
$$-\epsilon_0 \frac{\partial}{\partial t} (\vec{\mathbf{E}} \times \vec{\mathbf{B}})$$

$$= \epsilon_0 [(\nabla \cdot \vec{\mathbf{E}})\vec{\mathbf{E}} + (\vec{\mathbf{E}} \cdot \nabla)\vec{\mathbf{E}} - (\nabla \vec{\mathbf{E}}) \cdot \vec{\mathbf{E}}]$$

$$[\cdot \vec{\mathbf{B}}] - \epsilon_0 \frac{\partial}{\partial t} (\vec{\mathbf{E}} \times \vec{\mathbf{B}})$$

$$= \epsilon_{0}[(\nabla \cdot \mathbf{E})\mathbf{E} + (\mathbf{E} \cdot \nabla)\mathbf{E} - (\nabla \mathbf{E}) \cdot \mathbf{E}]$$

$$+ \frac{1}{\mu_{0}}[(\nabla \cdot \vec{\mathbf{B}})\vec{\mathbf{B}} + (\vec{\mathbf{B}} \cdot \nabla)\vec{\mathbf{B}} - (\nabla \vec{\mathbf{B}}) \cdot \vec{\mathbf{B}}] - \epsilon_{0}\frac{\partial}{\partial \mathbf{t}}(\vec{\mathbf{E}} \times \vec{\mathbf{B}})$$

## 动量密度与动量流密度 $\vec{\mathbf{f}} = \epsilon_0 [(\nabla \cdot \vec{\mathbf{E}}) \vec{\mathbf{E}} + (\vec{\mathbf{E}} \cdot \nabla) \vec{\mathbf{E}} - (\nabla \vec{\mathbf{E}}) \cdot \vec{\mathbf{E}}]$

 $+\frac{1}{\mu_{\mathbf{0}}}[(\nabla \cdot \vec{\mathbf{B}})\vec{\mathbf{B}} + (\vec{\mathbf{B}} \cdot \nabla)\vec{\mathbf{B}} - (\nabla \vec{\mathbf{B}}) \cdot \vec{\mathbf{B}}] - \epsilon_{\mathbf{0}}\frac{\partial}{\partial \mathbf{t}}(\vec{\mathbf{E}} \times \vec{\mathbf{B}})$ 











# <u>电磁相互作用能量动量的转化与守恒</u>: 动量密度与动量流密度

$$\begin{split} \vec{\mathbf{f}} &= \epsilon_0 [(\nabla \cdot \vec{\mathbf{E}}) \vec{\mathbf{E}} + (\vec{\mathbf{E}} \cdot \nabla) \vec{\mathbf{E}} - (\nabla \vec{\mathbf{E}}) \cdot \vec{\mathbf{E}}] \\ &+ \frac{1}{\mu_0} [(\nabla \cdot \vec{\mathbf{B}}) \vec{\mathbf{B}} + (\vec{\mathbf{B}} \cdot \nabla) \vec{\mathbf{B}} - (\nabla \vec{\mathbf{B}}) \cdot \vec{\mathbf{B}}] - \epsilon_0 \frac{\partial}{\partial t} (\vec{\mathbf{E}} \times \vec{\mathbf{B}}) \\ &= \epsilon_0 [\nabla \cdot (\vec{\mathbf{E}} \vec{\mathbf{E}}) - \frac{1}{2} \nabla \mathbf{E}^2] + \frac{1}{\mu_0} [\nabla \cdot (\vec{\mathbf{B}} \vec{\mathbf{B}}) - \frac{1}{2} \nabla \mathbf{B}^2] - \epsilon_0 \frac{\partial}{\partial t} (\vec{\mathbf{E}} \times \vec{\mathbf{B}}) \end{split}$$









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# 电磁相工作用能量动量的转化与守恒; 动量密度与动量流密度

$$\begin{split} \vec{\mathbf{f}} &= \epsilon_0 [(\nabla \cdot \vec{\mathbf{E}}) \vec{\mathbf{E}} + (\vec{\mathbf{E}} \cdot \nabla) \vec{\mathbf{E}} - (\nabla \vec{\mathbf{E}}) \cdot \vec{\mathbf{E}}] \\ &+ \frac{1}{\mu_0} [(\nabla \cdot \vec{\mathbf{B}}) \vec{\mathbf{B}} + (\vec{\mathbf{B}} \cdot \nabla) \vec{\mathbf{B}} - (\nabla \vec{\mathbf{B}}) \cdot \vec{\mathbf{B}}] - \epsilon_0 \frac{\partial}{\partial t} (\vec{\mathbf{E}} \times \vec{\mathbf{B}}) \\ &= \epsilon_0 [\nabla \cdot (\vec{\mathbf{E}} \vec{\mathbf{E}}) - \frac{1}{2} \nabla \mathbf{E}^2] + \frac{1}{\mu_0} [\nabla \cdot (\vec{\mathbf{B}} \vec{\mathbf{B}}) - \frac{1}{2} \nabla \mathbf{B}^2] - \epsilon_0 \frac{\partial}{\partial t} (\vec{\mathbf{E}} \times \vec{\mathbf{B}}) \\ &= \epsilon_0 \nabla \cdot (\vec{\mathbf{E}} \vec{\mathbf{E}} - \frac{1}{2} \overrightarrow{\mathbf{I}} \vec{\mathbf{E}}^2) + \frac{1}{\mu_0} \nabla \cdot (\vec{\mathbf{B}} \vec{\mathbf{B}} - \frac{1}{2} \overrightarrow{\mathbf{I}} \vec{\mathbf{B}}^2) - \epsilon_0 \frac{\partial}{\partial t} (\vec{\mathbf{E}} \times \vec{\mathbf{B}}) \end{split}$$



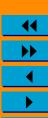






# 中磁和互作用能量动量的转化与守恒: 动量密度与动量流密度

$$\begin{split} \vec{\mathbf{f}} &= \epsilon_0 [(\nabla \cdot \vec{\mathbf{E}}) \vec{\mathbf{E}} + (\vec{\mathbf{E}} \cdot \nabla) \vec{\mathbf{E}} - (\nabla \vec{\mathbf{E}}) \cdot \vec{\mathbf{E}}] \\ &+ \frac{1}{\mu_0} [(\nabla \cdot \vec{\mathbf{B}}) \vec{\mathbf{B}} + (\vec{\mathbf{B}} \cdot \nabla) \vec{\mathbf{B}} - (\nabla \vec{\mathbf{B}}) \cdot \vec{\mathbf{B}}] - \epsilon_0 \frac{\partial}{\partial t} (\vec{\mathbf{E}} \times \vec{\mathbf{B}}) \\ &= \epsilon_0 [\nabla \cdot (\vec{\mathbf{E}} \vec{\mathbf{E}}) - \frac{1}{2} \nabla \mathbf{E}^2] + \frac{1}{\mu_0} [\nabla \cdot (\vec{\mathbf{B}} \vec{\mathbf{B}}) - \frac{1}{2} \nabla \mathbf{B}^2] - \epsilon_0 \frac{\partial}{\partial t} (\vec{\mathbf{E}} \times \vec{\mathbf{B}}) \\ &= \epsilon_0 \nabla \cdot (\vec{\mathbf{E}} \vec{\mathbf{E}} - \frac{1}{2} \vec{\mathbf{I}} \vec{\mathbf{E}}^2) + \frac{1}{\mu_0} \nabla \cdot (\vec{\mathbf{B}} \vec{\mathbf{B}} - \frac{1}{2} \vec{\mathbf{I}} \vec{\mathbf{B}}^2) - \epsilon_0 \frac{\partial}{\partial t} (\vec{\mathbf{E}} \times \vec{\mathbf{B}}) \\ &= -\nabla \cdot \vec{\mathcal{J}} - \frac{\partial \vec{\mathbf{g}}}{\partial t} \end{split}$$





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申磁相互作用能量动量的转化与守恒: 动量密度与动量流密度  $\overrightarrow{\mathbf{f}} = \epsilon_{\mathbf{0}} [(\nabla \cdot \overrightarrow{\mathbf{E}}) \overrightarrow{\mathbf{E}} + (\overrightarrow{\mathbf{E}} \cdot \nabla) \overrightarrow{\mathbf{E}} - (\nabla \overrightarrow{\mathbf{E}}) \cdot \overrightarrow{\mathbf{E}}]$ 

$$+\frac{1}{\mu_{0}}[(\nabla \cdot \vec{\mathbf{B}})\vec{\mathbf{B}} + (\vec{\mathbf{B}} \cdot \nabla)\vec{\mathbf{B}} - (\nabla \vec{\mathbf{B}}) \cdot \vec{\mathbf{B}}] - \epsilon_{0}\frac{\partial}{\partial \mathbf{t}}(\vec{\mathbf{E}} \times \vec{\mathbf{B}})$$

$$= \epsilon_{0}[\nabla \cdot (\vec{\mathbf{E}}\vec{\mathbf{E}}) - \frac{1}{2}\nabla\vec{\mathbf{E}}^{2}] + \frac{1}{\mu_{0}}[\nabla \cdot (\vec{\mathbf{B}}\vec{\mathbf{B}}) - \frac{1}{2}\nabla\vec{\mathbf{B}}^{2}] - \epsilon_{0}\frac{\partial}{\partial \mathbf{t}}(\vec{\mathbf{E}} \times \vec{\mathbf{B}})$$

$$= \epsilon_{0}\nabla \cdot (\vec{\mathbf{E}}\vec{\mathbf{E}} - \frac{1}{2}\vec{\mathbf{I}}\vec{\mathbf{E}}^{2}) + \frac{1}{\mu_{0}}\nabla \cdot (\vec{\mathbf{B}}\vec{\mathbf{B}} - \frac{1}{2}\vec{\mathbf{I}}\vec{\mathbf{B}}^{2}) - \epsilon_{0}\frac{\partial}{\partial \mathbf{t}}(\vec{\mathbf{E}} \times \vec{\mathbf{B}})$$

$$= -\nabla \cdot \stackrel{\rightharpoonup}{\mathcal{J}} - \frac{\partial \vec{\mathbf{g}}}{\partial \mathbf{t}}$$
$$\vec{\mathbf{g}} = \epsilon_0 \vec{\mathbf{E}} \times \vec{\mathbf{B}} = \epsilon_0 \mu_0 \vec{\mathbf{S}}$$

$$\vec{\mathcal{J}} = -\epsilon_0 \vec{\mathbf{E}} \vec{\mathbf{E}} - \frac{1}{\mu_0} \vec{\mathbf{B}} \vec{\mathbf{B}} + \frac{1}{2} \vec{\mathbf{I}} (\epsilon_0 \mathbf{E^2} + \frac{1}{\mu_0} \mathbf{B^2})$$

$$\mu_0$$
  $\mu_0$   $\mu_0$   $\mu_0$  两带电体之间的牛顿第三定律一般不成立!  $\vec{f}_{21}=\frac{dP_1}{dt}$   $\vec{f}_{12}=\frac{dP_2}{dt}$ 

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# 张量力 选体积为V的内部区域受外部电磁力作用:



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选体积为V的内部区域受外部电磁力作用:

$$-\oint_{\mathbf{V}}\mathbf{d}ec{\sigma}\cdot\stackrel{\rightharpoonup}{\mathcal{J}}=\int_{\mathbf{V}}\mathbf{d}\mathbf{V}\vec{\mathbf{f}}+rac{\mathbf{d}}{\mathbf{d}t}\int_{\mathbf{V}}\mathbf{d}\mathbf{V}\vec{\mathbf{g}}$$



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选体积为V的内部区域受外部电磁力作用:

$$-\oint_{\mathbf{V}}\mathbf{d}ec{\sigma}\cdot\stackrel{
ightharpoonup}{\mathcal{J}}=\int_{\mathbf{V}}\mathbf{d}\mathbf{V}ec{\mathbf{f}}+rac{\mathbf{d}}{\mathbf{dt}}\int_{\mathbf{V}}\mathbf{d}\mathbf{V}ec{\mathbf{g}}$$







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选体积为V的内部区域受外部电磁力作用:

$$\begin{split} -\oint_{V} d\vec{\sigma} \cdot \stackrel{\rightharpoonup}{\mathcal{J}} &= \int_{V} dV \vec{f} + \frac{d}{dt} \int_{V} dV \vec{g} \\ \int_{V} dV \vec{f} &= \frac{d\vec{P}_{\text{Mid}}}{dt} \quad \vec{P}_{\text{Hdd}} \equiv & \int_{V} dV \vec{g} \quad \rightarrow \quad \underbrace{\frac{d}{dt} [\vec{P}_{\text{Mid}} + \vec{P}_{\text{Hdd}}] = -\oint_{V} d\vec{\sigma} \cdot \vec{\mathcal{J}}}_{\text{Hdd} + \hat{\mathbf{J}} + \hat{\mathbf{$$

 $-\mathcal{J}$  是V单位表面所受外面的力.



**>>** 

选体积为V的内部区域受外部电磁力作用:

$$\begin{split} -\oint_{V} d\vec{\sigma} \cdot \stackrel{\rightharpoonup}{\mathcal{J}} &= \int_{V} dV \vec{f} + \frac{d}{dt} \int_{V} dV \vec{g} \\ \int_{V} dV \vec{f} &= \frac{d\vec{P}_{\text{MM}}}{dt} \quad \vec{P}_{\text{MM}} \equiv & \int_{V} dV \vec{g} \ \rightarrow \ \frac{d}{dt} [\vec{P}_{\text{MM}} + \vec{P}_{\text{MM}}] = -\oint_{V} d\vec{\sigma} \cdot \stackrel{\rightharpoonup}{\mathcal{J}} \end{split}$$

 $-\mathcal{J}$  是V单位表面所受外面的力. $\overline{\mathbf{k}}$  面有电磁场就要受力.



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#### 张量力

选体积为V的内部区域受外部电磁力作用:

$$-\oint_{V} d\vec{\sigma} \cdot \stackrel{\rightharpoonup}{\mathcal{J}} = \int_{V} dV \vec{f} + \frac{d}{dt} \int_{V} dV \vec{g} \qquad V$$
 
$$\int_{V} dV \vec{f} = \frac{d\vec{P}_{\text{flikk}}}{dt} \quad \vec{P}_{\text{flikk}} \equiv \int_{V} dV \vec{g} \quad \rightarrow \quad \frac{d}{dt} [\vec{P}_{\text{flikk}} + \vec{P}_{\text{flikk}}] = -\oint_{V} d\vec{\sigma} \cdot \vec{\mathcal{J}}$$

 $-\mathcal{J}$  是V单位表面所受外面的力.表面有电磁场就要受力.

表面法向方向单位矢量 $\vec{n}$ , 电场与 $\vec{n}$ 的夹角 $\theta_E$ , 电场投影所在的切向方向单位矢量 $\vec{e}_E$ , 磁 场与 $\vec{n}$ 的夹角 $\theta_B$ , 磁场投影所在切向方向单位矢量 $\vec{e}_B$ .







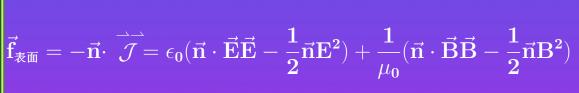
#### 张量力

选体积为V的内部区域受外部电磁力作用:

$$\begin{split} -\oint_{V} d\vec{\sigma} \cdot \stackrel{\rightharpoonup}{\overrightarrow{\mathcal{J}}} &= \int_{V} dV \vec{f} + \frac{d}{dt} \int_{V} dV \vec{g} \\ \int_{V} dV \vec{f} &= \frac{d\vec{P}_{\text{Mid}}}{dt} \quad \vec{P}_{\text{Hdd}} \equiv & \int_{V} dV \vec{g} \quad \rightarrow \quad \underbrace{\frac{d}{dt} [\vec{P}_{\text{Mid}} + \vec{P}_{\text{Hdd}}]}_{\text{V}} &= -\oint_{V} d\vec{\sigma} \cdot \stackrel{\rightharpoonup}{\mathcal{J}} \end{split}$$

 $-\mathcal{J}$  是V单位表面所受外面的力.表面有电磁场就要受力.

表面法向方向单位矢量 $\vec{n}$ , 电场与 $\vec{n}$ 的夹角 $\theta_E$ , 电场投影所在的切向方向单位矢量 $\vec{e}_E$ , 磁 场与 $\vec{n}$ 的夹角 $\theta_B$ ,磁场投影所在切向方向单位矢量 $\vec{e}_B$ . 此表面单位表面所受力:









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#### 张量力

选体积为1/的内部区域受外部电磁力作用:

$$\begin{split} -\oint_{V} d\vec{\sigma} \cdot \stackrel{\rightharpoonup}{\mathcal{J}} &= \int_{V} dV \vec{f} + \frac{d}{dt} \int_{V} dV \vec{g} \\ \int_{V} dV \vec{f} &= \frac{d\vec{P}_{\text{Mid}}}{dt} \quad \vec{P}_{\text{Mid}} \equiv & \int_{V} dV \vec{g} \quad \rightarrow \quad \frac{d}{dt} [\vec{P}_{\text{Mid}} + \vec{P}_{\text{Mid}}] = -\oint_{V} d\vec{\sigma} \cdot \stackrel{\rightharpoonup}{\mathcal{J}} \end{split}$$

 $-\mathcal{J}$  是V单位表面所受外面的力.表面有电磁场就要受力.

表面法向方向单位矢量 $\vec{n}$ , 电场与 $\vec{n}$ 的夹角 $\theta_E$ , 电场投影所在的切向方向单位矢量 $\vec{e}_E$ , 磁 场与 $\vec{n}$ 的夹角 $\theta_B$ , 磁场投影所在切向方向单位矢量 $\vec{e}_B$ . 此表面单位表面所受力:

$$ec{\mathbf{f}}_{ar{k}ar{\mathbf{n}}} = -ec{\mathbf{n}}\cdot\stackrel{
ightharpoonup}{\mathcal{J}} = \epsilon_{\mathbf{0}}(ec{\mathbf{n}}\cdotec{\mathbf{E}}ec{\mathbf{E}} - rac{1}{2}ec{\mathbf{n}}\mathbf{E^2}) + rac{1}{\mu_{\mathbf{0}}}(ec{\mathbf{n}}\cdotec{\mathbf{B}}ec{\mathbf{B}} - rac{1}{2}ec{\mathbf{n}}\mathbf{B^2}) + rac{1}{\mu_{\mathbf{0}}}(ec{\mathbf{n}}\cdotec{\mathbf{B}}ec{\mathbf{B}} - rac{1}{2}ec{\mathbf{n}}\mathbf{B^2})$$

$$= (\mathbf{E}\cos\theta_{\mathbf{E}}\vec{\mathbf{n}} + \mathbf{E}\sin\theta_{\mathbf{E}}\vec{\mathbf{e}}_{\mathbf{E}})\epsilon_{\mathbf{0}}\mathbf{E}\cos\theta_{\mathbf{E}} - \frac{1}{2}\epsilon_{\mathbf{0}}\vec{\mathbf{n}}\mathbf{E}^{\mathbf{2}}$$
$$(\mathbf{B}\cos\theta_{\mathbf{B}}\vec{\mathbf{n}} + \mathbf{B}\sin\theta_{\mathbf{B}}\vec{\mathbf{e}}_{\mathbf{B}})\frac{\mathbf{B}}{\mu_{\mathbf{0}}}\cos\theta_{\mathbf{B}} - \frac{1}{2\mu_{\mathbf{0}}}\vec{\mathbf{n}}\mathbf{B}^{\mathbf{2}}$$





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选体积为V的内部区域受外部电磁力作用:

$$\begin{split} -\oint_{V} d\vec{\sigma} \cdot \stackrel{\rightharpoonup}{\mathcal{J}} &= \int_{V} dV \vec{f} + \frac{d}{dt} \int_{V} dV \vec{g} \\ \int_{V} dV \vec{f} &= \frac{d\vec{P}_{\text{Mid}}}{dt} \quad \vec{P}_{\text{Mid}} \equiv & \int_{V} dV \vec{g} \quad \rightarrow \quad \underbrace{\frac{d}{dt} [\vec{P}_{\text{Mid}} + \vec{P}_{\text{Mid}}] = -\oint_{V} d\vec{\sigma} \cdot \vec{\mathcal{J}}}_{\text{CP}} \end{split}$$

 $-\mathcal{I}$  是V单位表面所受外面的力. $\overline{z}$  直有电磁场就要受力.

表面法向方向单位矢量 $\vec{n}$ ,电场与 $\vec{n}$ 的夹角 $\theta_E$ ,电场投影所在的切向方向单位矢量 $\vec{e}_E$ ,磁场与 $\vec{n}$ 的夹角 $\theta_B$ ,磁场投影所在切向方向单位矢量 $\vec{e}_B$ . 此表面单位表面所受力:

$$\begin{split} \vec{\mathbf{f}}_{\overline{\xi}\overline{\mathbf{m}}} &= -\vec{\mathbf{n}} \cdot \stackrel{\rightharpoonup}{\mathcal{J}} = \epsilon_{\mathbf{0}} (\vec{\mathbf{n}} \cdot \vec{\mathbf{E}}\vec{\mathbf{E}} - \frac{1}{2}\vec{\mathbf{n}}\mathbf{E}^{2}) + \frac{1}{\mu_{\mathbf{0}}} (\vec{\mathbf{n}} \cdot \vec{\mathbf{B}}\vec{\mathbf{B}} - \frac{1}{2}\vec{\mathbf{n}}\mathbf{B}^{2}) \\ &= (\mathbf{E}\cos\theta_{\mathbf{E}}\vec{\mathbf{n}} + \mathbf{E}\sin\theta_{\mathbf{E}}\vec{\mathbf{e}}_{\mathbf{E}})\epsilon_{\mathbf{0}}\mathbf{E}\cos\theta_{\mathbf{E}} - \frac{1}{2}\epsilon_{\mathbf{0}}\vec{\mathbf{n}}\mathbf{E}^{2} \end{split}$$

$$(\mathbf{B}\cos heta_{\mathbf{B}}\vec{\mathbf{n}} + \mathbf{B}\sin heta_{\mathbf{B}}\vec{\mathbf{e}}_{\mathbf{B}})rac{\mathbf{B}}{\mu_{\mathbf{0}}}\cos heta_{\mathbf{B}} - rac{\mathbf{I}}{2\mu_{\mathbf{0}}}\vec{\mathbf{n}}\mathbf{B}^{2} = \vec{\mathbf{f}}_{\mathbf{E}} + \vec{\mathbf{f}}_{\mathbf{B}}$$



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选体积为V的内部区域受外部电磁力作用:

$$-\oint_{V} d\vec{\sigma} \cdot \stackrel{\rightharpoonup}{\mathcal{J}} = \int_{V} dV \vec{f} + \frac{d}{dt} \int_{V} dV \vec{g}$$
 
$$\int_{V} dV \vec{f} = \frac{d\vec{P}_{\text{MM}}}{dt} \quad \vec{P}_{\text{HM}} \equiv \int_{V} dV \vec{g} \quad \rightarrow \quad \frac{d}{dt} [\vec{P}_{\text{MM}} + \vec{P}_{\text{HM}}] = -\oint_{V} d\vec{\sigma} \cdot \vec{\mathcal{J}}$$

 $-\mathcal{J}$  是V单位表面所受外面的力. $\frac{1}{2}$  起V 单位表面所受外面的力.

表面法向方向单位矢量 $\vec{n}$ ,电场与 $\vec{n}$ 的夹角 $\theta_E$ ,电场投影所在的切向方向单位矢量 $\vec{e}_E$ ,磁场与 $\vec{n}$ 的夹角 $\theta_B$ ,磁场投影所在切向方向单位矢量 $\vec{e}_B$ . 此表面单位表面所受力:

场与前的夹角
$$heta_{
m B}$$
,磁场投影所在切向方向单位矢量 $ec{
m e}_{
m B}$ .此表面单位表面所受力: $ec{
m f}_{
m ar{z}}=-ec{
m r}\cdot \overset{
ightharpoonup}{\mathcal{J}}=\epsilon_0(ec{
m r}\cdot ec{
m E}ec{
m E}-rac{1}{2}ec{
m r}{
m E}^2)+rac{1}{2}(ec{
m r}\cdot ec{
m B}ec{
m B}-rac{1}{2}ec{
m r}{
m B}^2)$ 

$$= (\mathbf{E}\cos\theta_{\mathbf{E}}\vec{\mathbf{n}} + \mathbf{E}\sin\theta_{\mathbf{E}}\vec{\mathbf{e}}_{\mathbf{E}})\epsilon_{\mathbf{0}}\mathbf{E}\cos\theta_{\mathbf{E}} - \frac{1}{2}\epsilon_{\mathbf{0}}\vec{\mathbf{n}}\mathbf{E}^{2}$$

$$(\mathbf{B}\cos\theta_{\mathbf{B}}\vec{\mathbf{n}} + \mathbf{B}\sin\theta_{\mathbf{B}}\vec{\mathbf{e}}_{\mathbf{B}})\frac{\mathbf{B}}{\mu_{\mathbf{0}}}\cos\theta_{\mathbf{B}} - \frac{\mathbf{1}}{2\mu_{\mathbf{0}}}\vec{\mathbf{n}}\mathbf{B}^{2} = \vec{\mathbf{f}}_{\mathbf{E}} + \vec{\mathbf{f}}_{\mathbf{B}}$$

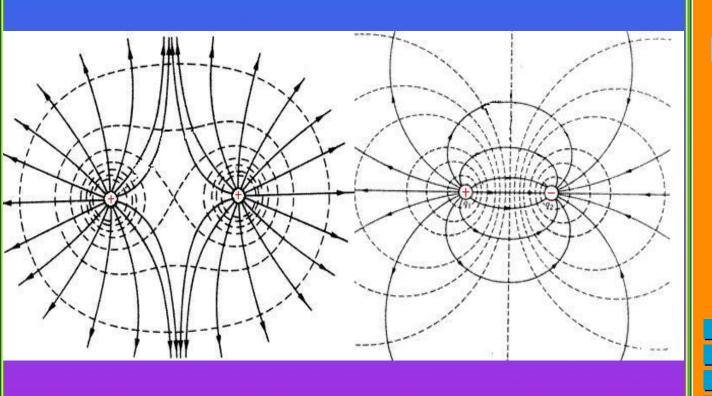
$$\vec{\mathbf{f}}_{\mathbf{E}} = \frac{1}{2} \epsilon_0 \mathbf{E}^2 (\vec{\mathbf{n}} \cos 2\theta_{\mathbf{E}} + \vec{\mathbf{e}}_{\mathbf{E}} \sin 2\theta_{\mathbf{E}}) \qquad \vec{\mathbf{f}}_{\mathbf{B}} = \frac{1}{2\mu_0} \mathbf{B}^2 (\vec{\mathbf{n}} \cos 2\theta_{\mathbf{B}} + \vec{\mathbf{e}}_{\mathbf{B}} \sin 2\theta_{\mathbf{B}})$$



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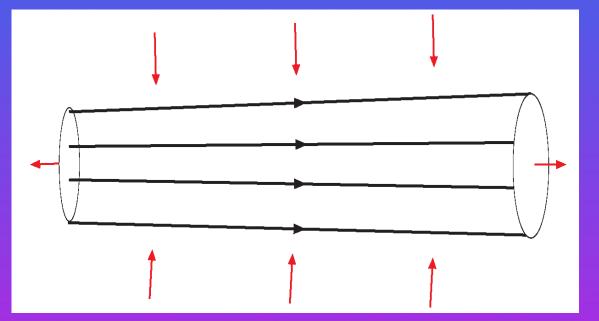






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#### 法拉第提出电力线、磁力线的橡皮盖受力模型



如前所述,目前场是没有受力的概念的,是什么受力呢?



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#### 电磁场出现在物质表面.单位表面受的力为

$$egin{aligned} ec{\mathbf{f}} &= -ec{\mathbf{n}} \cdot \stackrel{
ightharpoonup}{\mathcal{J}} = ec{\mathbf{f}}_{\mathrm{E}} + ec{\mathbf{f}}_{\mathrm{B}} \ ec{\mathbf{f}}_{\mathrm{E}} &= rac{1}{2} \epsilon_0 \mathbf{E}^2 (ec{\mathbf{n}} \cos 2 heta_{\mathrm{E}} + ec{\mathbf{e}}_{\mathrm{E}} \sin 2 heta_{\mathrm{E}}) \ ec{\mathbf{f}}_{\mathrm{B}} &= rac{1}{2\mu_0} \mathbf{B}^2 (ec{\mathbf{n}} \cos 2 heta_{\mathrm{B}} + ec{\mathbf{e}}_{\mathrm{B}} \sin 2 heta_{\mathrm{B}}) \end{aligned}$$



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#### 辐射压力(光压)

#### 电磁场出现在物质表面. 单位表面受的力为

$$egin{aligned} ec{\mathbf{f}} &= -ec{\mathbf{n}} \cdot \stackrel{
ightharpoonup}{\mathcal{J}} = ec{\mathbf{f}}_{\mathrm{E}} + ec{\mathbf{f}}_{\mathrm{B}} \ ec{\mathbf{f}}_{\mathrm{E}} &= rac{1}{2} \epsilon_0 \mathbf{E}^2 (ec{\mathbf{n}} \cos 2 heta_{\mathrm{E}} + ec{\mathbf{e}}_{\mathrm{E}} \sin 2 heta_{\mathrm{E}}) \ ec{\mathbf{f}}_{\mathrm{B}} &= rac{1}{2\mu_0} \mathbf{B}^2 (ec{\mathbf{n}} \cos 2 heta_{\mathrm{B}} + ec{\mathbf{e}}_{\mathrm{B}} \sin 2 heta_{\mathrm{B}}) \end{aligned}$$

若电磁场的方向不确定,在所有方向上都等几率出现:



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#### 电磁场出现在物质表面. 单位表面受的力为

$$egin{aligned} ec{\mathbf{f}} &= -ec{\mathbf{n}} \cdot \stackrel{
ightharpoonup}{\mathcal{J}} = ec{\mathbf{f}}_{\mathrm{E}} + ec{\mathbf{f}}_{\mathrm{B}} \ ec{\mathbf{f}}_{\mathrm{E}} &= rac{1}{2} \epsilon_0 \mathbf{E}^2 (ec{\mathbf{n}} \cos 2 heta_{\mathrm{E}} + ec{\mathbf{e}}_{\mathrm{E}} \sin 2 heta_{\mathrm{E}}) \ ec{\mathbf{f}}_{\mathrm{B}} &= rac{1}{2\mu_0} \mathbf{B}^2 (ec{\mathbf{n}} \cos 2 heta_{\mathrm{B}} + ec{\mathbf{e}}_{\mathrm{B}} \sin 2 heta_{\mathrm{B}}) \end{aligned}$$

若电磁场的方向不确定,在所有方向上都等几率出现:

$$\frac{1}{\cos 2\theta} = \frac{\int d\Omega \cos 2\theta}{\int d\Omega} = \frac{\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin \theta \cos 2\theta}{\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin \theta} = -\frac{1}{3}$$







#### 电磁场出现在物质表面. 单位表面受的力为

$$egin{aligned} ec{\mathbf{f}} &= -ec{\mathbf{n}} \cdot \stackrel{
ightharpoonup}{\mathcal{J}} = ec{\mathbf{f}}_{\mathrm{E}} + ec{\mathbf{f}}_{\mathrm{B}} \ ec{\mathbf{f}}_{\mathrm{E}} &= rac{1}{2} \epsilon_0 \mathbf{E}^2 (ec{\mathbf{n}} \cos 2 heta_{\mathrm{E}} + ec{\mathbf{e}}_{\mathrm{E}} \sin 2 heta_{\mathrm{E}}) \ ec{\mathbf{f}}_{\mathrm{B}} &= rac{1}{2\mu_0} \mathbf{B}^2 (ec{\mathbf{n}} \cos 2 heta_{\mathrm{B}} + ec{\mathbf{e}}_{\mathrm{B}} \sin 2 heta_{\mathrm{B}}) \end{aligned}$$

若电磁场的方向不确定,在所有方向上都等几率出现:

$$\frac{1}{\cos 2\theta} = \frac{\int d\Omega \cos 2\theta}{\int d\Omega} = \frac{\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin \theta \cos 2\theta}{\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin \theta} = -\frac{1}{3}$$

$$\frac{1}{\sin 2\theta} = \frac{\int d\Omega \sin 2\theta}{\int d\Omega} = \frac{\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin \theta \sin 2\theta}{\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin \theta} = 0$$



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#### 电磁场出现在物质表面. 单位表面受的力为

$$egin{aligned} ec{\mathbf{f}} &= -ec{\mathbf{n}} \cdot \stackrel{
ightharpoonup}{\mathcal{J}} = ec{\mathbf{f}}_{\mathbf{E}} + ec{\mathbf{f}}_{\mathbf{B}} \ ec{\mathbf{f}}_{\mathbf{E}} &= rac{1}{2} \epsilon_0 \mathbf{E}^2 (ec{\mathbf{n}} \cos 2 heta_{\mathbf{E}} + ec{\mathbf{e}}_{\mathbf{E}} \sin 2 heta_{\mathbf{E}}) \ ec{\mathbf{f}}_{\mathbf{E}} &= rac{1}{2} \epsilon_0 \mathbf{E}^2 (ec{\mathbf{n}} \cos 2 heta_{\mathbf{E}} + ec{\mathbf{e}}_{\mathbf{E}} \sin 2 heta_{\mathbf{E}}) \end{aligned}$$

 $\vec{\mathbf{f}}_{\mathbf{B}} = \frac{1}{2\mu_{\mathbf{0}}} \mathbf{B}^{2} (\vec{\mathbf{n}} \cos 2\theta_{\mathbf{B}} + \vec{\mathbf{e}}_{\mathbf{B}} \sin 2\theta_{\mathbf{B}})$ 

$$\frac{\cos 2\theta}{\cos 2\theta} = \frac{\int d\Omega \cos 2\theta}{\int d\Omega} = \frac{\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin \theta \cos 2\theta}{\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin \theta} = -\frac{1}{3}$$

$$\frac{\sin 2\theta}{\int d\Omega} = \frac{\int d\Omega \sin 2\theta}{\int d\Omega} = \frac{\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin \theta \sin 2\theta}{\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin \theta} = 0$$

若电磁场的方向不确定,在所有方向上都等几率出现:

$$\overline{\vec{\mathbf{f}}} = -\frac{\vec{\mathbf{n}}}{3}(\frac{1}{2}\epsilon_0\mathbf{E}^2 + \frac{1}{2}\mu_0\mathbf{H}^2) = -\frac{\vec{\mathbf{n}}}{3}\mathbf{W}$$

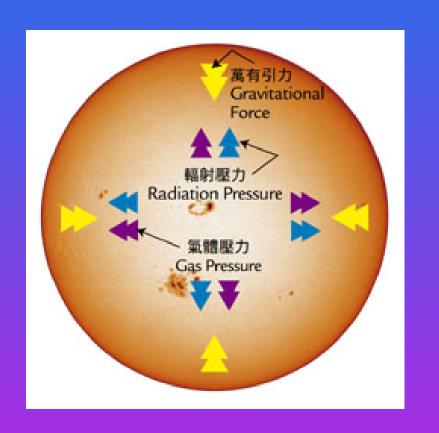


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# 二. 从作用量出发构建

协变的经典电磁学理论







伽利略变换 假设: 时空是均匀的! 后面性细讨论



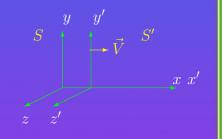
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#### 相对论基本原理,洛伦兹变换: 伽利略变换

假设: 时空是均匀的! 后而详细讨论

 $\overline{(\mathbf{t}, \mathbf{x}, \mathbf{y}, \mathbf{z})}$ 和 $(\mathbf{t}', \mathbf{x}', \mathbf{y}', \mathbf{z}')$ 之间的关系必须是线性的. 为什么?

可以选择:两个相对运动速度为 $\vec{V}$ 的惯性系S和S',两系的坐标系坐标轴方向相同,x轴取在 $\vec{V}$ 方向,并设t=0, t'=0时两坐标系重合.





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#### 相对论基本原理,洛伦兹变换: 伽利略变换

#### 假设: 时空是均匀的! 后而详细讨论

(t, x, y, z)和(t', x', y', z')之间的关系必须是线性的. 为什么?

可以选择:两个相对运动速度为 $\vec{\mathbf{v}}$ 的惯性系S和S',两系的坐标系坐标轴方向相同, $\mathbf{x}$ 轴取在 $\vec{\mathbf{v}}$ 方向,并设 $\mathbf{t}=\mathbf{0}$ ,  $\mathbf{t}'=\mathbf{0}$ 时两坐标系重合.

$$\mathbf{x}' = \alpha \mathbf{x} + \beta \mathbf{t}$$
  $\mathbf{y}' = \mathbf{y}$   $\mathbf{z}' = \mathbf{z}$   $\mathbf{t}' = \gamma \mathbf{x} + \delta \mathbf{t}'$ 



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#### 伽利略变换

#### 假设: 时空是均匀的! 后面详细讨论

(t, x, y, z)和(t', x', y', z')之间的关系必须是线性的. 为什么?

可以选择:两个相对运动速度为V的惯性系S和S',两系的坐标系坐标轴方向相同, x轴取在 $\vec{V}$ 方向,并设t = 0, t' = 0时两坐标系重合.

$$\mathbf{x}' = \alpha \mathbf{x} + \beta \mathbf{t}$$
  $\mathbf{y}' = \mathbf{y}$   $\mathbf{z}' = \mathbf{z}$   $\mathbf{t}' = \gamma \mathbf{x} + \delta \mathbf{t}^S$   $\vec{v}$   $\vec{v}$ 



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#### 相对论基本原理、洛伦兹变换:伽利略变换

#### 假设: 时空是均匀的! 后面详细讨论

(t, x, y, z)和(t', x', y', z')之间的关系必须是线性的. 为什么?

可以选择:两个相对运动速度为 $\vec{V}$ 的惯性系S和S<sup>2</sup>,两系的坐标系坐标轴方向相同, x轴取在 $\vec{V}$ 方向,并设t = 0, t' = 0时两坐标系重合.

$$\mathbf{x}' = \alpha \mathbf{x} + \beta \mathbf{t}$$
  $\mathbf{y}' = \mathbf{y}$   $\mathbf{z}' = \mathbf{z}$   $\mathbf{t}' = \gamma \mathbf{x} + \delta \mathbf{t}^S$   $\vec{v}$   $\vec{v}$ 

• 从S看,S'有运动速度 $V \Rightarrow dx' = 0$   $\frac{dx}{dt} = V$ 



#### 相对论基本原理,洛伦兹变换:伽利略变换

#### 假设: 时空是均匀的! 后面详细讨论

(t, x, y, z)和(t', x', y', z')之间的关系必须是线性的. 为什么?

可以选择:两个相对运动速度为 $\vec{V}$ 的惯性系S和S',两系的坐标系坐标轴方向相同,x轴取在 $\vec{V}$ 方向,并设t=0, t'=0时两坐标系重合.

$$\mathbf{x}' = \alpha \mathbf{x} + \beta \mathbf{t}$$
  $\mathbf{y}' = \mathbf{y}$   $\mathbf{z}' = \mathbf{z}$   $\mathbf{t}' = \gamma \mathbf{x} + \delta \mathbf{t}^S$   $\vec{v}$   $\vec{v}$ 

• 从S看,S'有运动速度V  $\Rightarrow$  dx' = 0  $\frac{dx}{dt} = V \Rightarrow \beta = -\alpha V$ 



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## 相对论基本原理、洛伦兹变换:伽利略变换

#### 假设: 时空是均匀的! 后面详细讨论

(t, x, y, z)和(t', x', y', z')之间的关系必须是线性的. 为什么?

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$$\mathbf{x}' = \alpha \mathbf{x} + \beta \mathbf{t}$$
  $\mathbf{y}' = \mathbf{y}$   $\mathbf{z}' = \mathbf{z}$   $\mathbf{t}' = \gamma \mathbf{x} + \delta \mathbf{t}^S$   $\vec{v}$   $\vec{v}$ 

- 从S看,S'有运动速度 $V \Rightarrow dx' = 0$   $\frac{dx}{dt} = V \Rightarrow \beta = -\alpha V$
- $\bullet$  S'看,S有运动速度-V 一定这样? 单向光速不变假设或空间各向同性  $\Rightarrow$   $\mathrm{dx} = 0$   $\frac{\mathrm{dx}'}{\mathrm{dt}'} = -\mathrm{V}$



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# 相对论基本原理,洛伦兹变换: 伽利略变换

## 假设: 时空是均匀的! 后面详细讨论

(t,x,y,z)和(t',x',y',z')之间的关系必须是线性的. 为什么?

可以选择:两个相对运动速度为 $\vec{V}$ 的惯性系S和S',两系的坐标系坐标轴方向相同, x轴取在 $\vec{V}$ 方向,并设t=0,t'=0时两坐标系重合.

$$\mathbf{x}' = \alpha \mathbf{x} + \beta \mathbf{t}$$
  $\mathbf{y}' = \mathbf{y}$   $\mathbf{z}' = \mathbf{z}$   $\mathbf{t}' = \gamma \mathbf{x} + \delta \mathbf{t}^S$   $\mathbf{z}'$   $\mathbf{z}'$   $\mathbf{z}'$   $\mathbf{z}'$ 

ullet S'看,S有运动速度- $\overline{
m V}$  一定这样? 单向光速不变假设或空间各向同性  $\Rightarrow$   ${
m d} {
m x}={
m O} {
m d} {
m d}$ 

- 从S看,S'有运动速度 $V \Rightarrow dx' = 0$   $\frac{dx}{dt} = V \Rightarrow \beta = -\alpha V$
- $\mathbf{x}' = \alpha(\mathbf{x} \mathbf{V}\mathbf{t})$   $\mathbf{y}' = \mathbf{y}$   $\mathbf{z}' = \mathbf{z}$   $\mathbf{t}' = \gamma\mathbf{x} + \alpha\mathbf{t}$



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# 相对论基本原理、洛伦兹变换: 伽利略变换

假设: 时空是均匀的! 后面详细讨论

(t, x, y, z)和(t', x', y', z')之间的关系必须是线性的. 为什么?

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$$\mathbf{x}' = \alpha \mathbf{x} + \beta \mathbf{t}$$
  $\mathbf{y}' = \mathbf{y}$   $\mathbf{z}' = \mathbf{z}$   $\mathbf{t}' = \gamma \mathbf{x} + \delta \mathbf{t}^S$   $\vec{v}$   $\vec{v}$ 

- 从S看,S'有运动速度 $V \Rightarrow dx' = 0$   $\frac{dx}{dt} = V \Rightarrow \beta = -\alpha V$
- ullet S'看,S有运动速度-V 一定这样? 单向光速不变假设或空间各向同性  $\Rightarrow$   $\mathrm{d} \mathbf{x} = \mathbf{0}$   $\overline{\mathrm{d} \mathbf{x}'} = -\mathbf{V}$   $\Rightarrow$   $eta = -\delta \mathbf{V}$  $\mathbf{x}' = \alpha(\mathbf{x} - \mathbf{V}\mathbf{t})$   $\mathbf{y}' = \mathbf{y}$   $\mathbf{z}' = \mathbf{z}$   $\mathbf{t}' = \gamma \mathbf{x} + \alpha \mathbf{t}$

$$\underline{m$$
 和略变换:  $t' = t \Rightarrow \gamma = 0, \alpha = 1$ 



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# 相对论基本原理,洛伦兹变换: 伽利略变换

假设: 时空是均匀的! 后面详细讨论

(t,x,y,z)和(t',x',y',z')之间的关系必须是线性的. 为什么?

可以选择:两个相对运动速度为 $\vec{V}$ 的惯性系S和S',两系的坐标系坐标轴方向相同,x轴取在 $\vec{V}$ 方向,并设t=0,t'=0时两坐标系重合.

$$\mathbf{x}' = \alpha \mathbf{x} + \beta \mathbf{t}$$
  $\mathbf{y}' = \mathbf{y}$   $\mathbf{z}' = \mathbf{z}$   $\mathbf{t}' = \gamma \mathbf{x} + \delta \mathbf{t}^S$   $\vec{v}$   $\vec{v}$ 

- 从S看,S'有运动速度 $V \Rightarrow dx' = 0$   $\frac{dx}{dt} = V \Rightarrow \beta = -\alpha V$
- S'看,S有运动速度- $\mathbf{V}$  一定这样? 单向光速不变假设或空间各向同性  $\Rightarrow$   $\mathrm{d}\mathbf{x} = \mathbf{0}$   $\mathbf{x}' = \alpha(\mathbf{x} \mathbf{V}\mathbf{t})$   $\mathbf{y}' = \mathbf{y}$   $\mathbf{z}' = \mathbf{z}$   $\mathbf{t}' = \gamma\mathbf{x} + \alpha\mathbf{t}$

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  - 伽利略变换:  $\mathbf{t}'=\mathbf{t}$   $\Rightarrow$   $\gamma=\mathbf{0},\ \alpha=\mathbf{1}$  绝对时间! 否定伽利略变换必须否定绝对时间

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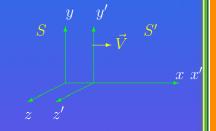
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#### 相对论基本原理,洛伦兹变换: 基本洛伦兹变换

两个相对运动速度为 $\vec{V}$ 的惯性系S和S',两系的坐标系坐标轴方向相同,x轴取在 $\vec{V}$ 方向,并设t=0, t'=0时两坐系标重合.





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# 基本洛伦兹变换 两个相对运动速度为V的惯性系S和S',两系的坐标系坐标轴 方向 相同,x轴取在 $\vec{V}$ 方向,并设t = 0, t' = 0时两坐系标重合. 一个事件在S和S'系中的时空坐标分别为 $(\mathbf{t}, \mathbf{x}, \mathbf{y}, \mathbf{z})$ 和 $(\mathbf{t}', \mathbf{x}', \mathbf{y}', \mathbf{z}')$



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两个相对运动速度为V的惯性系S和S',两系的坐标系坐标轴

方向 相同,x轴取在 $\vec{V}$ 方向,并设t = 0, t' = 0时两坐系标重合.

一个事件在S和S'系中的时空坐标分别为(t, x, y, z)和 (t', x', y', z') $\mathbf{x}' = \alpha(\mathbf{x} - \mathbf{V}\mathbf{t})$   $\mathbf{y}' = \mathbf{y}$   $\mathbf{z}' = \mathbf{z}$   $\mathbf{t}' = \gamma\mathbf{x} + \alpha\mathbf{t}$ 





两个相对运动速度为V的惯性系S和S',两系的坐标系坐标轴 方向 相同,x轴取在 $\vec{V}$ 方向,并设t = 0, t' = 0时两坐系标重合.

一个事件在S和S'系中的时空坐标分别为(t, x, y, z)和 (t', x', y', z')

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 $\Delta t = 0, t' = 0$ 时原点发出一个球面电磁波, 在t时刻电磁波达到以原点O为中心的某 个球波阵面上,相应 在t/时刻电磁波达到以原点O/为中心的某个球波阵面上,

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#### 相对论基本原理,洛伦兹变换:基本洛伦兹变换

两个相对运动速度为 $\vec{V}$ 的惯性系 $\vec{S}$ 和 $\vec{S}$ ',两系的坐标系坐标轴方向 相同, $\vec{x}$ 轴取在 $\vec{V}$ 方向,并设 $\vec{t}$  = 0. $\vec{t}$ ′ = 0时两坐系标重合.

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$$\left( egin{array}{l} {\mathbf{x}^2 + \mathbf{y}^2 + \mathbf{z}^2 = \mathbf{c^2} \mathbf{t}^2} \ {\mathbf{x}'^2 + \mathbf{y}'^2 + \mathbf{z}'^2 = \mathbf{c^2} \mathbf{t}'^2} \end{array} 
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两个相对运动速度为V的惯性系S和S',两系的坐标系坐标轴 方向 相同,x轴取在 $\vec{V}$ 方向,并设t = 0. t' = 0时两坐系标重合.

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$$\begin{cases} \mathbf{x^2 + y^2 + z^2} = \mathbf{c^2 t^2} \\ \mathbf{x'^2 + y'^2 + z'^2} = \mathbf{c^2 t'^2} \end{cases} \rightarrow \begin{cases} \mathbf{x^2 + y^2 + z^2} = \mathbf{c^2 t^2} \\ \alpha^2 (\mathbf{x - V t})^2 + \mathbf{y^2 + z^2} = \mathbf{c^2} (\gamma \mathbf{x} + \alpha \mathbf{t})^2 \end{cases}$$





两个相对运动速度为V的惯性系S和S,,两系的坐标系坐标轴 方向 相同,x轴取在 $\vec{V}$ 方向,并设t = 0. t' = 0时两坐系标重合.

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$$\rightarrow \begin{cases} \alpha^2 - \mathbf{c}^2 \gamma^2 = 1 \\ -2\mathbf{V}\alpha^2 - 2\mathbf{c}^2 \gamma \alpha = 0 \\ \mathbf{c}^2 \alpha^2 - \mathbf{V}^2 \alpha^2 = \mathbf{c}^2 \end{cases}$$



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两个相对运动速度为V的惯性系S和S,,两系的坐标系坐标轴 方向 相同,x轴取在 $\vec{V}$ 方向,并设t=0, t'=0时两坐系标重合.

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$$\begin{cases} \alpha^2 - \mathbf{c}^2 \gamma^2 = 1 \\ \alpha^2 \mathbf{x} - \mathbf{c}^2 \gamma^2 = 1 \end{cases}$$

$$\Rightarrow \begin{cases} \alpha^2 - \mathbf{c}^2 \gamma^2 = 1 \\ -2\mathbf{V}\alpha^2 - 2\mathbf{c}^2 \gamma \alpha = 0 \\ \mathbf{c}^2 \alpha^2 - \mathbf{V}^2 \alpha^2 = \mathbf{c}^2 \end{cases} \quad \Rightarrow \quad \alpha = \frac{1}{\sqrt{1 - \frac{\mathbf{V}^2}{\mathbf{c}^2}}} \quad \gamma = -\frac{\frac{\mathbf{V}}{\mathbf{c}^2}}{\sqrt{1 - \frac{\mathbf{V}^2}{\mathbf{c}^2}}}$$





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$$\mathbf{x}' = \frac{\mathbf{x} \! - \! V t}{\sqrt{1 \! - \! \frac{V^2}{c^2}}} \quad \mathbf{y}' = \mathbf{y} \quad \mathbf{z}' = \mathbf{z} \quad t' = \frac{t \! - \! \frac{V}{c^2} \mathbf{x}}{\sqrt{1 \! - \! \frac{V^2}{c^2}}}$$







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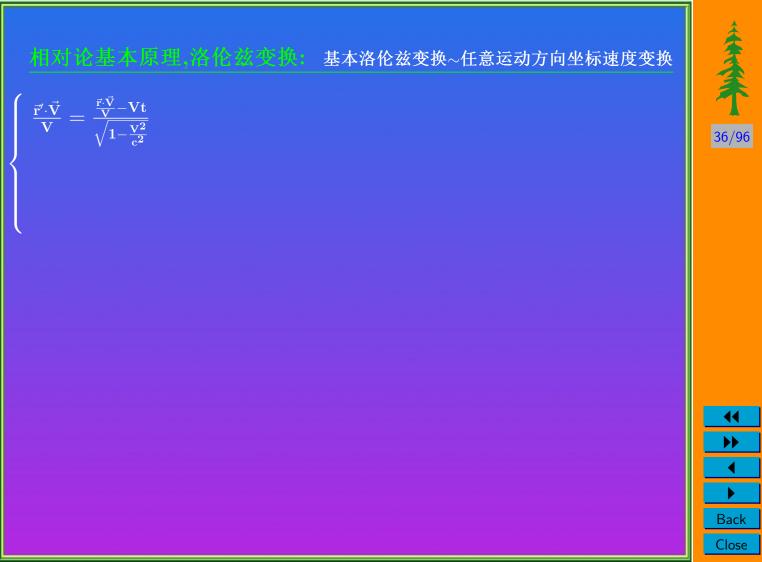
$$\begin{cases} \mathbf{x}^2 + \mathbf{y}^2 + \mathbf{z}^2 = \mathbf{c^2} \mathbf{t}^2 \\ \mathbf{x'}^2 + \mathbf{y'}^2 + \mathbf{z'}^2 = \mathbf{c^2} \mathbf{t'}^2 \end{cases} \rightarrow \begin{cases} \mathbf{x}^2 + \mathbf{y}^2 + \mathbf{z}^2 = \mathbf{c}^2 \mathbf{t}^2 \\ \alpha^2 (\mathbf{x} - \mathbf{V} \mathbf{t})^2 + \mathbf{y}^2 + \mathbf{z}^2 = \mathbf{c}^2 (\gamma \mathbf{x} + \alpha \mathbf{t})^2 \end{cases}$$

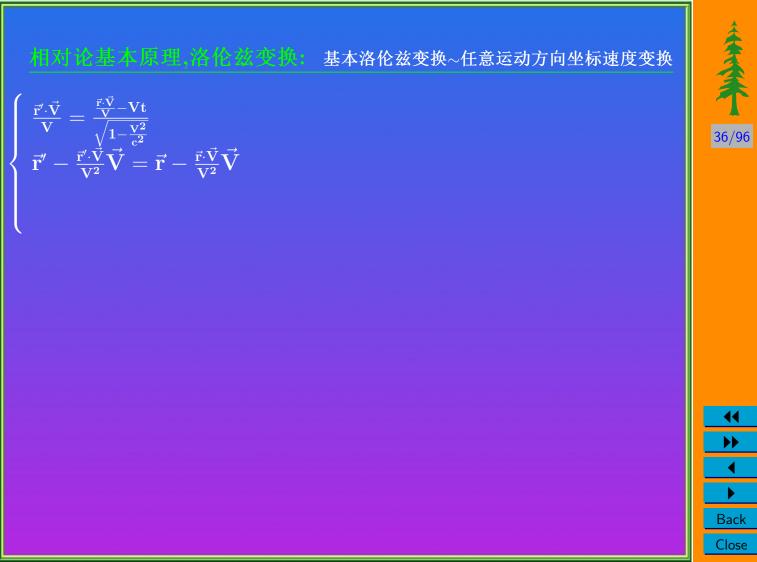
$$\rightarrow \begin{cases} \alpha^2 - \mathbf{c}^2 \gamma^2 = 1 \\ -2\mathbf{V}\alpha^2 - 2\mathbf{c}^2 \gamma \alpha = 0 \end{cases} \rightarrow \alpha = \frac{1}{\sqrt{1 - \mathbf{V}^2}} \quad \gamma = -\frac{\frac{\mathbf{V}}{\mathbf{c}^2}}{\sqrt{1 - \mathbf{V}^2}}$$

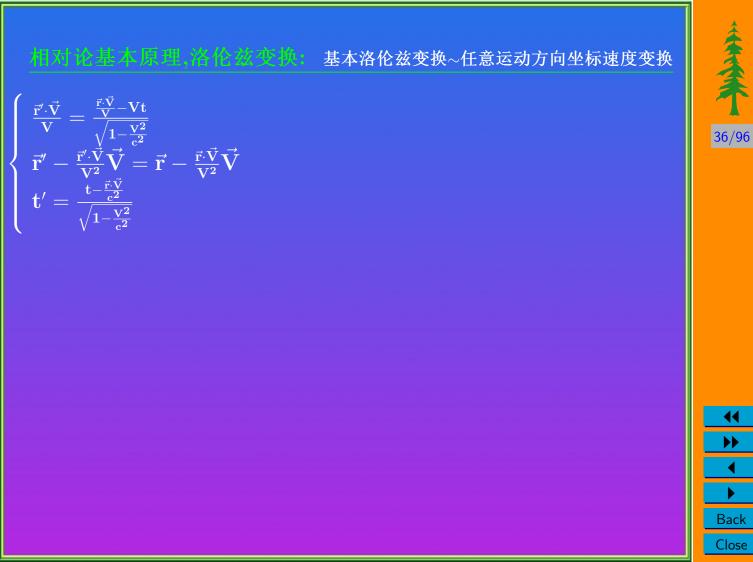
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基本洛伦兹变换~任意运动方向坐标速度变换  $egin{aligned} ec{\mathbf{r}'} &= ec{\mathbf{r}} - rac{ec{\mathbf{r}}.ec{\mathbf{V}}}{V^2}ec{\mathbf{V}} + rac{ec{\mathbf{r}}.ec{\mathbf{V}}}{V}-Vt}rac{ec{\mathbf{V}}}{V} rac{ec{\mathbf{V}}}{V} &= ec{\mathbf{r}} - ec{\mathbf{V}}t + ig(rac{ec{\mathbf{r}}.ec{\mathbf{V}}}{\sqrt{1-rac{V^2}{c^2}}}-1ig)ig(ec{\mathbf{r}} - ec{\mathbf{V}}tig) \cdot rac{ec{\mathbf{V}}}{V} rac{ec{\mathbf{V}}}{V} &= rac{t-rac{ec{\mathbf{r}}.ec{\mathbf{V}}}{c^2}}{\sqrt{1-rac{V^2}{c^2}}} & & & & \qquad \qquad \uparrow \ ec{\mathbf{r}} - ec{\mathbf{V}} t ec{ec{\mathbf{V}}} rac{ec{\mathbf{V}}}{V} aggreen t agg$  $egin{aligned} rac{ec{\mathbf{r}}'\cdotec{\mathbf{V}}}{\mathbf{V}} &= rac{ec{ec{\mathbf{r}}\cdotec{\mathbf{V}}}-\mathbf{V}\mathbf{t}}{\sqrt{1-rac{\mathbf{V}^2}{\mathbf{c}^2}}} \ ec{\mathbf{r}}' &- rac{ec{\mathbf{r}}'\cdotec{\mathbf{V}}}{\mathbf{V}^2}ec{\mathbf{V}} &= ec{\mathbf{r}} - rac{ec{\mathbf{r}}\cdotec{\mathbf{V}}}{\mathbf{V}^2}ec{\mathbf{V}} &
ightarrow \left\{ egin{aligned} ec{\mathbf{r}} \ t' &= rac{ec{\mathbf{t}}-rac{ec{\mathbf{r}}\cdotec{\mathbf{V}}}{\mathbf{c}^2}}{\sqrt{1-rac{\mathbf{V}^2}{\mathbf{c}^2}}} \end{aligned} 
ight. \end{aligned}$ 



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基本洛伦兹变换~任意运动方向坐标速度变换  $egin{align*} & rac{ec{r}'\cdotec{v}}{v} = rac{ec{r}\cdotec{v}}{\sqrt{1-rac{v^2}{c^2}}} \ ec{r}' - rac{ec{r}\cdotec{v}}{v^2}ec{V} + rac{ec{r}\cdotec{v}}{\sqrt{1-rac{v^2}{c^2}}}ec{V} \ & = ec{r} - rac{ec{r}\cdotec{v}}{v^2}ec{V} + rac{ec{r}\cdotec{v}}{\sqrt{1-rac{v^2}{c^2}}}ec{V} \ & = ec{r} - rac{ec{r}\cdotec{v}}{v^2}ec{V} + rac{ec{r}\cdotec{v}}{\sqrt{1-rac{v^2}{c^2}}}ec{V} \ & = ec{r} - rac{ec{r}\cdotec{v}}{v^2}ec{V} + rac{ec{r}\cdotec{v}}{\sqrt{1-rac{v^2}{c^2}}}ec{V} \ & = ec{r} - rac{ec{r}\cdotec{v}}{v^2}ec{V} + rac{ec{r}\cdotec{v}}{\sqrt{1-rac{v^2}{c^2}}}ec{V} \ & = ec{r} - rac{ec{r}\cdotec{v}}{v^2}ec{V} + rac{ec{r}\cdotec{v}}{\sqrt{1-rac{v^2}{c^2}}} \ & = ec{r} - rac{ec{r}\cdotec{v}}{v^2}ec{V} \ & = rac{ec{r}\cdotec{v}}{\sqrt{1-rac{v^2}{c^2}}} \ & \Leftrightarrow \ & = rac{ec{r}\cdotec{v}\cdotec{v}}{\sqrt{1-rac{v^2}{c^2}}} \ & \Leftrightarrow \ & = ec{r}\cdotec{v}\cdotec{v} + rac{ec{v}\cdotec{v}\cdotec{v}\cdotec{v}}{\sqrt{1-rac{v^2}{c^2}}} \ & \Leftrightarrow \ & = ec{v}\cdotec{v}\cdotec{v}\cdotec{v} + rac{ec{v}\cdotec{v}\cdotec{v}\cdotec{v}\cdotec{v}\cdotec{v}\cdotec{v}\cdotec{v}\cdotec{v}\cdotec{v}}{\sqrt{1-rac{v^2}{c^2}}} \ & \Leftrightarrow \ & = ec{v}\cdotec{v$ 



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基本洛伦兹变换~任意运动方向坐标速度变换 36/96  $ec{u} = rac{dec{r}}{dt} \quad ec{u}' \equiv rac{dec{r}'}{dt'}$ 







基本洛伦兹变换~任意运动方向坐标速度变换  $egin{align*} egin{align*} & rac{ec{r}'\cdotec{v}}{v} = rac{ec{r}\cdotec{v}}{\sqrt{1-rac{v^2}{c^2}}} \ ec{r}' - rac{ec{r}'\cdotec{v}}{v^2}ec{v} + rac{ec{r}\cdotec{v}}{\sqrt{1-rac{v^2}{c^2}}}ec{v} \ & = ec{r} - rac{ec{r}\cdotec{v}}{v^2}ec{v} + rac{ec{r}\cdotec{v}}{\sqrt{1-rac{v^2}{c^2}}}ec{v} \ & = ec{r} - rac{ec{r}\cdotec{v}}{v^2}ec{v} + rac{ec{r}\cdotec{v}}{\sqrt{1-rac{v^2}{c^2}}}ec{v} \ & = ec{r} - rac{ec{r}\cdotec{v}}{v^2}ec{v} + rac{ec{r}\cdotec{v}}{\sqrt{1-rac{v^2}{c^2}}} - 1)(ec{r} - ec{v}t) \cdot rac{ec{v}}{v} rac{ec{v}}{v} \ & = rac{t - rac{ec{r}\cdotec{v}}{v^2}}{\sqrt{1-rac{v^2}{c^2}}} & & \uparrow \ r - ec{v} = rac{ec{v}\cdotec{v}\cdotec{v}\cdotec{v}\cdotec{v}\cdotec{v}\cdotec{v}\cdotec{v} + rac{ec{v}\cdote$ 36/96  $ec{\mathbf{u}} = rac{dec{\mathbf{r}}}{dt} \quad ec{\mathbf{u}}' \equiv rac{dec{\mathbf{r}}'}{dt'} = rac{rac{dec{\mathbf{r}}'}{dt}}{rac{d\mathbf{t}'}{dt'}}$ 







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和对论基本原理,洛伦兹变换。任意运动方向坐标速度变换  $\begin{cases} \frac{\vec{r}' \cdot \vec{V}}{V} = \frac{\vec{r} \cdot \vec{V} - Vt}{\sqrt{1 - \frac{V^2}{c^2}}} \\ \vec{r}' - \frac{\vec{r}' \cdot \vec{V}}{V^2} \vec{V} = \vec{r} - \frac{\vec{r} \cdot \vec{V}}{V^2} \vec{V} \end{cases} \rightarrow \begin{cases} \vec{r}' = \vec{r} - \frac{\vec{r} \cdot \vec{V}}{V^2} \vec{V} + \frac{\vec{r} \cdot \vec{V}}{V^2} - Vt}{\sqrt{1 - \frac{V^2}{c^2}}} \\ = \vec{r} - \vec{V}t + (\frac{1}{\sqrt{1 - \frac{V^2}{c^2}}} - 1)(\vec{r} - \vec{V}t) \cdot \frac{\vec{V}}{V} \cdot \frac{\vec{V}}{V} \\ t' = \frac{t - \frac{\vec{r} \cdot \vec{V}}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} \end{cases} \uparrow \vec{r} - \vec{V}t + \vec{V} \cdot \vec$ 

 $ec{\mathbf{u}} = rac{dec{\mathbf{r}}}{dt} \quad ec{\mathbf{u}}' \equiv rac{dec{\mathbf{r}}'}{dt'} = rac{rac{dec{\mathbf{r}}'}{dt}}{rac{d\mathbf{t}'}{dt}} = rac{ec{\mathbf{u}} \cdot ec{\mathbf{V}}}{V^2} ec{\mathbf{V}} + rac{rac{ec{\mathbf{u}} \cdot ec{\mathbf{V}}}{V} - V}{\sqrt{1 - rac{V^2}{\mathbf{c}^2}}} rac{ec{\mathbf{V}}}{V}}{rac{1 - rac{ec{\mathbf{V}} \cdot ec{\mathbf{u}}}{\mathbf{c}^2}}{\sqrt{1 - rac{V^2}{\mathbf{c}^2}}}}$ 









# 基本洛伦兹变换~任意运动方向坐标速度变换

$$egin{dcases} rac{ec{\mathbf{r}}'\cdotec{\mathbf{V}}}{\mathbf{V}} = rac{ec{ec{\mathbf{r}}\cdotec{\mathbf{V}}}-\mathbf{V}\mathbf{t}}{\sqrt{1-rac{\mathbf{V}^2}{\mathbf{c}^2}}} \ ec{\mathbf{r}}' - rac{ec{\mathbf{r}}'\cdotec{\mathbf{V}}}{\mathbf{V}^2}ec{\mathbf{V}} = ec{\mathbf{r}} - rac{ec{\mathbf{r}}\cdotec{\mathbf{V}}}{\mathbf{V}^2}ec{\mathbf{V}} + rac{ec{ec{\mathbf{r}}\cdotec{\mathbf{V}}}-\mathbf{V}\mathbf{t}}{\sqrt{1-rac{\mathbf{V}^2}{\mathbf{c}^2}}} ec{\mathbf{V}} \ t' = rac{ec{\mathbf{r}}-ec{\mathbf{V}}\cdotec{\mathbf{V}}}{\sqrt{1-rac{\mathbf{V}^2}{\mathbf{c}^2}}} & \qquad \qquad \qquad \qquad \begin{cases} ec{\mathbf{r}}' &= ec{\mathbf{r}} - rac{ec{\mathbf{r}}\cdotec{\mathbf{V}}}{\mathbf{V}^2} ec{\mathbf{V}} + rac{ec{ec{\mathbf{r}}\cdotec{\mathbf{V}}}-\mathbf{V}\mathbf{t}}{\sqrt{1-rac{\mathbf{V}^2}{\mathbf{c}^2}}} ec{\mathbf{V}} \ t' &= rac{ec{\mathbf{r}}-ec{\mathbf{V}}\cdotec{\mathbf{V}}}{\sqrt{1-rac{\mathbf{V}^2}{\mathbf{c}^2}}} & \qquad \qquad \qquad \qquad \end{cases} egin{matrix} ec{\mathbf{r}} - ec{\mathbf{V}}\cdotec{\mathbf{V}} &= rac{ec{\mathbf{r}}\cdotec{\mathbf{V}}\cdotec{\mathbf{V}}}{\sqrt{1-rac{\mathbf{V}^2}{\mathbf{c}^2}}} & \qquad \qquad \qquad \end{cases} egin{matrix} ec{\mathbf{r}} - ec{\mathbf{V}}\cdotec{\mathbf{V}}\cdotec{\mathbf{V}} &= rac{ec{\mathbf{r}}\cdotec{\mathbf{V}}\cdotec{\mathbf{V}}}{\sqrt{1-rac{\mathbf{V}^2}{\mathbf{c}^2}}} & \qquad \qquad \qquad \end{cases} egin{matrix} ec{\mathbf{r}} - ec{\mathbf{V}}\cdotec{\mathbf{V}}\cdotec{\mathbf{V}} &= rac{ec{\mathbf{r}}\cdotec{\mathbf{V}}\cdotec{\mathbf{V}}\cdotec{\mathbf{V}} &= rac{ec{\mathbf{V}}\cdotec{\mathbf{V}}\cdotec{\mathbf{V}}\cdotec{\mathbf{V}}\cdotec{\mathbf{V}} &= rac{ec{\mathbf{V}}\cdotec{\mathbf{V}}\cdotec{\mathbf{V}}\cdotec{\mathbf{V}}\cdotec{\mathbf{V}} &= rac{ec{\mathbf{V}}\cdotec{\mathbf{V}}\cdotec{\mathbf{V}}\cdotec{\mathbf{V}}\cdotec{\mathbf{V}} &= rac{ec{\mathbf{V}}\cdotec{\mathbf{V}}\cdotec{\mathbf{V}}\cdotec{\mathbf{V}}\cdotec{\mathbf{V}}\cdotec{\mathbf{V}} &= rac{ec{\mathbf{V}}\cdotec{\mathbf{V}}\cdotec{\mathbf{V}}\cdotec{\mathbf{V}}\cdotec{\mathbf{V}}\cdotec{\mathbf{V}} &= rac{ec{\mathbf{V}}\cdotec{\mathbf{V}\cdotec{\mathbf{V}}\cdotec{\mathbf{V}}\cdotec{\mathbf{V}\cdotec{\mathbf{V}}\cdotec{\mathbf{V}}\cdotec{\mathbf{V}}\cdotec{\mathbf{V}}\cdotec{\mathbf{V}}\cdotec{\mathbf{V}}\cdotec{\mathbf{V}}\cdotec{\mathbf$$

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 $egin{split} ec{\mathbf{u}}' = rac{\sqrt{1 - rac{\mathbf{V}^2}{\mathbf{c}^2}} (ec{\mathbf{u}} - ec{\mathbf{V}}) + (1 - \sqrt{1 - rac{\mathbf{V}^2}{\mathbf{c}^2}}) (ec{\mathbf{u}} - ec{\mathbf{V}}) \cdot rac{ec{\mathbf{v}}}{\mathbf{V}} rac{ec{\mathbf{V}}}{\mathbf{V}}}{1 - rac{ec{\mathbf{V}} \cdot ec{\mathbf{u}}}{\mathbf{c}^2}} \end{split}$ 

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# 基本洛伦兹变换~任意运动方向坐标速度变换

$$ec{\mathbf{u}} = rac{\mathbf{d}ec{\mathbf{r}}}{\mathbf{d}\mathbf{t}} \quad ec{\mathbf{u}}' \equiv rac{\mathbf{d}ec{\mathbf{r}}'}{\mathbf{d}\mathbf{t}'} = rac{rac{\mathbf{d}ec{\mathbf{r}}'}{\mathbf{d}\mathbf{t}}}{rac{\mathbf{d}\mathbf{t}'}{\mathbf{d}\mathbf{t}}} = rac{ec{\mathbf{u}} - rac{ec{\mathbf{u}}\cdotec{\mathbf{V}}}{\mathbf{V}^2}ec{\mathbf{V}} + rac{rac{ec{\mathbf{u}}\cdotec{\mathbf{V}}}{\mathbf{V}} - \mathbf{V}}{\sqrt{1 - rac{\mathbf{V}^2}{\mathbf{c}^2}}} rac{ec{\mathbf{V}}}{\mathbf{V}}}{rac{1 - rac{ec{\mathbf{V}}\cdotec{\mathbf{u}}}{\mathbf{V}^2}}{\sqrt{1 - rac{\mathbf{V}^2}{\mathbf{c}^2}}}}$$

 $egin{split} ec{\mathbf{u}}' = rac{\sqrt{1 - rac{\mathbf{V}^2}{\mathbf{c}^2}} (ec{\mathbf{u}} - ec{\mathbf{V}}) + (1 - \sqrt{1 - rac{\mathbf{V}^2}{\mathbf{c}^2}}) (ec{\mathbf{u}} - ec{\mathbf{V}}) \cdot rac{ec{\mathbf{v}}}{\mathbf{V}} rac{ec{\mathbf{V}}}{\mathbf{V}}}{1 - rac{ec{\mathbf{V}} \cdot ec{\mathbf{u}}}{\mathbf{c}^2}} \end{split}$  $\mathrm{U}^lpha \equiv rac{\mathrm{d} \mathbf{x}^lpha}{\mathrm{d} au}$ 

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$$ec{\mathbf{u}} = rac{dec{\mathbf{r}}}{dt} \quad ec{\mathbf{u}}' \equiv rac{dec{\mathbf{r}}'}{dt'} = rac{\dfrac{dec{\mathbf{r}}'}{dt}}{\dfrac{dt'}{dt}} = rac{ec{\mathbf{u}} \cdot ec{\mathbf{V}}}{V^2} ec{\mathbf{V}} + rac{\dfrac{ec{\mathbf{u}} \cdot ec{\mathbf{V}}}{V} - V}{\sqrt{1 - \dfrac{\mathbf{V}^2}{\mathbf{c}^2}}} \dfrac{ec{\mathbf{V}}}{V}}{\dfrac{1 - \dfrac{ec{\mathbf{V}} \cdot ec{\mathbf{u}}}{c^2}}{\sqrt{1 - \dfrac{\mathbf{V}^2}{\mathbf{c}^2}}}}$$
  $ec{\mathbf{u}}' = rac{\sqrt{1 - \dfrac{\mathbf{V}^2}{\mathbf{c}^2}} (ec{\mathbf{u}} - ec{\mathbf{V}}) + (1 - \sqrt{1 - \dfrac{\mathbf{V}^2}{\mathbf{c}^2}}) (ec{\mathbf{u}} - ec{\mathbf{V}}) \cdot \dfrac{ec{\mathbf{V}}}{V} \dfrac{ec{\mathbf{V}}}{V}}{V}}$ 

 $\mathbf{U}^lpha \equiv rac{\mathrm{d}\mathbf{x}^lpha}{\mathrm{d} au} \!=\! rac{1}{\sqrt{1-rac{\mathbf{V}^2}{\mathrm{c}^2}}} rac{\mathrm{d}\mathbf{x}^lpha}{\mathrm{d}t}$ 

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# 基本洛伦兹变换~任意运动方向坐标速度变换

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 $\mathbf{U}^lpha \equiv rac{\mathrm{d}\mathbf{x}^lpha}{\mathrm{d} au} \!=\! rac{1}{\sqrt{1-rac{\mathrm{V}^2}{\mathrm{c}^2}}} rac{\mathrm{d}\mathbf{x}^lpha}{\mathrm{d}t} 
ightarrow \left\{egin{array}{c} \mathbf{U^i} \!=\! rac{\mathbf{v^i}}{\sqrt{1-rac{\mathrm{v}^2}{\mathrm{c}^2}}} \ \mathbf{U^4} \!=\! rac{\mathrm{ic}}{\sqrt{1-rac{\mathrm{v}^2}{2}}} \end{array}
ight.$  $\mathbf{x}^{1} = \mathbf{x}, \mathbf{x}^{2} = \mathbf{y}, \mathbf{x}^{3} = \mathbf{z}$  $x^4 = ict$ 

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# 百对论的时空理论: 间隔的不变性

c有限,钟和尺子测量的结果与参考系的选择有关,它们不能再作为绝对标准来衡量事件之间的时空关系.



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c有限,钟和尺子测量的结果与参考 系的选择有关,它们不能再作为绝对标准来衡量 事件之间的时空关系. 需要寻找新的与参考系选择无关的量,即在洛伦兹变换下不 变的量来作为绝对标准.









#### 相对论的时空理论:间隔的不变性

c有限,钟和尺子测量的结果与参考 系的选择有关,它们不能再作为绝对标准来衡量事件之间的时空关系. 需要寻找新的与 参考系选择无关的量,即在洛伦兹变换下不变的量来作为绝对标准.

对一个参考系中的两个事件 $(x_1,y_1,z_1,t_1),(x_2,y_2,z_2,t_2),$ 定义它们之间的 间隔为:

$$\Delta s^2 \equiv c^2 (t_1 - t_2)^2 - (x_1 - x_2)^2 - (y_1 - y_2)^2 - (z_1 - z_2)^2$$







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# 国对论的时空理论: 间隔的不变性

c有限,钟和尺子测量的结果与参考 系的选择有关,它们不能再作为绝对标准来衡量事件之间的时空关系. 需要寻找新的与 参考系选择无关的量,即在洛伦兹变换下不变的量来作为绝对标准.

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$$egin{aligned} \Delta \mathbf{s}^2 &\equiv \mathbf{c}^2 (\mathbf{t_1} - \mathbf{t_2})^2 - (\mathbf{x_1} - \mathbf{x_2})^2 - (\mathbf{y_1} - \mathbf{y_2})^2 - (\mathbf{z_1} - \mathbf{z_2})^2 \ &= \mathbf{c}^2 (\Delta \mathbf{t})^2 - (\Delta \mathbf{x})^2 - (\Delta \mathbf{y})^2 - (\Delta \mathbf{z})^2 \end{aligned}$$







# **计对论的时空理论**: 间隔的不变性

c有限,钟和尺子测量的结果与参考系的选择有关,它们不能再作为绝对标准来衡量事件之间的时空关系. 需要寻找新的与参考系选择无关的量,即在洛伦兹变换下不变的量来作为绝对标准.

对一个参考系中的两个事件 $(x_1, y_1, z_1, t_1), (x_2, y_2, z_2, t_2),$ 定义它们之间的 间隔为:

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 $\Delta s'^2 \equiv c^2 (t'_1 - t'_2)^2 - (x'_1 - x'_2)^2 - (y'_1 - y'_2)^2 - (z'_1 - z'_2)^2$ 



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c有限,钟和尺子测量的结果与参考 系的选择有关,它们不能再作为绝对标准来衡量 事件之间的时空关系. 需要寻找新的与参考系选择无关的量.即在洛伦兹变换下不 变的量来作为绝对标准.

对一个参考系中的两个事件 $(x_1, y_1, z_1, t_1), (x_2, y_2, z_2, t_2),$ 定义它们之间的 间隔为:

$$egin{aligned} \Delta \mathbf{s^2} &\equiv \mathbf{c^2} (\mathbf{t_1} - \mathbf{t_2})^2 - (\mathbf{x_1} - \mathbf{x_2})^2 - (\mathbf{y_1} - \mathbf{y_2})^2 - (\mathbf{z_1} - \mathbf{z_2})^2 \ &= \mathbf{c^2} (\Delta \mathbf{t})^2 - (\Delta \mathbf{x})^2 - (\Delta \mathbf{y})^2 - (\Delta \mathbf{z})^2 \end{aligned}$$

$$egin{aligned} \Delta {f s'}^2 &\equiv {f c}^2 ({f t}_1' - {f t}_2')^2 - ({f x}_1' - {f x}_2')^2 - ({f y}_1' - {f y}_2')^2 - ({f z}_1' - {f z}_2')^2 \ & = {f t}_1 - {f t}_2 - rac{{f V}}{{f c}^2} ({f x}_1 - {f x}_2)_{12} & {f x}_1 - {f x}_2 - {f V} ({f t}_1 - {f t}_2)_{12} & {f v}_2 \end{aligned}$$

$$egin{aligned} \Delta \mathbf{s} &\equiv \mathbf{c}^{\mathtt{-}}(\mathbf{t}_{1} - \mathbf{t}_{2})^{\mathtt{-}} - (\mathbf{x}_{1} - \mathbf{x}_{2})^{\mathtt{-}} - (\mathbf{y}_{1} - \mathbf{y}_{2})^{\mathtt{-}} - (\mathbf{z}_{1} - \mathbf{z}_{2})^{\mathtt{-}} \ =& \mathbf{c}^{2} [rac{\mathbf{t}_{1} - \mathbf{t}_{2} - rac{\mathbf{V}}{\mathbf{c}^{2}}(\mathbf{x}_{1} - \mathbf{x}_{2})}{\sqrt{1 - rac{\mathbf{V}^{2}}{\mathbf{c}^{2}}}}]^{2} - [rac{\mathbf{x}_{1} - \mathbf{x}_{2} - \mathbf{V}(\mathbf{t}_{1} - \mathbf{t}_{2})}{\sqrt{1 - rac{\mathbf{V}^{2}}{\mathbf{c}^{2}}}}]^{2} - (\mathbf{y}_{1} - \mathbf{y}_{2})^{2} - (\mathbf{z}_{1} - \mathbf{z}_{2})^{2} \end{aligned}$$





c有限,钟和尺子测量的结果与参考 系的选择有关,它们不能再作为绝对标准来衡量 事件之间的时空关系. 需要寻找新的与参考系选择无关的量.即在洛伦兹变换下不 变的量来作为绝对标准.

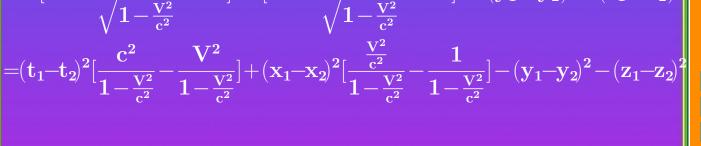
对一个参考系中的两个事件 $(x_1, y_1, z_1, t_1), (x_2, y_2, z_2, t_2),$ 定义它们之间的 间隔为:

$$egin{aligned} \Delta \mathbf{s^2} &\equiv \mathbf{c^2} (\mathbf{t_1} - \mathbf{t_2})^2 - (\mathbf{x_1} - \mathbf{x_2})^2 - (\mathbf{y_1} - \mathbf{y_2})^2 - (\mathbf{z_1} - \mathbf{z_2})^2 \ &= \mathbf{c^2} (\Delta \mathbf{t})^2 - (\Delta \mathbf{x})^2 - (\Delta \mathbf{y})^2 - (\Delta \mathbf{z})^2 \end{aligned}$$

$$egin{aligned} \Delta {f s'}^2 &\equiv {f c}^2 ({f t}_1' - {f t}_2')^2 - ({f x}_1' - {f x}_2')^2 - ({f y}_1' - {f y}_2')^2 - ({f z}_1' - {f z}_2')^2 \ &= {f c}^2 [rac{{f t}_1 - {f t}_2 - rac{{f v}}{{f c}^2} ({f x}_1 - {f x}_2)}{2}]^2 - [rac{{f x}_1 - {f x}_2 - {f V} ({f t}_1 - {f t}_2)}{2}]^2 - ({f v}_1 - {f v}_2)^2 - {f v}_1' - {f v}_2' - {f$$

$$= c^{2} \left[ \frac{t_{1} - t_{2} - \frac{V}{c^{2}}(x_{1} - x_{2})}{\sqrt{1 - \frac{V^{2}}{c^{2}}}} \right]^{2} - \left[ \frac{x_{1} - x_{2} - V(t_{1} - t_{2})}{\sqrt{1 - \frac{V^{2}}{c^{2}}}} \right]^{2} - (y_{1} - y_{2})^{2} - (z_{1} - z_{2})^{2}$$

$$= (t_{1} - t_{2})^{2} \left[ \frac{c^{2}}{c^{2}} - \frac{V^{2}}{c^{2}} \right] + (x_{1} - x_{2})^{2} \left[ \frac{\frac{V^{2}}{c^{2}}}{c^{2}} - \frac{1}{c^{2}} \right] - (y_{1} - y_{2})^{2} - (z_{1} - z_{2})^{2}$$









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c有限,钟和尺子测量的结果与参考 系的选择有关,它们不能再作为绝对标准来衡量 事件之间的时空关系. 需要寻找新的与参考系选择无关的量.即在洛伦兹变换下不 变的量来作为绝对标准.

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$$\begin{split} \Delta s^2 & \equiv c^2 (t_1 - t_2)^2 - (x_1 - x_2)^2 - (y_1 - y_2)^2 - (z_1 - z_2)^2 \\ & = c^2 (\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2 \end{split}$$

$$egin{aligned} \Delta {f s'}^2 &\equiv {f c}^2 (t_1' - t_2')^2 - ({f x}_1' - {f x}_2')^2 - ({f y}_1' - {f y}_2')^2 - ({f z}_1' - {f z}_2')^2 \ &= {f c}^2 [rac{{f t}_1 - {f t}_2 - rac{{f v}}{{f c}^2} ({f x}_1 - {f x}_2)}{\sqrt{{f v}_1 - {f v}_2}}]^2 - [rac{{f x}_1 - {f x}_2 - {f V} ({f t}_1 - {f t}_2)}{\sqrt{{f v}_1 - {f v}_2}}]^2 - ({f y}_1 - {f y}_2)^2 - {f v}_1' - {f v}_2' - {f v}_$$

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m V}^2}{{
m c}^2}}}]^2 \! - ({
m y_1} \! - \! {
m y_2})^2 \! - ({
m z_1} \! - \! {
m z_2})^2$$

$$egin{align*} & \sqrt{1-rac{c^2}{c^2}} & \sqrt{1-rac{c^2}{c^2}} \ = & (t_1-t_2)^2 [rac{c^2}{1-rac{V^2}{c^2}} - rac{V^2}{1-rac{V^2}{c^2}}] + (\mathbf{x}_1-\mathbf{x}_2)^2 [rac{rac{V^2}{c^2}}{1-rac{V^2}{c^2}} - rac{1}{1-rac{V^2}{c^2}}] - (\mathbf{y}_1-\mathbf{y}_2)^2 - (\mathbf{z}_1-\mathbf{z}_2)^2 \ = & \Delta \mathbf{s}^2 \end{aligned}$$







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#### 间隔的不变性

c有限,钟和尺子测量的结果与参考 系的选择有关,它们不能再作为绝对标准来衡量 事件之间的时空关系. 需要寻找新的与参考系选择无关的量,即在洛伦兹变换下不 变的量来作为绝对标准.

对一个参考系中的两个事件 $(x_1,y_1,z_1,t_1),(x_2,y_2,z_2,t_2),$ 定义它们之间的 间隔为:

$$\begin{split} \Delta s^2 & \equiv c^2 (t_1 - t_2)^2 - (x_1 - x_2)^2 - (y_1 - y_2)^2 - (z_1 - z_2)^2 \\ & = c^2 (\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2 \end{split}$$

$$oldsymbol{\Delta s'}^2 \equiv \mathbf{c^2} (\mathbf{t_1'} - \mathbf{t_2'})^2 - (\mathbf{x_1'} - \mathbf{x_2'})^2 - (\mathbf{y_1'} - \mathbf{y_2'})^2 - (\mathbf{z_1'} - \mathbf{z_2'})^2 \ \mathbf{t_1} - \mathbf{t_2} - \mathbf{v_2'} (\mathbf{x_1} - \mathbf{x_2})_{12} \mathbf{v_1} - \mathbf{x_2} - \mathbf{V} (\mathbf{t_1} - \mathbf{t_2})_{12} \mathbf{v_2} \mathbf{v_2} \mathbf{v_2}^2 - \mathbf{v_2} \mathbf{v_2} \mathbf{v_2}^2 \mathbf{v_$$

$$egin{aligned} egin{aligned} egi$$

$$egin{align*} &\sqrt{1-rac{\dot{\mathbf{c}}^2}{\mathbf{c}^2}} &\sqrt{1-rac{\dot{\mathbf{c}}^2}{\mathbf{c}^2}} \ = &(\mathbf{t_1} - \mathbf{t_2})^2 [rac{\mathbf{c}^2}{1-rac{\mathbf{V}^2}{\mathbf{c}^2}} - rac{\mathbf{V}^2}{1-rac{\mathbf{V}^2}{\mathbf{c}^2}}] + (\mathbf{x_1} - \mathbf{x_2})^2 [rac{rac{\dot{\mathbf{V}}^2}{\mathbf{c}^2}}{1-rac{\dot{\mathbf{V}}^2}{\mathbf{c}^2}} - rac{1}{1-rac{\dot{\mathbf{V}}^2}{\mathbf{c}^2}}] - (\mathbf{y_1} - \mathbf{y_2})^2 - (\mathbf{z_1} - \mathbf{z_2})^2 . \end{align}$$

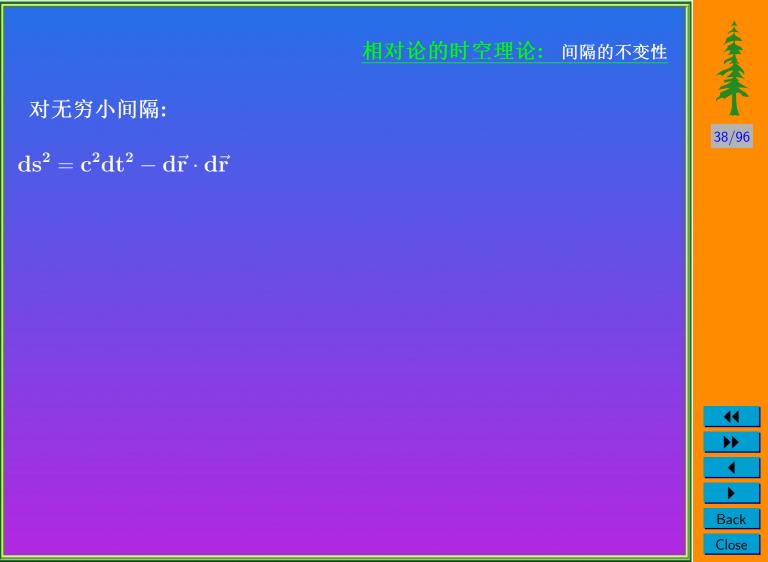
一般的洛伦兹变换在数学上定义为保持间隔不变的 (x, y, z, t)的齐次线性变换.

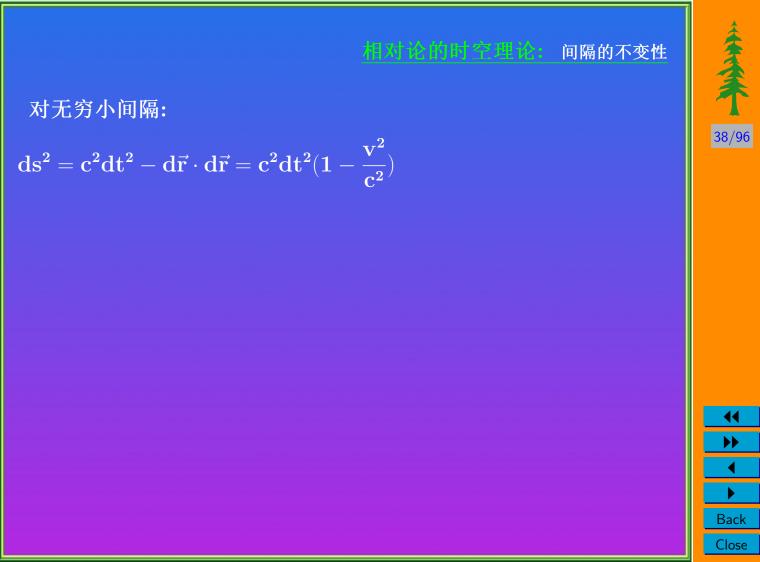


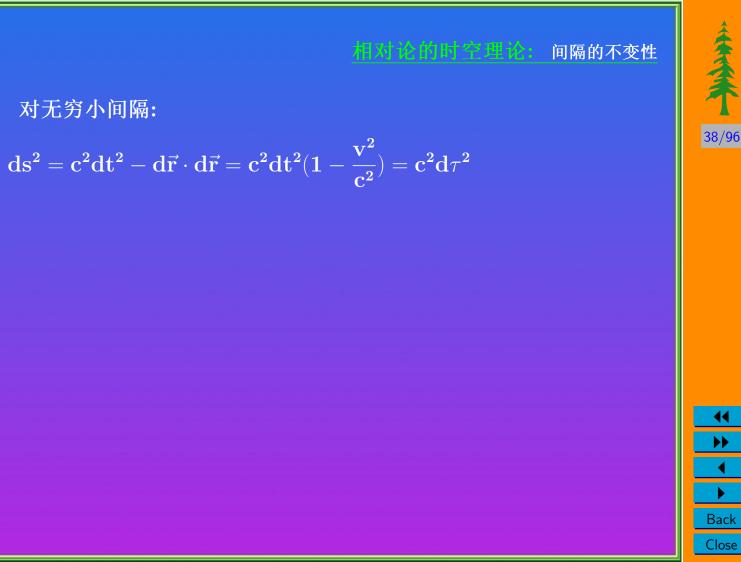


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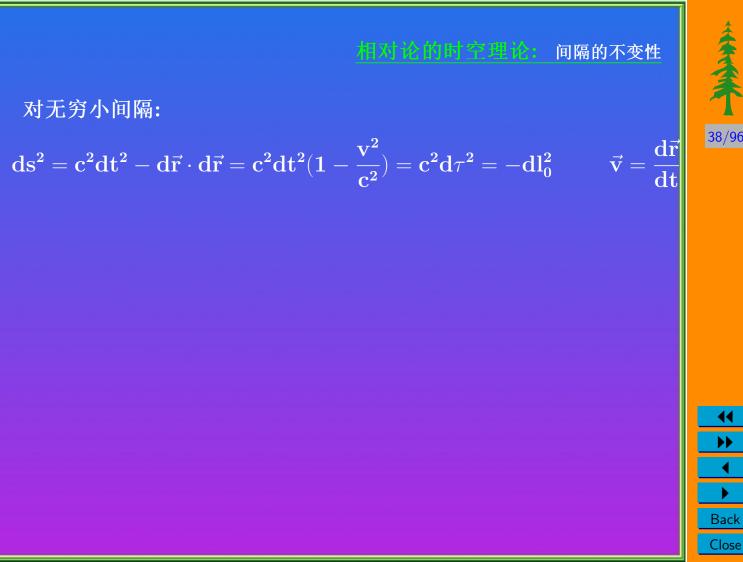








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## 相对论的时空理论:间隔的不变性

对无穷小间隔:

$$\mathrm{d}s^2 = \mathrm{c}^2\mathrm{d}t^2 - \mathrm{d}ec{r}\cdot\mathrm{d}ec{r} = \mathrm{c}^2\mathrm{d}t^2(1-rac{\mathrm{v}^2}{\mathrm{c}^2}) = \mathrm{c}^2\mathrm{d} au^2 = -\mathrm{d}l_0^2$$
  $\vec{\mathrm{v}} = rac{\mathrm{d}ec{r}}{\mathrm{d}t}$ 

● 对 $ds^2 > 0$ , $d\tau \equiv \frac{1}{c}\sqrt{ds^2}$ 是钟的固有时(原时), 它是由间隔引出的具有时间量纲的不变量.



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对无穷小间隔:

$$egin{aligned} \mathrm{d} s^2 &= \mathrm{c}^2 \mathrm{d} t^2 - \mathrm{d} ec{r} \cdot \mathrm{d} ec{r} = \mathrm{c}^2 \mathrm{d} t^2 (1 - rac{\mathrm{v}^2}{\mathrm{c}^2}) = \mathrm{c}^2 \mathrm{d} au^2 = - \mathrm{d} \mathrm{l}_0^2 \qquad ec{v} = rac{\mathrm{d} ec{r}}{\mathrm{d} t} \end{aligned}$$
 $\bullet \ \, orall \mathrm{d} s^2 > 0, \mathrm{d} au \equiv rac{1}{c} \sqrt{\mathrm{d} s^2}$  是钟的固有时(原时),它是由间隔

- 引出的具有时间量纲的不变量.
- 对 $ds^2 < 0$ , $dl_0 \equiv \sqrt{-ds^2}$ 是尺的固有长度,它是由间隔引出的具有长度量纲的不变量.

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对无穷小间隔:

- 对 $ds^2 < 0$ , $dl_0 \equiv \sqrt{-ds^2}$ 是尺的固有长度,它是由间隔引出的具有长度量纲的不变量.
- 两事件关系的分类:

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对无穷小间隔:

$$egin{aligned} \mathrm{d} s^2 &= \mathrm{c}^2 \mathrm{d} t^2 - \mathrm{d} ec{r} \cdot \mathrm{d} ec{r} = \mathrm{c}^2 \mathrm{d} t^2 (1 - rac{\mathrm{v}^2}{\mathrm{c}^2}) = \mathrm{c}^2 \mathrm{d} au^2 = - \mathrm{d} l_0^2 \qquad ec{\mathrm{v}} = rac{\mathrm{d} ec{r}}{\mathrm{d} t} \end{aligned}$$
 $ullet \ \, orall \ \, \mathrm{d} s^2 > 0, \ \, \mathrm{d} au \equiv rac{1}{c} \sqrt{\mathrm{d} s^2} \ \, \mathrm{E} \ \, \mathrm{end} \ \, \mathrm{fh} \ \, \mathrm{fh}$ 

- 引出的具有时间量纲的不变量.
- 对 $ds^2 < 0$ , $dl_0 \equiv \sqrt{-ds^2}$ 是尺的固有长度,它是由间隔引出的具有长度量纲的不变量.

#### 两事件关系的分类:

•  $\Delta s^2 > 0$ : 类时事件,类时间隔

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对无穷小间隔:

 $\mathrm{d} s^2 = c^2 \mathrm{d} t^2 - \mathrm{d} \vec{r} \cdot \mathrm{d} \vec{r} = c^2 \mathrm{d} t^2 (1 - rac{v^2}{c^2}) = c^2 \mathrm{d} au^2 = - \mathrm{d} l_0^2 \qquad \vec{v} = rac{\mathrm{d} \vec{r}}{\mathrm{d} t}$ 

- 对 $ds^2 > 0$ , $d\tau \equiv \frac{1}{c}\sqrt{ds^2}$ 是钟的固有时(原时), 它是由间隔引出的具有时间量纲的不变量.
- 对 $ds^2 < 0$ , $dl_0 \equiv \sqrt{-ds^2}$ 是尺的固有长度,它是由间隔引出的具有长度量纲的不变量.

#### 两事件关系的分类:

- $\Delta s^2 > 0$ : 类时事件,类时间隔
- $\Delta s^2 < 0$ : 类空事件,类空间隔

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# 秦季

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### 对无穷小间隔:

 $\mathrm{d}s^2 = \mathrm{c}^2\mathrm{d}t^2 - \mathrm{d}ec{r}\cdot\mathrm{d}ec{r} = \mathrm{c}^2\mathrm{d}t^2(1-rac{\mathrm{v}^2}{\mathrm{c}^2}) = \mathrm{c}^2\mathrm{d} au^2 = -\mathrm{d}l_0^2 \qquad ec{v} = rac{\mathrm{d}ec{r}}{\mathrm{d}t}$   $\bullet \; \forall \mathrm{d}s^2 > 0, \mathrm{d} au \equiv frac{1}{c}\sqrt{\mathrm{d}s^2}$ 是钟的固有时(原时),它是由间隔

- 对 $ds^2 < 0$ , $dl_0 \equiv \sqrt{-ds^2}$ 是尺的固有长度,它是由间隔引出的具有长度量纲的不变量。
- 两事件关系的分类:
- $\Delta s^2 > 0$ : 类时事件,类时间隔
  - $\Delta s^2 < 0$ : 类空事件,类空间隔

引出的具有时间量纲的不变量.

•  $\Delta s^2 = 0$ : 类光事件,类光间隔

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一般洛伦兹变换有六个独立自由度,三个相对运动,三个坐标轴相对转动.



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## 四维时空坐标变换 一般洛伦兹变换有六个独立自由度,三个相对运动,三个坐标轴相对转动. $c^2t^2 - (x^2 + y^2 + z^2) = c^2{t'}^2 - ({x'}^2 + {y'}^2 + {z'}^2)$ 39/96 Back Close

#### 四维时空坐标变换 一般洛伦兹变换有六个独立自由度,三个相对运动,三个坐标轴相对转动. $c^2t^2-(x^2+y^2+z^2)=c^2{t'}^2-({x'}^2+{y'}^2+{z'}^2)$ $x_1=x, x_2=y, x_3=z, x_4=ict \rightarrow$ 39/96 $x_1^2 + x_2^2 + x_3^2 + x_4^2 = x_1'^2 + x_2'^2 + x_3'^2 + x_4'^2$







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一般洛伦兹变换有六个独立自由度,三个相对运动,三个坐标轴相对转动.

$$c^2t^2\!\!-\!\!(x^2\!\!+\!\!y^2\!\!+\!\!z^2)\!\!=\!\!c^2{t'}^2\!\!-\!\!({x'}^2\!\!+\!\!{y'}^2\!\!+\!\!{z'}^2) \qquad x_1\!\!=\!\!x,x_2\!\!=\!\!y,x_3\!\!=\!\!z,x_4\!\!=\!\!\mathbf{ict} \rightarrow$$



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一般洛伦兹变换有六个独立自由度,三个相对运动,三个坐标轴相对转动.

$${
m c^2t^2}\!\!-\!\!({
m x^2}\!\!+\!\!{
m y^2}\!\!+\!\!{
m z^2})\!\!=\!\!{
m c^2t'^2}\!\!-\!\!({
m x'}^2\!\!+\!\!{
m y'}^2\!\!+\!\!{
m z'}^2) \hspace{0.5cm} {
m x_1}\!\!=\!\!{
m x},{
m x_2}\!\!=\!\!{
m y},{
m x_3}\!\!=\!\!{
m z},{
m x_4}\!\!=\!\!{
m ict} o 1$$



已在理论中出现付)措述时间分量外,不用上下标和度规(这新增了东西)的张量方式反映时间分量的特殊性

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一般洛伦兹变换有六个独立自由度,三个相对运动,三个坐标轴相对转动.

$$c^2t^2\!\!-\!\!(x^2\!\!+\!\!y^2\!\!+\!\!z^2)\!\!=\!\!c^2t'^2\!\!-\!\!(x'^2\!\!+\!\!y'^2\!\!+\!\!z'^2) \qquad x_1\!\!=\!\!x, x_2\!\!=\!\!y, x_3\!\!=\!\!z, x_4\!\!=\!\!ict \rightarrow$$

$$\mathbf{x}_1^2\!+\!\mathbf{x}_2^2\!+\!\mathbf{x}_3^2\!+\!\mathbf{x}_4^2\!=\!\mathbf{x}_1^{\prime\,2}\!+\!\mathbf{x}_2^{\prime\,2}\!+\!\mathbf{x}_3^{\prime\,2}\!+\!\mathbf{x}_4^{\prime\,2}$$
 或  $\sum_{\mu=1}^{}\!\!\mathbf{x}_{\mu}\mathbf{x}_{\mu}=\sum_{\mu=1}^{}\!\!\mathbf{x}_{\mu}^{\prime}\mathbf{x}_{\mu}^{\prime}$ 

 $\overline{\mu=1}$   $\overline{\mu=1}$  除用复数(己在理论中出现过)描述时间分量外,不用上下标和度规(**这新增了东西**)的张量方式反映时间分量的特殊性

时间的复数表示还特别提供了在实际的Minkovski空间的场论被看成是有更好数学性质的**欧式空间场论**的解析延拓的可能

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一般洛伦兹变换有六个独立自由度,三个相对运动,三个坐标轴相对转动.

$$c^2t^2\!\!-\!\!(x^2\!\!+\!\!y^2\!\!+\!\!z^2)\!\!=\!\!c^2t'^2\!\!-\!\!(x'^2\!\!+\!\!y'^2\!\!+\!\!z'^2) \qquad x_1\!\!=\!\!x,x_2\!\!=\!\!y,x_3\!\!=\!\!z,x_4\!\!=\!\!ict\rightarrow$$

$$oxed{x_1^2 + x_2^2 + x_3^2 + x_4^2 = {x_1'}^2 + {x_2'}^2 + {x_3'}^2 + {x_4'}^2} \quad 
oxed{oxed{x}} \quad 
oxed{\sum_{\mu=1}^{\mu}} x_\mu x_\mu = \sum_{\mu=1}^{\mu} x_\mu' x_\mu' x_\mu' = \sum_{\mu=1}^{\mu} x_\mu' x_\mu' x_\mu' = \sum_{\mu=1}^{\mu} x_\mu' x_\mu' = \sum_{\mu=1}^{\mu} x_\mu' x_\mu' x_\mu' = \sum_{$$

$$\mu$$
= $f 1$   $\mu$ = $f 1$  除用复数(己在理论中出现过)描述时间分量外。不用上下标和度规(**这新增了东西**)的张量方式反映时间分量的特殊性

数学性质的欧式空间场论的解析

满足
$$x_{\mu}=0$$
与 $x_{\mu}'=0$ 对应的一般线性变换为:  $\mathbf{x}_{\mu}'=\sum_{\nu=1}\mathbf{a}_{\mu\nu}\mathbf{x}_{\nu}$ 







一般洛伦兹变换有六个独立自由度,三个相对运动,三个坐标轴相对转动.

$$c^2t^2\!\!-\!\!(x^2\!\!+\!\!y^2\!\!+\!\!z^2)\!\!=\!\!c^2{t'}^2\!\!-\!\!({x'}^2\!\!+\!\!{y'}^2\!\!+\!\!{z'}^2) \qquad x_1\!\!=\!\!x,x_2\!\!=\!\!y,x_3\!\!=\!\!z,x_4\!\!=\!\!\mathbf{ict} \rightarrow$$

$$\mathbf{x}_1^2\!+\!\mathbf{x}_2^2\!+\!\mathbf{x}_3^2\!+\!\mathbf{x}_4^2\!=\!{\mathbf{x}_1'}^2\!+\!{\mathbf{x}_2'}^2\!+\!{\mathbf{x}_3'}^2\!+\!{\mathbf{x}_4'}^2$$
 或  $\sum_{\mu=1}\!\mathbf{x}_\mu\mathbf{x}_\mu=\sum_{\mu=1}\!\mathbf{x}_\mu'\mathbf{x}_\mu'$ 

$$\mu{=}1$$
  $\mu{=}1$  除用复数(已在理论中出现过)描述时间分量外。不用上下标泊度规(**这新增了东西**)的张量方式反映时间分量的赞殊性

满足
$$x_{\mu}=0$$
与 $x'_{\mu}=0$ 对应的一般线性变换为:  $\mathbf{x}'_{\mu}=\sum\mathbf{a}_{\mu\nu}\mathbf{x}_{\nu}$ 

$$\sum_{\mu=1}^4 \mathbf{x}_\mu' \mathbf{x}_\mu' = \sum_{\mu,
u,\lambda=1}^4 \mathbf{a}_{\mu
u} \mathbf{x}_
u \mathbf{a}_{\mu\lambda} \mathbf{x}_\lambda$$







一般洛伦兹变换有六个独立自由度,三个相对运动,三个坐标轴相对转动.

$$c^2t^2\!\!-\!\!(x^2\!\!+\!\!y^2\!\!+\!\!z^2)\!\!=\!\!c^2t'^2\!\!-\!\!(x'^2\!\!+\!\!y'^2\!\!+\!\!z'^2) \qquad x_1\!\!=\!\!x, x_2\!\!=\!\!y, x_3\!\!=\!\!z, x_4\!\!=\!\!ict \rightarrow$$

$$\mathbf{x}_1^2\!+\!\mathbf{x}_2^2\!+\!\mathbf{x}_3^2\!+\!\mathbf{x}_4^2\!=\!\mathbf{x}_1^{\prime}{}^2\!+\!\mathbf{x}_2^{\prime}{}^2\!+\!\mathbf{x}_3^{\prime}{}^2\!+\!\mathbf{x}_4^{\prime}{}^2 \quad ext{id} \quad \sum_{\mu=1}^{2}\!\!\mathbf{x}_{\mu}^{\prime}\mathbf{x}_{\mu}^{\prime} = \sum_{\mu=1}^{2}\!\!\mathbf{x}_{\mu}^{\prime}\mathbf{x}_{\mu}^{\prime} \, .$$

(已在理论中出现过) 描述时间分量外。不用上下标和度规(**这新增了东西**) 的张量方式反映时间分量的特殊件

满足
$$x_{\mu}=0$$
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$$\sum_{\mu=1}^4 \mathbf{x}_\mu' \mathbf{x}_\mu' = \sum_{\mu,
u,\lambda=1}^4 \mathbf{a}_{\mu
u} \mathbf{x}_
u \mathbf{a}_{\mu\lambda} \mathbf{x}_\lambda = \sum_{\mu,
u,\lambda=1}^4 \mathbf{a}_{\mu
u} \mathbf{a}_{\mu\lambda} \mathbf{x}_
u \mathbf{x}_\lambda$$









一般洛伦兹变换有六个独立自由度,三个相对运动,三个坐标轴相对转动.

$$c^2t^2\!\!-\!\!(x^2\!\!+\!\!y^2\!\!+\!\!z^2)\!\!=\!\!c^2t'^2\!\!-\!\!(x'^2\!\!+\!\!y'^2\!\!+\!\!z'^2) \qquad x_1\!\!=\!\!x,x_2\!\!=\!\!y,x_3\!\!=\!\!z,x_4\!\!=\!\!ict\rightarrow$$

$$\mathbf{x}_1^2\!+\!\mathbf{x}_2^2\!+\!\mathbf{x}_3^2\!+\!\mathbf{x}_4^2\!=\!\mathbf{x}_1^{\prime\,\,2}\!+\!\mathbf{x}_2^{\prime\,\,2}\!+\!\mathbf{x}_3^{\prime\,\,2}\!+\!\mathbf{x}_4^{\prime\,\,2}$$
 或  $\sum_{\mu=1}^{}\!\!\mathbf{x}_{\mu}^{}\mathbf{x}_{\mu}^{}=\sum_{\mu=1}^{}\!\!\mathbf{x}_{\mu}^{\prime}\mathbf{x}_{\mu}^{\prime}$ 

时间的复数表示还特别提供了在实际的Minkovski空间的场论被看成是有更好数学性质的**欧式空间场论**的解析延拓的可能件!

满足
$$x_{\mu}=0$$
与 $x_{\mu}'=0$ 对应的一般线性变换为:  $\mathbf{x}_{\mu}'=\sum_{\nu=1}\mathbf{a}_{\mu\nu}\mathbf{x}_{\nu}$ 

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u,\lambda=1}^4 \mathbf{a}_{\mu
u} \mathbf{x}_
u \mathbf{a}_{\mu\lambda} \mathbf{x}_\lambda = \sum_{\mu,
u,\lambda=1}^4 \mathbf{a}_{\mu
u} \mathbf{a}_{\mu\lambda} \mathbf{x}_
u \mathbf{x}_\lambda = \sum_{\mu=1}^4 \mathbf{x}_\mu \mathbf{x}_\mu$$



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一般洛伦兹变换有六个独立自由度,三个相对运动,三个坐标轴相对转动.

$$\mathbf{c^2t^2} \!\!-\!\! (\mathbf{x^2} \!\!+\!\! \mathbf{y^2} \!\!+\!\! \mathbf{z^2}) \!\!=\!\! \mathbf{c^2t'^2} \!\!-\!\! (\mathbf{x'^2} \!\!+\!\! \mathbf{y'^2} \!\!+\!\! \mathbf{z'^2}) \qquad \mathbf{x_1} \!\!=\!\! \mathbf{x}, \mathbf{x_2} \!\!=\!\! \mathbf{y}, \mathbf{x_3} \!\!=\!\! \mathbf{z}, \mathbf{x_4} \!\!=\!\! \mathbf{ict} \rightarrow \mathbf{x_1} \!\!=\!\! \mathbf{x}, \mathbf{x_2} \!\!=\!\! \mathbf{y}, \mathbf{x_3} \!\!=\!\! \mathbf{z}, \mathbf{x_4} \!\!=\!\! \mathbf{ict} \rightarrow \mathbf{x_5} \!\!=\!\! \mathbf{ict} \rightarrow \mathbf{ict}$$

$$egin{align*} \mathbf{x}_1^2\!+\!\mathbf{x}_2^2\!+\!\mathbf{x}_3^2\!+\!\mathbf{x}_4^2\!=\!{\mathbf{x}_1'}^2\!+\!{\mathbf{x}_2'}^2\!+\!{\mathbf{x}_3'}^2\!+\!{\mathbf{x}_4'}^2 & ext{gl} & \sum_{\mu=1}^{2}\!\!\mathbf{x}_{\mu}^{\prime}\mathbf{x}_{\mu}^{\prime} & = \sum_{\mu=1}^{2}\!\!\mathbf{x}_{\mu}^{\prime}\mathbf{x}_{\mu}^{\prime} & & & \end{aligned}$$

$$\mu{=}1$$
  $\mu{=}1$  除用复数(已在理论中出现过)描述时间分量外,不用上下标和度想(**这新增了东西**)的张景方式反映时间分量的影

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满足
$$x_{\mu}=0$$
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u,\lambda=1}^4 \mathbf{a}_{\mu
u} \mathbf{x}_
u \mathbf{a}_{\mu\lambda} \mathbf{x}_\lambda = \sum_{\mu,
u,\lambda=1}^4 \mathbf{a}_{\mu
u} \mathbf{a}_{\mu\lambda} \mathbf{x}_
u \mathbf{x}_\lambda = \sum_{\mu=1}^4 \mathbf{x}_\mu \mathbf{x}_\mu$$

$$\sum_{\mu=1}^{n} \mathbf{a}_{\mu
u} \mathbf{a}_{\mu\lambda} = \delta_{
u\lambda}$$



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一般洛伦兹变换有六个独立自由度,三个相对运动,三个坐标轴相对转动.

$$c^2t^2\!\!-\!\!(x^2\!\!+\!\!y^2\!\!+\!\!z^2)\!\!=\!\!c^2{t'}^2\!\!-\!\!(x'^2\!\!+\!\!y'^2\!\!+\!\!z'^2) \qquad x_1\!\!=\!\!x,x_2\!\!=\!\!y,x_3\!\!=\!\!z,x_4\!\!=\!\!ict \rightarrow$$

$$\mathbf{x}_1^2\!+\!\mathbf{x}_2^2\!+\!\mathbf{x}_3^2\!+\!\mathbf{x}_4^2\!=\!\mathbf{x}_1^{\prime\,2}\!+\!\mathbf{x}_2^{\prime\,2}\!+\!\mathbf{x}_3^{\prime\,2}\!+\!\mathbf{x}_4^{\prime\,2}$$
 或  $\sum_{\mu=1}\mathbf{x}_{\mu}\mathbf{x}_{\mu}=\sum_{\mu=1}\mathbf{x}_{\mu}^{\prime}\mathbf{x}_{\mu}^{\prime}$ 

$$\mu{=}\mathbf{1}$$
  $\mu{=}\mathbf{1}$  除用复数(已在理论中出现过)描述时间分量外。不用上下标泊度规(**这新增了东西**)的张量方式反映时间分量的影

$$\frac{4}{1}$$

满足
$$x_{\mu}=0$$
与 $x_{\mu}'=0$ 对应的一般线性变换为:  $\mathbf{x}_{\mu}'=\sum_{\nu=1}\mathbf{a}_{\mu\nu}\mathbf{x}_{\nu}$ 

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u,\lambda=1}^4 \mathbf{a}_{\mu
u} \mathbf{x}_
u \mathbf{a}_{\mu\lambda} \mathbf{x}_\lambda = \sum_{\mu,
u,\lambda=1}^4 \mathbf{a}_{\mu
u} \mathbf{a}_{\mu\lambda} \mathbf{x}_
u \mathbf{x}_\lambda = \sum_{\mu=1}^4 \mathbf{x}_\mu \mathbf{x}_\mu$$

$$\sum \mathbf{a}_{\mu\nu}\mathbf{a}_{\mu\lambda}=\delta_{
u\lambda}$$
 给出10个约束方程



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一般洛伦兹变换有六个独立自由度,三个相对运动,三个坐标轴相对转动.

$$c^2t^2\!\!-\!\!(x^2\!\!+\!\!y^2\!\!+\!\!z^2)\!\!=\!\!c^2{t'}^2\!\!-\!\!({x'}^2\!\!+\!\!{y'}^2\!\!+\!\!{z'}^2) \qquad x_1\!\!=\!\!x, x_2\!\!=\!\!y, x_3\!\!=\!\!z, x_4\!\!=\!\!ict \rightarrow$$

$$\mathbf{x}_1^2\!+\!\mathbf{x}_2^2\!+\!\mathbf{x}_3^2\!+\!\mathbf{x}_4^2\!=\!{\mathbf{x}_1'}^2\!+\!{\mathbf{x}_2'}^2\!+\!{\mathbf{x}_3'}^2\!+\!{\mathbf{x}_4'}^2$$
 或  $\sum_{\mu=1}\!\mathbf{x}_\mu\mathbf{x}_\mu=\sum_{\mu=1}\!\mathbf{x}_\mu'\mathbf{x}_\mu'$ 

$$\mu{=}\mathbf{1}$$
  $\mu{=}\mathbf{1}$  除用复数(已在理论中出现过)错误时间分量的影

可间的复数表示还特别提供了在实际的Minkovski空间的场论被看成是有更好数学性质的**欧式空间场论**的解析延拓的可能

满足
$$x_{\mu}=0$$
与 $x_{\mu}'=0$ 对应的一般线性变换为:  $\mathbf{x}_{\mu}'=\sum_{\nu=1}\mathbf{a}_{\mu\nu}\mathbf{x}_{\nu}$ 

$$\sum_{\mu=1}^4 \mathbf{x}_\mu' \mathbf{x}_\mu' = \sum_{\mu,
u,\lambda=1}^4 \mathbf{a}_{\mu
u} \mathbf{x}_
u \mathbf{a}_{\mu\lambda} \mathbf{x}_\lambda = \sum_{\mu,
u,\lambda=1}^4 \mathbf{a}_{\mu
u} \mathbf{a}_{\mu\lambda} \mathbf{x}_
u \mathbf{x}_\lambda = \sum_{\mu=1}^4 \mathbf{x}_\mu \mathbf{x}_\mu'$$

$$\sum_{\mu=1}^4 \mathbf{a}_{\mu
u} \mathbf{a}_{\mu\lambda} = \delta_{
u\lambda}$$
 给出10个约束方程  $\mathbf{x}_{\mu} = \sum_{
u=1}^4 \delta_{\mu
u} \mathbf{x}_{
u}$ 



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**>>** 

一般洛伦兹变换有六个独立自由度,三个相对运动,三个坐标轴相对转动.

$$c^2t^2\!\!-\!\!(x^2\!\!+\!\!y^2\!\!+\!\!z^2)\!\!=\!\!c^2{t'}^2\!\!-\!\!({x'}^2\!\!+\!\!{y'}^2\!\!+\!\!{z'}^2) \qquad x_1\!\!=\!\!x, x_2\!\!=\!\!y, x_3\!\!=\!\!z, x_4\!\!=\!\!ict \rightarrow$$

$$\mathbf{x}_1^2\!+\!\mathbf{x}_2^2\!+\!\mathbf{x}_3^2\!+\!\mathbf{x}_4^2\!=\!\mathbf{x}_1^{\prime}{}^2\!+\!\mathbf{x}_2^{\prime}{}^2\!+\!\mathbf{x}_3^{\prime}{}^2\!+\!\mathbf{x}_4^{\prime}{}^2 \quad ext{if} \quad \sum_{\mu=1}^{2}\!\!\mathbf{x}_{\mu}^{\prime}\mathbf{x}_{\mu}^{\prime} = \sum_{\mu=1}^{2}\!\!\mathbf{x}_{\mu}^{\prime}\mathbf{x}_{\mu}^{\prime}$$

$$\mu{=}\mathbf{1}$$
  $\mu{=}\mathbf{1}$  除用复数(已在理论中出现过)措述时间分量外。不用上下标和度规(**这新增了东西**)的张量方式反映时间分量的接近

时间的复数表示还特别提供了在实际的Minkovski空间的场论被看就是有更好数学性质的**欧式空间场论**的解析延振的可能

满足
$$x_{\mu}=0$$
与 $x_{\mu}'=0$ 对应的一般线性变换为:  $\mathbf{x}_{\mu}'=\sum_{\nu=1}\mathbf{a}_{\mu\nu}\mathbf{x}_{\nu}$ 

$$\sum_{\mu=1}^4 \mathbf{x}_\mu' \mathbf{x}_\mu' = \sum_{\mu,
u,\lambda=1}^4 \mathbf{a}_{\mu
u} \mathbf{x}_
u \mathbf{a}_{\mu\lambda} \mathbf{x}_\lambda = \sum_{\mu,
u,\lambda=1}^4 \mathbf{a}_{\mu
u} \mathbf{a}_{\mu\lambda} \mathbf{x}_
u \mathbf{x}_\lambda = \sum_{\mu=1}^4 \mathbf{x}_\mu \mathbf{x}_\mu'$$

$$\sum_{\mu=1}^4 \mathbf{a}_{\mu\nu} \mathbf{a}_{\mu\lambda} = \delta_{\nu\lambda}$$
 给出10个约束方程  $\mathbf{x}_{\mu} = \sum_{\nu=1}^4 \delta_{\mu\nu} \mathbf{x}_{\nu} = \sum_{\nu,\lambda=1}^4 \mathbf{a}_{\lambda\mu} \mathbf{a}_{\lambda\nu} \mathbf{x}_{\nu}$ 



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44

1

Back

一般洛伦兹变换有六个独立自由度,三个相对运动,三个坐标轴相对转动.

$$c^2t^2\!\!-\!\!(x^2\!\!+\!\!y^2\!\!+\!\!z^2)\!\!=\!\!c^2{t'}^2\!\!-\!\!({x'}^2\!\!+\!\!{y'}^2\!\!+\!\!{z'}^2) \qquad x_1\!\!=\!\!x, x_2\!\!=\!\!y, x_3\!\!=\!\!z, x_4\!\!=\!\!ict \rightarrow$$

$$\mathbf{x}_1^2\!+\!\mathbf{x}_2^2\!+\!\mathbf{x}_3^2\!+\!\mathbf{x}_4^2\!=\!\mathbf{x}_1^{\prime\,2}\!+\!\mathbf{x}_2^{\prime\,2}\!+\!\mathbf{x}_3^{\prime\,2}\!+\!\mathbf{x}_4^{\prime\,2}$$
 或  $\sum_{\mu=1}\!\mathbf{x}_\mu\mathbf{x}_\mu=\sum_{\mu=1}\!\mathbf{x}_\mu^{\prime}\mathbf{x}_\mu^{\prime}$ 

 $\mu{=}\mathbf{1}$   $\mu{=}\mathbf{1}$  除用复数(已在理论中出现过)描述时间分量外。不用上下标泊度规(**这新增了东西**)的张量方式反映时间分量的影

 $\frac{4}{2}$ 

满足
$$x_{\mu}=0$$
与 $x_{\mu}'=0$ 对应的一般线性变换为:  $\mathbf{x}_{\mu}'=\sum_{\nu=1}\mathbf{a}_{\mu\nu}\mathbf{x}_{\nu}$ 

$$\sum_{\mu=1}^4 \mathbf{x}_\mu' \mathbf{x}_\mu' = \sum_{\mu,
u,\lambda=1}^4 \mathbf{a}_{\mu
u} \mathbf{x}_
u \mathbf{a}_{\mu\lambda} \mathbf{x}_\lambda = \sum_{\mu,
u,\lambda=1}^4 \mathbf{a}_{\mu
u} \mathbf{a}_{\mu\lambda} \mathbf{x}_
u \mathbf{x}_\lambda = \sum_{\mu=1}^4 \mathbf{x}_\mu \mathbf{x}_\mu \mathbf{x}_\mu$$

$$\sum_{\mu=1}^4 \mathbf{a}_{\mu
u} \mathbf{a}_{\mu\lambda} = \delta_{
u\lambda}$$
 给出10个约束方程  $\mathbf{x}_{\mu} = \sum_{
u=1}^4 \delta_{\mu
u} \mathbf{x}_{
u} = \sum_{
u,\lambda=1}^4 \mathbf{a}_{\lambda\mu} \mathbf{a}_{\lambda
u} \mathbf{x}_{
u} = \sum_{\lambda=1}^4 \mathbf{a}_{\lambda\mu} \mathbf{a}_{\lambda
u} \mathbf{x}_{
u} = \sum_{\lambda=1}^4 \mathbf{a}_{\lambda\mu} \mathbf{a}_{\lambda
u} \mathbf{x}_{
u}$ 



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**→** 

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#### 起对沙理论的协变形式: 四维时空坐标变换

一般洛伦兹变换有六个独立自由度,三个相对运动,三个坐标轴相对转动.

$${
m c^2t^2}\!\!-\!\!({
m x^2}\!\!+\!\!{
m y^2}\!\!+\!\!{
m z^2})\!\!=\!\!{
m c^2t'^2}\!\!-\!\!({
m x'}^2\!\!+\!\!{
m y'}^2\!\!+\!\!{
m z'}^2) \hspace{0.5cm} {
m x_1}\!\!=\!\!{
m x},{
m x_2}\!\!=\!\!{
m y},{
m x_3}\!\!=\!\!{
m z},{
m x_4}\!\!=\!\!{
m ict} o 1$$

$$\mathbf{x}_1^2\!+\!\mathbf{x}_2^2\!+\!\mathbf{x}_3^2\!+\!\mathbf{x}_4^2\!=\!\mathbf{x}_1^{\prime}{}^2\!+\!\mathbf{x}_2^{\prime}{}^2\!+\!\mathbf{x}_3^{\prime}{}^2\!+\!\mathbf{x}_4^{\prime}{}^2 \quad ext{ if } \quad \sum_{\mu=1}^{}\!\!\mathbf{x}_{\mu}^{}\mathbf{x}_{\mu}^{} = \sum_{\mu=1}^{}\!\!\mathbf{x}_{\mu}^{\prime}\mathbf{x}_{\mu}^{\prime}$$

论中出现过)描述时间分量外,不用上下标和度规(**这新增了东西**)的张量方式反

满足
$$x_{\mu}=0$$
与 $x_{\mu}'=0$ 对应的一般线性变换为:  $\mathbf{x}_{\mu}'=\sum_{\nu=1}\mathbf{a}_{\mu\nu}\mathbf{x}_{\nu}$ 

$$\sum_{\mu=1}^4 \mathbf{x}_\mu' \mathbf{x}_\mu' = \sum_{\mu,
u,\lambda=1}^4 \mathbf{a}_{\mu
u} \mathbf{x}_
u \mathbf{a}_{\mu\lambda} \mathbf{x}_\lambda = \sum_{\mu,
u,\lambda=1}^4 \mathbf{a}_{\mu
u} \mathbf{a}_{\mu\lambda} \mathbf{x}_
u \mathbf{x}_\lambda = \sum_{\mu=1}^4 \mathbf{x}_\mu \mathbf{x}_\mu \mathbf{x}_\mu \mathbf{x}_\lambda$$

$$\sum_{\mu=1}^4 \mathbf{a}_{\mu
u} \mathbf{a}_{\mu\lambda} = \delta_{
u\lambda}$$
 给出10个约束方程  $\mathbf{x}_{\mu} = \sum_{
u=1}^4 \delta_{\mu
u} \mathbf{x}_{
u} = \sum_{
u,\lambda=1}^4 \mathbf{a}_{\lambda\mu} \mathbf{a}_{\lambda
u} \mathbf{x}_{
u} = \sum_{\lambda=1}^4 \mathbf{a}_{\lambda
u} \mathbf{a}_{
\mu} \mathbf{a}_{
\nu} \mathbf{a}_{$ 

另一种表达形式: 
$$\sum_{
u=1}^4 a_{
u\mu} a_{\lambda\mu} = \delta_{
u\lambda}$$



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一般洛伦兹变换有六个独立自由度,三个相对运动,三个坐标轴相对转动.

$$\mathbf{x}_{\mu}' = \sum_{
u=1}^4 \mathbf{a}_{\mu
u} \mathbf{x}_{
u} \quad \sum_{\mu=1}^4 \mathbf{a}_{\mu
u} \mathbf{a}_{\mu\lambda} = \delta_{
u\lambda} \quad \mathbf{x}_{\mu} = \sum_{\lambda=1}^4 \mathbf{a}_{\lambda\mu} \mathbf{x}_{\lambda}' \quad \sum_{\mu=1}^4 \mathbf{a}_{
u\mu} \mathbf{a}_{\lambda\mu} = \delta_{
u\lambda}$$









一般洛伦兹变换有六个独立自由度,三个相对运动,三个坐标轴相对转动.

$$\mathbf{x}_{\mu}' = \sum_{
u=1}^4 \mathbf{a}_{\mu
u} \mathbf{x}_{
u} \quad \sum_{\mu=1}^4 \mathbf{a}_{\mu
u} \mathbf{a}_{\mu\lambda} = \delta_{
u\lambda} \quad \mathbf{x}_{\mu} = \sum_{\lambda=1}^4 \mathbf{a}_{\lambda\mu} \mathbf{x}_{\lambda}' \quad \sum_{\mu=1}^4 \mathbf{a}_{
u\mu} \mathbf{a}_{\lambda\mu} = \delta_{
u\lambda}$$

$$\mathbf{X} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \qquad \mathbf{A} = \begin{pmatrix} \mathbf{a}_{11} & \mathbf{a}_{12} & \mathbf{a}_{13} & \mathbf{a}_{14} \\ \mathbf{a}_{21} & \mathbf{a}_{22} & \mathbf{a}_{23} & \mathbf{a}_{24} \\ \mathbf{a}_{31} & \mathbf{a}_{32} & \mathbf{a}_{33} & \mathbf{a}_{34} \\ \mathbf{a}_{41} & \mathbf{a}_{42} & \mathbf{a}_{43} & \mathbf{a}_{44} \end{pmatrix}$$









一般洛伦兹变换有六个独立自由度,三个相对运动,三个坐标轴相对转动.

$$\mathbf{x}_{\mu}' = \sum_{
u=1}^4 \mathbf{a}_{\mu
u} \mathbf{x}_{
u} \quad \sum_{\mu=1}^4 \mathbf{a}_{\mu
u} \mathbf{a}_{\mu\lambda} = \delta_{
u\lambda} \quad \mathbf{x}_{\mu} = \sum_{\lambda=1}^4 \mathbf{a}_{\lambda\mu} \mathbf{x}_{\lambda}' \quad \sum_{\mu=1}^4 \mathbf{a}_{
u\mu} \mathbf{a}_{\lambda\mu} = \delta_{
u\lambda}'$$

$$\mathbf{X} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$
  $\mathbf{A} = \begin{pmatrix} \mathbf{a_{11}} & \mathbf{a_{12}} & \mathbf{a_{13}} & \mathbf{a_{14}} \\ \mathbf{a_{21}} & \mathbf{a_{22}} & \mathbf{a_{23}} & \mathbf{a_{24}} \\ \mathbf{a_{31}} & \mathbf{a_{32}} & \mathbf{a_{33}} & \mathbf{a_{34}} \\ \mathbf{a_{41}} & \mathbf{a_{42}} & \mathbf{a_{43}} & \mathbf{a_{44}} \end{pmatrix}$   $\mathbf{X}' = \mathbf{A}\mathbf{X}$   $\mathbf{X} = \mathbf{A}^{\mathrm{T}}\mathbf{X}'$   $\mathbf{A}^{\mathrm{T}}\mathbf{A} = \mathbf{I}$   $\mathbf{A}\mathbf{A}^{\mathrm{T}} = \mathbf{I}$ 

44

**>>** 



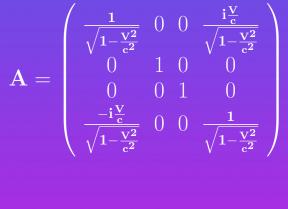
### 起对 企理 论的 协变形式: 四维时空坐标变换

一般洛伦兹变换有六个独立自由度,三个相对运动,三个坐标轴相对转动.

$$\mathbf{x}_{\mu}' = \sum_{
u=1}^4 \mathbf{a}_{\mu
u} \mathbf{x}_{
u} \quad \sum_{\mu=1}^4 \mathbf{a}_{\mu
u} \mathbf{a}_{\mu\lambda} = \delta_{
u\lambda} \quad \mathbf{x}_{\mu} = \sum_{\lambda=1}^4 \mathbf{a}_{\lambda\mu} \mathbf{x}_{\lambda}' \quad \sum_{\mu=1}^4 \mathbf{a}_{
u\mu} \mathbf{a}_{\lambda\mu} = \delta_{
u\lambda}$$

$$\mathbf{X} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \qquad \mathbf{A} = \begin{pmatrix} \mathbf{a}_{11} & \mathbf{a}_{12} & \mathbf{a}_{13} & \mathbf{a}_{14} \\ \mathbf{a}_{21} & \mathbf{a}_{22} & \mathbf{a}_{23} & \mathbf{a}_{24} \\ \mathbf{a}_{31} & \mathbf{a}_{32} & \mathbf{a}_{33} & \mathbf{a}_{34} \\ \mathbf{a}_{41} & \mathbf{a}_{42} & \mathbf{a}_{43} & \mathbf{a}_{44} \end{pmatrix}$$
 $\mathbf{X}' = \mathbf{A}\mathbf{X} \qquad \mathbf{X} = \mathbf{A}^{\mathrm{T}}\mathbf{X}' \qquad \mathbf{A}^{\mathrm{T}}\mathbf{A} = \mathbf{I} \qquad \mathbf{A}\mathbf{A}^{\mathrm{T}} = \mathbf{I}$ 

$$\begin{pmatrix} \frac{1}{\sqrt{1-\mathbf{Y}^2}} & 0 & 0 & \frac{\mathbf{i}^{\mathbf{Y}}_{\mathbf{c}}}{\sqrt{1-\mathbf{Y}^2}} \end{pmatrix}$$











一般洛伦兹变换有六个独立自由度,三个相对运动,三个坐标轴相对转动.

$$\mathbf{x}_{\mu}' = \sum_{
u=1}^4 \mathbf{a}_{\mu
u} \mathbf{x}_{
u} \quad \sum_{\mu=1}^4 \mathbf{a}_{\mu
u} \mathbf{a}_{\mu\lambda} = \delta_{
u\lambda} \quad \mathbf{x}_{\mu} = \sum_{\lambda=1}^4 \mathbf{a}_{\lambda\mu} \mathbf{x}_{\lambda}' \quad \sum_{\mu=1}^4 \mathbf{a}_{
u\mu} \mathbf{a}_{\lambda\mu} = \delta_{
u\lambda}$$

$$\mathbf{X} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \qquad \mathbf{A} = \begin{pmatrix} \mathbf{a}_{11} & \mathbf{a}_{12} & \mathbf{a}_{13} & \mathbf{a}_{14} \\ \mathbf{a}_{21} & \mathbf{a}_{22} & \mathbf{a}_{23} & \mathbf{a}_{24} \\ \mathbf{a}_{31} & \mathbf{a}_{32} & \mathbf{a}_{33} & \mathbf{a}_{34} \\ \mathbf{a}_{41} & \mathbf{a}_{42} & \mathbf{a}_{43} & \mathbf{a}_{44} \end{pmatrix}$$

$$\mathbf{X}' = \mathbf{A}\mathbf{X} \qquad \mathbf{X} = \mathbf{A}^{\mathsf{T}}\mathbf{X}' \qquad \mathbf{A}^{\mathsf{T}}\mathbf{A} = \mathbf{I} \qquad \mathbf{A}\mathbf{A}^{\mathsf{T}} = \mathbf{I}$$

$$\mathbf{A} = \begin{pmatrix} \frac{1}{\sqrt{1 - \frac{\mathbf{V}^2}{c^2}}} & 0 & 0 & \frac{\mathbf{i}\frac{\mathbf{V}}{c}}{\sqrt{1 - \frac{\mathbf{V}^2}{c^2}}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{-\mathbf{i}\frac{\mathbf{V}}{c}}{\sqrt{1 - \frac{\mathbf{V}^2}{c^2}}} & 0 & 0 & \frac{1}{\sqrt{1 - \frac{\mathbf{V}^2}{c^2}}} \end{pmatrix}$$

 $\mathbf{1} = \det \mathbf{I} = \det(\mathbf{A}^{\mathrm{T}}\mathbf{A})$ 









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一般洛伦兹变换有六个独立自由度,三个相对运动,三个坐标轴相对转动.

$$\mathbf{x}_{\mu}' = \sum_{
u=1}^4 \mathbf{a}_{\mu
u} \mathbf{x}_{
u} \quad \sum_{\mu=1}^4 \mathbf{a}_{\mu
u} \mathbf{a}_{\mu\lambda} = \delta_{
u\lambda} \quad \mathbf{x}_{\mu} = \sum_{\lambda=1}^4 \mathbf{a}_{\lambda\mu} \mathbf{x}_{\lambda}' \quad \sum_{\mu=1}^4 \mathbf{a}_{
u\mu} \mathbf{a}_{\lambda\mu} = \delta_{
u\lambda}$$

$$\mathbf{X} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \qquad \mathbf{A} = \begin{pmatrix} \mathbf{a}_{11} & \mathbf{a}_{12} & \mathbf{a}_{13} & \mathbf{a}_{14} \\ \mathbf{a}_{21} & \mathbf{a}_{22} & \mathbf{a}_{23} & \mathbf{a}_{24} \\ \mathbf{a}_{31} & \mathbf{a}_{32} & \mathbf{a}_{33} & \mathbf{a}_{34} \\ \mathbf{a}_{41} & \mathbf{a}_{42} & \mathbf{a}_{43} & \mathbf{a}_{44} \end{pmatrix}$$

$$\mathbf{X}' = \mathbf{A}\mathbf{X} \qquad \mathbf{X} = \mathbf{A}^{\mathsf{T}}\mathbf{X}' \qquad \mathbf{A}^{\mathsf{T}}\mathbf{A} = \mathbf{I} \qquad \mathbf{A}\mathbf{A}^{\mathsf{T}} = \mathbf{I}$$

$$\mathbf{A} = \begin{pmatrix} \frac{1}{\sqrt{1 - \frac{\mathbf{V}^2}{c^2}}} & 0 & 0 & \frac{\mathbf{i}\frac{\mathbf{V}}{c}}{\sqrt{1 - \frac{\mathbf{V}^2}{c^2}}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\mathbf{i}\frac{\mathbf{V}}{c} & 0 & 0 & \frac{\mathbf{1}}{\sqrt{1 - \frac{\mathbf{V}^2}{c^2}}} \end{pmatrix}$$

 $\mathbf{1} = \det \mathbf{I} = \det(\mathbf{A}^{\mathbf{T}}\mathbf{A}) = (\det \mathbf{A}^{\mathbf{T}})(\det \mathbf{A})$ 



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一般洛伦兹变换有六个独立自由度,三个相对运动,三个坐标轴相对转动.

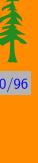
$$\mathbf{x}_{\mu}' = \sum_{
u=1}^4 \mathbf{a}_{\mu
u} \mathbf{x}_{
u} \quad \sum_{\mu=1}^4 \mathbf{a}_{\mu
u} \mathbf{a}_{\mu\lambda} = \delta_{
u\lambda} \quad \mathbf{x}_{\mu} = \sum_{\lambda=1}^4 \mathbf{a}_{\lambda\mu} \mathbf{x}_{\lambda}' \quad \sum_{\mu=1}^4 \mathbf{a}_{
u\mu} \mathbf{a}_{\lambda\mu} = \delta_{
u\lambda}$$

$$\mathbf{X} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \qquad \mathbf{A} = \begin{pmatrix} \mathbf{a}_{11} & \mathbf{a}_{12} & \mathbf{a}_{13} & \mathbf{a}_{14} \\ \mathbf{a}_{21} & \mathbf{a}_{22} & \mathbf{a}_{23} & \mathbf{a}_{24} \\ \mathbf{a}_{31} & \mathbf{a}_{32} & \mathbf{a}_{33} & \mathbf{a}_{34} \\ \mathbf{a}_{41} & \mathbf{a}_{42} & \mathbf{a}_{43} & \mathbf{a}_{44} \end{pmatrix}$$

$$\mathbf{X}' = \mathbf{A}\mathbf{X} \qquad \mathbf{X} = \mathbf{A}^{\mathsf{T}}\mathbf{X}' \qquad \mathbf{A}^{\mathsf{T}}\mathbf{A} = \mathbf{I} \qquad \mathbf{A}\mathbf{A}^{\mathsf{T}} = \mathbf{I}$$

$$\mathbf{A} = \begin{pmatrix} \frac{1}{\sqrt{1 - \frac{\mathbf{V}^2}{c^2}}} & 0 & 0 & \frac{\mathbf{i}^{\mathsf{V}}_{\mathsf{C}}}{\sqrt{1 - \frac{\mathbf{V}^2}{c^2}}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{-\mathbf{i}^{\mathsf{V}}_{\mathsf{C}}}{\sqrt{1 - \frac{\mathbf{V}^2}{c^2}}} & 0 & 0 & \frac{1}{\sqrt{1 - \frac{\mathbf{V}^2}{c^2}}} \end{pmatrix}$$

 $\mathbf{1} = \det \mathbf{I} = \det(\mathbf{A}^{T}\mathbf{A}) = (\det \mathbf{A}^{T})(\det \mathbf{A}) = (\det \mathbf{A})^{2}$ 









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一般洛伦兹变换有六个独立自由度,三个相对运动,三个坐标轴相对转动.

$$\mathbf{x}_{\mu}' = \sum_{
u=1}^4 \mathbf{a}_{\mu
u} \mathbf{x}_{
u} \quad \sum_{\mu=1}^4 \mathbf{a}_{\mu
u} \mathbf{a}_{\mu\lambda} = \delta_{
u\lambda} \quad \mathbf{x}_{\mu} = \sum_{\lambda=1}^4 \mathbf{a}_{\lambda\mu} \mathbf{x}_{\lambda}' \quad \sum_{\mu=1}^4 \mathbf{a}_{
u\mu} \mathbf{a}_{\lambda\mu} = \delta_{
u\lambda}$$

 $\mathbf{1} = \det \mathbf{I} = \det(\mathbf{A}^{\mathbf{T}}\mathbf{A}) = (\det \mathbf{A}^{\mathbf{T}})(\det \mathbf{A}) = (\det \mathbf{A})^{\mathbf{2}} \to (\det \mathbf{A}) = \pm \mathbf{1}$ 

$$\mathbf{X} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \qquad \mathbf{A} = \begin{pmatrix} \mathbf{a}_{11} & \mathbf{a}_{12} & \mathbf{a}_{13} & \mathbf{a}_{14} \\ \mathbf{a}_{21} & \mathbf{a}_{22} & \mathbf{a}_{23} & \mathbf{a}_{24} \\ \mathbf{a}_{31} & \mathbf{a}_{32} & \mathbf{a}_{33} & \mathbf{a}_{34} \\ \mathbf{a}_{41} & \mathbf{a}_{42} & \mathbf{a}_{43} & \mathbf{a}_{44} \end{pmatrix}$$

$$\mathbf{X}' = \mathbf{A}\mathbf{X} \qquad \mathbf{X} = \mathbf{A}^{\mathsf{T}}\mathbf{X}' \qquad \mathbf{A}^{\mathsf{T}}\mathbf{A} = \mathbf{I} \qquad \mathbf{A}\mathbf{A}^{\mathsf{T}} = \mathbf{I}$$

$$\mathbf{A} = \begin{pmatrix} \frac{1}{\sqrt{1 - \frac{\mathbf{V}^2}{c^2}}} & 0 & 0 & \frac{\mathbf{i}\frac{\mathbf{V}}{c}}{\sqrt{1 - \frac{\mathbf{V}^2}{c^2}}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\mathbf{i}\frac{\mathbf{V}}{c} & 0 & 0 & \frac{\mathbf{I}}{\sqrt{1 - \frac{\mathbf{V}^2}{c^2}}} \end{pmatrix}$$



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# 相对论理论的协变形式: 四维时空坐标变换

一般洛伦兹变换有六个独立自由度,三个相对运动,三个坐标轴相对转动.

$$\mathbf{x}_{\mu}' = \sum_{
u=1}^4 \mathbf{a}_{\mu
u} \mathbf{x}_{
u} \quad \sum_{\mu=1}^4 \mathbf{a}_{\mu
u} \mathbf{a}_{\mu\lambda} = \delta_{
u\lambda} \quad \mathbf{x}_{\mu} = \sum_{\lambda=1}^4 \mathbf{a}_{\lambda\mu} \mathbf{x}_{\lambda}' \quad \sum_{\mu=1}^4 \mathbf{a}_{
u\mu} \mathbf{a}_{\lambda\mu} = \delta_{
u\lambda}$$

$$\mathbf{X} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \qquad \mathbf{A} = \begin{pmatrix} \mathbf{a}_{11} & \mathbf{a}_{12} & \mathbf{a}_{13} & \mathbf{a}_{14} \\ \mathbf{a}_{21} & \mathbf{a}_{22} & \mathbf{a}_{23} & \mathbf{a}_{24} \\ \mathbf{a}_{31} & \mathbf{a}_{32} & \mathbf{a}_{33} & \mathbf{a}_{34} \\ \mathbf{a}_{41} & \mathbf{a}_{42} & \mathbf{a}_{43} & \mathbf{a}_{44} \end{pmatrix}$$

$$\mathbf{X}' = \mathbf{A}\mathbf{X} \qquad \mathbf{X} = \mathbf{A}^{\mathsf{T}}\mathbf{X}' \qquad \mathbf{A}^{\mathsf{T}}\mathbf{A} = \mathbf{I} \qquad \mathbf{A}\mathbf{A}^{\mathsf{T}} = \mathbf{I}$$

$$\mathbf{A} = \begin{pmatrix} \frac{1}{\sqrt{1 - \frac{\mathbf{V}^2}{c^2}}} & 0 & 0 & \frac{\mathbf{i} \frac{\mathbf{V}}{c}}{\sqrt{1 - \frac{\mathbf{V}^2}{c^2}}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{-\mathbf{i} \frac{\mathbf{V}}{c}}{\sqrt{1 - \frac{\mathbf{V}^2}{c^2}}} & 0 & 0 & \frac{1}{\sqrt{1 - \frac{\mathbf{V}^2}{c^2}}} \end{pmatrix}$$

 $\begin{pmatrix} \frac{c}{\sqrt{1-\frac{\mathbf{V}^2}{c^2}}} & 0 & 0 & \frac{1}{\sqrt{1-\frac{\mathbf{V}^2}{c^2}}} \end{pmatrix}$   $\mathbf{1} = \det \mathbf{I} = \det(\mathbf{A^T}\mathbf{A}) = (\det \mathbf{A^T})(\det \mathbf{A}) = (\det \mathbf{A})^2 \to (\det \mathbf{A}) = \pm \mathbf{1}$   $\det \mathbf{A} = \mathbf{1}$ 由纯转动构成









# 相对论理论的协变形式:四维时空坐标变换

一般洛伦兹变换有六个独立自由度,三个相对运动,三个坐标轴相对转动.

$$\mathbf{x}_{\mu}' = \sum_{
u=1}^4 \mathbf{a}_{\mu
u} \mathbf{x}_{
u} \quad \sum_{\mu=1}^4 \mathbf{a}_{\mu
u} \mathbf{a}_{\mu\lambda} = \delta_{
u\lambda} \quad \mathbf{x}_{\mu} = \sum_{\lambda=1}^4 \mathbf{a}_{\lambda\mu} \mathbf{x}_{\lambda}' \quad \sum_{\mu=1}^4 \mathbf{a}_{
u\mu} \mathbf{a}_{\lambda\mu} = \delta_{
u\lambda} \quad \mathbf{x}_{\mu} = \mathbf{a}_{\mu\lambda} \mathbf{a}_{\lambda\mu} \mathbf{a}_{\lambda\mu} = \delta_{
u\lambda} \quad \mathbf{x}_{\mu} = \mathbf{a}_{\mu\lambda} \mathbf{a}_{\lambda\mu} \mathbf{a}_{\lambda\mu} = \delta_{
u\lambda} \quad \mathbf{a}_{\mu\lambda} \mathbf{a}_{\mu\lambda} = \delta_{
u\lambda} \quad \mathbf{a}_{\mu\lambda} \quad \mathbf{a}_{\mu\lambda} \quad \mathbf{a}_{\mu\lambda} \quad$$

$$\mathbf{X} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \qquad \mathbf{A} = \begin{pmatrix} \mathbf{a}_{11} & \mathbf{a}_{12} & \mathbf{a}_{13} & \mathbf{a}_{14} \\ \mathbf{a}_{21} & \mathbf{a}_{22} & \mathbf{a}_{23} & \mathbf{a}_{24} \\ \mathbf{a}_{31} & \mathbf{a}_{32} & \mathbf{a}_{33} & \mathbf{a}_{34} \\ \mathbf{a}_{41} & \mathbf{a}_{42} & \mathbf{a}_{43} & \mathbf{a}_{44} \end{pmatrix}$$

$$\mathbf{X}' = \mathbf{A}\mathbf{X} \qquad \mathbf{X} = \mathbf{A}^{\mathsf{T}}\mathbf{X}' \qquad \mathbf{A}^{\mathsf{T}}\mathbf{A} = \mathbf{I} \qquad \mathbf{A}\mathbf{A}^{\mathsf{T}} = \mathbf{I}$$

$$\mathbf{A} = \begin{pmatrix} \frac{1}{\sqrt{1 - \frac{\mathbf{V}^2}{c^2}}} & 0 & 0 & \frac{\mathbf{i}^{\mathsf{V}}_{\mathsf{C}}}{\sqrt{1 - \frac{\mathbf{V}^2}{c^2}}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\mathbf{i}^{\mathsf{V}}_{\mathsf{C}} & 0 & 0 & \frac{\mathbf{1}}{\sqrt{1 - \frac{\mathbf{V}^2}{c^2}}} \end{pmatrix}$$









相对性原理的数学表达: 3+1维时空张量方程



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#### 相对性原理的数学表达: 3+1维时空张量方程

• 0阶张量:(标量,洛伦兹不变量)  $\phi' = \phi$ 

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#### 相对性原理的数学表达: 3+1维时空张量方程

- 0阶张量:(标量,洛伦兹不变量)  $\phi' = \phi$
- 1阶张量:(矢量)  $\mathbf{A}'_{\mu} = \sum_{
  u=1}^{n} \mathbf{a}_{\mu
  u} \mathbf{A}_{
  u}$







#### 相对性原理的数学表达: 3+1维时空张量方程

- 0阶张量:(标量,洛伦兹不变量)  $\phi' = \phi$
- ullet 1阶张量:(矢量)  $\mathbf{A}'_{\mu} = \sum_{
  u=1}^{n} \mathbf{a}_{\mu
  u} \mathbf{A}_{
  u}$
- 2阶张量: $\mathbf{B}'_{\mu 
  u} = \sum_{\mu', 
  u'=1} \mathbf{a}_{\mu \mu'} \mathbf{a}_{
  u 
  u'} \mathbf{B}_{\mu' 
  u'}$

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#### 相对性原理的数学表达: 3+1维时空张量方程

- 0阶张量:(标量,洛伦兹不变量)  $\phi' = \phi$
- 1阶张量:(矢量)  $\mathbf{A}'_{\mu} = \sum_{
  u=1}^{n} \mathbf{a}_{\mu
  u} \mathbf{A}_{
  u}$
- 2阶张量: $\mathbf{B}'_{\mu 
  u} = \overline{\sum_{\mu', 
  u'=1}} \mathbf{a}_{\mu \mu'} \mathbf{a}_{
  u 
  u'} \mathbf{B}_{\mu' 
  u'}$
- • •
- $\mathbf{n}$ 所张量:  $\mathbf{T}'_{\mu_1\cdots\mu_\mathbf{n}} = \sum_{
  u_1,\cdots,
  u_\mathbf{n}=1} \mathbf{a}_{\mu_1
  u_1}\cdots\mathbf{a}_{\mu_\mathbf{n}
  u_\mathbf{n}} \mathbf{T}_{
  u_1\cdots
  u_\mathbf{n}}$

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#### 相对性原理的数学表达: 3+1维时空张量方程

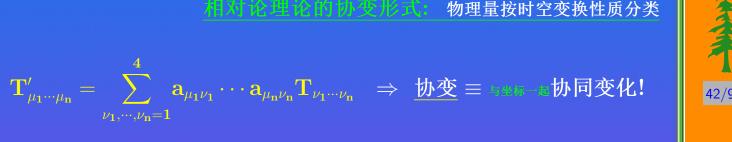
- 0阶张量:(标量,洛伦兹不变量)  $\phi' = \phi$
- 1阶张量:(矢量)  $\mathbf{A}'_{\mu} = \sum_{\nu=1}^{r} \mathbf{a}_{\mu\nu} \mathbf{A}_{\nu}$
- 2阶张量: $\mathrm{B}'_{\mu
  u} = \sum_{\mu',
  u'=1}^{} \mathrm{a}_{\mu\mu'} \mathrm{a}_{
  u
  u'} \mathrm{B}_{\mu'
  u'}$
- •
- п阶张量:  $\mathbf{T}'_{\mu_1\cdots\mu_{\mathbf{n}}} = \sum_{
  u_1,\dots,
  u_{\mathbf{n}}=1} \mathbf{a}_{\mu_1
  u_1}\cdots\mathbf{a}_{\mu_{\mathbf{n}}
  u_{\mathbf{n}}} \mathbf{T}_{
  u_1\cdots
  u_{\mathbf{n}}}$

n阶张量的特点是有 $4^n$ 个分量,在洛伦兹变换下,按n个坐标乘积的变换方式变换.

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 $\mathbf{T}'_{\mu_1\cdots\mu_n} = \sum_{
u_1,\cdots,
u_n=1} \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_n
u_n}\mathbf{T}_{
u_1\cdots
u_n} \quad \Rightarrow \quad \underline{b}$  <u>协变</u> = 与型层一定协同变化!

$$\mathbf{T}_{\mu_{\mathbf{1}} \cdots \mu_{\mathbf{n}}} = \mathbf{F}_{\mu_{\mathbf{1}} \cdots \mu_{\mathbf{n}}}$$





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 $\mathbf{T}'_{\mu_1\cdots\mu_n} = \sum_{\mathbf{a}}^{\mathbf{a}} \mathbf{a}_{\mu_1\nu_1}\cdots\mathbf{a}_{\mu_n\nu_n}\mathbf{T}_{\nu_1\cdots\nu_n} \Rightarrow \underline{\mathbf{b}}\underline{\mathfrak{T}} \equiv \mathbf{b}$  证据一是协同变化!

$$\mathbf{T}_{\mu_1 \cdots \mu_{\mathbf{n}}} = \mathbf{F}_{\mu_1 \cdots \mu_{\mathbf{n}}}$$









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 $\mathbf{T}'_{\mu_1\cdots\mu_n} = \sum_{\mathbf{a}_{\mu_1\nu_1}\cdots\mathbf{a}_{\mu_n\nu_n}} \mathbf{T}_{\nu_1\cdots\nu_n} \quad \Rightarrow \quad \underline{b}$  要 与选版一定协同变化!

$$\mu_{\mathbf{n}}$$

 $\mathbf{T}_{\mu_1\cdots\mu_{\mathbf{n}}}=\mathbf{F}_{\mu_1\cdots\mu_{\mathbf{n}}}$ 













 $\mathbf{T}'_{\mu_1\cdots\mu_n} = \sum_{\mathbf{a}} \mathbf{a}_{\mu_1\nu_1}\cdots\mathbf{a}_{\mu_n\nu_n}\mathbf{T}_{\nu_1\cdots\nu_n} \quad \Rightarrow \quad \underline{b}$  <u>协变</u>  $\equiv 5$  基本。协同变化!

$$\mathrm{T}_{u_1\cdots u_n}=\mathrm{F}_{u_1\cdots u_n}$$

$$\mathbf{\Gamma} \mu_{\mathbf{1}} \cdots \mu_{\mathbf{n}}$$

在新坐标系中的方程式形式上与旧坐标系中的方程式一样—相对性原理









 $\mathbf{T}'_{\mu_1\cdots\mu_{\mathbf{n}}}=\sum \mathbf{a}_{\mu_1\nu_1}\cdots\mathbf{a}_{\mu_{\mathbf{n}}\nu_{\mathbf{n}}}\mathbf{T}_{\nu_1\cdots\nu_{\mathbf{n}}} \ \Rightarrow \ \underline{b}\underline{\mathfrak{G}}\equiv\mathbf{b}$  场际一地协同变化!

$$\mathbf{T}_{\mu_{\mathbf{1}} \cdots \mu_{\mathbf{n}}} = \mathbf{F}_{\mu_{\mathbf{1}} \cdots \mu_{\mathbf{n}}}$$

在新坐标系中的方程式形式上与旧坐标系中的方程式一样—相对性原理

利用张量,可将描述物理规律的方程写成满足相对性原理的形式——物理规律的协变表达

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 $\mathbf{T}'_{\mu_1\cdots\mu_n} = \sum \mathbf{a}_{\mu_1\nu_1}\cdots\mathbf{a}_{\mu_n\nu_n}\mathbf{T}_{\nu_1\cdots\nu_n} \Rightarrow \underline{b}\underline{\mathfrak{Y}} \equiv \mathbf{b}$  点版 一起协同变化!

$$\mathbf{T}_{\mu_{\mathbf{1}} \cdots \mu_{\mathbf{n}}} = \mathbf{F}_{\mu_{\mathbf{1}} \cdots \mu_{\mathbf{n}}}$$

在新坐标系中的方程式形式上与旧坐标系中的方程式一样—相对性原理

利用张量,可将描述物理规律的方程写成满足相对性原理的形式—物理规律的协变表达

将所有物理规律都用张量表达,需知道:



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$$\mathbf{T}'_{\mu_1\cdots\mu_n}=\sum_{\mathbf{a}}^{\mathbf{a}}\mathbf{a}_{\mu_1\nu_1}\cdots\mathbf{a}_{\mu_n\nu_n}\mathbf{T}_{\nu_1\cdots\nu_n}$$
  $\Rightarrow$  协变  $\equiv$  与处际一足协同变化!

$$\mathbf{T}_{\mu_{\mathbf{1}} \cdots \mu_{\mathbf{n}}} = \mathbf{F}_{\mu_{\mathbf{1}} \cdots \mu_{\mathbf{n}}}$$

在新坐标系中的方程式形式上与旧坐标系中的方程式一样—相对性原理

利用张量,可将描述物理规律的方程写成满足相对性原理的形式—物理规律的协变表达

将所有物理规律都用张量表达,需知道:

• 张量之间的运算法则.





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# $\mathbf{T}'_{\mu_1\cdots\mu_n} = \sum_{\mathbf{a}}^{\mathbf{4}} \mathbf{a}_{\mu_1\nu_1}\cdots\mathbf{a}_{\mu_n\nu_n}\mathbf{T}_{\nu_1\cdots\nu_n} \Rightarrow \underline{b}\underline{\mathfrak{V}} \equiv \mathbf{b}$ **b b b b b b c b b c c c b d c c c c d d c d c d c d c d c d c d c d c d c d c d c d c d d c d d c d**

$$\mathbf{T}_{\mu_{\mathbf{1}}\cdots\mu_{\mathbf{n}}}=\mathbf{F}_{\mu_{\mathbf{1}}\cdots\mu_{\mathbf{n}}}$$

在新坐标系中的方程式形式上与旧坐标系中的方程式一样—相对性原理

利用张量,可将描述物理规律的方程写成满足相对性原理的形式—物理规律的协变表达

将所有物理规律都用张量表达,需知道:

- 张量之间的运算法则.
- 完备的张量集有多少种张量.

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#### 和对论理论的协变形式。物理量按时空变换性质分类



 $\mathbf{T}'_{\mu_1\cdots\mu_{\mathbf{n}}}=\sum_{\mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_{\mathbf{n}}
u_{\mathbf{n}}}}\mathbf{T}_{
u_1\cdots
u_{\mathbf{n}}} \ \Rightarrow \ \underline{b}$ 要  $\equiv$  5年第一起协同变化!

$$\mathbf{T}_{\mu_{\mathbf{1}}\cdots\mu_{\mathbf{n}}}=\mathbf{F}_{\mu_{\mathbf{1}}\cdots\mu_{\mathbf{n}}}$$

在新坐标系中的方程式形式上与旧坐标系中的方程式一样—相对性原理

利用张量,可将描述物理规律的方程写成满足相对性原理的形式—物理规律的协变表达

将所有物理规律都用张显表达,需知道:

- 张量之间的运算法则.
- 完备的张量集有多少种张量.
- 具体的物理规律如何用张量表达.

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# 43/96 相对论理论的协变形式: 张量运算



$$\mathbf{F}'_{\mu_1\cdots\mu_\mathbf{n}} = \sum_{
u_1,\cdots,
u_\mathbf{n}=1}^4 \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_\mathbf{n}
u_\mathbf{n}} \mathbf{F}_{
u_1\cdots
u_\mathbf{n}} \quad \mathbf{G}'_{\mu_1\cdots\mu_\mathbf{n}} = \sum_{
u_1,\cdots,
u_\mathbf{n}=1}^4 \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_\mathbf{n}
u_\mathbf{n}} \mathbf{G}_{
u_1\cdots
u_\mathbf{n}}$$











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# 相对论理论的协变形式: 张量运算

加减法: 同阶张量加减后仍是这阶张量

 $oxed{\mathbf{F}'_{\mu_1\cdots\mu_{\mathbf{n}}}} = oxed{\sum} egin{align*} \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_{\mathbf{n}}
u_{\mathbf{n}}} \mathbf{F}_{
u_1\cdots
u_{\mathbf{n}}} & \mathbf{G}'_{\mu_1\cdots\mu_{\mathbf{n}}} \end{bmatrix}} = oxed{\sum} egin{align*} \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_{\mathbf{n}}
u_{\mathbf{n}}} \mathbf{G}_{
u_1\cdots
u_{\mathbf{n}}} \end{pmatrix}$ 

$$\mathbf{F}'_{\mu_1\cdots\mu_{\mathbf{n}}}\pm\mathbf{G}'_{\mu_1\cdots\mu_{\mathbf{n}}}=\sum^{\mathbf{4}}\ \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_{\mathbf{n}}
u_{\mathbf{n}}}[\mathbf{F}_{
u_1\cdots
u_{\mathbf{n}}}\pm\mathbf{G}_{\mu_1\cdots\mu_{\mathbf{n}}}]$$

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#### 相对论理论的协变形式:张量运算

加减法:同阶张量加减后仍是这阶张量

 $\mathbf{F}'_{\mu_1\cdots\mu_\mathbf{n}} = \sum_{
u_1,\cdots,
u_\mathbf{n}=1} \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_\mathbf{n}}
olimits_\mathbf{n} \mathbf{F}_{
u_1\cdots
u_\mathbf{n}} \quad \mathbf{G}'_{\mu_1\cdots\mu_\mathbf{n}} = \sum_{
u_1,\cdots,
u_\mathbf{n}=1} \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_\mathbf{n}
u_\mathbf{n}} \mathbf{G}_{
u_1\cdots
u_\mathbf{n}}$ 

$$\mathbf{F}'_{\mu_1\cdots\mu_\mathbf{n}}\pm\mathbf{G}'_{\mu_1\cdots\mu_\mathbf{n}}=\sum_{
u_1,\cdots,
u_\mathbf{n}=1}^4\mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_\mathbf{n}
u_\mathbf{n}}[\mathbf{F}_{
u_1\cdots
u_\mathbf{n}}\pm\mathbf{G}_{\mu_1\cdots\mu_\mathbf{n}}]$$

乘法: m阶张量与n阶张量的乘积为m + n阶张量



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#### 和对论理论的协变形式: 张量运算

加减法: 同阶张量加减后仍是这阶张量

$$\mathbf{F}'_{\mu_1\cdots\mu_\mathbf{n}} = \sum^4 \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_\mathbf{n}
u_\mathbf{n}}\mathbf{F}_{
u_1\cdots
u_\mathbf{n}} \quad \mathbf{G}'_{\mu_1\cdots\mu_\mathbf{n}} = \sum^4 \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_\mathbf{n}
u_\mathbf{n}}\mathbf{G}_{
u_1\cdots
u_\mathbf{n}}$$

 $\mathbf{F}'_{\mu_1\cdots\mu_\mathbf{n}}\pm\mathbf{G}'_{\mu_1\cdots\mu_\mathbf{n}}=\sum_{\mathbf{a}_{\mu_1
u_1}}\mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_\mathbf{n}
u_\mathbf{n}}[\mathbf{F}_{
u_1\cdots
u_\mathbf{n}}\pm\mathbf{G}_{\mu_1\cdots\mu_\mathbf{n}}]$ 

乘法: m阶张量与n阶张量的乘积为m + n阶张量  $\mathbf{A}'_{\mu_1\cdots\mu_{\mathbf{m}}} = \sum \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_{\mathbf{m}}
u_{\mathbf{m}}}\mathbf{A}_{
u_1\cdots
u_{\mathbf{m}}} \quad \mathbf{B}'_{\mu_1\cdots\mu_{\mathbf{n}}} = \sum \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_{\mathbf{n}}
u_{\mathbf{n}}}\mathbf{B}_{
u_1\cdots
u_{\mathbf{n}}}$ 









加减法:同阶张量加减后仍是这阶张量

$$\mathbf{F}'_{\mu_1\cdots\mu_{\mathbf{n}}} = \sum^4 \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_{\mathbf{n}}
u_{\mathbf{n}}}\mathbf{F}_{
u_1\cdots
u_{\mathbf{n}}} \quad \mathbf{G}'_{\mu_1\cdots\mu_{\mathbf{n}}} = \sum^4 \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_{\mathbf{n}}
u_{\mathbf{n}}}\mathbf{G}_{
u_1\cdots
u_{\mathbf{n}}}$$

$$\mathbf{F}'_{\mu_1\cdots\mu_\mathbf{n}}\pm\mathbf{G}'_{\mu_1\cdots\mu_\mathbf{n}}=\sum_{
u_1,\cdots,
u_\mathbf{n}=1}\mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_\mathbf{n}
u_\mathbf{n}}[\mathbf{F}_{
u_1\cdots
u_\mathbf{n}}\pm\mathbf{G}_{\mu_1\cdots\mu_\mathbf{n}}]$$

乘法: m阶张量与n阶张量的乘积为m + n阶张量

$$\mathbf{A}'_{\mu_1\cdots\mu_{\mathbf{m}}} = \sum^4 \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_{\mathbf{m}}
u_{\mathbf{m}}}\mathbf{A}_{
u_1\cdots
u_{\mathbf{m}}} \quad \mathbf{B}'_{\mu_1\cdots\mu_{\mathbf{n}}} = \sum^4 \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_{\mathbf{n}}
u_{\mathbf{n}}}\mathbf{B}_{
u_1\cdots
u_{\mathbf{n}}}$$

 $\mathrm{T}_{\mu_{1}\cdots\mu_{\mathrm{m+n}}}\equiv \mathrm{A}_{\mu_{1}\cdots\mu_{\mathrm{m}}}\mathrm{B}_{\mu_{\mathrm{m+1}}\cdots\mu_{\mathrm{m+n}}}$ 









#### 相对论理论的协变形式: 张量运算

加减法: 同阶张量加减后仍是这阶张量

$$\mathbf{F}'_{\mu_1\cdots\mu_{\mathbf{n}}} = \sum^4 \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_{\mathbf{n}}
u_{\mathbf{n}}} \mathbf{F}_{
u_1\cdots
u_{\mathbf{n}}} \quad \mathbf{G}'_{\mu_1\cdots\mu_{\mathbf{n}}} = \sum^4 \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_{\mathbf{n}}
u_{\mathbf{n}}} \mathbf{G}_{
u_1\cdots
u_{\mathbf{n}}}$$

$$\mathbf{F}'_{\mu_1\cdots\mu_\mathbf{n}}\pm\mathbf{G}'_{\mu_1\cdots\mu_\mathbf{n}}=\sum_{
u_1,\cdots,
u_\mathbf{n}=1}\mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_\mathbf{n}
u_\mathbf{n}}[\mathbf{F}_{
u_1\cdots
u_\mathbf{n}}\pm\mathbf{G}_{\mu_1\cdots\mu_\mathbf{n}}]$$

乘法: m阶张量与n阶张量的乘积为m + n阶张量

$$\mathbf{A}'_{\mu_1\cdots\mu_{\mathbf{m}}} = \sum^4 \ \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_{\mathbf{m}}
u_{\mathbf{m}}} \mathbf{A}_{
u_1\cdots
u_{\mathbf{m}}} \quad \mathbf{B}'_{\mu_1\cdots\mu_{\mathbf{n}}} = \sum^4 \ \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_{\mathbf{n}}
u_{\mathbf{n}}} \mathbf{B}_{
u_1\cdots
u_{\mathbf{n}}}$$

$$\mathbf{T}_{\mu_1 ... \mu_{\mathbf{m}+\mathbf{n}}} \equiv \mathbf{A}_{\mu_1 ... \mu_{\mathbf{m}}} \mathbf{B}_{\mu_{\mathbf{m}+1} ... \mu_{\mathbf{m}+\mathbf{n}}}$$

$$\mathbf{T}'_{\mu_1 \cdots \mu_{\mathbf{m}+\mathbf{n}}} = \mathbf{A}'_{\mu_1 \cdots \mu_{\mathbf{m}}} \mathbf{B}'_{\mu_{\mathbf{m}+1} \cdots \mu_{\mathbf{m}+\mathbf{n}}}$$









加减法: 同阶张量加减后仍是这阶张量

$$\mathbf{E}'$$

$$\mathbf{F}'_{\mu_1\cdots\mu_\mathbf{n}} \!=\! \sum_{
u_1,\cdots,
u_\mathbf{n}=1} \! \mathbf{a}_{\mu_1
u_1}\cdots \mathbf{a}_{\mu_\mathbf{n}
u_\mathbf{n}} \mathbf{F}_{
u_1\cdots
u_\mathbf{n}} \quad \mathbf{G}'_{\mu_1\cdots\mu_\mathbf{n}} \!=\! \sum_{
u_1,\cdots,
u_\mathbf{n}=1} \! \mathbf{a}_{\mu_1
u_1}\cdots \mathbf{a}_{\mu_\mathbf{n}
u_\mathbf{n}} \mathbf{G}_{
u_1\cdots
u_\mathbf{n}}$$

乘法: m阶张量与n阶张量的乘积为m + n阶张量

$$\frac{\mathcal{K}\mathcal{K}}{\mathcal{K}}$$
: m阶张量与n阶张量的乘积为 $\mathbf{m} + \mathbf{n}$ 阶张量

$$u_1, \cdots, 
u_{
m m} = 1 \qquad \qquad 
u_1, \cdots, 
u_{
m m} = 1 \qquad \qquad$$

 $\mathbf{F}'_{\mu_1\cdots\mu_\mathbf{n}}\pm\mathbf{G}'_{\mu_1\cdots\mu_\mathbf{n}}=\sum_{\mathbf{a}_{\mu_1
u_1}}\mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_\mathbf{n}
u_\mathbf{n}}[\mathbf{F}_{
u_1\cdots
u_\mathbf{n}}\pm\mathbf{G}_{\mu_1\cdots\mu_\mathbf{n}}]$ 

$$\mathbf{T}_{\mu_{\mathbf{1}}\cdots\mu_{\mathbf{m}+\mathbf{n}}}\equiv\mathbf{A}_{\mu_{\mathbf{1}}\cdots\mu_{\mathbf{m}}}\mathbf{B}_{\mu_{\mathbf{m}+\mathbf{1}}\cdots\mu_{\mathbf{m}+\mathbf{n}}}$$

 $\mathbf{A}'_{\mu_1\cdots\mu_{\mathbf{m}}} = \sum \ \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_{\mathbf{m}}
u_{\mathbf{m}}} \mathbf{A}_{
u_1\cdots
u_{\mathbf{m}}} \quad \mathbf{B}'_{\mu_1\cdots\mu_{\mathbf{n}}} = \sum \ \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_{\mathbf{n}}
u_{\mathbf{n}}} \mathbf{B}_{
u_1\cdots
u_{\mathbf{n}}}$ 

$$\mathbf{T}'_{\mu_1\cdots\mu_{\mathbf{m}+\mathbf{n}}} = \mathbf{A}'_{\mu_1\cdots\mu_{\mathbf{m}}} \mathbf{B}'_{\mu_{\mathbf{m}+1}\cdots\mu_{\mathbf{m}+\mathbf{n}}}$$

 $\overline{\nu_1,\cdots,\nu_m}=1$ 

$$egin{align*} & = \sum_{\mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_{\mathbf{m}+1} \cdots \mu_{\mathbf{m}+\mathbf{n}} \ & = \sum_{\mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_{\mathbf{m}+1} \cdots \mu_{\mathbf{m}+\mathbf{n}} \ & = \sum_{\mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}+\mathbf{n}} \ & = \sum_{\mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}+\mathbf{n}} \ & = \sum_{\mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}+\mathbf{n}} \ & = \sum_{\mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \ & = \sum_{\mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \ & = \sum_{\mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \ & = \sum_{\mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \ & = \sum_{\mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \ & = \sum_{\mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \ & = \sum_{\mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \ & = \sum_{\mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \ & = \sum_{\mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \ & = \sum_{\mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \ & = \sum_{\mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \ & = \sum_{\mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \ & = \sum_{\mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \ & = \sum_{\mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \ & = \sum_{\mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \ & = \sum_{\mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \ & = \sum_{\mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \ & = \sum_{\mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \ & = \sum_{\mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \ & = \sum_{\mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \ & = \sum_{\mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \ & = \sum_{\mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \ & = \sum_{\mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \ & = \sum_{\mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \ & = \sum_{\mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \ & = \sum_{\mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \ & = \sum_{\mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \rightarrow \mu_1 \cdots \mu_{\mathbf{m}} \ & = \sum_{\mu_1 \cdots \mu_1 \ & = \sum_{\mu_1 \cdots \mu_1 \ & = \sum_{\mu_1 \cdots \mu_1 \cdots \mu_1 \cdots$$

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#### 相对论理论的协变形式: 张量运算

加减法: 同阶张量加减后仍是这阶张量

乘法: m阶张量与n阶张量的乘积为m + n阶张量

$$\frac{2\pi \alpha}{4}$$
。 MM 派里与NM 派里的来依为 $\frac{1}{4}$ 

 $\mathbf{A}'_{\mu_1\cdots\mu_{\mathbf{m}}} = \sum \ \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_{\mathbf{m}}
u_{\mathbf{m}}}\mathbf{A}_{
u_1\cdots
u_{\mathbf{m}}} \quad \mathbf{B}'_{\mu_1\cdots\mu_{\mathbf{n}}} = \sum \ \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_{\mathbf{n}}
u_{\mathbf{n}}}\mathbf{B}_{
u_1\cdots
u_{\mathbf{n}}}$ 

 $\mathbf{T}_{\mu_1 \cdots \mu_{\mathbf{m}+\mathbf{n}}} \equiv \mathbf{A}_{\mu_1 \cdots \mu_{\mathbf{m}}} \mathbf{B}_{\mu_{\mathbf{m}+1} \cdots \mu_{\mathbf{m}+\mathbf{n}}}$ 







#### 相对分理论的协变形式: 张量运算

加减法: 同阶张量加减后仍是这阶张量

乘法: m阶张量与n阶张量的乘积为m + n阶张量

 $\mathbf{T}_{\mu_1 \cdots \mu_{\mathbf{m}+\mathbf{n}}} \equiv \overline{\mathbf{A}_{\mu_1 \cdots \mu_{\mathbf{m}}} \mathbf{B}_{\mu_{\mathbf{m}+1} \cdots \mu_{\mathbf{m}+\mathbf{n}}}}$ 

 $\mathbf{A}'_{\mu_1\cdots\mu_{\mathbf{m}}} = \sum \ \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_{\mathbf{m}}
u_{\mathbf{m}}} \mathbf{A}_{
u_1\cdots
u_{\mathbf{m}}} \quad \mathbf{B}'_{\mu_1\cdots\mu_{\mathbf{n}}} = \sum \ \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_{\mathbf{n}}
u_{\mathbf{n}}} \mathbf{B}_{
u_1\cdots
u_{\mathbf{n}}}$ 

 $\mathbf{T}'_{\mu_1\cdots\mu_{\mathbf{m}+\mathbf{n}}} = \mathbf{A}'_{\mu_1\cdots\mu_{\mathbf{m}}} \mathbf{B}'_{\mu_{\mathbf{m}+1}\cdots\mu_{\mathbf{m}+\mathbf{n}}}$ 







#### 相对论理论的协变形式: 张量运算

加减法: 同阶张量加减后仍是这阶张量

乘法: m阶张量与n阶张量的乘积为m + n阶张量

$$\mathbf{A}_{\mu_1\cdots\mu_{\mathbf{m}}}'=\sum^4 \ \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_{\mathbf{m}}
u_{\mathbf{m}}}\mathbf{A}_{
u_1\cdots
u_{\mathbf{m}}} \quad \mathbf{B}_{\mu_1\cdots\mu_{\mathbf{n}}}'=\sum^4 \ \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_{\mathbf{n}}
u_{\mathbf{n}}}\mathbf{B}_{
u_1\cdots
u_{\mathbf{n}}}$$

 $\mathbf{T}_{\mu_1 \cdots \mu_{\mathbf{m}+\mathbf{n}}} \equiv \mathbf{A}_{\mu_1 \cdots \mu_{\mathbf{m}}} \mathbf{B}_{\mu_{\mathbf{m}+1} \cdots \mu_{\mathbf{m}+\mathbf{n}}}$ 

$$oldsymbol{\mu}_{\mu_1\cdots\mu_{\mathbf{m}+\mathbf{n}}}=oldsymbol{A}_{\mu_1\cdots\mu_{\mathbf{m}}}oldsymbol{
u}_{\mathbf{m}+1\cdots\mu_{\mathbf{n}}}$$

 $\mathbf{T}'_{\mu_1\cdots\mu_{\mathbf{m}+\mathbf{n}}} = \mathbf{A}'_{\mu_1\cdots\mu_{\mathbf{m}}} \mathbf{B}'_{\mu_{\mathbf{m}+1}\cdots\mu_{\mathbf{m}+\mathbf{n}}}$ 

$$oldsymbol{\mu_1}^{\mu_1\cdots\mu_{m+n}} - oldsymbol{A}_{\mu_1\cdots\mu_m}^{\mu_1\cdots\mu_{m+n}}^{\mu_1\cdots\mu_{m+n}}$$

$$-\mu_1 \cdots \mu_{m+n} \qquad -\mu_1 \cdots \mu_m - \mu_{m+1} \cdots \mu_{m+n}$$

$$=\sum_{\mathbf{a}_{1},\ldots,\mathbf{a}_{m}}^{\mathbf{a}_{1},\ldots,\mathbf{a}_{m}}\mathbf{a}_{1},\ldots,\mathbf{a}_{m}$$

 $\nu_1, \dots, \nu_m = 1$ 

 $\overline{a} = \sum \overline{a_{\mu_1 
u_1} \cdots a_{\mu_m 
u_m}} A_{
u_1 \cdots 
u_m} \sum \overline{a_{\mu_{m+1} 
u_{m+1}} \cdots a_{\mu_{m+n} 
u_{m+n}}} B_{
u_{m+1} \cdots 
u_{m+n}}$ 

 $\nu_{m+1}, \dots, \nu_{m+n} = 1$ 











加减法: 同阶张量加减后仍是这阶张量

乘法: m阶张量与n阶张量的乘积为m + n阶张量

$$\mathbf{A}'_{\mu_1\cdots\mu_{\mathbf{m}}} = \sum^4 \ \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_{\mathbf{m}}
u_{\mathbf{m}}} \mathbf{A}_{
u_1\cdots
u_{\mathbf{m}}} \quad \mathbf{B}'_{\mu_1\cdots\mu_{\mathbf{n}}} = \sum^4 \ \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_{\mathbf{n}}
u_{\mathbf{n}}} \mathbf{B}_{
u_1\cdots
u_{\mathbf{n}}}$$

 $\left\langle \mathrm{T}_{\mu_{1}\cdots\mu_{\mathrm{m+n}}} \equiv \mathrm{A}_{\mu_{1}\cdots\mu_{\mathrm{m}}} \mathrm{B}_{\mu_{\mathrm{m+1}}\cdots\mu_{\mathrm{m+n}}} 
ight
angle$ 

$$\mathbf{L}_{\mu_1\cdots\mu_{\mathbf{m}+\mathbf{n}}} = \mathbf{A}_{\mu_1\cdots\mu_{\mathbf{m}}}\mathbf{D}_{\mu_{\mathbf{m}+1}\cdots\mu_{\mathbf{m}+\mathbf{n}}}$$

 $\overline{\mathbf{T}'_{\mu_1\cdots\mu_{\mathbf{m}+\mathbf{n}}}} = \overline{\mathbf{A}'_{\mu_1\cdots\mu_{\mathbf{m}}}} \overline{\mathbf{B}'_{\mu_{\mathbf{m}+1}\cdots\mu_{\mathbf{m}+\mathbf{n}}}}$ 

$$\mathbf{a}_{\mu_1\cdots\mu_{\mathbf{m}+\mathbf{n}}} = \mathbf{a}_{\mu_1\cdots\mu_{\mathbf{m}}} \mathbf{a}_{\mu_{\mathbf{m}+\mathbf{1}}\cdots\mu_{\mathbf{m}+\mathbf{n}}} \mathbf{a}_{\mu_{\mathbf{m}+\mathbf{1}}\cdots\mu_{\mathbf{m}+\mathbf{n}}} \mathbf{a}_{\mu_{\mathbf{m}+\mathbf{1}}\cdots\mu_{\mathbf{m}}} \mathbf{a}_{\mu_{\mathbf{m}+\mathbf{1}}\nu_{\mathbf{m}+\mathbf{n}}} \mathbf{a}_{\mu_{\mathbf{m}+\mathbf{1}}\nu_{\mathbf{m}+$$

 $\nu_1, \dots, \nu_m = 1$ 

 $\mathbf{a} = \sum \mathbf{a}_{\mu_1 
u_1} \! \cdots \! \mathbf{a}_{\mu_{\mathbf{m}+\mathbf{n}} 
u_{\mathbf{m}+\mathbf{n}}} \mathbf{A}_{
u_1 \cdots 
u_{\mathbf{m}}} \mathbf{B}_{
u_{\mathbf{m}+1} \cdots 
u_{\mathbf{m}+\mathbf{n}}} \mathbf{A}_{
u_1 \cdots 
u_{\mathbf{m}}} \mathbf{B}_{
u_1 \cdots 
u_{\mathbf{m}$  $\nu_1, \dots, \nu_{m+n} = 1$ 



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加减法:同阶张量加减后仍是这阶张量

乘法: m阶张量与n阶张量的乘积为m + n阶张量

$$\mathbf{A}'_{\mu_1\cdots\mu_{\mathbf{m}}} = \sum^4 \ \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_{\mathbf{m}}
u_{\mathbf{m}}} \mathbf{A}_{
u_1\cdots
u_{\mathbf{m}}} \quad \mathbf{B}'_{\mu_1\cdots\mu_{\mathbf{n}}} = \sum^4 \ \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_{\mathbf{n}}
u_{\mathbf{n}}} \mathbf{B}_{
u_1\cdots
u_{\mathbf{n}}}$$

 $\mathbf{T}_{\mu_1 \cdots \mu_{\mathbf{m}+\mathbf{n}}} \equiv \mathbf{A}_{\mu_1 \cdots \mu_{\mathbf{m}}} \mathbf{B}_{\mu_{\mathbf{m}+1} \cdots \mu_{\mathbf{m}+\mathbf{n}}}$ 

$$\mathbf{I}_{\mu_1...\mu_{\mathbf{m}+\mathbf{n}}} \equiv \mathbf{A}_{\mu_1...\mu_{\mathbf{m}}} \mathbf{D}_{\mu_{\mathbf{m}+1}...\mu_{\mathbf{m}+\mathbf{n}}}$$

 $\overline{\Gamma'_{\mu_1\cdots\mu_{\mathbf{m}+\mathbf{n}}}} = \overline{\mathbf{A}'_{\mu_1\cdots\mu_{\mathbf{m}}}} \overline{\mathbf{B}'_{\mu_{\mathbf{m}+1}\cdots\mu_{\mathbf{m}+\mathbf{n}}}}$ 

$$\mathbf{a}_{\mu_{\mathbf{m}}, \mu_{\mathbf{m}}, \dots, \mu_{\mathbf{m}}, \mu_{\mathbf{m}}, \mu_{\mathbf{m}}, \dots, \mu_{\mathbf{m}}, \mu_$$









加减法:同阶张量加减后仍是这阶张量

乘法: m阶张量与n阶张量的乘积为m + n阶张量

$$\mathbf{A}_{\mu_1\cdots\mu_{\mathbf{m}}}'=\sum^4 \ \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_{\mathbf{m}}
u_{\mathbf{m}}}\mathbf{A}_{
u_1\cdots
u_{\mathbf{m}}} \quad \mathbf{B}_{\mu_1\cdots\mu_{\mathbf{n}}}'=\sum^4 \ \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_{\mathbf{n}}
u_{\mathbf{n}}}\mathbf{B}_{
u_1\cdots
u_{\mathbf{n}}}$$

$$\mathbf{T}_{\mu_1 \cdots \mu_{\mathbf{m}+\mathbf{n}}} \equiv \mathbf{A}_{\mu_1 \cdots \mu_{\mathbf{m}}} \mathbf{B}_{\mu_{\mathbf{m}+1} \cdots \mu_{\mathbf{m}+\mathbf{n}}}$$

$$\mathbf{T}'_{\mu_{\mathbf{1}}\cdots\mu_{\mathbf{m}+\mathbf{n}}}=\mathbf{A}'_{\mu_{\mathbf{1}}\cdots\mu_{\mathbf{m}}}\mathbf{B}'_{\mu_{\mathbf{m}+\mathbf{1}}\cdots\mu_{\mathbf{m}+\mathbf{n}}}$$

$$egin{align*} & \stackrel{\mu_1\cdots\mu_{\mathbf{m}}}{=} \stackrel{\mu_1\cdots\mu_{\mathbf{m}}}{=} \stackrel{\mu_1\cdots\mu_{\mathbf{m}}}{=} \stackrel{\mu_1\cdots\mu_{\mathbf{m}+1}\cdots\mu_{\mathbf{m}+n}}{=} & rac{4}{=} & \sum & a_{\mu_1
u_1}\cdots a_{\mu_{\mathbf{m}}
u_{\mathbf{m}}} A_{
u_1\cdots
u_{\mathbf{m}}} & \sum & a_{\mu_{\mathbf{m}+1}
u_{\mathbf{m}+1}} \cdots a_{\mu_{\mathbf{m}+n}
u_{\mathbf{m}+n}} B_{
u_{\mathbf{m}+1}\cdots
u_{\mathbf{m}+n}} & \sum & a_{\mu_{\mathbf{m}+1}
u_{\mathbf{m}+1}} \cdots a_{\mu_{\mathbf{m}+n}
u_{\mathbf{m}+n}} & \sum & a_{\mu_{\mathbf{m}+1}
u_{\mathbf{m}+1}} \cdots a_{\mu_{\mathbf{m}+1}
u_{\mathbf{m}+n}} & \sum & a_{\mu_{\mathbf{m}+1}
u_{\mathbf{m}+1}} \cdots a_{\mu_{\mathbf{m}+1}
u_{\mathbf{m}+n}} & \sum & a_{\mu_{\mathbf{m}+1}
u_{\mathbf{m}+1}} \cdots a_{\mu_{\mathbf{m}+1}
u_{\mathbf{m}+n}} & \sum & a_{\mu_{\mathbf{m}+1}
u_{\mathbf{m}+n}
u_{\mathbf{m}+n} & \sum & a_{\mu_{\mathbf{m}+1}
u_{\mathbf{m}+n}} & \sum & a_{\mu_{\mathbf{m}+1}
u_{\mathbf{m}+n} & \sum & a_{\mu_{\mathbf{m}+1}
u_{\mathbf{m}+n}} & \sum & a_{\mu_{\mathbf{m}+1}
u_{\mathbf{m}+n} & \sum & a_{\mu_{\mathbf{m}+1}
u_{\mathbf{m}+n}$$

缩并: n阶张量收缩一次变成n-2阶张量







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#### 相对论理论的协变形式:张量运算

加减法:同阶张量加减后仍是这阶张量

乘法:m阶张量与n阶张量的乘积为m+n阶张量

$$\mathbf{A}'_{\mu_1\cdots\mu_{\mathbf{m}}} = \sum^4 \; \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_{\mathbf{m}}
u_{\mathbf{m}}} \mathbf{A}_{
u_1\cdots
u_{\mathbf{m}}} \quad \mathbf{B}'_{\mu_1\cdots\mu_{\mathbf{n}}} = \sum^4 \; \mathbf{a}_{\mu_1
u_1}\cdots\mathbf{a}_{\mu_{\mathbf{n}}
u_{\mathbf{n}}} \mathbf{B}_{
u_1\cdots
u_{\mathbf{n}}}$$

$$\mathbf{T}_{\mu_1 \cdots \mu_{\mathbf{m}+\mathbf{n}}} \equiv \mathbf{A}_{\mu_1 \cdots \mu_{\mathbf{m}}} \mathbf{B}_{\mu_{\mathbf{m}+1} \cdots \mu_{\mathbf{m}+\mathbf{n}}}$$

 $\mathbf{T}'_{\mu_{\mathbf{1}}\cdots\mu_{\mathbf{m}+\mathbf{n}}} = \mathbf{A}'_{\mu_{\mathbf{1}}\cdots\mu_{\mathbf{m}}} \mathbf{B}'_{\mu_{\mathbf{m}+\mathbf{1}}\cdots\mu_{\mathbf{m}+\mathbf{n}}}$ 

$$oldsymbol{\mu_1}_{\mu_1\cdots\mu_{\mathbf{m}+\mathbf{n}}} - oldsymbol{A}_{\mu_1\cdots\mu_{\mathbf{m}}} oldsymbol{D}_{\mu_{\mathbf{m}+1}\cdots\mu_{\mathbf{m}+\mathbf{n}}}$$

缩并: n阶张量收缩一次变成n-2阶张量

$$\sum_{\mu_{\mathbf{i}},\mu_{\mathbf{i}}=\mathbf{1}} \delta_{\mu_{\mathbf{i}}\mu_{\mathbf{j}}} \mathbf{T}_{\mu_{\mathbf{1}}\cdots\mu_{\mathbf{i}}\cdots\mu_{\mathbf{j}}\cdots\mu_{\mathbf{n}}} 称为 \mathbf{T}_{\mu_{\mathbf{1}}\cdots\mu_{\mathbf{i}}\cdots\mu_{\mathbf{j}}\cdots\mu_{\mathbf{n}}}$$
对指标 $\mu_{\mathbf{i}},\mu_{\mathbf{j}}$ 的缩并.

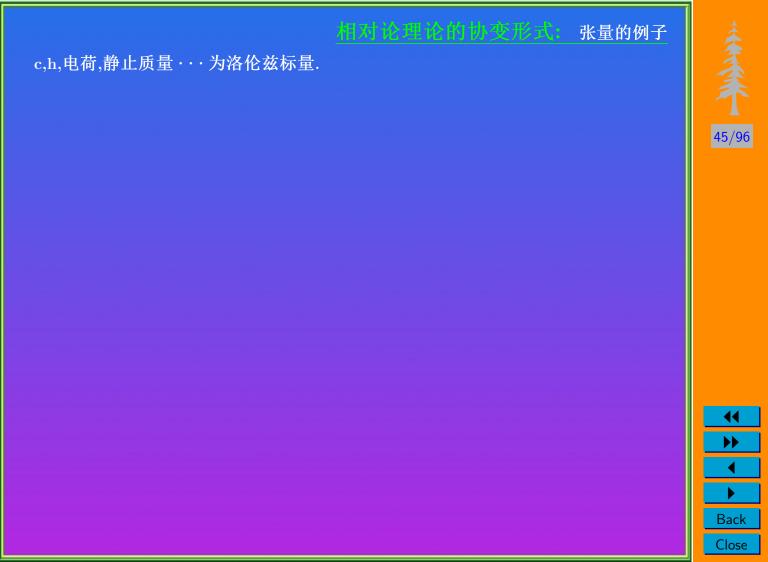


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#### 张量的例子 c,h,电荷,静止质量···为洛伦兹标量. 四维体积元 $d^4x \equiv dx_1 dx_2 dx_3 dx_4 = icdVdt$ 的变换关系为: 45/96









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#### 张量的例子 c,h,电荷,静止质量···为洛伦兹标量. 四维体积元 $d^4x \equiv dx_1 dx_2 dx_3 dx_4 = icdVdt$ 的变换关系为: $\mathbf{d}^4\mathbf{x}' = ||\det \mathbf{A}||\mathbf{d}^4\mathbf{x} = \mathbf{d}^4\mathbf{x}$



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c,h,电荷,静止质量···为洛伦兹标量.

四维体积元 $\mathbf{d^4x} \equiv \mathbf{dx_1} \mathbf{dx_2} \mathbf{dx_3} \mathbf{dx_4} = \mathbf{icdV} \mathbf{dt}$ 的变换关系为:  $\mathbf{d^4x'} = ||\det \mathbf{A}||\mathbf{d^4x} = \mathbf{d^4x} \rightarrow$ 洛伦兹标量 $\mathbf{dV'} \mathbf{dt'} = \mathbf{dV} \mathbf{dt}$ 



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# 张量的例子

c,h,电荷,静止质量···为洛伦兹标量.

四维体积元 $ext{d}^4 ext{x} \equiv ext{d} ext{x}_1 ext{d} ext{x}_2 ext{d} ext{x}_3 ext{d} ext{x}_4 = ext{icd} ext{V} ext{d} ext{t}$ 的变换关系为:  $\mathbf{d^4x'} = ||\det \mathbf{A}||\mathbf{d^4x} = \mathbf{d^4x} \rightarrow$  洛伦兹标量 $\mathbf{dV'dt'} = \mathbf{dVdt}$ 

四维速度 $\mathbf{u}_{\mu}=rac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d} au}$ 为四矢量.









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# 图为 论理论的协变形式: 张量的例子

c,h,电荷,静止质量···为洛伦兹标量.

四维体积元 $\mathbf{d^4x} \equiv \mathbf{dx_1} \mathbf{dx_2} \mathbf{dx_3} \mathbf{dx_4} = \mathbf{icdVdt}$ 的变换关系为:  $\mathbf{d^4x'} = ||\det \mathbf{A}||\mathbf{d^4x} = \mathbf{d^4x} \rightarrow$  洛伦兹标量 $\mathbf{dV'dt'} = \mathbf{dVdt}$ 

四维速度 $\mathbf{u}_{\mu}=rac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d} au}$ 为四矢量. 因 $\mathbf{d}\mathbf{x}_{\mu}$ 为四矢量,  $\mathbf{d} au$ 是不变量.



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c,h,电荷,静止质量···为洛伦兹标量.

四维体积元 $\mathbf{d^4x} \equiv \mathbf{dx_1}\mathbf{dx_2}\mathbf{dx_3}\mathbf{dx_4} = \mathbf{icdVdt}$ 的变换关系为:  $\mathbf{d^4x'} = ||\det \mathbf{A}||\mathbf{d^4x} = \mathbf{d^4x} \rightarrow$  洛伦兹标量 $\mathbf{dV'dt'} = \mathbf{dVdt}$ 

四维速度 $\mathbf{u}_{\mu}=rac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d} au}$ 为四矢量. 因 $\mathbf{d}\mathbf{x}_{\mu}$ 为四矢量,  $\mathbf{d} au$ 是不变量.

协变微商 $\dfrac{\partial}{\partial \mathbf{x}_{\mu}}$ 是一阶张量







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# **一对论理论的协变形式:** 张量的例子

c,h,电荷,静止质量···为洛伦兹标量.

四维体积元 $\mathbf{d^4x} \equiv \mathbf{dx_1}\mathbf{dx_2}\mathbf{dx_3}\mathbf{dx_4} = \mathbf{icdVdt}$ 的变换关系为:  $\mathbf{d^4x'} = ||\det \mathbf{A}||\mathbf{d^4x} = \mathbf{d^4x} \rightarrow$  洛伦兹标量 $\mathbf{dV'dt'} = \mathbf{dVdt}$ 

四维速度 $\mathbf{u}_{\mu}=rac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d} au}$ 为四矢量. 因 $\mathbf{d}\mathbf{x}_{\mu}$ 为四矢量,  $\mathbf{d} au$ 是不变量.

协变微商 $\dfrac{\partial}{\partial \mathbf{x}_{\mu}}$ 是一阶张量

$$\frac{\partial}{\partial \mathbf{x}'_{\mu}} = \sum_{\nu=1}^{4} \frac{\partial \mathbf{x}_{\nu}}{\partial \mathbf{x}'_{\mu}} \frac{\partial}{\partial \mathbf{x}_{\nu}}$$



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c,h,电荷,静止质量···为洛伦兹标量.

四维体积元 $d^4x \equiv dx_1 dx_2 dx_3 dx_4 = icdVdt$ 的变换关系为:  $\mathbf{d^4x'} = ||\det \mathbf{A}||\mathbf{d^4x} = \mathbf{d^4x} \rightarrow$  洛伦兹标量 $\mathbf{dV'dt'} = \mathbf{dVdt}$ 

四维速度 $\mathbf{u}_{\mu}=rac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d} au}$ 为四矢量,因 $\mathbf{d}\mathbf{x}_{\mu}$ 为四矢量, $\mathbf{d} au$ 是不变量.

协变微商 $\frac{\partial}{\partial \mathbf{x}_{\mu}}$ 是一阶张量

$$\frac{\partial}{\partial \mathbf{x}'_{\mu}} = \sum_{\nu=1}^{4} \frac{\partial \mathbf{x}_{\nu}}{\partial \mathbf{x}'_{\mu}} \frac{\partial}{\partial \mathbf{x}_{\nu}} = \sum_{\nu=1}^{4} \mathbf{a}_{\mu\nu} \frac{\partial}{\partial \mathbf{x}_{\nu}}$$





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c,h,电荷,静止质量···为洛伦兹标量.

四维体积元 $d^4x \equiv dx_1 dx_2 dx_3 dx_4 = icdVdt$ 的变换关系为:  $\mathbf{d^4x'} = ||\det \mathbf{A}||\mathbf{d^4x} = \mathbf{d^4x} \rightarrow$  洛伦兹标量 $\mathbf{dV'dt'} = \mathbf{dVdt}$ 

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协变微商 $\dfrac{\partial}{\partial \mathbf{x}_{\mu}}$ 是一阶张量

$$rac{\partial}{\partial \mathbf{x}'_{\mu}} = \sum_{
u=1}^{4} rac{\partial \mathbf{x}_{
u}}{\partial \mathbf{x}'_{\mu}} rac{\partial}{\partial \mathbf{x}_{
u}} = \sum_{
u=1}^{4} \mathbf{a}_{\mu
u} rac{\partial}{\partial \mathbf{x}_{
u}}$$

$$\sum_{\mu=1}^{4} \frac{\partial}{\partial \mathbf{x}_{\mu}} \frac{\partial}{\partial \mathbf{x}_{\mu}} = \frac{\partial}{\partial \mathbf{x}} \frac{\partial}{\partial \mathbf{x}} + \frac{\partial}{\partial \mathbf{y}} \frac{\partial}{\partial \mathbf{y}} + \frac{\partial}{\partial \mathbf{z}} \frac{\partial}{\partial \mathbf{z}} - \frac{1}{\mathbf{c}^{2}} \frac{\partial}{\partial \mathbf{t}} \frac{\partial}{\partial \mathbf{t}}$$





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秦季

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c,h,电荷,静止质量···为洛伦兹标量.

四维体积元 $\mathrm{d}^4\mathrm{x} \equiv \mathrm{d}\mathrm{x}_1\mathrm{d}\mathrm{x}_2\mathrm{d}\mathrm{x}_3\mathrm{d}\mathrm{x}_4 = \mathrm{i}\mathrm{c}\mathrm{d}\mathrm{V}\mathrm{d}\mathrm{t}$ 的变换关系为:

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四维体积元 $\mathbf{d}^4\mathbf{x} \equiv \mathbf{d}\mathbf{x}_1\mathbf{d}\mathbf{x}_2\mathbf{d}\mathbf{x}_3\mathbf{d}\mathbf{x}_4 = \mathbf{i}\mathbf{c}\mathbf{d}\mathbf{V}\mathbf{d}\mathbf{t}$ 的变换关系为:  $\mathbf{d}^4\mathbf{x}' = ||\det\mathbf{A}||\mathbf{d}^4\mathbf{x} = \mathbf{d}^4\mathbf{x} \rightarrow \text{洛伦兹标量}\mathbf{d}\mathbf{V}'\mathbf{d}\mathbf{t}' = \mathbf{d}\mathbf{V}\mathbf{d}\mathbf{t}$ 

 $\mathbf{d}\mathbf{x} = \prod_{\mathbf{d} \in \mathbf{d}} \mathbf{A} \prod_{\mathbf{d}} \mathbf{x} = \mathbf{d} \mathbf{x}$  不相比然你重**d** $\mathbf{v}$  **d** $\mathbf{t} = \mathbf{d} \mathbf{v}$  **d** $\mathbf{t}$ 

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c,h,电荷,静止质量···为洛伦兹标量.

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$$\nabla^2 - \frac{1}{\mathbf{c}^2} \frac{\partial^2}{\partial \mathbf{t}^2} = \nabla^2 - \frac{1}{\mathbf{c}^2} \frac{\partial^2}{\partial \mathbf{t}^2}$$

 $\delta_{\mu
u}$ 是二阶单位张量:

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四维体积元 $d^4x \equiv dx_1 dx_2 dx_3 dx_4 = icdVdt$ 的变换关系为:  $\mathbf{d^4x'} = ||\det \mathbf{A}||\mathbf{d^4x} = \mathbf{d^4x} \rightarrow$  洛伦兹标量 $\mathbf{dV'dt'} = \mathbf{dVdt}$ 

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u}=\sum_{\lambda=1}\mathbf{a}_{\mu\lambda}\mathbf{a}_{
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c,h,电荷,静止质量···为洛伦兹标量.

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u} = \sum_{\lambda=1}^4 \mathbf{a}_{\mu\lambda} \mathbf{a}_{
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u\lambda'} \delta_{\lambda\lambda'}$ 





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#### 相对论性物理学

- 已知存在一套张量理论可以从数学上明显地表达相对性原理
- ullet 只要把物理方程写成是四维时空中的 $\underline{\mathbb{R}}$   $\underline{\mathbb{R}}$   $\underline{\mathbb{R}}$   $\underline{\mathbb{R}}$   $\underline{\mathbb{R}}$   $\underline{\mathbb{R}}$



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#### 相对论理论的协变形式:相对论性物理学

- 已知存在一套张量理论可以从数学上明显地表达相对性原理
- ullet 只要把物理方程写成是四维时空中的张量方程:  $\mathbf{T}_{\mu_1\cdots\mu_n}=\mathbf{F}_{\mu_1\cdots\mu_n}$
- 我们怎么知道学过的物理方程能否写成张量方程?







## 推对论理论的协变形式: 相对论性物理学

- 已知存在一套张量理论可以从数学上明显地表达相对性原理
- ullet 只要把物理方程写成是四维时空中的 ${f R}$ 量方程:  ${f T}_{\mu_1\cdots\mu_n}={f F}_{\mu_1\cdots\mu_n}$
- 我们怎么知道学过的物理方程能否写成张量方程?
- ●除坐标外,我们不知道其它所有物理量和四维张量有什么关系?







#### 相对论理论的协变形式:相对论性物理学

- 已知存在一套张量理论可以从数学上明显地表达相对性原理
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#### 相对论理论的协变形式: 相对论性物理学

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- 重新审视我们的力学和电动力学理论
- 要求理论在一开始就满足相对性原理导出物理量和四维张量的关系!







#### 相对论理论的协变形式: 相对论性物理学

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- 并建立用四维张量表达的物理方程
- 副产品: 不从实验,而从理论原则导出了力学和电动力学的所有方程







对于每一个力学体系,存在一个叫作用量的积分S,它对于实际运动有最小值.









Back Close

对一个自由度的体系: 
$$\mathbf{S} = \int_{\alpha}^{\beta} \mathbf{dt} \ \mathbf{L}(\mathbf{q}.\dot{\mathbf{q}})$$

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**◀** 

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对于每一个力学体系,存在一个叫作用量的积分S,它对于实际运动有最小值.

对一个自由度的体系: 
$$\mathbf{S} = \int_{lpha}^{eta} \mathbf{dt} \; \mathbf{L}(\mathbf{q}.\mathbf{\dot{q}})$$

$$\mathbf{0} = \delta \mathbf{S}$$



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Back

对于每一个力学体系,存在一个叫作用量的积分S,它对于实际运动有最小值.

对一个自由度的体系: 
$$\mathbf{S} = \int_{lpha}^{eta} \mathbf{dt} \; \mathbf{L}(\mathbf{q}.\mathbf{\dot{q}})$$

$$\mathbf{0} = \delta \mathbf{S} = \int_{lpha}^{eta} \mathbf{dt} [rac{\partial \mathbf{L}}{\partial \dot{\mathbf{q}}} \delta \dot{\mathbf{q}} + rac{\partial \mathbf{L}}{\partial \mathbf{q}} \delta \mathbf{q}]$$



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1

对于每一个力学体系,存在一个叫作用量的积分S,它对于实际运动有最小值.

对一个自由度的体系: 
$$\mathbf{S} = \int_{lpha}^{eta} \mathbf{dt} \; \mathbf{L}(\mathbf{q}.\mathbf{\dot{q}})$$

$$\mathbf{0} = \delta \mathbf{S} = \int_{\alpha}^{\beta} \mathbf{dt} \left[ \frac{\partial \mathbf{L}}{\partial \dot{\mathbf{q}}} \delta \dot{\mathbf{q}} + \frac{\partial \mathbf{L}}{\partial \mathbf{q}} \delta \mathbf{q} \right] = \int_{\alpha}^{\beta} \mathbf{dt} \left[ \frac{\partial \mathbf{L}}{\partial \dot{\mathbf{q}}} \frac{\mathbf{d}}{\mathbf{dt}} \delta \mathbf{q} + \frac{\partial \mathbf{L}}{\partial \mathbf{q}} \delta \mathbf{q} \right]$$



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**→** 

对于每一个力学体系,存在一个叫作用量的积分S,它对于实际运动有最小值.

对一个自由度的体系: 
$$\mathbf{S} = \int_{lpha}^{eta} \mathbf{dt} \; \mathbf{L}(\mathbf{q}.\mathbf{\dot{q}})$$

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$$= \int_{\alpha}^{\beta} \mathbf{dt} \left[ \frac{\mathbf{d}}{\mathbf{dt}} \left( \frac{\partial \mathbf{L}}{\partial \dot{\mathbf{q}}} \delta \mathbf{q} \right) + \left( \frac{\partial \mathbf{L}}{\partial \mathbf{q}} - \frac{\mathbf{d}}{\mathbf{dt}} \frac{\partial \mathbf{L}}{\partial \dot{\mathbf{q}}} \right) \delta \mathbf{q} \right]$$



**44** 

**♦** 

对于每一个力学体系,存在一个叫作用量的积分S,它对于实际运动有最小值.

对一个自由度的体系:  $\mathbf{S} = \int_{lpha}^{eta} \mathbf{dt} \; \mathbf{L}(\mathbf{q}.\mathbf{\dot{q}})$ 

$$\begin{aligned} \mathbf{0} &= \delta \mathbf{S} = \int_{\alpha}^{\beta} \mathbf{d} \mathbf{t} \left[ \frac{\partial \mathbf{L}}{\partial \dot{\mathbf{q}}} \delta \dot{\mathbf{q}} + \frac{\partial \mathbf{L}}{\partial \mathbf{q}} \delta \mathbf{q} \right] = \int_{\alpha}^{\beta} \mathbf{d} \mathbf{t} \left[ \frac{\partial \mathbf{L}}{\partial \dot{\mathbf{q}}} \frac{\mathbf{d}}{\partial \mathbf{t}} \delta \mathbf{q} + \frac{\partial \mathbf{L}}{\partial \mathbf{q}} \delta \mathbf{q} \right] \\ &= \int_{\alpha}^{\beta} \mathbf{d} \mathbf{t} \left[ \frac{\mathbf{d}}{\mathbf{d} \mathbf{t}} \left( \frac{\partial \mathbf{L}}{\partial \dot{\mathbf{q}}} \delta \mathbf{q} \right) + \left( \frac{\partial \mathbf{L}}{\partial \mathbf{q}} - \frac{\mathbf{d}}{\mathbf{d} \mathbf{t}} \frac{\partial \mathbf{L}}{\partial \dot{\mathbf{q}}} \right) \delta \mathbf{q} \right] \quad \rightarrow \quad -\frac{\partial \mathbf{L}}{\partial \mathbf{q}} + \frac{\mathbf{d}}{\mathbf{d} \mathbf{t}} \frac{\partial \mathbf{L}}{\partial \dot{\mathbf{q}}} = \mathbf{0} \end{aligned}$$



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**→** 

**▶** Back

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$$\mathbf{p} = \frac{\partial \mathbf{L}}{\partial \mathbf{\dot{q}}}$$



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# 相对 沙力学: 最小作用量原理

对于每一个力学体系,存在一个叫作用量的积分S,它对于实际运动有最小值.

对一个自由度的体系:  $\mathbf{S} = \int^{eta} \mathbf{dt} \; \mathbf{L}(\mathbf{q}.\dot{\mathbf{q}})$ 

$$\begin{aligned} \mathbf{0} &= \delta \mathbf{S} = \int_{\alpha}^{\beta} \mathbf{dt} \left[ \frac{\partial \mathbf{L}}{\partial \dot{\mathbf{q}}} \delta \dot{\mathbf{q}} + \frac{\partial \mathbf{L}}{\partial \mathbf{q}} \delta \mathbf{q} \right] = \int_{\alpha}^{\beta} \mathbf{dt} \left[ \frac{\partial \mathbf{L}}{\partial \dot{\mathbf{q}}} \frac{\mathbf{d}}{\mathbf{dt}} \delta \mathbf{q} + \frac{\partial \mathbf{L}}{\partial \mathbf{q}} \delta \mathbf{q} \right] \\ &= \int_{\alpha}^{\beta} \mathbf{dt} \left[ \frac{\mathbf{d}}{\mathbf{dt}} \left( \frac{\partial \mathbf{L}}{\partial \dot{\mathbf{q}}} \delta \mathbf{q} \right) + \left( \frac{\partial \mathbf{L}}{\partial \mathbf{q}} - \frac{\mathbf{d}}{\mathbf{dt}} \frac{\partial \mathbf{L}}{\partial \dot{\mathbf{q}}} \right) \delta \mathbf{q} \right] \quad \rightarrow \quad -\frac{\partial \mathbf{L}}{\partial \mathbf{q}} + \frac{\mathbf{d}}{\mathbf{dt}} \frac{\partial \mathbf{L}}{\partial \dot{\mathbf{q}}} = \mathbf{0} \end{aligned}$$

$$\mathbf{p} = rac{\partial \mathbf{L}}{\partial \dot{\mathbf{q}}} \hspace{1cm} \mathbf{H} = \mathbf{p}\dot{\mathbf{q}} - \mathbf{L}$$



对于每一个力学体系,存在一个叫作用量的积分S,它对于实际运动有最小值.

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$$\mathbf{p} = rac{\partial \mathbf{L}}{\partial \dot{\mathbf{q}}}$$
  $rac{\mathbf{dp}}{\mathbf{dt}} = rac{\partial \mathbf{L}}{\partial \mathbf{q}}$ 

 $old H = {f p} old {f q} - {f L}$ 

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对一个自由度的体系:  $\mathbf{S} = \int_{lpha}^{eta} \mathbf{dt} \; \mathbf{L}(\mathbf{q}.\dot{\mathbf{q}})$ 

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$$= \int_{\alpha}^{\beta} \mathbf{dt} \left[ \frac{\mathbf{d}}{\mathbf{dt}} \left( \frac{\partial \mathbf{L}}{\partial \dot{\mathbf{q}}} \delta \mathbf{q} \right) + \left( \frac{\partial \mathbf{L}}{\partial \mathbf{q}} - \frac{\mathbf{d}}{\mathbf{dt}} \frac{\partial \mathbf{L}}{\partial \dot{\mathbf{q}}} \right) \delta \mathbf{q} \right] \rightarrow -\frac{\partial \mathbf{L}}{\partial \mathbf{q}} + \frac{\mathbf{d}}{\mathbf{dt}} \frac{\partial \mathbf{L}}{\partial \dot{\mathbf{q}}} = \mathbf{0}$$

$$\mathbf{p} = rac{\partial \mathbf{L}}{\partial \dot{\mathbf{q}}}$$
  $\mathbf{H} = \mathbf{p}\dot{\mathbf{q}} - \mathbf{L}$   $rac{\mathbf{d}\mathbf{p}}{\mathbf{d}t} = rac{\partial \mathbf{L}}{\partial \mathbf{q}}$   $rac{\mathbf{d}\mathbf{H}}{\mathbf{d}t} = rac{\mathbf{d}\mathbf{p}}{\mathbf{d}t}\dot{\mathbf{q}} + \mathbf{p}rac{\mathbf{d}\dot{\mathbf{q}}}{\mathbf{d}t} - rac{\partial \mathbf{L}}{\partial \mathbf{q}}rac{\mathbf{d}\mathbf{q}}{\mathbf{d}t} - rac{\partial \mathbf{L}}{\partial \dot{\mathbf{q}}}rac{\mathbf{d}\dot{\mathbf{q}}}{\mathbf{d}t}$ 



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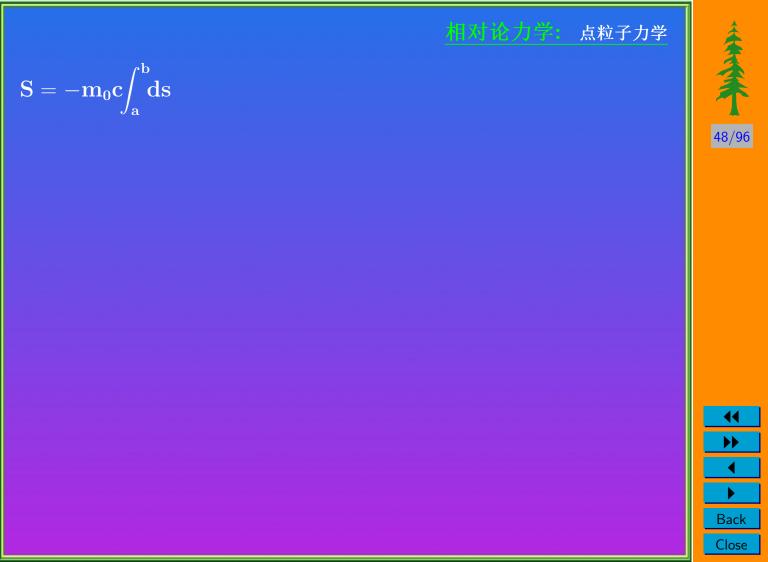
$$= \int_{\alpha}^{\beta} \mathbf{dt} \left[ \frac{\mathbf{d}}{\mathbf{dt}} \left( \frac{\partial \mathbf{L}}{\partial \dot{\mathbf{q}}} \delta \mathbf{q} \right) + \left( \frac{\partial \mathbf{L}}{\partial \mathbf{q}} - \frac{\mathbf{d}}{\mathbf{dt}} \frac{\partial \mathbf{L}}{\partial \dot{\mathbf{q}}} \right) \delta \mathbf{q} \right] \rightarrow -\frac{\partial \mathbf{L}}{\partial \mathbf{q}} + \frac{\mathbf{d}}{\mathbf{dt}} \frac{\partial \mathbf{L}}{\partial \dot{\mathbf{q}}} = \mathbf{0}$$

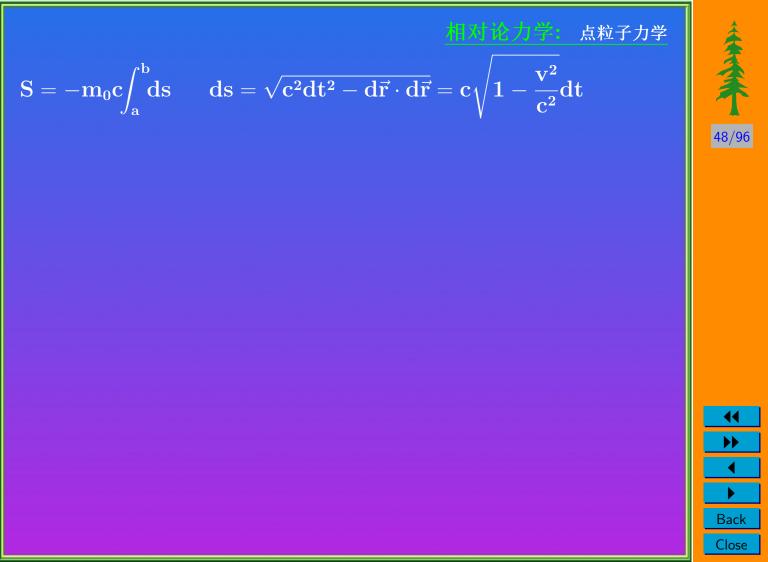
$$\begin{split} \mathbf{p} &= \frac{\partial L}{\partial \dot{\mathbf{q}}} \qquad \mathbf{H} = \mathbf{p}\dot{\mathbf{q}} - \mathbf{L} \\ &\frac{d\mathbf{p}}{dt} = \frac{\partial L}{\partial \mathbf{q}} \qquad \frac{d\mathbf{H}}{dt} = \frac{d\mathbf{p}}{dt}\dot{\mathbf{q}} + \mathbf{p}\frac{d\dot{\mathbf{q}}}{dt} - \frac{\partial L}{\partial \mathbf{q}}\frac{d\mathbf{q}}{dt} - \frac{\partial L}{\partial \dot{\mathbf{q}}}\frac{d\dot{\mathbf{q}}}{dt} = \mathbf{0} \end{split}$$

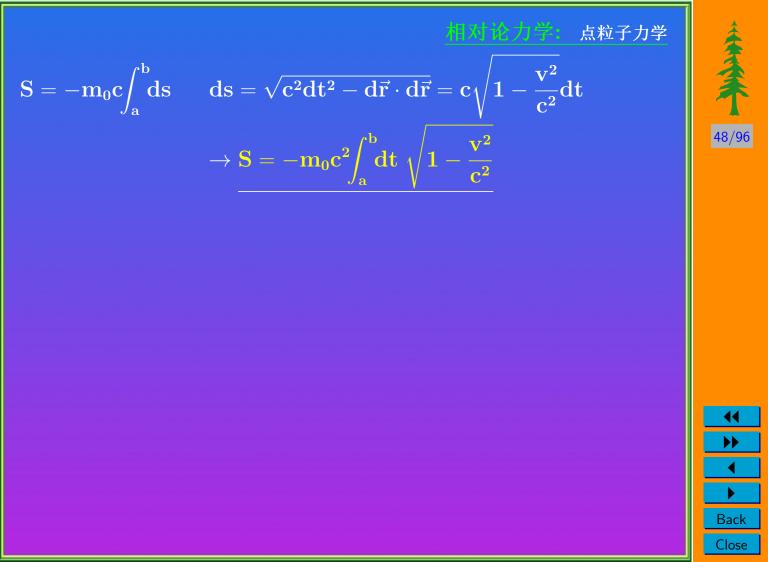


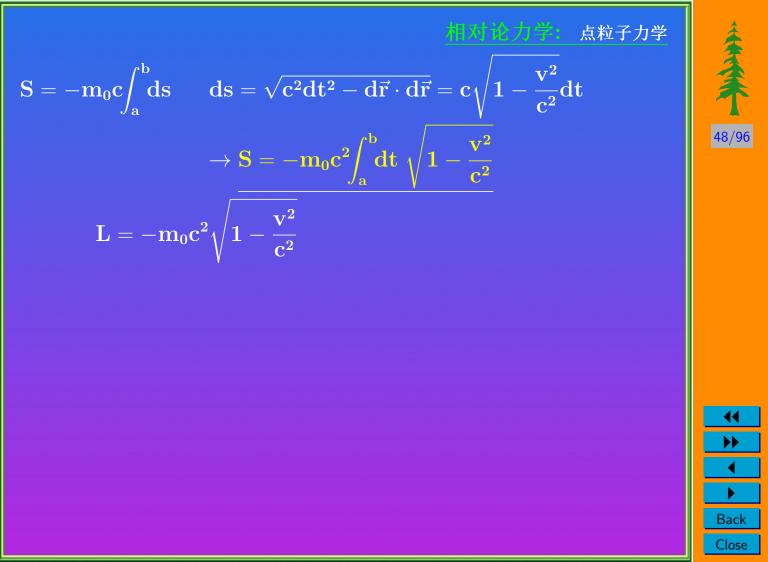


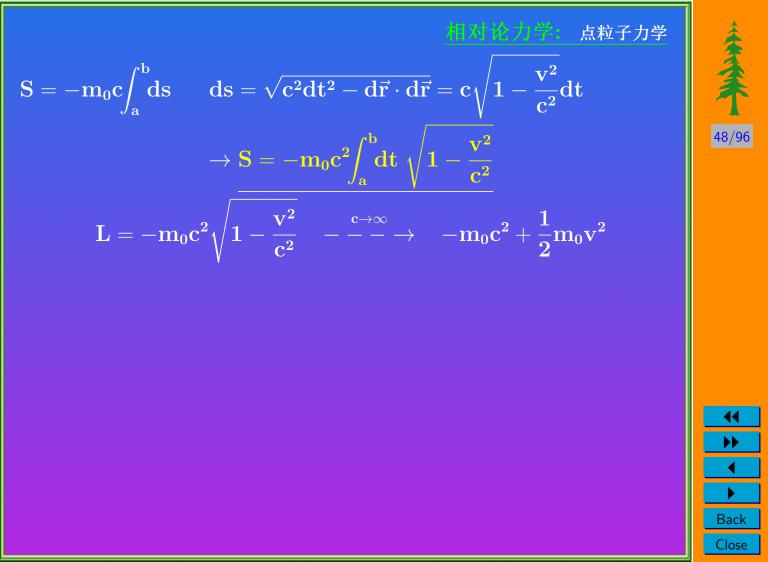


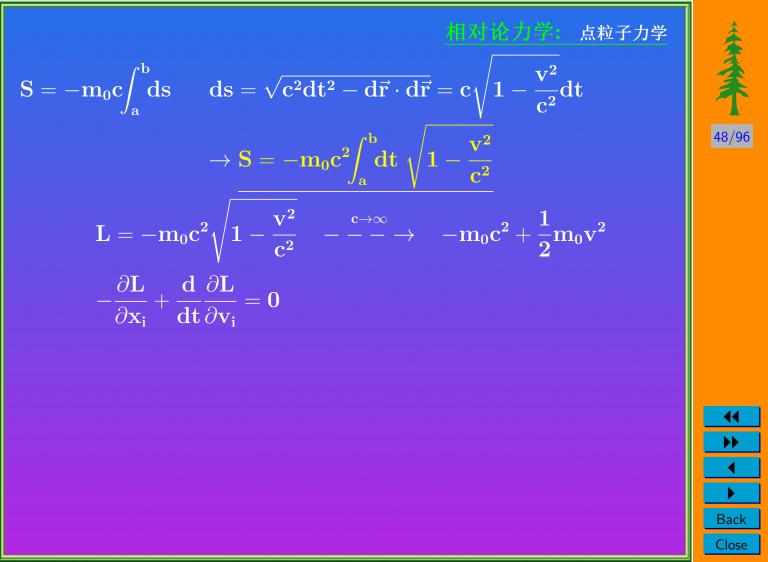


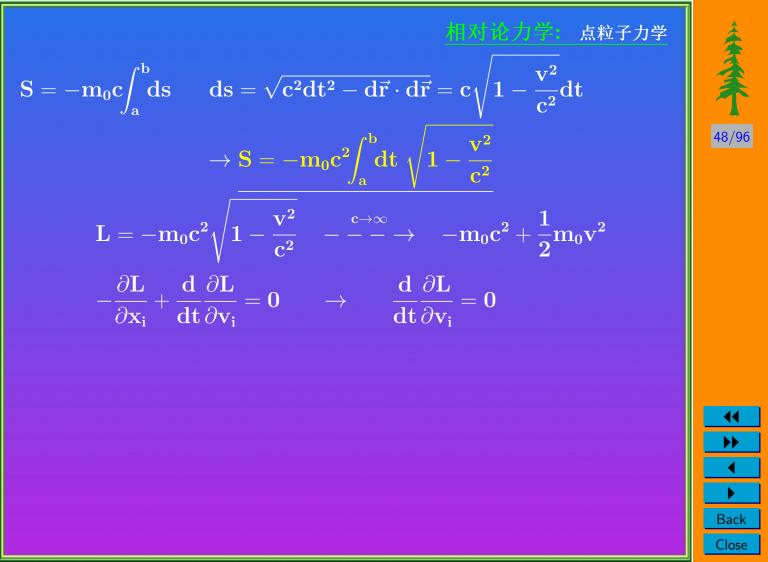


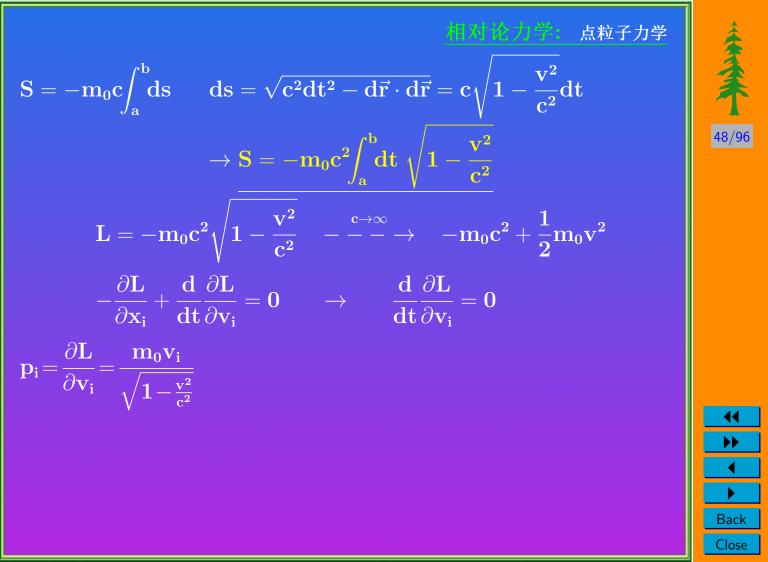


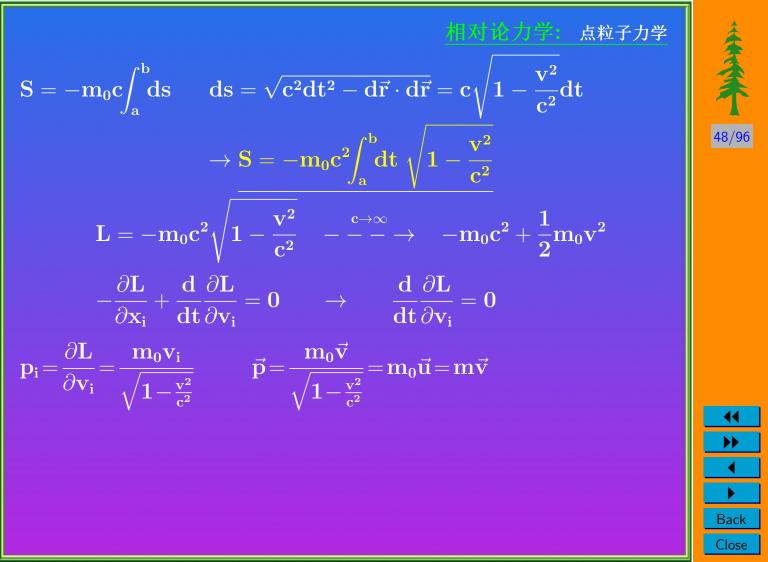


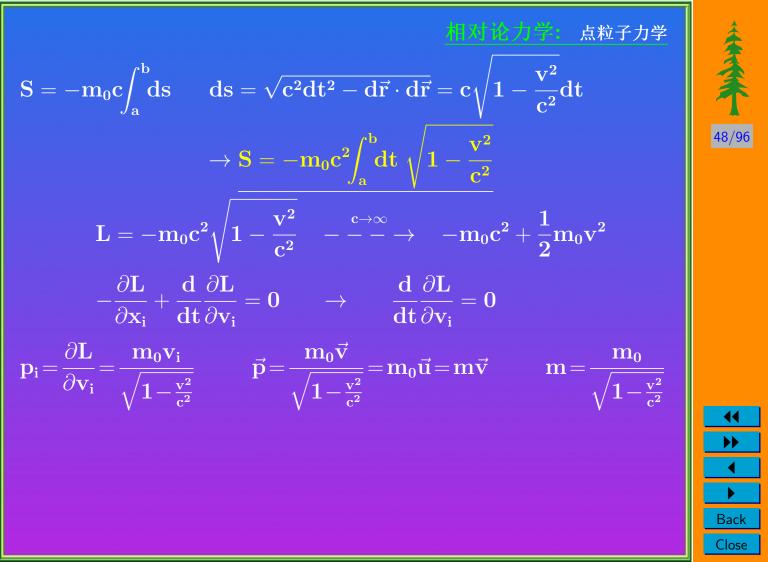


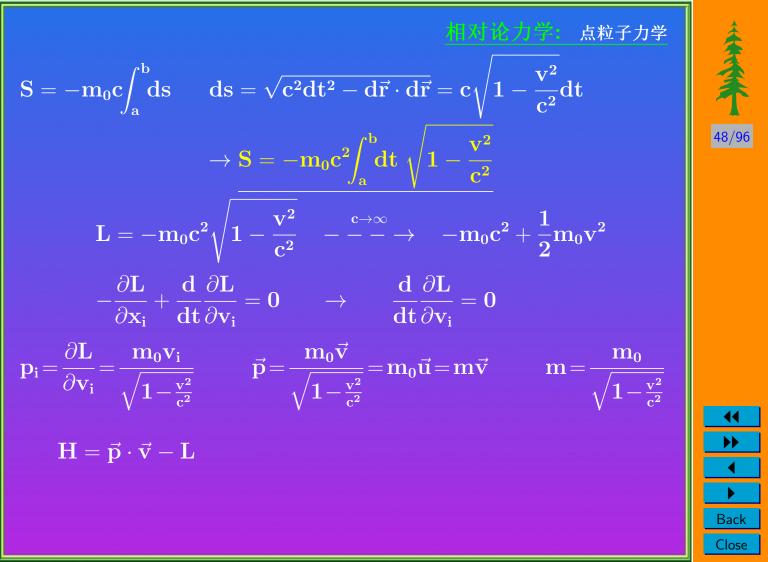


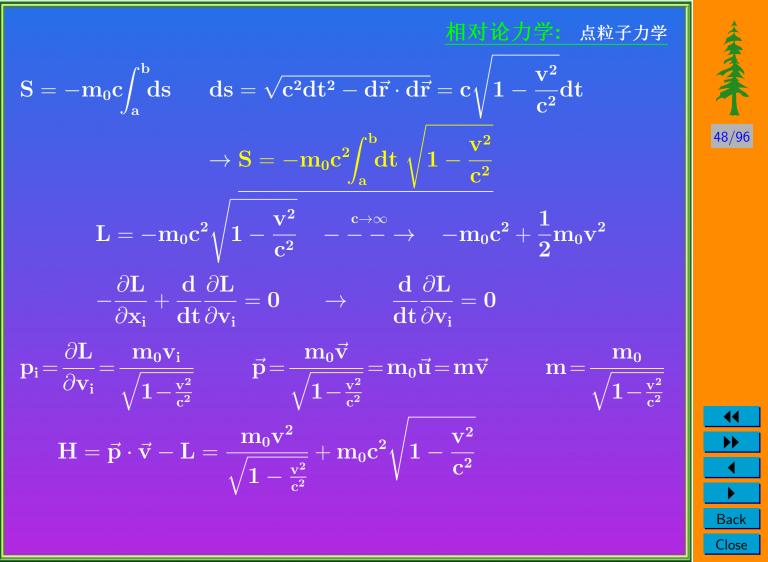


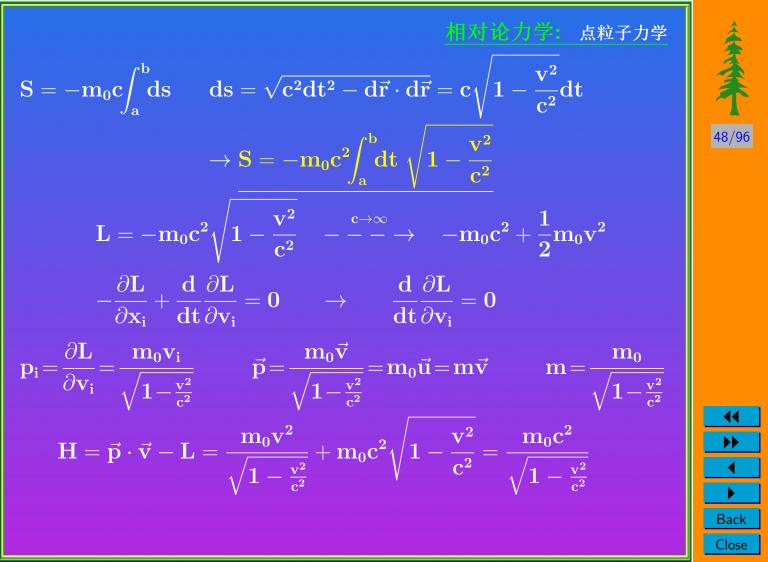


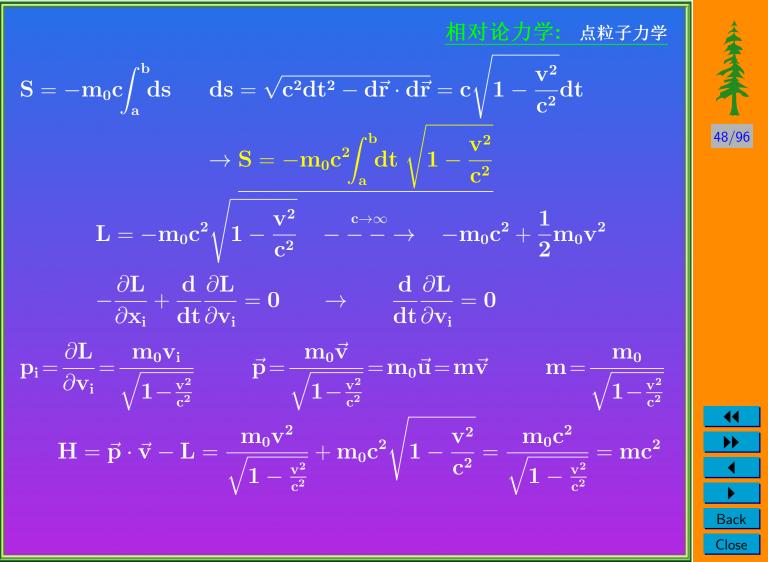


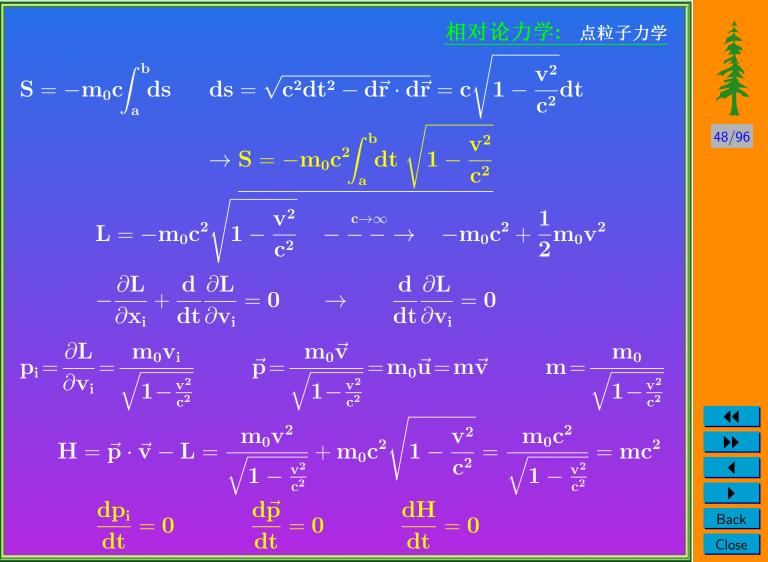


















在对论力学: 点粒子力学

$$egin{align} \mathbf{S} &= -m_0 c^2 \!\!\int_a^b \!\!\!\! dt \, \sqrt{1-rac{\mathbf{v}^2}{c^2}} & m = rac{m_0}{\sqrt{1-rac{\mathbf{v}^2}{c^2}}} \end{array}$$
 where  $\mathbf{m} = \frac{m_0}{\sqrt{1-rac{\mathbf{v}^2}{c^2}}}$  and  $\mathbf{m} = \frac{m_0 \vec{\mathbf{v}}}{\sqrt{1-rac{\mathbf{v}^2}{c^2}}}$ 

$$ec{p} = rac{m_0 ec{v}}{\sqrt{1 - rac{v^2}{c^2}}} = m ec{v} \qquad rac{dec{p}}{dt} = 0 \qquad \qquad H = m c^2 \ u^\mu = rac{dx^\mu}{d au} = rac{1}{\sqrt{1 - rac{v^2}{c^2}}} rac{dx^\mu}{dt} = (rac{ec{v}}{\sqrt{1 - rac{v^2}{c^2}}}, rac{ic}{\sqrt{1 - rac{v^2}{c^2}}})$$





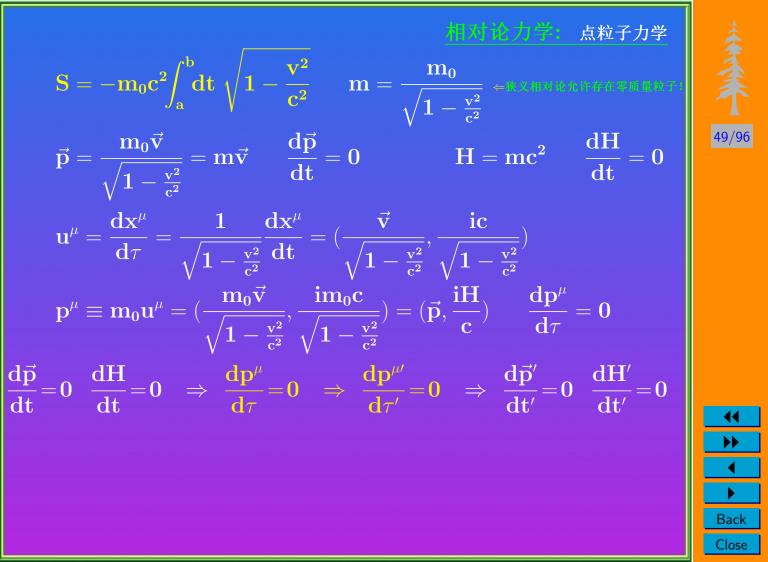


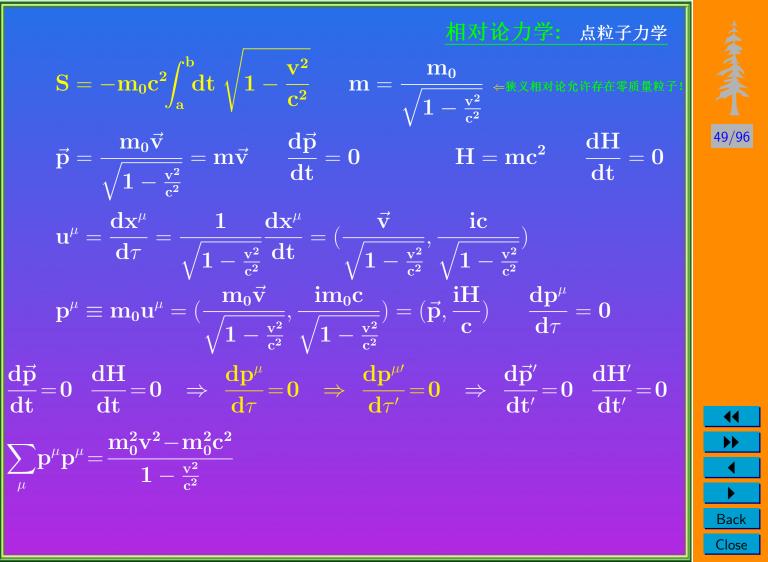




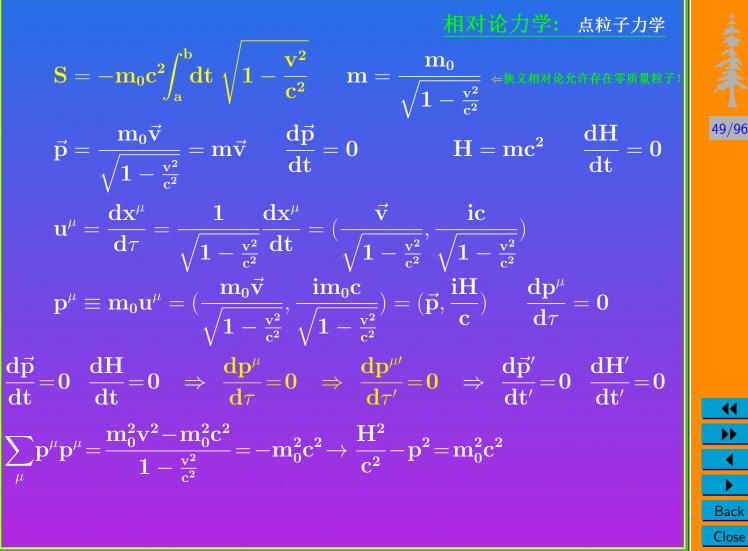












相对论力学: 点粒子力学  ${f S} = -{f m}_0 {f c}^2 {f \int}_2^{f b} \! dt \, \sqrt{1 - {{f v}^2 \over {f c}^2}} \, \, \, \, \, \, \, {f m} = {{f m}_0 \over \sqrt{1 - {{f v}^2 \over {f c}^2}}} \in {f r}$  $ec{
m p}=rac{{
m m_0}ec{
m v}}{\sqrt{1-rac{{
m v}^2}{2}}}={
m m}ec{
m v} \hspace{0.5cm} rac{{
m d}ec{
m p}}{{
m d}t}=0 \hspace{1cm} {
m H}={
m mc}^2 \hspace{0.5cm} rac{{
m d}H}{{
m d}t}=0$ 49/96  $\mathbf{u}^{\mu}=\overline{rac{\mathbf{d}\mathbf{x}^{\mu}}{\mathbf{d} au}}=rac{\mathbf{1}}{\sqrt{1-rac{\mathbf{v}^2}{\mathbf{c}^2}}}rac{\mathbf{d}\mathbf{x}^{\mu}}{\mathbf{d}\mathbf{t}}=(\overline{rac{\mathbf{ar{v}}}{\sqrt{1-rac{\mathbf{v}^2}{\mathbf{c}^2}}}}, \overline{rac{\mathbf{i}\mathbf{c}}{\sqrt{1-rac{\mathbf{v}^2}{\mathbf{c}^2}}}})$  $\mathbf{p}^{\mu} \equiv \mathbf{m_0} \mathbf{u}^{\mu} = (rac{\mathbf{m_0} ec{\mathbf{v}}}{\sqrt{1 - rac{\mathbf{v}^2}{c^2}}}, rac{ec{\mathbf{i} \mathbf{m_0} \mathbf{c}}}{\sqrt{1 - rac{\mathbf{v}^2}{c^2}}}) = (ec{\mathbf{p}}, rac{ec{\mathbf{i} H}}{\mathbf{c}}) \qquad rac{d\mathbf{p}^{\mu}}{d au} = \mathbf{0}$  $egin{aligned} rac{ ext{d}ec{ ext{p}}}{ ext{dt}} = 0 & rac{ ext{d} ext{p}^{\mu}}{ ext{d} au} = 0 & \Rightarrow & rac{ ext{d} ext{p}^{\mu\prime}}{ ext{d} au'} = 0 & \Rightarrow & rac{ ext{d}ec{ ext{p}}'}{ ext{d} au'} = 0 & rac{ ext{d} ext{p}'}{ ext{d} au'} = 0 \end{aligned}$ Back Close

# 相对论力学: 带电点粒子及电荷分布在外电磁场中

● 电磁场如何描述?



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#### 相对论力学。带电点粒子及电荷分布在外电磁场中

- 电磁场如何描述?
- 物理量可按其时空变换性质分类, 不同张量代表不同物理



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- 电磁场如何描述?
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- 电磁场应该属于某一张量, 假设 其为四矢量 $\mathbf{A}_{\mu} = (\vec{\mathbf{A}}, \frac{\mathbf{i}}{\mathbf{c}}\phi)$







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- 对比时空坐标矢量 $\mathbf{x}_{\mu} = (\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{ict})$ 及其洛伦兹变换关系

$$\mathbf{x}' = rac{\mathbf{x} - \mathbf{v}\mathbf{t}}{\sqrt{1 - rac{\mathbf{v}^2}{\mathbf{c}^2}}} \qquad \mathbf{y}' = \mathbf{y} \qquad \mathbf{z}' = \mathbf{z} \qquad \mathbf{t}' = rac{\mathbf{t} - rac{\mathbf{v}}{\mathbf{c}^2}\mathbf{x}}{\sqrt{1 - rac{\mathbf{v}^2}{\mathbf{c}^2}}}$$



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Back Close

- 电磁场如何描述?
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  $\mathbf{y}' = \mathbf{y}$   $\mathbf{z}' = \mathbf{z}$   $\mathbf{t}' = rac{\mathbf{t} - rac{\dot{\mathbf{c}}^2}{\mathbf{c}^2}}{\sqrt{1 - rac{\mathbf{v}^2}{\mathbf{c}^2}}}$ 

可得 $(\vec{A}, \phi)$ 的洛伦兹变换关系

$$\mathbf{A}_{\mathbf{x}}' = rac{\mathbf{A}_{\mathbf{x}} - rac{\mathbf{v}}{\mathbf{c}^2}\phi}{\sqrt{1 - rac{\mathbf{v}^2}{\mathbf{c}^2}}} \qquad \mathbf{A}_{\mathbf{y}}' = \mathbf{A}_{\mathbf{y}} \qquad \mathbf{A}_{\mathbf{z}}' = \mathbf{A}_{\mathbf{z}} \qquad \phi' = rac{\phi - \mathbf{v}\mathbf{A}_{\mathbf{x}}}{\sqrt{1 - rac{\mathbf{v}^2}{\mathbf{c}^2}}}$$



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$$\mathbf{x}' = rac{\mathbf{x} - \mathbf{v}\mathbf{t}}{\sqrt{1 - rac{\mathbf{v}^2}{\mathbf{c}^2}}} \qquad \mathbf{y}' = \mathbf{y} \qquad \mathbf{z}' = \mathbf{z} \qquad \mathbf{t}' = rac{\mathbf{t} - rac{\mathbf{c}^2}{\mathbf{c}^2}\mathbf{x}}{\sqrt{1 - rac{\mathbf{v}^2}{\mathbf{c}^2}}}$$

可得 $(\vec{A}, \phi)$ 的洛伦兹变换关系

$$A'_{x} = \frac{A_{x} - \frac{v}{c^{2}}\phi}{\sqrt{1 - \frac{v^{2}}{c^{2}}}} \qquad A'_{y} = A_{y} \qquad A'_{z} = A_{z} \qquad \phi' = \frac{\phi - vA_{x}}{\sqrt{1 - \frac{v^{2}}{c^{2}}}}$$
 为什么要用四度矢量场来描述电磁场?

● 为什么要用四度矢量场来描述电磁场?





## 相对论力学:带电点粒子及电荷分布在外电磁场中

假设矢量场 $A_{\mu} = (\vec{A}, \frac{i}{c}\phi)$ 描述电磁场,规定了  $(\vec{A}, \phi)$ 的洛伦兹变换关系,

$$\mathbf{A}_{\mathrm{x}}' = rac{\mathbf{A}_{\mathrm{x}} - rac{\mathrm{v}}{\mathrm{c}^2}\phi}{\sqrt{1 - rac{\mathrm{v}^2}{\mathrm{c}^2}}} \qquad \mathbf{A}_{\mathrm{y}}' = \mathbf{A}_{\mathrm{y}} \qquad \mathbf{A}_{\mathrm{z}}' = \mathbf{A}_{\mathrm{z}} \qquad \phi' = rac{\phi - \mathbf{v}\mathbf{A}_{\mathrm{x}}}{\sqrt{1 - rac{\mathrm{v}^2}{\mathrm{c}^2}}}$$









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为描述带电点粒子在外电磁场的行为,必须在自由点粒子作用量加上一个反应电磁 场与带电粒子发生相互作用的项。







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为描述带电点粒子在外电磁场的行为,必须在自由点粒子作用量加上一 个反应电磁场与带电粒子发生相互作用的项。 这一项至少应包含点粒子的性质和电磁场的性质,并且是<mark>洛伦兹变换标量</mark>:

这样的电磁相互作用具有规范不变性:  $\mathbf{A}_{\mu} o \mathbf{A}'_{\mu} = \mathbf{A}_{\mu} + \partial_{\mu}\chi$   $\chi$  的意义以后讨论!







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假设矢量场 $\mathbf{A}_{\mu} = (\vec{\mathbf{A}}, \frac{\mathbf{i}}{\mathbf{c}} \phi)$ 描述电磁场,规定了  $(\vec{\mathbf{A}}, \phi)$ 的洛伦兹变换关系,

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为描述带电点粒子在外电磁场的行为,必须在自由点粒子作用量加上一 个反应电磁场与带电粒子发生相互作用的项。 这一项至少应包含点粒子的性质和电磁场的性质,并且是<u>洛伦兹变换标量</u>:

这样的电磁相互作用具有<u>规范不变性</u>:  $\mathbf{A}_{\mu} \to \mathbf{A}_{\mu}' = \mathbf{A}_{\mu} + \partial_{\mu}\chi$  x的意义以后讨论!

$$\mathbf{S} = \int_{2}^{\mathbf{D}} (-\mathbf{m_0} \mathbf{cds} + \mathbf{e} \vec{\mathbf{A}} \cdot \mathbf{d} \vec{\mathbf{r}} - \mathbf{e} \phi \mathbf{dt})$$



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### 相对论力学: 带电点粒子及电荷分布在外电磁场中

假设矢量场 $A_{\mu} = (\vec{A}, \frac{i}{c}\phi)$ 描述电磁场,规定了  $(\vec{A}, \phi)$ 的洛伦兹变换关系,

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$$S = \int_{a}^{b} (-m_{0}cds + e\vec{A} \cdot d\vec{r} - e\phi dt) = \int_{a}^{b} dt \ (-m_{0}c^{2}\sqrt{1 - \frac{v^{2}}{c^{2}}} + e\vec{A} \cdot \vec{v} - e\phi) \ \bigg|$$



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# 52/96 相对论力学: 带电点粒子及电荷分布在外电磁场中 $\mathbf{S} = \int_{\mathbf{a}}^{\mathbf{b}} d\mathbf{t} (-\mathbf{m}_{\mathbf{0}} \mathbf{c}^{2} \sqrt{1 - \frac{\mathbf{v}^{2}}{\mathbf{c}^{2}}} + \mathbf{e} \vec{\mathbf{A}} \cdot \vec{\mathbf{v}} - \mathbf{e} \phi)$







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## 相对论力学: 带电点粒子及电荷分布在外电磁场中















## 相对论力学: 带电点粒子及电荷分布在外电磁场中



 $S = \int_{\mathbf{a}}^{\mathbf{b}} \!\! \mathbf{dt} (-\mathbf{m}_0 \mathbf{c^2} \! \sqrt{1 - \frac{\mathbf{v^2}}{\mathbf{c^2}}} + \mathbf{e} \vec{\mathbf{A}} \cdot \vec{\mathbf{v}} - \mathbf{e} \phi) \quad L = -\mathbf{m}_0 \mathbf{c^2} \! \sqrt{1 - \frac{\mathbf{v^2}}{\mathbf{c^2}}} + \mathbf{e} \vec{\mathbf{A}} \cdot \vec{\mathbf{v}} - \mathbf{e} \phi$  $ec{f p} = rac{{f m_0 ec {f v}}}{\sqrt{1-rac{{f v}^2}{c^2}}} + e ec {f A}$  正则动量 eq 机械动量





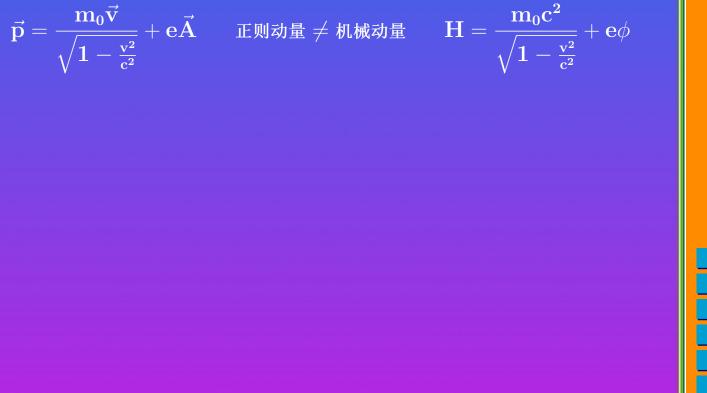




## 相对论力学。带电点粒子及电荷分布在外电磁场中



 $S = \int_{\mathbf{a}}^{\mathbf{b}} \!\! \mathbf{dt} (-\mathbf{m_0} \mathbf{c^2} \! \sqrt{1 - \frac{\mathbf{v^2}}{\mathbf{c^2}}} + \mathbf{e} \vec{\mathbf{A}} \cdot \vec{\mathbf{v}}} - \mathbf{e} \phi) \quad L = -\mathbf{m_0} \mathbf{c^2} \! \sqrt{1 - \frac{\mathbf{v^2}}{\mathbf{c^2}}} + \mathbf{e} \vec{\mathbf{A}} \cdot \vec{\mathbf{v}} - \mathbf{e} \phi$ 

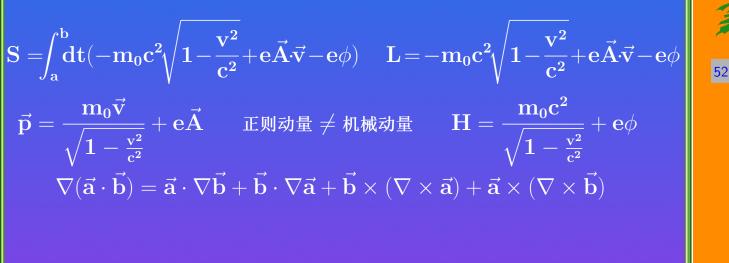








## 相对论力学 带电点粒子及电荷分布在外电磁场中











相对论力学 带电点粒子及电荷分布在外电磁场中

$$\mathbf{F}$$
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 $\sum_{i=1}^{3} \frac{\partial \mathbf{L}}{\partial \mathbf{x_i}} \vec{\mathbf{e}_i} = \nabla \mathbf{L}$ 









李季 相对论力学 带电点粒子及电荷分布在外电磁场中 52/96

$$\begin{split} \mathbf{S} = & \int_{\mathbf{a}}^{\mathbf{b}} \! dt (-\mathbf{m}_0 \mathbf{c}^2 \! \sqrt{1 - \frac{\mathbf{v}^2}{\mathbf{c}^2}} \! + \mathbf{e} \vec{\mathbf{A}} \cdot \vec{\mathbf{v}} \! - \! \mathbf{e} \phi) \quad \mathbf{L} \! = \! -\mathbf{m}_0 \mathbf{c}^2 \! \sqrt{1 - \frac{\mathbf{v}^2}{\mathbf{c}^2}} \! + \! \mathbf{e} \vec{\mathbf{A}} \cdot \vec{\mathbf{v}} \! - \! \mathbf{e} \phi \end{split}$$
 
$$\vec{\mathbf{p}} = \frac{\mathbf{m}_0 \vec{\mathbf{v}}}{\sqrt{1 - \frac{\mathbf{v}^2}{\mathbf{c}^2}}} + \mathbf{e} \vec{\mathbf{A}} \quad \text{E则动量} \neq \mathbf{M} \text{M} \text{动量} \quad \mathbf{H} = \frac{\mathbf{m}_0 \mathbf{c}^2}{\sqrt{1 - \frac{\mathbf{v}^2}{\mathbf{c}^2}}} + \mathbf{e} \phi$$
 
$$\nabla (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}) = \vec{\mathbf{a}} \cdot \nabla \vec{\mathbf{b}} + \vec{\mathbf{b}} \cdot \nabla \vec{\mathbf{a}} + \vec{\mathbf{b}} \times (\nabla \times \vec{\mathbf{a}}) + \vec{\mathbf{a}} \times (\nabla \times \vec{\mathbf{b}})$$
 
$$\sum_{i=1}^3 \frac{\partial \mathbf{L}}{\partial \mathbf{x}_i} \vec{\mathbf{e}}_i = \nabla \mathbf{L} \! = \! \mathbf{e} \nabla (\vec{\mathbf{A}} \cdot \vec{\mathbf{v}}) - \mathbf{e} \nabla \phi$$









相对论方学。带电点粒子及电荷分布在外电磁场中

$$\begin{split} \mathbf{S} = & \int_{\mathbf{a}}^{\mathbf{b}} \! dt (-m_0 c^2 \! \sqrt{1 - \frac{\mathbf{v}^2}{\mathbf{c}^2}} \! + e \vec{\mathbf{A}} \cdot \vec{\mathbf{v}} \! - e \phi) \quad \mathbf{L} \! = \! -m_0 c^2 \! \sqrt{1 - \frac{\mathbf{v}^2}{\mathbf{c}^2}} \! + e \vec{\mathbf{A}} \cdot \vec{\mathbf{v}} \! - e \phi \end{split}$$
 
$$\vec{\mathbf{p}} = \frac{m_0 \vec{\mathbf{v}}}{\sqrt{1 - \frac{\mathbf{v}^2}{\mathbf{c}^2}}} + e \vec{\mathbf{A}} \quad \text{E则动量} \neq \text{机械动量} \quad \mathbf{H} = \frac{m_0 c^2}{\sqrt{1 - \frac{\mathbf{v}^2}{\mathbf{c}^2}}} + e \phi$$
 
$$\nabla (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}) = \vec{\mathbf{a}} \cdot \nabla \vec{\mathbf{b}} + \vec{\mathbf{b}} \cdot \nabla \vec{\mathbf{a}} + \vec{\mathbf{b}} \times (\nabla \times \vec{\mathbf{a}}) + \vec{\mathbf{a}} \times (\nabla \times \vec{\mathbf{b}})$$
 
$$\sum_{i=1}^{3} \frac{\partial \mathbf{L}}{\partial \mathbf{x}_i} \vec{\mathbf{e}}_i = \nabla \mathbf{L} \! = \! e \nabla (\vec{\mathbf{A}} \cdot \vec{\mathbf{v}}) - e \nabla \phi \! = \! e \vec{\mathbf{v}} \cdot \nabla \vec{\mathbf{A}} + e \vec{\mathbf{v}} \times (\nabla \times \vec{\mathbf{A}}) - e \nabla \phi \end{split}$$









相对论方学。带电点粒子及电荷分布在外电磁场中 52/96

$$\begin{split} \mathbf{S} = & \int_{\mathbf{a}}^{b} \!\! \mathbf{d}t (-m_{0}c^{2}\!\sqrt{1\!-\!\frac{\mathbf{v}^{2}}{c^{2}}}\!+\!e\vec{\mathbf{A}}\vec{\mathbf{v}}\!-\!e\phi) \quad \mathbf{L} = \!-m_{0}c^{2}\!\sqrt{1\!-\!\frac{\mathbf{v}^{2}}{c^{2}}}\!+\!e\vec{\mathbf{A}}\vec{\mathbf{v}}\!-\!e\phi \\ \vec{p} = & \frac{m_{0}\vec{\mathbf{v}}}{\sqrt{1-\frac{\mathbf{v}^{2}}{c^{2}}}} + e\vec{\mathbf{A}} \quad \text{ E则动量} \neq \text{ 机械动量} \quad \mathbf{H} = \frac{m_{0}c^{2}}{\sqrt{1-\frac{\mathbf{v}^{2}}{c^{2}}}} + e\phi \\ & \nabla(\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}) = \vec{\mathbf{a}}\cdot\nabla\vec{\mathbf{b}} + \vec{\mathbf{b}}\cdot\nabla\vec{\mathbf{a}} + \vec{\mathbf{b}}\times(\nabla\times\vec{\mathbf{a}}) + \vec{\mathbf{a}}\times(\nabla\times\vec{\mathbf{b}}) \\ \sum_{i=1}^{3} \frac{\partial\mathbf{L}}{\partial\mathbf{x}_{i}}\vec{e}_{i} = \nabla\mathbf{L} \!=\! e\nabla(\vec{\mathbf{A}}\cdot\vec{\mathbf{v}}) - e\nabla\phi \!=\! e\vec{\mathbf{v}}\cdot\nabla\vec{\mathbf{A}} + e\vec{\mathbf{v}}\times(\nabla\times\vec{\mathbf{A}}) - e\nabla\phi \\ \frac{\mathbf{d}}{\mathbf{d}t}\frac{m_{0}\vec{\mathbf{v}}}{\sqrt{1-\frac{\mathbf{v}^{2}}{c^{2}}}} = \frac{d\vec{p}}{dt} - e\frac{d\vec{\mathbf{A}}}{dt} \end{split}$$







相对 论 力学: 带电点粒子及电荷分布在外电磁场中









相对论力学: 带电点粒子及电荷分布在外电磁场中  $+e\vec{A}\cdot\vec{v}-e\phi$   $L=-m_0c^2\sqrt{1-\frac{v^2}{v^2}}+e\vec{A}\cdot\vec{v}-e\phi$ 

$$\begin{split} \mathbf{S} = & \int_{\mathbf{a}}^{b} \!\! \mathbf{d}t (-\mathbf{m}_{0}\mathbf{c}^{2} \! \sqrt{1 - \frac{\mathbf{v}^{2}}{\mathbf{c}^{2}}} \! + e\vec{\mathbf{A}}\vec{\mathbf{v}} \! - e\phi) \quad \mathbf{L} = -\mathbf{m}_{0}\mathbf{c}^{2} \! \sqrt{1 - \frac{\mathbf{v}^{2}}{\mathbf{c}^{2}}} \! + e\vec{\mathbf{A}}\vec{\mathbf{v}} \! - e\phi \\ \vec{\mathbf{p}} = & \frac{\mathbf{m}_{0}\vec{\mathbf{v}}}{\sqrt{1 - \frac{\mathbf{v}^{2}}{\mathbf{c}^{2}}}} + e\vec{\mathbf{A}} \quad \text{E则动量} \neq \mathbf{M} \text{M} \text{动量} \quad \mathbf{H} = \frac{\mathbf{m}_{0}\mathbf{c}^{2}}{\sqrt{1 - \frac{\mathbf{v}^{2}}{\mathbf{c}^{2}}}} + e\phi \\ & \nabla(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}) = \vec{\mathbf{a}} \cdot \nabla\vec{\mathbf{b}} + \vec{\mathbf{b}} \cdot \nabla\vec{\mathbf{a}} + \vec{\mathbf{b}} \times (\nabla \times \vec{\mathbf{a}}) + \vec{\mathbf{a}} \times (\nabla \times \vec{\mathbf{b}}) \\ \sum_{i=1}^{3} \frac{\partial \mathbf{L}}{\partial \mathbf{x}_{i}} \vec{e}_{i} = \nabla \mathbf{L} = e\nabla(\vec{\mathbf{A}} \cdot \vec{\mathbf{v}}) - e\nabla\phi = e\vec{\mathbf{v}} \cdot \nabla\vec{\mathbf{A}} + e\vec{\mathbf{v}} \times (\nabla \times \vec{\mathbf{A}}) - e\nabla\phi \\ \frac{\mathbf{d}}{\mathbf{d}t} \frac{\mathbf{m}_{0}\vec{\mathbf{v}}}{\sqrt{1 - \frac{\mathbf{v}^{2}}{\mathbf{c}^{2}}}} = \frac{d\vec{\mathbf{p}}}{\mathbf{d}t} - e\frac{d\vec{\mathbf{A}}}{\mathbf{d}t} = \frac{-\frac{2\mathbf{L}}{2\mathbf{A}} + \frac{d}{\mathbf{d}}\frac{\partial \vec{\mathbf{L}}}{\partial \mathbf{N}}}{2\mathbf{d}t} = 0}{\mathbf{D}} \nabla \mathbf{L} - e\frac{d\vec{\mathbf{A}}}{\mathbf{d}t} \\ = e\vec{\mathbf{v}} \cdot \nabla\vec{\mathbf{A}} + e\vec{\mathbf{v}} \times (\nabla \times \vec{\mathbf{A}}) - e\nabla\phi - e(\frac{\partial\vec{\mathbf{A}}}{\partial t} + \frac{\partial\vec{\mathbf{r}}}{\partial t} \cdot \nabla\vec{\mathbf{A}}) \end{split}$$

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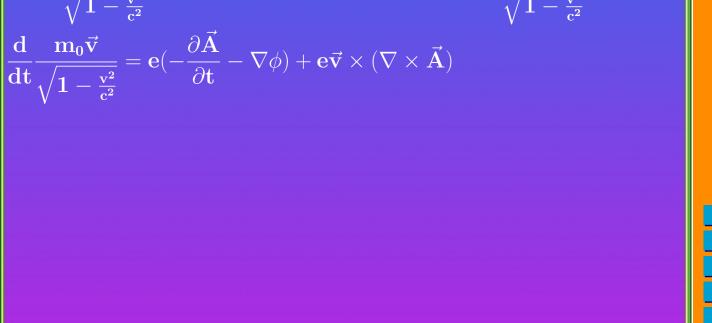
相对论力学: 带电点粒子及电荷分布在外电磁场中 $+\mathbf{e}ec{\mathbf{A}}ec{\mathbf{v}}-\mathbf{e}\phi$ )  $\mathbf{L}=-\mathbf{m}_0\mathbf{c}^2\sqrt{\mathbf{1}-rac{\mathbf{v}^2}{2}}+\mathbf{e}ec{\mathbf{A}}ec{\mathbf{v}}-\mathbf{e}\phi$ 

$$\begin{split} \mathbf{S} = & \int_{a}^{b} \!\! dt (-m_{0}c^{2}\!\sqrt{1\!-\!\frac{v^{2}}{c^{2}}}\!+\!e\vec{A}\vec{v}\!-\!e\phi) \quad L = \!-m_{0}c^{2}\!\sqrt{1\!-\!\frac{v^{2}}{c^{2}}}\!+\!e\vec{A}\vec{v}\!-\!e\phi \\ \vec{p} = & \frac{m_{0}\vec{v}}{\sqrt{1-\frac{v^{2}}{c^{2}}}} + e\vec{A} \quad \text{E则动量} \neq \text{机械动量} \quad H = \frac{m_{0}c^{2}}{\sqrt{1-\frac{v^{2}}{c^{2}}}} + e\phi \\ & \nabla(\vec{a}\cdot\vec{b}) = \vec{a}\cdot\nabla\vec{b} + \vec{b}\cdot\nabla\vec{a} + \vec{b}\times(\nabla\times\vec{a}) + \vec{a}\times(\nabla\times\vec{b}) \\ \sum_{i=1}^{3} & \frac{\partial L}{\partial x_{i}} \vec{e}_{i} = \nabla L = e\nabla(\vec{A}\cdot\vec{v}) - e\nabla\phi = e\vec{v}\cdot\nabla\vec{A} + e\vec{v}\times(\nabla\times\vec{A}) - e\nabla\phi \\ \frac{d}{dt} & \frac{m_{0}\vec{v}}{\sqrt{1-\frac{v^{2}}{c^{2}}}} = \frac{d\vec{p}}{dt} - e\frac{d\vec{A}}{dt} = = \frac{2\vec{L}}{2} + \frac{d}{dt} \frac{d\vec{L}}{2} = 0 \\ = e\vec{v}\cdot\nabla\vec{A} + e\vec{v}\times(\nabla\times\vec{A}) - e\nabla\phi - e(\frac{\partial\vec{A}}{\partial t} + \frac{\partial\vec{r}}{\partial t}\cdot\nabla\vec{A}) \\ = e(-\frac{\partial\vec{A}}{\partial t} - \nabla\phi) + e\vec{v}\times(\nabla\times\vec{A}) \end{split}$$



相对论力学 带电点粒子及电荷分布在外电磁场中  $\mathbf{S} = \int_{a}^{b} \!\! \mathrm{dt} (-\mathbf{m}_0 \mathbf{c^2} \sqrt{1 - \frac{\mathbf{v^2}}{\mathbf{c^2}}} + \mathbf{e} \vec{\mathbf{A}} \cdot \vec{\mathbf{v}} - \mathbf{e} \phi) \quad \mathbf{L} = -\mathbf{m}_0 \mathbf{c^2} \sqrt{1 - \frac{\mathbf{v^2}}{\mathbf{c^2}}} + \mathbf{e} \vec{\mathbf{A}} \cdot \vec{\mathbf{v}} - \mathbf{e} \phi$ 

$$ec{f p}=rac{{f m}_0ec{f v}}{\sqrt{1-rac{{f v}^2}{c^2}}}+eec{f A}$$
 正则动量  $eq$  机械动量  $extbf{H}=rac{{f m}_0{f c}^2}{\sqrt{1-rac{{f v}^2}{c^2}}}+e\phi$ 











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$$ec{f p}=rac{{f m_0ec v}}{\sqrt{1-rac{{f v}^2}{c^2}}}+e{f A}$$
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 $egin{aligned} rac{\mathrm{d}}{\mathrm{dt}} rac{\mathrm{m_0} ec{\mathbf{v}}}{\sqrt{1 - rac{\mathrm{v}^2}{\mathrm{c}^2}}} = \mathbf{e}(-rac{\partial ec{\mathbf{A}}}{\partial \mathrm{t}} - 
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相对论力学 带电点粒子及电荷分布在外电磁场中  $\mathbf{S} = \int_{a}^{b} \!\! \mathrm{dt} (-\mathbf{m}_{0} \mathbf{c}^{2} \sqrt{1 - \frac{\mathbf{v}^{2}}{\mathbf{c}^{2}}} + e \vec{\mathbf{A}} \cdot \vec{\mathbf{v}} - e \phi) \quad \mathbf{L} = -\mathbf{m}_{0} \mathbf{c}^{2} \sqrt{1 - \frac{\mathbf{v}^{2}}{\mathbf{c}^{2}}} + e \vec{\mathbf{A}} \cdot \vec{\mathbf{v}} - e \phi$ 

$$ec{\mathbf{p}}=rac{\mathbf{m}_0ec{\mathbf{v}}}{\sqrt{1-rac{\mathbf{v}^2}{c^2}}}+\mathbf{e}ec{\mathbf{A}}$$
 正则动量  $eq$  机械动量  $\mathbf{H}=rac{\mathbf{m}_0\mathbf{c}^2}{\sqrt{1-rac{\mathbf{v}^2}{c^2}}}+\mathbf{e}\phi$ 

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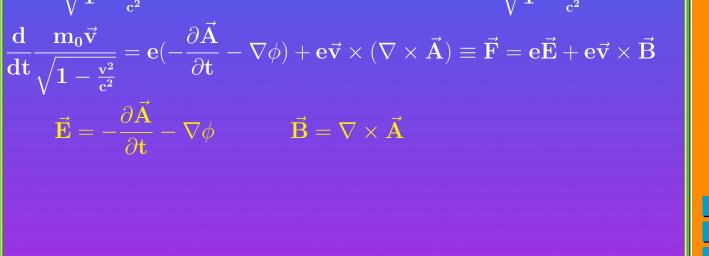






相对论力学 带电点粒子及电荷分布在外电磁场中

$$\mathbf{S} = \int_{\mathbf{a}}^{\mathbf{b}} \! \mathrm{dt}(-\mathbf{m}_0 \mathbf{c}^2 \sqrt{1 - \frac{\mathbf{v}^2}{\mathbf{c}^2}} + \mathbf{e} \vec{\mathbf{A}} \cdot \vec{\mathbf{v}} - \mathbf{e} \phi) \quad \mathbf{L} = -\mathbf{m}_0 \mathbf{c}^2 \sqrt{1 - \frac{\mathbf{v}^2}{\mathbf{c}^2}} + \mathbf{e} \vec{\mathbf{A}} \cdot \vec{\mathbf{v}} - \mathbf{e} \phi$$
 
$$\vec{\mathbf{p}} = \frac{\mathbf{m}_0 \vec{\mathbf{v}}}{\sqrt{1 - \frac{\mathbf{v}^2}{\mathbf{c}^2}}} + \mathbf{e} \vec{\mathbf{A}} \quad \text{ E则动量} \neq \mathbf{M} \vec{\mathbf{M}} \vec{\mathbf{m}} \vec{\mathbf{b}} \quad \mathbf{H} = \frac{\mathbf{m}_0 \mathbf{c}^2}{\sqrt{1 - \frac{\mathbf{v}^2}{\mathbf{c}^2}}} + \mathbf{e} \phi$$
 
$$\frac{\mathbf{d}}{\mathbf{d}} = \frac{\mathbf{m}_0 \vec{\mathbf{v}}}{\sqrt{1 - \frac{\mathbf{v}^2}{\mathbf{c}^2}}} = \mathbf{e}(-\frac{\partial \vec{\mathbf{A}}}{\partial t} - \nabla \phi) + \mathbf{e} \vec{\mathbf{v}} \times (\nabla \times \vec{\mathbf{A}}) \equiv \vec{\mathbf{F}} = \mathbf{e} \vec{\mathbf{E}} + \mathbf{e} \vec{\mathbf{v}} \times \vec{\mathbf{B}}$$









相文论力学。带电点粒子及电荷分布在外电磁场中 53/96

$$ec{\mathbf{p}}=rac{\mathbf{m}_0ec{\mathbf{v}}}{\sqrt{1-rac{\mathbf{v}^2}{\mathbf{c}^2}}}+\mathbf{e}ec{\mathbf{A}}$$
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abla \phi & ec{\mathbf{E}} = -rac{\partial ec{\mathbf{A}}}{\partial \mathbf{t}} - 
abla \phi & ec{\mathbf{E}} = -rac{\partial ec{\mathbf{A}}}{\partial \mathbf{t}} - 
abla \phi & ec{\mathbf{E}} = -rac{\partial ec{\mathbf{A}}}{\partial \mathbf{t}} - 
abla \phi & ec{\mathbf{E}} = -rac{\partial ec{\mathbf{A}}}{\partial \mathbf{t}} - 
abla \phi & ec{\mathbf{E}} = -rac{\partial ec{\mathbf{A}}}{\partial \mathbf{t}} - 
abla \phi & ec{\mathbf{E}} = -rac{\partial ec{\mathbf{A}}}{\partial \mathbf{t}} - 
abla \phi & ec{\mathbf{E}} = -rac{\partial ec{\mathbf{A}}}{\partial \mathbf$$















相对论力学: 带电点粒子及电荷分布在外电磁场中

 $S = \int_{a}^{b} dt (-m_{0}c^{2}\sqrt{1 - \frac{v^{2}}{c^{2}}} + e\vec{A}\cdot\vec{v} - e\phi) \quad L = -m_{0}c^{2}\sqrt{1 - \frac{v^{2}}{c^{2}}} + e\vec{A}\cdot\vec{v} - e\phi$ 

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 $ec{\mathbf{p}}=rac{\mathbf{m}_0ec{\mathbf{v}}}{\sqrt{1-rac{\mathbf{v}^2}{2}}}+\mathbf{e}ec{\mathbf{A}}$  正则动量 eq 机械动量  $\mathbf{H}=rac{\mathbf{m}_0\mathbf{c}^2}{\sqrt{1-rac{\mathbf{v}^2}{2}}}+\mathbf{e}\phi$ 

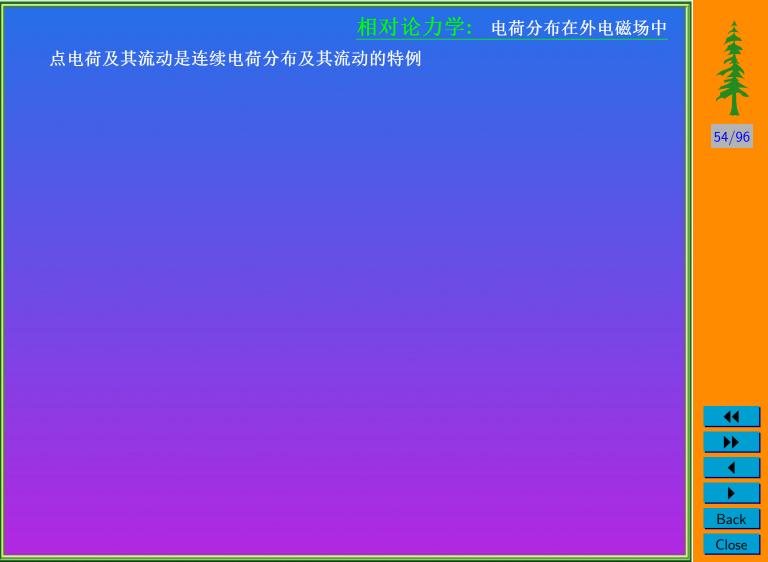
 $egin{aligned} \sqrt{1-rac{\dot{ au}^2}{c^2}} & \sqrt{1-rac{\dot{ au}^2}{c^2}} \ rac{d}{dt}rac{\mathbf{m_0}ec{\mathbf{v}}}{\sqrt{1-rac{\mathbf{v}^2}{c^2}}} = \mathbf{e}(-rac{\partialec{\mathbf{A}}}{\partial t}abla\phi) + \mathbf{e}ec{\mathbf{v}} imes(
abla imes\mathbf{A}) \equiv ec{\mathbf{F}} = \mathbf{e}ec{\mathbf{E}} + \mathbf{e}ec{\mathbf{v}} imesec{\mathbf{B}} \end{aligned}$ 

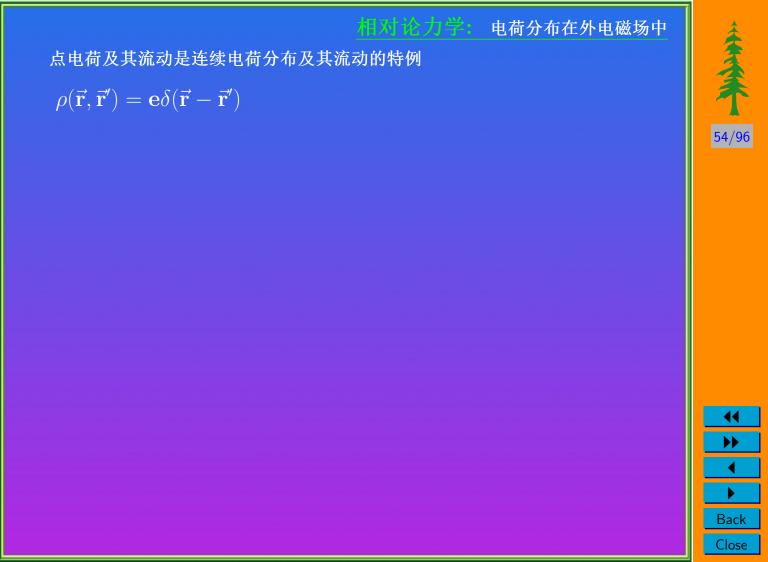
 $ec{\mathbf{E}} = -rac{\partial ec{\mathbf{A}}}{\partial \mathbf{t}} - 
abla \phi \qquad \qquad ec{\mathbf{B}} = 
abla imes ec{\mathbf{A}}$ 

 $\mathbf{E_x'} = \mathbf{E_x} \qquad \mathbf{E_y'} = rac{\mathbf{E_y - vB_z}}{\sqrt{1 - rac{\mathbf{v^2}}{\mathbf{c^2}}}} \qquad \mathbf{E_z'} = rac{\mathbf{E_z + vB_y}}{\sqrt{1 - rac{\mathbf{v^2}}{\mathbf{c^2}}}}$ 

 $\mathbf{B}_{\mathrm{x}}' = \mathbf{B}_{\mathrm{x}} \qquad \mathbf{B}_{\mathrm{y}}' = rac{\dot{\mathbf{B}}_{\mathrm{y}} + rac{\mathrm{v}}{\mathrm{c}^2}\mathbf{E}_{\mathrm{z}}}{\sqrt{1 - rac{\mathrm{v}^2}{\mathrm{c}^2}}} \qquad \mathbf{B}_{\mathrm{z}}' = rac{\dot{\mathbf{B}}_{\mathrm{z}} - rac{\mathrm{v}}{\mathrm{c}^2}\mathbf{E}_{\mathrm{y}}}{\sqrt{1 - rac{\mathrm{v}^2}{\mathrm{c}^2}}}$ 

电场强度和磁感应强度不各自形成单独的四矢量的空间分量,而是联合形成二阶反对称张量,凸显<u>电和磁的一体4</u>





## 电荷分布在外电磁场中 点电荷及其流动是连续电荷分布及其流动的特例 $\vec{\mathbf{j}}(\vec{\mathbf{r}}, \vec{\mathbf{r}}') = \rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}')\vec{\mathbf{v}}$ $\rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}') = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}')$ 54/96









## 加入 论力学 电荷分布在外电磁场中 点电荷及其流动是连续电荷分布及其流动的特例

$$\rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}') = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \qquad \qquad \vec{\mathbf{j}}(\vec{\mathbf{r}}, \vec{\mathbf{r}}') = \rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}')\vec{\mathbf{v}}$$

$$\mathbf{e}\mathbf{d}\mathbf{x}_{\mu}=\mathbf{e}\delta(ec{\mathbf{r}}-ec{\mathbf{r}}')\mathbf{d} au\mathbf{d}\mathbf{x}_{\mu}$$



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$$\rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}') = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \qquad \qquad \vec{\mathbf{j}}(\vec{\mathbf{r}}, \vec{\mathbf{r}}') = \rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}')\vec{\mathbf{v}}$$

$$\mathbf{e}\mathbf{d}\mathbf{x}_{\mu}=\mathbf{e}\delta(\vec{\mathbf{r}}-\vec{\mathbf{r}}')\mathbf{d} au\mathbf{d}\mathbf{x}_{\mu}=
ho\mathbf{d} au\mathbf{d}\mathbf{x}_{\mu}$$



Back

$$\begin{split} \rho(\vec{\mathbf{r}},\vec{\mathbf{r}}') &= \mathbf{e}\delta(\vec{\mathbf{r}}-\vec{\mathbf{r}}') & \vec{\mathbf{j}}(\vec{\mathbf{r}},\vec{\mathbf{r}}') = \rho(\vec{\mathbf{r}},\vec{\mathbf{r}}')\vec{\mathbf{v}} \\ \mathbf{e}\mathbf{d}\mathbf{x}_{\mu} &= \mathbf{e}\delta(\vec{\mathbf{r}}-\vec{\mathbf{r}}')\mathbf{d}\tau\mathbf{d}\mathbf{x}_{\mu} = \rho\mathbf{d}\tau\mathbf{d}\mathbf{x}_{\mu} = \rho\frac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}t}\mathbf{d}t\mathbf{d}\tau \end{split}$$



Back

$$\rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}') = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \qquad \qquad \vec{\mathbf{j}}(\vec{\mathbf{r}}, \vec{\mathbf{r}}') = \rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}')\vec{\mathbf{v}}$$

$$\mathbf{e}\mathbf{d}\mathbf{x}_{\mu} = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}')\mathbf{d} au\mathbf{d}\mathbf{x}_{\mu} = 
ho\mathbf{d} au\mathbf{d}\mathbf{x}_{\mu} = 
horac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}\mathbf{t}}\mathbf{d}\mathbf{t}\mathbf{d} au = \mathbf{j}_{\mu}\mathbf{d}\mathbf{t}\mathbf{d} au$$



Back Close 电荷分布在外电磁场中

点电荷及其流动是连续电荷分布及其流动的特例

$$\rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}') = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \qquad \qquad \vec{\mathbf{j}}(\vec{\mathbf{r}}, \vec{\mathbf{r}}') = \rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}')\vec{\mathbf{v}}$$

$$\mathbf{d}\mathbf{x}_{\mu}$$

$$-\vec{\mathbf{r}}')\mathbf{d} au\mathbf{d}\mathbf{x}_{\mu}=
ho\mathbf{d} au\mathbf{d}\mathbf{x}_{\mu}=
horac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}\mathbf{t}}$$

$$\begin{aligned} \mathbf{e} \mathbf{d} \mathbf{x}_{\mu} &= \mathbf{e} \delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \mathbf{d} \tau \mathbf{d} \mathbf{x}_{\mu} = \rho \mathbf{d} \tau \mathbf{d} \mathbf{x}_{\mu} = \rho \frac{\mathbf{d} \mathbf{x}_{\mu}}{\mathbf{d} t} \mathbf{d} t \mathbf{d} \tau = \mathbf{j}_{\mu} \mathbf{d} t \mathbf{d} \tau \\ \mathbf{j}_{\mu} &\equiv \rho \frac{\mathbf{d} \mathbf{x}_{\mu}}{\mathbf{d} t} = (\vec{\mathbf{j}}, \mathbf{i} \mathbf{c} \rho) \end{aligned}$$







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电荷分布在外电磁场中 点电荷及其流动是连续电荷分布及其流动的特例  $ho(ec{\mathbf{r}},ec{\mathbf{r}}') = \mathbf{e}\delta(ec{\mathbf{r}}-ec{\mathbf{r}}')$   $ec{\mathbf{j}}(ec{\mathbf{r}},ec{\mathbf{r}}') = 
ho(ec{\mathbf{r}},ec{\mathbf{r}}')ec{\mathbf{v}}$  $\mathbf{e}\mathbf{d}\mathbf{x}_{\mu} = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}')\mathbf{d} au\mathbf{d}\mathbf{x}_{\mu} = 
ho\mathbf{d} au\mathbf{d}\mathbf{x}_{\mu} = 
ho\frac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}\mathbf{t}}\mathbf{d}\mathbf{t}\mathbf{d} au = \mathbf{j}_{\mu}\mathbf{d}\mathbf{t}\mathbf{d} au$  $\mathbf{j}_{\mu} \equiv 
ho rac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}\mathbf{t}} = (\mathbf{ec{j}}, \mathbf{ic}
ho) \quad \Rightarrow \quad$ 对比  $\mathbf{x}_{\mu} = (\mathbf{ec{r}}, \mathbf{ict})$ 

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点电荷及其流动是连续电荷分布及其流动的特例  $\rho(\vec{\mathbf{r}},\vec{\mathbf{r}}') = \mathbf{e}\delta(\vec{\mathbf{r}}-\vec{\mathbf{r}}') \qquad \vec{\mathbf{i}}(\vec{\mathbf{r}},\vec{\mathbf{r}}') = \rho(\vec{\mathbf{r}},\vec{\mathbf{r}}')\vec{\mathbf{v}}$ 

$$\begin{split} \rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}') &= \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') & \vec{\mathbf{j}}(\vec{\mathbf{r}}, \vec{\mathbf{r}}') = \rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}')\vec{\mathbf{v}} \\ \mathbf{e}\mathbf{d}\mathbf{x}_{\mu} &= \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}')\mathbf{d}\tau\mathbf{d}\mathbf{x}_{\mu} = \rho\mathbf{d}\tau\mathbf{d}\mathbf{x}_{\mu} = \rho\frac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}t}\mathbf{d}t\mathbf{d}\tau = \mathbf{j}_{\mu}\mathbf{d}t\mathbf{d}\tau \end{split}$$

$$\mathbf{j}_{\mu} \equiv 
ho rac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}\mathbf{t}} = (\mathbf{ec{j}}, \mathbf{ic}
ho) \;\;\; \Rightarrow \;\;\;$$
 对比  $\; \mathbf{x}_{\mu} = (\mathbf{ec{r}}, \mathbf{ict})$ 

$$\mathbf{j}_{\mathrm{x}}' = rac{\mathbf{j}_{\mathrm{x}} - \mathbf{v}
ho}{\sqrt{1 - rac{\mathbf{v}^2}{\mathrm{c}^2}}} \qquad \mathbf{j}_{\mathrm{y}}' = \mathbf{j}_{\mathrm{y}} \qquad \mathbf{j}_{\mathrm{z}}' = \mathbf{j}_{\mathrm{z}} \qquad 
ho' = rac{
ho - rac{\mathrm{v}}{\mathrm{c}^2}\mathbf{j}_{\mathrm{x}}}{\sqrt{1 - rac{\mathrm{v}^2}{\mathrm{c}^2}}}$$









$$\begin{split} \rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}') &= \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') & \vec{\mathbf{j}}(\vec{\mathbf{r}}, \vec{\mathbf{r}}') = \rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}')\vec{\mathbf{v}} \\ \mathbf{e}\mathbf{d}\mathbf{x}_{\mu} &= \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}')\mathbf{d}\tau\mathbf{d}\mathbf{x}_{\mu} = \rho\mathbf{d}\tau\mathbf{d}\mathbf{x}_{\mu} = \rho\frac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}t}\mathbf{d}t\mathbf{d}\tau = \mathbf{j}_{\mu}\mathbf{d}t\mathbf{d}\tau \end{split}$$

$$\mathbf{j}_{\mu} \equiv 
ho rac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}\mathbf{t}} = (\mathbf{ec{j}}, \mathbf{i}\mathbf{c}
ho) \quad \Rightarrow \quad$$
 对比  $\mathbf{x}_{\mu} = (\mathbf{ec{r}}, \mathbf{i}\mathbf{c}\mathbf{t})$ 

$$\mathbf{j}_{\mathrm{x}}' = rac{\mathbf{j}_{\mathrm{x}} - \mathrm{v}
ho}{\sqrt{1 - rac{\mathrm{v}^2}{\mathrm{c}^2}}} \quad \mathbf{j}_{\mathrm{y}}' = \mathbf{j}_{\mathrm{y}} \quad \mathbf{j}_{\mathrm{z}}' = \mathbf{j}_{\mathrm{z}} \qquad 
ho' = rac{
ho - rac{\mathrm{v}}{\mathrm{c}^2}\mathbf{j}_{\mathrm{x}}}{\sqrt{1 - rac{\mathrm{v}^2}{\mathrm{c}^2}}} \ \sum_{\mu=1}^4 \int_{\mathbf{a}}^{\mathbf{b}} \mathrm{d}\mathbf{x}_{\mu} \ \mathbf{e}\mathbf{A}_{\mu} = \sum_{\mu=1}^4 \int \mathrm{d}\mathbf{t}\mathrm{d} au \mathbf{A}_{\mu}\mathbf{j}_{\mu}$$

$$\mathbf{e}\mathbf{A}_{\mu}=\sum_{\mu=1}^{4}\int\mathbf{d}\mathbf{t}\mathbf{d} au\mathbf{A}_{\mu}\mathbf{j}_{\mu}$$



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$$\rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}') = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \qquad \qquad \vec{\mathbf{j}}(\vec{\mathbf{r}}, \vec{\mathbf{r}}') = \rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}')\vec{\mathbf{v}}$$

$$\mathbf{e} \mathbf{d} \mathbf{x}_{\mu} = \mathbf{e} \delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \mathbf{d} \tau \mathbf{d} \mathbf{x}_{\mu} = \rho \mathbf{d} \tau \mathbf{d} \mathbf{x}_{\mu} = \rho \frac{\mathbf{d} \mathbf{x}_{\mu}}{\mathbf{d} t} \mathbf{d} t \mathbf{d} \tau = \mathbf{j}_{\mu} \mathbf{d} t \mathbf{d} \tau$$

$$\mathbf{j}_{\mu} \equiv 
ho rac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}\mathbf{t}} = (\mathbf{\vec{j}}, \mathbf{ic}
ho) \quad \Rightarrow \quad$$
 对比  $\mathbf{x}_{\mu} = (\mathbf{\vec{r}}, \mathbf{ict})$ 

$$\mathbf{j}_{\mathbf{x}}' = \frac{\mathbf{j}_{\mathbf{x}} - \mathbf{v}\rho}{\sqrt{1 - \frac{\mathbf{v}^2}{\mathbf{c}^2}}} \qquad \mathbf{j}_{\mathbf{y}}' = \mathbf{j}_{\mathbf{y}} \qquad \mathbf{j}_{\mathbf{z}}' = \mathbf{j}_{\mathbf{z}} \qquad \rho' = \frac{\rho - \frac{\mathbf{v}}{\mathbf{c}^2} \mathbf{j}_{\mathbf{x}}}{\sqrt{1 - \frac{\mathbf{v}^2}{\mathbf{c}^2}}}$$

$$\sum_{\mu=1}^4 \int_{\mathbf{a}}^{\mathbf{b}} \mathbf{d}\mathbf{x}_{\mu} \; \mathbf{e}\mathbf{A}_{\mu} = \sum_{\mu=1}^4 \int \mathbf{d}\mathbf{t}\mathbf{d} au \mathbf{A}_{\mu} \mathbf{j}_{\mu} = \int \mathbf{d}\mathbf{t}\mathbf{d} au (\mathbf{ec{A}}\cdot\mathbf{ec{j}} - \phi
ho)$$









$$\rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}') = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \qquad \qquad \vec{\mathbf{j}}(\vec{\mathbf{r}}, \vec{\mathbf{r}}') = \rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}')\vec{\mathbf{v}}$$

 $\mathbf{edx}_{\mu} = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}')\mathbf{d}\tau\mathbf{dx}_{\mu} = \rho\mathbf{d}\tau\mathbf{dx}_{\mu} = \rho\frac{\mathbf{dx}_{\mu}}{\mathbf{dt}}\mathbf{dt}\mathbf{d}\tau = \mathbf{j}_{\mu}\mathbf{dt}\mathbf{d}\tau$ 

$$\mathbf{j}_{\mathrm{u}} \equiv 
ho rac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}\mathbf{t}} = (\mathbf{ar{j}},\mathbf{i}\mathbf{c}
ho) \Rightarrow$$
 对比  $\mathbf{x}_{\mu} = (\mathbf{ar{r}},\mathbf{i}\mathbf{c}\mathbf{t})$   $\mathbf{j}_{\mathrm{x}}' = rac{\mathbf{j}_{\mathrm{x}} - \mathbf{v}
ho}{\sqrt{1 - rac{\mathbf{v}^2}{2}}}$   $\mathbf{j}_{\mathrm{y}}' = \mathbf{j}_{\mathrm{y}}$   $\mathbf{j}_{\mathrm{z}}' = \mathbf{j}_{\mathrm{z}}$   $ho' = rac{
ho - rac{\mathbf{v}}{\mathbf{c}^2}\mathbf{j}_{\mathrm{x}}}{\sqrt{1 - rac{\mathbf{v}^2}{2}}}$ 

$$\sum_{\mu=1}^4 \int_{\mathbf{a}}^{\mathbf{b}} \mathbf{d}\mathbf{x}_{\mu} \; \mathbf{e}\mathbf{A}_{\mu} = \sum_{\mu=1}^4 \int \mathbf{d}\mathbf{t}\mathbf{d} au \mathbf{A}_{\mu} \mathbf{j}_{\mu} = \int \mathbf{d}\mathbf{t}\mathbf{d} au (ec{\mathbf{A}} \cdot ec{\mathbf{j}} - \phi
ho)$$

$$egin{aligned} \mu=1 & J ext{ a} & \mu=1 & J & J \ 0 & = \sum^4 \int ext{d} t ext{d} au extbf{j}_\mu \partial_\mu \chi & J \end{aligned}$$













$$\rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}') = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \qquad \qquad \vec{\mathbf{j}}(\vec{\mathbf{r}}, \vec{\mathbf{r}}') = \rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}')\vec{\mathbf{v}}$$

$$\mathbf{edx}_{\mu} = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}')\mathbf{d} au\mathbf{dx}_{\mu} = 
ho\mathbf{d} au\mathbf{dx}_{\mu} = 
ho\frac{\mathbf{dx}_{\mu}}{\mathbf{dt}}\mathbf{dt}\mathbf{d} au = \mathbf{j}_{\mu}\mathbf{dt}\mathbf{d} au$$

$$\mathbf{j}_{\mu} \equiv 
ho rac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}\mathbf{t}} = (\mathbf{ar{j}}, \mathbf{i}\mathbf{c}
ho)$$
 ⇒ 対比  $\mathbf{x}_{\mu} = (\mathbf{ar{r}}, \mathbf{i}\mathbf{c}\mathbf{t})$ 
 $\mathbf{j}_{\mathbf{x}}' = rac{\mathbf{j}_{\mathbf{x}} - \mathbf{v}
ho}{\sqrt{1 - rac{\mathbf{v}^2}{\mathbf{c}^2}}}$   $\mathbf{j}_{\mathbf{y}}' = \mathbf{j}_{\mathbf{y}}$   $\mathbf{j}_{\mathbf{z}}' = \mathbf{j}_{\mathbf{z}}$   $ho' = rac{
ho - rac{\mathbf{v}}{\mathbf{c}^2}\mathbf{j}_{\mathbf{x}}}{\sqrt{1 - rac{\mathbf{v}^2}{\mathbf{c}^2}}}$ 

$$egin{aligned} \sqrt{1-rac{ ext{v}^2}{ ext{c}^2}} & \sqrt{1-rac{ ext{v}^2}{ ext{c}^2}} \ \sum_{\mu=1}^4 \int_{\mathbf{a}}^{\mathbf{b}} \mathbf{d}\mathbf{x}_{\mu} \ \mathbf{e}\mathbf{A}_{\mu} = \sum_{\mu=1}^4 \int \mathbf{d}\mathbf{t}\mathbf{d} au \mathbf{A}_{\mu} \mathbf{j}_{\mu} = \int \mathbf{d}\mathbf{t}\mathbf{d} au (ec{\mathbf{A}}\cdotec{\mathbf{j}}-\phi
ho) \end{aligned}$$

$$\mathbf{0} = \sum_{\mu=1}^4 \int \mathbf{d} \mathbf{t} \mathbf{d} au \mathbf{j}_\mu \partial_\mu \chi = -\sum_{\mu=1}^4 \int \mathbf{d} \mathbf{t} \mathbf{d} au (\partial_\mu \mathbf{j}_\mu) \chi$$



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$$\rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}') = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \qquad \qquad \vec{\mathbf{j}}(\vec{\mathbf{r}}, \vec{\mathbf{r}}') = \rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}')\vec{\mathbf{v}}$$

$$\mathbf{edx}_{\mu} = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}')\mathbf{d} au\mathbf{dx}_{\mu} = 
ho\mathbf{d} au\mathbf{dx}_{\mu} = 
ho\frac{\mathbf{dx}_{\mu}}{\mathbf{dt}}\mathbf{dt}\mathbf{d} au = \mathbf{j}_{\mu}\mathbf{dt}\mathbf{d} au$$

$$\mathbf{j}_{\mu} \equiv 
ho\frac{\mathbf{dx}_{\mu}}{\mathbf{dt}} = (\vec{\mathbf{j}},\mathbf{ic}
ho) \quad \Rightarrow \quad \text{対比} \quad \mathbf{x}_{\mu} = (\vec{\mathbf{r}},\mathbf{ict})$$

$$\mathbf{j}_{\mathrm{x}}' = rac{\mathbf{j}_{\mathrm{x}} - \mathrm{v}
ho}{\sqrt{1 - rac{\mathrm{v}^2}{\mathrm{c}^2}}} \qquad \mathbf{j}_{\mathrm{y}}' = \mathbf{j}_{\mathrm{y}} \qquad \mathbf{j}_{\mathrm{z}}' = \mathbf{j}_{\mathrm{z}} \qquad 
ho' = rac{
ho - rac{\mathrm{v}}{\mathrm{c}^2}\mathbf{j}_{\mathrm{x}}}{\sqrt{1 - rac{\mathrm{v}^2}{\mathrm{c}^2}}}$$

$$egin{aligned} \sum_{\mu=1}^4 \int_{\mathbf{a}}^{\mathbf{b}} \mathbf{d}\mathbf{x}_{\mu} \ \mathbf{e}\mathbf{A}_{\mu} &= \sum_{\mu=1}^4 \int \mathbf{d}\mathbf{t}\mathbf{d} au \mathbf{A}_{\mu} \mathbf{j}_{\mu} = \int \mathbf{d}\mathbf{t}\mathbf{d} au (ec{\mathbf{A}}\cdotec{\mathbf{j}} - \phi
ho) \ \mathbf{0} &= \sum_{\mu=1}^4 \int \mathbf{d}\mathbf{t}\mathbf{d} au \mathbf{j}_{\mu} \partial_{\mu}\chi = -\sum_{\mu=1}^4 \int \mathbf{d}\mathbf{t}\mathbf{d} au (\partial_{\mu}\mathbf{j}_{\mu})\chi \end{aligned}$$

$$egin{aligned} \mathbf{0} &= \sum_{\mu=1} \int \mathbf{d} \mathbf{t} \mathbf{d} au \mathbf{j}_{\mu} \partial_{\mu} \chi = -\sum_{\mu=1} \int \mathbf{d} \mathbf{t} \mathbf{d} au (\partial_{\mu} \mathbf{j}_{\mu}) \chi \ &\sum_{\mu=1}^{4} \partial_{\mu} \mathbf{j}_{\mu} = \mathbf{0} \end{aligned}$$



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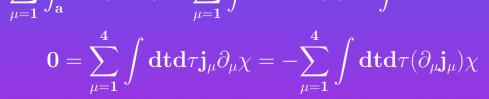
$$p(\mathbf{i}) = \rho(\mathbf{i})$$

 $\rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}') = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \qquad \qquad \vec{\mathbf{j}}(\vec{\mathbf{r}}, \vec{\mathbf{r}}') = \rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}')\vec{\mathbf{v}}$ 

$$\mathbf{e}\mathbf{d}\mathbf{x}_{\mu} = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}')\mathbf{d} au\mathbf{d}\mathbf{x}_{\mu} = 
ho\mathbf{d} au\mathbf{d}\mathbf{x}_{\mu} = 
ho\frac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}t}\mathbf{d}t\mathbf{d} au = \mathbf{j}_{\mu}\mathbf{d}t\mathbf{d} au$$

$$\mathbf{j}_{\mu} \equiv 
ho rac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}\mathbf{t}} = (\mathbf{ar{j}}, \mathbf{i}\mathbf{c}
ho) \Rightarrow$$
 対比  $\mathbf{x}_{\mu} = (\mathbf{ar{r}}, \mathbf{i}\mathbf{c}\mathbf{t})$   $\mathbf{v}
ho$   $\mathbf{z}'_{\mu} = \mathbf{z}'_{\mu} - \mathbf{z}'_{\mu}$   $\mathbf{z}'_{\mu} - \mathbf{z}'_{\mu} - \mathbf{z}'_{\mu}$ 

$$\mathbf{j}_{\mathrm{x}}' = rac{\mathbf{j}_{\mathrm{x}} - \mathbf{v}
ho}{\sqrt{1 - rac{\mathrm{v}^2}{\mathrm{c}^2}}} \quad \mathbf{j}_{\mathrm{y}}' = \mathbf{j}_{\mathrm{y}} \quad \mathbf{j}_{\mathrm{z}}' = \mathbf{j}_{\mathrm{z}} \qquad 
ho' = rac{
ho - rac{\mathrm{v}}{\mathrm{c}^2}\mathbf{j}_{\mathrm{x}}}{\sqrt{1 - rac{\mathrm{v}^2}{\mathrm{c}^2}}} \ \sum_{\mu=1}^4 \int_{\mathbf{a}}^{\mathbf{b}} \mathbf{d}\mathbf{x}_{\mu} \; \mathbf{e}\mathbf{A}_{\mu} = \sum_{\mu=1}^4 \int \mathbf{d}\mathbf{t}\mathbf{d} au \mathbf{A}_{\mu}\mathbf{j}_{\mu} = \int \mathbf{d}\mathbf{t}\mathbf{d} au (ec{\mathbf{A}} \cdot ec{\mathbf{j}} - \phi
ho)$$



$$egin{aligned} \sum_{\mu=1}^4 \partial_{\mu} \mathbf{j}_{\mu} &= \mathbf{0} \;\; \Rightarrow \;\; rac{\partial 
ho}{\partial \mathbf{t}} + 
abla \cdot ec{\mathbf{j}} &= \mathbf{0} \end{aligned}$$



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电荷分布在外电磁场中 点电荷及其流动是连续电荷分布及其流动的特例

$$ec{\mathbf{j}}(ec{\mathbf{r}},ec{\mathbf{r}}')=
ho(ec{\mathbf{i}})$$

$$\rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}') = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \qquad \qquad \vec{\mathbf{j}}(\vec{\mathbf{r}}, \vec{\mathbf{r}}') = \rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}')\vec{\mathbf{v}}$$

$$\mathbf{e}\mathbf{d}\mathbf{x} = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}')\mathbf{d}\mathbf{r}\mathbf{d}\mathbf{y} = \mathbf{e}\mathbf{d}\mathbf{r}\mathbf{d}\mathbf{y} = \mathbf{e}\mathbf{d}\mathbf{x}\mathbf{d}\mathbf{y} = \mathbf{e}\mathbf{d}\mathbf{x}\mathbf{d}\mathbf{y}$$

$$\mathbf{edx}_{\mu} = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}')\mathbf{d} au\mathbf{dx}_{\mu} = 
ho\mathbf{d} au\mathbf{dx}_{\mu} = 
horac{\mathbf{dx}_{\mu}}{\mathbf{dt}}\mathbf{dt}\mathbf{d} au = \mathbf{j}_{\mu}\mathbf{dt}\mathbf{d} au$$

$$\mathbf{j}_{\mu} \equiv 
horac{\mathbf{dx}_{\mu}}{\mathbf{dt}} = (\vec{\mathbf{j}},\mathbf{ic}
ho) \quad \Rightarrow \quad \text{对比} \quad \mathbf{x}_{\mu} = (\vec{\mathbf{r}},\mathbf{ict})$$

$$\mathbf{j}_{\mathrm{x}}' = rac{\mathbf{j}_{\mathrm{x}} - \mathbf{v}
ho}{\sqrt{1 - rac{\mathrm{v}^2}{\mathrm{c}^2}}} \quad \mathbf{j}_{\mathrm{y}}' = \mathbf{j}_{\mathrm{y}} \quad \mathbf{j}_{\mathrm{z}}' = \mathbf{j}_{\mathrm{z}} \qquad 
ho' = rac{
ho - rac{\mathrm{v}}{\mathrm{c}^2}\mathbf{j}_{\mathrm{x}}}{\sqrt{1 - rac{\mathrm{v}^2}{\mathrm{c}^2}}}$$

$$egin{aligned} \sum_{\mu=1}^4 \int_{\mathbf{a}}^{\mathbf{b}} \mathrm{d}\mathbf{x}_{\mu} \ \mathbf{e}\mathbf{A}_{\mu} &= \sum_{\mu=1}^4 \int \mathrm{d}\mathbf{t} \mathrm{d} au \mathbf{A}_{\mu} \mathbf{j}_{\mu} = \int \mathrm{d}\mathbf{t} \mathrm{d} au (ec{\mathbf{A}} \cdot ec{\mathbf{j}} - \phi 
ho) \ \mathbf{0} &= \sum_{\mu=1}^4 \int \mathrm{d}\mathbf{t} \mathrm{d} au \mathbf{i}_{\mu} \partial_{\mu} v = -\sum_{\mu=1}^4 \int \mathrm{d}\mathbf{t} \mathrm{d} au (\partial_{\mu} \mathbf{i}_{\mu}) v \end{aligned}$$

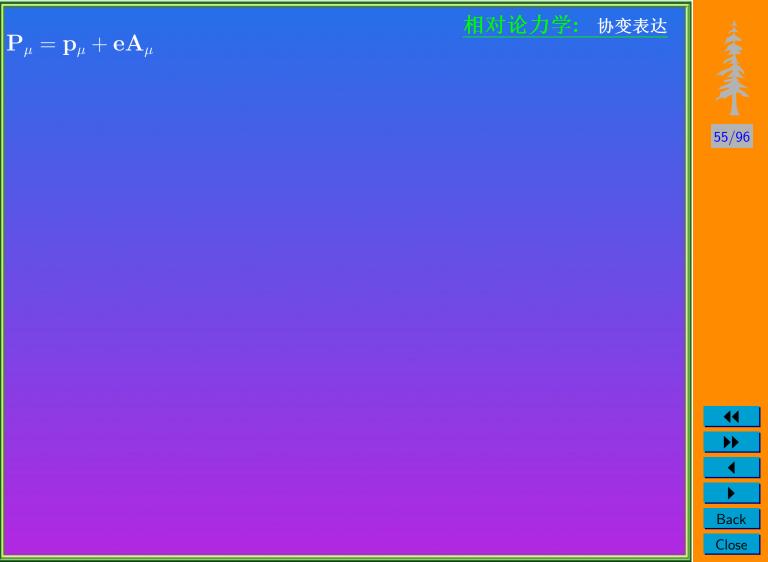
$$\mathbf{0} = \sum_{\mu=1}^4 \int \mathbf{d}t \mathbf{d} au \mathbf{j}_\mu \partial_\mu \chi = -\sum_{\mu=1}^4 \int \mathbf{d}t \mathbf{d} au (\partial_\mu \mathbf{j}_\mu) \chi$$

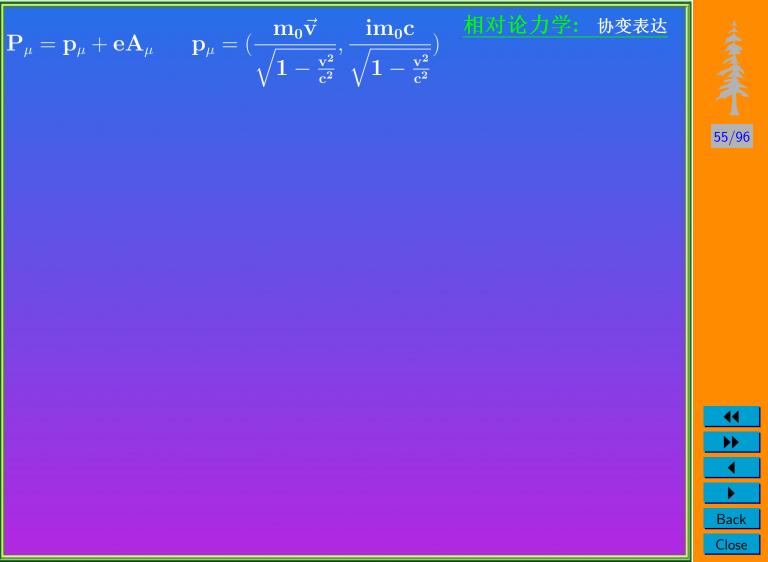
$$egin{align*} \sum_{\mu=1}^{4} J & \mu=1 \ J & \mu=1$$

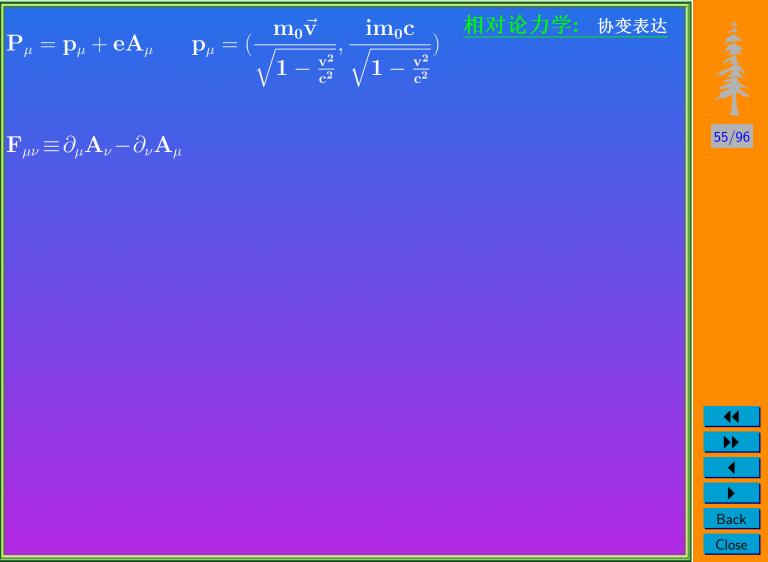
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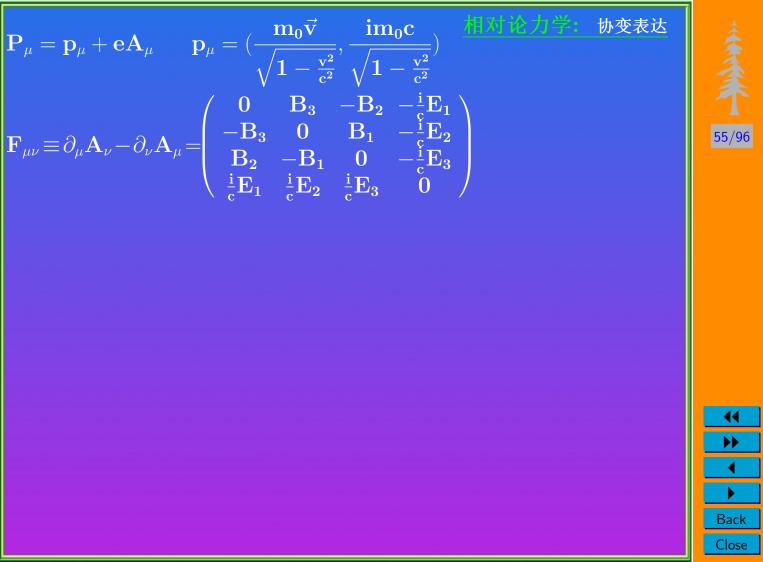


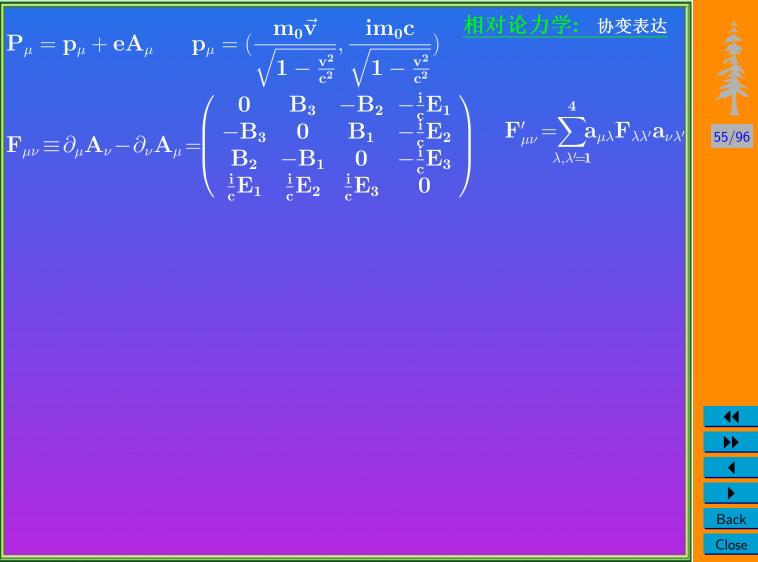
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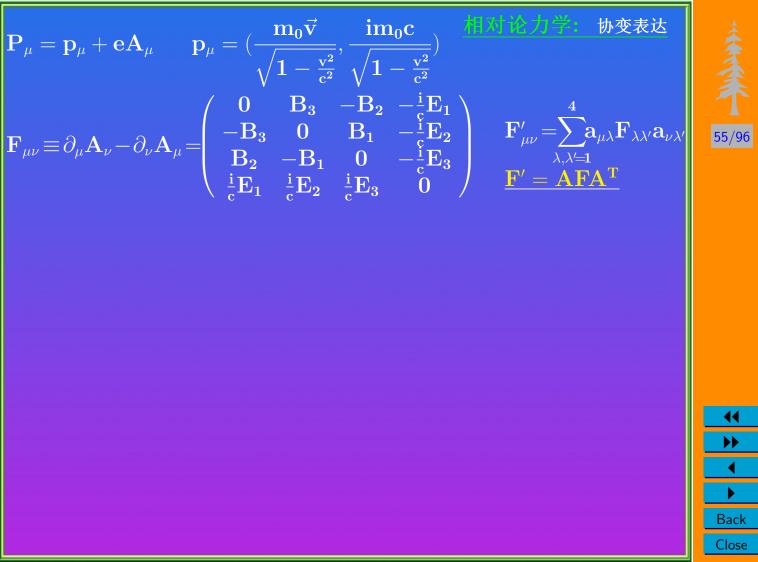


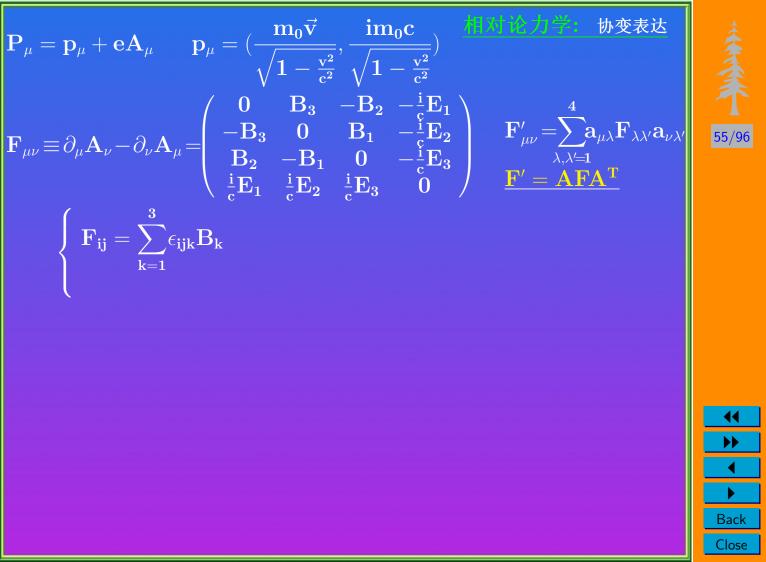


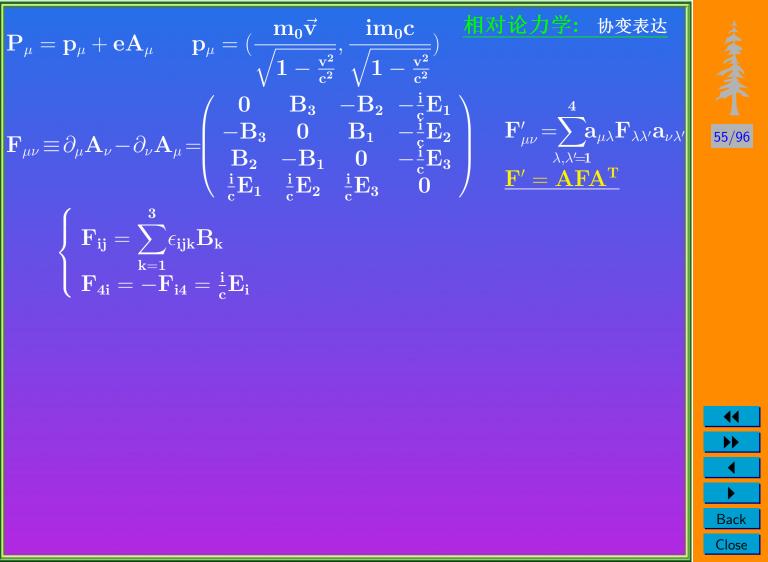


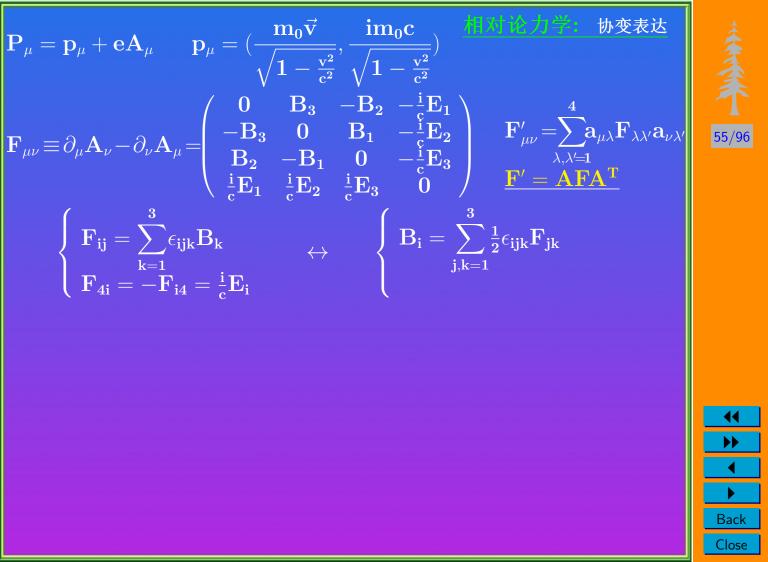


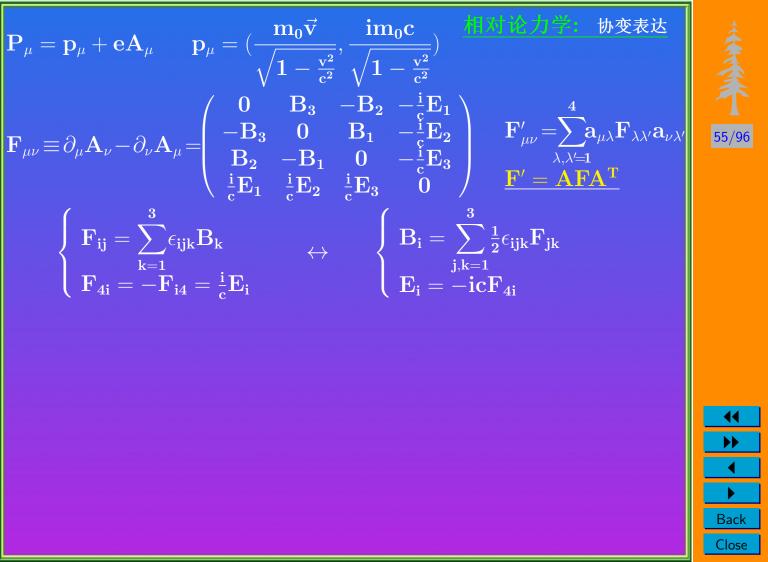


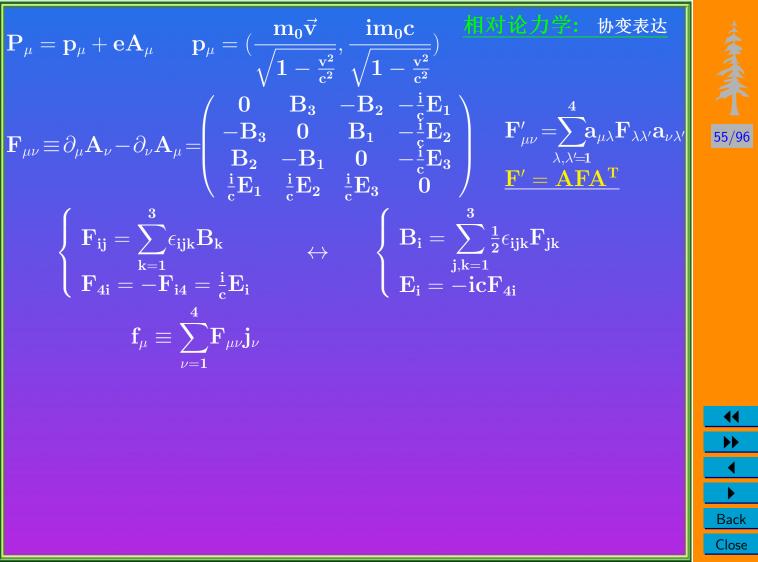












$$\begin{array}{c} P_{\mu} = p_{\mu} + eA_{\mu} & p_{\mu} = (\frac{m_{0}\vec{v}}{\sqrt{1-\frac{v^{2}}{c^{2}}}}, \frac{im_{0}c}{\sqrt{1-\frac{v^{2}}{c^{2}}}}) & \text{im}_{0}c \\ F_{\mu\nu} \equiv \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} = \begin{pmatrix} 0 & B_{3} & -B_{2} & -\frac{i}{c}E_{1} \\ -B_{3} & 0 & B_{1} & -\frac{i}{c}E_{2} \\ B_{2} & -B_{1} & 0 & -\frac{i}{c}E_{3} \end{pmatrix} & F'_{\mu\nu} = \sum_{\lambda,\lambda = 1}^{4} a_{\mu\lambda}F_{\lambda\lambda'}a_{\nu\lambda'} \\ F_{ij} = \sum_{k=1}^{3} \epsilon_{ijk}B_{k} & \longleftrightarrow & \begin{cases} B_{i} = \sum_{j,k=1}^{3} \frac{1}{2}\epsilon_{ijk}F_{jk} \\ E_{i} = -icF_{4i} \end{cases} \\ F_{4i} = -F_{i4} = \frac{i}{c}E_{i} & E_{i} = -icF_{4i} \end{cases} \\ f_{\mu} \equiv \sum_{\nu=1}^{4} F_{\mu\nu}j_{\nu} = (\rho\vec{E} + \vec{j} \times \vec{B}, \ \frac{i}{c}\vec{j} \cdot \vec{E}) \\ f_{i} = F_{i4}j_{4} + \sum_{k=1}^{3} F_{ik}j_{k} \end{cases} & \text{if} i = F_{i4}j_{4} + \sum_{k=1}^{3} F_{ik}j_{k} \end{cases}$$

$$\begin{split} \mathbf{P}_{\mu} &= \mathbf{p}_{\mu} + e\mathbf{A}_{\mu} &\quad \mathbf{p}_{\mu} = (\frac{\mathbf{m}_{0}\vec{\mathbf{v}}}{\sqrt{1-\frac{\mathbf{v}^{2}}{c^{2}}}}, \frac{i\mathbf{m}_{0}\mathbf{c}}{\sqrt{1-\frac{\mathbf{v}^{2}}{c^{2}}}}) &\quad \text{协変表达} \\ \mathbf{F}_{\mu\nu} &\equiv \partial_{\mu}\mathbf{A}_{\nu} - \partial_{\nu}\mathbf{A}_{\mu} = \begin{pmatrix} \mathbf{0} & \mathbf{B}_{3} & -\mathbf{B}_{2} & -\frac{\mathbf{i}}{c}\mathbf{E}_{1} \\ -\mathbf{B}_{3} & \mathbf{0} & \mathbf{B}_{1} & -\frac{\mathbf{i}}{c}\mathbf{E}_{2} \\ \mathbf{B}_{2} & -\mathbf{B}_{1} & \mathbf{0} & -\frac{\mathbf{i}}{c}\mathbf{E}_{3} \\ \frac{\mathbf{i}}{c}\mathbf{E}_{1} & \frac{\mathbf{i}}{c}\mathbf{E}_{2} & \frac{\mathbf{i}}{c}\mathbf{E}_{3} & \mathbf{0} \end{pmatrix} &\quad \mathbf{F}'_{\mu\nu} = \sum_{\lambda,\lambda'=1}^{4} \mathbf{a}_{\mu\lambda}\mathbf{F}_{\lambda\lambda'}\mathbf{a}_{\nu\lambda'} \\ \mathbf{F}_{\mu\nu} &= \sum_{i=1}^{3} \epsilon_{ijk}\mathbf{B}_{k} & \longleftrightarrow & \begin{cases} \mathbf{B}_{i} &= \sum_{j,k=1}^{3} \frac{1}{2} \epsilon_{ijk}\mathbf{F}_{jk} \\ \mathbf{E}_{i} &= -\mathbf{i}\mathbf{c}\mathbf{F}_{4i} \end{cases} \\ \mathbf{F}_{4i} &= -\mathbf{F}_{i4} &= \frac{\mathbf{i}}{c}\mathbf{E}_{i} & \mathbf{E}_{i} &= -\mathbf{i}\mathbf{c}\mathbf{F}_{4i} \end{cases} \\ \mathbf{f}_{\mu} &\equiv \sum_{\nu=1}^{4} \mathbf{F}_{\mu\nu}\mathbf{j}_{\nu} = (\rho\mathbf{E} + \mathbf{j} \times \mathbf{B}, \frac{\mathbf{i}}{c}\mathbf{j} \cdot \mathbf{E}) \end{cases} \\ \mathbf{f}_{i} &= \mathbf{F}_{i4}\mathbf{j}_{4} + \sum_{k=1}^{3} \mathbf{F}_{ik}\mathbf{j}_{k} = -\frac{\mathbf{i}}{c}\mathbf{E}_{i}\mathbf{i}\mathbf{c}\rho + \sum_{k,l=1}^{3} \epsilon_{ikl}\mathbf{j}_{k}\mathbf{B}_{l} \end{cases} \end{aligned}$$

$$\begin{split} \mathbf{P}_{\mu} &= \mathbf{p}_{\mu} + e\mathbf{A}_{\mu} &\quad \mathbf{p}_{\mu} = (\frac{\mathbf{m}_{0}\vec{\mathbf{v}}}{\sqrt{1 - \frac{\mathbf{v}^{2}}{c^{2}}}}, \frac{i\mathbf{m}_{0}\mathbf{c}}{\sqrt{1 - \frac{\mathbf{v}^{2}}{c^{2}}}}) &\quad \mathbf{b}$$
 协变表达 
$$\mathbf{F}_{\mu\nu} &\equiv \partial_{\mu}\mathbf{A}_{\nu} - \partial_{\nu}\mathbf{A}_{\mu} = \begin{pmatrix} \mathbf{0} & \mathbf{B}_{3} & -\mathbf{B}_{2} & -\frac{\mathbf{i}}{c}\mathbf{E}_{1} \\ -\mathbf{B}_{3} & \mathbf{0} & \mathbf{B}_{1} & -\frac{\mathbf{i}}{c}\mathbf{E}_{2} \\ \mathbf{B}_{2} & -\mathbf{B}_{1} & \mathbf{0} & -\frac{\mathbf{i}}{c}\mathbf{E}_{3} \\ \frac{\mathbf{i}}{c}\mathbf{E}_{1} & \frac{\mathbf{i}}{c}\mathbf{E}_{2} & \frac{\mathbf{i}}{c}\mathbf{E}_{3} & \mathbf{0} \end{pmatrix} &\quad \mathbf{F}'_{\mu\nu} = \sum_{\lambda,\lambda'=1}^{4}\mathbf{a}_{\mu\lambda}\mathbf{F}_{\lambda\lambda'}\mathbf{a}_{\nu\lambda'} \\ \mathbf{F}'_{\mu\nu} &= \sum_{\lambda,\lambda'=1}^{4}\mathbf{E}_{1} &\quad \mathbf{F}'_{\mu\nu} = \sum_{\lambda,\lambda'=1}^{4}\mathbf{E}_{1} \\ \mathbf{F}'_{ij} &= \sum_{k=1}^{3}\epsilon_{ijk}\mathbf{B}_{k} &\quad \Leftrightarrow &\quad \left\{ \mathbf{B}_{i} &= \sum_{j,k=1}^{3}\frac{1}{2}\epsilon_{ijk}\mathbf{F}_{jk} \\ \mathbf{E}_{i} &= -\mathbf{i}\mathbf{c}\mathbf{F}_{4i} \\ \mathbf{F}_{4i} &= -\mathbf{F}_{i4} &= \frac{\mathbf{i}}{c}\mathbf{E}_{i} \\ \mathbf{F}_{\mu\nu} &= \sum_{\nu=1}^{4}\mathbf{F}_{\mu\nu}\mathbf{j}_{\nu} &= (\rho\vec{\mathbf{E}} + \vec{\mathbf{j}} \times \vec{\mathbf{B}}, \frac{\mathbf{i}}{c}\vec{\mathbf{j}} \cdot \vec{\mathbf{E}}) \\ \mathbf{f}_{i} &= \mathbf{F}_{i4}\mathbf{j}_{4} + \sum_{k=1}^{3}\mathbf{F}_{ik}\mathbf{j}_{k} &= -\frac{\mathbf{i}}{c}\mathbf{E}_{i}\mathbf{i}\mathbf{c}\rho + \sum_{k,l=1}^{3}\epsilon_{ikl}\mathbf{j}_{k}\mathbf{B}_{l} &= (\rho\vec{\mathbf{E}} + \vec{\mathbf{j}} \times \vec{\mathbf{B}})_{i} \\ \mathbf{H}_{\mu\nu} &= \mathbf{E}_{\mu\nu} \mathbf{E}_{$$

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$$\begin{cases} \mathbf{F_{ij}} = \sum_{k=1}^{3} \epsilon_{ijk} \mathbf{B_k} \\ \mathbf{F_{4i}} = -\mathbf{F_{i4}} = \frac{\mathbf{i}}{\mathbf{c}} \mathbf{E_i} \end{cases} \Leftrightarrow \begin{cases} \mathbf{B_i} = \sum_{\mathbf{j}, k=1}^{3} \frac{1}{2} \epsilon_{ijk} \mathbf{F_{jk}} \\ \mathbf{E_i} = -\mathbf{i} \mathbf{c} \mathbf{F_{4i}} \end{cases}$$
$$\mathbf{f_{\mu}} \equiv \sum_{\nu=1}^{4} \mathbf{F_{\mu\nu}} \mathbf{j_{\nu}} = (\rho \vec{\mathbf{E}} + \vec{\mathbf{j}} \times \vec{\mathbf{B}}, \ \frac{\mathbf{i}}{\mathbf{c}} \vec{\mathbf{j}} \cdot \vec{\mathbf{E}})$$
$$\mathbf{f_i} = \mathbf{F_{i4}} \mathbf{j_4} + \sum_{k=1}^{3} \mathbf{F_{ik}} \mathbf{j_k} = -\frac{\mathbf{i}}{\mathbf{c}} \mathbf{E_i} \mathbf{i} \mathbf{c} \rho + \sum_{k,l=1}^{3} \epsilon_{ikl} \mathbf{j_k} \mathbf{B_l} = (\rho \vec{\mathbf{E}} + \vec{\mathbf{j}} \times \vec{\mathbf{B}})_i$$

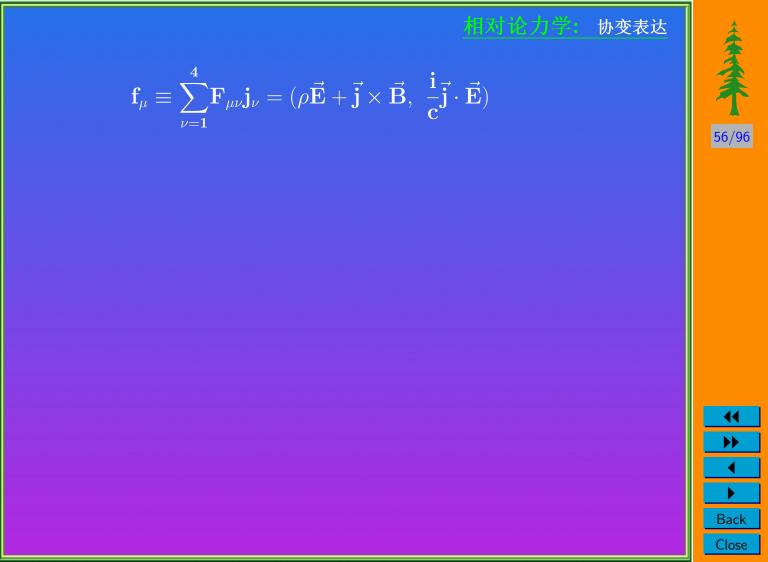
 $\mathbf{f_4} = \sum \mathbf{F_{4k}} \mathbf{j_k}$ 

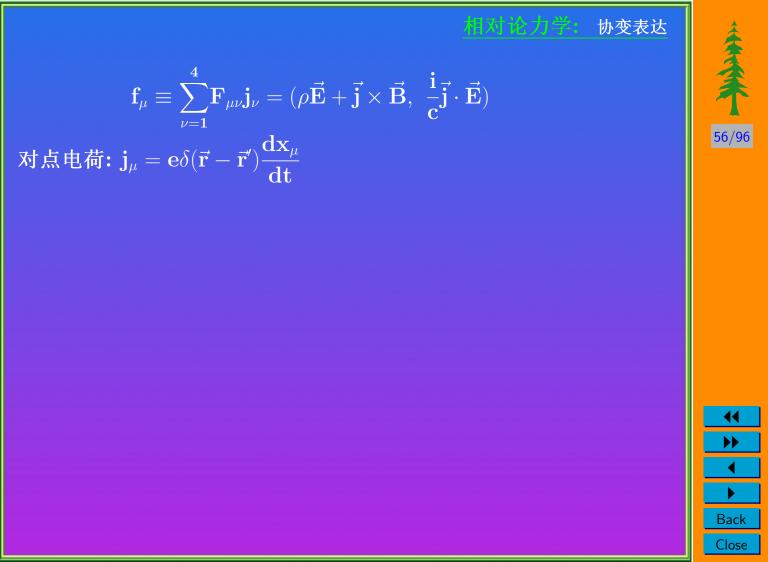
$$\begin{split} \mathbf{P}_{\mu} &= \mathbf{p}_{\mu} + e\mathbf{A}_{\mu} &\quad \mathbf{p}_{\mu} = (\frac{\mathbf{m}_{0}\vec{\mathbf{v}}}{\sqrt{1 - \frac{\mathbf{v}^{2}}{c^{2}}}}, \frac{i\mathbf{m}_{0}\mathbf{c}}{\sqrt{1 - \frac{\mathbf{v}^{2}}{c^{2}}}}) \\ \mathbf{F}_{\mu\nu} &\equiv \partial_{\mu}\mathbf{A}_{\nu} - \partial_{\nu}\mathbf{A}_{\mu} = \begin{pmatrix} \mathbf{0} & \mathbf{B}_{3} & -\mathbf{B}_{2} & -\frac{\mathbf{i}}{c}\mathbf{E}_{1} \\ -\mathbf{B}_{3} & \mathbf{0} & \mathbf{B}_{1} & -\frac{\mathbf{i}}{c}\mathbf{E}_{2} \\ \mathbf{B}_{2} & -\mathbf{B}_{1} & \mathbf{0} & -\frac{\mathbf{i}}{c}\mathbf{E}_{3} \\ \frac{\mathbf{i}}{c}\mathbf{E}_{1} & \frac{\mathbf{i}}{c}\mathbf{E}_{2} & \frac{\mathbf{i}}{c}\mathbf{E}_{3} & \mathbf{0} \end{pmatrix} \quad \mathbf{F}'_{\mu\nu} = \sum_{\lambda,\lambda'=1}^{4} \mathbf{a}_{\mu\lambda}\mathbf{F}_{\lambda\lambda'}\mathbf{a}_{\nu\lambda'} \\ \mathbf{F}'_{\mu\nu} &= \sum_{\lambda,\lambda'=1}^{4} \mathbf{F}_{\lambda\lambda'}\mathbf{a}_{\nu\lambda'} \\ \mathbf{F}'_{\mu\nu} &= \sum_{\lambda,\lambda'=1}^{4} \mathbf{F}_{\mu\nu}\mathbf{j}_{\nu} \\ \mathbf{F}'_{\mu\nu} &= \sum_{\lambda,\lambda'=1}^{4} \mathbf{F}_{\lambda\lambda'}\mathbf{a}_{\nu\lambda'} \\ \mathbf{F}'_{\mu\nu} &= \sum_{\lambda,\lambda'=1}^{4} \mathbf{F}_{\mu\nu}\mathbf{j}_{\nu} \\ \mathbf{F}'_{\mu\nu} &= \sum_{\lambda,\lambda'=1}^{4} \mathbf{F}_{\lambda\lambda'}\mathbf{F}_{\lambda\lambda'}\mathbf{a}_{\nu\lambda'} \\ \mathbf{F}'_{\mu\nu} &= \sum_{\lambda,\lambda'=1}^{4} \mathbf{F}_{\lambda\lambda'}\mathbf{F}_{\lambda\lambda'}\mathbf{a}_{\nu\lambda'} \\ \mathbf{F}'_{\mu\nu} &= \sum_{\lambda,\lambda'=1}^{4} \mathbf{F}_{\lambda\lambda'}\mathbf{F}_{\lambda\lambda'}\mathbf{a}_{\nu\lambda'} \\ \mathbf{F}'_{\mu\nu} &= \sum_{\lambda,\lambda'=1}^{4} \mathbf{F}_{\lambda\lambda'}\mathbf{F}_{\lambda\lambda'}\mathbf{A}_{\lambda\lambda'} \\ \mathbf{F}'_{\mu\nu} &= \sum_{\lambda,\lambda'=1}^{4} \mathbf{F}_{\lambda\lambda'}\mathbf{F}_{\lambda\lambda'}\mathbf{A}_{\lambda\lambda'} \\ \mathbf{F}'_{\mu\nu} &= \sum_{\lambda,\lambda'=1}^{4} \mathbf{F}_{\lambda\lambda'}\mathbf{F}_{\lambda\lambda'}\mathbf{F}_{\lambda\lambda'}\mathbf{A}_{\lambda\lambda'} \\ \mathbf{F}'_{\mu\nu} &= \sum_{\lambda,\lambda'=1}^{4} \mathbf{F}_{\lambda\lambda'}\mathbf{F}_{\lambda\lambda'}\mathbf{F}_{\lambda\lambda'}\mathbf{F}_{\lambda\lambda'}\mathbf{A}_{\lambda\lambda'} \\ \mathbf{F}'_{\mu\nu} &= \sum_{\lambda,\lambda'=1}^{4} \mathbf{F}_{\lambda\lambda'}\mathbf{F}_{\lambda$$

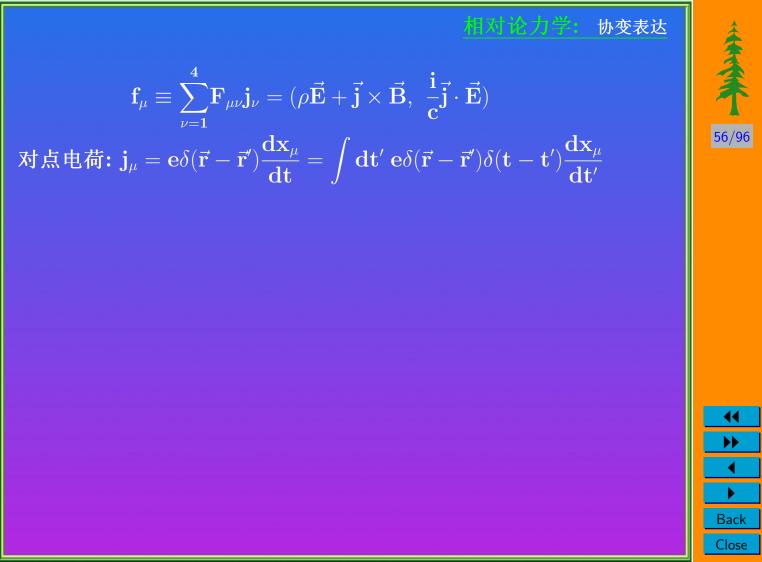
Close

$$\begin{split} \mathbf{P}_{\mu} &= \mathbf{p}_{\mu} + e\mathbf{A}_{\mu} &\quad \mathbf{p}_{\mu} = (\frac{\mathbf{m}_{0}\vec{\mathbf{v}}}{\sqrt{1 - \frac{\mathbf{v}^{2}}{c^{2}}}}, \frac{i\mathbf{m}_{0}\mathbf{c}}{\sqrt{1 - \frac{\mathbf{v}^{2}}{c^{2}}}}) \\ \mathbf{F}_{\mu\nu} &\equiv \partial_{\mu}\mathbf{A}_{\nu} - \partial_{\nu}\mathbf{A}_{\mu} = \begin{pmatrix} \mathbf{0} & \mathbf{B}_{3} & -\mathbf{B}_{2} & -\frac{\mathbf{i}}{c}\mathbf{E}_{1} \\ -\mathbf{B}_{3} & \mathbf{0} & \mathbf{B}_{1} & -\frac{\mathbf{i}}{c}\mathbf{E}_{2} \\ \mathbf{B}_{2} & -\mathbf{B}_{1} & \mathbf{0} & -\frac{\mathbf{i}}{c}\mathbf{E}_{3} \\ \frac{\mathbf{i}}{c}\mathbf{E}_{1} & \frac{\mathbf{i}}{c}\mathbf{E}_{2} & \frac{\mathbf{i}}{c}\mathbf{E}_{3} & \mathbf{0} \end{pmatrix} \quad \mathbf{F}'_{\mu\nu} = \sum_{\lambda,\lambda'=1}^{4} \mathbf{a}_{\mu\lambda}\mathbf{F}_{\lambda\lambda'}\mathbf{a}_{\nu\lambda'} \\ \mathbf{F}'_{\mu\nu} &= \sum_{\lambda,\lambda'=1}^{4} \mathbf{F}_{\lambda\lambda'}\mathbf{a}_{\nu\lambda'} \\ \mathbf{F}'_{\mu\nu} &= \sum_{\lambda,\lambda'=1}^{4} \mathbf{F}_{\mu\nu}\mathbf{j}_{\nu} \\ \mathbf{F}'_{\mu\nu} &= \sum_{\lambda,\lambda'=1}^{4} \mathbf{F}_{\lambda\lambda'}\mathbf{a}_{\nu\lambda'} \\ \mathbf{F}'_{\mu\nu} &= \sum_{\lambda,\lambda'=1}^{4} \mathbf{F}_{\mu\nu}\mathbf{j}_{\nu} \\ \mathbf{F}'_{\mu\nu} &= \sum_{\lambda,\lambda'=1}^{4} \mathbf{F}_{\lambda\lambda'}\mathbf{F}_{\lambda\lambda'}\mathbf{a}_{\nu\lambda'} \\ \mathbf{F}'_{\mu\nu} &= \sum_{\lambda,\lambda'=1}^{4} \mathbf{F}_{\lambda\lambda'}\mathbf{F}_{\lambda\lambda'}\mathbf{a}_{\nu\lambda'} \\ \mathbf{F}'_{\mu\nu} &= \sum_{\lambda,\lambda'=1}^{4} \mathbf{F}_{\lambda\lambda'}\mathbf{F}_{\lambda\lambda'}\mathbf{a}_{\nu\lambda'} \\ \mathbf{F}'_{\mu\nu} &= \sum_{\lambda,\lambda'=1}^{4} \mathbf{F}_{\lambda\lambda'}\mathbf{F}_{\lambda\lambda'}\mathbf{A}_{\lambda\lambda'} \\ \mathbf{F}'_{\mu\nu} &= \sum_{\lambda,\lambda'=1}^{4} \mathbf{F}_{\lambda\lambda'}\mathbf{F}_{\lambda\lambda'}\mathbf{A}_{\lambda\lambda'} \\ \mathbf{F}'_{\mu\nu} &= \sum_{\lambda,\lambda'=1}^{4} \mathbf{F}_{\lambda\lambda'}\mathbf{F}_{\lambda\lambda'}\mathbf{F}_{\lambda\lambda'}\mathbf{A}_{\lambda\lambda'} \\ \mathbf{F}'_{\mu\nu} &= \sum_{\lambda,\lambda'=1}^{4} \mathbf{F}_{\lambda\lambda'}\mathbf{F}_{\lambda\lambda'}\mathbf{F}_{\lambda\lambda'}\mathbf{F}_{\lambda\lambda'}\mathbf{A}_{\lambda\lambda'} \\ \mathbf{F}'_{\mu\nu} &= \sum_{\lambda,\lambda'=1}^{4} \mathbf{F}_{\lambda\lambda'}\mathbf{F}_{\lambda$$

Close







# 相对论力学: 协变表达 $\mathbf{f}_{\mu} \equiv \sum \mathbf{F}_{\mu u} \mathbf{j}_{ u} = ( ho \vec{\mathbf{E}} + \vec{\mathbf{j}} imes \vec{\mathbf{B}}, \ \ rac{\mathbf{i}}{\mathbf{c}} \vec{\mathbf{j}} \cdot \vec{\mathbf{E}})$ 对点电荷: $\mathbf{j}_{\mu} = \mathbf{e}\delta(\mathbf{\vec{r}} - \mathbf{\vec{r}}') \frac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}\mathbf{t}} = \int \mathbf{d}\mathbf{t}' \; \mathbf{e}\delta(\mathbf{\vec{r}} - \mathbf{\vec{r}}') \delta(\mathbf{t} - \mathbf{t}') \frac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}\mathbf{t}'}$ $=\int \mathbf{d} au' \mathbf{e}\delta^{(4)}(\mathbf{x}-\mathbf{x}') rac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d} au'}$











#### 相对论力学:协变表达 56/96

## $\mathbf{f}_{\mu} \equiv \sum \mathbf{F}_{\mu u} \mathbf{j}_{ u} = ( ho \vec{\mathbf{E}} + \vec{\mathbf{j}} imes \vec{\mathbf{B}}, \ \ \dot{\mathbf{f}} \vec{\mathbf{j}} \cdot \vec{\mathbf{E}})$

対点电荷: 
$$\mathbf{j}_{\mu} = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \frac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}\mathbf{t}} = \int \mathbf{d}\mathbf{t}' \, \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \delta(\mathbf{t} - \mathbf{t}') \frac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}\mathbf{t}'}$$

$$= \int \mathbf{d}\tau' \mathbf{e}\delta^{(4)}(\mathbf{x} - \mathbf{x}') \frac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}\tau'} = \int \mathbf{d}\tau' \mathbf{e}\delta^{(4)}(\mathbf{x} - \mathbf{x}') \mathbf{u}_{\mu}$$











# 相对论力学: 协变表达

## $\mathbf{f}_{\mu} \equiv \sum \mathbf{F}_{\mu u} \mathbf{j}_{ u} = ( ho \vec{\mathbf{E}} + \vec{\mathbf{j}} imes \vec{\mathbf{B}}, \ \ \dot{\mathbf{f}} \vec{\mathbf{j}} \cdot \vec{\mathbf{E}})$

对点电荷: 
$$\mathbf{j}_{\mu} = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \frac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}t} = \int \mathbf{d}t' \, \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \delta(\mathbf{t} - \mathbf{t}') \frac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}t'}$$

$$= \int \mathbf{d}\tau' \mathbf{e}\delta^{(4)}(\mathbf{x} - \mathbf{x}') \frac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}\tau'} = \int \mathbf{d}\tau' \mathbf{e}\delta^{(4)}(\mathbf{x} - \mathbf{x}') \mathbf{u}_{\mu}$$

$$\frac{\mathbf{d}\mathbf{p}_{\mu}}{\mathbf{d}\tau} = \mathbf{K}_{\mu}$$











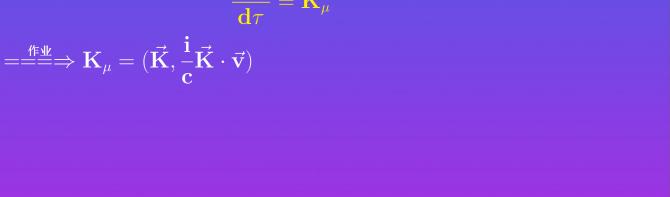
相对论力学: 协变表达

### $\mathbf{f}_{\mu} \equiv \sum \mathbf{F}_{\mu u} \mathbf{j}_{ u} = ( ho \vec{\mathbf{E}} + \vec{\mathbf{j}} imes \vec{\mathbf{B}}, \ \ \dot{\mathbf{f}} \vec{\mathbf{j}} \cdot \vec{\mathbf{E}})$

対点电荷: 
$$\mathbf{j}_{\mu} = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \frac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}\mathbf{t}} = \int \mathbf{d}\mathbf{t}' \, \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \delta(\mathbf{t} - \mathbf{t}') \frac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}\mathbf{t}'}$$

$$= \int \mathbf{d}\tau' \mathbf{e}\delta^{(4)}(\mathbf{x} - \mathbf{x}') \frac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}\tau'} = \int \mathbf{d}\tau' \mathbf{e}\delta^{(4)}(\mathbf{x} - \mathbf{x}') \mathbf{u}_{\mu}$$

$$\int \mathbf{d} \tau \, \mathbf{c} \sigma^{\prime} \, (\mathbf{A} - \mathbf{A}) \, \mathbf{d} au^{\prime} = \int \mathbf{d} \tau^{\prime} \, \mathbf{c} \sigma^{\prime} \, (\mathbf{A} - \mathbf{A}) \, \mathbf{c} \sigma^{\prime} \, \mathbf{d} au^{\prime} = \mathbf{K}_{\mu}$$













相对论力学: 协变表达

$$\mathbf{f}_{\mu} \equiv \sum_{
u=1}^{\mathbf{T}} \mathbf{F}_{\mu
u} \mathbf{j}_{
u} = (
ho \vec{\mathbf{E}} + \vec{\mathbf{j}} imes \vec{\mathbf{B}}, \ \ rac{\mathbf{i}}{\mathbf{c}} \vec{\mathbf{j}} \cdot \vec{\mathbf{E}})$$

対点电荷: 
$$\mathbf{j}_{\mu} = \mathbf{e}\delta(\mathbf{\vec{r}} - \mathbf{\vec{r}}') \frac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}\mathbf{t}} = \int \mathbf{d}\mathbf{t}' \, \mathbf{e}\delta(\mathbf{\vec{r}} - \mathbf{\vec{r}}') \delta(\mathbf{t} - \mathbf{t}') \frac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}\mathbf{t}'}$$

$$= \int \mathbf{d}\tau' \mathbf{e}\delta^{(4)}(\mathbf{x} - \mathbf{x}') \frac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}\tau'} = \int \mathbf{d}\tau' \mathbf{e}\delta^{(4)}(\mathbf{x} - \mathbf{x}') \mathbf{u}_{\mu}$$

$$egin{align} rac{\mathbf{d}\mathbf{p}_{\mu}}{\mathbf{d} au} &= \mathbf{K}_{\mu} \ = \stackrel{\text{fight}}{=} \Rightarrow \mathbf{K}_{\mu} &= (\vec{\mathbf{K}}, rac{\mathbf{i}}{\mathbf{c}} \vec{\mathbf{K}} \cdot \vec{\mathbf{v}}) &= \sum_{\mathbf{j}}^{4} \mathbf{e} \mathbf{F}_{\mu 
u} \mathbf{u}_{
u} \end{aligned}$$









相对论力学:协变表达

$$\mathbf{f}_{\mu} \equiv \sum_{
u=1}^{\mathbf{i}} \mathbf{F}_{\mu
u} \mathbf{j}_{
u} = (
ho \vec{\mathbf{E}} + \vec{\mathbf{j}} imes \vec{\mathbf{B}}, \; rac{\dot{\mathbf{i}}}{\mathbf{c}} \vec{\mathbf{j}} \cdot \vec{\mathbf{E}})$$

对点电荷: 
$$\mathbf{j}_{\mu} = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \frac{d\mathbf{x}_{\mu}}{dt} = \int dt' \, \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \delta(\mathbf{t} - \mathbf{t}') \frac{d\mathbf{x}_{\mu}}{dt'}$$

$$= \int d\tau' \mathbf{e}\delta^{(4)}(\mathbf{x} - \mathbf{x}') \frac{d\mathbf{x}_{\mu}}{d\tau'} = \int d\tau' \mathbf{e}\delta^{(4)}(\mathbf{x} - \mathbf{x}') \mathbf{u}_{\mu}$$

$$-\int \mathbf{d}\tau \, \mathbf{e}\sigma \, \nabla (\mathbf{x} - \mathbf{x}) \frac{1}{\mathbf{c}}$$
$$\frac{\mathbf{d}\mathbf{p}_{\mu}}{\mathbf{d}\tau} = \mathbf{K}_{\mu}$$

$$=\stackrel{f \in \Psi}{=} \Rightarrow \mathbf{K}_{\mu} = (\vec{\mathbf{K}}, \frac{\mathbf{i}}{\mathbf{c}} \vec{\mathbf{K}} \cdot \vec{\mathbf{v}}) = \sum_{
u=1}^{4} \mathbf{e} \mathbf{F}_{\mu
u} \mathbf{u}_{
u} \qquad \mathbf{f}_{\mu} = \int \mathbf{d} au \delta^{(4)}(\mathbf{x} - \mathbf{x}') \mathbf{K}_{\mu}$$









协变表达

$$\mathbf{f}_{\mu} \equiv \sum_{
u=1}^{\mathbf{T}} \mathbf{F}_{\mu
u} \mathbf{j}_{
u} = (
ho \vec{\mathbf{E}} + \vec{\mathbf{j}} imes \vec{\mathbf{B}}, \ \ rac{\dot{\mathbf{i}}}{\mathbf{c}} \vec{\mathbf{j}} \cdot \vec{\mathbf{E}})$$

对点电荷: 
$$\mathbf{j}_{\mu} = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \frac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}\mathbf{t}} = \int \mathbf{d}\mathbf{t}' \, \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \delta(\mathbf{t} - \mathbf{t}') \frac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}\mathbf{t}'}$$

$$= \int \mathbf{d}\tau' \mathbf{e}\delta^{(4)}(\mathbf{x} - \mathbf{x}') \frac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}\tau'} = \int \mathbf{d}\tau' \mathbf{e}\delta^{(4)}(\mathbf{x} - \mathbf{x}') \mathbf{u}_{\mu}$$

$$\int rac{\mathbf{d} \mathbf{p}_{\mu}}{\mathbf{d} au} = \mathbf{K}_{\mu}$$

$$egin{aligned} & \mathbf{d} au \ = & \stackrel{\text{\tiny (EML)}}{=} \Rightarrow \mathbf{K}_{\mu} = (\mathbf{\vec{K}}, rac{\mathbf{i}}{\mathbf{c}} \mathbf{\vec{K}} \cdot \mathbf{\vec{v}}) = \sum_{
u=1}^{4} \mathbf{e} \mathbf{F}_{\mu
u} \mathbf{u}_{
u} \qquad \mathbf{f}_{\mu} = \int \mathbf{d} au \delta^{(4)}(\mathbf{x} - \mathbf{x}') \mathbf{K}_{\mu} \ & = (rac{\mathbf{e}}{\sqrt{1 - rac{\mathbf{v}^{2}}{\mathbf{c}^{2}}}} (\mathbf{\vec{E}} + \mathbf{\vec{v}} imes \mathbf{\vec{B}}), rac{rac{\mathrm{i}\mathbf{e}}{\mathbf{c}} \mathbf{\vec{E}} \cdot \mathbf{\vec{v}}}{\sqrt{1 - rac{\mathbf{v}^{2}}{\mathbf{c}^{2}}}}) \end{aligned}$$









协变表达

$$\mathbf{f}_{\mu} \equiv \sum_{
u=1}^{\mathbf{j}} \mathbf{F}_{\mu
u} \mathbf{j}_{
u} = (
ho \vec{\mathbf{E}} + \vec{\mathbf{j}} imes \vec{\mathbf{B}}, \ \ rac{\mathbf{i}}{\mathbf{c}} \vec{\mathbf{j}} \cdot \vec{\mathbf{E}})$$

対点电荷: 
$$\mathbf{j}_{\mu} = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \frac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}\mathbf{t}} = \int \mathbf{d}\mathbf{t}' \, \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \delta(\mathbf{t} - \mathbf{t}') \frac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}\mathbf{t}'}$$

$$= \int \mathbf{d}\tau' \mathbf{e}\delta^{(4)}(\mathbf{x} - \mathbf{x}') \frac{\mathbf{d}\mathbf{x}_{\mu}}{\mathbf{d}\tau'} = \int \mathbf{d}\tau' \mathbf{e}\delta^{(4)}(\mathbf{x} - \mathbf{x}') \mathbf{u}_{\mu}$$

$$\mathbf{d} = \int \mathbf{d} au' \mathbf{e} \delta^{(4)}(\mathbf{x} - \mathbf{x}') rac{\mathbf{d}}{\mathbf{d}} \mathbf{e} \ rac{\mathbf{d}\mathbf{p}_{\mu}}{\mathbf{d} au} = \mathbf{K}_{\mu}$$

$$egin{aligned} rac{-\mathbf{F}\mu}{\mathbf{d} au} &= \mathbf{K}, \ \mathbf{v}, &= \sum_{i=1}^4 \mathbf{e}_i \end{aligned}$$

$$=\stackrel{\text{\tiny (F)}}{=}\Longrightarrow \mathbf{K}_{\mu}=(\vec{\mathbf{K}},rac{\mathbf{i}}{\mathbf{c}}\vec{\mathbf{K}}\cdot\vec{\mathbf{v}})=\sum_{
u=1}^{4}\mathbf{e}\mathbf{F}_{\mu
u}\mathbf{u}_{
u}\qquad \mathbf{f}_{\mu}=\int\mathbf{d} au\delta^{(4)}(\mathbf{x}-\mathbf{x}')\mathbf{K}_{\mu}$$

$$\mathbf{e} = (rac{\mathbf{e}}{\sqrt{1-rac{\mathbf{v}^2}{\mathbf{c}^2}}}(\vec{\mathbf{E}}+\vec{\mathbf{v}} imes\vec{\mathbf{B}}), rac{rac{\mathrm{ie}}{\mathbf{c}}\vec{\mathbf{E}}\cdot\vec{\mathbf{v}}}{\sqrt{1-rac{\mathbf{v}^2}{\mathbf{c}^2}}})$$

或: 
$$ec{ ext{F}} = ec{ ext{K}} \sqrt{1\!-\!rac{ ext{v}^2}{ ext{c}^2}}$$









协变表达

### $\overline{\left(\mathbf{f}_{\mu}\equiv\sum\mathbf{F}_{\mu u}\mathbf{j}_{ u}}=( ho\vec{\mathbf{E}}+\overrightarrow{\mathbf{j}} imes\vec{\mathbf{B}},\ \ \mathbf{\dot{c}}\overrightarrow{\mathbf{j}}\cdot\vec{\mathbf{E}})$

対点电荷: 
$$\mathbf{j}_{\mu} = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \frac{d\mathbf{x}_{\mu}}{dt} = \int dt' \ \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \delta(\mathbf{t} - \mathbf{t}') \frac{d\mathbf{x}_{\mu}}{dt'}$$

$$= \int d\tau' \mathbf{e}\delta^{(4)}(\mathbf{x} - \mathbf{x}') \frac{d\mathbf{x}_{\mu}}{d\tau'} = \int d\tau' \mathbf{e}\delta^{(4)}(\mathbf{x} - \mathbf{x}') \mathbf{u}_{\mu}$$

$$-\int \mathbf{d} au \, \mathbf{e} \sigma^{(\mathbf{x}-\mathbf{x})} \, \mathbf{d} \mathbf{p}_{\mu} \ rac{\mathbf{d}\mathbf{p}_{\mu}}{\mathbf{d} au} = \mathbf{K}_{\mu}$$

$$rac{\mathbf{p}_{\mu}}{\mathbf{l} au}=\mathbf{K}_{\mu}$$

$$\mathbf{e} = (rac{\mathbf{e}}{\sqrt{1-rac{\mathbf{v}^2}{\mathbf{c}^2}}}(ec{\mathbf{E}}+ec{\mathbf{v}} imesec{\mathbf{B}}), rac{rac{\mathrm{i}\mathbf{e}}{\mathbf{c}}ec{\mathbf{E}}\cdotec{\mathbf{v}}}{\sqrt{1-rac{\mathbf{v}^2}{\mathbf{c}^2}}})$$

$$egin{aligned} \vec{E} &= (rac{\vec{C}}{\sqrt{1-rac{\mathbf{v}^2}{\mathbf{c}^2}}} (\vec{E} + \vec{\mathbf{v}} imes \vec{B}), rac{\mathbf{c} \cdot \mathbf{F} \cdot \mathbf{v}}{\sqrt{1-rac{\mathbf{v}^2}{\mathbf{c}^2}}}) \ \vec{E} &= \vec{K} \sqrt{1-rac{\mathbf{v}^2}{\mathbf{c}^2}} \quad \vec{F} &= rac{d\vec{P}}{dt} \quad \vec{F} \cdot \vec{\mathbf{v}} = rac{dH}{dt} \quad \Rightarrow \quad \vec{F}' = rac{d\vec{P}'}{dt'} \quad \vec{F}' \cdot \vec{\mathbf{v}}' = rac{dH'}{dt'} \end{aligned}$$

$$egin{align*} & \overrightarrow{\mathbf{d} au} = \mathbf{K}_{\mu} \ = & \stackrel{\text{\tiny $\dagger$}}{=} \Rightarrow \mathbf{K}_{\mu} = (\vec{\mathbf{K}}, rac{\mathbf{i}}{\mathbf{c}} \vec{\mathbf{K}} \cdot \vec{\mathbf{v}}) = \sum_{
u=1}^{4} \mathbf{e} \mathbf{F}_{\mu
u} \mathbf{u}_{
u} \qquad \mathbf{f}_{\mu} = \int \mathbf{d} au \delta^{(4)}(\mathbf{x} - \mathbf{x}') \mathbf{K}_{\mu} \ & \qquad \qquad \mathbf{e} \qquad \vec{\mathbf{c}} \vec{\mathbf{E}} \cdot \vec{\mathbf{v}} \qquad \qquad \mathbf{e} \qquad \vec{\mathbf{c}} \vec{\mathbf{E}} \cdot \vec{\mathbf{v}} \qquad \qquad \mathbf{e} \qquad \vec{\mathbf{c}} \vec{\mathbf{e}} \vec{\mathbf{E}} \cdot \vec{\mathbf{v}} \end{pmatrix}$$





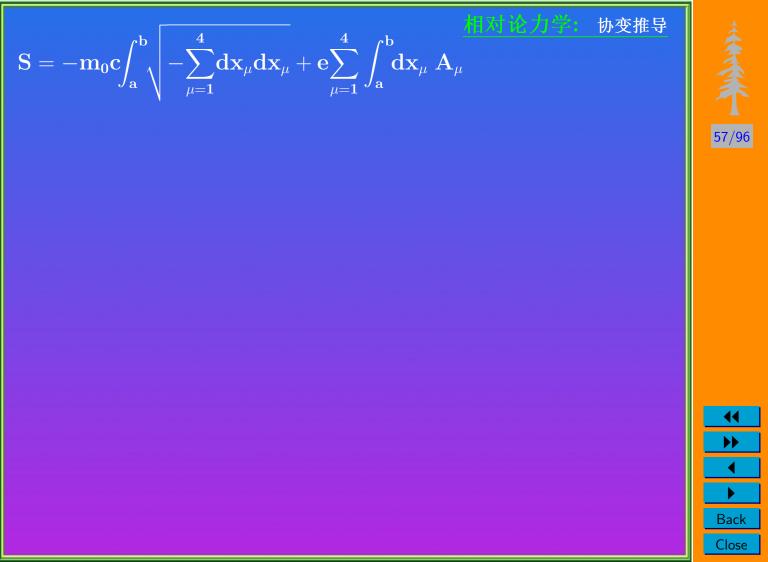


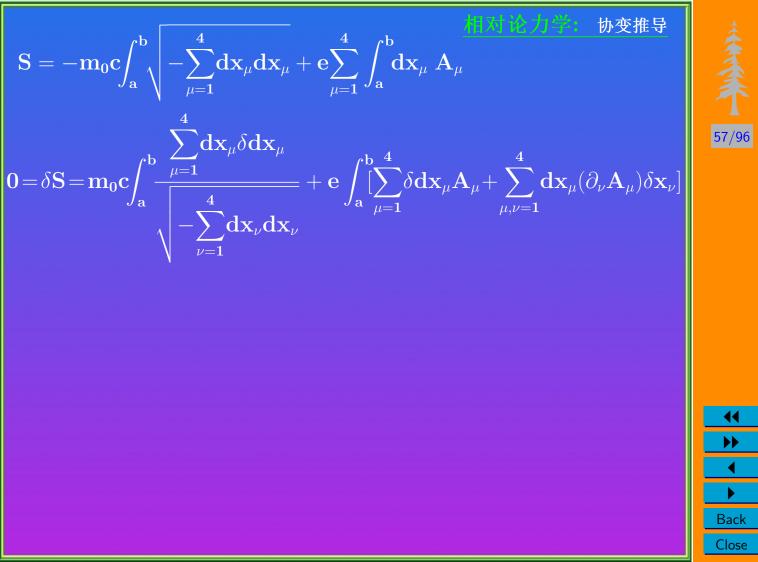


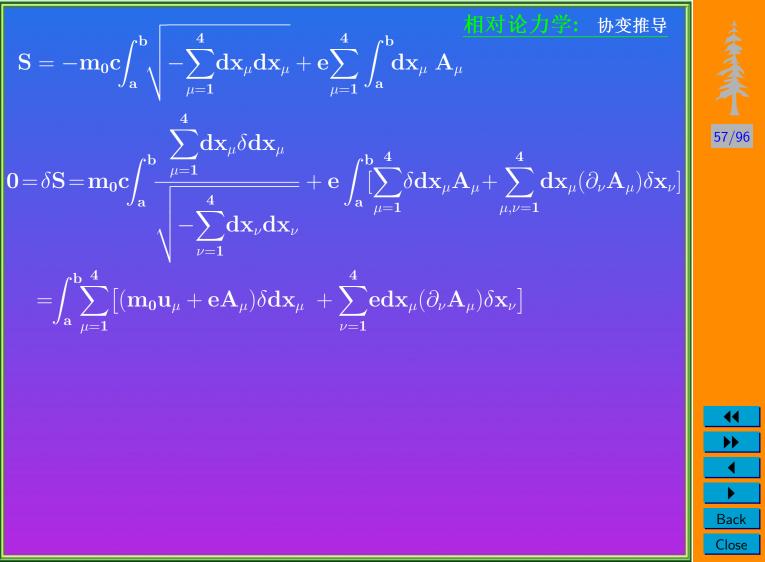














$$\begin{split} \mathbf{S} &= -\mathbf{m}_0 \mathbf{c} \int_{\mathbf{a}}^{\mathbf{b}} \sqrt{-\sum_{\mu=1}^4 \mathbf{d} \mathbf{x}_\mu \mathbf{d} \mathbf{x}_\mu} + \mathbf{e} \sum_{\mu=1}^4 \int_{\mathbf{a}}^{\mathbf{b}} \mathbf{d} \mathbf{x}_\mu \mathbf{A}_\mu \\ \mathbf{0} &= \delta \mathbf{S} = \mathbf{m}_0 \mathbf{c} \int_{\mathbf{a}}^{\mathbf{b}} \frac{\sum_{\mu=1}^4 \mathbf{d} \mathbf{x}_\mu \delta \mathbf{d} \mathbf{x}_\mu}{\sqrt{-\sum_{\nu=1}^4 \mathbf{d} \mathbf{x}_\nu d} \mathbf{x}_\nu} + \mathbf{e} \int_{\mathbf{a}}^{\mathbf{b}} \sum_{\mu=1}^4 \delta \mathbf{d} \mathbf{x}_\mu \mathbf{A}_\mu + \sum_{\mu,\nu=1}^4 \mathbf{d} \mathbf{x}_\mu (\partial_\nu \mathbf{A}_\mu) \delta \mathbf{x}_\nu \\ &= \int_{\mathbf{a}}^{\mathbf{b}} \sum_{\mu=1}^4 \left[ (\mathbf{m}_0 \mathbf{u}_\mu + \mathbf{e} \mathbf{A}_\mu) \delta \mathbf{d} \mathbf{x}_\mu + \sum_{\nu=1}^4 \mathbf{e} \mathbf{d} \mathbf{x}_\mu (\partial_\nu \mathbf{A}_\mu) \delta \mathbf{x}_\nu \right] \\ &= \int_{\mathbf{a}}^{\mathbf{b}} \sum_{\mu=1}^4 \left[ -\mathbf{m}_0 \mathbf{d} \mathbf{u}_\mu - \mathbf{e} \mathbf{d} \mathbf{A}_\mu + \sum_{\nu=1}^4 \mathbf{e} \mathbf{d} \mathbf{x}_\nu (\partial_\mu \mathbf{A}_\nu) \right] \delta \mathbf{x}_\mu \\ &= \int_{\mathbf{a}}^{\mathbf{b}} \sum_{\mu=1}^4 \left[ -\mathbf{m}_0 \mathbf{d} \mathbf{u}_\mu - \sum_{\nu=1}^4 \mathbf{e} \mathbf{d} \mathbf{x}_\nu (\partial_\nu \mathbf{A}_\mu) + \sum_{\nu=1}^4 \mathbf{e} \mathbf{d} \mathbf{x}_\nu (\partial_\mu \mathbf{A}_\nu) \right] \delta \mathbf{x}_\mu \\ &= \int_{\mathbf{a}}^{\mathbf{b}} \sum_{\mu=1}^4 \left[ -\mathbf{m}_0 \mathbf{d} \mathbf{u}_\mu - \sum_{\nu=1}^4 \mathbf{e} \mathbf{d} \mathbf{x}_\nu (\partial_\nu \mathbf{A}_\mu) + \sum_{\nu=1}^4 \mathbf{e} \mathbf{d} \mathbf{x}_\nu (\partial_\mu \mathbf{A}_\nu) \right] \delta \mathbf{x}_\mu \\ &= \int_{\mathbf{a}}^{\mathbf{b}} \sum_{\mu=1}^4 \left[ -\mathbf{m}_0 \mathbf{d} \mathbf{u}_\mu - \sum_{\nu=1}^4 \mathbf{e} \mathbf{d} \mathbf{x}_\nu (\partial_\nu \mathbf{A}_\mu) + \sum_{\nu=1}^4 \mathbf{e} \mathbf{d} \mathbf{x}_\nu (\partial_\mu \mathbf{A}_\nu) \right] \delta \mathbf{x}_\mu \\ &= \int_{\mathbf{a}}^{\mathbf{b}} \sum_{\mu=1}^4 \left[ -\mathbf{m}_0 \mathbf{d} \mathbf{u}_\mu - \sum_{\nu=1}^4 \mathbf{e} \mathbf{d} \mathbf{x}_\nu (\partial_\nu \mathbf{A}_\mu) + \sum_{\nu=1}^4 \mathbf{e} \mathbf{d} \mathbf{x}_\nu (\partial_\mu \mathbf{A}_\nu) \right] \delta \mathbf{x}_\mu \\ &= \int_{\mathbf{a}}^{\mathbf{b}} \sum_{\mu=1}^4 \left[ -\mathbf{m}_0 \mathbf{d} \mathbf{u}_\mu - \sum_{\nu=1}^4 \mathbf{e} \mathbf{d} \mathbf{x}_\nu (\partial_\nu \mathbf{A}_\mu) + \sum_{\nu=1}^4 \mathbf{e} \mathbf{d} \mathbf{x}_\nu (\partial_\mu \mathbf{A}_\nu) \right] \delta \mathbf{x}_\mu \\ &= \int_{\mathbf{a}}^{\mathbf{b}} \sum_{\mu=1}^4 \left[ -\mathbf{m}_0 \mathbf{d} \mathbf{u}_\mu - \sum_{\nu=1}^4 \mathbf{e} \mathbf{d} \mathbf{x}_\nu (\partial_\nu \mathbf{A}_\mu) \right] \delta \mathbf{x}_\mu \\ &= \int_{\mathbf{a}}^4 \sum_{\mu=1}^4 \left[ -\mathbf{m}_0 \mathbf{d} \mathbf{u}_\mu - \sum_{\nu=1}^4 \mathbf{e} \mathbf{d} \mathbf{x}_\nu (\partial_\nu \mathbf{A}_\mu) \right] \delta \mathbf{x}_\mu \\ &= \int_{\mathbf{a}}^4 \sum_{\mu=1}^4 \left[ -\mathbf{m}_0 \mathbf{d} \mathbf{u}_\mu - \sum_{\nu=1}^4 \mathbf{e} \mathbf{d} \mathbf{x}_\nu (\partial_\nu \mathbf{A}_\mu) \right] \delta \mathbf{x}_\mu \\ &= \int_{\mathbf{a}}^4 \sum_{\mu=1}^4 \left[ -\mathbf{m}_0 \mathbf{d} \mathbf{u}_\mu - \sum_{\nu=1}^4 \mathbf{e} \mathbf{d} \mathbf{x}_\nu (\partial_\nu \mathbf{A}_\mu) \right] \delta \mathbf{x}_\mu \\ &= \int_{\mathbf{a}}^4 \sum_{\mu=1}^4 \left[ -\mathbf{m}_0 \mathbf{d} \mathbf{u}_\mu - \sum_{\mu=1}^4 \mathbf{e} \mathbf{d} \mathbf{u}_\mu (\partial_\nu \mathbf{A}_\mu) \right] \delta \mathbf{x}_\mu \\ &= \int_{\mathbf{a}}^4 \sum_{\mu=1}^4 \left[ -\mathbf{m}_0 \mathbf{d} \mathbf{u}_\mu - \sum_{\mu=1}^4 \mathbf{e} \mathbf{d} \mathbf{u}_\mu (\partial_\nu \mathbf{A}_\mu) \right] \delta \mathbf{x}_\mu \\ &= \int_{\mathbf{a}}^4 \sum_{\mu=1}^4 \left[ -\mathbf{m}_0 \mathbf{d} \mathbf{u}_\mu - \sum_{\mu=1}^$$

$$\begin{split} \mathbf{S} &= -\mathbf{m}_0 \mathbf{c} \int_{\mathbf{a}}^{\mathbf{b}} \sqrt{-\sum_{\mu=1}^{4} \mathbf{d} \mathbf{x}_{\mu} \mathbf{d} \mathbf{x}_{\mu}} + \mathbf{e} \sum_{\mu=1}^{4} \int_{\mathbf{a}}^{\mathbf{b}} \mathbf{d} \mathbf{x}_{\mu} \, \mathbf{A}_{\mu} \\ \mathbf{0} &= \delta \mathbf{S} = \mathbf{m}_0 \mathbf{c} \int_{\mathbf{a}}^{\mathbf{b}} \frac{\sum_{\mu=1}^{4} \mathbf{d} \mathbf{x}_{\mu} \delta \mathbf{d} \mathbf{x}_{\mu}}{\sqrt{-\sum_{\nu=1}^{4} \mathbf{d} \mathbf{x}_{\nu} d \mathbf{x}_{\nu}}} + \mathbf{e} \int_{\mathbf{a}}^{\mathbf{b}} [\sum_{\mu=1}^{4} \delta \mathbf{d} \mathbf{x}_{\mu} \mathbf{A}_{\mu} + \sum_{\mu,\nu=1}^{4} \mathbf{d} \mathbf{x}_{\mu} (\partial_{\nu} \mathbf{A}_{\mu}) \delta \mathbf{x}_{\nu}] \\ &= \int_{\mathbf{a}}^{\mathbf{b}} \sum_{\mu=1}^{4} [(\mathbf{m}_0 \mathbf{u}_{\mu} + \mathbf{e} \mathbf{A}_{\mu}) \delta \mathbf{d} \mathbf{x}_{\mu} + \sum_{\nu=1}^{4} \mathbf{e} \mathbf{d} \mathbf{x}_{\nu} (\partial_{\nu} \mathbf{A}_{\mu}) \delta \mathbf{x}_{\nu}] \\ &= \int_{\mathbf{a}}^{\mathbf{b}} \sum_{\mu=1}^{4} [-\mathbf{m}_0 \mathbf{d} \mathbf{u}_{\mu} - \mathbf{e} \mathbf{d} \mathbf{A}_{\mu} + \sum_{\nu=1}^{4} \mathbf{e} \mathbf{d} \mathbf{x}_{\nu} (\partial_{\mu} \mathbf{A}_{\nu})] \delta \mathbf{x}_{\mu} \\ &= \int_{\mathbf{a}}^{\mathbf{b}} \sum_{\mu=1}^{4} [-\mathbf{m}_0 \mathbf{d} \mathbf{u}_{\mu} - \sum_{\nu=1}^{4} \mathbf{e} \mathbf{d} \mathbf{x}_{\nu} (\partial_{\nu} \mathbf{A}_{\mu}) + \sum_{\nu=1}^{4} \mathbf{e} \mathbf{d} \mathbf{x}_{\nu} (\partial_{\mu} \mathbf{A}_{\nu})] \delta \mathbf{x}_{\mu} \\ &= \int_{\mathbf{a}}^{\mathbf{b}} \sum_{\mu=1}^{4} [-\mathbf{m}_0 \frac{\mathbf{d} \mathbf{u}_{\mu}}{\mathbf{d} \tau} + \sum_{\nu=1}^{4} \mathbf{e} \frac{\mathbf{d} \mathbf{x}_{\nu}}{\mathbf{d} \tau} \mathbf{F}_{\mu\nu}] \mathbf{d} \tau \delta \mathbf{x}_{\mu} \end{split}$$

$$\begin{split} \mathbf{S} &= -m_0 c \int_{\mathbf{a}}^{\mathbf{b}} \sqrt{-\sum_{\mu=1}^4 d\mathbf{x}_\mu d\mathbf{x}_\mu + e \sum_{\mu=1}^4 \int_{\mathbf{a}}^{\mathbf{b}} d\mathbf{x}_\mu \, \mathbf{A}_\mu} \\ \mathbf{0} &= \delta \mathbf{S} = m_0 c \int_{\mathbf{a}}^{\mathbf{b}} \frac{\sum_{\mu=1}^4 d\mathbf{x}_\mu \delta d\mathbf{x}_\mu}{\sqrt{-\sum_{\nu=1}^4 d\mathbf{x}_\nu d\mathbf{x}_\nu}} + e \int_{\mathbf{a}}^{\mathbf{b}} [\sum_{\mu=1}^4 \delta d\mathbf{x}_\mu \mathbf{A}_\mu + \sum_{\mu,\nu=1}^4 d\mathbf{x}_\mu (\partial_\nu \mathbf{A}_\mu) \delta \mathbf{x}_\nu] \\ &= \int_{\mathbf{a}}^{\mathbf{b}} \sum_{\mu=1}^4 \left[ (m_0 \mathbf{u}_\mu + e \mathbf{A}_\mu) \delta d\mathbf{x}_\mu + \sum_{\nu=1}^4 e d\mathbf{x}_\mu (\partial_\nu \mathbf{A}_\mu) \delta \mathbf{x}_\nu \right] \\ &= \int_{\mathbf{a}}^{\mathbf{b}} \sum_{\mu=1}^4 \left[ -m_0 d\mathbf{u}_\mu - e d\mathbf{A}_\mu + \sum_{\nu=1}^4 e d\mathbf{x}_\nu (\partial_\mu \mathbf{A}_\nu) \right] \delta \mathbf{x}_\mu \\ &= \int_{\mathbf{a}}^{\mathbf{b}} \sum_{\mu=1}^4 \left[ -m_0 d\mathbf{u}_\mu - \sum_{\nu=1}^4 e d\mathbf{x}_\nu (\partial_\nu \mathbf{A}_\mu) + \sum_{\nu=1}^4 e d\mathbf{x}_\nu (\partial_\mu \mathbf{A}_\nu) \right] \delta \mathbf{x}_\mu \\ &= \int_{\mathbf{a}}^{\mathbf{b}} \sum_{\mu=1}^4 \left[ -m_0 \frac{d\mathbf{u}_\mu}{d\tau} + \sum_{\nu=1}^4 e \frac{d\mathbf{x}_\nu}{d\tau} \mathbf{F}_{\mu\nu} \right] d\tau \delta \mathbf{x}_\mu \quad \Rightarrow \quad \frac{d\mathbf{p}_\mu}{d\tau} = \sum_{\nu=1}^4 e \mathbf{F}_{\mu\nu} \mathbf{u}_\nu \end{split}$$

为描述电磁场,在作用量中加一纯含电磁场的项:标量、规范不变、最为简单.



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为描述电磁场,在作用量中加一纯含电磁场的项:标量、规范不变、最为简单.

$$\sum_{\mu,
u=1}^4 \mathbf{F}_{\mu
u} \mathbf{F}_{\mu
u} = \mathbf{2} (\mathbf{B^2} - rac{1}{\mathbf{c^2}} \mathbf{E^2})$$



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为描述电磁场,在作用量中加一纯含电磁场的项:标量、规范不变、最为简单.

$$\begin{split} \sum_{\mu,\nu=1}^{4} & \mathbf{F}_{\mu\nu} \mathbf{F}_{\mu\nu} = 2(\mathbf{B}^2 - \frac{1}{\mathbf{c}^2} \mathbf{E}^2) \\ \sum_{\mu,\nu,\sigma,\rho=1}^{4} & \epsilon_{\mu\nu\sigma\rho} \mathbf{F}_{\mu\nu} \mathbf{F}_{\sigma\rho} = -\frac{8\mathbf{i}}{\mathbf{c}} \vec{\mathbf{E}} \cdot \vec{\mathbf{B}} = \sum_{\mu,\nu,\sigma,\rho=1}^{4} 4\epsilon_{\mu\nu\sigma\rho} \partial_{\mu} (\mathbf{A}_{\nu} \partial_{\sigma} \mathbf{A}_{\rho}) \end{split}$$

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$$\sum_{\mu,
u=1}^4 \mathbf{F}_{\mu
u} \mathbf{F}_{\mu
u} = 2(\mathbf{B^2} - rac{1}{\mathbf{c^2}}\mathbf{E^2})$$

$$\sum_{\mu,
u,\sigma,
ho=1}^4 \epsilon_{\mu
u\sigma
ho} \mathbf{F}_{\mu
u} \mathbf{F}_{\sigma
ho} = -rac{8\mathbf{i}}{\mathbf{c}} ec{\mathbf{E}} \cdot ec{\mathbf{B}} = \sum_{\mu,
u,\sigma,
ho=1}^4 4\epsilon_{\mu
u\sigma
ho} \partial_\mu (\mathbf{A}_
u \partial_\sigma \mathbf{A}_
ho)$$

破坏空间反演;后面讨论!

$$\mathbf{S} = \int \mathbf{dt} \mathbf{d au} igl( -rac{\mathbf{1}}{4\mu_{\mathbf{0}}} igr) \sum_{\mu,
u=\mathbf{1}}^{4} \mathbf{F}_{\mu
u} \mathbf{F}_{\mu
u} + \sum_{\mu=\mathbf{1}}^{4} \mathbf{j}_{\mu} \mathbf{A}_{\mu} igr]$$



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为描述电磁场,在作用量中加一纯含电磁场的项:标量、规范不变、最为简单.

$$\sum_{\substack{\mu,
u=1\ }{}}^4 \mathrm{F}_{\mu
u} \mathrm{F}_{\mu
u} = 2 (\mathrm{B^2} - rac{1}{\mathrm{c^2}} \mathrm{E^2})$$

$$\underbrace{\sum_{\mu,\nu,\sigma,\rho=1}^4}_{\text{破坏空间反演: 后面讨论!}} \epsilon_{\mu\nu\sigma\rho} \mathbf{F}_{\mu\nu} \mathbf{F}_{\sigma\rho} = -\frac{8\mathbf{i}}{\mathbf{c}} \vec{\mathbf{E}} \cdot \vec{\mathbf{B}} = \sum_{\mu,\nu,\sigma,\rho=1}^4 4\epsilon_{\mu\nu\sigma\rho} \partial_\mu (\mathbf{A}_\nu \partial_\sigma \mathbf{A}_\rho)$$

$$\mathbf{S} = \int \mathbf{dt} \mathbf{d} au ig[ (-rac{1}{4\mu_{\mathbf{0}}}) \sum_{\mu,
u=1}^{4} \mathbf{F}_{\mu
u} \mathbf{F}_{\mu
u} + \sum_{\mu=1}^{4} \mathbf{j}_{\mu} \mathbf{A}_{\mu} ig]$$

 $\mathbf{0} = \delta \mathbf{S}$ 



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为描述电磁场,在作用量中加一纯含电磁场的项:标量、规范不变、最为简单.

$$\sum_{\mu,
u=1}^4 \mathbf{F}_{\mu
u} \mathbf{F}_{\mu
u} = 2(\mathbf{B^2} - rac{1}{\mathbf{c^2}}\mathbf{E^2})$$

$$\sum_{\mu,
u,\sigma,
ho=1}^4 \epsilon_{\mu
u\sigma
ho} \mathbf{F}_{\mu
u} \mathbf{F}_{\sigma
ho} = -rac{8\mathbf{i}}{\mathbf{c}} ec{\mathbf{E}} \cdot ec{\mathbf{B}} = \sum_{\mu,
u,\sigma,
ho=1}^4 4\epsilon_{\mu
u\sigma
ho} \partial_\mu (\mathbf{A}_
u \partial_\sigma \mathbf{A}_
ho)$$

$$\mathbf{S} = \int \mathbf{dt} \mathbf{d} au ig[ (-rac{1}{4\mu_{\mathbf{0}}}) \sum_{\mu,
u=\mathbf{1}}^{4} \mathbf{F}_{\mu
u} \mathbf{F}_{\mu
u} + \sum_{\mu=\mathbf{1}}^{4} \mathbf{j}_{\mu} \mathbf{A}_{\mu} ig]$$

$$\mathbf{0} = \delta \mathbf{S} = \int \mathbf{dt} \mathbf{d} au ig[ (-rac{1}{4\mu_{\mathbf{0}}}) \sum_{\mu,
u=1}^4 \delta(\mathbf{F}_{\mu
u}\mathbf{F}_{\mu
u}) + \sum_{\mu=1}^4 \mathbf{j}_{\mu}\delta\mathbf{A}_{\mu} ig]$$



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为描述电磁场,在作用量中加一纯含电磁场的项:标量、规范不变、最为简单.

$$\sum_{\mu,
u=1}^4 \mathbf{F}_{\mu
u} \mathbf{F}_{\mu
u} = \mathbf{2} (\mathbf{B^2} - rac{1}{\mathbf{c^2}} \mathbf{E^2})$$

$$\sum_{\substack{\mu,
u,\sigma,
ho=1}}^4 \epsilon_{\mu
u\sigma
ho} \mathbf{F}_{\mu
u} \mathbf{F}_{\sigma
ho} = -rac{8\mathbf{i}}{\mathbf{c}} ec{\mathbf{E}} \cdot ec{\mathbf{B}} = \sum_{\mu,
u,\sigma,
ho=1}^4 4\epsilon_{\mu
u\sigma
ho} \partial_\mu (\mathbf{A}_
u \partial_\sigma \mathbf{A}_
ho)$$

$$\mathbf{S} = \int \mathbf{dt} \mathbf{d} au ig[ (-rac{1}{4\mu_{\mathbf{0}}}) \sum_{\mu,
u=1}^{4} \mathbf{F}_{\mu
u} \mathbf{F}_{\mu
u} + \sum_{\mu=1}^{4} \mathbf{j}_{\mu} \mathbf{A}_{\mu} ig]$$

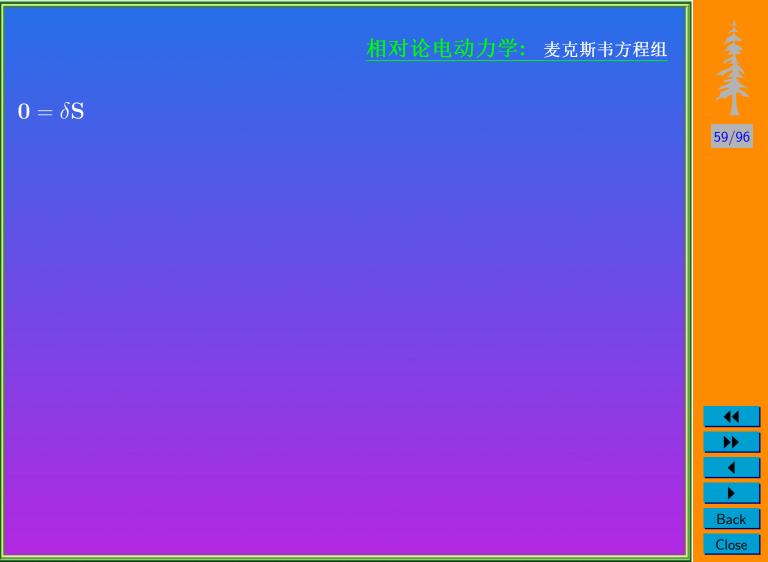
$$egin{aligned} \mathbf{0} &= \delta \mathbf{S} = \int \mathbf{d} \mathbf{t} \mathbf{d} au ig[ (-rac{1}{4\mu_{\mathbf{0}}}) \sum_{\mu,
u=1}^4 \delta(\mathbf{F}_{\mu
u}\mathbf{F}_{\mu
u}) + \sum_{\mu=1}^4 \mathbf{j}_{\mu}\delta\mathbf{A}_{\mu} ig] \ &= \int \mathbf{d} \mathbf{t} \mathbf{d} au ig[ (-rac{1}{2\mu_{\mathbf{0}}}) \sum_{\mu,
u=1}^4 \mathbf{F}_{\mu
u}\delta\mathbf{F}_{\mu
u} + \sum_{\mu=1}^4 \mathbf{j}_{\mu}\delta\mathbf{A}_{\mu} ig] \end{aligned}$$



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Back



## 相对论电动力学:麦克斯韦方程组 $\mathbf{0} = \delta \mathbf{S} = \int \mathbf{dt} \mathbf{d} au ig[ (-rac{1}{4\mu_{\mathbf{0}}}) \sum_{\mu, u=1}^{4} \delta(\mathbf{F}_{\mu u}\mathbf{F}_{\mu u}) + \sum_{\mu=1}^{4} \mathbf{j}_{\mu}\delta\mathbf{A}_{\mu} ig] ig]$





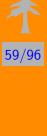




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$$egin{aligned} \mathbf{0} &= \delta \mathbf{S} = \int \mathbf{d} \mathbf{t} \mathbf{d} au ig[ (-rac{1}{4\mu_{\mathbf{0}}}) \sum_{\mu,
u=1}^4 \delta(\mathbf{F}_{\mu
u}\mathbf{F}_{\mu
u}) + \sum_{\mu=1}^4 \mathbf{j}_{\mu}\delta\mathbf{A}_{\mu} ig] \ &= \int \mathbf{d} \mathbf{t} \mathbf{d} au ig[ (-rac{1}{2\mu_{\mathbf{0}}}) \sum_{\mu,
u=1}^4 \mathbf{F}_{\mu
u}\delta\mathbf{F}_{\mu
u} + \sum_{\mu=1}^4 \mathbf{j}_{\mu}\delta\mathbf{A}_{\mu} ig] \end{aligned}$$









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## 相对论自动力学: 麦克斯韦方程组

$$\mathbf{0} = \delta \mathbf{S} = \int \mathbf{d} \mathbf{t} \mathbf{d} \tau \left[ \left( -\frac{1}{4\mu_0} \right) \sum_{\mu,\nu=1}^4 \delta(\mathbf{F}_{\mu\nu} \mathbf{F}_{\mu\nu}) + \sum_{\mu=1}^4 \mathbf{j}_{\mu} \delta \mathbf{A}_{\mu} \right]$$

$$= \int \mathbf{d} \mathbf{t} \mathbf{d} \tau \left[ \left( -\frac{1}{2\mu_0} \right) \sum_{\mu,\nu=1}^4 \mathbf{F}_{\mu\nu} \delta \mathbf{F}_{\mu\nu} + \sum_{\mu=1}^4 \mathbf{j}_{\mu} \delta \mathbf{A}_{\mu} \right]$$

$$= \int \mathbf{d} \mathbf{t} \mathbf{d} \tau \left[ \left( -\frac{1}{\mu_0} \right) \sum_{\mu,\nu=1}^4 \mathbf{F}_{\mu\nu} \delta(\partial_{\mu} \mathbf{A}_{\nu}) + \sum_{\mu=1}^4 \mathbf{j}_{\mu} \delta \mathbf{A}_{\mu} \right]$$









## 相对论自动力学: 麦克斯韦方程组

$$egin{aligned} \mathbf{0} &= \delta \mathbf{S} = \int \mathbf{d} \mathbf{t} \mathbf{d} au ig[ (-rac{1}{4\mu_{\mathbf{0}}}) \sum_{\mu,
u=1}^4 \delta(\mathbf{F}_{\mu
u}\mathbf{F}_{\mu
u}) + \sum_{\mu=1}^4 \mathbf{j}_{\mu}\delta\mathbf{A}_{\mu} ig] \ &= \int \mathbf{d} \mathbf{t} \mathbf{d} au ig[ (-rac{1}{2\mu_{\mathbf{0}}}) \sum_{\mu,
u=1}^4 \mathbf{F}_{\mu
u}\delta\mathbf{F}_{\mu
u} + \sum_{\mu=1}^4 \mathbf{j}_{\mu}\delta\mathbf{A}_{\mu} ig] \end{aligned}$$

$$= \int \mathbf{dt} \mathbf{d}\tau \left[ (-\frac{1}{\mu_0}) \sum_{\mu,\nu=1}^4 \mathbf{F}_{\mu\nu} \delta(\partial_{\mu} \mathbf{A}_{\nu}) + \sum_{\mu=1}^4 \mathbf{j}_{\mu} \delta \mathbf{A}_{\mu} \right]$$

$$= \int \mathbf{dt} \mathbf{d}\tau \left[ (-\frac{1}{\mu_0}) \sum_{\mu,\nu=1}^4 \mathbf{F}_{\mu\nu} (\partial_{\mu} \delta \mathbf{A}_{\nu}) + \sum_{\mu=1}^4 \mathbf{j}_{\mu} \delta \mathbf{A}_{\mu} \right]$$







## 相对论电动力学:麦克斯韦方程组

$$\begin{aligned} \mathbf{0} &= \delta \mathbf{S} = \int \mathbf{d}t \mathbf{d}\tau \big[ (-\frac{1}{4\mu_0}) \sum_{\mu,\nu=1}^4 \delta(\mathbf{F}_{\mu\nu} \mathbf{F}_{\mu\nu}) + \sum_{\mu=1}^4 \mathbf{j}_{\mu} \delta \mathbf{A}_{\mu} \big] \\ &= \int \mathbf{d}t \mathbf{d}\tau \big[ (-\frac{1}{2\mu_0}) \sum_{\mu,\nu=1}^4 \mathbf{F}_{\mu\nu} \delta \mathbf{F}_{\mu\nu} + \sum_{\mu=1}^4 \mathbf{j}_{\mu} \delta \mathbf{A}_{\mu} \big] \\ &= \int \mathbf{d}t \mathbf{d}\tau \big[ (-\frac{1}{\mu_0}) \sum_{\mu,\nu=1}^4 \mathbf{F}_{\mu\nu} \delta(\partial_{\mu} \mathbf{A}_{\nu}) + \sum_{\mu=1}^4 \mathbf{j}_{\mu} \delta \mathbf{A}_{\mu} \big] \\ &= \int \mathbf{d}t \mathbf{d}\tau \big[ (-\frac{1}{\mu_0}) \sum_{\mu,\nu=1}^4 \mathbf{F}_{\mu\nu} (\partial_{\mu} \delta \mathbf{A}_{\nu}) + \sum_{\mu=1}^4 \mathbf{j}_{\mu} \delta \mathbf{A}_{\mu} \big] \\ &= \int \mathbf{d}t \mathbf{d}\tau \big[ (-\frac{1}{\mu_0}) \sum_{\mu,\nu=1}^4 [\partial_{\mu} (\mathbf{F}_{\mu\nu} \delta \mathbf{A}_{\nu}) - (\partial_{\mu} \mathbf{F}_{\mu\nu}) \delta \mathbf{A}_{\nu} \big] + \sum_{\mu=1}^4 \mathbf{j}_{\mu} \delta \mathbf{A}_{\mu} \big] \end{aligned}$$











## 相对论电动力学: 麦克斯韦方程组

$$\begin{split} \mathbf{0} &= \delta \mathbf{S} = \int \mathbf{d}t \mathbf{d}\tau \big[ (-\frac{1}{4\mu_0}) \sum_{\mu,\nu=1}^4 \delta(\mathbf{F}_{\mu\nu}\mathbf{F}_{\mu\nu}) + \sum_{\mu=1}^4 \mathbf{j}_{\mu}\delta\mathbf{A}_{\mu} \big] \\ &= \int \mathbf{d}t \mathbf{d}\tau \big[ (-\frac{1}{2\mu_0}) \sum_{\mu,\nu=1}^4 \mathbf{F}_{\mu\nu}\delta\mathbf{F}_{\mu\nu} + \sum_{\mu=1}^4 \mathbf{j}_{\mu}\delta\mathbf{A}_{\mu} \big] \\ &= \int \mathbf{d}t \mathbf{d}\tau \big[ (-\frac{1}{\mu_0}) \sum_{\mu,\nu=1}^4 \mathbf{F}_{\mu\nu}\delta(\partial_{\mu}\mathbf{A}_{\nu}) + \sum_{\mu=1}^4 \mathbf{j}_{\mu}\delta\mathbf{A}_{\mu} \big] \\ &= \int \mathbf{d}t \mathbf{d}\tau \big[ (-\frac{1}{\mu_0}) \sum_{\mu,\nu=1}^4 \mathbf{F}_{\mu\nu}(\partial_{\mu}\delta\mathbf{A}_{\nu}) + \sum_{\mu=1}^4 \mathbf{j}_{\mu}\delta\mathbf{A}_{\mu} \big] \\ &= \int \mathbf{d}t \mathbf{d}\tau \big[ (-\frac{1}{\mu_0}) \sum_{\mu,\nu=1}^4 [\partial_{\mu}(\mathbf{F}_{\mu\nu}\delta\mathbf{A}_{\nu}) - (\partial_{\mu}\mathbf{F}_{\mu\nu})\delta\mathbf{A}_{\nu} \big] + \sum_{\mu=1}^4 \mathbf{j}_{\mu}\delta\mathbf{A}_{\mu} \big] \\ &= \int \mathbf{d}t \mathbf{d}\tau \big[ (-\frac{1}{\mu_0}) \sum_{\mu,\nu=1}^4 [\partial_{\mu}(\mathbf{F}_{\mu\nu}\delta\mathbf{A}_{\nu}) - (\partial_{\mu}\mathbf{F}_{\mu\nu})\delta\mathbf{A}_{\nu} \big] + \sum_{\mu=1}^4 \mathbf{j}_{\mu}\delta\mathbf{A}_{\mu} \big] \\ &= \int \partial_{\mu}\mathbf{F}_{\mu\nu} = -\mu_0 \mathbf{j}_{\nu} \qquad \partial_{\mu}\mathbf{F}_{\nu\lambda} + \partial_{\nu}\mathbf{F}_{\lambda\mu} + \partial_{\lambda}\mathbf{F}_{\mu\nu} = \mathbf{0} \end{split}$$

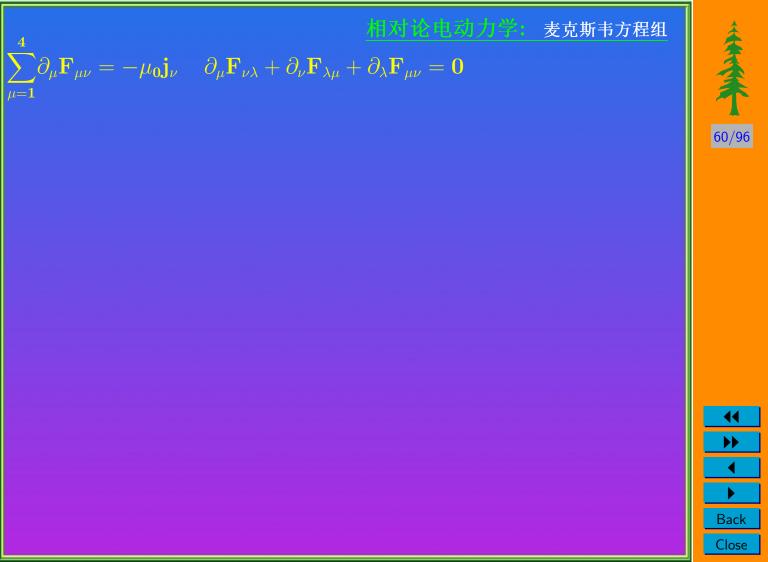


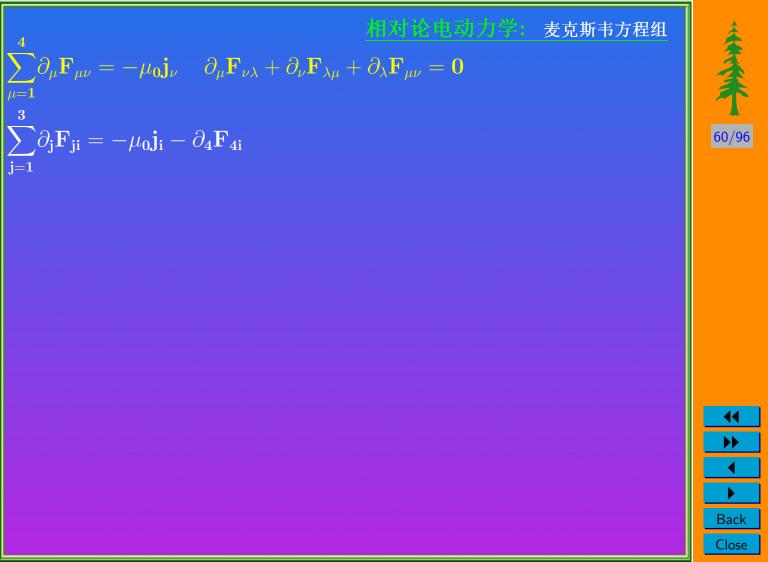
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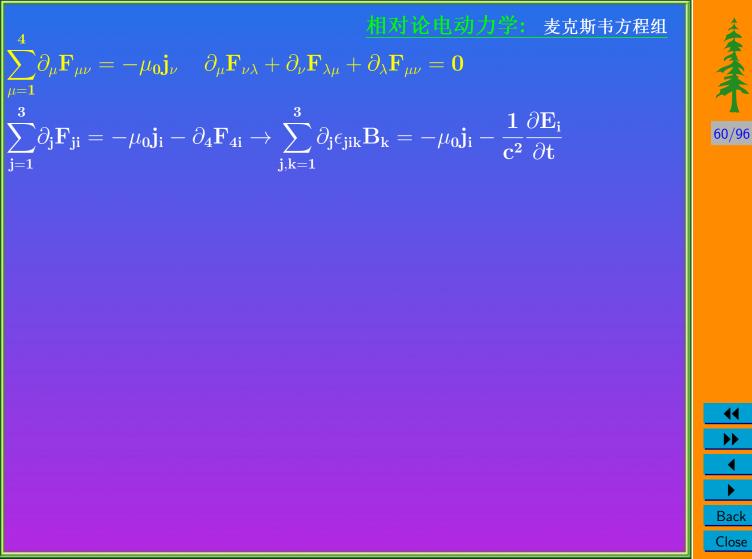
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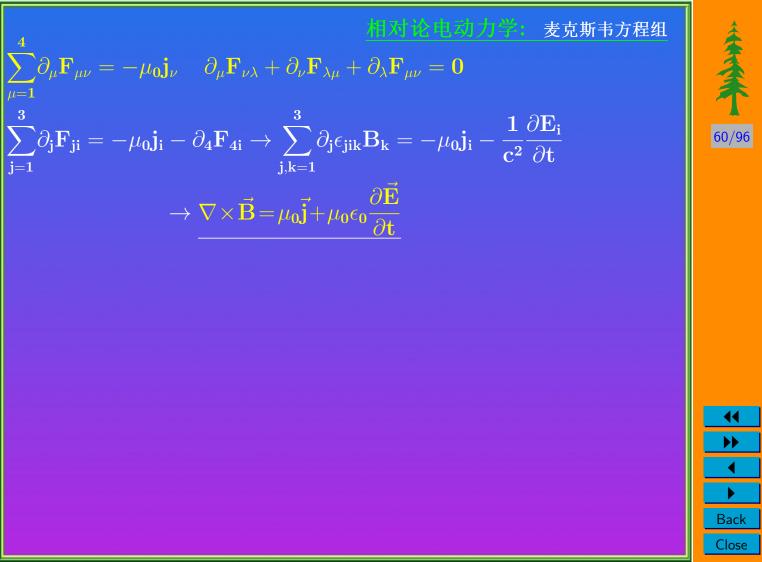
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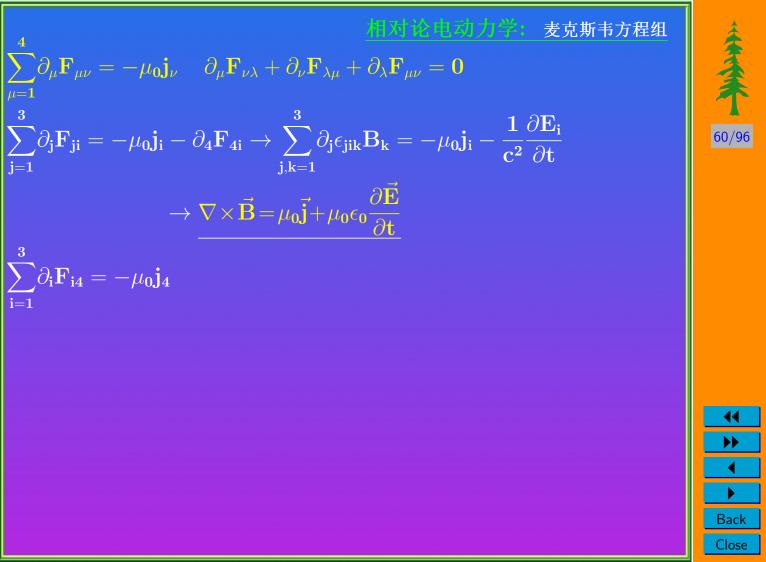


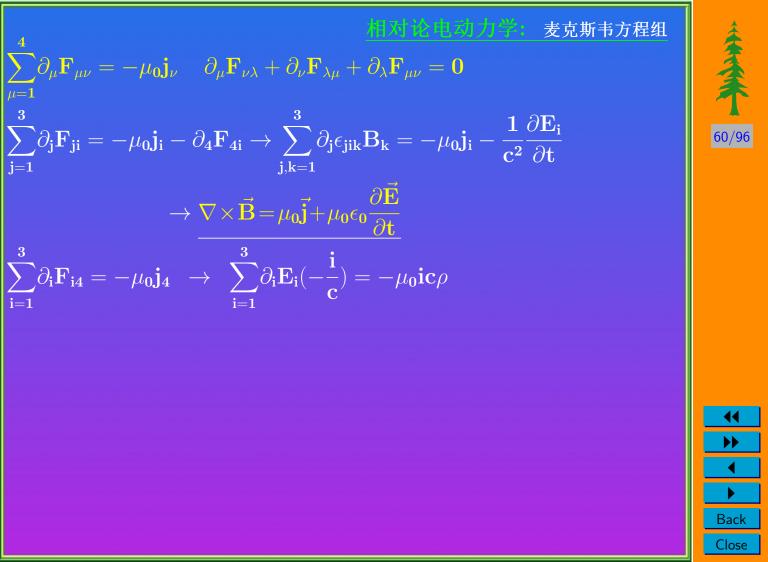


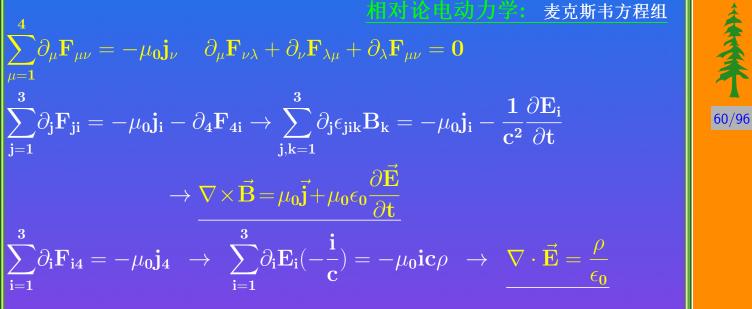


















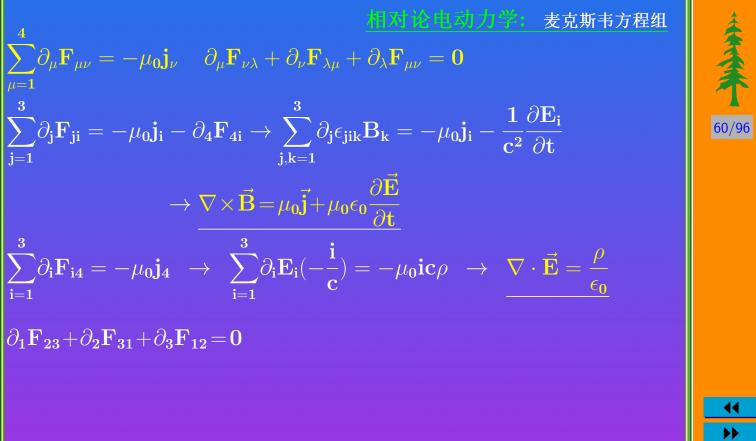














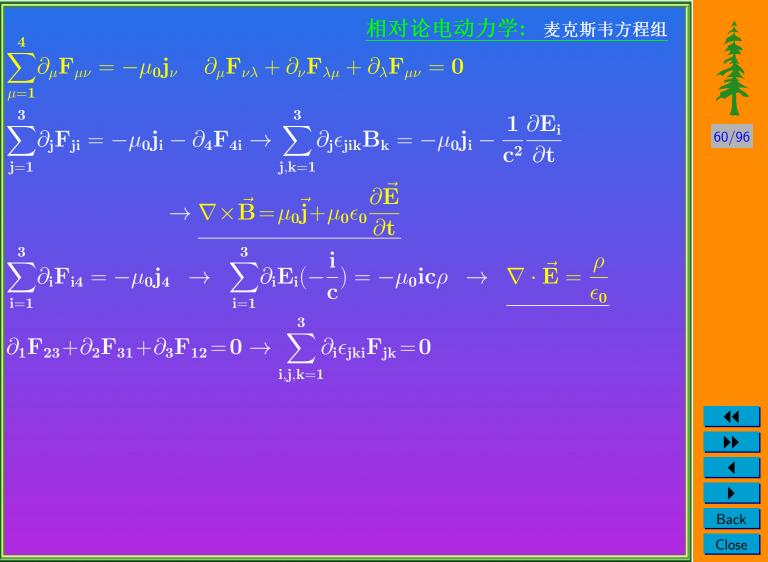


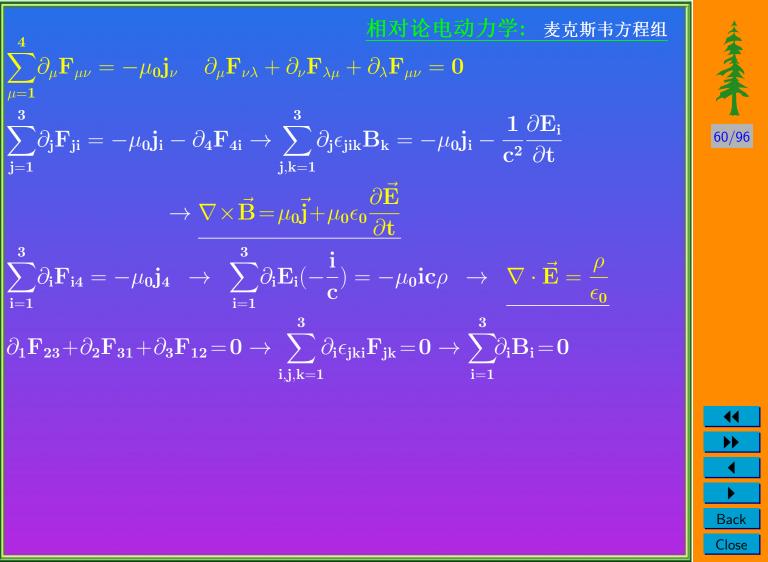


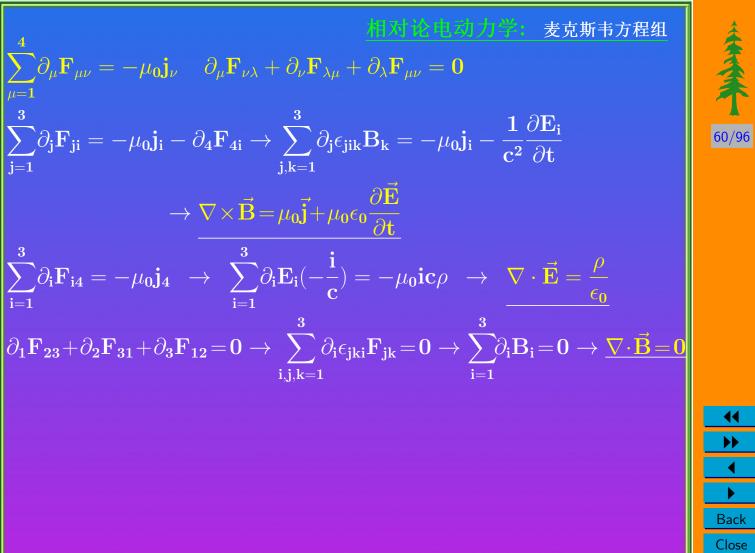


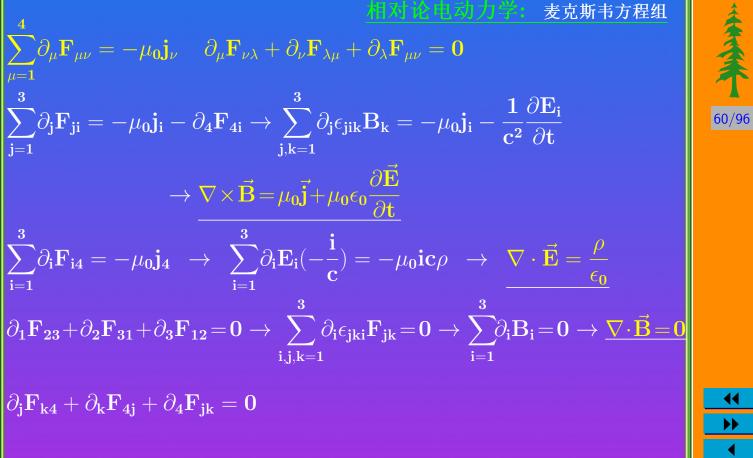












































麦克斯韦方程组  $\boxed{\sum} \partial_{\mu} \mathbf{F}_{\mu\nu} = -\mu_{\mathbf{0}} \mathbf{j}_{\nu} \quad \partial_{\mu} \mathbf{F}_{\nu\lambda} + \partial_{\nu} \mathbf{F}_{\lambda\mu} + \partial_{\lambda} \mathbf{F}_{\mu\nu} = \mathbf{0}$  $\sum_{\mathbf{j}=1}^{3} \partial_{\mathbf{j}} \mathbf{F}_{\mathbf{j}\mathbf{i}} = -\mu_{\mathbf{0}} \mathbf{j}_{\mathbf{i}} - \partial_{\mathbf{4}} \mathbf{F}_{\mathbf{4}\mathbf{i}} \rightarrow \sum_{\mathbf{i} \ \mathbf{k}=1}^{3} \partial_{\mathbf{j}} \epsilon_{\mathbf{j}\mathbf{i}\mathbf{k}} \mathbf{B}_{\mathbf{k}} = -\mu_{\mathbf{0}} \mathbf{j}_{\mathbf{i}} - \frac{1}{\mathbf{c}^{2}} \frac{\partial \mathbf{E}_{\mathbf{i}}}{\partial \mathbf{t}}$ 60/96  $\rightarrow \nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t}$  $\left| \sum_{i=1}^{3} \partial_{i} \mathbf{F}_{i4} = -\mu_{0} \mathbf{j}_{4} \right| \rightarrow \left| \sum_{i=1}^{3} \partial_{i} \mathbf{E}_{i} \left( -\frac{\mathbf{i}}{\mathbf{c}} \right) \right| = -\mu_{0} \mathbf{i} \mathbf{c} \rho \rightarrow \nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_{0}}$  $\partial_{1}F_{23} + \partial_{2}F_{31} + \partial_{3}F_{12} = 0 \rightarrow \sum \partial_{i}\epsilon_{jki}F_{jk} = 0 \rightarrow \sum \partial_{i}B_{i} = 0 \rightarrow \frac{\nabla \cdot \vec{B} = 0}{}$  $egin{aligned} \partial_{\mathbf{j}}\mathbf{F_{k4}} + \partial_{\mathbf{k}}\mathbf{F_{4j}} + \partial_{\mathbf{4}}\mathbf{F_{jk}} &= \mathbf{0} 
ightarrow \ \sum^{3} \epsilon_{\mathbf{ijk}}\partial_{\mathbf{j}}(\mathbf{icF_{k4}}) = -\mathbf{ic}\partial_{\mathbf{4}}(rac{1}{2}\sum^{3} \epsilon_{\mathbf{jki}}\mathbf{F_{jk}}) \end{aligned}$ i.k=1Back

麦克斯韦方程组  $\sum \partial_{\mu} \mathbf{F}_{\mu\nu} = -\mu_{\mathbf{0}} \mathbf{j}_{\nu} \quad \partial_{\mu} \mathbf{F}_{\nu\lambda} + \partial_{\nu} \mathbf{F}_{\lambda\mu} + \partial_{\lambda} \mathbf{F}_{\mu\nu} = \mathbf{0}$  $\sum_{\mathbf{j}=1}^{3} \partial_{\mathbf{j}} \mathbf{F}_{\mathbf{j}\mathbf{i}} = -\mu_{\mathbf{0}} \mathbf{j}_{\mathbf{i}} - \partial_{\mathbf{4}} \mathbf{F}_{\mathbf{4}\mathbf{i}} \rightarrow \sum_{\mathbf{i} \ \mathbf{k}=1}^{3} \partial_{\mathbf{j}} \epsilon_{\mathbf{j}\mathbf{i}\mathbf{k}} \mathbf{B}_{\mathbf{k}} = -\mu_{\mathbf{0}} \mathbf{j}_{\mathbf{i}} - \frac{1}{\mathbf{c}^{2}} \frac{\partial \mathbf{E}_{\mathbf{i}}}{\partial \mathbf{t}}$ 60/96  $\rightarrow \nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t}$  $\sum_{\mathbf{i}=1}^{\mathbf{j}} \partial_{\mathbf{i}} \mathbf{F}_{\mathbf{i}4} = -\mu_{\mathbf{0}} \mathbf{j}_{\mathbf{4}} \quad \rightarrow \quad \sum_{\mathbf{i}=1}^{\mathbf{3}} \partial_{\mathbf{i}} \mathbf{E}_{\mathbf{i}} (-\frac{\mathbf{i}}{\mathbf{c}}) = -\mu_{\mathbf{0}} \mathbf{i} \mathbf{c} \rho \quad \rightarrow \quad \nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\mathbf{c}}$  $\boxed{\partial_1 F_{23} + \partial_2 F_{31} + \partial_3 F_{12} \!=\! 0 \to \sum \partial_i \epsilon_{jki} F_{jk}} \!=\! 0 \to \sum \!\! \partial_i B_i \!=\! 0 \to \underline{\nabla} \cdot \underline{\vec{B}} \!=\! 0}$  $egin{aligned} \partial_{\mathbf{j}}\mathbf{F_{k4}} + \partial_{\mathbf{k}}\mathbf{F_{4j}} + \partial_{\mathbf{4}}\mathbf{F_{jk}} &= \mathbf{0} 
ightarrow \sum^{\mathbf{3}} \epsilon_{\mathbf{ijk}} \partial_{\mathbf{j}}(\mathbf{icF_{k4}}) = -\mathbf{ic}\partial_{\mathbf{4}}(rac{1}{2}\sum^{\mathbf{3}} \epsilon_{\mathbf{jki}}\mathbf{F_{jk}}) \end{aligned}$  $ightarrow \sum \epsilon_{\mathbf{i}\mathbf{j}\mathbf{k}} \partial_{\mathbf{j}} \mathbf{E}_{\mathbf{k}} = -rac{\partial}{\partial \mathbf{t}} \mathbf{B}_{\mathbf{i}}$ 













麦克斯韦方程组  $\sum \partial_{\mu} \mathbf{F}_{\mu\nu} = -\mu_{\mathbf{0}} \mathbf{j}_{\nu} \quad \partial_{\mu} \mathbf{F}_{\nu\lambda} + \partial_{\nu} \mathbf{F}_{\lambda\mu} + \partial_{\lambda} \mathbf{F}_{\mu\nu} = \mathbf{0}$  $\sum_{\mathbf{j}=1}^{3} \partial_{\mathbf{j}} \mathbf{F}_{\mathbf{j}\mathbf{i}} = -\mu_{\mathbf{0}} \mathbf{j}_{\mathbf{i}} - \partial_{\mathbf{4}} \mathbf{F}_{\mathbf{4}\mathbf{i}} \rightarrow \sum_{\mathbf{i} \ \mathbf{k}=1}^{3} \partial_{\mathbf{j}} \epsilon_{\mathbf{j}\mathbf{i}\mathbf{k}} \mathbf{B}_{\mathbf{k}} = -\mu_{\mathbf{0}} \mathbf{j}_{\mathbf{i}} - \frac{1}{\mathbf{c}^{2}} \frac{\partial \mathbf{E}_{\mathbf{i}}}{\partial \mathbf{t}}$ 60/96  $\rightarrow \nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t}$  $\sum_{\mathbf{i}=\mathbf{1}} \partial_{\mathbf{i}} \mathbf{F}_{\mathbf{i}\mathbf{4}} = -\mu_{\mathbf{0}} \mathbf{j}_{\mathbf{4}} \quad \rightarrow \quad \sum_{\mathbf{i}=\mathbf{1}}^{\mathbf{3}} \partial_{\mathbf{i}} \mathbf{E}_{\mathbf{i}} (-\frac{\mathbf{i}}{\mathbf{c}}) = -\mu_{\mathbf{0}} \mathbf{i} \mathbf{c} \rho \quad \rightarrow \quad \nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{c}$  $\boxed{\partial_1 F_{23} + \partial_2 F_{31} + \partial_3 F_{12} = 0 \rightarrow \sum \partial_i \epsilon_{jki} F_{jk}} = 0 \rightarrow \sum \partial_i B_i = 0 \rightarrow \boxed{\nabla \cdot \vec{B}} = 0}$  $egin{aligned} \partial_{\mathbf{j}}\mathbf{F_{k4}} + \partial_{\mathbf{k}}\mathbf{F_{4j}} + \partial_{\mathbf{4}}\mathbf{F_{jk}} &= \mathbf{0} 
ightarrow \ \sum^{3} \epsilon_{\mathbf{ijk}}\partial_{\mathbf{j}}(\mathbf{icF_{k4}}) = -\mathbf{ic}\partial_{\mathbf{4}}(rac{1}{2}\sum^{3} \epsilon_{\mathbf{jki}}\mathbf{F_{jk}}) \end{aligned}$  $ho + \sum_{i=1}^{3} \epsilon_{ijk} \partial_{j} \mathbf{E}_{k} = -\frac{\partial}{\partial t} \mathbf{B}_{i} 
ightarrow \mathbf{\nabla} imes \mathbf{\vec{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}$ Back Close

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# $abla imes \vec{\mathbf{E}} = -rac{\partial \vec{\mathbf{B}}}{\partial t}$ $abla imes \vec{\mathbf{E}} = -rac{\partial \vec{\mathbf{B}}}{\partial t}$ $abla imes \vec{\mathbf{E}} = 0$

$$\nabla \cdot (\epsilon_0 \vec{\mathbf{E}} + \vec{\mathbf{P}}) = \rho_f \qquad \qquad \nabla \times (\frac{1}{\mu_0} \vec{\mathbf{B}} - \vec{\mathbf{M}}) = \vec{\mathbf{j}}_c + \frac{\partial}{\partial t} (\epsilon_0 \vec{\mathbf{E}} + \vec{\mathbf{P}})$$

$$\vec{\mathbf{E}} \Rightarrow \frac{1}{\epsilon_0} \vec{\mathbf{P}}$$
  $\vec{\mathbf{B}} \Rightarrow -\mu_0 \vec{\mathbf{M}}$ 

$$\mathbf{F}_{\mu
u} = egin{pmatrix} 0 & \mathbf{B}_3 & -\mathbf{B}_2 & -rac{\mathrm{i}}{\mathrm{c}} \mathbf{E}_1 \ -\mathbf{B}_3 & 0 & \mathbf{B}_1 & -rac{\mathrm{i}}{\mathrm{c}} \mathbf{E}_2 \ \mathbf{B}_2 & -\mathbf{B}_1 & 0 & -rac{\mathrm{i}}{\mathrm{c}} \mathbf{E}_3 \ rac{\mathrm{i}}{\mathrm{c}} \mathbf{E}_1 & rac{\mathrm{i}}{\mathrm{c}} \mathbf{E}_2 & rac{\mathrm{i}}{\mathrm{c}} \mathbf{E}_3 & 0 \end{pmatrix} egin{pmatrix} \mathbf{F}'_{\mu
u} = & \sum_{\lambda,\lambda'=1}^{4} \mathbf{a}_{\mu\lambda} \mathbf{F}_{\lambda\lambda'} \mathbf{a}_{
u\lambda'} \ \mathbf{F}'_{\mu
u} = & \sum_{\lambda,\lambda'=1}^{4} \mathbf{a}_{\mu\lambda} \mathbf{F}_{\lambda\lambda'} \mathbf{a}_{
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u\lambda'} \ \mathbf{F}'_{\mu
u} = & \sum_{\lambda,\lambda'=1}^{4} \mathbf{a}_{\mu\lambda'} \mathbf{F}_{\lambda\lambda'} \mathbf{a}_{
u\lambda'} \ \mathbf{F}'_{\mu\nu} = & \sum_{\lambda,\lambda'=1}^{4} \mathbf{A}_{\mu\lambda'} \mathbf{F}$$

$${
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m M}_2 & -{
m ic}{
m P}_1 \ {
m M}_3 & 0 & -{
m M}_1 & -{
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m P}_2 \ -{
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m P}_3 \ {
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m ic}{
m P}_2 & {
m ic}{
m P}_3 & 0 \end{array}
ight)$$







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## 

$$\nabla \cdot (\epsilon_0 \vec{\mathbf{E}} + \vec{\mathbf{P}}) = \rho_f \qquad \qquad \nabla \times (\frac{1}{\mu_0} \vec{\mathbf{B}} - \vec{\mathbf{M}}) = \vec{\mathbf{j}}_c + \frac{\partial}{\partial t} (\epsilon_0 \vec{\mathbf{E}} + \vec{\mathbf{P}})$$

$$\vec{\mathbf{E}} \Rightarrow \frac{1}{\epsilon_0} \vec{\mathbf{P}}$$
  $\vec{\mathbf{B}} \Rightarrow -\mu_0 \vec{\mathbf{M}}$ 

 $abla imes ec{\mathbf{E}} = -rac{\partial ec{\mathbf{B}}}{\partial \mathbf{t}}$ 

$$F_{\mu\nu} = \begin{pmatrix} 0 & B_3 & -B_2 & -\frac{\mathrm{i}}{c}E_1 \\ -B_3 & 0 & B_1 & -\frac{\mathrm{i}}{c}E_2 \\ B_2 & -B_1 & 0 & -\frac{\mathrm{i}}{c}E_3 \\ \frac{\mathrm{i}}{c}E_1 & \frac{\mathrm{i}}{c}E_2 & \frac{\mathrm{i}}{c}E_3 & 0 \end{pmatrix} \quad \begin{matrix} F'_{\mu\nu} = \sum_{\lambda,\lambda'=1}^4 a_{\mu\lambda}F_{\lambda\lambda'}a_{\nu\lambda'} \\ F'_{\mu\nu} = \sum_{\lambda,\lambda'=1}^4 a_{\mu\lambda}F_{\lambda\lambda'}a_{\lambda\lambda'} \\ F'_{\mu\nu} = \sum_{\lambda,\lambda'=1}^4 a_{\lambda\lambda'}F_{\lambda\lambda'}a_{\lambda\lambda'} \\ F'_{\lambda\nu} = \sum_{\lambda,\lambda'=1}^4 a_{\lambda\lambda$$

$$egin{aligned} \mathbf{M}_{\mu
u} \equiv egin{pmatrix} 0 & -\mathbf{M}_3 & \mathbf{M}_2 & -\mathbf{i}\mathbf{c}\mathbf{P}_1 \ \mathbf{M}_3 & 0 & -\mathbf{M}_1 & -\mathbf{i}\mathbf{c}\mathbf{P}_2 \ -\mathbf{M}_2 & \mathbf{M}_1 & 0 & -\mathbf{i}\mathbf{c}\mathbf{P}_3 \ \mathbf{i}\mathbf{c}\mathbf{P}_1 & \mathbf{i}\mathbf{c}\mathbf{P}_2 & \mathbf{i}\mathbf{c}\mathbf{P}_3 & 0 \end{pmatrix} egin{pmatrix} \mathbf{M}_{\mu
u}' = \sum_{\lambda, \lambda' = 1} \mathbf{a}_{\mu\lambda} \mathbf{M}_{\lambda\lambda'} \mathbf{a}_{
u\lambda'} \ \mathbf{M}_{\mu\nu}' = \mathbf{A}\mathbf{M}\mathbf{A}^{\mathrm{T}} \ \mathbf{M}_{\mu\nu}' = \mathbf{A}\mathbf{M}\mathbf{A}^{\mathrm{T}} \end{pmatrix} \end{aligned}$$

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的麦克别韦万桂组: 极化强度和磁化强度  $\nabla imes \vec{\mathbf{E}} = -\frac{\partial \mathbf{B}}{\partial t}$  $\nabla \cdot \vec{\mathbf{B}} = \mathbf{0}$  $abla imes (rac{1}{\mu_0} ec{\mathbf{B}} - ec{\mathbf{M}}) = ec{\mathbf{j}_c} + rac{\partial}{\partial \mathbf{t}} (\epsilon_0 ec{\mathbf{E}} + ec{\mathbf{P}})$  $\nabla \cdot (\epsilon_0 \vec{\mathbf{E}} + \vec{\mathbf{P}}) = \rho_f$  $\vec{\mathbf{E}} \Rightarrow \frac{1}{\epsilon_0} \vec{\mathbf{P}}$  $|\vec{\mathrm{B}}| \Rightarrow -\mu_0 \vec{\mathrm{M}}$  $F_{\mu\nu} = \begin{pmatrix} 0 & B_3 & -B_2 & -\frac{\mathrm{i}}{c}E_1 \\ -B_3 & 0 & B_1 & -\frac{\mathrm{i}}{c}E_2 \\ B_2 & -B_1 & 0 & -\frac{\mathrm{i}}{c}E_3 \\ \frac{\mathrm{i}}{c}E_1 & \frac{\mathrm{i}}{c}E_2 & \frac{\mathrm{i}}{c}E_3 & 0 \end{pmatrix} \quad \begin{array}{c} \mathbf{F}'_{\mu\nu} = \sum_{\lambda,\lambda'=1}^{4} \mathbf{a}_{\mu\lambda}\mathbf{F}_{\lambda\lambda'}\mathbf{a}_{\nu\lambda'} \\ \mathbf{F}'_{\mu\nu} = \sum_{\lambda,\lambda'=1}^{4} \mathbf{a}_{\mu\lambda'}\mathbf{F}_{\lambda\lambda'}\mathbf{a}_{\nu\lambda'} \\ \mathbf{F}'_{\mu\nu} = \sum_{\lambda,\lambda'=1}^{4} \mathbf{a}_{\mu\lambda'}\mathbf{F}_{\lambda\lambda'}\mathbf{a}_{\lambda\lambda'} \\ \mathbf{F}'_{\mu\nu} = \sum_{\lambda,\lambda'=1}^{4} \mathbf{a}_{\lambda\lambda'}\mathbf{F}_{\lambda\lambda'}\mathbf{a}_{\lambda\lambda'} \\ \mathbf{F}'_{\mu\nu$  $\mathbf{M}_{\mu
u} \equiv egin{pmatrix} 0 & -\mathbf{M}_3 & \mathbf{M}_2 & -\mathrm{icP}_1 \ \mathbf{M}_3 & 0 & -\mathbf{M}_1 & -\mathrm{icP}_2 \ -\mathbf{M}_2 & \mathbf{M}_1 & 0 & -\mathrm{icP}_3 \ \mathrm{icP}_1 & \mathrm{icP}_2 & \mathrm{icP}_3 & 0 \end{pmatrix} egin{pmatrix} \mathbf{M}'_{\mu
u} = \sum_{\lambda, \lambda' = 1} \mathbf{a}_{\mu\lambda} \mathbf{M}_{\lambda\lambda'} \mathbf{a}_{
u\lambda'} \ \mathbf{M}' = \mathbf{A} \mathbf{M} \mathbf{A}^{\mathbf{T}} \ \mathbf{M}' = \mathbf{A} \mathbf{M} \mathbf{A}^{\mathbf{T}} \end{pmatrix}$  $\sum \partial_{\mu} [\mathbf{F}_{\mu\nu} + \mu_{\mathbf{0}} \mathbf{M}_{\mu\nu}] = -\mu_{\mathbf{0}} \mathbf{j}_{
u}$  $\partial_{\mu}\mathbf{F}_{
u\lambda}+\partial_{
u}\mathbf{F}_{\lambda\mu}+\partial_{\lambda}\mathbf{F}_{\mu
u}=\mathbf{0}$ 

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#### 电动力学作用量的更深层次含义

• 电磁场的作用量中出现了 $\sum_{\mu,\nu=1}^{\mathbf{F}}\mathbf{F}_{\mu\nu}\mathbf{F}_{\mu\nu}$ 和 $\sum_{\mu,\nu,\sigma,\rho=1}\epsilon_{\mu\nu\sigma\rho}\mathbf{F}_{\mu\nu}\mathbf{F}_{\sigma\rho}$ 



#### 电动力学作用量的更深层次含义

- 电磁场的作用量中出现了 $\sum_{\mu,\nu=1}^{\mathbf{\Gamma}}\mathbf{\Gamma}_{\mu\nu}\mathbf{\Gamma}_{\mu\nu}$ 和 $\sum_{\mu,\nu,\sigma,\rho=1}^{\mathbf{\Gamma}}\epsilon_{\mu\nu\sigma\rho}\mathbf{F}_{\mu\nu}\mathbf{F}_{\sigma\rho}$
- $\epsilon_{\mu\nu\sigma\rho}\mathbf{F}_{\mu\nu}\mathbf{F}_{\sigma\rho}$ 是 $^{*20hh}\mathbf{K}_{\mathbf{K}_{\mathbf{M}}}\mathbf{K}_{\mathbf{K}_{\mathbf{M}}}$ ,数学研究很深入



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#### 电动力学作用量的更深层次含义

杨振宁: 2012年6月21日在清华高等研究院Fujikawa报告后的评论:

- 电磁场的作用量中出现了 $\sum_{\mu,\nu=1}^{\bullet} \mathbf{F}_{\mu\nu} \mathbf{F}_{\mu\nu}$ 和  $\sum_{\mu,\nu,\sigma,\rho=1}^{\bullet} \epsilon_{\mu\nu\sigma\rho} \mathbf{F}_{\mu\nu} \mathbf{F}_{\sigma\rho}$
- $\sum_{\mu,\nu,\sigma,\rho=1}^{7} \epsilon_{\mu\nu\sigma\rho} \mathbf{F}_{\mu\nu} \mathbf{F}_{\sigma\rho}$ 是著2的拓扑<mark>陈数</mark>陈省身,数学研究很深入
- $\sum_{\mu,\nu=1}^{\infty} \mathbf{F}_{\mu\nu} \mathbf{E}_{\mu\nu}$ 数学家开始未关注,后受物理启发开始研究
- 由此建立了著名的 <u>Donalson理论</u>!

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#### 电动力学作用量的更深层次含义

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- $\sum_{\mu,\nu,\sigma,\rho=1}^{\mathbf{r}} \epsilon_{\mu\nu\sigma\rho} \mathbf{F}_{\mu\nu} \mathbf{F}_{\sigma\rho}$ 是著2015年<mark>陈数陈省身,数学研究很深入</mark>
- $\sum_{\mu,\nu=1}^{\infty} \mathbf{F}_{\mu\nu} \mathbf{E}_{\mu\nu}$ 数学家开始未关注,后受物理启发开始研究
- 由此建立了著名的 <u>Donalson理论</u>!
- 发现: 「<sup>2</sup>在非四维时空和<mark>陈数</mark>类似,无特殊结构

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#### 电动力学作用量的更深层次含义

杨振宁: 2012年6月21日在清华高等研究院Pujikawa报告后的评论:

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- $\sum_{\mu,\nu,\sigma,\rho=1}^{7} \epsilon_{\mu\nu\sigma\rho} \mathbf{F}_{\mu\nu} \mathbf{F}_{\sigma\rho}$ 是著2的拓扑<mark>陈数</mark>陈省身,数学研究很深入
- $\sum_{\mu,\nu=1}^{\infty} \mathbf{F}_{\mu\nu} \mathbf{E}_{\mu\nu}$ 数学家开始未关注,后受物理启发开始研究
- 由此建立了著名的 <u>Donalson理论</u>!
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 $\mathbf{W}$  Witten效应: 电荷磁单极共生 $\mathbf{Z}\theta/\pi$ 为分数

量子场论: 
$$\theta \to \theta + 2n\pi$$
  $\alpha \equiv \frac{e^2}{\epsilon_0 \hbar}$ 

$$\Delta \mathcal{L} = \frac{1\theta c e^2}{32\pi^2 h} \sum_{\mu\nu\sigma\rho} \epsilon_{\mu\nu\sigma\rho} \mathbf{F}_{\mu\nu} \mathbf{F}_{\sigma\rho} =$$

 $\Delta \mathcal{L} = rac{\mathrm{i} heta \mathrm{ce}^2}{32\pi^2 \hbar} \sum \overline{\epsilon_{\mu
u\sigma
ho}} \mathrm{F}_{\mu
u} \mathrm{F}_{\sigma
ho} = -rac{\mathrm{i} \mathrm{ce}^2}{8\pi^2 \hbar} \sum \epsilon_{\mu
u\sigma
ho} (\partial_{\mu} heta) \mathrm{A}_{
u} \partial_{\sigma} \mathrm{A}_{
ho} \, .$  $= \qquad \theta rac{\mathrm{e}^2}{2\pi\mathrm{h}} ec{\mathrm{B}} \cdot ec{\mathrm{E}}$  $\sum_{\mu} \mathbf{A}_{\mu} \mathbf{\Delta} \mathbf{j}_{\mu} \ \rightarrow \ \mathbf{\Delta} \mathbf{j}_{\mu} = -\frac{\mathbf{i} \epsilon \epsilon_{\mathbf{0}} \alpha}{8\pi^{2}} \sum_{\nu \sigma \rho} \epsilon_{\mu \nu \sigma \rho} (\partial_{\nu} \theta) \partial_{\sigma} \mathbf{A}_{\rho}$ 全微商导致的 拓扑项

$$\sum_{\mu} \mathbf{A}_{\mu} \mathbf{\Delta} \mathbf{j}_{\mu} \; 
ightarrow$$

$$\sum_{\mu} \mathbf{A}_{\mu} \Delta \mathbf{j}_{\mu} \, 
ightarrow \, \Delta \mathbf{j}_{\mu} = -rac{\mathrm{i} \epsilon_{0} \alpha}{8\pi^{2}} \sum_{
u\sigma
ho} \epsilon_{\mu
u\sigma
ho} (\partial_{
u} heta) \partial_{\sigma} \mathbf{A}_{
ho} \ 
ightarrow \, \partial_{\mu} \vec{\mathbf{E}} \, 
ightarrow \, \partial_{\mu} \vec{\mathbf{E}} \, 
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ightarrow \, \partial_{\sigma} \vec{\mathbf{E}} \, 
ightarrow \, \partial_{\sigma} \vec{\mathbf{E}} \, 
ightarrow \, \partial_{\sigma} \vec{\mathbf{E}} \, \ \partial_{\sigma} \vec{\mathbf{E}}$$

$$\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0} \underbrace{-\frac{\alpha}{4\pi^2} \nabla \theta \cdot \vec{\mathbf{B}}}_{\text{不匀}\theta + \mathbf{B}\mathbf{B}\mathbf{B}\mathbf{F}\mathbf{E} + \mathbf{b}\mathbf{d}} \qquad \nabla \times \vec{\mathbf{B}} = \mu_0 [\epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}} + \vec{\mathbf{j}} \underbrace{+\frac{\alpha \epsilon_0}{4\pi^2} (\nabla \theta \times \vec{\mathbf{E}} + \frac{\partial \theta}{\partial \mathbf{t}} \vec{\mathbf{B}})}_{\text{不匀}\theta + \mathbf{B}\mathbf{B}\mathbf{F}\mathbf{E} + \mathbf{b}\mathbf{d}\mathbf{d}}]$$







Back Close 例: Witten效应: 电荷磁单极共生及 $heta/\pi$ 为分数 $-rac{\mathrm{ice}^2}{8\pi^2\hbar}\sum\epsilon_{\mu
u\sigma
ho}(\partial_{\mu} heta)\mathbf{A}_{
u}\partial_{\sigma}\mathbf{A}_{
ho} = hetarac{\mathrm{e}^2}{2\pi\hbar}ec{\mathbf{B}}\cdotec{\mathbf{E}}$ 

$$\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0} \underbrace{-\frac{\alpha}{4\pi^2} \nabla \theta \cdot \vec{\mathbf{B}}}_{\text{TSIGNER FOR ELLER}} \qquad \nabla \times \vec{\mathbf{B}} = \mu_0 [\epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t} + \vec{\mathbf{j}} \underbrace{+\frac{\alpha \epsilon_0}{4\pi^2} (\nabla \theta \times \vec{\mathbf{E}} + \frac{\partial \theta}{\partial t} \vec{\mathbf{B}})}_{\text{TSIGNER FOR ELLER}}]$$

$$ec{\mathbf{P}} = rac{lpha \epsilon_{\mathbf{0}}}{4\pi^{2}} heta ec{\mathbf{B}} \quad -rac{\mathbf{i} \mathbf{c} \epsilon_{\mathbf{0}} lpha}{8\pi^{2}} \sum_{
u \sigma 
ho} \partial_{
u} (\epsilon_{\mu 
u \sigma 
ho} heta \partial_{\sigma} \mathbf{A}_{
ho}) = \Delta \mathbf{j}_{\mu} = \partial_{
u} \mathbf{M}_{
u \mu} \quad \vec{\mathbf{M}} = rac{lpha \epsilon_{\mathbf{0}}}{4\pi^{2}} heta ec{\mathbf{E}}$$

量子场论:  $\theta \to \theta + 2n\pi$   $\alpha \equiv \frac{e^2}{6\pi\hbar}$ 









Witten效应: 电荷磁单极共生及 $\theta/\pi$ 为分数  $-\frac{ice^2}{2}\sum_{\delta_{m,m}}(\partial_{\alpha}\theta)\Delta_{\alpha}\partial_{\alpha}\Delta_{\alpha} = \theta \frac{e^2}{2}\vec{B}\cdot\vec{E}$ 

$$\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0} - \frac{\alpha}{4\pi^2} \nabla \theta \cdot \vec{\mathbf{B}} \qquad \nabla \times \vec{\mathbf{B}} = \mu_0 \left[ \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t} + \vec{\mathbf{j}} + \frac{\alpha \epsilon_0}{4\pi^2} (\nabla \theta \times \vec{\mathbf{E}} + \frac{\partial \theta}{\partial t} \vec{\mathbf{B}}) \right]$$

$$ec{\mathbf{P}} = rac{lpha \epsilon_{\mathbf{0}}}{4\pi^2} heta ec{\mathbf{B}} \quad -rac{\mathbf{i} \mathbf{c} \epsilon_{\mathbf{0}} lpha}{8\pi^2} \sum_{
u} \partial_{
u} (\epsilon_{\mu
u\sigma
ho} heta \partial_{\sigma} \mathbf{A}_{
ho}) = \Delta \mathbf{j}_{\mu} = \partial_{
u} \mathbf{M}_{
u\mu} \quad \vec{\mathbf{M}} = rac{lpha \epsilon_{\mathbf{0}}}{4\pi^2} heta ec{\mathbf{E}}$$

$$\mu$$
世者介质中:  $\vec{\mathbf{j}} = \mathbf{0}$ ,  $\nabla \theta = \mathbf{0} \Rightarrow \mu_0 \epsilon_0 \frac{\partial \nabla \cdot \vec{\mathbf{E}}}{\partial \mathbf{t}} + \frac{\alpha \epsilon_0}{4\pi^2} \frac{\partial \theta \nabla \cdot \vec{\mathbf{B}}}{\partial \mathbf{t}} = \mathbf{0}$ 

量子场论:  $\theta \to \theta + 2n\pi$   $\alpha \equiv \frac{e^2}{6\pi\hbar}$ 









 $\mathbf{W}$  Witten效应: 电荷磁单极共生 $\mathbf{Z}\theta/\pi$ 为分数  $oxed{\Delta \mathcal{L} = rac{\mathrm{i} heta \mathrm{ce}^2}{32\pi^2 \hbar} \sum \epsilon_{\mu
u\sigma
ho} \mathrm{F}_{\mu
u} \mathrm{F}_{\sigma
ho} = -rac{\mathrm{i} \mathrm{ce}^2}{8\pi^2 \hbar} \sum \epsilon_{\mu
u\sigma
ho} (\partial_{\mu} heta) \mathrm{A}_{
u} \partial_{\sigma} \mathrm{A}_{
ho}}$  $= \theta rac{\mathrm{e}^2}{2\pi\mathrm{h}} ec{\mathrm{B}} \cdot ec{\mathrm{E}}$ 

$$\sum_{\mu} \mathbf{A}_{\mu} \Delta \mathbf{j}_{\mu} 
ightarrow \Delta \mathbf{j}_{\mu} = -rac{\mathrm{icc_0} lpha}{8\pi^2} \sum_{
u\sigma
ho} \epsilon_{\mu
u\sigma
ho} (\partial_{
u} heta) \partial_{\sigma} \mathbf{A}_{
ho}$$
  $\nabla \cdot \vec{\mathbf{E}} = rac{
ho}{\epsilon_0} - rac{lpha}{4\pi^2} 
abla heta \cdot \vec{\mathbf{B}}$   $\nabla imes \vec{\mathbf{B}} = \mu_0 [\epsilon_0 rac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}} + \vec{\mathbf{j}} + rac{lpha \epsilon_0}{4\pi^2} (
abla imes \vec{\mathbf{E}} + rac{\partial heta}{\partial \mathbf{t}} \vec{\mathbf{B}})]$ 

不匀
$$heta$$
中磁场产生电荷 不匀 $heta$ 中电场或变化 $heta$ 中的磁场产生电流  $ec{\mathbf{P}} = rac{lpha \epsilon_0}{4\pi^2} heta ec{\mathbf{B}} \qquad -rac{\mathbf{i} \mathbf{c} \epsilon_0 lpha}{\mathbf{c} \pi^2} \sum \partial_{
u} (\epsilon_{\mu 
u \sigma 
ho} heta \partial_{\sigma} \mathbf{A}_{
ho}) = \Delta \mathbf{j}_{\mu} = \partial_{
u} \mathbf{M}_{
u \mu} \quad \vec{\mathbf{M}} = rac{lpha \epsilon_0}{4\pi^2} heta \vec{\mathbf{E}}$ 

量子场论:  $\theta \to \theta + 2n\pi$   $\alpha \equiv \frac{e^2}{6\pi\hbar}$ 

進一步若介质中: 
$$\vec{\mathbf{j}} = \mathbf{0}, \quad \nabla \theta = \mathbf{0} \ \Rightarrow \ \mu_0 \epsilon_0 \frac{\partial \nabla \cdot \vec{\mathbf{E}}}{\partial \mathbf{t}} + \frac{\alpha \epsilon_0}{4\pi^2} \frac{\partial \theta \nabla \cdot \vec{\mathbf{B}}}{\partial \mathbf{t}} = \mathbf{0}$$

$$rac{\mathrm{e}^2}{2\pi\mathrm{h}} heta\mathrm{q}_\mathrm{m} + \mathrm{q} = \int_\mathrm{V}^\mathrm{d} \mathbf{x} \, \mathrm{V} \cdot \mathbf{D} 
ightarrow rac{\partial}{\partial \mathrm{t}} (rac{\partial}{4\pi^2} \ell \mathrm{q}_\mathrm{m} + \mathrm{q}) - rac{\mathrm{e}^2}{2\pi\mathrm{h}} heta\mathrm{q}_\mathrm{m} + \mathrm{q} = = \equiv \equiv = 0 \quad \Rightarrow \quad$$
电荷与磁单极共生:双荷子dyon



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 $\mathbf{W}$  Witten效应: 电荷磁单极共生 $\mathbf{Z} \theta / \pi$ 为分数  $\Delta \mathcal{L} = rac{\mathrm{i} heta \mathrm{ce}^2}{32 \pi^2 \hbar} \sum \epsilon_{\mu 
u \sigma 
ho} \mathrm{F}_{\mu 
u} \mathrm{F}_{\sigma 
ho} = - rac{\mathrm{i} \mathrm{ce}^2}{8 \pi^2 \hbar} \sum \epsilon_{\mu 
u \sigma 
ho} (\partial_{\mu} heta) \mathrm{A}_{
u} \partial_{\sigma} \mathrm{A}_{
ho} \, .$  $= heta rac{\mathrm{e}^2}{2\pi \mathrm{h}} ec{\mathrm{B}} \cdot ec{\mathrm{E}}$ 

$$\mathbf{x}_{\mu\nu\sigma\rho}$$
 全微商导致的  $\mathbf{x}_{\mu\nu\sigma\rho}$  全微商导致的  $\mathbf{x}_{\mu\nu\sigma\rho}$   $\mathbf{x}_{\mu\nu\sigma}$   $\mathbf{x}_{\mu\nu\sigma}$ 

$$abla\cdotec{\mathbf{E}} = rac{
ho}{\epsilon_0}\underbrace{-rac{lpha}{4\pi^2}
abla heta\cdot\mathbf{B}}_{\mathbf{K}eta heta heta}$$
 $abla imes\mathbf{B} = \mu_0[\epsilon_0rac{\partialec{\mathbf{E}}}{\partial\mathbf{t}} + ec{\mathbf{j}}\underbrace{+rac{lpha\epsilon_0}{4\pi^2}(
abla heta imesec{\mathbf{E}} + rac{\partial heta}{\partial\mathbf{t}}ec{\mathbf{B}})]}_{\mathbf{K}eta heta heta heta heta}$ 
 $abla\cdotec{\mathbf{P}} = rac{lpha\epsilon_0}{4\pi^2} hetaec{\mathbf{B}}$ 
 $abla\cdotec{\mathbf{B}} = \frac{\mathbf{D}\cdot\mathbf{B}}{\mathbf{B}} - rac{\mathbf{D}\cdot\mathbf{B}}{\mathbf{B}}$ 
 $abla\cdot\mathbf{B} = \frac{\mathbf{D}\cdot\mathbf{B}}{\mathbf{B}} - rac{\mathbf{D}\cdot\mathbf{B}}{\mathbf{B}}$ 
 $abla\cdot\mathbf{B} = \mathbf{D}\cdot\mathbf{B}$ 
 $abla\cdot\mathbf{B} = \mathbf{D}\cdot\mathbf{B}$ 

進一步若介质中: 
$$ec{\mathbf{j}} = \mathbf{0}, \quad 
abla heta = \mathbf{0} \quad \Rightarrow \quad \mu_0 \epsilon_0 \frac{\partial 
abla \cdot \vec{\mathbf{E}}}{\partial \mathbf{t}} + \frac{\alpha \epsilon_0}{4\pi^2} \frac{\partial \theta 
abla \cdot \vec{\mathbf{B}}}{\partial \mathbf{t}} = \mathbf{0}$$

量子场论:  $\theta \to \theta + 2n\pi$   $\alpha \equiv \frac{e^2}{6\pi\hbar}$ 









Back Close

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u \sigma 
ho} \mathrm{F}_{\mu 
u} \mathrm{F}_{\sigma 
ho} = - rac{\mathrm{i} \mathrm{ce}^2}{8 \pi^2 \hbar} \sum \epsilon_{\mu 
u \sigma 
ho} (\partial_{\mu} heta) \mathrm{A}_{
u} \partial_{\sigma} \mathrm{A}_{
ho} \, .$  $= heta rac{\mathrm{e}^2}{2\pi \mathrm{h}} ec{\mathrm{B}} \cdot ec{\mathrm{E}}$ 

$$\Sigma_{\mu}\mathbf{A}_{\mu}\mathbf{\Delta}\mathbf{j}_{\mu}$$
  $\rightarrow$   $\mathbf{\Delta}\mathbf{j}_{\mu}=-rac{\mathrm{icc_{0}lpha}}{8\pi^{2}}\sum_{
u\sigma
ho}\epsilon_{\mu
u\sigma
ho}(\partial_{
u} heta)\partial_{\sigma}\mathbf{A}_{
ho}$   $\nabla\cdot\vec{\mathbf{E}}=rac{
ho}{\epsilon_{0}}-rac{lpha}{4\pi^{2}}
abla heta\cdot\vec{\mathbf{B}}$   $\nabla imes\vec{\mathbf{B}}=\mu_{0}[\epsilon_{0}rac{\partialec{\mathbf{E}}}{\partial\mathbf{t}}+ec{\mathbf{j}}+rac{lpha\epsilon_{0}}{4\pi^{2}}(
abla heta imesec{\mathbf{E}}+rac{\partial heta}{\partial\mathbf{t}}ec{\mathbf{B}})]$ 

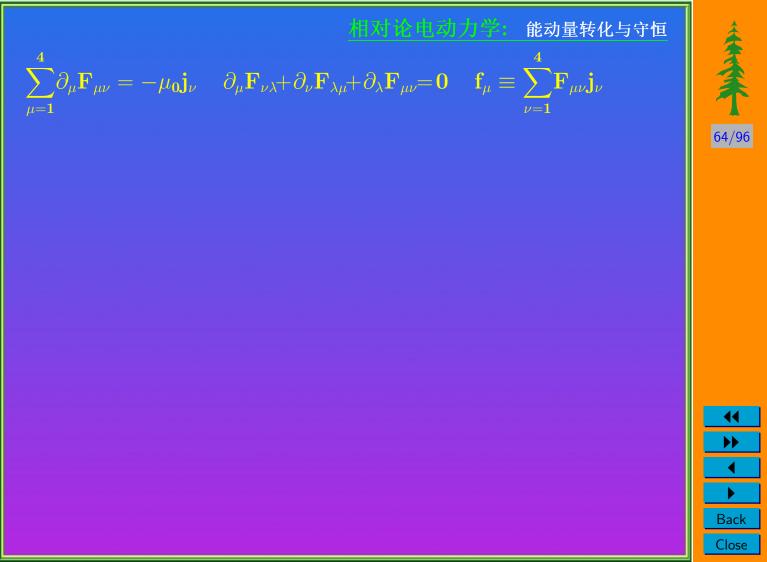
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$$heta$$
中藏场产生电荷 不句 $heta$ 中电场或变化 $heta$ 中的磁场产生电流  $ec{\mathbf{P}} = rac{lpha \epsilon_0}{4\pi^2} heta ec{\mathbf{B}} \quad -rac{\mathbf{i} \mathbf{c} \epsilon_0 lpha}{8\pi^2} \sum \partial_{
u} (\epsilon_{\mu
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ho} heta \partial_{\sigma} \mathbf{A}_{
ho}) = \Delta \mathbf{j}_{\mu} = \partial_{
u} \mathbf{M}_{
u\mu} \quad \vec{\mathbf{M}} = rac{lpha \epsilon_0}{4\pi^2} heta ec{\mathbf{E}}$ 

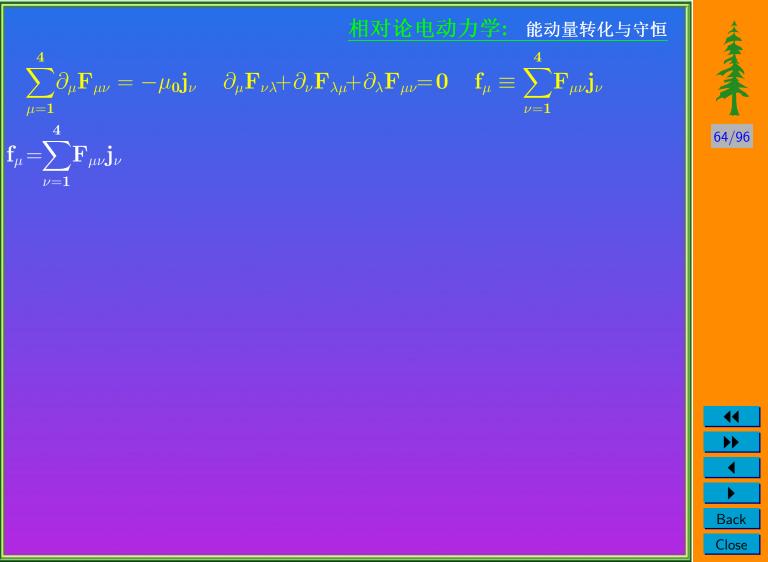
量子场论:  $\theta \to \theta + 2n\pi$   $\alpha \equiv \frac{e^2}{6\pi\hbar}$ 

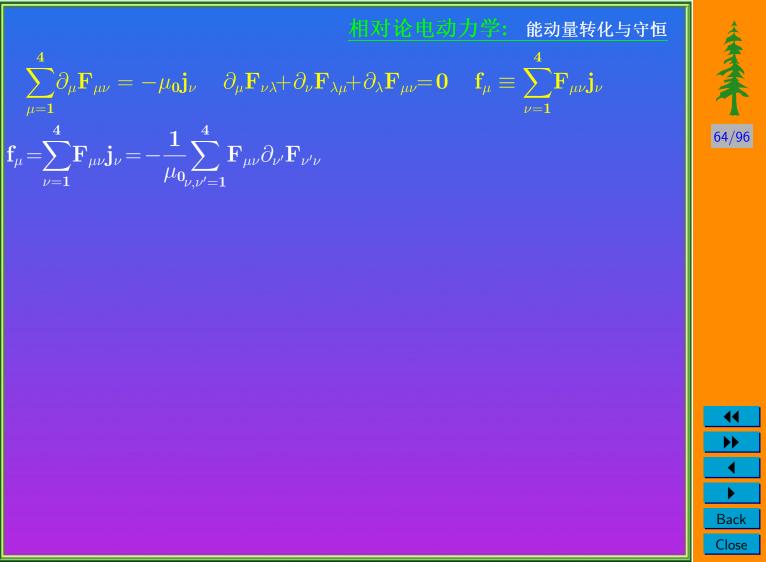
幾一步若介质中: 
$$ec{\mathbf{j}} = \mathbf{0}, \;\; 
abla heta = \mathbf{0} \;\; \Rightarrow \;\; \mu_0 \epsilon_0 \frac{\partial 
abla \cdot ec{\mathbf{E}}}{\partial \mathbf{t}} + \frac{\alpha \epsilon_0}{4\pi^2} \frac{\partial \theta 
abla \cdot ec{\mathbf{B}}}{\partial \mathbf{t}} = \mathbf{0}$$

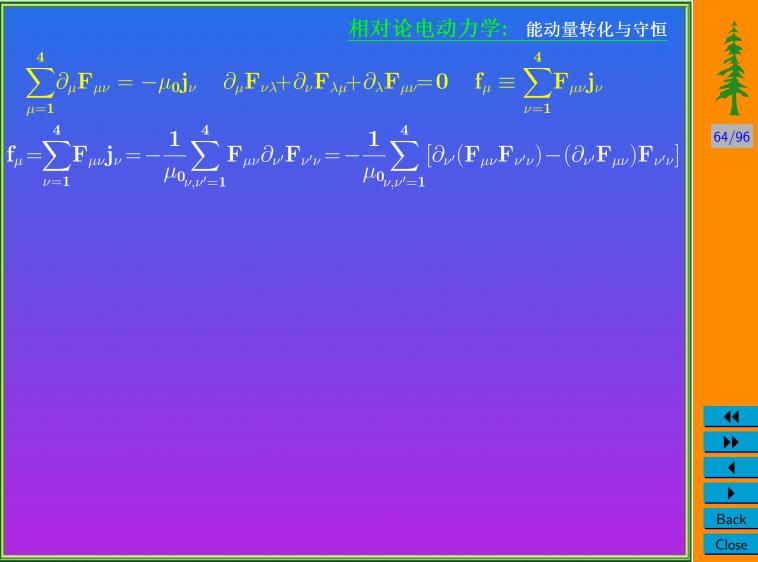
தெருந்ததாகள்  $\mathbf{q} \equiv \int_{\mathbf{V}} \mathbf{d}^3 \mathbf{x} \epsilon_0 
abla \cdot ec{\mathbf{E}} \;\;\; \mathbf{q}_{\mathrm{m}} \equiv \int_{\mathbf{V}} \mathbf{d}^3 \mathbf{x} 
abla \cdot ec{\mathbf{B}} \Rightarrow \frac{\partial}{\partial \mathbf{t}} (\frac{\alpha \epsilon_0}{4\pi^2} \theta \mathbf{q}_{\mathrm{m}} + \mathbf{q}) = \mathbf{0}$ 

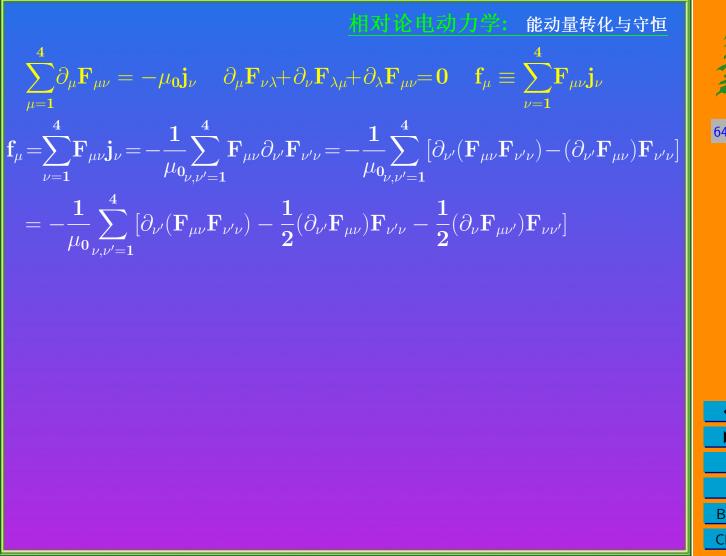
63/96



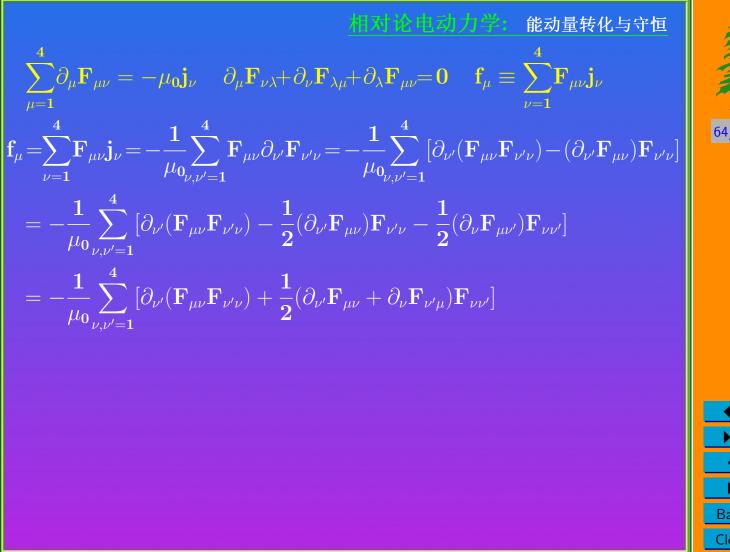




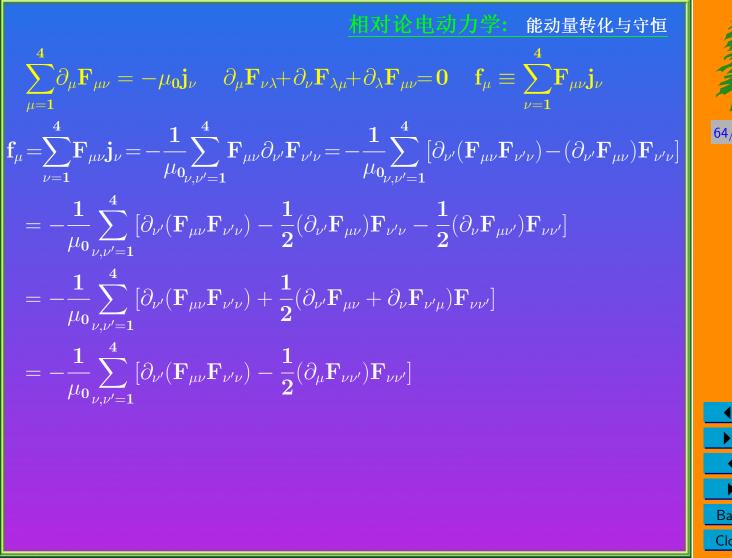














能动量转化与守恒  $\sum \partial_{\mu} \mathbf{F}_{\mu\nu} = -\mu_{\mathbf{0}} \mathbf{j}_{\nu} \quad \partial_{\mu} \mathbf{F}_{\nu\lambda} + \partial_{\nu} \mathbf{F}_{\lambda\mu} + \partial_{\lambda} \mathbf{F}_{\mu\nu} = \mathbf{0} \quad \mathbf{f}_{\mu} \equiv \sum \mathbf{F}_{\mu\nu} \mathbf{j}_{\nu}$ 64/96  $\left| \mathbf{f}_{\mu} = \sum_{\nu=1}^{4} \mathbf{F}_{\mu\nu} \mathbf{j}_{\nu} = -\frac{1}{\mu_{0,\nu,\nu'=1}} \sum_{\nu=1}^{4} \mathbf{F}_{\mu\nu} \partial_{\nu'} \mathbf{F}_{\nu'\nu} = -\frac{1}{\mu_{0,\nu,\nu'=1}} \sum_{\nu=1}^{4} \left[ \partial_{\nu'} (\mathbf{F}_{\mu\nu} \mathbf{F}_{\nu'\nu}) - (\partial_{\nu'} \mathbf{F}_{\mu\nu}) \mathbf{F}_{\nu'\nu} \right] \right|$ 

$$= -\frac{1}{\mu_0} \sum_{\nu,\nu'=1}^{4} \left[ \partial_{\nu'} (\mathbf{F}_{\mu\nu} \mathbf{F}_{\nu'\nu}) - \frac{1}{2} (\partial_{\nu'} \mathbf{F}_{\mu\nu}) \mathbf{F}_{\nu'\nu} - \frac{1}{2} (\partial_{\nu} \mathbf{F}_{\mu\nu'}) \mathbf{F}_{\nu\nu'} \right]$$

$$= -\frac{1}{\mu_0} \sum_{\nu,\nu'=1}^{4} \left[ \partial_{\nu'} (\mathbf{F}_{\mu\nu} \mathbf{F}_{\nu'\nu}) + \frac{1}{2} (\partial_{\nu'} \mathbf{F}_{\mu\nu} + \partial_{\nu} \mathbf{F}_{\nu'\mu}) \mathbf{F}_{\nu\nu'} \right]$$
1

 $=-rac{1}{\mu_0}\sum_{
u,
u'=1}^4[\partial_{
u'}(\mathbf{F}_{\mu
u}\mathbf{F}_{
u'
u})-rac{1}{2}(\partial_{\mu}\mathbf{F}_{
u
u'})\mathbf{F}_{
u
u'}]$  $=-rac{1}{\mu_0}\sum_{
u,
u'=1}^4\partial_{
u'}[\mathbf{F}_{\mu
u}\mathbf{F}_{
u'
u}-rac{1}{4}\delta_{\mu
u'}\sum_{
u''=1}^4\mathbf{F}_{
u
u''}\mathbf{F}_{
u
u''}]\equiv-\sum_{
u'=1}^4\partial_{
u'}\mathbf{T}_{\mu
u'}$ 

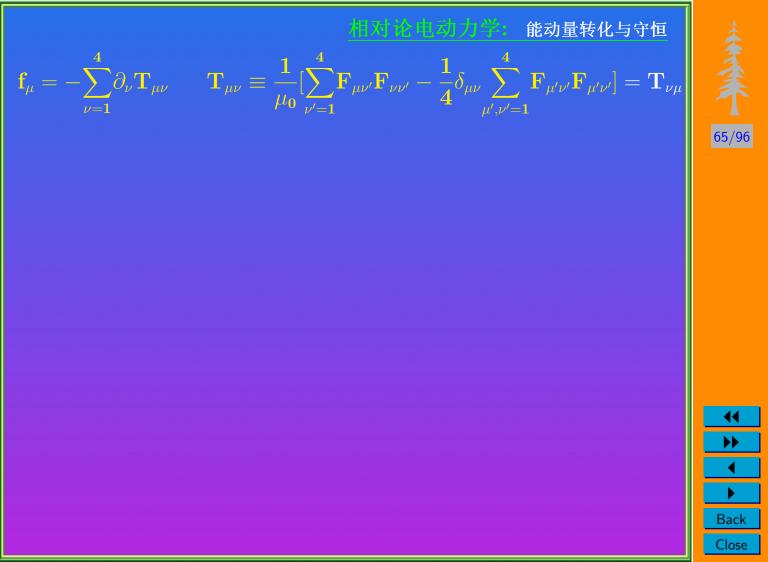


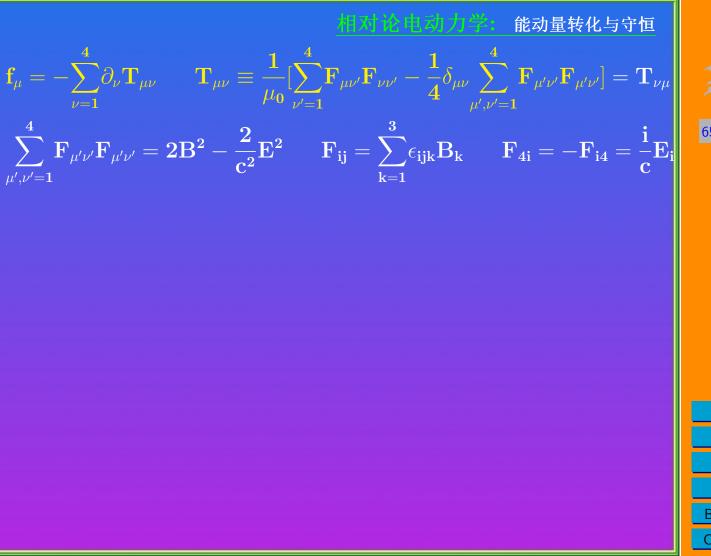




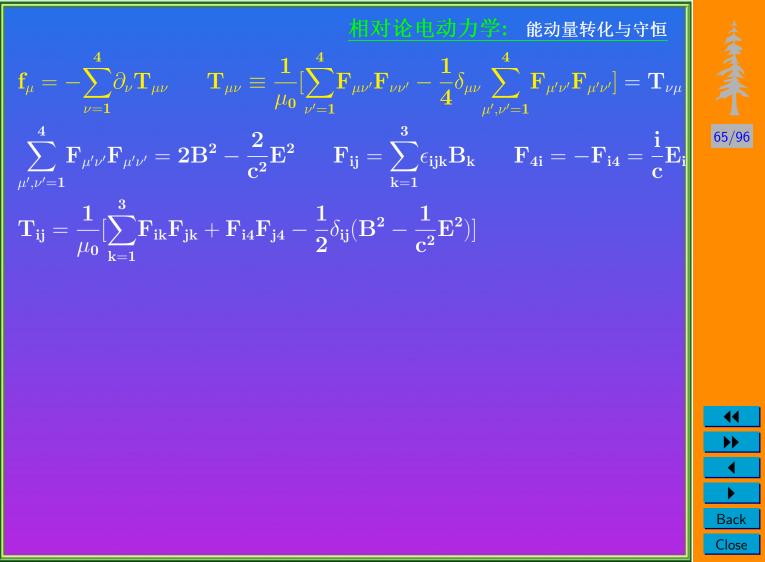


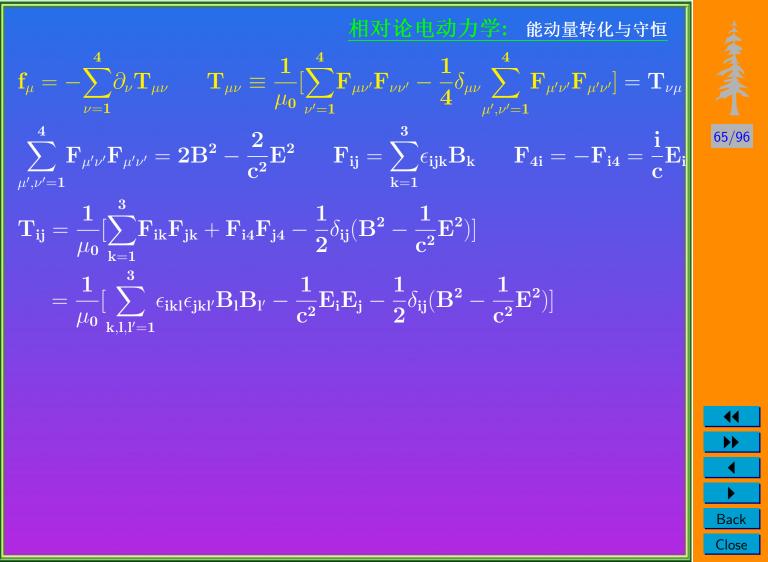


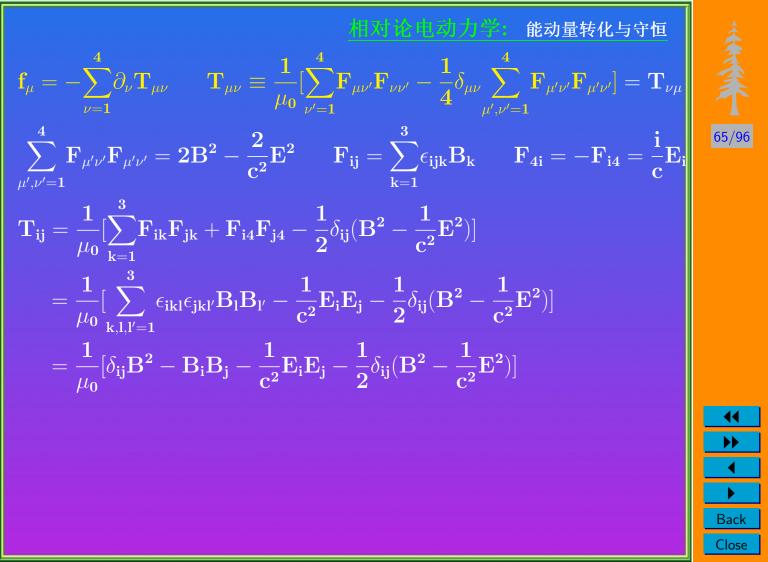


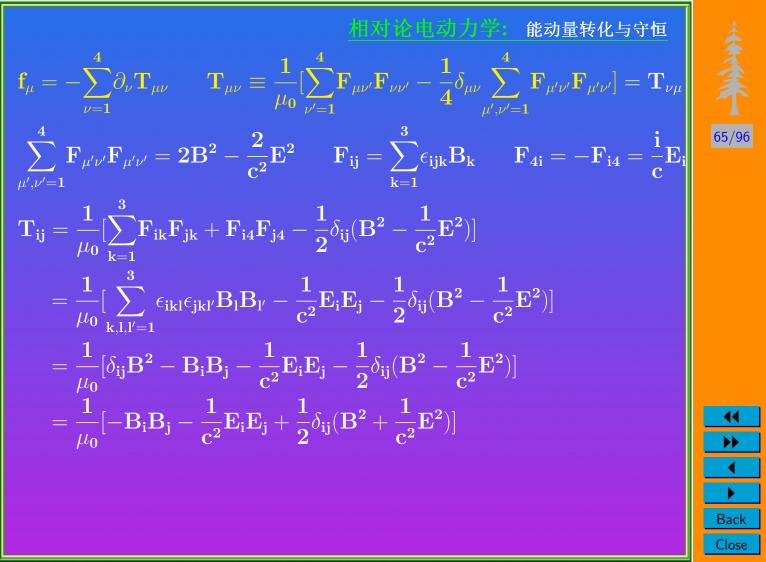












能动量转化与守恒  $\mathbf{f}_{\mu} = -\sum_{
u=1}^{4} \partial_{
u} \mathbf{T}_{\mu
u} \qquad \mathbf{T}_{\mu
u} \equiv rac{1}{\mu_{\mathbf{0}}} [\sum_{
u'=1}^{4} \mathbf{F}_{\mu
u'} \mathbf{F}_{
u
u'} - rac{1}{4} \delta_{\mu
u} \sum_{
u'=1}^{4} \mathbf{F}_{\mu'
u'} \mathbf{F}_{\mu'
u'}] = \mathbf{T}_{
u\mu}$ 65/96  $\sum_{i=1}^4 {
m F}_{\mu'
u'} {
m F}_{\mu'
u'} = 2 {
m B}^2 - rac{2}{{
m c}^2} {
m E}^2 \hspace{0.5cm} {
m F}_{ij} = \sum_{i=1}^3 \epsilon_{ijk} {
m B}_k \hspace{0.5cm} {
m F}_{4i} = - {
m F}_{i4} = rac{i}{{
m c}} {
m E}_i$  $egin{split} \mathbf{T_{ij}} &= rac{1}{\mu_0} [\sum_{\mathbf{k}=1}^3 \mathbf{F_{ik}} \mathbf{F_{jk}} + \mathbf{F_{i4}} \mathbf{F_{j4}} - rac{1}{2} \delta_{ij} (\mathbf{B^2} - rac{1}{\mathbf{c^2}} \mathbf{E^2})] \end{split}$  $=\frac{1}{\mu_0}[\sum_{\mathbf{k,l,l'}=1}^{3}\epsilon_{i\mathbf{k}\mathbf{l}}\epsilon_{j\mathbf{k}\mathbf{l'}}\mathbf{B_l}\mathbf{B_{l'}}-\frac{1}{\mathbf{c^2}}\mathbf{E_i}\mathbf{E_j}-\frac{1}{2}\delta_{i\mathbf{j}}(\mathbf{B^2}-\frac{1}{\mathbf{c^2}}\mathbf{E^2})]$  $\mathbf{E} = \frac{1}{\mu} [\delta_{ij} \mathbf{B^2} - \mathbf{B_i} \mathbf{B_j} - \frac{1}{\mathbf{c^2}} \mathbf{E_i} \mathbf{E_j} - \frac{1}{2} \delta_{ij} (\mathbf{B^2} - \frac{1}{\mathbf{c^2}} \mathbf{E^2})]$  $[=rac{1}{\mu_0}[-\mathbf{B_i}\mathbf{B_j}-rac{1}{\mathbf{c^2}}\mathbf{E_i}\mathbf{E_j}+rac{1}{2}\delta_{ij}(\mathbf{B^2}+rac{1}{\mathbf{c^2}}\mathbf{E^2})]$  $\mathbf{E} = \frac{1}{\mu_0} (-\mathbf{B_i}\mathbf{B_j} + \frac{1}{2}\delta_{ij}\mathbf{B^2}) + \epsilon_0 (-\mathbf{E_i}\mathbf{E_j} + \frac{1}{2}\delta_{ij}\mathbf{E^2})$ 

能动量转化与守恒  $\mathbf{f}_{\mu} = -\sum_{
u=1}^{3} \partial_{
u} \mathbf{T}_{\mu
u} \qquad \mathbf{T}_{\mu
u} \equiv rac{1}{\mu_{\mathbf{0}}} [\sum_{
u'=1}^{3} \mathbf{F}_{\mu
u'} \mathbf{F}_{
u
u'} - rac{1}{4} \delta_{\mu
u} \sum_{
u'=1}^{4} \mathbf{F}_{\mu'
u'} \mathbf{F}_{\mu'
u'}] = \mathbf{T}_{
u\mu}$ 65/96  $\sum_{i=1}^4 {
m F}_{\mu'
u'} {
m F}_{\mu'
u'} = 2 {
m B}^2 - rac{2}{{
m c}^2} {
m E}^2 \hspace{0.5cm} {
m F}_{ij} = \sum_{i=1}^3 \epsilon_{ijk} {
m B}_k \hspace{0.5cm} {
m F}_{4i} = - {
m F}_{i4} = rac{i}{{
m c}} {
m E}_i$  $egin{split} \mathbf{T_{ij}} &= rac{1}{\mu_0} [\sum_{\mathbf{k}=1}^3 \mathbf{F_{ik}} \mathbf{F_{jk}} + \mathbf{F_{i4}} \mathbf{F_{j4}} - rac{1}{2} \delta_{ij} (\mathbf{B^2} - rac{1}{\mathbf{c^2}} \mathbf{E^2})] \end{split}$  $=\frac{1}{\mu_0}[\sum_{\mathbf{k,l,l'}=1}^{3}\epsilon_{i\mathbf{k}\mathbf{l}}\epsilon_{j\mathbf{k}\mathbf{l'}}\mathbf{B_l}\mathbf{B_{l'}}-\frac{1}{\mathbf{c^2}}\mathbf{E_i}\mathbf{E_j}-\frac{1}{2}\delta_{i\mathbf{j}}(\mathbf{B^2}-\frac{1}{\mathbf{c^2}}\mathbf{E^2})]$  $=rac{1}{\mu_{\mathrm{c}}}[\delta_{\mathrm{ij}}\mathrm{B^2}-\mathrm{B_i}\mathrm{B_j}-rac{1}{c^2}\mathrm{E_i}\mathrm{E_j}-rac{1}{2}\delta_{\mathrm{ij}}(\mathrm{B^2}-rac{1}{c^2}\mathrm{E^2})]$  $[=rac{1}{\mu_0}[-\mathbf{B_i}\mathbf{B_j}-rac{1}{\mathbf{c^2}}\mathbf{E_i}\mathbf{E_j}+rac{1}{2}\delta_{ij}(\mathbf{B^2}+rac{1}{\mathbf{c^2}}\mathbf{E^2})]$  $\mathbf{E} = \frac{1}{\mu_0}(-\mathbf{B_i}\mathbf{B_j} + \frac{1}{2}\delta_{ij}\mathbf{B^2}) + \epsilon_0(-\mathbf{E_i}\mathbf{E_j} + \frac{1}{2}\delta_{ij}\mathbf{E^2}) = \mathcal{J}_{ij}$ 

### 能动量转化与守恒 $( ho\vec{\mathbf{E}} + \vec{\mathbf{j}} imes \vec{\mathbf{B}}, \ \frac{\mathbf{i}}{\mathbf{c}}\vec{\mathbf{j}} \cdot \vec{\mathbf{E}}) = \mathbf{f}_{\mu} = -\sum_{\nu=1}^{4} \partial_{\nu} \mathbf{T}_{\mu\nu}$

66/96

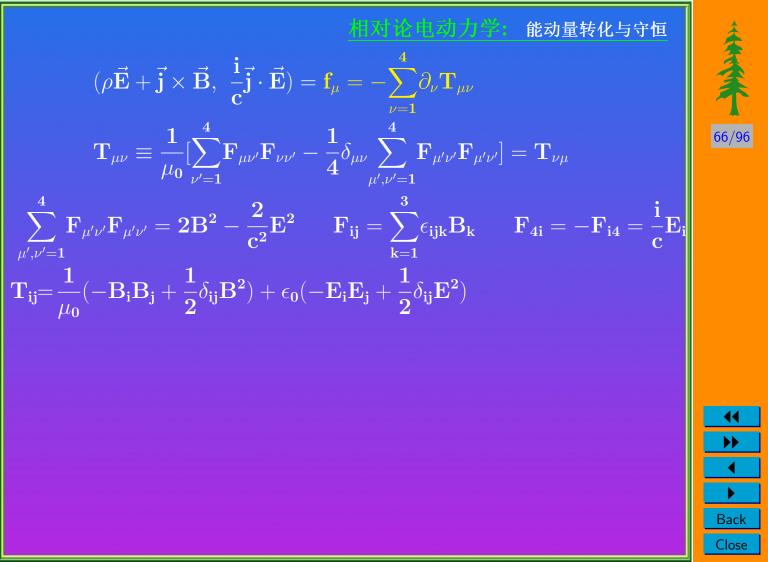
$\mathbf{T}_{\mu u} \equiv rac{\mathbf{T}}{\mu_{0}} [\sum_{ u'=1} \mathbf{F}_{\mu u'} \mathbf{F}_{ u u'} -$	$+rac{1}{4}\delta_{\mu u} {\displaystyle\sum_{\mu', u'=1}} \mathbf{F}_{\mu' u'} \mathbf{F}_{\mu' u'}] = \mathbf{T}_{ u\mu} + \mathbf{T}_{ u}$











#以下はよう方学。能动量转化与守恒 
$$(\rho\vec{E}+\vec{j}\times\vec{B},\ \frac{i}{c}\vec{j}\cdot\vec{E}) = f_{\mu} = -\sum_{\nu=1}^{4} \partial_{\nu} T_{\mu\nu}$$
 
$$T_{\mu\nu} \equiv \frac{1}{\mu_{0}} [\sum_{\nu'=1}^{4} F_{\mu\nu'} F_{\nu\nu'} - \frac{1}{4} \delta_{\mu\nu} \sum_{\mu',\nu'=1}^{4} F_{\mu'\nu'} F_{\mu'\nu'}] = T_{\nu\mu}$$
 
$$\sum_{\mu',\nu'=1}^{4} F_{\mu'\nu'} F_{\mu'\nu'} = 2B^{2} - \frac{2}{c^{2}}E^{2} \qquad F_{ij} = \sum_{k=1}^{3} \epsilon_{ijk} B_{k} \qquad F_{4i} = -F_{i4} = \frac{i}{c} E_{i}$$
 
$$T_{ij} = \frac{1}{\mu_{0}} (-B_{i}B_{j} + \frac{1}{2} \delta_{ij}B^{2}) + \epsilon_{0} (-E_{i}E_{j} + \frac{1}{2} \delta_{ij}E^{2}) = \mathcal{J}_{ij}$$

## 能动量转化与守恒 $( ho \vec{\mathbf{E}} + \vec{\mathbf{j}} imes \vec{\mathbf{B}}, \ \frac{\mathbf{i}}{\mathbf{c}} \vec{\mathbf{j}} \cdot \vec{\mathbf{E}}) = \mathbf{f}_{\mu} = -\sum \partial_{\nu} \mathbf{T}_{\mu\nu}$ $oxed{\mathbf{T}_{\mu u}\equivrac{1}{\mu_{oldsymbol{0}}}[\sum_{ u'=1}^{4}\!\mathbf{F}_{\mu u'}\mathbf{F}_{ u u'}-rac{1}{4}\delta_{\mu u}\sum_{\mu', u'=1}^{4}\!\mathbf{F}_{\mu' u'}\mathbf{F}_{\mu' u'}]=\mathbf{T}_{ u\mu}}$ 66/96 $\sum_{i=1}^4 \mathbf{F}_{\mu' u'} \mathbf{F}_{\mu' u'} = 2\mathbf{B^2} - rac{2}{\mathbf{c^2}} \mathbf{E^2} \qquad \mathbf{F_{ij}} = \sum_{i=1}^3 \epsilon_{ijk} \mathbf{B_k} \qquad \mathbf{F_{4i}} = -\mathbf{F_{i4}} = rac{i}{\mathbf{c}} \mathbf{E_{ij}}$ $\boxed{\mathbf{T_{ij}} = \frac{1}{\mu_0}(-\mathbf{B_i}\mathbf{B_j} + \frac{1}{2}\delta_{ij}\mathbf{B^2}) + \epsilon_0}(-\mathbf{E_i}\mathbf{E_j} + \frac{1}{2}\delta_{ij}\mathbf{E^2}) = \boxed{\mathcal{J}_{ij}}$ $\overline{\mathbf{T}_{44}} = \frac{1}{\mu_0} [\sum_{i=1}^{3} \mathbf{F}_{4i}^2 - \frac{1}{2} (\mathbf{B}^2 - \frac{1}{\mathbf{c}^2} \mathbf{E}^2)]$

$$(\rho\vec{E}+\vec{j}\times\vec{B},\,\,\dot{\vec{c}}\vec{j}\cdot\vec{E}) = f_{\mu} = -\sum_{\nu=1}^{4}\partial_{\nu}T_{\mu\nu}$$
 
$$T_{\mu\nu} \equiv \frac{1}{\mu_{0}}[\sum_{\nu'=1}^{4}F_{\mu\nu'}F_{\nu\nu'} - \frac{1}{4}\delta_{\mu\nu}\sum_{\mu',\nu'=1}^{4}F_{\mu'\nu'}F_{\mu'\nu'}] = T_{\nu\mu}$$
 
$$\sum_{\mu',\nu'=1}^{4}F_{\mu'\nu'}F_{\mu'\nu'} = 2B^{2} - \frac{2}{c^{2}}E^{2} \qquad F_{ij} = \sum_{k=1}^{3}\epsilon_{ijk}B_{k} \qquad F_{4i} = -F_{i4} = \frac{i}{c}E_{i}$$
 
$$T_{ij} = \frac{1}{\mu_{0}}(-B_{i}B_{j} + \frac{1}{2}\delta_{ij}B^{2}) + \epsilon_{0}(-E_{i}E_{j} + \frac{1}{2}\delta_{ij}E^{2}) = \mathcal{J}_{ij}$$
 
$$T_{44} = \frac{1}{\mu_{0}}[\sum_{i=1}^{3}F_{4i}^{2} - \frac{1}{2}(B^{2} - \frac{1}{c^{2}}E^{2})] = \frac{-1}{2\mu_{0}}(B^{2} + \frac{1}{c^{2}}E^{2}) = \frac{-1}{2\mu_{0}}B^{2} - \frac{\epsilon_{0}}{2}E^{2}$$
 44 ) Back Close

$$(\rho\vec{E}+\vec{j}\times\vec{B},\ \frac{i}{c}\vec{j}\cdot\vec{E})=f_{\mu}=-\sum_{\nu=1}^{4}\partial_{\nu}T_{\mu\nu}$$
 能动量转化与守恒 
$$T_{\mu\nu}\equiv\frac{1}{\mu_{0}}[\sum_{\nu'=1}^{4}F_{\mu\nu'}F_{\nu\nu'}-\frac{1}{4}\delta_{\mu\nu}\sum_{\mu',\nu'=1}^{4}F_{\mu'\nu'}F_{\mu'\nu'}]=T_{\nu\mu}$$
 
$$\sum_{\mu',\nu'=1}^{4}F_{\mu'\nu'}F_{\mu'\nu'}=2B^{2}-\frac{2}{c^{2}}E^{2}\qquad F_{ij}=\sum_{k=1}^{3}\epsilon_{ijk}B_{k}\qquad F_{4i}=-F_{i4}=\frac{i}{c}E_{i}$$
 
$$T_{ij}=\frac{1}{\mu_{0}}(-B_{i}B_{j}+\frac{1}{2}\delta_{ij}B^{2})+\epsilon_{0}(-E_{i}E_{j}+\frac{1}{2}\delta_{ij}E^{2})=\mathcal{J}_{ij}$$
 
$$T_{44}=\frac{1}{\mu_{0}}[\sum_{i=1}^{3}F_{4i}^{2}-\frac{1}{2}(B^{2}-\frac{1}{c^{2}}E^{2})]=\frac{-1}{2\mu_{0}}(B^{2}+\frac{1}{c^{2}}E^{2})=\frac{-1}{2\mu_{0}}B^{2}-\frac{\epsilon_{0}}{2}E^{2}=-W$$

能动量转化与守恒  $(
ho \vec{\mathbf{E}} + \vec{\mathbf{j}} imes \vec{\mathbf{B}}, \ \ \dot{\vec{\mathbf{f}}} \vec{\mathbf{j}} \cdot \vec{\mathbf{E}}) = \mathbf{f}_{\mu} = -\sum \partial_{
u} \mathbf{T}_{\mu
u}$  $\mathbf{T}_{\mu
u} \equiv rac{1}{\mu_0} [\sum_{
u'=1}^4 \mathbf{F}_{\mu
u'} \mathbf{F}_{
u
u'} - rac{1}{4} \delta_{\mu
u} \sum_{
u',
u'=1}^4 \mathbf{F}_{\mu'
u'} \mathbf{F}_{\mu'
u'}] = \mathbf{T}_{
u\mu}$ 66/96  $\sum_{
u',
u'=1}^{4} \mathrm{F}_{\mu'
u'} \mathrm{F}_{\mu'
u'} = 2 \mathrm{B}^2 - rac{2}{\mathrm{c}^2} \mathrm{E}^2 \qquad \mathrm{F}_{\mathbf{i}\mathbf{j}} = \sum_{\mathbf{k}=1}^{3} \epsilon_{\mathbf{i}\mathbf{j}\mathbf{k}} \mathrm{B}_{\mathbf{k}} \qquad \mathrm{F}_{4\mathbf{i}} = -\mathrm{F}_{\mathbf{i}4} = rac{\mathbf{i}}{\mathrm{c}} \mathrm{E}_{\mathbf{i}}$ 

$$\begin{split} &\sum_{\mu',\nu'=1}^{\mathbf{F}} \mathbf{F}_{\mu'\nu'} \mathbf{F}_{\mu'\nu'} = 2\mathbf{B}^{2} - \frac{1}{\mathbf{c}^{2}} \mathbf{E}^{2} \quad \mathbf{F}_{ij} = \sum_{k=1}^{\epsilon} \epsilon_{ijk} \mathbf{B}_{k} \quad \mathbf{F}_{4i} = -\mathbf{F}_{i4} = -\mathbf{E}_{i} \\ &\mathbf{T}_{ij} = \frac{1}{\mu_{0}} (-\mathbf{B}_{i} \mathbf{B}_{j} + \frac{1}{2} \delta_{ij} \mathbf{B}^{2}) + \epsilon_{0} (-\mathbf{E}_{i} \mathbf{E}_{j} + \frac{1}{2} \delta_{ij} \mathbf{E}^{2}) = \mathcal{J}_{ij} \\ &\mathbf{T}_{44} = \frac{1}{\mu_{0}} [\sum_{i=1}^{3} \mathbf{F}_{4i}^{2} - \frac{1}{2} (\mathbf{B}^{2} - \frac{1}{\mathbf{c}^{2}} \mathbf{E}^{2})] = \frac{-1}{2\mu_{0}} (\mathbf{B}^{2} + \frac{1}{\mathbf{c}^{2}} \mathbf{E}^{2}) = \frac{-1}{2\mu_{0}} \mathbf{B}^{2} - \frac{\epsilon_{0}}{2} \mathbf{E}^{2} = -\mathbf{W} \\ &\mathbf{T}_{i4} = \mathbf{T}_{4i} = \frac{1}{\mu_{0}} \sum_{i=1}^{3} \mathbf{F}_{ij} \mathbf{F}_{4j} = \frac{\mathbf{i}}{\mu_{0} \mathbf{c}} \sum_{i=1}^{3} \epsilon_{ijk} \mathbf{B}_{k} \mathbf{E}_{j} = \frac{\mathbf{i}}{\mu_{0} \mathbf{c}} (\vec{\mathbf{E}} \times \vec{\mathbf{B}})_{i} = \frac{\mathbf{i}}{\mathbf{c}} \mathbf{S}_{i} \end{split}$$

相対论电动力学: 能动量转化与守恒 
$$(\rho\vec{E}+\vec{j}\times\vec{B},\ \frac{i}{c}\vec{j}\cdot\vec{E}) = \mathbf{f}_{\mu} = -\sum_{\nu=1}^{4}\partial_{\nu}\mathbf{T}_{\mu\nu}$$
 
$$\mathbf{T}_{\mu\nu} \equiv \frac{1}{\mu_{0}}[\sum_{\nu'=1}^{4}\mathbf{F}_{\mu\nu'}\mathbf{F}_{\nu\nu'} - \frac{1}{4}\delta_{\mu\nu}\sum_{\mu',\nu'=1}^{4}\mathbf{F}_{\mu'\nu'}\mathbf{F}_{\mu'\nu'}] = \mathbf{T}_{\nu\mu}$$
 
$$\mathbf{T}_{ij} = \frac{1}{\mu_{0}}(-\mathbf{B}_{i}\mathbf{B}_{j} + \frac{1}{2}\delta_{ij}\mathbf{B}^{2}) + \epsilon_{0}(-\mathbf{E}_{i}\mathbf{E}_{j} + \frac{1}{2}\delta_{ij}\mathbf{E}^{2}) = \mathcal{J}_{ij}$$
 
$$\mathbf{T}_{44} = -\frac{1}{2\mu_{0}}\mathbf{B}^{2} - \frac{\epsilon_{0}}{2}\mathbf{E}^{2} = -\mathbf{W} \qquad \mathbf{T}_{i4} = \mathbf{T}_{4i} = \frac{i}{\mu_{0}\mathbf{c}}(\vec{\mathbf{E}}\times\vec{\mathbf{B}})_{i} = \frac{i}{\mathbf{c}}\mathbf{S}_{i} = \mathbf{i}\mathbf{c}\mathbf{g}_{i}$$
 Back the second state of the property of the proof of the proof

解析で用力す学:能効量转化与守恒 
$$(\rho\vec{E}+\vec{j}\times\vec{B},\,\,\frac{i}{c}\vec{j}\cdot\vec{E}) = f_{\mu} = -\sum_{\nu=1}^{4}\partial_{\nu}T_{\mu\nu}$$
 
$$T_{\mu\nu} \equiv \frac{1}{\mu_{0}}[\sum_{\nu'=1}^{4}F_{\mu\nu'}F_{\nu\nu'} - \frac{1}{4}\delta_{\mu\nu}\sum_{\mu',\nu'=1}^{4}F_{\mu'\nu'}F_{\mu'\nu'}] = T_{\nu\mu}$$
 
$$T_{ij} = \frac{1}{\mu_{0}}(-B_{i}B_{j} + \frac{1}{2}\delta_{ij}B^{2}) + \epsilon_{0}(-E_{i}E_{j} + \frac{1}{2}\delta_{ij}E^{2}) = \mathcal{J}_{ij}$$
 
$$T_{44} = -\frac{1}{2\mu_{0}}B^{2} - \frac{\epsilon_{0}}{2}E^{2} = -W \qquad T_{i4} = T_{4i} = \frac{i}{\mu_{0}c}(\vec{E}\times\vec{B})_{i} = \frac{i}{c}S_{i} = icg_{i}$$
 
$$T_{\mu\nu} = \mathbf{E}_{\mu}\mathbf{E}_{$$

能动量转化与守恒  $(
ho ec{\mathbf{E}} + ec{\mathbf{j}} imes ec{\mathbf{B}}, \ \ rac{\dot{\mathbf{I}}}{\mathbf{G}} ec{\mathbf{j}} \cdot ec{\mathbf{E}}) = \mathbf{f}_{\mu} = -\sum \partial_{
u} \mathbf{T}_{\mu
u}$ 

$$\mathbf{T}_{\mu
u} \equiv rac{1}{\mu_{\mathbf{0}}} [\sum_{
u'=1}^{4} \mathbf{F}_{\mu
u'} \mathbf{F}_{
u
u'} - rac{1}{4} \delta_{\mu
u} \sum_{
u'=1}^{4} \mathbf{F}_{\mu'
u'} \mathbf{F}_{\mu'
u'}] = \mathbf{T}_{
u\mu}$$

$$\mathbf{T_{ij}} = \frac{1}{\mu_0} (-\mathbf{B_i} \mathbf{B_j} + \frac{1}{2} \delta_{ij} \mathbf{B^2}) + \epsilon_0 (-\mathbf{E_i} \mathbf{E_j} + \frac{1}{2} \delta_{ij} \mathbf{E^2}) = \mathcal{J}_{ij}$$

$$egin{aligned} \mathbf{T_{44}} \!\!\!\! = \! -rac{1}{2\mu_0} \mathbf{B^2} \!\!\!\! - \! rac{\epsilon_0}{2} \mathbf{E^2} \!\!\! = \! -\mathbf{W} \qquad \mathbf{T_{i4}} \!\!\! = \! \mathbf{T_{4i}} = \! rac{\mathbf{i}}{\mu_0 \mathbf{c}} (\vec{\mathbf{E}} imes \vec{\mathbf{B}})_\mathbf{i} = \! rac{\mathbf{i}}{\mathbf{c}} \mathbf{S_i} = \mathbf{i} \mathbf{c} \mathbf{g_i} \\ \mathbf{T_{\mu\nu}} \quad ext{2.23} \end{aligned}$$

$$\begin{split} \mathbf{f_i} &= -\sum_{\nu=1}^4 \partial_{\nu} \mathbf{T_{i\nu}} = -\sum_{\mathbf{j}=1}^3 \partial_{\mathbf{j}} \mathbf{T_{ij}} - \frac{\partial \mathbf{T_{i4}}}{\mathbf{i} \mathbf{c} \partial \mathbf{t}} & \Rightarrow & \vec{\mathbf{f}} = -\nabla \cdot \overrightarrow{\mathcal{J}} - \frac{\partial \vec{\mathbf{g}}}{\partial \mathbf{t}} \\ \mathbf{f_4} &= -\sum_{\nu=1}^4 \partial_{\nu} \mathbf{T_{4\nu}} = -\sum_{\mathbf{i}=1}^3 \partial_{\mathbf{i}} \mathbf{T_{4i}} - \frac{\partial \mathbf{T_{44}}}{\mathbf{i} \mathbf{c} \partial \mathbf{t}} & \Rightarrow & \vec{\mathbf{f}} \cdot \vec{\mathbf{v}} = -\nabla \cdot \vec{\mathbf{S}} - \frac{\partial \mathbf{W}}{\partial \mathbf{t}} \end{split}$$















が起動力学。能动量转化与守恒 
$$(
ho \vec{\mathbf{E}} + \vec{\mathbf{j}} imes \vec{\mathbf{B}}, \ \frac{\mathbf{i}}{\mathbf{c}} \vec{\mathbf{j}} \cdot \vec{\mathbf{E}}) = \mathbf{f}_{\mu} = -\sum_{\nu=1}^{4} \partial_{\nu} \mathbf{T}_{\mu\nu}$$
 
$$\mathbf{T}_{\mu\nu} \equiv \frac{1}{\mu_{0}} [\sum_{\nu'=1}^{4} \mathbf{F}_{\mu\nu'} \mathbf{F}_{\nu\nu'} - \frac{1}{4} \delta_{\mu\nu} \sum_{\mu',\nu'=1}^{4} \mathbf{F}_{\mu'\nu'} \mathbf{F}_{\mu'\nu'}] = \mathbf{T}_{\nu\mu}$$
 
$$\mathbf{T}_{\mu\nu} = \frac{1}{\mu_{0}} [\mathbf{F}_{\mu\nu'} \mathbf{F}_{\nu\nu'} - \frac{1}{4} \delta_{\mu\nu} \sum_{\mu',\nu'=1}^{4} \mathbf{F}_{\mu'\nu'} \mathbf{F}_{\mu'\nu'}] = \mathbf{T}_{\nu\mu}$$

$$\begin{split} \mathbf{T}_{\mathbf{i}\mathbf{j}} &= \frac{1}{\mu_0} (-\mathbf{B}_{\mathbf{i}} \mathbf{B}_{\mathbf{j}} + \frac{1}{2} \delta_{\mathbf{i}\mathbf{j}} \mathbf{B}^2) + \epsilon_0 (-\mathbf{E}_{\mathbf{i}} \mathbf{E}_{\mathbf{j}} + \frac{1}{2} \delta_{\mathbf{i}\mathbf{j}} \mathbf{E}^2) = \mathcal{J}_{\mathbf{i}\mathbf{j}} \\ \mathbf{T}_{44} &= -\frac{1}{2\mu_0} \mathbf{B}^2 - \frac{\epsilon_0}{2} \mathbf{E}^2 = -\mathbf{W} \qquad \mathbf{T}_{\mathbf{i}4} = \mathbf{T}_{4\mathbf{i}} = \frac{\mathbf{i}}{\mu_0 \mathbf{c}} (\vec{\mathbf{E}} \times \vec{\mathbf{B}})_{\mathbf{i}} = \frac{\mathbf{i}}{\mathbf{c}} \mathbf{S}_{\mathbf{i}} = \mathbf{i} \mathbf{c} \mathbf{g}_{\mathbf{i}} \\ \mathbf{T}_{\mu\nu} &= -\sum_{\nu=1}^{4} \partial_{\nu} \mathbf{T}_{\mathbf{i}\nu} = -\sum_{\mathbf{j}=1}^{3} \partial_{\mathbf{j}} \mathbf{T}_{\mathbf{i}\mathbf{j}} - \frac{\partial \mathbf{T}_{\mathbf{i}4}}{\mathbf{i}\mathbf{c}\partial \mathbf{t}} & \Rightarrow & \vec{\mathbf{f}} = -\nabla \cdot \overrightarrow{\mathcal{J}} - \frac{\partial \vec{\mathbf{g}}}{\partial \mathbf{t}} \\ \mathbf{f}_4 &= -\sum_{\nu=1}^{4} \partial_{\nu} \mathbf{T}_{4\nu} = -\sum_{\mathbf{i}=1}^{3} \partial_{\mathbf{i}} \mathbf{T}_{4\mathbf{i}} - \frac{\partial \mathbf{T}_{44}}{\mathbf{i}\mathbf{c}\partial \mathbf{t}} & \Rightarrow & \vec{\mathbf{f}} \cdot \vec{\mathbf{v}} = -\nabla \cdot \vec{\mathbf{S}} - \frac{\partial \mathbf{W}}{\partial \mathbf{t}} \\ \vec{\mathbf{f}} \cdot \vec{\mathbf{v}} &= \rho \vec{\mathbf{v}} \cdot \vec{\mathbf{E}} = \vec{\mathbf{j}} \cdot \vec{\mathbf{E}} \end{split}$$

















#### 相对论电动力学: 能动量的洛伦兹变换 四度电磁能动量 $P_{\mu}$ 是能动量张量 $T_{\mu\nu}$ 的分量:

$$\mathbf{P}_{\mu}(\mathbf{t}) = -rac{\mathbf{i}}{\mathbf{c}}\int_{\pm 2\mathrm{i}}\mathbf{d}ec{\mathbf{r}} \; \mathbf{T}_{4\mu}(ec{\mathbf{r}},\mathbf{t})$$
 它的变换性质似乎不是四矢量类型的!









### 相对论里司力学。能动量的洛伦兹变换

四度电磁能动量 $P_{\mu}$ 是能动量张量 $T_{\mu\nu}$ 的分量:

 $\mathbf{P}_{\mu}(\mathbf{t}) = -rac{1}{\mathbf{c}}\int_{\mathrm{\hat{2}}\mathrm{\hat{2}}\mathrm{\hat{n}}}\mathbf{d}ec{\mathbf{r}}\;\mathbf{T}_{4\mu}(ec{\mathbf{r}},\mathbf{t})$  它的变换性质似乎不是四矢量类型的!

$$\mathbf{P}_{\mu}'(\mathbf{t}') = -rac{\mathbf{i}}{\mathbf{c}}\int_{\mathbf{\hat{z}}\hat{\mathbf{r}}\hat{\mathbf{n}}}\mathbf{d}ec{\mathbf{r}}' \; \mathbf{T}_{4\mu}'(ec{\mathbf{r}}',\mathbf{t}') = -rac{\mathbf{i}}{\mathbf{c}}\!\sum_{\lambda
u}\!\mathbf{a}_{4\lambda}\mathbf{a}_{\mu
u}\int_{\mathbf{\hat{z}}\hat{\mathbf{r}}\hat{\mathbf{n}}}\mathbf{d}ec{\mathbf{r}}' \; \mathbf{T}_{\lambda
u}(ec{\mathbf{r}},\mathbf{t})$$

$$egin{aligned} rac{\partial z}{\partial t} & = -rac{\dot{\mathbf{I}}}{\mathbf{C}} \sum_{
u} \mathbf{a}_{\mu
u} \int_{\pm \hat{\mathbf{C}} | \mathbf{I} |} dec{\mathbf{r}} \; \mathbf{T}_{4
u}(ec{\mathbf{r}}, \mathbf{t}) \ & = \sum_{
u} \mathbf{a}_{4\lambda} \int_{\pm \hat{\mathbf{C}} | \mathbf{I} |} dec{\mathbf{r}} \; \mathbf{T}_{\lambda
u}(ec{\mathbf{r}}, \mathbf{t}) \; 
otag \int_{\pm \hat{\mathbf{C}} | \mathbf{I} |} dec{\mathbf{r}} \; \mathbf{T}_{4
u}(ec{\mathbf{r}}, \mathbf{t}) \end{aligned}$$

对在S系静止的一团相对距离维持固定的电荷系统的四度能动量, $T_{4i}=T_{i4}=0$ 

$$\mathbf{P}_{\mu}=-rac{\mathbf{i}}{\mathbf{c}}\delta_{\mu 4}\!\!\int_{\Phi^{2}\mathbf{i}\mathbf{l}}\!\!\mathbf{d}ec{\mathbf{r}}\;\mathbf{T_{44}}(ec{\mathbf{r}})\equiv\mathbf{im_{0}}\mathbf{c}\delta_{\mu 4}$$
 在静止系无磁场因而无电磁动量,且不含于

$${f P}_{\mu}'({f t}') = -rac{{f i}}{{f c}}{f a}_{44}{f a}_{\mu 4}\int_{\pm 2{f ar n}}{f d}{f ar r}' \ {f T}_{44}({f ar r}) -rac{{f i}}{{f c}}{f \sum}_{{f ij}}{f a}_{4{f i}}{f a}_{\mu {f j}}\int_{\pm 2{f ar n}}{f d}{f ar r}' \ {f T}_{{f ij}}({f ar r})$$



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## 相对论电动力学: 能动量的洛伦兹变换

假设S'系相对S系以速度
$$\bar{v}$$
运动, $\gamma \equiv \frac{1}{\sqrt{1-\frac{\mathbf{v}^2}{c^2}}} = \mathbf{a}_{44}$  
$$\mathbf{P}'_{\mu}(\mathbf{t}') = -\frac{\mathbf{i}}{\mathbf{c}}\mathbf{a}_{44}\mathbf{a}_{\mu 4}\int_{\hat{\mathbf{c}}\hat{\mathbf{c}}|\mathbf{l}}\mathbf{d}\vec{\mathbf{r}}' \ \mathbf{T}_{44}(\gamma(\mathbf{x}'+\mathbf{v}\mathbf{t}'),\mathbf{y}',\mathbf{z}')$$

$$\mathbf{P}_{\mu}'(\mathbf{t}') = -\frac{1}{\mathbf{c}} \mathbf{a}_{44} \mathbf{a}_{\mu 4} \int_{\mathbf{\hat{z}\hat{z}\hat{u}}} \mathbf{d}\vec{\mathbf{r}}' \; \mathbf{T}_{44}(\gamma(\mathbf{x}'+\mathbf{v}\mathbf{t}'),\mathbf{y}',\mathbf{z}') 
onumber \ -\frac{\mathbf{i}}{\mathbf{c}} \sum_{\mathbf{i}\mathbf{j}} \mathbf{a}_{4\mathbf{i}} \mathbf{a}_{\mu \mathbf{j}} \int_{\mathbf{\hat{z}\hat{z}\hat{u}}} \mathbf{d}\vec{\mathbf{r}}' \; \mathbf{T}_{\mathbf{i}\mathbf{j}}(\gamma(\mathbf{x}'+\mathbf{v}\mathbf{t}'),\mathbf{y}',\mathbf{z}')$$



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## 相对论自动力学: 能动量的洛伦兹变换

假设S'系相对S系以速度
$$\vec{v}$$
运动, $\gamma \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = a_{44}$ 

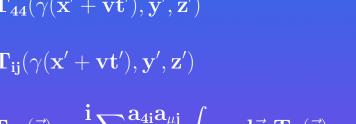
$$\mathbf{P}_{\mu}'(\mathbf{t}') = -rac{\mathbf{i}}{\mathbf{c}}\mathbf{a}_{44}\mathbf{a}_{\mu 4}\int_{\mathbf{\hat{z}}\hat{\mathbf{z}}|\mathbf{i}}\mathbf{d}ec{\mathbf{r}}' \; \mathbf{T}_{44}(\gamma(\mathbf{x}'+\mathbf{v}\mathbf{t}'),\mathbf{y}',\mathbf{z}')$$

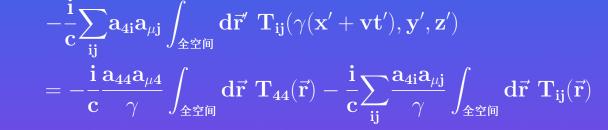
$$\mathbf{c} = -\mathbf{c}_{\mathbf{44}} \mathbf{a}_{\mu \mathbf{4}} \int_{\mathbf{\hat{z}}\hat{\mathbf{\hat{z}}}\hat{\mathbf{n}}} \mathbf{d}\mathbf{r}' \ \mathbf{T}_{\mathbf{44}} (\gamma(\mathbf{x}' + \mathbf{r}'))$$

$$\int_{\Phi^4}\int_{\Phi^2} \mathbf{d} ec{\mathbf{r}}' \; \mathbf{T_{44}} (\gamma (\mathbf{x}' + \mathbf{x}')) \, d\mathbf{r}' \, d\mathbf{r}' \, \mathbf{T_{44}} (\gamma (\mathbf{x}' + \mathbf{x}')) \, d\mathbf{r}' \, d\mathbf$$

$$\Gamma_{44}(\gamma(\mathbf{x}'+\mathbf{v}))$$

$$egin{aligned} & \sqrt{1-\mathbf{z}^2} \ & \mathbf{z}^4 + \mathbf{v} \mathbf{t}' +$$







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## 相对论自动力学: 能动量的洛伦兹变换

假设S'系相对S系以速度
$$\vec{v}$$
运动, $\gamma \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = a_{44}$ 

$$\mathbf{P}_{\mu}'(\mathbf{t}') = -rac{\mathbf{i}}{\mathbf{c}}\mathbf{a}_{44}\mathbf{a}_{\mu 4}\int_{\hat{\mathbf{z}}\hat{\mathbf{z}}\hat{\mathbf{n}}}\mathbf{d}ec{\mathbf{r}}' \; \mathbf{T}_{44}(\gamma(\mathbf{x}'+\mathbf{v}\mathbf{t}'),\mathbf{y}',\mathbf{z}')$$

$$\mathbf{c}^{\mathbf{i}}$$
  $\int_{\mathbf{\hat{z}}\hat{\mathbf{z}}\hat{\mathbf{n}}} \mathbf{d}\mathbf{\vec{r}}' \; \mathbf{T_{ij}}(\gamma(\mathbf{x}'+\mathbf{vt}'),\mathbf{y}',\mathbf{z}')$ 

$$\begin{split} &= -\frac{i}{c}\frac{a_{44}a_{\mu 4}}{\gamma}\int_{\underline{\hat{\mathbf{x}}}\underline{\hat{\mathbf{x}}}\underline{\hat{\mathbf{n}}}}d\vec{\mathbf{r}}\ \mathbf{T}_{44}(\vec{\mathbf{r}}) - \frac{i}{c}\underset{\mathbf{ij}}{\sum}\frac{a_{4i}a_{\mu \mathbf{j}}}{\gamma}\int_{\underline{\hat{\mathbf{x}}}\underline{\hat{\mathbf{x}}}\underline{\hat{\mathbf{n}}}}d\vec{\mathbf{r}}\ \mathbf{T}_{\mathbf{ij}}(\vec{\mathbf{r}})\\ &= a_{\mu 4}im_{0}c - \frac{i}{c\gamma}\underset{\mathbf{...}}{\sum}a_{4i}a_{\mu \mathbf{j}}\int_{\underline{\hat{\mathbf{x}}}\underline{\hat{\mathbf{x}}}\underline{\hat{\mathbf{n}}}}d\vec{\mathbf{r}}\ \mathbf{T}_{\mathbf{ij}}(\vec{\mathbf{r}}) \end{split}$$



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#### 担对论电动力学 能动量的洛伦兹变换

假设S'系相对S系以速度 $\overline{v}$ 运动, $\gamma \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \mathbf{a}_{44}$ 

$$egin{aligned} \mathbf{P}_{\mu}'(\mathbf{t}') &= -rac{\mathbf{i}}{\mathbf{c}}\mathbf{a}_{44}\mathbf{a}_{\mu 4}\int_{\pm 2 ext{R}} dec{\mathbf{r}}' \; \mathbf{T}_{44}(\gamma(\mathbf{x}'+\mathbf{v}\mathbf{t}'),\mathbf{y}',\mathbf{z}') \ &-rac{\mathbf{i}}{\mathbf{c}}\sum_{\mathbf{i}\mathbf{j}}\mathbf{a}_{4\mathbf{i}}\mathbf{a}_{\mu \mathbf{j}}\int_{\pm 2 ext{R}} dec{\mathbf{r}}' \; \mathbf{T}_{\mathbf{i}\mathbf{j}}(\gamma(\mathbf{x}'+\mathbf{v}\mathbf{t}'),\mathbf{y}',\mathbf{z}') \ &= -rac{\mathbf{i}}{\mathbf{c}}rac{\mathbf{a}_{44}\mathbf{a}_{\mu 4}}{\gamma}\int_{\pm 2 ext{R}} dec{\mathbf{r}} \; \mathbf{T}_{44}(ec{\mathbf{r}}) - rac{\mathbf{i}}{\mathbf{c}}\sum_{\mathbf{i}\mathbf{i}}rac{\mathbf{a}_{4\mathbf{i}}\mathbf{a}_{\mu \mathbf{j}}}{\gamma}\int_{\pm 2 ext{R}} dec{\mathbf{r}} \; \mathbf{T}_{\mathbf{i}\mathbf{j}}(ec{\mathbf{r}}) \end{aligned}$$

$$\mathbf{a} = \mathbf{a}_{\mu 4} \mathbf{i} \mathbf{m_0} \mathbf{c} - rac{\mathbf{i}}{\mathbf{c} \gamma} {\sum_{i:}} \mathbf{a_{4i}} \mathbf{a}_{\mu \mathbf{j}} \int_{\mathbf{\hat{z}} \mathbf{\hat{z}} \mathbf{\hat{u}}} \mathbf{d} \vec{\mathbf{r}} \; \mathbf{T_{ij}} (\vec{\mathbf{r}})$$

- 上式第二项贡献的是能动量偏离四矢量洛伦兹变换的行为
- 对在S系静止的一团相对距离维持固定的电荷系统,第二项一般不为零



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#### 相对论电动力学 能动量的洛伦兹变换

假设S'系相对S系以速度 $\vec{v}$ 运动, $\gamma \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \mathbf{a}_{44}$ 

$$egin{aligned} \mathbf{P}_{\mu}'(\mathbf{t}') &= -rac{\mathbf{i}}{\mathbf{c}} \mathbf{a}_{44} \mathbf{a}_{\mu 4} \int_{\hat{\pm}\hat{\Xi}\hat{\Pi}} dec{\mathbf{r}}' \; \mathbf{T}_{44}(\gamma(\mathbf{x}'+\mathbf{v}\mathbf{t}'),\mathbf{y}',\mathbf{z}') \ &- rac{\mathbf{i}}{\mathbf{c}} \sum_{\mathbf{i}\mathbf{j}} \mathbf{a}_{4\mathbf{i}} \mathbf{a}_{\mu\mathbf{j}} \int_{\hat{\pm}\hat{\Xi}\hat{\Pi}} dec{\mathbf{r}}' \; \mathbf{T}_{\mathbf{i}\mathbf{j}}(\gamma(\mathbf{x}'+\mathbf{v}\mathbf{t}'),\mathbf{y}',\mathbf{z}') \ &= -rac{\mathbf{i}}{\mathbf{c}} rac{\mathbf{a}_{44} \mathbf{a}_{\mu 4}}{\gamma} \int_{\hat{\pm}\hat{\Xi}\hat{\Pi}} dec{\mathbf{r}} \; \mathbf{T}_{44}(ec{\mathbf{r}}) - rac{\mathbf{i}}{\mathbf{c}} \sum_{\mathbf{i}\mathbf{i}} rac{\mathbf{a}_{4\mathbf{i}} \mathbf{a}_{\mu\mathbf{j}}}{\gamma} \int_{\hat{\pm}\hat{\Xi}\hat{\Pi}} dec{\mathbf{r}} \; \mathbf{T}_{\mathbf{i}\mathbf{j}}(ec{\mathbf{r}}) \end{aligned}$$

$$\mathbf{a}_{\mu \mathbf{i}} = \mathbf{a}_{\mu \mathbf{i}} \mathbf{i} \mathbf{m_0} \mathbf{c} - rac{\mathbf{i}}{\mathbf{c} \gamma} {\sum_{\mathbf{i}:}} \mathbf{a_{4i}} \mathbf{a}_{\mu \mathbf{j}} \int_{\hat{\mathbf{c}}\hat{\mathbf{c}}\hat{\mathbf{n}}} \mathbf{d}\vec{\mathbf{r}} \; \mathbf{T_{ij}}(\vec{\mathbf{r}})$$

- 上式第二项贡献的是能动量偏离四矢量洛伦兹变换的行为
- 对在S系静止的一团相对距离维持固定的电荷系统,第二项一般不为零
- 但这样的系统仅靠电磁力是维持不住的,需要外力!
- 如果考虑维持这个系统的外力–彭加莱应力:  $\mathbf{T}_{\mu\nu}^{\text{poincare}}$
- 它应被考虑进前面能动量的计算中

$$\sum \! \partial_{\mu} (\mathbf{T}_{\mu
u} + \mathbf{T}^{\mathbf{poincare}}_{\mu
u}) = \mathbf{0}$$
 它意味对体系每个时空点的合力为零!



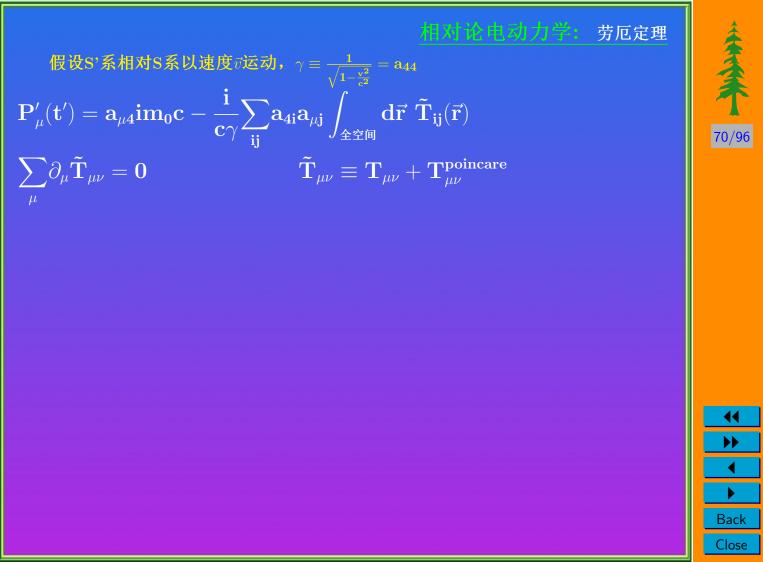
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# 相对论自动力学: 劳厄定理

假设S'系相对S系以速度 $\overline{v}$ 运动, $\gamma \equiv \frac{1}{\sqrt{1-\frac{\mathbf{v}^2}{2}}} = \mathbf{a}_{44}$ 

 $ilde{\mathbf{T}}_{\mu
u} \equiv \mathbf{T}_{\mu
u} + \mathbf{T}_{\mu
u}^{ ext{poincare}}$ 

 $\mathbf{P}_{\mu}'(\mathbf{t}') = \mathbf{a}_{\mu \mathbf{4}} \mathbf{i} \mathbf{m_0} \mathbf{c} - rac{\mathbf{i}}{\mathbf{c} \gamma} {\sum_{\mathbf{i}:}} \mathbf{a_{4i}} \mathbf{a}_{\mu \mathbf{j}} \int_{\mathbf{\hat{z}} \mathbf{\hat{z}} \mathbf{\hat{q}}} \mathbf{d} \vec{\mathbf{r}} \; \mathbf{ ilde{T}_{ij}}(\vec{\mathbf{r}})$ 

 $\sum\!\partial_{\mu} ilde{\mathbf{T}}_{\mu
u}=\mathbf{0}$ 

在S系 $T_{\mu\nu}$ 不显含时间;维持其稳定的 $T_{\mu\nu}^{
m poincare}$ 也会不显含时间,导致 $\tilde{T}_{\mu\nu}$ 不显含时间

 $\int_{\hat{\mathbf{x}}\hat{\mathbf{T}}\hat{\mathbf{i}}} dec{\mathbf{r}} \; ilde{\mathbf{T}}_{\mathbf{i}\mathbf{j}}(ec{\mathbf{r}}) = \sum_{\mathbf{k}} \int_{\hat{\mathbf{x}}\hat{\mathbf{T}}\hat{\mathbf{i}}} dec{\mathbf{r}} \; \partial_{\mathbf{k}}[\mathbf{x}_{\mathbf{i}} ilde{\mathbf{T}}_{\mathbf{k}\mathbf{j}}(ec{\mathbf{r}})] = \sum_{\mathbf{k}} \int_{\hat{\mathbf{x}}\hat{\mathbf{T}}\hat{\mathbf{i}}} dec{\mathbf{S}}_{\mathbf{k}} \mathbf{x}_{\mathbf{i}} ilde{\mathbf{T}}_{\mathbf{k}\mathbf{j}}(ec{\mathbf{r}}) = \mathbf{0}$ 

在 $\mathbf{s}$ 系, $ilde{\mathbf{T}}_{\mu
u}$ 不显含时间  $\Rightarrow$   $\partial_{\mathbf{0}} ilde{\mathbf{T}}_{\mu
u}=\mathbf{0}$   $\Rightarrow$   $\sum \partial_{\mathbf{k}} ilde{\mathbf{T}}_{\mathbf{k}
u}=\mathbf{0}$ 

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加入彭加莱应力的一团相对距离维持固定的电荷系统满足:  $\mathbf{P}_{\mu}'(\mathbf{t}') = \mathbf{a}_{\mu \mathbf{4}} \mathbf{im_0} \mathbf{c}$ 

 $\mathbf{E} = rac{\mathbf{m_0 c^2}}{\sqrt{1-rac{\mathbf{v^2}}{c^2}}} \qquad ec{\mathbf{P}} = rac{\mathbf{m_0 \vec{v}}}{\sqrt{1-rac{\mathbf{v^2}}{c^2}}} \qquad \mathbf{im_0 c} = -rac{\mathbf{i}}{\mathbf{c}} \int_{ ext{$\hat{\mathbf{c}}$} ext{$\hat{\mathbf{T}}$} ext{$\hat{\mathbf{T}}$}} \mathrm{d}ec{\mathbf{r}} \ ilde{\mathbf{T}}_{44}(ec{\mathbf{r}})$ 

# 相对论自动力学: 劳厄定理

假设S'系相对S系以速度 $\overline{v}$ 运动, $\gamma \equiv \frac{1}{\sqrt{1-\frac{\mathbf{v}^2}{2}}} = \mathbf{a}_{44}$ 

 $\mathbf{P}_{\mu}'(\mathbf{t}') = \mathbf{a}_{\mu \mathbf{4}} \mathbf{i} \mathbf{m_0} \mathbf{c} - rac{\mathbf{i}}{\mathbf{c} \gamma} \sum_{\mathbf{i}:} \mathbf{a_{4i}} \overline{\mathbf{a}_{\mu \mathbf{j}}} \int_{\mathbf{\hat{z}} \mathbf{\hat{z}} \mathbf{\hat{i}}} \mathbf{d} \vec{\mathbf{r}} \ \mathbf{ ilde{T}_{ij}} (\vec{\mathbf{r}})$ 

 $ilde{\mathbf{T}}_{\mu
u} \equiv \mathbf{T}_{\mu
u} + \mathbf{T}_{\mu
u}^{ ext{poincare}}$ 

在S系 $T_{\mu\nu}$ 不显含时间;维持其稳定的 $T_{\mu\nu}^{
m poincare}$ 也会不显含时间,导致 $\tilde{T}_{\mu\nu}$ 不显含时间 在 $\mathbf{s}$ 系, $\tilde{\mathbf{T}}_{\mu 
u}$ 不显含时间  $\Rightarrow$   $\partial_{\mathbf{0}} \tilde{\mathbf{T}}_{\mu 
u} = \mathbf{0}$   $\Rightarrow$   $\sum \partial_{\mathbf{k}} \tilde{\mathbf{T}}_{\mathbf{k} 
u} = \mathbf{0}$ 

 $\overline{\int_{\pm 2 ilde{ ext{l}}} dec{ ext{r}} \; ilde{ ext{T}}_{\mathbf{ij}}(ec{ ext{r}}) = \sum_{\mathbf{k}} \int_{\pm 2 ilde{ ext{l}}} dec{ ext{r}} \; \partial_{\mathbf{k}} [\mathbf{x_i} ilde{ ext{T}}_{\mathbf{kj}}(ec{ ext{r}})] = \sum_{\mathbf{k}} \int_{\pm 2 ilde{ ext{l}}} dec{ ext{S}}_{\mathbf{k}} \mathbf{x_i} ilde{ ext{T}}_{\mathbf{kj}}(ec{ ext{r}}) = 0$ 

加入彭加莱应力的一团相对距离维持固定的电荷系统满足:  $\mathbf{P}_{\mu}'(\mathbf{t}') = \mathbf{a}_{\mu \mathbf{4}} \mathbf{im_0} \mathbf{c}$ 

 $\sum\!\partial_{\mu} ilde{\mathbf{T}}_{\mu
u}=\mathbf{0}$ 

 ${
m E} = rac{{
m m_0 c^2}}{\sqrt{1 - rac{{
m v^2}}{c^2}}} \qquad ec{
m P} = rac{{
m m_0 ec{
m v}}}{\sqrt{1 - rac{{
m v}^2}{c^2}}} \qquad {
m im_0 c} = -rac{{
m i}}{{
m c}} \int_{rac{1}{2}} {
m d}ec{
m r} \; ilde{
m T}_{44}(ec{
m r})$ 

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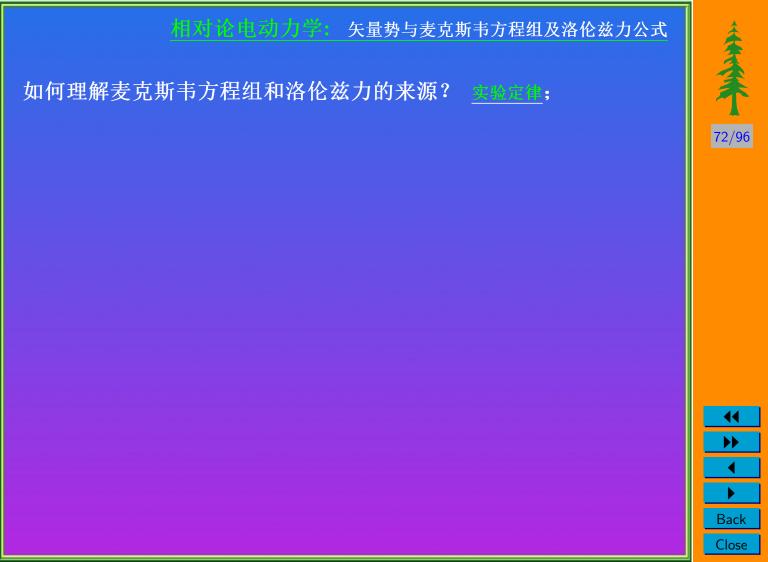
# 三. 经典电磁学理论

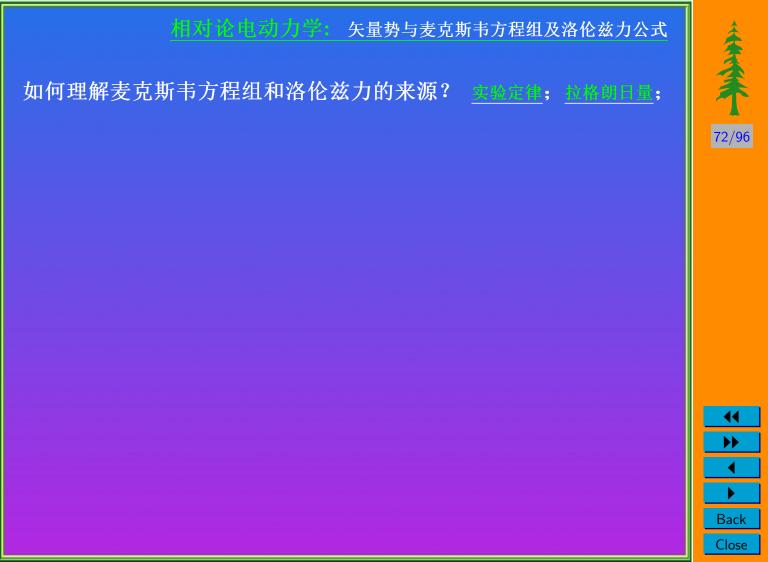
# 的另类理解及扩展











# 

$$\sum_{\mu=1}^{4} \partial_{\mu} \mathbf{F}_{\mu\nu} = -\mu_{0} \mathbf{j}_{\nu} \quad \partial_{\mu} \mathbf{F}_{\nu\lambda} + \partial_{\nu} \mathbf{F}_{\lambda\mu} + \partial_{\lambda} \mathbf{F}_{\mu\nu} = \mathbf{0}$$



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# 相对论电动力学。矢量势与麦克斯韦方程组及洛伦兹力公式

如何理解麦克斯韦方程组和洛伦兹力的来源? <del>实验定律</del>; <del>拉格朗日星</del>;

$$\sum_{\mu=1} \partial_{\mu} \mathbf{F}_{\mu\nu} = -\mu_{\mathbf{0}} \mathbf{j}_{\nu} \quad \partial_{\mu} \mathbf{F}_{\nu\lambda} + \partial_{\nu} \mathbf{F}_{\lambda\mu} + \partial_{\lambda} \mathbf{F}_{\mu\nu} = \mathbf{0}$$

ullet 只要定义 $_{\text{KMMP20}}$  先量势 $_{\mu}$  就可定义场强 $_{\mu}$  了。 $_{\mu}$  是 $_{\mu}$ 



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## 相对论电动力学: 矢量势与麦克斯韦方程组及洛伦兹力公式

$$\sum_{\mu=1} \partial_{\mu} \mathbf{F}_{\mu\nu} = -\mu_{\mathbf{0}} \mathbf{j}_{\nu} \quad \partial_{\mu} \mathbf{F}_{\nu\lambda} + \partial_{\nu} \mathbf{F}_{\lambda\mu} + \partial_{\lambda} \mathbf{F}_{\mu\nu} = \mathbf{0}$$

- 只要定义 $\frac{\mathbf{K}_{\mathbf{K}}\mathbf{M}\mathbf{H}\mathbf{P}\mathbf{M}}{\mathbf{K}}$  就可定义场强  $\mathbf{F}_{\mu\nu}$  =  $\partial_{\mu}\mathbf{A}_{\mu}$   $\partial_{\nu}\mathbf{A}_{\mu}$   $\partial_{\mu}\mathbf{A}_{\mu}$
- 场强的结构使方程  $\partial_{\mu}\mathbf{F}_{\nu\lambda} + \partial_{\nu}\mathbf{F}_{\lambda\mu} + \partial_{\lambda}\mathbf{F}_{\mu\nu} = \mathbf{0}$  自然成立

无源的麦克斯韦方程

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### 相对论电动力学: 矢量势与麦克斯韦方程组及洛伦兹力公式

$$\sum_{\mu=1}^{\mathbf{T}} \partial_{\mu} \mathbf{F}_{\mu\nu} = -\mu_{\mathbf{0}} \mathbf{j}_{\nu} \quad \partial_{\mu} \mathbf{F}_{\nu\lambda} + \partial_{\nu} \mathbf{F}_{\lambda\mu} + \partial_{\lambda} \mathbf{F}_{\mu\nu} = \mathbf{0}$$

- ullet 只要定义 $_{rac{k}{N}}$  大量势 $\underline{\mathbf{A}}_{\mu}$  就可定义场强 $_{rac{\mathbf{F}}{\mu}
  u}$  =  $\partial_{\mu}\mathbf{A}_{
  u}$   $-\partial_{
  u}\mathbf{A}_{
  u}$   $\partial_{
  u}\mathbf{A}_{
  u}$   $\partial_{
  u}\mathbf{A}_{
  u}$
- 场强的结构使方程  $\partial_{\mu}\mathbf{F}_{\nu\lambda} + \partial_{\nu}\mathbf{F}_{\lambda\mu} + \partial_{\lambda}\mathbf{F}_{\mu\nu} = \mathbf{0}$  自然成立
- 场强的结构还使方程  $\sum_{\mu,\nu} \partial_{\mu} \partial_{\nu} \mathbf{F}_{\mu\nu} = \mathbf{0}$  自然成立





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## 相对论电动力学: 矢量势与麦克斯韦方程组及洛伦兹力公式

● 只要定义 $\frac{1}{1}$  大量势 $\frac{1}{1}$  就可定义场强 $\frac{1}{1}$   $\frac{1}$   $\frac{1}{1}$   $\frac{1}{1}$   $\frac{1}{1}$   $\frac{1}{1}$   $\frac{1}{1}$   $\frac{1}{1}$ 

$$\sum_{\mu=1}^{4} \partial_{\mu} \mathbf{F}_{\mu\nu} = -\mu_{\mathbf{0}} \mathbf{j}_{
u} \quad \partial_{\mu} \mathbf{F}_{
u\lambda} + \partial_{
u} \mathbf{F}_{\lambda\mu} + \partial_{\lambda} \mathbf{F}_{\mu
u} = \mathbf{0}$$

- 场强的结构还使方程  $\sum_{\mu,\nu} \partial_{\mu} \partial_{\nu} \mathbf{F}_{\mu\nu} = \mathbf{0}$  自然成立
  - 它意味着可定义流  $\mathbf{j}_{\nu} \equiv -\frac{1}{\mu_0} \sum_{\mu} \partial_{\mu} \mathbf{F}_{\mu\nu}$ ,满足守恒方程  $\sum_{\nu} \partial_{\nu} \mathbf{j}_{\nu} = \mathbf{0}$

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## 相对论电动力学 矢量势与麦克斯韦方程组及洛伦兹力公式

实验定律; 拉格朗日量;

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$$\sum_{\mu=1} \partial_{\mu} \mathbf{F}_{\mu 
u} = -\mu_{\mathbf{0}} \mathbf{j}_{
u} \quad \partial_{\mu} \mathbf{F}_{
u \lambda} + \partial_{
u} \mathbf{F}_{\lambda \mu} + \partial_{\lambda} \mathbf{F}_{\mu 
u} = \mathbf{0}$$

如何理解麦克斯韦方程组和洛伦兹力的来源?

- ullet 只要定义 $_{(KM)MPSM}$  <mark>矢量勢 $_{(I)}$ </mark> 就可定义场强  $_{(I)}$   $_{(I)}$
- 场强的结构使方程  $\partial_{\mu}\mathbf{F}_{\nu\lambda} + \partial_{\nu}\mathbf{F}_{\lambda\mu} + \partial_{\lambda}\mathbf{F}_{\mu\nu} = \mathbf{0}$  自然成立
- 场强的结构还使方程  $\sum_{\mu,\nu} \partial_{\mu} \partial_{\nu} \mathbf{F}_{\mu\nu} = \mathbf{0}$  自然成立
- 它意味着可定义流  $\mathbf{j}_{\nu} \equiv -\frac{1}{\mu_0} \sum_{\mu} \partial_{\mu} \mathbf{F}_{\mu\nu}$ ,满足守恒方程  $\sum_{\nu} \partial_{\nu} \mathbf{j}_{\nu} = \mathbf{0}$

• 从  $\sum_{\nu} \overline{\mathbf{F}_{\mu\nu} \mathbf{j}_{\nu}} = -\sum_{\nu} \partial_{\nu} \mathbf{T}_{\mu\nu}$  可认定四度力:  $\mathbf{f}_{\mu} = \sum_{\nu} \overline{\mathbf{F}}_{\mu\nu} \overline{\mathbf{j}}_{\nu}$  絡伦兹力公式!



→

**▶** Back

#### **福对论电动力学** 矢量势与麦克斯韦方程组及洛伦兹力公式

你对上述对经典电动力学的"理解"如何评价?它靠谱吗?

• 你对四度电磁势在经典电动力学中的地位如何看?

特别对四度电流密度、四度力密度的定义方式如何评价?它会有什么引申的讨论吗?

规范对称性的作用又如何?

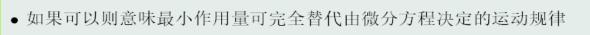
杨振宁在研读法拉第撰写的《电学的实验研究》后特别提到: 法拉第书中的电紧张状态频繁出现在书的各处,并且又被频繁赋子各种其他的名字, 诸如特殊态、强度态、特殊状态等,但从始至终未给出清晰的定义。 73/96

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# 磁单极与最小作用量的"逆问题"及磁荷量子化: • 从作用量求极值出发,可以得到 运动方程!

- 任一决定运动规律的 <u>方程</u>,都存在相应的作用量?
- 当初费曼发明"路径积分"就因有些系统哈密顿体系无法描写



自由质点被约束在单位球面上运动:  $S = \int dt \frac{1}{2} m v^2$   $\vec{r} \cdot \vec{r} = 1$ 

自田庾总被约果在单位球面上运动: 
$$S = \int dt \frac{\pi v^2}{2} \qquad r \cdot r = 1$$
 
$$S' = \int dt \left[ \frac{1}{2} m v^2 + \lambda(r^2 - 1) \right] \Rightarrow m\ddot{\vec{r}} - 2\lambda \vec{r} = 0 \quad \lambda = -\frac{m}{2} v^2 \Leftarrow \dot{\vec{r}} \cdot \dot{\vec{r}} + \vec{r} \cdot \ddot{\vec{r}} = 0 \Leftrightarrow \vec{r} = \vec{r} =$$

$$m[\ddot{\vec{r}} + \vec{r}(\dot{\vec{r}} \cdot \dot{\vec{r}})] = 0 \qquad r^2 = 1$$

现假设此质点带电电量q; 并且在球心放置一个单位磁单极:  $\vec{B} = \frac{\vec{r}}{r^3}$ 

场方程中应出现洛伦兹力项: 
$$m(\ddot{\vec{r}} + \vec{r}\dot{\vec{r}} \cdot \dot{\vec{r}}) = q\vec{v} \times \vec{B} = q\vec{r} \times \vec{r}$$

 $m[\ddot{r}+\vec{r}(\dot{r}\cdot\dot{r})]+q\vec{r}\times\dot{\vec{r}}=0 \qquad r^2=1$  洛伦兹力项如何在作用量中体现 ?  $\vec{r}\cdot(\vec{r}\times\dot{\vec{r}})=0$  例: tensionless string



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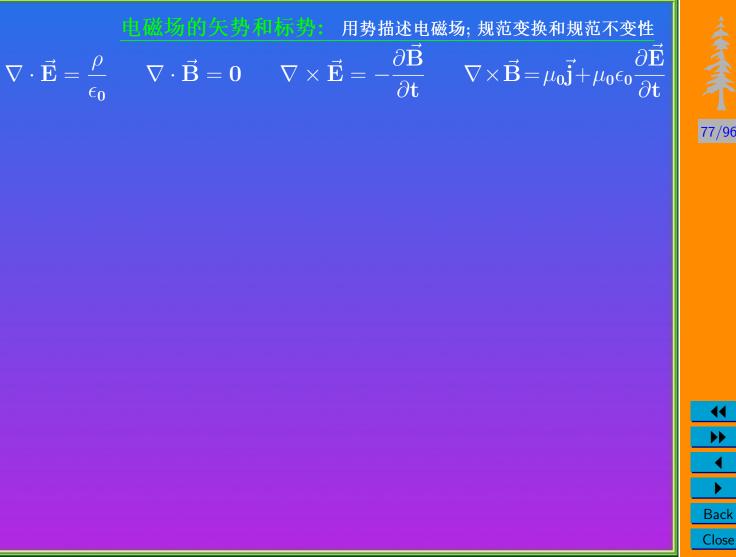
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## $S = \int dt \, \left[ \frac{1}{2} m v^2 + \lambda (r^2 - 1) + q \dot{\vec{r}} \cdot \vec{A} \right]$ 考虑带电粒子走闭合路径 $\gamma$ : $\vec{r}(t=\infty) = \vec{r}(t=0)$ 可以得到洛伦兹力,但矢量势在有磁单极时有奇异▽·(▽×Ā)≠0

路径在其球面上所包围的面积可用格林公式: 
$$\int dt \ q \vec{r} \cdot \vec{A} = q \oint_{D} d\vec{s} \cdot (\nabla \times \vec{A}) = q \int_{D} d\vec{s} \cdot \vec{B} \qquad \textbf{D: 所包围的面积}$$
 含磁单极的磁场作用量在带电粒子运动的一维曲线上无法很好定义: 但在以一维曲线为边界的 二维 球面上确可很好定义! 全息原理? 但故事没完... 面积D有两种取法D和D': 
$$\oint_{D+D'} \vec{B} \cdot d\vec{s} = -\oint_{D+D'} d\vec{s} \cdot \nabla \frac{1}{r} = -\int dV \nabla^2 \frac{1}{r} = 4\pi$$
 若要求经典自治: 
$$\int dt \ q \vec{r} \cdot \vec{A} = q \int_{D} d\vec{s} \cdot \vec{B} = -q \int_{D'} d\vec{s} \cdot \vec{B} + 4\pi q \Leftarrow \text{不能为零!}$$
  $e^{\frac{i\pi}{2} \int_{D} d\vec{s} \cdot \vec{B}} \Rightarrow 1 = e^{\frac{i\pi}{2} \frac{f}{2} \int_{D+D'} d\vec{s} \cdot \vec{B}} \stackrel{\text{单位磁荷磁通量}}{=} e^{4\pi \frac{i\pi}{2}} \Rightarrow q = \frac{n}{2} \hbar \text{Dirac} \text{wäd}$ 

 $S = \int dt \left[ \frac{1}{2} m v^2 + \lambda (r^2 - 1) \right] + q \int_{\mathbb{R}} d\vec{S} \cdot \vec{B}$ 若要求量子自洽↑

- 最小作用量的逆问题是一个真命题,还是一个伪命题?
- 如何理解在原空间不存在的作用量,扩充了空间就存在这个事实?
- 是否可能存在即使在扩充了的空间也不存在的作用量?如何理解?



# <u>电应场的失势和标</u>势。 用势描述电磁场; 规范变换和规范不变性 $egin{aligned} abla \cdot ec{\mathbf{E}} = rac{ ho}{\epsilon_{\mathbf{0}}} & abla \cdot ec{\mathbf{B}} = \mathbf{0} & abla imes ec{\mathbf{E}} = -rac{\partial ec{\mathbf{B}}}{\partial \mathbf{t}} & abla imes ec{\mathbf{E}} = \mu_{\mathbf{0}} ec{\mathbf{j}} + \mu_{\mathbf{0}} \epsilon_{\mathbf{0}} rac{\partial ec{\mathbf{E}}}{\partial \mathbf{t}} \end{aligned}$

$$abla imes ec{\mathbf{E}} = -rac{\partial}{\partial \mathbf{t}} 
abla imes ec{\mathbf{A}}$$













# <u>电磁场的大势和标势</u> 用势描述电磁场; 规范变换和规范不变性 $egin{aligned} egin{aligned} abla \cdot ec{\mathbf{E}} &= rac{ ho}{\epsilon_{\mathbf{0}}} & abla \cdot ec{\mathbf{B}} &= \mathbf{0} & abla imes ec{\mathbf{E}} &= -rac{\partial ec{\mathbf{B}}}{\partial \mathbf{t}} & abla imes ec{\mathbf{B}} &= \mu_{\mathbf{0}} ec{\mathbf{j}} + \mu_{\mathbf{0}} \epsilon_{\mathbf{0}} rac{\partial ec{\mathbf{E}}}{\partial \mathbf{t}} \end{aligned}$



$$abla imes \vec{\mathbf{E}} = -rac{\partial}{\partial \mathbf{t}} 
abla imes \vec{\mathbf{A}} = -
abla imes rac{\partial \vec{\mathbf{A}}}{\partial \mathbf{t}}$$











## 电磁场的失势和标势: 用势描述电磁场; 规范变换和规范不变性 $abla \cdot \vec{\mathbf{E}} = \frac{ ho}{\epsilon_0} \quad abla \cdot \vec{\mathbf{B}} = \mathbf{0} \quad abla imes \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}} \quad abla imes \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}}$

$$abla imes ilde{\mathbf{E}} = -rac{\partial}{\partial \mathbf{t}} 
abla imes ilde{\mathbf{A}} = -
abla imes rac{\partial ilde{\mathbf{A}}}{\partial \mathbf{t}} \qquad o \qquad 
abla imes ( ilde{\mathbf{E}} + rac{\partial ilde{\mathbf{A}}}{\partial \mathbf{t}}) = \mathbf{0}$$













# 电磁场的矢势和标势 用势描述电磁场;规范变换和规范不变性 $abla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0} \qquad \nabla \cdot \vec{\mathbf{B}} = \mathbf{0} \qquad \nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}} \qquad \nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}}$

$$\nabla \times \vec{\mathbf{E}} = -\frac{\partial}{\partial t} \nabla \times \vec{\mathbf{A}} = -\nabla \times \frac{\partial \vec{\mathbf{A}}}{\partial t} \longrightarrow \nabla \times (\vec{\mathbf{E}} + \frac{\partial \vec{\mathbf{A}}}{\partial t}) = \mathbf{0}$$

$$\vec{\mathbf{E}} + \frac{\partial \vec{\mathbf{A}}}{\partial t} = -\nabla \phi$$













# $abla \cdot \vec{\mathbf{E}} = \frac{ ho}{\epsilon_0} \qquad abla \cdot \vec{\mathbf{B}} = \mathbf{0} \qquad abla imes \vec{\mathbf{E}} = -rac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}} \qquad abla imes \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}} abla$

$$\nabla \times \vec{\mathbf{E}} = -\frac{\partial}{\partial t} \nabla \times \vec{\mathbf{A}} = -\nabla \times \frac{\partial \vec{\mathbf{A}}}{\partial t} \qquad \rightarrow \qquad \nabla \times (\vec{\mathbf{E}} + \frac{\partial \vec{\mathbf{A}}}{\partial t}) = \mathbf{0}$$















# <u>电磁场的 天勢 和标券</u>: 用势描述电磁场; 规范变换和规范不变性 $egin{aligned} egin{aligned} abla \cdot ec{\mathbf{E}} &= rac{ ho}{\epsilon_0} & abla \cdot ec{\mathbf{B}} &= \mathbf{0} & abla imes ec{\mathbf{E}} &= -rac{\partial ec{\mathbf{B}}}{\partial \mathbf{t}} & abla imes ec{\mathbf{E}} &= \mu_0 ec{\mathbf{j}} + \mu_0 \epsilon_0 rac{\partial ec{\mathbf{E}}}{\partial \mathbf{t}} \end{aligned}$

$$\nabla \times \vec{\mathbf{E}} = -\frac{\partial}{\partial t} \nabla \times \vec{\mathbf{A}} = -\nabla \times \frac{\partial \vec{\mathbf{A}}}{\partial t} \longrightarrow \nabla \times (\vec{\mathbf{E}} + \frac{\partial \vec{\mathbf{A}}}{\partial t}) = \mathbf{0}$$

$$\vec{\mathbf{E}} + \frac{\partial \vec{\mathbf{A}}}{\partial t} = -\nabla \phi \longrightarrow \vec{\mathbf{E}} = -\nabla \phi - \frac{\partial \vec{\mathbf{A}}}{\partial t}$$

$$\begin{cases} \vec{\mathbf{E}} = -\nabla \phi - \frac{\partial \vec{\mathbf{A}}}{\partial t} \\ \vec{\mathbf{B}} = \nabla \times \vec{\mathbf{A}} \end{cases}$$



















# 电磁场的矢势和标分。用势描述电磁场; 规范变换和规范不变性 $\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0} \qquad \nabla \cdot \vec{\mathbf{B}} = \mathbf{0} \qquad \nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}} \qquad \nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}}$



# 图 版场的 矢 男和 标要。 用势描述电磁场; 规范变换和规范不变性 $\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0} \qquad \nabla \cdot \vec{\mathbf{B}} = \mathbf{0} \qquad \nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}} \qquad \nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}}$

$$abla imes \mathbf{E} = -rac{\partial}{\partial \mathbf{t}} 
abla imes \mathbf{A} = -
abla imes rac{\partial \mathbf{A}}{\partial \mathbf{t}} \qquad o \qquad 
abla imes (\mathbf{E} + rac{\partial \mathbf{A}}{\partial \mathbf{t}}) = \mathbf{0}$$

$$\vec{\mathbf{E}} + \frac{\partial \vec{\mathbf{A}}}{\partial \mathbf{t}} = -\nabla \phi \qquad \rightarrow \qquad \vec{\mathbf{E}} = -\nabla \phi - \frac{\partial \vec{\mathbf{A}}}{\partial \mathbf{t}}$$

$$\begin{cases} \vec{\mathbf{E}} = -\nabla \phi - \frac{\partial \vec{\mathbf{A}}}{\partial \mathbf{t}} \\ \vec{\mathbf{B}} = \nabla \times \vec{\mathbf{A}} \end{cases} \qquad \vec{\mathbf{A}}, \phi \vec{\mathbf{A}} = \vec{\mathbf{A}} + \nabla \chi$$

$$\phi \rightarrow \phi' = \phi - \frac{\partial \chi}{\partial \mathbf{t}}$$



# <u>自磁场的矢势和标学</u> 用势描述电磁场; 规范变换和规范不变性

$$\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0} \qquad \nabla \cdot \vec{\mathbf{B}} = \mathbf{0} \qquad \nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}} \qquad \nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}}$$

$$\nabla \times \vec{\mathbf{E}} = -\frac{\partial}{\partial \mathbf{t}} \nabla \times \vec{\mathbf{A}} = -\nabla \times \frac{\partial \vec{\mathbf{A}}}{\partial \mathbf{t}} \quad \rightarrow \quad \nabla \times (\vec{\mathbf{E}} + \frac{\partial \vec{\mathbf{A}}}{\partial \mathbf{t}}) = \mathbf{0}$$

$$\vec{\mathbf{E}} + \frac{\partial \vec{\mathbf{A}}}{\partial \mathbf{t}} = -\nabla \phi \quad \rightarrow \quad \vec{\mathbf{E}} = -\nabla \phi - \frac{\partial \vec{\mathbf{A}}}{\partial \mathbf{t}}$$

$$\left\{ \begin{array}{l} \vec{\mathbf{E}} = -\nabla\phi - \frac{\partial\vec{\mathbf{A}}}{\partial t} \\ \vec{\mathbf{B}} = \nabla\times\vec{\mathbf{A}} \end{array} \right. \qquad \vec{\mathbf{A}}, \phi$$
不唯一 
$$\left\{ \begin{array}{l} \vec{\mathbf{A}} \to \vec{\mathbf{A}}' = \vec{\mathbf{A}} + \nabla\chi \\ \phi \to \phi' = \phi - \frac{\partial\chi}{\partial t} \end{array} \right.$$

每组 $(\vec{A}, \phi)$ 叫一种规范,不同规范对应同一组物理观察量 $\vec{E}, \vec{B}$ . W有证  $\vec{B}$  何要点  $\phi$ 2

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## 电磁场的矢势和标券: 用势描述电磁场; 规范变换和规范不变性

$$\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0} \qquad \nabla \cdot \vec{\mathbf{B}} = \mathbf{0} \qquad \nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}} \qquad \nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}}$$

$$abla imes ec{\mathbf{E}} = -rac{\partial}{\partial \mathbf{t}} 
abla imes ec{\mathbf{A}} = -
abla imes rac{\partial ec{\mathbf{A}}}{\partial \mathbf{t}} \qquad o \qquad 
abla imes (ec{\mathbf{E}} + rac{\partial ec{\mathbf{A}}}{\partial \mathbf{t}}) = \mathbf{0}$$

$$ec{\mathbf{E}} + rac{\partial ec{\mathbf{A}}}{\partial \mathbf{t}} = -
abla \phi \qquad \rightarrow \qquad ec{\mathbf{E}} = -
abla \phi - rac{\partial ec{\mathbf{A}}}{\partial \mathbf{t}}$$
 
$$\begin{cases} ec{\mathbf{E}} = -
abla \phi - rac{\partial ec{\mathbf{A}}}{\partial \mathbf{t}} & \vec{\mathbf{A}}, \phi \text{ with } \qquad \begin{cases} ec{\mathbf{A}} \to ec{\mathbf{A}}' = ec{\mathbf{A}} + 
abla \chi \\ \phi \to \phi' = \phi - rac{\partial \chi}{\partial \lambda} \end{cases}$$
 where  $\mathbf{E}$ 

每组 $(\vec{A}, \phi)$ 叫一种规范,不同规范对应同一组物理观察量 $\vec{E}, \vec{B}$ . 黑有 $\vec{E}, \vec{B}$  所有物理量和物理规律与特殊的规范选择无关—规范不变性!

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#### 电磁场的矢势和标势: 用势描述电磁场; 规范变换和规范不变性

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$$\left\{ egin{array}{ll} ec{\mathbf{E}} = -
abla \phi - rac{\partial ec{\mathbf{A}}}{\partial \mathbf{t}} & ec{\mathbf{A}}, \phi$$
不唯一  $\left\{ egin{array}{ll} ec{\mathbf{A}} 
ightarrow ec{\mathbf{A}}' = ec{\mathbf{A}} + 
abla \chi & \chi \\ \phi 
ightarrow \phi' = \phi - rac{\partial \chi}{\partial \mathbf{t}} & \chi \end{array} 
ight.$ 

每组 $(\vec{A}, \phi)$ 叫一种规范,不同规范对应同一组物理观察量 $\vec{E}, \vec{B}$ . Reference of  $\vec{B}$  所有物理量和物理规律与特殊的规范选择无关—规范不变性!

 $\vec{\mathbf{E}} + \frac{\partial \vec{\mathbf{A}}}{\partial t} = -\nabla \phi \qquad \rightarrow \qquad \vec{\mathbf{E}} = -\nabla \phi - \frac{\partial \vec{\mathbf{A}}}{\partial t}$ 

所有物理量和物理规律与特殊的规范选择无关—规范不变性! 在数学上一种不变性就对应一种对称性,与上面规范不变性对应的对称性叫U(1)规范对称性.



**↔** 



<u>电磁场的大势和标</u>势: 用势描述电磁场; 规范变换和规范不变性 →

$$\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0} \qquad \nabla \cdot \vec{\mathbf{B}} = \mathbf{0} \qquad \nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}} \qquad \nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}}$$

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$$\left\{egin{array}{ll} ec{\mathbf{E}} = -
abla \phi - rac{\partial ec{\mathbf{A}}}{\partial \mathbf{t}} & ec{\mathbf{A}}, \phi$$
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ightarrow ec{\mathbf{A}}' = ec{\mathbf{A}} + 
abla \chi \ \phi 
ightarrow \phi' = \phi - rac{\partial \chi}{\partial \mathbf{t}} \end{array}
ight.$ 

每组 $(\vec{A}, \phi)$ 叫一种规范,不同规范对应同一组物理观察量 $\vec{E}, \vec{B}$ . Explained or 所有物理量和物理规律与特殊的规范选择无关—规范不变性!

 $\vec{\mathbf{E}} + \frac{\partial \vec{\mathbf{A}}}{\partial t} = -\nabla \phi \qquad \rightarrow \qquad \vec{\mathbf{E}} = -\nabla \phi - \frac{\partial \vec{\mathbf{A}}}{\partial t}$ 

在数学上一种不变性就对应一种对称性,与上面规范不变性对应的对称性叫U(1)规范对称性. 因此, <u>电磁相互作用具有U(1)规范对称性</u>.



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电磁场的矢势和标势: 用势描述电磁场; 规范变换和规范不变性

$$\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0} \qquad \nabla \cdot \vec{\mathbf{B}} = \mathbf{0} \qquad \nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}} \qquad \nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}}$$

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abla imes (\vec{\mathbf{E}} + rac{\partial \vec{\mathbf{A}}}{\partial t}) = \mathbf{0}$$

$$\left\{ egin{array}{ll} ec{\mathbf{E}} = -
abla \phi - rac{\partial ec{\mathbf{A}}}{\partial \mathbf{t}} & ec{\mathbf{A}}, \phi$$
不唯一  $\left\{ egin{array}{ll} ec{\mathbf{A}} 
ightarrow ec{\mathbf{A}}' = ec{\mathbf{A}} + 
abla \chi \ \phi 
ightarrow \phi' = \phi - rac{\partial \chi}{\partial \mathbf{t}} \end{array} 
ight.$ 

每组 $(\vec{A}, \phi)$ 叫一种规范,不同规范对应同一组物理观察量 $\vec{E}, \vec{B}$ . 医有压  $\vec{B}$  所有物理量和物理规律与特殊的规范选择无关—规范不变性!

 $\vec{\mathbf{E}} + \frac{\partial \vec{\mathbf{A}}}{\partial t} = -\nabla \phi \qquad \rightarrow \qquad \vec{\mathbf{E}} = -\nabla \phi - \frac{\partial \vec{\mathbf{A}}}{\partial t}$ 

在数学上一种不变性就对应一种对称性,与上面规范不变性对应的对称性叫U(1)规范对称性. 因此,电磁相互作用具有U(1)规范对称性.

在实际计算中,选择一定的条件来把 $\vec{A}$ , $\phi$ 所具有的不确定的规范自由度限制住.



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电磁场的矢势和标务: 用势描述电磁场; 规范变换和规范不变性

$$\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0} \qquad \nabla \cdot \vec{\mathbf{B}} = \mathbf{0} \qquad \nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}} \qquad \nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}}$$

$$abla imes ec{\mathbf{E}} = -rac{\partial}{\partial \mathbf{t}} 
abla imes ec{\mathbf{A}} = -
abla imes rac{\partial ec{\mathbf{A}}}{\partial \mathbf{t}} \qquad o \qquad 
abla imes (ec{\mathbf{E}} + rac{\partial ec{\mathbf{A}}}{\partial \mathbf{t}}) = \mathbf{0}$$

$$\left\{ egin{array}{ll} ec{\mathbf{E}} = -
abla \phi - rac{\partial ec{\mathbf{A}}}{\partial \mathbf{t}} & ec{\mathbf{A}}, \phi$$
不唯一  $\left\{ egin{array}{ll} ec{\mathbf{A}} 
ightarrow ec{\mathbf{A}}' = ec{\mathbf{A}} + 
abla \chi \ \phi 
ightarrow \phi' = \phi - rac{\partial \chi}{\partial \mathbf{t}} \end{array} 
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每组 $(\vec{A}, \phi)$ 叫一种规范,不同规范对应同一组物理观察量 $\vec{E}, \vec{B}$ . Example of 所有物理量和物理规律与特殊的规范选择无关—规范不变性!

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**▲** 

**电磁场的矢势和标**势: 用势描述电磁场; 规范变换和规范不变性

$$\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0} \qquad \nabla \cdot \vec{\mathbf{B}} = \mathbf{0} \qquad \nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}} \qquad \nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}}$$

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库伦规范:  $\nabla \cdot \mathbf{A} = \mathbf{0}$ 

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<u>电磁场的矢势和标券</u>: 用势描述电磁场; 规范变换和规范不变性

$$\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0} \qquad \nabla \cdot \vec{\mathbf{B}} = \mathbf{0} \qquad \nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}} \qquad \nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}}$$

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库伦规范:  $\nabla \cdot \vec{A} = 0$  洛伦兹规范:  $\nabla \cdot \vec{A} + \frac{1}{c^2} \frac{\partial \phi}{\partial t} = 0$ 

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### 达朗伯(d'Alembert)方程 $\nabla \times \vec{\mathbf{B}} = \mu_{0}\vec{\mathbf{j}} + \mu_{0}\epsilon_{0}\frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}} \rightarrow \nabla \times (\nabla \times \vec{\mathbf{A}}) = \mu_{0}\vec{\mathbf{j}} + \mu_{0}\epsilon_{0}[-\nabla \frac{\partial \phi}{\partial \mathbf{t}} - \frac{\partial^{2}\vec{\mathbf{A}}}{\partial \mathbf{t}^{2}}]$ $= \nabla(\nabla \cdot \vec{\mathbf{A}}) - \nabla^{2}\vec{\mathbf{A}}$





## 达朗伯(d'Alembert)方程

$$\nabla \times \vec{\mathbf{B}} = \mu_{0}\vec{\mathbf{j}} + \mu_{0}\epsilon_{0}\frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}} \rightarrow \nabla \times (\nabla \times \vec{\mathbf{A}}) = \mu_{0}\vec{\mathbf{j}} + \mu_{0}\epsilon_{0}[-\nabla \frac{\partial \phi}{\partial \mathbf{t}} - \frac{\partial^{2}\vec{\mathbf{A}}}{\partial \mathbf{t}^{2}}]$$
$$= \nabla(\nabla \cdot \vec{\mathbf{A}}) - \nabla^{2}\vec{\mathbf{A}}$$
$$\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_{0}}$$











### 达朗伯(d'Alembert)方程 $\nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}} \rightarrow \nabla \times (\nabla \times \vec{\mathbf{A}}) = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 [-\nabla \frac{\partial \phi}{\partial \mathbf{t}} - \frac{\partial^2 \vec{\mathbf{A}}}{\partial \mathbf{t}^2}]$

$$\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0} \qquad \qquad \rightarrow \qquad -\nabla^2 \phi - \frac{\partial}{\partial t} \nabla \cdot \vec{\mathbf{A}} = \frac{\rho}{\epsilon_0}$$













达朗伯(d'Alembert)方程  $\nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}} \rightarrow \nabla \times (\nabla \times \vec{\mathbf{A}}) = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 [-\nabla \frac{\partial \phi}{\partial \mathbf{t}} - \frac{\partial^2 \vec{\mathbf{A}}}{\partial \mathbf{t}^2}]$ 

$$\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0} \qquad \rightarrow \qquad -\nabla^2 \phi - \frac{\partial}{\partial t} \nabla \cdot \vec{\mathbf{A}} = \frac{\rho}{\epsilon_0}$$

库伦规范:  $\nabla \cdot \vec{A} = 0$ 



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达朗伯(d'Alembert)方程  $\nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}} \rightarrow \nabla \times (\nabla \times \vec{\mathbf{A}}) = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \left[ -\nabla \frac{\partial \phi}{\partial \mathbf{t}} - \frac{\partial^2 \vec{\mathbf{A}}}{\partial \mathbf{t}^2} \right]$ 78/96

$$\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0} \qquad \qquad \rightarrow \qquad -\nabla^2 \phi - \frac{\partial}{\partial t} \nabla \cdot \vec{\mathbf{A}} = \frac{\rho}{\epsilon_0}$$

 $abla \cdot ec{ ext{A}} = 0$ 库伦规范:

$$\nabla^2 \vec{\mathbf{A}} - \frac{1}{\mathbf{c}^2} \frac{\partial^2 \vec{\mathbf{A}}}{\partial \mathbf{t}^2} = -\mu_0 \vec{\mathbf{j}} + \frac{1}{\mathbf{c}^2} \frac{\partial}{\partial \mathbf{t}} \nabla \phi = -\mu_0 \vec{\mathbf{j}}^* \qquad \nabla^2 \phi = -\frac{\rho}{\epsilon_0}$$





电磁场的矢势和标势: 达朗伯(d'Alembert)方程

$$\nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}} \rightarrow \nabla \times (\nabla \times \vec{\mathbf{A}}) = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \left[ -\nabla \frac{\partial \phi}{\partial \mathbf{t}} - \frac{\partial^2 \vec{\mathbf{A}}}{\partial \mathbf{t}^2} \right]$$
$$= \nabla (\nabla \cdot \vec{\mathbf{A}}) - \nabla^2 \vec{\mathbf{A}}$$
$$\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0} \rightarrow -\nabla^2 \phi - \frac{\partial}{\partial \mathbf{t}} \nabla \cdot \vec{\mathbf{A}} = \frac{\rho}{\epsilon_0}$$

库伦规范: 
$$abla \cdot \vec{\mathbf{A}} = \mathbf{0}$$

$$\nabla^2 \vec{\mathbf{A}} - \frac{1}{\mathbf{c}^2} \frac{\partial^2 \vec{\mathbf{A}}}{\partial \mathbf{t}^2} = -\mu_0 \vec{\mathbf{j}} + \frac{1}{\mathbf{c}^2} \frac{\partial}{\partial \mathbf{t}} \nabla \phi = -\mu_0 \vec{\mathbf{j}}^* \qquad \qquad \nabla^2 \phi = -\frac{\rho}{\epsilon_0}$$

洛伦兹规范: 
$$\nabla \cdot \vec{\mathbf{A}} + \frac{1}{\mathbf{c}^2} \frac{\partial \phi}{\partial \mathbf{t}} = \mathbf{0}$$
 实际上是  $\nabla \cdot \vec{\mathbf{A}}_{\mathbf{L}} + \frac{1}{\mathbf{c}^2} \frac{\partial \phi}{\partial \mathbf{t}} = \mathbf{0}$ 

势的时间分量的时间变化与空间分量的纵向相互抵消1









达朗伯(d'Alembert)方程

$$\nabla \times \vec{\mathbf{B}} = \mu_{0}\vec{\mathbf{j}} + \mu_{0}\epsilon_{0}\frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}} \rightarrow \nabla \times (\nabla \times \vec{\mathbf{A}}) = \mu_{0}\vec{\mathbf{j}} + \mu_{0}\epsilon_{0}[-\nabla \frac{\partial \phi}{\partial \mathbf{t}} - \frac{\partial^{2}\vec{\mathbf{A}}}{\partial \mathbf{t}^{2}}]$$

$$= \nabla(\nabla \cdot \vec{\mathbf{A}}) - \nabla^{2}\vec{\mathbf{A}}$$

$$\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_{0}} \rightarrow -\nabla^{2}\phi - \frac{\partial}{\partial \mathbf{t}}\nabla \cdot \vec{\mathbf{A}} = \frac{\rho}{\epsilon_{0}}$$

库伦规范: 
$$abla \cdot \vec{\mathbf{A}} = \mathbf{0}$$

$$\nabla^2 \vec{\mathbf{A}} - \frac{1}{\mathbf{c}^2} \frac{\partial^2 \vec{\mathbf{A}}}{\partial \mathbf{t}^2} = -\mu_0 \vec{\mathbf{j}} + \frac{1}{\mathbf{c}^2} \frac{\partial}{\partial \mathbf{t}} \nabla \phi = -\mu_0 \vec{\mathbf{j}}^* \qquad \qquad \nabla^2 \phi = -\frac{\rho}{\epsilon_0}$$

洛伦兹规范: 
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$$\nabla^2 \vec{\mathbf{A}} - \frac{1}{\mathbf{c}^2} \frac{\partial^2 \vec{\mathbf{A}}}{\partial \mathbf{t}^2} = -\mu_0 \vec{\mathbf{j}} \qquad \nabla^2 \phi - \frac{1}{\mathbf{c}^2} \frac{\partial^2 \phi}{\partial \mathbf{t}^2} = -\frac{\rho}{\epsilon_0}$$



电磁场的矢势和标势: 达朗伯(d'Alembert)方程

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$$\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_{0}} \rightarrow -\nabla^{2}\phi - \frac{\partial}{\partial \mathbf{t}}\nabla \cdot \vec{\mathbf{A}} = \frac{\rho}{\epsilon_{0}}$$

库伦规范: 
$$abla \cdot ec{\mathbf{A}} = \mathbf{0}$$

$$\nabla^2 \vec{\mathbf{A}} - \frac{1}{\mathbf{c}^2} \frac{\partial^2 \vec{\mathbf{A}}}{\partial \mathbf{t}^2} = -\mu_0 \vec{\mathbf{j}} + \frac{1}{\mathbf{c}^2} \frac{\partial}{\partial \mathbf{t}} \nabla \phi = -\mu_0 \vec{\mathbf{j}}^* \qquad \nabla^2 \phi = -\frac{\rho}{\epsilon_0}$$

 $\mathbf{c} \to \infty$  时,回到静电磁情形

洛伦兹规范: 
$$\nabla \cdot \vec{\mathbf{A}} + \frac{1}{\mathbf{c}^2} \frac{\partial \phi}{\partial \mathbf{t}} = \mathbf{0}$$
 实际上是  $\nabla \cdot \vec{\mathbf{A}}_{\mathbf{L}} + \vec{\mathbf{c}}_{\mathbf{c}}$  为证的 阿安化与全国分量的级的和互振讯 
$$\nabla^2 \vec{\mathbf{A}} - \frac{1}{\mathbf{c}^2} \frac{\partial^2 \vec{\mathbf{A}}}{\partial \mathbf{t}^2} = -\mu_0 \vec{\mathbf{j}} \qquad \nabla^2 \phi - \frac{1}{\mathbf{c}^2} \frac{\partial^2 \phi}{\partial \mathbf{t}^2} = -\frac{\rho}{\epsilon_0}$$

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● <mark>超导</mark>无电阻、迈斯纳效应 是物理学尚未了解清楚的介质 电磁性质 单个原子层的超导薄膜

● 还是 已为大众熟知,开始影响人们生活 现代科技电磁学常讲的例子

● 超导的标准理解需要 量子力学 及很多 凝聚态物理 的知识 各种奇怪的的假设和规则

● 早年超导的研究不仅对固体物理,也影响了粒子物理核物理08年诺贝尔物理奖

南部阳一郎 ■ 目前它仍是凝聚态最前沿和活跃的研究方向! 新的诺贝尔奖? 期待再次影响粒子物理?

- 我们期待着高温或室温超导未来可能对人类生活造成的重大影响
- ●问题: 电动力学能否不过多地依赖量子力学和凝聚态物理来介绍超导? →般不行
- 若可实现,则说明电磁场及麦克斯韦方程在超导现象中的核心支配作用图像更清晰
- 还说明涉及超导的所谓量子力学及凝聚态物理细节很多可有效地用 经典电动力学 描写 • 核心要说明从经典电动力学的水平上看是介质的什么物理机制及图像导致了超导

 $\vec{D} = \epsilon \vec{E}, \vec{B} = \mu \vec{H}, \vec{i} = \gamma \vec{E}$ ● 以下依据电动力学进行详细推导、演绎和诠释超导的另类推导:理论物理但非凝聚态人的观点 79/96

有双元子 质量 伦敦方程,理想导体及迈斯纳效应 真空中的场方程:  $\left[\nabla^2 - \frac{1}{\mathbf{c^2}} \frac{\partial^2}{\partial \mathbf{t^2}}\right] \vec{\mathbf{A}} - \nabla \left[\nabla \cdot \vec{\mathbf{A}} + \frac{1}{\mathbf{c^2}} \frac{\partial \phi}{\partial \mathbf{t}}\right] = -\mu_0 \vec{\mathbf{j}}$  $\mu_0 \epsilon_0$  $\left[\nabla^2 - \frac{1}{\mathbf{c}^2} \frac{\partial^2}{\partial \mathbf{t}^2}\right] \phi + \frac{\partial}{\partial \mathbf{t}} \left[\nabla \cdot \vec{\mathbf{A}} + \frac{1}{\mathbf{c}^2} \frac{\partial \phi}{\partial \mathbf{t}}\right] = -\frac{\rho}{\epsilon_0}\right]$ 80/96 它具有规范不变性: Back Close

伦敦方程,理想导体及迈斯纳效应 真空中的场方程:  $\left[\nabla^2 - \frac{1}{\mathbf{c}^2} \frac{\partial^2}{\partial \mathbf{t}^2}\right] \vec{\mathbf{A}} - \nabla \left[\nabla \cdot \vec{\mathbf{A}} + \frac{1}{\mathbf{c}^2} \frac{\partial \phi}{\partial \mathbf{t}}\right] = -\mu_0 \vec{\mathbf{j}}$  $\mu_0 \epsilon_0$  $\left[\nabla^2 - \frac{1}{\mathbf{c^2}} \frac{\partial^2}{\partial \mathbf{t^2}}\right] \phi + \frac{\partial}{\partial \mathbf{t}} \left[\nabla \cdot \vec{\mathbf{A}} + \frac{1}{\mathbf{c^2}} \frac{\partial \phi}{\partial \mathbf{t}}\right] = -\frac{\rho}{\epsilon_0}$ 80/96 它具有规范不变性:  $igg[
abla^2 - rac{1}{\mathbf{c^2}}rac{ar{\partial^2}}{\partial \mathbf{t^2}} - rac{\mathbf{m_{rac{2}{\mathcal{K}^2}}^2}\mathbf{c^2}}{\hbar^2}igg]ec{\mathbf{A}} - 
ablaig[
abla\cdotec{\mathbf{A}} + rac{1}{\mathbf{c^2}}rac{\partial\phi}{\partial\mathbf{t}}ig] = \mathbf{0}ig]$ 若在某种导电介质中:  $\left[\nabla^{2} - \frac{1}{\mathbf{c}^{2}} \frac{\partial^{2}}{\partial \mathbf{t}^{2}} - \frac{\mathbf{m}_{\mathcal{KF}}^{2} \mathbf{c}^{2}}{\hbar^{2}}\right] \phi + \frac{\partial}{\partial \mathbf{t}} \left[\nabla \cdot \vec{\mathbf{A}} + \frac{1}{\mathbf{c}^{2}} \frac{\partial \phi}{\partial \mathbf{t}}\right] = \mathbf{0}$ Back Close

伦敦方程,理想导体及迈斯纳效应 真空中的场方程:  $\left[\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\right] \vec{\mathbf{A}} - \nabla \left[\nabla \cdot \vec{\mathbf{A}} + \frac{1}{c^2} \frac{\partial \phi}{\partial t}\right] = -\mu_0 \vec{\mathbf{j}}$  $c^2 = \frac{1}{1}$  $\left[\nabla^2 - \frac{1}{\mathbf{c}^2} \frac{\partial^2}{\partial \mathbf{t}^2}\right] \phi + \frac{\partial}{\partial \mathbf{t}} \left[\nabla \cdot \vec{\mathbf{A}} + \frac{1}{\mathbf{c}^2} \frac{\partial \phi}{\partial \mathbf{t}}\right] = -\frac{\rho}{\epsilon_0}$ 它具有规范不变性: 者在某种导电介质中:  $\left[ \nabla^2 - \frac{1}{\mathbf{c}^2} \frac{\partial^2}{\partial \mathbf{t}^2} - \frac{\mathbf{m}_{\mathbb{H}^2}^2 \mathbf{c}^2}{\hbar^2} \right] \vec{\mathbf{A}} - \nabla \left[ \nabla \cdot \vec{\mathbf{A}} + \frac{1}{\mathbf{c}^2} \frac{\partial \phi}{\partial \mathbf{t}} \right] = \mathbf{0}$  $\left[\nabla^{2} - \frac{1}{\mathbf{c}^{2}} \frac{\partial^{2}}{\partial \mathbf{t}^{2}} - \frac{\mathbf{m}_{\mathcal{H}\vec{r}}^{2} \mathbf{c}^{2}}{\hbar^{2}}\right] \phi + \frac{\partial}{\partial \mathbf{t}} \left[\nabla \cdot \vec{\mathbf{A}} + \frac{1}{\mathbf{c}^{2}} \frac{\partial \phi}{\partial \mathbf{t}}\right] = \mathbf{0}$  $ec{\mathbf{j}} = -rac{\mathrm{m}_{\mathcal{H}}^2\mathrm{c}^2}{\mu_0\hbar^2}ec{\mathbf{A}}$  $\rho = -\frac{\mathrm{m}_{\mathsf{\#}\mathsf{f}}^2 \epsilon_0 \mathrm{c}^2}{\hbar^2} \phi$ 对比真空:介质中的电荷、电流完全由光子有效质量1114平产产生! Back Close

伦敦方程,理想导体及迈斯纳效应 真空中的场方程:  $\left[\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\right] \vec{\mathbf{A}} - \nabla \left[\nabla \cdot \vec{\mathbf{A}} + \frac{1}{c^2} \frac{\partial \phi}{\partial t}\right] = -\mu_0 \vec{\mathbf{j}}$  $c^2 = \frac{1}{}$  $\left[\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\right] \phi + \frac{\partial}{\partial t} \left[\nabla \cdot \vec{\mathbf{A}} + \frac{1}{c^2} \frac{\partial \phi}{\partial t}\right] = -\frac{\rho}{\epsilon_0}$ 它具有规范不变性: 若在某种导电介质中:  $\left[\nabla^2 - \frac{1}{\mathbf{c}^2} \frac{\partial^2}{\partial \mathbf{t}^2} - \frac{\mathbf{m}_{\mathcal{H}}^2 \mathbf{c}^2}{\hbar^2}\right] \vec{\mathbf{A}} - \nabla \left[\nabla \cdot \vec{\mathbf{A}} + \frac{1}{\mathbf{c}^2} \frac{\partial \phi}{\partial \mathbf{t}}\right] = \mathbf{0}$  $\left[\left[
abla^2 - rac{1}{\mathbf{c^2}} rac{\partial^2}{\partial \mathbf{t^2}} - rac{\mathbf{m}_{\mathcal{H}}^2 \mathbf{c^2}}{\hbar^2}
ight]\phi + rac{\partial}{\partial \mathbf{t}} \left[
abla \cdot \vec{\mathbf{A}} + rac{1}{\mathbf{c^2}} rac{\partial \phi}{\partial \mathbf{t}}
ight] = \mathbf{0}$  $oldsymbol{ec{j}} = -rac{oldsymbol{\mathrm{m}}_{\mathcal{H}}^2 oldsymbol{\mathrm{c}}^2}{u_0 \hbar^2} ec{\mathrm{A}}^{\mathrm{l}}$ 对比真空:介质中的电荷、电流完全由光子有效质量皿+2 产生!  $\rho = -\frac{\mathbf{m}_{\mathcal{K}}^2}{\epsilon_0 \mathbf{c}^2} \phi$ 这时 规范对称性是破缺的 ,且  $\mathbf{0} = \nabla \cdot \vec{\mathbf{j}} + \frac{\partial \rho}{\partial \mathbf{t}} = -\frac{\mathbf{m}_{\text{光子}}^2 \mathbf{c}^2}{\mu_0 \hbar^2} [\nabla \cdot \vec{\mathbf{A}} + \mu_0 \epsilon_0 \frac{\partial \phi}{\partial \mathbf{t}}]$ Back Close

 $egin{aligned} 
abla \cdot ec{\mathbf{A}} + rac{1}{\mathbf{c^2}} rac{\partial \phi}{\partial \mathbf{t}} &= \mathbf{0} \quad \left\{ egin{aligned} ec{\mathbf{j}} &= -rac{\mathbf{m}_{\mathcal{K} 
eta}^2 \mathbf{c^2}}{\mu_0 \hbar^2} ec{\mathbf{A}} \ 
ho &= -rac{\mathbf{m}_{\mathcal{K} 
eta}^2 \mathbf{c^2}}{\hbar^2} \phi \end{aligned} 
ight.$  $ig[
abla^2 - rac{1}{\mathrm{c}^2}rac{\partial^2}{\partial \mathrm{t}^2} - rac{\mathrm{m}_{\mathcal{H}}^2 - \mathrm{c}^2}{\hbar^2}ig]ec{\mathbf{A}} = \mathbf{0}$ 

 $igl[
abla^2 - rac{1}{\mathrm{c}^2} rac{\partial^2}{\partial \mathrm{t}^2} - rac{\mathrm{m}_{\mathcal{H}}^2 \mathrm{c}^2}{\hbar^2} igr] \phi = \mathbf{0}$ 









#### **有效光子质量**: 伦敦方程,理想导体及迈斯纳效应 $ig[ abla^2 - rac{1}{\mathrm{c}^2}rac{\partial^2}{\partial \mathrm{t}^2} - rac{\mathrm{m}_{\mathcal{H}}^2}{\hbar^2}ig]ec{\mathbf{A}} = \mathbf{0}$

Close

 $egin{aligned} 
abla \cdot ec{\mathbf{A}} + rac{1}{\mathbf{c^2}} rac{\partial \phi}{\partial \mathbf{t}} &= \mathbf{0} \quad \left\{ egin{aligned} ec{\mathbf{j}} &= -rac{\mathbf{m}_{\mathcal{K} 
eta}^2 \mathbf{c^2}}{\mu_0 \hbar^2} ec{\mathbf{A}} \ 
ho &= -rac{\mathbf{m}_{\mathcal{K} 
eta}^2 \epsilon_0 \mathbf{c^2}}{\hbar^2} \phi \end{aligned} 
ight.$ 

 $-\gamma \frac{\partial \vec{\mathbf{E}}}{\partial t} = \gamma \nabla \times \vec{\mathbf{E}} = \nabla \times \vec{\mathbf{j}} = -\frac{\mathbf{m}_{\mathcal{K}\vec{\mathcal{T}}}^2 \mathbf{c}^2}{\mu_0 \hbar^2} \nabla \times \vec{\mathbf{A}} = -\frac{\mathbf{m}_{\mathcal{K}\vec{\mathcal{T}}}^2 \mathbf{c}^2}{\mu_0 \hbar^2} \vec{\mathbf{B}} \text{ when the }$ 

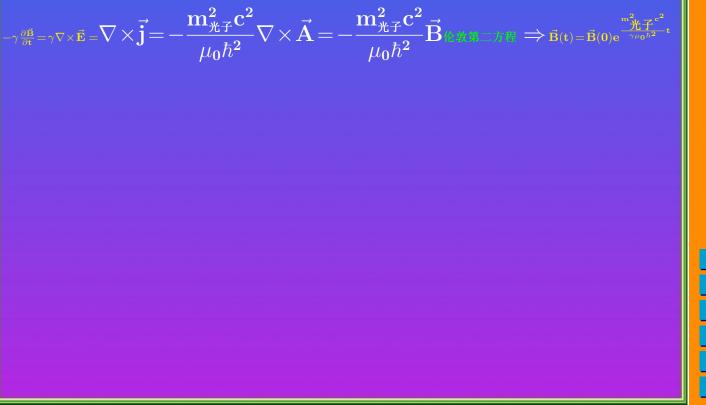
 $ar{\left[
abla^2-rac{1}{\mathrm{c}^2}rac{\partial^2}{\partial \mathrm{t}^2}-rac{\mathrm{m}_{\mathcal{H}}^2\mathrm{c}^2}{\hbar^2}
ight]}\phi=0$ 

$$\rho = -\frac{\mathbf{m}_{\mathcal{H}}^2 + \epsilon_0 \mathbf{c}^2}{\hbar^2} \phi$$

$$egin{aligned} egin{aligned} egin{aligned\\ egin{aligned} egi$$

$$ig[
abla^2 - rac{1}{\mathrm{c}^2}rac{\partial^2}{\partial \mathrm{t}^2} - rac{\mathrm{m}_{\mathcal{H}}^2\mathrm{c}^2}{\hbar^2}ig]\phi = 0$$

$$ho = -rac{\mathrm{m}_{\mathcal{H}}^2 \epsilon_0 \mathrm{c}^2}{\hbar^2} \phi$$





#### $igl[ abla^2 - rac{1}{\mathrm{c}^2} rac{\partial^2}{\partial \mathrm{t}^2} - rac{\mathrm{m}_{rac{3}{2}}^2 \mathrm{c}^2}{\hbar^2} igr] ec{\mathbf{A}} = \mathbf{0} igr]$

$$abla \cdot ec{\mathbf{A}} + rac{1}{\mathbf{c^2}} rac{\partial \phi}{\partial \mathbf{t}} = \mathbf{0} \quad \left\{ egin{array}{l} ec{\mathbf{j}} = -rac{\mathbf{m}_{\mathcal{H}}^2}{\mu_0 \hbar^2} ec{\mathbf{A}} \ 
ho = -rac{\mathbf{m}_{\mathcal{H}}^2}{\hbar^2} \phi \end{array} 
ight.$$

 $ig[
abla^2 - rac{1}{\mathrm{c}^2}rac{\partial^2}{\partial \mathrm{t}^2} - rac{\mathrm{m}_{\mathcal{H}\mathcal{F}}^2\mathrm{c}^2}{\hbar^2}ig]\phi = 0$ 

$$-\gamma^{\frac{\partial \mathbf{B}}{\partial t}} = \gamma \nabla \times \vec{\mathbf{E}} = \nabla \times \vec{\mathbf{j}} = -\frac{\mathbf{m}_{\mathcal{K}\mathcal{T}}^2 \mathbf{c}^2}{\mu_0 \hbar^2} \nabla \times \vec{\mathbf{A}} = -\frac{\mathbf{m}_{\mathcal{K}\mathcal{T}}^2 \mathbf{c}^2}{\mu_0 \hbar^2} \vec{\mathbf{B}} \text{ where } \vec{\mathbf{B}} \text{ is } \vec{\mathbf{B}} \text{$$

$$\frac{\partial \rho}{\partial \mathbf{t}} = -\nabla \cdot \vec{\mathbf{j}} = -\nabla \cdot (\gamma \vec{\mathbf{E}}) = -\frac{\gamma}{\epsilon_0} \rho \quad \Rightarrow \quad \rho(\mathbf{t}) = \rho(\mathbf{0}) \mathbf{e}^{-\frac{\gamma}{\epsilon_0} \mathbf{t}}$$







#### **有效光子质量** 伦敦方程,理想导体及迈斯纳效应 $ig|ig[ abla^2 - rac{1}{\mathrm{c}^2}rac{\partial^2}{\partial \mathrm{t}^2} - rac{\mathrm{m}_{\mathcal{H}}^2}{\hbar^2}ig|ec{\mathbf{A}} = \mathbf{0}ig|$

$$abla \cdot ec{\mathbf{A}} + rac{1}{\mathbf{c}^2} rac{\partial \phi}{\partial \mathbf{t}} = \mathbf{0} \quad \left\{ egin{array}{l} ec{\mathbf{j}} = -rac{\mathrm{m}_{\mathcal{H}}^2 \mathrm{c}^2}{\mu_0 \hbar^2} ec{\mathbf{A}} \ 
ho = -rac{\mathrm{m}_{\mathcal{H}}^2 \mathrm{c}^2}{\hbar^2} \phi \end{array} 
ight.$$

#### $ig|ig[ abla^2 - rac{1}{\mathrm{c}^2}rac{\partial^2}{\partial \mathrm{t}^2} - rac{\mathrm{m}_{rac{3}{2}}^2\mathrm{c}^2}{\hbar^2}ig]ec{\mathbf{A}} = \mathbf{0}ig|$

$$egin{align} 
abla \cdot ec{\mathbf{A}} + rac{1}{\mathbf{c}^2} rac{\partial \phi}{\partial \mathbf{t}} &= \mathbf{0} & \left\{ egin{align} ec{\mathbf{j}} = -rac{\mathbf{m}_{\mathcal{H}}^2 \mathbf{c}^2}{\mu_0 \hbar^2} ec{\mathbf{A}} \ & \ 
ho &= -rac{\mathbf{m}_{\mathcal{H}}^2 \mathbf{c}^2}{\hbar^2} \phi \end{array} 
ight. 
onumber \ 
onumber \ \mathbf{m}_{\mathbf{c}}^2 - \mathbf{c}^2 
ightarrow \mathbf{m}_{\mathbf{c}}^2 - \mathbf{c}^2 \ 
onumber \ \mathbf{m}_{\mathbf{c}}^2 - \mathbf{c}^2 
ight.$$

$$\begin{split} & \left[ \nabla^2 - \frac{1}{\mathbf{c}^2} \frac{\partial^2}{\partial \mathbf{t}^2} - \frac{\mathbf{m}_{\mathcal{H} \neq}^2 \mathbf{c}^2}{\hbar^2} \right] \phi = \mathbf{0} \\ & - \gamma_{\frac{\partial \mathbf{B}}{\partial \mathbf{t}}} = \gamma \nabla \times \vec{\mathbf{E}} = \nabla \times \vec{\mathbf{j}} = -\frac{\mathbf{m}_{\mathcal{H} \neq}^2 \mathbf{c}^2}{\mu_0 \hbar^2} \nabla \times \vec{\mathbf{A}} = -\frac{\mathbf{m}_{\mathcal{H} \neq}^2 \mathbf{c}^2}{\mu_0 \hbar^2} \vec{\mathbf{B}} \\ & \frac{\partial \rho}{\partial \mathbf{t}} = - \nabla \cdot \vec{\mathbf{j}} = - \nabla \cdot (\gamma \vec{\mathbf{E}}) = -\frac{\gamma}{\epsilon_0} \rho \quad \Rightarrow \quad \rho(\mathbf{t}) = \rho(\mathbf{0}) \mathbf{e}^{-\frac{\gamma}{\epsilon_0} \mathbf{t}} \\ & \gamma \neq \mathbf{0} \Rightarrow \quad \text{介质内} \rho \text{可取为零: } \underline{\nabla \cdot \vec{\mathbf{j}}} = \mathbf{0} \quad \mathbf{E} \vec{\mathbf{E}} = -(\nabla \phi + \frac{\partial \vec{\mathbf{A}}}{\partial \mathbf{t}}) = \frac{\mu_0 \hbar^2}{\mathbf{m}_{\mathcal{H} \neq}^2 \mathbf{c}^2} \frac{\partial \vec{\mathbf{j}}}{\partial \mathbf{t}} \\ & \frac{\partial \vec{\mathbf{j}}}{\partial \mathbf{t}} = -\mathbf{0} \vec{\mathbf{j}} + \mathbf{0} \vec{\mathbf$$



$$\vec{\mathbf{j}} = \gamma \vec{\mathbf{E}} = \frac{\gamma \mu_0 \hbar^2}{\mathbf{m}_{\mathcal{H}}^2 \mathbf{c}^2} \frac{\partial \vec{\mathbf{j}}}{\partial \mathbf{t}}$$







### <del>「效光子」」</del> 伦敦方程,理想导体及迈斯纳效应 $egin{aligned} abla \cdot ec{\mathbf{A}} + rac{\mathbf{1}}{\mathbf{c^2}} rac{\partial \phi}{\partial \mathbf{t}} &= \mathbf{0} \quad \left\{ egin{aligned} ec{\mathbf{j}} &= -rac{\mathbf{m}_{\mathcal{X} eg c^2}^2}{\mu_0 \hbar^2} ec{\mathbf{A}} \ ho &= -rac{\mathbf{m}_{\mathcal{X} eg c^2}^2}{\hbar^2} \phi \end{aligned} ight.$

$$egin{align*} igl[
abla^2 - rac{1}{\mathbf{c}^2}rac{\partial^2}{\partial \mathbf{t}^2} - rac{\mathbf{m}_{\mathcal{H} extcolored}^2\mathbf{c}^2}{\hbar^2}igr]\phi = \mathbf{0} \end{bmatrix} \phi = \mathbf{0} \ & \mathbf{c}^2\,\partial\mathbf{t} \ & -\gamma^{rac{\partial \mathbf{B}}{m}} = \gamma 
abla imes \mathbf{f f} = 
abla imes \mathbf{f j} = -rac{\mathbf{m}_{\mathcal{H} extcolored}^2\mathbf{c}^2}{\hbar^2}
abla imes \mathbf{f A} = -rac{\mathbf{m}_{\mathcal{H} extcolored}^2\mathbf{c}^2}{\hbar^2} \mathbf{f B}_{12} \end{aligned}$$

$$\begin{split} & \frac{\mathbf{m}_{\mathcal{H}}^{2}\mathbf{c}^{2}}{\rho_{\mathbf{m}}^{2}} = \gamma \nabla \times \vec{\mathbf{p}} = -\frac{\mathbf{m}_{\mathcal{H}}^{2}\mathbf{c}^{2}}{\mu_{0}\hbar^{2}} \nabla \times \vec{\mathbf{A}} = -\frac{\mathbf{m}_{\mathcal{H}}^{2}\mathbf{c}^{2}}{\mu_{0}\hbar^{2}} \vec{\mathbf{B}}_{\mathbf{E}}^{2} = \vec{\mathbf{B}}_{\mathbf{E}}^{2$$



$$ec{f j} = \gamma ec{f E} = rac{\gamma \mu_0 \hbar^2}{m_{\mathcal{H} extsf{C}^2}^2 c^2} rac{\partial ec{f j}}{\partial t}$$
  $ec{f j}(f t) = ec{f j}(f 0) e^{rac{m_{\mathcal{H} extsf{C}^2}^2 c^2}{\gamma \mu_0 \hbar^2} t} \Rightarrow ext{ 有限的初始电流随时间无穷增强!}$   $ec{f E} = f 0$   $ec{f E} = f 0$   $ec{f E} = f 0$   $ec{f E} = f 0$ 

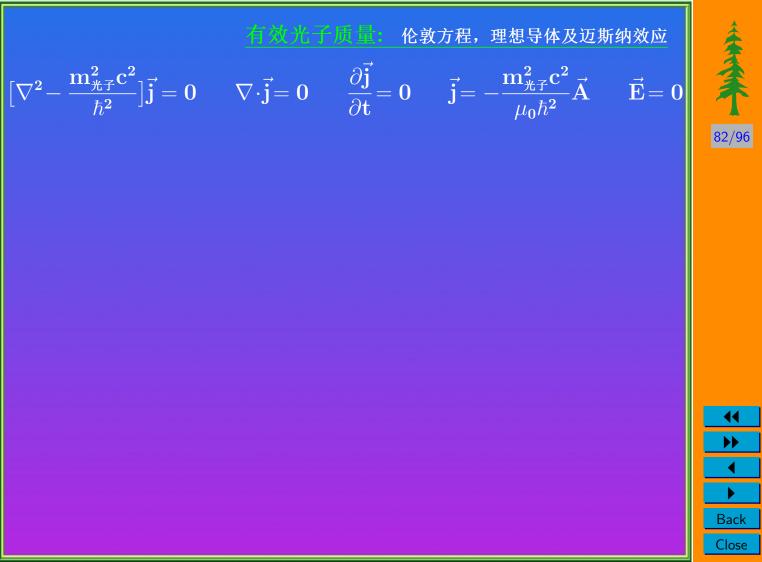


 $ec{\mathbf{E}} = \mathbf{0} \qquad rac{\partial ec{\mathbf{j}}}{\partial \mathbf{t}} = \mathbf{0}$ 

Back Close

 $ec{\mathbf{j}}(\mathbf{0}) = \mathbf{0} \; \Rightarrow \; ec{\mathbf{j}}(\mathbf{t}) = \mathbf{0}$  后面不讨论这种情形

 $ig[
abla^2 - rac{1}{c^2}rac{\partial^2}{\partial t^2} - rac{\mathrm{m}_{\mathcal{H}}^2c^2}{\hbar^2}ig]ec{\mathbf{A}} = 0$ 

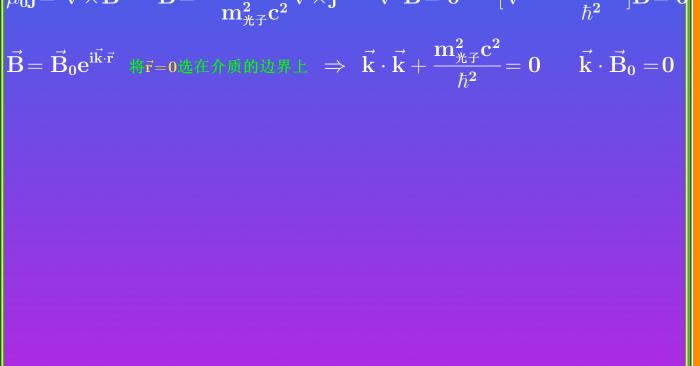


$$egin{aligned} egin{aligned} egin{aligned} \left[
abla^2 - rac{\mathbf{m}_{\mathcal{H}
eta}^2 \mathbf{c}^2}{\hbar^2}
ight] \ddot{\mathbf{j}} &= \mathbf{0} & 
abla \cdot \ddot{\mathbf{j}} &= \mathbf{0} & 
abla \cdot \ddot{\mathbf{j}} &= -rac{\mathbf{m}_{\mathcal{H}
eta}^2 \mathbf{c}^2}{\mu_0 \hbar^2} \ddot{\mathbf{A}} & \ddot{\mathbf{E}} &= \mathbf{0} \end{aligned} \ \mu_0 \ddot{\mathbf{j}} &= 
abla imes \ddot{\mathbf{J}} &= \mathbf{0} & 
abla \cdot \ddot{\mathbf{J}} &= -rac{\mathbf{m}_{\mathcal{H}
eta}^2 \mathbf{c}^2}{\mu_0 \hbar^2} \ddot{\mathbf{A}} & \ddot{\mathbf{E}} &= \mathbf{0} \end{aligned} \ \mu_0 \ddot{\mathbf{j}} &= 
abla imes \ddot{\mathbf{J}} &= \mathbf{0} & 
abla \cdot \ddot{\mathbf{J}} &= -rac{\mathbf{m}_{\mathcal{H}
eta}^2 \mathbf{c}^2}{\mu_0 \hbar^2} \ddot{\mathbf{J}} \ddot{\mathbf{B}} &= \mathbf{0} \end{aligned}$$





$$egin{align*} & \overrightarrow{A}$$
 死  $\overrightarrow{M}$   $\overrightarrow{J}$   $\overrightarrow{$ 















$$\begin{bmatrix} \nabla^2 - \frac{\mathbf{m}_{\mathcal{H}}^2 - \mathbf{c}^2}{\hbar^2} \end{bmatrix} \vec{\mathbf{j}} = \mathbf{0} \qquad \nabla \cdot \vec{\mathbf{j}} = \mathbf{0} \qquad \frac{\partial \vec{\mathbf{j}}}{\partial \mathbf{t}} = \mathbf{0} \qquad \vec{\mathbf{j}} = -\frac{\mathbf{m}_{\mathcal{H}}^2 - \mathbf{c}^2}{\mu_0 \hbar^2} \vec{\mathbf{A}} \qquad \vec{\mathbf{E}} = \mathbf{0} \\ \mu_0 \vec{\mathbf{j}} = \nabla \times \vec{\mathbf{B}} \qquad \vec{\mathbf{B}} = -\frac{\mu_0 \hbar^2}{\mathbf{m}_{\mathcal{H}}^2 - \mathbf{c}^2} \nabla \times \vec{\mathbf{j}} \qquad \nabla \cdot \vec{\mathbf{B}} = \mathbf{0} \qquad [\nabla^2 - \frac{\mathbf{m}_{\mathcal{H}}^2 - \mathbf{c}^2}{\hbar^2}] \vec{\mathbf{B}} = \mathbf{0} \\ \vec{\mathbf{D}} = \vec{\mathbf{D}} = i \vec{\mathbf{k}} \cdot \vec{\mathbf{f}} \qquad \vec{\mathbf{J}} = \vec{\mathbf{J}} + \frac{\mathbf{m}_{\mathcal{H}}^2 - \mathbf{c}^2}{\hbar^2} \qquad \vec{\mathbf{J}} = \vec{\mathbf{D}} \qquad \vec{\mathbf{D}} = \mathbf{0}$$

$$ec{f B}=ec{f B}_0{f e}^{iec{k}\cdotec{r}}$$
 இ $ec{f r}={f 0}$  கோத்திய தட $\Rightarrow$   $ec{k}\cdotec{k}+rac{m_{rac{k}{2}}^2c^2}{\hbar^2}=0$   $ec{k}\cdotec{f B}_0=0$   $ec{k}=i{f k}_Iec{f e}_k$   $\Rightarrow$   ${f k}_I=rac{m_{rac{k}{2}}c}{\hbar}>0$  நக்கத்திய இதைய









$$egin{align*} & [
abla^2 - rac{m_{\mathcal{H}}^2 - \mathbf{c}^2}{\hbar^2}] ec{\mathbf{j}} = \mathbf{0} \qquad 
abla \cdot ec{\mathbf{j}} = \mathbf{0} \qquad rac{\partial ec{\mathbf{j}}}{\partial \mathbf{t}} = \mathbf{0} \qquad ec{\mathbf{j}} = -rac{m_{\mathcal{H}}^2 - \mathbf{c}^2}{\mu_0 \hbar^2} ec{\mathbf{A}} \qquad ec{\mathbf{E}} = \mathbf{0} \ \mu_0 ec{\mathbf{j}} = 
abla imes ec{\mathbf{j}} = -rac{m_{\mathcal{H}}^2 - \mathbf{c}^2}{\mu_0 \hbar^2} ec{\mathbf{A}} \qquad ec{\mathbf{E}} = \mathbf{0} \ \mu_0 ec{\mathbf{j}} = -rac{\mu_0 \hbar^2}{\mu_0 \hbar^2} \nabla imes ec{\mathbf{j}} \qquad 
abla \cdot ec{\mathbf{j}} = \mathbf{0} \qquad [
abla^2 - rac{m_{\mathcal{H}}^2 - \mathbf{c}^2}{\mu_0 \hbar^2} ec{\mathbf{j}} = \mathbf{0} \ \mu_0 \$$

$$ec{\mathrm{B}}=ec{\mathrm{B}}_0\mathrm{e}^{\mathrm{i}ec{\mathrm{k}}\cdotec{\mathrm{r}}}$$
 கு $ec{\mathrm{gr}}=0$ க்காகள்கது. $ec{\mathrm{k}}\cdotec{\mathrm{k}}$   $\Rightarrow$   $ec{\mathrm{k}}\cdotec{\mathrm{k}}+rac{\mathrm{m}_{\mathcal{H}}^2\mathrm{c}^2}{\hbar^2}=0$   $ec{\mathrm{k}}\cdotec{\mathrm{B}}_0=0$ 

 $ec{k}=ik_{
m I}ec{e}_{
m k}\Rightarrow k_{
m I}=rac{m_{
m HF}c}{\epsilon}>0$  ក្រុកក្រុម្ពិប្បយៈប្រហែល គឺក្រុមក្រុម  $ightarrow ec{B}$   $ightarrow ec{B}$ 

在介质界面附近:  $\vec{\mathbf{e}}_{\mathbf{k}}$  与界面的外法线方向夹角只能是小于或大于 $90^{\circ}$ 







$$egin{align*} & \overline{A}$$
 放死于 质量 论数方程,理想导体及迈斯纲效应  $egin{align*} & \overline{C} &$ 

$$egin{align*} \mathbf{m}^2_{ ext{ iny 2}}\mathbf{c}^2 & \hbar^2 & eta^2 & eta$$

 $ec{f k}={f i}{f k}_{
m I}ec{f e}_{
m k} \Rightarrow {f k}_{
m I}=rac{{f m}_{{\cal H}}{
m c}{f c}}{f b}> 0$  எம்மத்தொல்லும் தெற்கிடி  $\Rightarrow ec{f B}\stackrel{{f r}_{
m k}\equiv ec{f r}\cdot ec{f e}_{
m k}}{=}ec{f B}_0{f e}^{-{f k}_{
m I}{f r}_{
m k}}$ 在介质界面附近:  $\vec{\mathbf{e}}_{\mathbf{k}}$  与界面的外法线方向夹角只能是小于或大于 $90^{\circ}$ 与外法线方向夹角小于 $90^\circ$   $\Rightarrow$   $\mathbf{e}^{-\mathbf{k_I r_k}}$  是增强因子

 $f e^{-{f k_Ir_k}}$  是 $f e^{-{f k_Ir_k}}$  是 $f k_{f k_I}$ 因子







Back

$$\begin{split} & \left[ \nabla^2 - \frac{\mathbf{m}_{\mathcal{H}\vec{\tau}}^2 \mathbf{c}^2}{\hbar^2} \right] \vec{\mathbf{j}} = \mathbf{0} \qquad \nabla \cdot \vec{\mathbf{j}} = \mathbf{0} \qquad \frac{\partial \vec{\mathbf{j}}}{\partial \mathbf{t}} = \mathbf{0} \qquad \vec{\mathbf{j}} = -\frac{\mathbf{m}_{\mathcal{H}\vec{\tau}}^2 \mathbf{c}^2}{\mu_0 \hbar^2} \vec{\mathbf{A}} \qquad \vec{\mathbf{E}} = \mathbf{0} \\ & \mu_0 \vec{\mathbf{j}} = \nabla \times \vec{\mathbf{B}} \qquad \vec{\mathbf{B}} = -\frac{\mu_0 \hbar^2}{\mathbf{m}_{\mathcal{H}\vec{\tau}}^2 \mathbf{c}^2} \nabla \times \vec{\mathbf{j}} \qquad \nabla \cdot \vec{\mathbf{B}} = \mathbf{0} \qquad [\nabla^2 - \frac{\mathbf{m}_{\mathcal{H}\vec{\tau}}^2 \mathbf{c}^2}{\hbar^2}] \vec{\mathbf{B}} = \mathbf{0} \end{split}$$

$$\mathbf{B} = \mathbf{B} - \frac{\mathbf{B} \cdot \mathbf{C}}{\mathbf{B}} \nabla \times \mathbf{B} \qquad \mathbf{B} = -\frac{\mathbf{B} \cdot \mathbf{C}}{\mathbf{B}} \nabla \times \mathbf{J} \qquad \nabla \cdot \mathbf{B} = \mathbf{0} \qquad [\nabla^2 - \frac{\mathbf{B} \cdot \mathbf{E} \cdot \mathbf{C}}{\hbar^2}] \mathbf{B} = \mathbf{0}$$

$$ec{f B}=ec{f B}_0{f e}^{iec{f k}\cdotec{f r}}$$
 கோ $={f 0}$ க்கோகிறப்கேட் $\Rightarrow$   $ec{f k}\cdotec{f k}+rac{m_{ ext{ iny T}}^2{f c}^2}{\hbar^2}=0$   $ec{f k}\cdotec{f B}_0=0$   $ec{f k}=i{f k}_1ec{f e}_k$   $\Rightarrow$   ${f k}_1=rac{m_{ ext{ iny T}}{f c}}{\hbar}>0$  என்றதோய்யும் தெய்யும்  $ec{f e}_k$ மைத்தா  $\Rightarrow$   $ec{f B}$   $\stackrel{{f r}_k\equivec{f r}\cdotec{f e}_k}{\equiv}=ec{f B}_0{f e}^{-{f k}_1{f r}_k}$ 

在介质界面附近: 
$$\vec{\mathbf{e}}_{\mathbf{k}}$$
 与界面的外法线方向夹角只能是小于或大于 $90^\circ$ 

$$egin{array}{lll} egin{array}{lll} egin{array} egin{array}{lll} egin{array}{lll} egin{array}{lll} egin{array}{lll} egin{array}{lll} egin{array}{lll} egin{array}{l$$





物理上只能选择衰减的情形!



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$$egin{align*} egin{align*} egin{align*}$$

$$ec{k}=ik_{
m I}ec{e}_{
m k}$$
  $\Rightarrow$   $k\cdot k+rac{33}{\hbar^2}=0$   $k\cdot B_0=0$   $ec{k}=ik_{
m I}ec{e}_{
m k}$   $\Rightarrow$   $k_{
m I}=rac{m_{
m HF}c}{\hbar}>0$  என்று தொல்லும் இதுவடை  $ec{e}_{
m k}$  இது  $ec{e}_{
m k}$   $ec{e}_{
m k}$  இது  $ec{e}_{
m k}$   $ec{e}_{
m k}$ 

在介质界面附近:  $\vec{\mathbf{e}}_{\mathbf{k}}$  与界面的外法线方向夹角只能是小于或大于 $90^{\circ}$ 

与外法线方向夹角小于 $90^\circ$   $\Rightarrow$   $\mathbf{e^{-\mathbf{k_I r_k}}}$  是增强因子

$$\left(\begin{array}{ccc} =& -1.5 & -$$

**介质内部无电场和电荷 磁场和电流随穿透距离指数衰减** 





物理上只能选择衰减的情形!



$$egin{aligned} egin{aligned} egin{aligned\\ egin{aligned} egi$$

 $ec{ ext{B}} = ec{ ext{B}}_0 ext{e}^{ ext{i} ec{ ext{k}} \cdot ec{ ext{r}}}$  将 $ec{ ext{r}} = ext{0}$ 选在介质的边界上。 $\Rightarrow ec{ ext{k}} \cdot ec{ ext{k}} + rac{ ext{m}_{ ext{2}}^2 ext{c}^2}{ ext{5}^2} = 0$   $ec{ ext{k}} \cdot ec{ ext{B}}_0 = 0$ 

$$ec{k}=ik_{
m I}ec{e}_{
m k}\Rightarrow k_{
m I}=rac{m_{ ext{#F}}c}{\hbar}>0$$
 ការរបស់ខែការរបស់ខេត្តការបស់ខេត្តការរបស់ខេត្តការរបស់ខេត្តការរបស់ខេត្តការរបស់ខេត្តការរបស់ខេត្តការរបស់ខេត្តការបស់ខេត្ត

在介质界面附近:  $\vec{\mathbf{e}}_{\mathbf{k}}$  与界面的外法线方向夹角只能是小于或大于 $90^{\circ}$ 

与外法线方向夹角小于 $90^\circ$   $\Rightarrow$   $\mathbf{e^{-\mathbf{k_I r_k}}}$  是增强因子

介质内部无电场和电荷 磁场和电流随穿透距离指数衰减 超导体 近期的效应!

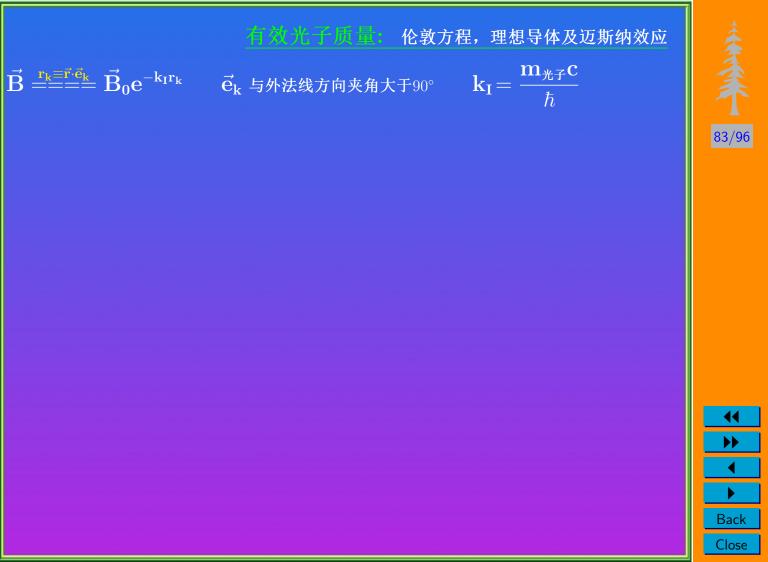












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$$ec{\mathbf{e}}_{\mathbf{k}}$$
 与外法线方向夹角大于 $90^\circ$   $\mathbf{k}_{\mathbf{I}} = \frac{\mathbf{m}_{\mathcal{H}} \mathbf{c}}{\hbar}$ 

对这些  $\vec{j} \neq 0$  的区域,前面的  $\vec{j}(0) = 0$  的情形不存在。即只能有  $\gamma = \infty$  的情形!

 $\vec{\mathbf{B}} \stackrel{\mathbf{r_k} \equiv \vec{\mathbf{r}} \cdot \vec{\mathbf{e}}_k}{===} \vec{\mathbf{B}}_0 e^{-\mathbf{k}_I \mathbf{r}_k}$ 

 $ec{\mathbf{e}}_{\mathbf{k}}$  与外法线方向夹角大于 $90^\circ$   $\mathbf{k}_{\mathbf{I}} = rac{\mathbf{m}_{\mathcal{K} extstyle T} \mathbf{c}}{\hbar}$ 

定义磁场衰减为边界值的1/e时深度为 $\overline{m{g}$ 透深度 $}$   $\lambda$  :  $\lambda = m{rac{1}{k_{
m I}}} = m{rac{\hbar}{m_{
m H} + c}}$ 

对这些  $\vec{j} \neq 0$  的区域,前面的  $\vec{j}(0) = 0$  的情形不存在。即只能有  $\gamma = \infty$  的情形!

 $\vec{\mathbf{B}} \stackrel{\mathbf{r_k} \equiv \vec{\mathbf{r}} \cdot \vec{\mathbf{e}}_k}{====} \vec{\mathbf{B}}_0 e^{-\mathbf{k}_I \mathbf{r}_k}$ 

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 $ec{\mathbf{e}}_{\mathbf{k}}$  与外法线方向夹角大于 $90^\circ$   $\mathbf{k}_{\mathbf{I}} = \frac{\mathbf{m}_{\mathcal{H}} \mathbf{c}}{\hbar}$ 

对这些  $\vec{j} \neq 0$  的区域,前面的  $\vec{j}(0) = 0$  的情形不存在。即只能有  $\gamma = \infty$  的情形!

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定义磁场衰减为边界值的
$$1/e$$
时深度为 $rac{f g}{f g}$ 透深度 $eta$ 。 $eta=rac{1}{f k_I}=rac{\hbar}{f m_{
m H}f g}$  电流具有同样的变迹深度







$$rac{\mathbf{m}_{\mathcal{H}}$$
子 $\mathbf{c}}{\hbar}$ 

 $ec{f B} \stackrel{{f r_k} \equiv {f r_{f e_k}}}{=\!=\!=\!=\!=\!=\!=} ec{f B_0} {f e^{-k_I {f r_k}}} \qquad ec{f e_k}$  与外法线方向夹角大于 $90^\circ \qquad {f k_I} = rac{{f m_{ ext{ iny M-S}} {f c}}}{\hbar}$ 

对这些  $\vec{j} \neq 0$  的区域,前面的  $\vec{j}(0) = 0$  的情形不存在。即只能有  $\gamma = \infty$  的情形!

定义磁场衰减为边界值的
$$1/e$$
时深度为穿透深度  $\lambda$ 。 $\lambda=rac{\mathbf{1}}{\mathbf{k_I}}=rac{\hbar}{\mathbf{m}_{\mathtt{H},\mathbf{r}}}$  电流具有同样的穿透深度

$$\mathbf{0} = \vec{\mathbf{B}}_{\mathsf{H}} = \mu_{\mathbf{0}}(\vec{\mathbf{H}}_{\mathsf{H}} + \vec{\mathbf{M}}_{\mathsf{H}}) \; \Rightarrow \; \vec{\mathbf{M}}_{\mathsf{H}} = -\vec{\mathbf{H}}_{\mathsf{H}} = \chi_{\mathbf{m}}\vec{\mathbf{H}}_{\mathsf{H}} \; \Rightarrow \; \chi_{\mathbf{m}} = -\mathbf{1}$$



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$$ec{\mathbf{e}}_{\mathbf{k}}$$
 与外法线方向夹角大于 $90^\circ$   $\mathbf{k}_{\mathbf{I}} = \frac{\mathbf{m}_{\mathcal{H}} \mathbf{c}}{\hbar}$ 

对这些  $\vec{j} \neq 0$  的区域,前面的  $\vec{j}(0) = 0$  的情形不存在。即只能有  $\gamma = \infty$  的情形!

 $\vec{\mathbf{B}} \stackrel{\mathbf{r_k} \equiv \vec{\mathbf{r}} \cdot \vec{\mathbf{e}_k}}{==} \vec{\mathbf{B}}_0 e^{-\mathbf{k_I} \mathbf{r_k}}$ 

定义磁场衰减为边界值的
$$1/e$$
时深度为穿透深度  $\lambda$ 。 $\lambda=rac{\mathbf{1}}{\mathbf{k_I}}=rac{\hbar}{\mathbf{m}_{f x o c}}$ 电流具有同样的穿透深度

$$\mathbf{0} = \vec{\mathbf{B}}_{\mathsf{P}} = \mu_{\mathbf{0}}(\vec{\mathbf{H}}_{\mathsf{P}} + \vec{\mathbf{M}}_{\mathsf{P}}) \Rightarrow \vec{\mathbf{M}}_{\mathsf{P}} = -\vec{\mathbf{H}}_{\mathsf{P}} = \chi_{\mathbf{m}}\vec{\mathbf{H}}_{\mathsf{P}} \Rightarrow \chi_{\mathbf{m}} = -\mathbf{1}$$





 $ec{\mathbf{e}}_{\mathbf{k}}$  与外法线方向夹角大于 $90^\circ$   $\mathbf{k}_{\mathbf{I}} = rac{\mathbf{m}_{\mathcal{H} oldsymbol{ iny}} \mathbf{c}}{\hbar}$ 

对这些  $\vec{j} \neq 0$  的区域,前面的  $\vec{j}(0) = 0$  的情形不存在。即只能有  $\gamma = \infty$  的情形!

 $\vec{\mathbf{B}} \stackrel{\mathbf{r_k} \equiv \vec{\mathbf{r}} \cdot \vec{\mathbf{e}_k}}{===} \vec{\mathbf{B}}_0 e^{-\mathbf{k_I} \mathbf{r_k}}$ 

定义磁场衰减为边界值的1/e时深度为<u>穿透深度</u>  $\lambda$ :  $\lambda = \frac{1}{\mathbf{k_I}} = \frac{\hbar}{\mathbf{m_{**}} \cdot \mathbf{c}}$ 

$$\mathbf{0} = \vec{\mathbf{B}}_{oldsymbol{eta}} = \mu_{\mathbf{0}}(\vec{\mathbf{H}}_{oldsymbol{eta}} + \vec{\mathbf{M}}_{oldsymbol{eta}}) \;\; \Rightarrow \;\; \vec{\mathbf{M}}_{oldsymbol{eta}} = -\vec{\mathbf{H}}_{oldsymbol{eta}} = \chi_{\mathbf{m}} \vec{\mathbf{H}}_{oldsymbol{eta}} \;\; \Rightarrow \;\; \chi_{\mathbf{m}} = -\mathbf{1}$$
  $\Rightarrow \;\; \mu = \mu_{\mathbf{0}}(\mathbf{1} + \chi_{\mathbf{m}}) = \mathbf{0}$  தென்றாகும் ப

$$rac{\mathbf{1} + \chi_{\mathbf{e}}}{\chi_{\mathbf{e}}} ec{\mathbf{P}} = \epsilon_{\mathbf{0}} (\mathbf{1} + \chi_{\mathbf{e}}) ec{\mathbf{E}}_{oldsymbol{eta}} = ec{\mathbf{D}}_{oldsymbol{eta}} = \epsilon_{\mathbf{0}} ec{\mathbf{E}}_{oldsymbol{eta}} + ec{\mathbf{P}} = ec{\mathbf{P}} \;\; \Rightarrow \;\;\; \Rightarrow \;\; \chi_{\mathbf{e}} = \infty$$



 $ec{\mathbf{e}}_{\mathbf{k}}$  与外法线方向夹角大于 $90^\circ$   $\mathbf{k}_{\mathbf{I}} = rac{\mathbf{m}_{\mathcal{H}} \mathbf{c}}{\hbar}$ 

对这些  $\vec{j} \neq 0$  的区域,前面的  $\vec{j}(0) = 0$  的情形不存在。即只能有  $\gamma = \infty$  的情形!

 $\vec{\mathbf{B}} \stackrel{\mathbf{r_k} \equiv \vec{\mathbf{r}} \cdot \vec{\mathbf{e}_k}}{==} \vec{\mathbf{B}}_0 e^{-\mathbf{k_I} \mathbf{r_k}}$ 

定义磁场衰减为边界值的1/e时深度为<u>穿透深度</u>  $\lambda$ 。 $\lambda = rac{1}{\mathbf{k_I}} = rac{\hbar}{\mathbf{m}_{*:T}}\mathbf{c}$ 

$$\mathbf{0}=\mathbf{ec{B}}_{oldsymbol{eta}}=\mu_{\mathbf{0}}(\mathbf{ec{H}}_{oldsymbol{eta}}+\mathbf{ec{M}}_{oldsymbol{eta}})\ \Rightarrow\ \mathbf{ec{M}}_{oldsymbol{eta}}=-\mathbf{ec{H}}_{oldsymbol{eta}}=\chi_{\mathbf{m}}\mathbf{ec{H}}_{oldsymbol{eta}}\ \Rightarrow\ \chi_{\mathbf{m}}=-\mathbf{1}$$

$$\frac{1+\chi_{\mathbf{e}}}{\chi_{\mathbf{e}}}\vec{\mathbf{P}} = \epsilon_{\mathbf{0}}(1+\chi_{\mathbf{e}})\vec{\mathbf{E}}_{\mathsf{M}} = \vec{\mathbf{D}}_{\mathsf{M}} = \epsilon_{\mathbf{0}}\vec{\mathbf{E}}_{\mathsf{M}} + \vec{\mathbf{P}} = \vec{\mathbf{P}} \quad \Rightarrow \quad \chi_{\mathbf{e}} = \infty$$





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 $ec{\mathbf{e}}_{\mathbf{k}}$  与外法线方向夹角大于 $90^\circ$   $\mathbf{k}_{\mathbf{I}} = \frac{\mathbf{m}_{\mathcal{H}} \mathbf{c}}{\hbar}$ 

对这些  $\vec{j} \neq 0$  的区域,前面的  $\vec{j}(0) = 0$  的情形不存在。即只能有  $\gamma = \infty$  的情形!

定义磁场衰减为边界值的1/e时深度为<u>穿透深度</u>  $\lambda$ 。 $\lambda = rac{1}{\mathbf{k_I}} = rac{\hbar}{\mathbf{m}_{*^2}\mathbf{c}}$ 

$$\mathbf{0} = \vec{\mathbf{B}}_{\mathsf{P}} = \mu_{\mathbf{0}}(\vec{\mathbf{H}}_{\mathsf{P}} + \vec{\mathbf{M}}_{\mathsf{P}}) \Rightarrow \vec{\mathbf{M}}_{\mathsf{P}} = -\vec{\mathbf{H}}_{\mathsf{P}} = \chi_{\mathbf{m}}\vec{\mathbf{H}}_{\mathsf{P}} \Rightarrow \chi_{\mathbf{m}} = -\mathbf{1}$$

$$\Rightarrow \ \mu = \mu_0 (\mathbf{1} + \chi_{\mathbf{m}}) = \mathbf{0}$$
 完全的抗酸体1

$$\frac{1+\chi_{\mathbf{e}}}{\chi_{\mathbf{e}}}\vec{\mathbf{P}} = \epsilon_{\mathbf{0}}(1+\chi_{\mathbf{e}})\vec{\mathbf{E}}_{\mathbf{p}} = \vec{\mathbf{D}}_{\mathbf{p}} = \epsilon_{\mathbf{0}}\vec{\mathbf{E}}_{\mathbf{p}} + \vec{\mathbf{P}} = \vec{\mathbf{P}} \quad \Rightarrow \quad \chi_{\mathbf{e}} = \infty$$

$$\Rightarrow \ \epsilon = \epsilon_0 (1 + \chi_e) = \infty \quad \underline{\text{skl}}$$

若导体上电荷、电流完全由光子的有效质量产生

 $\vec{\mathbf{B}} \stackrel{\mathbf{r_k} \equiv \vec{\mathbf{r}} \cdot \vec{\mathbf{e}}_k}{===} \vec{\mathbf{B}}_0 e^{-\mathbf{k}_{\mathbf{I}} \mathbf{r}_k}$ 









 $ec{\mathbf{e}}_{\mathbf{k}}$  与外法线方向夹角大于 $90^\circ$   $\mathbf{k}_{\mathbf{I}} = rac{\mathbf{m}_{\mathcal{H}} \mathbf{c}}{\hbar}$ 

对这些  $\vec{j} \neq 0$  的区域,前面的  $\vec{j}(0) = 0$  的情形不存在。即只能有  $\gamma = \infty$  的情形!

 $\vec{\mathbf{B}} \stackrel{\mathbf{r_k} \equiv \vec{\mathbf{r}} \cdot \vec{\mathbf{e}}_k}{===} \vec{\mathbf{B}}_0 e^{-\mathbf{k}_I \mathbf{r}_k}$ 

定义磁场衰减为边界值的1/e时深度为<u>穿透深度</u>  $\lambda$ :  $\lambda = \frac{1}{\mathbf{k_I}} = \frac{\hbar}{\mathbf{m_{**}} \cdot \mathbf{c}}$ 

 $\mathbf{\hat{G}} = \mathbf{\hat{H}}_{\mathsf{M}} = \mu_{\mathsf{O}}(\mathbf{\hat{H}}_{\mathsf{M}} + \mathbf{\hat{M}}_{\mathsf{M}}) \ \Rightarrow \ \mathbf{\hat{M}}_{\mathsf{M}} = -\mathbf{\hat{H}}_{\mathsf{M}} = \chi_{\mathbf{m}}\mathbf{\hat{H}}_{\mathsf{M}} \ \Rightarrow \ \chi_{\mathbf{m}} = -\mathbf{\hat{H}}_{\mathsf{M}}$ 

 $rac{1+\chi_{\mathbf{e}}}{\mathbf{P}} ec{\mathbf{P}} = \epsilon_{\mathbf{0}} (1+\chi_{\mathbf{e}}) ec{\mathbf{E}}_{\mathbf{P}} = ec{\mathbf{D}}_{\mathbf{P}} = \epsilon_{\mathbf{0}} ec{\mathbf{E}}_{\mathbf{P}} + ec{\mathbf{P}} = ec{\mathbf{P}} \quad \Rightarrow \quad \Rightarrow \quad \chi_{\mathbf{e}} = \infty$ 

 $\Rightarrow \epsilon = \epsilon_0 (1 + \chi_e) = \infty$ 

若导体上电荷、电流完全由光子的有效质量产生→麦克斯韦方程组、欧姆定律加有效光子质量。 电场和电荷;介质 内磁感和电流指数衰减;电导率和介电常数为无穷大;磁导











● 关于有效光子质量:

$$egin{align*} igl[
abla^2 - rac{1}{\mathbf{c}^2}rac{\partial^2}{\partial \mathbf{t}^2} - rac{\mathbf{m}_{\mathcal{H}}^2\mathbf{c}^2}{\hbar^2}igr] ec{\mathbf{A}} = \mathbf{0} \ = & = = = = = \Rightarrow -ec{\mathbf{k}} \cdot ec{\mathbf{k}} + rac{\omega^2}{\mathbf{c}^2} - rac{\mathbf{m}_{\mathcal{H}}^2\mathbf{c}^2}{\hbar^2} = \mathbf{0} \ igl[
abla^2 - rac{1}{\mathbf{c}^2}rac{\partial^2}{\partial \mathbf{t}^2} - rac{\mathbf{m}_{\mathcal{H}}^2\mathbf{c}^2}{\hbar^2}igr]\phi = \mathbf{0} \end{split}$$

 $\hbar \vec{\mathbf{k}} 
ightharpoonup \vec{\mathbf{p}} \qquad \hbar \omega 
ightharpoonup \mathbf{E} \qquad \Rightarrow - \vec{\mathbf{p}} \cdot \vec{\mathbf{p}} + rac{\mathbf{E}^2}{\mathbf{c}^2} - \mathbf{m}_{\mathfrak{KF}}^2 \mathbf{c}^2 = \mathbf{0}$ 

关于质量与规范对称性及规范自由度:

$$egin{aligned} igl[
abla^2 - rac{1}{\mathbf{c}^2} rac{\partial^2}{\partial \mathbf{t}^2} - rac{\mathbf{m}_{rac{\mathcal{X}}{\mathcal{X}}}^2 \mathbf{c}^2}{\hbar^2} igr] ec{\mathbf{A}} - 
abla igl[
abla \cdot ec{\mathbf{A}} + rac{1}{\mathbf{c}^2} rac{\partial \phi}{\partial \mathbf{t}} igr] = \mathbf{0} \ igl[
abla^2 - rac{1}{\mathbf{c}^2} rac{\partial^2}{\partial \mathbf{t}^2} - rac{\mathbf{m}_{rac{\mathcal{X}}{\mathcal{X}}}^2 \mathbf{c}^2}{\hbar^2} igr] \phi + rac{\partial}{\partial \mathbf{t}} igl[
abla \cdot ec{\mathbf{A}} + rac{1}{\mathbf{c}^2} rac{\partial \phi}{\partial \mathbf{t}} igr] = \mathbf{0} \end{aligned}$$

只有质量项破坏规范对称性! 反过来规范对称性要求质量为零!

不有灰里坝似外就把对你住。及这未就把对你住安
$$ec{\mathbf{A}}' = ec{\mathbf{A}} + \nabla \chi$$
  $\phi' = \phi - \frac{\partial \chi}{\partial t}$ 

它使纵场变成不可观察! 失去它(破缺或隐藏)则纵场可观察!









### 有效光子质量与超导,零磁场与超导

$$\int ec{f j} = -rac{m_{ extcolored{MT}}^2 c^2}{\mu_0 \hbar^2} ec{f A}$$

●若假设

则导致 超导

$$ho = -rac{\mathrm{m}_{\mathrm{光}}^2 \epsilon_0 \mathrm{c}^2}{\hbar^2} \phi$$



Back

$$egin{aligned} \vec{\mathbf{j}} = -rac{\mathbf{m}_{\mathcal{H}\mathcal{F}}^2\mathbf{c}^2}{\mu_0\hbar^2} \vec{\mathbf{A}} \end{aligned}$$

$$\rho = -\frac{m_{\mathcal{H}} + c_0 c}{\hbar^2} \phi$$

$$egin{aligned} \mathbf{J} &= -rac{\kappa_1}{\mu_0 h^2} \mathbf{A} \ & ? \$$
 充分必要 $ho &= -rac{\mathbf{m}_{\mathcal{H},\Gamma}^2 \epsilon_1 \mathbf{c}^2}{h^2} \phi \end{aligned}$ 



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・若假设 
$$\begin{cases} \vec{\mathbf{j}} = -\frac{\mathbf{m}_{\mathcal{K}\mathcal{T}}^2 \mathbf{c}^2}{\mu_0 h^2} \vec{\mathbf{A}} \\ \rho = -\frac{\mathbf{m}_{\mathcal{K}\mathcal{T}}^2 \epsilon_0 \mathbf{c}^2}{h^2} \phi \end{cases}$$
 则导致 超导

• 若超导一定有極端上流。
$$\nabla \times \vec{\mathbf{j}} = -\frac{\mathbf{m}_{\mathcal{H}}^2 \mathbf{c}^2}{\mu_0 \hbar^2} \vec{\mathbf{B}} = -\frac{\mathbf{m}_{\mathcal{H}}^2 \mathbf{c}^2}{\mu_0 \hbar^2} \nabla \times \vec{\mathbf{A}}$$

$$ullet$$
 则一定有  $ec{f j}=-rac{{f m}_{{\cal H}{f T}}^2{f c}^2}{\mu_0h^2}[ec{f A}+
abla\chi]$  可通过规范选择将  $\chi$  场去掉,虽规范对称性已失去





$$\rho = -\frac{\mathbf{m}_{\mathcal{H}}^2 + \epsilon_0 \mathbf{c}^2}{\hbar^2} \phi$$

$$ullet$$
 反过来 招导 是否一定导致 
$$\begin{cases} \vec{j} = -\frac{m_{\mathcal{H}_{2}}^{2}c^{2}}{\mu_{0}h^{2}}\vec{A} \\ & ?$$
 充分必要 
$$\rho = -\frac{m_{\mathcal{H}_{2}}^{2}c_{0}c^{2}}{h^{2}} \rho \end{cases}$$

$$ullet$$
 若超导一定有 $\frac{\mathbf{m}_{\mathcal{H}}^{2}\mathbf{r}^{2}}{\mu_{0}\hbar^{2}}\nabla \times \vec{\mathbf{j}} = -\frac{\mathbf{m}_{\mathcal{H}}^{2}\mathbf{r}^{2}}{\mu_{0}\hbar^{2}}\vec{\mathbf{B}} = -\frac{\mathbf{m}_{\mathcal{H}}^{2}\mathbf{r}^{2}}{\mu_{0}\hbar^{2}}\nabla \times \vec{\mathbf{A}}$ 

$$ullet$$
 则一定有  $ec{f j}=-rac{{f m}_{{\mathcal H}{f f}}^2{f c}^2}{\mu_0h^2}[ec{f A}+
abla\chi]$  可通过规范选择符  $\chi$  场去掉,且规范对称性已失去

• 剩下 
$$\rho = -\frac{m_{\mathcal{H}}^2 \epsilon_0 c^2}{\hbar^2} [\phi - \frac{\partial \chi}{\partial t}]$$
 可通过协变性 җ  $\chi$  相对论 要求得到!

• 若假设 
$$\begin{cases} \vec{\mathbf{j}} = -\frac{m_{\mathcal{H}}^2 c^2}{\mu_0 h^2} \vec{\mathbf{A}} \\ \rho = -\frac{m_{\mathcal{H}}^2 \epsilon_0 c^2}{\hbar^2} \phi \end{cases}$$
 则导致 超导

$$\int$$
 反过来  $\frac{1}{12}$  是否一定导致 
$$\begin{cases} \vec{J} = -\frac{m_{JLJ}^2 c^2}{\mu_0 h^2} \vec{A} \\ &?$$
 充分必要 
$$\rho = -\frac{m_{JLJ}^2 c_0 c^2}{h^2} \phi \end{cases}$$

• 若超导一定有
$$\frac{\mathbf{m}_{\mathcal{H}}^{2}\mathbf{r}^{2}}{\mu_{0}\hbar^{2}}$$
  $\nabla \times \vec{\mathbf{j}} = -\frac{\mathbf{m}_{\mathcal{H}}^{2}\mathbf{r}^{2}}{\mu_{0}\hbar^{2}} \vec{\mathbf{B}} = -\frac{\mathbf{m}_{\mathcal{H}}^{2}\mathbf{r}^{2}}{\mu_{0}\hbar^{2}} \nabla \times \vec{\mathbf{A}}$ 

$$ullet$$
 则一定有  $ec{f j}=-rac{{f m}_{{\cal H}{f T}}^2{f c}^2}{\mu_0h^2}[ec{f A}+
abla\chi]$  可通过规范选择将  $\chi$  汤去掉,虽规范对称性已失去

• 剩下 
$$\rho = -\frac{\mathbf{m}_{\mathcal{H}}^2 \epsilon_0 \mathbf{c}^2}{\hbar^2} [\phi - \frac{\partial \chi}{\partial \mathbf{t}}]$$
 可通过协变性  $\chi_{\chi}$  要求得到!

• 它相当 此广版化第一方形 
$$\frac{\partial \vec{\mathbf{j}}}{\partial \mathbf{t}} + \mathbf{c^2} \nabla \rho = -\frac{\mathbf{m}_{\mathcal{H}}^2 \mathbf{c^2}}{\mu_0 \hbar^2} (\nabla \phi + \frac{\partial \vec{\mathbf{A}}}{\partial \mathbf{t}}) = \frac{\mathbf{m}_{\mathcal{H}}^2 \mathbf{c^2}}{\mu_0 \hbar^2} \vec{\mathbf{E}}$$



### 有效光子质量: 有效光子质量与超导, 零磁场与超导 无磁场。 \*\*\*\* 内部无磁场的介质一定是超导休吗?

● 超导体内无磁场, ∞ → 内部无磁场的介质一定是超导体吗?



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**>>** 

- 超导体内无磁场,፳፫素内部无磁场的介质一定是超导体吗?
  - 辰//- 大磁导率的绝缘磁介质内部也无磁场 展展, 不是超导体!



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- 超导体内无磁场,∞∞\*内部无磁场的介质一定是超导体吗?
- 退一步内部无磁场的 导体 一定是超导体吗?



00/90





- 超导体内无磁场, 處 內部 无磁场的介质一定是超导体吗?
- ৣৢৢৢৢৢ。大磁导率的绝缘磁介质内部也无磁场;;;;;,不是超导体!
- 및一步内部无磁场的 <mark>导体</mark> 一定是超导体吗? <u>是!</u> 主要看内部无磁场的导体中是否也无电场,或电导率是否为无穷大 j = γĒι



86/96





- 超导体内无磁场, ∞ ∞ 水内部无磁场的介质一定是超导体吗?
  - № 大磁导率的绝缘磁介质内部也无磁场\*\*\*\*,不是超导体!
- 退一步内部无磁场的 <mark>导体</mark> 一定是超导体吗? <u>是</u>! 主要看内部无磁场的导体中是否也无电场,或电导率是否为无穷大 i= ¬Ē!

ጽሮጀመጀ: 
$$\left[ 
abla^2 - rac{1}{c^2} rac{\partial^2}{\partial t^2} 
ight] ec{\mathbf{B}} = -\mu_0 
abla imes ec{\mathbf{j}} = -\mu_0 \gamma 
abla imes ec{\mathbf{E}} = \mu_0 \gamma rac{\partial ec{\mathbf{B}}}{\partial t}$$



00/90





- 超导体内无磁场, ∞ ∞ 水内部无磁场的介质一定是超导体吗?
- ፫灿 大磁导率的绝缘磁介质内部也无磁场 ≝元元,不是超导体!
- 및一步内部无磁场的 <mark>导体</mark> 一定是超导体吗? <u>是!</u> 主要看内部无磁场的导体中是否也无电场,或电导率是否为无穷大 i= √E!

$$\begin{split} \mathbf{\vec{B}} &= \mathbf{\vec{B}}_0 \mathbf{e}^{\mathbf{i}\vec{k} \cdot \vec{r} - \mathbf{i}\omega \mathbf{t}} \ = \mathbf{\vec{B}}_0 \mathbf{e}^{\mathbf{i}\vec{k} \cdot \vec{r} - \mathbf{i}\omega \mathbf{t}} \ = \mathbf{\vec{B}}_0 \mathbf{e}^{\mathbf{i}\vec{k} \cdot \vec{r} - \mathbf{i}\omega \mathbf{t}} \end{split} = \mathbf{\vec{B}}_0 \mathbf{\vec{B}} \\ \mathbf{\vec{B}} &= \mathbf{\vec{B}}_0 \mathbf{e}^{\mathbf{i}\vec{k} \cdot \vec{r} - \mathbf{i}\omega \mathbf{t}} \ = \mathbf{\vec{E}} \\ \mathbf{\vec{B}} &= \mathbf{\vec{B}}_0 \mathbf{\vec{C}} \\ \mathbf{\vec{C}} &= \mathbf{\vec{C}} \\ \mathbf{\vec{C}} \\ \mathbf{\vec{C}} &= \mathbf{\vec{C}} \\ \mathbf{\vec{C}} \\ \mathbf{\vec{C}} &= \mathbf{\vec{C}} \\ \mathbf{\vec{C}}$$







### 

- 超导体内无磁场, 處應 内部无磁场的介质一定是超导体吗?
- № 大磁导率的绝缘磁介质内部也无磁场 7 7 7 2 超导体!
- 退一步内部无磁场的 <mark>导体</mark> 一定是超导体吗? <u>是</u>!

  主要看内部无磁场的导体中是否也无电场。或电导率是否为无穷大了= ~配

\*\*\* 
$$\begin{split} \begin{bmatrix} \nabla^2 - \frac{1}{\mathbf{c}^2} \frac{\partial^2}{\partial \mathbf{t}^2} \end{bmatrix} \vec{\mathbf{B}} &= -\mu_0 \nabla \times \vec{\mathbf{j}} = -\mu_0 \gamma \nabla \times \vec{\mathbf{E}} = \mu_0 \gamma \frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}} \\ \vec{\mathbf{B}} &= \vec{\mathbf{B}}_0 \mathbf{e}^{\mathbf{i} \vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \mathbf{i} \omega \mathbf{t}} &= \stackrel{\text{Uniform}}{=====} \Rightarrow \quad \vec{\mathbf{k}} \cdot \vec{\mathbf{k}} - \frac{\omega^2}{\mathbf{c}^2} = \mathbf{i} \mu_0 \gamma \omega \end{split}$$
 
$$\vec{\mathbf{k}}_{\mathbf{R}} \cdot \vec{\mathbf{k}}_{\mathbf{R}} - \vec{\mathbf{k}}_{\mathbf{I}} \cdot \vec{\mathbf{k}}_{\mathbf{I}} = \frac{\omega^2}{\mathbf{c}^2} \qquad \vec{\mathbf{k}}_{\mathbf{R}} \cdot \vec{\mathbf{k}}_{\mathbf{I}} = \frac{1}{2} \mu_0 \gamma \omega \end{split}$$



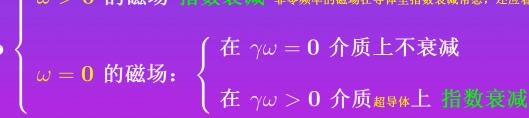
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**√**(

**←** 

- 超导体内无磁场,底进入内部无磁场的介质一定是超导体吗?
- № 大磁导率的绝缘磁介质内部也无磁场 , 不是超导体!
- 週一步内部无磁场的 <mark>导体</mark> 一定是超导体吗? <u>是</u>

後後数規范: 
$$\left[ 
abla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right] \vec{\mathbf{B}} = -\mu_0 \nabla \times \vec{\mathbf{j}} = -\mu_0 \gamma \nabla \times \vec{\mathbf{E}} = \mu_0 \gamma \frac{\partial \vec{\mathbf{B}}}{\partial t}$$
  $\vec{\mathbf{B}} = \vec{\mathbf{B}}_0 \mathbf{e}^{i\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}-i\omega t} = \stackrel{\Box \dot{\mathbf{M}} \dot{\mathbf{M}} \dot{\mathbf{M}} \dot{\mathbf{M}}}{=====} \Rightarrow \vec{\mathbf{k}} \cdot \vec{\mathbf{k}} - \frac{\omega^2}{c^2} = i\mu_0 \gamma \omega$   $\vec{\mathbf{k}}_{\mathbf{R}} \cdot \vec{\mathbf{k}}_{\mathbf{R}} - \vec{\mathbf{k}}_{\mathbf{I}} \cdot \vec{\mathbf{k}}_{\mathbf{I}} = \frac{\omega^2}{c^2}$   $\vec{\mathbf{k}}_{\mathbf{R}} \cdot \vec{\mathbf{k}}_{\mathbf{I}} = \frac{1}{2}\mu_0 \gamma \omega$  沿位相体形方面设施  $\omega > 0$  的磁场 指数衰减 非零频率的磁场在导体里指数衰减常态。还应看静磁场  $\psi$ 





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**←** 

Back

### 有效光子质量: 超越洛伦兹规范,规范不变描写 $egin{aligned} ilde{\phi} \equiv rac{-1}{\left[ abla^2 - rac{1}{\mathbf{c^2}}rac{\partial^2}{\partial \mathbf{t^2}} ight]} \left[ abla \cdot ec{\mathbf{A}} + rac{1}{\mathbf{c^2}}rac{\partial \phi}{\partial \mathbf{t}} ight] \end{aligned}$









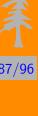




### <u>有效光子质量</u>: 超越洛伦兹规范,规范不变描写



 $egin{aligned} ar{\phi} &\equiv rac{-1}{[
abla^2 - rac{1}{\mathbf{c^2}}rac{\partial^2}{\partial \mathbf{t}^2}]} [
abla \cdot ec{\mathbf{A}} + rac{1}{\mathbf{c^2}}rac{\partial \phi}{\partial \mathbf{t}}] \end{aligned}$ 





$$egin{align*} ilde{\phi} &\equiv rac{-1}{\left[
abla^2 - rac{1}{\mathbf{c^2}}rac{\partial^2}{\partial \mathbf{t^2}}
ight]} \left[
abla \cdot ec{\mathbf{A}} + rac{1}{\mathbf{c^2}}rac{\partial \phi}{\partial \mathbf{t}}
ight] & \underbrace{\left[
abla^2 - rac{1}{\mathbf{c^2}}rac{\partial^2}{\partial \mathbf{t^2}}
ight] ilde{\phi} = -\left[
abla \cdot ec{\mathbf{A}} + rac{1}{\mathbf{c^2}}rac{\partial \phi}{\partial \mathbf{t}}
ight]}_{ ext{NNSELVOT: Struckelbergilik}} \end{aligned}$$

规范变换:  $\phi \rightarrow \phi' = \phi - \chi$ 









$$ilde{\phi} \equiv rac{-1}{\left[
abla^2 - rac{1}{\mathbf{c}^2}rac{\partial^2}{\partial \mathbf{t}^2}
ight]} \left[
abla \cdot \mathbf{A} + rac{1}{\mathbf{c}^2}rac{\partial \phi}{\partial \mathbf{t}}
ight] \qquad \underbrace{\left[
abla^2 - rac{1}{\mathbf{c}^2}rac{\partial^2}{\partial \mathbf{t}^2}
ight] ilde{\phi} = -\left[
abla \cdot \mathbf{A} + rac{1}{\mathbf{c}^2}rac{\partial \phi}{\partial \mathbf{t}}
ight]}_{NNSMEMT: Stuckelberg Eigen}$$

规范变换:  $\tilde{\phi} \rightarrow \tilde{\phi}' = \tilde{\phi} - \chi$ 

$$\mu_0 \vec{\mathbf{j}} = \frac{\mathbf{m^2 c^2}}{\hbar^2} [\vec{\mathbf{A}} + \nabla \tilde{\phi}] \qquad \qquad \frac{\rho}{\epsilon_0} = \frac{\mathbf{m^2 c^2}}{\hbar^2} [\phi - \frac{\partial \tilde{\phi}}{\partial \mathbf{t}}] \quad \text{for all } \mathbf{m}_{\chi = \tilde{\phi}}$$







**美** 

$$ilde{\phi} \equiv rac{-1}{\left[
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ight]} \left[
abla \cdot \vec{\mathbf{A}} + rac{1}{\mathbf{c}^2}rac{\partial \phi}{\partial \mathbf{t}}
ight] \qquad \underbrace{\left[
abla^2 - rac{1}{\mathbf{c}^2}rac{\partial^2}{\partial \mathbf{t}^2}
ight] ilde{\phi} = -\left[
abla \cdot \vec{\mathbf{A}} + rac{1}{\mathbf{c}^2}rac{\partial \phi}{\partial \mathbf{t}}
ight]}_{ imes \mathbf{M} \otimes \mathbf{M} \otimes \mathbf{M} \otimes \mathbf{T}^* \cdot \mathbf{Streckelberg} \otimes \mathbf{M} \otimes$$

规范变换:  $\phi o ilde{\phi}' = ilde{\phi} - \chi$ 

$$\mu_0 \vec{\mathbf{j}} = rac{\mathbf{m^2 c^2}}{\hbar^2} [\vec{\mathbf{A}} + 
abla ilde{\phi}] \qquad \qquad rac{
ho}{\epsilon_0} = rac{\mathbf{m^2 c^2}}{\hbar^2} [\phi - rac{\partial ilde{\phi}}{\partial \mathbf{t}}] \qquad \qquad \vec{\mathbf{m}} = \mathbf{v} = \mathbf{v}$$

$$\nabla \cdot \vec{\mathbf{j}} + \frac{\partial \rho}{\partial \mathbf{t}} = \nabla \cdot \left[ \frac{\mathbf{m^2 c^2}}{\mu_0 \hbar^2} [\vec{\mathbf{A}} + \nabla \tilde{\phi}] \right] + \frac{\partial}{\partial \mathbf{t}} \left[ \frac{\epsilon_0 \mathbf{m^2 c^2}}{\hbar^2} [\phi - \frac{\partial \tilde{\phi}}{\partial \mathbf{t}}] \right]$$







Back

$$\tilde{\phi} \equiv \frac{-1}{\left[\nabla^2 - \frac{1}{\mathbf{c}^2} \frac{\partial^2}{\partial \mathbf{t}^2}\right]} \left[\nabla \cdot \vec{\mathbf{A}} + \frac{1}{\mathbf{c}^2} \frac{\partial \phi}{\partial \mathbf{t}}\right] \qquad \underbrace{\left[\nabla^2 - \frac{1}{\mathbf{c}^2} \frac{\partial^2}{\partial \mathbf{t}^2}\right] \tilde{\phi} = -\left[\nabla \cdot \vec{\mathbf{A}} + \frac{1}{\mathbf{c}^2} \frac{\partial \phi}{\partial \mathbf{t}}\right]}_{\text{XINGE for the results}}$$

规范变换:  $\phi \rightarrow \phi = \phi - \gamma$ 

$$\mu_0 ec{\mathbf{j}} = rac{\mathbf{m^2 c^2}}{\hbar^2} [ec{\mathbf{A}} + 
abla ilde{\phi}] \qquad \qquad rac{
ho}{\epsilon_0} = rac{\mathbf{m^2 c^2}}{\hbar^2} [\phi - rac{\partial ilde{\phi}}{\partial \mathbf{t}}] \quad \text{for all } \mathbf{x} = ilde{\phi}$$

$$\nabla \cdot \vec{\mathbf{j}} + \frac{\partial \rho}{\partial \mathbf{t}} = \nabla \cdot \left[ \frac{\mathbf{m}^2 \mathbf{c}^2}{\mu_0 \hbar^2} [\vec{\mathbf{A}} + \nabla \tilde{\phi}] \right] + \frac{\partial}{\partial \mathbf{t}} \left[ \frac{\epsilon_0 \mathbf{m}^2 \mathbf{c}^2}{\hbar^2} [\phi - \frac{\partial \tilde{\phi}}{\partial \mathbf{t}}] \right]$$

$$\frac{\mathbf{m}^2 \mathbf{c}^2}{\hbar^2} \mathbf{1} = \mathbf{3}^2$$

$$=rac{rac{ ext{m}^2 ext{c}^2}{\mu_0\hbar^2}}{\left[
abla^2-rac{1}{ ext{c}^2}rac{\partial^2}{\partial ext{t}^2}
ight]}\left[\left[
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ight]
abla\cdotec{ ext{A}}-
abla^2\left[
abla\cdotec{ ext{A}}+rac{1}{ ext{c}^2}rac{\partial\phi}{\partial ext{t}}
ight]
ight]$$

$$egin{aligned} & rac{\mu_0\hbar^2}{\left[
abla^2-rac{1}{\mathbf{c}^2}rac{\partial^2}{\partial \mathbf{t}^2}
ight]} \left[\left[
abla^2-rac{\mathbf{r}}{\mathbf{c}^2}rac{\partial}{\partial \mathbf{t}^2}
ight]
abla\cdotec{\mathbf{A}} -
abla^2\left[
abla\cdotec{\mathbf{A}} +rac{\mathbf{r}}{\mathbf{c}^2}rac{\partial\phi}{\partial \mathbf{t}}
ight] \\ & +rac{rac{\epsilon_0\mathbf{m}^2\mathbf{c}^2}{\hbar^2}}{\left[
abla^2-rac{1}{\mathbf{c}^2}rac{\partial^2}{\partial \mathbf{t}^2}
ight]} \left[\left[
abla^2-rac{1}{\mathbf{c}^2}rac{\partial^2}{\partial \mathbf{t}^2}
ight]rac{\partial\phi}{\partial \mathbf{t}} +rac{\partial^2}{\partial \mathbf{t}^2}\left[
abla\cdotec{\mathbf{A}} +rac{1}{\mathbf{c}^2}rac{\partial\phi}{\partial \mathbf{t}}
ight] = \mathbf{0} \end{aligned}$$









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$$egin{align*} ilde{\phi} &\equiv rac{-1}{\left[
abla^2 - rac{1}{\mathbf{c^2}}rac{\partial^2}{\partial \mathbf{t^2}}
ight]} \left[
abla \cdot \mathbf{A} + rac{1}{\mathbf{c^2}}rac{\partial \phi}{\partial \mathbf{t}}
ight] &\qquad \underbrace{\left[
abla^2 - rac{1}{\mathbf{c^2}}rac{\partial^2}{\partial \mathbf{t^2}}
ight] ilde{\phi} = -\left[
abla \cdot \mathbf{A} + rac{1}{\mathbf{c^2}}rac{\partial \phi}{\partial \mathbf{t}}
ight]}_{NNSMED (T)} & \qquad \underbrace{\left[
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ight] ilde{\phi} = -\left[
abla \cdot \mathbf{A} + rac{1}{\mathbf{c^2}}rac{\partial \phi}{\partial \mathbf{t}}
ight]}_{NNSMED (T)} & \qquad \underbrace{\left[
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ight] ilde{\phi} = -\left[
abla \cdot \mathbf{A} + rac{1}{\mathbf{c^2}}rac{\partial \phi}{\partial \mathbf{t}}
ight]}_{NNSMED (T)} & \qquad \underbrace{\left[
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ight] ilde{\phi} = -\left[
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ight]}_{NNSMED (T)} & \qquad \underbrace{\left[
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ight] ilde{\phi}}_{NNSMED (T)} & \qquad \underbrace{\left[
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ight] ilde{\phi}}_{NNSMED (T)} & \qquad \underbrace{\left[
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ight]}_{NNSMED (T)} & \qquad \underbrace{\left[
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ight]}_{NNSMED (T)} & \qquad \underbrace{\left[
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ight]}_{NNSMED (T)} & \qquad \underbrace{\left[
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ight]}_{NNSMED (T)} & \qquad \underbrace{\left[
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ight]}_{NNSMED (T)} & \qquad \underbrace{\left[
abla^$$

规范变换:  $\phi \rightarrow \phi = \phi - \gamma$ 

$$\mu_0 \vec{\mathbf{j}} = rac{\mathbf{m^2 c^2}}{\hbar^2} [\vec{\mathbf{A}} + \nabla \tilde{\phi}] \qquad \qquad rac{
ho}{\epsilon_0} = rac{\mathbf{m^2 c^2}}{\hbar^2} [\phi - rac{\partial \tilde{\phi}}{\partial t}] \quad \text{for } t = \delta$$

$$\nabla \cdot \vec{\mathbf{j}} + \frac{\partial \rho}{\partial \mathbf{t}} = \nabla \cdot \left[ \frac{\mathbf{m}^2 \mathbf{c}^2}{\mu_0 \hbar^2} [\vec{\mathbf{A}} + \nabla \tilde{\phi}] \right] + \frac{\partial}{\partial \mathbf{t}} \left[ \frac{\epsilon_0 \mathbf{m}^2 \mathbf{c}^2}{\hbar^2} [\phi - \frac{\partial \tilde{\phi}}{\partial \mathbf{t}}] \right]$$

$$\mathbf{m}^2 \mathbf{c}^2$$

$$= \frac{\frac{\mathbf{m}^2 \mathbf{c}^2}{\mu_0 \hbar^2}}{\left[\nabla^2 - \frac{1}{\mathbf{c}^2} \frac{\partial^2}{\partial \mathbf{t}^2}\right]} \left[ \left[\nabla^2 - \frac{1}{\mathbf{c}^2} \frac{\partial^2}{\partial \mathbf{t}^2}\right] \nabla \cdot \vec{\mathbf{A}} - \nabla^2 \left[\nabla \cdot \vec{\mathbf{A}} + \frac{1}{\mathbf{c}^2} \frac{\partial \phi}{\partial \mathbf{t}}\right] \right]$$

$$+\frac{\frac{\epsilon_0 m^2 c^2}{\hbar^2}}{\left[\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\right]} \left[ \left[\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\right] \frac{\partial \phi}{\partial t} + \frac{\partial^2}{\partial t^2} \left[\nabla \cdot \vec{A} + \frac{1}{c^2} \frac{\partial \phi}{\partial t}\right] \right] = 0$$

Close

可通过选择 洛伦兹规范 或 规范变换 使 场为零!

## 有效光子质量: 有效光子质量起源 $\left|rac{ ho}{\epsilon_0} = rac{\mathbf{m^2c^2}}{\hbar^2} [\phi - rac{\partial ilde{\phi}}{\partial \mathbf{t}}] ight|$ $\mu_0 ec{\mathbf{j}} = rac{\mathrm{m^2 c^2}}{\hbar^2} [ec{\mathbf{A}} + abla ilde{\phi}]$ $rac{\mathbf{mc}}{\hbar} \equiv ar{\phi}$ उन्नहंस्त्रेला $\left[ rac{ ho}{\epsilon_{\mathbf{0}}} = \overline{\phi}^{\mathbf{2}} [\phi - rac{\partial ilde{\phi}}{\partial \mathbf{t}}] ight]$ $oxed{\mu_0 ec{\mathbf{j}}} = ar{\phi}^{\mathbf{2}} [ ec{\mathbf{A}} + abla ilde{\phi} ]$ Back Close

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有效光子质量: 有效光子质量起源 秦華  $egin{aligned} rac{
ho}{\epsilon_0} = rac{\mathbf{m^2 c^2}}{\hbar^2} [\phi - rac{\partial ilde{\phi}}{\partial t}] \end{aligned}$  $\mu_0 ec{\mathbf{j}} = rac{\mathrm{m^2 c^2}}{\hbar^2} [ec{\mathbf{A}} + 
abla ilde{\phi}]$  $rac{
m mc}{\hbar}\equivar{\phi}$  shares 88/96  $[rac{
ho}{\epsilon_0} = ar{\phi}^2 [\phi - rac{\partial ilde{\phi}}{\partial t}]$  $\mu_{\mathbf{0}}\vec{\mathbf{j}} = \overline{\phi}^{\mathbf{2}}[\vec{\mathbf{A}} + \nabla \widetilde{\phi}]$  $\mu_0 \vec{\mathbf{j}} = (\overline{\phi} \mathbf{e}^{-\mathbf{i} ilde{\phi}})(\overline{\phi} \mathbf{e}^{\mathbf{i} ilde{\phi}}) \overline{\mathbf{A}} - \frac{\mathbf{i}}{2} (\overline{\phi} \mathbf{e}^{-\mathbf{i} ilde{\phi}}) \nabla (\overline{\phi} \mathbf{e}^{\mathbf{i} ilde{\phi}}) + \frac{\mathbf{i}}{2} (\overline{\phi} \mathbf{e}^{\mathbf{i} ilde{\phi}}) \nabla (\overline{\phi} \mathbf{e}^{-\mathbf{i} ilde{\phi}})$  $=\Phi^*\Phiec{f A} \ - \ rac{f i}{2}\Phi^*
abla\Phi \ + \ rac{f i}{2}\Phi
abla\Phi^*$  $\Phi \equiv ar{\phi} {
m e}^{{
m i} ilde{\phi}}$  $\left|\frac{\rho}{\epsilon_{\mathbf{0}}} = (\bar{\phi}\mathbf{e}^{-\mathbf{i}\tilde{\phi}})(\bar{\phi}\mathbf{e}^{\mathbf{i}\tilde{\phi}})\phi + \frac{\mathbf{i}}{2}(\bar{\phi}\mathbf{e}^{-\mathbf{i}\tilde{\phi}})\frac{\partial}{\partial t}(\bar{\phi}\mathbf{e}^{\mathbf{i}\tilde{\phi}}) - \frac{\mathbf{i}}{2}(\bar{\phi}\mathbf{e}^{\mathbf{i}\tilde{\phi}})\frac{\partial}{\partial t}(\bar{\phi}\mathbf{e}^{\mathbf{i}\tilde{\phi}})\right|$  $=\Phi^*\Phi\phi + \frac{\mathrm{i}}{2}\Phi^*\frac{\partial\Phi}{\partial t} - \overline{\frac{\mathrm{i}}{2}\Phi\frac{\partial\Phi^*}{\partial t}}$ 

#### 有效光子质量: 有效光子质量起源 秦奉 $egin{aligned} rac{ ho}{\epsilon_0} = rac{\mathbf{m^2c^2}}{\hbar^2} [\phi - rac{\partial ilde{\phi}}{\partial t}] \end{aligned}$ $\mu_0 ec{\mathbf{j}} = rac{\mathrm{m^2 c^2}}{\hbar^2} [ec{\mathbf{A}} + abla ilde{\phi}]$ 88/96

$$rac{\mathbf{mc}}{\hbar}\equivar{\phi}$$
 success  $rac{\mathbf{mc}}{\hbar}\equivar{\phi}$  success  $rac{
ho}{\epsilon_0}=ar{\phi}^{\mathbf{2}}[\phi-rac{\partial ilde{\phi}}{\partial \mathbf{t}}]$ 

$$\begin{split} \mu_0 \vec{\mathbf{j}} &= (\bar{\phi} \mathrm{e}^{-\mathrm{i}\bar{\phi}}) (\bar{\phi} \mathrm{e}^{\mathrm{i}\bar{\phi}}) \vec{\mathrm{A}} - \frac{\mathrm{i}}{2} (\bar{\phi} \mathrm{e}^{-\mathrm{i}\tilde{\phi}}) \nabla (\bar{\phi} \mathrm{e}^{\mathrm{i}\tilde{\phi}}) + \frac{\mathrm{i}}{2} (\bar{\phi} \mathrm{e}^{\mathrm{i}\tilde{\phi}}) \nabla (\bar{\phi} \mathrm{e}^{-\mathrm{i}\tilde{\phi}}) \\ &= \Phi^* \Phi \vec{\mathrm{A}} - \frac{\mathrm{i}}{2} \Phi^* \nabla \Phi + \frac{\mathrm{i}}{2} \Phi \nabla \Phi^* \\ \frac{\rho}{\epsilon_0} &= (\bar{\phi} \mathrm{e}^{-\mathrm{i}\tilde{\phi}}) (\bar{\phi} \mathrm{e}^{\mathrm{i}\tilde{\phi}}) \phi + \frac{\mathrm{i}}{2} (\bar{\phi} \mathrm{e}^{-\mathrm{i}\tilde{\phi}}) \frac{\partial}{\partial t} (\bar{\phi} \mathrm{e}^{\mathrm{i}\tilde{\phi}}) - \frac{\mathrm{i}}{2} (\bar{\phi} \mathrm{e}^{\mathrm{i}\tilde{\phi}}) \frac{\partial}{\partial t} (\bar{\phi} \mathrm{e}^{-\mathrm{i}\tilde{\phi}}) \\ &= \Phi^* \Phi \phi + \frac{\mathrm{i}}{2} \Phi^* \frac{\partial \Phi}{\partial t} - \frac{\mathrm{i}}{2} \Phi \frac{\partial \Phi^*}{\partial t} \\ &= \Phi^* \Phi \phi + \frac{\mathrm{i}}{2} \Phi^* \frac{\partial \Phi}{\partial t} - \frac{\mathrm{i}}{2} \Phi \frac{\partial \Phi^*}{\partial t} \end{split}$$









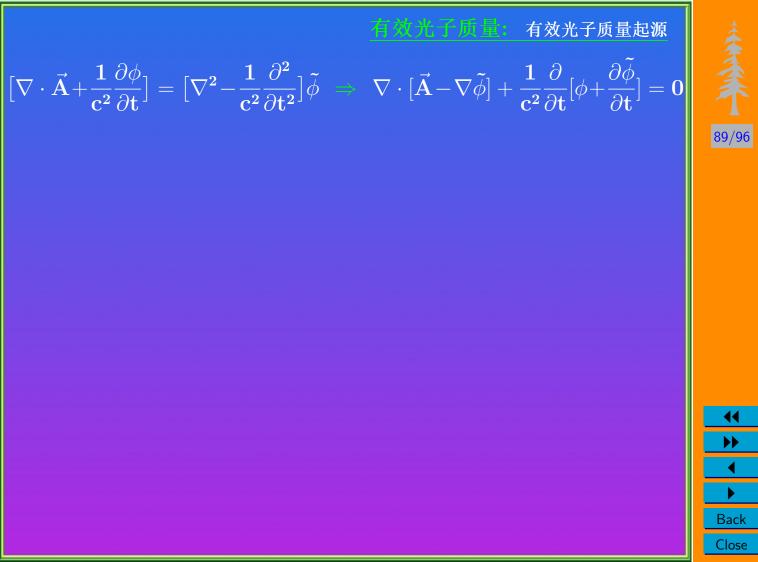






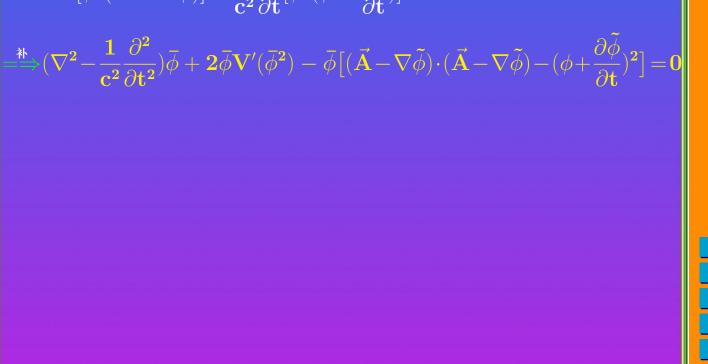






### 有效光子质量: 有效光子质量起源 $\left[ \left[ abla \cdot ec{\mathbf{A}} + rac{1}{\mathbf{c^2}} rac{\partial \phi}{\partial \mathbf{t}} ight] = \left[ abla^2 - rac{1}{\mathbf{c^2}} rac{\partial^2}{\partial \mathbf{t^2}} ight] ilde{\phi} \ \Rightarrow \ abla \cdot \left[ ec{\mathbf{A}} - abla ilde{\phi} ight] + rac{1}{\mathbf{c^2}} rac{\partial}{\partial \mathbf{t}} \left[ \phi + rac{\partial ilde{\phi}}{\partial \mathbf{t}} ight] = 0$

$$\Rightarrow \nabla \cdot [\vec{\phi}^{2}(\vec{\mathbf{A}} - \nabla \tilde{\phi})] + \frac{1}{\mathbf{c}^{2}} \frac{\partial}{\partial \mathbf{t}} [\vec{\phi}^{2}(\phi + \frac{\partial \tilde{\phi}}{\partial \mathbf{t}})] = \mathbf{0}$$















# $egin{align*} egin{align*} egin{align*}$

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 $egin{aligned} \Rightarrow & 
abla \cdot [oldsymbol{ar{\phi^2}}(ar{\mathbf{A}} - 
abla ar{\phi})] + rac{\mathbf{1}}{\mathbf{c^2}} rac{\partial}{\partial \mathbf{t}} [oldsymbol{ar{\phi^2}}(\phi + rac{\partial ilde{\phi}}{\partial \mathbf{t}})] = \mathbf{0} \end{aligned}$ 

$$= \nabla^2 - \frac{1}{\mathbf{c}^2} \frac{\partial^2}{\partial \mathbf{t}^2} \bar{\phi} + 2\bar{\phi} \mathbf{V}'(\bar{\phi}^2) - \bar{\phi} \left[ (\vec{\mathbf{A}} - \nabla \tilde{\phi}) \cdot (\vec{\mathbf{A}} - \nabla \tilde{\phi}) - (\phi + \frac{\partial \phi}{\partial \mathbf{t}})^2 \right] = 0$$

 $egin{align*} \Phi \equiv ar{\phi} \mathrm{e}^{\mathrm{i} ilde{\phi}} & [(
abla + \mathrm{i} A) \cdot (
abla + \mathrm{i} A) - rac{1}{c^2} (rac{\partial}{\partial \mathbf{t}} - \mathrm{i} \phi)^2] \Phi + 2 \mathrm{V}'(\Phi^* \Phi) \Phi = 0 \end{aligned}$ 

 $\mathbf{c^2}$   $\partial \mathbf{t}$   $\partial \mathbf{t}$   $\partial \mathbf{t}$   $\partial \mathbf{t}$   $\partial \mathbf{t}$   $\partial \mathbf{t}$  Higgs 因提出上面理论并特别是指出希格斯粒子而获2013年诺贝尔物理奖!

什 <u>Higgs</u> 因提出上而理论并特别是指出希格斯粒子而获2013年诺贝尔物理等







**有效光子质量**: 有效光子质量起源  $\left[ \left[ 
abla \cdot ec{\mathbf{A}} + rac{1}{\mathbf{c^2}} rac{\partial \phi}{\partial \mathbf{t}} 
ight] = \left[ 
abla^2 - rac{1}{\mathbf{c^2}} rac{\partial^2}{\partial \mathbf{t^2}} 
ight] ilde{\phi} \ \Rightarrow \ 
abla \cdot \left[ ec{\mathbf{A}} - 
abla ilde{\phi} 
ight] + rac{1}{\mathbf{c^2}} rac{\partial}{\partial \mathbf{t}} \left[ \phi + rac{\partial \phi}{\partial \mathbf{t}} 
ight] = 0$ 

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$$\Rightarrow \nabla \cdot [\overline{\phi}^{2}(\vec{\mathbf{A}} - \nabla \tilde{\phi})] + \frac{1}{\mathbf{c}^{2}} \frac{\partial}{\partial \mathbf{t}} [\overline{\phi}^{2}(\phi + \frac{\partial \tilde{\phi}}{\partial \mathbf{t}})] = \mathbf{0}$$

 $[(\nabla + i\vec{A}) \cdot (\nabla + i\vec{A}) - \frac{1}{c^2} (\frac{\partial}{\partial t} - i\phi)^2]\Phi + 2V'(\Phi^*\Phi)\Phi = 0$  $\Phi \equiv ar{\phi} {
m e}^{{
m i} ilde{\phi}}$ 

 $= (\nabla^2 - \frac{1}{\mathbf{c}^2} \frac{\partial^2}{\partial \mathbf{t}^2}) \overline{\phi} + 2 \overline{\phi} \mathbf{V}' (\overline{\phi}^2) - \overline{\phi} [(\overrightarrow{\mathbf{A}} - \nabla \widetilde{\phi}) \cdot (\overrightarrow{\mathbf{A}} - \nabla \widetilde{\phi}) - (\phi + \frac{\partial \phi}{\partial \mathbf{t}})^2] = \mathbf{0}$ 

 $\Phi = \sqrt{rac{\mu_0 {
m e}^* \hbar}{{
m m}^*}} \Psi$ 静态,寒电势。 $\vec{\mathbf{A}} \to -\frac{\mathbf{e}^*}{\mathbf{c}^b} \vec{\mathbf{A}}$ 



 $\mathbf{V}'(\mid \mathbf{\Phi} \mid^{\mathbf{2}}) = -\frac{\mathbf{2m}^{*}}{\hbar^{\mathbf{2}}} \left[ \alpha + \frac{\beta \mathbf{m}^{*}}{\mu_{\mathbf{0}} \mathbf{e}^{*} \hbar} \mid \mathbf{\Phi} \mid^{\mathbf{2}} \right]$ 

 $(
abla - rac{\mathbf{e}^*}{\mathbf{c}\hbar}\mathbf{i}ec{\mathbf{A}}) \cdot (
abla - rac{\mathbf{e}^*}{\mathbf{c}\hbar}\mathbf{i}ec{\mathbf{A}})\Psi - rac{\mathbf{2m}^*}{\hbar^2}[lpha + eta \mid \Psi \mid^2]\Psi = \mathbf{0} \Leftarrow \mathbf{Ginzberg-Landau}\pi \mathbf{E}$ 

 $\frac{7 \times \vec{A} + \vec{D}}{[\nabla \cdot \vec{A} + \frac{1}{c^2} \frac{\partial \phi}{\partial t}]} = [\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}] \tilde{\phi} \implies \nabla \cdot [\vec{A} - \nabla \tilde{\phi}] + \frac{1}{c^2} \frac{\partial}{\partial t} [\phi + \frac{\partial \tilde{\phi}}{\partial t}] = 0$ 

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$$\Rightarrow \nabla \cdot [\overline{\phi}^{2}(\vec{\mathbf{A}} - \nabla \tilde{\phi})] + \frac{1}{\mathbf{c}^{2}} \frac{\partial}{\partial \mathbf{t}} [\overline{\phi}^{2}(\phi + \frac{\partial \tilde{\phi}}{\partial \mathbf{t}})] = \mathbf{0}$$

 $\begin{array}{c} \stackrel{\text{\tiny{$\dag$}$}}{\Longrightarrow} (\nabla^2 - \frac{1}{\mathbf{c}^2} \frac{\partial^2}{\partial \mathbf{t}^2}) \bar{\phi} + 2 \bar{\phi} \mathbf{V}'(\bar{\phi}^2) - \bar{\phi} \big[ (\vec{\mathbf{A}} - \nabla \tilde{\phi}) \cdot (\vec{\mathbf{A}} - \nabla \tilde{\phi}) - (\phi + \frac{\partial \phi}{\partial \mathbf{t}})^2 \big] = 0 \\ \text{\tiny{$\dag$}$} \text$ 

游杰,琴电游: $ec{\mathbf{A}} o - rac{\mathbf{e}^*}{\mathbf{c}\hbar} ec{\mathbf{A}}$   $\Phi = \sqrt{rac{\mu_0 \mathbf{e}^* \hbar}{\mathbf{m}^*}} \Psi \qquad \qquad \mathbf{V}'(\mid \Phi \mid^2) = - rac{2\mathbf{m}^*}{\hbar^2} \left[ \alpha + rac{\beta \mathbf{m}^*}{\mu_0 \mathbf{e}^* \hbar} \mid \Phi \mid^2 \right]$ 

$$(
abla - rac{\mathbf{e}^*}{\mathbf{c}\hbar}\mathbf{i}ec{\mathbf{A}}) \cdot (
abla - rac{\mathbf{e}^*}{\mathbf{c}\hbar}\mathbf{i}ec{\mathbf{A}})\Psi - rac{2\mathbf{m}^*}{\hbar^2}[lpha + eta \mid \Psi \mid^2]\Psi = \mathbf{0} \Leftarrow Ginzberg-Landau \pi \#!$$

brikosov 因在CL方程中求出涡旋解并应用于超导研究而获得03年诺贝尔物理奖, 16年诺贝尔物理奖则与正反涡旋对有关!

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into the form  $\nabla_{\mu}\varphi_{1} = \partial_{\mu}\varphi_{1} - eA_{\mu}\varphi_{2}, \qquad \qquad \partial_{\mu}B^{\mu} = 0, \quad \partial_{\nu}G^{\mu\nu} + e^{2}\varphi_{0}^{2}B^{\mu} = 0. \tag{4}$  Equation (4) describes vector waves whose quanta have (bare) mass  $e\varphi_{0}$ . In the absence of the gauge field coupling (e=0) the situation is quite different: Equations (2a) and (2c) describe zero-mass

scalar and vector bosons, respectively. In pass-

ing, we note that the right-hand side of (2c) is

 $F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$ 

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# $\partial_{\mu}\partial^{\mu}A^{\nu} - \partial^{\nu}\partial_{\mu}A^{\mu} = \mu_{0}j^{\nu}$

$$\partial^{\mu}\partial_{\mu}\tilde{\phi} \equiv \partial^{\mu}A_{\mu} = = = = = = = \Rightarrow \qquad \tilde{\phi} \rightarrow \tilde{\phi}' = \tilde{\phi} - \chi$$

$$\mathcal{L}_{\text{Goldstone}} = \frac{1}{2}(\partial^{\mu}\tilde{\phi})(\partial_{\mu}\tilde{\phi}) + \tilde{\phi}\partial^{\mu}A_{\mu}$$

 $\mathcal{L}_{\text{Electrodynamics}} = -\frac{1}{4} (\partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu}) (\partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}) - \mu_{0} j^{\mu} A_{\mu}$ 

$$j^{\mu} = -\frac{m^2 c^2}{\mu_0 \hbar^2} [A^{\mu} - \partial^{\mu} \tilde{\phi}]$$

 $f = \frac{mc}{c\hbar}$   $U \equiv e^{i\tilde{\phi}}$   $D_{\mu}U \equiv (\partial_{\mu} - iA_{\mu})U$ 

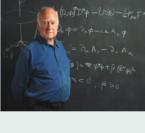
$$\mu_0\hbar^2$$
  $\mu_0\hbar^2$   $\mu_0\mu_0$ 

$$\mu_0\hbar^2 = -\frac{1}{4}(\partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu})(\partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}) + \underbrace{\frac{m^2c^2}{2\hbar^2}[A^{\mu} - (\partial^{\mu}\tilde{\phi})][A_{\mu} - (\partial_{\mu}\tilde{\phi})]}_{\text{交叉项: 希格斯机制}}$$

$$\mathcal{L}_{\text{Stueckelberg}} = -\frac{1}{4} (\partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu}) (\partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}) + \underbrace{\frac{m^{2} c^{2}}{2\hbar^{2}} [A^{\mu} - (\partial^{\mu} \tilde{\phi})] [A_{\mu} - (\partial_{\mu} \tilde{\phi})]}_{\text{ZZT}: \hat{\pi} \text{Khhhh}}$$

$$\mathcal{L}_{\text{Chiral Lagrangian}} = -\frac{1}{4} (\partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu}) (\partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}) + \frac{f^{2}}{2} (D^{\mu} U)^{*} (D_{\mu} U)$$

手征拉氏量;非线性 $\sigma$ 模型



 $f = \frac{mc}{e\hbar} \equiv \langle \bar{\phi} \rangle \qquad fU = \langle \bar{\phi} \rangle e^{i\tilde{\phi}} \Rightarrow \bar{\phi}e^{i\tilde{\phi}} \equiv \Phi$ 物理理论中的这种扩充或推广比比皆是!

 $\mathcal{L}_{\text{Abel Higgs}} = -\frac{1}{4} (\partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu})(\partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}) + \frac{1}{2} (D^{\mu} \Phi)^* (D_{\mu} \Phi) - V(\Phi^* \Phi)$ 

当年希格斯的诺奖工作模型,当年他用实部虚部(线性实现)进行讨论

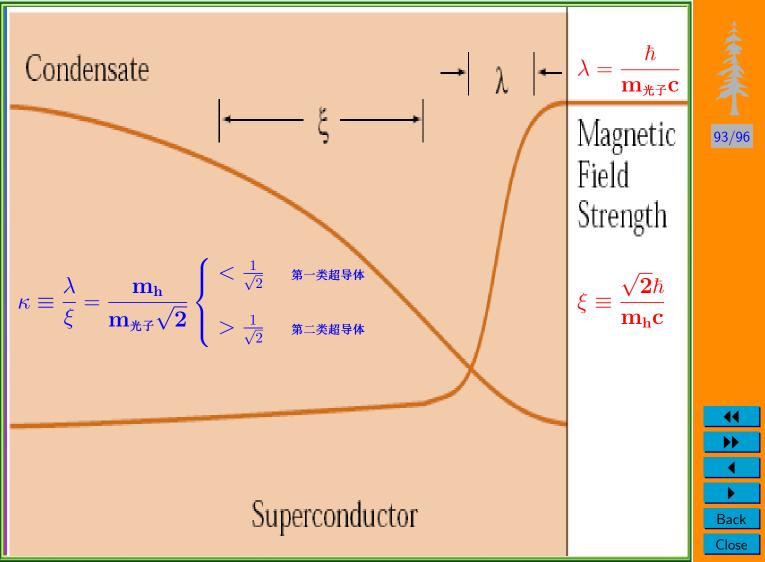
光子场求极值:  $\partial_{\mu}\partial^{\mu}A^{\nu} - \partial^{\nu}\partial_{\mu}A^{\mu} - \bar{\phi}^2[A^{\nu} - (\partial^{\nu}\tilde{\phi})] = 0$   $j^{\mu} = -\frac{\phi^2}{\mu}[A^{\mu} - \partial^{\mu}\tilde{\phi}]$ 

Goldstone场求极值:  $\partial^{\mu}[\bar{\phi}(A_{\mu}-\partial_{\mu}\tilde{\phi}]=0 \quad \Rightarrow \quad \partial^{\mu}\partial_{\mu}\tilde{\phi}=\partial^{\mu}A_{\mu}+\frac{\mu_{0}}{3}(\partial^{\mu}\frac{1}{\bar{\phi}^{2}})j_{\mu}$ 

Higgs场求极值:  $\partial^{\mu}\partial_{\mu}\bar{\phi} - 2\bar{\phi}V'(\bar{\phi}^2) - \bar{\phi}(D^{\mu}U)^*(D_{\mu}U) = 0$   $m_h^2 = 4f^2V''(f^2)$ 

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当年Higgs的结果: Equation (2b) describes wave whose quanta have (bare) mass  $2\phi_0\{V^*(\phi_0^2)\}^{\frac{1}{2}} \Leftarrow$  南部的提醒?



### Light-induced collective pseudospin precession resonating with Higgs mode in a superconductor

Ryusuke Matsunaga, 1\* Naoto Tsuji, 1 Hiroyuki Fujita, 1 Arata Sugioka, 1 Kazumasa Makise, 2 Yoshinori Uzawa, 3+ Hirotaka Terai, 2 Zhen Wang, 2+ Hideo Aoki, 1,4 Ryo Shimano 1,5 \*

Superconductors host collective modes that can be manipulated with light. We show that a strong terahertz light field can induce oscillations of the superconducting order parameter in NbN with twice the frequency of the terahertz field. The result can be captured as a collective precession of Anderson's pseudospins in ac driving fields. A resonance between the field and the Higgs amplitude mode of the superconductor then results in large terahertz third-harmonic generation. The method we present here paves a way toward nonlinear quantum optics in superconductors with driving the pseudospins collectively and can be potentially extended to exotic superconductors for shedding light on the character of order parameters and their coupling to other degrees of freedom.

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One manifestation of the macroscopic quantum

nature is the appearance of characteristic collective excitations. Indeed, phenomena associated with collective modes, such as second sound and spin waves in condensates, have been revealed in superfluid helium (1, 2) and in ultracold atomic gases (3, 4).

#### SUPERCONDUCTIVITY

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