经典电磁学 基本理论 构架

清华大学物理系

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理解经典电磁学理论

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摘 要 本文先回顾和评述传统对经典麦克斯韦方程组及洛伦兹力公式通过实验定律和作用量的两种理解方式，再给出一种以四度电磁势 A_μ 及规范对称为基础的另类理解方式。

关键词 麦克斯韦方程组；洛伦兹力；四度电磁势

UNDERSTANDING CLASSICAL ELECTROMAGNETIC THEORY

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Abstract Based on the review and comment on two kinds traditional understandings by experiment laws and actions for the classical Maxwell equations and Lorentz force formula, we present an alternative understanding in terms of four-vector electromagnetic potential A_μ and gauge invariance.

Key words Maxwell equations; Lorentz force; four-vector electromagnetic potential

经典电磁学理论的核心是麦克斯韦方程组和洛伦兹力公式。前者确定了电场强度和磁感应强度对给定的电荷电流源密度分布的依赖方程组(或者说已知电荷电流源的分布决定了电磁场)，后者则对给定的电场强度和磁感应强度给出了电荷电流源所受的电磁力(已知电磁场的分布决定了源的受力)，若再辅以牛顿第二定律并加上源所受的其他可能的非电磁力，则可以写出电荷电流源的运动方程(或者说已知电磁场决定了电荷电流源的运动)，场方程和运动方程两者结合起来就确定了一个电磁系统的完整运动规律。本文只限于讨论真空中的电荷电流源及电磁场，如若考虑介质材料，须要在上面所讨论的电荷电流源中加入由于介质的极化和磁化导致的极化磁化电荷电流的贡献。

在学习、研究和应用电磁学理论时我们总希望能尽量深入地理解它，特别是能有直观图像的理解，麦克斯韦方程组和洛伦兹力公式为什么会是这个样子？一定是需要目前电磁学理论所给出的这种结构而不可能有所变化吗？这个问题在一般的教科书和研究论文中进行深入论述的比较少，本文就此作一讨论。

谈论理解一个事物总要依据一些出发点，理解麦克斯韦方程组和洛伦兹力公式同样需要有出发点，教科书里一般有两类理解方式：一是从实验定律出发，另一是从作用量出发。对这两种做法本文将分别在第1和第2节作一系统回顾和评述，在第3节给出一种自认为更优的新的理解方法，并进行讨论。著名物理学家费曼曾经说过"从一个新角度看待旧问题是很有意思的"。如果觉得本文前面的基本内容过于基本和简单，读者可以直接从2.3节的对作用量做法的评述开始阅读。

1 从实验定律推导经典电磁学理论

通常的静态和稳恒情形的麦克斯韦方程组和洛伦兹力公式可以从实验给出的电荷源之间相互作用力的库仑定律、电流源之间相互作用力的毕奥-萨伐尔定律出发，将电场强度和磁感应强度分别定义为单位电量的电荷电流源所受的电力和磁力得到。具体地，若一团由电荷密度 $\rho_1(r)$ 电流密

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章节 ...力学 5.6 相对论电动力学 5.7 磁单极-规范不变性-Witten效应 5.8 第五章作业 5.6 相对论电动力学...

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（2-1）电荷的理解
创建者王青 回答1 查看6 2020-02-05

（2-2）电荷密度、电流
创建者王青 回答0 查看5 2020-02-05

（2-3）电荷与场
创建者王青 回答0 查看4 2020-02-05

（2-4）平方反比律
创建者王青 回答0 查看3 2020-02-05

（2-5）叠加原理
创建者王青 回答2 查看7 2020-02-05

（2-6）极矢量、轴矢量与电磁对偶
创建者王青 回答1 查看5 2020-02-05

（2-7）磁单极与电磁对偶
创建者王青 回答2 查看13 2020-02-05

- 从实验定律出发构建经典电磁学理论 6-32
- 从作用量出发构建协变的经典电磁学理论 33-70
- 经典电磁学理论的另类理解及扩展 71-94

Back
Close

一. 从实验定律出发

构建经典电磁学理论

电荷是电相互作用之源. 物质的电相互作用强度与其携带的电荷成正比. 电荷可正可负, 也可为零(即不带电). 电荷的起源? 摩擦生电

- 点电荷: 无体积, 带电量, 是基本电量单位的整数倍 $(1.6021917733(49) \times 10^{-19}$库伦) 分立性在宏观物理中可忽略。
- 电荷密度: 电荷分布在空间体积中.

$$\rho = \lim_{\Delta\tau \to 0} \frac{\Delta \mathbf{q}}{\Delta \tau} = \text{场点处单位体积的电量}$$

$$\sigma = \lim_{\Delta \mathbf{S} \to 0} \frac{\Delta \mathbf{q}}{\Delta \mathbf{S}} = \text{场点处单位面积的电量}$$

$$\eta = \lim_{\Delta \mathbf{l} \to 0} \frac{\Delta \mathbf{q}}{\Delta \mathbf{l}} = \text{场点处单位长度的电量}$$

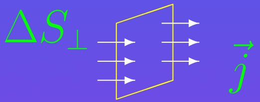
- 点电荷的电荷密度: 位于$\vec{\mathbf{r}}_0$处电量为$\mathbf{q}$ 的点电荷在空间的电荷密度分布为$\rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}_0)$

$$\int_\tau \mathbf{d}\tau\ \rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}_0) = \mathbf{q} \ \rightarrow \rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}_0) = \begin{cases} 0 & \vec{r} \neq \vec{r}_0 \\ \infty & \vec{r} = \vec{r}_0 \end{cases} = \mathbf{q}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}_0)$$

运动电荷形成电流,有方向和电量大小。

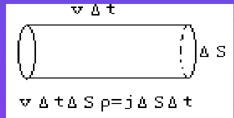
电流密度矢量$\vec{\mathrm{j}}$的大小为单位时间流过垂直于电流方向单位面积的电量，方向为电流方向。

对任意一个不一定垂直于电流方向的面元$\Delta\vec{\mathrm{S}}$，单位时间通过的电量为$\mathrm{j}\Delta \mathrm{S}_\perp = \vec{\mathrm{j}}\cdot\Delta\vec{\mathrm{S}}$. 对有限大小面元,单位时间流过的电量叫电流强度



$$\mathbf{J} = \int_{\mathrm{S}} \mathrm{d}\vec{\mathbf{S}}\cdot\vec{\mathbf{j}}$$

以速度$\vec{\mathrm{v}}$运动的电荷具有电流密度 $\vec{\mathbf{j}} = \rho\vec{\mathbf{v}}$



电流密度用图画出为电流线,它方向为$\vec{\mathrm{j}}$方向,
密度为$\vec{\mathrm{j}}$大小。在电流线图上沿$\vec{\mathrm{j}}$方向作一小电流管:

$$\mathbf{J}\mathrm{dl} = \mathrm{j}\Delta\mathrm{S}\mathrm{d}\vec{\mathrm{l}} = \vec{\mathrm{j}}\mathrm{d}\tau = \rho\vec{\mathrm{v}}\mathrm{d}\tau \overset{\text{多种速度}}{=\!=\!=\!=} \sum_{\mathrm{i}}\rho_{\mathrm{i}}\vec{\mathrm{v}}_{\mathrm{i}}\mathrm{d}\tau = \sum_{\mathrm{i}}\mathrm{q}_{\mathrm{i}}\vec{\mathrm{v}}_{\mathrm{i}}$$

$$\rho_- = -\rho_+ \to \rho = \rho_- + \rho_+ = 0 \qquad \vec{\mathbf{j}} = \rho_+\vec{\mathbf{v}}_+ + \rho_-\vec{\mathbf{v}}_- = \rho_+(\vec{\mathbf{v}}_+ - \vec{\mathbf{v}}_-)$$

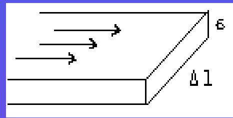
Back
Close

电磁相互作用的源: 电流,电流密度; 磁单极

电流分布在表面上用面电流密度$\vec{i}$,方向为电流方向,大小为 单位时间流过垂直于电流方向单位长度的电量。 对任一线元$\Delta\vec{l}$,单位 时间通过的电量$\vec{i}\cdot\Delta\vec{l}$。

对以速度$\vec{v}_i$运动的面电荷 σ_i, $\vec{i}=\sum_i \sigma_i\vec{v}_i$ 面电流密度 与体电流密度有关, $\vec{j}=\frac{\vec{i}}{\delta}$,

$$J d\vec{l} = i\Delta l d\vec{l} = \vec{i} dS = \sum_i \sigma_i \vec{v}_i dS = \sum_i q_i \vec{v}_i$$



磁单极:

磁相互作用产生源的认识历史:

- 磁铁 $\Rightarrow$ 磁荷(磁单极) Q_m; 磁荷密度ρ_m; 磁流密度$\vec{j}_m$
- 磁荷之间遵从类似于库仑定律的平方反比率
- 电流也可以产生磁作用
- 安培提出分子电流自旋假说$\Rightarrow$磁相互作用由电流运动电荷产生
- 还有没有磁单极?Dirac Strings and Magnetic Monopoles in Spin Ice $Dy_2Ti_2O_7$, Science.1178868,2009.9.3

电荷守恒定律:

电荷不会自动产生或湮灭,除非碰上其它电荷

$$\oint_\tau \mathrm{d}\vec{\mathrm{S}}\cdot\vec{\mathrm{j}} = -\frac{\mathrm{d}}{\mathrm{dt}}\int_\tau \mathrm{d}\tau\rho$$

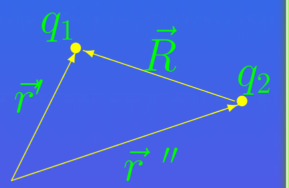
对一个任意体积τ,若在某一段时间内其电量减少,则减少的电量一定是 从此体积的表面S流出(或流进等量的负电荷)。

叠加原理:

电磁相互作用力具有可迭加性质

$$\vec{\mathrm{f}} = \sum_{\mathrm{i}} \vec{\mathrm{f}}_{\mathrm{i}}$$

库仑定律:



对静止点电荷,同号相斥,异号相吸,相互作用力沿电荷连线方向,大小正比于电荷 的电量,反比于电荷之间距离平方

$$\vec{f}_{21} = k_1 \frac{q_1 q_2}{R^3} \vec{R} \qquad k_1 = \frac{1}{4\pi\epsilon_0} \qquad \epsilon_0 = 8.854 \times 10^{-12} \text{库伦}^2 / \text{牛顿} \cdot \text{米}^2$$

$\vec{f}_{21}$是q_2对q_1的作用力, q_1对q_2的作用力为$\vec{f}_{12} = -\vec{f}_{21}$, $\vec{R}$是从q_2指向q_1的矢量。

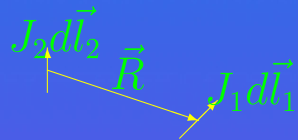
♣平方反比? 两来源 $3 \to 3 + \epsilon \quad \epsilon < 3 \times 10^{-16}$ ♠方向?对称性 ¶$\vec{R} = 0$?

对现实世界存在的两块带电体,τ_2对τ_1的作用力$\vec{F}_{21}$

$$\vec{F}_{21} = \int d\vec{f}_{21} = \int_{\tau_1} \int_{\tau_2} \frac{k_1 \rho_1(\vec{r}') d\tau' \rho_2(\vec{r}'') d\tau''}{R^3} \vec{R} \qquad \vec{R} = \vec{r}' - \vec{r}''$$

$$\vec{f}_{21} = \int_{\tau_1} d\tau' \int_{\tau_2} d\tau'' \frac{k_1 q_1 \delta(\vec{r}' - \vec{r}_1) q_2 \delta(\vec{r}'' - \vec{r}_2)}{|\vec{r}' - \vec{r}''|^3} (\vec{r}' - \vec{r}'') = \frac{k_1 q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3} (\vec{r}_1 - \vec{r}_2)$$

电流元的相互作用：



恒定电流元$\mathbf{J_2 d\vec{l}_2}$对$\mathbf{J_1 d\vec{l}_1}$的作用力为

$$\mathbf{d^2\vec{F}_{21} = k_2 J_1 d\vec{l}_1 \times \left(\frac{J_2 d\vec{l}_2 \times \vec{R}}{R^3}\right) = \frac{k_2 J_1 J_2}{R^3}[(\vec{R}\cdot d\vec{l}_1)d\vec{l}_2 - (d\vec{l}_1 \cdot d\vec{l}_2)\vec{R}]}$$

电流元$\mathbf{J_1 d\vec{l}_1}$对 $\mathbf{J_2 d\vec{l}_2}$的作用力为

$$\mathbf{d^2\vec{F}_{12} = \frac{k_2 J_1 J_2}{R^3}[-(\vec{R}\cdot d\vec{l}_2)d\vec{l}_1 + (d\vec{l}_1 \cdot d\vec{l}_2)\vec{R}]}$$ 极矢量

其中： 安培最早的工作测量的不是电流元，而是电流圈之间的相互作用力！

$$\mathbf{\vec{R} = \vec{r}_1 - \vec{r}_2} \qquad \mathbf{k_2 = \frac{\mu_0}{4\pi}} \qquad \mu_0 = 4\pi \times 10^{-7}\text{公斤}\cdot\text{米}/\text{库伦}^2$$

两电流元之间的作用力与反作用力不一样,$\mathbf{\vec{F}_1 + \vec{F}_2 \neq 0}$
(例:两运动点电荷$\mathbf{\vec{v}_2 \parallel \vec{R} \perp \vec{v}_1}$) $\rightarrow \mathbf{\frac{d\vec{P}_1}{dt} + \frac{d\vec{P}_2}{dt} \neq 0}$或 $\mathbf{\vec{P}_1 + \vec{P}_2 \neq 0}$,
动量守恒不再成立。

场：

场：

两个电流元的相互作用动量守恒不成立的现象启示两个电流元构成的 体系并不是一个封闭体系,还有其它携带动量的物质实在。

场的概念最早是法拉第提出的： 场是局域的；弥漫于整个空间。源产生场，场反作用于源。场是传递源之间相互作用的载体。

$$\vec{F}_{21}=\int_{\tau_1}d\tau'\ \Big[\ \rho_1(\vec{r}')\int_{\tau_2}d\tau''\frac{k_1\rho_2(\vec{r}'')}{R^3}\vec{R}\ +\ \vec{j}_1(\vec{r}')\times\int_{\tau_2}d\tau''\frac{k_2\vec{j}_2(\vec{r}'')\times\vec{R}}{R^3}\ \Big]$$

$$=\int_{\tau_1}d\tau'\ [\ \rho_1(\vec{r}')\vec{E}_1(\vec{r}')\ +\ \vec{j}_1(\vec{r}')\times\vec{B}_1(\vec{r}')\ ]\qquad \vec{R}=\vec{r}'-\vec{r}''$$

$$\vec{E}_1(\vec{r}')=k_1\int d\tau''\frac{\rho_2(\vec{r}'')}{R^3}\vec{R}\qquad \vec{B}_1(\vec{r}')=k_2\int d\tau''\frac{\vec{j}_2(\vec{r}'')}{R^3}\times\vec{R}$$

库伦定律和安培比萨定律中源受力的公式分解为 源产生场公式和源在场中受力公式的结合！

电磁力之间的迭加原理即转化为场的迭加原理： • 场方程是线性的！

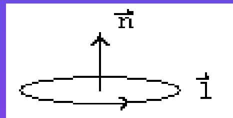
如此引进的场在一开始没有场受力的概念，只有源产生场，场作用源的概念

$$\vec{F}_{11}\equiv\int d\tau[\rho_1(\vec{r})\vec{E}_s(\vec{r})+\vec{j}_1(\vec{r})\times\vec{B}_s(\vec{r})]=\int_\tau\int_\tau d\tau'd\tau''[k_1\rho_1(\vec{r}')\rho_1(\vec{r}'')+k_2\vec{j}_1(\vec{r}')\times\vec{j}_1(\vec{r}'')\times]\frac{(\vec{r}'-\vec{r}'')}{|\vec{r}'-\vec{r}''|^3}=0$$

两粒子交换　　闭合回路

电场由电荷产生,磁场由电流即运动的电荷产生,因而电场与磁场的关系一定与运动即时间的变化有关.

一根闭合导线所包围的磁通的改变,将在此闭合导线上产生感应电动势,其数值 等于单位时间内磁通的变化率,由此产生的感应电动势所决定的感应电流方向 阻止导线内磁通的变化.



$$\varepsilon = -\frac{d\Phi}{dt} \qquad \varepsilon = \oint_S d\vec{l}\cdot\vec{E} \qquad \Phi = \int_S d\vec{S}\cdot\vec{B}$$

$\vec{S}$方向与$\vec{l}$方向成右手螺旋关系。

此定律早期在很低频电磁场中建立，后来发现居然对很高频的电磁场仍然成立！

真空中电磁相互作用的场方程： 麦克斯韦方程组的积分形式

电荷守恒: $$\oint_\tau \mathrm{d}\vec{\mathrm{S}} \cdot \vec{\mathrm{j}} = -\int_\tau \mathrm{d}\tau \frac{\partial \rho}{\partial \mathrm{t}}$$

电荷守恒: $\oint_\tau \mathrm{d}\vec{\mathrm{S}}\cdot\vec{\mathrm{j}} = -\int_\tau \mathrm{d}\tau\frac{\partial\rho}{\partial \mathrm{t}}$

库伦定律:

$$\vec{\mathrm{E}} = \mathrm{k}_1\int \mathrm{d}\tau'\frac{\rho(\vec{\mathrm{r}}')}{\mathrm{R}^3}\vec{\mathrm{R}}$$

真空中电磁相互作用的场方程: 麦克斯韦方程组的积分形式

电荷守恒: $\oint_\tau d\vec{S}\cdot\vec{j} = -\int_\tau d\tau\frac{\partial\rho}{\partial t}$

库伦定律:

$$\vec{E} = k_1\int d\tau'\frac{\rho(\vec{r}')}{R^3}\vec{R} = -k_1\int d\tau'\rho(\vec{r}')\nabla\frac{1}{R}$$

真空中电磁相互作用的场方程：麦克斯韦方程组的积分形式

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电荷守恒： $\oint_{\tau} \mathrm{d}\vec{\mathbf{S}} \cdot \vec{\mathbf{j}} = -\int_{\tau} \mathrm{d}\tau \frac{\partial \rho}{\partial \mathbf{t}}$ 库伦定律：

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$$\phi(\vec{\mathbf{r}}) = \mathbf{k}_1 \int \mathrm{d}\tau' \frac{\rho(\vec{\mathbf{r}}')}{\mathbf{R}} + \text{常数}$$

真空中电磁相互作用的场方程：麦克斯韦方程组的积分形式

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$$\phi(\vec{r}) = k_1\int d\tau'\frac{\rho(\vec{r}')}{R} + 常数 \qquad \vec{E}(\vec{r}) = -\nabla\phi(\vec{r})$$

Back
Close

真空中电磁相互作用的场方程：麦克斯韦方程组的积分形式

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$$\oint_{\mathbf{S}} \mathbf{d\vec{l}}\cdot\vec{\mathbf{E}} = \int_{\mathbf{S}} \mathbf{d\vec{S}}\cdot(\nabla\times\vec{\mathbf{E}})$$

Back
Close

真空中电磁相互作用的场方程: 麦克斯韦方程组的积分形式

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$$\oint_S d\vec{l}\cdot\vec{E} = \int_S d\vec{S}\cdot(\nabla\times\vec{E}) = -\int_S d\vec{S}\cdot[\nabla\times(\nabla\phi)]$$

Back

Close

真空中电磁相互作用的场方程: 麦克斯韦方程组的积分形式

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$$\oint_\tau d\vec{S}\cdot\vec{E} = \int_\tau d\tau\nabla\cdot\vec{E}$$

Back

Close

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$$= -\int_\tau \mathrm{d}\tau\int \mathrm{d}\tau'\mathrm{k}_1\rho(\vec{\mathrm{r}}')\nabla^2\frac{1}{\mathrm{R}}$$

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$$= -\int_\tau d\tau\int d\tau' k_1\rho(\vec{r}')\nabla^2\frac{1}{R} = -\int_\tau d\tau\int d\tau' k_1\rho(\vec{r}')(-4\pi)\delta(\vec{r}-\vec{r}')$$

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$$= \frac{1}{\epsilon_0}\int_\tau d\tau\rho(\vec{r})$$

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$$= -\int_\tau d\tau\int d\tau' k_1\rho(\vec{r}')\nabla^2\frac{1}{R} = -\int_\tau d\tau\int d\tau' k_1\rho(\vec{r}')(-4\pi)\delta(\vec{r}-\vec{r}')$$

$$= \frac{1}{\epsilon_0}\int_\tau d\tau\rho(\vec{r}) = \frac{1}{\epsilon_0}Q_{内}$$

Back

Close

比萨定律：$\vec{B} = \int \frac{k_2 J(\vec{r}')d\vec{l}' \times \vec{R}}{R^3}$

比萨定律： $$\vec{B} = \int \frac{k_2 J(\vec{r}')d\vec{l}' \times \vec{R}}{R^3} = -\int k_2 J(\vec{r}')d\vec{l}' \times \nabla \frac{1}{R}$$

真空中电磁相互作用的场方程：麦克斯韦方程组的积分形式

比萨定律：
$$\vec{B} = \int \frac{k_2 J(\vec{r}')d\vec{l}' \times \vec{R}}{R^3} = -\int k_2 J(\vec{r}')d\vec{l}' \times \nabla \frac{1}{R}$$
$$= \nabla \times \int \frac{k_2 J(\vec{r}')d\vec{l}'}{R}$$

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$$\vec{B} = \int \frac{k_2 J(\vec{r}')d\vec{l}' \times \vec{R}}{R^3} = -\int k_2 J(\vec{r}')d\vec{l}' \times \nabla \frac{1}{R}$$
$$= \nabla \times \int \frac{k_2 J(\vec{r}')d\vec{l}'}{R}$$

$$\vec{A}(\vec{r}) = k_2 \int d\vec{l} \frac{J(\vec{r}')}{R}$$

Back
Close

比萨定律：
$$\vec{B} = \int \frac{k_2 J(\vec{r}')d\vec{l}' \times \vec{R}}{R^3} = -\int k_2 J(\vec{r}')d\vec{l}' \times \nabla\frac{1}{R}$$
$$= \nabla \times \int \frac{k_2 J(\vec{r}')d\vec{l}'}{R}$$

$$\vec{A}(\vec{r}) = k_2 \int d\vec{l}\frac{J(\vec{r}')}{R} + \nabla\chi$$

Back
Close

真空中电磁相互作用的场方程： 麦克斯韦方程组的积分形式

比萨定律： $$\vec{B} = \int \frac{k_2 J(\vec{r}')d\vec{l}' \times \vec{R}}{R^3} = -\int k_2 J(\vec{r}')d\vec{l}' \times \nabla\frac{1}{R}$$

$$= \nabla \times \int \frac{k_2 J(\vec{r}')d\vec{l}'}{R}$$

$$\vec{A}(\vec{r}) = k_2 \int d\vec{l}\frac{J(\vec{r}')}{R} + \nabla\chi = \int d\tau'\frac{\vec{j}(\vec{r}')}{R} + \nabla\chi$$ χ的意义以后讨论！

真空中电磁相互作用的场方程：麦克斯韦方程组的积分形式

比萨定律：
$$\vec{B} = \int \frac{k_2 J(\vec{r}')d\vec{l}' \times \vec{R}}{R^3} = -\int k_2 J(\vec{r}')d\vec{l}' \times \nabla\frac{1}{R}$$
$$= \nabla \times \int \frac{k_2 J(\vec{r}')d\vec{l}'}{R}$$

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$$\vec{B}(\vec{r}) = \nabla \times \vec{A}(\vec{r})$$

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$$\vec{B}(\vec{r}) = \nabla \times \vec{A}(\vec{r})$$

$$\oint_\tau d\vec{S}\cdot\vec{B} = \int_\tau d\tau\nabla\cdot\vec{B} = \int_\tau d\tau\nabla\cdot(\nabla\times\vec{A}) = 0$$

$$\oint_S d\vec{l} \cdot \vec{B}$$

真空中电磁相互作用的场方程： 麦克斯韦方程组的积分形式

$$\oint_S d\vec{l} \cdot \vec{B} = \int_S d\vec{S} \cdot (\nabla \times \vec{B})$$

真空中电磁相互作用的场方程: 麦克斯韦方程组的积分形式

$$\oint_S d\vec{l} \cdot \vec{B} = \int_S d\vec{S} \cdot (\nabla \times \vec{B}) = -\int_S d\vec{S} \cdot \nabla \times \left[\int d\vec{l}' \times k_2 J(\vec{r}') \nabla \frac{1}{R}\right]$$

$$\oint_S d\vec{l}\cdot\vec{B} = \int_S d\vec{S}\cdot(\nabla\times\vec{B}) = -\int_S d\vec{S}\cdot\nabla\times[\int d\vec{l}'\times k_2 J(\vec{r}')\nabla\frac{1}{R}]$$

$$= -\int_S d\vec{S}\cdot\int_\infty d\tau' k_2\nabla\times[\vec{j}(\vec{r}')\times\nabla\frac{1}{R}]$$

真空中电磁相互作用的场方程: 麦克斯韦方程组的积分形式

$$
\begin{aligned}
\oint_S d\vec{l}\cdot\vec{B} &= \int_S d\vec{S}\cdot(\nabla\times\vec{B}) = -\int_S d\vec{S}\cdot\nabla\times\left[\int d\vec{l}'\times k_2 J(\vec{r}')\nabla\frac{1}{R}\right] \\
&= -\int_S d\vec{S}\cdot\int_\infty d\tau' k_2\nabla\times\left[\vec{j}(\vec{r}')\times\nabla\frac{1}{R}\right] \\
&= k_2\int_S d\vec{S}\cdot\int_\infty d\tau'\left[\vec{j}(\vec{r}')\cdot\nabla\nabla-\vec{j}(\vec{r}')\nabla^2\right]\frac{1}{R}
\end{aligned}
$$

真空中电磁相互作用的场方程：麦克斯韦方程组的积分形式

$$\oint_S d\vec{l}\cdot\vec{B} = \int_S d\vec{S}\cdot(\nabla\times\vec{B}) = -\int_S d\vec{S}\cdot\nabla\times[\int d\vec{l}'\times k_2 J(\vec{r}')\nabla\frac{1}{R}]$$

$$= -\int_S d\vec{S}\cdot\int_\infty d\tau' k_2\nabla\times[\vec{j}(\vec{r}')\times\nabla\frac{1}{R}]$$

$$= k_2\int_S d\vec{S}\cdot\int_\infty d\tau'[\vec{j}(\vec{r}')\cdot\nabla\nabla-\vec{j}(\vec{r}')\nabla^2]\frac{1}{R}$$

$$\int_\infty d\tau'\vec{j}(\vec{r}')\cdot\nabla\nabla\frac{1}{R}$$

真空中电磁相互作用的场方程：麦克斯韦方程组的积分形式

$$
\begin{aligned}
\oint_S d\vec{l}\cdot\vec{B} &= \int_S d\vec{S}\cdot(\nabla\times\vec{B}) = -\int_S d\vec{S}\cdot\nabla\times\left[\int d\vec{l}'\times k_2 J(\vec{r}')\nabla\frac{1}{R}\right] \\
&= -\int_S d\vec{S}\cdot\int_\infty d\tau' k_2\nabla\times\left[\vec{j}(\vec{r}')\times\nabla\frac{1}{R}\right] \\
&= k_2\int_S d\vec{S}\cdot\int_\infty d\tau'\left[\vec{j}(\vec{r}')\cdot\nabla\nabla-\vec{j}(\vec{r}')\nabla^2\right]\frac{1}{R}
\end{aligned}
$$

$$
\int_\infty d\tau'\vec{j}(\vec{r}')\cdot\nabla\nabla\frac{1}{R} = -\int_\infty d\tau'\vec{j}(\vec{r}')\cdot\nabla'\nabla\frac{1}{R}
$$

真空中电磁相互作用的场方程: 麦克斯韦方程组的积分形式

$$\oint_S \mathrm{d}\vec{l}\cdot\vec{B} = \int_S \mathrm{d}\vec{S}\cdot(\nabla\times\vec{B}) = -\int_S \mathrm{d}\vec{S}\cdot\nabla\times[\int \mathrm{d}\vec{l}'\times k_2 J(\vec{r}')\nabla\frac{1}{R}]$$

$$= -\int_S \mathrm{d}\vec{S}\cdot\int_\infty \mathrm{d}\tau' k_2\nabla\times[\vec{j}(\vec{r}')\times\nabla\frac{1}{R}]$$

$$= k_2\int_S \mathrm{d}\vec{S}\cdot\int_\infty \mathrm{d}\tau'[\vec{j}(\vec{r}')\cdot\nabla\nabla-\vec{j}(\vec{r}')\nabla^2]\frac{1}{R}$$

$$\int_\infty \mathrm{d}\tau'\vec{j}(\vec{r}')\cdot\nabla\nabla\frac{1}{R} = -\int_\infty \mathrm{d}\tau'\vec{j}(\vec{r}')\cdot\nabla'\nabla\frac{1}{R}$$

$$= \int_\infty \mathrm{d}\tau'[\nabla'\cdot[-\vec{j}(\vec{r}')\nabla\frac{1}{R}]+[\nabla'\cdot\vec{j}(\vec{r}')]\nabla\frac{1}{R}]$$

$$\oint_S d\vec{l}\cdot\vec{B} = \int_S d\vec{S}\cdot(\nabla\times\vec{B}) = -\int_S d\vec{S}\cdot\nabla\times[\int d\vec{l}'\times k_2 J(\vec{r}')\nabla\frac{1}{R}]$$
$$= -\int_S d\vec{S}\cdot\int_\infty d\tau' k_2\nabla\times[\vec{j}(\vec{r}')\times\nabla\frac{1}{R}]$$
$$= k_2\int_S d\vec{S}\cdot\int_\infty d\tau'[\vec{j}(\vec{r}')\cdot\nabla\nabla - \vec{j}(\vec{r}')\nabla^2]\frac{1}{R}$$

$$\int_\infty d\tau'\vec{j}(\vec{r}')\cdot\nabla\nabla\frac{1}{R} = -\int_\infty d\tau'\vec{j}(\vec{r}')\cdot\nabla'\nabla\frac{1}{R}$$
$$= \int_\infty d\tau'[\nabla'\cdot[-\vec{j}(\vec{r}')\nabla\frac{1}{R}] + [\nabla'\cdot\vec{j}(\vec{r}')]\nabla\frac{1}{R}]$$
$$= -\int_\infty d\vec{S}'\cdot\vec{j}(\vec{r}')\nabla\frac{1}{R} + \int_\infty d\tau'[\nabla'\cdot\vec{j}(\vec{r}')]\nabla\frac{1}{R}$$

真空中电磁相互作用的场方程: 麦克斯韦方程组的积分形式

$$
\begin{aligned}
\oint_S d\vec{l}\cdot\vec{B} &= \int_S d\vec{S}\cdot(\nabla\times\vec{B}) = -\int_S d\vec{S}\cdot\nabla\times[\int d\vec{l}'\times k_2 J(\vec{r}')\nabla\frac{1}{R}]\\
&= -\int_S d\vec{S}\cdot\int_\infty d\tau' k_2\nabla\times[\vec{j}(\vec{r}')\times\nabla\frac{1}{R}]\\
&= k_2\int_S d\vec{S}\cdot\int_\infty d\tau'[\vec{j}(\vec{r}')\cdot\nabla\nabla-\vec{j}(\vec{r}')\nabla^2]\frac{1}{R}
\end{aligned}
$$

$$
\begin{aligned}
\int_\infty d\tau'\vec{j}(\vec{r}')\cdot\nabla\nabla\frac{1}{R} &= -\int_\infty d\tau'\vec{j}(\vec{r}')\cdot\nabla'\nabla\frac{1}{R}\\
&= \int_\infty d\tau'[\nabla'\cdot[-\vec{j}(\vec{r}')\nabla\frac{1}{R}]+[\nabla'\cdot\vec{j}(\vec{r}')]\nabla\frac{1}{R}]\\
&= -\int_\infty d\vec{S}'\cdot\vec{j}(\vec{r}')\nabla\frac{1}{R}+\int_\infty d\tau'[\nabla'\cdot\vec{j}(\vec{r}')]\nabla\frac{1}{R}\\
&= 0
\end{aligned}
$$

真空中电磁相互作用的场方程： 麦克斯韦方程组的积分形式

$$\oint_S d\vec{l}\cdot\vec{B} = \int_S d\vec{S}\cdot(\nabla\times\vec{B}) = -\int_S d\vec{S}\cdot\nabla\times[\int d\vec{l}'\times k_2 J(\vec{r}')\nabla\frac{1}{R}]$$

$$= -\int_S d\vec{S}\cdot\int_\infty d\tau' k_2\nabla\times[\vec{j}(\vec{r}')\times\nabla\frac{1}{R}]$$

$$= k_2\int_S d\vec{S}\cdot\int_\infty d\tau'[\vec{j}(\vec{r}')\cdot\nabla\nabla - \vec{j}(\vec{r}')\nabla^2]\frac{1}{R}$$

$$\int_\infty d\tau'\vec{j}(\vec{r}')\cdot\nabla\nabla\frac{1}{R} = -\int_\infty d\tau'\vec{j}(\vec{r}')\cdot\nabla'\nabla\frac{1}{R}$$

$$= \int_\infty d\tau'[\nabla'\cdot[-\vec{j}(\vec{r}')\nabla\frac{1}{R}] + [\nabla'\cdot\vec{j}(\vec{r}')]\nabla\frac{1}{R}]$$

$$= -\int_\infty d\vec{S}'\cdot\vec{j}(\vec{r}')\nabla\frac{1}{R} + \int_\infty d\tau'[\nabla'\cdot\vec{j}(\vec{r}')]\nabla\frac{1}{R}$$

$$= 0$$

$$0 = \oint_\tau d\vec{S}\cdot\vec{j} = \int_\tau d\tau\nabla\cdot\vec{j}$$

真空中电磁相互作用的场方程：麦克斯韦方程组的积分形式

$$\oint_S d\vec{l}\cdot\vec{B} = \int_S d\vec{S}\cdot(\nabla\times\vec{B}) = -\int_S d\vec{S}\cdot\nabla\times[\int d\vec{l}'\times k_2 J(\vec{r}')\nabla\frac{1}{R}]$$

$$= -\int_S d\vec{S}\cdot\int_\infty d\tau' k_2\nabla\times[\vec{j}(\vec{r}')\times\nabla\frac{1}{R}]$$

$$= k_2\int_S d\vec{S}\cdot\int_\infty d\tau'[\vec{j}(\vec{r}')\cdot\nabla\nabla-\vec{j}(\vec{r}')\nabla^2]\frac{1}{R}$$

$$\int_\infty d\tau'\vec{j}(\vec{r}')\cdot\nabla\nabla\frac{1}{R} = -\int_\infty d\tau'\vec{j}(\vec{r}')\cdot\nabla'\nabla\frac{1}{R}$$

$$= \int_\infty d\tau'[\nabla'\cdot[-\vec{j}(\vec{r}')\nabla\frac{1}{R}]+[\nabla'\cdot\vec{j}(\vec{r}')]\nabla\frac{1}{R}]$$

$$= -\int_\infty d\vec{S}'\cdot\vec{j}(\vec{r}')\nabla\frac{1}{R}+\int_\infty d\tau'[\nabla'\cdot\vec{j}(\vec{r}')]\nabla\frac{1}{R}$$

$$= 0$$

$$0 = \oint_\tau d\vec{S}\cdot\vec{j} = \int_\tau d\tau\nabla\cdot\vec{j} \rightarrow \qquad \nabla\cdot\vec{j} = 0$$

真空中电磁相互作用的场方程：麦克斯韦方程组的积分形式

$$\oint_S d\vec{l} \cdot \vec{B}$$

真空中电磁相互作用的场方程：麦克斯韦方程组的积分形式

$$\oint_S \mathrm{d}\vec{\mathrm{l}} \cdot \vec{\mathrm{B}} = \mathrm{k}_2 \int_S \mathrm{d}\vec{\mathrm{S}} \cdot \int \mathrm{d}\tau' [\vec{\mathrm{j}}(\vec{\mathrm{r}}') \cdot \nabla\nabla - \vec{\mathrm{j}}(\vec{\mathrm{r}}')\nabla^2] \frac{1}{\mathrm{R}}$$

真空中电磁相互作用的场方程：麦克斯韦方程组的积分形式

$$\oint_S d\vec{l} \cdot \vec{B} = k_2 \int_S d\vec{S} \cdot \int d\tau' [\vec{j}(\vec{r}') \cdot \nabla\nabla - \vec{j}(\vec{r}')\nabla^2] \frac{1}{R}$$

$$= -k_2 \int_S d\vec{S} \cdot \int d\tau' \vec{j}(\vec{r}') \nabla^2 \frac{1}{R}$$

真空中电磁相互作用的场方程：麦克斯韦方程组的积分形式

$$
\begin{aligned}
\oint_S d\vec{l}\cdot\vec{B} &= k_2\int_S d\vec{S}\cdot\int d\tau'[\vec{j}(\vec{r}')\cdot\nabla\nabla-\vec{j}(\vec{r}')\nabla^2]\frac{1}{R} \\
&= -k_2\int_S d\vec{S}\cdot\int d\tau'\vec{j}(\vec{r}')\nabla^2\frac{1}{R} \\
&= 4\pi k_2\int_S d\vec{S}\cdot\int d\tau'\vec{j}(\vec{r}')\delta(\vec{r}-\vec{r}')
\end{aligned}
$$

真空中电磁相互作用的场方程： 麦克斯韦方程组的积分形式

$$
\begin{aligned}
\oint_S d\vec{l}\cdot\vec{B} &= k_2\int_S d\vec{S}\cdot\int d\tau'[\vec{j}(\vec{r}')\cdot\nabla\nabla-\vec{j}(\vec{r}')\nabla^2]\frac{1}{R} \\
&= -k_2\int_S d\vec{S}\cdot\int d\tau'\vec{j}(\vec{r}')\nabla^2\frac{1}{R} \\
&= 4\pi k_2\int_S d\vec{S}\cdot\int d\tau'\vec{j}(\vec{r}')\delta(\vec{r}-\vec{r}') \\
&= 4\pi k_2\int_S d\vec{S}\cdot\vec{j}(\vec{r})
\end{aligned}
$$

真空中电磁相互作用的场方程: 麦克斯韦方程组的积分形式

$$
\begin{aligned}
\oint_S \mathrm{d}\vec{l}\cdot\vec{B} &= k_2\int_S \mathrm{d}\vec{S}\cdot\int \mathrm{d}\tau'[\vec{j}(\vec{r}')\cdot\nabla\nabla - \vec{j}(\vec{r}')\nabla^2]\frac{1}{R} \\
&= -k_2\int_S \mathrm{d}\vec{S}\cdot\int \mathrm{d}\tau'\vec{j}(\vec{r}')\nabla^2\frac{1}{R} \\
&= 4\pi k_2\int_S \mathrm{d}\vec{S}\cdot\int \mathrm{d}\tau'\vec{j}(\vec{r}')\delta(\vec{r}-\vec{r}') \\
&= 4\pi k_2\int_S \mathrm{d}\vec{S}\cdot\vec{j}(\vec{r}) = \mu_0 J
\end{aligned}
$$

真空中电磁相互作用的场方程: 麦克斯韦方程组的积分形式

$$
\begin{aligned}
\oint_S d\vec{l}\cdot\vec{B} &= k_2\int_S d\vec{S}\cdot\int d\tau'[\vec{j}(\vec{r}')\cdot\nabla\nabla-\vec{j}(\vec{r}')\nabla^2]\frac{1}{R}\\
&= -k_2\int_S d\vec{S}\cdot\int d\tau'\vec{j}(\vec{r}')\nabla^2\frac{1}{R}\\
&= 4\pi k_2\int_S d\vec{S}\cdot\int d\tau'\vec{j}(\vec{r}')\delta(\vec{r}-\vec{r}')\\
&= 4\pi k_2\int_S d\vec{S}\cdot\vec{j}(\vec{r}) = \mu_0 J
\end{aligned}
$$

法拉第电磁感应定律: $\oint_S d\vec{l}\cdot\vec{E} = -\int_S d\vec{S}\cdot\frac{\partial\vec{B}}{\partial t}$

$$\oint_S d\vec{l}\cdot\vec{B} = k_2\int_S d\vec{S}\cdot\int d\tau'[\vec{j}(\vec{r}')\cdot\nabla\nabla - \vec{j}(\vec{r}')\nabla^2]\frac{1}{R}$$

$$= -k_2\int_S d\vec{S}\cdot\int d\tau'\vec{j}(\vec{r}')\nabla^2\frac{1}{R}$$

$$= 4\pi k_2\int_S d\vec{S}\cdot\int d\tau'\vec{j}(\vec{r}')\delta(\vec{r}-\vec{r}')$$

$$= 4\pi k_2\int_S d\vec{S}\cdot\vec{j}(\vec{r}) = \mu_0 J$$

法拉第电磁感应定律： $\oint_S d\vec{l}\cdot\vec{E} = -\int_S d\vec{S}\cdot\frac{\partial\vec{B}}{\partial t}$

库伦定律 $\oint_S d\vec{l}\cdot\vec{E} = 0$的推广！

$$\oint_{\tau} \mathrm{d}\vec{\mathbf{S}} \cdot \vec{\mathbf{j}} = -\int_{\tau} \mathrm{d}\tau \frac{\partial \rho}{\partial \mathrm{t}}$$

真空中电磁相互作用的场方程：麦克斯韦方程组的微分形式

$$\oint_\tau \mathrm{d}\vec{\mathbf{S}} \cdot \vec{\mathbf{j}} = -\int_\tau \mathrm{d}\tau \frac{\partial \rho}{\partial \mathbf{t}} \rightarrow \int_\tau \mathrm{d}\tau (\nabla \cdot \vec{\mathbf{j}} + \frac{\partial \rho}{\partial \mathbf{t}}) = \mathbf{0}$$

真空中电磁相互作用的场方程：麦克斯韦方程组的微分形式

$$\oint_\tau \mathrm{d}\vec{\mathbf{S}} \cdot \vec{\mathbf{j}} = -\int_\tau \mathrm{d}\tau \frac{\partial \rho}{\partial \mathbf{t}} \to \int_\tau \mathrm{d}\tau (\nabla \cdot \vec{\mathbf{j}} + \frac{\partial \rho}{\partial \mathbf{t}}) = \mathbf{0} \to \frac{\partial \rho}{\partial \mathbf{t}} + \nabla \cdot \vec{\mathbf{j}} = \mathbf{0}$$

真空中电磁相互作用的场方程：麦克斯韦方程组的微分形式

$$\oint_\tau \mathrm{d}\vec{\mathbf{S}} \cdot \vec{\mathbf{j}} = -\int_\tau \mathbf{d}\tau \frac{\partial \rho}{\partial \mathbf{t}} \rightarrow \int_\tau \mathbf{d}\tau (\nabla \cdot \vec{\mathbf{j}} + \frac{\partial \rho}{\partial \mathbf{t}}) = \mathbf{0} \rightarrow \frac{\partial \rho}{\partial \mathbf{t}} + \nabla \cdot \vec{\mathbf{j}} = \mathbf{0}$$

$$\oint_\tau \mathrm{d}\vec{\mathbf{S}} \cdot \vec{\mathbf{E}} = \frac{\mathbf{1}}{\epsilon_\mathbf{0}} \int_\tau \mathbf{d}\tau \rho$$

真空中电磁相互作用的场方程：麦克斯韦方程组的微分形式

$$\oint_\tau d\vec{S}\cdot\vec{j} = -\int_\tau d\tau\frac{\partial\rho}{\partial t} \to \int_\tau d\tau(\nabla\cdot\vec{j}+\frac{\partial\rho}{\partial t}) = 0 \to \frac{\partial\rho}{\partial t}+\nabla\cdot\vec{j} = 0$$

$$\oint_\tau d\vec{S}\cdot\vec{E} = \frac{1}{\epsilon_0}\int_\tau d\tau\rho \to \int_\tau d\tau(\nabla\cdot\vec{E}-\frac{1}{\epsilon_0}\rho) = 0$$

真空中电磁相互作用的场方程：麦克斯韦方程组的微分形式

$$\oint_\tau d\vec{S}\cdot\vec{j} = -\int_\tau d\tau\frac{\partial\rho}{\partial t} \rightarrow \int_\tau d\tau(\nabla\cdot\vec{j}+\frac{\partial\rho}{\partial t}) = 0 \rightarrow \frac{\partial\rho}{\partial t}+\nabla\cdot\vec{j} = 0$$

$$\oint_\tau d\vec{S}\cdot\vec{E} = \frac{1}{\epsilon_0}\int_\tau d\tau\rho \rightarrow \int_\tau d\tau(\nabla\cdot\vec{E}-\frac{1}{\epsilon_0}\rho) = 0 \rightarrow \nabla\cdot\vec{E} = \frac{1}{\epsilon_0}\rho$$

Back
Close

真空中电磁相互作用的场方程：麦克斯韦方程组的微分形式

$$\oint_\tau d\vec{S}\cdot\vec{j} = -\int_\tau d\tau\frac{\partial\rho}{\partial t} \rightarrow \int_\tau d\tau(\nabla\cdot\vec{j}+\frac{\partial\rho}{\partial t}) = 0 \rightarrow \frac{\partial\rho}{\partial t}+\nabla\cdot\vec{j} = 0$$

$$\oint_\tau d\vec{S}\cdot\vec{E} = \frac{1}{\epsilon_0}\int_\tau d\tau\rho \rightarrow \int_\tau d\tau(\nabla\cdot\vec{E}-\frac{1}{\epsilon_0}\rho) = 0 \rightarrow \nabla\cdot\vec{E} = \frac{1}{\epsilon_0}\rho$$

$$\oint_S d\vec{l}\cdot\vec{B} = \mu_0\int_S d\vec{S}\cdot\vec{j}$$

真空中电磁相互作用的场方程： 麦克斯韦方程组的微分形式

$$\oint_\tau \mathrm{d}\vec{\mathrm{S}}\cdot\vec{\mathrm{j}} = -\int_\tau \mathrm{d}\tau\frac{\partial\rho}{\partial \mathrm{t}} \rightarrow \int_\tau \mathrm{d}\tau(\nabla\cdot\vec{\mathrm{j}}+\frac{\partial\rho}{\partial \mathrm{t}}) = 0 \rightarrow \frac{\partial\rho}{\partial \mathrm{t}}+\nabla\cdot\vec{\mathrm{j}} = 0$$

$$\oint_\tau \mathrm{d}\vec{\mathrm{S}}\cdot\vec{\mathrm{E}} = \frac{1}{\epsilon_0}\int_\tau \mathrm{d}\tau\rho \rightarrow \int_\tau \mathrm{d}\tau(\nabla\cdot\vec{\mathrm{E}}-\frac{1}{\epsilon_0}\rho) = 0 \rightarrow \nabla\cdot\vec{\mathrm{E}} = \frac{1}{\epsilon_0}\rho$$

$$\oint_\mathrm{S} \mathrm{d}\vec{\mathrm{l}}\cdot\vec{\mathrm{B}} = \mu_0\int_\mathrm{S} \mathrm{d}\vec{\mathrm{S}}\cdot\vec{\mathrm{j}} \rightarrow \int_\mathrm{S} \mathrm{d}\vec{\mathrm{S}}\cdot(\nabla\times\vec{\mathrm{B}}-\mu_0\vec{\mathrm{j}}) = 0$$

真空中电磁相互作用的场方程：麦克斯韦方程组的微分形式

$$\oint_\tau d\vec{S}\cdot\vec{j} = -\int_\tau d\tau\frac{\partial\rho}{\partial t} \rightarrow \int_\tau d\tau(\nabla\cdot\vec{j}+\frac{\partial\rho}{\partial t}) = 0 \rightarrow \frac{\partial\rho}{\partial t}+\nabla\cdot\vec{j} = 0$$

$$\oint_\tau d\vec{S}\cdot\vec{E} = \frac{1}{\epsilon_0}\int_\tau d\tau\rho \rightarrow \int_\tau d\tau(\nabla\cdot\vec{E}-\frac{1}{\epsilon_0}\rho) = 0 \rightarrow \nabla\cdot\vec{E} = \frac{1}{\epsilon_0}\rho$$

$$\oint_S d\vec{l}\cdot\vec{B} = \mu_0\int_S d\vec{S}\cdot\vec{j} \rightarrow \int_S d\vec{S}\cdot(\nabla\times\vec{B}-\mu_0\vec{j}) = 0 \rightarrow \nabla\times\vec{B} = \mu_0\vec{j}$$

真空中电磁相互作用的场方程：麦克斯韦方程组的微分形式

$$\oint_\tau \mathrm{d}\vec{\mathrm{S}}\cdot\vec{\mathrm{j}} = -\int_\tau \mathrm{d}\tau\frac{\partial\rho}{\partial \mathrm{t}} \rightarrow \int_\tau \mathrm{d}\tau(\nabla\cdot\vec{\mathrm{j}}+\frac{\partial\rho}{\partial \mathrm{t}}) = 0 \rightarrow \frac{\partial\rho}{\partial \mathrm{t}}+\nabla\cdot\vec{\mathrm{j}} = 0$$

$$\oint_\tau \mathrm{d}\vec{\mathrm{S}}\cdot\vec{\mathrm{E}} = \frac{1}{\epsilon_0}\int_\tau \mathrm{d}\tau\rho \rightarrow \int_\tau \mathrm{d}\tau(\nabla\cdot\vec{\mathrm{E}}-\frac{1}{\epsilon_0}\rho) = 0 \rightarrow \nabla\cdot\vec{\mathrm{E}} = \frac{1}{\epsilon_0}\rho$$

$$\oint_\mathrm{S} \mathrm{d}\vec{\mathrm{l}}\cdot\vec{\mathrm{B}} = \mu_0\int_\mathrm{S} \mathrm{d}\vec{\mathrm{S}}\cdot\vec{\mathrm{j}} \rightarrow \int_\mathrm{S} \mathrm{d}\vec{\mathrm{S}}\cdot(\nabla\times\vec{\mathrm{B}}-\mu_0\vec{\mathrm{j}}) = 0 \rightarrow \nabla\times\vec{\mathrm{B}} = \mu_0\vec{\mathrm{j}}$$

$$\oint_\tau \mathrm{d}\vec{\mathrm{S}}\cdot\vec{\mathrm{B}} = 0$$

真空中电磁相互作用的场方程：麦克斯韦方程组的微分形式

$$\oint_\tau \mathrm{d}\vec{\mathrm{S}}\cdot\vec{\mathrm{j}} = -\int_\tau \mathrm{d}\tau\frac{\partial\rho}{\partial \mathrm{t}} \rightarrow \int_\tau \mathrm{d}\tau(\nabla\cdot\vec{\mathrm{j}}+\frac{\partial\rho}{\partial \mathrm{t}}) = 0 \rightarrow \frac{\partial\rho}{\partial \mathrm{t}}+\nabla\cdot\vec{\mathrm{j}} = 0$$

$$\oint_\tau \mathrm{d}\vec{\mathrm{S}}\cdot\vec{\mathrm{E}} = \frac{1}{\epsilon_0}\int_\tau \mathrm{d}\tau\rho \rightarrow \int_\tau \mathrm{d}\tau(\nabla\cdot\vec{\mathrm{E}}-\frac{1}{\epsilon_0}\rho) = 0 \rightarrow \nabla\cdot\vec{\mathrm{E}} = \frac{1}{\epsilon_0}\rho$$

$$\oint_{\mathrm{S}} \mathrm{d}\vec{\mathrm{l}}\cdot\vec{\mathrm{B}} = \mu_0\int_{\mathrm{S}} \mathrm{d}\vec{\mathrm{S}}\cdot\vec{\mathrm{j}} \rightarrow \int_{\mathrm{S}} \mathrm{d}\vec{\mathrm{S}}\cdot(\nabla\times\vec{\mathrm{B}}-\mu_0\vec{\mathrm{j}}) = 0 \rightarrow \nabla\times\vec{\mathrm{B}} = \mu_0\vec{\mathrm{j}}$$

$$\oint_\tau \mathrm{d}\vec{\mathrm{S}}\cdot\vec{\mathrm{B}} = 0 \rightarrow \int_\tau \mathrm{d}\tau\nabla\cdot\vec{\mathrm{B}} = 0$$

真空中电磁相互作用的场方程: 麦克斯韦方程组的微分形式

$$\oint_\tau d\vec{S}\cdot\vec{j} = -\int_\tau d\tau\frac{\partial\rho}{\partial t} \rightarrow \int_\tau d\tau(\nabla\cdot\vec{j}+\frac{\partial\rho}{\partial t}) = 0 \rightarrow \frac{\partial\rho}{\partial t}+\nabla\cdot\vec{j} = 0$$

$$\oint_\tau d\vec{S}\cdot\vec{E} = \frac{1}{\epsilon_0}\int_\tau d\tau\rho \rightarrow \int_\tau d\tau(\nabla\cdot\vec{E}-\frac{1}{\epsilon_0}\rho) = 0 \rightarrow \nabla\cdot\vec{E} = \frac{1}{\epsilon_0}\rho$$

$$\oint_S d\vec{l}\cdot\vec{B} = \mu_0\int_S d\vec{S}\cdot\vec{j} \rightarrow \int_S d\vec{S}\cdot(\nabla\times\vec{B}-\mu_0\vec{j}) = 0 \rightarrow \nabla\times\vec{B} = \mu_0\vec{j}$$

$$\oint_\tau d\vec{S}\cdot\vec{B} = 0 \rightarrow \int_\tau d\tau\nabla\cdot\vec{B} = 0 \rightarrow \nabla\cdot\vec{B} = 0$$

真空中电磁相互作用的场方程：麦克斯韦方程组的微分形式

$$\oint_\tau d\vec{S}\cdot\vec{j} = -\int_\tau d\tau\frac{\partial\rho}{\partial t} \;\rightarrow\; \int_\tau d\tau(\nabla\cdot\vec{j}+\frac{\partial\rho}{\partial t}) = 0 \;\rightarrow\; \frac{\partial\rho}{\partial t}+\nabla\cdot\vec{j} = 0$$

$$\oint_\tau d\vec{S}\cdot\vec{E} = \frac{1}{\epsilon_0}\int_\tau d\tau\rho \;\rightarrow\; \int_\tau d\tau(\nabla\cdot\vec{E}-\frac{1}{\epsilon_0}\rho) = 0 \;\rightarrow\; \nabla\cdot\vec{E} = \frac{1}{\epsilon_0}\rho$$

$$\oint_S d\vec{l}\cdot\vec{B} = \mu_0\int_S d\vec{S}\cdot\vec{j} \rightarrow \int_S d\vec{S}\cdot(\nabla\times\vec{B}-\mu_0\vec{j}) = 0 \rightarrow \nabla\times\vec{B} = \mu_0\vec{j}$$

$$\oint_\tau d\vec{S}\cdot\vec{B} = 0 \;\rightarrow\; \int_\tau d\tau\nabla\cdot\vec{B} = 0 \;\rightarrow\; \nabla\cdot\vec{B} = 0$$

$$\oint_S d\vec{l}\cdot\vec{E} = -\int_S d\vec{S}\cdot\frac{\partial\vec{B}}{\partial t}$$

真空中电磁相互作用的场方程：麦克斯韦方程组的微分形式

$$\oint_\tau \mathrm{d}\vec{S}\cdot\vec{j} = -\int_\tau \mathrm{d}\tau\frac{\partial\rho}{\partial t} \rightarrow \int_\tau \mathrm{d}\tau(\nabla\cdot\vec{j}+\frac{\partial\rho}{\partial t}) = 0 \rightarrow \frac{\partial\rho}{\partial t}+\nabla\cdot\vec{j} = 0$$

$$\oint_\tau \mathrm{d}\vec{S}\cdot\vec{E} = \frac{1}{\epsilon_0}\int_\tau \mathrm{d}\tau\rho \rightarrow \int_\tau \mathrm{d}\tau(\nabla\cdot\vec{E}-\frac{1}{\epsilon_0}\rho) = 0 \rightarrow \nabla\cdot\vec{E} = \frac{1}{\epsilon_0}\rho$$

$$\oint_S \mathrm{d}\vec{l}\cdot\vec{B} = \mu_0\int_S \mathrm{d}\vec{S}\cdot\vec{j} \rightarrow \int_S \mathrm{d}\vec{S}\cdot(\nabla\times\vec{B}-\mu_0\vec{j}) = 0 \rightarrow \nabla\times\vec{B} = \mu_0\vec{j}$$

$$\oint_\tau \mathrm{d}\vec{S}\cdot\vec{B} = 0 \rightarrow \int_\tau \mathrm{d}\tau\nabla\cdot\vec{B} = 0 \rightarrow \nabla\cdot\vec{B} = 0$$

$$\oint_S \mathrm{d}\vec{l}\cdot\vec{E} = -\int_S \mathrm{d}\vec{S}\cdot\frac{\partial\vec{B}}{\partial t} \rightarrow \int_S \mathrm{d}\vec{S}\cdot(\nabla\times\vec{E}+\frac{\partial\vec{B}}{\partial t}) = 0$$

真空中电磁相互作用的场方程：麦克斯韦方程组的微分形式

$$\oint_\tau d\vec{S}\cdot\vec{j} = -\int_\tau d\tau\frac{\partial\rho}{\partial t} \to \int_\tau d\tau(\nabla\cdot\vec{j}+\frac{\partial\rho}{\partial t}) = 0 \to \frac{\partial\rho}{\partial t}+\nabla\cdot\vec{j} = 0$$

$$\oint_\tau d\vec{S}\cdot\vec{E} = \frac{1}{\epsilon_0}\int_\tau d\tau\rho \to \int_\tau d\tau(\nabla\cdot\vec{E}-\frac{1}{\epsilon_0}\rho) = 0 \to \nabla\cdot\vec{E} = \frac{1}{\epsilon_0}\rho$$

$$\oint_S d\vec{l}\cdot\vec{B} = \mu_0\int_S d\vec{S}\cdot\vec{j} \to \int_S d\vec{S}\cdot(\nabla\times\vec{B}-\mu_0\vec{j}) = 0 \to \nabla\times\vec{B} = \mu_0\vec{j}$$

$$\oint_\tau d\vec{S}\cdot\vec{B} = 0 \to \int_\tau d\tau\nabla\cdot\vec{B} = 0 \to \nabla\cdot\vec{B} = 0$$

$$\oint_S d\vec{l}\cdot\vec{E} = -\int_S d\vec{S}\cdot\frac{\partial\vec{B}}{\partial t} \to \int_S d\vec{S}\cdot(\nabla\times\vec{E}+\frac{\partial\vec{B}}{\partial t}) = 0$$

$$\to \nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

真空中电磁相互作用的场方程：麦克斯韦方程组的微分形式

$$\oint_\tau d\vec{S}\cdot\vec{j} = -\int_\tau d\tau\frac{\partial\rho}{\partial t} \rightarrow \int_\tau d\tau(\nabla\cdot\vec{j}+\frac{\partial\rho}{\partial t}) = 0 \rightarrow \frac{\partial\rho}{\partial t}+\nabla\cdot\vec{j} = 0$$

$$\oint_\tau d\vec{S}\cdot\vec{E} = \frac{1}{\epsilon_0}\int_\tau d\tau\rho \rightarrow \int_\tau d\tau(\nabla\cdot\vec{E}-\frac{1}{\epsilon_0}\rho) = 0 \rightarrow \nabla\cdot\vec{E} = \frac{1}{\epsilon_0}\rho$$

$$\oint_S d\vec{l}\cdot\vec{B} = \mu_0\int_S d\vec{S}\cdot\vec{j} \rightarrow \int_S d\vec{S}\cdot(\nabla\times\vec{B}-\mu_0\vec{j}) = 0 \rightarrow \nabla\times\vec{B} = \mu_0\vec{j}$$

$$\oint_\tau d\vec{S}\cdot\vec{B} = 0 \rightarrow \int_\tau d\tau\nabla\cdot\vec{B} = 0 \rightarrow \nabla\cdot\vec{B} = 0$$

$$\oint_S d\vec{l}\cdot\vec{E} = -\int_S d\vec{S}\cdot\frac{\partial\vec{B}}{\partial t} \rightarrow \int_S d\vec{S}\cdot(\nabla\times\vec{E}+\frac{\partial\vec{B}}{\partial t}) = 0$$

$$\rightarrow \nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{F} = \int_\tau d\tau[\rho(\vec{r})\vec{E}(\vec{r})+\vec{j}\times\vec{B}(\vec{r})]$$

真空中电磁相互作用的场方程： 麦克斯韦方程组的微分形式

$$\oint_\tau d\vec{S}\cdot\vec{j} = -\int_\tau d\tau\frac{\partial\rho}{\partial t} \to \int_\tau d\tau(\nabla\cdot\vec{j}+\frac{\partial\rho}{\partial t}) = 0 \to \frac{\partial\rho}{\partial t}+\nabla\cdot\vec{j}=0$$

$$\oint_\tau d\vec{S}\cdot\vec{E} = \frac{1}{\epsilon_0}\int_\tau d\tau\rho \to \int_\tau d\tau(\nabla\cdot\vec{E}-\frac{1}{\epsilon_0}\rho) = 0 \to \nabla\cdot\vec{E}=\frac{1}{\epsilon_0}\rho$$

$$\oint_S d\vec{l}\cdot\vec{B} = \mu_0\int_S d\vec{S}\cdot\vec{j} \to \int_S d\vec{S}\cdot(\nabla\times\vec{B}-\mu_0\vec{j}) = 0 \to \nabla\times\vec{B}=\mu_0\vec{j}$$

$$\oint_\tau d\vec{S}\cdot\vec{B} = 0 \to \int_\tau d\tau\nabla\cdot\vec{B} = 0 \to \nabla\cdot\vec{B}=0$$

$$\oint_S d\vec{l}\cdot\vec{E} = -\int_S d\vec{S}\cdot\frac{\partial\vec{B}}{\partial t} \to \int_S d\vec{S}\cdot(\nabla\times\vec{E}+\frac{\partial\vec{B}}{\partial t}) = 0$$

$$\to \nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{F} = \int_\tau d\tau[\rho(\vec{r})\vec{E}(\vec{r})+\vec{j}\times\vec{B}(\vec{r})] \to \int_\tau d\tau(\vec{f}-\rho\vec{E}-\vec{j}\times\vec{B}) = 0$$

真空中电磁相互作用的场方程：麦克斯韦方程组的微分形式

$$\oint_\tau d\vec{S}\cdot\vec{j} = -\int_\tau d\tau\frac{\partial\rho}{\partial t} \to \int_\tau d\tau(\nabla\cdot\vec{j}+\frac{\partial\rho}{\partial t}) = 0 \to \frac{\partial\rho}{\partial t}+\nabla\cdot\vec{j}=0$$

$$\oint_\tau d\vec{S}\cdot\vec{E} = \frac{1}{\epsilon_0}\int_\tau d\tau\rho \to \int_\tau d\tau(\nabla\cdot\vec{E}-\frac{1}{\epsilon_0}\rho) = 0 \to \nabla\cdot\vec{E} = \frac{1}{\epsilon_0}\rho$$

$$\oint_S d\vec{l}\cdot\vec{B} = \mu_0\int_S d\vec{S}\cdot\vec{j} \to \int_S d\vec{S}\cdot(\nabla\times\vec{B}-\mu_0\vec{j}) = 0 \to \nabla\times\vec{B} = \mu_0\vec{j}$$

$$\oint_\tau d\vec{S}\cdot\vec{B} = 0 \to \int_\tau d\tau\nabla\cdot\vec{B} = 0 \to \nabla\cdot\vec{B} = 0$$

$$\oint_S d\vec{l}\cdot\vec{E} = -\int_S d\vec{S}\cdot\frac{\partial\vec{B}}{\partial t} \to \int_S d\vec{S}\cdot(\nabla\times\vec{E}+\frac{\partial\vec{B}}{\partial t}) = 0$$

$$\to \nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{F} = \int_\tau d\tau[\rho(\vec{r})\vec{E}(\vec{r})+\vec{j}\times\vec{B}(\vec{r})] \to \int_\tau d\tau(\vec{f}-\rho\vec{E}-\vec{j}\times\vec{B}) = 0$$

$$\to \vec{f} = \rho\vec{E}+\vec{j}\times\vec{B}$$

真空中电磁相互作用的场方程: 麦克斯韦方程组的微分形式

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{j} = 0 \qquad \nabla \cdot \vec{E} = \frac{1}{\epsilon_0}\rho \qquad \nabla \times \vec{B} = \mu_0 \vec{j}$$

$$\frac{\partial \rho}{\partial t} = \epsilon_0 \frac{\partial}{\partial t} \nabla \cdot \vec{E} = \epsilon_0 \nabla \cdot \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \cdot \vec{j} = \frac{1}{\mu_0} \nabla \cdot (\nabla \times \vec{B}) = \frac{1}{\mu_0} (\nabla \times \nabla) \cdot \vec{B} = 0$$

$$\nabla \times \vec{B} \ \rightarrow \nabla \times \vec{B} = \mu_0 \vec{j} + \underline{\mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}} \leftarrow \text{位移电流项(电生磁)}$$

真空中电磁相互作用的场方程：麦克斯韦方程组的微分形式

$$\nabla\cdot\vec{E}=\frac{1}{\epsilon_0}\rho \qquad \sum_{i=1}^{3}\partial_i E_i=\frac{1}{\epsilon_0}\rho$$

电荷是电场发散聚敛的源

$$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t} \qquad \sum_{i,j=1}^{3}\epsilon_{ijk}\partial_i E_j=-\frac{\partial B_k}{\partial t}$$

磁场的时间变化率是电场旋转的源

$$\nabla\cdot\vec{B}=0 \qquad \sum_{i=1}^{3}\partial_i B_i=0$$

磁场不发散和聚敛

$$\nabla\times\vec{B}=\mu_0\vec{j}+\mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t} \qquad \sum_{i,j=1}^{3}\epsilon_{ijk}\partial_i B_j=\mu_0 j_k+\mu_0\epsilon_0\frac{\partial E_k}{\partial t}$$

电流密度和电场的时间变化率是磁场旋转的源

$$\vec{f}=\rho\vec{E}+\vec{j}\times\vec{B} \qquad f_k=\rho E_k+\sum_{i,j=1}^{3}\epsilon_{ijk}j_i B_j$$

能动量的转化与守恒： 场和电荷系统能动量转化和守恒定律的一般形式

- 场的能量密度w：单位体积的场所带的能量,是时空坐标的函数 $\mathbf{w} = \mathbf{w}(\vec{\mathbf{r}}, \mathbf{t})$

- 场的能量密度w: 单位体积的场所带的能量,是时空坐标的函数 $w = w(\vec{r}, t)$

- 场的动量密度$\vec{g}$: 单位体积的场所带的动量,是时空坐标的函数 $\vec{g} = \vec{g}(\vec{r}, t)$

能动量的转化与守恒: 场和电荷系统能动量转化和守恒定律的一般形式

- 场的能量密度w: 单位体积的场所带的能量,是时空坐标的函数 $w = w(\vec{r}, t)$

- 场的动量密度$\vec{g}$: 单位体积的场所带的动量,是时空坐标的函数 $\vec{g} = \vec{g}(\vec{r}, t)$

- 场的能流密度$\vec{S}$: 描述能量的传播,大小等于单位时间垂直流过单位横 截面的能量,方向指向能量传播的方向,是时空坐标的函数 $\vec{S} = \vec{S}(\vec{r}, t)$

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- 场的动量流密度 $\overleftrightarrow{\mathcal{J}}$: 它描述动量的传播, $\Delta\vec{S}\cdot\overleftrightarrow{\mathcal{J}} = \Delta\vec{p}$定义为单位时间通过面元 $\Delta\vec{S}$流出的动量,是时空坐标的函数 $\overleftrightarrow{\mathcal{J}} = \overleftrightarrow{\mathcal{J}}(\vec{r}, t)$

Back
Close

- 能量转化与守恒定律:

- 能量转化与守恒定律：单位时间通过表面流入V的能量等于单位时间场对 V内电荷所做的功(即功率)与V内单位时间电磁场能量的增加之和。

Back
Close

- 能量转化与守恒定律: 单位时间通过表面流入V的能量等于单位时间场对 V内电荷所做的功(即功率)与V内单位时间电磁场能量的增加之和。

$$-\oint_{V} d\vec{\sigma}\cdot\vec{S}=\int_{V} dV\vec{f}\cdot\vec{v}+\frac{d}{dt}\int_{V} dVw$$

$\vec{f}$:场对电荷作用力密度　　$\vec{v}$:电荷运动速度

- 能量转化与守恒定律: 单位时间通过表面流入V的能量等于单位时间场对 V内电荷所做的功(即功率)与V内单位时间电磁场能量的增加之和。

$$-\oint_V \mathrm{d}\vec{\sigma}\cdot\vec{\mathrm{S}} = \int_V \mathrm{dV}\vec{\mathrm{f}}\cdot\vec{\mathrm{v}} + \frac{\mathrm{d}}{\mathrm{dt}}\int_V \mathrm{dVw}$$

$\vec{f}$:场对电荷作用力密度　　$\vec{v}$:电荷运动速度

$$-\nabla\cdot\vec{\mathrm{S}} = \vec{\mathrm{f}}\cdot\vec{\mathrm{v}} + \frac{\partial \mathrm{w}}{\partial \mathrm{t}}$$

- 能量转化与守恒定律：单位时间通过表面流入V的能量等于单位时间场对 V内电荷所做的功(即功率)与V内单位时间电磁场能量的增加之和。

$$-\oint_V d\vec{\sigma}\cdot\vec{S} = \int_V dV\vec{f}\cdot\vec{v} + \frac{d}{dt}\int_V dVw$$

$\vec{f}$:场对电荷作用力密度　　$\vec{v}$:电荷运动速度

$$-\nabla\cdot\vec{S} = \vec{f}\cdot\vec{v} + \frac{\partial w}{\partial t}$$

- 动量转化与守恒定律： 单位时间通过表面流入V的动量等于V 内电荷所受的力与V内单位时间电磁场动量的增加之和。

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$$-\oint_V d\vec{\sigma}\cdot\vec{S} = \int_V dV\vec{f}\cdot\vec{v} + \frac{d}{dt}\int_V dVw$$

$\vec{f}$:场对电荷作用力密度　　$\vec{v}$:电荷运动速度

$$-\nabla\cdot\vec{S} = \vec{f}\cdot\vec{v} + \frac{\partial w}{\partial t}$$

- 动量转化与守恒定律： 单位时间通过表面流入V的动量等于V 内电荷所受的力与V内单位时间电磁场动量的增加之和。

$$-\oint_V d\vec{\sigma}\cdot\overleftrightarrow{\mathcal{T}} = \int_V dV\vec{f} + \frac{d}{dt}\int_V dV\vec{g}$$

- **能量转化与守恒定律:** 单位时间通过表面流入V的能量等于单位时间场对 V内电荷所做的功(即功率)与V内单位时间电磁场能量的增加之和。

$$-\oint_V d\vec{\sigma}\cdot\vec{S}=\int_V dV\vec{f}\cdot\vec{v}+\frac{d}{dt}\int_V dVw$$

$\vec{f}$:场对电荷作用力密度　　$\vec{v}$:电荷运动速度

$$-\nabla\cdot\vec{S}=\vec{f}\cdot\vec{v}+\frac{\partial w}{\partial t}$$

- **动量转化与守恒定律:** 单位时间通过表面流入V的动量等于V 内电荷所受的力与V内单位时间电磁场动量的增加之和。

$$-\oint_V d\vec{\sigma}\cdot\vec{\vec{\mathcal{J}}}=\int_V dV\vec{f}+\frac{d}{dt}\int_V dV\vec{g}$$

$$-\nabla\cdot\vec{\vec{\mathcal{J}}}=\vec{f}+\frac{\partial\vec{g}}{\partial t}$$

能动量的转化与守恒：场和电荷系统能动量转化和守恒定律的一般形式

物理量	密度	流密度	守恒定律	守恒荷
电荷	ρ	$\vec{j}$	$\nabla\cdot\vec{j}+\frac{\partial\rho}{\partial t}=0$	$Q=\int dV\rho$
能量	w	$\vec{S}$	$\nabla\cdot\vec{S}+\frac{\partial w}{\partial t}+\vec{f}\cdot\vec{v}=0$	$E_{能量}=\int dVw$
动量	$\vec{g}$	$\overleftrightarrow{\mathcal{J}}$	$\nabla\cdot\overleftrightarrow{\mathcal{J}}+\frac{\partial\vec{g}}{\partial t}+\vec{f}=0$	$\vec{P}_{动量}=\int dV\vec{g}$

$\vec{f} \cdot \vec{v}$

$$\vec{\mathbf{f}} \cdot \vec{\mathbf{v}} = (\rho_{\mathrm{f}} \vec{\mathbf{E}} + \rho_{\mathrm{f}} \vec{\mathbf{v}} \times \vec{\mathbf{B}}) \cdot \vec{\mathbf{v}}$$

$$\vec{\mathbf{f}} \cdot \vec{\mathbf{v}} = (\rho_{\mathrm{f}} \vec{\mathbf{E}} + \rho_{\mathrm{f}} \vec{\mathbf{v}} \times \vec{\mathbf{B}}) \cdot \vec{\mathbf{v}} = \rho_{\mathrm{f}} \vec{\mathbf{v}} \cdot \vec{\mathbf{E}}$$

$$\vec{\mathbf{f}} \cdot \vec{\mathbf{v}} = (\rho_{\mathrm{f}}\vec{\mathbf{E}} + \rho_{\mathrm{f}}\vec{\mathbf{v}} \times \vec{\mathbf{B}}) \cdot \vec{\mathbf{v}} = \rho_{\mathrm{f}}\vec{\mathbf{v}} \cdot \vec{\mathbf{E}} = \vec{\mathbf{j}}_{\mathrm{c}} \cdot \vec{\mathbf{E}}$$

$$\vec{\mathbf{f}} \cdot \vec{\mathbf{v}} = (\rho_{\mathrm{f}} \vec{\mathbf{E}} + \rho_{\mathrm{f}} \vec{\mathbf{v}} \times \vec{\mathbf{B}}) \cdot \vec{\mathbf{v}} = \rho_{\mathrm{f}} \vec{\mathbf{v}} \cdot \vec{\mathbf{E}} = \vec{\mathbf{j}}_{\mathbf{c}} \cdot \vec{\mathbf{E}} = \vec{\mathbf{E}} \cdot (\nabla \times \frac{\vec{\mathbf{B}}}{\mu_0} - \frac{\partial \epsilon_0 \vec{\mathbf{E}}}{\partial \mathbf{t}})$$

$$\vec{\mathbf{f}}\cdot\vec{\mathbf{v}} = (\rho_{\mathrm{f}}\vec{\mathbf{E}} + \rho_{\mathrm{f}}\vec{\mathbf{v}}\times\vec{\mathbf{B}})\cdot\vec{\mathbf{v}} = \rho_{\mathrm{f}}\vec{\mathbf{v}}\cdot\vec{\mathbf{E}} = \vec{\mathbf{j}}_{\mathbf{c}}\cdot\vec{\mathbf{E}} = \vec{\mathbf{E}}\cdot(\nabla\times\frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}} - \frac{\partial\epsilon_{\mathbf{0}}\vec{\mathbf{E}}}{\partial\mathbf{t}})$$

$$= \vec{\mathbf{E}}\cdot(\nabla\times\frac{\vec{\mathbf{B}}}{\mu_{\mathbf{0}}}) - \vec{\mathbf{E}}\cdot\frac{\partial\epsilon_{\mathbf{0}}\vec{\mathbf{E}}}{\partial\mathbf{t}}$$

$$\vec{\mathbf{f}} \cdot \vec{\mathbf{v}} = (\rho_{\mathrm{f}} \vec{\mathbf{E}} + \rho_{\mathrm{f}} \vec{\mathbf{v}} \times \vec{\mathbf{B}}) \cdot \vec{\mathbf{v}} = \rho_{\mathrm{f}} \vec{\mathbf{v}} \cdot \vec{\mathbf{E}} = \vec{\mathbf{j}}_{\mathbf{c}} \cdot \vec{\mathbf{E}} = \vec{\mathbf{E}} \cdot (\nabla \times \frac{\vec{\mathbf{B}}}{\mu_0} - \frac{\partial \epsilon_0 \vec{\mathbf{E}}}{\partial \mathbf{t}})$$

$$= \vec{\mathbf{E}} \cdot (\nabla \times \frac{\vec{\mathbf{B}}}{\mu_0}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_0 \vec{\mathbf{E}}}{\partial \mathbf{t}} = -\nabla \cdot (\vec{\mathbf{E}} \times \frac{\vec{\mathbf{B}}}{\mu_0}) + \frac{\vec{\mathbf{B}}}{\mu_0} \cdot (\nabla \times \vec{\mathbf{E}}) - \vec{\mathbf{E}} \cdot \frac{\partial \epsilon_0 \vec{\mathbf{E}}}{\partial \mathbf{t}}$$

$$\vec{f}\cdot\vec{v} = (\rho_f\vec{E} + \rho_f\vec{v}\times\vec{B})\cdot\vec{v} = \rho_f\vec{v}\cdot\vec{E} = \vec{j}_c\cdot\vec{E} = \vec{E}\cdot(\nabla\times\frac{\vec{B}}{\mu_0} - \frac{\partial\epsilon_0\vec{E}}{\partial t})$$

$$= \vec{E}\cdot(\nabla\times\frac{\vec{B}}{\mu_0}) - \vec{E}\cdot\frac{\partial\epsilon_0\vec{E}}{\partial t} = -\nabla\cdot(\vec{E}\times\frac{\vec{B}}{\mu_0}) + \frac{\vec{B}}{\mu_0}\cdot(\nabla\times\vec{E}) - \vec{E}\cdot\frac{\partial\epsilon_0\vec{E}}{\partial t}$$

$$= -\nabla\cdot(\vec{E}\times\frac{\vec{B}}{\mu_0}) - \vec{E}\cdot\frac{\partial\epsilon_0\vec{E}}{\partial t} - \frac{\vec{B}}{\mu_0}\cdot\frac{\partial\vec{B}}{\partial t}$$

Back
Close

$$\vec{f}\cdot\vec{v} = (\rho_f\vec{E} + \rho_f\vec{v}\times\vec{B})\cdot\vec{v} = \rho_f\vec{v}\cdot\vec{E} = \vec{j}_c\cdot\vec{E} = \vec{E}\cdot(\nabla\times\frac{\vec{B}}{\mu_0} - \frac{\partial\epsilon_0\vec{E}}{\partial t})$$

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$$= -\nabla\cdot(\vec{E}\times\frac{\vec{B}}{\mu_0}) - \vec{E}\cdot\frac{\partial\epsilon_0\vec{E}}{\partial t} - \frac{\vec{B}}{\mu_0}\cdot\frac{\partial\vec{B}}{\partial t} = -\nabla\cdot\vec{S} - \frac{\partial w}{\partial t}$$

$$\vec{f}\cdot\vec{v} = (\rho_f\vec{E} + \rho_f\vec{v}\times\vec{B})\cdot\vec{v} = \rho_f\vec{v}\cdot\vec{E} = \vec{j}_c\cdot\vec{E} = \vec{E}\cdot(\nabla\times\frac{\vec{B}}{\mu_0} - \frac{\partial\epsilon_0\vec{E}}{\partial t})$$

$$= \vec{E}\cdot(\nabla\times\frac{\vec{B}}{\mu_0}) - \vec{E}\cdot\frac{\partial\epsilon_0\vec{E}}{\partial t} = -\nabla\cdot(\vec{E}\times\frac{\vec{B}}{\mu_0}) + \frac{\vec{B}}{\mu_0}\cdot(\nabla\times\vec{E}) - \vec{E}\cdot\frac{\partial\epsilon_0\vec{E}}{\partial t}$$

$$= -\nabla\cdot(\vec{E}\times\frac{\vec{B}}{\mu_0}) - \vec{E}\cdot\frac{\partial\epsilon_0\vec{E}}{\partial t} - \frac{\vec{B}}{\mu_0}\cdot\frac{\partial\vec{B}}{\partial t} = -\nabla\cdot\vec{S} - \frac{\partial w}{\partial t}$$

$$\vec{S} = \vec{E}\times\frac{\vec{B}}{\mu_0} \qquad \frac{\partial w}{\partial t} = \vec{E}\cdot\frac{\partial\epsilon_0\vec{E}}{\partial t} + \frac{\vec{B}}{\mu_0}\cdot\frac{\partial\vec{B}}{\partial t}$$

Back
Close

$$\vec{f}\cdot\vec{v} = (\rho_f\vec{E} + \rho_f\vec{v}\times\vec{B})\cdot\vec{v} = \rho_f\vec{v}\cdot\vec{E} = \vec{j}_c\cdot\vec{E} = \vec{E}\cdot(\nabla\times\frac{\vec{B}}{\mu_0} - \frac{\partial\epsilon_0\vec{E}}{\partial t})$$

$$= \vec{E}\cdot(\nabla\times\frac{\vec{B}}{\mu_0}) - \vec{E}\cdot\frac{\partial\epsilon_0\vec{E}}{\partial t} = -\nabla\cdot(\vec{E}\times\frac{\vec{B}}{\mu_0}) + \frac{\vec{B}}{\mu_0}\cdot(\nabla\times\vec{E}) - \vec{E}\cdot\frac{\partial\epsilon_0\vec{E}}{\partial t}$$

$$= -\nabla\cdot(\vec{E}\times\frac{\vec{B}}{\mu_0}) - \vec{E}\cdot\frac{\partial\epsilon_0\vec{E}}{\partial t} - \frac{\vec{B}}{\mu_0}\cdot\frac{\partial\vec{B}}{\partial t} = -\nabla\cdot\vec{S} - \frac{\partial w}{\partial t}$$

$$\vec{S} = \vec{E}\times\frac{\vec{B}}{\mu_0} \qquad\qquad \frac{\partial w}{\partial t} = \vec{E}\cdot\frac{\partial\epsilon_0\vec{E}}{\partial t} + \frac{\vec{B}}{\mu_0}\cdot\frac{\partial\vec{B}}{\partial t}$$

$$\frac{\partial w}{\partial t}$$

电磁相互作用能量动量的转化与守恒： 能量密度与能流密度

$$\vec{f}\cdot\vec{v} = (\rho_f\vec{E} + \rho_f\vec{v}\times\vec{B})\cdot\vec{v} = \rho_f\vec{v}\cdot\vec{E} = \vec{j}_c\cdot\vec{E} = \vec{E}\cdot(\nabla\times\frac{\vec{B}}{\mu_0} - \frac{\partial\epsilon_0\vec{E}}{\partial t})$$

$$= \vec{E}\cdot(\nabla\times\frac{\vec{B}}{\mu_0}) - \vec{E}\cdot\frac{\partial\epsilon_0\vec{E}}{\partial t} = -\nabla\cdot(\vec{E}\times\frac{\vec{B}}{\mu_0}) + \frac{\vec{B}}{\mu_0}\cdot(\nabla\times\vec{E}) - \vec{E}\cdot\frac{\partial\epsilon_0\vec{E}}{\partial t}$$

$$= -\nabla\cdot(\vec{E}\times\frac{\vec{B}}{\mu_0}) - \vec{E}\cdot\frac{\partial\epsilon_0\vec{E}}{\partial t} - \frac{\vec{B}}{\mu_0}\cdot\frac{\partial\vec{B}}{\partial t} = -\nabla\cdot\vec{S} - \frac{\partial w}{\partial t}$$

$$\vec{S} = \vec{E}\times\frac{\vec{B}}{\mu_0} \qquad \frac{\partial w}{\partial t} = \vec{E}\cdot\frac{\partial\epsilon_0\vec{E}}{\partial t} + \frac{\vec{B}}{\mu_0}\cdot\frac{\partial\vec{B}}{\partial t}$$

$$\frac{\partial w}{\partial t} = \epsilon_0\vec{E}\cdot\frac{\partial\vec{E}}{\partial t} + \frac{1}{\mu_0}\vec{B}\cdot\frac{\partial\vec{B}}{\partial t}$$

电磁相互作用能量动量的转化与守恒： 能量密度与能流密度

$$\vec{f}\cdot\vec{v} = (\rho_f\vec{E} + \rho_f\vec{v}\times\vec{B})\cdot\vec{v} = \rho_f\vec{v}\cdot\vec{E} = \vec{j}_c\cdot\vec{E} = \vec{E}\cdot(\nabla\times\frac{\vec{B}}{\mu_0} - \frac{\partial\epsilon_0\vec{E}}{\partial t})$$

$$= \vec{E}\cdot(\nabla\times\frac{\vec{B}}{\mu_0}) - \vec{E}\cdot\frac{\partial\epsilon_0\vec{E}}{\partial t} = -\nabla\cdot(\vec{E}\times\frac{\vec{B}}{\mu_0}) + \frac{\vec{B}}{\mu_0}\cdot(\nabla\times\vec{E}) - \vec{E}\cdot\frac{\partial\epsilon_0\vec{E}}{\partial t}$$

$$= -\nabla\cdot(\vec{E}\times\frac{\vec{B}}{\mu_0}) - \vec{E}\cdot\frac{\partial\epsilon_0\vec{E}}{\partial t} - \frac{\vec{B}}{\mu_0}\cdot\frac{\partial\vec{B}}{\partial t} = -\nabla\cdot\vec{S} - \frac{\partial w}{\partial t}$$

$$\vec{S} = \vec{E}\times\frac{\vec{B}}{\mu_0} \qquad \frac{\partial w}{\partial t} = \vec{E}\cdot\frac{\partial\epsilon_0\vec{E}}{\partial t} + \frac{\vec{B}}{\mu_0}\cdot\frac{\partial\vec{B}}{\partial t}$$

$$\frac{\partial w}{\partial t} = \epsilon_0\vec{E}\cdot\frac{\partial\vec{E}}{\partial t} + \frac{1}{\mu_0}\vec{B}\cdot\frac{\partial\vec{B}}{\partial t} = \frac{\partial}{\partial t}\frac{1}{2}(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2)$$

$$\vec{f}\cdot\vec{v} = (\rho_f\vec{E} + \rho_f\vec{v}\times\vec{B})\cdot\vec{v} = \rho_f\vec{v}\cdot\vec{E} = \vec{j}_c\cdot\vec{E} = \vec{E}\cdot(\nabla\times\frac{\vec{B}}{\mu_0} - \frac{\partial\epsilon_0\vec{E}}{\partial t})$$

$$= \vec{E}\cdot(\nabla\times\frac{\vec{B}}{\mu_0}) - \vec{E}\cdot\frac{\partial\epsilon_0\vec{E}}{\partial t} = -\nabla\cdot(\vec{E}\times\frac{\vec{B}}{\mu_0}) + \frac{\vec{B}}{\mu_0}\cdot(\nabla\times\vec{E}) - \vec{E}\cdot\frac{\partial\epsilon_0\vec{E}}{\partial t}$$

$$= -\nabla\cdot(\vec{E}\times\frac{\vec{B}}{\mu_0}) - \vec{E}\cdot\frac{\partial\epsilon_0\vec{E}}{\partial t} - \frac{\vec{B}}{\mu_0}\cdot\frac{\partial\vec{B}}{\partial t} = -\nabla\cdot\vec{S} - \frac{\partial w}{\partial t}$$

$$\vec{S} = \vec{E}\times\frac{\vec{B}}{\mu_0} \qquad \frac{\partial w}{\partial t} = \vec{E}\cdot\frac{\partial\epsilon_0\vec{E}}{\partial t} + \frac{\vec{B}}{\mu_0}\cdot\frac{\partial\vec{B}}{\partial t}$$

$$\frac{\partial w}{\partial t} = \epsilon_0\vec{E}\cdot\frac{\partial\vec{E}}{\partial t} + \frac{1}{\mu_0}\vec{B}\cdot\frac{\partial\vec{B}}{\partial t} = \frac{\partial}{\partial t}\frac{1}{2}(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2) = \frac{\partial}{\partial t}\frac{1}{2}(\vec{E}\cdot\vec{D} + \vec{H}\cdot\vec{B})$$

Back
Close

电磁相互作用能量动量的转化与守恒：能量密度与能流密度

$$\vec{f}\cdot\vec{v} = (\rho_f\vec{E}+\rho_f\vec{v}\times\vec{B})\cdot\vec{v} = \rho_f\vec{v}\cdot\vec{E} = \vec{j}_c\cdot\vec{E} = \vec{E}\cdot(\nabla\times\frac{\vec{B}}{\mu_0} - \frac{\partial\epsilon_0\vec{E}}{\partial t})$$

$$= \vec{E}\cdot(\nabla\times\frac{\vec{B}}{\mu_0}) - \vec{E}\cdot\frac{\partial\epsilon_0\vec{E}}{\partial t} = -\nabla\cdot(\vec{E}\times\frac{\vec{B}}{\mu_0}) + \frac{\vec{B}}{\mu_0}\cdot(\nabla\times\vec{E}) - \vec{E}\cdot\frac{\partial\epsilon_0\vec{E}}{\partial t}$$

$$= -\nabla\cdot(\vec{E}\times\frac{\vec{B}}{\mu_0}) - \vec{E}\cdot\frac{\partial\epsilon_0\vec{E}}{\partial t} - \frac{\vec{B}}{\mu_0}\cdot\frac{\partial\vec{B}}{\partial t} = -\nabla\cdot\vec{S} - \frac{\partial w}{\partial t}$$

$$\vec{S} = \vec{E}\times\frac{\vec{B}}{\mu_0} \qquad \frac{\partial w}{\partial t} = \vec{E}\cdot\frac{\partial\epsilon_0\vec{E}}{\partial t} + \frac{\vec{B}}{\mu_0}\cdot\frac{\partial\vec{B}}{\partial t}$$

$$\frac{\partial w}{\partial t} = \epsilon_0\vec{E}\cdot\frac{\partial\vec{E}}{\partial t} + \frac{1}{\mu_0}\vec{B}\cdot\frac{\partial\vec{B}}{\partial t} = \frac{\partial}{\partial t}\frac{1}{2}(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2) = \frac{\partial}{\partial t}\frac{1}{2}(\vec{E}\cdot\vec{D}+\vec{H}\cdot\vec{B})$$

$$w = \frac{1}{2}(\vec{E}\cdot\vec{D}+\vec{H}\cdot\vec{B})$$

场具有能量说明场是物质的一种形态！

$\vec{f}$

Back
Close

$$\vec{\mathbf{f}} = \rho_{\mathrm{f}}\vec{\mathbf{E}} + \vec{\mathbf{j}}_{\mathrm{c}} \times \vec{\mathbf{B}}$$

$$\vec{\mathbf{f}} = \rho_{\mathrm{f}}\vec{\mathbf{E}} + \vec{\mathbf{j}}_{\mathrm{c}} \times \vec{\mathbf{B}} = (\nabla \cdot \epsilon_0 \vec{\mathbf{E}})\vec{\mathbf{E}} + (\nabla \times \frac{1}{\mu_0}\vec{\mathbf{B}} - \frac{\partial \epsilon_0 \vec{\mathbf{E}}}{\partial \mathbf{t}}) \times \vec{\mathbf{B}}$$

$$\vec{\mathbf{f}} = \rho_{\mathrm{f}}\vec{\mathbf{E}} + \vec{\mathbf{j}}_{\mathrm{c}} \times \vec{\mathbf{B}} = (\nabla \cdot \epsilon_0\vec{\mathbf{E}})\vec{\mathbf{E}} + (\nabla \times \frac{1}{\mu_0}\vec{\mathbf{B}} - \frac{\partial \epsilon_0\vec{\mathbf{E}}}{\partial \mathbf{t}}) \times \vec{\mathbf{B}}$$

$$= \epsilon_0(\nabla \cdot \vec{\mathbf{E}})\vec{\mathbf{E}} + \frac{1}{\mu_0}(\nabla \times \vec{\mathbf{B}}) \times \vec{\mathbf{B}} - \epsilon_0\frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}} \times \vec{\mathbf{B}}$$

电磁相互作用能量动量的转化与守恒: 动量密度与动量流密度

$$\vec{f} = \rho_f \vec{E} + \vec{j}_c \times \vec{B} = (\nabla \cdot \epsilon_0 \vec{E})\vec{E} + (\nabla \times \frac{1}{\mu_0}\vec{B} - \frac{\partial \epsilon_0 \vec{E}}{\partial t}) \times \vec{B}$$

$$= \epsilon_0 (\nabla \cdot \vec{E})\vec{E} + \frac{1}{\mu_0}(\nabla \times \vec{B}) \times \vec{B} - \epsilon_0 \frac{\partial \vec{E}}{\partial t} \times \vec{B}$$

$$= \epsilon_0 (\nabla \cdot \vec{E})\vec{E} + \frac{1}{\mu_0}[(\nabla \cdot \vec{B})\vec{B} + (\nabla \times \vec{B}) \times \vec{B}]$$

$$-\epsilon_0 [\frac{\partial \vec{E}}{\partial t} \times \vec{B} + \vec{E} \times \frac{\partial \vec{B}}{\partial t} + \vec{E} \times (\nabla \times \vec{E})]$$

$$\vec{\mathbf{f}} = \rho_{\mathrm{f}}\vec{\mathbf{E}} + \vec{\mathbf{j}}_{\mathrm{c}} \times \vec{\mathbf{B}} = (\nabla \cdot \epsilon_0\vec{\mathbf{E}})\vec{\mathbf{E}} + (\nabla \times \frac{1}{\mu_0}\vec{\mathbf{B}} - \frac{\partial \epsilon_0\vec{\mathbf{E}}}{\partial \mathrm{t}}) \times \vec{\mathbf{B}}$$

$$= \epsilon_0(\nabla \cdot \vec{\mathbf{E}})\vec{\mathbf{E}} + \frac{1}{\mu_0}(\nabla \times \vec{\mathbf{B}}) \times \vec{\mathbf{B}} - \epsilon_0\frac{\partial \vec{\mathbf{E}}}{\partial \mathrm{t}} \times \vec{\mathbf{B}}$$

$$= \epsilon_0(\nabla \cdot \vec{\mathbf{E}})\vec{\mathbf{E}} + \frac{1}{\mu_0}[(\nabla \cdot \vec{\mathbf{B}})\vec{\mathbf{B}} + (\nabla \times \vec{\mathbf{B}}) \times \vec{\mathbf{B}}]$$

$$-\epsilon_0[\frac{\partial \vec{\mathbf{E}}}{\partial \mathrm{t}} \times \vec{\mathbf{B}} + \vec{\mathbf{E}} \times \frac{\partial \vec{\mathbf{B}}}{\partial \mathrm{t}} + \vec{\mathbf{E}} \times (\nabla \times \vec{\mathbf{E}})]$$

$$= \epsilon_0[(\nabla \cdot \vec{\mathbf{E}})\vec{\mathbf{E}} + (\nabla \times \vec{\mathbf{E}}) \times \vec{\mathbf{E}}] + \frac{1}{\mu_0}[(\nabla \cdot \vec{\mathbf{B}})\vec{\mathbf{B}} + (\nabla \times \vec{\mathbf{B}}) \times \vec{\mathbf{B}}]$$

$$-\epsilon_0\frac{\partial}{\partial \mathrm{t}}(\vec{\mathbf{E}} \times \vec{\mathbf{B}})$$

$$
\begin{aligned}
\vec{\mathrm{f}} &= \rho_{\mathrm{f}}\vec{\mathrm{E}} + \vec{\mathrm{j}}_{\mathrm{c}} \times \vec{\mathrm{B}} = (\nabla \cdot \epsilon_0 \vec{\mathrm{E}})\vec{\mathrm{E}} + (\nabla \times \frac{1}{\mu_0}\vec{\mathrm{B}} - \frac{\partial \epsilon_0 \vec{\mathrm{E}}}{\partial \mathrm{t}}) \times \vec{\mathrm{B}} \\
&= \epsilon_0(\nabla \cdot \vec{\mathrm{E}})\vec{\mathrm{E}} + \frac{1}{\mu_0}(\nabla \times \vec{\mathrm{B}}) \times \vec{\mathrm{B}} - \epsilon_0 \frac{\partial \vec{\mathrm{E}}}{\partial \mathrm{t}} \times \vec{\mathrm{B}} \\
&= \epsilon_0(\nabla \cdot \vec{\mathrm{E}})\vec{\mathrm{E}} + \frac{1}{\mu_0}[(\nabla \cdot \vec{\mathrm{B}})\vec{\mathrm{B}} + (\nabla \times \vec{\mathrm{B}}) \times \vec{\mathrm{B}}] \\
&\quad -\epsilon_0[\frac{\partial \vec{\mathrm{E}}}{\partial \mathrm{t}} \times \vec{\mathrm{B}} + \vec{\mathrm{E}} \times \frac{\partial \vec{\mathrm{B}}}{\partial \mathrm{t}} + \vec{\mathrm{E}} \times (\nabla \times \vec{\mathrm{E}})] \\
&= \epsilon_0[(\nabla \cdot \vec{\mathrm{E}})\vec{\mathrm{E}} + (\nabla \times \vec{\mathrm{E}}) \times \vec{\mathrm{E}}] + \frac{1}{\mu_0}[(\nabla \cdot \vec{\mathrm{B}})\vec{\mathrm{B}} + (\nabla \times \vec{\mathrm{B}}) \times \vec{\mathrm{B}}] \\
&\quad -\epsilon_0 \frac{\partial}{\partial \mathrm{t}}(\vec{\mathrm{E}} \times \vec{\mathrm{B}}) \\
&= \epsilon_0[(\nabla \cdot \vec{\mathrm{E}})\vec{\mathrm{E}} + (\vec{\mathrm{E}} \cdot \nabla)\vec{\mathrm{E}} - (\nabla \vec{\mathrm{E}}) \cdot \vec{\mathrm{E}}] \\
&\quad + \frac{1}{\mu_0}[(\nabla \cdot \vec{\mathrm{B}})\vec{\mathrm{B}} + (\vec{\mathrm{B}} \cdot \nabla)\vec{\mathrm{B}} - (\nabla \vec{\mathrm{B}}) \cdot \vec{\mathrm{B}}] - \epsilon_0 \frac{\partial}{\partial \mathrm{t}}(\vec{\mathrm{E}} \times \vec{\mathrm{B}})
\end{aligned}
$$

Back
Close

$$\vec{\mathbf{f}} = \epsilon_0[(\nabla \cdot \vec{\mathbf{E}})\vec{\mathbf{E}} + (\vec{\mathbf{E}} \cdot \nabla)\vec{\mathbf{E}} - (\nabla\vec{\mathbf{E}}) \cdot \vec{\mathbf{E}}]$$
$$+\frac{1}{\mu_0}[(\nabla \cdot \vec{\mathbf{B}})\vec{\mathbf{B}} + (\vec{\mathbf{B}} \cdot \nabla)\vec{\mathbf{B}} - (\nabla\vec{\mathbf{B}}) \cdot \vec{\mathbf{B}}] - \epsilon_0\frac{\partial}{\partial \mathbf{t}}(\vec{\mathbf{E}} \times \vec{\mathbf{B}})$$

$$\vec{\mathbf{f}} = \epsilon_0[(\nabla\cdot\vec{\mathbf{E}})\vec{\mathbf{E}} + (\vec{\mathbf{E}}\cdot\nabla)\vec{\mathbf{E}} - (\nabla\vec{\mathbf{E}})\cdot\vec{\mathbf{E}}]$$

$$+\frac{1}{\mu_0}[(\nabla\cdot\vec{\mathbf{B}})\vec{\mathbf{B}} + (\vec{\mathbf{B}}\cdot\nabla)\vec{\mathbf{B}} - (\nabla\vec{\mathbf{B}})\cdot\vec{\mathbf{B}}] - \epsilon_0\frac{\partial}{\partial \mathbf{t}}(\vec{\mathbf{E}}\times\vec{\mathbf{B}})$$

$$= \epsilon_0[\nabla\cdot(\vec{\mathbf{E}}\vec{\mathbf{E}}) - \frac{1}{2}\nabla\mathbf{E}^2] + \frac{1}{\mu_0}[\nabla\cdot(\vec{\mathbf{B}}\vec{\mathbf{B}}) - \frac{1}{2}\nabla\mathbf{B}^2] - \epsilon_0\frac{\partial}{\partial \mathbf{t}}(\vec{\mathbf{E}}\times\vec{\mathbf{B}})$$

Back
Close

电磁相互作用能量动量的转化与守恒： 动量密度与动量流密度

$$
\begin{aligned}
\vec{\mathbf{f}} &= \epsilon_0[(\nabla\cdot\vec{\mathbf{E}})\vec{\mathbf{E}} + (\vec{\mathbf{E}}\cdot\nabla)\vec{\mathbf{E}} - (\nabla\vec{\mathbf{E}})\cdot\vec{\mathbf{E}}] \\
&\quad + \frac{1}{\mu_0}[(\nabla\cdot\vec{\mathbf{B}})\vec{\mathbf{B}} + (\vec{\mathbf{B}}\cdot\nabla)\vec{\mathbf{B}} - (\nabla\vec{\mathbf{B}})\cdot\vec{\mathbf{B}}] - \epsilon_0\frac{\partial}{\partial \mathbf{t}}(\vec{\mathbf{E}}\times\vec{\mathbf{B}}) \\
&= \epsilon_0[\nabla\cdot(\vec{\mathbf{E}}\vec{\mathbf{E}}) - \frac{1}{2}\nabla\mathbf{E}^2] + \frac{1}{\mu_0}[\nabla\cdot(\vec{\mathbf{B}}\vec{\mathbf{B}}) - \frac{1}{2}\nabla\mathbf{B}^2] - \epsilon_0\frac{\partial}{\partial \mathbf{t}}(\vec{\mathbf{E}}\times\vec{\mathbf{B}}) \\
&= \epsilon_0\nabla\cdot(\vec{\mathbf{E}}\vec{\mathbf{E}} - \frac{1}{2}\overset{\rightharpoonup\rightharpoonup}{\mathbf{I}}\,\mathbf{E}^2) + \frac{1}{\mu_0}\nabla\cdot(\vec{\mathbf{B}}\vec{\mathbf{B}} - \frac{1}{2}\overset{\rightharpoonup\rightharpoonup}{\mathbf{I}}\,\mathbf{B}^2) - \epsilon_0\frac{\partial}{\partial \mathbf{t}}(\vec{\mathbf{E}}\times\vec{\mathbf{B}})
\end{aligned}
$$

Back
Close

电磁相互作用能量动量的转化与守恒：动量密度与动量流密度

$$
\begin{aligned}
\vec{f} &= \epsilon_0[(\nabla\cdot\vec{E})\vec{E} + (\vec{E}\cdot\nabla)\vec{E} - (\nabla\vec{E})\cdot\vec{E}] \\
&\quad + \frac{1}{\mu_0}[(\nabla\cdot\vec{B})\vec{B} + (\vec{B}\cdot\nabla)\vec{B} - (\nabla\vec{B})\cdot\vec{B}] - \epsilon_0\frac{\partial}{\partial t}(\vec{E}\times\vec{B}) \\
&= \epsilon_0[\nabla\cdot(\vec{E}\vec{E}) - \frac{1}{2}\nabla E^2] + \frac{1}{\mu_0}[\nabla\cdot(\vec{B}\vec{B}) - \frac{1}{2}\nabla B^2] - \epsilon_0\frac{\partial}{\partial t}(\vec{E}\times\vec{B}) \\
&= \epsilon_0\nabla\cdot(\vec{E}\vec{E} - \frac{1}{2}\overleftrightarrow{I}\,E^2) + \frac{1}{\mu_0}\nabla\cdot(\vec{B}\vec{B} - \frac{1}{2}\overleftrightarrow{I}\,B^2) - \epsilon_0\frac{\partial}{\partial t}(\vec{E}\times\vec{B}) \\
&= -\nabla\cdot\overleftrightarrow{\mathcal{J}} - \frac{\partial\vec{g}}{\partial t}
\end{aligned}
$$

$$\vec{f} = \epsilon_0[(\nabla\cdot\vec{E})\vec{E} + (\vec{E}\cdot\nabla)\vec{E} - (\nabla\vec{E})\cdot\vec{E}] + \frac{1}{\mu_0}[(\nabla\cdot\vec{B})\vec{B} + (\vec{B}\cdot\nabla)\vec{B} - (\nabla\vec{B})\cdot\vec{B}] - \epsilon_0\frac{\partial}{\partial t}(\vec{E}\times\vec{B})$$

$$= \epsilon_0[\nabla\cdot(\vec{E}\vec{E}) - \frac{1}{2}\nabla E^2] + \frac{1}{\mu_0}[\nabla\cdot(\vec{B}\vec{B}) - \frac{1}{2}\nabla B^2] - \epsilon_0\frac{\partial}{\partial t}(\vec{E}\times\vec{B})$$

$$= \epsilon_0\nabla\cdot(\vec{E}\vec{E} - \frac{1}{2}\overleftrightarrow{I}E^2) + \frac{1}{\mu_0}\nabla\cdot(\vec{B}\vec{B} - \frac{1}{2}\overleftrightarrow{I}B^2) - \epsilon_0\frac{\partial}{\partial t}(\vec{E}\times\vec{B})$$

$$= -\nabla\cdot\overleftrightarrow{\mathcal{J}} - \frac{\partial\vec{g}}{\partial t}$$

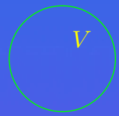
$$\vec{g} = \epsilon_0\vec{E}\times\vec{B} = \epsilon_0\mu_0\vec{S}$$

$$\overleftrightarrow{\mathcal{J}} = -\epsilon_0\vec{E}\vec{E} - \frac{1}{\mu_0}\vec{B}\vec{B} + \frac{1}{2}\overleftrightarrow{I}(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2)$$

两带电体之间的牛顿第三定律一般不成立！ $\vec{f}_{21} = \frac{dP_1}{dt}$ $\vec{f}_{12} = \frac{dP_2}{dt}$

Back
Close

选体积为V的内部区域受外部电磁力作用：



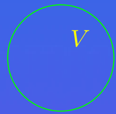
电磁相互作用能量动量的转化与守恒：张量力

选体积为V的内部区域受外部电磁力作用：

$$-\oint_{\mathrm{V}} \mathrm{d}\vec{\sigma}\cdot \overleftrightarrow{\mathcal{T}} = \int_{\mathrm{V}} \mathrm{dV}\vec{\mathrm{f}} + \frac{\mathrm{d}}{\mathrm{dt}}\int_{\mathrm{V}} \mathrm{dV}\vec{\mathrm{g}}$$

选体积为V的内部区域受外部电磁力作用：

$$-\oint_{V} \mathrm{d}\vec{\sigma}\cdot \overset{\rightharpoonup\rightharpoonup}{\mathcal{T}} = \int_{V} \mathrm{dV}\vec{\mathrm{f}} + \frac{\mathrm{d}}{\mathrm{dt}}\int_{V} \mathrm{dV}\vec{\mathrm{g}}$$



$$\int_{V} \mathrm{dV}\vec{\mathrm{f}} = \frac{\mathrm{d}\vec{\mathrm{P}}_{\text{机械}}}{\mathrm{dt}} \qquad \vec{\mathrm{P}}_{\text{电磁}} \equiv \int_{V} \mathrm{dV}\vec{\mathrm{g}}$$

选体积为V的内部区域受外部电磁力作用:

$$-\oint_V \mathrm{d}\vec{\sigma}\cdot \overset{\rightharpoonup\rightharpoonup}{\mathcal{T}} = \int_V \mathrm{dV}\vec{\mathrm{f}} + \frac{\mathrm{d}}{\mathrm{dt}}\int_V \mathrm{dV}\vec{\mathrm{g}}$$

$$\int_V \mathrm{dV}\vec{\mathrm{f}} = \frac{\mathrm{d}\vec{\mathrm{P}}_{\text{机械}}}{\mathrm{dt}} \qquad \vec{\mathrm{P}}_{\text{电磁}} \equiv \int_V \mathrm{dV}\vec{\mathrm{g}} \;\rightarrow\; \underbrace{\frac{\mathrm{d}}{\mathrm{dt}}[\vec{\mathrm{P}}_{\text{机械}}+\vec{\mathrm{P}}_{\text{电磁}}] = -\oint_V \mathrm{d}\vec{\sigma}\cdot\overset{\rightharpoonup\rightharpoonup}{\mathcal{T}}}_{\text{电磁力的全息性?}}$$

$-\overset{\rightharpoonup\rightharpoonup}{\mathcal{T}}$是$V$单位表面所受外面的力.

选体积为V的内部区域受外部电磁力作用：

$$-\oint_V \mathrm{d}\vec{\sigma}\cdot\overleftrightarrow{\mathcal{J}} = \int_V \mathrm{dV}\vec{\mathrm{f}} + \frac{\mathrm{d}}{\mathrm{dt}}\int_V \mathrm{dV}\vec{\mathrm{g}}$$

$$\int_V \mathrm{dV}\vec{\mathrm{f}} = \frac{\mathrm{d}\vec{\mathrm{P}}_{\text{机械}}}{\mathrm{dt}} \qquad \vec{\mathrm{P}}_{\text{电磁}} \equiv \int_V \mathrm{dV}\vec{\mathrm{g}} \;\to\; \underbrace{\frac{\mathrm{d}}{\mathrm{dt}}[\vec{\mathrm{P}}_{\text{机械}}+\vec{\mathrm{P}}_{\text{电磁}}] = -\oint_V \mathrm{d}\vec{\sigma}\cdot\overleftrightarrow{\mathcal{J}}}_{\text{电磁力的全息性？}}$$

$-\overleftrightarrow{\mathcal{J}}$ 是V单位表面所受外面的力.表面有电磁场就要受力.

电磁相互作用能量动量的转化与守恒: 张量力

选体积为V的内部区域受外部电磁力作用:

$$-\oint_V d\vec{\sigma}\cdot\overrightarrow{\overrightarrow{\mathcal{J}}} = \int_V dV\vec{f} + \frac{d}{dt}\int_V dV\vec{g}$$

$$\int_V dV\vec{f} = \frac{d\vec{P}_{机械}}{dt} \qquad \vec{P}_{电磁} \equiv \int_V dV\vec{g} \;\rightarrow\; \underbrace{\frac{d}{dt}[\vec{P}_{机械}+\vec{P}_{电磁}] = -\oint_V d\vec{\sigma}\cdot\overrightarrow{\overrightarrow{\mathcal{J}}}}_{\text{电磁力的全息性?}}$$

$-\overrightarrow{\overrightarrow{\mathcal{J}}}$是$V$单位表面所受外面的力.表面有电磁场就要受力.

表面法向方向单位矢量$\vec{n}$, 电场与$\vec{n}$的夹角θ_E, 电场投影所在的切向方向单位矢量$\vec{e}_E$, 磁场与$\vec{n}$的夹角θ_B, 磁场投影所在切向方向单位矢量$\vec{e}_B$.

电磁相互作用能量动量的转化与守恒：张量力

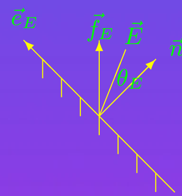
选体积为V的内部区域受外部电磁力作用：

$$-\oint_V d\vec{\sigma}\cdot\overset{\rightharpoonup\rightharpoonup}{\mathcal{T}}=\int_V dV\vec{f}+\frac{d}{dt}\int_V dV\vec{g}$$

$$\int_V dV\vec{f}=\frac{d\vec{P}_{\text{机械}}}{dt}\qquad \vec{P}_{\text{电磁}}\equiv\int_V dV\vec{g}\ \rightarrow\ \underbrace{\frac{d}{dt}[\vec{P}_{\text{机械}}+\vec{P}_{\text{电磁}}]=-\oint_V d\vec{\sigma}\cdot\overset{\rightharpoonup\rightharpoonup}{\mathcal{T}}}_{\text{电磁力的全息性？}}$$

$-\overset{\rightharpoonup\rightharpoonup}{\mathcal{T}}$是$V$单位表面所受外面的力.表面有电磁场就要受力.

表面法向方向单位矢量$\vec{n}$, 电场与$\vec{n}$的夹角θ_E, 电场投影所在的切向方向单位矢量$\vec{e}_E$, 磁场与$\vec{n}$的夹角θ_B, 磁场投影所在切向方向单位矢量$\vec{e}_B$. 此表面单位表面所受力:



$$\vec{f}_{\text{表面}}=-\vec{n}\cdot\overset{\rightharpoonup\rightharpoonup}{\mathcal{T}}=\epsilon_0(\vec{n}\cdot\vec{E}\vec{E}-\frac{1}{2}\vec{n}E^2)+\frac{1}{\mu_0}(\vec{n}\cdot\vec{B}\vec{B}-\frac{1}{2}\vec{n}B^2)$$

电磁相互作用能量动量的转化与守恒: 张量力

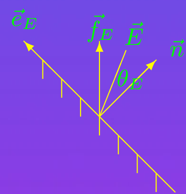
选体积为V的内部区域受外部电磁力作用:

$$-\oint_V d\vec{\sigma}\cdot \overset{\rightharpoonup\rightharpoonup}{\mathcal{T}} = \int_V dV\vec{f} + \frac{d}{dt}\int_V dV\vec{g}$$

$$\int_V dV\vec{f} = \frac{d\vec{P}_{\text{机械}}}{dt} \qquad \vec{P}_{\text{电磁}} \equiv \int_V dV\vec{g} \rightarrow \underbrace{\frac{d}{dt}[\vec{P}_{\text{机械}}+\vec{P}_{\text{电磁}}] = -\oint_V d\vec{\sigma}\cdot\overset{\rightharpoonup\rightharpoonup}{\mathcal{T}}}_{\text{电磁力的全息性?}}$$

$-\overset{\rightharpoonup\rightharpoonup}{\mathcal{T}}$是$V$单位表面所受外面的力.表面有电磁场就要受力.

表面法向方向单位矢量$\vec{n}$, 电场与$\vec{n}$的夹角θ_E, 电场投影所在的切向方向单位矢量$\vec{e}_E$, 磁场与$\vec{n}$的夹角θ_B, 磁场投影所在切向方向单位矢量$\vec{e}_B$. 此表面单位表面所受力:



$$\vec{f}_{\text{表面}} = -\vec{n}\cdot\overset{\rightharpoonup\rightharpoonup}{\mathcal{T}} = \epsilon_0(\vec{n}\cdot\vec{E}\vec{E} - \frac{1}{2}\vec{n}E^2) + \frac{1}{\mu_0}(\vec{n}\cdot\vec{B}\vec{B} - \frac{1}{2}\vec{n}B^2)$$

$$= (E\cos\theta_E\vec{n} + E\sin\theta_E\vec{e}_E)\epsilon_0 E\cos\theta_E - \frac{1}{2}\epsilon_0\vec{n}E^2$$

$$(B\cos\theta_B\vec{n} + B\sin\theta_B\vec{e}_B)\frac{B}{\mu_0}\cos\theta_B - \frac{1}{2\mu_0}\vec{n}B^2$$

电磁相互作用能量动量的转化与守恒：张量力

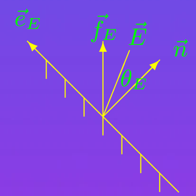
选体积为V的内部区域受外部电磁力作用：

$$-\oint_{V} d\vec{\sigma}\cdot\overset{\rightharpoonup\rightharpoonup}{\mathcal{J}}=\int_{V}dV\vec{f}+\frac{d}{dt}\int_{V}dV\vec{g}$$

$$\int_{V}dV\vec{f}=\frac{d\vec{P}_{\text{机械}}}{dt}\qquad \vec{P}_{\text{电磁}}\equiv\int_{V}dV\vec{g}\ \rightarrow\ \underbrace{\frac{d}{dt}[\vec{P}_{\text{机械}}+\vec{P}_{\text{电磁}}]=-\oint_{V}d\vec{\sigma}\cdot\overset{\rightharpoonup\rightharpoonup}{\mathcal{J}}}_{\text{电磁力的全息性？}}$$

$-\overset{\rightharpoonup\rightharpoonup}{\mathcal{J}}$是$V$单位表面所受外面的力.表面有电磁场就要受力.

表面法向方向单位矢量$\vec{n}$，电场与$\vec{n}$的夹角θ_E，电场投影所在的切向方向单位矢量$\vec{e}_E$，磁场与$\vec{n}$的夹角θ_B，磁场投影所在切向方向单位矢量$\vec{e}_B$. 此表面单位表面所受力：



$$\vec{f}_{\text{表面}}=-\vec{n}\cdot\overset{\rightharpoonup\rightharpoonup}{\mathcal{J}}=\epsilon_0(\vec{n}\cdot\vec{E}\vec{E}-\frac{1}{2}\vec{n}E^2)+\frac{1}{\mu_0}(\vec{n}\cdot\vec{B}\vec{B}-\frac{1}{2}\vec{n}B^2)$$

$$=(E\cos\theta_E\vec{n}+E\sin\theta_E\vec{e}_E)\epsilon_0E\cos\theta_E-\frac{1}{2}\epsilon_0\vec{n}E^2$$

$$(B\cos\theta_B\vec{n}+B\sin\theta_B\vec{e}_B)\frac{B}{\mu_0}\cos\theta_B-\frac{1}{2\mu_0}\vec{n}B^2=\vec{f}_E+\vec{f}_B$$

电磁相互作用能量动量的转化与守恒：张量力

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$$-\oint_V d\vec{\sigma}\cdot\overleftrightarrow{\mathcal{J}}=\int_V dV\vec{f}+\frac{d}{dt}\int_V dV\vec{g}$$

$$\int_V dV\vec{f}=\frac{d\vec{P}_{机械}}{dt}\qquad \vec{P}_{电磁}\equiv\int_V dV\vec{g}\ \rightarrow\ \underbrace{\frac{d}{dt}[\vec{P}_{机械}+\vec{P}_{电磁}]=-\oint_V d\vec{\sigma}\cdot\overleftrightarrow{\mathcal{J}}}_{电磁力的全息性?}$$

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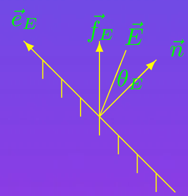


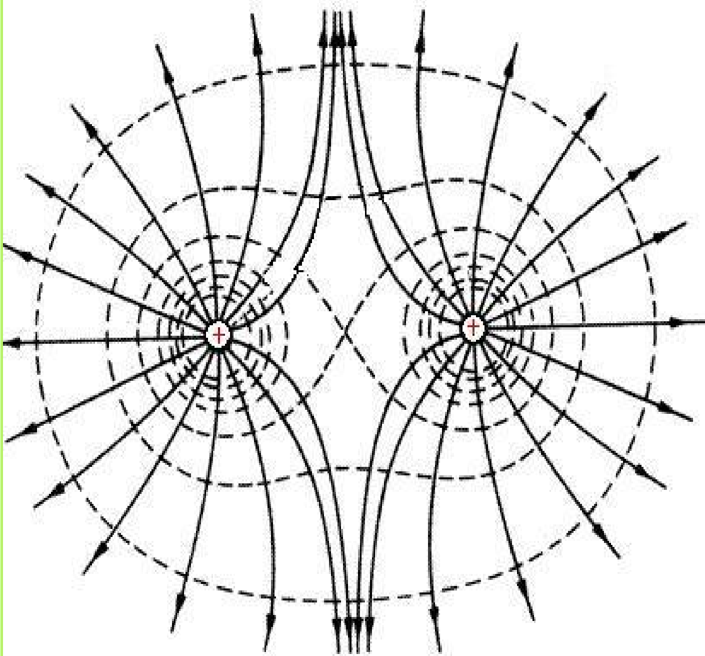
$$\vec{f}_{表面}=-\vec{n}\cdot\overleftrightarrow{\mathcal{J}}=\epsilon_0(\vec{n}\cdot\vec{E}\vec{E}-\frac{1}{2}\vec{n}E^2)+\frac{1}{\mu_0}(\vec{n}\cdot\vec{B}\vec{B}-\frac{1}{2}\vec{n}B^2)$$

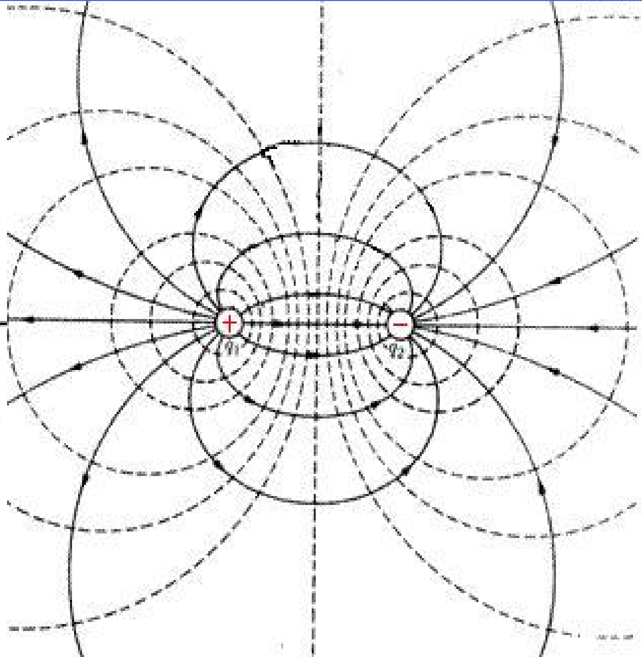
$$=(E\cos\theta_E\vec{n}+E\sin\theta_E\vec{e}_E)\epsilon_0E\cos\theta_E-\frac{1}{2}\epsilon_0\vec{n}E^2$$

$$(B\cos\theta_B\vec{n}+B\sin\theta_B\vec{e}_B)\frac{B}{\mu_0}\cos\theta_B-\frac{1}{2\mu_0}\vec{n}B^2=\vec{f}_E+\vec{f}_B$$

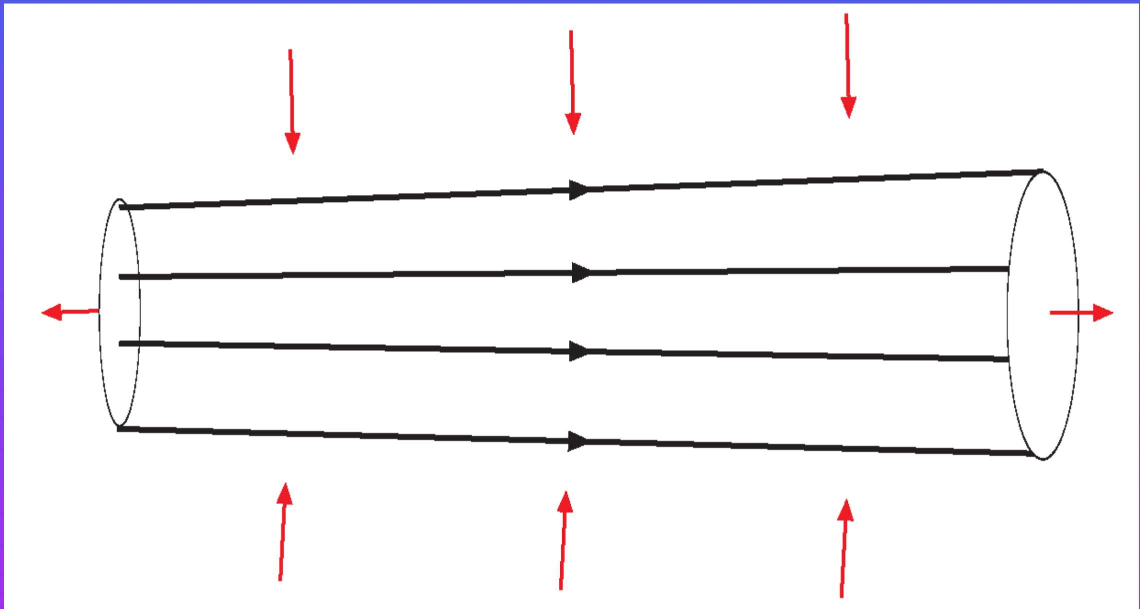
$$\vec{f}_E=\frac{1}{2}\epsilon_0E^2(\vec{n}\cos 2\theta_E+\vec{e}_E\sin 2\theta_E)\qquad \vec{f}_B=\frac{1}{2\mu_0}B^2(\vec{n}\cos 2\theta_B+\vec{e}_B\sin 2\theta_B)$$

电磁相互作用能量动量的转化与守恒： 张量力





法拉第提出电力线、磁力线的橡皮筋受力模型



如前所述，目前场是没有受力的概念的，是什么受力呢？

电磁场出现在物质表面. 单位表面受的力为

$$\vec{\mathbf{f}} = -\vec{\mathbf{n}} \cdot \overleftrightarrow{\mathcal{J}} = \vec{\mathbf{f}}_{\mathbf{E}} + \vec{\mathbf{f}}_{\mathbf{B}}$$

$$\vec{\mathbf{f}}_{\mathbf{E}} = \frac{1}{2}\epsilon_0 \mathbf{E}^2 (\vec{\mathbf{n}} \cos 2\theta_{\mathbf{E}} + \vec{\mathbf{e}}_{\mathbf{E}} \sin 2\theta_{\mathbf{E}})$$

$$\vec{\mathbf{f}}_{\mathbf{B}} = \frac{1}{2\mu_0} \mathbf{B}^2 (\vec{\mathbf{n}} \cos 2\theta_{\mathbf{B}} + \vec{\mathbf{e}}_{\mathbf{B}} \sin 2\theta_{\mathbf{B}})$$

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$$\vec{\mathbf{f}}_{\mathbf{B}} = \frac{1}{2\mu_0}\mathbf{B}^2(\vec{\mathbf{n}}\cos 2\theta_{\mathbf{B}} + \vec{\mathbf{e}}_{\mathbf{B}}\sin 2\theta_{\mathbf{B}})$$

若电磁场的方向不确定,在所有方向上都等几率出现:

电磁场出现在物质表面. 单位表面受的力为

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$$\vec{\mathbf{f}}_{\mathbf{B}} = \frac{1}{2\mu_0}\mathbf{B}^2(\vec{\mathbf{n}}\cos 2\theta_{\mathbf{B}} + \vec{\mathbf{e}}_{\mathbf{B}}\sin 2\theta_{\mathbf{B}})$$

若电磁场的方向不确定,在所有方向上都等几率出现:

$$\overline{\cos 2\theta} = \frac{\int d\Omega \cos 2\theta}{\int d\Omega} = \frac{\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta \cos 2\theta}{\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta} = -\frac{1}{3}$$

Back

Close

电磁场出现在物质表面. 单位表面受的力为

$$\vec{\mathbf{f}} = -\vec{\mathbf{n}}\cdot \overleftrightarrow{\mathcal{J}} = \vec{\mathbf{f}}_{\mathbf{E}} + \vec{\mathbf{f}}_{\mathbf{B}}$$

$$\vec{\mathbf{f}}_{\mathbf{E}} = \frac{1}{2}\epsilon_0 \mathbf{E}^2(\vec{\mathbf{n}}\cos 2\theta_{\mathbf{E}} + \vec{\mathbf{e}}_{\mathbf{E}}\sin 2\theta_{\mathbf{E}})$$

$$\vec{\mathbf{f}}_{\mathbf{B}} = \frac{1}{2\mu_0}\mathbf{B}^2(\vec{\mathbf{n}}\cos 2\theta_{\mathbf{B}} + \vec{\mathbf{e}}_{\mathbf{B}}\sin 2\theta_{\mathbf{B}})$$

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$$\overline{\sin 2\theta} = \frac{\int d\Omega \sin 2\theta}{\int d\Omega} = \frac{\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta \sin 2\theta}{\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta} = 0$$

电磁场出现在物质表面. 单位表面受的力为

$$\vec{f} = -\vec{n}\cdot \overleftrightarrow{\mathcal{T}} = \vec{f}_E + \vec{f}_B$$

$$\vec{f}_E = \frac{1}{2}\epsilon_0 E^2(\vec{n}\cos 2\theta_E + \vec{e}_E \sin 2\theta_E)$$

$$\vec{f}_B = \frac{1}{2\mu_0} B^2(\vec{n}\cos 2\theta_B + \vec{e}_B \sin 2\theta_B)$$

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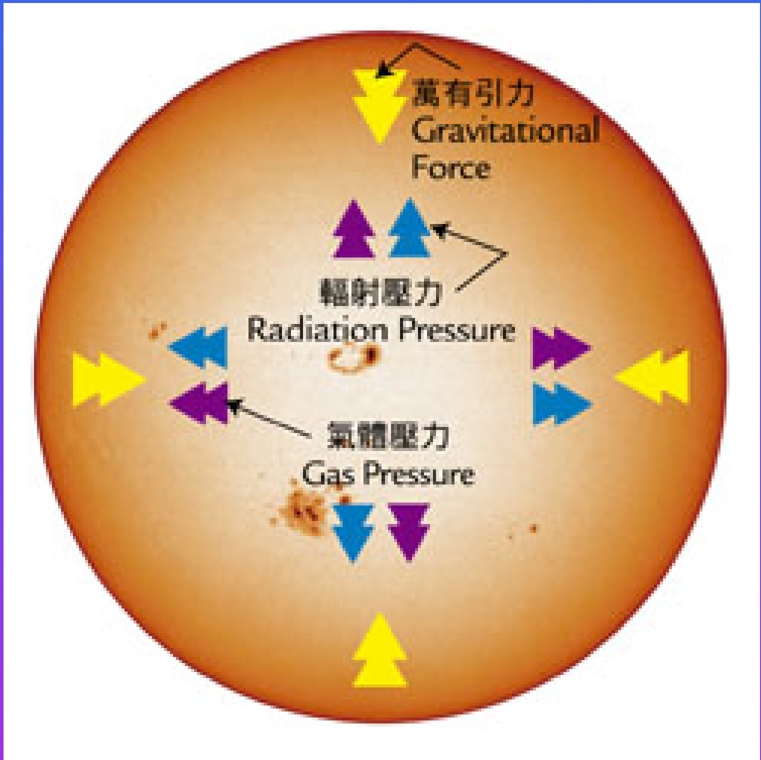
$$\overline{\sin 2\theta} = \frac{\int d\Omega \sin 2\theta}{\int d\Omega} = \frac{\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta \sin 2\theta}{\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta} = 0$$

$$\overline{\vec{f}} = -\frac{\vec{n}}{3}\left(\frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\mu_0 H^2\right) = -\frac{\vec{n}}{3} W$$

电磁相互作用能量动量的转化与守恒： 张量力



二. 从作用量出发构建

协变的经典电磁学理论

假设: 时空是均匀的! 后面详细讨论

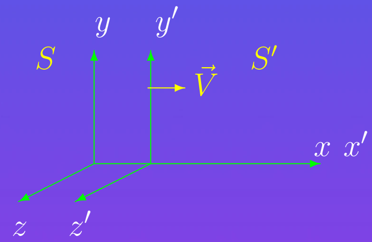
Back
Close

假设: 时空是均匀的! 后面详细讨论

(t, x, y, z)和(t', x', y', z')之间的关系必须是线性的. 为什么?

可以选择:两个相对运动速度为$\vec{V}$的惯性系S和S',两系的坐标系坐标轴方向相同,

x轴取在$\vec{V}$方向,并设$t=0, t'=0$时两坐标系重合.



相对论基本原理,洛伦兹变换: 伽利略变换

<u>假设</u>: 时空是均匀的! 后面详细讨论

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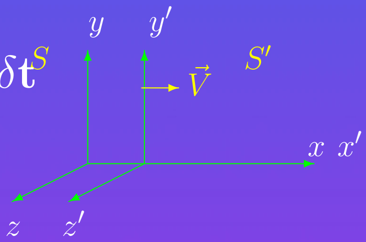
$$\mathbf{x}'=\alpha\mathbf{x}+\beta\mathbf{t}\qquad\mathbf{y}'=\mathbf{y}\qquad\mathbf{z}'=\mathbf{z}\qquad\mathbf{t}'=\gamma\mathbf{x}+\delta\mathbf{t}$$



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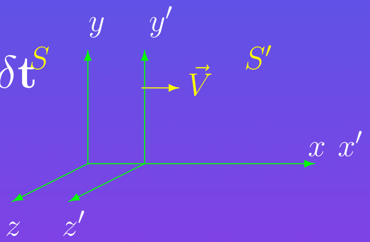

假设: 运动的相对性:

相对论基本原理,洛伦兹变换: 伽利略变换

假设: 时空是均匀的! 后面详细讨论

$(\mathrm{t}, \mathrm{x}, \mathrm{y}, \mathrm{z})$和$(\mathrm{t}', \mathrm{x}', \mathrm{y}', \mathrm{z}')$之间的关系必须是线性的. 为什么?

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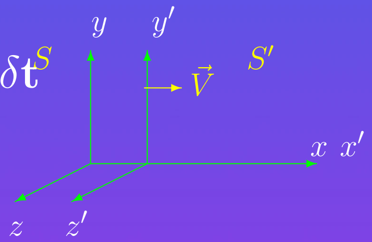
$$\mathbf{x}' = \alpha \mathbf{x} + \beta \mathbf{t} \qquad \mathbf{y}' = \mathbf{y} \qquad \mathbf{z}' = \mathbf{z} \qquad \mathbf{t}' = \gamma \mathbf{x} + \delta \mathbf{t}$$



假设: 运动的相对性:

- 从S看,S'有运动速度V $\Rightarrow$ $\mathrm{dx}' = 0 \quad \frac{\mathrm{dx}}{\mathrm{dt}} = \mathrm{V}$

假设: 时空是均匀的! 后面详细讨论

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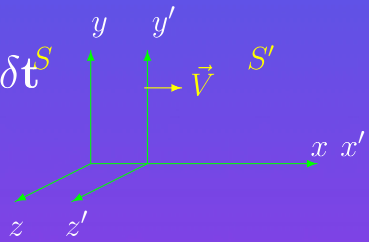

假设: 运动的相对性:

- 从S看,S'有运动速度V $\Rightarrow$ $\mathrm{dx}'=0$ $\frac{\mathrm{dx}}{\mathrm{dt}}=\mathrm{V}$ $\Rightarrow$ $\beta=-\alpha\mathrm{V}$

假设: 时空是均匀的! 后面详细讨论

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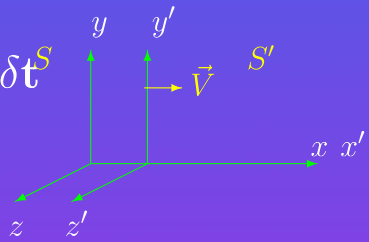

假设: 运动的相对性:

- 从S看,S'有运动速度V $\Rightarrow$ $dx' = 0$ $\frac{dx}{dt} = V \Rightarrow \beta = -\alpha V$
- S'看,S有运动速度-V 一定这样? 单向光速不变假设或空间各向同性 $\Rightarrow$ $dx = 0$ $\frac{dx'}{dt'} = -V$

相对论基本原理,洛伦兹变换: 伽利略变换

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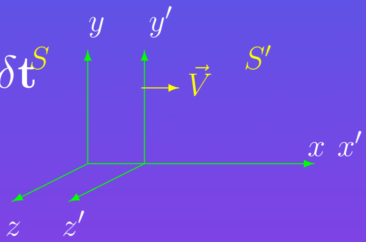
$$x' = \alpha x + \beta t \qquad y' = y \qquad z' = z \qquad t' = \gamma x + \delta t$$



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$$x' = \alpha(x - Vt) \qquad y' = y \qquad z' = z \qquad t' = \gamma x + \alpha t$$

相对论基本原理,洛伦兹变换: 伽利略变换

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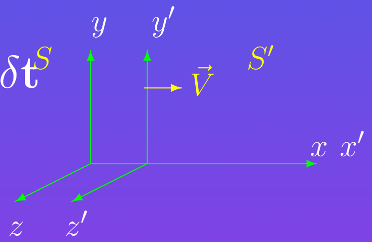

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- 从S看,S'有运动速度V $\Rightarrow$ $\mathrm{dx}'=0$ $\quad \frac{\mathrm{dx}}{\mathrm{dt}}=\mathrm{V}\Rightarrow\beta=-\alpha\mathrm{V}$
- S'看,S有运动速度-V 一定这样? 单向光速不变假设或空间各向同性 $\Rightarrow \mathrm{dx}=0\quad \frac{\mathrm{dx}'}{\mathrm{dt}'}=-\mathrm{V}\Rightarrow\beta=-\delta\mathrm{V}$

$$\mathbf{x}'=\alpha(\mathbf{x}-\mathbf{V}\mathbf{t})\qquad \mathbf{y}'=\mathbf{y}\qquad \mathbf{z}'=\mathbf{z}\qquad \mathbf{t}'=\gamma\mathbf{x}+\alpha\mathbf{t}$$

伽利略变换: $\mathrm{t}'=\mathrm{t}\quad\Rightarrow\quad\gamma=0,\ \alpha=1$

相对论基本原理,洛伦兹变换: 伽利略变换

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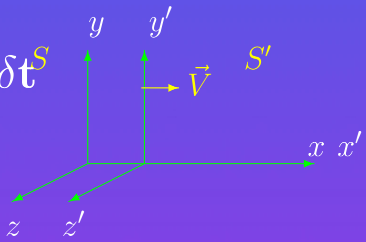

假设: 运动的相对性:

- 从S看,S'有运动速度V $\Rightarrow$ $dx' = 0$ $\frac{dx}{dt} = V \Rightarrow \beta = -\alpha V$
- S'看,S有运动速度-V 一定这样? 单向光速不变假设或空间各向同性 $\Rightarrow dx=0$ $\frac{dx'}{dt'} = -V \Rightarrow \beta = -\delta V$

$$x' = \alpha(x - Vt) \qquad y' = y \qquad z' = z \qquad t' = \gamma x + \alpha t$$

伽利略变换: $t' = t \quad \Rightarrow \quad \gamma = 0, \ \alpha = 1$ 　绝对时间!

相对论基本原理,洛伦兹变换: 伽利略变换

假设: 时空是均匀的! 后面详细讨论

(t, x, y, z)和(t', x', y', z')之间的关系必须是线性的. 为什么?

可以选择:两个相对运动速度为$\vec{V}$的惯性系S和S',两系的坐标系坐标轴方向相同,

x轴取在$\vec{V}$方向,并设$t = 0, t' = 0$时两坐标系重合.

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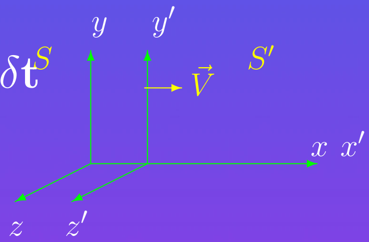

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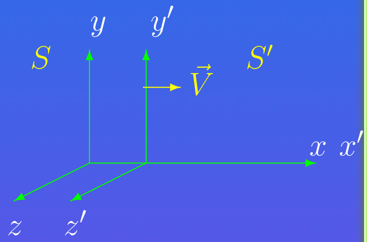
伽利略变换: $t' = t \quad \Rightarrow \quad \gamma = 0, \ \alpha = 1$ 绝对时间! 否定伽利略变换必须否定绝对时间

Back
Close

两个相对运动速度为$\vec{\mathrm{V}}$的惯性系S和S',两系的坐标系坐标轴方向 相同,x轴取在$\vec{\mathrm{V}}$方向,并设$\mathrm{t}=0, \mathrm{t}'=0$时两坐标系标重合.



相对论基本原理,洛伦兹变换: 基本洛伦兹变换

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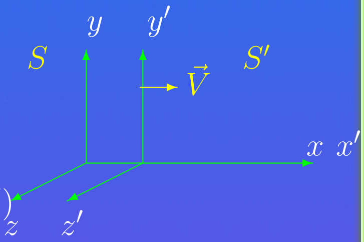

一个事件在S和S'系中的时空坐标分别为(t,x,y,z)和 (t',x',y',z')

相对论基本原理,洛伦兹变换: 基本洛伦兹变换

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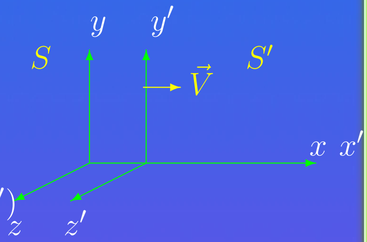

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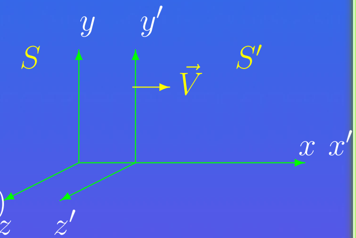

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在$\mathrm{t}=0, \mathrm{t}'=0$时原点发出一个球面电磁波, 在$t$时刻电磁波达到以原点O为中心的某个球波阵面上,相应 在t'时刻电磁波达到以原点O'为中心的某个球波阵面上,

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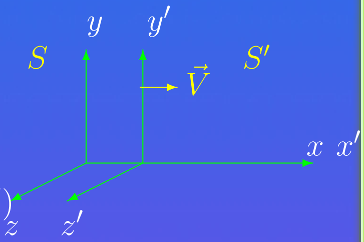

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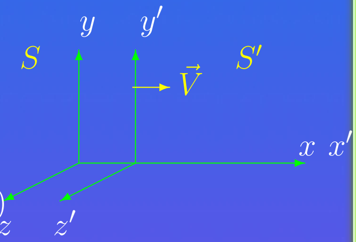
相对论基本原理,洛伦兹变换: 基本洛伦兹变换



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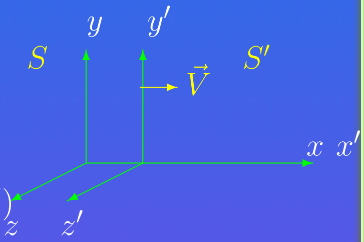

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相对论基本原理,洛伦兹变换: 基本洛伦兹变换

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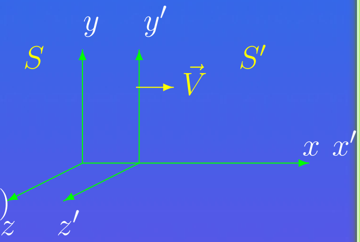

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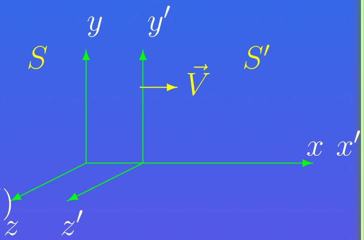

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$$\mathrm{x}'=\frac{\mathrm{x}-\mathrm{Vt}}{\sqrt{1-\frac{\mathrm{V}^2}{\mathrm{c}^2}}}\qquad \mathrm{y}'=\mathrm{y}\qquad \mathrm{z}'=\mathrm{z}\qquad \mathrm{t}'=\frac{\mathrm{t}-\frac{\mathrm{V}}{\mathrm{c}^2}\mathrm{x}}{\sqrt{1-\frac{\mathrm{V}^2}{\mathrm{c}^2}}}$$

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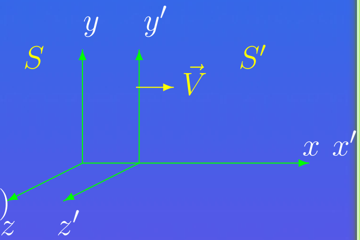

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Back
Close

相对论基本原理,洛伦兹变换: 基本洛伦兹变换~任意运动方向坐标速度变换

$$\begin{cases} \dfrac{\vec{r}'\cdot\vec{V}}{V} = \dfrac{\frac{\vec{r}\cdot\vec{V}}{V} - Vt}{\sqrt{1-\frac{V^2}{c^2}}} \\ \\ \\ \end{cases}$$

相对论基本原理,洛伦兹变换: 基本洛伦兹变换~任意运动方向坐标速度变换

$$\begin{cases} \dfrac{\vec{r}'\cdot\vec{V}}{V} = \dfrac{\frac{\vec{r}\cdot\vec{V}}{V} - Vt}{\sqrt{1-\frac{V^2}{c^2}}} \\ \vec{r}' - \dfrac{\vec{r}'\cdot\vec{V}}{V^2}\vec{V} = \vec{r} - \dfrac{\vec{r}\cdot\vec{V}}{V^2}\vec{V} \\ \\ \end{cases}$$

相对论基本原理,洛伦兹变换: 基本洛伦兹变换~任意运动方向坐标速度变换

$$
\begin{cases}
\dfrac{\vec{r}'\cdot\vec{V}}{V} = \dfrac{\frac{\vec{r}\cdot\vec{V}}{V} - Vt}{\sqrt{1-\frac{V^2}{c^2}}} \\
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t' = \dfrac{t - \frac{\vec{r}\cdot\vec{V}}{c^2}}{\sqrt{1-\frac{V^2}{c^2}}}
\end{cases}
$$

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$\Uparrow$ $\vec{r}-\vec{V}t$沿$\vec{V}$方向收缩;垂直$\vec{V}$方向不变!

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$$\vec{u} = \frac{d\vec{r}}{dt}$$

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$$\left\{\begin{aligned} \frac{\vec{r}'\cdot\vec{V}}{V} &= \frac{\frac{\vec{r}\cdot\vec{V}}{V}-Vt}{\sqrt{1-\frac{V^2}{c^2}}} \\ \vec{r}'-\frac{\vec{r}'\cdot\vec{V}}{V^2}\vec{V} &= \vec{r}-\frac{\vec{r}\cdot\vec{V}}{V^2}\vec{V} \\ t' &= \frac{t-\frac{\vec{r}\cdot\vec{V}}{c^2}}{\sqrt{1-\frac{V^2}{c^2}}} \end{aligned}\right. \rightarrow \left\{\begin{aligned} \vec{r}' &= \vec{r}-\frac{\vec{r}\cdot\vec{V}}{V^2}\vec{V}+\frac{\frac{\vec{r}\cdot\vec{V}}{V}-Vt}{\sqrt{1-\frac{V^2}{c^2}}}\frac{\vec{V}}{V} \\ &= \vec{r}-\vec{V}t+\Big(\frac{1}{\sqrt{1-\frac{V^2}{c^2}}}-1\Big)(\vec{r}-\vec{V}t)\cdot\frac{\vec{V}}{V}\frac{\vec{V}}{V} \\ t' &= \frac{t-\frac{\vec{r}\cdot\vec{V}}{c^2}}{\sqrt{1-\frac{V^2}{c^2}}} \end{aligned}\right.$$

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$$\vec{u}=\frac{d\vec{r}}{dt} \qquad \vec{u}'\equiv\frac{d\vec{r}'}{dt'}$$

Back

Close

相对论基本原理,洛伦兹变换: 基本洛伦兹变换~任意运动方向坐标速度变换

$$\left\{\begin{aligned} \frac{\vec{r}'\cdot\vec{V}}{V} &= \frac{\frac{\vec{r}\cdot\vec{V}}{V}-Vt}{\sqrt{1-\frac{V^2}{c^2}}} \\ \vec{r}' - \frac{\vec{r}'\cdot\vec{V}}{V^2}\vec{V} &= \vec{r} - \frac{\vec{r}\cdot\vec{V}}{V^2}\vec{V} \\ t' &= \frac{t-\frac{\vec{r}\cdot\vec{V}}{c^2}}{\sqrt{1-\frac{V^2}{c^2}}} \end{aligned}\right. \rightarrow \left\{\begin{aligned} \vec{r}' &= \vec{r} - \frac{\vec{r}\cdot\vec{V}}{V^2}\vec{V} + \frac{\frac{\vec{r}\cdot\vec{V}}{V}-Vt}{\sqrt{1-\frac{V^2}{c^2}}}\frac{\vec{V}}{V} \\ &= \vec{r}-\vec{V}t + \Big(\frac{1}{\sqrt{1-\frac{V^2}{c^2}}}-1\Big)(\vec{r}-\vec{V}t)\cdot\frac{\vec{V}}{V}\frac{\vec{V}}{V} \\ t' &= \frac{t-\frac{\vec{r}\cdot\vec{V}}{c^2}}{\sqrt{1-\frac{V^2}{c^2}}} \end{aligned}\right.$$

$\Uparrow$ $\vec{r}-\vec{V}t$沿$\vec{V}$方向收缩;垂直$\vec{V}$方向不变!

$$\vec{u} = \frac{d\vec{r}}{dt} \qquad \vec{u}' \equiv \frac{d\vec{r}'}{dt'} = \frac{\frac{d\vec{r}'}{dt}}{\frac{dt'}{dt}}$$

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$$\begin{cases}\dfrac{\vec{r}'\cdot\vec{V}}{V}=\dfrac{\frac{\vec{r}\cdot\vec{V}}{V}-Vt}{\sqrt{1-\frac{V^2}{c^2}}}\\ \vec{r}'-\dfrac{\vec{r}'\cdot\vec{V}}{V^2}\vec{V}=\vec{r}-\dfrac{\vec{r}\cdot\vec{V}}{V^2}\vec{V}\\ t'=\dfrac{t-\frac{\vec{r}\cdot\vec{V}}{c^2}}{\sqrt{1-\frac{V^2}{c^2}}}\end{cases}\rightarrow\begin{cases}\vec{r}' &=\vec{r}-\dfrac{\vec{r}\cdot\vec{V}}{V^2}\vec{V}+\dfrac{\frac{\vec{r}\cdot\vec{V}}{V}-Vt}{\sqrt{1-\frac{V^2}{c^2}}}\dfrac{\vec{V}}{V}\\ &=\vec{r}-\vec{V}t+\Big(\dfrac{1}{\sqrt{1-\frac{V^2}{c^2}}}-1\Big)(\vec{r}-\vec{V}t)\cdot\dfrac{\vec{V}}{V}\dfrac{\vec{V}}{V}\\ t' &=\dfrac{t-\frac{\vec{r}\cdot\vec{V}}{c^2}}{\sqrt{1-\frac{V^2}{c^2}}}\end{cases}$$

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Back
Close

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$$\left\{\begin{array}{l} \frac{\vec{r}'\cdot\vec{V}}{V} = \frac{\frac{\vec{r}\cdot\vec{V}}{V} - Vt}{\sqrt{1-\frac{V^2}{c^2}}} \\ \vec{r}' - \frac{\vec{r}'\cdot\vec{V}}{V^2}\vec{V} = \vec{r} - \frac{\vec{r}\cdot\vec{V}}{V^2}\vec{V} \\ t' = \frac{t-\frac{\vec{r}\cdot\vec{V}}{c^2}}{\sqrt{1-\frac{V^2}{c^2}}} \end{array}\right. \rightarrow \left\{\begin{array}{ll} \vec{r}' & = \vec{r} - \frac{\vec{r}\cdot\vec{V}}{V^2}\vec{V} + \frac{\frac{\vec{r}\cdot\vec{V}}{V} - Vt}{\sqrt{1-\frac{V^2}{c^2}}}\frac{\vec{V}}{V} \\ & = \vec{r} - \vec{V}t + \left(\frac{1}{\sqrt{1-\frac{V^2}{c^2}}} - 1\right)(\vec{r} - \vec{V}t)\cdot\frac{\vec{V}}{V}\frac{\vec{V}}{V} \\ t' & = \frac{t-\frac{\vec{r}\cdot\vec{V}}{c^2}}{\sqrt{1-\frac{V^2}{c^2}}} \end{array}\right.$$

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相对论中速度合成关系!

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$$U^\alpha \equiv \frac{dx^\alpha}{d\tau}$$

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相对论中速度合成关系!

$$U^\alpha \equiv \frac{dx^\alpha}{d\tau} = \frac{1}{\sqrt{1-\frac{V^2}{c^2}}}\frac{dx^\alpha}{dt} \rightarrow \left\{\begin{aligned} U^i &= \frac{v^i}{\sqrt{1-\frac{v^2}{c^2}}} \\ U^4 &= \frac{ic}{\sqrt{1-\frac{v^2}{c^2}}} \end{aligned}\right. \qquad \begin{aligned} &x^1=x, x^2=y, x^3=z \\ &x^4 = ict \end{aligned}$$

c有限,钟和尺子测量的结果与参考 系的选择有关,它们不能再作为绝对标准来衡量事件之间的时空关系.

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对一个参考系中的两个事件$(x_1, y_1, z_1, t_1), (x_2, y_2, z_2, t_2)$,定义它们之间的 间隔为:

$$\Delta s^2 \equiv c^2(t_1 - t_2)^2 - (x_1 - x_2)^2 - (y_1 - y_2)^2 - (z_1 - z_2)^2$$

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\Delta s^2 &\equiv c^2(t_1 - t_2)^2 - (x_1 - x_2)^2 - (y_1 - y_2)^2 - (z_1 - z_2)^2 \\
&= c^2(\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2
\end{aligned}$$

$$\Delta s'^2 \equiv c^2(t'_1 - t'_2)^2 - (x'_1 - x'_2)^2 - (y'_1 - y'_2)^2 - (z'_1 - z'_2)^2$$

Back
Close

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$$\Delta s'^2 \equiv c^2(t_1' - t_2')^2 - (x_1' - x_2')^2 - (y_1' - y_2')^2 - (z_1' - z_2')^2$$

$$= c^2\left[\frac{t_1 - t_2 - \frac{V}{c^2}(x_1 - x_2)}{\sqrt{1 - \frac{V^2}{c^2}}}\right]^2 - \left[\frac{x_1 - x_2 - V(t_1 - t_2)}{\sqrt{1 - \frac{V^2}{c^2}}}\right]^2 - (y_1 - y_2)^2 - (z_1 - z_2)^2$$

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对一个参考系中的两个事件$(x_1,y_1,z_1,t_1),(x_2,y_2,z_2,t_2)$,定义它们之间的 间隔为:

$$\Delta s^2 \equiv c^2(t_1-t_2)^2-(x_1-x_2)^2-(y_1-y_2)^2-(z_1-z_2)^2$$
$$=c^2(\Delta t)^2-(\Delta x)^2-(\Delta y)^2-(\Delta z)^2$$
$$\Delta s'^2 \equiv c^2(t'_1-t'_2)^2-(x'_1-x'_2)^2-(y'_1-y'_2)^2-(z'_1-z'_2)^2$$
$$=c^2\left[\frac{t_1-t_2-\frac{V}{c^2}(x_1-x_2)}{\sqrt{1-\frac{V^2}{c^2}}}\right]^2-\left[\frac{x_1-x_2-V(t_1-t_2)}{\sqrt{1-\frac{V^2}{c^2}}}\right]^2-(y_1-y_2)^2-(z_1-z_2)^2$$
$$=(t_1-t_2)^2\left[\frac{c^2}{1-\frac{V^2}{c^2}}-\frac{V^2}{1-\frac{V^2}{c^2}}\right]+(x_1-x_2)^2\left[\frac{\frac{V^2}{c^2}}{1-\frac{V^2}{c^2}}-\frac{1}{1-\frac{V^2}{c^2}}\right]-(y_1-y_2)^2-(z_1-z_2)^2$$

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$$= c^2\left[\frac{t_1 - t_2 - \frac{V}{c^2}(x_1 - x_2)}{\sqrt{1 - \frac{V^2}{c^2}}}\right]^2 - \left[\frac{x_1 - x_2 - V(t_1 - t_2)}{\sqrt{1 - \frac{V^2}{c^2}}}\right]^2 - (y_1 - y_2)^2 - (z_1 - z_2)^2$$
$$= (t_1 - t_2)^2\left[\frac{c^2}{1 - \frac{V^2}{c^2}} - \frac{V^2}{1 - \frac{V^2}{c^2}}\right] + (x_1 - x_2)^2\left[\frac{\frac{V^2}{c^2}}{1 - \frac{V^2}{c^2}} - \frac{1}{1 - \frac{V^2}{c^2}}\right] - (y_1 - y_2)^2 - (z_1 - z_2)^2$$
$$= \Delta s^2$$

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对一个参考系中的两个事件(x_1, y_1, z_1, t_1), (x_2, y_2, z_2, t_2),定义它们之间的 间隔为:

$$\begin{aligned}\Delta s^2 &\equiv c^2(t_1-t_2)^2-(x_1-x_2)^2-(y_1-y_2)^2-(z_1-z_2)^2\\&=c^2(\Delta t)^2-(\Delta x)^2-(\Delta y)^2-(\Delta z)^2\end{aligned}$$

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$$=(t_1-t_2)^2\left[\frac{c^2}{1-\frac{V^2}{c^2}}-\frac{V^2}{1-\frac{V^2}{c^2}}\right]+(x_1-x_2)^2\left[\frac{\frac{V^2}{c^2}}{1-\frac{V^2}{c^2}}-\frac{1}{1-\frac{V^2}{c^2}}\right]-(y_1-y_2)^2-(z_1-z_2)^2$$

$$=\Delta s^2$$

一般的洛伦兹变换在数学上定义为保持间隔不变的 (x, y, z, t)的齐次线性变换.

对无穷小间隔:

对无穷小间隔:

$$\mathrm{ds}^2 = \mathrm{c}^2\mathrm{dt}^2 - \mathrm{d}\vec{\mathrm{r}} \cdot \mathrm{d}\vec{\mathrm{r}}$$

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$$ds^2 = c^2dt^2 - d\vec{r} \cdot d\vec{r} = c^2dt^2(1 - \frac{v^2}{c^2})$$

对无穷小间隔：

$$ds^2 = c^2dt^2 - d\vec{r} \cdot d\vec{r} = c^2dt^2(1 - \frac{v^2}{c^2}) = c^2d\tau^2$$

Back
Close

对无穷小间隔:

$$\mathrm{ds^2 = c^2dt^2 - d\vec{r}\cdot d\vec{r} = c^2dt^2(1-\frac{v^2}{c^2}) = c^2d\tau^2 = -dl_0^2} \qquad \mathrm{\vec{v} = \frac{d\vec{r}}{dt}}$$

对无穷小间隔:

$$ds^2 = c^2dt^2 - d\vec{r}\cdot d\vec{r} = c^2dt^2(1-\frac{v^2}{c^2}) = c^2d\tau^2 = -dl_0^2 \qquad \vec{v} = \frac{d\vec{r}}{dt}$$

- 对$ds^2 > 0, d\tau \equiv \frac{1}{c}\sqrt{ds^2}$是钟的固有时(原时), 它是由间隔引出的具有时间量纲的不变量.

对无穷小间隔:

$$\mathrm{ds}^2 = \mathrm{c}^2\mathrm{dt}^2 - \mathrm{d}\vec{r}\cdot\mathrm{d}\vec{r} = \mathrm{c}^2\mathrm{dt}^2(1-\frac{\mathrm{v}^2}{\mathrm{c}^2}) = \mathrm{c}^2\mathrm{d}\tau^2 = -\mathrm{dl}_0^2 \qquad \vec{\mathrm{v}} = \frac{\mathrm{d}\vec{r}}{\mathrm{dt}}$$

- 对$\mathrm{ds}^2 > 0, \mathrm{d}\tau \equiv \frac{1}{\mathrm{c}}\sqrt{\mathrm{ds}^2}$是钟的固有时(原时), 它是由间隔引出的具有时间量纲的不变量.
- 对$\mathrm{ds}^2 < 0, \mathrm{dl}_0 \equiv \sqrt{-\mathrm{ds}^2}$是尺的固有长度, 它是由间隔引出的具有长度量纲的不变量.

对无穷小间隔:

$$ds^2 = c^2dt^2 - d\vec{r}\cdot d\vec{r} = c^2dt^2(1-\frac{v^2}{c^2}) = c^2d\tau^2 = -dl_0^2 \qquad \vec{v} = \frac{d\vec{r}}{dt}$$

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两事件关系的分类:

对无穷小间隔:

$$\mathrm{ds}^2 = \mathrm{c}^2\mathrm{dt}^2 - \mathrm{d}\vec{r}\cdot\mathrm{d}\vec{r} = \mathrm{c}^2\mathrm{dt}^2\left(1-\frac{\mathrm{v}^2}{\mathrm{c}^2}\right) = \mathrm{c}^2\mathrm{d}\tau^2 = -\mathrm{dl}_0^2 \qquad \vec{v} = \frac{\mathrm{d}\vec{r}}{\mathrm{dt}}$$

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两事件关系的分类:

- $\Delta \mathrm{s}^2 > 0$: 类时事件,类时间隔

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两事件关系的分类:

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- $\Delta s^2 = 0$: 类光事件,类光间隔

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Back

Close

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Back

Close

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另一种表达形式： $\quad \sum_{\mu=1}^{4}a_{\nu\mu}a_{\lambda\mu}=\delta_{\nu\lambda}$

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Back
Close

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Back
Close

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Back
Close

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$\det \mathbf{A} = \mathbf{1}$由纯转动构成

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$\det \mathbf{A} = \mathbf{1}$由纯转动构成 $\det \mathbf{A} = -\mathbf{1}$由纯转动迭加上空间反射或时间反演构成

相对性原理的数学表达： 3+1维时空张量方程

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n阶张量的特点是有4^n个分量,在洛伦兹变换下,按n个坐标乘积的变换方式变换.

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协变 $\equiv$ 与坐标一起协同变化!

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$$\mathbf{T}_{\mu_1\cdots\mu_n} = \mathbf{F}_{\mu_1\cdots\mu_n}$$

Back
Close

相对论理论的协变形式: 物理量按时空变换性质分类

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Back
Close

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在新坐标系中的方程式形式上与旧坐标系中的方程式一样—相对性原理

Back
Close

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$$\mathbf{T}'_{\mu_1\cdots\mu_n} = \sum_{\nu_1,\cdots,\nu_n=1}^{4} \mathbf{a}_{\mu_1\nu_1}\cdots\mathbf{a}_{\mu_n\nu_n}\mathbf{T}_{\nu_1\cdots\nu_n} \quad \Rightarrow \quad \text{协变} \equiv {}_{\text{与坐标一起}}\text{协同变化!}$$

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在新坐标系中的方程式形式上与旧坐标系中的方程式一样—相对性原理

利用张量,可将描述物理规律的方程写成满足相对性原理的形式 —物理规律的协变表达.

相对论理论的协变形式: 物理量按时空变换性质分类

$$\mathbf{T}'_{\mu_1\cdots\mu_n} = \sum_{\nu_1,\cdots,\nu_n=1}^{4} \mathbf{a}_{\mu_1\nu_1}\cdots\mathbf{a}_{\mu_n\nu_n}\mathbf{T}_{\nu_1\cdots\nu_n} \quad \Rightarrow \quad \text{协变} \equiv \text{与坐标一起协同变化!}$$

$$\mathbf{T}_{\mu_1\cdots\mu_n} = \mathbf{F}_{\mu_1\cdots\mu_n}$$

$$\sum_{\mu_1,\cdot,\mu_n=1}^{4} \mathbf{a}_{\nu_1\mu_1}\cdots\mathbf{a}_{\nu_n\mu_n}\mathbf{T}_{\mu_1\cdots\mu_n} = \sum_{\mu_1,\cdot,\mu_n=1}^{4} \mathbf{a}_{\nu_1\mu_1}\cdots\mathbf{a}_{\nu_n\mu_n}\mathbf{F}_{\mu_1\cdots\mu_n} \rightarrow \mathbf{T}'_{\mu_1\cdots\mu_n} = \mathbf{F}'_{\mu_1\cdots\mu_n}$$

在新坐标系中的方程式形式上与旧坐标系中的方程式一样—相对性原理

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将所有物理规律都用张量表达,需知道:

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将所有物理规律都用张量表达,需知道:

- 张量之间的运算法则.
- 完备的张量集有多少种张量.

Back
Close

相对论理论的协变形式: 物理量按时空变换性质分类

$$\mathbf{T}'_{\mu_1\cdots\mu_n} = \sum_{\nu_1,\cdots,\nu_n=1}^{4} \mathbf{a}_{\mu_1\nu_1}\cdots\mathbf{a}_{\mu_n\nu_n}\mathbf{T}_{\nu_1\cdots\nu_n} \quad \Rightarrow \quad \text{协变} \equiv_{\text{与坐标一起}} \text{协同变化!}$$

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- 张量之间的运算法则.
- 完备的张量集有多少种张量.
- 具体的物理规律如何用张量表达.

加减法：同阶张量加减后仍是这阶张量

$$\mathbf{F}'_{\mu_1\cdots\mu_n} = \sum_{\nu_1,\cdots,\nu_n=1}^{4} \mathbf{a}_{\mu_1\nu_1}\cdots\mathbf{a}_{\mu_n\nu_n}\mathbf{F}_{\nu_1\cdots\nu_n} \qquad \mathbf{G}'_{\mu_1\cdots\mu_n} = \sum_{\nu_1,\cdots,\nu_n=1}^{4} \mathbf{a}_{\mu_1\nu_1}\cdots\mathbf{a}_{\mu_n\nu_n}\mathbf{G}_{\nu_1\cdots\nu_n}$$

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$$\mathbf{F}'_{\mu_1\cdots\mu_n}\pm\mathbf{G}'_{\mu_1\cdots\mu_n}=\sum_{\nu_1,\cdots,\nu_n=1}^{4}\mathbf{a}_{\mu_1\nu_1}\cdots\mathbf{a}_{\mu_n\nu_n}[\mathbf{F}_{\nu_1\cdots\nu_n}\pm\mathbf{G}_{\mu_1\cdots\mu_n}]$$

加减法：同阶张量加减后仍是这阶张量

$$F'_{\mu_1\cdots\mu_n} = \sum_{\nu_1,\cdots,\nu_n=1}^{4} a_{\mu_1\nu_1}\cdots a_{\mu_n\nu_n} F_{\nu_1\cdots\nu_n} \qquad G'_{\mu_1\cdots\mu_n} = \sum_{\nu_1,\cdots,\nu_n=1}^{4} a_{\mu_1\nu_1}\cdots a_{\mu_n\nu_n} G_{\nu_1\cdots\nu_n}$$

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乘法：m阶张量与n阶张量的乘积为$m+n$阶张量

Back
Close

加减法：同阶张量加减后仍是这阶张量

$$F'_{\mu_1\cdots\mu_n} = \sum_{\nu_1,\cdots,\nu_n=1}^{4} a_{\mu_1\nu_1}\cdots a_{\mu_n\nu_n} F_{\nu_1\cdots\nu_n} \qquad G'_{\mu_1\cdots\mu_n} = \sum_{\nu_1,\cdots,\nu_n=1}^{4} a_{\mu_1\nu_1}\cdots a_{\mu_n\nu_n} G_{\nu_1\cdots\nu_n}$$

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乘法：m阶张量与n阶张量的乘积为$m+n$阶张量

$$A'_{\mu_1\cdots\mu_m} = \sum_{\nu_1,\cdots,\nu_m=1}^{4} a_{\mu_1\nu_1}\cdots a_{\mu_m\nu_m} A_{\nu_1\cdots\nu_m} \qquad B'_{\mu_1\cdots\mu_n} = \sum_{\nu_1,\cdots,\nu_n=1}^{4} a_{\mu_1\nu_1}\cdots a_{\mu_n\nu_n} B_{\nu_1\cdots\nu_n}$$

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乘法： m阶张量与n阶张量的乘积为$\mathbf{m}+\mathbf{n}$阶张量

$$\mathbf{A}'_{\mu_1\cdots\mu_m} = \sum_{\nu_1,\cdots,\nu_m=1}^{4} \mathbf{a}_{\mu_1\nu_1}\cdots\mathbf{a}_{\mu_m\nu_m}\mathbf{A}_{\nu_1\cdots\nu_m} \qquad \mathbf{B}'_{\mu_1\cdots\mu_n} = \sum_{\nu_1,\cdots,\nu_n=1}^{4} \mathbf{a}_{\mu_1\nu_1}\cdots\mathbf{a}_{\mu_n\nu_n}\mathbf{B}_{\nu_1\cdots\nu_n}$$

$$\mathbf{T}_{\mu_1\cdots\mu_{m+n}} \equiv \mathbf{A}_{\mu_1\cdots\mu_m}\mathbf{B}_{\mu_{m+1}\cdots\mu_{m+n}}$$

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$$T_{\mu_1\cdots\mu_{m+n}} \equiv A_{\mu_1\cdots\mu_m} B_{\mu_{m+1}\cdots\mu_{m+n}}$$

$$T'_{\mu_1\cdots\mu_{m+n}} = A'_{\mu_1\cdots\mu_m} B'_{\mu_{m+1}\cdots\mu_{m+n}}$$

Back

Close

加减法： 同阶张量加减后仍是这阶张量

$$F'_{\mu_1\cdots\mu_n} = \sum_{\nu_1,\cdots,\nu_n=1}^{4} a_{\mu_1\nu_1}\cdots a_{\mu_n\nu_n} F_{\nu_1\cdots\nu_n} \qquad G'_{\mu_1\cdots\mu_n} = \sum_{\nu_1,\cdots,\nu_n=1}^{4} a_{\mu_1\nu_1}\cdots a_{\mu_n\nu_n} G_{\nu_1\cdots\nu_n}$$

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$$\mathbf{A}'_{\mu_1\cdots\mu_m}=\sum_{\nu_1,\cdots,\nu_m=1}^{4}\mathbf{a}_{\mu_1\nu_1}\cdots\mathbf{a}_{\mu_m\nu_m}\mathbf{A}_{\nu_1\cdots\nu_m}\qquad \mathbf{B}'_{\mu_1\cdots\mu_n}=\sum_{\nu_1,\cdots,\nu_n=1}^{4}\mathbf{a}_{\mu_1\nu_1}\cdots\mathbf{a}_{\mu_n\nu_n}\mathbf{B}_{\nu_1\cdots\nu_n}$$

$$\mathbf{T}_{\mu_1\cdots\mu_{m+n}}\equiv\mathbf{A}_{\mu_1\cdots\mu_m}\mathbf{B}_{\mu_{m+1}\cdots\mu_{m+n}}$$

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$$\mathbf{T}'_{\mu_1\cdots\mu_{m+n}}=\mathbf{A}'_{\mu_1\cdots\mu_m}\mathbf{B}'_{\mu_{m+1}\cdots\mu_{m+n}}$$

Back
Close

加减法： 同阶张量加减后仍是这阶张量

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加减法: 同阶张量加减后仍是这阶张量

乘法: m阶张量与n阶张量的乘积为m + n阶张量

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$$= \sum_{\nu_1,\cdots,\nu_{m+n}=1}^{4} a_{\mu_1\nu_1}\cdots a_{\mu_{m+n}\nu_{m+n}} A_{\nu_1\cdots\nu_m} B_{\nu_{m+1}\cdots\nu_{m+n}}$$

Back
Close

加减法： 同阶张量加减后仍是这阶张量

乘法： m阶张量与n阶张量的乘积为$\mathbf{m}+\mathbf{n}$阶张量

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$$=\sum_{\nu_1,\cdots,\nu_{m+n}=1}^{4}a_{\mu_1\nu_1}\cdots a_{\mu_{m+n}\nu_{m+n}}A_{\nu_1\cdots\nu_m}B_{\nu_{m+1}\cdots\nu_{m+n}}=\sum_{\nu_1,\cdots,\nu_{m+n}=1}^{4}a_{\mu_1\nu_1}\cdots a_{\mu_{m+n}\nu_{m+n}}T_{\nu_1\cdots\nu_{m+n}}$$

加减法: 同阶张量加减后仍是这阶张量

乘法: m阶张量与n阶张量的乘积为$\mathbf{m+n}$阶张量

$$A'_{\mu_1\cdots\mu_m}=\sum_{\nu_1,\cdots,\nu_m=1}^{4}a_{\mu_1\nu_1}\cdots a_{\mu_m\nu_m}A_{\nu_1\cdots\nu_m}\qquad B'_{\mu_1\cdots\mu_n}=\sum_{\nu_1,\cdots,\nu_n=1}^{4}a_{\mu_1\nu_1}\cdots a_{\mu_n\nu_n}B_{\nu_1\cdots\nu_n}$$

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$$=\sum_{\nu_1,\cdots,\nu_{m+n}=1}^{4}a_{\mu_1\nu_1}\cdots a_{\mu_{m+n}\nu_{m+n}}A_{\nu_1\cdots\nu_m}B_{\nu_{m+1}\cdots\nu_{m+n}}=\sum_{\nu_1,\cdots,\nu_{m+n}=1}^{4}a_{\mu_1\nu_1}\cdots a_{\mu_{m+n}\nu_{m+n}}T_{\nu_1\cdots\nu_{m+n}}$$

缩并: n阶张量收缩一次变成n-2阶张量

加减法： 同阶张量加减后仍是这阶张量

乘法： m阶张量与n阶张量的乘积为$\mathbf{m+n}$阶张量

$$A'_{\mu_1\cdots\mu_m} = \sum_{\nu_1,\cdots,\nu_m=1}^{4} a_{\mu_1\nu_1}\cdots a_{\mu_m\nu_m} A_{\nu_1\cdots\nu_m} \qquad B'_{\mu_1\cdots\mu_n} = \sum_{\nu_1,\cdots,\nu_n=1}^{4} a_{\mu_1\nu_1}\cdots a_{\mu_n\nu_n} B_{\nu_1\cdots\nu_n}$$

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$$= \sum_{\nu_1,\cdots,\nu_{m+n}=1}^{4} a_{\mu_1\nu_1}\cdots a_{\mu_{m+n}\nu_{m+n}} A_{\nu_1\cdots\nu_m} B_{\nu_{m+1}\cdots\nu_{m+n}} = \sum_{\nu_1,\cdots,\nu_{m+n}=1}^{4} a_{\mu_1\nu_1}\cdots a_{\mu_{m+n}\nu_{m+n}} T_{\nu_1\cdots\nu_{m+n}}$$

缩并： n阶张量收缩一次变成n-2阶张量

$\sum_{\mu_i,\mu_j=1}^{4} \delta_{\mu_i\mu_j} T_{\mu_1\cdots\mu_i\cdots\mu_j\cdots\mu_n}$称为$T_{\mu_1\cdots\mu_i\cdots\mu_j\cdots\mu_n}$对指标$\mu_i$, μ_j的缩并.

Back
Close

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Back
Close

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Back
Close

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Back
Close

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$$\mathbf{0} = \delta\mathbf{S} = \int_\alpha^\beta \mathbf{dt}[\frac{\partial\mathbf{L}}{\partial\dot{\mathbf{q}}}\delta\dot{\mathbf{q}} + \frac{\partial\mathbf{L}}{\partial\mathbf{q}}\delta\mathbf{q}] = \int_\alpha^\beta \mathbf{dt}[\frac{\partial\mathbf{L}}{\partial\dot{\mathbf{q}}}\frac{\mathbf{d}}{\mathbf{dt}}\delta\mathbf{q} + \frac{\partial\mathbf{L}}{\partial\mathbf{q}}\delta\mathbf{q}]$$

$$= \int_\alpha^\beta \mathbf{dt}\ [\frac{\mathbf{d}}{\mathbf{dt}}(\frac{\partial\mathbf{L}}{\partial\dot{\mathbf{q}}}\delta\mathbf{q}) + (\frac{\partial\mathbf{L}}{\partial\mathbf{q}} - \frac{\mathbf{d}}{\mathbf{dt}}\frac{\partial\mathbf{L}}{\partial\dot{\mathbf{q}}})\delta\mathbf{q}] \quad\rightarrow\quad -\frac{\partial\mathbf{L}}{\partial\mathbf{q}} + \frac{\mathbf{d}}{\mathbf{dt}}\frac{\partial\mathbf{L}}{\partial\dot{\mathbf{q}}} = \mathbf{0}$$

Back
Close

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Back
Close

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$$\mathrm{S} = -\mathrm{m}_0\mathrm{c}\int_{\mathrm{a}}^{\mathrm{b}} \mathrm{ds}$$

$$S = -m_0 c \int_a^b ds \qquad ds = \sqrt{c^2 dt^2 - d\vec{r} \cdot d\vec{r}} = c\sqrt{1 - \frac{v^2}{c^2}} dt$$

Back
Close

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Back
Close

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Back
Close

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Back

Close

$$S=-m_0c\int_a^b ds \qquad ds=\sqrt{c^2dt^2-d\vec{r}\cdot d\vec{r}}=c\sqrt{1-\frac{v^2}{c^2}}dt$$

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$$H = \vec{p}\cdot\vec{v} - L$$

Back
Close

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$$H = \vec{p} \cdot \vec{v} - L = \frac{m_0 v^2}{\sqrt{1 - \frac{v^2}{c^2}}} + m_0 c^2 \sqrt{1 - \frac{v^2}{c^2}}$$

Back
Close

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$$\frac{dp_i}{dt} = 0 \qquad \frac{d\vec{p}}{dt} = 0 \qquad \frac{dH}{dt} = 0$$

$$S = -m_0 c^2 \int_a^b dt \sqrt{1 - \frac{v^2}{c^2}} \qquad m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

⇐狭义相对论允许存在零质量粒子！

$$\vec{p} = \frac{m_0 \vec{v}}{\sqrt{1 - \frac{v^2}{c^2}}} = m\vec{v} \qquad \frac{d\vec{p}}{dt} = 0 \qquad H = mc^2 \qquad \frac{dH}{dt} = 0$$

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$$u^\mu = \frac{dx^\mu}{d\tau}$$

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$$S = -m_0c^2\int_a^b dt\sqrt{1-\frac{v^2}{c^2}} \qquad m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$ ⇐狭义相对论允许存在零质量粒子！

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$$p^\mu \equiv m_0u^\mu$$

$$S = -m_0c^2\int_a^b dt\sqrt{1-\frac{v^2}{c^2}} \qquad m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

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Back
Close

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虚粒子不满足质壳条件 ！

Back
Close

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Back
Close

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- 为什么要用四度矢量场来描述电磁场?

假设矢量场$A_\mu = (\vec{A}, \frac{i}{c}\phi)$描述电磁场,规定了 $(\vec{A}, \phi)$的洛伦兹变换关系,

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为描述带电点粒子在外电磁场的行为,必须在自由点粒子作用量加上一 个反应电磁场与带电粒子发生相互作用的项。

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Back
Close

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Back
Close

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Back
Close

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相对论力学: 带电点粒子及电荷分布在外电磁场中

$$S = \int_a^b dt\left(-m_0 c^2\sqrt{1-\frac{v^2}{c^2}} + e\vec{A}\cdot\vec{v} - e\phi\right) \qquad L = -m_0 c^2\sqrt{1-\frac{v^2}{c^2}} + e\vec{A}\cdot\vec{v} - e\phi$$

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$$\sum_{i=1}^{3}\frac{\partial L}{\partial x_i}\vec{e}_i = \nabla L$$

相对论力学: 带电点粒子及电荷分布在外电磁场中

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正则动量 $\neq$ 机械动量

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Back

Close

相对论力学: 带电点粒子及电荷分布在外电磁场中

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$$\frac{d}{dt}\frac{m_0\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{d\vec{p}}{dt} - e\frac{d\vec{A}}{dt}$$

Back
Close

相对论力学：带电点粒子及电荷分布在外电磁场中

$$S=\int_a^b dt\left(-m_0c^2\sqrt{1-\frac{v^2}{c^2}}+e\vec{A}\cdot\vec{v}-e\phi\right)\qquad L=-m_0c^2\sqrt{1-\frac{v^2}{c^2}}+e\vec{A}\cdot\vec{v}-e\phi$$

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相对论力学: 带电点粒子及电荷分布在外电磁场中

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$$\frac{d}{dt}\frac{m_0\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{d\vec{p}}{dt} - e\frac{d\vec{A}}{dt} \xlongequal{-\frac{\partial L}{\partial x_i} + \frac{d}{dt}\frac{\partial L}{\partial v_i} = 0} \nabla L - e\frac{d\vec{A}}{dt}$$

$$= e\vec{v}\cdot\nabla\vec{A} + e\vec{v}\times(\nabla\times\vec{A}) - e\nabla\phi - e\left(\frac{\partial\vec{A}}{\partial t} + \frac{\partial\vec{r}}{\partial t}\cdot\nabla\vec{A}\right)$$

相对论力学：带电点粒子及电荷分布在外电磁场中

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相对论力学：带电点粒子及电荷分布在外电磁场中

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$$E'_x = E_x \qquad E'_y = \frac{E_y - vB_z}{\sqrt{1-\frac{v^2}{c^2}}} \qquad E'_z = \frac{E_z + vB_y}{\sqrt{1-\frac{v^2}{c^2}}}$$

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$$\vec{E}=-\frac{\partial\vec{A}}{\partial t}-\nabla\phi\qquad \vec{B}=\nabla\times\vec{A}$$

$$E'_x=E_x\qquad E'_y=\frac{E_y-vB_z}{\sqrt{1-\frac{v^2}{c^2}}}\qquad E'_z=\frac{E_z+vB_y}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$B'_x=B_x\qquad B'_y=\frac{B_y+\frac{v}{c^2}E_z}{\sqrt{1-\frac{v^2}{c^2}}}\qquad B'_z=\frac{B_z-\frac{v}{c^2}E_y}{\sqrt{1-\frac{v^2}{c^2}}}$$

电场强度和磁感应强度不各自形成单独的四矢量的空间分量，而是联合形成二阶反对称张量，凸显电和磁的一体化！

点电荷及其流动是连续电荷分布及其流动的特例

Back
Close

点电荷及其流动是连续电荷分布及其流动的特例

$$\rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}') = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}')$$

点电荷及其流动是连续电荷分布及其流动的特例

$$\rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}') = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \qquad \vec{\mathbf{j}}(\vec{\mathbf{r}}, \vec{\mathbf{r}}') = \rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}')\vec{\mathbf{v}}$$

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Back
Close

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点电荷及其流动是连续电荷分布及其流动的特例

$$\rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}') = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \qquad \vec{\mathbf{j}}(\vec{\mathbf{r}}, \vec{\mathbf{r}}') = \rho(\vec{\mathbf{r}}, \vec{\mathbf{r}}')\vec{\mathbf{v}}$$

$$\mathbf{e}\mathbf{d}\mathbf{x}_\mu = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}')\mathbf{d}\tau\mathbf{d}\mathbf{x}_\mu = \rho\mathbf{d}\tau\mathbf{d}\mathbf{x}_\mu = \rho\frac{\mathbf{d}\mathbf{x}_\mu}{\mathbf{d}\mathbf{t}}\mathbf{d}\mathbf{t}\mathbf{d}\tau = \mathbf{j}_\mu\mathbf{d}\mathbf{t}\mathbf{d}\tau$$

$$\mathbf{j}_\mu \equiv \rho\frac{\mathbf{d}\mathbf{x}_\mu}{\mathbf{d}\mathbf{t}} = (\vec{\mathbf{j}}, \mathbf{i}\mathbf{c}\rho)$$

点电荷及其流动是连续电荷分布及其流动的特例

$$\rho(\vec{r}, \vec{r}') = e\delta(\vec{r} - \vec{r}') \qquad \vec{j}(\vec{r}, \vec{r}') = \rho(\vec{r}, \vec{r}')\vec{v}$$

$$e dx_\mu = e\delta(\vec{r} - \vec{r}') d\tau dx_\mu = \rho d\tau dx_\mu = \rho \frac{dx_\mu}{dt} dt d\tau = j_\mu dt d\tau$$

$$j_\mu \equiv \rho \frac{dx_\mu}{dt} = (\vec{j}, ic\rho) \quad \Rightarrow \quad \text{对比} \quad x_\mu = (\vec{r}, ict)$$

$$j'_x = \frac{j_x - v\rho}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Back

Close

点电荷及其流动是连续电荷分布及其流动的特例

$$\rho(\vec{\mathbf{r}},\vec{\mathbf{r}}')=\mathbf{e}\delta(\vec{\mathbf{r}}-\vec{\mathbf{r}}')\qquad\qquad \vec{\mathbf{j}}(\vec{\mathbf{r}},\vec{\mathbf{r}}')=\rho(\vec{\mathbf{r}},\vec{\mathbf{r}}')\vec{\mathbf{v}}$$

$$\mathbf{edx}_\mu=\mathbf{e}\delta(\vec{\mathbf{r}}-\vec{\mathbf{r}}')\mathbf{d}\tau\mathbf{dx}_\mu=\rho\mathbf{d}\tau\mathbf{dx}_\mu=\rho\frac{\mathbf{dx}_\mu}{\mathbf{dt}}\mathbf{dtd}\tau=\mathbf{j}_\mu\mathbf{dtd}\tau$$

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$$\mathbf{j}'_\mathbf{x}=\frac{\mathbf{j}_\mathbf{x}-\mathbf{v}\rho}{\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}}\qquad \mathbf{j}'_\mathbf{y}=\mathbf{j}_\mathbf{y}\qquad \mathbf{j}'_\mathbf{z}=\mathbf{j}_\mathbf{z}\qquad \rho'=\frac{\rho-\frac{\mathbf{v}}{\mathbf{c}^2}\mathbf{j}_\mathbf{x}}{\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}}$$

点电荷及其流动是连续电荷分布及其流动的特例

$$\rho(\vec{r},\vec{r}') = e\delta(\vec{r}-\vec{r}') \qquad \vec{j}(\vec{r},\vec{r}') = \rho(\vec{r},\vec{r}')\vec{v}$$

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$$j_\mu \equiv \rho\frac{dx_\mu}{dt} = (\vec{j}, ic\rho) \quad \Rightarrow \quad \text{对比} \quad x_\mu = (\vec{r}, ict)$$

$$j'_x = \frac{j_x - v\rho}{\sqrt{1-\frac{v^2}{c^2}}} \qquad j'_y = j_y \qquad j'_z = j_z \qquad \rho' = \frac{\rho - \frac{v}{c^2}j_x}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\sum_{\mu=1}^{4}\int_a^b dx_\mu\, eA_\mu = \sum_{\mu=1}^{4}\int dtd\tau A_\mu j_\mu$$

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$$j_x' = \frac{j_x - v\rho}{\sqrt{1-\frac{v^2}{c^2}}} \qquad j_y' = j_y \qquad j_z' = j_z \qquad \rho' = \frac{\rho - \frac{v}{c^2}j_x}{\sqrt{1-\frac{v^2}{c^2}}}$$

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点电荷及其流动是连续电荷分布及其流动的特例

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Back
Close

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Back
Close

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Back
Close

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$$\mathbf{P}_\mu = \mathbf{p}_\mu + \mathbf{e}\mathbf{A}_\mu$$

Back
Close

$$\mathbf{P}_\mu = \mathbf{p}_\mu + \mathbf{e}\mathbf{A}_\mu \qquad \mathbf{p}_\mu = \left(\frac{\mathbf{m}_0\vec{\mathbf{v}}}{\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}}, \frac{\mathbf{i}\mathbf{m}_0\mathbf{c}}{\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}}\right)$$

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$$\mathbf{F}_{\mu\nu} \equiv \partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu$$

Back
Close

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Back
Close

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$$\underline{F' = AFA^T}$$

Back
Close

相对论力学：协变表达

$$\mathbf{P}_\mu = \mathbf{p}_\mu + \mathbf{e}\mathbf{A}_\mu \qquad \mathbf{p}_\mu = \left(\frac{\mathbf{m_0}\vec{\mathbf{v}}}{\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}}, \frac{\mathbf{i}\mathbf{m_0}\mathbf{c}}{\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}}\right)$$

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$$\underline{\mathbf{F}' = \mathbf{A}\mathbf{F}\mathbf{A}^{\mathrm{T}}}$$

$$\left\{ \begin{array}{l} \mathbf{F}_{\mathbf{ij}} = \sum_{\mathbf{k}=1}^{3} \epsilon_{\mathbf{ijk}}\mathbf{B}_{\mathbf{k}} \\ \\ \end{array} \right.$$

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Back
Close

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$$\mathbf{f}_\mu \equiv \sum_{\nu=1}^{4} \mathbf{F}_{\mu\nu}\mathbf{j}_\nu = \left(\rho\vec{\mathbf{E}} + \vec{\mathbf{j}} \times \vec{\mathbf{B}},\ \frac{\mathbf{i}}{\mathbf{c}}\vec{\mathbf{j}}\cdot\vec{\mathbf{E}}\right)$$

$$\mathbf{f}_{\mathbf{i}} = \mathbf{F}_{\mathbf{i}4}\mathbf{j}_4 + \sum_{\mathbf{k}=1}^{3} \mathbf{F}_{\mathbf{ik}}\mathbf{j}_{\mathbf{k}}$$

Back
Close

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$$f_i = F_{i4} j_4 + \sum_{k=1}^{3} F_{ik} j_k = -\frac{i}{c} E_i ic\rho + \sum_{k,l=1}^{3} \epsilon_{ikl} j_k B_l$$

Back
Close

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$$f_i = F_{i4}j_4 + \sum_{k=1}^{3}F_{ik}j_k = -\frac{i}{c}E_i ic\rho + \sum_{k,l=1}^{3}\epsilon_{ikl}j_kB_l = (\rho\vec{E} + \vec{j}\times\vec{B})_i$$

Back
Close

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$$\left\{\begin{array}{l} F_{ij} = \sum_{k=1}^{3} \epsilon_{ijk} B_k \\ F_{4i} = -F_{i4} = \frac{i}{c}E_i \end{array}\right. \leftrightarrow \left\{\begin{array}{l} B_i = \sum_{j,k=1}^{3} \frac{1}{2}\epsilon_{ijk} F_{jk} \\ E_i = -icF_{4i} \end{array}\right.$$

$$f_\mu \equiv \sum_{\nu=1}^{4} F_{\mu\nu} j_\nu = (\rho\vec{E} + \vec{j}\times\vec{B},\ \frac{i}{c}\vec{j}\cdot\vec{E})$$

$$f_i = F_{i4}j_4 + \sum_{k=1}^{3} F_{ik}j_k = -\frac{i}{c}E_i ic\rho + \sum_{k,l=1}^{3} \epsilon_{ikl} j_k B_l = (\rho\vec{E} + \vec{j}\times\vec{B})_i$$

$$f_4 = \sum_{k=1}^{3} F_{4k} j_k$$

Back

Close

$$P_\mu = p_\mu + eA_\mu \qquad p_\mu = \left(\frac{m_0\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}, \frac{im_0c}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu = \begin{pmatrix} 0 & B_3 & -B_2 & -\frac{i}{c}E_1 \\ -B_3 & 0 & B_1 & -\frac{i}{c}E_2 \\ B_2 & -B_1 & 0 & -\frac{i}{c}E_3 \\ \frac{i}{c}E_1 & \frac{i}{c}E_2 & \frac{i}{c}E_3 & 0 \end{pmatrix} \qquad F'_{\mu\nu} = \sum_{\lambda,\lambda'=1}^{4} a_{\mu\lambda}F_{\lambda\lambda'}a_{\nu\lambda'}$$

$$\underline{F' = AFA^T}$$

$$\begin{cases} F_{ij} = \sum_{k=1}^{3}\epsilon_{ijk}B_k \\ F_{4i} = -F_{i4} = \frac{i}{c}E_i \end{cases} \leftrightarrow \begin{cases} B_i = \sum_{j,k=1}^{3}\frac{1}{2}\epsilon_{ijk}F_{jk} \\ E_i = -icF_{4i} \end{cases}$$

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$$f_i = F_{i4}j_4 + \sum_{k=1}^{3}F_{ik}j_k = -\frac{i}{c}E_i ic\rho + \sum_{k,l=1}^{3}\epsilon_{ikl}j_kB_l = (\rho\vec{E} + \vec{j}\times\vec{B})_i$$

$$f_4 = \sum_{k=1}^{3}F_{4k}j_k = \frac{i}{c}E_kj_k = \frac{i}{c}\vec{E}\cdot\vec{j}$$

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$$f_4 = \sum_{k=1}^{3}F_{4k}j_k = \frac{i}{c}E_kj_k = \frac{i}{c}\vec{E}\cdot\vec{j}$$

Back
Close

$$\mathbf{f}_\mu \equiv \sum_{\nu=1}^{4} \mathbf{F}_{\mu\nu}\mathbf{j}_\nu = (\rho\vec{\mathbf{E}} + \vec{\mathbf{j}} \times \vec{\mathbf{B}},\ \frac{\mathbf{i}}{\mathbf{c}}\vec{\mathbf{j}} \cdot \vec{\mathbf{E}})$$

$$\mathbf{f}_\mu \equiv \sum_{\nu=1}^{4} \mathbf{F}_{\mu\nu}\mathbf{j}_\nu = (\rho\vec{\mathbf{E}} + \vec{\mathbf{j}} \times \vec{\mathbf{B}},\ \frac{\mathbf{i}}{\mathbf{c}}\vec{\mathbf{j}} \cdot \vec{\mathbf{E}})$$

对点电荷: $\mathbf{j}_\mu = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}')\dfrac{\mathbf{dx}_\mu}{\mathbf{dt}}$

$$\mathbf{f}_\mu \equiv \sum_{\nu=1}^{4} \mathbf{F}_{\mu\nu}\mathbf{j}_\nu = (\rho\vec{\mathbf{E}} + \vec{\mathbf{j}} \times \vec{\mathbf{B}},\ \frac{\mathbf{i}}{\mathbf{c}}\vec{\mathbf{j}} \cdot \vec{\mathbf{E}})$$

对点电荷: $\mathbf{j}_\mu = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}')\frac{\mathbf{dx}_\mu}{\mathbf{dt}} = \int \mathbf{dt}'\ \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}')\delta(\mathbf{t} - \mathbf{t}')\frac{\mathbf{dx}_\mu}{\mathbf{dt}'}$

$$\mathbf{f}_\mu \equiv \sum_{\nu=1}^{4} \mathbf{F}_{\mu\nu}\mathbf{j}_\nu = (\rho\vec{\mathbf{E}} + \vec{\mathbf{j}} \times \vec{\mathbf{B}},\ \frac{\mathbf{i}}{\mathbf{c}}\vec{\mathbf{j}} \cdot \vec{\mathbf{E}})$$

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$$= \int \mathbf{d}\tau'\mathbf{e}\delta^{(4)}(\mathbf{x} - \mathbf{x}')\frac{\mathbf{dx}_\mu}{\mathbf{d}\tau'}$$

Back
Close

$$\mathbf{f}_\mu \equiv \sum_{\nu=1}^{4} \mathbf{F}_{\mu\nu}\mathbf{j}_\nu = (\rho\vec{\mathbf{E}} + \vec{\mathbf{j}} \times \vec{\mathbf{B}},\ \frac{\mathbf{i}}{\mathbf{c}}\vec{\mathbf{j}} \cdot \vec{\mathbf{E}})$$

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$$= \int \mathbf{d}\tau'\mathbf{e}\delta^{(4)}(\mathbf{x} - \mathbf{x}')\frac{\mathbf{dx}_\mu}{\mathbf{d}\tau'} = \int \mathbf{d}\tau'\mathbf{e}\delta^{(4)}(\mathbf{x} - \mathbf{x}')\mathbf{u}_\mu$$

$$\mathbf{f}_\mu \equiv \sum_{\nu=1}^{4} \mathbf{F}_{\mu\nu}\mathbf{j}_\nu = (\rho\vec{\mathbf{E}} + \vec{\mathbf{j}} \times \vec{\mathbf{B}},\ \frac{\mathbf{i}}{\mathbf{c}}\vec{\mathbf{j}} \cdot \vec{\mathbf{E}})$$

对点电荷: $\mathbf{j}_\mu = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}')\frac{\mathbf{dx}_\mu}{\mathbf{dt}} = \int \mathbf{dt}'\ \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}')\delta(\mathbf{t} - \mathbf{t}')\frac{\mathbf{dx}_\mu}{\mathbf{dt}'}$

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$$\frac{\mathbf{dp}_\mu}{\mathbf{d}\tau} = \mathbf{K}_\mu$$

$$\mathbf{f}_\mu \equiv \sum_{\nu=1}^{4} \mathbf{F}_{\mu\nu}\mathbf{j}_\nu = (\rho\vec{\mathbf{E}} + \vec{\mathbf{j}} \times \vec{\mathbf{B}},\ \frac{\mathbf{i}}{\mathbf{c}}\vec{\mathbf{j}} \cdot \vec{\mathbf{E}})$$

对点电荷: $\mathbf{j}_\mu = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}')\frac{\mathbf{dx}_\mu}{\mathbf{dt}} = \int \mathbf{dt}'\ \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}')\delta(\mathbf{t} - \mathbf{t}')\frac{\mathbf{dx}_\mu}{\mathbf{dt}'}$

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$$\frac{\mathbf{dp}_\mu}{\mathbf{d}\tau} = \mathbf{K}_\mu$$

$\overset{\text{作业}}{\Longrightarrow} \mathbf{K}_\mu = (\vec{\mathbf{K}}, \frac{\mathbf{i}}{\mathbf{c}}\vec{\mathbf{K}} \cdot \vec{\mathbf{v}})$

$$\mathbf{f}_\mu \equiv \sum_{\nu=1}^{4} \mathbf{F}_{\mu\nu}\mathbf{j}_\nu = (\rho\vec{\mathbf{E}} + \vec{\mathbf{j}} \times \vec{\mathbf{B}},\ \frac{\mathbf{i}}{\mathbf{c}}\vec{\mathbf{j}} \cdot \vec{\mathbf{E}})$$

对点电荷: $\mathbf{j}_\mu = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}')\dfrac{\mathbf{dx}_\mu}{\mathbf{dt}} = \displaystyle\int \mathbf{dt}'\ \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}')\delta(\mathbf{t} - \mathbf{t}')\frac{\mathbf{dx}_\mu}{\mathbf{dt}'}$

$$= \int \mathbf{d}\tau'\mathbf{e}\delta^{(4)}(\mathbf{x} - \mathbf{x}')\frac{\mathbf{dx}_\mu}{\mathbf{d}\tau'} = \int \mathbf{d}\tau'\mathbf{e}\delta^{(4)}(\mathbf{x} - \mathbf{x}')\mathbf{u}_\mu$$

$$\frac{\mathbf{dp}_\mu}{\mathbf{d}\tau} = \mathbf{K}_\mu$$

$$\xRightarrow{\text{作业}} \mathbf{K}_\mu = (\vec{\mathbf{K}}, \frac{\mathbf{i}}{\mathbf{c}}\vec{\mathbf{K}} \cdot \vec{\mathbf{v}}) = \sum_{\nu=1}^{4} \mathbf{e}\mathbf{F}_{\mu\nu}\mathbf{u}_\nu$$

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$$\mathbf{f}_\mu \equiv \sum_{\nu=1}^{4} \mathbf{F}_{\mu\nu}\mathbf{j}_\nu = (\rho\vec{\mathbf{E}} + \vec{\mathbf{j}} \times \vec{\mathbf{B}},\ \frac{\mathbf{i}}{\mathbf{c}}\vec{\mathbf{j}} \cdot \vec{\mathbf{E}})$$

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$$= (\frac{\mathbf{e}}{\sqrt{1 - \frac{\mathbf{v}^2}{\mathbf{c}^2}}}(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}), \frac{\frac{\mathbf{ie}}{\mathbf{c}}\vec{\mathbf{E}} \cdot \vec{\mathbf{v}}}{\sqrt{1 - \frac{\mathbf{v}^2}{\mathbf{c}^2}}})$$

Back
Close

$$\mathbf{f}_\mu \equiv \sum_{\nu=1}^{4} \mathbf{F}_{\mu\nu}\mathbf{j}_\nu = (\rho\vec{\mathbf{E}} + \vec{\mathbf{j}} \times \vec{\mathbf{B}},\ \frac{\mathbf{i}}{\mathbf{c}}\vec{\mathbf{j}} \cdot \vec{\mathbf{E}})$$

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$$\xRightarrow{\text{作业}} \mathbf{K}_\mu = (\vec{\mathbf{K}}, \frac{\mathbf{i}}{\mathbf{c}}\vec{\mathbf{K}} \cdot \vec{\mathbf{v}}) = \sum_{\nu=1}^{4} \mathbf{e}\mathbf{F}_{\mu\nu}\mathbf{u}_\nu \qquad \mathbf{f}_\mu = \int \mathbf{d}\tau\delta^{(4)}(\mathbf{x} - \mathbf{x}')\mathbf{K}_\mu$$

$$= (\frac{\mathbf{e}}{\sqrt{1 - \frac{\mathbf{v}^2}{\mathbf{c}^2}}}(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}), \frac{\frac{\mathbf{ie}}{\mathbf{c}}\vec{\mathbf{E}} \cdot \vec{\mathbf{v}}}{\sqrt{1 - \frac{\mathbf{v}^2}{\mathbf{c}^2}}})$$

或: $\vec{\mathbf{F}} = \vec{\mathbf{K}}\sqrt{1 - \frac{\mathbf{v}^2}{\mathbf{c}^2}}$

$$\mathbf{f}_\mu \equiv \sum_{\nu=1}^{4} \mathbf{F}_{\mu\nu}\mathbf{j}_\nu = (\rho\vec{\mathbf{E}} + \vec{\mathbf{j}} \times \vec{\mathbf{B}},\ \frac{\mathbf{i}}{\mathbf{c}}\vec{\mathbf{j}} \cdot \vec{\mathbf{E}})$$

对点电荷: $\mathbf{j}_\mu = \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}')\frac{\mathbf{dx}_\mu}{\mathbf{dt}} = \int \mathbf{dt}'\ \mathbf{e}\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}')\delta(\mathbf{t} - \mathbf{t}')\frac{\mathbf{dx}_\mu}{\mathbf{dt}'}$

$$= \int \mathbf{d}\tau'\mathbf{e}\delta^{(4)}(\mathbf{x} - \mathbf{x}')\frac{\mathbf{dx}_\mu}{\mathbf{d}\tau'} = \int \mathbf{d}\tau'\mathbf{e}\delta^{(4)}(\mathbf{x} - \mathbf{x}')\mathbf{u}_\mu$$

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$$\xRightarrow{\text{作业}} \mathbf{K}_\mu = (\vec{\mathbf{K}}, \frac{\mathbf{i}}{\mathbf{c}}\vec{\mathbf{K}} \cdot \vec{\mathbf{v}}) = \sum_{\nu=1}^{4} \mathbf{eF}_{\mu\nu}\mathbf{u}_\nu \qquad \mathbf{f}_\mu = \int \mathbf{d}\tau\delta^{(4)}(\mathbf{x} - \mathbf{x}')\mathbf{K}_\mu$$

$$= (\frac{\mathbf{e}}{\sqrt{1 - \frac{\mathbf{v}^2}{\mathbf{c}^2}}}(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}), \frac{\frac{\mathbf{ie}}{\mathbf{c}}\vec{\mathbf{E}} \cdot \vec{\mathbf{v}}}{\sqrt{1 - \frac{\mathbf{v}^2}{\mathbf{c}^2}}})$$

或: $\vec{\mathbf{F}} = \vec{\mathbf{K}}\sqrt{1 - \frac{\mathbf{v}^2}{\mathbf{c}^2}} \quad \vec{\mathbf{F}} = \frac{\mathbf{d}\vec{\mathbf{P}}}{\mathbf{dt}} \quad \vec{\mathbf{F}} \cdot \vec{\mathbf{v}} = \frac{\mathbf{dH}}{\mathbf{dt}} \Rightarrow \vec{\mathbf{F}}' = \frac{\mathbf{d}\vec{\mathbf{P}}'}{\mathbf{dt}'} \quad \vec{\mathbf{F}}' \cdot \vec{\mathbf{v}}' = \frac{\mathbf{dH}'}{\mathbf{dt}'}$

$$\mathbf{S} = -\mathbf{m_0 c}\int_{\mathbf{a}}^{\mathbf{b}}\sqrt{-\sum_{\mu=1}^{4}\mathbf{dx}_\mu\mathbf{dx}_\mu} + \mathbf{e}\sum_{\mu=1}^{4}\int_{\mathbf{a}}^{\mathbf{b}}\mathbf{dx}_\mu\ \mathbf{A}_\mu$$

$$\mathbf{S} = -\mathbf{m_0 c}\int_{\mathbf{a}}^{\mathbf{b}}\sqrt{-\sum_{\mu=1}^{4}\mathbf{dx}_\mu\mathbf{dx}_\mu} + \mathbf{e}\sum_{\mu=1}^{4}\int_{\mathbf{a}}^{\mathbf{b}}\mathbf{dx}_\mu\ \mathbf{A}_\mu$$

$$\mathbf{0}=\delta\mathbf{S}=\mathbf{m_0 c}\int_{\mathbf{a}}^{\mathbf{b}}\frac{\sum_{\mu=1}^{4}\mathbf{dx}_\mu\delta\mathbf{dx}_\mu}{\sqrt{-\sum_{\nu=1}^{4}\mathbf{dx}_\nu\mathbf{dx}_\nu}} + \mathbf{e}\int_{\mathbf{a}}^{\mathbf{b}}[\sum_{\mu=1}^{4}\delta\mathbf{dx}_\mu\mathbf{A}_\mu + \sum_{\mu,\nu=1}^{4}\mathbf{dx}_\mu(\partial_\nu\mathbf{A}_\mu)\delta\mathbf{x}_\nu]$$

$$\mathbf{S} = -\mathbf{m_0 c}\int_{\mathbf{a}}^{\mathbf{b}}\sqrt{-\sum_{\mu=1}^{4}\mathbf{dx}_\mu\mathbf{dx}_\mu} + \mathbf{e}\sum_{\mu=1}^{4}\int_{\mathbf{a}}^{\mathbf{b}}\mathbf{dx}_\mu\ \mathbf{A}_\mu$$

$$\mathbf{0} = \delta\mathbf{S} = \mathbf{m_0 c}\int_{\mathbf{a}}^{\mathbf{b}}\frac{\sum_{\mu=1}^{4}\mathbf{dx}_\mu\delta\mathbf{dx}_\mu}{\sqrt{-\sum_{\nu=1}^{4}\mathbf{dx}_\nu\mathbf{dx}_\nu}} + \mathbf{e}\int_{\mathbf{a}}^{\mathbf{b}}[\sum_{\mu=1}^{4}\delta\mathbf{dx}_\mu\mathbf{A}_\mu + \sum_{\mu,\nu=1}^{4}\mathbf{dx}_\mu(\partial_\nu\mathbf{A}_\mu)\delta\mathbf{x}_\nu]$$

$$= \int_{\mathbf{a}}^{\mathbf{b}}\sum_{\mu=1}^{4}[(\mathbf{m_0 u}_\mu + \mathbf{eA}_\mu)\delta\mathbf{dx}_\mu + \sum_{\nu=1}^{4}\mathbf{edx}_\mu(\partial_\nu\mathbf{A}_\mu)\delta\mathbf{x}_\nu]$$

Back
Close

$$\mathbf{S} = -\mathbf{m_0 c}\int_{\mathbf{a}}^{\mathbf{b}} \sqrt{-\sum_{\mu=1}^{4} \mathbf{dx}_\mu \mathbf{dx}_\mu} + \mathbf{e}\sum_{\mu=1}^{4}\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{dx}_\mu\ \mathbf{A}_\mu$$

$$\mathbf{0} = \delta\mathbf{S} = \mathbf{m_0 c}\int_{\mathbf{a}}^{\mathbf{b}} \frac{\sum_{\mu=1}^{4} \mathbf{dx}_\mu \delta\mathbf{dx}_\mu}{\sqrt{-\sum_{\nu=1}^{4} \mathbf{dx}_\nu \mathbf{dx}_\nu}} + \mathbf{e}\int_{\mathbf{a}}^{\mathbf{b}} [\sum_{\mu=1}^{4} \delta\mathbf{dx}_\mu \mathbf{A}_\mu + \sum_{\mu,\nu=1}^{4} \mathbf{dx}_\mu (\partial_\nu \mathbf{A}_\mu)\delta\mathbf{x}_\nu]$$

$$= \int_{\mathbf{a}}^{\mathbf{b}} \sum_{\mu=1}^{4} [(\mathbf{m_0 u}_\mu + \mathbf{eA}_\mu)\delta\mathbf{dx}_\mu + \sum_{\nu=1}^{4} \mathbf{edx}_\mu (\partial_\nu \mathbf{A}_\mu)\delta\mathbf{x}_\nu]$$

$$= \int_{\mathbf{a}}^{\mathbf{b}} \sum_{\mu=1}^{4} [-\mathbf{m_0 du}_\mu - \mathbf{edA}_\mu + \sum_{\nu=1}^{4} \mathbf{edx}_\nu (\partial_\mu \mathbf{A}_\nu)]\delta\mathbf{x}_\mu$$

$$\mathbf{S} = -\mathbf{m_0 c}\int_a^b \sqrt{-\sum_{\mu=1}^{4} \mathbf{dx}_\mu \mathbf{dx}_\mu} + \mathbf{e}\sum_{\mu=1}^{4}\int_a^b \mathbf{dx}_\mu\ \mathbf{A}_\mu$$

$$\mathbf{0} = \delta\mathbf{S} = \mathbf{m_0 c}\int_a^b \frac{\sum_{\mu=1}^{4}\mathbf{dx}_\mu \delta\mathbf{dx}_\mu}{\sqrt{-\sum_{\nu=1}^{4}\mathbf{dx}_\nu \mathbf{dx}_\nu}} + \mathbf{e}\int_a^b [\sum_{\mu=1}^{4}\delta\mathbf{dx}_\mu \mathbf{A}_\mu + \sum_{\mu,\nu=1}^{4}\mathbf{dx}_\mu(\partial_\nu \mathbf{A}_\mu)\delta\mathbf{x}_\nu]$$

$$= \int_a^b \sum_{\mu=1}^{4}[(\mathbf{m_0 u}_\mu + \mathbf{eA}_\mu)\delta\mathbf{dx}_\mu + \sum_{\nu=1}^{4}\mathbf{edx}_\mu(\partial_\nu \mathbf{A}_\mu)\delta\mathbf{x}_\nu]$$

$$= \int_a^b \sum_{\mu=1}^{4}[-\mathbf{m_0 du}_\mu - \mathbf{edA}_\mu + \sum_{\nu=1}^{4}\mathbf{edx}_\nu(\partial_\mu \mathbf{A}_\nu)]\delta\mathbf{x}_\mu$$

$$= \int_a^b \sum_{\mu=1}^{4}[-\mathbf{m_0 du}_\mu - \sum_{\nu=1}^{4}\mathbf{edx}_\nu(\partial_\nu \mathbf{A}_\mu) + \sum_{\nu=1}^{4}\mathbf{edx}_\nu(\partial_\mu \mathbf{A}_\nu)]\delta\mathbf{x}_\mu$$

$$\mathbf{S} = -\mathbf{m_0 c}\int_a^b \sqrt{-\sum_{\mu=1}^{4}\mathbf{dx}_\mu \mathbf{dx}_\mu} + \mathbf{e}\sum_{\mu=1}^{4}\int_a^b \mathbf{dx}_\mu\ \mathbf{A}_\mu$$

$$\mathbf{0} = \delta\mathbf{S} = \mathbf{m_0 c}\int_a^b \frac{\sum_{\mu=1}^{4}\mathbf{dx}_\mu \delta\mathbf{dx}_\mu}{\sqrt{-\sum_{\nu=1}^{4}\mathbf{dx}_\nu \mathbf{dx}_\nu}} + \mathbf{e}\int_a^b [\sum_{\mu=1}^{4}\delta\mathbf{dx}_\mu \mathbf{A}_\mu + \sum_{\mu,\nu=1}^{4}\mathbf{dx}_\mu(\partial_\nu \mathbf{A}_\mu)\delta\mathbf{x}_\nu]$$

$$= \int_a^b \sum_{\mu=1}^{4}[(\mathbf{m_0 u}_\mu + \mathbf{eA}_\mu)\delta\mathbf{dx}_\mu + \sum_{\nu=1}^{4}\mathbf{edx}_\mu(\partial_\nu \mathbf{A}_\mu)\delta\mathbf{x}_\nu]$$

$$= \int_a^b \sum_{\mu=1}^{4}[-\mathbf{m_0 du}_\mu - \mathbf{edA}_\mu + \sum_{\nu=1}^{4}\mathbf{edx}_\nu(\partial_\mu \mathbf{A}_\nu)]\delta\mathbf{x}_\mu$$

$$= \int_a^b \sum_{\mu=1}^{4}[-\mathbf{m_0 du}_\mu - \sum_{\nu=1}^{4}\mathbf{edx}_\nu(\partial_\nu \mathbf{A}_\mu) + \sum_{\nu=1}^{4}\mathbf{edx}_\nu(\partial_\mu \mathbf{A}_\nu)]\delta\mathbf{x}_\mu$$

$$= \int_a^b \sum_{\mu=1}^{4}[-\mathbf{m_0}\frac{\mathbf{du}_\mu}{\mathbf{d}\tau} + \sum_{\nu=1}^{4}\mathbf{e}\frac{\mathbf{dx}_\nu}{\mathbf{d}\tau}\mathbf{F}_{\mu\nu}]\mathbf{d}\tau\delta\mathbf{x}_\mu$$

$$\mathbf{S} = -\mathbf{m_0 c}\int_a^b \sqrt{-\sum_{\mu=1}^{4} \mathbf{dx}_\mu \mathbf{dx}_\mu} + \mathbf{e}\sum_{\mu=1}^{4}\int_a^b \mathbf{dx}_\mu\ \mathbf{A}_\mu$$

$$\mathbf{0} = \delta\mathbf{S} = \mathbf{m_0 c}\int_a^b \frac{\sum_{\mu=1}^{4}\mathbf{dx}_\mu \delta\mathbf{dx}_\mu}{\sqrt{-\sum_{\nu=1}^{4}\mathbf{dx}_\nu\mathbf{dx}_\nu}} + \mathbf{e}\int_a^b [\sum_{\mu=1}^{4}\delta\mathbf{dx}_\mu\mathbf{A}_\mu + \sum_{\mu,\nu=1}^{4}\mathbf{dx}_\mu(\partial_\nu\mathbf{A}_\mu)\delta\mathbf{x}_\nu]$$

$$= \int_a^b \sum_{\mu=1}^{4}[(\mathbf{m_0 u}_\mu + \mathbf{eA}_\mu)\delta\mathbf{dx}_\mu + \sum_{\nu=1}^{4}\mathbf{edx}_\mu(\partial_\nu\mathbf{A}_\mu)\delta\mathbf{x}_\nu]$$

$$= \int_a^b \sum_{\mu=1}^{4}[-\mathbf{m_0 du}_\mu - \mathbf{edA}_\mu + \sum_{\nu=1}^{4}\mathbf{edx}_\nu(\partial_\mu\mathbf{A}_\nu)]\delta\mathbf{x}_\mu$$

$$= \int_a^b \sum_{\mu=1}^{4}[-\mathbf{m_0 du}_\mu - \sum_{\nu=1}^{4}\mathbf{edx}_\nu(\partial_\nu\mathbf{A}_\mu) + \sum_{\nu=1}^{4}\mathbf{edx}_\nu(\partial_\mu\mathbf{A}_\nu)]\delta\mathbf{x}_\mu$$

$$= \int_a^b \sum_{\mu=1}^{4}[-\mathbf{m_0}\frac{\mathbf{du}_\mu}{\mathbf{d}\tau} + \sum_{\nu=1}^{4}\mathbf{e}\frac{\mathbf{dx}_\nu}{\mathbf{d}\tau}\mathbf{F}_{\mu\nu}]\mathbf{d}\tau\delta\mathbf{x}_\mu \quad \Rightarrow \quad \frac{\mathbf{dp}_\mu}{\mathbf{d}\tau} = \sum_{\nu=1}^{4}\mathbf{eF}_{\mu\nu}\mathbf{u}_\nu$$

为描述电磁场,在作用量中加一纯含电磁场的项：标量、规范不变、最为简单.

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$$\underbrace{\sum_{\mu,\nu=1}^{4} \mathbf{F}_{\mu\nu}\mathbf{F}_{\mu\nu}}_{\text{后面专门讨论！}} = 2(\mathbf{B}^2 - \frac{1}{\mathbf{c}^2}\mathbf{E}^2)$$

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$$\underbrace{\sum_{\mu,\nu,\sigma,\rho=1}^{4} \epsilon_{\mu\nu\sigma\rho}\mathbf{F}_{\mu\nu}\mathbf{F}_{\sigma\rho}}_{\text{破坏空间反演；后面讨论！}} = -\frac{8\mathbf{i}}{\mathbf{c}}\vec{\mathbf{E}}\cdot\vec{\mathbf{B}} = \sum_{\mu,\nu,\sigma,\rho=1}^{4} 4\epsilon_{\mu\nu\sigma\rho}\partial_\mu(\mathbf{A}_\nu\partial_\sigma\mathbf{A}_\rho)$$

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$$\mathbf{S} = \int \mathbf{d}\mathbf{t}\mathbf{d}\tau\Big[(-\frac{1}{4\mu_0})\sum_{\mu,\nu=1}^{4}\mathbf{F}_{\mu\nu}\mathbf{F}_{\mu\nu} + \sum_{\mu=1}^{4}\mathbf{j}_\mu\mathbf{A}_\mu\Big]$$

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$$\mathbf{S} = \int \mathbf{dtd}\tau\left[(-\frac{1}{4\mu_0})\sum_{\mu,\nu=1}^{4}\mathbf{F}_{\mu\nu}\mathbf{F}_{\mu\nu} + \sum_{\mu=1}^{4}\mathbf{j}_\mu\mathbf{A}_\mu\right]$$

$$\mathbf{0} = \delta\mathbf{S}$$

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$$\mathbf{0} = \delta\mathbf{S} = \int \mathbf{dtd}\tau[(-\frac{1}{4\mu_0})\sum_{\mu,\nu=1}^{4}\delta(\mathbf{F}_{\mu\nu}\mathbf{F}_{\mu\nu}) + \sum_{\mu=1}^{4}\mathbf{j}_\mu\delta\mathbf{A}_\mu]$$

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$$\mathbf{0} = \delta\mathbf{S} = \int \mathbf{dtd}\tau\Big[(-\frac{1}{4\mu_0})\sum_{\mu,\nu=1}^{4}\delta(\mathbf{F}_{\mu\nu}\mathbf{F}_{\mu\nu}) + \sum_{\mu=1}^{4}\mathbf{j}_\mu\delta\mathbf{A}_\mu\Big]$$

$$= \int \mathbf{dtd}\tau\Big[(-\frac{1}{2\mu_0})\sum_{\mu,\nu=1}^{4}\mathbf{F}_{\mu\nu}\delta\mathbf{F}_{\mu\nu} + \sum_{\mu=1}^{4}\mathbf{j}_\mu\delta\mathbf{A}_\mu\Big]$$

$$\mathbf{0} = \delta \mathbf{S}$$

Back
Close

$$\mathbf{0} = \delta \mathbf{S} = \int \mathbf{d t d}\tau \left[\left(-\frac{\mathbf{1}}{\mathbf{4}\mu_{\mathbf{0}}}\right)\sum_{\mu,\nu=\mathbf{1}}^{\mathbf{4}} \delta(\mathbf{F}_{\mu\nu}\mathbf{F}_{\mu\nu}) + \sum_{\mu=\mathbf{1}}^{\mathbf{4}} \mathbf{j}_{\mu}\delta\mathbf{A}_{\mu}\right]$$

$$\mathbf{0} = \delta\mathbf{S} = \int \mathbf{dtd}\tau[(-\frac{\mathbf{1}}{\mathbf{4}\mu_0})\sum_{\mu,\nu=\mathbf{1}}^{\mathbf{4}}\delta(\mathbf{F}_{\mu\nu}\mathbf{F}_{\mu\nu}) + \sum_{\mu=\mathbf{1}}^{\mathbf{4}}\mathbf{j}_\mu\delta\mathbf{A}_\mu]$$

$$= \int \mathbf{dtd}\tau[(-\frac{\mathbf{1}}{\mathbf{2}\mu_0})\sum_{\mu,\nu=\mathbf{1}}^{\mathbf{4}}\mathbf{F}_{\mu\nu}\delta\mathbf{F}_{\mu\nu} + \sum_{\mu=\mathbf{1}}^{\mathbf{4}}\mathbf{j}_\mu\delta\mathbf{A}_\mu]$$

$$\mathbf{0} = \delta\mathbf{S} = \int \mathbf{dtd}\tau[(-\frac{\mathbf{1}}{\mathbf{4}\mu_\mathbf{0}})\sum_{\mu,\nu=\mathbf{1}}^{\mathbf{4}}\delta(\mathbf{F}_{\mu\nu}\mathbf{F}_{\mu\nu}) + \sum_{\mu=\mathbf{1}}^{\mathbf{4}}\mathbf{j}_\mu\delta\mathbf{A}_\mu]$$

$$= \int \mathbf{dtd}\tau[(-\frac{\mathbf{1}}{\mathbf{2}\mu_\mathbf{0}})\sum_{\mu,\nu=\mathbf{1}}^{\mathbf{4}}\mathbf{F}_{\mu\nu}\delta\mathbf{F}_{\mu\nu} + \sum_{\mu=\mathbf{1}}^{\mathbf{4}}\mathbf{j}_\mu\delta\mathbf{A}_\mu]$$

$$= \int \mathbf{dtd}\tau[(-\frac{\mathbf{1}}{\mu_\mathbf{0}})\sum_{\mu,\nu=\mathbf{1}}^{\mathbf{4}}\mathbf{F}_{\mu\nu}\delta(\partial_\mu\mathbf{A}_\nu) + \sum_{\mu=\mathbf{1}}^{\mathbf{4}}\mathbf{j}_\mu\delta\mathbf{A}_\mu]$$

$$
\begin{aligned}
\mathbf{0} = \delta \mathbf{S} &= \int \mathbf{dtd}\tau\Big[\Big(-\frac{\mathbf{1}}{\mathbf{4}\mu_\mathbf{0}}\Big)\sum_{\mu,\nu=1}^{4}\delta(\mathbf{F}_{\mu\nu}\mathbf{F}_{\mu\nu}) + \sum_{\mu=1}^{4}\mathbf{j}_\mu\delta\mathbf{A}_\mu\Big] \\
&= \int \mathbf{dtd}\tau\Big[\Big(-\frac{\mathbf{1}}{\mathbf{2}\mu_\mathbf{0}}\Big)\sum_{\mu,\nu=1}^{4}\mathbf{F}_{\mu\nu}\delta\mathbf{F}_{\mu\nu} + \sum_{\mu=1}^{4}\mathbf{j}_\mu\delta\mathbf{A}_\mu\Big] \\
&= \int \mathbf{dtd}\tau\Big[\Big(-\frac{\mathbf{1}}{\mu_\mathbf{0}}\Big)\sum_{\mu,\nu=1}^{4}\mathbf{F}_{\mu\nu}\delta(\partial_\mu\mathbf{A}_\nu) + \sum_{\mu=1}^{4}\mathbf{j}_\mu\delta\mathbf{A}_\mu\Big] \\
&= \int \mathbf{dtd}\tau\Big[\Big(-\frac{\mathbf{1}}{\mu_\mathbf{0}}\Big)\sum_{\mu,\nu=1}^{4}\mathbf{F}_{\mu\nu}(\partial_\mu\delta\mathbf{A}_\nu) + \sum_{\mu=1}^{4}\mathbf{j}_\mu\delta\mathbf{A}_\mu\Big]
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{0} = \delta\mathbf{S} &= \int \mathbf{dt}\mathbf{d}\tau[(-\frac{\mathbf{1}}{\mathbf{4}\mu_{\mathbf{0}}})\sum_{\mu,\nu=\mathbf{1}}^{\mathbf{4}}\delta(\mathbf{F}_{\mu\nu}\mathbf{F}_{\mu\nu}) + \sum_{\mu=\mathbf{1}}^{\mathbf{4}}\mathbf{j}_{\mu}\delta\mathbf{A}_{\mu}] \\
&= \int \mathbf{dt}\mathbf{d}\tau[(-\frac{\mathbf{1}}{\mathbf{2}\mu_{\mathbf{0}}})\sum_{\mu,\nu=\mathbf{1}}^{\mathbf{4}}\mathbf{F}_{\mu\nu}\delta\mathbf{F}_{\mu\nu} + \sum_{\mu=\mathbf{1}}^{\mathbf{4}}\mathbf{j}_{\mu}\delta\mathbf{A}_{\mu}] \\
&= \int \mathbf{dt}\mathbf{d}\tau[(-\frac{\mathbf{1}}{\mu_{\mathbf{0}}})\sum_{\mu,\nu=\mathbf{1}}^{\mathbf{4}}\mathbf{F}_{\mu\nu}\delta(\partial_{\mu}\mathbf{A}_{\nu}) + \sum_{\mu=\mathbf{1}}^{\mathbf{4}}\mathbf{j}_{\mu}\delta\mathbf{A}_{\mu}] \\
&= \int \mathbf{dt}\mathbf{d}\tau[(-\frac{\mathbf{1}}{\mu_{\mathbf{0}}})\sum_{\mu,\nu=\mathbf{1}}^{\mathbf{4}}\mathbf{F}_{\mu\nu}(\partial_{\mu}\delta\mathbf{A}_{\nu}) + \sum_{\mu=\mathbf{1}}^{\mathbf{4}}\mathbf{j}_{\mu}\delta\mathbf{A}_{\mu}] \\
&= \int \mathbf{dt}\mathbf{d}\tau[(-\frac{\mathbf{1}}{\mu_{\mathbf{0}}})\sum_{\mu,\nu=\mathbf{1}}^{\mathbf{4}}[\partial_{\mu}(\mathbf{F}_{\mu\nu}\delta\mathbf{A}_{\nu}) - (\partial_{\mu}\mathbf{F}_{\mu\nu})\delta\mathbf{A}_{\nu}] + \sum_{\mu=\mathbf{1}}^{\mathbf{4}}\mathbf{j}_{\mu}\delta\mathbf{A}_{\mu}]
\end{aligned}
$$

$$\mathbf{0} = \delta\mathbf{S} = \int \mathbf{dtd}\tau\Big[(-\frac{\mathbf{1}}{\mathbf{4}\mu_\mathbf{0}})\sum_{\mu,\nu=\mathbf{1}}^{\mathbf{4}}\delta(\mathbf{F}_{\mu\nu}\mathbf{F}_{\mu\nu}) + \sum_{\mu=\mathbf{1}}^{\mathbf{4}}\mathbf{j}_\mu\delta\mathbf{A}_\mu\Big]$$

$$= \int \mathbf{dtd}\tau\Big[(-\frac{\mathbf{1}}{\mathbf{2}\mu_\mathbf{0}})\sum_{\mu,\nu=\mathbf{1}}^{\mathbf{4}}\mathbf{F}_{\mu\nu}\delta\mathbf{F}_{\mu\nu} + \sum_{\mu=\mathbf{1}}^{\mathbf{4}}\mathbf{j}_\mu\delta\mathbf{A}_\mu\Big]$$

$$= \int \mathbf{dtd}\tau\Big[(-\frac{\mathbf{1}}{\mu_\mathbf{0}})\sum_{\mu,\nu=\mathbf{1}}^{\mathbf{4}}\mathbf{F}_{\mu\nu}\delta(\partial_\mu\mathbf{A}_\nu) + \sum_{\mu=\mathbf{1}}^{\mathbf{4}}\mathbf{j}_\mu\delta\mathbf{A}_\mu\Big]$$

$$= \int \mathbf{dtd}\tau\Big[(-\frac{\mathbf{1}}{\mu_\mathbf{0}})\sum_{\mu,\nu=\mathbf{1}}^{\mathbf{4}}\mathbf{F}_{\mu\nu}(\partial_\mu\delta\mathbf{A}_\nu) + \sum_{\mu=\mathbf{1}}^{\mathbf{4}}\mathbf{j}_\mu\delta\mathbf{A}_\mu\Big]$$

$$= \int \mathbf{dtd}\tau\Big[(-\frac{\mathbf{1}}{\mu_\mathbf{0}})\sum_{\mu,\nu=\mathbf{1}}^{\mathbf{4}}[\partial_\mu(\mathbf{F}_{\mu\nu}\delta\mathbf{A}_\nu) - (\partial_\mu\mathbf{F}_{\mu\nu})\delta\mathbf{A}_\nu] + \sum_{\mu=\mathbf{1}}^{\mathbf{4}}\mathbf{j}_\mu\delta\mathbf{A}_\mu\Big]$$

$$\sum_{\mu=\mathbf{1}}^{\mathbf{4}}\partial_\mu\mathbf{F}_{\mu\nu} = -\mu_\mathbf{0}\mathbf{j}_\nu \qquad\qquad \partial_\mu\mathbf{F}_{\nu\lambda} + \partial_\nu\mathbf{F}_{\lambda\mu} + \partial_\lambda\mathbf{F}_{\mu\nu} = \mathbf{0}$$

$$\sum_{\mu=1}^{4}\partial_\mu \mathbf{F}_{\mu\nu} = -\mu_0 \mathbf{j}_\nu \quad \partial_\mu \mathbf{F}_{\nu\lambda} + \partial_\nu \mathbf{F}_{\lambda\mu} + \partial_\lambda \mathbf{F}_{\mu\nu} = \mathbf{0}$$

$$\sum_{\mu=1}^{4} \partial_\mu \mathbf{F}_{\mu\nu} = -\mu_0 \mathbf{j}_\nu \qquad \partial_\mu \mathbf{F}_{\nu\lambda} + \partial_\nu \mathbf{F}_{\lambda\mu} + \partial_\lambda \mathbf{F}_{\mu\nu} = \mathbf{0}$$

$$\sum_{\mathrm{j}=1}^{3} \partial_{\mathrm{j}} \mathbf{F}_{\mathrm{ji}} = -\mu_0 \mathbf{j}_{\mathrm{i}} - \partial_4 \mathbf{F}_{4\mathrm{i}}$$

$$\sum_{\mu=1}^{4} \partial_\mu \mathbf{F}_{\mu\nu} = -\mu_0 \mathbf{j}_\nu \qquad \partial_\mu \mathbf{F}_{\nu\lambda} + \partial_\nu \mathbf{F}_{\lambda\mu} + \partial_\lambda \mathbf{F}_{\mu\nu} = \mathbf{0}$$

$$\sum_{j=1}^{3} \partial_j \mathbf{F}_{ji} = -\mu_0 \mathbf{j}_i - \partial_4 \mathbf{F}_{4i} \rightarrow \sum_{j,k=1}^{3} \partial_j \epsilon_{jik} \mathbf{B}_k = -\mu_0 \mathbf{j}_i - \frac{1}{c^2} \frac{\partial \mathbf{E}_i}{\partial t}$$

$$\sum_{\mu=1}^{4}\partial_\mu \mathbf{F}_{\mu\nu} = -\mu_0 \mathbf{j}_\nu \qquad \partial_\mu \mathbf{F}_{\nu\lambda} + \partial_\nu \mathbf{F}_{\lambda\mu} + \partial_\lambda \mathbf{F}_{\mu\nu} = 0$$

$$\sum_{j=1}^{3}\partial_j \mathbf{F}_{ji} = -\mu_0 \mathbf{j}_i - \partial_4 \mathbf{F}_{4i} \rightarrow \sum_{j,k=1}^{3}\partial_j \epsilon_{jik}\mathbf{B}_k = -\mu_0 \mathbf{j}_i - \frac{1}{c^2}\frac{\partial \mathbf{E}_i}{\partial t}$$

$$\rightarrow \underline{\nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t}}$$

Back
Close

$$\sum_{\mu=1}^{4}\partial_\mu \mathbf{F}_{\mu\nu} = -\mu_0 \mathbf{j}_\nu \qquad \partial_\mu \mathbf{F}_{\nu\lambda} + \partial_\nu \mathbf{F}_{\lambda\mu} + \partial_\lambda \mathbf{F}_{\mu\nu} = \mathbf{0}$$

$$\sum_{j=1}^{3}\partial_j \mathrm{F}_{ji} = -\mu_0 \mathrm{j}_i - \partial_4 \mathrm{F}_{4i} \rightarrow \sum_{j,k=1}^{3}\partial_j \epsilon_{jik}\mathrm{B}_k = -\mu_0 \mathrm{j}_i - \frac{1}{c^2}\frac{\partial \mathrm{E}_i}{\partial t}$$

$$\rightarrow \underline{\nabla \times \vec{\mathrm{B}} = \mu_0 \vec{\mathrm{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathrm{E}}}{\partial t}}$$

$$\sum_{i=1}^{3}\partial_i \mathrm{F}_{i4} = -\mu_0 \mathrm{j}_4$$

$$\sum_{\mu=1}^{4}\partial_\mu \mathbf{F}_{\mu\nu} = -\mu_0 \mathbf{j}_\nu \qquad \partial_\mu \mathbf{F}_{\nu\lambda} + \partial_\nu \mathbf{F}_{\lambda\mu} + \partial_\lambda \mathbf{F}_{\mu\nu} = \mathbf{0}$$

$$\sum_{j=1}^{3}\partial_j F_{ji} = -\mu_0 j_i - \partial_4 F_{4i} \rightarrow \sum_{j,k=1}^{3}\partial_j \epsilon_{jik} B_k = -\mu_0 j_i - \frac{1}{c^2}\frac{\partial E_i}{\partial t}$$

$$\rightarrow \underline{\nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}}$$

$$\sum_{i=1}^{3}\partial_i F_{i4} = -\mu_0 j_4 \quad \rightarrow \quad \sum_{i=1}^{3}\partial_i E_i\left(-\frac{i}{c}\right) = -\mu_0 i c \rho$$

$$\sum_{\mu=1}^{4}\partial_\mu \mathbf{F}_{\mu\nu} = -\mu_0 \mathbf{j}_\nu \qquad \partial_\mu \mathbf{F}_{\nu\lambda} + \partial_\nu \mathbf{F}_{\lambda\mu} + \partial_\lambda \mathbf{F}_{\mu\nu} = \mathbf{0}$$

$$\sum_{j=1}^{3}\partial_j F_{ji} = -\mu_0 j_i - \partial_4 F_{4i} \rightarrow \sum_{j,k=1}^{3}\partial_j \epsilon_{jik} B_k = -\mu_0 j_i - \frac{1}{c^2}\frac{\partial E_i}{\partial t}$$

$$\rightarrow \underline{\nabla\times\vec{B} = \mu_0\vec{j} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}}$$

$$\sum_{i=1}^{3}\partial_i F_{i4} = -\mu_0 j_4 \rightarrow \sum_{i=1}^{3}\partial_i E_i\left(-\frac{i}{c}\right) = -\mu_0 i c\rho \rightarrow \underline{\nabla\cdot\vec{E} = \frac{\rho}{\epsilon_0}}$$

$$\sum_{\mu=1}^{4}\partial_\mu \mathbf{F}_{\mu\nu} = -\mu_0 \mathbf{j}_\nu \qquad \partial_\mu \mathbf{F}_{\nu\lambda} + \partial_\nu \mathbf{F}_{\lambda\mu} + \partial_\lambda \mathbf{F}_{\mu\nu} = \mathbf{0}$$

$$\sum_{\mathrm{j}=1}^{3}\partial_{\mathrm{j}} \mathbf{F}_{\mathrm{ji}} = -\mu_0 \mathbf{j}_{\mathrm{i}} - \partial_4 \mathbf{F}_{4\mathrm{i}} \rightarrow \sum_{\mathrm{j,k}=1}^{3}\partial_{\mathrm{j}}\epsilon_{\mathrm{jik}}\mathbf{B}_{\mathrm{k}} = -\mu_0 \mathbf{j}_{\mathrm{i}} - \frac{1}{\mathbf{c}^2}\frac{\partial \mathbf{E}_{\mathrm{i}}}{\partial \mathrm{t}}$$

$$\rightarrow \underline{\nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial \mathrm{t}}}$$

$$\sum_{\mathrm{i}=1}^{3}\partial_{\mathrm{i}} \mathbf{F}_{\mathrm{i}4} = -\mu_0 \mathbf{j}_4 \quad \rightarrow \quad \sum_{\mathrm{i}=1}^{3}\partial_{\mathrm{i}}\mathbf{E}_{\mathrm{i}}(-\frac{\mathbf{i}}{\mathbf{c}}) = -\mu_0 \mathbf{i}\mathbf{c}\rho \quad \rightarrow \quad \underline{\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0}}$$

$$\partial_1 \mathbf{F}_{23} + \partial_2 \mathbf{F}_{31} + \partial_3 \mathbf{F}_{12} = \mathbf{0}$$

$$\sum_{\mu=1}^{4}\partial_\mu \mathbf{F}_{\mu\nu} = -\mu_0 \mathbf{j}_\nu \quad \partial_\mu \mathbf{F}_{\nu\lambda} + \partial_\nu \mathbf{F}_{\lambda\mu} + \partial_\lambda \mathbf{F}_{\mu\nu} = \mathbf{0}$$

$$\sum_{\mathrm{j}=1}^{3}\partial_\mathrm{j} \mathbf{F}_{\mathrm{ji}} = -\mu_0 \mathbf{j}_\mathrm{i} - \partial_4 \mathbf{F}_{4\mathrm{i}} \rightarrow \sum_{\mathrm{j,k}=1}^{3}\partial_\mathrm{j}\epsilon_{\mathrm{jik}}\mathbf{B}_\mathrm{k} = -\mu_0 \mathbf{j}_\mathrm{i} - \frac{1}{\mathbf{c}^2}\frac{\partial \mathbf{E}_\mathrm{i}}{\partial \mathrm{t}}$$

$$\rightarrow \underline{\nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0\epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial \mathrm{t}}}$$

$$\sum_{\mathrm{i}=1}^{3}\partial_\mathrm{i} \mathbf{F}_{\mathrm{i}4} = -\mu_0 \mathbf{j}_4 \rightarrow \sum_{\mathrm{i}=1}^{3}\partial_\mathrm{i}\mathbf{E}_\mathrm{i}\left(-\frac{\mathbf{i}}{\mathbf{c}}\right) = -\mu_0 \mathbf{i}\mathbf{c}\rho \rightarrow \underline{\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0}}$$

$$\partial_1 \mathbf{F}_{23} + \partial_2 \mathbf{F}_{31} + \partial_3 \mathbf{F}_{12} = \mathbf{0} \rightarrow \sum_{\mathrm{i,j,k}=1}^{3}\partial_\mathrm{i}\epsilon_{\mathrm{jki}}\mathbf{F}_{\mathrm{jk}} = \mathbf{0}$$

$$\sum_{\mu=1}^{4}\partial_\mu \mathbf{F}_{\mu\nu} = -\mu_0 \mathbf{j}_\nu \qquad \partial_\mu \mathbf{F}_{\nu\lambda} + \partial_\nu \mathbf{F}_{\lambda\mu} + \partial_\lambda \mathbf{F}_{\mu\nu} = \mathbf{0}$$

$$\sum_{j=1}^{3}\partial_j \mathbf{F}_{ji} = -\mu_0 \mathbf{j}_i - \partial_4 \mathbf{F}_{4i} \rightarrow \sum_{j,k=1}^{3}\partial_j \epsilon_{jik}\mathbf{B}_k = -\mu_0 \mathbf{j}_i - \frac{1}{c^2}\frac{\partial \mathbf{E}_i}{\partial t}$$

$$\rightarrow \underline{\nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t}}$$

$$\sum_{i=1}^{3}\partial_i \mathbf{F}_{i4} = -\mu_0 \mathbf{j}_4 \ \rightarrow \ \sum_{i=1}^{3}\partial_i \mathbf{E}_i\left(-\frac{i}{c}\right) = -\mu_0 i c \rho \ \rightarrow \ \underline{\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0}}$$

$$\partial_1 \mathbf{F}_{23} + \partial_2 \mathbf{F}_{31} + \partial_3 \mathbf{F}_{12} = \mathbf{0} \rightarrow \sum_{i,j,k=1}^{3}\partial_i \epsilon_{jki}\mathbf{F}_{jk} = \mathbf{0} \rightarrow \sum_{i=1}^{3}\partial_i \mathbf{B}_i = \mathbf{0}$$

$$\sum_{\mu=1}^{4}\partial_\mu \mathbf{F}_{\mu\nu} = -\mu_0 \mathbf{j}_\nu \qquad \partial_\mu \mathbf{F}_{\nu\lambda} + \partial_\nu \mathbf{F}_{\lambda\mu} + \partial_\lambda \mathbf{F}_{\mu\nu} = \mathbf{0}$$

$$\sum_{j=1}^{3}\partial_j \mathbf{F}_{ji} = -\mu_0 \mathbf{j}_i - \partial_4 \mathbf{F}_{4i} \to \sum_{j,k=1}^{3}\partial_j \epsilon_{jik}\mathbf{B}_k = -\mu_0 \mathbf{j}_i - \frac{1}{c^2}\frac{\partial \mathbf{E}_i}{\partial t}$$

$$\to \underline{\nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t}}$$

$$\sum_{i=1}^{3}\partial_i \mathbf{F}_{i4} = -\mu_0 \mathbf{j}_4 \to \sum_{i=1}^{3}\partial_i \mathbf{E}_i\left(-\frac{i}{c}\right) = -\mu_0 i c \rho \to \underline{\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0}}$$

$$\partial_1 \mathbf{F}_{23} + \partial_2 \mathbf{F}_{31} + \partial_3 \mathbf{F}_{12} = \mathbf{0} \to \sum_{i,j,k=1}^{3}\partial_i \epsilon_{jki}\mathbf{F}_{jk} = \mathbf{0} \to \sum_{i=1}^{3}\partial_i \mathbf{B}_i = \mathbf{0} \to \underline{\nabla \cdot \vec{\mathbf{B}} = \mathbf{0}}$$

$$\sum_{\mu=1}^{4}\partial_\mu \mathbf{F}_{\mu\nu} = -\mu_0 \mathbf{j}_\nu \qquad \partial_\mu \mathbf{F}_{\nu\lambda} + \partial_\nu \mathbf{F}_{\lambda\mu} + \partial_\lambda \mathbf{F}_{\mu\nu} = \mathbf{0}$$

$$\sum_{j=1}^{3}\partial_j \mathbf{F}_{ji} = -\mu_0 \mathbf{j}_i - \partial_4 \mathbf{F}_{4i} \rightarrow \sum_{j,k=1}^{3}\partial_j \epsilon_{jik}\mathbf{B}_k = -\mu_0 \mathbf{j}_i - \frac{1}{c^2}\frac{\partial \mathbf{E}_i}{\partial t}$$

$$\rightarrow \underline{\nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t}}$$

$$\sum_{i=1}^{3}\partial_i \mathbf{F}_{i4} = -\mu_0 \mathbf{j}_4 \rightarrow \sum_{i=1}^{3}\partial_i \mathbf{E}_i\left(-\frac{i}{c}\right) = -\mu_0 i c \rho \rightarrow \underline{\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0}}$$

$$\partial_1 \mathbf{F}_{23} + \partial_2 \mathbf{F}_{31} + \partial_3 \mathbf{F}_{12} = \mathbf{0} \rightarrow \sum_{i,j,k=1}^{3}\partial_i \epsilon_{jki}\mathbf{F}_{jk} = \mathbf{0} \rightarrow \sum_{i=1}^{3}\partial_i \mathbf{B}_i = \mathbf{0} \rightarrow \underline{\nabla \cdot \vec{\mathbf{B}} = \mathbf{0}}$$

$$\partial_j \mathbf{F}_{k4} + \partial_k \mathbf{F}_{4j} + \partial_4 \mathbf{F}_{jk} = \mathbf{0}$$

$$\sum_{\mu=1}^{4}\partial_\mu \mathbf{F}_{\mu\nu} = -\mu_0 \mathbf{j}_\nu \qquad \partial_\mu \mathbf{F}_{\nu\lambda} + \partial_\nu \mathbf{F}_{\lambda\mu} + \partial_\lambda \mathbf{F}_{\mu\nu} = 0$$

$$\sum_{j=1}^{3}\partial_j \mathbf{F}_{ji} = -\mu_0 \mathbf{j}_i - \partial_4 \mathbf{F}_{4i} \to \sum_{j,k=1}^{3}\partial_j \epsilon_{jik} \mathbf{B}_k = -\mu_0 \mathbf{j}_i - \frac{1}{c^2}\frac{\partial \mathbf{E}_i}{\partial t}$$

$$\to \underline{\nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t}}$$

$$\sum_{i=1}^{3}\partial_i \mathbf{F}_{i4} = -\mu_0 \mathbf{j}_4 \quad \to \quad \sum_{i=1}^{3}\partial_i \mathbf{E}_i\left(-\frac{i}{c}\right) = -\mu_0 i c \rho \quad \to \quad \underline{\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0}}$$

$$\partial_1 \mathbf{F}_{23} + \partial_2 \mathbf{F}_{31} + \partial_3 \mathbf{F}_{12} = 0 \to \sum_{i,j,k=1}^{3}\partial_i \epsilon_{jki} \mathbf{F}_{jk} = 0 \to \sum_{i=1}^{3}\partial_i \mathbf{B}_i = 0 \to \underline{\nabla \cdot \vec{\mathbf{B}} = 0}$$

$$\partial_j \mathbf{F}_{k4} + \partial_k \mathbf{F}_{4j} + \partial_4 \mathbf{F}_{jk} = 0 \to \sum_{j,k=1}^{3}\epsilon_{ijk}\partial_j(i c \mathbf{F}_{k4}) = -ic\partial_4\left(\frac{1}{2}\sum_{j,k=1}^{3}\epsilon_{jki}\mathbf{F}_{jk}\right)$$

$$\sum_{\mu=1}^{4}\partial_\mu \mathbf{F}_{\mu\nu} = -\mu_0 \mathbf{j}_\nu \qquad \partial_\mu \mathbf{F}_{\nu\lambda} + \partial_\nu \mathbf{F}_{\lambda\mu} + \partial_\lambda \mathbf{F}_{\mu\nu} = \mathbf{0}$$

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$$\sum_{i=1}^{3}\partial_i \mathbf{F}_{i4} = -\mu_0 \mathbf{j}_4 \ \rightarrow \ \sum_{i=1}^{3}\partial_i \mathbf{E}_i\left(-\frac{i}{c}\right) = -\mu_0 \mathbf{i} c\rho \ \rightarrow \ \underline{\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0}}$$

$$\partial_1 \mathbf{F}_{23} + \partial_2 \mathbf{F}_{31} + \partial_3 \mathbf{F}_{12} = \mathbf{0} \rightarrow \sum_{i,j,k=1}^{3}\partial_i \epsilon_{jki}\mathbf{F}_{jk} = \mathbf{0} \rightarrow \sum_{i=1}^{3}\partial_i \mathbf{B}_i = \mathbf{0} \rightarrow \underline{\nabla \cdot \vec{\mathbf{B}} = \mathbf{0}}$$

$$\partial_j \mathbf{F}_{k4} + \partial_k \mathbf{F}_{4j} + \partial_4 \mathbf{F}_{jk} = \mathbf{0} \rightarrow \sum_{j,k=1}^{3}\epsilon_{ijk}\partial_j(\mathbf{i}c\mathbf{F}_{k4}) = -\mathbf{i}c\partial_4\left(\frac{1}{2}\sum_{j,k=1}^{3}\epsilon_{jki}\mathbf{F}_{jk}\right)$$

$$\rightarrow \sum_{j,k=1}^{3}\epsilon_{ijk}\partial_j \mathbf{E}_k = -\frac{\partial}{\partial t}\mathbf{B}_i$$

$$\sum_{\mu=1}^{4}\partial_\mu F_{\mu\nu} = -\mu_0 j_\nu \qquad \partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} + \partial_\lambda F_{\mu\nu} = 0$$

$$\sum_{j=1}^{3}\partial_j F_{ji} = -\mu_0 j_i - \partial_4 F_{4i} \rightarrow \sum_{j,k=1}^{3}\partial_j \epsilon_{jik} B_k = -\mu_0 j_i - \frac{1}{c^2}\frac{\partial E_i}{\partial t}$$

$$\rightarrow \underline{\nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}}$$

$$\sum_{i=1}^{3}\partial_i F_{i4} = -\mu_0 j_4 \rightarrow \sum_{i=1}^{3}\partial_i E_i\left(-\frac{i}{c}\right) = -\mu_0 i c \rho \rightarrow \underline{\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}}$$

$$\partial_1 F_{23} + \partial_2 F_{31} + \partial_3 F_{12} = 0 \rightarrow \sum_{i,j,k=1}^{3}\partial_i \epsilon_{jki} F_{jk} = 0 \rightarrow \sum_{i=1}^{3}\partial_i B_i = 0 \rightarrow \underline{\nabla \cdot \vec{B} = 0}$$

$$\partial_j F_{k4} + \partial_k F_{4j} + \partial_4 F_{jk} = 0 \rightarrow \sum_{j,k=1}^{3}\epsilon_{ijk}\partial_j (icF_{k4}) = -ic\partial_4\left(\frac{1}{2}\sum_{j,k=1}^{3}\epsilon_{jki}F_{jk}\right)$$

$$\rightarrow \sum_{j,k=1}^{3}\epsilon_{ijk}\partial_j E_k = -\frac{\partial}{\partial t}B_i \rightarrow \underline{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

Back
Close

介质中的麦克斯韦方程组：极化强度和磁化强度

$$\nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}} \qquad \nabla \cdot \vec{\mathbf{B}} = \mathbf{0}$$

$$\nabla \cdot (\epsilon_0 \vec{\mathbf{E}} + \vec{\mathbf{P}}) = \rho_\mathrm{f} \qquad \nabla \times (\frac{1}{\mu_0}\vec{\mathbf{B}} - \vec{\mathbf{M}}) = \vec{\mathbf{j}}_\mathbf{c} + \frac{\partial}{\partial \mathbf{t}}(\epsilon_0 \vec{\mathbf{E}} + \vec{\mathbf{P}})$$

$$\vec{\mathbf{E}} \;\Rightarrow\; \frac{1}{\epsilon_0}\vec{\mathbf{P}} \qquad \vec{\mathbf{B}} \;\Rightarrow\; -\mu_0 \vec{\mathbf{M}}$$

$$\mathbf{F}_{\mu\nu} = \begin{pmatrix} 0 & \mathrm{B}_3 & -\mathrm{B}_2 & -\frac{\mathrm{i}}{\mathrm{c}}\mathrm{E}_1 \\ -\mathrm{B}_3 & 0 & \mathrm{B}_1 & -\frac{\mathrm{i}}{\mathrm{c}}\mathrm{E}_2 \\ \mathrm{B}_2 & -\mathrm{B}_1 & 0 & -\frac{\mathrm{i}}{\mathrm{c}}\mathrm{E}_3 \\ \frac{\mathrm{i}}{\mathrm{c}}\mathrm{E}_1 & \frac{\mathrm{i}}{\mathrm{c}}\mathrm{E}_2 & \frac{\mathrm{i}}{\mathrm{c}}\mathrm{E}_3 & 0 \end{pmatrix} \qquad \mathbf{F}'_{\mu\nu} = \sum_{\lambda,\lambda'=1}^{4} \mathbf{a}_{\mu\lambda}\mathbf{F}_{\lambda\lambda'}\mathbf{a}_{\nu\lambda'}$$

$$\underline{\mathbf{F}' = \mathbf{A}\mathbf{F}\mathbf{A}^\mathrm{T}}$$

$$\mathbf{M}_{\mu\nu} \equiv \begin{pmatrix} 0 & -\mathrm{M}_3 & \mathrm{M}_2 & -\mathrm{icP}_1 \\ \mathrm{M}_3 & 0 & -\mathrm{M}_1 & -\mathrm{icP}_2 \\ -\mathrm{M}_2 & \mathrm{M}_1 & 0 & -\mathrm{icP}_3 \\ \mathrm{icP}_1 & \mathrm{icP}_2 & \mathrm{icP}_3 & 0 \end{pmatrix}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \cdot (\epsilon_0 \vec{E} + \vec{P}) = \rho_f \qquad \nabla \times (\frac{1}{\mu_0}\vec{B} - \vec{M}) = \vec{j}_c + \frac{\partial}{\partial t}(\epsilon_0 \vec{E} + \vec{P})$$

$$\vec{E} \ \Rightarrow \ \frac{1}{\epsilon_0}\vec{P} \qquad \vec{B} \ \Rightarrow \ -\mu_0 \vec{M}$$

$$F_{\mu\nu} = \begin{pmatrix} 0 & B_3 & -B_2 & -\frac{i}{c}E_1 \\ -B_3 & 0 & B_1 & -\frac{i}{c}E_2 \\ B_2 & -B_1 & 0 & -\frac{i}{c}E_3 \\ \frac{i}{c}E_1 & \frac{i}{c}E_2 & \frac{i}{c}E_3 & 0 \end{pmatrix} \qquad F'_{\mu\nu} = \sum_{\lambda,\lambda'=1}^{4} a_{\mu\lambda} F_{\lambda\lambda'} a_{\nu\lambda'}$$

$$\underline{F' = AFA^T}$$

$$M_{\mu\nu} \equiv \begin{pmatrix} 0 & -M_3 & M_2 & -icP_1 \\ M_3 & 0 & -M_1 & -icP_2 \\ -M_2 & M_1 & 0 & -icP_3 \\ icP_1 & icP_2 & icP_3 & 0 \end{pmatrix} \qquad M'_{\mu\nu} = \sum_{\lambda,\lambda'=1}^{4} a_{\mu\lambda} M_{\lambda\lambda'} a_{\nu\lambda'}$$

$$\underline{M' = AMA^T}$$

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$$\nabla\times\vec{\mathbf{E}}=-\frac{\partial\vec{\mathbf{B}}}{\partial t}\qquad \nabla\cdot\vec{\mathbf{B}}=0$$

$$\nabla\cdot(\epsilon_0\vec{\mathbf{E}}+\vec{\mathbf{P}})=\rho_f\qquad \nabla\times(\frac{1}{\mu_0}\vec{\mathbf{B}}-\vec{\mathbf{M}})=\vec{\mathbf{j}}_c+\frac{\partial}{\partial t}(\epsilon_0\vec{\mathbf{E}}+\vec{\mathbf{P}})$$

$$\vec{\mathbf{E}}\ \Rightarrow\ \frac{1}{\epsilon_0}\vec{\mathbf{P}}\qquad \vec{\mathbf{B}}\ \Rightarrow\ -\mu_0\vec{\mathbf{M}}$$

$$\mathbf{F}_{\mu\nu}=\begin{pmatrix} 0 & \mathbf{B}_3 & -\mathbf{B}_2 & -\frac{i}{c}\mathbf{E}_1\\ -\mathbf{B}_3 & 0 & \mathbf{B}_1 & -\frac{i}{c}\mathbf{E}_2\\ \mathbf{B}_2 & -\mathbf{B}_1 & 0 & -\frac{i}{c}\mathbf{E}_3\\ \frac{i}{c}\mathbf{E}_1 & \frac{i}{c}\mathbf{E}_2 & \frac{i}{c}\mathbf{E}_3 & 0\end{pmatrix}\qquad \mathbf{F}'_{\mu\nu}=\sum_{\lambda,\lambda'=1}^{4}\mathbf{a}_{\mu\lambda}\mathbf{F}_{\lambda\lambda'}\mathbf{a}_{\nu\lambda'}$$

$$\underline{\mathbf{F}'=\mathbf{A}\mathbf{F}\mathbf{A}^{\mathrm{T}}}$$

$$\mathbf{M}_{\mu\nu}\equiv\begin{pmatrix} 0 & -\mathbf{M}_3 & \mathbf{M}_2 & -ic\mathbf{P}_1\\ \mathbf{M}_3 & 0 & -\mathbf{M}_1 & -ic\mathbf{P}_2\\ -\mathbf{M}_2 & \mathbf{M}_1 & 0 & -ic\mathbf{P}_3\\ ic\mathbf{P}_1 & ic\mathbf{P}_2 & ic\mathbf{P}_3 & 0\end{pmatrix}\qquad \mathbf{M}'_{\mu\nu}=\sum_{\lambda,\lambda'=1}^{4}\mathbf{a}_{\mu\lambda}\mathbf{M}_{\lambda\lambda'}\mathbf{a}_{\nu\lambda'}$$

$$\underline{\mathbf{M}'=\mathbf{A}\mathbf{M}\mathbf{A}^{\mathrm{T}}}$$

$$\sum_{\mu=1}^{4}\partial_\mu[\mathbf{F}_{\mu\nu}+\mu_0\mathbf{M}_{\mu\nu}]=-\mu_0\mathbf{j}_\nu\qquad \partial_\mu\mathbf{F}_{\nu\lambda}+\partial_\nu\mathbf{F}_{\lambda\mu}+\partial_\lambda\mathbf{F}_{\mu\nu}=0$$

$$\mathbf{j}_\nu\to\mathbf{j}_\nu+\partial_\mu\mathbf{M}_{\mu\nu}$$

电动力学作用量的更深层次含义

杨振宁：2012年6月21日在清华高等研究院Fujikawa报告后的评论：

- 电磁场的作用量中出现了$\sum_{\mu,\nu=1}^{4} \mathbf{F}_{\mu\nu}\mathbf{F}_{\mu\nu}$和$\sum_{\mu,\nu,\sigma,\rho=1}^{4} \epsilon_{\mu\nu\sigma\rho}\mathbf{F}_{\mu\nu}\mathbf{F}_{\sigma\rho}$

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- $\sum_{\mu,\nu,\sigma,\rho=1}^{4}\epsilon_{\mu\nu\sigma\rho}\mathbf{F}_{\mu\nu}\mathbf{F}_{\sigma\rho}$ 是著名的拓扑陈数陈省身，数学研究很深入

Back

Close

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- $\sum_{\mu,\nu,\sigma,\rho=1}^{4} \epsilon_{\mu\nu\sigma\rho}\mathbf{F}_{\mu\nu}\mathbf{F}_{\sigma\rho}$ 是著名的拓扑陈数陈省身，数学研究很深入
- $\sum_{\mu,\nu=1}^{4} \mathbf{F}_{\mu\nu}\mathbf{F}_{\mu\nu}$ 数学家开始未关注，后受物理启发开始研究
- 由此建立了著名的 Donalson理论 ！

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- $\sum_{\mu,\nu,\sigma,\rho=1}^{4} \epsilon_{\mu\nu\sigma\rho}\mathbf{F}_{\mu\nu}\mathbf{F}_{\sigma\rho}$ 是著名的拓扑陈数陈省身，数学研究很深入
- $\sum_{\mu,\nu=1}^{4} \mathbf{F}_{\mu\nu}\mathbf{F}_{\mu\nu}$ 数学家开始未关注，后受物理启发开始研究
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- $\sum_{\mu,\nu,\sigma,\rho=1}^{4}\epsilon_{\mu\nu\sigma\rho}\mathbf{F}_{\mu\nu}\mathbf{F}_{\sigma\rho}$是著名的拓扑陈数陈省身，数学研究很深入
- $\sum_{\mu,\nu=1}^{4}\mathbf{F}_{\mu\nu}\mathbf{F}_{\mu\nu}$数学家开始未关注，后受物理启发开始研究
- 由此建立了著名的 Donalson理论 ！
- 发现：$\mathbf{F}^2$在非四维时空和陈数类似，无特殊结构
- 在四维时空：$\mathbf{F}^2$和陈数不同，有特殊结构理解四位时空? ！

例: Witten效应: 电荷磁单极共生及θ/π为分数

量子场论: $\theta \to \theta + 2n\pi \quad \alpha \equiv \frac{e^2}{\epsilon_0 \hbar}$

$$\Delta\mathcal{L} = \frac{i\theta c e^2}{32\pi^2\hbar}\sum_{\mu\nu\sigma\rho}\epsilon_{\mu\nu\sigma\rho}F_{\mu\nu}F_{\sigma\rho} = \underbrace{-\frac{ice^2}{8\pi^2\hbar}\sum_{\mu\nu\sigma\rho}\epsilon_{\mu\nu\sigma\rho}(\partial_\mu\theta)A_\nu\partial_\sigma A_\rho}_{\sum_\mu A_\mu \Delta j_\mu \ \to\ \Delta j_\mu = -\frac{ic\epsilon_0\alpha}{8\pi^2}\sum_{\nu\sigma\rho}\epsilon_{\mu\nu\sigma\rho}(\partial_\nu\theta)\partial_\sigma A_\rho} = \underbrace{\theta\frac{e^2}{2\pi h}\vec{B}\cdot\vec{E}}_{\text{全微商导致的 拓扑项}}$$

$$\nabla\cdot\vec{E} = \frac{\rho}{\epsilon_0}\underbrace{-\frac{\alpha}{4\pi^2}\nabla\theta\cdot\vec{B}}_{\text{不匀}\theta\text{中磁场产生电荷}} \qquad \nabla\times\vec{B} = \mu_0[\epsilon_0\frac{\partial\vec{E}}{\partial t} + \vec{j} + \underbrace{\frac{\alpha\epsilon_0}{4\pi^2}(\nabla\theta\times\vec{E} + \frac{\partial\theta}{\partial t}\vec{B})}_{\text{不匀}\theta\text{中电场或变化}\theta\text{中的磁场产生电流}}]$$

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$$\vec{P}=\frac{\alpha\epsilon_0}{4\pi^2}\theta\vec{B} \qquad -\frac{ic\epsilon_0\alpha}{8\pi^2}\sum_{\nu\sigma\rho}\partial_\nu(\epsilon_{\mu\nu\sigma\rho}\theta\partial_\sigma A_\rho)=\Delta j_\mu=\partial_\nu M_{\nu\mu} \qquad \vec{M}=\frac{\alpha\epsilon_0}{4\pi^2}\theta\vec{E}$$

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$$\sum_\mu A_\mu \Delta j_\mu \to \Delta j_\mu = -\frac{ic\epsilon_0\alpha}{8\pi^2}\sum_{\nu\sigma\rho}\epsilon_{\mu\nu\sigma\rho}(\partial_\nu\theta)\partial_\sigma A_\rho$$

$$\nabla\cdot\vec{E} = \frac{\rho}{\epsilon_0}\underbrace{-\frac{\alpha}{4\pi^2}\nabla\theta\cdot\vec{B}}_{\text{不匀}\theta\text{中磁场产生电荷}} \qquad \nabla\times\vec{B} = \mu_0[\epsilon_0\frac{\partial\vec{E}}{\partial t} + \vec{j} + \underbrace{\frac{\alpha\epsilon_0}{4\pi^2}(\nabla\theta\times\vec{E} + \frac{\partial\theta}{\partial t}\vec{B})}_{\text{不匀}\theta\text{中电场或变化}\theta\text{中的磁场产生电流}}]$$

$$\vec{P} = \frac{\alpha\epsilon_0}{4\pi^2}\theta\vec{B} \qquad -\frac{ic\epsilon_0\alpha}{8\pi^2}\sum_{\nu\sigma\rho}\partial_\nu(\epsilon_{\mu\nu\sigma\rho}\theta\partial_\sigma A_\rho) = \Delta j_\mu = \partial_\nu M_{\nu\mu} \qquad \vec{M} = \frac{\alpha\epsilon_0}{4\pi^2}\theta\vec{E}$$

进一步若介质中：$\vec{j} = 0, \quad \nabla\theta = 0 \Rightarrow \mu_0\epsilon_0\frac{\partial\nabla\cdot\vec{E}}{\partial t} + \frac{\alpha\epsilon_0}{4\pi^2}\frac{\partial\theta\nabla\cdot\vec{B}}{\partial t} = 0$

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进一步若介质中: $\vec{j} = 0, \quad \nabla\theta = 0 \;\Rightarrow\; \mu_0\epsilon_0\frac{\partial\nabla\cdot\vec{E}}{\partial t} + \frac{\alpha\epsilon_0}{4\pi^2}\frac{\partial\theta\nabla\cdot\vec{B}}{\partial t} = 0$

介质上某小体积V $q \equiv \int_V d^3x\,\epsilon_0\nabla\cdot\vec{E} \qquad q_m \equiv \int_V d^3x\,\nabla\cdot\vec{B} \Rightarrow \frac{\partial}{\partial t}\left(\frac{\alpha\epsilon_0}{4\pi^2}\theta q_m + q\right) = 0$

$$\frac{e^2}{2\pi h}\theta q_m + q \overset{\text{不随时间改变}}{=\!=\!=\!=\!=\!=\!=\!=} 0 \quad\Rightarrow$$ <u>电荷与磁单极共生：双荷子dyon</u>

例：Witten效应：电荷磁单极共生及θ/π为分数

量子场论：$\theta \to \theta + 2n\pi \quad \alpha \equiv \frac{e^2}{\epsilon_0 \hbar}$

$$\Delta\mathcal{L} = \frac{i\theta ce^2}{32\pi^2\hbar}\sum_{\mu\nu\sigma\rho}\epsilon_{\mu\nu\sigma\rho}F_{\mu\nu}F_{\sigma\rho} = \underbrace{-\frac{ice^2}{8\pi^2\hbar}\sum_{\mu\nu\sigma\rho}\epsilon_{\mu\nu\sigma\rho}(\partial_\mu\theta)A_\nu\partial_\sigma A_\rho}_{} = \underbrace{\theta\frac{e^2}{2\pi h}\vec{B}\cdot\vec{E}}_{\text{全微商导致的 拓扑项}}$$

$$\sum_\mu A_\mu \Delta j_\mu \to \Delta j_\mu = -\frac{ic\epsilon_0\alpha}{8\pi^2}\sum_{\nu\sigma\rho}\epsilon_{\mu\nu\sigma\rho}(\partial_\nu\theta)\partial_\sigma A_\rho$$

$$\nabla\cdot\vec{E} = \frac{\rho}{\epsilon_0}\underbrace{-\frac{\alpha}{4\pi^2}\nabla\theta\cdot\vec{B}}_{\text{不匀}\theta\text{中磁场产生电荷}} \qquad \nabla\times\vec{B} = \mu_0[\epsilon_0\frac{\partial\vec{E}}{\partial t} + \vec{j} + \underbrace{\frac{\alpha\epsilon_0}{4\pi^2}(\nabla\theta\times\vec{E} + \frac{\partial\theta}{\partial t}\vec{B})}_{\text{不匀}\theta\text{中电场或变化}\theta\text{中的磁场产生电流}}]$$

$$\vec{P} = \frac{\alpha\epsilon_0}{4\pi^2}\theta\vec{B} \qquad -\frac{ic\epsilon_0\alpha}{8\pi^2}\sum_{\nu\sigma\rho}\partial_\nu(\epsilon_{\mu\nu\sigma\rho}\theta\partial_\sigma A_\rho) = \Delta j_\mu = \partial_\nu M_{\nu\mu} \qquad \vec{M} = \frac{\alpha\epsilon_0}{4\pi^2}\theta\vec{E}$$

进一步若介质中：$\vec{j} = 0, \quad \nabla\theta = 0 \Rightarrow \mu_0\epsilon_0\frac{\partial\nabla\cdot\vec{E}}{\partial t} + \frac{\alpha\epsilon_0}{4\pi^2}\frac{\partial\theta\nabla\cdot\vec{B}}{\partial t} = 0$

介质上某小体积V $q \equiv \int_V d^3x\epsilon_0\nabla\cdot\vec{E} \qquad q_m \equiv \int_V d^3x\nabla\cdot\vec{B} \Rightarrow \frac{\partial}{\partial t}(\frac{\alpha\epsilon_0}{4\pi^2}\theta q_m + q) = 0$

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$$eq_m = \pm 4\pi n\hbar \ \text{见后} \quad \Rightarrow \quad \frac{q}{e} = \mp n\frac{\theta}{\pi} \xRightarrow{q=me} \frac{\theta}{\pi} = \mp\frac{n}{m}$$

例：Witten效应：电荷磁单极共生及θ/π为分数

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$$\nabla\cdot\vec{E} = \frac{\rho}{\epsilon_0}\underbrace{-\frac{\alpha}{4\pi^2}\nabla\theta\cdot\vec{B}}_{\text{不匀}\theta\text{中磁场产生电荷}} \qquad \nabla\times\vec{B} = \mu_0[\epsilon_0\frac{\partial\vec{E}}{\partial t} + \vec{j} + \underbrace{\frac{\alpha\epsilon_0}{4\pi^2}(\nabla\theta\times\vec{E} + \frac{\partial\theta}{\partial t}\vec{B})}_{\text{不匀}\theta\text{中电场或变化}\theta\text{中的磁场产生电流}}]$$

$$\vec{P} = \frac{\alpha\epsilon_0}{4\pi^2}\theta\vec{B} \qquad -\frac{ic\epsilon_0\alpha}{8\pi^2}\sum_{\nu\sigma\rho}\partial_\nu(\epsilon_{\mu\nu\sigma\rho}\theta\partial_\sigma A_\rho) = \Delta j_\mu = \partial_\nu M_{\nu\mu} \qquad \vec{M} = \frac{\alpha\epsilon_0}{4\pi^2}\theta\vec{E}$$

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$eq_m = \pm 4\pi n\hbar$ 见后 $\;\Rightarrow\; \frac{q}{e} = \mp n\frac{\theta}{\pi} \xRightarrow{q=me} \frac{\theta}{\pi} = \mp\frac{n}{m}$ Axion：θ是动力学场

$$\sum_{\mu=1}^{4}\partial_\mu \mathbf{F}_{\mu\nu} = -\mu_0 \mathbf{j}_\nu \quad \partial_\mu \mathbf{F}_{\nu\lambda}+\partial_\nu \mathbf{F}_{\lambda\mu}+\partial_\lambda \mathbf{F}_{\mu\nu}=\mathbf{0} \quad \mathbf{f}_\mu \equiv \sum_{\nu=1}^{4}\mathbf{F}_{\mu\nu}\mathbf{j}_\nu$$

$$\sum_{\mu=1}^{4}\partial_\mu \mathbf{F}_{\mu\nu} = -\mu_0 \mathbf{j}_\nu \quad \partial_\mu \mathbf{F}_{\nu\lambda}+\partial_\nu \mathbf{F}_{\lambda\mu}+\partial_\lambda \mathbf{F}_{\mu\nu}=\mathbf{0} \quad \mathbf{f}_\mu \equiv \sum_{\nu=1}^{4}\mathbf{F}_{\mu\nu}\mathbf{j}_\nu$$

$$\mathbf{f}_\mu = \sum_{\nu=1}^{4}\mathbf{F}_{\mu\nu}\mathbf{j}_\nu$$

$$\sum_{\mu=1}^{4}\partial_\mu \mathbf{F}_{\mu\nu} = -\mu_0 \mathbf{j}_\nu \qquad \partial_\mu \mathbf{F}_{\nu\lambda}+\partial_\nu \mathbf{F}_{\lambda\mu}+\partial_\lambda \mathbf{F}_{\mu\nu}=\mathbf{0} \qquad \mathbf{f}_\mu \equiv \sum_{\nu=1}^{4}\mathbf{F}_{\mu\nu}\mathbf{j}_\nu$$

$$\mathbf{f}_\mu = \sum_{\nu=1}^{4}\mathbf{F}_{\mu\nu}\mathbf{j}_\nu = -\frac{1}{\mu_0}\sum_{\nu,\nu'=1}^{4}\mathbf{F}_{\mu\nu}\partial_{\nu'}\mathbf{F}_{\nu'\nu}$$

$$\sum_{\mu=1}^{4}\partial_\mu \mathbf{F}_{\mu\nu} = -\mu_0 \mathbf{j}_\nu \quad \partial_\mu \mathbf{F}_{\nu\lambda} + \partial_\nu \mathbf{F}_{\lambda\mu} + \partial_\lambda \mathbf{F}_{\mu\nu} = 0 \quad \mathbf{f}_\mu \equiv \sum_{\nu=1}^{4}\mathbf{F}_{\mu\nu}\mathbf{j}_\nu$$

$$\mathbf{f}_\mu = \sum_{\nu=1}^{4}\mathbf{F}_{\mu\nu}\mathbf{j}_\nu = -\frac{1}{\mu_0}\sum_{\nu,\nu'=1}^{4}\mathbf{F}_{\mu\nu}\partial_{\nu'}\mathbf{F}_{\nu'\nu} = -\frac{1}{\mu_0}\sum_{\nu,\nu'=1}^{4}\left[\partial_{\nu'}(\mathbf{F}_{\mu\nu}\mathbf{F}_{\nu'\nu}) - (\partial_{\nu'}\mathbf{F}_{\mu\nu})\mathbf{F}_{\nu'\nu}\right]$$

Back
Close

$$\sum_{\mu=1}^{4}\partial_\mu \mathbf{F}_{\mu\nu} = -\mu_0 \mathbf{j}_\nu \quad \partial_\mu \mathbf{F}_{\nu\lambda}+\partial_\nu \mathbf{F}_{\lambda\mu}+\partial_\lambda \mathbf{F}_{\mu\nu}=\mathbf{0} \quad \mathbf{f}_\mu \equiv \sum_{\nu=1}^{4}\mathbf{F}_{\mu\nu}\mathbf{j}_\nu$$

$$\mathbf{f}_\mu = \sum_{\nu=1}^{4}\mathbf{F}_{\mu\nu}\mathbf{j}_\nu = -\frac{1}{\mu_0}\sum_{\nu,\nu'=1}^{4}\mathbf{F}_{\mu\nu}\partial_{\nu'}\mathbf{F}_{\nu'\nu} = -\frac{1}{\mu_0}\sum_{\nu,\nu'=1}^{4}\left[\partial_{\nu'}(\mathbf{F}_{\mu\nu}\mathbf{F}_{\nu'\nu}) - (\partial_{\nu'}\mathbf{F}_{\mu\nu})\mathbf{F}_{\nu'\nu}\right]$$

$$= -\frac{1}{\mu_0}\sum_{\nu,\nu'=1}^{4}\left[\partial_{\nu'}(\mathbf{F}_{\mu\nu}\mathbf{F}_{\nu'\nu}) - \frac{1}{2}(\partial_{\nu'}\mathbf{F}_{\mu\nu})\mathbf{F}_{\nu'\nu} - \frac{1}{2}(\partial_{\nu}\mathbf{F}_{\mu\nu'})\mathbf{F}_{\nu\nu'}\right]$$

Back
Close

$$\sum_{\mu=1}^{4}\partial_\mu \mathbf{F}_{\mu\nu} = -\mu_0 \mathbf{j}_\nu \quad \partial_\mu \mathbf{F}_{\nu\lambda}+\partial_\nu \mathbf{F}_{\lambda\mu}+\partial_\lambda \mathbf{F}_{\mu\nu}=\mathbf{0} \quad \mathbf{f}_\mu \equiv \sum_{\nu=1}^{4}\mathbf{F}_{\mu\nu}\mathbf{j}_\nu$$

$$\mathbf{f}_\mu = \sum_{\nu=1}^{4}\mathbf{F}_{\mu\nu}\mathbf{j}_\nu = -\frac{1}{\mu_0}\sum_{\nu,\nu'=1}^{4}\mathbf{F}_{\mu\nu}\partial_{\nu'}\mathbf{F}_{\nu'\nu} = -\frac{1}{\mu_0}\sum_{\nu,\nu'=1}^{4}\left[\partial_{\nu'}(\mathbf{F}_{\mu\nu}\mathbf{F}_{\nu'\nu}) - (\partial_{\nu'}\mathbf{F}_{\mu\nu})\mathbf{F}_{\nu'\nu}\right]$$

$$= -\frac{1}{\mu_0}\sum_{\nu,\nu'=1}^{4}\left[\partial_{\nu'}(\mathbf{F}_{\mu\nu}\mathbf{F}_{\nu'\nu}) - \frac{1}{2}(\partial_{\nu'}\mathbf{F}_{\mu\nu})\mathbf{F}_{\nu'\nu} - \frac{1}{2}(\partial_{\nu}\mathbf{F}_{\mu\nu'})\mathbf{F}_{\nu\nu'}\right]$$

$$= -\frac{1}{\mu_0}\sum_{\nu,\nu'=1}^{4}\left[\partial_{\nu'}(\mathbf{F}_{\mu\nu}\mathbf{F}_{\nu'\nu}) + \frac{1}{2}(\partial_{\nu'}\mathbf{F}_{\mu\nu} + \partial_{\nu}\mathbf{F}_{\nu'\mu})\mathbf{F}_{\nu\nu'}\right]$$

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$$\mathbf{f}_\mu = \sum_{\nu=1}^{4}\mathbf{F}_{\mu\nu}\mathbf{j}_\nu = -\frac{1}{\mu_0}\sum_{\nu,\nu'=1}^{4}\mathbf{F}_{\mu\nu}\partial_{\nu'}\mathbf{F}_{\nu'\nu} = -\frac{1}{\mu_0}\sum_{\nu,\nu'=1}^{4}\left[\partial_{\nu'}(\mathbf{F}_{\mu\nu}\mathbf{F}_{\nu'\nu}) - (\partial_{\nu'}\mathbf{F}_{\mu\nu})\mathbf{F}_{\nu'\nu}\right]$$

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$$= -\frac{1}{\mu_0}\sum_{\nu,\nu'=1}^{4}\left[\partial_{\nu'}(\mathbf{F}_{\mu\nu}\mathbf{F}_{\nu'\nu}) + \frac{1}{2}(\partial_{\nu'}\mathbf{F}_{\mu\nu} + \partial_{\nu}\mathbf{F}_{\nu'\mu})\mathbf{F}_{\nu\nu'}\right]$$

$$= -\frac{1}{\mu_0}\sum_{\nu,\nu'=1}^{4}\left[\partial_{\nu'}(\mathbf{F}_{\mu\nu}\mathbf{F}_{\nu'\nu}) - \frac{1}{2}(\partial_{\mu}\mathbf{F}_{\nu\nu'})\mathbf{F}_{\nu\nu'}\right]$$

Back
Close

$$\sum_{\mu=1}^{4}\partial_\mu \mathbf{F}_{\mu\nu} = -\mu_0 \mathbf{j}_\nu \qquad \partial_\mu \mathbf{F}_{\nu\lambda}+\partial_\nu \mathbf{F}_{\lambda\mu}+\partial_\lambda \mathbf{F}_{\mu\nu}=\mathbf{0} \qquad \mathbf{f}_\mu \equiv \sum_{\nu=1}^{4}\mathbf{F}_{\mu\nu}\mathbf{j}_\nu$$

$$\mathbf{f}_\mu = \sum_{\nu=1}^{4}\mathbf{F}_{\mu\nu}\mathbf{j}_\nu = -\frac{1}{\mu_0}\sum_{\nu,\nu'=1}^{4}\mathbf{F}_{\mu\nu}\partial_{\nu'}\mathbf{F}_{\nu'\nu} = -\frac{1}{\mu_0}\sum_{\nu,\nu'=1}^{4}\left[\partial_{\nu'}(\mathbf{F}_{\mu\nu}\mathbf{F}_{\nu'\nu}) - (\partial_{\nu'}\mathbf{F}_{\mu\nu})\mathbf{F}_{\nu'\nu}\right]$$

$$= -\frac{1}{\mu_0}\sum_{\nu,\nu'=1}^{4}\left[\partial_{\nu'}(\mathbf{F}_{\mu\nu}\mathbf{F}_{\nu'\nu}) - \frac{1}{2}(\partial_{\nu'}\mathbf{F}_{\mu\nu})\mathbf{F}_{\nu'\nu} - \frac{1}{2}(\partial_{\nu}\mathbf{F}_{\mu\nu'})\mathbf{F}_{\nu\nu'}\right]$$

$$= -\frac{1}{\mu_0}\sum_{\nu,\nu'=1}^{4}\left[\partial_{\nu'}(\mathbf{F}_{\mu\nu}\mathbf{F}_{\nu'\nu}) + \frac{1}{2}(\partial_{\nu'}\mathbf{F}_{\mu\nu} + \partial_{\nu}\mathbf{F}_{\nu'\mu})\mathbf{F}_{\nu\nu'}\right]$$

$$= -\frac{1}{\mu_0}\sum_{\nu,\nu'=1}^{4}\left[\partial_{\nu'}(\mathbf{F}_{\mu\nu}\mathbf{F}_{\nu'\nu}) - \frac{1}{2}(\partial_{\mu}\mathbf{F}_{\nu\nu'})\mathbf{F}_{\nu\nu'}\right]$$

$$= -\frac{1}{\mu_0}\sum_{\nu,\nu'=1}^{4}\partial_{\nu'}\left[\mathbf{F}_{\mu\nu}\mathbf{F}_{\nu'\nu} - \frac{1}{4}\delta_{\mu\nu'}\sum_{\nu''=1}^{4}\mathbf{F}_{\nu\nu''}\mathbf{F}_{\nu\nu''}\right] \equiv -\sum_{\nu'=1}^{4}\partial_{\nu'}\mathbf{T}_{\mu\nu'}$$

Back
Close

$$\mathbf{f}_\mu = -\sum_{\nu=1}^{4} \partial_\nu \mathbf{T}_{\mu\nu} \qquad \mathbf{T}_{\mu\nu} \equiv \frac{1}{\mu_0}\Big[\sum_{\nu'=1}^{4} \mathbf{F}_{\mu\nu'}\mathbf{F}_{\nu\nu'} - \frac{1}{4}\delta_{\mu\nu}\sum_{\mu',\nu'=1}^{4} \mathbf{F}_{\mu'\nu'}\mathbf{F}_{\mu'\nu'}\Big] = \mathbf{T}_{\nu\mu}$$

$$f_\mu = -\sum_{\nu=1}^{4} \partial_\nu T_{\mu\nu} \qquad T_{\mu\nu} \equiv \frac{1}{\mu_0}\Big[\sum_{\nu'=1}^{4} F_{\mu\nu'}F_{\nu\nu'} - \frac{1}{4}\delta_{\mu\nu}\sum_{\mu',\nu'=1}^{4} F_{\mu'\nu'}F_{\mu'\nu'}\Big] = T_{\nu\mu}$$

$$\sum_{\mu',\nu'=1}^{4} F_{\mu'\nu'}F_{\mu'\nu'} = 2B^2 - \frac{2}{c^2}E^2 \qquad F_{ij} = \sum_{k=1}^{3} \epsilon_{ijk}B_k \qquad F_{4i} = -F_{i4} = \frac{i}{c}E_i$$

Back
Close

$$\mathbf{f}_{\mu} = -\sum_{\nu=1}^{4}\partial_{\nu}\mathbf{T}_{\mu\nu} \qquad \mathbf{T}_{\mu\nu} \equiv \frac{1}{\mu_0}[\sum_{\nu'=1}^{4}\mathbf{F}_{\mu\nu'}\mathbf{F}_{\nu\nu'} - \frac{1}{4}\delta_{\mu\nu}\sum_{\mu',\nu'=1}^{4}\mathbf{F}_{\mu'\nu'}\mathbf{F}_{\mu'\nu'}] = \mathbf{T}_{\nu\mu}$$

$$\sum_{\mu',\nu'=1}^{4}\mathbf{F}_{\mu'\nu'}\mathbf{F}_{\mu'\nu'} = 2\mathbf{B}^2 - \frac{2}{\mathbf{c}^2}\mathbf{E}^2 \qquad \mathbf{F}_{\mathrm{ij}} = \sum_{\mathrm{k}=1}^{3}\epsilon_{\mathrm{ijk}}\mathbf{B}_{\mathrm{k}} \qquad \mathbf{F}_{4\mathrm{i}} = -\mathbf{F}_{\mathrm{i}4} = \frac{\mathrm{i}}{\mathbf{c}}\mathbf{E}_{\mathrm{i}}$$

$$\mathbf{T}_{\mathrm{ij}} = \frac{1}{\mu_0}[\sum_{\mathrm{k}=1}^{3}\mathbf{F}_{\mathrm{ik}}\mathbf{F}_{\mathrm{jk}} + \mathbf{F}_{\mathrm{i}4}\mathbf{F}_{\mathrm{j}4} - \frac{1}{2}\delta_{\mathrm{ij}}(\mathbf{B}^2 - \frac{1}{\mathbf{c}^2}\mathbf{E}^2)]$$

$$\mathbf{f}_\mu = -\sum_{\nu=1}^{4}\partial_\nu \mathbf{T}_{\mu\nu} \qquad \mathbf{T}_{\mu\nu} \equiv \frac{1}{\mu_0}[\sum_{\nu'=1}^{4}\mathbf{F}_{\mu\nu'}\mathbf{F}_{\nu\nu'} - \frac{1}{4}\delta_{\mu\nu}\sum_{\mu',\nu'=1}^{4}\mathbf{F}_{\mu'\nu'}\mathbf{F}_{\mu'\nu'}] = \mathbf{T}_{\nu\mu}$$

$$\sum_{\mu',\nu'=1}^{4}\mathbf{F}_{\mu'\nu'}\mathbf{F}_{\mu'\nu'} = 2\mathbf{B}^2 - \frac{2}{\mathbf{c}^2}\mathbf{E}^2 \qquad \mathbf{F}_{\mathbf{ij}} = \sum_{\mathbf{k}=1}^{3}\epsilon_{\mathbf{ijk}}\mathbf{B}_{\mathbf{k}} \qquad \mathbf{F}_{4\mathbf{i}} = -\mathbf{F}_{\mathbf{i}4} = \frac{\mathbf{i}}{\mathbf{c}}\mathbf{E}_{\mathbf{i}}$$

$$\mathbf{T}_{\mathbf{ij}} = \frac{1}{\mu_0}[\sum_{\mathbf{k}=1}^{3}\mathbf{F}_{\mathbf{ik}}\mathbf{F}_{\mathbf{jk}} + \mathbf{F}_{\mathbf{i}4}\mathbf{F}_{\mathbf{j}4} - \frac{1}{2}\delta_{\mathbf{ij}}(\mathbf{B}^2 - \frac{1}{\mathbf{c}^2}\mathbf{E}^2)]$$

$$= \frac{1}{\mu_0}[\sum_{\mathbf{k},\mathbf{l},\mathbf{l}'=1}^{3}\epsilon_{\mathbf{ikl}}\epsilon_{\mathbf{jkl}'}\mathbf{B}_{\mathbf{l}}\mathbf{B}_{\mathbf{l}'} - \frac{1}{\mathbf{c}^2}\mathbf{E}_{\mathbf{i}}\mathbf{E}_{\mathbf{j}} - \frac{1}{2}\delta_{\mathbf{ij}}(\mathbf{B}^2 - \frac{1}{\mathbf{c}^2}\mathbf{E}^2)]$$

$$\mathbf{f}_\mu = -\sum_{\nu=1}^{4}\partial_\nu \mathbf{T}_{\mu\nu} \qquad \mathbf{T}_{\mu\nu} \equiv \frac{1}{\mu_0}\Big[\sum_{\nu'=1}^{4}\mathbf{F}_{\mu\nu'}\mathbf{F}_{\nu\nu'} - \frac{1}{4}\delta_{\mu\nu}\sum_{\mu',\nu'=1}^{4}\mathbf{F}_{\mu'\nu'}\mathbf{F}_{\mu'\nu'}\Big] = \mathbf{T}_{\nu\mu}$$

$$\sum_{\mu',\nu'=1}^{4}\mathbf{F}_{\mu'\nu'}\mathbf{F}_{\mu'\nu'} = 2\mathbf{B}^2 - \frac{2}{\mathbf{c}^2}\mathbf{E}^2 \qquad \mathbf{F}_{\mathbf{ij}} = \sum_{\mathbf{k}=1}^{3}\epsilon_{\mathbf{ijk}}\mathbf{B}_{\mathbf{k}} \qquad \mathbf{F}_{4\mathbf{i}} = -\mathbf{F}_{\mathbf{i}4} = \frac{\mathbf{i}}{\mathbf{c}}\mathbf{E}_{\mathbf{i}}$$

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$$= \frac{1}{\mu_0}\Big[\sum_{\mathbf{k},\mathbf{l},\mathbf{l}'=1}^{3}\epsilon_{\mathbf{ikl}}\epsilon_{\mathbf{jkl}'}\mathbf{B}_{\mathbf{l}}\mathbf{B}_{\mathbf{l}'} - \frac{1}{\mathbf{c}^2}\mathbf{E}_{\mathbf{i}}\mathbf{E}_{\mathbf{j}} - \frac{1}{2}\delta_{\mathbf{ij}}\Big(\mathbf{B}^2 - \frac{1}{\mathbf{c}^2}\mathbf{E}^2\Big)\Big]$$

$$= \frac{1}{\mu_0}\Big[\delta_{\mathbf{ij}}\mathbf{B}^2 - \mathbf{B}_{\mathbf{i}}\mathbf{B}_{\mathbf{j}} - \frac{1}{\mathbf{c}^2}\mathbf{E}_{\mathbf{i}}\mathbf{E}_{\mathbf{j}} - \frac{1}{2}\delta_{\mathbf{ij}}\Big(\mathbf{B}^2 - \frac{1}{\mathbf{c}^2}\mathbf{E}^2\Big)\Big]$$

$$\mathbf{f}_\mu = -\sum_{\nu=1}^{4} \partial_\nu \mathbf{T}_{\mu\nu} \qquad \mathbf{T}_{\mu\nu} \equiv \frac{1}{\mu_0}[\sum_{\nu'=1}^{4} \mathbf{F}_{\mu\nu'}\mathbf{F}_{\nu\nu'} - \frac{1}{4}\delta_{\mu\nu} \sum_{\mu',\nu'=1}^{4} \mathbf{F}_{\mu'\nu'}\mathbf{F}_{\mu'\nu'}] = \mathbf{T}_{\nu\mu}$$

$$\sum_{\mu',\nu'=1}^{4} \mathbf{F}_{\mu'\nu'}\mathbf{F}_{\mu'\nu'} = 2\mathrm{B}^2 - \frac{2}{\mathrm{c}^2}\mathrm{E}^2 \qquad \mathrm{F}_{\mathrm{ij}} = \sum_{\mathrm{k}=1}^{3} \epsilon_{\mathrm{ijk}}\mathrm{B}_{\mathrm{k}} \qquad \mathrm{F}_{4\mathrm{i}} = -\mathrm{F}_{\mathrm{i}4} = \frac{\mathrm{i}}{\mathrm{c}}\mathrm{E}_{\mathrm{i}}$$

$$\mathrm{T}_{\mathrm{ij}} = \frac{1}{\mu_0}[\sum_{\mathrm{k}=1}^{3} \mathrm{F}_{\mathrm{ik}}\mathrm{F}_{\mathrm{jk}} + \mathrm{F}_{\mathrm{i}4}\mathrm{F}_{\mathrm{j}4} - \frac{1}{2}\delta_{\mathrm{ij}}(\mathrm{B}^2 - \frac{1}{\mathrm{c}^2}\mathrm{E}^2)]$$

$$= \frac{1}{\mu_0}[\sum_{\mathrm{k,l,l'}=1}^{3} \epsilon_{\mathrm{ikl}}\epsilon_{\mathrm{jkl'}}\mathrm{B}_{\mathrm{l}}\mathrm{B}_{\mathrm{l'}} - \frac{1}{\mathrm{c}^2}\mathrm{E}_{\mathrm{i}}\mathrm{E}_{\mathrm{j}} - \frac{1}{2}\delta_{\mathrm{ij}}(\mathrm{B}^2 - \frac{1}{\mathrm{c}^2}\mathrm{E}^2)]$$

$$= \frac{1}{\mu_0}[\delta_{\mathrm{ij}}\mathrm{B}^2 - \mathrm{B}_{\mathrm{i}}\mathrm{B}_{\mathrm{j}} - \frac{1}{\mathrm{c}^2}\mathrm{E}_{\mathrm{i}}\mathrm{E}_{\mathrm{j}} - \frac{1}{2}\delta_{\mathrm{ij}}(\mathrm{B}^2 - \frac{1}{\mathrm{c}^2}\mathrm{E}^2)]$$

$$= \frac{1}{\mu_0}[-\mathrm{B}_{\mathrm{i}}\mathrm{B}_{\mathrm{j}} - \frac{1}{\mathrm{c}^2}\mathrm{E}_{\mathrm{i}}\mathrm{E}_{\mathrm{j}} + \frac{1}{2}\delta_{\mathrm{ij}}(\mathrm{B}^2 + \frac{1}{\mathrm{c}^2}\mathrm{E}^2)]$$

Back
Close

$$\mathbf{f}_\mu = -\sum_{\nu=1}^{4} \partial_\nu \mathbf{T}_{\mu\nu} \qquad \mathbf{T}_{\mu\nu} \equiv \frac{1}{\mu_0}\Big[\sum_{\nu'=1}^{4} \mathbf{F}_{\mu\nu'}\mathbf{F}_{\nu\nu'} - \frac{1}{4}\delta_{\mu\nu}\sum_{\mu',\nu'=1}^{4} \mathbf{F}_{\mu'\nu'}\mathbf{F}_{\mu'\nu'}\Big] = \mathbf{T}_{\nu\mu}$$

$$\sum_{\mu',\nu'=1}^{4} \mathbf{F}_{\mu'\nu'}\mathbf{F}_{\mu'\nu'} = 2\mathbf{B}^2 - \frac{2}{\mathbf{c}^2}\mathbf{E}^2 \qquad \mathbf{F}_{\mathrm{ij}} = \sum_{\mathrm{k}=1}^{3} \epsilon_{\mathrm{ijk}}\mathbf{B}_{\mathrm{k}} \qquad \mathbf{F}_{4\mathrm{i}} = -\mathbf{F}_{\mathrm{i}4} = \frac{\mathrm{i}}{\mathbf{c}}\mathbf{E}_{\mathrm{i}}$$

$$\begin{aligned}
\mathbf{T}_{\mathrm{ij}} &= \frac{1}{\mu_0}\Big[\sum_{\mathrm{k}=1}^{3} \mathbf{F}_{\mathrm{ik}}\mathbf{F}_{\mathrm{jk}} + \mathbf{F}_{\mathrm{i}4}\mathbf{F}_{\mathrm{j}4} - \frac{1}{2}\delta_{\mathrm{ij}}\Big(\mathbf{B}^2 - \frac{1}{\mathbf{c}^2}\mathbf{E}^2\Big)\Big] \\
&= \frac{1}{\mu_0}\Big[\sum_{\mathrm{k,l,l'}=1}^{3} \epsilon_{\mathrm{ikl}}\epsilon_{\mathrm{jkl'}}\mathbf{B}_{\mathrm{l}}\mathbf{B}_{\mathrm{l'}} - \frac{1}{\mathbf{c}^2}\mathbf{E}_{\mathrm{i}}\mathbf{E}_{\mathrm{j}} - \frac{1}{2}\delta_{\mathrm{ij}}\Big(\mathbf{B}^2 - \frac{1}{\mathbf{c}^2}\mathbf{E}^2\Big)\Big] \\
&= \frac{1}{\mu_0}\Big[\delta_{\mathrm{ij}}\mathbf{B}^2 - \mathbf{B}_{\mathrm{i}}\mathbf{B}_{\mathrm{j}} - \frac{1}{\mathbf{c}^2}\mathbf{E}_{\mathrm{i}}\mathbf{E}_{\mathrm{j}} - \frac{1}{2}\delta_{\mathrm{ij}}\Big(\mathbf{B}^2 - \frac{1}{\mathbf{c}^2}\mathbf{E}^2\Big)\Big] \\
&= \frac{1}{\mu_0}\Big[-\mathbf{B}_{\mathrm{i}}\mathbf{B}_{\mathrm{j}} - \frac{1}{\mathbf{c}^2}\mathbf{E}_{\mathrm{i}}\mathbf{E}_{\mathrm{j}} + \frac{1}{2}\delta_{\mathrm{ij}}\Big(\mathbf{B}^2 + \frac{1}{\mathbf{c}^2}\mathbf{E}^2\Big)\Big] \\
&= \frac{1}{\mu_0}\Big(-\mathbf{B}_{\mathrm{i}}\mathbf{B}_{\mathrm{j}} + \frac{1}{2}\delta_{\mathrm{ij}}\mathbf{B}^2\Big) + \epsilon_0\Big(-\mathbf{E}_{\mathrm{i}}\mathbf{E}_{\mathrm{j}} + \frac{1}{2}\delta_{\mathrm{ij}}\mathbf{E}^2\Big)
\end{aligned}$$

$$f_\mu = -\sum_{\nu=1}^{4}\partial_\nu T_{\mu\nu} \qquad T_{\mu\nu} \equiv \frac{1}{\mu_0}[\sum_{\nu'=1}^{4}F_{\mu\nu'}F_{\nu\nu'} - \frac{1}{4}\delta_{\mu\nu}\sum_{\mu',\nu'=1}^{4}F_{\mu'\nu'}F_{\mu'\nu'}] = T_{\nu\mu}$$

$$\sum_{\mu',\nu'=1}^{4}F_{\mu'\nu'}F_{\mu'\nu'} = 2B^2 - \frac{2}{c^2}E^2 \qquad F_{ij} = \sum_{k=1}^{3}\epsilon_{ijk}B_k \qquad F_{4i} = -F_{i4} = \frac{i}{c}E_i$$

$$\begin{aligned} T_{ij} &= \frac{1}{\mu_0}[\sum_{k=1}^{3}F_{ik}F_{jk} + F_{i4}F_{j4} - \frac{1}{2}\delta_{ij}(B^2 - \frac{1}{c^2}E^2)] \\ &= \frac{1}{\mu_0}[\sum_{k,l,l'=1}^{3}\epsilon_{ikl}\epsilon_{jkl'}B_lB_{l'} - \frac{1}{c^2}E_iE_j - \frac{1}{2}\delta_{ij}(B^2 - \frac{1}{c^2}E^2)] \\ &= \frac{1}{\mu_0}[\delta_{ij}B^2 - B_iB_j - \frac{1}{c^2}E_iE_j - \frac{1}{2}\delta_{ij}(B^2 - \frac{1}{c^2}E^2)] \\ &= \frac{1}{\mu_0}[-B_iB_j - \frac{1}{c^2}E_iE_j + \frac{1}{2}\delta_{ij}(B^2 + \frac{1}{c^2}E^2)] \\ &= \frac{1}{\mu_0}(-B_iB_j + \frac{1}{2}\delta_{ij}B^2) + \epsilon_0(-E_iE_j + \frac{1}{2}\delta_{ij}E^2) = \mathcal{J}_{ij} \end{aligned}$$

$$(\rho\vec{\mathbf{E}}+\vec{\mathbf{j}}\times\vec{\mathbf{B}},\ \frac{\mathbf{i}}{\mathbf{c}}\vec{\mathbf{j}}\cdot\vec{\mathbf{E}}) = \mathbf{f}_{\mu} = -\sum_{\nu=1}^{4}\partial_{\nu}\mathbf{T}_{\mu\nu}$$

$$\mathbf{T}_{\mu\nu} \equiv \frac{1}{\mu_0}[\sum_{\nu'=1}^{4}\mathbf{F}_{\mu\nu'}\mathbf{F}_{\nu\nu'} - \frac{1}{4}\delta_{\mu\nu}\sum_{\mu',\nu'=1}^{4}\mathbf{F}_{\mu'\nu'}\mathbf{F}_{\mu'\nu'}] = \mathbf{T}_{\nu\mu}$$

$$(\rho\vec{\mathbf{E}}+\vec{\mathbf{j}}\times\vec{\mathbf{B}},\ \frac{\mathrm{i}}{\mathrm{c}}\vec{\mathbf{j}}\cdot\vec{\mathbf{E}})=\mathbf{f}_\mu=-\sum_{\nu=1}^{4}\partial_\nu\mathbf{T}_{\mu\nu}$$

$$\mathbf{T}_{\mu\nu}\equiv\frac{1}{\mu_0}[\sum_{\nu'=1}^{4}\mathbf{F}_{\mu\nu'}\mathbf{F}_{\nu\nu'}-\frac{1}{4}\delta_{\mu\nu}\sum_{\mu',\nu'=1}^{4}\mathbf{F}_{\mu'\nu'}\mathbf{F}_{\mu'\nu'}]=\mathbf{T}_{\nu\mu}$$

$$\sum_{\mu',\nu'=1}^{4}\mathbf{F}_{\mu'\nu'}\mathbf{F}_{\mu'\nu'}=2\mathbf{B}^2-\frac{2}{\mathbf{c}^2}\mathbf{E}^2\qquad \mathbf{F}_{\mathrm{ij}}=\sum_{\mathrm{k}=1}^{3}\epsilon_{\mathrm{ijk}}\mathbf{B}_{\mathrm{k}}\qquad \mathbf{F}_{4\mathrm{i}}=-\mathbf{F}_{\mathrm{i}4}=\frac{\mathrm{i}}{\mathrm{c}}\mathbf{E}_{\mathrm{i}}$$

$$\mathbf{T}_{\mathrm{ij}}=\frac{1}{\mu_0}(-\mathbf{B}_{\mathrm{i}}\mathbf{B}_{\mathrm{j}}+\frac{1}{2}\delta_{\mathrm{ij}}\mathbf{B}^2)+\epsilon_0(-\mathbf{E}_{\mathrm{i}}\mathbf{E}_{\mathrm{j}}+\frac{1}{2}\delta_{\mathrm{ij}}\mathbf{E}^2)$$

Back
Close

$$(\rho\vec{\mathbf{E}} + \vec{\mathbf{j}} \times \vec{\mathbf{B}},\ \frac{\mathrm{i}}{\mathrm{c}}\vec{\mathbf{j}} \cdot \vec{\mathbf{E}}) = \mathbf{f}_\mu = -\sum_{\nu=1}^{4} \partial_\nu \mathbf{T}_{\mu\nu}$$

$$\mathbf{T}_{\mu\nu} \equiv \frac{1}{\mu_0}\Big[\sum_{\nu'=1}^{4} \mathbf{F}_{\mu\nu'}\mathbf{F}_{\nu\nu'} - \frac{1}{4}\delta_{\mu\nu} \sum_{\mu',\nu'=1}^{4} \mathbf{F}_{\mu'\nu'}\mathbf{F}_{\mu'\nu'}\Big] = \mathbf{T}_{\nu\mu}$$

$$\sum_{\mu',\nu'=1}^{4} \mathbf{F}_{\mu'\nu'}\mathbf{F}_{\mu'\nu'} = 2\mathbf{B}^2 - \frac{2}{\mathbf{c}^2}\mathbf{E}^2 \qquad \mathbf{F}_{\mathrm{ij}} = \sum_{\mathrm{k}=1}^{3} \epsilon_{\mathrm{ijk}}\mathbf{B}_{\mathrm{k}} \qquad \mathbf{F}_{4\mathrm{i}} = -\mathbf{F}_{\mathrm{i}4} = \frac{\mathrm{i}}{\mathrm{c}}\mathbf{E}_{\mathrm{i}}$$

$$\mathbf{T}_{\mathrm{ij}} = \frac{1}{\mu_0}(-\mathbf{B}_{\mathrm{i}}\mathbf{B}_{\mathrm{j}} + \frac{1}{2}\delta_{\mathrm{ij}}\mathbf{B}^2) + \epsilon_0(-\mathbf{E}_{\mathrm{i}}\mathbf{E}_{\mathrm{j}} + \frac{1}{2}\delta_{\mathrm{ij}}\mathbf{E}^2) = \mathcal{J}_{\mathrm{ij}}$$

$$(\rho\vec{\mathbf{E}}+\vec{\mathbf{j}}\times\vec{\mathbf{B}},\ \frac{\mathrm{i}}{\mathrm{c}}\vec{\mathbf{j}}\cdot\vec{\mathbf{E}})=\mathbf{f}_\mu=-\sum_{\nu=1}^{4}\partial_\nu\mathbf{T}_{\mu\nu}$$

$$\mathbf{T}_{\mu\nu}\equiv\frac{1}{\mu_0}[\sum_{\nu'=1}^{4}\mathbf{F}_{\mu\nu'}\mathbf{F}_{\nu\nu'}-\frac{1}{4}\delta_{\mu\nu}\sum_{\mu',\nu'=1}^{4}\mathbf{F}_{\mu'\nu'}\mathbf{F}_{\mu'\nu'}]=\mathbf{T}_{\nu\mu}$$

$$\sum_{\mu',\nu'=1}^{4}\mathbf{F}_{\mu'\nu'}\mathbf{F}_{\mu'\nu'}=2\mathbf{B}^2-\frac{2}{\mathbf{c}^2}\mathbf{E}^2 \qquad \mathbf{F}_{\mathrm{ij}}=\sum_{\mathrm{k}=1}^{3}\epsilon_{\mathrm{ijk}}\mathbf{B}_{\mathrm{k}} \qquad \mathbf{F}_{4\mathrm{i}}=-\mathbf{F}_{\mathrm{i}4}=\frac{\mathrm{i}}{\mathrm{c}}\mathbf{E}_{\mathrm{i}}$$

$$\mathbf{T}_{\mathrm{ij}}=\frac{1}{\mu_0}(-\mathbf{B}_{\mathrm{i}}\mathbf{B}_{\mathrm{j}}+\frac{1}{2}\delta_{\mathrm{ij}}\mathbf{B}^2)+\epsilon_0(-\mathbf{E}_{\mathrm{i}}\mathbf{E}_{\mathrm{j}}+\frac{1}{2}\delta_{\mathrm{ij}}\mathbf{E}^2)=\mathscr{T}_{\mathrm{ij}}$$

$$\mathbf{T}_{44}=\frac{1}{\mu_0}[\sum_{\mathrm{i}=1}^{3}\mathbf{F}_{4\mathrm{i}}^2-\frac{1}{2}(\mathbf{B}^2-\frac{1}{\mathbf{c}^2}\mathbf{E}^2)]$$

Back
Close

$$(\rho\vec{\mathbf{E}}+\vec{\mathbf{j}}\times\vec{\mathbf{B}},\ \frac{\mathrm{i}}{\mathrm{c}}\vec{\mathbf{j}}\cdot\vec{\mathbf{E}})=\mathbf{f}_\mu=-\sum_{\nu=1}^{4}\partial_\nu\mathbf{T}_{\mu\nu}$$

$$\mathbf{T}_{\mu\nu}\equiv\frac{1}{\mu_0}[\sum_{\nu'=1}^{4}\mathbf{F}_{\mu\nu'}\mathbf{F}_{\nu\nu'}-\frac{1}{4}\delta_{\mu\nu}\sum_{\mu',\nu'=1}^{4}\mathbf{F}_{\mu'\nu'}\mathbf{F}_{\mu'\nu'}]=\mathbf{T}_{\nu\mu}$$

$$\sum_{\mu',\nu'=1}^{4}\mathbf{F}_{\mu'\nu'}\mathbf{F}_{\mu'\nu'}=2\mathbf{B}^2-\frac{2}{\mathbf{c}^2}\mathbf{E}^2\qquad \mathbf{F}_{\mathrm{ij}}=\sum_{\mathrm{k}=1}^{3}\epsilon_{\mathrm{ijk}}\mathbf{B}_{\mathrm{k}}\qquad \mathbf{F}_{4\mathrm{i}}=-\mathbf{F}_{\mathrm{i}4}=\frac{\mathrm{i}}{\mathbf{c}}\mathbf{E}_{\mathrm{i}}$$

$$\mathbf{T}_{\mathrm{ij}}=\frac{1}{\mu_0}(-\mathbf{B}_{\mathrm{i}}\mathbf{B}_{\mathrm{j}}+\frac{1}{2}\delta_{\mathrm{ij}}\mathbf{B}^2)+\epsilon_0(-\mathbf{E}_{\mathrm{i}}\mathbf{E}_{\mathrm{j}}+\frac{1}{2}\delta_{\mathrm{ij}}\mathbf{E}^2)=\mathcal{J}_{\mathrm{ij}}$$

$$\mathbf{T}_{44}=\frac{1}{\mu_0}[\sum_{\mathrm{i}=1}^{3}\mathbf{F}_{4\mathrm{i}}^2-\frac{1}{2}(\mathbf{B}^2-\frac{1}{\mathbf{c}^2}\mathbf{E}^2)]=\frac{-1}{2\mu_0}(\mathbf{B}^2+\frac{1}{\mathbf{c}^2}\mathbf{E}^2)$$

Back
Close

$$(\rho\vec{\mathbf{E}}+\vec{\mathbf{j}}\times\vec{\mathbf{B}},\ \frac{\mathrm{i}}{\mathrm{c}}\vec{\mathbf{j}}\cdot\vec{\mathbf{E}})=\mathbf{f}_\mu=-\sum_{\nu=1}^{4}\partial_\nu\mathbf{T}_{\mu\nu}$$

$$\mathbf{T}_{\mu\nu}\equiv\frac{1}{\mu_0}[\sum_{\nu'=1}^{4}\mathbf{F}_{\mu\nu'}\mathbf{F}_{\nu\nu'}-\frac{1}{4}\delta_{\mu\nu}\sum_{\mu',\nu'=1}^{4}\mathbf{F}_{\mu'\nu'}\mathbf{F}_{\mu'\nu'}]=\mathbf{T}_{\nu\mu}$$

$$\sum_{\mu',\nu'=1}^{4}\mathbf{F}_{\mu'\nu'}\mathbf{F}_{\mu'\nu'}=2\mathbf{B}^2-\frac{2}{\mathrm{c}^2}\mathbf{E}^2 \qquad \mathbf{F}_{\mathrm{ij}}=\sum_{\mathrm{k}=1}^{3}\epsilon_{\mathrm{ijk}}\mathbf{B}_{\mathrm{k}} \qquad \mathbf{F}_{4\mathrm{i}}=-\mathbf{F}_{\mathrm{i}4}=\frac{\mathrm{i}}{\mathrm{c}}\mathbf{E}_{\mathrm{i}}$$

$$\mathbf{T}_{\mathrm{ij}}=\frac{1}{\mu_0}(-\mathbf{B}_{\mathrm{i}}\mathbf{B}_{\mathrm{j}}+\frac{1}{2}\delta_{\mathrm{ij}}\mathbf{B}^2)+\epsilon_0(-\mathbf{E}_{\mathrm{i}}\mathbf{E}_{\mathrm{j}}+\frac{1}{2}\delta_{\mathrm{ij}}\mathbf{E}^2)=\mathcal{J}_{\mathrm{ij}}$$

$$\mathbf{T}_{44}=\frac{1}{\mu_0}[\sum_{\mathrm{i}=1}^{3}\mathbf{F}_{4\mathrm{i}}^2-\frac{1}{2}(\mathbf{B}^2-\frac{1}{\mathrm{c}^2}\mathbf{E}^2)]=\frac{-1}{2\mu_0}(\mathbf{B}^2+\frac{1}{\mathrm{c}^2}\mathbf{E}^2)=\frac{-1}{2\mu_0}\mathbf{B}^2-\frac{\epsilon_0}{2}\mathbf{E}^2$$

Back
Close

$$(\rho\vec{\mathbf{E}}+\vec{\mathbf{j}}\times\vec{\mathbf{B}},\ \frac{\mathbf{i}}{\mathbf{c}}\vec{\mathbf{j}}\cdot\vec{\mathbf{E}}) = \mathbf{f}_\mu = -\sum_{\nu=1}^{4}\partial_\nu \mathbf{T}_{\mu\nu}$$

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$$\sum_{\mu',\nu'=1}^{4}\mathbf{F}_{\mu'\nu'}\mathbf{F}_{\mu'\nu'} = 2\mathbf{B}^2 - \frac{2}{\mathbf{c}^2}\mathbf{E}^2 \qquad \mathbf{F}_{\mathbf{ij}} = \sum_{\mathbf{k}=1}^{3}\epsilon_{\mathbf{ijk}}\mathbf{B}_{\mathbf{k}} \qquad \mathbf{F}_{4\mathbf{i}} = -\mathbf{F}_{\mathbf{i}4} = \frac{\mathbf{i}}{\mathbf{c}}\mathbf{E}_{\mathbf{i}}$$

$$\mathbf{T}_{\mathbf{ij}} = \frac{1}{\mu_0}(-\mathbf{B}_{\mathbf{i}}\mathbf{B}_{\mathbf{j}} + \frac{1}{2}\delta_{\mathbf{ij}}\mathbf{B}^2) + \epsilon_0(-\mathbf{E}_{\mathbf{i}}\mathbf{E}_{\mathbf{j}} + \frac{1}{2}\delta_{\mathbf{ij}}\mathbf{E}^2) = \mathcal{J}_{\mathbf{ij}}$$

$$\mathbf{T}_{44} = \frac{1}{\mu_0}[\sum_{\mathbf{i}=1}^{3}\mathbf{F}_{4\mathbf{i}}^2 - \frac{1}{2}(\mathbf{B}^2 - \frac{1}{\mathbf{c}^2}\mathbf{E}^2)] = \frac{-1}{2\mu_0}(\mathbf{B}^2 + \frac{1}{\mathbf{c}^2}\mathbf{E}^2) = \frac{-1}{2\mu_0}\mathbf{B}^2 - \frac{\epsilon_0}{2}\mathbf{E}^2 = -\mathbf{W}$$

$$(\rho\vec{E} + \vec{j} \times \vec{B},\ \frac{i}{c}\vec{j} \cdot \vec{E}) = f_\mu = -\sum_{\nu=1}^{4} \partial_\nu T_{\mu\nu}$$

$$T_{\mu\nu} \equiv \frac{1}{\mu_0}[\sum_{\nu'=1}^{4} F_{\mu\nu'} F_{\nu\nu'} - \frac{1}{4}\delta_{\mu\nu} \sum_{\mu',\nu'=1}^{4} F_{\mu'\nu'} F_{\mu'\nu'}] = T_{\nu\mu}$$

$$\sum_{\mu',\nu'=1}^{4} F_{\mu'\nu'} F_{\mu'\nu'} = 2B^2 - \frac{2}{c^2}E^2 \qquad F_{ij} = \sum_{k=1}^{3} \epsilon_{ijk} B_k \qquad F_{4i} = -F_{i4} = \frac{i}{c}E_i$$

$$T_{ij} = \frac{1}{\mu_0}(-B_i B_j + \frac{1}{2}\delta_{ij} B^2) + \epsilon_0(-E_i E_j + \frac{1}{2}\delta_{ij} E^2) = \mathcal{J}_{ij}$$

$$T_{44} = \frac{1}{\mu_0}[\sum_{i=1}^{3} F_{4i}^2 - \frac{1}{2}(B^2 - \frac{1}{c^2}E^2)] = \frac{-1}{2\mu_0}(B^2 + \frac{1}{c^2}E^2) = \frac{-1}{2\mu_0}B^2 - \frac{\epsilon_0}{2}E^2 = -W$$

$$T_{i4} = T_{4i} = \frac{1}{\mu_0}\sum_{j=1}^{3} F_{ij} F_{4j} = \frac{i}{\mu_0 c}\sum_{j,k=1}^{3} \epsilon_{ijk} B_k E_j = \frac{i}{\mu_0 c}(\vec{E} \times \vec{B})_i = \frac{i}{c} S_i$$

$$(\rho\vec{\mathbf{E}}+\vec{\mathbf{j}}\times\vec{\mathbf{B}},\ \frac{\mathrm{i}}{\mathrm{c}}\vec{\mathbf{j}}\cdot\vec{\mathbf{E}}) = \mathbf{f}_\mu = -\sum_{\nu=1}^{4}\partial_\nu \mathbf{T}_{\mu\nu}$$

$$\mathbf{T}_{\mu\nu} \equiv \frac{1}{\mu_0}[\sum_{\nu'=1}^{4}\mathbf{F}_{\mu\nu'}\mathbf{F}_{\nu\nu'} - \frac{1}{4}\delta_{\mu\nu}\sum_{\mu',\nu'=1}^{4}\mathbf{F}_{\mu'\nu'}\mathbf{F}_{\mu'\nu'}] = \mathbf{T}_{\nu\mu}$$

$$\mathbf{T}_{\mathrm{ij}} = \frac{1}{\mu_0}(-\mathbf{B}_\mathrm{i}\mathbf{B}_\mathrm{j} + \frac{1}{2}\delta_{\mathrm{ij}}\mathbf{B}^2) + \epsilon_0(-\mathbf{E}_\mathrm{i}\mathbf{E}_\mathrm{j} + \frac{1}{2}\delta_{\mathrm{ij}}\mathbf{E}^2) = \mathcal{J}_{\mathrm{ij}}$$

$$\mathbf{T}_{44} = -\frac{1}{2\mu_0}\mathbf{B}^2 - \frac{\epsilon_0}{2}\mathbf{E}^2 = -\mathbf{W} \qquad \mathbf{T}_{\mathrm{i}4} = \mathbf{T}_{4\mathrm{i}} = \frac{\mathrm{i}}{\mu_0 \mathrm{c}}(\vec{\mathbf{E}}\times\vec{\mathbf{B}})_\mathrm{i} = \frac{\mathrm{i}}{\mathrm{c}}\mathbf{S}_\mathrm{i} = \mathrm{i}\mathbf{c}\mathbf{g}_\mathrm{i}$$

$$(\rho\vec{E}+\vec{j}\times\vec{B},\ \frac{i}{c}\vec{j}\cdot\vec{E}) = f_\mu = -\sum_{\nu=1}^{4}\partial_\nu T_{\mu\nu}$$

$$T_{\mu\nu} \equiv \frac{1}{\mu_0}[\sum_{\nu'=1}^{4} F_{\mu\nu'}F_{\nu\nu'} - \frac{1}{4}\delta_{\mu\nu}\sum_{\mu',\nu'=1}^{4} F_{\mu'\nu'}F_{\mu'\nu'}] = T_{\nu\mu}$$

$$T_{ij} = \frac{1}{\mu_0}(-B_iB_j + \frac{1}{2}\delta_{ij}B^2) + \epsilon_0(-E_iE_j + \frac{1}{2}\delta_{ij}E^2) = \mathcal{J}_{ij}$$

$$T_{44} = -\frac{1}{2\mu_0}B^2 - \frac{\epsilon_0}{2}E^2 = -W \qquad T_{i4} = T_{4i} = \frac{i}{\mu_0 c}(\vec{E}\times\vec{B})_i = \frac{i}{c}S_i = icg_i$$

$T_{\mu\nu}$ 是无迹张量

$$f_i = -\sum_{\nu=1}^{4}\partial_\nu T_{i\nu} = -\sum_{j=1}^{3}\partial_j T_{ij} - \frac{\partial T_{i4}}{ic\partial t} \qquad \Rightarrow \qquad \vec{f} = -\nabla\cdot\overleftrightarrow{\mathcal{J}} - \frac{\partial\vec{g}}{\partial t}$$

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$$f_4 = -\sum_{\nu=1}^{4}\partial_\nu T_{4\nu} = -\sum_{i=1}^{3}\partial_i T_{4i} - \frac{\partial T_{44}}{ic\partial t} \quad \Rightarrow \quad \vec{f}\cdot\vec{v} = -\nabla\cdot\vec{S} - \frac{\partial W}{\partial t}$$

Back
Close

$$\left(\rho\vec{E}+\vec{j}\times\vec{B},\ \frac{i}{c}\vec{j}\cdot\vec{E}\right)=f_\mu=-\sum_{\nu=1}^{4}\partial_\nu T_{\mu\nu}$$

$$T_{\mu\nu}\equiv\frac{1}{\mu_0}\Big[\sum_{\nu'=1}^{4}F_{\mu\nu'}F_{\nu\nu'}-\frac{1}{4}\delta_{\mu\nu}\sum_{\mu',\nu'=1}^{4}F_{\mu'\nu'}F_{\mu'\nu'}\Big]=T_{\nu\mu}$$

$$T_{ij}=\frac{1}{\mu_0}\left(-B_iB_j+\frac{1}{2}\delta_{ij}B^2\right)+\epsilon_0\left(-E_iE_j+\frac{1}{2}\delta_{ij}E^2\right)=\mathcal{J}_{ij}$$

$$T_{44}=-\frac{1}{2\mu_0}B^2-\frac{\epsilon_0}{2}E^2=-W\qquad T_{i4}=T_{4i}=\frac{i}{\mu_0 c}(\vec{E}\times\vec{B})_i=\frac{i}{c}S_i=icg_i$$

$T_{\mu\nu}$ 是无迹张量

$$f_i=-\sum_{\nu=1}^{4}\partial_\nu T_{i\nu}=-\sum_{j=1}^{3}\partial_j T_{ij}-\frac{\partial T_{i4}}{ic\partial t}\qquad\Rightarrow\qquad\vec{f}=-\nabla\cdot\overleftrightarrow{\mathcal{J}}-\frac{\partial\vec{g}}{\partial t}$$

$$f_4=-\sum_{\nu=1}^{4}\partial_\nu T_{4\nu}=-\sum_{i=1}^{3}\partial_i T_{4i}-\frac{\partial T_{44}}{ic\partial t}\qquad\Rightarrow\qquad\vec{f}\cdot\vec{v}=-\nabla\cdot\vec{S}-\frac{\partial W}{\partial t}$$

$$\vec{f}\cdot\vec{v}=\rho\vec{v}\cdot\vec{E}=\vec{j}\cdot\vec{E}$$

四度电磁能动量P_μ是能动量张量$T_{\mu\nu}$的分量:

$$\mathbf{P}_\mu(\mathbf{t}) = -\frac{\mathbf{i}}{\mathbf{c}}\int_{\text{全空间}} \mathbf{d}\vec{\mathbf{r}}\ \mathbf{T}_{4\mu}(\vec{\mathbf{r}},\mathbf{t})$$

它的变换性质似乎不是四矢量类型的!

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它的变换性质似乎不是四矢量类型的！

$$\mathbf{P}'_\mu(\mathbf{t}') = -\frac{\mathbf{i}}{\mathbf{c}}\int_{\text{全空间}} \mathbf{d}\vec{\mathbf{r}}'\ \mathbf{T}'_{4\mu}(\vec{\mathbf{r}}',\mathbf{t}') = -\frac{\mathbf{i}}{\mathbf{c}}\sum_{\lambda\nu}\mathbf{a}_{4\lambda}\mathbf{a}_{\mu\nu}\int_{\text{全空间}} \mathbf{d}\vec{\mathbf{r}}'\ \mathbf{T}_{\lambda\nu}(\vec{\mathbf{r}},\mathbf{t})$$

$$\overset{?}{=} \sum_\nu \mathbf{a}_{\mu\nu}\mathbf{P}_\nu(\mathbf{t}) = -\frac{\mathbf{i}}{\mathbf{c}}\sum_\nu \mathbf{a}_{\mu\nu}\int_{\text{全空间}} \mathbf{d}\vec{\mathbf{r}}\ \mathbf{T}_{4\nu}(\vec{\mathbf{r}},\mathbf{t})$$

$$\sum_\lambda \mathbf{a}_{4\lambda}\int_{\text{全空间}} \mathbf{d}\vec{\mathbf{r}}'\ \mathbf{T}_{\lambda\nu}(\vec{\mathbf{r}},\mathbf{t}) \overset{?}{=} \int_{\text{全空间}} \mathbf{d}\vec{\mathbf{r}}\ \mathbf{T}_{4\nu}(\vec{\mathbf{r}},\mathbf{t})$$

对在S系静止的一团相对距离维持固定的电荷系统的四度能动量，$\mathbf{T}_{4\mathbf{i}} = \mathbf{T}_{\mathbf{i}4} = 0$

$$\mathbf{P}_\mu = -\frac{\mathbf{i}}{\mathbf{c}}\delta_{\mu 4}\int_{\text{全空间}} \mathbf{d}\vec{\mathbf{r}}\ \mathbf{T}_{44}(\vec{\mathbf{r}}) \equiv \mathbf{i}\mathbf{m}_0\mathbf{c}\delta_{\mu 4}$$

在静止系无磁场因而无电磁动量,且不含t

$$\mathbf{P}'_\mu(\mathbf{t}') = -\frac{\mathbf{i}}{\mathbf{c}}\mathbf{a}_{44}\mathbf{a}_{\mu 4}\int_{\text{全空间}} \mathbf{d}\vec{\mathbf{r}}'\ \mathbf{T}_{44}(\vec{\mathbf{r}}) - \frac{\mathbf{i}}{\mathbf{c}}\sum_{\mathbf{ij}}\mathbf{a}_{4\mathbf{i}}\mathbf{a}_{\mu\mathbf{j}}\int_{\text{全空间}} \mathbf{d}\vec{\mathbf{r}}'\ \mathbf{T}_{\mathbf{ij}}(\vec{\mathbf{r}})$$

假设S'系相对S系以速度$\vec{v}$运动，$\gamma \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = a_{44}$

$$
\begin{aligned}
P'_\mu(t') = & -\frac{i}{c} a_{44} a_{\mu 4} \int_{\text{全空间}} d\vec{r}' \; T_{44}(\gamma(x' + vt'), y', z') \\
& -\frac{i}{c} \sum_{ij} a_{4i} a_{\mu j} \int_{\text{全空间}} d\vec{r}' \; T_{ij}(\gamma(x' + vt'), y', z')
\end{aligned}
$$

假设S'系相对S系以速度$\vec{v}$运动，$\gamma \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = a_{44}$

$$
\begin{aligned}
P'_\mu(t') &= -\frac{i}{c}a_{44}a_{\mu 4}\int_{\text{全空间}} d\vec{r}'\ T_{44}(\gamma(x'+vt'), y', z') \\
&\quad -\frac{i}{c}\sum_{ij} a_{4i}a_{\mu j}\int_{\text{全空间}} d\vec{r}'\ T_{ij}(\gamma(x'+vt'), y', z') \\
&= -\frac{i}{c}\frac{a_{44}a_{\mu 4}}{\gamma}\int_{\text{全空间}} d\vec{r}\ T_{44}(\vec{r}) - \frac{i}{c}\sum_{ij}\frac{a_{4i}a_{\mu j}}{\gamma}\int_{\text{全空间}} d\vec{r}\ T_{ij}(\vec{r})
\end{aligned}
$$

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&\quad -\frac{i}{c} \sum_{ij} a_{4i} a_{\mu j} \int_{\text{全空间}} d\vec{r}' \; T_{ij}(\gamma(x'+vt'), y', z') \\
&= -\frac{i}{c} \frac{a_{44} a_{\mu 4}}{\gamma} \int_{\text{全空间}} d\vec{r} \; T_{44}(\vec{r}) - \frac{i}{c} \sum_{ij} \frac{a_{4i} a_{\mu j}}{\gamma} \int_{\text{全空间}} d\vec{r} \; T_{ij}(\vec{r}) \\
&= a_{\mu 4} i m_0 c - \frac{i}{c\gamma} \sum_{ij} a_{4i} a_{\mu j} \int_{\text{全空间}} d\vec{r} \; T_{ij}(\vec{r})
\end{aligned}
$$

假设S'系相对S系以速度$\vec{v}$运动，$\gamma \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = a_{44}$

$$
\begin{aligned}
P'_\mu(t') = & -\frac{i}{c} a_{44} a_{\mu 4} \int_{\text{全空间}} d\vec{r}' \; T_{44}(\gamma(x' + vt'), y', z') \\
& -\frac{i}{c} \sum_{ij} a_{4i} a_{\mu j} \int_{\text{全空间}} d\vec{r}' \; T_{ij}(\gamma(x' + vt'), y', z') \\
= & -\frac{i}{c} \frac{a_{44} a_{\mu 4}}{\gamma} \int_{\text{全空间}} d\vec{r} \; T_{44}(\vec{r}) - \frac{i}{c} \sum_{ij} \frac{a_{4i} a_{\mu j}}{\gamma} \int_{\text{全空间}} d\vec{r} \; T_{ij}(\vec{r}) \\
= & \; a_{\mu 4} i m_0 c - \frac{i}{c\gamma} \sum_{ij} a_{4i} a_{\mu j} \int_{\text{全空间}} d\vec{r} \; T_{ij}(\vec{r})
\end{aligned}
$$

- 上式第二项贡献的是能动量偏离四矢量洛伦兹变换的行为
- 对在S系静止的一团相对距离维持固定的电荷系统，第二项一般不为零

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$$
\begin{aligned}
P'_\mu(t') &= -\frac{i}{c} a_{44} a_{\mu 4} \int_{\text{全空间}} d\vec{r}'\ T_{44}(\gamma(x'+vt'), y', z') \\
&\quad -\frac{i}{c} \sum_{ij} a_{4i} a_{\mu j} \int_{\text{全空间}} d\vec{r}'\ T_{ij}(\gamma(x'+vt'), y', z') \\
&= -\frac{i}{c} \frac{a_{44} a_{\mu 4}}{\gamma} \int_{\text{全空间}} d\vec{r}\ T_{44}(\vec{r}) - \frac{i}{c} \sum_{ij} \frac{a_{4i} a_{\mu j}}{\gamma} \int_{\text{全空间}} d\vec{r}\ T_{ij}(\vec{r}) \\
&= a_{\mu 4} i m_0 c - \frac{i}{c\gamma} \sum_{ij} a_{4i} a_{\mu j} \int_{\text{全空间}} d\vec{r}\ T_{ij}(\vec{r})
\end{aligned}
$$

- 上式第二项贡献的是能动量偏离四矢量洛伦兹变换的行为
- 对在S系静止的一团相对距离维持固定的电荷系统，第二项一般不为零
- 但这样的系统仅靠电磁力是维持不住的，需要外力！
- 如果考虑维持这个系统的外力–彭加莱应力：$T_{\mu\nu}^{\text{poincare}}$
- 它应被考虑进前面能动量的计算中

$$\sum_\mu \partial_\mu (T_{\mu\nu} + T_{\mu\nu}^{\text{poincare}}) = 0$$

它意味对体系每个时空点的合力为零！

假设S'系相对S系以速度$\vec{v}$运动， $\gamma \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = a_{44}$

$$P'_\mu(t') = a_{\mu 4} i m_0 c - \frac{i}{c\gamma}\sum_{ij} a_{4i} a_{\mu j} \int_{全空间} d\vec{r}\ \tilde{T}_{ij}(\vec{r})$$

$$\sum_\mu \partial_\mu \tilde{T}_{\mu\nu} = 0 \qquad \tilde{T}_{\mu\nu} \equiv T_{\mu\nu} + T^{poincare}_{\mu\nu}$$

假设S'系相对S系以速度$\vec{v}$运动，$\gamma \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = a_{44}$

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$$\sum_\mu \partial_\mu \tilde{T}_{\mu\nu} = 0 \qquad \tilde{T}_{\mu\nu} \equiv T_{\mu\nu} + T_{\mu\nu}^{\text{poincare}}$$

在S系$T_{\mu\nu}$不显含时间；维持其稳定的$T_{\mu\nu}^{\text{poincare}}$也会不显含时间，导致$\tilde{T}_{\mu\nu}$不显含时间

在S系，$\tilde{T}_{\mu\nu}$不显含时间 $\Rightarrow\ \partial_0 \tilde{T}_{\mu\nu} = 0 \ \Rightarrow\ \sum_\mu \partial_k \tilde{T}_{k\nu} = 0$

$$\int_{\text{全空间}} d\vec{r}\ \tilde{T}_{ij}(\vec{r}) = \sum_k \int_{\text{全空间}} d\vec{r}\ \partial_k [x_i \tilde{T}_{kj}(\vec{r})] = \sum_k \int_{\text{全空间}} d\vec{S}_k x_i \tilde{T}_{kj}(\vec{r}) = 0$$

加入彭加莱应力的一团相对距离维持固定的电荷系统满足：$P'_\mu(t') = a_{\mu 4} i m_0 c$

$$E = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \vec{P} = \frac{m_0 \vec{v}}{\sqrt{1-\frac{v^2}{c^2}}} \qquad i m_0 c = -\frac{i}{c}\int_{\text{全空间}} d\vec{r}\ \tilde{T}_{44}(\vec{r})$$

假设S'系相对S系以速度$\vec{v}$运动，$\gamma \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = a_{44}$

$$P'_\mu(t') = a_{\mu 4} i m_0 c - \frac{i}{c\gamma}\sum_{ij} a_{4i} a_{\mu j} \int_{全空间} d\vec{r}\ \tilde{T}_{ij}(\vec{r})$$

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另类总能动量的定义朗道：$P_\mu(t) \overset{四度超曲面上x_4值相同}{=\!=\!=\!=\!=\!=} \sum_\nu \int_{全空间} dS_\nu\ T_{\nu\mu}(\vec{r}, t)$

三. 经典电磁学理论

的另类理解及扩展

如何理解麦克斯韦方程组和洛伦兹力的来源？ 实验定律；

如何理解麦克斯韦方程组和洛伦兹力的来源？ 实验定律； 拉格朗日量；

如何理解麦克斯韦方程组和洛伦兹力的来源？ 实验定律； 拉格朗日量；

$$\sum_{\mu=1}^{4}\partial_\mu \mathbf{F}_{\mu\nu} = -\mu_0 \mathbf{j}_\nu \qquad \partial_\mu \mathbf{F}_{\nu\lambda} + \partial_\nu \mathbf{F}_{\lambda\mu} + \partial_\lambda \mathbf{F}_{\mu\nu} = \mathbf{0}$$

如何理解麦克斯韦方程组和洛伦兹力的来源？ 实验定律； 拉格朗日量；

$$\sum_{\mu=1}^{4}\partial_\mu \mathbf{F}_{\mu\nu} = -\mu_0 \mathbf{j}_\nu \qquad \partial_\mu \mathbf{F}_{\nu\lambda} + \partial_\nu \mathbf{F}_{\lambda\mu} + \partial_\lambda \mathbf{F}_{\mu\nu} = \mathbf{0}$$

- 只要定义依赖时空的矢量势 $\mathbf{A}_\mu$ 就可定义场强 $\mathbf{F}_{\mu\nu} = \partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu$ 势比场强更基本

如何理解麦克斯韦方程组和洛伦兹力的来源？ 实验定律； 拉格朗日量；

$$\sum_{\mu=1}^{4}\partial_\mu \mathbf{F}_{\mu\nu} = -\mu_0 \mathbf{j}_\nu \qquad \partial_\mu \mathbf{F}_{\nu\lambda} + \partial_\nu \mathbf{F}_{\lambda\mu} + \partial_\lambda \mathbf{F}_{\mu\nu} = \mathbf{0}$$

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- 场强的结构使方程 $\underbrace{\partial_\mu \mathbf{F}_{\nu\lambda} + \partial_\nu \mathbf{F}_{\lambda\mu} + \partial_\lambda \mathbf{F}_{\mu\nu} = \mathbf{0}}_{\text{无源的麦克斯韦方程组}}$ 自然成立

如何理解麦克斯韦方程组和洛伦兹力的来源？ 实验定律； 拉格朗日量；

$$\sum_{\mu=1}^{4}\partial_\mu \mathbf{F}_{\mu\nu} = -\mu_0 \mathbf{j}_\nu \qquad \partial_\mu \mathbf{F}_{\nu\lambda} + \partial_\nu \mathbf{F}_{\lambda\mu} + \partial_\lambda \mathbf{F}_{\mu\nu} = \mathbf{0}$$

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- 场强的结构还使方程 $\sum_{\mu,\nu}\partial_\mu\partial_\nu \mathbf{F}_{\mu\nu} = \mathbf{0}$ 自然成立

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- 场强的结构还使方程 $\sum_{\mu,\nu}\partial_\mu\partial_\nu \mathbf{F}_{\mu\nu} = \mathbf{0}$ 自然成立
- 它意味着可定义流 $\underbrace{\mathbf{j}_\nu \equiv -\frac{1}{\mu_0}\sum_\mu \partial_\mu \mathbf{F}_{\mu\nu}}_{\text{有源的麦克斯韦方程组}}$，满足守恒方程 $\sum_\nu \partial_\nu \mathbf{j}_\nu = \mathbf{0}$

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$$\sum_{\mu=1}^{4}\partial_\mu \mathbf{F}_{\mu\nu} = -\mu_0 \mathbf{j}_\nu \qquad \partial_\mu \mathbf{F}_{\nu\lambda} + \partial_\nu \mathbf{F}_{\lambda\mu} + \partial_\lambda \mathbf{F}_{\mu\nu} = \mathbf{0}$$

- 只要定义依赖时空的矢量势 $\mathbf{A}_\mu$ 就可定义场强 $\mathbf{F}_{\mu\nu}=\partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu$ 势比场强更基本
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- 从 $\sum_\nu \mathbf{F}_{\mu\nu}\mathbf{j}_\nu = -\sum_\nu \partial_\nu \mathbf{T}_{\mu\nu}$ 可认定四度力： $\mathbf{f}_\mu = \sum_\nu \mathbf{F}_{\mu\nu}\mathbf{j}_\nu$ 洛伦兹力公式！

- 你对上述对经典电动力学的“理解”如何评价？

  它靠谱吗？

- 特别对四度电流密度、四度力密度的定义方式如何评价？

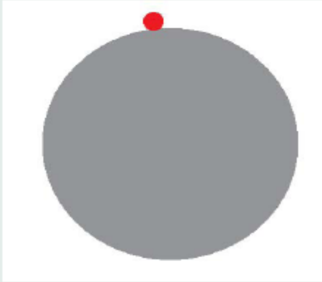
  它会有什么引申的讨论吗？

- 你对四度电磁势在经典电动力学中的地位如何看？

  规范对称性的作用又如何？

杨振宁在研读法拉第撰写的《电学的实验研究》后特别提到：
法拉第书中的电紧张状态频繁出现在书的各处，并且又被频繁赋予各种其他的名字，诸如特殊态、强度态、特殊状态等，但从始至终未给出清晰的定义。

磁单极与最小作用量的“逆问题”及磁荷量子化：



- 从作用量求极值出发，可以得到 运动方程！
- 任一决定运动规律的 方程，都存在相应的作用量？
- 当初费曼发明“路径积分”就因有些系统哈密顿体系无法描写
- 如果可以则意味最小作用量可完全替代由微分方程决定的运动规律

自由质点被约束在单位球面上运动：$S = \int dt\, \frac{1}{2} m v^2$ $\qquad \vec{r}\cdot\vec{r} = 1$

$$S' = \int dt\, [\frac{1}{2} m v^2 + \lambda(r^2 - 1)] \Rightarrow m\ddot{\vec{r}} - 2\lambda\vec{r} = 0 \quad \lambda = -\frac{m}{2} v^2 \Leftarrow \dot{\vec{r}}\cdot\dot{\vec{r}} + \vec{r}\cdot\ddot{\vec{r}} = 0 \xLeftarrow{\text{约束条件微商两次}}$$

$$m[\ddot{\vec{r}} + \vec{r}(\dot{\vec{r}}\cdot\dot{\vec{r}})] = 0 \qquad r^2 = 1$$

现假设此质点带电电量q；并且在球心放置一个单位磁单极：$\vec{B} = \frac{\vec{r}}{r^3}$

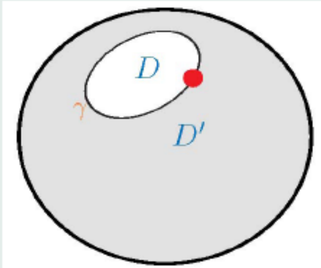
场方程中应出现洛伦兹力项：$m(\ddot{\vec{r}} + \vec{r}\dot{\vec{r}}\cdot\dot{\vec{r}}) = q\vec{v}\times\vec{B} = q\dot{\vec{r}}\times\vec{r}$

$$m[\ddot{\vec{r}} + \vec{r}(\dot{\vec{r}}\cdot\dot{\vec{r}})] + q\vec{r}\times\dot{\vec{r}} = 0 \qquad r^2 = 1$$

洛伦兹力项如何在作用量中体现？$\vec{r}\cdot(\vec{r}\times\dot{\vec{r}}) = 0$ 例：tensionless string

$$S=\int dt\,[\frac{1}{2}mv^2+\lambda(r^2-1)+q\dot{\vec{r}}\cdot\vec{A}]$$

考虑带电粒子走闭合路径γ： $\vec{r}(t=\infty)=\vec{r}(t=0)$ $\vec{v}=\dot{\vec{r}}$



可以得到洛伦兹力，但矢量势在有磁单极时有奇异$\nabla\cdot(\nabla\times\vec{A})\neq 0$

路径在其球面上所包围的面积可用格林公式：

$$\int dt\,q\dot{\vec{r}}\cdot\vec{A}=q\oint_\gamma d\vec{r}\cdot\vec{A}=q\int_D d\vec{S}\cdot(\nabla\times\vec{A})=q\int_D d\vec{S}\cdot\vec{B}$$

D： γ所包围的面积

含磁单极的磁场作用量在带电粒子运动的一维曲线上无法很好定义；

但在以一维曲线为边界的 二维 球面上确可很好定义！ 全息原理？ 但故事没完...

面积D有两种取法D和D’： $\oint_{D+D'}\vec{B}\cdot d\vec{S}=-\oint_{D+D'}d\vec{S}\cdot\nabla\frac{1}{r}=-\int dV\nabla^2\frac{1}{r}=4\pi$

若要求经典自洽： $\int dt\,q\dot{\vec{r}}\cdot\vec{A}=q\int_D d\vec{S}\cdot\vec{B}=-q\int_{D'}d\vec{S}\cdot\vec{B}+4\pi q\Leftarrow$ 不能为零！

$$e^{\frac{iq}{\hbar}\int_D d\vec{S}\cdot\vec{B}}\overset{\text{路径积分}}{=\!=\!=}e^{-\frac{iq}{\hbar}\int_{D'}d\vec{S}\cdot\vec{B}}\Rightarrow 1=e^{\frac{iq}{\hbar}\oint_{D+D'}d\vec{S}\cdot\vec{B}}\overset{\text{单位磁荷磁通量}}{=\!=\!=}e^{4\pi\frac{iq}{\hbar}}\Rightarrow q=\frac{n}{2}\hbar$$ Dirac磁荷量子化

若要求量子自洽⇑ $$S=\int dt\,[\frac{1}{2}mv^2+\lambda(r^2-1)]+q\int_D d\vec{S}\cdot\vec{B}$$

最小作用量原理

- 最小作用量中的“最小”的来源及含义到底是什么？

- 最小作用量的逆问题是一个真命题，还是一个伪命题？

- 如何理解在原空间不存在的作用量，扩充了空间就存在这个事实？

- 是否可能存在即使在扩充了的空间也不存在的作用量？如何理解？

电磁场的矢势和标势: 用势描述电磁场; 规范变换和规范不变性

$$\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0} \qquad \nabla \cdot \vec{\mathbf{B}} = 0 \qquad \nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}} \qquad \nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}}$$

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$$\nabla\times\vec{\mathbf{E}}=-\frac{\partial}{\partial\mathbf{t}}\nabla\times\vec{\mathbf{A}}$$

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$$\nabla \times \vec{\mathbf{E}} = -\frac{\partial}{\partial \mathbf{t}} \nabla \times \vec{\mathbf{A}} = -\nabla \times \frac{\partial \vec{\mathbf{A}}}{\partial \mathbf{t}}$$

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电磁场的矢势和标势: 用势描述电磁场; 规范变换和规范不变性

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$$\nabla\times\vec{\mathbf{E}}=-\frac{\partial}{\partial \mathbf{t}}\nabla\times\vec{\mathbf{A}}=-\nabla\times\frac{\partial\vec{\mathbf{A}}}{\partial \mathbf{t}}\qquad\rightarrow\qquad \nabla\times(\vec{\mathbf{E}}+\frac{\partial\vec{\mathbf{A}}}{\partial \mathbf{t}})=\mathbf{0}$$

$$\vec{\mathbf{E}}+\frac{\partial\vec{\mathbf{A}}}{\partial \mathbf{t}}=-\nabla\phi$$

电磁场的矢势和标势: 用势描述电磁场; 规范变换和规范不变性

$$\nabla\cdot\vec{\mathbf{E}}=\frac{\rho}{\epsilon_0}\qquad \nabla\cdot\vec{\mathbf{B}}=\mathbf{0}\qquad \nabla\times\vec{\mathbf{E}}=-\frac{\partial\vec{\mathbf{B}}}{\partial\mathbf{t}}\qquad \nabla\times\vec{\mathbf{B}}=\mu_0\vec{\mathbf{j}}+\mu_0\epsilon_0\frac{\partial\vec{\mathbf{E}}}{\partial\mathbf{t}}$$

$$\nabla\times\vec{\mathbf{E}}=-\frac{\partial}{\partial\mathbf{t}}\nabla\times\vec{\mathbf{A}}=-\nabla\times\frac{\partial\vec{\mathbf{A}}}{\partial\mathbf{t}}\qquad\rightarrow\qquad\nabla\times(\vec{\mathbf{E}}+\frac{\partial\vec{\mathbf{A}}}{\partial\mathbf{t}})=\mathbf{0}$$

$$\vec{\mathbf{E}}+\frac{\partial\vec{\mathbf{A}}}{\partial\mathbf{t}}=-\nabla\phi\qquad\rightarrow\qquad\vec{\mathbf{E}}=-\nabla\phi-\frac{\partial\vec{\mathbf{A}}}{\partial\mathbf{t}}$$

电磁场的矢势和标势： 用势描述电磁场; 规范变换和规范不变性

$$\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0} \qquad \nabla \cdot \vec{\mathbf{B}} = \mathbf{0} \qquad \nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial \mathbf{t}} \qquad \nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}}$$

$$\nabla \times \vec{\mathbf{E}} = -\frac{\partial}{\partial \mathbf{t}} \nabla \times \vec{\mathbf{A}} = -\nabla \times \frac{\partial \vec{\mathbf{A}}}{\partial \mathbf{t}} \quad \rightarrow \quad \nabla \times \left(\vec{\mathbf{E}} + \frac{\partial \vec{\mathbf{A}}}{\partial \mathbf{t}}\right) = \mathbf{0}$$

$$\vec{\mathbf{E}} + \frac{\partial \vec{\mathbf{A}}}{\partial \mathbf{t}} = -\nabla \phi \quad \rightarrow \quad \vec{\mathbf{E}} = -\nabla \phi - \frac{\partial \vec{\mathbf{A}}}{\partial \mathbf{t}}$$

$$\begin{cases} \vec{\mathbf{E}} = -\nabla \phi - \frac{\partial \vec{\mathbf{A}}}{\partial \mathbf{t}} \\ \vec{\mathbf{B}} = \nabla \times \vec{\mathbf{A}} \end{cases}$$

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$\vec{\mathbf{A}},\ \phi$不唯一

Back

Close

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$$\nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Back
Close

电磁场的矢势和标势： 达朗伯(d'Alembert)方程

$$\nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial \mathbf{t}} \;\rightarrow\; \nabla \times (\nabla \times \vec{\mathbf{A}}) = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \left[-\nabla \frac{\partial \phi}{\partial \mathbf{t}} - \frac{\partial^2 \vec{\mathbf{A}}}{\partial \mathbf{t}^2}\right]$$

$$= \nabla(\nabla \cdot \vec{\mathbf{A}}) - \nabla^2 \vec{\mathbf{A}}$$

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Back
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库伦规范: $\nabla\cdot\vec{\mathbf{A}}=\mathbf{0}$

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$$\nabla^2\vec{A}-\frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2}=-\mu_0\vec{j}+\frac{1}{c^2}\frac{\partial}{\partial t}\nabla\phi=-\mu_0\vec{j}^*\qquad\nabla^2\phi=-\frac{\rho}{\epsilon_0}$$

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洛伦兹规范: $\nabla\cdot\vec{A}+\frac{1}{c^2}\frac{\partial\phi}{\partial t}=0$ 实际上是 $\nabla\cdot\vec{A}_L+\frac{1}{c^2}\frac{\partial\phi}{\partial t}=0$

势的时间分量的时间变化与空间分量的纵向相互抵消!

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库伦规范： $\nabla\cdot\vec{A}=0$

$$\nabla^2\vec{A}-\frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2}=-\mu_0\vec{j}+\frac{1}{c^2}\frac{\partial}{\partial t}\nabla\phi=-\mu_0\vec{j}^*\qquad\nabla^2\phi=-\frac{\rho}{\epsilon_0}$$

洛伦兹规范： $\nabla\cdot\vec{A}+\frac{1}{c^2}\frac{\partial\phi}{\partial t}=0$ 实际上是 $\nabla\cdot\vec{A}_L+\frac{1}{c^2}\frac{\partial\phi}{\partial t}=0$

势的时间分量的时间变化与空间分量的纵向相互抵消！

$$\nabla^2\vec{A}-\frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2}=-\mu_0\vec{j}\qquad\nabla^2\phi-\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}=-\frac{\rho}{\epsilon_0}$$

电磁场的矢势和标势：达朗伯(d'Alembert)方程

$$\nabla\times\vec{\mathbf{B}}=\mu_0\vec{\mathbf{j}}+\mu_0\epsilon_0\frac{\partial\vec{\mathbf{E}}}{\partial \mathbf{t}} \quad\rightarrow\quad \nabla\times(\nabla\times\vec{\mathbf{A}})=\mu_0\vec{\mathbf{j}}+\mu_0\epsilon_0[-\nabla\frac{\partial\phi}{\partial \mathbf{t}}-\frac{\partial^2\vec{\mathbf{A}}}{\partial \mathbf{t}^2}]$$

$$=\nabla(\nabla\cdot\vec{\mathbf{A}})-\nabla^2\vec{\mathbf{A}}$$

$$\nabla\cdot\vec{\mathbf{E}}=\frac{\rho}{\epsilon_0} \quad\rightarrow\quad -\nabla^2\phi-\frac{\partial}{\partial \mathbf{t}}\nabla\cdot\vec{\mathbf{A}}=\frac{\rho}{\epsilon_0}$$

库伦规范： $\nabla\cdot\vec{\mathbf{A}}=\mathbf{0}$

$$\nabla^2\vec{\mathbf{A}}-\frac{\mathbf{1}}{\mathbf{c}^2}\frac{\partial^2\vec{\mathbf{A}}}{\partial \mathbf{t}^2}=-\mu_0\vec{\mathbf{j}}+\frac{\mathbf{1}}{\mathbf{c}^2}\frac{\partial}{\partial \mathbf{t}}\nabla\phi=-\mu_0\vec{\mathbf{j}}^* \qquad \nabla^2\phi=-\frac{\rho}{\epsilon_0}$$

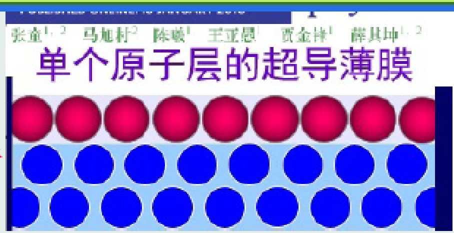
洛伦兹规范： $\nabla\cdot\vec{\mathbf{A}}+\frac{\mathbf{1}}{\mathbf{c}^2}\frac{\partial\phi}{\partial \mathbf{t}}=\mathbf{0}$ 实际上是 $\nabla\cdot\vec{\mathbf{A}}_{\mathbf{L}}+\frac{\mathbf{1}}{\mathbf{c}^2}\frac{\partial\phi}{\partial \mathbf{t}}=\mathbf{0}$

势的时间分量的时间变化与空间分量的纵向相互抵消！

$$\nabla^2\vec{\mathbf{A}}-\frac{\mathbf{1}}{\mathbf{c}^2}\frac{\partial^2\vec{\mathbf{A}}}{\partial \mathbf{t}^2}=-\mu_0\vec{\mathbf{j}} \qquad \nabla^2\phi-\frac{\mathbf{1}}{\mathbf{c}^2}\frac{\partial^2\phi}{\partial \mathbf{t}^2}=-\frac{\rho}{\epsilon_0}$$

$\mathbf{c}\rightarrow\infty$ 时,回到静电磁情形

- 超导无电阻、迈斯纳效应 是物理学尚未了解清楚的介质电磁性质



- 还是 已为大众熟知，开始影响人们生活 现代科技电磁学常讲的例子
- 超导的标准理解需要 量子力学 及很多 凝聚态物理 的知识 各种奇怪的的假设和规则
- 早年超导的研究不仅对固体物理，也影响了粒子物理核物理08年诺贝尔物理奖

南部阳一郎

- 目前它仍是凝聚态最前沿和活跃的研究方向！ 新的诺贝尔奖？ 期待再次影响粒子物理？
- 我们期待着高温或室温超导未来可能对人类生活造成的重大影响
- 问题： 电动力学能否不过多地依赖量子力学和凝聚态物理来介绍超导？ 一般不行
- 若可实现，则说明电磁场及麦克斯韦方程在超导现象中的核心支配作用图像更清晰
- 还说明涉及超导的所谓量子力学及凝聚态物理细节很多可有效地用 经典电动力学 描写
- 核心要说明从经典电动力学的水平上看是介质的什么物理机制及图像导致了超导！

$\vec{D} = \epsilon \vec{E}$, $\vec{B} = \mu \vec{H}$, $\vec{j} = \gamma \vec{E}$

- 以下依据电动力学进行详细推导、演绎和诠释超导的另类推导：理论物理但非凝聚态人的观点

Back
Close

真空中的场方程：

$$\mathbf{c}^2 = \frac{1}{\mu_0\epsilon_0}$$

$$[\nabla^2 - \frac{1}{\mathbf{c}^2}\frac{\partial^2}{\partial \mathbf{t}^2}]\vec{\mathbf{A}} - \nabla[\nabla\cdot\vec{\mathbf{A}} + \frac{1}{\mathbf{c}^2}\frac{\partial\phi}{\partial \mathbf{t}}] = -\mu_0\vec{\mathbf{j}}$$

$$[\nabla^2 - \frac{1}{\mathbf{c}^2}\frac{\partial^2}{\partial \mathbf{t}^2}]\phi + \frac{\partial}{\partial \mathbf{t}}[\nabla\cdot\vec{\mathbf{A}} + \frac{1}{\mathbf{c}^2}\frac{\partial\phi}{\partial \mathbf{t}}] = -\frac{\rho}{\epsilon_0}$$

它具有规范不变性： $\vec{\mathbf{A}}' = \vec{\mathbf{A}} + \nabla\chi$ $\quad \phi' = \phi - \frac{\partial\chi}{\partial \mathbf{t}}$

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$$\vec{\mathbf{A}}' = \vec{\mathbf{A}} + \nabla\chi \qquad \phi' = \phi - \frac{\partial\chi}{\partial \mathbf{t}}$$

若在某种导电介质中：

$$[\nabla^2 - \frac{1}{\mathbf{c}^2}\frac{\partial^2}{\partial \mathbf{t}^2} - \frac{\mathbf{m}^2_{光子}\mathbf{c}^2}{\hbar^2}]\vec{\mathbf{A}} - \nabla[\nabla\cdot\vec{\mathbf{A}} + \frac{1}{\mathbf{c}^2}\frac{\partial\phi}{\partial \mathbf{t}}] = \mathbf{0}$$

$-\mathbf{i}\hbar\nabla \Leftrightarrow \vec{\mathbf{p}} \qquad \mathbf{i}\hbar\frac{\partial}{\partial \mathbf{t}} \Leftrightarrow \mathbf{E}$

$$[\underbrace{\nabla^2 - \frac{1}{\mathbf{c}^2}\frac{\partial^2}{\partial \mathbf{t}^2}}_{-\frac{\vec{\mathbf{p}}\cdot\vec{\mathbf{p}}}{\hbar^2} \quad + \quad \frac{\mathbf{E}^2}{\mathbf{c}^2\hbar^2}} - \frac{\mathbf{m}^2_{光子}\mathbf{c}^2}{\hbar^2}]\phi + \frac{\partial}{\partial \mathbf{t}}[\nabla\cdot\vec{\mathbf{A}} + \frac{1}{\mathbf{c}^2}\frac{\partial\phi}{\partial \mathbf{t}}] = \mathbf{0}$$

真空中的场方程：

$$\mathbf{c}^2 = \frac{1}{\mu_0\epsilon_0}$$

$$\left[\nabla^2 - \frac{1}{\mathbf{c}^2}\frac{\partial^2}{\partial \mathbf{t}^2}\right]\vec{\mathbf{A}} - \nabla\left[\nabla\cdot\vec{\mathbf{A}} + \frac{1}{\mathbf{c}^2}\frac{\partial\phi}{\partial \mathbf{t}}\right] = -\mu_0\vec{\mathbf{j}}$$

$$\left[\nabla^2 - \frac{1}{\mathbf{c}^2}\frac{\partial^2}{\partial \mathbf{t}^2}\right]\phi + \frac{\partial}{\partial \mathbf{t}}\left[\nabla\cdot\vec{\mathbf{A}} + \frac{1}{\mathbf{c}^2}\frac{\partial\phi}{\partial \mathbf{t}}\right] = -\frac{\rho}{\epsilon_0}$$

它具有规范不变性： $\vec{\mathbf{A}}' = \vec{\mathbf{A}} + \nabla\chi \qquad \phi' = \phi - \frac{\partial\chi}{\partial \mathbf{t}}$

若在某种导电介质中：

$$\left[\nabla^2 - \frac{1}{\mathbf{c}^2}\frac{\partial^2}{\partial \mathbf{t}^2} - \frac{\mathbf{m}^2_{光子}\mathbf{c}^2}{\hbar^2}\right]\vec{\mathbf{A}} - \nabla\left[\nabla\cdot\vec{\mathbf{A}} + \frac{1}{\mathbf{c}^2}\frac{\partial\phi}{\partial \mathbf{t}}\right] = \mathbf{0}$$

$-\mathbf{i}\hbar\nabla \Leftrightarrow \vec{\mathbf{p}} \qquad \mathbf{i}\hbar\frac{\partial}{\partial \mathbf{t}} \Leftrightarrow \mathbf{E}$

$$\left[\underbrace{\nabla^2 - \frac{1}{\mathbf{c}^2}\frac{\partial^2}{\partial \mathbf{t}^2}}_{-\frac{\vec{\mathbf{p}}\cdot\vec{\mathbf{p}}}{\hbar^2} + \frac{\mathbf{E}^2}{\mathbf{c}^2\hbar^2}} - \frac{\mathbf{m}^2_{光子}\mathbf{c}^2}{\hbar^2}\right]\phi + \frac{\partial}{\partial \mathbf{t}}\left[\nabla\cdot\vec{\mathbf{A}} + \frac{1}{\mathbf{c}^2}\frac{\partial\phi}{\partial \mathbf{t}}\right] = \mathbf{0}$$

对比真空：介质中的电荷、电流完全由光子有效质量$\mathbf{m}_{光子}$ 产生！

$$\begin{cases} \vec{\mathbf{j}} = -\dfrac{\mathbf{m}^2_{光子}\mathbf{c}^2}{\mu_0\hbar^2}\vec{\mathbf{A}} \\[2ex] \rho = -\dfrac{\mathbf{m}^2_{光子}\epsilon_0\mathbf{c}^2}{\hbar^2}\phi \end{cases}$$

有效光子质量：伦敦方程，理想导体及迈斯纳效应

真空中的场方程：

$$\mathbf{c}^2 = \frac{1}{\mu_0\epsilon_0}$$

$$[\nabla^2 - \frac{1}{\mathbf{c}^2}\frac{\partial^2}{\partial \mathbf{t}^2}]\vec{\mathbf{A}} - \nabla[\nabla\cdot\vec{\mathbf{A}} + \frac{1}{\mathbf{c}^2}\frac{\partial\phi}{\partial \mathbf{t}}] = -\mu_0\vec{\mathbf{j}}$$

$$[\nabla^2 - \frac{1}{\mathbf{c}^2}\frac{\partial^2}{\partial \mathbf{t}^2}]\phi + \frac{\partial}{\partial \mathbf{t}}[\nabla\cdot\vec{\mathbf{A}} + \frac{1}{\mathbf{c}^2}\frac{\partial\phi}{\partial \mathbf{t}}] = -\frac{\rho}{\epsilon_0}$$

它具有规范不变性： $\vec{\mathbf{A}}' = \vec{\mathbf{A}} + \nabla\chi$ $\qquad \phi' = \phi - \frac{\partial\chi}{\partial \mathbf{t}}$

若在某种导电介质中：
$$[\nabla^2 - \frac{1}{\mathbf{c}^2}\frac{\partial^2}{\partial \mathbf{t}^2} - \frac{\mathbf{m}^2_{\text{光子}}\mathbf{c}^2}{\hbar^2}]\vec{\mathbf{A}} - \nabla[\nabla\cdot\vec{\mathbf{A}} + \frac{1}{\mathbf{c}^2}\frac{\partial\phi}{\partial \mathbf{t}}] = 0$$

$-i\hbar\nabla \Leftrightarrow \vec{\mathbf{p}}$ $\qquad i\hbar\frac{\partial}{\partial \mathbf{t}} \Leftrightarrow \mathbf{E}$

$$[\underbrace{\nabla^2 - \frac{1}{\mathbf{c}^2}\frac{\partial^2}{\partial \mathbf{t}^2}}_{-\frac{\vec{\mathbf{p}}\cdot\vec{\mathbf{p}}}{\hbar^2} + \frac{\mathbf{E}^2}{\mathbf{c}^2\hbar^2}} - \frac{\mathbf{m}^2_{\text{光子}}\mathbf{c}^2}{\hbar^2}]\phi + \frac{\partial}{\partial \mathbf{t}}[\nabla\cdot\vec{\mathbf{A}} + \frac{1}{\mathbf{c}^2}\frac{\partial\phi}{\partial \mathbf{t}}] = 0$$

对比真空：介质中的电荷、电流完全由光子有效质量$\mathbf{m}_{\text{光子}}$ 产生！
$$\begin{cases} \vec{\mathbf{j}} = -\frac{\mathbf{m}^2_{\text{光子}}\mathbf{c}^2}{\mu_0\hbar^2}\vec{\mathbf{A}} \\ \rho = -\frac{\mathbf{m}^2_{\text{光子}}\epsilon_0\mathbf{c}^2}{\hbar^2}\phi \end{cases}$$

这时 $\underbrace{\text{规范对称性是破缺的}}_{\text{以后详细讨论}}$ ，且 $\mathbf{0} = \nabla\cdot\vec{\mathbf{j}} + \frac{\partial\rho}{\partial \mathbf{t}} = -\frac{\mathbf{m}^2_{\text{光子}}\mathbf{c}^2}{\mu_0\hbar^2}[\underbrace{\nabla\cdot\vec{\mathbf{A}} + \mu_0\epsilon_0\frac{\partial\phi}{\partial \mathbf{t}}}_{\text{洛伦兹规范}}]$

有效光子质量：伦敦方程，理想导体及迈斯纳效应

$$[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{m^2_{光子}c^2}{\hbar^2}]\vec{A} = 0$$

$$[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{m^2_{光子}c^2}{\hbar^2}]\phi = 0$$

$$\nabla \cdot \vec{A} + \frac{1}{c^2}\frac{\partial \phi}{\partial t} = 0$$

$$\begin{cases} \vec{j} = -\frac{m^2_{光子}c^2}{\mu_0 \hbar^2}\vec{A} \\ \rho = -\frac{m^2_{光子}\epsilon_0 c^2}{\hbar^2}\phi \end{cases}$$

有效光子质量: 伦敦方程，理想导体及迈斯纳效应

$$\left[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{m_{光子}^2 c^2}{\hbar^2}\right]\vec{A} = 0$$

$$\left[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{m_{光子}^2 c^2}{\hbar^2}\right]\phi = 0$$

$$\nabla\cdot\vec{A} + \frac{1}{c^2}\frac{\partial\phi}{\partial t} = 0$$

$$\begin{cases} \vec{j} = -\dfrac{m_{光子}^2 c^2}{\mu_0\hbar^2}\vec{A} \\ \rho = -\dfrac{m_{光子}^2 \epsilon_0 c^2}{\hbar^2}\phi \end{cases}$$

$$-\gamma\frac{\partial\vec{B}}{\partial t} = \gamma\nabla\times\vec{E} = \nabla\times\vec{j} = -\frac{m_{光子}^2 c^2}{\mu_0\hbar^2}\nabla\times\vec{A} = -\frac{m_{光子}^2 c^2}{\mu_0\hbar^2}\vec{B}$$ 伦敦第二方程

有效光子质量: 伦敦方程，理想导体及迈斯纳效应

$$[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{m^2_{\text{光子}}c^2}{\hbar^2}]\vec{A} = 0$$

$$[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{m^2_{\text{光子}}c^2}{\hbar^2}]\phi = 0$$

$$\nabla\cdot\vec{A} + \frac{1}{c^2}\frac{\partial\phi}{\partial t} = 0$$

$$\begin{cases} \vec{j} = -\dfrac{m^2_{\text{光子}}c^2}{\mu_0\hbar^2}\vec{A} \\ \rho = -\dfrac{m^2_{\text{光子}}\epsilon_0 c^2}{\hbar^2}\phi \end{cases}$$

$$-\gamma\frac{\partial\vec{B}}{\partial t} = \gamma\nabla\times\vec{E} = \nabla\times\vec{j} = -\frac{m^2_{\text{光子}}c^2}{\mu_0\hbar^2}\nabla\times\vec{A} = -\frac{m^2_{\text{光子}}c^2}{\mu_0\hbar^2}\vec{B}_{\text{伦敦第二方程}} \Rightarrow \vec{B}(t) = \vec{B}(0)e^{\frac{m^2_{\text{光子}}c^2}{\gamma\mu_0\hbar^2}t}$$

有效光子质量: 伦敦方程，理想导体及迈斯纳效应

$$[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{m^2_{\text{光子}}c^2}{\hbar^2}]\vec{A} = 0$$

$$\nabla \cdot \vec{A} + \frac{1}{c^2}\frac{\partial \phi}{\partial t} = 0$$

$$\begin{cases} \vec{j} = -\dfrac{m^2_{\text{光子}}c^2}{\mu_0\hbar^2}\vec{A} \\ \rho = -\dfrac{m^2_{\text{光子}}\epsilon_0 c^2}{\hbar^2}\phi \end{cases}$$

$$[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{m^2_{\text{光子}}c^2}{\hbar^2}]\phi = 0$$

$$-\gamma\frac{\partial \vec{B}}{\partial t} = \gamma\nabla\times\vec{E} = \nabla\times\vec{j} = -\frac{m^2_{\text{光子}}c^2}{\mu_0\hbar^2}\nabla\times\vec{A} = -\frac{m^2_{\text{光子}}c^2}{\mu_0\hbar^2}\vec{B}_{\text{伦敦第二方程}} \Rightarrow \vec{B}(t) = \vec{B}(0)e^{\frac{m^2_{\text{光子}}c^2}{\gamma\mu_0\hbar^2}t}$$

$$\frac{\partial \rho}{\partial t} = -\nabla\cdot\vec{j} = -\nabla\cdot(\gamma\vec{E}) = -\frac{\gamma}{\epsilon_0}\rho \quad \Rightarrow \quad \rho(t) = \rho(0)e^{-\frac{\gamma}{\epsilon_0}t}$$

有效光子质量：伦敦方程，理想导体及迈斯纳效应

$$\left[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{m^2_{\text{光子}}c^2}{\hbar^2}\right]\vec{A} = 0$$

$$\left[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{m^2_{\text{光子}}c^2}{\hbar^2}\right]\phi = 0$$

$$\nabla\cdot\vec{A} + \frac{1}{c^2}\frac{\partial\phi}{\partial t} = 0 \qquad \begin{cases} \vec{j} = -\dfrac{m^2_{\text{光子}}c^2}{\mu_0\hbar^2}\vec{A} \\ \rho = -\dfrac{m^2_{\text{光子}}\epsilon_0 c^2}{\hbar^2}\phi \end{cases}$$

$$-\gamma\frac{\partial\vec{B}}{\partial t} = \gamma\nabla\times\vec{E} = \nabla\times\vec{j} = -\frac{m^2_{\text{光子}}c^2}{\mu_0\hbar^2}\nabla\times\vec{A} = -\frac{m^2_{\text{光子}}c^2}{\mu_0\hbar^2}\vec{B}\text{伦敦第二方程} \Rightarrow \vec{B}(t) = \vec{B}(0)e^{\frac{m^2_{\text{光子}}c^2}{\gamma\mu_0\hbar^2}t}$$

$$\frac{\partial\rho}{\partial t} = -\nabla\cdot\vec{j} = -\nabla\cdot(\gamma\vec{E}) = -\frac{\gamma}{\epsilon_0}\rho \quad \Rightarrow \quad \rho(t) = \rho(0)e^{-\frac{\gamma}{\epsilon_0}t}$$

$\gamma\neq 0 \Rightarrow$ 介质内ρ可取为零：$\underline{\nabla\cdot\vec{j}=0}$ 且 $\vec{E} = -(\nabla\phi + \dfrac{\partial\vec{A}}{\partial t}) = \dfrac{\mu_0\hbar^2}{m^2_{\text{光子}}c^2}\dfrac{\partial\vec{j}}{\partial t}$ 伦敦第一方程

有效光子质量: 伦敦方程，理想导体及迈斯纳效应

$$\left[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{m^2_{光子}c^2}{\hbar^2}\right]\vec{A} = 0$$

$$\left[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{m^2_{光子}c^2}{\hbar^2}\right]\phi = 0$$

$$\nabla\cdot\vec{A} + \frac{1}{c^2}\frac{\partial\phi}{\partial t} = 0$$

$$\begin{cases} \vec{j} = -\dfrac{m^2_{光子}c^2}{\mu_0\hbar^2}\vec{A} \\ \rho = -\dfrac{m^2_{光子}\epsilon_0 c^2}{\hbar^2}\phi \end{cases}$$

$$-\gamma\frac{\partial\vec{B}}{\partial t} = \gamma\nabla\times\vec{E} = \nabla\times\vec{j} = -\frac{m^2_{光子}c^2}{\mu_0\hbar^2}\nabla\times\vec{A} = -\frac{m^2_{光子}c^2}{\mu_0\hbar^2}\vec{B}_{\text{伦敦第二方程}} \Rightarrow \vec{B}(t) = \vec{B}(0)e^{\frac{m^2_{光子}c^2}{\gamma\mu_0\hbar^2}t}$$

$$\frac{\partial\rho}{\partial t} = -\nabla\cdot\vec{j} = -\nabla\cdot(\gamma\vec{E}) = -\frac{\gamma}{\epsilon_0}\rho \quad \Rightarrow \quad \rho(t) = \rho(0)e^{-\frac{\gamma}{\epsilon_0}t}$$

$\gamma \neq 0 \Rightarrow$ 介质内ρ可取为零: $\underline{\nabla\cdot\vec{j} = 0}$ 且 $\vec{E} = -\left(\nabla\phi + \dfrac{\partial\vec{A}}{\partial t}\right) = \dfrac{\mu_0\hbar^2}{m^2_{光子}c^2}\dfrac{\partial\vec{j}}{\partial t}$ 伦敦第一方程

$$\vec{j} = \gamma\vec{E} = \frac{\gamma\mu_0\hbar^2}{m^2_{光子}c^2}\frac{\partial\vec{j}}{\partial t}$$

有效光子质量：伦敦方程，理想导体及迈斯纳效应

$$\left[\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\frac{m^2_{光子}c^2}{\hbar^2}\right]\vec{A}=0$$

$$\left[\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\frac{m^2_{光子}c^2}{\hbar^2}\right]\phi=0$$

$$\nabla\cdot\vec{A}+\frac{1}{c^2}\frac{\partial\phi}{\partial t}=0$$

$$\begin{cases}\vec{j}=-\dfrac{m^2_{光子}c^2}{\mu_0\hbar^2}\vec{A}\\[2ex] \rho=-\dfrac{m^2_{光子}\epsilon_0c^2}{\hbar^2}\phi\end{cases}$$

$$-\gamma\frac{\partial\vec{B}}{\partial t}=\gamma\nabla\times\vec{E}=\nabla\times\vec{j}=-\frac{m^2_{光子}c^2}{\mu_0\hbar^2}\nabla\times\vec{A}=-\frac{m^2_{光子}c^2}{\mu_0\hbar^2}\vec{B}\ \text{伦敦第二方程}\ \Rightarrow\vec{B}(t)=\vec{B}(0)e^{\frac{m^2_{光子}c^2}{\gamma\mu_0\hbar^2}t}$$

$$\frac{\partial\rho}{\partial t}=-\nabla\cdot\vec{j}=-\nabla\cdot(\gamma\vec{E})=-\frac{\gamma}{\epsilon_0}\rho\quad\Rightarrow\quad\rho(t)=\rho(0)e^{-\frac{\gamma}{\epsilon_0}t}$$

$\gamma\neq0\Rightarrow$ 介质内ρ可取为零：$\underline{\nabla\cdot\vec{j}=0}$ 且 $\vec{E}=-\left(\nabla\phi+\dfrac{\partial\vec{A}}{\partial t}\right)=\dfrac{\mu_0\hbar^2}{m^2_{光子}c^2}\dfrac{\partial\vec{j}}{\partial t}$ 伦敦第一方程

$$\vec{j}=\gamma\vec{E}=\frac{\gamma\mu_0\hbar^2}{m^2_{光子}c^2}\frac{\partial\vec{j}}{\partial t}\qquad\vec{j}(t)=\vec{j}(0)e^{\frac{m^2_{光子}c^2}{\gamma\mu_0\hbar^2}t}\Rightarrow$$ 有限的初始电流随时间无穷增强!

$$\xRightarrow{\text{有限的电流密度}}\begin{cases}\gamma=\infty\ \ \underline{\text{理想导体}}\ \ \vec{j}(t)=\vec{j}(0)\ \ \text{恒定的稳恒电流}\\ \vec{j}(0)=0\ \Rightarrow\ \vec{j}(t)=0\ \ \text{后面不讨论这种情形}\end{cases}$$

$$\underline{\vec{E}=0\qquad\frac{\partial\vec{j}}{\partial t}=0}$$

有效光子质量: 伦敦方程，理想导体及迈斯纳效应

$$\left[\nabla^2 - \frac{\mathbf{m}_{光子}^2 \mathbf{c}^2}{\hbar^2}\right]\vec{\mathbf{j}} = 0 \qquad \nabla \cdot \vec{\mathbf{j}} = 0 \qquad \frac{\partial \vec{\mathbf{j}}}{\partial \mathbf{t}} = 0 \qquad \vec{\mathbf{j}} = -\frac{\mathbf{m}_{光子}^2 \mathbf{c}^2}{\mu_0 \hbar^2}\vec{\mathbf{A}} \qquad \vec{\mathbf{E}} = 0$$

有效光子质量: 伦敦方程，理想导体及迈斯纳效应

$$[\nabla^2 - \frac{\mathbf{m}^2_{光子}\mathbf{c}^2}{\hbar^2}]\vec{\mathbf{j}} = 0 \qquad \nabla\cdot\vec{\mathbf{j}} = 0 \qquad \frac{\partial\vec{\mathbf{j}}}{\partial \mathbf{t}} = 0 \qquad \vec{\mathbf{j}} = -\frac{\mathbf{m}^2_{光子}\mathbf{c}^2}{\mu_0\hbar^2}\vec{\mathbf{A}} \qquad \vec{\mathbf{E}} = 0$$

$$\mu_0\vec{\mathbf{j}} = \nabla\times\vec{\mathbf{B}} \qquad \vec{\mathbf{B}} = -\frac{\mu_0\hbar^2}{\mathbf{m}^2_{光子}\mathbf{c}^2}\nabla\times\vec{\mathbf{j}} \qquad \nabla\cdot\vec{\mathbf{B}} = 0 \qquad [\nabla^2 - \frac{\mathbf{m}^2_{光子}\mathbf{c}^2}{\hbar^2}]\vec{\mathbf{B}} = 0$$

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$\vec{\mathbf{B}} = \vec{\mathbf{B}}_0\mathbf{e}^{\mathbf{i}\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}}$ 将$\vec{\mathbf{r}}=\mathbf{0}$选在介质的边界上 $\Rightarrow \vec{\mathbf{k}}\cdot\vec{\mathbf{k}} + \dfrac{\mathbf{m}^2_{光子}\mathbf{c}^2}{\hbar^2} = 0 \qquad \vec{\mathbf{k}}\cdot\vec{\mathbf{B}}_0 = 0$

有效光子质量: 伦敦方程，理想导体及迈斯纳效应

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$\vec{k} = ik_I\vec{e}_k \Rightarrow k_I = \frac{m_{光子}c}{\hbar} > 0$ 可能的负号可被吸收到 $\vec{e}_k$的定义中

Back
Close

有效光子质量：伦敦方程，理想导体及迈斯纳效应

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在介质界面附近：$\vec{e}_k$ 与界面的外法线方向夹角只能是小于或大于90°

Back
Close

有效光子质量: 伦敦方程，理想导体及迈斯纳效应

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$$\begin{cases} \text{与外法线方向夹角小于}90^\circ \quad \Rightarrow \quad e^{-k_I r_k} \quad \text{是增强因子} \\ \text{与外法线方向夹角大于}90^\circ \quad \Rightarrow \quad e^{-k_I r_k} \quad \text{是衰减因子} \end{cases}$$

有效光子质量: 伦敦方程，理想导体及迈斯纳效应

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物理上只能选择衰减的情形!

有效光子质量：伦敦方程，理想导体及迈斯纳效应

$$[\nabla^2-\frac{m^2_{光子}c^2}{\hbar^2}]\vec{j}=0 \qquad \nabla\cdot\vec{j}=0 \qquad \frac{\partial\vec{j}}{\partial t}=0 \qquad \vec{j}=-\frac{m^2_{光子}c^2}{\mu_0\hbar^2}\vec{A} \qquad \vec{E}=0$$

$$\mu_0\vec{j}=\nabla\times\vec{B} \qquad \vec{B}=-\frac{\mu_0\hbar^2}{m^2_{光子}c^2}\nabla\times\vec{j} \qquad \nabla\cdot\vec{B}=0 \qquad [\nabla^2-\frac{m^2_{光子}c^2}{\hbar^2}]\vec{B}=0$$

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介质内部无电场和电荷 磁场和电流随穿透距离指数衰减

有效光子质量：伦敦方程，理想导体及迈斯纳效应

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对这些 $\vec{j} \neq 0$ 的区域，前面的 $\vec{j}(0) = 0$ 的情形不存在。即只能有 $\gamma = \infty$ 的情形！

有效光子质量: 伦敦方程，理想导体及迈斯纳效应

$\vec{\mathrm{B}} \xlongequal{\mathrm{r_k}\equiv\vec{\mathrm{r}}\cdot\vec{\mathrm{e}}_\mathrm{k}} \vec{\mathrm{B}}_0 \mathrm{e}^{-\mathrm{k_I r_k}}$ $\vec{\mathrm{e}}_\mathrm{k}$ 与外法线方向夹角大于90° $\mathrm{k_I} = \dfrac{\mathrm{m}_{\text{光子}}\mathrm{c}}{\hbar}$

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定义磁场衰减为边界值的$1/e$时深度为穿透深度 λ： $\lambda = \dfrac{1}{\mathrm{k_I}} = \dfrac{\hbar}{\mathrm{m}_{\text{光子}}\mathrm{c}}$

Back
Close

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$$0 = \vec{B}_{内} = \mu_0(\vec{H}_{内} + \vec{M}_{内}) \ \Rightarrow\ \vec{M}_{内} = -\vec{H}_{内} = \chi_m \vec{H}_{内} \ \Rightarrow\ \chi_m = -1$$

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完全的抗磁体！

有效光子质量：伦敦方程，理想导体及迈斯纳效应

$\vec{B} \xlongequal{r_k \equiv \vec{r}\cdot\vec{e}_k} \vec{B}_0 e^{-k_I r_k}$　　$\vec{e}_k$ 与外法线方向夹角大于$90°$　　$k_I = \dfrac{m_{光子} c}{\hbar}$

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若导体上电荷、电流完全由光子的有效质量产生

有效光子质量：伦敦方程，理想导体及迈斯纳效应

$$\vec{B} \overset{r_k\equiv\vec{r}\cdot\vec{e}_k}{=\!=\!=\!=} \vec{B}_0 e^{-k_I r_k} \qquad \vec{e}_k \text{ 与外法线方向夹角大于}90^\circ \qquad k_I = \frac{m_{光子}c}{\hbar}$$

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若导体上电荷、电流完全由光子的有效质量产生⇒麦克斯韦方程组、欧姆定律加有效光子质量：

介质上无电场和电荷；介质 内磁感和电流指数衰减；电导率和介电常数为无穷大；磁导率为零

- 关于有效光子质量：自由空间的光子没有质量，但介质上的光子可以有有效质量！

$$[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{m_{光子}^2 c^2}{\hbar^2}]\vec{A} = 0$$

$$[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{m_{光子}^2 c^2}{\hbar^2}]\phi = 0$$

$$\xRightarrow{\vec{A},\phi\ \sim\ e^{i\vec{k}\cdot\vec{r}-i\omega t}} -\vec{k}\cdot\vec{k} + \frac{\omega^2}{c^2} - \frac{m_{光子}^2 c^2}{\hbar^2} = 0$$

$$\hbar\vec{k} \rightleftharpoons \vec{p} \qquad \hbar\omega \rightleftarrows E \qquad \Rightarrow -\vec{p}\cdot\vec{p} + \frac{E^2}{c^2} - m_{光子}^2 c^2 = 0$$

- 关于质量与规范对称性及规范自由度：

$$[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{m_{光子}^2 c^2}{\hbar^2}]\vec{A} - \nabla[\nabla\cdot\vec{A} + \frac{1}{c^2}\frac{\partial\phi}{\partial t}] = 0$$

$$[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{m_{光子}^2 c^2}{\hbar^2}]\phi + \frac{\partial}{\partial t}[\nabla\cdot\vec{A} + \frac{1}{c^2}\frac{\partial\phi}{\partial t}] = 0$$

只有质量项破坏规范对称性！反过来规范对称性要求质量为零！

$$\vec{A}' = \vec{A} + \nabla\chi \qquad \phi' = \phi - \frac{\partial\chi}{\partial t}$$

它使纵场变成不可观察！失去它（破缺或隐藏）则纵场可观察！

- 若假设 $\begin{cases} \vec{\mathbf{j}} = -\dfrac{\mathbf{m}^2_{\text{光子}}\mathbf{c}^2}{\mu_0\hbar^2}\vec{\mathbf{A}} \\ \\ \rho = -\dfrac{\mathbf{m}^2_{\text{光子}}\epsilon_0\mathbf{c}^2}{\hbar^2}\phi \end{cases}$ 则导致 超导

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- 若超导一定有伦敦第二方程: $\nabla\times\vec{\mathbf{j}} = -\dfrac{\mathbf{m}^2_{\text{光子}}\mathbf{c}^2}{\mu_0\hbar^2}\vec{\mathbf{B}} = -\dfrac{\mathbf{m}^2_{\text{光子}}\mathbf{c}^2}{\mu_0\hbar^2}\nabla\times\vec{\mathbf{A}}$

- 则一定有 $\vec{\mathbf{j}} = -\dfrac{\mathbf{m}^2_{\text{光子}}\mathbf{c}^2}{\mu_0\hbar^2}[\vec{\mathbf{A}}+\nabla\chi]$ 可通过规范选择将 χ 场去掉,虽规范对称性已失去

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- 它相当推广的伦敦第一方程: $\frac{\partial\vec{j}}{\partial t} + c^2\nabla\rho = -\frac{m_{光子}^2 c^2}{\mu_0 \hbar^2}(\nabla\phi + \frac{\partial\vec{A}}{\partial t}) = \frac{m_{光子}^2 c^2}{\mu_0 \hbar^2}\vec{E}$

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Back
Close

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- $\omega>0$ 的磁场 指数衰减 非零频率的磁场在导体里指数衰减常态，还应看静磁场 $\Downarrow$

  $\omega=0$ 的磁场：
  - 在 $\gamma\omega=0$ 介质上不衰减
  - 在 $\gamma\omega>0$ 介质超导体上 指数衰减

有效光子质量: 超越洛伦兹规范，规范不变描写

$$\tilde{\phi} \equiv \frac{-1}{[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}]}[\nabla \cdot \vec{A} + \frac{1}{c^2}\frac{\partial \phi}{\partial t}]$$

Back
Close

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$$\nabla\cdot\vec{j} + \frac{\partial\rho}{\partial t} = \nabla\cdot[\frac{m^2c^2}{\mu_0\hbar^2}[\vec{A} + \nabla\tilde{\phi}]] + \frac{\partial}{\partial t}[\frac{\epsilon_0 m^2c^2}{\hbar^2}[\phi - \frac{\partial\tilde{\phi}}{\partial t}]]$$

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$$= \frac{\frac{m^2c^2}{\mu_0\hbar^2}}{[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}]}[[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}]\nabla\cdot\vec{A} - \nabla^2[\nabla\cdot\vec{A} + \frac{1}{c^2}\frac{\partial\phi}{\partial t}]]$$

$$+ \frac{\frac{\epsilon_0 m^2c^2}{\hbar^2}}{[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}]}[[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}]\frac{\partial\phi}{\partial t} + \frac{\partial^2}{\partial t^2}[\nabla\cdot\vec{A} + \frac{1}{c^2}\frac{\partial\phi}{\partial t}]] = 0$$

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可通过选择 洛伦兹规范 或 规范变换 使$\tilde{\phi}$场为零！

Back

Close

$$\mu_0\vec{\mathbf{j}} = \frac{\mathbf{m^2c^2}}{\hbar^2}[\vec{\mathbf{A}}+\nabla\tilde{\phi}] \qquad\qquad \frac{\rho}{\epsilon_0} = \frac{\mathbf{m^2c^2}}{\hbar^2}[\phi-\frac{\partial\tilde{\phi}}{\partial \mathbf{t}}]$$

推广 $\Downarrow$ $\qquad \frac{\mathbf{mc}}{\hbar} \equiv \bar{\phi}$ $\quad$ $\bar{\phi}$现在是常数哟! $\qquad \Downarrow$ 推广

$$\mu_0\vec{\mathbf{j}} = \bar{\phi}^2[\vec{\mathbf{A}} + \nabla\tilde{\phi}] \qquad\qquad \frac{\rho}{\epsilon_0} = \bar{\phi}^2[\phi - \frac{\partial\tilde{\phi}}{\partial \mathbf{t}}]$$

Back
Close

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$$= \mathbf{\Phi}^*\mathbf{\Phi}\vec{\mathbf{A}} - \frac{\mathbf{i}}{\mathbf{2}}\mathbf{\Phi}^*\nabla\mathbf{\Phi} + \frac{\mathbf{i}}{\mathbf{2}}\mathbf{\Phi}\nabla\mathbf{\Phi}^*$$

$$\underline{\mathbf{\Phi} \equiv \bar{\phi}\mathbf{e}^{\mathbf{i}\tilde{\phi}}}$$

$$\frac{\rho}{\epsilon_0} = (\bar{\phi}\mathbf{e}^{-\mathbf{i}\tilde{\phi}})(\bar{\phi}\mathbf{e}^{\mathbf{i}\tilde{\phi}})\phi + \frac{\mathbf{i}}{\mathbf{2}}(\bar{\phi}\mathbf{e}^{-\mathbf{i}\tilde{\phi}})\frac{\partial}{\partial \mathbf{t}}(\bar{\phi}\mathbf{e}^{\mathbf{i}\tilde{\phi}}) - \frac{\mathbf{i}}{\mathbf{2}}(\bar{\phi}\mathbf{e}^{\mathbf{i}\tilde{\phi}})\frac{\partial}{\partial \mathbf{t}}(\bar{\phi}\mathbf{e}^{-\mathbf{i}\tilde{\phi}})$$

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Back
Close

$$\mu_0\vec{\mathbf{j}} = \frac{\mathbf{m^2c^2}}{\hbar^2}[\vec{\mathbf{A}}+\nabla\tilde{\phi}] \qquad \frac{\rho}{\epsilon_0} = \frac{\mathbf{m^2c^2}}{\hbar^2}[\phi-\frac{\partial\tilde{\phi}}{\partial \mathbf{t}}]$$

推广 $\Downarrow$ $\qquad \frac{\mathbf{mc}}{\hbar} \equiv \bar{\phi}$ $\bar{\phi}$现在是常数哟！ $\qquad \Downarrow$ 推广

$$\mu_0\vec{\mathbf{j}} = \bar{\phi}^2[\vec{\mathbf{A}} + \nabla\tilde{\phi}] \qquad \frac{\rho}{\epsilon_0} = \bar{\phi}^2[\phi - \frac{\partial\tilde{\phi}}{\partial \mathbf{t}}]$$

$$\mu_0\vec{\mathbf{j}} = (\bar{\phi}\mathbf{e}^{-\mathbf{i}\tilde{\phi}})(\bar{\phi}\mathbf{e}^{\mathbf{i}\tilde{\phi}})\vec{\mathbf{A}} - \frac{\mathbf{i}}{\mathbf{2}}(\bar{\phi}\mathbf{e}^{-\mathbf{i}\tilde{\phi}})\nabla(\bar{\phi}\mathbf{e}^{\mathbf{i}\tilde{\phi}}) + \frac{\mathbf{i}}{\mathbf{2}}(\bar{\phi}\mathbf{e}^{\mathbf{i}\tilde{\phi}})\nabla(\bar{\phi}\mathbf{e}^{-\mathbf{i}\tilde{\phi}})$$

$$= \mathbf{\Phi}^*\mathbf{\Phi}\vec{\mathbf{A}} - \frac{\mathbf{i}}{\mathbf{2}}\mathbf{\Phi}^*\nabla\mathbf{\Phi} + \frac{\mathbf{i}}{\mathbf{2}}\mathbf{\Phi}\nabla\mathbf{\Phi}^* \qquad \underline{\mathbf{\Phi} \equiv \bar{\phi}\mathbf{e}^{\mathbf{i}\tilde{\phi}}}$$

$$\frac{\rho}{\epsilon_0} = (\bar{\phi}\mathbf{e}^{-\mathbf{i}\tilde{\phi}})(\bar{\phi}\mathbf{e}^{\mathbf{i}\tilde{\phi}})\phi + \frac{\mathbf{i}}{\mathbf{2}}(\bar{\phi}\mathbf{e}^{-\mathbf{i}\tilde{\phi}})\frac{\partial}{\partial \mathbf{t}}(\bar{\phi}\mathbf{e}^{\mathbf{i}\tilde{\phi}}) - \frac{\mathbf{i}}{\mathbf{2}}(\bar{\phi}\mathbf{e}^{\mathbf{i}\tilde{\phi}})\frac{\partial}{\partial \mathbf{t}}(\bar{\phi}\mathbf{e}^{-\mathbf{i}\tilde{\phi}})$$

$$= \mathbf{\Phi}^*\mathbf{\Phi}\phi + \frac{\mathbf{i}}{\mathbf{2}}\mathbf{\Phi}^*\frac{\partial\mathbf{\Phi}}{\partial \mathbf{t}} - \frac{\mathbf{i}}{\mathbf{2}}\mathbf{\Phi}\frac{\partial\mathbf{\Phi}^*}{\partial \mathbf{t}}$$

$\bar{\phi}$不是常数时(超出传统的伦敦理论)上面两个公式仍然成立哟！

$$[\nabla\cdot\vec{\mathbf{A}}+\frac{\mathbf{1}}{\mathbf{c^2}}\frac{\partial\phi}{\partial\mathbf{t}}]=[\nabla^2-\frac{\mathbf{1}}{\mathbf{c^2}}\frac{\partial^2}{\partial\mathbf{t}^2}]\tilde{\phi}\ \Rightarrow\ \nabla\cdot[\vec{\mathbf{A}}-\nabla\tilde{\phi}]+\frac{\mathbf{1}}{\mathbf{c^2}}\frac{\partial}{\partial\mathbf{t}}[\phi+\frac{\partial\tilde{\phi}}{\partial\mathbf{t}}]=\mathbf{0}$$

$$[\nabla\cdot\vec{\mathbf{A}}+\frac{\mathbf{1}}{\mathbf{c^2}}\frac{\partial\phi}{\partial\mathbf{t}}]=[\nabla^2-\frac{\mathbf{1}}{\mathbf{c^2}}\frac{\partial^2}{\partial\mathbf{t}^2}]\tilde{\phi}\;\Rightarrow\;\nabla\cdot[\vec{\mathbf{A}}-\nabla\tilde{\phi}]+\frac{\mathbf{1}}{\mathbf{c^2}}\frac{\partial}{\partial\mathbf{t}}[\phi+\frac{\partial\tilde{\phi}}{\partial\mathbf{t}}]=\mathbf{0}$$

$$\Rightarrow\;\nabla\cdot[\bar{\phi}^{\mathbf{2}}(\vec{\mathbf{A}}-\nabla\tilde{\phi})]+\frac{\mathbf{1}}{\mathbf{c^2}}\frac{\partial}{\partial\mathbf{t}}[\bar{\phi}^{\mathbf{2}}(\phi+\frac{\partial\tilde{\phi}}{\partial\mathbf{t}})]=\mathbf{0}$$

$$\overset{\text{补}}{\Longrightarrow}(\nabla^2-\frac{\mathbf{1}}{\mathbf{c^2}}\frac{\partial^2}{\partial\mathbf{t}^2})\bar{\phi}+\mathbf{2}\bar{\phi}\mathbf{V}'(\bar{\phi}^{\mathbf{2}})-\bar{\phi}[(\vec{\mathbf{A}}-\nabla\tilde{\phi})\cdot(\vec{\mathbf{A}}-\nabla\tilde{\phi})-(\phi+\frac{\partial\tilde{\phi}}{\partial\mathbf{t}})^{\mathbf{2}}]=\mathbf{0}$$

有效光子质量: 有效光子质量起源

$$[\nabla\cdot\vec{A}+\frac{1}{c^2}\frac{\partial\phi}{\partial t}] = [\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}]\tilde{\phi} \Rightarrow \nabla\cdot[\vec{A}-\nabla\tilde{\phi}]+\frac{1}{c^2}\frac{\partial}{\partial t}[\phi+\frac{\partial\tilde{\phi}}{\partial t}]=0$$

$$\Rightarrow \nabla\cdot[\bar{\phi}^2(\vec{A}-\nabla\tilde{\phi})]+\frac{1}{c^2}\frac{\partial}{\partial t}[\bar{\phi}^2(\phi+\frac{\partial\tilde{\phi}}{\partial t})]=0$$

$$\xrightarrow{\text{补}}(\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2})\bar{\phi}+2\bar{\phi}V'(\bar{\phi}^2)-\bar{\phi}[(\vec{A}-\nabla\tilde{\phi})\cdot(\vec{A}-\nabla\tilde{\phi})-(\phi+\frac{\partial\tilde{\phi}}{\partial t})^2]=0$$

类似于光子场的质量，从中可以读出$\bar{\phi}$场的质量平方为 $m_h^2 = 4\phi_0^2 V''(\phi_0^2)$ $\Downarrow$ $\phi_0 = \frac{mc}{\hbar}$

$\Phi \equiv \bar{\phi}e^{i\tilde{\phi}}$

$$[(\nabla+i\vec{A})\cdot(\nabla+i\vec{A})-\frac{1}{c^2}(\frac{\partial}{\partial t}-i\phi)^2]\Phi+2V'(\Phi^*\Phi)\Phi=0$$

$\Uparrow$ Higgs 因提出上面理论并特别是指出希格斯粒子而获2013年诺贝尔物理奖！

$$[\nabla\cdot\vec{A}+\frac{1}{c^2}\frac{\partial\phi}{\partial t}]=[\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}]\tilde{\phi}\Rightarrow\nabla\cdot[\vec{A}-\nabla\tilde{\phi}]+\frac{1}{c^2}\frac{\partial}{\partial t}[\phi+\frac{\partial\tilde{\phi}}{\partial t}]=0$$

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静态，零电势：$\vec{A}\to-\frac{e^*}{c\hbar}\vec{A}$ $\quad\Phi=\sqrt{\frac{\mu_0e^*\hbar}{m^*}}\Psi$ $\quad V'(|\Phi|^2)=-\frac{2m^*}{\hbar^2}\left[\alpha+\frac{\beta m_*}{\mu_0e^*\hbar}|\Phi|^2\right]$

$$(\nabla-\frac{e^*}{c\hbar}i\vec{A})\cdot(\nabla-\frac{e^*}{c\hbar}i\vec{A})\Psi-\frac{2m^*}{\hbar^2}[\alpha+\beta|\Psi|^2]\Psi=0\Leftarrow$$ Ginzberg-Landau方程！

有效光子质量：有效光子质量起源

$$[\nabla\cdot\vec{A}+\frac{1}{c^2}\frac{\partial\phi}{\partial t}]=[\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}]\tilde{\phi}\;\Rightarrow\;\nabla\cdot[\vec{A}-\nabla\tilde{\phi}]+\frac{1}{c^2}\frac{\partial}{\partial t}[\phi+\frac{\partial\tilde{\phi}}{\partial t}]=0$$

$$\Rightarrow\;\nabla\cdot[\bar{\phi}^2(\vec{A}-\nabla\tilde{\phi})]+\frac{1}{c^2}\frac{\partial}{\partial t}[\bar{\phi}^2(\phi+\frac{\partial\tilde{\phi}}{\partial t})]=0$$

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$$(\nabla-\frac{e^*}{c\hbar}i\vec{A})\cdot(\nabla-\frac{e^*}{c\hbar}i\vec{A})\Psi-\frac{2m^*}{\hbar^2}[\alpha+\beta|\Psi|^2]\Psi=0\;\Leftarrow$$ Ginzberg-Landau方程！

Abrikosov 因在GL方程中求出涡旋解并应用于超导研究而获得03年诺贝尔物理奖， 16年诺贝尔物理奖则与正反涡旋对有关！

BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

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(Received 31 August 1964)

In a recent note[1] it was shown that the Goldstone theorem,[2] that Lorentz-covariant field theories in which spontaneous breakdown of symmetry under an internal Lie group occurs contain zero-mass particles, fails if and only if the conserved currents associated with the internal group are coupled to gauge fields. The purpose of the present note is to report that, as a consequence of this coupling, the spin-one quanta of some of the gauge fields acquire mass; the longitudinal degrees of freedom of these particles (which would be absent if their mass were

$$L = -\tfrac{1}{2}(\nabla\varphi_1)^2 - \tfrac{1}{2}(\nabla\varphi_2)^2 - V(\varphi_1^{\,2}+\varphi_2^{\,2}) - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}, \qquad (1)$$

where

$$\nabla_\mu\varphi_1 = \partial_\mu\varphi_1 - eA_\mu\varphi_2,$$

$$\nabla_\mu\varphi_2 = \partial_\mu\varphi_2 + eA_\mu\varphi_1,$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

about the "vacuum" solution $\varphi_1(x)=0$, $\varphi_2(x)=\varphi_0$:

$$\partial^\mu\{\partial_\mu(\Delta\varphi_1) - e\varphi_0 A_\mu\} = 0, \qquad (2a)$$

$$\{\partial^2 - 4\varphi_0^{\,2}V''(\varphi_0^{\,2})\}(\Delta\varphi_2) = 0, \qquad (2b)$$

$$\partial_\nu F^{\mu\nu} = e\varphi_0\{\partial^\mu(\Delta\varphi_1) - e\varphi_0 A_\mu\}. \qquad (2c)$$

Equation (2b) describes waves whose quanta have (bare) mass $2\varphi_0\{V''(\varphi_0^{\,2})\}^{1/2}$; Eqs. (2a) and (2c) may be transformed, by the introduction of new variables

$$B_\mu = A_\mu - (e\varphi_0)^{-1}\partial_\mu(\Delta\varphi_1),$$

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu = F_{\mu\nu}, \qquad (3)$$

into the form

$$\partial_\mu B^\mu = 0, \quad \partial_\nu G^{\mu\nu} + e^2\varphi_0^{\,2}B^\mu = 0. \qquad (4)$$

Equation (4) describes vector waves whose quanta have (bare) mass $e\varphi_0$. In the absence of the gauge field coupling ($e=0$) the situation is quite different: Equations (2a) and (2c) describe zero-mass scalar and vector bosons, respectively. In passing, we note that the right-hand side of (2c) is

$$\mathcal{L}_{\text{Electrodynamics}} = -\frac{1}{4}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) - \mu_0 j^\mu A_\mu$$

$$\partial_\mu \partial^\mu A^\nu - \partial^\nu \partial_\mu A^\mu = \mu_0 j^\nu$$

$$\partial^\mu \partial_\mu \tilde{\phi} \equiv \partial^\mu A_\mu \quad \xRightarrow{A^\mu \to A^\mu - \partial^\mu \chi} \quad \tilde{\phi} \to \tilde{\phi}' = \tilde{\phi} - \chi$$

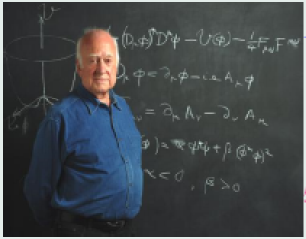
$\mathcal{L}_{\text{Goldstone}} = \frac{1}{2}(\partial^\mu \tilde{\phi})(\partial_\mu \tilde{\phi}) + \tilde{\phi} \partial^\mu A_\mu$

$$j^\mu = -\frac{m^2 c^2}{\mu_0 \hbar^2}[A^\mu - \partial^\mu \tilde{\phi}]$$

$$\mathcal{L}_{\text{Stueckelberg}} = -\frac{1}{4}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) + \underbrace{\frac{m^2 c^2}{2\hbar^2}[A^\mu - (\partial^\mu \tilde{\phi})][A_\mu - (\partial_\mu \tilde{\phi})]}_{\text{交叉项：希格斯机制}}$$

$$\mathcal{L}_{\text{Chiral Lagrangian}} = -\frac{1}{4}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) + \frac{f^2}{2}(D^\mu U)^*(D_\mu U)$$

$f = \frac{mc}{e\hbar}$ $\quad U \equiv e^{i\tilde{\phi}}$ $\quad D_\mu U \equiv (\partial_\mu - iA_\mu)U$ 手征拉氏量；非线性σ模型



嵌入有效光子质量起源的相互作用从手征拉氏量的角度看就是从非线性实现到线性实现的扩充！

$$f = \frac{mc}{e\hbar} \equiv <\bar{\phi}> \qquad fU = <\bar{\phi}> e^{i\tilde{\phi}} \Rightarrow \bar{\phi}e^{i\tilde{\phi}} \equiv \Phi$$

物理理论中的这种扩充或推广比比皆是！

$$\mathcal{L}_{\text{Abel Higgs}} = -\tfrac{1}{4}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) + \frac{1}{2}(D^\mu\Phi)^*(D_\mu\Phi) - V(\Phi^*\Phi)$$

当年希格斯的诺奖工作模型，当年他用实部虚部（线性实现）进行讨论 $V'(f^2)=0 \quad V''(f^2)>0$

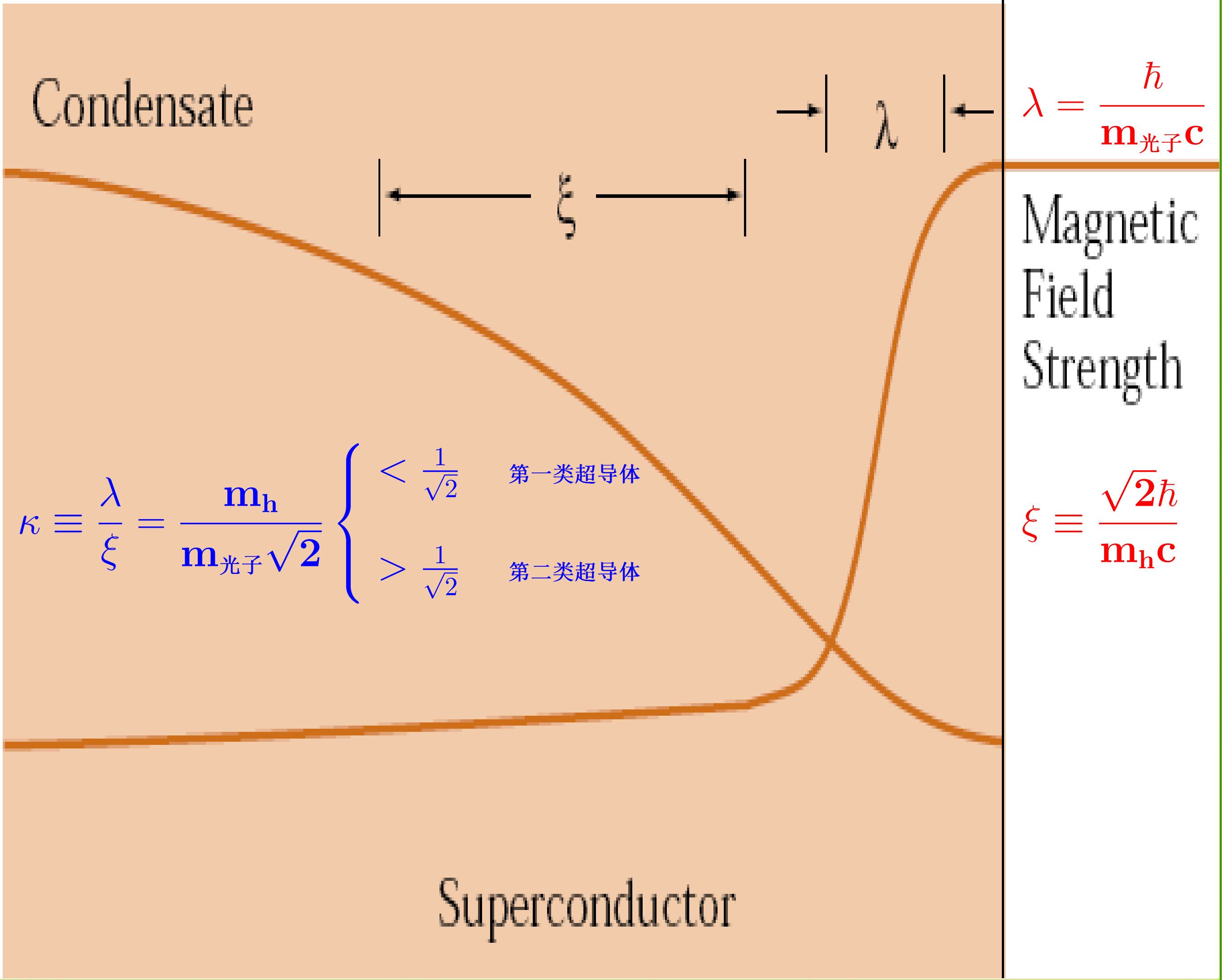
光子场求极值：$\partial_\mu\partial^\mu A^\nu - \partial^\nu\partial_\mu A^\mu - \bar{\phi}^2[A^\nu - (\partial^\nu\tilde{\phi})] = 0$ $\quad \underline{j^\mu = -\frac{\bar{\phi}^2}{\mu_0}[A^\mu - \partial^\mu\tilde{\phi}]}$

超越伦敦理论（见后）

Goldstone场求极值：$\partial^\mu[\bar{\phi}(A_\mu - \partial_\mu\tilde{\phi}] = 0 \quad \Rightarrow \quad \partial^\mu\partial_\mu\tilde{\phi} = \partial^\mu A_\mu + \frac{\mu_0}{3}(\partial^\mu\frac{1}{\bar{\phi}^2})j_\mu$

Higgs场求极值：$\partial^\mu\partial_\mu\bar{\phi} - 2\bar{\phi}V'(\bar{\phi}^2) - \bar{\phi}(D^\mu U)^*(D_\mu U) = 0$ $\quad \underline{m_h^2 = 4f^2V''(f^2)}$

当年Higgs的结果：Equation (2b) describes wave whose quanta have (bare) mass $2\phi_0\{V''(\phi_0^2)\}^{\frac{1}{2}}$ $\Leftarrow$ 南部的提醒？



$$\lambda = \frac{\hbar}{\mathbf{m}_{光子}\mathbf{c}}$$

$$\kappa \equiv \frac{\lambda}{\xi} = \frac{\mathbf{m_h}}{\mathbf{m}_{光子}\sqrt{\mathbf{2}}} \begin{cases} < \frac{1}{\sqrt{2}} & \text{第一类超导体} \\ > \frac{1}{\sqrt{2}} & \text{第二类超导体} \end{cases}$$

$$\xi \equiv \frac{\sqrt{\mathbf{2}}\hbar}{\mathbf{m_h c}}$$

SUPERCONDUCTIVITY

Light-induced collective pseudospin precession resonating with Higgs mode in a superconductor

Ryusuke Matsunaga,[1*] Naoto Tsuji,[1] Hiroyuki Fujita,[1] Arata Sugioka,[1] Kazumasa Makise,[2] Yoshinori Uzawa,[3†] Hirotaka Terai,[2] Zhen Wang,[2‡] Hideo Aoki,[1,4] Ryo Shimano[1,5*]

Superconductors host collective modes that can be manipulated with light. We show that a strong terahertz light field can induce oscillations of the superconducting order parameter in NbN with twice the frequency of the terahertz field. The result can be captured as a collective precession of Anderson's pseudospins in ac driving fields. A resonance between the field and the Higgs amplitude mode of the superconductor then results in large terahertz third-harmonic generation. The method we present here paves a way toward nonlinear quantum optics in superconductors with driving the pseudospins collectively and can be potentially extended to exotic superconductors for shedding light on the character of order parameters and their coupling to other degrees of freedom.

Macroscopic quantum phenomena, such as superconductivity and superfluidity, emerge in a variety of physical systems, such as metals, liquid helium, ultracold atomic quantum gases, and neutron stars. One manifestation of the macroscopic quantum nature is the appearance of characteristic collective excitations. Indeed, phenomena associated with collective modes, such as second sound and spin waves in condensates, have been revealed in superfluid helium (*1*, *2*) and in ultracold atomic gases (*3*, *4*).

这个理论的另一种可能的用途：暗光子！

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预祝大家竞赛出好成绩!