

Markscheme

November 2019

Extended mathematics

On-screen examination

23 pages

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Error Carried Forward (ECF) marks

Errors made at any step of a solution affect all working that follows. In general, **Error Carried Forward (ECF)** marks are awarded after an error.

- a) **ECF** applies from one part of a question to a subsequent part of the question and also applies within the same part.
- b) If an answer resulting from **ECF** is inappropriate (eg, negative distances or $\sin x > 1$) then subsequent marks should not be awarded.
- c) If a question is transformed by an error into a **simpler question** then **ECF** may not be fully awarded.
- d) To award **ECF** marks for a question part, **there must be working present for that part**.
- e) **ECF** is only applied to working which is correct. This means that all working subsequent to an error must be checked for accuracy.
- f) A misread (**MR**) is an error. **ECF** is normally awarded.

General points

- a) As this is an international examination, accept all alternative forms of **notation**, for example 1.9 and 1,9 or 1 000 or 1.000. However **DO NOT ACCEPT** incorrect mathematical notation e.g x^2 for x^2 unless noted otherwise in the MS.
- b) Accept notation errors in intermediate steps.
- c) Ignore further working after a correct answer **unless** it indicates a lack of mathematical understanding **i.e. if the further working contradicts the correct answer**, then the last mark cannot be awarded.
- d) In the case when a correct result is obtained using incorrect seen method, do not award the mark for the result.
- e) Where candidates have written two solutions to a question, mark the first solution.
- f) In the markscheme, equivalent examples of **numerical** and **algebraic** forms or **simplified** answers will generally be written in the notes preceded by **OE** (or equivalent) e.g. $\frac{1}{2}$ **OR** $1/2$ **OR** $1 \div 2$ and $\frac{x}{2}$ **OR** $x/2$
- g) In the markscheme, information provided in brackets indicate detail that may be seen in a candidate response but is not necessary to award the marks.
- h) Special case marks **SC** can be allocated instead of but not in addition to the marks prescribed in the markscheme.
- i) Accept seeing equation not in-line.
- j) Calculator screenshots are accepted as working steps. And when a calculator screenshot is taken, accept not seeing the whole operation.
- k) In task 2 and 3 where the markscheme is set out in a table then, unless noted otherwise, awarding the highest mark in a category includes all the lower marks in that category. It is probably best to look for the top category mark answer and if you don't find it look at the next mark down.
- l) **ACCEPT** using the correct values regardless their previous result

Question	Answers	Notes	Total
1 a	<p>AM1 (adding volumes)</p> <p>.1 correct Volume for one part .2 correct Volume for second part .3 correctly add their volumes</p> <p>AM2 (Subtracting volumes)</p> <p>.1 correct volume including empty part .2 correct volume of empty part .3 correctly subtract their volumes</p> <p>AM3 (Area x depth)</p> <p>.1 correct Area for one part .2 correct Area for the face .3 correctly multiply the total of their areas by 30</p>	<p>AM1 (adding volumes)</p> <p>.1 $50 \times 20 \times 30 (=30000)$ OR $35 \times 20 \times 30 (=21000)$.2 $20 \times 15 \times 30 (=9000)$ OR $40 \times 15 \times 30 (=18000)$.3 $(their30000+their9000)$ OR $(their21000+their18000) =39000 (\text{ft}^3)$</p> <p>AM2 (Subtracting volumes)</p> <p>.1 $50 \times 40 \times 30 (=60000)$.2 $35 \times 20 \times 30 (=21000)$.3 $(their60000-their21000) =39000 (\text{ft}^3)$</p> <p>AM3 (Area x depth)</p> <p>.1 $50 \times 20 (=1000)$ OR $20 \times 15 (=300)$.2 $(50 \times 20 + 20 \times 15) = 1300$.3 $(their1300 \times 30) =39000 (\text{ft}^3)$</p>	3
b	.1 correct working step .2 correct sum of employees in working days .3 correct solution of their equation	.1 Ex: setting equation $5 \times w + 2 \times 3 = 7 \times 58$ OR 6 and 406 seen .2 $(5w) = 400$.3 $(w =) 80$	3
c	.1 correctly substitute their39000 and their80 .2 correctly calculate their P after their substitution involving at least one multiplication in numerator	.1 $(P =) \frac{(6 \times their39000 + 500 \times their80)}{9000}$.2 their 30(.444...)	2

Question	Answers	Notes	Total
2	a Correct vectors have been dragged		1
b	.1 correct path in un-simplified form .1 correct path in simplest form	.1 $-2b + 3.5a - 4b$ or $4a - 2b - 0.5a - 4b$.1 ACCEPT any path starting from M, passing by P and ending at conveyor belt .2 $1.5a - 4b + 4c$ ACCEPT $3.5a - 6b$	2
c	.1 correct working step .2 another correct working step .3 correct answer Examples can be seen below: AM1 .1 correctly calculate the number of minutes for 4 robots to prepare 300 orders .2 correctly calculate the number of minutes for 1 robot to prepare 300 orders .3 correct answer	.3 24 (min) AM1 .1 60 (min) .2 240 (min) .3 24 (min)	3

	<p>AM2</p> <p>.1 correctly calculate the number of orders prepared by 1 robot in 3 minutes and 10 robots in 3 minutes .2 correctly calculate the number of orders prepared by 10 robots in 3 minutes .3 correct answer</p> <p>AM3</p> <p>.1 correctly calculate the number of robots or minutes to prepare 300 orders .2 correctly multiply the number of robots by the correct constant of proportionality .3 correct answer</p> <p>AM4</p> <p>.1 correctly calculate the number of orders or minutes prepared by 10 robots .2 correctly multiply the number of orders by the correct constant of proportionality .3 correct answer</p> <p>AM5</p> <p>.1 correctly calculate the number of minutes to prepare 1 order OR finding the number of orders prepared in 1 minute .2 correctly calculate the number of robots or minutes to prepare 300 orders .3 correct answer</p> <p>AM6</p> <p>.1 correctly calculate the number of orders prepared by 1 robot .2 correctly calculate the number of orders prepared by 10 robots .3 correct answer</p>	<p>AM2</p> <p>.1 1 robot 3.75 (orders) and 10 robots 37.5 (orders) .2 $300/37.5 = 8$.3 24 (min)</p> <p>AM3</p> <p>.1 80 (robots) or 60 (min) .2 0.125 or 2.5 .3 24 (min)</p> <p>AM4</p> <p>.1 37.5 (orders) or 1.2 (min) .2 20 or 8 .3 $(1.2 \times 20 =) 24$ (min)</p> <p>AM5</p> <p>.1 0.2 (min) or 5 (orders) .2 60 (min) or 240 (robots) .3 $(60/2.5 =) 24$ (min) OR $(1 \times 24 =) 24$ (min)</p> <p>AM6</p> <p>.1 3.75 (order) .2 37.5 (order) .3 $(3 \times 8 =) 24$ (min)</p>	
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Question	Answers	Notes	Total
3 a	correctly place 4	<p>ACCEPT 4%</p>	1
b	correctly state a reason for mutually exclusive related to sets representation	<p>No intersection ACCEPT they are separate or apart WTTE $A \cap B \cap C = 0$ or \emptyset DO NOT ACCEPT No girl selected both</p>	1
c	.1 correct working step .2 correct value of cat only .3 correct value of intersection .4 correct value of dog only .5 correct value of rabbit only	<p>.1 16+39+50+23(-8-100) OE or 128 seen .2 18 .3 20 .4 23 .5 15</p>	5

	d	<p>.1 0.84 seen .2 probability of adopt from both branches seen .3 their result correct after adding their multiplied values</p>	<p>.1 ACCEPT 84 seen on tree diagram .2 $0.6 \times 0.96(=0.576)$ OE, ACCEPT 0.58 and $0.4 \times 0.84(=0.336)$OE, ACCEPT 0.34 .3 their 0.912 OE, ACCEPT 0.91</p> <pre> graph LR Root(()) -- "60 %" --> Girls[Girls] Root -- "40 %" --> Boys[Boys] Girls -- "96 %" --> Adopt1[Adopt] Girls -- "4 %" --> NotAdopt1[Not adopt] Boys -- "84 %" --> Adopt2[Adopt] Boys -- "16 %" --> NotAdopt2[Not adopt] </pre>	3
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Question	Answers	Notes	Total
4	a .1 (0, -2)	ACCEPT 0;-2 X=0, y=-2 DO NOT ACCEPT -2 C=-2	1
	b .1 correctly set equation to solve .2 correctly factorize their equation or substitute into quadratic formula .3 correct coordinates of their point A .4 correct coordinates of their point B	.1 $3x^2 - 5x - 2 = 0$.2 their $(3x+1)(x-2)$.3 their (A =) (-1/3,0) OE, ACCEPT -0.3 .3 DO NOT ACCEPT if positive .4 Their (B =) (2,0) .4 DO NOT ACCEPT if negative	4
	c correctly write coordinates of their point B after reflection on the y-axis	ACCEPT their-2,0 x=their-2 , y=0 DO NOT ACCEPT -2 Their(-2,0) if positive	1

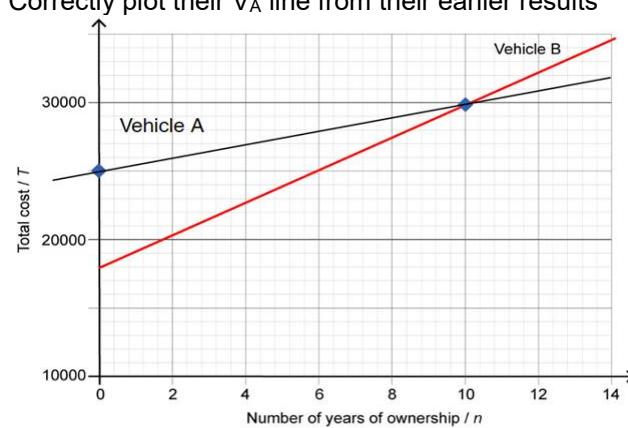
Question		Answers	Notes	Total
5	a	.1 correctly write as a single log .2 remove logs AG $x = 2y$.1 $(\log 2x =) \log \frac{12y}{3}$ or $\log 4y$ OR $\log 2 \times 3x = \log 12y$.2 $2x = \frac{12y}{3}$ or $2x = 4y$ or $6x = 12y$	2
	b	.1 correctly remove log from first equation .2 correct equation in one unknown .3 correct value of their x after substituting $x=2y$.4 correct value of their y after substituting $x=2y$.1 $3x - 2y = 16$.2 $6y - 2y = 16$ or $3x - x = 16$ OE or $\log_4(6y - 2y) = 2$ OE .3 their $y = 4$.4 their $x = 8$	4

Question	Answers	Notes	Total
6 a	400 + 6372 AG 6772	ACCEPT 6772-400=6372 or 6772-6372=400	1
b	.1 recognising that angle XON is the same as angle XOS .2 recognising that angle ONX is equal to angle OXS AG similar triangles	.1 Shared angle .2 Both 90° , ACCEPT perpendicular	2
c	AM1 .1 equate ratios .2 rearrange correctly their equated ratios .3 correct answer .4 correctly round their answer to the nearest km AM2 .1 correctly find value of angle XOS .2 correctly substitute into trig ratio or cosine rule their angle and 6372 .3 correct answer .4 correctly round their answer to the nearest km	AM1 .1 $\frac{r}{OS} = \frac{ON}{r}$ or $\frac{6372}{6772} = \frac{ON}{6372}$.2 $(ON =) \frac{r^2}{OS}$ or $(ON =) \frac{6372^2}{6772}$.3 5995.626... .4 their 5996 AM2 .1 $(\cos^{-1} \frac{6372}{6772}) = 19.791...$.2 $\cos(\text{their } 19.791..) = \frac{ON}{6372}$, DO NOT ACCEPT their19.7911.. unless .1 seen .3 5995.626... .4 their 5996	4
d	.1 correctly calculate their MN .2 correctly substitute theirMN and 6372 into the surface area of spherical cap formula .3 correctly calculate their result after substitution into the surface area of spherical cap formula .4 correctly write their answer rounded to 2 sf and in standard form	.1 $(6372 - \text{their } ON) = \text{their } 376$, ACCEPT answers in range [376,376.4] .2 $2\pi \times 6372 \times \text{their } 376$ or their 4791744π .3 their 15 068 653(.43...) .3 ACCEPT answers in the range [15 053 707.75,15 069 722.33] .4 their 1.5×10^7	4

	e	<p>.1 correctly substitute 6372 into the Surface Area of sphere formula</p> <p>.2 divide their $6d$ by their surface area of sphere</p> <p>.3 correctly calculate their ratio as percentage</p>	<ul style="list-style-type: none"> •1 $4\pi \times 6372^2$ or 510 224 605(.2..) •2 $\frac{\text{their } 1.5 \times 10^7}{\text{their } 510 224 605}$ OE e.g. $\frac{\text{their } 1.5}{\text{their } 51}$ or 0.029 •3 their 2.9(...) ACCEPT 3(%) 	3
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Question		Answers	Notes	Total
7	a	<p>AM1</p> <p>.1 recognise 80</p> <p>.2 multiply 80 % by 31250</p> <p>.3 correctly calculate their result after multiplying 31250 by their percentage</p> <p>AM2</p> <p>.1 multiply 20 % by 31250</p> <p>.2 subtract 31250×0.2 from 31250</p> <p>.3 correctly calculate their result after subtraction of their reduction from 31250</p>	<p>AM1</p> <p>.1 80 or 0.8 seen</p> <p>.2 0.8×31250 OE</p> <p>.3 their25000</p> <p>AM2</p> <p>.1 31250×0.2 or 6250 seen</p> <p>.2 $31250 - 31250 \times 0.2$ or $31250 - 6250$</p> <p>.3 their25000</p>	3
	b	.1 multiply 14000 by 0.035 .2 correct answer	.1 14000×0.035 seen .2 490	2
	c	<p>AM1</p> <p>.1 correctly calculate the difference of the annual fuel costs</p> <p>.2 divide their result by 1190</p> <p>.3 correctly write their ratio as percentage</p>	<p>AM1</p> <p>.1 $1190 - \text{their } 490$ or $\text{their } 490 - 1190$ or 700</p> <p>.2 $\frac{\text{their } 700}{1190}$ or 0.588235... OE</p> <p>.3 their58.8235... (%) ACCEPT 58.8 or 59 or 60</p>	3

	AM2 .1 divide their (7b) by 1190 .2 correctly write their ratio as percentage .3 correctly subtract their result from 100	AM2 .1 $\frac{\text{their}490}{1190}$.2 their41.17... .3 $(100 - \text{their}41.17\ldots =)\text{their}58.8235\ldots (\%)$ ACCEPT 58.8 or 59 or 60	
d	$T = 490n + 25000$.1 their25 000 as the y intercept .2 their490 as the gradient	ACCEPT $y = 490x + 25000$, $y = (14000 \times 0.035)x + 25000$	2

7	e	(1 mark)	(2 marks)	10
		Factors (F) Two factors from: Vehicle cost; ACCEPT Total cost Fuel (cost or usage of per mile) The number of miles driven per year Number of years of ownership or distances travelled maintenance cost or long term cost Effect on the environment	Three factors from: Vehicle cost; ACCEPT Total cost Fuel (cost or usage of per mile) The number of miles driven per year Number of years of ownership or distances travelled maintenance cost or long term cost Effect on the environment	
		Graph (G) Attempt to plot their V_A line from earlier results Ex: correct slope or T-intercept from their earlier results or At least two points plotted satisfy their earlier results with acceptable accuracy	Correctly plot their V_A line from their earlier results  ACCEPT T-intercept $\in [24000, 26000]$ and n at intersection $\in [9.5, 10.5]$	
		Number of years (N) Attempt to determine after how many years their V_A and V_B will have the same total cost Ex: Attempt to solve simultaneously or trial and improvement OR Correctly calculate the total cost after a specific number of years for both vehicles	Correctly determine after how many years their V_A and V_B will have the same total cost Ex: Correctly calculate $n=10$ after solving simultaneously or trial and improvement OR $n=10$ and their graph intersects at $n=10$	
		Justify (J) Weak justification Ex: V_A is better for the environment even if its initial cost is more OR V_A is better for the environment AND it will cost less on the long run or overall cost will be less OR	Good justification Ex: V_A is better for the environment AND V_A is better if years of ownership are more than their n at point of intersection or V_B is better if years of ownership are less than their n point of intersection on the graph	

		V _A is better for the environment AND a justification matching their graph OR V _B is better as their calculations show V _B costs less even if not good for environment		
	Comment on Accuracy (A)	Not very accurate with weak reason. Ex: I rounded the number of years to get the total cost Didn't include other factors of owning a vehicle e.g. running costs OR Accurate with good reason. Ex: The values did not require rounding so the total cost is accurate. The values were whole numbers OR Sensible rounding used	Not very accurate with good reason. Ex: Average mileage 14 000 may vary and will affect the total cost	

Question	Answers	Notes	Total														
8 a	.1 identify the base of the triangle .2 correctly substitute into Pythagoras OR trig ratio AG $\sqrt{2}$.1 2-1 or 1 seen .1 ACCEPT triangle indicated on diagram .2 $\sqrt{1^2 + 1^2}$ OR $\sin 45 = \frac{1}{\sqrt{2}}$ or $\cos 45 = \frac{1}{\sqrt{2}}$.2 ACCEPT $\sin 45$ and $1, 1, \sqrt{2}$ seen .2 ACCEPT explanation in words	2														
b	correctly place $16\sqrt{2}$ and $32\sqrt{2}$	<table border="1"> <thead> <tr> <th>Stage (n)</th> <th>Diagonal (D)</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>$\sqrt{2}$</td> </tr> <tr> <td>2</td> <td>$2\sqrt{2}$</td> </tr> <tr> <td>3</td> <td>$4\sqrt{2}$</td> </tr> <tr> <td>4</td> <td>$8\sqrt{2}$</td> </tr> <tr> <td>5</td> <td>$16\sqrt{2}$</td> </tr> <tr> <td>6</td> <td>$32\sqrt{2}$</td> </tr> </tbody> </table>	Stage (n)	Diagonal (D)	1	$\sqrt{2}$	2	$2\sqrt{2}$	3	$4\sqrt{2}$	4	$8\sqrt{2}$	5	$16\sqrt{2}$	6	$32\sqrt{2}$	1
Stage (n)	Diagonal (D)																
1	$\sqrt{2}$																
2	$2\sqrt{2}$																
3	$4\sqrt{2}$																
4	$8\sqrt{2}$																
5	$16\sqrt{2}$																
6	$32\sqrt{2}$																
c	.1 correctly describe one pattern for D in words with correct terminology .2 correctly describe another pattern for D in words with correct terminology	ACCEPT Multiplying by 2, or doubles WTTF $\sqrt{2}$ is always there or number under root always the same WTTF Increasing by 1, 2, 4... Powers of 2	2														

		<p>DO NOT ACCEPT All even numbers or multiples of 2 Exponential or geometric General rule in words $2^{n-1}\sqrt{2}$ OE D is increasing Note for more than two different patterns All correct award 2 marks Ex: D is increasing, even numbers and $\sqrt{2}$ is always there At least one correct award 1 mark Ex: even numbers and $\sqrt{2}$ is always there and add 2</p>	
	d	<p>.1 the correct general rule .2 the correct simplified general rule with correct notation</p> <p>.1 $2^{n-1}\sqrt{2}$ OE .2 $D = 2^{n-1}\sqrt{2}$ OE ACCEPT using D instead of D</p>	2
	e	<p>.1 correctly substitute $n \geq 5$ into their general rule .2 correctly calculate their value of S after substituting $n \geq 5$.3 recognise that their correctly calculated value of S is the same as their predicted value</p> <p>.1 Ex: $2^{5-1}\sqrt{2}$.2 Ex: $16\sqrt{2}$ (for n=5) •3 Same as value I predicted in table (and we find the candidate has $16\sqrt{2}$ in the table for $n = 5$) OR same as when we continue the pattern and explains how $16\sqrt{2}$ is obtained from pattern of multiplying the 8 by 2 •3 ACCEPT seeing the $16\sqrt{2}$ in the table and seeing their calculated $D = 16\sqrt{2}$ when $n = 5$</p>	3

8	g	Mark	1	2	3	4	
		Predictions (P)	<p>Correctly predict two terms for P OR Correctly predict 4 terms without square root provided that $\sqrt{2}$ is in their response ACCEPT whether in the table or in the response box</p>	<p>Correctly predict four terms for P ACCEPT whether in the table or in the response box</p>			
		Description (D)	<p>Attempt to describe a pattern in words for P OR a rule in words Ex: Multiplying by 2 every time Related to powers of 2 OR Attempt to describe pattern for P as general rule Ex for rule attempt: $2^{n-1}\sqrt{2}$ or $2^{n+1} + D$ $4s + s\sqrt{2}$ OR Correctly describe in words their pattern</p>	<p>Correctly describe one pattern in words for P Ex: The coefficient of $\sqrt{2}$ doubles each time adds to number multiplied by 2 OR Correctly describe the pattern for P as a general rule Ex: $(P =) 2^{n+1} + 2^{n-1}\sqrt{2}$ OE ACCEPT the general rule completely not simplified</p>	<p>Correctly describe one pattern in words for P Ex: The coefficient of $\sqrt{2}$ doubles each time AND Correctly describe the pattern for P as a general rule Ex: $(P =) 2^{n+1} + 2^{n-1}\sqrt{2}$ OE ACCEPT the general rule completely not simplified</p>		20
		Testing (T)	<p>Attempt to test their general rule for P using $n \leq 4$ Ex: correctly substitute in their general rule value of $n \leq 4$</p>	<p>Correctly test their general rule for P using $n \leq 4$ Ex:</p>			

		OR Correctly test their described pattern or their rule (e.g. recursive rule)	Correctly calculate their value for P in their general rule using $n \leq 4$ AND Recognise that their correctly calculated value for P is the same as the given value. ACCEPT seeing their correctly calculated value for P and the given value in the table being equal		
	Verifying (V)	Attempt to verify their general rule for P using $n \geq 5$ Ex: correctly substitute in their general rule value of $n \geq 5$ OR Correctly verify their described pattern or their rule (e.g. recursive rule)	Correctly calculate their value for P in their general rule using $n \geq 5$	Correctly calculate their value for P in their general rule using $n \geq 5$ AND Recognise that their correctly calculated value for P is the same as their predicted value obtained by continuing the pattern ACCEPT seeing their correctly calculated value for P and their predicted value in the table being equal	
	Justify/proof (J)	Attempt to justify their described pattern or their general rule Examples: trying at least two more values and arguing as justification that they are the same or rule works OR It is a geometric sequence or exponential used as justification OR	Justify their general rule arithmetically Examples: It is a geometric sequence with first term $4 + \sqrt{2}$ and ratio 2 Setting equation for a given term and solve it correctly for n OR Attempt to justify their general rule geometrically Ex:	Good attempt to justify the general rule for P geometrically by using incorrect four lengths in terms of n Ex: adding incorrect sides in terms of n $2^{n+1} + 2^{n+1} + 2^n + 2^{n-1}\sqrt{2}$ OE	Correctly justify the general rule for P geometrically by using correct lengths in terms of n Ex: $2^{n-1} + 2^{n-1} + 2^n + 2^{n-1}\sqrt{2}$ or $4(2^{n-1}) + 2^{n-1}\sqrt{2}$ OE J4 automatically gains T2 and V3

		Recognise that P is the sum of 4S+D used as justification	weak attempt to get perimeter using three lengths in terms of n			
	Notation and terminology (N)	<p>Correct notation of <u>their</u> rule OR Correct terminology describing at least one pattern</p> <p>DO NOT ACCEPT if they don't have any rules and they don't describe any patterns</p> <p>ACCEPT $(P =) 2^{n+1} + 2^{n-1} \times \sqrt{2}$ OE</p>	<p>Correct notation of <u>the general</u> rule for P OR The notation of <u>the general</u> rule includes errors AND Correct terminology describing at least one pattern</p> <p>DO NOT ACCEPT if they don't have a general rule</p>	<p>Correct notation of <u>the general</u> rule for P AND Correct terminology describing at least one pattern</p> <p>The general rule: $P = 2^{n+1} + 2^{n-1} \sqrt{2}$ OE</p> <p>ACCEPT the general rule partially simplified but DO NOT ACCEPT the general rule completely not simplified</p> <p>For notation of general rule,</p> <p>DO NOT ACCEPT $P = 2^{n+1} + 2^{n-1} \sqrt{2}$ OE $P = 2^n * 2 + 2^n / \sqrt{2}$ OE $(P =) 2^{n+1} + 2^{n-1} \times \sqrt{2}$ OE</p> <p>The rule for P is $2^{n+1} + 2^{n-1} \sqrt{2}$ OE</p>		
	Communication (L)	<p>Very weak communication Two or three lines of communication OR Only calculations or algebraic steps</p>	<p>Weak communication More than three lines of communication but lack coherence</p>	<p>Good communication More than three lines of coherent communication</p> <p>Can be awarded only if J2 is achieved</p>		

Predictions

Stage (n)	Side (S)	Diagonal (D)	Perimeter (P)
1	1	$\sqrt{2}$	$4 + \sqrt{2}$
2	2	$2\sqrt{2}$	$8 + 2\sqrt{2}$
3	4	$4\sqrt{2}$	$16 + 4\sqrt{2}$
4	8	$8\sqrt{2}$	$32 + 8\sqrt{2}$
5	16	$16\sqrt{2}$	$64 + 16\sqrt{2}$
6	32	$32\sqrt{2}$	$128 + 32\sqrt{2}$
7	64	$64\sqrt{2}$	$256 + 64\sqrt{2}$
8	128	$128\sqrt{2}$	$512 + 128\sqrt{2}$

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The markscheme may make use of the following abbreviations:

RM Assessor has the following annotations that should be used to award marks:

A0 only use to award a zero mark for an answer that has no merit e.g. awarded for the candidate that has a wrong answer with no working

NR only use when the candidate has not made any response also stamp the response with 

 Marks awarded by stamping the tick

 Seen; must be stamped on all blank response areas and on concatenated responses

 unclear

- Bullet notation means award 1 mark – see example 1 below

ECF Marks that can be awarded as **error carried forward** from previous results in the question

BOD Benefit of the doubt

MR misread

NWS no working shown

SC special case

OE or equivalent

WTTE or words to that effect or accept incomplete calculator display

AG Answer given

Example 1

- 1 mark awarded and corresponding notes are aligned

b	<ul style="list-style-type: none">• Show clear line of reasoning in the method• 4	45 & 49 seen OE eg, $49 = 45 + x$ ACCEPT $45 + X/10 = 4.9$ and Ans 4	2
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Task 1

		Answers	Notes	Total
1		<ul style="list-style-type: none"> •¹ set up of one correct equation •² set up another correct equation •³ correct working elimination or substitution or trial and improvement •⁴ $b = 35$ •⁵ $w = 100$ 	<ul style="list-style-type: none"> •¹ $w + b = 135$ •² $2w + 3b = 305$ •³ $w + 2b = 270$ or $2(135 - b) + 3b = 305$ An elimination or substitution step or evidence of trial and improvement must be seen <p>For correct answers with evidences of trial and improvement method: Award 3 marks (marks •³ •⁴ •⁵) Marks •¹ and •² can still be awarded for two equations seen</p> <p>For correct answers with no working: Award 2 marks For •¹ and •² accept equations in words OE Equations seen (•¹ and •² awarded), then for correct answers with no further working award 4 marks in total</p>	5

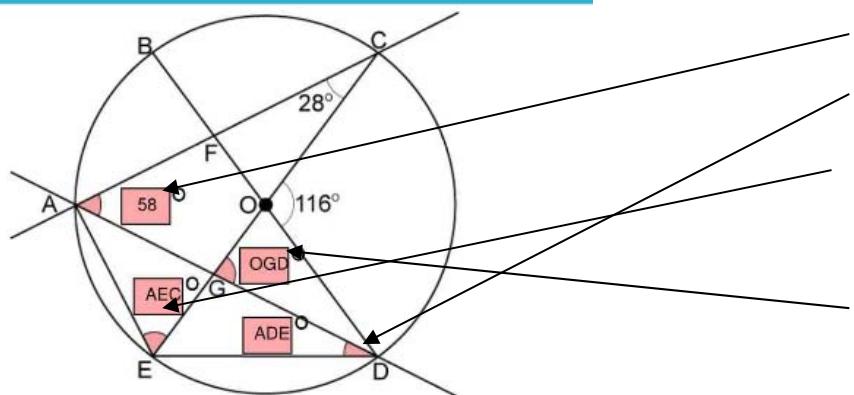
2	<p>a</p> <ul style="list-style-type: none"> •¹ two correct •² three correct •³ four correct 		3
b	<ul style="list-style-type: none"> •¹ correct expression for the total area •² equating their expression with 210 •³ their expression - 210 = 0 •⁴ correct factorisation of their equation, or correct substitution into the quadratic formula for their equation •⁵ their correctly calculated positive value of x selected 	<ul style="list-style-type: none"> •¹ $x^2 + 3x + 2x + 6$ OR $x^2 + 5x + 6$ OR $(x+3)*(x+2)$ •² $x^2 + 3x + 2x + 6 = 210$ OR $x^2 + 5x + 6 = 210$ •³ $x^2 + 5x - 204 = 0$ •⁴ $(x + 17)(x - 12) = 0$ •⁵ $x = 12$ Footnote: The positive x value has to be selected in order to award •⁵ $x = 12$ without working award 4 marks $x = 12$ with trial and error working award 4 marks $x = 12$ with incorrect working award 0 marks $x = 12$ with one correct algebraic step award 5 marks SC: Candidate who does not put their expression =210 and then solves $x^2 + 5x + 6 = 0$ award 2 marks 	5

3	a	$\frac{2 \times 1 + 3 \times 4 + 4 \times 3 + 5 \times 6 + 6 \times 4 + 7 \times 2}{20} = \frac{94}{20}$ $\frac{2 \times 1 + 3 \times 4 + 4 \times 3 + 5 \times 6 + 6 \times 4 + 7 \times 2}{20} = \frac{94}{20}$ <p> \bullet^1 multiplying at least three grades by their frequency OR evidence of adding repeated grades \bullet^2 adding correctly the values of grade multiplied by the frequency or adding correctly the repeated grades \bullet^3 dividing by 20 \bullet^4 94/20 4.7 AG </p>	<ul style="list-style-type: none"> \bullet^1 seeing 2×1 and 3×4 and 4×3 for example \bullet^2 adding $2 \times 1 + 3 \times 4 + 4 \times 3 \dots$ for example \bullet^4 do not accept any other value but 94/20 (OE) <p>Footnote: When a calculator screenshot is taken, accept not seeing the whole fraction (WTTE)</p> <p>94/20 (OE) with any evidence of correct working from the above award 4 marks</p> <p>94/20 (OE) with no working award 3 marks</p>
	b	<p> \bullet^1 Evidence of working to find the middle value. \bullet^2 5 </p>	<ul style="list-style-type: none"> \bullet^1 attempt to add $1 + 4 + 3$ or $2 + 4 + 6$ or writing the repeated grades in order \bullet^2 5 with no working award 2 marks
	c	$\frac{6}{20} \times \frac{8}{20} = 0.12$ <p> \bullet^1 6/20 OR \bullet^1 8/20 \bullet^2 multiplies by $\frac{8}{20}$ OR \bullet^2 multiplies by 6/20 \bullet^3 their value after multiplying fractions </p>	<p> \bullet^3 $\frac{3}{25}$ OR $\frac{48}{400}$ OR 0.12 OR their values correctly multiplied $\frac{48}{400}$ or 0.12 with no working award 2 marks </p> <p>Footnote: All fractions must be less than 1. Adding $6/20 + 8/20 = 14/20$ award 1 mark only (for the 6/20 seen)</p>

	d $\frac{\text{their}(48 / 400)}{\text{their}(6 / 20)} = \frac{8}{20}$ $\frac{\text{their}(48 / 400)}{\text{their}(6 / 20)} = \frac{8}{20} \text{ OR } \frac{2}{5} \text{ OR } 0.4$ <ul style="list-style-type: none">•¹ seeing their 6/20•² dividing their (48/400) by their (6/20)•³ the correct value after division	ECF from part (c) $\frac{8}{20}$ OR 0.4 Correct answer without working award 2 marks	3
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4	a	• (DAC =) 58 (degrees)		1
	b	• (ADE =) 28		1
	c	• ¹ $180 - (90 + 28)$ OR $90 - 28$ • ² (AEC =) 62	62 with no working award 2 marks	2
	d	• ¹ (OED =) 58 • ² (OGD =) sum of their 58 OED and their 28 ADE • ³ Their = 86 OR Their 58(their DAC)+28 = 86 • ¹ Seeing 58 • ² Adding 28 • ³ 86	• ¹ seeing 58 • ² (OGD =) $58 + 28$ 86 with no working award 2 marks	3

s Object



- a) 58
b) 28
c) 62
d) 86 $180 - (58 + 28) = 180 - 86$
or exterior angle = $58 + 28$

<p>5</p> <ul style="list-style-type: none"> •¹ $\tan 22$ •² substitutes correctly into tan ratio •³ calculates the height above the theodolite •⁴ (their) height + 1.2 •⁵ rounds their height correctly to the nearest cm <p>Alternatively</p> <ul style="list-style-type: none"> •¹ Substitutes correctly into cosine ratio •² Calculates the length of the hypotenuse •³ calculates the height above the theodolite (using Pythagoras) •⁴ (their) height + 1.2 •⁵ rounds their height correctly to the nearest cm 	<ul style="list-style-type: none"> •¹ accept not seeing this step, can be implied by •² •² $\tan 22 = \frac{\text{height}}{57.25}$ •³ height = $57.25 \times \tan 22 = 23.1305\dots$ •⁴ $23.1305\dots + 1.2 = 24.3305\dots$ •⁵ 24.33 (m). Accept 2433 (cm) Accept seeing rounding to nearest cm in earlier step Footnote: WTTE accept incomplete calculator display <p>Alternatively</p> <ul style="list-style-type: none"> •¹ $\cos(22) = \frac{57.25}{H}$ •² hypotenuse=61.746... •³ $\sqrt{(\text{their } 61.746\dots)^2 - 57.25^2} = 23.1305\dots$ •⁴ (their 23.1305)+1.2 •⁵ 24.33 (m). Accept 2433 (cm) Accept seeing rounding to nearest cm in earlier step <p>23.1305 with no working award 2 marks 23.13 with no working award 3 marks 24.33 with no working award 4 marks 24.33 with one correct step award 5 marks Footnote: WTTE accept incomplete calculator display</p>	<p>5</p>
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	b	<ul style="list-style-type: none"> •¹ attempt to use cosine rule (the selection of appropriate mathematics) •² correct substitution in cosine rule •³ correctly calculating BC from their rule •⁴ correctly approximating their value to the nearest cm 	<ul style="list-style-type: none"> •¹ Evidence of using cosine rule even if not completely correct •² $BC^2 = 62.31^2 + 71.54^2 - 2 \times 62.31 \times 71.54 \times \cos 10.2$ •³ $BC = 15.036423\dots$ •⁴ $BC = 15.04\text{m}$ OR 1504cm <p>Incorrect cosine rule can be awarded •¹. •⁴ can also be awarded for correct rounding. For incorrect use of Pythagoras only •⁴ can be awarded.</p> <p>15.04 with no working award 3 marks 15.036423 with no working award 2 marks</p>	4
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Task 2		Answers	Notes	Total
6	a	<ul style="list-style-type: none"> •¹ substitutes $y = 65000$ into the equation •² solving their equation correctly for x •³ their value approximated correctly to the nearest million 	<ul style="list-style-type: none"> •¹ $65000 = 2110 x - 52818$ •² $x = 55.8379\dots$ •³ 56 million 55 with no working award 0 marks 55.8379... with no working award 1 mark 56 million with no working award 2 marks 	3
	b	<ul style="list-style-type: none"> •¹ substituting correctly their value in 6(a) into the percentage error formula •² their percentage correctly calculated •³ their negative percentage written as +ve 	<ul style="list-style-type: none"> •¹ (<i>percentage error</i> =) $\frac{\text{their } 56000000 - 68488000}{68488000} \times 100$ •² -18 % Accept not seeing this step. And award it if they make the correct calculation for their formula •³ 18 % Final answer must be positive and does not need to be rounded Footnote: Award •³ only if their calculation gives a negative value Accept evidence of substitution seen in calculator screenshot (even if incomplete due to screenshot limitations) (WTTE) -18% with no working award 1 mark 18% with no working award 2 marks 	3
	c	<ul style="list-style-type: none"> •¹ referring to gradient or slope or constant increase or steady increase •² referring to gradient (or slope) being 2110 which is nearly 2000 	<ul style="list-style-type: none"> •¹ Accept: constant line steepness or proportional or rise over run DO NOT ACCEPT just referring to increase or positive relationship •² SC: comparing with a numerical example (other than the given 1000000 allows 2000 jobs) and showing that it is approximately true: Award 1 mark. The increase in passengers must be accompanied by a corresponding increase in jobs to award this SC. 	2
	d	<ul style="list-style-type: none"> •¹ substitutes ($r =$) 1.5 into the equation •² re-arranges the equation for x or x^2 OR $3.548\dots$ seen •³ ($x =$) 4 million 	<ul style="list-style-type: none"> •¹ Accept not seeing this step. $1.5 = 2\log(x) + 0.4$ •² Award 2 marks for $10^{\frac{1.5-0.4}{2}}$ OR 3.55 million 4 with no working award 2 marks 	3

6	e						10
		Aspect	1 mark	2 marks	3 marks	4 marks	
IR: Identification of relevant information	One numerical factor mentioned from: Length of runway Number of jobs Economic activity Size of aircraft Number of passengers Safety considering the measured distance from houses	More than one numerical factor mentioned Length of runway Number of jobs Economic activity Size of aircraft Number of passengers Safety considering the measured distance from houses					
CM: Calculations	showing approximately correct numbers without showing calculations	Attempts to use the runway length (approximately 3.5) to find number of passengers OR Attempts to use the number of jobs assumed (18000) to find the number of passengers SC: writing approximate values with reference to tab1 and tab2	Uses correctly the runway length (approximately 3.5) to find number of passengers OR Uses correctly the number of jobs assumed (25000) to find the number of passengers	Uses correctly the runway length (approximately 3.5) to find number of passengers AND Uses correctly the number of jobs assumed (25000) to find the number of passengers			
JD: Justification of degree of accuracy	Rounding used in any element (rounding to nearest million or 1 dp is accepted but not to 2dp) OR Referring to percentage error	Justifies their choice of rounding OR the implications of working with rounded values OR refers to the limitations of the data by attempting to calculate possible percentage error					

		<p>PD: sustainability of proposed airport</p> <ul style="list-style-type: none"> • Safety: It is in a safe area and referring to distance from houses or distance from river or both (even if they measure the distance). Environmental impact (pollution) • Length of runway: around 3.5 and possible increase in the future based on the map • Room for expansion: there is land to add more runways in the future and increase the number of jobs and/or economic activity. • Economical expansion outside the airport • Facilities and services: availability of transport networks 	<p>It is sustainable or it is not sustainable or a balanced discussion around sustainability supported by a relevant comment from:</p> <ul style="list-style-type: none"> • Safety: It is in a safe area and referring to distance from houses or distance from river or both (even if they measure the distance). Environmental impact (pollution) • Length of runway: around 3.5 and possible increase in the future based on the map • Room for expansion: there is land to add more runways in the future and increase the number of jobs and/or economic activity. • Economical expansion outside the airport • Facilities and services: availability of transport networks 		
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mathemoeengtz0xxxx

Further guidance:

For 25000 jobs, the number of passengers is $(25000+52818)/2110 = 36.88$ millions

From the length of runway 3.5, the number of passengers is 35.481m... and hence number of jobs 22046

7	a	<ul style="list-style-type: none"> •¹ substituting 65 correctly into the area formula •² 4225π or 13273.2289... •³ substituting their Area and 30 correctly into the P_A formula •⁴ calculating correctly their value of P_A •⁵ substituting correctly their value of P_A into the P_E formula •⁶ their value of P_E calculated correctly in watt •⁷ writing their value of P_E to nearest kw 	<p>Allow their area using π or 3.14 or 22/7 •¹ (Area =) $\pi \times 65^2$ · Accept not seeing this step. 4225π with no working award 2 marks</p> <p>•³ $P_A = 0.6$ (their area)(30³)</p> <p>•⁴ (=) 215026309.2 or 214917300 or 215112857.1</p> <p>•⁵ $P_E = 0.45$(their 215026309.2 or 214917300 or 215112857)</p> <p>•⁶ (=) 96761839.13 or 96712785 or 96800785.71 (w)</p> <p>•⁷ (=) 96762 or 96713 or 96801 (kw)</p> <p>96762000 or 96713000 or 96801000 do not allow the •⁷ mark</p> <p>96761839.13 or 96712785 or 96800785.71 with no working award 5 marks</p> <p>96762 or 96713 or 96801 with no working award 6 marks</p> <p>96762 or 96713 or 96801 with one correct step seen award 7 marks</p> <p>Footnote: Substituting their area into incorrect formula does not allow •³ mark. •⁴ can be awarded as ECF only if their formula is not becoming easier (example: If they do not cube the velocity then the formula is easier and hence they are not awarded •⁴) The rest of the bullets (•⁵, •⁶, •⁷) can be awarded as ECF if appropriate</p>	7
b		<ul style="list-style-type: none"> •¹ 5/2 or 2.5 or 650 seen or 5×130 or $130/2$ or 65 •² For multiplying 2.5 by 130 or dividing the 650 by 2 or multiplying the radius of one ($130/2$ or 65) by 5 <p>325 AG</p>	Accept showing that half way is $130+130+65=325$ for 2 marks	2

7	c				10
		Aspect	1 mark	2 marks	
IR: Identification of strategy	One mentioned from: <ul style="list-style-type: none"> Consider the length or width and the diameter of turning zone SC not numerical: bases need to be staggered. Or accept "diagonally placed" Fitting three rows in the 1800 instead of only two Making turning zones closest possible Consider total area and area of one turbine Refer to turning zones 	More than one mentioned from <ul style="list-style-type: none"> Consider the length or width and the diameter of turning zone SC not numerical: bases need to be staggered. Or accept "diagonally placed" Fitting three rows in the 1800 instead of only two Making turning zones closest possible Consider total area and area of one turbine Refer to turning zones 			
CM a: Calculates number of wind turbines	Number of wind turbines : 10–13 Or can be seen on the canvas 10–13 turbines fitting inside the area OR more than 13 turbines but overlapping inside the area	Number of wind turbines 14 or more Or can be seen on the canvas 14 or more turbines fitting inside the area Accept seeing zones of turbines partially outside the area (strictly speaking the centre should be inside the area)			
CM b: Estimate Power Output	Attempts to substitute their area into formula of P_A and P_E	Calculates correctly their P_E = their number of turbines from their diagram x their P_E of one turbine			

7	c	PD: sustainability of wind farm	<p>A relevant comment from:</p> <ul style="list-style-type: none"> • The diameter of the turbine: For better use of this land maybe we need turbines with different diameter • The average wind speed: This area experiences an average wind speed 30 ms^{-1} and since the turbines do not operate beyond this wind speed of 30 then this means that the turbines will not be operating about half the time (at least). • Possibility of expanding the area: When providing an area there has to be consideration of possible extension and extra area added. OR as there is no more land to add more wind turbines so output cannot be increased above the maximum • Turning/turbulence zones of wind turbines: There should be no problem if the circular zone of turbines on the extremities go beyond the land because this will not affect their effectiveness but the problem is if they overlap inside the land because this will affect their effectiveness • Maintenance or environmental impacts or general relevant points 	<p>At least two relevant comments from:</p> <ul style="list-style-type: none"> • The diameter of the turbine: For better use of this land maybe we need turbines with different diameter • The average wind speed: This area experiences an average wind speed 30 ms^{-1} and since the turbines do not operate beyond this wind speed of 30 then this means that the turbines will not be operating about half the time (at least). • Possibility of expanding the area: When providing an area there has to be consideration of possible extension and extra area added. OR as there is no more land to add more wind turbines so output cannot be increased above the maximum • Turning/turbulence zones of wind turbines: There should be no problem if the circular zone of turbines on the extremities go beyond the land because this will not affect their effectiveness but the problem is if they overlap inside the land because this will affect their effectiveness • Maintenance or environmental impacts or general relevant points 	
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7	c	JD: Justification of degree of accuracy	Rounding used in any element OR Simple comment like: <ul style="list-style-type: none">• I used the actual answer not the rounded one from 7a) so the accuracy is exact• My answer is not accurate as I think I can fit more turbines in the area	Justifies their choice of rounding OR the implications of working with rounded values Justified comment like: <ul style="list-style-type: none">• Wind speed cannot be guaranteed so the P_E is not reliable• I used rounded answer form 7a) and the power output may differ by ... (and they calculate the value) if the answer was not rounded		
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Task 3 (total 39 marks)

Answers			Notes	Total
8	a	<ul style="list-style-type: none"> •¹ relation between x-coordinates: states the x values of A is the same as C but negative (or opposite) •² relation between y-coordinates: states the y values of A and C are the same OR both 0 	<p>SC: Any correct comment(s) about the location award 1 mark Example: The x coordinates of both are equally apart from the y-axis OR They are both on the x-axis and equally distant from y-axis OR A and C are reflection of each other on the y-axis</p> <p>SC: Comment on both coordinates saying they (or both) are opposite signs or they are multiplied by -1 award 2 marks</p>	2
	b	<ul style="list-style-type: none"> the x coordinate of Q is half the x coordinate of C (or x-coordinate of C double x-coordinate of Q) 		1
	c	<ul style="list-style-type: none"> •¹ ($Q_4 =$) (5,3) •² ($Q_5 =$) (6,3) •³ ($Q_6 =$) (7,3) 		3
	d	<ul style="list-style-type: none"> •¹ one correct term •² fully correct $X_c = 2n + 2$	Accept equivalent expressions with any correct rearrangement or not simplified for 2 marks example: $4+2(n-1)$ OE award 2 marks Award 1 mark for each correct term	2
	e	<ul style="list-style-type: none"> • $X_Q = (2n + 2)/2$ or $n + 1$ OE 	Allow ECF from (d)	1
	f	<ul style="list-style-type: none"> •¹ substitutes a number $n \geq 4$ into their equation from (e) •² compares with the corresponding value in the table for $4 \leq n \leq 6$ or compares with predictions for $n \geq 7$ •³ acknowledges that the two values above are equal 	SC if “tested” correctly with a value of $n \leq 3$ award 1 mark	3
	g	<ul style="list-style-type: none"> • (Midpoint =) $(y_1 + y_2)/2$ AND $(0 + 6)/2$ 	Formula must be seen	1

	h	$\begin{array}{r} 3-0 \\ \hline 2-4 \end{array}$ <ul style="list-style-type: none">•¹ for numerator 3 – 0 or 0-3 or vertical distance correctly described in words•² for denominator 2– 4 or -4-2 or horizontal difference correctly described in words	Accept inappropriate notation (example: 3-0/2-4)	2
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8	i									24
		Mark	Predictions (P)	Description (D)	Testing (T)	Verifying (V)	Justify/pro of (J)	Notation and terminology (N)	Communication (L)	
		1	Attempted to make predictions for any set of data	Attempted to describe a pattern	Attempted to test their described pattern or general rule for $n \leq 3$	Attempted to verify their general rule for $n \geq 4$ (ex: substitutes in their formula $n \geq 4$)	Attempted to justify their described pattern or general rule	The notation and terminology have significant errors	No communication Only calculations or algebraic steps	
		2	Correctly predicted terms for different sets of data	Correctly described one pattern (or a simple pattern)	Tested correctly their general rule for $n \leq 3$	Calculates correctly their value for an $n \geq 4$ and mentions the corresponding value in the table	Justified the general rule for the square root or the length AQ Award only if D4 is achieved	The notation and terminology are mostly correct Award only if D4 is achieved	Weak communication	
		3	Correctly predicted most of terms for all sets of data	Correctly described pattern for the square root or the length AQ		Comment comparing the values above to verify	Correctly proved the general rule for the square root		Good communication Award only if J2 is achieved	
		4	Correctly predicted up to $n=6$ for all sets of data	Attempted to describe a correct pattern as general rule for the square root or the length AQ			Correctly proved the general rule for the length AQ			
		5		Correctly described pattern as general rule for the square root						
		6		Correctly described pattern as general rule for the length AQ						
SC If only the set of lengths is predicted correctly and all others are empty or wrong award (2 marks).										

Markscheme

November 2023

Extended mathematics

On-screen examination

28 pages

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The markscheme may make use of the following abbreviation: OE – ‘or equivalent’

The markscheme abbreviations:

- Bullet notation means award 1 mark – see example below

Example 1 .1 mark awarded and corresponding notes are aligned			
b	.1 Show clear line of reasoning in the method .2 4	.1 45 and 49 seen OE Ex: $49 = 45 + x$.2	2

Error Carried Forward (ECF) marks

Errors made at any step of a solution affect all working that follows. In general, **Error Carried Forward (ECF)** marks are awarded after an error.

- a) **ECF** applies from one part of a question to a subsequent part of the question and also applies within the same part.
- b) If an answer resulting from **ECF** is inappropriate (eg, negative distances or $\sin x > 1$) then subsequent marks should not be awarded.
- c) If a question is transformed by an error into a **simpler question** then **ECF** may not be fully awarded.
- d) To award **ECF** marks for a question part, **there must be working present for that part**.
- e) **ECF** is only applied to working which is correct. This means that all working subsequent to an error must be checked for accuracy.
- f) A misread (**MR**) is an error. **ECF** is normally awarded.

General points

- a) As this is an international examination, accept all **alternative forms of notation**, for example 1,9 as 1.9 ; 1,000 or 1.000. However, **DO NOT ACCEPT** incorrect mathematical notation x^2 for x^2 unless noted otherwise in the MS.
- b) **ACCEPT** notation errors in intermediate steps.
- c) Ignore further working after a correct answer **unless** it indicates a lack of mathematical understanding **i.e. if the further working contradict the correct answer**, then that last mark cannot be awarded.
- d) In the case when a correct result is obtained using incorrect seen method, do not award the mark for the result.
- e) Where candidates have written two solutions to a question, mark the first solution.
- f) In the markscheme, equivalent examples of **numerical** and **algebraic** forms or **simplified** answers will generally be written in the notes preceded by **OE** (Or Equivalent) e.g. $\frac{1}{2}$ **OR** 1/2 **OR** 0.5 **OR** $2 \div 4$; $\frac{x}{2}$ **OR** $x / 2$ or $x \div 2$; 0.23 **OR** 23%
- g) In the markscheme, information provided in brackets indicate detail that may be seen in a candidate response but is not necessary to award the marks. However, it indicates what the candidate's result represents. Ex: if last mark is for the result: (AB)=5; this means we award the mark for seeing 5 as the result of calculating AB without necessarily seeing AB=5, but it does not mean we award the mark for seeing 5 representing another length
- h) Special case marks **SC** can be allocated instead of but not in addition to the marks prescribed in the markscheme.
- i) Accept seeing equation not in-line.
- j) Calculator screenshots are accepted as working steps. And when a calculator screenshot is taken, accept not seeing the whole operation.
- k) In task 2 and 3 where the markscheme is set out in a table then, unless noted otherwise, awarding the highest mark in a category includes all the lower marks in that category. It is probably best to look for the top category mark answer and if you don't find it look at the next mark down.
- l) **ACCEPT** using the correct values or working regardless their previous result.
- m) Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. **If in doubt, contact your team leader for advice.**
- n) Unless noted otherwise, if a note in a part says to accept the answer without working for 1 mark less than total marks, then seeing the correct answer with any acceptable working step, award full marks. Example: If the note in a part worth 3 marks says "4.3(3...) without working award 2 marks", then seeing any acceptable working step and seeing 4.3(3...) as the answer award the 3 marks.
- o) For "**show that**" questions, unless otherwise noted, every bullet point has to be seen in order to be awarded.

Q1	Answers	Notes	Total
a	<p>AM1</p> <p>.1 Two from:<ul style="list-style-type: none"> • Subtract correct y-coordinates • Subtract correct x-coordinates • Divide their difference in y by their difference in x </p> <p>.2 Correctly calculate gradient of AB .3 Correctly calculate gradient of AD .4 Showing that the two gradients are opposite reciprocals</p> <p>AM2</p> <p>.1 Correct length of AB .2 Correct length of AD .3 Correct length of BD .4 Show that the value of AB^2+AD^2 is equal value of BD^2</p> <p>AM3</p> <p>.1 Two from:<ul style="list-style-type: none"> • Subtract correct y-coordinates • Subtract correct x-coordinates • Write vector as (x,y) OE </p> <p>.2 Correct vector AB .3 Correct vector AD .4 Show that the dot product =0</p>	<p>AM1</p> <p>.1 Two from (for AB or AD). Ex for AB:<ul style="list-style-type: none"> • 10-4 or 4-10 or 6 or -6 • 9- -3 or -3-9 or 12 or -12 • $\frac{\text{their}(10-4)}{\text{their}(9--3)}$ </p> <p>.2 6/12 OE .3 -2 OE .4 "they are <u>opposite reciprocals</u>" WTTE or $0.5x-2=-1$.4 ACCEPT negative instead of opposite</p> <p>AM2</p> <p>.1 $\sqrt{6^2 + 12^2}$ OE or 13.4(16...) OE .2 $\sqrt{4^2 + 8^2}$ OE or 8.9(44...)OE .3 $\sqrt{8^2 + 14^2}$ OE or 16.1(2...) (OE) .4 Ex: $260=180+80$</p> <p>AM3</p> <p>.1 Two from:<ul style="list-style-type: none"> • 10-4 or 4-10 or 6 or -6 • 9- -3 or -3-9 or 12 or -12 • (Their12,their6) OE </p> <p>.2 (12,6) .3 (4,-8) .4 $12 \times 4 - 6 \times 8 = 0$</p>	4

	b	<p>.1 Correct equation involving x and their gradient of AB used in a) .2 Correct value of x</p>	<p>.1 their $\frac{6}{12} = \frac{2-4}{x-1}$ OE or $\frac{8}{-4} = \frac{10-2}{9-x}$ OE or $9-x = -4$ OE .2 ($x = 13$)</p>	2
	c	<p>.1 Correctly substitute into distance formula .2 Correctly calculate their length of AB after substitution into distance formula .3 Correctly write their AB in simplest surd form</p>	<p>.1 $\sqrt{(9-3)^2 + (10-4)^2}$ OE .2 their $\sqrt{180}$ or their 13.4(164...) .2 ACCEPT their AB correctly calculated after one mistake examples: $(\sqrt{(9-3)^2 + (10-4)^2}) = 12.2(47...)$ or $(\sqrt{(9-3)^2 + (10-4)^2}) = 48$.3 their $6\sqrt{5}$.3 DO NOT ACCEPT if their surd does not need simplification</p>	3
	d	<p>.1 Correctly substitute into mid-point formula for x OR y coordinates .2 Correct coordinates of point E</p>	<p>.1 $\frac{1+9}{2}$ or $\frac{-3+13}{2}$ OR $\frac{10-4}{2}$ or $\frac{4+2}{2}$.2 (5, 3) OE ACCEPT 5,3</p>	2

Q2	Answers	Notes	Total
a	<p>.1 Correctly write the first simultaneous equation</p> <p>.2 Correctly write the second simultaneous equation</p> <p>.3 Correct steps to solve the system of equations for one unknown</p> <p>.4 Correctly substitute $x=0.5$ or $y=3$ into one of the equations and show calculation of the other AG $x=0.5$ and $y = 3$</p>	<p>.1 $6x+3y = 12$ ACCEPT using inequality</p> <p>.2 $6x + 8x + 4y = 19$ or $14x + 4y = 19$ ACCEPT using inequality</p> <p>.3 By elimination. Example: $4 \times (6x + 3y = 12)$ and $3 \times (14x + 4y = 19)$ ACCEPT “4 times eq1 and 3 times eq2” And $42x-24x=57-48$ OE Or by substitution $y = 4-2x$ or $x = 2-0.5y$ or $4y = 19-14x$ or $14x = 19-4y$ OE And $6x+3(4-2x)=12$ OE .3 DO NOT ACCEPT if the solution is incorrect</p> <p>.4 Ex: $6(0.5)+3y=12$ and $y=9/3$ OE ACCEPT showing simplified equality with one variable .4 ACCEPT substituting 0.5 and 3 into one equation Ex: $6(0.5)+3(3)=12$</p>	4
b	<p>.1 Correct route</p> <p>.2 Correct minimum distance for the correct path</p>	<p>.1 $3+4+12+8+7+9+5+14$ OE ACCEPT in letters .1 ACCEPT <u>passing by all paths</u> Ex: ABDECEBCA or ACEDBECBA .1 ACCEPT if only 6 of the paths are added Ex: $3+4+12+8+5+14$.1 ACCEPT use of the same value (I.e. repeating a paths)</p> <p>.2 62 (km)</p>	2
c	<p>.1 Correctly add a route with adjacent nodes and passing by all the nodes</p> <p>.2 Correct route</p> <p>.3 Correct minimum distance</p>	<p>.1 Ex: $3+4+12+8+7+9+5+14 = 62$ ACCEPT in letters .2 ABDECBA or ABCEDBA OE ACCEPT $3+4+12+7+5+3$ or $3+5+7+12+4+3$.2 ACCEPT rout passing by all nodes and with a seen <u>total of 40 or less</u> Ex: the route ABDECA (40) or ABCEBDBA (36) for bp2</p> <p>.3 $(3+4+12+7+5+3=)34$ (km)</p>	3

Q3 Ext		Answers	Notes	Total
	a	Apply power rule on at least one term and show equality or ratio 2 AG	Ex: $2x\log_2 = \log_2^2 = \log_4$ ACCEPT $\log_4/\log_2 = 2\log_2/\log_2$ ACCEPT using other terms Ex: $\log_{256}/\log_{16} = 8\log_2/4\log_2$ DO NOT ACCEPT only \log_4 / \log_2	1
	b	AM1 .1 Correctly apply law of product or law of quotient .2 Correctly write the total of left-hand side as one log .3 Correct write the fifth term in log form AM2 .1 Correctly write <u>two</u> of the three terms in terms of \log_2 .2 Correctly write the total of left-hand side in terms of \log_2 .3 Correct write the fifth term in log form	AM1 .1 $\log(1024*256)$ or $\log(256/4)$ OE .2 \log_{65536} ACCEPT $\log(1024x256/4)$.3 $\log 256^2$ OE .3 ACCEPT $\log_{65536}/\log_{256}=2$.3 ACCEPT calculating $n=5$ using the GS general rule and $r=2$ only if bp2 is awarded AM2 .1 $10\log_2 + 8\log_2 - 2\log_2$.2 $16 \log_2$ ACCEPT $10\log_2 + 8\log_2 - 2\log_2$.3 \log_2^{16} OE	3
	c	AM1 .1 Correctly write the first 6 terms in terms of \log_2 .2 Correctly write down the sum of first 6 terms in terms of \log_2 AM2 .1 Correctly substitute into the sum of GP formula .2 Correctly write down the sum of first 6 terms in terms of \log_2	AM1 .1 $\log_2, 2\log_2, 4\log_2, 8\log_2, 16\log_2, 32\log_2$.1 ACCEPT only 4 correct terms .2 $63\log_2$ AM2 .1 $(S_6 =) \log_2 \times (2^6 - 1)/(2 - 1)$ ACCEPT seeing only $(2^6 - 1)/1$ OE .2 $63\log_2$	2

Q4		Answers	Notes	Total
	a	<p>.1 Correctly substitute into the sine rule</p> <p>.2 Correctly calculate their s for their trig ratio</p> <p>.3 Their s correct to three significant figures</p>	<p>.1 $s/\sin 60 = 3/\sin 105$ $s = 3\sin 60/\sin 105$ OE</p> <p>.2 their 2.689(726...) ACCEPT 2.7</p> <p>.2 ACCEPT only if their seen trigonometric sine rule operation involves side=3 and two of the angles: 105, 60, 15</p> <p>.2 ACCEPT using the 3 sig fig rounded value of the trig function only if bp1 is awarded</p> <p>.3 their 2.69</p> <p>.3 ACCEPT their 2.69 only if their used sine rule is with maximum two mistakes and their bp2 can be rounded</p>	3
	b	1	ACCEPT (2,1) OE	1
	c	<p>.1 Correctly substitute 0 and 2.6 into the parabola</p> <p>.2 Correctly re-arrange their equation to find a AG 0.4</p>	<p>.1 $a(0 - 2)^2 + 1 = 2.6$ or $2.6 = 4a + 1$ ACCEPT substituting 4 and 2.6</p> <p>.2 $(a =) 1.6/4$ ACCEPT $1.6 = 4a$</p>	2

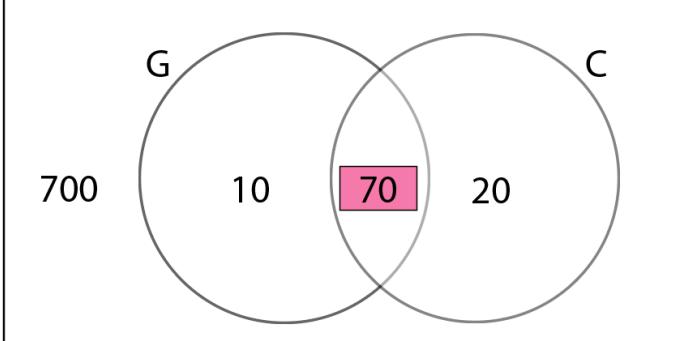
d	<p>AM1 (working with $(x - 2)$)</p> <p>.1 Correctly substitute 1.4 into the equation .2 Correctly rearrange to have $(x-2)^2$ on one side .3 Correctly square root both sides of their quadratic from .2 .4 Correctly identify 3</p> <p>AM2 (expanding $(x - 2)^2$)</p> <p>.1 Correctly substitute 1.4 into the equation .2 Correct quadratic after expansion and multiplying by 0.4 .3 Correctly factorise their quadratic from .2 or correctly substitute their coefficients into quadratic formula .4 Correctly identify 3</p> <p>AM3 (working with numbers)</p> <p>.1 Correctly substitute a value for x into the equation and calculate corresponding value of y .2 Correctly substitute 1 into the equation and equate with 1.4 .3 Correctly substitute 3 into the equation and equate with 1.4 .4 Correctly identify 3</p>	<p>AM1</p> <p>.1 $1.4 = 0.4(x-2)^2 + 1$ or $0.4 = 0.4(x-2)^2$.2 $0.4/0.4 = (x-2)^2$ or $1 = (x-2)^2$.3 $x-2 = \pm\sqrt{1}$ OE ACCEPT $x-2 = \text{their1}$ or $x-2 = -\text{their1}$.4 (L=) 3 (m)</p> <p>AM2</p> <p>.1 $1.4 = 0.4(x-2)^2 + 1$.2 $0.4x^2 - 1.6x + 1.6 + 1 = 1.4$ or $x^2 - 4x + 3 = 0$ OE .3 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ or $x = \frac{-1.6 \pm \sqrt{1.6^2 - 4(0.4 \times 1)}}{2(0.4)}$ OE .3 In the case when their bp2 is in the form $ax^2 + c = 0$, ACCEPT correctly <u>solving</u> their quadratic for x^2 Ex: if their bp2 is $0.4x^2 + 10 = 0$ and $x^2 = \frac{-10}{0.4}$.4 (L=) 3 (m)</p> <p>AM3</p> <p>.1 Ex: $0.4(5-2)^2 + 1 = 4.6$.2 $0.4(1-2)^2 + 1 = 1.4$.3 $0.4(3-2)^2 + 1 = 1.4$.4 (L=) 3 (m)</p> <p>For Any AM:</p> <p>Seeing only bp2 implies bp1 (so award 2 marks) Seeing only bp3 implies bp1 and bp2 (so award 3 marks)</p>	4
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Q5		Answers	Notes	Total
	a	$P(G' \cap C)$ $P(G' \cap C')$	<p>Probability of selecting a person who does not have two red hair genes and has red hair</p> <p>Probability of selecting a person who does not have two red hair genes and does not have red hair</p>	2

- .1 correct probability
- .2 correct statement

- .1 $P(G' \cap C)$ ACCEPT $P(C \cap G')$ ACCEPT using g instead of G and c instead of C
- .2 (Probability of selecting a person who) does not have two red hair genes and does not have red hair
 - .2 ACCEPT no red hair genes and no red hair WTTE
 - .2 ACCEPT not have two genes and not have red hair WTTE
 - .2 ACCEPT no red hair genes nor red hair WTTE
 - .2 ACCEPT no red hair genes or red hair WTTE
 - .2 ACCEPT neither red hair genes nor red hair WTTE
 - .2 ACCEPT neither red hair genes and red hair WTTE

b	<p>.1 Correctly write 0.85 and the complementary in the 2nd branch (their 0.1 and 0.94)</p> <p>.2 Correctly write the $P(C G)$ as 0.9</p> <p>.3 Correctly write their $P(G' \cap C)$ ACCEPT rounded to 2dp, Ex: their 0.05</p>	<p style="text-align: right;">3</p>	
c	<p>.1 Correctly add 0.135 to their 0.051 from b) to find $P(C)$</p> <p>.2 Correct reason based on their bp1 AG independent</p>	<p>.1 $(P(C)) = 0.135 + \text{their } 0.051 = 0.186$</p> <p>.2 Either $P(C) \times P(G)$ not equal $P(C \cap G)$ ie: $0.15 \times \text{their } 0.186$ is not equal 0.135 OR $P(C G)$ is not equal $P(C)$ i.e. $\text{their } 0.9$ is not equal $0.15 \times \text{their } 0.186$</p>	2
d	<p>.1 Correct lower bound</p> <p>.2 Multiply their 0.186 seen in c) or here in d) by their 15450</p> <p>.3 correctly write their result as a whole number after multiplying a probability by their 15450</p>	<p>.1 15450</p> <p>.2 $\text{their } 0.186 \times \text{their } 15450$ ACCEPT $\text{their } 0.186 \times 15500$</p> <p>.3 $\text{their } 2873$ OE ACCEPT 2874 .3 ACCEPT only if bp1 or bp2 are awarded .3 ACCEPT their whole number to be a rounding up or down for their result</p>	3

	e	correctly write 70 in the appropriate place		1
	f	.1 correct P(C) OR ratio .2 correctly write their N that satisfies their ratio $\frac{\text{their90}}{800} = \frac{1}{N}$ before rounding .3 correctly round up their N	.1 $(\frac{1}{N} =) \frac{90}{800}$ or 0.1125 OE OR 90:800 .1 ACCEPT ratio in any notation Ex: 90 to 800 .1 ACCEPT their90 used in bp1 instead of 90 only if it is equal to 20+their70 from (g) .2 $(\frac{800}{\text{their90}} =) \text{their8.88(8...)}$ ACCEPT $\frac{\text{their90}}{800} = \frac{1}{\text{their8.88(8...)}}$ OE .2 ACCEPT (N=)8.88 or 8.9 .2 ACCEPT their90 used in bp2 being any number .2 ACCEPT if their N satisfies their ratio $\frac{90}{\text{their800}} = \frac{1}{N}$.3 their 9 .3 ACCEPT only if their bp2 needs rounding .3 ACCEPT only if bp1 or bp2 awarded	3

	<p>g correctly subtract their11.25 from their18.6 seen in c) or d) or calculated here</p>	<p>(their18.6-their11.25=) their7.(35)(%) ACCEPT their11.25 even if it is incorrect ACCEPT only if percentage ACCEPT positive or negative Ex: $(18.6-20=-)1.4(%)$ DO NOT ACCEPT their7.35(%) without working</p>	<p>1</p>
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Q6	Answers	Notes	Total
a	30<A ≤40	ACCEPT in any notation Ex: 30<A<40 or 30≤A<40 or 30≤A≤40 30 – 40 30 to 40 DO NOT ACCEPT 31≤A≤40 OE	1
b	.1 Correct mid-interval values .2 Add the product of their mid-interval values by frequency .3 Divide their sum by 331 .4 Correctly divide the sum of their products by 331	.1 5,15,25,35,45,55 ACCEPT seeing only four correct .2 $5 \times 59 + 15 \times 72 + 25 \times 54 + 35 \times 126 + 45 \times 15 + 55 \times 5$ OE or 8085 .2 ACCEPT seeing only four correct products OE added .3 $\frac{\text{their sum}}{331}$ ACCEPT their sum being any sum .4 their 24.(425...) ACCEPT only if $0 < \text{their } 24.(425...) < 60$.4 DO NOT ACCEPT if it is the result of a sum divided by 331 Ex: $((5+15+25+35+45+55)/331)=0.54(38...)$	4
c	.1 Apply the ratio on 72 .2 Correct operation based on the data 115 AG	.1 $72 \times \frac{7}{9}$ OE OR $72 \times \frac{2}{9}$ OE .1 ACCEPT “9 is 72 so 7 is” OE .2 56+59 or 59+72-16 or 131-16 ACCEPT 115-59=56 or 131-115=16 .2 ACCEPT in words. Ex: add 56 to 59 Note: Seeing only bp2 award 1 mark Ex: seeing only $56+59=115$ or $115-59=56$ award 1 mark	2
d	.1 Multiply 1400 or 90 by 15% OE .2 Correct result	.1 0.15×1400 or 210 or 0.15×90 or 13.5 OE ACCEPT 1400×90 or 126000 .2 $(0.15 \times 1400 \times 90) = (\$)18900$	2

Mark	1	2
Identify factors (F)	<p>Two from the four factors below mentioned in the factors box</p> <ul style="list-style-type: none"> Number of children or number of users of playground ACCEPT under 14 ACCEPT using numbers even if incorrect. Ex: there are 56 children Number of adults or number of users of fitness centre ACCEPT above 14 ACCEPT using numbers even if incorrect. Ex: there are 200 adults Needed area per child or children per m² ACCEPT area or size of playground ACCEPT using numbers even if incorrect. Ex: 50m² for 24 children or 3 m² per child Cost ACCEPT money spent ACCEPT using numbers even if incorrect. Ex: (\$)^{15 per m² for playground or (\$)^{1000 for fitness}} 	<p>Three from the four factors mentioned in the factors box</p>
ACCEPT		
<p>Factors that WTTE</p> <p>Ex: "not everyone is going to use the playground" is WTTE for the 1st factor</p> <p>Ex: "not everyone uses the fitness" is WTTE for the 2nd factor</p> <p>The use of "residents scheduled for" instead of "number of". The use of the word "gym" instead of "fitness centre".</p> <p>"Number of people" or "Number of residents in apartments" or "Number of users" instead of either the 1st factor or the 2nd factor but not both</p> <p>"Number of children in 15m²" instead of either the 1st factor or the 3rd factor but not both</p> <p>"People under 14 and older" as the 1st factor only</p> <p>"cost of maintenance for the playground/recreational area" as the 4th factor only</p> <p>"Residents scheduled for the playground and for the gym" as the 1st and 2nd factors</p>		
DO NOT ACCEPT		
<p>Incomplete factors (using just a word). Examples: Number or Age or Area or recreational area or maintenance or children or adults</p> <p>Factors related to budget. Example: Maintenance budget or number of apartments or the (\$)1400 or the 15% or (\$)18900</p> <p>"Budget of the recreational area"</p>		

Calc. (C) ACCEPT rounding in calculations	<p>Weak attempt to calculate maintenance cost for area of playground (involves one correct operation) Ex:<ul style="list-style-type: none"> • 6 in 15 so 115 is... • $\frac{6}{15} = 0.4$ OE • $\frac{15}{6} = 2.5$ OE • $\frac{115}{6} = 19.16\dots$ OE or 20 • $15 \times 115 = 1725$ • Multiply area by 2 Ex: $15 \times 2 = 30$ Ex: $115 \times 2 = 230$ </p>	<p>Good attempt to calculate maintenance cost for area of playground (involves two correct operations) Ex:<ul style="list-style-type: none"> • $(\frac{15}{6} \times 115 \text{ or } \frac{115}{0.4}) = 287.5$ ACCEPT [285,300] • $(\frac{115}{6} \times 2) = 38.33\dots$ ACCEPT [38,40] • $(15 \times 115 \times 2) = (\\$)3450$ • $(30/6) = (\\$)5$ </p> <p>Note: For all the above, only one more remaining operation is needed to reach the maintenance cost.</p>	<p>Correctly calculate the maintenance cost for the area of playground</p> $(\frac{15}{6} \times 115 \times 2) = (\$)575$	<p>Correctly calculate the maintenance cost for the area of playground monthly or yearly</p> <p>(monthly) ACCEPT [570,600]</p>	<p>Correctly calculate the total cost for recreation area monthly or yearly</p> <p>(monthly) (600+1200=) ACCEPT [1770,1800]</p>
	<p>OR Correct number of fitness centre users $(331 - 115) = 216$</p> <p>OR Correctly identify the maintenance cost for area of fitness based on their number of adults Their(\$1200 (for their 216 adult)</p>	<p>OR Correctly identify the maintenance cost for area of fitness (\$1200)</p>	<p>OR Good attempt to calculate maintenance cost for area of playground (involves two correct operations) AND Correctly identify the maintenance cost for area of fitness (\$1200)</p>		

Mark	1	2
Accuracy (A)	<p>Correct sensible rounding seen in any calculation and ACCEPT not seeing the value before rounding Ex: Round their 19.1666 to their 19.2 or their 19 or their 20 ACCEPT $115/6=19.2$ Round their 287.5 to 285 or 288 or 290 or 300</p> <p style="text-align: center;">ACCEPT</p> <p>Rounding up or down No rounding of exact calculations leading to whole numbers only if C3 is achieved “my calculations do not need rounding” WTTE and C1 or C2 achieved with a result of a whole number</p> <p style="text-align: center;">DO NOT ACCEPT</p> <p>In sensible rounding. In particular, rounding to two decimal places. Ex: $19.1666 = 19.17$</p>	

Mark	1	2
Justify (J)	<p>ACCEPT ONLY IF C1 is achieved</p> <p>Justify suitability by comparing their calculations of maintenance cost to their budget from d) Ex: Maintenance cost exceeds the budget so not good WTTE Maintenance cost is less than budget so it is fine WTTE The suitable budget should be $1800 \times 12 = (\\$)21600$ per year WTTE The suitable budget should be $(\frac{1800}{90}) = (\\$)20$ per apartment per month WTTE ACCEPT <u>Incorrect justification</u> by comparing correct monthly maintenance cost with their yearly budget Ex: 1800 less than 18900, <u>so it is suitable</u></p> <p>OR The correct cost and their corresponding (monthly or yearly) budget seen OE without comment (at least C4 achieved) Ex: seeing the cost as $(\\$)1800$ and the budget as $(18900/12) = (\\$)1575$ Ex: seeing the cost as $(\\$)21240$ and the budget as $(\\$)18900$ Ex: the remaining for fitness is $1575 - 575 = (\\$)1000$ and we see cost fitness $(\\$)1200$</p> <p>OR General justification of suitability Ex: -The spaces calculated are just estimates so calculations are estimates -Collect more money for maintenance just in case -Number of persons in each category will vary overtime and hence the areas need to consider being a bit bigger for the future use. WTTE - Some families may get new babies and children number increases so we need to have the recreation spaces a bit bigger -Some of the children will become adults soon so we need to have the fitness spaces a bit bigger</p>	<p>ACCEPT ONLY IF C4 is achieved</p> <p>Justify suitability by comparing the correct calculations of maintenance cost to their budget from d) The maintenance cost exceeds budget so not good WTTE The maintenance cost is less than budget so it is fine WTTE The suitable budget should be $1800 \times 12 = (\\$)21600$ per year WTTE The suitable budget should be $(\frac{1800}{90}) = (\\$)20$ per apartment per month WTTE ACCEPT <u>Correct justification implied</u> by comparing correct monthly maintenance cost with their yearly budget Ex: The monthly cost is 1800 and the budget is 18900 <u>so not suitable</u></p> <p>DO NOT ACCEPT Justification by comparing monthly value with yearly value Ex: 1800 less than 18900 so it is suitable</p>
	NOTE	
	ACCEPT Seeing just the word "budget" when comparing instead of their actual value in part (d). Check their value in part (d) when marking the justification	

Q7		Answers	Notes	Total														
a	Correctly place 9 and 11	<table border="1"> <thead> <tr> <th>Stage (n)</th> <th>Number of squares (Q)</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>1</td> </tr> <tr> <td>2</td> <td>3</td> </tr> <tr> <td>3</td> <td>5</td> </tr> <tr> <td>4</td> <td>7</td> </tr> <tr> <td>5</td> <td>9</td> </tr> <tr> <td>6</td> <td>11</td> </tr> </tbody> </table>	Stage (n)	Number of squares (Q)	1	1	2	3	3	5	4	7	5	9	6	11		1
Stage (n)	Number of squares (Q)																	
1	1																	
2	3																	
3	5																	
4	7																	
5	9																	
6	11																	
b	<p>.1 correctly describe one pattern for Q in words</p> <p>.2 correctly describe a second pattern for Q in words</p>	<p>ACCEPT complete terminology only, for example (below are four different descriptions) (The) odd numbers increasing by 2, adds 2, goes up by 2, moving up by 2 difference 2, common difference 2, linear with difference 2, arithmetic with difference 2 second difference is zero</p> <p>DO NOT ACCEPT incomplete terminology, for example: Arithmetic, linear, increasing by a constant, constant difference</p> <p>DO NOT ACCEPT general description, for example: Integers, whole numbers, positive</p> <p>DO NOT ACCEPT the rule in words or description related to n for example: 2 times n minus 1, n multiplied by 2 minus 1, WTTE DO NOT ACCEPT n goes up by 1 It is increasing general rules in terms of n, example: $Q = 2n - 1$</p> <p>Note, in the case when they have more than two different patterns:</p> <p>If two are accepted and the rest are all correct: award 2 marks Ex: odd numbers and adds 2 and Second difference is zero and it is 2 times n then subtract 1. Ex: odd numbers and goes up by 2 and linear and difference is 2</p> <p>If two are accepted and any of the rest is incorrect: award 1 mark Ex: increases by 2 and Second difference is zero and it is divisible by 2 If only one is accepted, ignore the rest and award 1 mark</p>		2														

c	<ul style="list-style-type: none"> .1 the correct general rule .2 the correct simplified general rule with correct notation 	<p>.1 ($Q = 2n-1$ or $(Q =) (2n-1)$ or $Q = 2x-1$ or $Q = 1+2(n-1)$ $Q=2xn-1$ or $Q=2^*n-1$)</p> <p>.2 $Q = 2n-1$ ACCEPT $Q=(2n-1)$ ACCEPT using Q_n instead of Q ACCEPT $Q=n2-1$ ACCEPT using q and N DO NOT ACCEPT description in words</p> <p>SC 1 mark in 8d and consequently mark 8e as appropriate If NR in 8d and correct general rule seen in 8c or 8e If they write in 8d the rule in terms of u_1 and d Ex: $u_1 + (n - 1)d$ then in 8e, when they test or verify, they directly use 1 instead of u_1 and 2 instead of d</p>	2
d	<ul style="list-style-type: none"> .1 Correctly substitute $n \geq 5$ into their general rule .2 Correctly calculate their value of P after substituting $n \geq 5$.3 Recognize that their result is the same as the correctly predicted value 	<p>.1 Ex: $2 \times 5 - 1$.2 Ex: 9</p> <p>.3 “the same as when we continue the pattern” WTTE and states how Ex: For $n=7$, 13 is obtained from pattern of adding 2 to 11 For $n=7$, $13 - 2 = 11$</p> <p>.3 ACCEPT if their value from .2 is the same as their value in the table in part a) or seen here in part d) Ex: we see their calculated $P=9$ and we see $P=9$ in their table</p> <p>SC for 1 mark Correctly test their general rule by applying the steps of verification mentioned in the left column with a value of $n \leq 4$</p> <p>SC for 1 mark verify with $n \geq 5$ the correctly described pattern or recursive rule from part b) or c)</p>	3
e	Correctly write the value of u_1 and the value of d	$u_1=2$ and $d=3$ ACCEPT $2+(n-1)x3$	1
f	Correct substitution of 7 into the $2+(n-1)x3$	$2+(7-1)x3$ or $2+6x3$ ACCEPT incorrect notation ACCEPT in words DO NOT ACCEPT other operations DO NOT ACCEPT substitution in the recursive rule	1

Glossary for task 3

Term used	Clarification
General rule	Rule in terms of only n (if they use x, it is still general rule but penalise in notation)
The general rule	The correct general rule in terms of only n (if they use x, it is still the general rule but penalise in notation)
Their general rule	Incorrect rule but in terms of only n (if they use x, it is still their general rule but penalise in notation)
Their rule	Correct rule not in terms of only n
Recursive rule	$U_n = U_{n-1} + d$ or $U_{n+1} = U_n + d$ or $B_{n+1} = B_n + 12n + 1$

We accept subsequent use of their general rule (when marking D,T,V,J,N, and L) provided it is of similar complexity. In general, the complexity of the rule depends on its form. The table below shows examples.

The general rule	ACCEPT for their general rule	DO NOT ACCEPT for their general rule
$Q = 2n - 1$	linear	constant
$B = 6n^2 - 5n + 1$	Quadratic or exponential	Linear or constant

Stage number (n)	Area of shape (B)
1	2
2	15
3	40
4	77
5	126
6	187
7	260
8	345

Mark	1	2	3	4
Predictions (P)	Correctly predict two terms for B ACCEPT whether in the table or in the response box			
Description (D) ACCEPT incorrect terminologies, notation errors, non-simplified rule, or rule in words but penalize in notation (N)	Correctly describe a pattern in words or recursive rule for B ($B_{n+1} = B_n + 12n + 1$ or $B_n = Q(n + Q)$) Examples of different patterns: <u>(can be seen anywhere in the response)</u> even then odd then even then odd difference is odd the increase increases by 12 first difference increases by 12 second difference 12 It is quadratic ACCEPT patterns described with incorrect terminologies or recursive rule with incorrect notation but penalise in notation (N).	Correctly describe two patterns in words for B (or one pattern and the recursive rule for B)	Correctly describe two patterns in words for B (or one pattern and the recursive rule for B) AND Valid attempt to write down a general rule for B	Correctly describe two patterns in words for B (or one pattern and the recursive rule for B) AND Correctly write down the general rule for B
Ignore additional incorrect patterns	OR Valid attempt to write down a general rule for B Ex: $B = 6n^2$ Ex: An exponential rule that satisfies one value for n	OR Correctly describe a pattern in words for A (or the recursive rule for B) AND valid attempt to write down a general rule for B	OR Correctly write down the general rule for B $B = 6n^2 - 5n + 1$	
	DO NOT ACCEPT			
	Descriptions of how bricks are added Goes up by 13,25,37,... Recursive rule in words Second difference constant			

Mark	1	2	3
Testing (T)	<p>Attempt to test their general rule for B using $n \leq 4$ Ex: Substitute in their general rule for B value of $n \leq 4$</p> <p>OR Correctly test their described pattern or their rule (e.g. the recursive rule for B or their linear rule for B)</p>	<p>Correctly test their general rule for B using $n \leq 4$ Ex: Correctly calculate their value for B in their general rule using $n \leq 4$ AND Recognise that their correctly calculated value for B is the same as the given value.</p>	
		<p>ACCEPT seeing their correctly calculated value for B and the given value in the table being equal</p>	
Verifying (V)	<p>Attempt to verify their general rule for B using $n \geq 5$ Ex: Correctly substitute in their general rule for B value of $n \geq 5$</p> <p>OR Correctly verify their described pattern or their rule (e.g. the recursive rule for B or their linear rule for B)</p>	<p>Correctly calculate their value for B in their general rule using $n \geq 5$</p>	<p>Correctly calculate their value for B in <u>the general rule</u> using $n \geq 5$ AND Recognise that their correctly calculated value for B is the same as the correct predicted value obtained by continuing the pattern</p>
			<p>ACCEPT seeing their correctly calculated value for B and the correctly predicted value in the table being equal</p>

Mark	1	2	3	4
Justify/ proof (J)	ACCEPT only if D1 is achieved	ACCEPT only if they have a general rule for B	ACCEPT only if they have the correct general rule for B	ACCEPT only if they have the correct general rule for B
	Attempt to justify <u>their</u> general rule aligned with their notation for <u>their</u> general rule or recursive rule by attempting to find coefficients of quadratic using any method	Correctly justify <u>the</u> general rule aligned with their notation for <u>the</u> general rule for A by correctly substituting and finding parameters	Attempt to justify the general rule for B in relation to the context Ex: $B = Q(n+Q)$ And $(2n-1)x(3n-1)$ <i>Ex :</i> $B = 1x2, 3x5, 5x8, 7x11, \dots$ etc And $(2n-1)x(3n-1)$	Correctly justify the general rule for B in relation to the context Ex: B=number of Squares times number of bricks placed on the square And $(2n-1)x(3n-1)$ <i>Ex:</i> Substitute $N=2n-1$ and $d=3$ in $S_N = \frac{N}{2}(2u_1 + (N-1)d)$ using $N=2n-1$ and $d=3$ OR Good attempt to prove by induction
	OR Weak attempt to justify <u>their</u> general rule for B in relation to arithmetic series Ex: Ex: (what is added forms arithmetic sequence with $u_1=13$ and $d=12$) Ex: realizing B is $S_N = \frac{N}{2}(2u_1 + (N-1)d)$ but using N and d incorrect	OR Weak attempt to justify <u>the</u> general rule for B in relation to arithmetic series or using numbers in the context Ex: $S_N = \frac{N}{2}(2u_1 + (N-1)d)$ but either N incorrect or d incorrect <i>Ex :</i> $B = 1x2, 3x5, 5x8, \dots$ etc And either $(2n-1)$ or $(3n-1)$ in the product of the rules		OR Correct proof by induction

Mark	1	2	3
Notation and terminology (N)	ACCEPT only if D1 achieved Correct notation of <u>their</u> general rule for B	ACCEPT only if D3 achieved Correct notation of <u>the general</u> rule for A in simplest form $B = 6n^2 - 5n + 1$ ACCEPT using B_n or A_n instead of B	ACCEPT only if D3 achieved Correct notation of <u>the general</u> rule for B in simplest form (see examples in N2) AND Correctly describe one pattern in words for B using correct terminology or correct recursive rule for B using correct notation
	OR The notation of <u>the general</u> rule includes errors or not simplified or in words (see examples in N1) AND Correctly describe one pattern in words for A using correct terminology or correct recursive rule for A using correct notation	OR The notation of <u>the general</u> rule includes errors or not simplified or in words (see examples in N1) AND Correctly describe one pattern in words for A using correct terminology or correct recursive rule for A using correct notation	ACCEPT using U_n instead of B only if they mention that $B = U_n$
	OR The notation of <u>the general</u> rule includes errors or not simplified or in words Ex: $B=6xn^2-5n+1$ The rule for B is $B = 6n^2 - 5n + 1$ $B = (2n-1)(3n-1)$	ACCEPT using U_n instead of B only if they mention that $B = U_n$	DO NOT ACCEPT using * for multiplication using / for division using ^ for power using x instead of n
	OR Correctly describe one pattern in words for B using correct terminology or correct recursive rule for B using correct notation $B_{n+1} = B_n + 12n + 1 \quad \text{or} \quad B_n = B_{n-1} + 12n - 11 \quad \text{or}$ $B_n = Q(n + Q)$	DO NOT ACCEPT using * for multiplication using / for division using ^ for power using x instead of n	DO NOT ACCEPT if they don't have the general rule for B

Mark	1	2	3
<p>Communication (L)</p> <p>Organisation and coherence Can be awarded even if there are errors</p> <p>For items: Describing pattern and writing rule can be considered an item even if D0 awarded</p>	<p>At least three from the following items are seen:</p> <ul style="list-style-type: none"> • describe a pattern in words • write a rule • test their general rule or rule or recursive rule or pattern (at least T1) • verify their general rule or rule or recursive rule or pattern (at least V1) • justify their general rule or rule or recursive rule or pattern (at least J1) 	<p>ACCEPT only if they have a general rule</p> <p>At least four of the following items are seen:</p> <ul style="list-style-type: none"> • describe a pattern in words • write a general rule • test their general rule (at least T1) • verify their general rule (at least V1) • justify their general rule (at least J1) <p>AND</p> <p>For coherence, they identify the processes correctly.</p> <p>At least one from the following:</p> <ul style="list-style-type: none"> • test • verify • justify <p>Ex:</p> <ul style="list-style-type: none"> • For test: they say “test” and they substitute in their general rule value(s) of $n \leq 4$ only • For verify: they say “verify” and they substitute in their general rule value(s) of $n \geq 5$ only • For justify: They say “justify” and they write a justification <p>Note for coherence: If they say “test and verify” and they substitute in their general rule value(s) of $n \leq 4$ <u>followed by</u> value(s) of $n \geq 5$, consider it as only one identified process</p>	<p>ACCEPT only if they have the general rule for B</p> <p>The following two items must be seen :</p> <ul style="list-style-type: none"> • write <u>the general rule for B</u> • justify <u>the general rule</u> (at least J2) <p>AND</p> <p>At least two of the following items are seen:</p> <ul style="list-style-type: none"> • describe a pattern or rule in words • test <u>the general rule</u> (at least T1) • verify <u>the general rule</u> (at least V1) <p>AND</p> <p>For coherence, they identify the processes correctly.</p> <p>At least two from the following:</p> <ul style="list-style-type: none"> • test • verify • justify <p>Ex:</p> <ul style="list-style-type: none"> • For test: they say “test” and they substitute in the general rule for B value(s) of $n \leq 4$ only • For verify: they say “verify” and they substitute in the general rule for B value(s) of $n \geq 5$ only • For justify: They say “justify” and they write a justification <p>Note for coherence: If they say “test and verify” and they substitute in the general rule value(s) of $n \leq 4$ <u>followed by</u> value(s) of $n \geq 5$, consider it as only one identified process</p>

Markscheme

November 2022

Extended Mathematics

On-screen examination

25 pages

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The markscheme abbreviations:

The markscheme may make use of the following abbreviation: **OE** – ‘or equivalent’

- Bullet notation means award 1 mark – see example below

Example 1

.1 mark awarded and corresponding notes are aligned

b	<p>.1 Show clear line of reasoning in the method</p> <p>.2 4</p>	<p>.1 45 and 49 seen OE</p> <p>Ex: $49 = 45 + x$</p> <p>.2</p>	2
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Error Carried Forward (ECF) marks

Errors made at any step of a solution affect all working that follows. In general, **Error Carried Forward (ECF)** marks are awarded after an error.

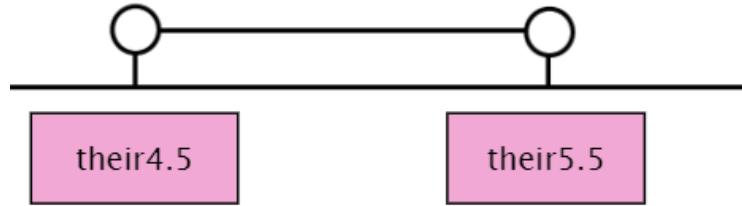
- a. **ECF** applies from one part of a question to a subsequent part of the question and also applies within the same part.
- b. If an answer resulting from **ECF** is inappropriate (eg, negative distances or $\sin x > 1$) then subsequent marks should not be awarded.
- c. If a question is transformed by an error into a **simpler question** then **ECF** may not be fully awarded.
- d. To award **ECF** marks for a question part, **there must be working present for that part**.
- e. **ECF** is only applied to working which is correct. This means that all working subsequent to an error must be checked for accuracy.
- f. A misread (**MR**) is an error. **ECF** is normally awarded.

General points

- a. As this is an international examination, accept all **alternative forms of notation**, for example 1,9 as 1.9 ; 1,000 or 1.000. However **DO NOT ACCEPT** incorrect mathematical notation x^2 for x^2 unless noted otherwise in the MS
- b. Accept notation errors in intermediate steps.
- c. Ignore further working after a correct answer **unless** it indicates a lack of mathematical understanding **i.e. if the further working contradict the correct answer**, then that last mark cannot be awarded.
- d. In the case when a correct result is obtained using incorrect seen method, do not award the mark for the result.
- e. Where candidates have written two solutions to a question, mark the first solution.
- f. In the markscheme, equivalent examples of **numerical** and **algebraic** forms or **simplified** answers will generally be written in the notes preceded by **OE** (Or Equivalent) e.g. **OR 1/2 OR 0.5 OR** ; **OR x/2 OR** ; **0.23 OR 23%**
- g. In the markscheme, information provided in brackets indicate detail that may be seen in a candidate response but is not necessary to award the marks. However, these results need to be seen in the appropriate context to award the mark.
- h. Special case marks **SC** can be allocated instead of but not in addition to the marks prescribed in the markscheme.
- i. Accept seeing equation not in-line.
- j. Calculator screenshots are accepted as working steps. And when a calculator screenshot is taken, accept not seeing the whole operation.
- k. In task 2 and 3 where the markscheme is set out in a table then, unless noted otherwise, awarding the highest mark in a category includes all the lower marks in that category. It is probably best to look for the top category mark answer and if you don't find it look at the next mark down.
- l. **ACCEPT** using the correct values regardless their previous result.
- m. Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.
- n. Unless noted otherwise, if a note in a part says to accept the answer without working for 1 mark less than total marks, then seeing the correct answer with any acceptable working step, award full marks. Example: If the note in a part worth 3 marks says "4.3(3...) without working award 2 marks", then seeing any acceptable working step and seeing 4.3(3...) as the answer award the 3 marks.

Q1		Answers	Notes				Total	
	a	.1 Correctly write two missing values .2 Correctly write the third missing value	Students	Bus	Train	Car	Total	2
			Teachers	12	29	15	56	
			Total	78	37	35	150	
	b	.1 Correct number of students who travel by bus or car .2 Correctly divide their 86 by their total number of students in part (a)	.1 86 .2 ($\frac{86}{150}$) or $\frac{43}{75}$ or 0.57(333...) OE					2
	c	$\frac{15}{56}$ or 0.267(857...) OE						1
	d	.1 Correct initial probability .2 Multiply three probabilities without OR with replacement .3 Correctly multiply their three probabilities without replacement .4 Correctly multiply their product of probabilities by 3	.1 $\frac{12}{56}$ OE or $\frac{44}{56}$ OE ACCEPT seeing it embedded in calculations .2 their($\frac{12}{56} \times \frac{11}{55} \times \frac{44}{54}$) OR their($\frac{12}{56} \times \frac{12}{56} \times \frac{44}{56}$) or $\frac{99}{2744}$ or 0.036(07..) .3 their($\frac{5808}{166320}$) or their($\frac{11}{315}$) or their 0.0349(2...) OE ACCEPT 0.035 .4 their($\frac{17424}{166320}$) or their($\frac{33}{315}$) or their 0.1047(61..) ACCEPT 0.105 Accept only if $0 \leq \text{probability} \leq 1$					4

Q2		Answers	Notes	Total
	a	.1 Correctly state 1 st transformation .2 Correctly state 2 nd transformation	.1 vertical stretch or dilation by a factor of 4 ACCEPT enlargement .2 vertical translation of -1 or shift down 1 unit WTTE	2
	b	AM1 .1 Correctly rearrange the equation to have $(x - 5)^2$ on one side .2 Correctly square root .3 Correctly calculate their two values of x AM2 .1 Correctly expand the equation to have all terms on one side equal zero .2 Correctly factorise their quadratic or correctly substitute their coefficients into quadratic formula .3 Correctly calculate their two values of x	AM1 .1 $(x - 5)^2 = \frac{1}{4}$ OE .2 $x - 5 = \pm \frac{1}{2}$ OE ACCEPT $x - 5 = \frac{1}{2}$.3 $x = \text{their}4.5$ AND $x = \text{their}5.5$ OE AM2 .1 $4x^2 - 40x + 99 = 0$ ACCEPT $4x^2 - 40x + 100 - 1 = 0$.2 $(2x - 9)(2x - 11)$ or $\frac{40 \pm \sqrt{40^2 - 4 \times 4 \times 99}}{2 \times 4}$ OE ACCEPT solving correctly their simpler quadratic for one value of x .3 $x = \text{their}4.5$ AND $x = \text{their}5.5$ OE ACCEPT their correctly calculated values provided only one mistake in substitution into the quadratic formula	3

c	<p>.1 Correctly place two items .2 Correctly place the rest of the five items</p>	 <p>DO NOT ACCEPT if their 4.5 and their 5 are switched in places DO NOT ACCEPT their inequalities in the boxes e.g. $x > 4.5$</p>	2
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Q3		Answers	Notes	Total
	a	.1 Correct use of Pythagoras .2 Correct value of height .3 Correct comparison of their 230.48... and 230 Storage unit cannot be lifted into place AG	.1 $50^2 + 225^2 = h^2$ OE .2 ($h =$) 230.48(...) ACCEPT 230.5 .3 their230.48(...) > 230 or anything that WTTE Ex: 230.48 is greater than height of ceiling .3 ACCEPT 53125 > 52900 or anything that WTTE .3 DO NOT ACCEPT unless .1 or .2 is awarded	3
	b	.1 Correctly substitute h , x and 230 into the Pythagoras's theorem .2 Correctly rearrange the formula to have h on one side	.1 $h^2 + x^2 = 230^2$ OE .1 ACCEPT using inequality sign .2 ($h =$) $\sqrt{52900 - x^2}$ OE .2 ACCEPT using inequality sign	2
	c	Correctly calculate their h using $x=50$ and 230 into their formula for h	Their 224(.4994432) ACCEPT in the working ex: $50^2 + 224.(499)^2 = hyp^2$ DO NOT ACCEPT 224.49	1
	d	.1 Correctly substitute 50 and 220 into volume of cuboid .2 Correct calculate their volume for their height	.1 $50 \times 50 \times 220$.2 their550 000 (cm^3)	2

Q4		Answers	Notes	Total
	a	<p>.1 A correct reason for a pair of congruent angles</p> <p>.2 A correct reason for another pair of congruent angles (AA)</p> <p>◆ABC ~ ◆ADE AG</p>	<p>Examples of correct reasons for pair of congruent angles:</p> <ul style="list-style-type: none"> - Angle BAC = Angle DAE or Angle A is common angle or same angle A WTTE. (They have to mention the angle A). ACCEPT angle A = angle A DO NOT ACCEPT they share same angle or they have a common angle - Angle ABC= Angle ADE with reason Ex: BC parallel to DE or parallel lines or corresponding angles ACCEPT Angle B = Angle D instead of Angle ABC= Angle ADE - Angle ACB= Angle AED with reason Ex: BC parallel to DE or parallel lines or corresponding angles ACCEPT Angle C = Angle E instead of Angle ACB= Angle AED DO NOT ACCEPT Angle ABC and Angle ADE are similar or Angle ACB and AED are similar DO NOT ACCEPT seeing only Angle ABC= Angle ADE or Angle ACB= Angle AED without correct reasoning 	2

<p>b</p> <p>AM1</p> <ul style="list-style-type: none"> .1 Correct ratio .2 Their correct ratio of area (squaring their ratio) .3 Multiply their ratio of area by 9 .4 Correctly subtract their area of triangle ADE by 9 <p>AM2</p> <ul style="list-style-type: none"> .1 Correct ratio .2 Correctly calculate length BC .3 Correctly calculate their length of DE .4 Correctly calculate their area of trapezium 	<p>AM1</p> <ul style="list-style-type: none"> .1 $\frac{7}{3}$ or $0.428(57\dots)$ OE ACCEPT $\frac{3}{7}$ OE .2 $(\text{their } \frac{7}{3})^2$ OE .3 $\text{their } (\frac{7}{3})^2 \times 9$ or 49 .3 ACCEPT seeing their $\frac{7}{3} \times 9$ or 21 BUT DO NOT ACCEPT <i>their</i> $\frac{3}{7} \times 9$.4 $(\text{their } 49 - 9 =)$ their 40 (cm^2) Accept only if positive <p>AM2</p> <ul style="list-style-type: none"> .1 $\frac{7}{3}$ or $0.428(57\dots)$ OE ACCEPT $\frac{3}{7}$ OE .2 $(9 = \frac{1}{2} \times BC \times 3 \times \sin 96^\circ)$, $(BC =)6.03(3\dots)$ OR $(\frac{3 \times \sin 59^\circ}{\sin 96^\circ} =)6.084(69\dots)$.3 $(\text{their } \frac{7}{3} \times \text{their } 6.03(\dots) =)14.077(1\dots)$ ACCEPT 14 .4 $(\frac{1}{2} \times \text{their } 14.077(1\dots) \times 7 \times \sin 96^\circ - 9) = \text{their } 39.99$ ACCEPT 39.98 <p>OR</p> $\left(\frac{6.03(3\dots) + \text{their } 14.077(1\dots)}{2} \times \left(\frac{9}{0.5 \times 6.03(3\dots)} \right) \times \frac{7}{3} - \frac{9}{0.5 \times 6.03(3\dots)} \right) = 40(.03\dots)$ <p>.4 ACCEPT their 40 only if .2 and .3 are awarded</p>	<p>4</p>
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c	<p>AM1</p> <p>.1 Correct substitution into sine rule .2 Correctly rearranging to make $\sin \angle ABC$ the subject .3 Correctly calculate the angle ABC using their sine rule</p> <p>AM2</p> <p>.1 Correctly calculate length BC .2 Correctly substitute their sides of triangle ABC into cosine rule .3 Correctly calculate the angle ABC using their cosine rule</p> <p>25 AG</p>	<p>AM1</p> <p>.1 $\frac{\sin \angle ABC}{3} = \frac{\sin 96}{7}$ OE .2 $((\sin \angle ABC) =) \frac{3 \sin 96}{7}$.3 25.2(...)</p> <p>AM2</p> <p>.1 $(9 = \frac{1}{2} \times BC \times 3 \times \sin 96)$, $(BC =) 6.03(3\dots)$ OR $(\frac{3 \times \sin 59}{\sin 96} =) 6.084(69\dots)$.2 $(\cos B =) \frac{\text{their } 6.03(3\dots)^2 + 7^2 - 3^2}{2 \times \text{their } 6.03(3\dots) \times 7}$ OE .3 25.2(...) ACCEPT their 25.2 if it rounds to 25</p>	3
d	<p>AM1</p> <p>.1 Correctly calculate angle BAC .2 Correctly substitute their angle BAC into cosine rule .3 Correctly calculate their BE after substitution into cosine rule</p> <p>AM2</p> <p>.1 Correctly calculate angle BAC .2 Correctly calculate angle ABE (or AEB) AND correctly substitute their values into sine rule .3 Correctly calculate their BE after substitution into sine rule</p> <p>AM3</p> <p>.1 Correctly calculate angle BAC .2 Correctly calculate angles of triangle BCE AND correctly substitute their values into sine rule or cosine rule .3 Correctly calculate their BE after substitution into sine rule</p>	<p>AM1</p> <p>.1 $(180 - 96 - 25 =) 59$ (degrees) ACCEPT their 25 .2 $(BE^2) = 7^2 + 7^2 - 2 \times 7 \times 7 \times \cos(\text{their } 59)$ or $47.5(2626\dots)$.3 their $6.89(39\dots)$ (cm) ACCEPT 7 only if .2 or .1 is awarded</p> <p>AM2</p> <p>.1 $(180 - 96 - 25 =) 59$ (degrees) .2 $(ABE \text{ or } AEB) = (\frac{180 - 59}{2} =) 60.5$ AND $\frac{7}{\sin 60.5} = \frac{BE}{\sin 59}$.3 their $6.89(39\dots)$ (cm) ACCEPT 7 only if .2 or .1 is awarded</p> <p>AM3</p> <p>.1 $(180 - 96 - 25 =) 59$ (degrees) .2 $(BCE =) 84$ and $(CBE = \frac{180 - 59}{2} - 25) = 35.5$ AND $\frac{4}{\sin 35.5} = \frac{BE}{\sin 84}$ or $6.(033\dots)^2 + 4^2 - 2 \times 4 \times 6.(033\dots) \times \cos 84$.3 their $6.85(04\dots)$ (cm) ACCEPT 7 only if .2 or .1 is awarded</p>	3

Q5		Answers	Notes	Total
	a	Correct mass of phone case	($1.25 \times 29 =$) 36(.25) (g)	1
	b	.1 Divide 1000 by their mass .2 Correctly rounds their answer to nearest whole number	.1 $\frac{1000}{\text{their } 36.25}$ or 27.586.. .2 their 27 ACCEPT only if $\frac{1000}{\text{their } 36.25}$ is not a whole number	2
	c	.1 Correct method to calculate the mean .2 Correct value of mean before rounding 19.9 AG	.1 $\frac{18.3 + 18.5 + 19.1 + \dots + 21.1}{11}$ or $\frac{218.7}{11}$ ACCEPT $18.3 + 18.5 + 19.1 + \dots + 21.1 / 11$.1 ACCEPT if they divide at least sum of eight values by 10 or 11 .2 19.88(18....) DO NOT ACCEPT 19.89	2
	d	.1 two correct values .2 another two correct values .3 another two correct values	<p>Printer A</p> <p>Min 18.3, Q1 19.1, Median 19.9, Q3 20.7, Max 21.1 and Mode 19.8</p> <p>Mean: 19.9 Mode: 19.8</p>	3

	e	<p>One fully correct reason for Printer A referring to a measure of dispersion or mode Printer A is better AG .</p> <p>Examples (WTTE): has smaller IQR has smaller range less spread Mode is less ACCEPT most recurring time or most common or most frequent ACCEPT Printer A is better because it is more reliable or more consistent ACCEPT using the descriptor 'better' for 'less/lower/smaller' WTTE DO NOT ACCEPT comments about skewness ex: less skewed DO NOT ACCEPT the length of the box is smaller DO NOT ACCEPT comments just about the maximum Ex: The maximum is less</p>	1
	f	<p>One fully correct reason for Printer B referring to a measure of central tendency Printer B is better AG</p> <p>Examples (WTTE): has a lower median has smaller average ACCEPT using the descriptor 'better' for 'less/lower/smaller' WTTE ACCEPT "smaller Q1 and Q3" DO NOT ACCEPT smaller Q1 only or smaller Q3 only DO NOT ACCEPT comments just about the minimum Ex: The minimum is less</p>	1

Q6		Answers	Notes	Total
	a	.1 b = 2.5 OE ACCEPT [2,2.5] OE .2 c = 50 OE	ACCEPT using capital letters ACCEPT seeing values replaced in the equation	2
	b	.1 (n =)8 and (Fmax) = 96 (%)		1
	c	<p>AM1 (Solving)</p> <p>.1 Equate their E with $-1.5(n-8)^2 + 96$</p> <p>.2 Correctly expand $(n-8)^2$ and multiply by -1.5</p> <p>.3 Correctly rearrange their quadratic equation on one side = 0</p> <p>.4 Correctly substitute into quadratic formula</p> <p>.5 Correctly calculate their n</p> <p>AM2 (substitution trials)</p> <p>.1 Correctly substitute in their E and the F same value of n, accept decimal within domain</p> <p>.2 Correctly calculate their value of E and the value of F for their value of n</p> <p>.3 Correctly calculate their value of E and the value of F for another value of n</p> <p>.4 Compare value of E and F</p> <p>.5 Correctly identify their n within the given domain</p>	<p>AM1 (Solving)</p> <p>.1 their $2.5n + 50 = -1.5(n-8)^2 + 96$ ACCEPT using < instead of =</p> <p>.1 ACCEPT their E to be 70</p> <p>.2 $-1.5n^2 + 24n - 96$ OE SEEN</p> <p>.3 their $1.5n^2 - 21.5n + 50 = 0$ ACCEPT using < instead of =</p> <p>.4 their $\frac{21.5 \pm \sqrt{21.5^2 - 4 \times 1.5 \times 50}}{2 \times 1.5}$</p> <p>.5 (n=) Their 2.92 or 3</p> <p>.5 ACCEPT their positive value of n correctly rounded regardless the domain</p> <p>AM2 (substitution trials)</p> <p>.1 Ex: $2.5(2) + 50$ and $-1.5(2-8)^2 + 96$</p> <p>.2 Ex: their 55 and 42 when n=2 ACCEPT values from graph not exact</p> <p>.3 Ex: their 60 and 72 when n=4 ACCEPT values from graph not exact</p> <p>.4 evidence of comparing values of their E and F at same value of n Ex: 72>their 60 ACCEPT argument based on the graph</p> <p>.5 (n=) Their 3</p> <p>.5 DO NOT ACCEPT if n>8 .5 DO NOT ACCEPT their 3 if they use only values from graph</p>	5

d	<p>.1 Correct value of h .2 Correct value of k .3 Substitute point (2,50) into conic form .4 Correct value of a</p>	<p>.1 h=8 ACCEPT seeing 8 in the expression ($L =)a(n - 8)^2 + k$.2 k=86 ACCEPT seeing 86 in the expression ($L =)a(n - h)^2 + 86$.3 $50 = a(2 - h)^2 + k$ ACCEPT substituting n=2 and any values for a,f, and k that gives Ex: $4(2-5)^2+14=50$.4 $a = -1$ ACCEPT seeing -1 in the expression ($L =) - (n - h)^2 + k$.4 evidence of comparing values of their E and F at same value of n Ex: $72 > \text{their}60$ ACCEPT argument based on the graph .5 (n=)their3 5 DO NOT ACCEPT if $n > 8$.5 DO NOT ACCEPT their3 if they use only values from graph</p>	4
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6	e	1	2	3	4	5	10
Mark							
Identify factors (F)		State one factor from: - Number of observations - The y-intercept Ignore additional irrelevant factors					
Calculation (C)		Equate their L with $-1.5(n-8)^2 + 96$ OR Correctly substitute in their L and the F same value of n	Correctly Expand both quadratics Ex: $-n^2 + 16n - 64 + 86 = -1.5n^2 + 24n$ OR Correctly calculate their value of L and the value of F for a certain value of n	Correctly rearrange their quadratics on one side = 0 Ex: $0.5n^2 - 8n + 22 = 0$ OR Correctly calculate their value of L and the value of F for another value of n	Correctly substitute into quadratic formula OR Compare value of L and F	Correctly calculate their value of n Ex: (n =) 3.53 OR Correctly identify their n within the given domain Accept their correctly calculated n only if between 2 and 5	
Accuracy (A)		Recognizing it is not accurate with weak justification Examples: (WTTE) these are approximate calculations and not accurate Inaccurate because of rounding Inaccurate because equations not accurate OR Sensible rounding used in calculations without a comment ex: 2.55 is 3 DO NOT ACCEPT: my results are accurate WTTE	Recognizing it is not accurate with acceptable justification Examples: The equations we are using are not exactly the models that represent the data OR Accurate with strong justification Ex: Regardless of the equation I use the intersection between both will still be in the range 2 to 3	—	—	—	—

Justify (J)	<p>Weak Justification</p> <p>Appropriate comment on one of the methods Examples: (WTTE) - Correct comment on the graphical behaviour of a method - The success rate of E is 50% even with zero observations</p> <p>OR</p> <p>Attempt to comment on two or more methods Examples: (WTTE) - The Fisherface is most accurate (or highest success rate) - The maximum success rate for F is 96% while for L only 86%</p>	<p>Good Justification</p> <p>Appropriate comment/comparison on at least two methods Examples: (WTTE) - Fisherface is better than LBPH in the long run - When n = 2 or less use LBPH and when n = 4 or more use Fisherface - The maximum success rate for F is 96% while for L only 86% for the maximum n - The success rate of E is 50% even with zero observations while for others it is less</p> <p>ACCEPT: The Fisherface is most accurate (or highest success rate) for the maximum n.</p> <p>DO NOT ACCEPT if they do not have a choice of method</p>		
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Q7		Answers	Notes	Total
a	<p>AM1 .1 correctly substitute 4 or -4 into the correct equation .2 subtract 2 from 16</p> <p>AM2 .1 equate 14 to g(x)-2 or f(x)-2 .2 evidence of determining x=4 using g(x) or x=-4 using f(x) 14 AG</p>	<p>AM1 .1 $y = 2^4$ or $(1/2)^{-4}$.2 (height =) $16 - 2$ or $2^4 - 2$</p> <p>AM2 .1 $2^x - 2 = 14$ or $(\frac{1}{2})^x - 2 = 14$.2 $x=4$ or $x=-4$ ACCEPT $2^4 = 16$ or $(\frac{1}{2})^{-4} = 16$</p>		2

b	Correctly place 64 AND 128	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr style="background-color: #00aaff; color: white;"> <th style="padding: 5px;">Stage (n)</th><th style="padding: 5px;">Number of new squares added (S)</th></tr> </thead> <tbody> <tr><td style="padding: 5px;">1</td><td style="padding: 5px;">4</td></tr> <tr><td style="padding: 5px;">2</td><td style="padding: 5px;">8</td></tr> <tr><td style="padding: 5px;">3</td><td style="padding: 5px;">16</td></tr> <tr><td style="padding: 5px;">4</td><td style="padding: 5px;">32</td></tr> <tr><td style="padding: 5px;">5</td><td style="padding: 5px; color: red;">64</td></tr> <tr><td style="padding: 5px;">6</td><td style="padding: 5px; color: red;">128</td></tr> </tbody> </table>	Stage (n)	Number of new squares added (S)	1	4	2	8	3	16	4	32	5	64	6	128	1
Stage (n)	Number of new squares added (S)																
1	4																
2	8																
3	16																
4	32																
5	64																
6	128																
c	<p>.1 correctly describe one pattern for S in words</p> <p>.2 correctly describe a second pattern for S in words</p>	<p>Examples of accepted terminology:</p> <ul style="list-style-type: none"> - Even numbers, multiples of 2 - Multiples of 8 except the first row OE - Multiples of 4, divisible by 4 - Multiplies by 2, doubles, ratio 2, powers of 2, twice of previous, added to itself -powers of 2 - Geometric with ratio 2 <p>DO NOT ACCEPT incomplete terminology, for example: Geometric, multiplies by a constant, has a common ratio, the even numbers or the multiples of 4</p> <p>DO NOT ACCEPT the rule in words, for example: $2^{\text{power } n+1}$</p> <p>DO NOT ACCEPT n goes up by 1 It is increasing general rules in terms of n, example: $S = 2^{n+1}$</p> <p>More than two different patterns, all correct award (2 marks) Ex: multiplies by 2, and multiples of 4 and it is 2 to the power $n+1$</p> <p>More than two different patterns, with any incorrect award (1 mark) Ex: multiplies by 2, and multiples of 4 and it is 2 to the power n</p>	2														

	d	.1 the correct general rule .2 the correct simplified general rule with correct notation for S in terms of n .	.1 ($S = 2^{n+1}$ or $S = 2 \times 2^n$ ACCEPT $S = 4 \times 2^{n-1}$ OE .2 $S = 2^{n+1}$ DO NOT ACCEPT description in words	2
	e	.1 correctly substitute $n \geq 5$ into their general rule .1 Correctly substitute $n \geq 5$ into their general rule .2 Correctly calculate their value of S after substituting $n \geq 5$.3 Recognize that their result is the same as the correctly predicted value	.1 Ex: 2^{5+1} .2 Ex: 64 (for $n = 5$) .3 “the same as when we continue the pattern” WTTE and states how Ex: For $n=5$, 64 is obtained from pattern of multiplying 32 by 2 .3 ACCEPT if their value from .2 is the same as their value in the table in part b) or seen here in part e)	3

7	f					22
Mark	1	2	3	4	5	
Predictions (P)	Correctly predict one term for T ACCEPT whether in the table or in the response box	Correctly predict two terms for T ACCEPT whether in the table or in the response box				
Description (D)	Correctly describe a pattern in words or recursive rule for T (Ignore additional incorrect patterns) Ex : even numbers, multiples of 2 Multiples of 4 Increases by even numbers Increases by 8, 16, 32.. Adds up by 8, 16, 32... First difference doubles Difference is geometric Difference is 2^{n+1} Increases by 2S $T=2S-4$ or 2 power n Or Recursive rule: $T_n = T_{n-1} + S_n$ or $T_n = T_{n-1} + 4 \times 2^{n-1}$ or $T_n = 2T_{n-1} + 4$ OE DO NOT ACCEPT recursive rule in words or double S subtract 4	Correctly describe two patterns in words or recursive rule for T (Ignore additional incorrect patterns) OR Valid attempt to write general rule for T Ex: number times 2^n or number plus 2^n provided it is not the same S rule	Correctly describe one pattern in words or recursive rule for T (Ignore additional incorrect patterns) AND Valid attempt to write general rule for T OR Correctly describe the pattern for T as a general rule $T = 2^{n+2} - 4$ OE	Correctly describe one patterns in words or recursive rule for T (Ignore additional incorrect patterns) AND Correctly write down the general rule for T OR Correctly describe two patterns in words or recursive rule for T (Ignore additional incorrect patterns) AND Valid attempt to write general rule for T	Correctly describe two patterns in words or recursive rule for T (Ignore additional incorrect patterns) AND Correctly write down the general rule for T ACCEPT the general rule in words but penalise in notation (for Notation see N)	

Testing (T)	<p>Attempt to test their general rule for T using $n \leq 4$</p> <p>Correctly substitute in their general rule for T value of $n \leq 4$</p> <p>OR</p> <p>Correctly test their rule for T or described pattern or recursive rule</p>	<p>Correctly test their general rule for T using $n \leq 4$</p> <p>Correctly calculate their value for T in their general rule using $n \leq 4$</p> <p>AND</p> <p>Recognize that their correctly calculated value for T is the same as the given value.</p> <p>ACCEPT testing correctly $T_n = T_{n-1} + 4 \times 2^{n-1}$</p> <p>ACCEPT seeing their correctly calculated value for T and the given value in the table being equal</p>			
Verifying (V)	<p>Attempt to verify their general rule for T using $n \geq 5$</p> <p>Correctly substitute in their general rule for T value of $n \geq 5$</p> <p>OR</p> <p>Correctly verify their rule for T or described pattern or recursive rule</p>	<p>Correctly calculate their value for T in their general rule using $n \geq 5$</p>	<p>Correctly calculate their value for T in their general rule using $n \geq 5$</p> <p>AND</p> <p>Recognise that their correctly calculated value for T is the same as their predicted value obtained by continuing the pattern</p> <p>ACCEPT verifying correctly $T_n = T_{n-1} + 4 \times 2^{n-1}$</p> <p>ACCEPT seeing their correctly calculated value for T and their predicted value in the table being equal</p>		

Justify/ proof (J)	<p>Attempt to justify their described pattern or their general rule Ex: trying at least two more values and arguing as justification that they are the same or rule works or seen as justification Ex: It is partially geometric, the increase doubles so rule includes exponential WTTE Ex: The difference is geometric Weak attempt to justify their general rule geometrically Ex: $T=2S-4$ (or in words)</p>	<p>Justify their general rule arithmetically Ex: Seen as justification Part of it is geometric with first term 4 and ratio 2 then we subtract 4</p> <p>OR Attempt to justify their general rule geometrically Ex: Try to calculate area rectangle Adding $2^{n+1} + 2^{n+2} + \dots$ $T=2S-4$ (or in words) and $2^{n+2} - 4 = 2 \times 2^{n+1} - 4$</p>	<p>Good attempt to justify the general rule for T geometrically by using incorrect equations or information.</p> <p>height of $y = 2^x$ minus the blank space under the rectangle of area 4 (4 squares) and $x=n+2$</p> <p>OR Good attempt to proof by induction</p>	<p>Correctly justify the general rule for T geometrically by using area of rectangle with base 2 and functions for the height.</p> <p>Total number of squares is the area of a rectangle with base 2 and height of $y = 2^x$ minus the blank space under the rectangle of area 4 (4 squares) $x = n + 1$ (because of starting at $x=2$) $A = b \cdot h - 4$ $A = 2 \cdot (2)^{n+1} - 4$ $A = 2^{n+2} - 4$</p> <p>OR Correctly proof by induction</p> <p>J4 automatically gains T2 and V3</p>	
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Communication criteria

Mark	1	2	3
Notation and terminology (N)	<p>Correct notation of <u>their general rule</u> for T</p> <p>OR The notation of <u>the general rule</u> for T includes errors, ex: $2^{(n+2)} - 4$ or non-simplified general rule Ex: $T = 2(2^n + 2^n) - 4$</p> <p>OR Correctly describe at least one pattern in words for T DO NOT ACCEPT if they don't have any rules and they don't describe any patterns</p>	<p>Correct notation of <u>the general rule</u> for T $T = 2^{n+2} - 4$ ACCEPT $T = 4 \times 2^n - 4$ or $T = 4(2^n - 1)$ ACCEPT the use of brackets</p> <p>OR The notation of <u>the general rule</u> includes errors (see examples in N1) AND Correctly describe at least one pattern in words for T ACCEPT using U_n instead of T only if they mention that $T = U_n$ DO NOT ACCEPT using * for multiplication using / for division using ^ for power Using x instead of n</p> <p>DO NOT ACCEPT if they don't have the general rule for T</p>	<p>Correct notation of <u>the general rule</u> for T (see examples in N2)</p> <p>AND Correctly describe at least one pattern in words for T ACCEPT using U_n instead of T only if they mention that $T = U_n$ DO NOT ACCEPT using * for multiplication using / for division using ^ for power Using x instead of n</p> <p>DO NOT ACCEPT if they don't have the general rule for T</p>

Continued on next page

Mark	1	2	3
<p>Communication (L)</p> <p>Organisation and coherence</p> <p>Can be awarded even if there are errors</p> <p>Describing pattern and writing rule can be considered an item even if D0 awarded</p> <p>Test, verify, and justify may include errors but cannot be considered item (or identified for coherence) if they are zeros</p>	<p>At least three from the following items are seen:</p> <ul style="list-style-type: none"> · describe a pattern in words · write a rule · test their general rule or rule or recursive rule or pattern (at least T1) · verify their general rule or rule or recursive rule or pattern (at least V1) · justify their general rule or rule or recursive rule or pattern (at least J1) 	<p>DO NOT ACCEPT if they don't have a general rule</p> <p>At least four of the following items are seen:</p> <ul style="list-style-type: none"> · describe a pattern in words · write a general rule · test their general rule (at least T1) · verify their general rule (at least V1) · justify their general rule (at least J1) <p>AND</p> <p>For coherence, they identify the processes correctly. At least one from the following:</p> <ul style="list-style-type: none"> · test (at least T1) · verify (at least V1) · justify (at least J1) <p>Ex:</p> <ul style="list-style-type: none"> • For test: they say “test” and they test using value(s) of $n \leq 4$ only • For verify: they say “verify” and they verify using value(s) of $n \geq 5$ only • For test and for verify: they say ‘test and verify’ and they test using value(s) of $n \leq 4$ and then verify using value(s) of $n \geq 5$ <p>For justify: At least J1 awarded</p>	<p>The following two items must be seen :</p> <ul style="list-style-type: none"> • write <u>the general rule for T</u> • justify <u>the general rule</u> (at least J2) <p>AND</p> <p>At least two of the following items are seen:</p> <ul style="list-style-type: none"> • describe a pattern or rule in words • test <u>the general rule</u> (at least T1) • verify <u>the general rule</u> (at least V1) <p>AND</p> <p>For coherence, they identify the processes correctly. At least one from the following:</p> <ul style="list-style-type: none"> • test (at least T1) • verify (at least V1) <p>Ex:</p> <ul style="list-style-type: none"> • For test: they say “test” and they test using value(s) of $n \leq 4$ only • For verify: they say “verify” and they verify using value(s) of $n \geq 5$ only • For test and for verify: they say ‘test and verify’ and they test using value(s) of $n \leq 4$ and then verify using value(s) of $n \geq 5$

Stage (n)	Number of new squares (S)	Total number of squares (T)
1	4	4
2	8	12
3	16	28
4	32	60
5	64	124
6	128	252
7	256	508
8	512	1020

Markscheme

November 2023

Mathematics

On-screen examination

33 pages

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The markscheme may make use of the following abbreviation: **OE** – ‘or equivalent’

The markscheme abbreviations:

- Bullet notation means award 1 mark – see example below

Example 1 .1 mark awarded and corresponding notes are aligned			
b	.1 Show clear line of reasoning in the method .2 4	.1 45 and 49 seen OE Ex: $49 = 45 + x$.2	2

Error Carried Forward (ECF) marks

Errors made at any step of a solution affect all working that follows. In general, **Error Carried Forward (ECF)** marks are awarded after an error.

- a) **ECF** applies from one part of a question to a subsequent part of the question and also applies within the same part.
- b) If an answer resulting from **ECF** is inappropriate (eg, negative distances or $\sin x > 1$) then subsequent marks should not be awarded.
- c) If a question is transformed by an error into a **simpler question** then **ECF** may not be fully awarded.
- d) To award **ECF** marks for a question part, **there must be working present for that part**.
- e) **ECF** is only applied to working which is correct. This means that all working subsequent to an error must be checked for accuracy.
- f) A misread (**MR**) is an error. **ECF** is normally awarded.

General points

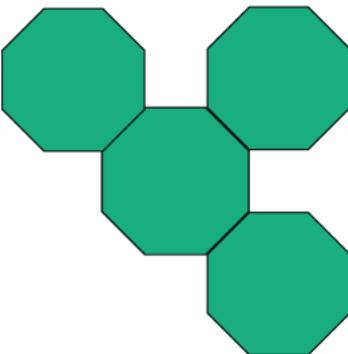
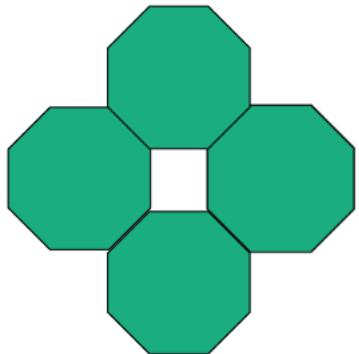
- a) As this is an international examination, accept all **alternative forms of notation**, for example 1,9 as 1.9 ; 1,000 or 1.000. However, **DO NOT ACCEPT** incorrect mathematical notation x^2 for x^2 unless noted otherwise in the MS.
- b) **ACCEPT** notation errors in intermediate steps.
- c) Ignore further working after a correct answer **unless** it indicates a lack of mathematical understanding **i.e. if the further working contradict the correct answer**, then that last mark cannot be awarded.
- d) In the case when a correct result is obtained using incorrect seen method, do not award the mark for the result.
- e) Where candidates have written two solutions to a question, mark the first solution.
- f) In the markscheme, equivalent examples of **numerical** and **algebraic** forms or **simplified** answers will generally be written in the notes preceded by **OE** (Or Equivalent) e.g. $\frac{1}{2}$ **OR** 1/2 **OR** 0.5 **OR** $2 \div 4$; $\frac{x}{2}$ **OR** $x / 2$ or $x \div 2$; 0.23 **OR** 23%
- g) In the markscheme, information provided in brackets indicate detail that may be seen in a candidate response but is not necessary to award the marks. However, it indicates what the candidate's result represents. Ex: if last mark is for the result: (AB)=5; this means we award the mark for seeing 5 as the result of calculating AB without necessarily seeing AB=5, but it does not mean we award the mark for seeing 5 representing another length
- h) Special case marks **SC** can be allocated instead of but not in addition to the marks prescribed in the markscheme.
- i) Accept seeing equation not in-line.
- j) Calculator screenshots are accepted as working steps. And when a calculator screenshot is taken, accept not seeing the whole operation.
- k) In task 2 and 3 where the markscheme is set out in a table then, unless noted otherwise, awarding the highest mark in a category includes all the lower marks in that category. It is probably best to look for the top category mark answer and if you don't find it look at the next mark down.
- l) **ACCEPT** using the correct values or working regardless their previous result.
- m) Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. **If in doubt, contact your team leader for advice.**
- n) Unless noted otherwise, if a note in a part says to accept the answer without working for 1 mark less than total marks, then seeing the correct answer with any acceptable working step, award full marks. Example: If the note in a part worth 3 marks says "4.3(3...) without working award 2 marks", then seeing any acceptable working step and seeing 4.3(3...) as the answer award the 3 marks.
- o) For "**show that**" questions, unless otherwise noted, every bullet point has to be seen in order to be awarded.

Q1		Answers	Notes	Total
	a	Correct angle	60 (degrees)	1
	b	Correct angle	120 (degrees) ACCEPT -120 (degrees)	1
	c	AM1 .1 Correctly write the horizontal translation .2 Correctly write the vertical translation AM2 .1 Correctly write the first rotation point and angle .2 Correctly write the second rotation and angle	AM1 .1 to the right WTTE and 3 (units) .1 ACCEPT X+3 .2 upwards WTTE and $\sqrt{3}$ (units) .2 ACCEPT up WTTE and 1.7(32...) or root 3 .2 ACCEPT Y + $\sqrt{3}$ AM2 .1 rotation about point $(2, \sqrt{3})$ of -120(degrees) or 120 clockwise .2 rotation about point $(3, 2\sqrt{3})$ of 120(degrees) or 120 anticlockwise	2

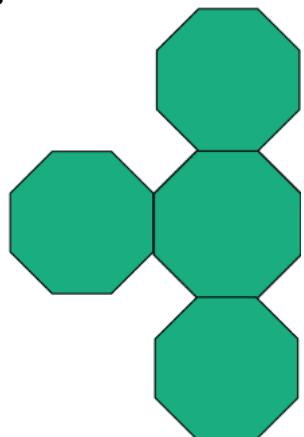
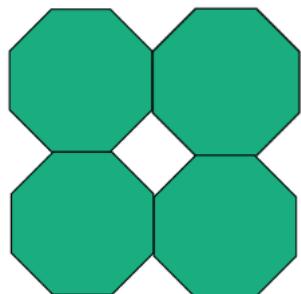
	d	<p>.1 Correctly relate interior angle to 360 for two regular shapes</p> <p>.2 Correctly relate interior angle to 360 for the third regular shape</p>	<p>.1 two from: (For the hexagon) $120 \times 3 = 360$ or $\frac{360}{120} = 3$ or $\frac{360}{3} = 120$ or 120 is a factor of 360 WTTE (For the square) $90 \times 4 = 360$ or $\frac{360}{90} = 4$ or $\frac{360}{4} = 90$ or 90 is a factor of 360 WTTE (For the triangle) $60 \times 6 = 360$ or $\frac{360}{60} = 6$ or $\frac{360}{6} = 60$ or 60 is a factor of 360 WTTE</p> <p>.2 the third from the above</p>	2
	e	<p>.1 4 connected octagons correctly touching in two directions</p> <p>.2 Exactly 8 connected octagons correctly touching in two directions</p>	<p>ACCEPT touching if the octagons are reasonably close (cannot take the square in-between) ACCEPT not seeing the squares (ignore the squares)</p> <p>.1 Ignore additional incorrectly touching octagons. See examples on the next pages</p> <p>.2 See examples on the next pages</p> <p>For bp1 and bp2: Count the connected octagons correctly touching either touching on the horizontal and vertical sides OR on the diagonal sides but not both. See examples on the next pages</p>	2

4 octagons correctly touching in two directions

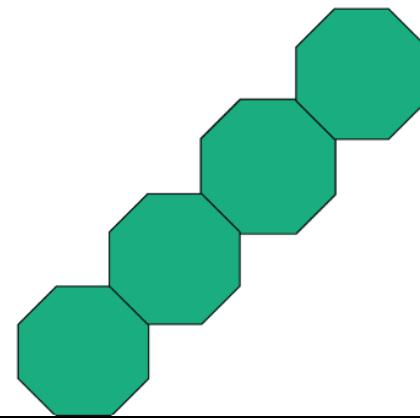
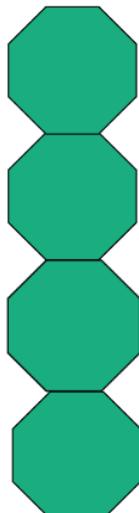
4 octagons correctly touching on diagonal sides



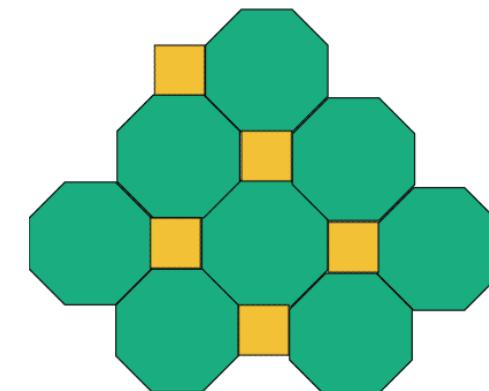
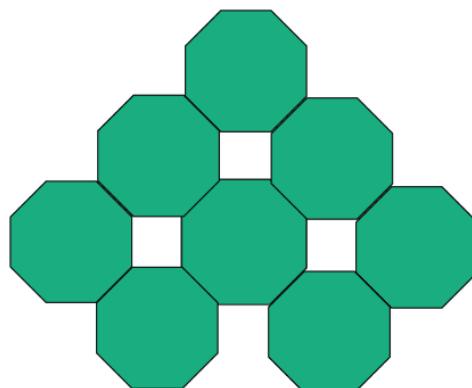
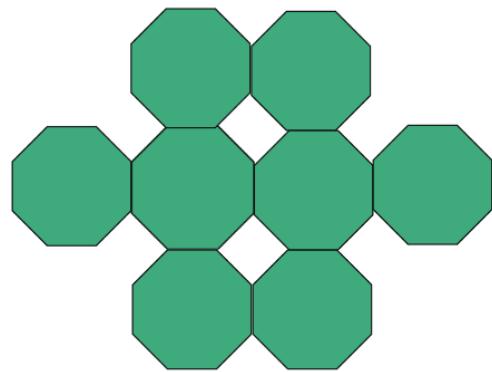
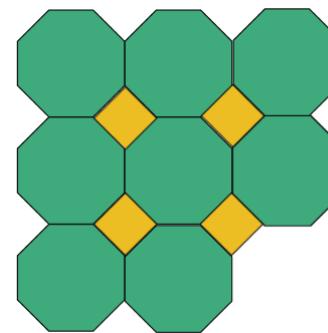
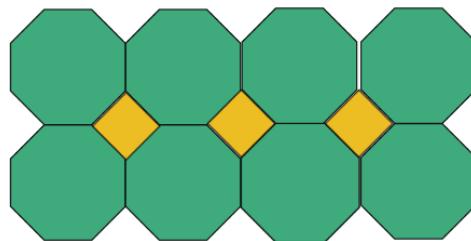
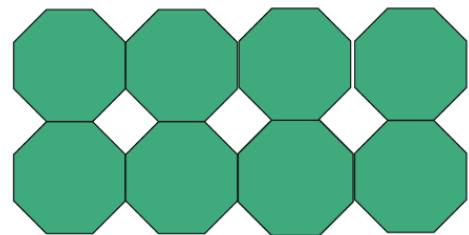
4 octagons correctly touching on the horizontal and vertical sides



4 octagons correctly touching but in one direction only

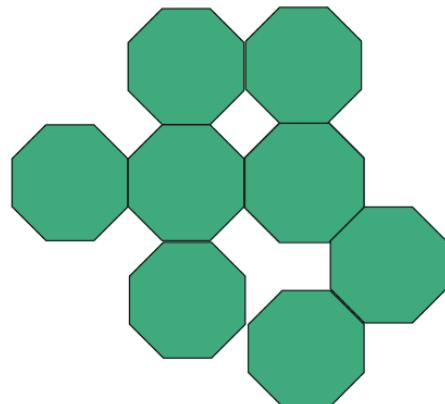


Examples for 2 marks

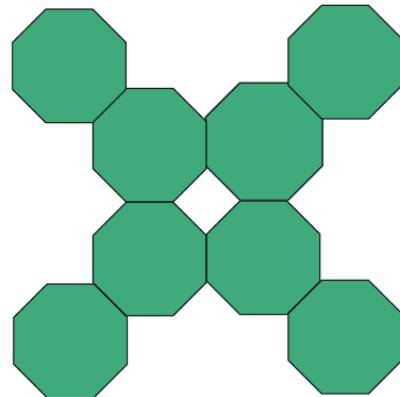
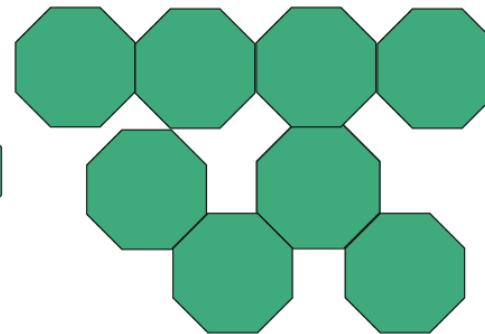
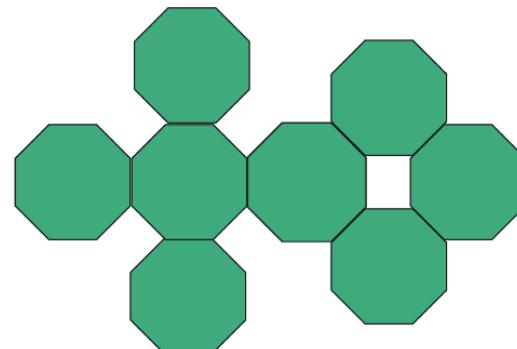


Examples for 1 mark**6 octagons correctly touching in two directions.**

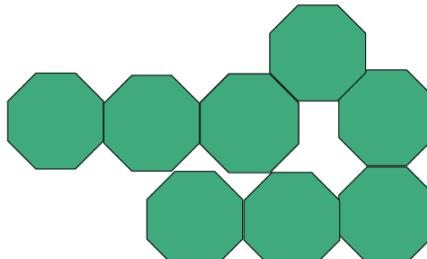
(The 6 touching on horizontal and vertical sides and cannot add to them the 2 octagons touching on diagonal sides)

**4 octagons correctly touching in two directions.**

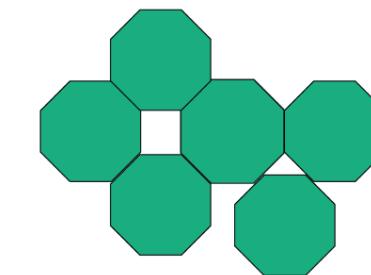
(The 4 touching on horizontal and vertical sides and cannot add to them the 4 octagons touching on diagonal sides)

**5 correctly touching octagons in two directions. (The 5 touching on horizontal and vertical sides and cannot add to them the three octagons touching on diagonal sides)****4 octagons correctly touching in two directions. (Cannot count touching horizontally and vertically with the ones touching diagonally)**

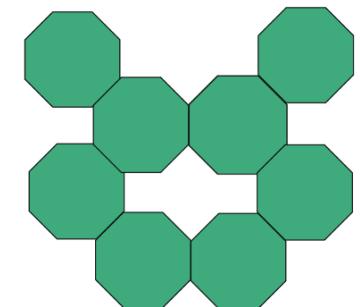
(1)



(2)



(3)



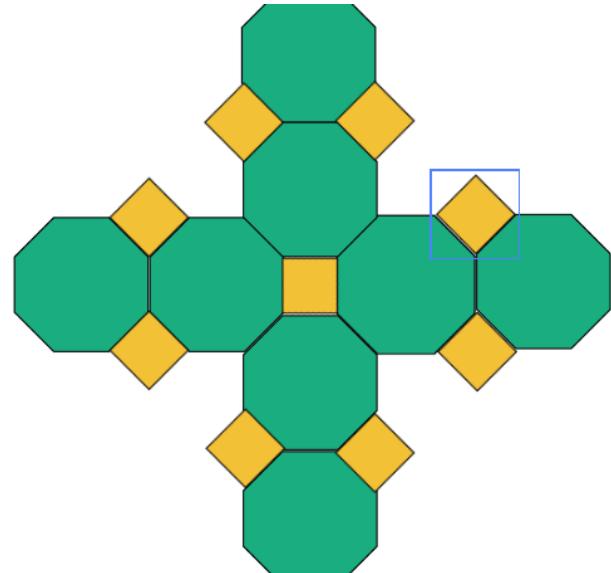
1) 4 touching horizontally and vertically

2) 4 touching diagonally

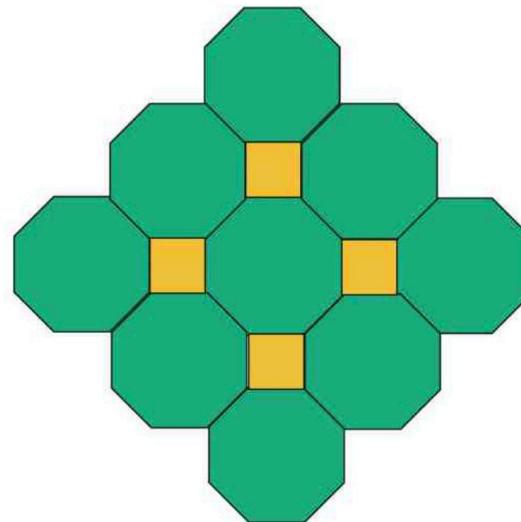
3) Two sets of 4 toucing diagonally

Further examples for 1 mark

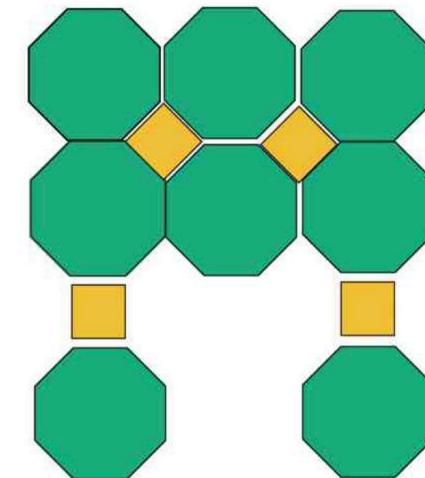
4 Octagons correctly touching diagonally in two directions



9 Octagons correctly touching diagonally in two directions

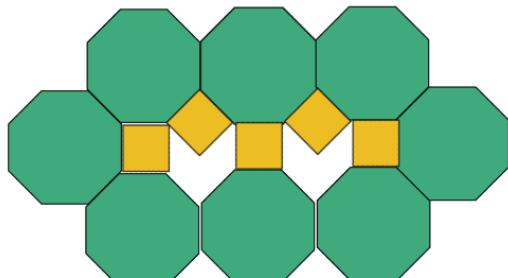
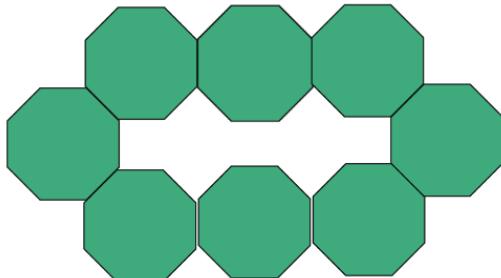
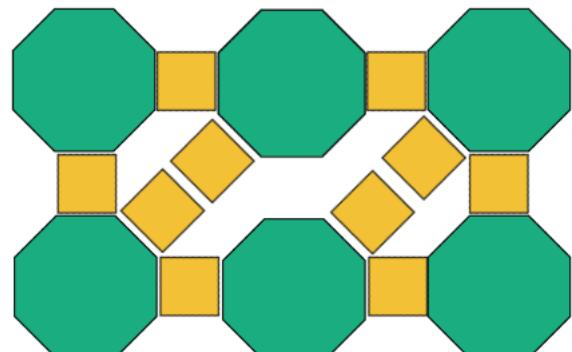
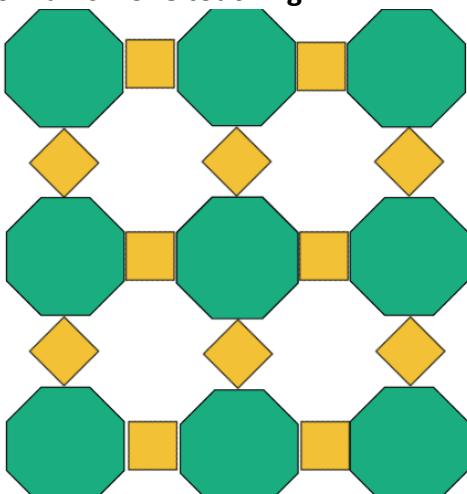


6 Octagons correctly touching horizontally and vertically in two directions

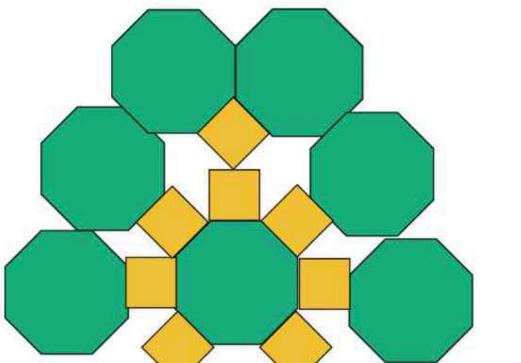


Examples for 0 marks

Two sets of 3 octagons correctly touching Horizontally and vertically
And two sets of 3 octagons correctly touching diagonally

**0 marks None touching**

2 correctly touching horizontally and vertically.



Q2	Answers	Notes	Total
a	(SAS) AE = BE, ED = EC, and $\angle AED = \angle BEC$ WTTE OE (SSS) AE = BE, ED = EC, and AD = BC OE $\angle A = \angle B$, $\angle D = \angle C$, and AE = BE OE .1 Two correct reasons .2 Third correct reason AG AED and BEC are congruent	SC 1 mark for any of the following (sides are not named) SAS SSS (they have) equal sides or same sides or same length WTTE (they have) equal angles and one equal corresponding side WTTE (they are) reflection (or mirror) of each other or symmetrical WTTE	2
b	.1 Two from: <ul style="list-style-type: none">• Subtract correct y-coordinates• Subtract correct x-coordinates• Divide their difference in y by their difference in x .2 The correct gradient	.1 Two from:: <ul style="list-style-type: none">• $10-4$ or $4-10$ or 6 or -6• $9-(-3)$ or $-3-9$ or 12 or -12• $\frac{6}{12}$ OE .2 $\frac{6}{12}$ OE	2
c	.1 Correct equation involving x and their gradient from b) .2 Correct value of x	.1 their $\frac{6}{12} = \frac{2-x}{x-1}$ OE or $\frac{8}{-4} = \frac{10-2}{9-x}$ OE or $9-x = -4$ OE .2 $(x=)13$	2
d	.1 Correctly substitute into distance formula .2 Correctly calculate their length of AB after substitution into distance formula .3 Correctly write their AB in simplest surd form	.1 $\sqrt{(9-(-3))^2 + (10-4)^2}$ OE .2 their $\sqrt{180}$ or their $13.4(164\dots)$.2 ACCEPT their AB correctly calculated after one mistake examples: $(\sqrt{(9-(-3))^2 + (10-4)^2}) = 12.2(47\dots)$ or $(\sqrt{(9-(-3))^2 + (10-4)^2}) = 48$.3 their $6\sqrt{5}$.3 DO NOT ACCEPT if their surd does not need simplification	3
e	.1 Correctly substitute into mid-point formula for x OR y coordinates .2 Correct coordinates of point E	.1 $\frac{1+9}{2}$ or $\frac{-3+\text{their } 13}{2}$ OR $\frac{10-4}{2}$ or $\frac{4+2}{2}$.2 $(5, 3)$ OE ACCEPT 5,3	2

Q3	Answers	Notes	Total
a	<p>.1 Correctly write the first simultaneous equation</p> <p>.2 Correctly write the second simultaneous equation</p> <p>.3 Correct step towards solving their system including <u>at least one correct equation</u></p> <p>.4 Correctly solve their equations for one unknown</p> <p>.5 Correctly write their corresponding unknown using one of their equations</p>	<p>.1 $6x+3y = 12$ ACCEPT using inequality</p> <p>.2 $6x + 8x + 4y = 19$ or $14x + 4y = 19$ ACCEPT using inequality</p> <p>.3 correct operator for elimination. Example: $4 \times (6x + 3y = 12)$ and $3 \times (14x + 4y = 19)$ ACCEPT "4 times eq1 and 3 times eq2" or correct step for substitution $y = 4-2x$ or $x = 2-0.5y$ or $4y = 19-14x$ or $14x = 19-4y$ OE .3 DO NOT ACCEPT if their incorrect equation is not of similar complexity Ex: if their incorrect equation has coefficients of x and y both 1 $6x+3y=6$ and $3 \times (x + y = 5)$ OR substitute $y=5 - x$ into $14x+4y=19$.3 DO NOT ACCEPT if they don't have two different equations</p> <p>.4 ($x =$) their0.5 OE OR ($y =$) their3 OE</p> <p>.5 ($x =$) their0.5 for their ($y =$)their3 OR their3 for their ($x =$)0.5 OE .5 ACCEPT seeing substitution of values for x and y that satisfy one of their equations Ex: $6 \times 1 + 3 \times 2 = 12$</p> <p>Note: every bp has to be seen to award its mark Ex:</p>	5
b	<p>.1 Correct time in hours OR in minutes</p> <p>.2 Correctly write their time in hours and minutes</p>	<p>.1 3.3 (hrs) ACCEPT $\frac{33}{10}$ OR 198 (mins)</p> <p>.2 their 3hours and their18 minutes .2 ACCEPT only if their bp1 is a decimal</p>	2

Q4		Answers	Notes	Total
	a	2 (m)	ACCEPT (20,2) OE	1
	b	.1 Correctly substitute 0 and 18 into the parabola .2 Correctly re-arrange their equation to find a AG 0.04	.1 $a(0 - 20)^2 + 2 = 18$ or $18 = 400a + 2$.2 ($a = \frac{16}{400}$) ACCEPT $16 = 400a$	2

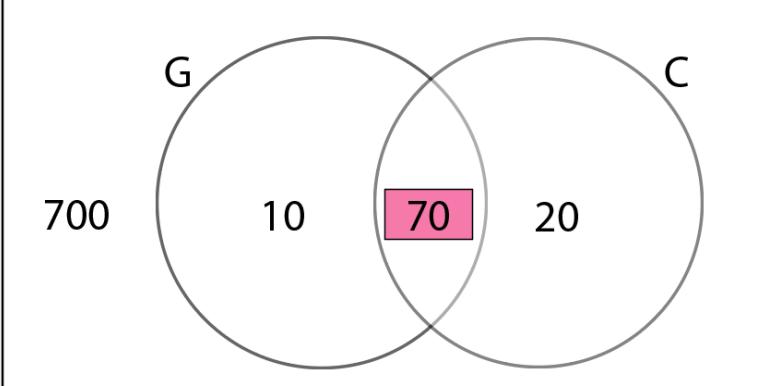
c	<p>AM1 (working with $(x-20)$)</p> <p>.1 Correctly substitute 3 into the equation</p> <p>.2 Correctly rearrange to have $(x-20)^2$ on one side</p> <p>.3 Correctly square root both sides of their quadratic from .2</p> <p>.4 Correctly identify 25</p> <p>AM2 (expanding $(x-20)^2$)</p> <p>.1 Correctly substitute 3 into the equation</p> <p>.2 Correct quadratic after expansion and multiplying by 0.04</p> <p>.3 Correctly factorise their quadratic from .2 or correctly substitute their coefficients into quadratic formula</p> <p>.4 Correctly identify 25</p> <p>AM3 (working with numbers)</p> <p>.1 Correctly substitute a value for x into the equation and calculate corresponding value of y</p> <p>.2 Correctly substitute 15 into the equation and equate with 3</p> <p>.3 Correctly substitute 25 into the equation and equate with 3</p> <p>.4 Correctly identify 25</p>	<p>AM1</p> <p>.1 $3 = 0.04(x-20)^2 + 2$ or $1 = 0.04(x-20)^2$</p> <p>.2 $\frac{1}{0.04} = (x-20)^2$ or $25 = (x-20)^2$</p> <p>.3 $x-20 = \pm\sqrt{25}$ or $x-20 = \pm\sqrt{\frac{1}{0.04}}$ or $x-20 = \pm\sqrt{25}$</p> <p>.4 (L=) 25 (m)</p> <p>AM2</p> <p>.1 $3 = 0.04(x-20)^2 + 2$</p> <p>.2 $0.04x^2 - 1.6x + 16 + 2 = 3$ or $x^2 - 40x + 375 = 0$ OE</p> <p>.3 their $(x-15)(x-25)$ or $\frac{-\text{their } 1.6 \pm \sqrt{\text{their } 1.6^2 - 4(0.04 \times \text{their } 15)}}{2(0.04)}$ OE</p> <p>.3 In the case when their bp2 is in the form $ax^2 + c = 0$, ACCEPT correctly <u>solving</u> their quadratic for x^2 Ex: if their bp2 is $0.04x^2 + 10 = 0$ and $x^2 = \frac{-10}{0.04}$</p> <p>.4 (L=) 25 (m)</p> <p>AM3</p> <p>.1 Ex: $0.04(10-20)^2 + 2 = 6$</p> <p>.2 $0.04(15-20)^2 + 2 = 3$</p> <p>.3 $0.04(25-20)^2 + 2 = 3$</p> <p>.4 (L=) 25 (m)</p> <p>For Any AM: Seeing only bp2 implies bp1 (so award 2 marks) Seeing only bp3 implies bp1 and bp2 (so award 3 marks)</p>	4

d	.1 Correctly substitute into the correct trig ratio .2 Correctly calculate their d for their trig ratio	.1 $\sin 40 = \frac{25}{d}$ OE or $\cos 50 = \frac{25}{d}$ OE .2 their 38.89(30...) ACCEPT 38.9(...) or 39(.06) (m) .2 ACCEPT only if their seen trigonometric operation involves side=25 and angle=40 or 50 .2 ACCEPT using the rounded value of the trig function only if bp1 is awarded Ex: 41.6(...) or 42		2

Q5	Answers	Notes	Total				
a	<p>.1 correct probability .2 correct statement</p>	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="text-align: center; padding: 5px;">$P(G' \cap C)$</td> <td style="padding: 5px;">Probability of selecting a person who does not have two red hair genes and has red hair</td> </tr> <tr> <td style="text-align: center; padding: 5px;">$P(G' \cap C')$</td> <td style="padding: 5px;">Probability of selecting a person who does not have two red hair genes and does not have red hair</td> </tr> </table> <p>.1 P($G' \cap C$) ACCEPT $P(G \cap C')$ ACCEPT using g instead of G and c instead of C</p> <p>.2 (Probability of selecting a person who) does not have two red hair genes and does not have red hair .2 ACCEPT no red hair genes and no red hair WTTE .2 ACCEPT not have two genes and not have red hair WTTE .2 ACCEPT no red hair genes nor red hair WTTE .2 ACCEPT <u>no</u> red hair genes or red hair WTTE .2 ACCEPT neither red hair genes nor red hair WTTE .2 ACCEPT neither red hair genes and red hair WTTE</p>	$P(G' \cap C)$	Probability of selecting a person who does not have two red hair genes and has red hair	$P(G' \cap C')$	Probability of selecting a person who does not have two red hair genes and does not have red hair	2
$P(G' \cap C)$	Probability of selecting a person who does not have two red hair genes and has red hair						
$P(G' \cap C')$	Probability of selecting a person who does not have two red hair genes and does not have red hair						

b	correctly write 0.85, 0.1 and 0.94 in the appropriate place	<pre> graph LR G -- "0.15" --> G_prime[G'] G -- "0.85" --> G G_prime -- "0.94" --> C_prime_prime[C''] G_prime -- "0.06" --> C_prime[C'] G -- "0.9" --> C[C] G -- "0.06" --> C_prime[C'] </pre>	1
c	.1 multiply the $P(G)$ by $P(C)$.2 the correct result	.1 0.15×0.9 OE .2 0.135 OE	2
d	.1 multiply their $P(G')$ by $P(C)$.2 add their 0.135 to their bp1 .3 correctly calculate their result after adding their probabilities	.1 their 0.85×0.06 or 0.051 .2 their $0.135 + $ their bp1 .2 ACCEPT their $0.135 + $ their bp1 + other probabilities .2 ACCEPT their 0.135 from part c) or calculated here in part d) .2 ACCEPT their bp1 being a probability being without working .3 their 0.186 OE .3 ACCEPT only if bp1 or bp2 are awarded	3

e	.1 multiply their d) by 15500	.1 Their 0.186×15500	2
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	.2 correctly write their result as a whole number after multiplying a probability by 15500	.2 Their 2883 .2 ACCEPT their whole number to be a rounding up or down for their result	
f	A with correct reason	A (is better) because it has a <u>larger sample</u> or more values WTTE (so is more reliable) ACCEPT "the other" or "the first" instead of A DO NOT ACCEPT reasoning related to irrelevant measures Ex: mean, mode, median, IQR, or range	1
g	correctly write 70 in the appropriate place		1
h	<p>.1 correct P(C) OR ratio $\frac{1}{N} = \frac{90}{800}$ or 0.1125 OE OR 90:800</p> <p>.1 ACCEPT ratio in any notation Ex: 90 to 800</p> <p>.1 ACCEPT their 90 used in bp1 instead of 90 only if it is equal to 20+their 70 from (g)</p> <p>.2 correctly write their N that satisfies their ratio $\frac{\text{their } 90}{800} = \frac{1}{N}$ before rounding</p> <p>.3 correctly round up their N</p>	$\frac{800}{\text{their } 90} = \text{their } 8.88(8\dots)$ $\text{ACCEPT } \frac{\text{their } 90}{800} = \frac{1}{\text{their } 8.88(8\dots)} \text{ OE}$ <p>.2 ACCEPT (N=)8.88 or 8.9 .2 ACCEPT their 90 used in bp2 being any number</p> <p>.3 their 9 .3 ACCEPT only if their bp2 needs rounding .3 ACCEPT only if bp1 or bp2 awarded</p>	3
I	correctly subtract their 11.25 from their 18.6 from d)	(their 18.6-their 11.25=) their 7.(35)(%) ACCEPT their 11.25 even if it is incorrect ACCEPT only if percentage ACCEPT positive or negative Ex: (18.6-20=)-1.4(%) DO NOT ACCEPT their 7.35(%) without working	1

Q6	Answers	Notes	Total
a	$30 < A \leq 40$	ACCEPT in any notation Ex: $30 < A < 40$ or $30 \leq A < 40$ or $30 \leq A \leq 40$ $30 - 40$ 30 to 40 DO NOT ACCEPT $31 \leq A \leq 40$ OE	1
b	.1 Correct mid-interval values .2 Add the product of their mid-interval values by frequency .3 Divide their sum by 331 .4 Correctly divide the sum of their products by 331	.1 $5, 15, 25, 35, 45, 55$ ACCEPT seeing only four correct .2 $5 \times 59 + 15 \times 72 + 25 \times 54 + 35 \times 126 + 45 \times 15 + 55 \times 5$ OE or 8085 .2 ACCEPT seeing only four correct products OE added .3 $\frac{\text{their sum}}{331}$ ACCEPT their sum being any sum .4 $24.425\dots$ ACCEPT only if $0 < \text{their } 24.425\dots < 60$.4 DO NOT ACCEPT if it is the result of a sum divided by 331 Ex: $((5+15+25+35+45+55)/331=)0.54(38\dots)$	4
c	.1 Apply the ratio on 72 .2 Correct operation based on the data 115 AG	.1 $72 \times \frac{7}{9}$ OE OR $72 \times \frac{2}{9}$ OE .1 ACCEPT "9 is 72 so 7 is" OE .2 $56+59$ or $59+72-16$ or $131-16$ ACCEPT $115-59=56$ or $131-115=16$.2 ACCEPT in words. Ex: add 56 to 59 Note: Seeing only bp2 award 1 mark	2
d	.1 Multiply 1400 or 90 by 15% OE .2 Correct result	.1 0.15×1400 or 210 or 0.15×90 or 13.5 OE ACCEPT 1400×90 or 126000 .2 $(0.15 \times 1400 \times 90 =) (\$)18900$	2

6	e		10
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Mark	1	2
Identify factors (F)	<p>Two from the four factors below mentioned in the factors box</p> <ul style="list-style-type: none"> • Number of children or number of users of playground ACCEPT under 14 ACCEPT using numbers even if incorrect. Ex: there are 56 children • Number of adults or number of users of fitness centre ACCEPT above 14 ACCEPT using numbers even if incorrect. Ex: there are 200 adults • Needed area per child or children per m² ACCEPT area or size of playground ACCEPT using numbers even if incorrect. Ex: 50m² for 24 children or 3 m² per child • Cost ACCEPT money spent ACCEPT using numbers even if incorrect. Ex: (\$)¹⁵ per m² for playground or (\$)¹⁰⁰⁰ for fitness 	<p>Three from the four factors mentioned in the factors box</p>
ACCEPT		
<p>Factors that WTTE</p> <p>Ex: "not everyone is going to use the playground" is WTTE for the 1st factor</p> <p>Ex: "not everyone uses the fitness" is WTTE for the 2nd factor</p> <p>The use of "residents scheduled for" instead of "number of". The use of the word "gym" instead of "fitness centre".</p> <p>"Number of people" or "Number of residents in apartments" or "Number of users" instead of either the 1st factor or the 2nd factor but not both</p> <p>"Number of children in 15m²" instead of either the 1st factor or the 3rd factor but not both</p> <p>"People under 14 and older" as the 1st factor only</p> <p>"cost of maintenance for the playground/recreational area" as the 4th factor only</p> <p>"Residents scheduled for the playground and for the gym" as the 1st and 2nd factors</p>		
DO NOT ACCEPT		
<p>Incomplete factors (using just a word). Examples: Number or Age or Area or recreational area or maintenance or children or adults</p> <p>Factors related to budget. Example: Maintenance budget or number of apartments or the (\$)¹⁴⁰⁰ or the 15% or (\$)¹⁸⁹⁰⁰</p> <p>"Budget of the recreational area"</p>		

	1	2	3	4	5
Calc. (C) ACCEPT rounding in calculations	<p>Weak attempt to calculate maintenance cost for area of playground (involves one correct operation)</p> <p>Ex:</p> <ul style="list-style-type: none"> • 6 in 15 so 115 is... • $\frac{6}{15} = 0.4$ OE • $\frac{15}{6} = 2.5$ OE • $\frac{115}{6} = 19.16\dots$ OE or 20 • $15 \times 115 = 1725$ • Multiply area by 2 Ex: $15 \times 2 = 30$ Ex: $115 \times 2 = 230$ 	<p>Good attempt to calculate maintenance cost for area of playground (involves two correct operations)</p> <p>Ex:</p> <ul style="list-style-type: none"> • $(\frac{15}{6} \times 115 \text{ or } \frac{115}{0.4}) = 287.5$ ACCEPT [285,300] • $(\frac{115}{6} \times 2) = 38.33\dots$ ACCEPT [38,40] • $(15 \times 115 \times 2) = (\\$)3450$ • $(30/6) = (\\$)5$ <p>Note: For all the above, only one more remaining operation is needed to reach the maintenance cost.</p>	<p>Correctly calculate the maintenance cost for the area of playground</p> $(\frac{15}{6} \times 115 \times 2) = (\$)575$ <p>Ex:</p> $(15 \times 2 \times 19) = (\$)570$ $(15 \times 2 \times 20) = (\$)600$ $(290 \times 2) = (\$)580$ <p>ACCEPT [570,600]</p>	<p>Correctly calculate the maintenance cost for the area of playground monthly or yearly</p> <p>(monthly) ACCEPT [570,600]</p> <p>(yearly) ACCEPT [1770,1800]</p> <p>AND</p> <p>Correctly identify the maintenance cost for area of fitness (\$1200)</p>	<p>Correctly calculate the total cost for recreation area monthly or yearly</p> <p>(monthly) (600+1200=) ACCEPT [1770,1800]</p> <p>(yearly) (600\times 12 + 1200\times 12=) ACCEPT [21240,21600]</p>
	<p>OR Correct number of fitness centre users $(331 - 115) = 216$</p> <p>OR Correctly identify the maintenance cost for area of fitness based on their number of adults Their (\$1200) (for their 216 adult)</p>	<p>OR Correctly identify the maintenance cost for area of fitness (\$1200)</p>	<p>OR Good attempt to calculate maintenance cost for area of playground (involves two correct operations)</p> <p>AND</p> <p>Correctly identify the maintenance cost for area of fitness (\$1200)</p>		

Mark	1	2
Accuracy (A)	<p>Correct sensible rounding seen in any calculation and ACCEPT not seeing the value before rounding Ex: Round their 19.1666 to their 19.2 or their 19 or their 20 ACCEPT $115/6=19.2$ Round their 287.5 to 285 or 288 or 290 or 300</p>	
	ACCEPT	
	<p>Rounding up or down No rounding of exact calculations leading to whole numbers only if C3 is achieved "my calculations do not need rounding" WTTE and C1 or C2 achieved with a result of a whole number</p>	
	DO NOT ACCEPT	
	<p>In sensible rounding. In particular, rounding to two decimal places. Ex: $19.1666 = 19.17$</p>	

Mark	1	2
Justify (J)	<p>ACCEPT ONLY IF C1 is achieved</p> <p>Justify suitability by comparing their calculations of maintenance cost to their budget from d) Ex: Maintenance cost exceeds the budget so not good WTTE Maintenance cost is less than budget so it is fine WTTE The suitable budget should be $\text{their } 1800 \times 12 = (\\$)\text{their } 21600$ per year WTTE The suitable budget should be $(\frac{\text{their } 1800}{90} =)(\\$)\text{their } 20$ per apartment per month WTTE</p> <p style="text-align: center;">ACCEPT</p> <p><u>Incorrect justification</u> by comparing correct monthly maintenance cost with their yearly budget Ex: 1800 less than 18900, <u>so it is suitable</u></p>	<p>ACCEPT ONLY IF C4 is achieved</p> <p>Justify suitability by comparing the correct calculations of maintenance cost to their budget from d) The maintenance cost exceeds budget so not good WTTE The maintenance cost is less than budget so it is fine WTTE The suitable budget should be $1800 \times 12 = (\\$)21600$ per year WTTE The suitable budget should be $(\frac{1800}{90} =)(\\$)20$ per apartment per month WTTE</p> <p style="text-align: center;">ACCEPT</p> <p><u>Correct justification implied</u> by comparing correct monthly maintenance cost with their yearly budget Ex: The monthly cost is 1800 and the budget is 18900 <u>so not suitable</u></p>
	<p style="text-align: center;">OR</p> <p>The correct cost and their corresponding (monthly or yearly) budget seen OE without comment (at least C4 achieved) Ex: seeing the cost as $(\\$)1800$ and the budget as $(\text{their } 18900 / 12 =)(\\$)\text{their } 1575$ Ex: seeing the cost as $(\\$)21240$ and the budget as $\text{their } (\\$)18900$ Ex: the remaining for fitness is $1575 - 575 = (\\$)1000$ and we see cost fitness $(\\$)1200$</p>	<p style="text-align: center;">DO NOT ACCEPT</p> <p>Justification by comparing monthly value with yearly value Ex: 1800 less than 18900 so it is suitable</p>
	<p style="text-align: center;">OR</p> <p>General justification of suitability Ex: -The spaces calculated are just estimates so calculations are estimates -Collect more money for maintenance just in case -Number of persons in each category will vary overtime and hence the areas need to consider being a bit bigger for the future use. WTTE - Some families may get new babies and children number increases so we need to have the recreation spaces a bit bigger -Some of the children will become adults soon so we need to have the fitness spaces a bit bigger</p>	<p style="text-align: center;">NOTE</p> <p>ACCEPT Seeing just the word "budget" when comparing instead of their actual value in part (d). Check their value in part (d) when marking the justification</p>

Q7	Answers	Notes	Total
a	The correct operations on the numbers 28 AG	4x7 or $7+7+7+7$ ACCEPT incorrect notation Ex: $7*7$ ACCEPT in words, Ex: four times seven or four by 7 WTTE ACCEPT $28/4=7$ only if they mention that 7 is the side of the square WTTE DO NOT ACCEPT other operations Ex: $20+8$ or $14+14$ DO NOT ACCEPT substitution in the general rule: $4+3\times 8$ or $8\times 4-4$ DO NOT ACCEPT substitution in the recursive rule: $28=20+8$	1

b	Correctly place 36 and 44	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr style="background-color: #00AEEF; color: white;"> <th style="text-align: center;">Stage (n)</th><th style="text-align: center;">Number of new squares added (P)</th></tr> </thead> <tbody> <tr><td style="text-align: center;">1</td><td style="text-align: center;">4</td></tr> <tr><td style="text-align: center;">2</td><td style="text-align: center;">12</td></tr> <tr><td style="text-align: center;">3</td><td style="text-align: center;">20</td></tr> <tr><td style="text-align: center;">4</td><td style="text-align: center;">28</td></tr> <tr><td style="text-align: center;">5</td><td style="text-align: center;">36</td></tr> <tr><td style="text-align: center;">6</td><td style="text-align: center;">44</td></tr> </tbody> </table>	Stage (n)	Number of new squares added (P)	1	4	2	12	3	20	4	28	5	36	6	44	1
Stage (n)	Number of new squares added (P)																
1	4																
2	12																
3	20																
4	28																
5	36																
6	44																
c	<p>.1 correctly describe one pattern for P in words</p> <p>.2 correctly describe a second pattern for P in words</p>	<p>ACCEPT complete terminology only, for example (below are four different descriptions) multiples of 4, divisible by 4 increasing by 8, adds 8, goes up by 8, moving up by 8 difference 8, common difference 8, linear with difference 8, arithmetic with difference 8 second difference is zero</p> <p>DO NOT ACCEPT incomplete terminology, for example: Arithmetic, linear, increasing by a constant, constant difference</p> <p>DO NOT ACCEPT general description, for example:</p> <p>Integers, whole numbers, positive, even numbers, divisible by 2, multiples of 2</p> <p>DO NOT ACCEPT the rule in words or description related to n for example: 8 times n minus 4, n multiplied by 8 minus 4, WTTE</p> <p>DO NOT ACCEPT</p> <p>n goes up by 1</p> <p>It is increasing</p> <p>general rules in terms of n, example: $P = 8n-4$</p> <p>Note, in the case when they have more than two different patterns: If two are accepted and the rest are all correct: award 2 marks Ex: even numbers and adds 8 and Second difference is zero and it is 8 times n then subtract 4. Ex: multiples of 2 and 4 and linear and increases by 8 and divisible by 4</p> <p>If two are accepted and any of the rest is incorrect: award 1 mark Ex: increases by 4 and Second difference is zero and it is divisible by 8</p> <p>If only one is accepted, ignore the rest and award 1 mark</p>	2														

d	<ul style="list-style-type: none"> .1 the correct general rule .2 the correct simplified general rule with correct notation 	<p>.1 ($P =$) $8n-4$ or ($P =$) $4(2n-1)$ or $P = 8x-4$ or $P = 4+8(n-1)$ or $P = 8xn-4$ or $P = 8^*n-4$</p> <p>.2 $P = 8n-4$ or $P = 4(2n-1)$ ACCEPT $P = (8n-4)$ or $P = -4+8n$</p> <p>ACCEPT using P_n instead of P</p> <p>ACCEPT $P = n8-4$</p> <p>ACCEPT using p and N</p> <p>DO NOT ACCEPT description in words</p> <p>SC 1 mark in 8d and consequently mark 8e as appropriate</p> <p>If NR in 8d and correct general rule seen in 8c or 8e</p> <p>If they write in 8d the rule in terms of u_1 and d Ex: $u_1 + (n-1)d$ then in 8e, when they test or verify, they directly use 4 instead of u_1 and 8 instead of d</p>	2
e	<ul style="list-style-type: none"> .1 Correctly substitute $n \geq 5$ into their general rule .2 Correctly calculate their value of P after substituting $n \geq 5$.3 Recognize that their result is the same as the correctly predicted value 	<p>.1 Ex: $8x5-4$</p> <p>.2 Ex: 36</p> <p>.3 "the same as when we continue the pattern" WTTE and states how. Ex: For $n=7$, 52 is obtained from pattern of adding 8 to 44 For $n=7$, $52-8=44$</p> <p>.3 ACCEPT if their value from .2 is the same as their value in the table in part b) or seen here in part e) Ex: we see their calculated $P=36$ and we see $P=36$ in their table</p> <p>SC for 1 mark Correctly test their general rule by applying the steps of verification mentioned in the left column with a value of $n \leq 4$</p> <p>SC for 1 mark verify with $n \geq 5$ the correctly described pattern or recursive rule from part c) or d)</p>	3
f	The correct operations on the numbers 53 AG	$7x7+4$ or 7^2+4 ACCEPT incorrect notation Ex: 7^A2+4 ACCEPT in words Ex: seven times seven plus four or square of side 7 then add 4 WTTE DO NOT ACCEPT other operations Ex: $53x1$ or $29+24$ DO NOT ACCEPT substitution in the general rule: $4(4^2)-4(4)+5$ DO NOT ACCEPT substitution in the recursive rule: $29+8x4$ OE	1

Glossary for task 3

Term used	Clarification
General rule	Rule in terms of only n (if they use x , it is still general rule but penalise in notation)
The general rule	The correct general rule in terms of only n (if they use x , it is still the general rule but penalise in notation)
Their general rule	Incorrect rule but in terms of only n (if they use x , it is still their general rule but penalise in notation)
Their rule	Correct rule not in terms of only n
Recursive rule	$U_n = U_{n-1} + d$ or $U_{n+1} = U_n + d$ Ex: $A_n = A_{n-1} + 8n$

We accept subsequent use of their general rule (when marking D,T, V,J, N, and L) provided it is of similar complexity. In general, the complexity of the rule depends on its form. The table below shows examples.

The general rule	ACCEPT for their general rule	DO NOT ACCEPT for their general rule
$P = 8n - 4$	linear	constant
$A = (2n-1)^2 + 4$	Quadratic or exponential	Linear or constant

Stage number (n)	Area of shape (A)
1	5
2	13
3	29
4	53
5	85
6	125
7	173
8	229

Mark	1	2	3	4
Predictions (P)	Correctly predict two terms for A ACCEPT whether in the table or in the response box ACCEPT additional incorrect predictions			
Description (D) ACCEPT incorrect terminologies, notation errors, non-simplified rule, or rule in words but penalize in notation (N)	Correctly describe a pattern in words for A (or the recursive rule for A : $A_{n+1} = A_n + 8n$) Examples of different patterns: <u>(can be seen anywhere in the response)</u> Odd numbers Difference (or increase) is multiple of 8 Difference (or the increase) is even or multiple of 4 The increase increases by 8 or what I add increases by 8 Difference increases by 8, increase of the difference by 8 Second difference 8 It is quadratic ACCEPT patterns described with incorrect terminologies or recursive rule with incorrect notation but penalise in notation (N). Ex: the odd numbers uneven numbers increases by the 8 times table ($A=$) $A+8n$ or $X=X+8n$ or the rule is $A+8n$	Correctly describe two patterns in words for A (or one pattern and the recursive rule for A)	Correctly describe two patterns in words for A (or one pattern and the recursive rule for A) AND Valid attempt to write down a general rule for A	Correctly describe two patterns in words for A (or one pattern and the recursive rule for A) AND Correctly write down the general rule for A
Ignore additional incorrect patterns	OR Valid attempt to write down a general rule for A Ex: $A = (2n-1)^2$ or $4n^2$ Ex: An exponential rule that satisfies one value for n	OR Correctly describe a pattern in words for A (or the recursive rule for A) AND valid attempt to write down a general rule for A .	OR Correctly write down the general rule for A	
	DO NOT ACCEPT			
	Descriptions of how squares are added Goes up by 8,16,24,... Recursive rule in words Second difference constant			

Mark	1	2	3
Testing (T)	Attempt to test their general rule for A using $n \leq 4$ Ex: Substitute in their general rule for A value of $n \leq 4$	Correctly test their general rule for A using $n \leq 4$ Ex: Correctly calculate their value for A in their general rule using $n \leq 4$ AND Recognise that their correctly calculated value for A is the same as the given value.	
	OR Correctly test their described pattern or their rule (e.g. the recursive rule for A or their linear rule for A)		
		ACCEPT seeing their correctly calculated value for A and the given value in the table being equal	
Verifying (V)	Attempt to verify their general rule for A using $n \geq 5$ Ex: Correctly substitute in their general rule for A value of $n \geq 5$	Correctly calculate their value for A in their general rule using $n \geq 5$	Correctly calculate their value for A in <u>the general rule</u> using $n \geq 5$ AND Recognise that their correctly calculated value for A is the same as the correct predicted value obtained by continuing the pattern
	OR Correctly verify their described pattern or their rule (e.g. the recursive rule for A or their linear rule for A)		ACCEPT seeing their correctly calculated value for A and the correctly predicted value in the table being equal

Mark	1	2	3	4
Justify/ proof (J)	ACCEPT only if D1 is achieved	ACCEPT only if they have a general rule for A	ACCEPT only if they have the correct general rule for A	ACCEPT only if they have the correct general rule for A
	Attempt to justify <u>their</u> general rule aligned with their notation for <u>their</u> general rule or recursive rule by attempting to find coefficients using any method	Correctly justify <u>the</u> general rule aligned with their notation for <u>the</u> general rule for A by correctly finding coefficients of quadratic using any method	Attempt to justify geometrically the general rule for A Divide the general rule of P by 4 or recognize ($L=$) $2n-1$ then add 4 Divide the general rule of P by 4 or recognize ($L=$) $2n-1$ and square it or multiply by itself but forget to add 4	Correctly justify geometrically the general rule for A Divide the general rule of P by 4 or recognize ($L=$) $2n-1$, square it or multiply by itself, then add 4
	OR Weak attempt to justify <u>their</u> general rule for A geometrically by saying big square added to 4 squares WTTE and we see 4 added in their general rule	OR Weak attempt to justify <u>the</u> general rule for A geometrically by Squaring incorrect length of square in terms of n and add 4 or using numbers Ex: in 5 th stage, area= $9^2 + 4 = 85$ which is the same as using my rule $4(5)^2 - 4(5) + 5 = 85$		
	OR Substitute at least two other values of n in their general rule for A and say they are the same or hence the rule works WTTE			

Mark	1	2	3
Notation and terminology (N)	ACCEPT only if D1 achieved Correct notation of <u>their</u> general rule for A	ACCEPT only if D3 achieved Correct notation of <u>the general</u> rule for A in simplest form $A = (2n - 1)^2 + 4$ or $A = 4n^2 - 4n + 5$ ACCEPT using A_n or A_n instead of A	ACCEPT only if D3 achieved Correct notation of <u>the general</u> rule for A in simplest form (see examples in N2) AND Correctly describe one pattern in words for A using correct terminology or correct recursive rule for A using correct notation
		OR The notation of <u>the general</u> rule includes errors or not simplified or in words (see examples in N1) AND Correctly describe one pattern in words for A using correct terminology or correct recursive rule for A using correct notation	ACCEPT using U_n instead of A only if they mention that $A = U_n$
	OR The notation of <u>the general</u> rule includes errors or not simplified or in words Ex: $A=(2n-1)^2+4$ or $A=4xn^2-4n+5$ The rule for A is $(2n - 1)^2 + 4$ $A = (2n - 1)(2n - 1) + 4$ or the square of $(2n - 1)$ then add 4	ACCEPT using U_n instead of A only if they mention that $A = U_n$	DO NOT ACCEPT using * for multiplication using / for division using ^ for power using x instead of n
	OR Correctly describe one pattern in words for A using correct terminology or correct recursive rule for A using correct notation $A_n = A_{n-1} + 8n$ or $A_{n+1} = A_n + 8n$	DO NOT ACCEPT using * for multiplication using / for division using ^ for power using x instead of n	DO NOT ACCEPT if they don't have the general rule for A

Mark	1	2	3
<p>Communication (L)</p> <p>Organisation and coherence Can be awarded even if there are errors</p> <p>For items: Describing pattern and writing rule can be considered an item even if D0 awarded</p>	<p>At least three from the following items are seen:</p> <ul style="list-style-type: none"> • describe a pattern in words • write a rule • test their general rule or rule or recursive rule or pattern (at least T1) • verify their general rule or rule or recursive rule or pattern (at least V1) • justify their general rule or rule or recursive rule or pattern (at least J1) 	<p>ACCEPT only if they have a general rule</p> <p>At least four of the following items are seen:</p> <ul style="list-style-type: none"> • describe a pattern in words • write a general rule • test their general rule (at least T1) • verify their general rule (at least V1) • justify their general rule (at least J1) <p>AND</p> <p>For coherence, they identify the processes correctly.</p> <p>At least one from the following:</p> <ul style="list-style-type: none"> • test • verify • justify <p>Ex:</p> <ul style="list-style-type: none"> • For test: they say “test” and they substitute in their general rule value(s) of $n \leq 4$ only • For verify: they say “verify” and they substitute in their general rule value(s) of $n \geq 5$ only • For justify: They say “justify” and they write a justification <p>Note for coherence: If they say “test and verify” and they substitute in their general rule value(s) of $n \leq 4$ <u>followed by</u> value(s) of $n \geq 5$, consider it as only one identified process</p>	<p>ACCEPT only if they have the general rule for A</p> <p>The following two items must be seen :</p> <ul style="list-style-type: none"> • write <u>the general rule for A</u> • justify <u>the general rule</u> (at least J2) <p>AND</p> <p>At least two of the following items are seen:</p> <ul style="list-style-type: none"> • describe a pattern or rule in words • test <u>the general rule</u> (at least T1) • verify <u>the general rule</u> (at least V1) <p>AND</p> <p>For coherence, they identify the processes correctly.</p> <p>At least two from the following:</p> <ul style="list-style-type: none"> • test • verify • justify <p>Ex:</p> <ul style="list-style-type: none"> • For test: they say “test” and they substitute in the general rule for A value(s) of $n \leq 4$ only • For verify: they say “verify” and they substitute in the general rule for A value(s) of $n \geq 5$ only • For justify: They say “justify” and they write a justification <p>Note for coherence: If they say “test and verify” and they substitute in the general rule value(s) of $n \leq 4$ <u>followed by</u> value(s) of $n \geq 5$, consider it as only one identified process</p>

Markscheme

May 2024

Extended mathematics

On-screen examination

41 pages

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The markscheme may make use of the following abbreviation: OE – ‘or equivalent’

The markscheme abbreviations:

- Bullet notation means award 1 mark – see example below

Example 1
.1 mark awarded and corresponding notes are aligned

b	.1 Show clear line of reasoning in the method .2 4	.1 45 and 49 seen OE Ex: $49 = 45 + x$.2	2
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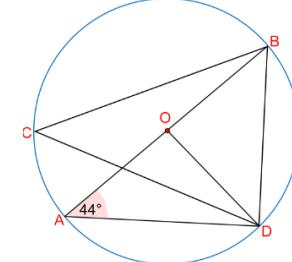
Error Carried Forward (ECF) marks

Errors made at any step of a solution affect all working that follows. In general, **Error Carried Forward (ECF)** marks are awarded after an error.

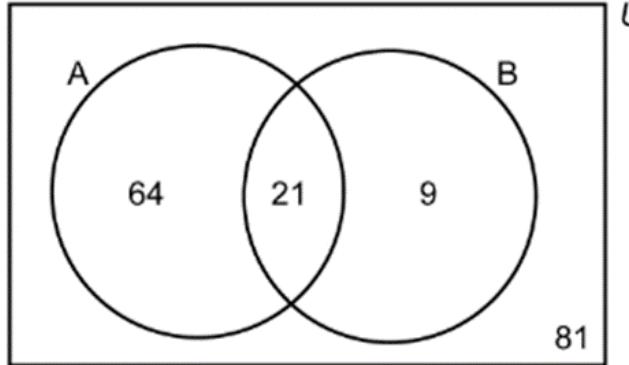
- ECF** applies from one part of a question to a subsequent part of the question and also applies within the same part.
- If an answer resulting from **ECF** is inappropriate (eg, negative distances or $\sin x > 1$) then subsequent marks should not be awarded.
- If a question is transformed by an error into a **simpler question** then **ECF** may not be fully awarded.
- To award **ECF** marks for a question part, **there must be working present for that part**.
- ECF** is only applied to working which is correct. This means that all working subsequent to an error must be checked for accuracy.
- A misread (**MR**) is considered an error that allows ECF afterwards even if the rest of the question requires “the” result and not “their” result.

General notes

- As this is an international examination, accept all **alternative forms of notation**, for example 1,9 as 1.9 ; 1,000 or 1.000. However, **DO NOT ACCEPT** incorrect mathematical notation x^2 for x^2 unless noted otherwise in the MS.
- ACCEPT** notation errors in intermediate steps.
- Unless noted otherwise, ignore further working after a correct answer even if further working is incorrect.
- In the case when a correct result is obtained using incorrect seen method, do not award the mark for the result.
- Where candidates have written two solutions to a question, mark the first solution.
- In the markscheme, equivalent examples of **numerical** and **algebraic** forms or **simplified** answers will generally be written in the notes preceded by **OE** (Or Equivalent) e.g. $\frac{1}{2}$ or $1/2$ or 0.5 or $2 \div 4$; $\frac{x}{2}$ or $x/2$ or $x \div 2$; 0.23 or 23%
- In the markscheme, information provided in brackets indicate detail that may be seen in a candidate response but is not necessary to award the marks. However, it indicates what the candidate's result represents. Ex: if last mark is for the result: (AB)=5; this means we award the mark for seeing 5 as the result of calculating AB without necessarily seeing AB=5, but it does not mean we award the mark for seeing 5 representing another length.
- Special case marks **SC** can be allocated instead of but not in addition to the marks prescribed in the markscheme.
- ACCEPT** seeing equation not in-line or the fraction line missing.
- Calculator screenshots are accepted as working steps. And when a calculator screenshot is taken, accept not seeing the whole operation.
- In task 2 and 3 where the markscheme is set out in a table then, unless noted otherwise, awarding the highest mark in a category includes all the lower marks in that category. It is probably best to look for the top category mark answer and if you don't find it look at the next mark down.
- Unless noted otherwise, **ACCEPT** using the correct values or working regardless their previous result.
- Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. **If in doubt, contact your team leader for advice.**
- Unless noted otherwise, if a note in a part says to accept the answer without working for 1 mark less than total marks, then seeing the correct answer with any acceptable working step, award full marks. Example: If the note in a part worth 3 marks says “4.3(3...) without working award 2 marks”, then seeing any acceptable working step and seeing 4.3(3...) as the answer award the 3 marks.
- For “**show that**” questions, unless otherwise noted, every bullet point has to be seen in order to be awarded.
- When a result is written as “their5.7(37...)” it means accept their result if its minimum accuracy is 1 dp. Providing higher accuracy is of course accepted but not required. Rounding their result incorrectly to nearest 1 dp is not accepted.
- When there are multiple alternative methods (multiple AM), mark the response using one specific AM. Do not add different marks from different AM.

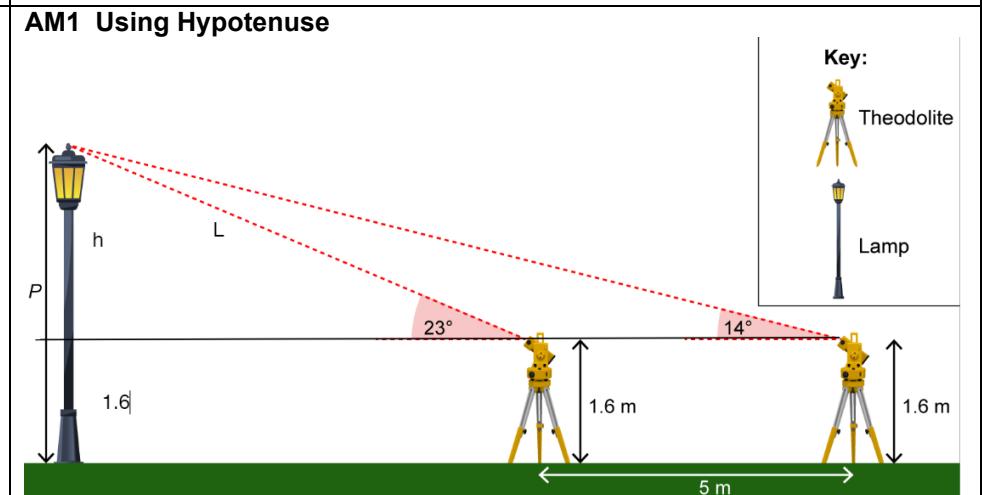
Q1		Answers	Notes	Total
	a	Angle AOD 92 Angle DOB 88 Angle DCB 44	<p>Angle AOD <input type="text"/> 92 °</p> <p>Angle DOB <input type="text"/> 88 °</p> <p>Angle DCB <input type="text"/> 44 °</p> 	3
	b	The correct reason	<p>ACCEPT (AB) diameter, passes by the centre, $AB=2r$ A, O, and B are on the same line semi-circle, half circle, cuts the middle of the circle (angle) ADB is half (angle) AOB WTTE</p> <p>DO NOT ACCEPT seeing only 90 is half 180 AB passes by the origin O is midpoint , OA and OB are radius AB doesn't move regardless how we move D Triangle inside a circle A and B positioned at both ends</p>	1

	c	<p>.1 correctly substitute into Pythagoras theorem or cosine rule</p> <p>.2 correctly write the result in simplest surd form</p> <p>For reference:</p>	<p>.1 $(AB^2 =) 12^2 + 6^2$ or 180 or $\sqrt{180}$ OE $or (AB^2 =) 12^2 + 6^2 - 2(12)(6) \cos 90$ OE .1 ACCEPT $13(.4164\dots)$</p> <p>.2 $6\sqrt{5}$.2 ACCEPT $6\sqrt{3}$ only if their bp1 is $12^2 - 6^2$ or 108 or $\sqrt{108}$.2 DO NOT ACCEPT $6\sqrt{3}$ coming from $1: 2: \sqrt{3}$.2 DO NOT ACCEPT $13\sqrt{1}$</p>	2
	d	<p>.1 Correctly substitute their radius as half their diameter from c) into area of circle formula</p> <p>.2 Correct value of their area in terms of π after squaring their radius seen in bp1.</p>	<p>.1 $\pi \left(\frac{\text{their}6\sqrt{5}}{2}\right)^2$ or $\pi \times (\text{their}3\sqrt{5})^2$ or $\pi \times \left(\frac{\text{their}13(.416\dots)}{2}\right)^2$ or $\pi \times (\text{their}6.5)^2$ OE .1 ACCEPT using 3.14 or $\frac{22}{7}$ instead of π</p> <p>.2 $\text{their}45\pi$.2 ACCEPT $\text{their}45$ rounded as decimals to at least 1 dp (Ex.42.3π)</p>	2

Q2 using		Answers	Notes	Total
	a	.1 The correct probability	<p>.1 $\frac{21}{30}$ OE</p> 	1
	b	<p>.1 Correctly write their two probabilities without replacement based on their probability in part a) OR correctly multiply their two probabilities with replacement based on their probability in part a).</p> <p>.2 Correct probability after multiplying their two probabilities without replacement from bp1.</p>	<p>.1 Seeing their $\frac{21}{30}$ and their $\frac{20}{29}$ OR their $\frac{21}{30} \times$ their $\frac{21}{30}$ or $\left(\text{their } \frac{21}{30}\right)^2$ or their $\frac{49}{100}$ or their 0.49</p> <p>.2 $\left(\text{their } \frac{21}{30} \times \frac{20}{29} = \right)$ their $\frac{14}{29}$ OE or their 0.48(2758621)</p> <p>.2 DO NOT ACCEPT the multiplication of any two probabilities</p>	2

	c	Using $P(A \cap B) = P(A) \times P(B)$ or $P(A B) = P(A)$ or $P(B A) = P(B)$	
	.1	Correctly calculate one from P(A) × P(B) OR P(A B) OR P(B A)	.1 $\left(\frac{30}{175} \times \frac{85}{175} = \right) \frac{2550}{30625}$ or 0.08(3...) OE $\frac{21}{30}$ OE or 0.7 $\frac{21}{85}$ OE or 0.247(0588..) or 0.25
	.2	Correct explicit argument from P(A \cap B) is not equal to their P(A) × P(B) OR Their P(A B) is not equal to P(A) OR Their P(B A) is not equal to P(B)	.2 0.12 ≠ their 0.08(3...) or $\frac{3675}{30625} \neq$ their $\frac{2550}{30625}$ (from $\frac{21}{175} \neq \frac{2550}{30625}$) their 0.7 ≠ 0.49 or their $\frac{735}{1050} \neq \frac{510}{1050}$ (from $\frac{21}{30} \neq \frac{85}{175}$) their 0.25 ≠ 0.17 or their $\frac{735}{2975} \neq \frac{510}{2975}$ (from $\frac{21}{85} \neq \frac{30}{175}$) .2 DO NOT ACCEPT P(A) × P(B) without seeing the calculated product .2 DO NOT ACCEPT an argument of comparison without a similar base of comparison (same denominator or decimals) .2 DO NOT ACCEPT any explanation around the independency. .1 AND .2 DO NOT ACCEPT probabilities more than 1.
	AG They are not independent		SC for 1 mark when correctly discounting the students outside A and B $P(A \cap B) = P(A) \times P(B)$ $\frac{21}{94} = \frac{85}{94} \times \frac{30}{94}$ $\frac{21}{94} = \frac{2550}{8836}$ 0.22(34 ...) = 0.29 Not equal, hence not independent.

2

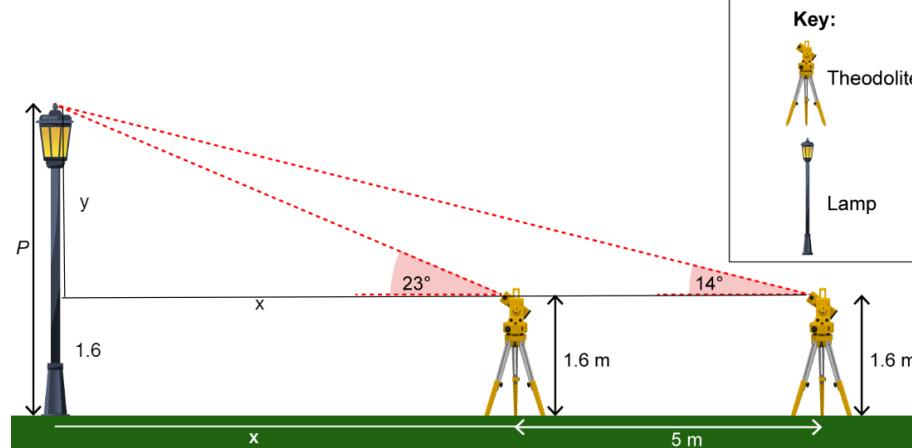
Q3	Answers	Notes	Total
EXT	<p>AM1 Using Hypotenuse (Here we're using L and h, but candidates may use different variables)</p> <p>.1 Correctly calculate top angle .2 Correctly substitute their angle into sine rule .3 Their correct value for L OR hypotenuse .4 Correctly substitute their L or their hypotenuse into correct trig ratio or sine rule to find h. .5 Correct value for their P after adding 1.6 to their h.</p> <p>AM2 on next page</p>	<p>AM1 Using Hypotenuse</p>  <p>Key:</p> <ul style="list-style-type: none"> Theodolite Lamp <p>.1 (180-14-157=) 9 (degrees) seen either calculated or seen in working.</p> <p>.2 $\frac{L}{\sin 14} = \frac{5}{\sin 9}$ OR $\frac{\text{hyp}}{\sin 14} = \frac{5}{\sin 9}$ OE</p> <p>.3 ($L=$) their7.7(323...) OR ($\text{hyp}=$) their12.48865(231...) or 12.5</p> <p>.4 $\sin 23 = \frac{h}{7.7}$ or $\cos 67 = \frac{h}{7.7}$ OE OR $\sin 14 = \frac{h}{12.5}$ or $\cos 76 = \frac{h}{12.5}$ OE</p> <p>.5 (their3(.0086...)+ 1.6) = their4.6(086...) or (their3(.02402...)+1.6) = 4.6(240...) .5 DO NOT ACCEPT adding 1.6 to their bp3.</p> <p>AM2 on next page</p>	5

AM2 Using simultaneous equations

(here we're using x and y, but candidate may use different variables)

.1 Correctly substitute 14 AND 23 into *tan* ratio or *sine* rule

.2 Correctly rearrange to solve simultaneous equations
(either one equation equal to the other OR one equation substituted into the other)

AM2 Using simultaneous equations with x

.1 Examples of correct substitutions

Tan ratio	$\tan 14 = \frac{P-1.6}{5+x}$ OE AND $\tan 23 = \frac{P-1.6}{x}$ OE
-----------	---

Sine rule	$\frac{P-1.6}{\sin 14} = \frac{x+5}{\sin 76}$ OE AND $\frac{P-1.6}{\sin 23} = \frac{x}{\sin 67}$
-----------	--

.2 Examples of rearrangements. ACCEPT decimals with correct roundings.

Tan ratio	$\frac{\tan 14}{\tan 23} = \frac{x}{5+x}$ or $(5+x) \tan 14 = x \tan 23$ ACCEPT $0.25(x+5) = 0.42x$ OE
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Sine rule	$\frac{x \sin 23}{\sin 67} = \frac{(x+5) \sin 14}{\sin 76}$ or $\frac{x}{x+5} = \frac{\sin 14 \sin 67}{\sin 23 \sin 76}$ OE
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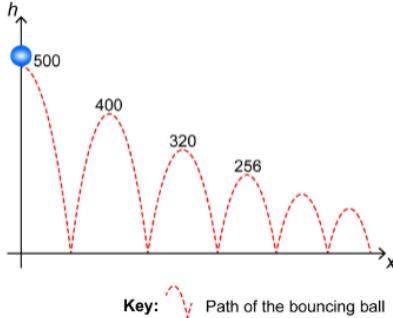
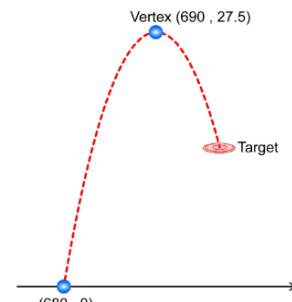
The correct bp2 implies bp1

bp3 and bp4 continued on the next page

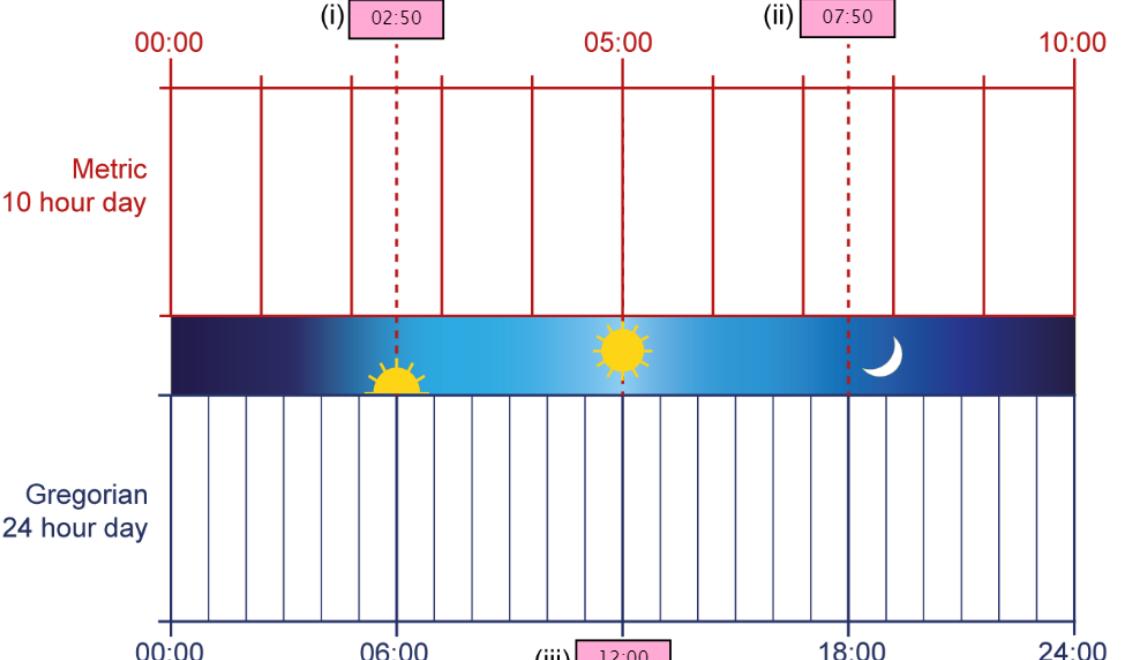
	<p>.3 Correctly solve for their x</p> <p>.4 Correctly substitute their x from bp3 into correct trig ratio or correct sine rule to find vertical length (y or $P-1.6$) OR correctly substitute into an equation from the start to find vertical length (y or $P-1.6$).</p> <p>.5 Correct value for their P after taking into account 1.6</p>	<p>.3 their7.1(1768...) ACCEPT 7.35(2941..) or 7.17(0731...) or 7.2 .3 ACCEPT their 12.1(153...) coming from x being the total and $(x-5)$ the other horizontal length. .3 The correct bp3 implies bp2 and bp1</p> <p>.4 $\tan 23 = \frac{P-1.6}{\text{their7.1}}$ or $\tan 67 = \frac{\text{their7.1}}{P-1.6}$ or $\frac{\text{their7.1}}{\sin 67} = \frac{P-1.6}{\sin 23}$ OR $\tan 14 = \frac{P-1.6}{5+\text{their7.1}}$ OE or $\tan 23 = \frac{P-1.6}{\text{their7.1}}$ OE or $\frac{P-1.6}{\sin 14} = \frac{\text{their7.1}+5}{\sin 76}$ OE or $\frac{P-1.6}{\sin 23} = \frac{\text{their7.1}}{\sin 67}$</p> <p>.4 ACCEPT incorrect use of P (they use P rather than $P-1.6$). Ex. $\tan 23 = \frac{P}{\text{their7.1}}$</p> <p>.5 (their3.02...+1.6) = their4.6(21...) ACCEPT 4.7(2...) DO NOT ACCEPT adding 1.6 to their x from bp3.</p> <p>ACCEPT correct rounding in any step DO NOT ACCEPT rounding errors</p>
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Q4		Answers	Notes	Total
a	$\frac{1-3x}{x-2} = 4x + 1$ <p>.1 Correctly multiply $(4x+1)$ and $(x-2)$</p> <p>.2 Correctly rearrange and simplify their quadratic to have all variables on one side after equating to $(1-3x)$</p> <p>.3 Correctly factorize their rearranged quadratic after bp2</p> <p>OR</p> <p>Correctly substitute their coefficients of their quadratic after bp2 into quadratic formula</p> <p>.4 The correct two values for x.</p>	<p>.1 $(1-3x =) 4x^2 + x - 8x - 2$ or $(1-3x =) 4x^2 - 7x - 2$</p> <p>.1 DO NOT ACCEPT seeing only $(4x + 1)(x - 2) = 1 - 3x$</p> <p>.2 their $4x^2 - 4x - 3$ or their $4x^2 - 4x = 3$</p> <p>.2 the correct .2 implies .1</p> <p>.2 DO NOT ACCEPT other quadratic without seeing .1</p> <p>.3 their $(2x + 1)(2x - 3)$ or $\left(x - \frac{3}{2}\right)\left(x + \frac{1}{2}\right)$ or $(4x - 6)\left(x + \frac{1}{2}\right)$ OE</p> <p>OR</p> <p>their $\frac{4 \pm \sqrt{(-4)^2 - 4(4)(-3)}}{2(4)}$</p> <p>.3 ACCEPT one notation mistake in substitution into quad formula</p> <p>Ex : $\frac{4 \pm \sqrt{-4^2 - 4(4)(-3)}}{2(4)}$ or $\frac{4 + \sqrt{(-4)^2 - 4(4)(-3)}}{2(4)}$</p> <p>.3 DO NOT ACCEPT their $(2x+1)(2x-3)$ without seeing their quadratic</p> <p>.4 $(x =) \frac{-1}{2}$ and $(x =) \frac{3}{2}$ OE</p>	4	

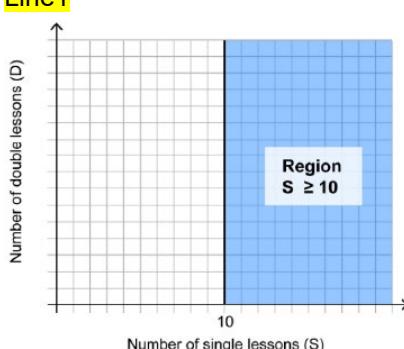
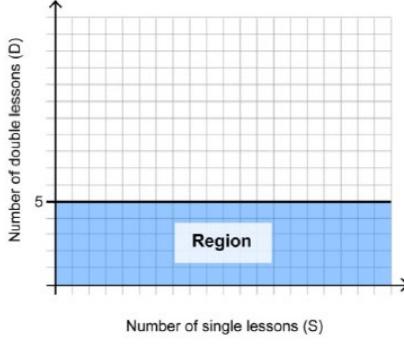
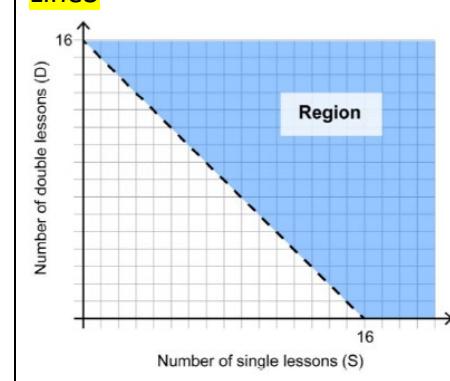
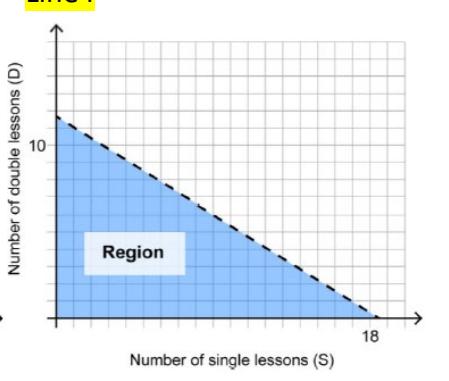
	b	<p>For reference $f(x) = 4x + 1$ and $g(x) = \frac{1-3x}{x-2}$</p> <p>.1 Correctly substitute their -0.5 or their 1.5 into $f(x)$ or $g(x)$</p> <p>.2 Correct value of their y-coordinate for A after substituting into $f(x)$ or $g(x)$</p> <p>.3 Correct value of their y-coordinate for B after substituting into $f(x)$ or $g(x)$</p>	<p>.1 $4(\text{their} - 0.5) + 1$ or $4(\text{their}1.5) + 1$ or $\frac{1-3(\text{their}-0.5)}{\text{their}-0.5-2}$ or $\frac{1-3(\text{their}1.5)}{\text{their}1.5-2}$</p> <p>.2 their -1 .2 ACCEPT only if negative. DO NOT ACCEPT 0. .2 DO NOT ACCEPT their-1 without working</p> <p>.3 their 7 .3 ACCEPT only if positive. DO NOT ACCEPT 0. .3 DO NOT ACCEPT their 7 without working</p> <p>ACCEPT -1 and 7 not labeled as A or B.</p>	3
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Q5		Answers	Notes	Total
	a	<p>At least two ratios showed equals OR at least two ratios equals to 0.8 showed</p>  <p>Key: Path of the bouncing ball</p>	<p>Two from:</p> $\frac{256}{320} = \frac{320}{400} = \frac{400}{500}$ <p>OR two from:</p> $\frac{256}{320} = 0.8, \frac{320}{400} = 0.8, \frac{400}{500} = 0.8$ <p>ACCEPT at least two from $500 \times 0.8 = 400, 400 \times 0.8 = 320, 320 \times 0.8 = 256$</p> <p>DO NOT ACCEPT using the geometric sequence formula. EX. $500 \times 0.8^2 = 320$.</p>	1
	b	<p>.1 Correctly substitute 10 into correct geometric sequence formula</p> <p>.2 Correctly calculate their U_{10} after substituting 10 into their geometric sequence formula</p>	<p>.1 ($U_{10} =) 500 \times 0.8^{(10-1)}$ OE ACCEPT $n = 10$ AND $500 \times 0.8^{(n-1)}$</p> <p>.2 ($U_{10} =$ their 67(.108864)(cm)</p> <p>.2 DO NOT ACCEPT 67.12</p> <p>.2 ACCEPT 65 (coming from $0.8^9 = 0.13$)</p>	2
	c	<p>.1 Correctly substitute (690, 27.5) into correct formula</p> <p>.2 Correctly substitute (680,0) into their formula from bp1</p> <p>.3 Correct value of their a after their bp2.</p>	<p>.1 ($h(x) =) a(L - 690)^2 + 27.5$</p> <p>.1 ACCEPT a quadratic equation in the form $a(x-b)^2+c$ with incorrect "a", but (690,27.5) correctly substituted. Ex: $h(x) = -1(x-690)^2 + 27.5$</p> <p>.2 their ($0 = a(680 - 690)^2 + 27.5$)</p> <p>.3 ($a =$ their -0.275)</p> <p>.3 ACCEPT only if their "a" is negative</p> <p>Accept the use of x instead of L</p>	 <p>3</p>

<p>d</p> <p>AM1 When they don't expand $(L-690)^2$</p> <p>.1 Equate their equation to 17.6</p> <p>.2 Correctly rearrange the quadratic for $(L - 690)^2$</p> <p>.3 Correctly square root both sides from their bp2.</p> <p>.4 Correctly identify the value for L as 696</p> <p>AM2 When they expand $(L-690)^2$</p> <p>.1 Equate their equation to 17.6</p> <p>.2 Correctly expand $(L-690)^2$ AND rearrange the quadratic = 0</p> <p>.3 Correctly substitute their coefficients of their quadratic from bp2 into quadratic formula</p> <p>.4 Correctly identify the value for L as 696</p>	<p>AM1 When they don't expand $(L-690)^2$</p> <p>.1 $17.6 = \text{their } (-0.275(L-690)^2+27.5)$ or $\text{their } -9.9 = \text{their } (-0.275(L-690)^2)$</p> <p>.2 $(L - 690)^2 = 36$ OE</p> <p>.3 $L-690 = \text{their}+/-6$ ACCEPT $L-690 = \text{their}6$ ACCEPT not seeing their negative root.</p> <p>.3 DO NOT ACCEPT negative under the root or as the answer</p> <p>.4 $(L) = 696$</p> <p>AM2 When they expand $(L-690)^2$</p> <p>.1 $17.6 = \text{their}(-0.275(L-690)^2+27.5)$ or $\text{their } -9.9 = \text{their } (-0.275(L-690)^2)$</p> <p>.2 $0 = -0.275L^2 + 379.5L - 130917.6$ or $0 = 0.275L^2 - 379.5L + 130917.6$ OE</p> <p>.3 $\text{their } \frac{379.5 \pm \sqrt{(-379.5)^2 - 4(0.275)(130917.6)}}{2(0.275)}$</p> <p>.3 ACCEPT $(-b)^2$ written as b^2 under the square root</p> <p>.3 ACCEPT their simpler quadratic $ax^2 + c = 0$ or $ax^2 + bx = 0$, only if they solve it correctly at least for the positive value.</p> <p>Ex: if their bp2 is $-9.9 = 0.275L^2 - 130900$ and $L = 689.9$</p> <p>.4 $(L) = 696$</p> <p>For any AM, bp2 implies bp1 DO NOT ACCEPT if candidate found both values (684 and 696) and didn't identify 696 as the one.</p> <p>Accept the use of x instead of L</p>	<p>4</p>
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Q6	Answers	Notes	Total
6a	<p>.1 Correctly write one time</p> <p>.2 Correctly write another two times</p>	<p>ACCEPT 2:50 or 2h50 or 250 or 2.50 or 2.5 2.5:00 OE ACCEPT 7:50 or 7h50 or 750 or 7.50 or 7.5 7.5:00 OE ACCEPT 12:00 or 12h00 or 1200 or 12.00 or 12 or 12pm OE</p> <p>DO NOT ACCEPT 2:5 or 25 or 2:30 OE DO NOT ACCEPT 7:5 or 75 or 7:30 OE</p>  <p>(i) 02:50 (ii) 07:50 (iii) 12:00</p>	2
6b	<p>.1 Correct number of hours in the hours box</p> <p>.2 Correct number of minutes in the minutes box</p>	<p>.1 2 in the hours box .1 ACCEPT $\frac{24}{10}$ or 2.4 OE in the hours box .1 ACCEPT 2 or 2.4 hours in the response box <u>only if the hours box is empty</u></p> <p>.2 24 in the minutes box .2 ACCEPT 24 minutes in the response box <u>only if the minutes box is empty</u></p>	2

6c	School event	Metric time	Metric seconds in standard form	Examples of correct values with incorrect form	
				9×10^3	90×10^2
	Examination	90 minutes	9×10^3	90×10^2	0.9×10^4
	School day	2 hours and 40 minutes	2.4×10^4	0.24×10^5	24×10^3
	Time to run 10 kilometres	53 minutes	5.3×10^3	240×10^2	
	Climbing mount Kilimanjaro	4 days, 2 hours and 5 minutes	4.205×10^5	0.4205×10^6	42.05×10^4
				420.5×10^3	4205×10^2
.1	Correctly write first row	.1	9×10^3 ACCEPT 9.0×10^3		
.2	Correctly write second row	.2	2.4×10^4 ACCEPT 2.40×10^4		
.3	Correctly write third row	.3	53 min .3 ACCEPT 53.00 mins or half hour and 3 mins or 50 minutes and 300 sec OE .3 DO NOT ACCEPT without units Ex: 53		
.4	Correctly write fourth row	.4	4.205×10^5		
			For bp1, bp2, and bp4 ACCEPT if the value in seconds is <u>correct</u> but written in <u>incorrect standard form</u> , and deduct only one mark from the total Do not deduct if the value incorrectly written in standard form is already 0 marks (incorrect value of seconds)		
			Examples: 90×10^2 , 24×10^3 , 53 minutes, 4205×10^2 → Award $(1 + 1 + 1 + 1) - 1 = 3$ marks 90×10^3 , 2.4×10^4 , 5 hours and 3 minutes, 4205×10^2 → Award $(0 + 1 + 0 + 1) - 1 = 1$ mark 9×10^3 , 24×10^4 , 53 mins, 4.2×10^5 → Award $(1 + 0 + 1 + 0) - 0 = 2$ marks		
					4

<p>6 d .1 Two correct .2 Another two correct .3 Another two correct</p> <table border="1" style="margin-top: 10px; border-collapse: collapse;"> <tr><td style="background-color: #f2e0c7;">0 – 1 correct</td><td style="background-color: #f2e0c7;">0 marks</td></tr> <tr><td style="background-color: #ffd700;">2 – 3 correct</td><td style="background-color: #ffd700;">1 mark</td></tr> <tr><td style="background-color: #d9eaf7;">4 – 5 correct</td><td style="background-color: #d9eaf7;">2 marks</td></tr> <tr><td style="background-color: #a9f5e0;">6 correct</td><td style="background-color: #a9f5e0;">3 marks</td></tr> </table> <p style="margin-top: 20px;">For reference Line1</p>  <p style="margin-top: 20px;">Line2</p> 	0 – 1 correct	0 marks	2 – 3 correct	1 mark	4 – 5 correct	2 marks	6 correct	3 marks	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="text-align: center; padding: 5px;">Inequalities</th> <th style="text-align: center; padding: 5px;">Description of region in words</th> </tr> </thead> <tbody> <tr> <td style="padding: 5px;">$S \geq 10$</td> <td style="padding: 5px;">The number of single lessons is at least 10 WTTE</td> </tr> <tr> <td style="padding: 5px;">$D \leq 5$</td> <td style="padding: 5px;">The number of double lessons is at most 5 WTTE</td> </tr> <tr> <td style="padding: 5px;">$S + D > 16$</td> <td style="padding: 5px;">The total number of single and double lessons is more than 16 WTTE</td> </tr> <tr> <td style="padding: 5px;">$5S + 8D < 92$</td> <td style="padding: 5px;"></td> </tr> </tbody> </table> <p style="margin-top: 20px;">For reference: Line3</p>  <p style="margin-top: 20px;">Line4</p> 	Inequalities	Description of region in words	$S \geq 10$	The number of single lessons is at least 10 WTTE	$D \leq 5$	The number of double lessons is at most 5 WTTE	$S + D > 16$	The total number of single and double lessons is more than 16 WTTE	$5S + 8D < 92$		3
0 – 1 correct	0 marks																			
2 – 3 correct	1 mark																			
4 – 5 correct	2 marks																			
6 correct	3 marks																			
Inequalities	Description of region in words																			
$S \geq 10$	The number of single lessons is at least 10 WTTE																			
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$S + D > 16$	The total number of single and double lessons is more than 16 WTTE																			
$5S + 8D < 92$																				

e	<p>.1 One correct pair of integer values of S and D. .2 Correctly calculate number of minutes using one of their (S,D) .3 Explicitly identify 15 and 2 OR 15 and 2 seen as the only final answer.</p> <p>.1 (16,1) or (S=16 and D=1) OR (15,2) or (S=15 and D=2) .2 (20xtheir16 + 40xtheir1=) their360 or (their15x20 + 40xtheir2=) their380 Examples of minutes</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tbody> <tr> <td>(12,4)</td> <td>(13,3)</td> <td>(14,2)</td> <td>(15,1)</td> <td>(15,2)</td> <td>(16,0)</td> <td>(16,1)</td> <td>(18,0)</td> </tr> <tr> <td>400</td> <td>380</td> <td>360</td> <td>340</td> <td>380</td> <td>320</td> <td>360</td> <td>360</td> </tr> </tbody> </table> <p>.3 (S=15) and (D=2)</p>	(12,4)	(13,3)	(14,2)	(15,1)	(15,2)	(16,0)	(16,1)	(18,0)	400	380	360	340	380	320	360	360	3
(12,4)	(13,3)	(14,2)	(15,1)	(15,2)	(16,0)	(16,1)	(18,0)											
400	380	360	340	380	320	360	360											

The MYP 5 chemistry class is creating copper sulphate crystals from a solution.

It takes **one** metric day for the solution to form 2.987 grams of copper sulphate crystal.

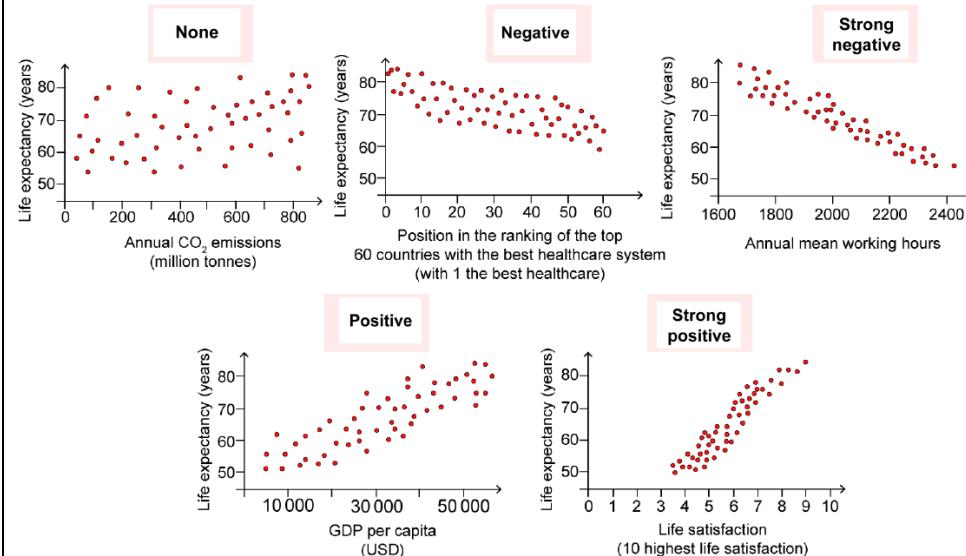
Show that the rate of production is 2.987 milligrams per minute.

<p>6f Correctly apply on 2.987 g per day both conversions: from g to mg and from day to minutes</p> <p>AG 2.987 (mg/min)</p>	<p>Conversions with correct mass units mentioned and divide by 1000. Example: 2987 mg (per day) and 2987/1000 OR Conversions with correct time units mentioned and multiply by 1000. Example: 0.002987 (grams) per min and 0.002987x1000</p> <p>ACCEPT</p> <p>Multiply 2.987 by 1000 and divide by 1000. Ex: $2.987 \times \frac{1000}{1000}$</p> <p>DO NOT ACCEPT any of the following to replace any part from the accepted above 2.987 mg per minute (it is the required AG) 2.987 g per day 1 day=10 hours or 1000 mins 1g=1000mg</p> <p>DO NOT ACCEPT (in any order): $\frac{2.987}{1000} = 0.002987$ g/min and 0.002987g/min=2.987mg/min (because the second part is 2.987 mg per minute AG)</p>	<p>1</p>
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Calculate the maximum number of grams that can be produced from 04:00 on Day 1 to 04:90 on Day 2. Give your answer to the nearest gram.

g)

<p>6g</p> <p>AM1</p> <p>.1 Correct total duration in days OR in hours OR in minutes</p> <p>.2 Correct ratio seen in any notation or applied on their time</p> <p>.3 Correct result of 3 after the correct calculation</p> <p>AM2</p> <p>.1 Correct additional duration in days OR in hours OR in minutes</p> <p>.2 Correct ratio seen in any notation or applied on their time</p> <p>.3 Correct result of 3 after the correct calculation</p>	<p>AM1</p> <p>.1 $\frac{10.9}{10}$ or 1.09 (days) OR 10.9 (hours) OR 1090 (mins)</p> <p>.2 "if in 1 day 2.987 gm then in their1.09 days it is or " $\frac{1}{\text{their1.09}} = \frac{2.987}{x}$ OE OR "if in 10 hours 2.987 gm then in 10.9 it is " or $\frac{10}{\text{their10.9}} = \frac{2.987}{x}$ OE OR "if in 1000 min 2.987 gm then in their1090 it is" or $\frac{1000}{\text{their1090}} = \frac{2.987}{x}$ OE</p> <p>.2 ACCEPT $2.987 \times \text{their1090}$</p> <p>.2 DO NOT ACCEPT their time being 10 or 100 or 1000</p> <p>.3 2.987×1.09 or $\frac{2.987 \times 10.9}{10}$ or $\frac{2.987 \times 1090}{1000}$ OE or $3.25583 \cong 3$ (gm)</p> <p>.3 ACCEPT only if the calculation is correct</p> <p>AM2</p> <p>.1 0.09 (days) OR 0.9 (hours) OR 90 (mins)</p> <p>.2 "if in 1 day 2.987 gm then in their0.09 days it is or " $\frac{1}{\text{their0.09}} = \frac{2.987}{x}$ OE OR "if in 10 hours 2.987 gm then in their0.9 it is" or $\frac{10}{\text{their0.9}} = \frac{2.987}{x}$ OE OR "if in 1000 min 2.987 gm then in their90 it is" or $\frac{1000}{\text{their90}} = \frac{2.987}{x}$ OE</p> <p>.2 ACCEPT $2.987 \times \text{their90}$</p> <p>.2 DO NOT ACCEPT their time being 10 or 100 or 1000</p> <p>.3 2.987×0.09 or $\frac{2.987 \times 0.9}{10}$ or $\frac{2.987 \times 90}{1000}$ or $0.26883 + 2.987$ or $3.25583 \cong 3$ (gm)</p> <p>.3 ACCEPT only if the calculation is correct</p>	<p>3</p>
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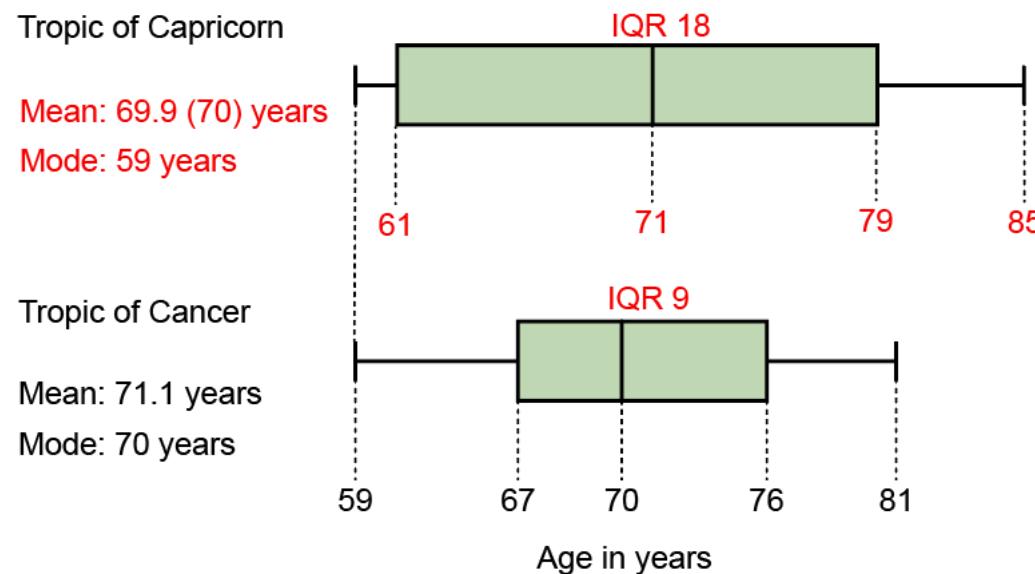
Q7	Answers	Notes	Total
a	<p>.1 Correctly identify 3 scatter plots</p> <p>.2 Correctly identify another 2 scatter plots</p>	 <p>The figure consists of four scatter plots, each with 'Life expectancy (years)' on the vertical axis (ranging from 50 to 80).</p> <ul style="list-style-type: none"> Plot 1: X-axis is 'Annual CO₂ emissions (million tonnes)'. A pink box labeled 'None' is placed above the plot. Plot 2: X-axis is 'Position in the ranking of the top 60 countries with the best healthcare system (with 1 the best healthcare)'. A pink box labeled 'Negative' is placed above the plot. Plot 3: X-axis is 'Annual mean working hours'. A pink box labeled 'Strong negative' is placed above the plot. Plot 4: X-axis is 'GDP per capita (USD)'. A pink box labeled 'Positive' is placed above the plot. Plot 5: X-axis is 'Life satisfaction (10 highest life satisfaction)'. A pink box labeled 'Strong positive' is placed above the plot. <p>Below the plots, the text 'ACCEPT identifying on the graph instead of the box above it. Ex:' is followed by a small scatter plot of its own, identical to Plot 4 but with a pink box labeled 'Positive' placed directly on the graph area.</p>	2

b	<ul style="list-style-type: none"> .1 Two correct .2 Third correct .3 fourth and fifth correct 	<p>Mode 59 LQ 61 Median 71 UQ79 Max 85 Tropic of Capricorn</p> <p>A box plot with the following data points: - Minimum (notched): 59 - Lower Quartile (LQ): 61 - Median: 71 - Upper Quartile (UQ): 79 - Maximum: 85 - Mode: 59</p>	3
c	<p>Range:</p> <ul style="list-style-type: none"> .1 Correctly subtract 59 from 85 <p>Interquartile range (IQR):</p> <ul style="list-style-type: none"> .2 Correctly subtract 61 from 79 	<p>Range:</p> <ul style="list-style-type: none"> .1 26 .1 ACCEPT correctly subtracting 59 from their max in their box plot in part b) i.e (their85–59=) their26 <p>Interquartile range (IQR):</p> <ul style="list-style-type: none"> .2 18 .2 ACCEPT correctly subtracting their LQ from their UQ on their box plot in part b) i.e (their79–their61=) their18 	2

7	d		10
Mark	1	2	
Identify factors (F)	<p>Note: Award F1 automatically if C3 is achieved</p> <p>OR</p> <p>Identify two from the elements below <u>in the elements box</u> (ACCEPT WTTE):</p> <p>1) Mean or Mode or Median 2) Range or IQR 3) Trend over the years or line of best fit or any description for the graph 4) Max or Min or UQ or LQ or quartiles 5) Health care or mean working hours or GDP or life satisfaction or the correlation</p> <p>DO NOT ACCEPT</p> <p>General terms like: Measures of central tendency or measures of dispersion or statistics or Box and whisker or the graph for life expectancy or the scatter graph Number of countries in each tropic (11 countries on Capricorn and 17 on Cancer) Location of countries Annual CO₂ emissions Just values for the elements</p>	<p>Identify three from the elements below <u>in the elements box</u> (ACCEPT WTTE):</p> <p>1) Mean or Mode or Median 2) Range or IQR 3) Trend over the years or line of best fit or any description for the graph</p> <p>DO NOT ACCEPT Any element from 4) or 5) (mentioned for F1)</p>	

In 2021	Mean	Mode	IQR	Range	Min	LQ	Median	UQ	Max
Capricorn	69.9.. (or 70)	59	18	26	59	61	71	79	85
Cancer	71	70	9	22	59	67	70	76	81

Image 2

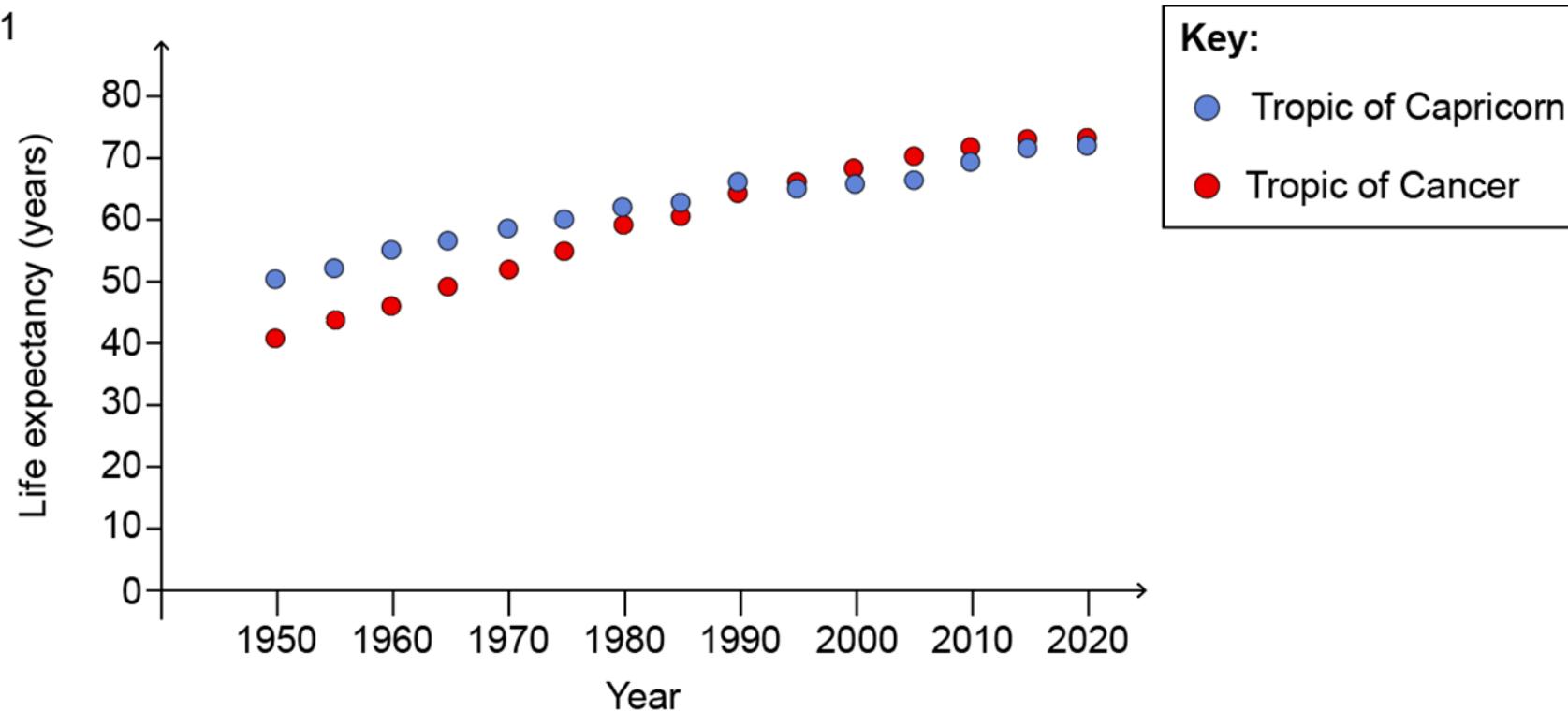
**In 2021**

Capricorn has bigger/larger: Median, IQR and Range

Cancer has bigger/larger: Mean and Mode

Mark	1	2	3	4
Comparative calculations (C) ACCEPT seeing in the elements response box or under any title in their response box	<p>Correctly compare any two from their statistics: Their Mean Their Mode Their Median Their Range Their IQR Their Max or Min or LQ or UQ Example: They have similar mean and median Highest value in Capricorn is 85 while Cancer 81 Cancer mode is 70 and compared to Capricorn 59 Capricorn less Q1 but higher median</p> <p style="text-align: center;">OR</p> <p>Attempt to calculate the mean for the tropic of Capricorn Ex: adding the values incorrectly and divide by 11 their$769/11$</p>	<p>Correctly compare any three from their statistics: Their Mean Their Mode Their Median Their Range Their IQR Their Max or Min or LQ or UQ Example: Capricorn higher median and IQR but same min</p> <p style="text-align: center;">OR</p> <p>Correctly calculate The mean for the tropic of Capricorn $(769/11=)69.9$ or 70</p>	<p>Values must be seen</p> <p>Correctly compare: The Mean: 69.9 or 70 AND One from Their Range or Their IQR</p> <p>Note: When comparing their range or their IQR ACCEPT seeing only their values for cancer (their22 or their9)</p> <p style="text-align: center;">OR</p> <p>Correctly compare: The Mean AND The Mode and The Median ACCEPT only one with their value but not both</p>	<p>Values must be seen</p> <p>Correctly compare: The Mean: 69.9 or 70 AND One from The Range or The IQR AND One from The Mode or The Median</p> <p>Note: When comparing the range or the IQR ACCEPT seeing only the values for cancer (22 or 9) Note: If they compare by writing directly the difference then it has to be correct (4 for range or 9 for IQR)</p>
	ACCEPT			
	<p>Spelling mistakes Ex: “medium” instead of “median” “similar” WTTE when comparing Mean or Median Q1 instead of LQ, Q3 instead of UQ, “Middle” age or “Q2” instead of “median” “most common” instead of “mode” “lowest” or “smallest” OE instead of “minimum, “highest” or “largest” OE instead of “Maximum” IQR “more concentrated” instead of smaller</p> <p style="text-align: center;">DO NOT ACCEPT</p> <p>Cancer is higher WTTE when comparing their median, their IQR, or their range Using “central tendency” or “average” instead of “mean or mode or median” Using “measures of dispersion” or “spread” instead of “range or IQR”</p>			

Image 1



Mark	1	2
Comment on the Headline (H) ACCEPT seeing in the elements response box or under any title in their response box	<p>Correct comment on the headline referring to the trend over the years with <u>specific years</u> (based on the graph in image 1)</p> <p>Ex: WTTE Correct up to [1980 , 2000] Incorrect after [1980 , 2000] After 1950 Capricorn higher for [30,50] years In the last [20,40] years cancer increased further There was a gap but it narrowed in [1980 , 2000] There was a gap for [30,50] years after 1950 then it narrowed The headline was written in the [1980 , 2000] It was true up to [1980 , 2000] but after that the opposite Capricorn had higher life expectancy until [1980 , 2000]</p> <p style="text-align: center;">OR</p> <p>C1 is achieved AND Correct comment on the headline referring to <u>general</u> trend over the years (based on the graph in image 1)</p> <p>Ex: WTTE Correct in the past Incorrect recently Somehow correct but they are becoming closer Over the years the gap has narrowed The Headline is outdated Cancer has higher slope (or steeper) Capricorn has lower slope Cancer was lower but increased more Recently Capricorn became closer to Cancer and expected to increase</p>	<p>C3 is achieved AND Correct comment on the headline referring to trend over the years with <u>specific years</u> (based on the graph in image 1)</p>

Mark	1	2
	ACCEPT only if C1 is achieved	ACCEPT only if C3 is achieved
Justify (J) ACCEPT seeing in the elements response box or under any title in their response box	<p>One reason Implying accurate or Inaccurate related to the data given WTTE</p> <p>1) Inaccurate as Capricorn only 11 countries or (tropics) don't have same number of countries or (tropics) don't have same number of people or we don't know the number of people</p> <p>2) Accurate as the data given in the graph "starts from 1950" or "is until 2020" or "until 2021" OR as they are based on [70,73] years of data</p> <p>3) Accurate as I compared <u>both</u> measures of central tendency and measures of dispersion OR mean (or median or mode) and range (or IQR)</p> <p>4) Inaccurate as life expectancies were rounded OR accurate/inaccurate as some (accept all) data is given to nearest 1 decimal place</p> <p>5) Inaccurate as image 1 (or the graph) does not indicate which average is graphed. e.g. image 1 does not show mean or median for life expectancy</p> <p>DO NOT ACCEPT other reasoning Ex: Inaccurate because diseases (or pandemic or storm or, wars...etc) Accurate/Inaccurate as data is from a (given) graph Inaccurate because image 2 is only about one year (2021) Inaccurate as we don't know the source of data Inaccurate as I am not sure of my calculations Accurate as I am sure of what I calculated or I used the right formula Inaccurate as I had to guess values Accurate as the data is provided</p>	<p>Two reasons from the list (1 to 5) in J1</p> <p>OR</p> <p>Implying inaccurate with the reason related to population of each country WTTE</p> <p>Ex: Inaccurate as we do not know the number of people <u>in each country</u> Inaccurate as the population of <u>each country</u> is different Inaccurate as <u>countries</u> don't have same number of people</p>

Task 3

Glossary for task 3

Term used	Clarification
General rule	Rule in terms of only n (if they use x , it is still general rule but penalise in notation)
The general rule	The correct general rule in terms of only n (if they use x , it is still the general rule but penalise in notation)
Their general rule	valid attempt for the general rule but in terms of only n (if they use x , it is still their general rule but penalise in notation)
Their rule	Correct rule not in terms of only n
Recursive rule	$U_n = \frac{(U_{n-1}-1)}{n}$ or $U_{n+1} = \frac{(U_n-1)}{n+1}$ or $U_n = \sqrt{(U_{n-1} + 4n)}$ or $U_{n+1} = \sqrt{(U_n + 4n + 4)}$

We accept subsequent use of their general rule provided it is of similar complexity. In general, the complexity of the rule depends on its form. The table below shows examples.

The general rule accepted simplified forms	ACCEPT for their general rule	DO NOT ACCEPT for their general rule
$G = \frac{-(n+1)}{n}$ or $G = \frac{-n-1}{n}$ or $G = \frac{(-n-1)}{n}$	Quadratic or exponential or rational	Linear
$L = \sqrt{2n^2 + 2n + 1}$ or $L = \sqrt{n^2 + (n + 1)^2}$	Quadratic or exponential or $\sqrt{\text{quadratic}}$	Linear or $\sqrt{\text{linear}}$

Question 8 EXT	Answers	Notes	Total
a	<p>Correctly substitute (4,0) and (0,5) into gradient formula</p> $\frac{5-0}{0-4} \text{ OR } \frac{0-5}{4-0}$ $\frac{-5}{4} \text{ AG}$ <p>For reference</p>	<p>ACCEPT With or without brackets.</p> <p>ACCEPT Substituting (4,0) into line equation $y=ax+b$ where b is 5 AND correctly calculate "a".</p> <p>Ex: $0=a(4)+5$ and $-5 = 4a$ or $a = \frac{0-5}{4}$</p> <p>ACCEPT inappropriate notation Ex:</p> <p>$5-0/0-4$ or $0-5/4-0$</p> <p>$5-0$ over $0-4$</p> <p>Numerator $5-0$ and denominator $0-4$</p> <p>DO NOT ACCEPT</p> <p>$x=4$ and $y=-5$</p> <p>(4,0) and (0,5) with formula $\frac{y_2-y_1}{x_2-x_1}$</p> <p>Only seeing $5-0$ and $0-4$</p> <p>Rise -5 over run 4</p> <p>Using the formula from part d)</p>	1

	b	<p>.1 Correctly write at least one value for G</p> <p>.2 Correctly write all six values of b, h, and G for n=5 and 6.</p>	<table border="1"> <thead> <tr> <th>Stage (n)</th><th>Base (b)</th><th>Height (h)</th><th>Gradient (G)</th></tr> </thead> <tbody> <tr><td>1</td><td>1</td><td>2</td><td>-2/1</td></tr> <tr><td>2</td><td>2</td><td>3</td><td>-3/2</td></tr> <tr><td>3</td><td>3</td><td>4</td><td>-4/3</td></tr> <tr><td>4</td><td>4</td><td>5</td><td>-5/4</td></tr> <tr><td>5</td><td>5</td><td>6</td><td>-6/5</td></tr> <tr><td>6</td><td>6</td><td>7</td><td>-7/6</td></tr> </tbody> </table> <table border="1"> <tr> <td style="text-align: center;">5</td><td style="text-align: center;">5</td><td style="text-align: center;">6</td><td style="text-align: center;">$\frac{-6}{5}$ or -6/5</td></tr> <tr> <td style="text-align: center;">6</td><td style="text-align: center;">6</td><td style="text-align: center;">7</td><td style="text-align: center;">$\frac{-7}{6}$ or -7/6</td></tr> </table> <p>ACCEPT</p> <table border="1"> <tr> <td style="text-align: center;">5</td><td style="text-align: center;">6</td><td style="text-align: center;">$\frac{-6}{5}$ 5</td></tr> <tr> <td style="text-align: center;">6</td><td style="text-align: center;">7</td><td style="text-align: center;">$\frac{-7}{6}$ 6</td></tr> </table> <p>ACCEPT equivalent values for all</p>	Stage (n)	Base (b)	Height (h)	Gradient (G)	1	1	2	-2/1	2	2	3	-3/2	3	3	4	-4/3	4	4	5	-5/4	5	5	6	-6/5	6	6	7	-7/6	5	5	6	$\frac{-6}{5}$ or -6/5	6	6	7	$\frac{-7}{6}$ or -7/6	5	6	$\frac{-6}{5}$ 5	6	7	$\frac{-7}{6}$ 6	2
Stage (n)	Base (b)	Height (h)	Gradient (G)																																											
1	1	2	-2/1																																											
2	2	3	-3/2																																											
3	3	4	-4/3																																											
4	4	5	-5/4																																											
5	5	6	-6/5																																											
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5	6	$\frac{-6}{5}$ 5																																												
6	7	$\frac{-7}{6}$ 6																																												

c	<p>.1 correctly describe one pattern for G in words</p> <p>.2 correctly describe a second pattern for G in words</p> <p>For reference</p> <table border="1" data-bbox="393 509 579 1283"> <tr><td>G</td></tr> <tr><td>-2</td></tr> <tr><td>1</td></tr> <tr><td>-3</td></tr> <tr><td>2</td></tr> <tr><td>-4</td></tr> <tr><td>3</td></tr> <tr><td>-5</td></tr> <tr><td>4</td></tr> <tr><td>-6</td></tr> <tr><td>5</td></tr> <tr><td>-7</td></tr> <tr><td>6</td></tr> </table>	G	-2	1	-3	2	-4	3	-5	4	-6	5	-7	6	<p>ACCEPT complete terminology only, for example (below are seven different descriptions)</p> <p>Numerator decreases by one or goes down by one or subtract 1 or absolute value increases by one WTTE Denominator increases by 1 or adds 1 or goes up by 1 WTTE Numerator difference -1, common difference -1, linear with difference -1, arithmetic with difference -1 Denominator difference 1, common difference 1, linear with difference 1, arithmetic with difference 1 Numerator or denominator second difference zero Absolute value of numerator is always 1 more than absolute value of denominator Rational</p> <p>ACCEPT small spelling errors. Examples: Using “Numoroter” or “Nomurator” instead of “Numerator”</p> <p>DO NOT ACCEPT Using “nominator” instead of “numerator” Using “rise” instead of numerator or using “run” instead of “denominator” General description, for example: (Num. or Denom.) Integers, negative, arithmetic, decreasing, composed of only fractions The rule in words or description related to n for example: negative n minus one over n, denominator is always n, n goes up by 1 General rules in terms of n, example: $G=-(n+1)/n$ Description in terms of h and b, example: gradient is always $-h/b$</p> <p>Note, in the case when they have more than two different patterns: If two are accepted and the rest are all correct: award 2 marks Ex: Denominator integers and adds 1 and Second difference is zero Ex: Denominator linear and increases by 1 and has difference of 1</p> <p>If two are accepted and any of the rest is incorrect: award 1 mark Ex: Denominator increases by 1 and Second difference is zero and G decreases by one.</p> <p>If only one is accepted, ignore the rest and award 1 mark</p> <p>SC for 1 mark Two different <u>correct</u> descriptions using incorrect terminology. Examples The top decreases by 1 and bottom increases by 1 The rise has difference -1 and the run increases by 1</p>	2
G																
-2																
1																
-3																
2																
-4																
3																
-5																
4																
-6																
5																
-7																
6																

	d	<p>.1 The correct general rule</p> <p>.1 $(G =) \frac{-(n+1)}{n}$ OE or $G = - (n+1)/n$</p> <p>.1 ACCEPT incorrect rule because of brackets only for bp1. Ex: $G = -n-1 / n$</p> <p>.2 The correct simplified general rule with correct notation for G in terms of n</p> <p>.2 $G = \frac{-(n+1)}{n}$ or $G = \frac{-n-1}{n}$ or $G = \frac{(-n-1)}{n}$ ACCEPT $G = - \frac{n+1}{n}$</p> <p>.2 DO NOT ACCEPT incorrect rules because of brackets. Ex: $G = -n-1 / n$ ACCEPT using $G(n)$ or G_n or g or N.</p> <p>DO NOT ACCEPT description in words</p> <p>SC for 1 mark if NR in 8d and correct general rule seen in 8c condone incorrect notation award 1 mark</p>	2
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<p>e</p> <p>.1 correctly substitute $n \geq 5$ into their general rule (from 8c or 8d)</p> <p>.2 correctly calculate their value of G after their bp1</p> <p>.3 recognise that their correctly calculated value of G is the same as their predicted value</p>	<p>.1 Ex: $\frac{-(5+1)}{5}$ ACCEPT seeing their equation AND their chosen n. Ex: $n = 5$ AND $(G=) -\frac{(n+1)}{n}$.1 DO NOT ACCEPT if their G is linear.</p> <p>.2 Ex: $\frac{-6}{5}$ (for $n=5$) bp1 must be awarded .2 DO NOT ACCEPT if their G is linear. .2 DO NOT ACCEPT brackets error. Ex: their $-6-1 / 6$ is not $-7/6$, it gives $-37/6$</p> <p>.3 If they use $n=5$ or $n=6$ ACCEPT only if they say "as seen in the table" WTTE or "correct" .3 If they use $n \geq 6$ ACCEPT only if they say "the same as when we continue the pattern" WTTE and state how. Examples: for $\frac{-8}{7}$; "it is obtained by subtracting 1 to -7 and adding 1 to 6" OR they show the further predictions including $\frac{-8}{7}$ here in part e)</p> <p>SC for 1 mark Correctly test their general rule by applying the steps of verification mentioned in the left column, including bp3, with a value of $n \leq 4$</p> <p>SC for 1 mark Correctly verify, with a value ≥ 5, including bp3, the correctly described pattern or recursive rule or rule from part c) or d). Ex.: If they have two rules, one for numerator and one for denominator Numerator : $U_n = -2 + (n-1)x-1$ $U_6 = -2 + (6-1)x-1 = -7$ and they say it's the same as the table WTTE AND Denominator $U_n = 1 + (n-1)x1$ $U_6 = 1 + (6-1)x1 = 6$ and they say it's the same as the table WTTE</p>	<p>3</p>
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Question 8f (20 marks)

Mark	1
Predictions (P)	<p>Correctly predict two terms for L ACCEPT whether in the table or in the response box.</p> <p>$\sqrt{61}$</p> <p>$\sqrt{85}$</p> <p>ACCEPT square root of 61 or square root of 85</p> <p>DO NOT ACCEPT decimals</p>

Mark	1	2	3	4
Description (D)	<p>Attempt to describe a pattern in words or recursive rule for L</p> <p>Examples for $(2n^2 + 2n + 1)$:</p> <p>Difference (or the increase) increases by 4 WTTE It increases by multiples of 4 WTTE Second difference 4 Quadratic (Recursive rule) $U_n = U_{n-1} + 4n - 4$ or $U_{n+1} = U_n + 4n + 4$</p> <p>DO NOT ACCEPT incorrect or too general patterns Ex: second difference is $\sqrt{4}$ Ex: L is a surd Ex: Under the square root are odd numbers</p>	<p>Attempt to describe a pattern in words or recursive rule for L</p> <p>AND</p> <p>Their general rule for L a <u>quadratic</u> satisfying at least one value for n in their $(2n^2 + 2n + 1)$</p>	<p>Attempt to describe a pattern in words or recursive rule for L</p> <p>AND</p> <p>Attempt the general rule for L DO NOT ACCEPT in words</p>	<p>Attempt to describe a pattern in words or recursive rule for L</p> <p>AND</p> <p>The general rule for L</p>
ACCEPT incorrect terminologies, notation errors, non-simplified rule, or rule in words but penalize in notation (N)	<p>OR</p> <p>Their general rule for L a <u>quadratic</u> satisfying at least one value for n in their $(2n^2 + 2n + 1)$</p>	<p>OR</p> <p>Attempt the general rule for L $(L =) n(n + 1) \times 2 + 1$ or $(L =) n^2 + (n + 1)^2$ or $(L =) 2n^2 + 2n + 1$ OE or A quadratic satisfying at least two values for n in their $(2n^2 + 2n + 1)$</p>	<p>OR</p> <p>The general rule for L $(L =) \sqrt{2n^2 + 2n + 1}$ or $(L =) \sqrt{n^2 + (n + 1)^2}$ or $(L =) \sqrt{n(n + 1) \times 2 + 1}$ OE</p>	
Ignore additional incorrect patterns	<p>OR</p> <p>The rule for L in terms of b and h. $(L =) \sqrt{b^2 + h^2}$</p> <p>ACCEPT $L^2 = b^2 + h^2$</p>	<p>OR</p> <p>The general rule for L in words or partially in words</p>		

Mark	1	2
Testing (T)	<p>Attempt to test their general rule for L or their $(2n^2 + 2n + 1)$ using $n \leq 4$</p> <p>Ex: Correctly substitute in their general rule for L or their $(2n^2 + 2n + 1)$ value of $n \leq 4$</p> <p style="text-align: center;">OR</p> <p>Correctly test their described pattern or their rule (e.g. recursive rule, or rule in terms of b and h)</p> <p>Ex: Testing $L = 4n + 1$, when $n=1$, $L = 4(1) + 1 = 5$</p> <p>Ex: Testing $L = \sqrt{b^2 + h^2}$ when $n=2$, $b=2$, $h=3$, $L = \sqrt{2^2 + 3^2} L = \sqrt{13}$,</p>	<p>Correctly test their general rule for L or their $(2n^2 + 2n + 1)$ using $n \leq 4$ Ex: Correctly calculate their value for L or their $(2n^2 + 2n + 1)$ in their general rule for L or their $(2n^2 + 2n + 1)$ using $n \leq 4$</p> <p>AND</p> <p>Recognise that <u>their</u> correctly calculated value for L or their $(2n^2 + 2n + 1)$ is the same as <u>the</u> given value.</p> <p>ACCEPT Seeing their correctly calculated value for L or their $(2n^2 + 2n + 1)$ and the given value in the table being equal</p>

Mark	1	2	3
Verifying (V)	<p>Attempt to verify their general rule for L or their $(2n^2 + 2n + 1)$ using $n \geq 5$</p> <p>Ex: Correctly substitute in their general rule for L or their $(2n^2 + 2n + 1)$ value of $n \geq 5$</p> <p style="text-align: center;">OR</p> <p>Correctly verify their described pattern or their rule (e.g. recursive rule or rule in terms of b and h)</p> <p>Ex: Verifying $L = 5n + 26$ $L = 5(5) + 26 = 61$</p> <p>Ex.: Verifying $L = \sqrt{b^2 + h^2}$ when $n=5$, $b=5$, $h=6$ $L = \sqrt{5^2 + 6^2} L = \sqrt{61}$</p>	<p>Correctly calculate their value for L or their $(2n^2 + 2n + 1)$ in their general rule or their $(2n^2 + 2n + 1)$ using $n \geq 5$</p>	<p>ACCEPT only if they have the general rule for L</p> <p>Correctly calculate their value for L in the general rule using $n \geq 5$</p> <p>AND</p> <p>Recognise that <u>their</u> correctly calculated value for L is the same as <u>the</u> correctly predicted value obtained by continuing the pattern</p> <p>ACCEPT Seeing their correctly calculated value for L and the correctly predicted value in the table being equal</p>

Mark	1	2	3	4
	ACCEPT only if D1 is achieved	ACCEPT only if D2 is achieved	ACCEPT only if they have <u>the general rule for L</u>	ACCEPT only if they have <u>the general rule for L</u>
Justify (J) ACCEPT seeing anywhere in the response	<p>Attempt to justify their general rule for L</p> <p>ACCEPT justification for their $(2n^2 + 2n + 1)$ provided it's quadratic</p> <p>Ex: Quadratic model and valid attempt to find coefficient using any method</p> <p>OR</p> <p>Weak attempt to justify their general rule for L</p> <p>Writing down Pythagoras theorem in terms of b and h.</p> $(L =) \sqrt{b^2 + h^2}$ <p>ACCEPT only if it is not their rule for L.</p> <p>ACCEPT writing down Pythagoras using values for a specific n provided the value is the same than the table.</p> $\sqrt{25} = \sqrt{3^2 + 4^2}$ <p>OR</p> <p>Substitute at least two other values of n in their general rule for L or their $(2n^2 + 2n + 1)$ and say they are the same or hence the rule works WTTE</p>	<p>Attempt to justify $(2n^2 + 2n + 1)$ algebraically.</p> <p>Ex: Quadratic model and find the correct values of coefficients using any method</p> <p>OR</p> <p>Attempt to justify their general rule for L geometrically with incorrect L or incorrect rules for b and h.</p> <p>Ex1: Seeing $(L =) b^2 + h^2$ AND Rules $b = n$ and $h = n+1$</p> <p>Ex2: Seeing $(L =) \sqrt{b^2 + h^2}$ AND Rules $b = n$ and $h = n-1$</p>	<p>Good attempt to justify <u>the</u> general rule for L geometrically using <u>the</u> rules for b and h into Pythagoras</p> <p>Showing the rules for b and h that could be substituted into Pythagoras</p> <p>Ex: Seeing $(L =) \sqrt{b^2 + h^2}$ AND Rules $b = n$ and $h = n+1$</p>	<p>Correctly justify <u>the</u> general rule for L geometrically using <u>the</u> rules for b and h into Pythagoras :</p> <p>Substituting the rules for b and h into Pythagoras with <u>correct simplification</u> leading to one of the two forms</p> <p>When their form of the general rule is $(L =) \sqrt{n^2 + (n + 1)^2}$</p> <p>Seeing $(L =) \sqrt{b^2 + h^2}$ AND Rules $b = n$ and $h = n+1$ AND $(L =) \sqrt{n^2 + (n + 1)^2}$</p> <p>When their form of the general rule is $(L =) \sqrt{2n^2 + 2n + 1}$</p> <p>Seeing $(L =) \sqrt{b^2 + h^2}$ AND Rules $b = n$ and $h = n+1$ AND $(L =) \sqrt{n^2 + n^2 + 2n + 1}$</p>

NOTE	The following are considered errors in notation		
	using * for multiplication or using / for division using ^ for power using x instead of n using U_n instead of L without mentioning that $L = U_n$	1	2
Mark	ACCEPT only if D1 achieved	ACCEPT only if they have the general rule for L	ACCEPT only if they have the general rule for L
Notation and terminology (N)	Correctly describe a pattern for L in words <u>using correct terminology</u> Ex: The terms under the root form a quadratic The root of a quadratic What's under the root has a second difference of 4 The second difference between values squared is 4 ACCEPT radical or surd WTTE DO NOT ACCEPT a quadratic then “add” root DO NOT ACCEPT if the pattern is not accepted in D1	Correctly describe a pattern for L in words <u>using correct terminology</u> (see examples in N1) AND The notation of the general rule includes errors or not simplified or in words (see examples in N1)	Correctly describe a pattern for L in words <u>using correct terminology</u> (see examples in N1) AND Correct notation of <u>the general rule</u> for L in simplest form (see N2)
Ignore additional incorrect patterns	OR The notation of <u>the general rule</u> includes errors or not simplified or in words. Ex: $y = \sqrt{2n^2 + 2n + 1}$ The rule for L is $\sqrt{2n^2 + 2n + 1}$ $L = \sqrt{n(n+1) \times 2 + 1}$ $L = \sqrt{(2n^2 + 2n + 1)}$, $L = \sqrt{2n^2 + 2n + 1}$ L is the square root of $2n^2 + 2n + 1$ Under the root, it is the square of n plus the square of $n + 1$ ACCEPT $(\)^{\frac{1}{2}}$ instead of $\sqrt{\ }$ OR Correct notation or <u>their general rule</u> for L . ACCEPT if not simplified DO NOT ACCEPT rules for L^2 . DO NOT ACCEPT without the root $\sqrt{\ }$ Ex : $L = \sqrt{2n^2 + 1}$, $L = \sqrt{n^2 + 2n + 1}$, $L = \sqrt{b^2 + h^2}$	OR Correct notation of <u>the general rule</u> for L in simplest form $L = \sqrt{n^2 + (n+1)^2}$ OR $L = \sqrt{2n^2 + 2n + 1}$ ACCEPT $L = \sqrt{(2n^2 + 2n + 1)}$ ACCEPT $(\)^{\frac{1}{2}}$ instead of $\sqrt{\ }$ ACCEPT L_n or L_n or $L(n)$	

Mark	1	2	3
		ACCEPT only if they have a general rule for L or for their($2n^2 + 2n + 1$)	ACCEPT only if they have <u>the</u> general rule for L
Communication (L) Organisation and coherence Can be awarded even if there are errors For items: Describing pattern and writing rule can be considered an item even if D0 and N0 awarded	<p>At least three from the following items are seen:</p> <ul style="list-style-type: none"> • describe a pattern in words • write a rule • test their general rule or rule or recursive rule or pattern (at least T1) • verify their general rule or rule or recursive rule or pattern (at least V1) • justify their general rule or rule or recursive rule or pattern (at least J1) 	<p>At least four of the following items are seen:</p> <ul style="list-style-type: none"> • describe a pattern in words • write a general rule • test their general rule (at least T1) • verify their general rule (at least V1) • justify their general rule (at least J1) <p>AND</p> <p>For coherence, they identify the processes correctly.</p> <p>At least one from the following:</p> <ul style="list-style-type: none"> • test • verify • justify <p>Ex:</p> <ul style="list-style-type: none"> • For test: they say “test” and they substitute in their general rule value(s) of $n \leq 4$ only • For verify: they say “verify” and they substitute in their general rule value(s) of $n \geq 5$ only • For justify: They say “justify” and they write a justification <p>Note for coherence: If they say “test and verify” and they substitute in their general rule value(s) of $n \leq 4$ <u>followed by</u> value(s) of $n \geq 5$, consider it as only one identified process</p>	<p>The following two items must be seen :</p> <ul style="list-style-type: none"> • write <u>the general rule for L</u> • justify <u>the general rule</u> (at least J2) <p>AND</p> <p>At least two of the following items are seen:</p> <ul style="list-style-type: none"> • describe a pattern or rule in words • test <u>the general rule</u> (at least T1) • verify <u>the general rule</u> (at least V1) <p>AND</p> <p>For coherence, they identify the processes correctly. At least two from the following:</p> <ul style="list-style-type: none"> • test • verify • justify <p>Ex:</p> <ul style="list-style-type: none"> • For test: they say “test” and they substitute in the general rule for A value(s) of $n \leq 4$ only • For verify: they say “verify” and they substitute in the general rule for A value(s) of $n \geq 5$ only • For justify: They say “justify” and they write a justification <p>Note for coherence: If they say “test and verify” and they substitute in the general rule value(s) of $n \leq 4$ <u>followed by</u> value(s) of $n \geq 5$, consider it as only one identified process</p>

Markscheme

May 2023

Extended Mathematics

On-screen examination

30 pages

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The markscheme abbreviations:

- Bullet notation means award 1 mark – see example below

Example 1

.1 mark awarded and corresponding notes are aligned

b	<p>.1 Show clear line of reasoning in the method</p> <p>.2 4</p>	<p>.1 45 and 49 seen OE</p> <p>Ex: $49 = 45 + x$</p> <p>.2</p>	2
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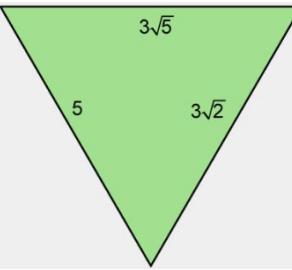
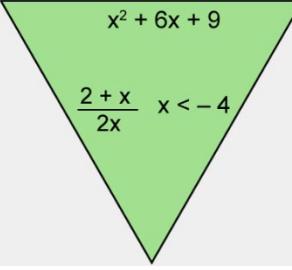
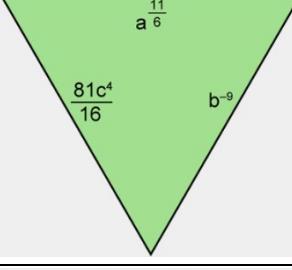
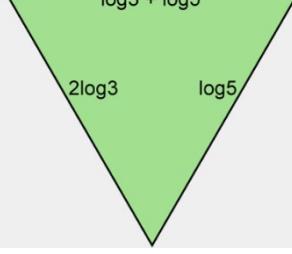
Error Carried Forward (ECF) marks

Errors made at any step of a solution affect all working that follows. In general, **Error Carried Forward (ECF)** marks are awarded after an error.

- a) **ECF** applies from one part of a question to a subsequent part of the question and also applies within the same part.
- b) If an answer resulting from **ECF** is inappropriate (eg, negative distances or $\sin x > 1$) then subsequent marks should not be awarded.
- c) If a question is transformed by an error into a **simpler question** then **ECF** may not be fully awarded.
- d) To award **ECF** marks for a question part, **there must be working present for that part**.
- e) **ECF** is only applied to working which is correct. This means that all working subsequent to an error must be checked for accuracy.
- f) A misread (**MR**) is an error. **ECF** is normally awarded.

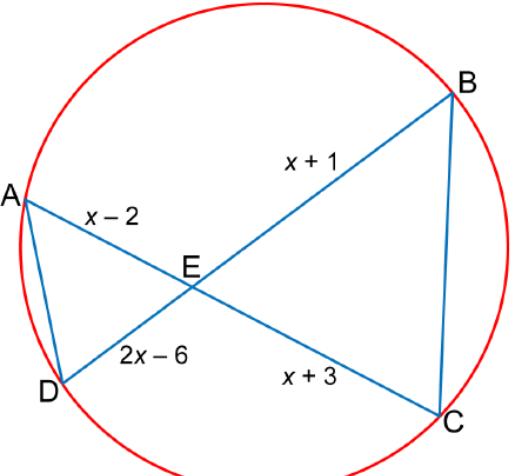
General points

- a) As this is an international examination, accept all **alternative forms of notation**, for example 1,9 as 1.9 ; 1,000 or 1.000. However, **DO NOT ACCEPT** incorrect mathematical notation x^2 for x^2 unless noted otherwise in the MS.
- b) **ACCEPT** notation errors in intermediate steps.
- c) Ignore further working after a correct answer **unless** it indicates a lack of mathematical understanding **i.e. if the further working contradict the correct answer**, then that last mark cannot be awarded.
- d) In the case when a correct result is obtained using incorrect seen method, do not award the mark for the result.
- e) Where candidates have written two solutions to a question, mark the first solution.
- f) In the markscheme, equivalent examples of **numerical** and **algebraic** forms or **simplified** answers will generally be written in the notes preceded by **OE** (Or Equivalent) e.g. $\frac{1}{2}$ **OR** 1/2 **OR** 0.5 **OR** $2 \div 4$; $\frac{x}{2}$ **OR** $x / 2$ or $x \div 2$; 0.23 **OR** 23%
- g) In the markscheme, information provided in brackets indicate detail that may be seen in a candidate response but is not necessary to award the marks. However, it indicates what the candidate's result represents. Ex: if last mark is for the result: (AB)=5; this means we award the mark for seeing 5 as the result of calculating AB without necessarily seeing AB=5, but it does not mean we award the mark for seeing 5 representing another length
- h) Special case marks **SC** can be allocated instead of but not in addition to the marks prescribed in the markscheme.
- i) Accept seeing equation not in-line.
- j) Calculator screenshots are accepted as working steps. And when a calculator screenshot is taken, accept not seeing the whole operation.
- k) In task 2 and 3 where the markscheme is set out in a table then, unless noted otherwise, awarding the highest mark in a category includes all the lower marks in that category. It is probably best to look for the top category mark answer and if you don't find it look at the next mark down.
- l) **ACCEPT** using the correct values or working regardless their previous result.
- m) Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. **If in doubt, contact your team leader for advice.**
- n) Unless noted otherwise, if a note in a part says to accept the answer without working for 1 mark less than total marks, then seeing the correct answer with any acceptable working step, award full marks. Example: If the note in a part worth 3 marks says "4.3(3...) without working award 2 marks", then seeing any acceptable working step and seeing 4.3(3...) as the answer award the 3 marks.
- o) For "**show that**" questions, unless otherwise noted, every bullet point has to be seen in order to be awarded.

Q1		Answers	Notes	Total
	a	Place the correct triangle		1
	b	Place the correct triangle		1
	c	Place the correct triangle		1
	d	Place the correct triangle		1

Q2		Answers	Notes	Total
	a	<p>.1 Three correct numerators</p> <p>.2 All correct numerators</p>		2

b	<p>.1 One correct product of three probabilities (DDW or DWD or WDD)</p> <p>.2 The other two correct products of three probabilities OR 3 × their .1</p> <p>.3 Correctly add their three probabilities OR their correct probability after multiplying by 3</p>	<p>.1 $\frac{10}{25} \times \frac{9}{24} \times$ their $\frac{15}{23}$ OR $\frac{10}{25} \times$ their $\frac{15}{24} \times \frac{9}{23}$ OR $\frac{15}{25} \times \frac{10}{24} \times \frac{9}{23}$ OR $\frac{9}{92}$ OR 0.0978(3...) ACCEPT 0.098 or 0.1</p> <p>.2 $\frac{10}{25} \times \frac{9}{24} \times$ their $\frac{15}{23}$ AND $\frac{10}{25} \times$ their $\frac{15}{24} \times \frac{9}{23}$ AND $\frac{15}{25} \times \frac{10}{24} \times \frac{9}{23}$ OR 3 × their $\frac{9}{92}$ OR 3 × 0.0978(3...)</p> <p>.3 their $\frac{27}{92}$ or 0.29(3478...) OE ACCEPT 27/92 .3 DO NOT ACCEPT if they add more than three .3 ONLY ACCEPT if their probability is less than 1</p>	3
c	<p>.1 Correctly substitute $P(A \cap B)$ and their part b) into the correct conditional probability formula</p> <p>.2 Correctly determine their $P(A B)$</p>	<p>.1 $\frac{15}{25} \times \frac{10}{24} \times \frac{9}{23}$ or $0.0978(260...)$ or $\frac{9}{92}$ or $0.0978(260...)$ ACCEPT $\frac{0.1}{\text{their part b}}$.1 DO NOT ACCEPT $P(A \cap B)$ being $P(A) \times$ their $P(B)$</p> <p>.2 their $\frac{1}{3}$.2 ACCEPT only if their $\frac{1}{3}$ is less than 1</p>	2
d	<p>.1 Correctly multiply $P(A)$ and their $P(B)$ from part b)</p> <p>.2 Correctly state their .1 does not equal $P(A \cap B)$</p> <p>Event A and B are not independent AG</p>	<p>.1 $(\frac{15}{25} \times \text{their } \frac{27}{92}) =$ their $\frac{81}{460}$ or their 0.176(08695...) ACCEPT 0.18 .1 DO NOT ACCEPT if they don't have a calculated product</p> <p>.2 Their .1 does not equal to their $\frac{9}{92}$ WTTE .2 DO NOT ACCEPT if they don't have a calculated product from .1</p> <p>DO NOT ACCEPT any explanation about independency of events even though it seems correct</p>	2

Q3	Answers	Notes	Total
a	<p>AM1</p> <p>.1 Recognize a first pair of congruent angles (with or without reason)</p> <p>.2 Recognize a second pair of congruent angles (with or without reason)</p> <p>AM2</p> <p>.1 Correctly apply chords inside circle theorem to recognize that sides are in the same proportion.</p> <p>.2 Recognize vertically opposite angles</p> 	<p>AM1</p> <p>.1 One from the list of possibilities</p> <p>.2 Another from the list of possibilities</p> <p>List of possibilities :</p> <ul style="list-style-type: none"> - inscribed angles subtended by the same arc $\angle ADE = \angle BCE$ or $\angle DAE = \angle CBE$ - the vertically opposite angles $\angle AED = \angle BEC$ <p>AM2</p> <p>.1 $\overline{BE} \times \overline{ED} = \overline{AE} \times \overline{EC}$ OE AND $\frac{\overline{BE}}{\overline{EC}} = \frac{\overline{AE}}{\overline{ED}}$ or $\frac{\overline{BE}}{\overline{AE}} = \frac{\overline{CE}}{\overline{DE}}$ OE</p> <p>.2 $\angle AED = \angle BEC$</p> <p>ACCEPT correct angles using different labels</p> <p>CAD or A for $\angle EAD$ DAC or A for $\angle DAE$ DBC or B for $\angle EBC$ CBD or B for $\angle CBE$ ACB or C for $\angle ECB$ BCA or C for $\angle BCE$ ADB or D for $\angle ADE$ BDA or D for $\angle EDA$</p> <p>ACCEPT saying equal WTTE ex: congruent, the same, etc</p> <p>DO NOT ACCEPT E for any angle around E DO NOT ACCEPT $\angle E = \angle E$ or lines create two congruent angles at E DO NOT ACCEPT AC and BD are chords on the circumference DO NOT ACCEPT AED similar to BEC</p>	2

<p>b</p> <p>.1 Correctly apply the similarity</p> <p>.2 Correctly cross multiply their ratios</p> <p>.3 Correctly rearrange their correct quadratic</p> <p>.4 Correctly factorize $x^2 - 5x$ OE OR substitute the correct coefficients into the quadratic formula OR divide by x.</p> <p>$(x =) 5$ AG</p>	<p>.1 $\frac{2x-6}{x+3} = \frac{x-2}{x+1}$ OE or $\frac{x+1}{x+3} = \frac{x-2}{2x-6}$ OE ACCEPT $\frac{DE}{CE} = \frac{AE}{BE}$ OE or $\frac{DE}{AE} = \frac{CE}{BE}$ OE .1 ACCEPT without brackets written as $2x-6 / x+3 = x-2/x+1$ OE .1 ACCEPT $(x-2):(x+1) = (2x-6):(x+3)$ OE or $(x-2):(2x-6)=(x+1):(x+3)$ OE .1 DO NOT ACCEPT using ratio 0.5 or 2. Ex: $(x+3)=2(2x-6)$</p> <p>.2 their$(2x-6)(x+1)=$their$(x-2)(x+3)$ OR their$2x^2-4x-6=x^2+x-6$.2 Accept not seeing this step .2 DO NOT ACCEPT incorrect cross multiplication without seeing their ratios Seeing $(2x-6)(x+1)=(x-2)(x+3)$ implies .1</p> <p>.3 $(0=)$ their $x^2 - 5x$ or their $(0 =) -x^2 + 5x$ OR $x^2 = 5x$</p> <p>.4 $(0 =) x(x-5)$ OE OR $\frac{5 \pm \sqrt{(-5)^2 - 4(1)(0)}}{2(1)}$ OE OR $\frac{-5 \pm \sqrt{(5)^2 - 4(-1)(0)}}{2(-1)}$ OE OR from $x^2 = 5x$ to $x = 5$</p> <p>.4 ACCEPT seeing $x = 0$ AND 5 $(x=) 5$ AG</p> <p>For 4 marks .1.3.4 or .2.3.4 must be seen</p>	<p>4</p>
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c <ul style="list-style-type: none"> .1 Correctly substitute using $x=5$ into cosine rule .2 Correctly calculate their BC^2 after substituting into cosine rule .3 Correctly calculate their BC from their BC^2 .4 Their correct values of m and n 	<ul style="list-style-type: none"> .1 $(BC^2 =) 6^2 + 8^2 - 2 \times 6 \times 8 \times \cos 60$ OE .2 $(BC =)$ their 52 .3 their $\sqrt{52}$ ACCEPT decimals their 7.2(111) .3 Seeing $\sqrt{52}$ or 7.2(111) implies .2 .4 $m=2$ and $n = 13$ ACCEPT seeing $2\sqrt{13}$.4 ACCEPT $m=1$ and $n=\text{their}52$ only if $\text{their}52$ is an integer .4 ACCEPT their correct m and n from their 52 	4
d <ul style="list-style-type: none"> .1 Correct ratio .2 Correctly apply their ratio to their $2\sqrt{13}$ to find AD .3 Their perimeter correct after adding 21 to their $2\sqrt{13}$ and their $\sqrt{13}$ in surd form 	<ul style="list-style-type: none"> .1 0.5 or 2 OE seen .1 ACCEPT seeing $2 \times \text{their } 2\sqrt{13}$.2 their $\sqrt{13}$.2 implies .1 .2 ACCEPT in decimals i.e. their 3.6(0555...) .2 DO NOT ACCEPT if $\text{theirAD} = \text{theirBC}$.3 $21 + \text{their } 3\sqrt{13}$.3 DO NOT ACCEPT decimals .3 DO NOT ACCEPT if answer does not include surd .3 DO NOT ACCEPT the use of "root" in words. .3 DO NOT ACCEPT a perimeter that does not include theirAD 	3

Q4		Answers	Notes	Total
a	<p>AM1 (Using slant height)</p> <p>.1 Correctly substitute into Pythagoras OR correctly substitute into correct trig ratio OR correctly use area of triangle formula to find slant height</p> <p>.2 Correct value for slant height</p> <p>.3 Correctly substitute their .2 into Pythagoras to find h</p> <p>.4 Correct value of h before simplifying $12\sqrt{2}$ AG</p> <p>AM2 (Using square base)</p> <p>.1 Correctly substitute into Pythagoras OR correctly use trig ratio to find diagonal of the base or half diagonal of the base</p> <p>.2 Correct value for diagonal of the base or half diagonal of the base</p> <p>.3 Correctly substitute half of base into Pythagoras to find h</p> <p>.4 Correct value of h before simplifying $12\sqrt{2}$ AG</p>	<p>AM1</p> <p>.1 $\sqrt{24^2 - 12^2}$ OR $12 \times \tan 60$ OE OR $(\frac{1}{2} \times 24 \times 24 \times \sin 60) \div 12$ OE ACCEPT $24^2 = 12^2 + L^2$ OE or $(h^2 =) 24^2 - 12^2$</p> <p>.2 $\sqrt{432}$ or $12\sqrt{3}$ or $20.78(4609\dots)$ OE</p> <p>.3 $\sqrt{(\text{their } \sqrt{432})^2 - 12^2}$ or $\sqrt{\text{their } (12\sqrt{3})^2 - 12^2}$ or $h^2 + 12^2 = \text{their } 432$ OE or their $20.78(4609)^2 - 12^2$ OE</p> <p>.4 $\sqrt{288}$.4 ACCEPT 16.97 AND $12\sqrt{2} = 16.97$</p> <p>AM2</p> <p>.1 $\sqrt{24^2 + 24^2}$ OE OR $\frac{24}{\sin 45}$ OE ACCEPT $24^2 + 24^2$ or $12^2 + 12^2$ OE</p> <p>.2 $\sqrt{1152}$ or $24\sqrt{2}$ or $33.9(4111)$ OE .2 ACCEPT 16.97 or $12\sqrt{2}$</p> <p>.3 $\sqrt{24^2 - (12\sqrt{2})^2}$ OE or $h^2 + (12\sqrt{2})^2 = 24^2$ OE .3 ACCEPT in decimals .3 DO NOT ACCEPT trig ratio using angle 45</p> <p>.4 $\sqrt{288}$.4 ACCEPT 16.97 AND $12\sqrt{2} = 16.97$.4 ACCEPT only if .2 and .3 are awarded Reaching $\sqrt{288}$ coming from half of 576, award 0 marks See notes on next page</p>	4	

		<p>Seeing pyramid made with equilateral triangles so height is half of side $\sqrt{2}$, award 0 marks</p> <p>Calculating the height using the volume of part b, award 0 marks.</p>		
	b	<p>.1 Correctly substitute $12\sqrt{2}$ and 24 into volume of pyramid formula</p> <p>.2 Correctly calculate their result after substituting into their volume formula</p> <p>.3 Correctly rounded their volume to the nearest integer</p>	<p>.1 $\frac{1}{3} \times 24 \times 24 \times 12\sqrt{2}$ ACCEPT $\frac{1}{3} \times 24 \times 24 \times 16.97$</p> <p>.1 ACCEPT $\frac{1}{3} \times 576 \times 12\sqrt{2}$</p> <p>.2 their3258.34(80...)</p> <p>.2 ACCEPT incorrect volume formula providing it includes $24 \times 24 \times 12\sqrt{2}$</p> <p>.2 DO NOT ACCEPT their result if it comes from $\frac{1}{3} \times \text{length} \times 12\sqrt{2}$</p> <p>.3 their3258</p>	3
	c	<p>.1 Correctly calculate the modified ratio for the height</p> <p>.2 The correct value of h</p>	<p>.1 $\sqrt[3]{0.75}$ or 0.9(0856...) OE seen ACCEPT $\frac{1}{\sqrt[3]{0.75}}$ or 1.1 (the inverse ratio) ACCEPT $0.75 = \left(\frac{h}{12\sqrt{2}}\right)^3$ OE</p> <p>.2 $(\sqrt[3]{0.75} \times 12\sqrt{2}) = 15.4(187...)$</p>	2

Q5		Answers	Notes	Total
	a	Correct result $(0.16 \times 4500 =)720(t)$		1
	b	.1 Correct ratio .2 Correctly apply <u>the correct</u> ratio on their720	.1 $\frac{3}{8}$ (=0.375) OE or $\frac{8}{3}$ or 3:8 or 8:3 OE or seen within calculations Ex: $\frac{\text{their720}}{2.67}$ OE .1 ACCEPT seeing $\frac{3}{8}$ as 0.4 or $\frac{8}{3}$ as 2.6 or 2.7 in bp1 only .1 DO NOT ACCEPT 3×8 as ratio or seen within calculations .2 $(\frac{3}{8} \times \text{their720} \text{ or } \text{their720} \div \frac{8}{3} =) \text{their270}$	2

	c	<p>.1 Correctly write two values .2 Correctly write a third values .3 Correctly write a fourth value .4 Correctly write fifth and sixth values</p> <table border="1"> <thead> <tr> <th rowspan="2">Category</th><th rowspan="2"></th><th>Weight, tonnes (t)</th><th>Production ratio Energy per tonne (TJ / t)</th><th>Energy Tera-joules (TJ)</th></tr> <tr> <th></th><th></th><th></th></tr> </thead> <tbody> <tr> <td></td><td>Domestic</td><td>1350</td><td>$r =$ 0.1 or 1/10 or 1:10 or "1 to 10"</td><td>135</td></tr> <tr> <td></td><td>Commercial</td><td>Answer from (a) their 720</td><td>$r =$ their 0.1 (above)</td><td>(0.1 × their720=) their72</td></tr> <tr> <td></td><td>Industrial</td><td>Answer from (b) their 270</td><td>$2r =$ (2 × their 0.1 =) their 0.2 OE</td><td>(0.2 × their270=) their54</td></tr> </tbody> </table> <p style="text-align: right;">Total energy produced by the organization this months (135 + their72 + their54=) their261</p> <p>Answers from (a) and (b) are not counted as one of the six values <u>The six values are:</u> $r_{\text{domestic}}=0.1 \text{ OE}$ ACCEPT 1/10 or "1:10" or "1 to 10" OE $r_{\text{commercial}}= \text{their } r$ for domestic $2r=(2\times \text{their } r_{\text{domestic}})=\text{their } 0.2$ ACCEPT their2/10 or their "2:10" or their"2 to 10" OE $\text{Energy commercial}=(0.1 \times \text{their720}=) \text{their72}$ $\text{Energy industrial}=(0.2 \times \text{their270}=) \text{their54}$ $\text{Total Energy}=(135+\text{their72}+\text{their54}=) \text{their261}$ ACCEPT Energy Commercial 72 or Energy industrial 54 regardless their r value</p>	Category		Weight, tonnes (t)	Production ratio Energy per tonne (TJ / t)	Energy Tera-joules (TJ)					Domestic	1350	$r =$ 0.1 or 1/10 or 1:10 or "1 to 10"	135		Commercial	Answer from (a) their 720	$r =$ their 0.1 (above)	(0.1 × their720=) their72		Industrial	Answer from (b) their 270	$2r =$ (2 × their 0.1 =) their 0.2 OE	(0.2 × their270=) their54	4
Category		Weight, tonnes (t)			Production ratio Energy per tonne (TJ / t)	Energy Tera-joules (TJ)																				
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	Industrial	Answer from (b) their 270	$2r =$ (2 × their 0.1 =) their 0.2 OE	(0.2 × their270=) their54																						
	d	<p>.1 Correct mid-interval values .2 Add the product of their mid-interval values by frequency .3 Divide their sum by 12 .4 Correct mean before rounding AG 900</p>	<p>.1 860, 880, 900, 920, 940 ACCEPT seeing only three correct .2 $2 \times \text{their860} + 4 \times \text{their880} + 2 \times \text{their900} + 1 \times \text{their920} + 3 \times \text{their940}$ OE or 10780 .2 ACCEPT seeing only three correct products OE added .3 Ex: $\frac{\text{their bp2}}{2 + 4 + 2 + 1 + 3}$.4 898(.333...)</p>	4																						

e	<p>.1 Correct ratio seen <u>in any notation</u></p> <p>.1 "if 68420 is 900 then their261 is" WTTE or 68420:900 or $\frac{68420}{900}$ OE ($= \frac{3421}{45} = 76.022\dots$) ACCEPT 76</p> <p>.1 ACCEPT 900:68420 or $\frac{900}{68420}$ OE ($= \frac{45}{3421} = 0.013(154\dots)$)</p> <p>.2 Correctly apply <u>the correct ratio</u> on their total energy from c)</p> <p>.2 $\frac{68420}{900}$ OE \times their261 or their261 $\div \frac{900}{68420}$ OE or their19841.8</p> <p>.2 ACCEPT using 76 instead $\frac{68420}{900}$ or using 0.013 instead of $\frac{900}{68420}$</p> <p>.3 Correctly write their result as an integer</p> <p>.3 their 19842</p> <p>.3 ACCEPT their rounding being up or down</p> <p>.3 ACCEPT only if .1 or .2 is awarded</p>	3
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Q6		Answers	Notes	Total
	a	<p>.1 Correctly substitute (0,120) into the equation</p> <p>.1 $120 = \frac{a}{(0+6)} + 65$ or $120 = \frac{a}{6} + 65$</p> <p>ACCEPT substitution of other points (n,E) from the graph in .1</p> <p>Accepted points (n,E) from the graph: $(0,120), (1,112), (2,106), (3,102), (4,98), (5,95), (6, 93), (7,90), (8, 89)$</p> <p>ACCEPT E being ± 5 the values indicated above</p> <p>.2 Correct value of a as a product after the substitution of $(0,120)$</p> <p>$a = 330$ AG</p>		2
	b	<p>.1 Correctly substitute 2 into the equation</p> <p>.1 $E = \frac{330}{(2+6)} + 65$ or $E = \frac{330}{8} + 65$</p> <p>.2 Correct answer after their substitution</p> <p>.2 their106.25 OE</p> <p>.2 ACCEPT 106.3 or 106 or 107</p>		2
	c	<p>.1 Correctly subtract their result in b) from 120</p> <p>.1 (their120 – their part b) = their13.75</p> <p>.1 ACCEPT their part b) being correctly rounded up or down as an integer Ex: $(120-\text{their106}) = \text{their14}$ or $(120-\text{their107}) = \text{their13}$</p> <p>.2 (their13.75 \times 60000 =) their825 000</p> <p>825 000 without working, award 2 marks $(\text{their part b}) \times 60000$ or their6 375 000, award 1 mark</p> <p>120×60000 or $7 200 000$, award 0 marks</p>		2

d	<p>AM1 (Solving)</p> <p>.1 Correctly equate to 95</p> <p>.2 Correctly rearrange for their n</p> <p>.3 The correct value of n</p> <p>AM2 (using numbers)</p> <p>.1 Correctly substitute one number $n \neq 5$ and correctly calculate corresponding E</p> <p>.2 Correctly substitute 5 and equate to 95</p> <p>.3 Correctly identify the value of n</p>	<p>AM1 (Solving)</p> <p>.1 $95 = \frac{330}{(n+6)} + 65$ or $30 = \frac{330}{(n+6)}$ or $30(n+6) = 330$ OE</p> <p>.1 ACCEPT the use of inequalities</p> <p>.2 $n = \frac{330}{\text{their}30} - 6$ OE</p> <p>.2 ACCEPT $(n+6) = \frac{330}{\text{their}30}$</p> <p>.2 ACCEPT their30n=their150</p> <p>.2 implies .1</p> <p>.3 $(n=)5$</p> <p>.3 ACCEPT 2027</p> <p>AM2 (using numbers)</p> <p>.1 Ex: $\frac{330}{(1+6)} + 65 = 112.14$ ACCEPT 112</p> <p>.2 $\frac{330}{(5+6)} + 65 = 95$</p> <p>.3 $(n=) 5$ ACCEPT 2027</p>	3
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Q6e total 10 marks

Mark	1	2		
Factors (F)	<p>Two from the keywords/phrases below mentioned in the factors box</p> <ul style="list-style-type: none"> • filter(s) • carbon or unit(s) or kt or emission(s) • limit(s) or restriction(s) or requirement(s) or regulation(s) or “difference in emission(s)” or “value(s) of difference” <p>DO NOT ACCEPT other similar keywords Ex: “excess” or “reduction”</p> <p>ACCEPT multiple keywords in one sentence and ignore additional irrelevant factors</p>	<p>The three keywords/phrases below mentioned in the factors box</p> <ul style="list-style-type: none"> • filter(s) • carbon or unit(s) or kt or emission(s) • limit(s) or restriction(s) or requirement(s) or regulation(s) or “difference in emission(s)” or “value(s) of difference” <p>DO NOT ACCEPT other similar keywords Ex: “excess” or “reduction”</p> <p>ACCEPT multiple keywords in one sentence and ignore additional irrelevant factors</p>		

Key**DC:** The missing values in the row of Difference in carbon emissions**VC:** The missing values in the row of Value of difference in carbon emissions **ACCEPT** negative values for buying (Ex: -0.48,-0.63,-0.72)

Year	2022	2023	2024	2025	2026	2027	2028	2029	2030
Difference in carbon emissions (kt)	28	19.5	12	5.5	0	4.5	8	10.5	12
Value of difference in carbon emissions (\$ million)	1.68	1.17	0.72	0.33		0.27	0.48	0.63	0.72
Buy or sell	sell	sell	sell	sell		buy	buy	buy	buy

Total of Sell is: $1.68+1.17+0.72+0.33=3.9$ Total of Buy is: $0.27+0.48+0.63+0.72=2.1$

Mark	1	2	3	4
Calculate (C)	<p>One from: (see table above)</p> <ul style="list-style-type: none"> • Correctly write three DC • Correctly calculate four of <u>their</u> VC • Correctly calculate cost if no filters installed: $(222 \times 60k =) 13.32$ (\$ mil) ACCEPT – 13.32 • Weak attempt to calculate <u>their</u> cost if filters installed: They forgot the cost of filters or income from selling or price of buying <p>Ex: their3.9 – their2.1 or their1.8 Ex: 14 – their3.9 or their10.1 Ex: 14 + their2.1 or their16.1</p>	<p>Two from: (see table above)</p> <ul style="list-style-type: none"> • Correctly write three DC • Correctly calculate four of <u>their</u> VC • Correctly calculate cost if no filters installed : $(222 \times 60k =) 13.32$ (\$ mil) ACCEPT – 13.32 • Good attempt to calculate their cost if filters installed: They include cost of filters, income from selling, and price of buying but incorrect final result <p>Ex: $14 + \text{their}3.9 - \text{their}2.1 \text{ or their } 15.8 (\\$ \text{mil})$</p> <p>ACCEPT <u>the</u> correct costs without final result Ex: filters 14 then sell for 3.9 then buy for 2.1 WTTE</p>	<p>Three from: (see table above)</p> <ul style="list-style-type: none"> • Correctly write all DC • Correctly calculate all of <u>their</u> VC • Correctly calculate cost if no filters installed: $(222 \times 60k =) 13.32$ (\$ mil) ACCEPT – 13.32 • Correctly calculate <u>their</u> cost if filters installed $(14 - \text{their}3.9 + \text{their}2.1 =) \text{their}12.2$ <p>ACCEPT – their12.2(\$mil)</p>	<p>The four: (see table above)</p> <ul style="list-style-type: none"> • Correctly write all DC • Correctly calculate all of <u>the</u> VC • Correctly calculate cost if no filters installed: $(222 \times 60k =) 13.32$ (\$ mil) ACCEPT – 13.32 • Correctly calculate <u>the</u> cost if filters installed: $(14 - 3.9 + 2.1 =) 12.2 (\\$ \text{mil})$ <p>ACCEPT – 12.2(\$mil)</p>

Mark	1	2
Recommendation (J)	<p>ACCEPT only if C1 is achieved</p> <p>Recommendation with one weak justification considering the 8 years period or beyond the 8 years period</p> <p style="text-align: center;">Examples considering the 8 years period</p> <p>Ex: Filters will save money WTTE (and we see values aligned with this recommendation even if incorrectly calculated)</p> <p>Ex: Should install filters but in 4 years they will have to buy new ones WTTE</p> <p>Ex: Should not install filters because after 2026 (or starting 2027) filters will not be efficient enough WTTE</p> <p>Ex: Should not install because it is not economic (and we see values aligned with this recommendation even if incorrectly calculated)</p> <p>Ex: should not install filters because the price of filters is paid upfront WTTE.</p> <p style="text-align: center;">DO NOT ACCEPT seeing only price of filters too high</p> <p style="text-align: center;">OR</p> <p style="text-align: center;">Examples considering beyond 8 years period</p> <p>Ex: After 2030 (or after 8 years), installing filters will be no good or they will have to buy new filters or take new measures WTTE</p> <p>Ex: On the long run (in the future) they will have to buy more filters WTTE</p> <p style="text-align: center;">ACCEPT not recommending filters if their justification supports it</p> <p style="text-align: center;">DO NOT ACCEPT</p> <p>“install filters because it is better for environment” WTTE</p>	<p>ACCEPT only if C4 is achieved</p> <p><u>Recommendation to install filters with one good justification aligned with the correct calculations</u></p> <p>Ex: Should install filters because they will save 1.12 (mil\$) WTTE</p> <p>Ex: Should install filters because without filters they will spend 13.32(mil) while with filters only 12.2 WTTE</p> <p>ACCEPT Should install filters because it will save money WTTE, and we see 13.32(mil\$) and 12.2(mil\$) in the calculations</p>

Mark	1	2
Accuracy (A)	<p>Implies inaccurate with one justification</p> <ul style="list-style-type: none"> • Price of buying/selling (60000) carbon units is approximate WTTE OR <u>Limits</u>/restrictions could change (based on world events or to decrease pollution or because climate change). WTTE <p style="text-align: center;">OR</p> <ul style="list-style-type: none"> • <u>(Carbon) emissions</u> are predictions/estimates or might change over the years WTTE 	<p>Implies inaccurate with two justifications</p> <ul style="list-style-type: none"> • Price of buying/selling (60000) carbon units is approximate WTTE OR <u>Limits</u>/restrictions could change (based on world events or to decrease pollution or because climate change). WTTE <p style="text-align: center;">AND</p> <ul style="list-style-type: none"> • <u>(Carbon) emissions</u> are predictions/estimates or might change over the years WTTE
	<p style="text-align: center;">ACCEPT</p> <p>“values used are estimations or approximated” WTTE for A1</p> <p style="text-align: center;">DO NOT ACCEPT</p> <p>“inaccurate because I rounded” WTTE</p> <p>accurate regardless their justification</p> <p>just rounding their results</p>	<p style="text-align: center;">DO NOT ACCEPT</p> <p>“inaccurate because I rounded” WTTE</p> <p>accurate regardless their justification</p> <p>just rounding their results</p>

Q7 Task 3

Glossary for task 3

Term used	Clarification
General rule	Rule in terms of only n (if they use x , it is still general rule but penalise in notation)
The general rule	The correct general rule in terms of only n (if they use x , it is still the general rule but penalise in notation)
Their general rule	valid attempt for the general rule but in terms of only n (if they use x , it is still their general rule but penalise in notation)
Their rule	Correct rule not in terms of only n
Recursive rule	$A = \frac{A_{n-1}}{4} OE$

We accept subsequent use of their general rule provided it is of similar complexity. In general, the complexity of the rule depends on its form. The table below shows examples.

The general rule	ACCEPT for their general rule	DO NOT ACCEPT for their general rule
$T = 8n - 8$	Linear	
$A = \frac{8}{2^n}$	Quadratic or exponential	Linear

7	Answers	Notes	Total
a	Correctly place 32 and 40	5 32 6 40	1

b .1 correctly describe one pattern for T in words .2 correctly describe a second pattern for T in words	ACCEPT complete terminology only, for example (below are four different descriptions) divisible by 8, multiples of 8 Increasing by 8, adds 8, goes up by 8, moving up by 8 difference 8, common difference 8, linear with difference 8, arithmetic with difference 8 Second difference zero DO NOT ACCEPT incomplete terminology, for example: Arithmetic, linear, increasing by a constant, constant difference DO NOT ACCEPT general description, for example: Integers, whole numbers, positive, even numbers, divisible by 2, multiples of 2, multiples of 4 DO NOT ACCEPT the rule in words or description related to n for example: 8 times n minus 8, n multiplied by 8 minus 8, WTTE Note, in the case when they have more than two different patterns: If two are accepted and the rest are all correct: award 2 marks Ex: even numbers and adds 8 and Second difference is zero and it is 8 times n then subtract 8. Ex: multiples of 2 and 4 and linear and increases by 8 and divisible by 8 If two are accepted and any of the rest is incorrect: award 1 mark Ex: <u>increases by 2</u> and Second difference is zero and it is divisible by 8 If only one is accepted, ignore the rest and award 1 mark	2
c .1 the correct general rule .2 the correct simplified general rule with correct notation for T in terms of n .	.1 $8n - 8$ or $8(n - 1)$ or $T = 8 \times n - 8$ or $(T =)8 * n - 8$ or $T = 8 \times (n - 1)$ or $T = 0 + 8(n - 1)$.2 $T = 8n - 8$ or $T = 8(n - 1)$.2 ACCEPT using T_n instead of T .2 ACCEPT $T = n8 - 8$ or $T = (n - 1)8$.2 ACCEPT using t and N DO NOT ACCEPT description in words	2

d	<p>.1 correctly substitute $n \geq 5$ into their general rule (from 7c or 7b)</p> <p>.2 correctly calculate their value of T after substituting $n \geq 5$</p> <p>.3 recognise that their correctly calculated value of T is the same as their predicted value</p>	<p>.1 Ex: $8 \times 5 - 8$</p> <p>.2 Ex: 32 (for $n = 5$)</p> <p>.3 "the same as when we continue the pattern" WTTE and states how Ex: for $n=7$, 48 is obtained from pattern of adding 8 to 40</p> <p>.3 ACCEPT if their value from .2 is the same as their value in the table in part a) or seen here in part d) Ex: we see their calculated $T=32$ and we see $T=32$ in their table</p>	3
e	<p>AM1 (rhombus) Correctly half the product of the two diagonals</p> <p>AM2 (rectangle – triangles) Correctly subtract area of 4 triangles from the rectangle</p> <p>AM3 (4 triangles) Correct area of 4 triangles or 2 big triangles that form 1 rhombus</p> <p>$\frac{1}{16}$ AG</p>	<p>AM1 $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{4}$</p> <p>AM2 $\frac{1}{8} - \frac{1}{16}$ Comes from 4 triangles each being $\frac{1}{2} \times \frac{1}{4} \times \frac{1}{8} = \frac{1}{64}$ and 1 rectangle $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$</p> <p>AM3 $4 \times \frac{1}{64} \text{ or } 2 \times \frac{1}{32}$ Comes from area of 4 triangles each being $\frac{1}{2} \times \frac{1}{4} \times \frac{1}{8}$ or 2 triangles each being $\frac{1}{2} \times \frac{1}{8} \times \frac{1}{2}$ or $\frac{1}{2} \times \frac{1}{4} \times \frac{1}{4}$</p> <p>ACCEPT notation errors ACCEPT lengths in decimals only if $\frac{1}{16} = 0.0625$ is seen Ex : $0.5 \times 0.5 \times 0.25 = 0.625$ AND $\frac{1}{16} = 0.0625$</p> <p>DO NOT ACCEPT $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{4}$ that comes from $A = b^2 \times h$ DO NOT ACCEPT $\frac{1}{4} \times \frac{1}{4}$ DO NOT ACCEPT working only with denominators Ex : denominators $2 \times 2 \times 4 = 16$</p>	1

Q7e 23 marks

Mark	1	2	3
Predictions (P) ACCEPT whether in the table or in the response box	Correctly predict two terms for L or W	Correctly predict two terms for L or W AND Correctly predict one term for A	Correctly predict two terms for L and two terms for W AND Correctly predict two terms for A
	OR Correctly predict one term for A	OR Correctly predict two terms for A	
	Ignore additional incorrect predictions ACCEPT fractions or index form ACCEPT in the table or in the response box DO NOT ACCEPT decimals	Ignore additional incorrect predictions ACCEPT fractions or index form ACCEPT in the table or in the response box DO NOT ACCEPT decimals	Ignore additional incorrect predictions ACCEPT fractions or index form ACCEPT in the table or in the response box DO NOT ACCEPT decimals

Stage number (n)	Length of big diagonal (L)	Length of small diagonal (W)	Area of rhombus (A)
5	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{256}$
6	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{1024}$
7	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{4096}$
8	$\frac{1}{64}$	$\frac{1}{128}$	$\frac{1}{16384}$

Examples of rules in equivalent forms (accept decimals) :

$$L = \frac{4}{2^n} = 2 \times \left(\frac{1}{2}\right)^{n-1} = \frac{1}{2^{n-2}} = 2^{2-n}$$

$$W = \frac{2}{2^n} = 1 \times \left(\frac{1}{2}\right)^{n-1} = \frac{1}{2^{n-1}} = 2^{1-n}$$

$$A = \frac{1}{4^{n-1}} \text{ or } A = \frac{1}{2^{2n-2}} \text{ or } A = 4^{1-n} \text{ or } A = 2^{2-2n}$$

Mark	1	2	3	4	5
Description (D) ACCEPT notation errors but penalized in notation (N)	<p>Correctly describe a pattern in words for A Ex: Multiply by $\frac{1}{4}$ OE, Divide by 4, Divide by 2 two times (Geometric with) ratio $\frac{1}{4}$ OE. Denominators multiply by 4 or they are divisible by 4 or they all have 4 as a factor or they are all powers of 4. Denominators <u>except first</u> are multiples of 4.</p> <p style="text-align: center;">OR</p> <p>Recursive rule for A $A = \frac{A_{n-1}}{4}$ OE or the rule $A=4^n$</p>	<p>Correctly describe a pattern in words for A AND</p> <p>Correct recursive rule for A or rule $A=4^n$ or Valid attempt to write down general rule for A (at least seeing $(an+b)$ as an exponent)</p>	<p>Correctly describe a pattern in words or recursive rule for A AND</p> <p>Valid attempt to write down a general rule for A (at least seeing $(an+b)$ as an exponent)</p>	<p>Correctly describe a pattern in words or recursive rule for A AND</p> <p>Correctly write down the general rule for L or W</p>	<p>Correctly describe a pattern in words or recursive rule for A AND</p> <p>Correctly write down the general rule for A</p>
	<p>Valid attempt to write down a general rule for L or W (at least seeing n as an exponent) ACCEPT if they correctly describe a pattern in words or recursive rule for L or W Ex: Multiply by $\frac{1}{2}$ OE, Divide by 2 (Geometric with) ratio $\frac{1}{2}$ OE Denominators are multiples of 2 or divisible by 2 or even numbers or powers of 2 $L = \frac{L_{n-1}}{2}$ or $W = \frac{W_{n-1}}{2}$ OE</p> <p>IGNORE additional incorrect patterns</p> <p>DO NOT ACCEPT</p> <p>Exponential, geometric, arithmetic sequence, denominators are multiples of 2 or even numbers, the square numbers, A is decreasing $A = \frac{1}{2}(L \times W)$</p> <p>The rule for A in words as a pattern.</p>	<p>Valid attempt to write down a general rule for A (may be incorrect but contains at least $(an+b)$ as an exponent)</p> <p>OR</p> <p>Valid attempt to write down a general rule for A (may be incorrect but contains at least $(an+b)$ as an exponent)</p>	<p>OR</p> <p>Correctly write down the general rule for L or W $L = \frac{4}{2^n}$ OE $W = \frac{2}{2^n}$ OE</p>	<p>OR</p> <p>Correctly write down the general rule for A $A = \frac{1}{4^{n-1}}$ OE</p>	

Mark	1	2	3
Testing (T) ACCEPT transforming into decimals when testing	<p>Attempt to test their general rule for A using $n \leq 4$</p> <p>Ex: Substitute in their general rule for A value of $n \leq 4$</p> <p>OR</p> <p>Correctly test their described pattern or their rule (e.g. recursive rule) or their rule for L or W in terms of n</p> <p>DO NOT ACCEPT testing their rule of $A=0.5xLxW$</p>	<p>Correctly test their general rule for A, using $n \leq 4$</p> <p>Ex: Correctly calculate their value for A in their general rule using $n \leq 4$</p> <p>AND</p> <p>Recognise that their correctly calculated value for A is the same as the given value.</p> <p>ACCEPT seeing their correctly calculated value for A and the given value in the table being equal</p>	
Verifying (V) ACCEPT transforming into decimals when verifying	<p>Attempt to verify their general rule for A using $n \geq 5$</p> <p>Ex: Correctly substitute in their general rule for A value of $n \geq 5$</p> <p>OR</p> <p>Correctly verify their described pattern or their rule (e.g. recursive rule) or their rule for L or W in terms of n</p> <p>DO NOT ACCEPT verifying their rule of $A=0.5xLxW$</p>	<p>Correctly calculate their value for A in their general rule, using $n \geq 5$</p>	<p>Correctly calculate their value for A in the <u>general rule</u> using $n \geq 5$</p> <p>AND</p> <p>Recognise that their correctly calculated value for A is the same as the correct predicted value obtained by continuing the pattern</p> <p>ACCEPT seeing their correctly calculated value for A and the correctly predicted value in the table being equal</p>

Mark	1	2	3	4
Justify (J)	ACCEPT only if D1 is achieved Attempt to use geometric sequence to justify <u>their</u> general rule for A or described pattern or rule or recursive rule Ex : Every time we multiply by $\frac{1}{4}$ so it makes sense to have 4 in my general rule WTTE and we see 4 or $\frac{1}{4}$ in <u>their</u> general rule for A	ACCEPT only if D4 is achieved Correctly use geometric sequence to justify <u>the</u> general rule aligned with their notation for <u>the</u> general rule for A Ex : Assume $a \times r^n$ and substitute $r = \frac{1}{4}$ and a value for A and n then find $a = 4$	ACCEPT only if D4 is achieved Attempt to justify geometrically the general rule for A Half of the product of correct L and W rules without complete simplification Seeing $\frac{L \times W}{2} = \frac{4 \times 2}{2 \times 2^n \times 2^n} \text{ OE}$ or $\frac{4 \times 2}{2 \times 2^n \times 2^n} = \frac{4}{4^n} \text{ OE}$	ACCEPT only if D4 is achieved Correctly justify geometrically the general rule for A Half the product of correct L and W with simplification. ACCEPT simplification using factor 4 instead of 2. They show how it simplifies to $A = \frac{1}{4^{n-1}} \text{ or } A = 4^{1-n}$ $A = \frac{1}{4^{n-1}} \text{ or } A = \frac{1}{2^{2n-2}} \text{ or }$ $A = 4^{1-n} \text{ or } A = 2^{2-2n}$
	OR using parameters Assume $a \times r^n$ and substitute $r = \frac{1}{4}$ and a value for A and n then find incorrect a	OR Weak attempt to justify <u>the</u> general rule for A geometrically by using half the product of incorrect L and W rules or by using correct L and W rules but incorrectly halved or with mistakes in simplification		
	OR Weak attempt to justify <u>their</u> general rule for A geometrically by using the product of <u>incorrect</u> L and W <u>rules</u> OR L is halved and W is halved so A is divide 4 WTTE			
	OR Substitute at least two other values of n in their general rule for A, L or W and say they are the same or the rule works WTTE			
	OR Recognize it is geometric progression and define first term 1 and ratio $\frac{1}{4}$ WTTE			
	DO NOT ACCEPT A is square of W			

Mark	1	2	3
Notation and terminology (N)	<p>CORRECT NOTATION</p> <ul style="list-style-type: none"> - using U_n instead of A only if they mention that $A = U_n$ - Using decimals 	<p>NOTATION WITH ERRORS</p> <ul style="list-style-type: none"> - using * for multiplication - using / for division - using ^ for power - using x instead of n - “the rule for A is:” instead of “A=” 	
	<p>ACCEPT only if D1 is achieved</p> <p>Correct notation of <u>their general rule for A or rule for L or W</u> Ex: $A = 4^n$ or $L = \frac{4}{2^n}$ or $W = \frac{2}{2^n}$</p>	<p>ACCEPT only if D4 is achieved</p> <p>Correct notation of <u>the general rule for A in simplest form (see examples below and above)</u> ACCEPT simplification using factor 4 instead of 2. Ex: $A = \frac{1}{4^{n-1}}$ or $A = \frac{1}{2^{2n-2}}$ or $A = 4^{1-n}$ or $A = 2^{2-2n}$</p>	<p>ACCEPT only if D5 is achieved</p> <p>Correct notation of <u>the general rule for A in simplest form (see examples in N2 and above)</u> AND Correctly describe a pattern in words for A</p>
	<p>OR</p> <p>The notation of <u>the general rule includes errors or non-simplified or in words</u> Ex: $A = 1/4^{(n-1)}$ or $A = 4/4^n$ or $A = \frac{4}{4^n}$ or Missing the “A=” Ex: The rule for A is $\frac{1}{4^{n-1}}$</p>	<p>And OE for decimals Ex: $A = 0.25^{n-1}$ or $A = 0.5^{2n-2}$</p> <p>OR</p> <p>The notation of <u>the general rule includes errors or not simplified or in words (see examples in N1 and above)</u> AND Correctly describe a pattern in words for A</p>	
	<p>OR</p> <p>Correctly describe a pattern or recursive rule in words for A Ex: $A = \frac{A_{n-1}}{4}$</p>		

<p>Communication (L)</p> <p>Organisation and coherence can be awarded even if there are errors</p> <p>For items: Describing pattern and writing rule can be considered an item even if D0 awarded Test, verify, and justify may include errors but cannot be considered item (or identified for coherence) if they are zeros</p>	<p>At least three of the following items are seen:</p> <ul style="list-style-type: none"> - describe a pattern in words - write a rule - test their general rule or rule or recursive rule or pattern (at least T1) - verify their general rule or rule or recursive rule or pattern (at least V1) - justify their general rule or rule or recursive rule or pattern (at least J1) 	<p>At least four of the following items are seen:</p> <ul style="list-style-type: none"> - describe a pattern in words - write a general rule - test their general rule - verify their general rule - justify their general rule <p>AND</p> <p>For coherence, they identify the processes correctly. At least one from the following:</p> <ul style="list-style-type: none"> - test - verify - justify <p>Ex:</p> <ul style="list-style-type: none"> • For test: they say “test” and they substitute in their general rule value(s) of $n \leq 4$ only • For verify: they say “verify” and they substitute in their general rule value(s) of $n \geq 5$ only • For justify: They say “justify” and they write a justification <p>Note for coherence: If they say “test and verify” and they substitute in their general rule value(s) of $n \leq 4$ followed by value(s) of $n \geq 5$, consider it as only one identified process</p>	<p>ACCEPT only if D4 and J2 are achieved</p> <p>At least four of the following are seen:</p> <ul style="list-style-type: none"> - describe a pattern or rule in words - write <u>the general rule</u> - test <u>the general rule</u> - verify <u>the general rule</u> - justify <u>the general rule</u> <p>AND</p> <p>For coherence, they identify the processes correctly. At least two from the following:</p> <ul style="list-style-type: none"> · test · verify · justify <p>Ex:</p> <ul style="list-style-type: none"> • For test: they say “test” and they substitute in their general rule value(s) of $n \leq 4$ only • For verify: they say “verify” and they substitute in their general rule value(s) of $n \geq 5$ only • For justify: They say “justify” and they write a justification <p>Note for coherence: If they say “test and verify” and they substitute in their general rule value(s) of $n \leq 4$ followed by value(s) of $n \geq 5$, consider it as only one identified process</p>
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Markscheme

May 2022

Extended mathematics

On-screen examination

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The markscheme abbreviations:

- Bullet notation means award 1 mark – see example below

Example 1

.1 mark awarded and corresponding notes are aligned

b	<p>.1 Show clear line of reasoning in the method</p> <p>.2 4</p>	<p>.1 45 and 49 seen OE Ex: $49 = 45 + x$</p> <p>.2</p>	2
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Error Carried Forward (ECF) marks

Errors made at any step of a solution affect all working that follows. In general, **Error Carried Forward (ECF)** marks are awarded after an error.

- a) **ECF** applies from one part of a question to a subsequent part of the question and also applies within the same part.
- b) If an answer resulting from **ECF** is inappropriate (eg, negative distances or $\sin x > 1$) then subsequent marks should not be awarded.
- c) If a question is transformed by an error into a **simpler question** then **ECF** may not be fully awarded.
- d) To award **ECF** marks for a question part, **there must be working present for that part**.
- e) **ECF** is only applied to working which is correct. This means that all working subsequent to an error must be checked for accuracy.
- f) A misread (**MR**) is an error. **ECF** is normally awarded.

General points

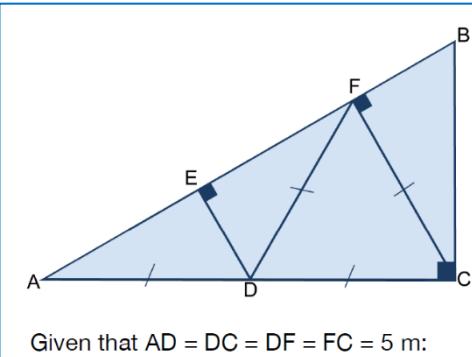
- a) As this is an international examination, accept all **alternative forms of notation**, for example 1,9 as 1.9 ; 1,000 or 1.000. However, **DO NOT ACCEPT** incorrect mathematical notation x^2 for x^2 unless noted otherwise in the MS
- b) Accept notation errors in intermediate steps.
- c) Ignore further working after a correct answer **unless** it indicates a lack of mathematical understanding **i.e. if the further working contradict the correct answer**, then that last mark cannot be awarded.
- d) In the case when a correct result is obtained using incorrect seen method, do not award the mark for the result.
- e) Where candidates have written two solutions to a question, mark the first solution.
- f) In the markscheme, equivalent examples of **numerical** and **algebraic** forms or **simplified** answers will generally be written in the notes preceded by **OE** (Or Equivalent) e.g. $\frac{1}{2}$ **OR** 0.5 **OR** 0.5 **OR** $2 \div 4$; $\frac{x}{2}$ **OR** $x / 2$ or $x \div 2$; 0.23 **OR** 23%
- g) In the markscheme, information provided in brackets indicate detail that may be seen in a candidate response but is not necessary to award the marks.
- h) Special case marks **SC** can be allocated instead of but not in addition to the marks prescribed in the markscheme.
- i) Accept seeing equation not in-line.
- j) Calculator screenshots are accepted as working steps. And when a calculator screenshot is taken, accept not seeing the whole operation.
- k) In task 2 and 3 where the markscheme is set out in a table then, unless noted otherwise, awarding the highest mark in a category includes all the lower marks in that category. It is probably best to look for the top category mark answer and if you don't find it look at the next mark down.
- l) **ACCEPT** using the correct values regardless their previous result.
- m) Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.
- n) Unless noted otherwise, if a note in a part says to accept the answer without working for 1 mark less than total marks, then seeing the correct answer with any acceptable working step, award full marks. Example: If the note in a part worth 3 marks says “4.3(3...) without working award 2 marks”, then seeing any acceptable working step and seeing 4.3(3...) as the answer award the 3 marks.

Q1	Answers	Notes	Total
	<p>.1 First and Second graph descriptions correct</p> <p>.2 First comparison (third and fourth graphs) both correct</p> <p>.3 Second comparison (fifth and sixth graphs) both correct</p> <p>.4 Third comparison (seventh and eighth graphs) both correct</p>	<p>.1 <input type="button" value="Text/MCQ/Mini-Cloze Object"/> <input type="button" value="Large"/> standard deviation</p> <p>.2 <input type="button" value="Text/MCQ/Mini-Cloze Object"/> <input type="button" value="Zero"/> standard deviation</p> <p>.3 <input type="button" value="Text/MCQ/Mini-Cloze Object"/> <input type="button" value="Different"/> mean <input type="button" value="Same"/> standard deviation</p> <p>.4 <input type="button" value="Text/MCQ/Mini-Cloze Object"/> <input type="button" value="Different"/> mean <input type="button" value="Different"/> standard deviation</p> <p><input type="button" value="Text/MCQ/Mini-Cloze Object"/> <input type="button" value="Same"/> mean <input type="button" value="Different"/> standard deviation</p>	4

Q2	Answers	Notes	Total
	$P(A) = \frac{18}{25}$, $P(A B) = \frac{2}{3}$ and $P(A \cap B) = \frac{8}{25}$		
a	<p>.1 Correctly substitute into the conditional probability formula $P(A B) = \frac{P(A \cap B)}{P(B)}$</p> <p>OR recognize that $\frac{8}{25}$ is $\frac{2}{3}$ of $P(B)$</p> <p>.2 Correct working step using their substitution into conditional probability formula OR correct calculations from $8/25$ being the $2/3$ of $P(B)$.</p> <p>.3 Their correct answer after working steps using the conditional probability formula or the logic behind $\frac{8}{25}$ being the $\frac{2}{3}$ of $P(B)$</p>	<p>.1 $\frac{2}{3} = \frac{8}{25} \div P(B)$ OE OR $\frac{8}{25}$ is $\frac{2}{3}$ of $P(B)$</p> <p>.2 $(P(B) =) \frac{8}{25} \div \frac{2}{3}$ OE</p> <p>.3 $(P(B) =)$ their $\frac{12}{25}$ OE .3 ACCEPT only if their P is less than 1.</p>	3
b	<p>AM1</p> <p>.1 Evidence of using the rule for independent events</p> <p>.2 Correctly calculate $P(A) \times$ their $P(B)$</p> <p>.3 Correctly state their product does not equal to $P(A \cap B)$</p> <p>AM2</p> <p>.1 Evidence of using $P(A)$ and $P(A B)$</p> <p>.2 $\frac{18}{25}$ and $\frac{2}{3}$ seen</p> <p>.3 Acknowledge that $\frac{18}{25}$ and $\frac{2}{3}$ are not equal</p>	<p>AM1</p> <p>.1 $P(A \cap B)$ and $P(A) \times P(B)$ seen or $\frac{8}{25}$ and $\frac{18}{25} \times$ their $\frac{12}{25}$ seen OE</p> <p>.2 their $\frac{216}{625}$ OE</p> <p>.3 their $\frac{216}{625} \neq \frac{8}{25}$ OE ACCEPT their $\frac{216}{625} \neq P(A \cap B)$</p> <p>.3 DO NOT ACCEPT if there is no calculated product</p> <p>AM2</p> <p>.1 $P(A) = P(A B)$ ACCEPT not seeing this step</p> <p>.2 $\frac{18}{25}$ and $\frac{2}{3}$ seen</p> <p>.3 $\frac{18}{25} \neq \frac{2}{3}$ or $\frac{18}{25}$ not equal to $\frac{2}{3}$ ACCEPT $P(A) \neq P(A B)$</p>	3
c			2

	<p>.1 Correctly substitute into the addition rule $P(A \cup B) = P(A) + P(B) - P(A \cap B)$</p> <p>.2 Their correct answer after using addition rule</p>	<p>.1 $\frac{18}{25} + \text{their } \frac{12}{25} - \frac{8}{25}$</p> <p>.2 their $\frac{22}{25}$ OE</p> <p>.2 ACCEPT only if their P is less than 1 or equal to 1.</p>	
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Q3		Answers	Notes	Total
	a	<p>AM1</p> <p>.1 Correctly substitute into Pythagoras</p> <p>.2 The correct value of AF</p> <p>AM2</p> <p>.1 Correctly substitute into trigonometric ratio OR sine rule OR cosine rule</p> <p>.2 The correct value of AF</p>	<p>AM1</p> <p>.1 $(AF =) \sqrt{10^2 - 5^2}$ or $AF^2 + 5^2 = 10^2$ OE</p> <p>.2 ACCEPT 8.7 or $\sqrt{75}$ or $5\sqrt{3}$</p> <p>.2 DO NOT ACCEPT 8.67 or 8.6 or 9</p> <p>AM2</p> <p>.1 $\tan 30 = \frac{5}{AF}$ or $(AF =) 10 \times \cos 30$ or $(AF =) 10 \sin 60$ OE</p> <p>OR $\frac{AF}{\sin 120} = \frac{5}{\sin 30}$ or $\frac{AF}{\sin 60} = \frac{5}{\sin 30}$ OE</p> <p>OR $(AF^2 =) 5^2 + 5^2 - 2(5)(5)\cos 120$ OE or $(AF^2 =) 10^2 + 5^2 - 2(10)(5)\cos 60$ OE</p> <p>.1 ACCEPT $(AF =) 2 \times 5 \cos 30$ or $(AF =) 2 \times 5 \sin 60$ or $AE = 4.33(\dots)$ or $EF = 4.33(\dots)$</p> <p>.2 $(AF =) 8.66(\dots)$ ACCEPT 8.7 or $\sqrt{75}$ or $5\sqrt{3}$</p> <p>.2 DO NOT ACCEPT 8.67 or 8.6 or 9</p>	2

<p>b .1 A correct reason for a pair of congruent angles .2 A correct reason for another pair of congruent angles</p> <div style="border: 1px solid blue; padding: 10px; margin-top: 20px;">  <p>Given that $AD = DC = DF = FC = 5 \text{ m}$:</p> </div>	<p>Examples of correct reasons for pair of congruent angles:</p> <p>For angle A:</p> <ul style="list-style-type: none"> → Angle $BAC = \text{Angle } FAC$ → Angle A is common angle → They share same angle A WTTE (they have to mention angle A) <p>ACCEPT angle $A = \text{angle } A$</p> <p>DO NOT ACCEPT they share same angle or they have a common angle</p> <p>DO NOT ACCEPT Angle BAC is similar to Angle FAC</p> <p>For the 90 degrees angle</p> <ul style="list-style-type: none"> → Angle $ACB = \text{Angle } AFC$ → They both have a 90 degrees angle → Both right angle triangles WTTE <p>ACCEPT they are both 30-60-90 triangles</p> <p>DO NOT ACCEPT angle $F = \text{angle } C$</p> <p>For the 60 degrees angle</p> <ul style="list-style-type: none"> → FC is half AC so angle $A=30$ hence $ABC = \text{Angle } ACF=60$ <p>DO NOT ACCEPT seeing only Angle $ABC = \text{Angle } ACF$</p> <p>DO NOT ACCEPT without correct justification</p>	<p>2</p>
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c	<p>AM1 (Using proportions)</p> <p>.1 Their correct ratio using AC and theirAF seen or used .2 Correctly apply their ratio on AC .3 Their correct value for AB</p> <p>AM2 (trigonometry using AC)</p> <p>.1 Correct angle ABC OR BAC seen or used .2 Correctly substitute their angle into trigonometric ratio .3 Their correct value for AB</p> <p>AM3 (trigonometry FB and AF or Pythagoras AC)</p> <p>.1 Correct angle BCF OR CBF seen or used .2 Correctly substitute their angle into trigonometric ratio .3 Their correct answer for AB using theirBF added to theirAF or using Pythagoras with AC and theirBC.</p>	<p>AM1</p> <p>.1 $\frac{\text{theirAF}}{\text{AC}} = \frac{\text{their8.66}}{10} (= 0.866...) \text{ or } \frac{\text{AC}}{\text{theirAF}} = \frac{10}{\text{their8.66}} (= 1.1547...) \text{ OE}$</p> <p>.2 $10 \times \text{their1.1547...} \text{ or } \frac{10}{\text{their0.866}}$</p> <p>.2 DO NOT ACCEPT if their ratio is 2 or $\frac{1}{2}$</p> <p>.3 $(AB =) \text{ their11.547(005...)} \text{ ACCEPT 11.5 or 11.55}$</p> <p>AM2</p> <p>.1 $(BAC=)30 \text{ OR } (ABC=)60$</p> <p>.2 $\sin \text{their60} = \frac{10}{AB} \text{ or } \cos \text{their30} = \frac{10}{AB} \text{ OE}$</p> <p>ACCEPT $\frac{10}{\sin \text{their60}} \text{ or } \frac{10}{\cos \text{their30}} \text{ OE}$</p> <p>.3 $(AB =) \text{ their11.547(005...)} \text{ ACCEPT 11.5 or 11.55}$</p> <p>AM3</p> <p>.1 $(BCF=)30 \text{ or } (CBF=)60$</p> <p>.2 $\tan \text{their30} = \frac{BF}{5} \text{ OE or } \sin \text{their60} = \frac{5}{BC} \text{ OE or } BC = \frac{5}{\cos \text{their30}} \text{ OE}$</p> <p>.2 ACCEPT $(BF=) 2.89 \text{ or } 2.9 \text{ or } (BC=) 5.77$</p> <p>.3 $(\text{their8.66} + \text{their2.89} =) \text{ their11.547(005...)} \text{ ACCEPT 11.5 or 11.55}$</p>	<p>3</p>
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Q4	Answers	Notes				Total
	.1 One correct .2 A second correct .3 A third correct .4 The fourth and fifth correct	$f(x)$	4	0.5	1	4
		$g(x)$	8	[REDACTED]	-3	

Q5		Answers	Notes	Total
	a	<p>Important note: Candidates may use a correct method but not listed below. Mark it in-line with any of these methods. If in doubt contact your team leader for advice.</p> <p>AM1 (55 as an alternate interior angle)</p> <ul style="list-style-type: none"> .1 Correctly subtract 55 from 180 .2 Subtract the sum of 125 and 140 from 360 <p>95 AG</p> <p>AM2 (55 as an alternate interior angle)</p> <ul style="list-style-type: none"> .1 Correctly subtract 140 from 360 .2 Subtract the difference of 180 and 55 from 220 <p>95 AG</p> <p>AM3 (supplementary angle and alternate interior angle)</p> <ul style="list-style-type: none"> .1 Correctly subtract 140 from 180 .2 Add 55 and 40 <p>95 AG</p> <p>AM4 (complementary angle and alternate interior angle)</p> <ul style="list-style-type: none"> .1 Correctly subtract 55 from 90 .2 Correctly subtract the sum of 35, 90 and 140 from 360 <p>95 AG</p> <p>AM5 (bearing of H from A)</p> <ul style="list-style-type: none"> .1 Correctly determine bearing of H from A .2 Correctly subtract 140 from 235 <p>95 AG</p>	<p>AM1</p> <ul style="list-style-type: none"> .1 $180 - 55$ or 125 .2 $360 - (125 + 140)$ ACCEPT $140 + 125 + 95 = 360$.2 ACCEPT $360 - 265$ only if .1 is awarded <p>AM2</p> <ul style="list-style-type: none"> .1 $360 - 140$ or 220 .2 $220 - (180 - 55)$ or $220 - 125$ <p>AM3</p> <ul style="list-style-type: none"> .1 $180 - 140$ or 40 .2 $55 + 40$ <p>AM4</p> <ul style="list-style-type: none"> .1 $90 - 55$ or 35 .2 $360 - (35 + 90 + 140)$ ACCEPT $270 - (150 + 35)$.2 ACCEPT $360 - 265$ only if .1 is awarded <p>AM5 (bearing of H from A)</p> <ul style="list-style-type: none"> .1 $180 + 55 = 235$ DO NOT ACCEPT only seeing 235 .2 $235 - 140$ 	2

	b	<ul style="list-style-type: none">·1 Correctly substitute into cosine rule·2 Correctly calculate the square of BH·3 Correctly square root their result	<ul style="list-style-type: none">·1 $(x^2 =)100^2 + 250^2 - 2 \times 100 \times 250 \times \cos 95$ OE·2 76857(.787...) ACCEPT not seeing this step·3 their 277(.232...)	3
	c	<ul style="list-style-type: none">·1 Correctly identify the shortest route	<ul style="list-style-type: none">·1 HABDCH or HCDBAH ACCEPT HBACDH or HDCABH	1

5	d	<p>SPECIAL NOTE 1 Even though candidate may have a route that does not include BD, marking to ensure fairness and consistency must be made following all the steps.</p> <p>SPECIAL NOTE 2 If candidates use both AC and BD in c) they will make calculations for both distances in d) – identify the distance that will give the most marks and ignore the other distance calculation. However for .5 when they find the total distance, they must calculate the sum using their AC and BD found in d).</p>						
5	d	<p>.1 Correct method to calculate angle AHB or angle ABH</p> <p>AHB</p> <p>ABH</p> <p>.2 Their correct angle BHD (bottom)</p> <table border="1"> <tr> <td>.1</td> <td> AHB $\frac{\sin AHB}{250} = \frac{\sin 95}{\text{their277}}$ OE $250^2 = 100^2 + \text{their277}^2 - 2(100)(\text{their277})\cos(\text{AHB})$ $\text{AHB} = 64(.0390602)$ </td> <td> ABH $\frac{\sin ABH}{100} = \frac{\sin 95}{\text{their277}}$ OE $100^2 = 250^2 + \text{their277}^2 - 2(250)(\text{their277})\cos(\text{ABH})$ $\text{ABH} = 21(.0779...)$ </td> </tr> </table> <table border="1"> <tr> <td>.2</td> <td> Using their AHB $360 - 110 - 2(\text{their64})$ or $(\text{BHD}) = \text{their122}$ </td> <td> Using their ABH (Trapezium total :360) (Upper angles 130) (Lower angles $\frac{1}{2}(360 - 2(130)) = 50$) $(\text{HBD}) = 50 - \text{their21} = \text{their29}$ $180 - 2(\text{their29})$ or $(\text{BHD}) = \text{their122}$ </td> </tr> </table> <p>.2 DO NOT ACCEPT 110 for angle BHD for .2</p>	.1	AHB $\frac{\sin AHB}{250} = \frac{\sin 95}{\text{their277}}$ OE $250^2 = 100^2 + \text{their277}^2 - 2(100)(\text{their277})\cos(\text{AHB})$ $\text{AHB} = 64(.0390602)$	ABH $\frac{\sin ABH}{100} = \frac{\sin 95}{\text{their277}}$ OE $100^2 = 250^2 + \text{their277}^2 - 2(250)(\text{their277})\cos(\text{ABH})$ $\text{ABH} = 21(.0779...)$.2	Using their AHB $360 - 110 - 2(\text{their64})$ or $(\text{BHD}) = \text{their122}$	Using their ABH (Trapezium total :360) (Upper angles 130) (Lower angles $\frac{1}{2}(360 - 2(130)) = 50$) $(\text{HBD}) = 50 - \text{their21} = \text{their29}$ $180 - 2(\text{their29})$ or $(\text{BHD}) = \text{their122}$
.1	AHB $\frac{\sin AHB}{250} = \frac{\sin 95}{\text{their277}}$ OE $250^2 = 100^2 + \text{their277}^2 - 2(100)(\text{their277})\cos(\text{AHB})$ $\text{AHB} = 64(.0390602)$	ABH $\frac{\sin ABH}{100} = \frac{\sin 95}{\text{their277}}$ OE $100^2 = 250^2 + \text{their277}^2 - 2(250)(\text{their277})\cos(\text{ABH})$ $\text{ABH} = 21(.0779...)$						
.2	Using their AHB $360 - 110 - 2(\text{their64})$ or $(\text{BHD}) = \text{their122}$	Using their ABH (Trapezium total :360) (Upper angles 130) (Lower angles $\frac{1}{2}(360 - 2(130)) = 50$) $(\text{HBD}) = 50 - \text{their21} = \text{their29}$ $180 - 2(\text{their29})$ or $(\text{BHD}) = \text{their122}$						

	<p>.3 Correctly substitute into cosine rule to get BD or AC</p> <p>OR</p> <p>Correctly substitute into a correct trig ratio to get half BD or half AC</p> <p>OR</p> <p>correctly substitute into sine rule to get BD or AC</p> <p>.4 Correctly calculate their BD or AC</p> <p>.5 Correctly add their values for their route</p>	<p>.3 $(BD^2 =) \text{their}277^2 + \text{their}277^2 - 2(\text{their}277)(\text{their}277)\cos \text{their}122$ or $(AC^2 =) 100^2 + 100^2 - 2(100)(100)\cos 110$</p> <p>ACCEPT use of 110 in .3 for their BHD or 55 for half BHD $(BD^2 =) \text{their}277^2 + \text{their}277^2 - 2(\text{their}277)(\text{their}277)\cos 110$</p> <p>OR</p> <p>$\sin 61 = \frac{\text{halfBD}}{\text{their}277}$ or $\sin 55 = \frac{\text{halfBD}}{\text{their}277}$ or $\sin 55 = \frac{\text{halfAC}}{100}$ OE</p> <p>OR</p> <p>$\frac{BD}{\sin 122} = \frac{\text{their}277}{\sin 29}$ or $\frac{BD}{\sin 110} = \frac{\text{their}277}{\sin 35}$ or $\frac{AC}{\sin 110} = \frac{100}{\sin 35}$ OE</p> <p>.4 (BD=) Their484.5(393178...) or 453.81(02325...) seen (AC=) 163.8(30...) Correct .4 implies .3</p> <p>.5 Correct answer 1184.5 or 1185 (100+250+their484.5+250+100=) their1184.5 or their 1185 (100+250+their453.8+250+100=) their1153.8 or their 1154 (their 277+250+their163.8+250+their 277=) their1217.8 or their 1218 .5 ACCEPT values for their route in part c) even if route is incorrect .5 DO NOT ACCEPT if their route is incomplete (ex : doesn't end at H)</p>
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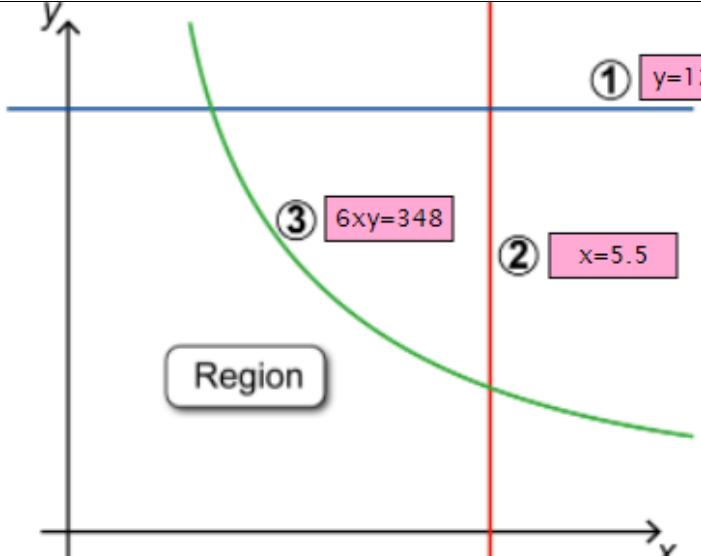
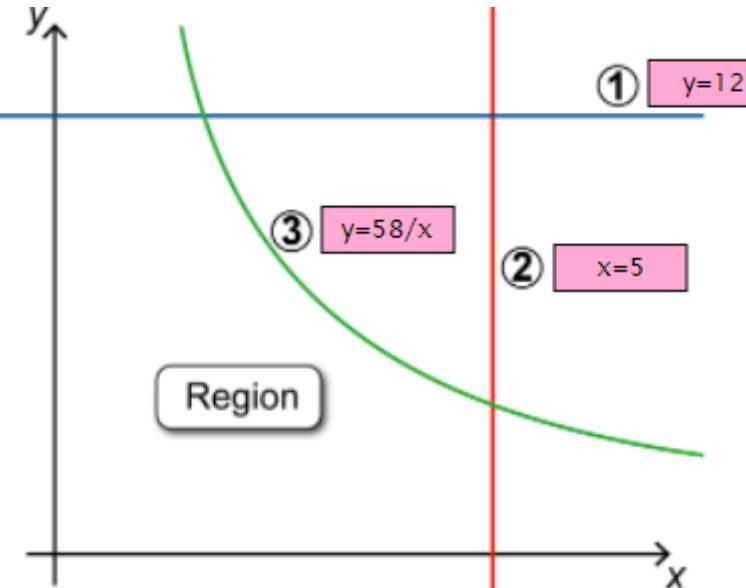
Q6		Answers	Notes	Total
	a	<p>.1 Correct area for two from: square, circle or half mouth seen</p> <p>.2 Correct area for third seen</p> <p>.3 Subtract, at least, their circle from their square</p> <p>.4 The correct answer before rounding</p> <p>AG 196</p>	<p>.1 Two from: 20×20 or 400 , $7^2\pi$ or 153.9(...) or 153.86 or 154, $\frac{100}{2}$ or 50</p> <p>.2 Third from: 20×20 or 400 , $7^2\pi$ or 153.9(...) or 153.86 or 154, $\frac{100}{2}$ or 50</p> <p>.3 their400 - their153.9(...) or their400 - their153.9(...) - their50 OE</p> <p>.3 ACCEPT only if .1 is awarded</p> <p>.4 196.06(...) or 196.1 ACCEPT using $\pi = 3.14$ and reaching 196.14 or 196.1</p> <p>.4 ACCEPT seeing evidence of correct rounding in intermediate steps</p> <p>Examples: $400 - 154 - 50 = 196$ if 153.9(...) is seen in their response $400 - 204 = 196$ if 203.9(...) is seen in their response</p>	4

	b	<p>AM1</p> <p>.1 Correct length ratio</p> <p>.2 Correct area ratio</p> <p>.3 The correct fraction after applying the area ratio</p> <p>AG 18.5</p> <p>AM2</p> <p>.1 Correct length ratio</p> <p>.2 Correctly dividing areas</p> <p>.3 Correctly show that area ratio is the square of length ratio</p> <p>AG 18.5</p>	<p>AM1</p> <p>.1 $\frac{37.6}{4.7}$ or 8 or $\frac{1}{8}$ OE</p> <p>.2 $(\frac{1}{8^2} =) \frac{1}{64}$ OE or $(8^2 =) 64$ OE ACCEPT $(\frac{37.6}{4.7})^2$</p> <p>.3 $\frac{1184}{64} (= 18.5)$ or $\frac{37}{2}$ OE ACCEPT $64 \times 18.5 = 1184$</p> <p>.3 ACCEPT $\frac{1184}{(\frac{37.6}{4.7})^2}$</p> <p>AM2</p> <p>.1 $\frac{37.6}{4.7}$ or 8 or $\frac{1}{8}$ OE</p> <p>.2 $(\frac{1184}{18.5} =) 64$ or $(\frac{18.5}{1184} =) \frac{1}{64}$ OE ACCEPT $\sqrt{\frac{1184}{18.5}} = 8$</p> <p>.3 $64 = 8^2$ or $\frac{1}{64} = (\frac{1}{8})^2$ or 64 is square of 8 WTTE</p> <p>.3 ACCEPT ratio of area is the square of ratio of side or length ratio is the square root of area ratio WTTE</p> <p>.3 DO NOT ACCEPT 64 is a multiple of 8</p>	3
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	c	<p>AM1 (using equations)</p> <p>.1 Correctly write the second equation</p> <p>.2 Correct step towards solving the correct equations</p> <p>.3 Correctly solve their equations for one unknown x OR y</p> <p>.4 Correctly write their corresponding value of the other unknown satisfying one of their equations</p> <p>.5 Correctly identify x=24 and y=6</p> <p>AM2 (using numbers)</p> <p>.1 Correctly calculate their area using their numbers of small and big triangles</p> <p>.2 Correctly calculate the area using 24 small triangles and 6 big triangles</p> <p>.3 Correctly calculate the area of painting left</p> <p>.4 Seeing their value of x is 4 times their value of y</p> <p>.5 Correctly identify x=24 and y=6</p>	<p>AM1 (using equations) Note: only .3 and .4 are using their</p> <p>.1 $2x + 5y = 80$ ACCEPT using inequality</p> <p>.2 correct substitution: $2x + 5 \times \frac{x}{4} = 80$ or $2 \times 4y + 5y = 80$ OR correct coefficients for elimination. Example: $2x + 5y = 80$ and $2x - 8y = 0$ or $8x + 20y = 320$ and $-5x + 20y = 0$</p> <p>.2 DO NOT ACCEPT working with their equations from .1</p> <p>.3 (x =) their $\frac{320}{13}$ or 24.61(...) OE OR (y =) their $\frac{80}{13}$ or 6.15(...) OE</p> <p>.4 (x =) their $\frac{320}{13}$ or 24.61(...) OE OR (y =) their $\frac{80}{13}$ or 6.15(...) OE</p> <p>.5 x = 24 and y = 6 or 24 small (triangles) and 6 big (triangles)</p> <p>AM2 (using numbers) Note: Only .1 and .4 are using their</p> <p>.1 $(2 \times \text{their}24 + 5 \times \text{their}6 =) \text{their}78$</p> <p>.2 $(2 \times 24 + 5 \times 6 =) = 78$</p> <p>.3 $(80 - 78 =) 2 \text{ (cm}^2\text{)}$</p> <p>.4 their24 is 4 times their6</p> <p>.5 x = 24 and y = 6 or 24 small (triangles) and 6 big (triangles)</p>	5
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Q7		Answers	Notes	Total
	a	.1 Correctly substitute into Arithmetic Progression .2 The correct common difference	.1 $3.5 = 1.5 + 5d$ or $\frac{(3.5 - 1.5)}{5}$ OE .2 0.4 OE	2
	b	Correctly substitute their 0.4 AND 1.5 into the correct arithmetic sequence formula	($H_n =$) $1.5 + \text{their}0.4(n-1)$ OE ACCEPT seeing $u_1 + \text{their}0.4(n-1)$ if $u_1=1.5$ is used in part c)	1
	c	.1 Correctly substitute $n = 12$ in their H_n .2 Their correct answer	.1 ($H_5 =$) $1.5 + \text{their}0.4(12 - 1)$ OE .2 their 5.9 (m) .2 ACCEPT only if their 5.9 is in the range]3.5 , 10]	2
	d	AM1 (solving for x) .1 Correctly set an equation in terms of x .2 Correctly rearrange the equation .3 The correct value for x before rounding down $x=5$ AG	AM1 (solving for x) .1 $3.5x + 1.5(x-1) + 2 \times 2 = 30$ OE ACCEPT using inequality .2 $5x = 27.5$ ACCEPT $5x + 2.5=30$ ACCEPT using inequality .3 $(x = \frac{27.5}{5} =) 5.5$	3

	<p>AM2 (using x=5)</p> <p>.1 Correctly set calculations for width</p> <p>.2 Correctly calculate one of :</p> <ul style="list-style-type: none"> → width of sections and gaps → total width excluding gaps → the remaining distance <p>.3 Correct argument that x = 5 is the maximum possible in the width of 30</p>	<p>AM2 (using x=5)</p> <p>.1 Two from</p> <ul style="list-style-type: none"> → 5×3.5 OE → 4×1.5 OE → 2×2 OE or $30 - 2 \times 2$ OE <p>.2 One of the following is seen</p> <ul style="list-style-type: none"> → $(5 \times 3.5 + 4 \times 1.5 + 2 \times 2 =) 27.5$ ACCEPT width of sections and in-between gaps $(5 \times 3.5 + 4 \times 1.5 =) 23.5$ → $(30 - 4 \times 1.5 - 2 \times 2 =) 20$ → $(30 - 5 \times 3.5 - 4 \times 1.5 - 2 \times 2 =) 2.5$ <p>.3 A corresponding argument from:</p> <ul style="list-style-type: none"> → $\frac{27.5}{5} = 5.5$ or making calculations with x=6 and showing that total 32.5 is more than 30 → $\frac{20}{3.5} = 5.7(1\dots)$ → Recognise 2.5 remaining WTTE <p>DO NOT ACCEPT $\frac{30}{6} = 5$</p>
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e	<p>.1 Two correct from: $x = 5.5, y = 12.25, 6xy = 348$ OE , Region ACCEPT $x = 5, y = 12$</p> <p>.2 A third correct from: $x = 5.5, y = 12.25, 6xy = 348$ OE , Region ACCEPT $x=5, y=12$</p> <p>.3 A fourth correct from: $x = 5.5, y= 12.25, 6xy = 348$ OE , Region ACCEPT $x = 5, y = 12$</p> <p>DO NOT ACCEPT inequalities Ex: $x < 5.5$ and $y < 12.25$ or $x \leq 5.5$ and $y \leq 12.25$</p> <p>DO NOT ACCEPT numbers not simplified Ex: $y = \frac{6 - 1.1}{0.4}$</p>	 	3
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	f	See table below	10
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7f	(1 mark)	(2 marks)	(3 marks)	(4 marks)
Factors (F)	<p>Two factors seen in factors box from:</p> <ul style="list-style-type: none"> - number of seats in a row or in a section, number of sections, or width of section(s) - number of tiers or height - <u>total</u> number of seats or capacity of theatre - width of theatre or space available - gaps that must be left (either in-between or at the ends) <p>ACCEPT using values for any of the above ACCEPT WTTE DO NOT ACCEPT only ‘number of seats’ as a factor</p>	<p>Three factors seen in factors box from:</p> <ul style="list-style-type: none"> - number of seats in a row or in a section, number of sections, or width of section(s) - number of tiers or height - <u>total</u> number of seats or capacity of theatre - width of theatre or space available - gaps that must be left (either in-between or at the ends) <p>ACCEPT using values for any of the above ACCEPT WTTE DO NOT ACCEPT only ‘number of seats’ as a factor</p>		

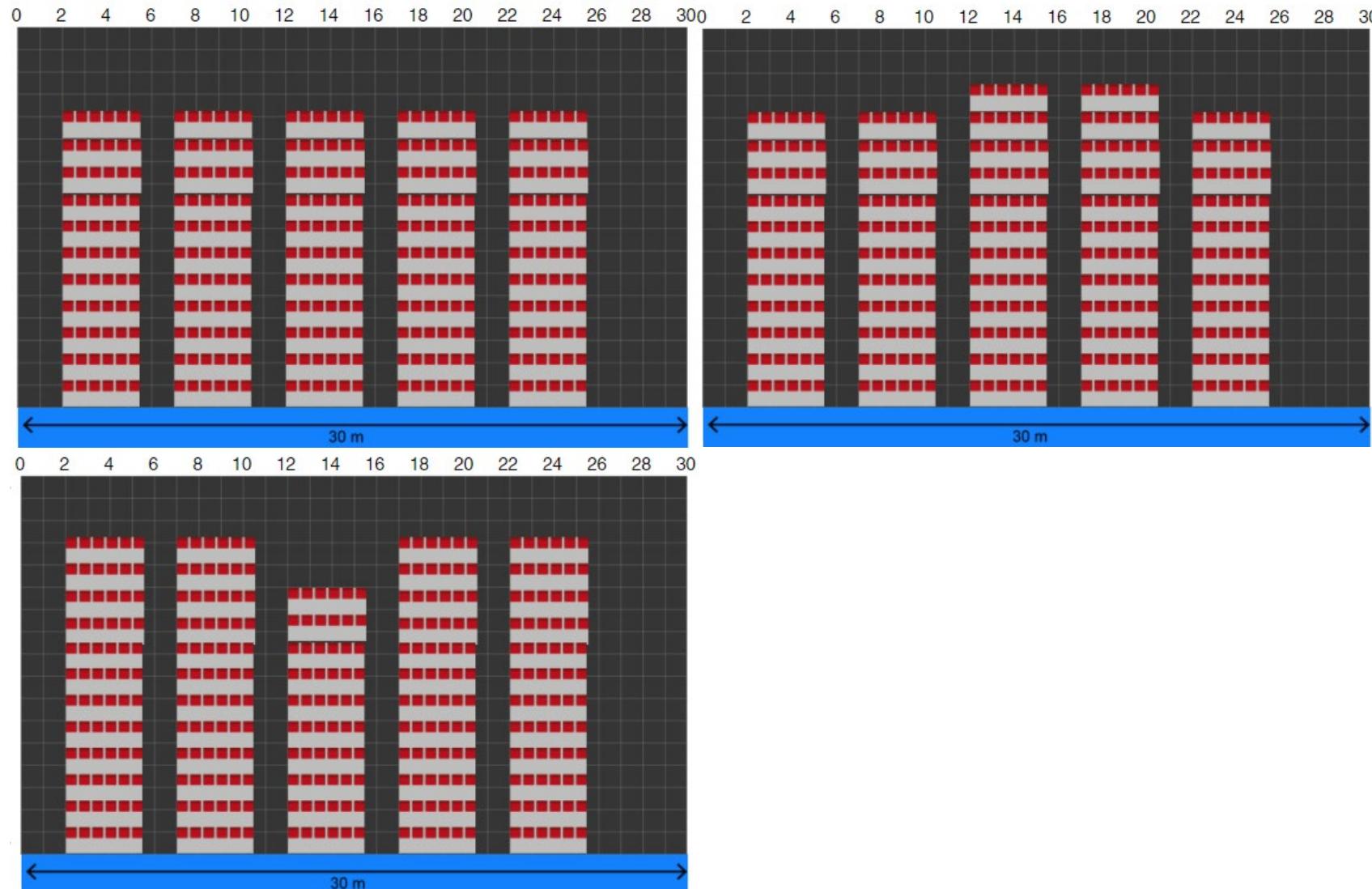
Cal (C)	Correct calculations for their number of seats in a row or in a block Total seats per row: 5×6 or 30 ACCEPT $348/12 = 29$ OE OR Total seats in one of their blocks Ex: 8×6 or 48 12×6 or 72 OR Number of sections of seats: $348/6=58$	Correct calculations related to the total number of seats in the range [120,420] Total number of seats 30 multiplied by their number of tiers: Example: $30 \times 4=120$ OR Number of tiers: Their total number of seats divided by 30 Ex: $348/30=11.6$ OR Number of blocks of seats: Their total number of seats divided by 6 times their number of tiers Ex: $348/78$ ($78=6 \times 13$) ACCEPT correct combinations using blocks for a total in the range [120,420] Ex: $5 \times 24 = 120$ ACCEPT listing number of seats per block for a total in the range [120,420] Ex: 48,48,48,48,24,24,24,24,12	Correct calculations for total number of seats in the range [330,360] EITHER $11 \times 30 = 330$ OR $12 \times 30 = 360$ ACCEPT if they make further step and reach total number of seats in the range [330,360] Ex: $12 \times 30 - 10 = 350$ ACCEPT correct combinations using blocks for a total of 330 or 360 $66 \times 5=330$ or $72 \times 5=360$ OE ACCEPT listing number of seats per block only if total is 348 Ex: 48,48,48,48,48,24,24,24,24,12 ACCEPT if they make further step and reach total number of seats in the range [330,360] Ex: $12 \times 30 - 10 = 350$ OE DO NOT ACCEPT $29 \times 12=348$	Correct calculations for a total number of 348 seats EITHER $11 \times 30 = 330$ then $330+3 \times 6=348$ OR $12 \times 30 = 360$ then $360 - 2 \times 6 = 348$ OR ACCEPT correct combinations using blocks for a total of 348 Ex: $72 \times 3 + 66 \times 2 = 348$ OE or $72 \times 4 + 60 = 348$ OE or $48 \times 5 + 24 \times 4 + 12 = 348$ OE DO NOT ACCEPT $29 \times 12=348$
		DO NOT ACCEPT $29 \times 12=348$	DO NOT ACCEPT $29 \times 12=348$	

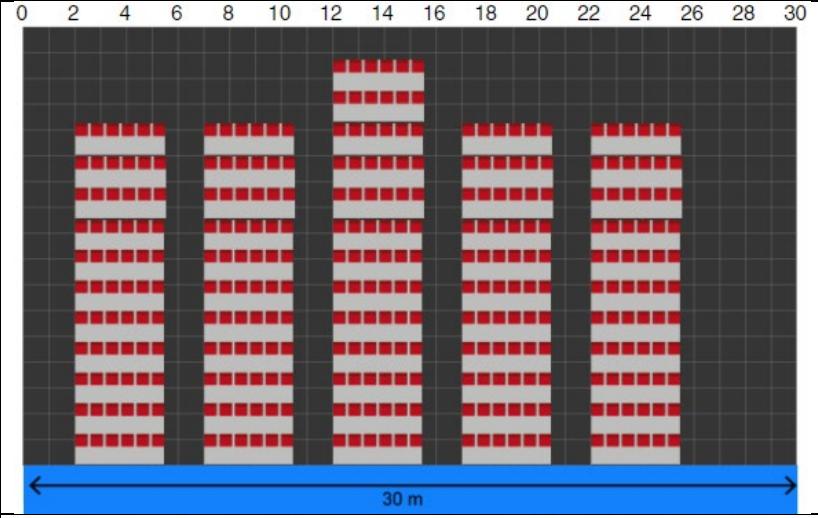
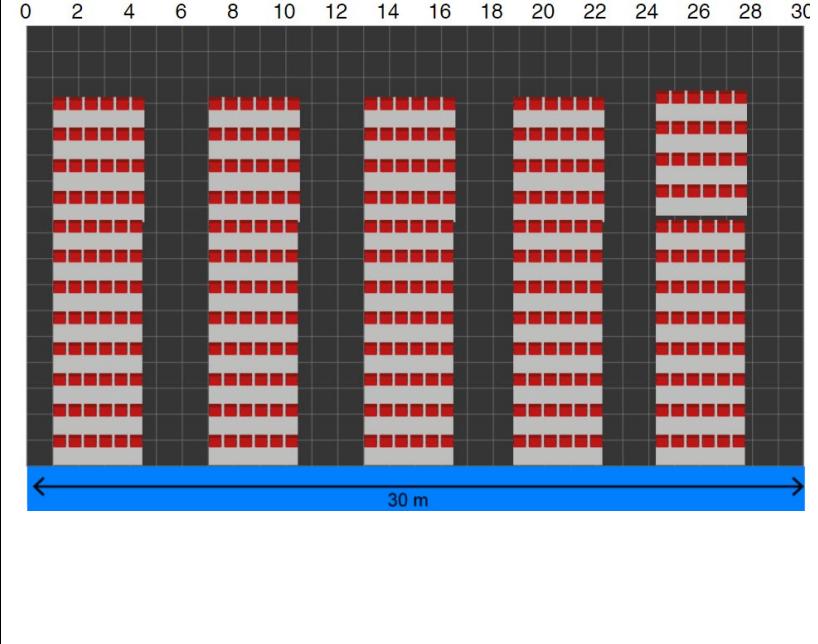
Justify (J)	<p>Weak justification</p> <p>Awareness of effect of constraints</p> <p>I made the best use (or didn't make the best use) of space within the constraint(s) WTTE</p> <p>ACCEPT for the constraints if they list at least one constraint</p> <p>DO NOT ACCEPT if C0 and D0 are awarded</p>	<p>Good justification</p> <p>I made the best use (or didn't make the best use) of space within the constraint(s) WTTE</p> <p>ACCEPT for the constraints if they list at least one constraint</p> <p>AND</p> <p>I have blocks of different sizes that do not exceed 12 tiers WTTE (seen on canvas or in calculations)</p> <p>DO NOT ACCEPT if C0 and D0 are awarded</p>		
Design (D)	<p>Two from:</p> <p>Gaps between sections ≥ 1.5</p> <p>Gaps left and right ≥ 2</p> <p>$330 \leq$ Total number of seats ≤ 348</p> <p>Tiers ≤ 12</p> <p>ACCEPT number of seats seen in response box different from number on canvas</p> <p>ACCEPT error in the gaps up to 0.25m</p> <p>DO NOT ACCEPT if their total number of seats on canvas is less than 120</p>	<p>The four of:</p> <p>Gaps between sections ≥ 1.5</p> <p>Gaps left and right ≥ 2</p> <p>$330 \leq$ Total number of seats ≤ 348</p> <p>Tiers ≤ 12</p> <p>ACCEPT number of seats seen in response box different from number on canvas</p> <p>A</p> <p>CCEPT error in the gaps up to 0.25m</p>		

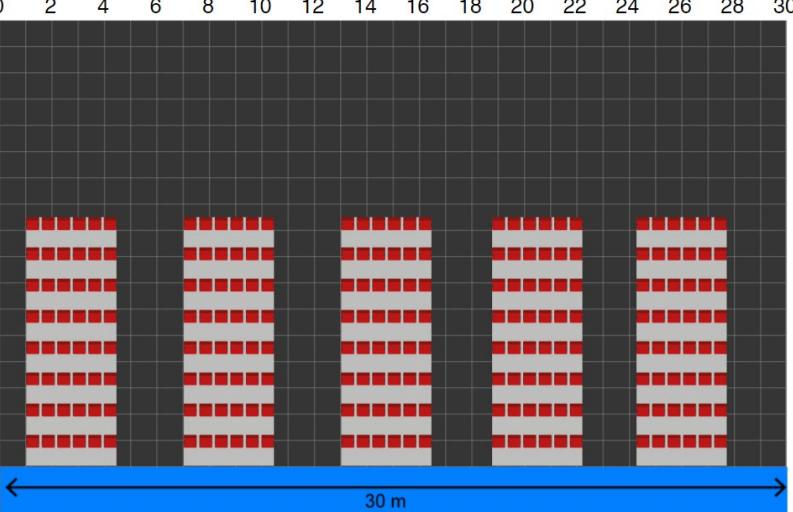
For examples of marks for the design (D) scroll down

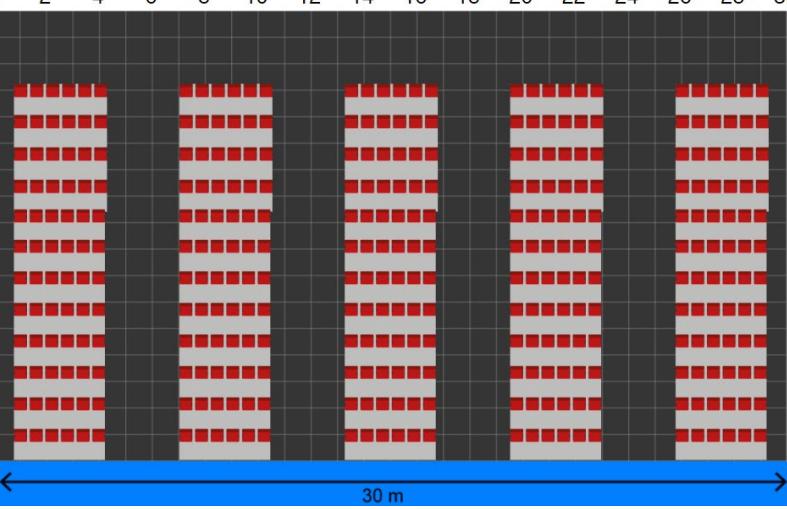
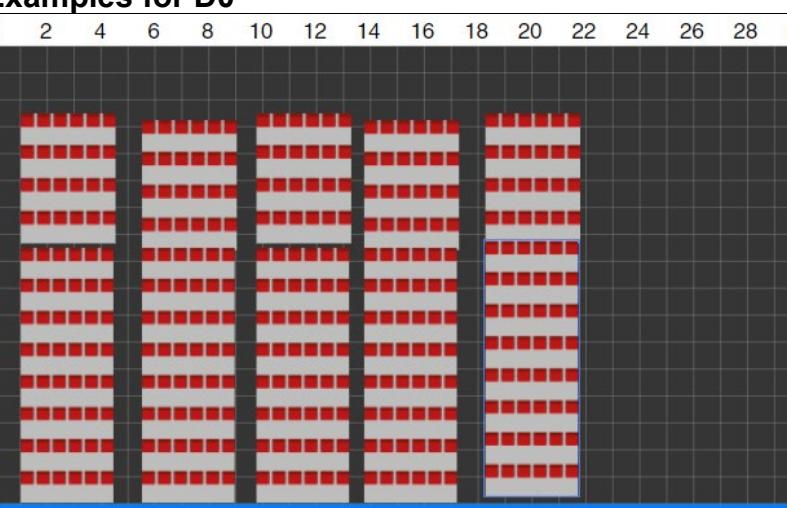
Accepted for D2

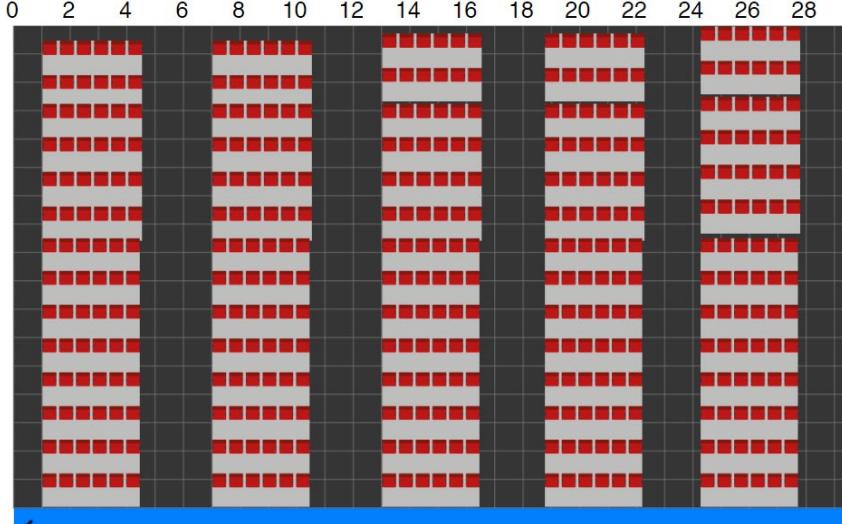
Gaps between sections ≥ 1.5 AND Gaps left and right ≥ 2 AND $330 \leq \text{Total number of seats} \leq 348$ AND tiers ≤ 12



Examples for accepted D1	Reason
 <p>Diagram showing a stadium section with 5 sections. Each section contains 6 rows of red seats. The total width is 30 meters. The diagram includes a horizontal blue bar at the bottom labeled "30 m". Above the diagram is a horizontal axis with tick marks from 0 to 30 in increments of 2.</p>	<p>Gaps between sections ≥ 1.5</p> <p>Gaps left and right ≥ 2</p> <p>$330 \leq \text{Total number of seats} \leq 348$</p>
 <p>Diagram showing a stadium section with 5 sections. Each section contains 6 rows of red seats. The total width is 30 meters. The diagram includes a horizontal blue bar at the bottom labeled "30 m". Above the diagram is a horizontal axis with tick marks from 0 to 30 in increments of 2.</p>	<p>Gaps between sections ≥ 1.5</p> <p>tiers ≤ 12</p>

Examples for accepted D1	Reason
 <p>Diagram showing a stadium section with 5 sections of 12 tiers each, separated by gaps of 1.5 meters or more. The total width is 30 meters.</p>	<p>Gaps between sections ≥ 1.5 tiers ≤ 12</p>
 <p>Diagram showing a stadium section with 4 sections of 12 tiers each, separated by gaps of 2 meters or more. The total width is 30 meters.</p>	<p>Gaps left and right ≥ 2 tiers ≤ 12</p>

Examples for accepted D1	Reason
 <p>0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30</p> <p>30 m</p>	<p>Gaps between sections ≥ 1.5</p> <p>tiers ≤ 12</p>
Examples for D0	Reason
 <p>0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30</p> <p>30 m</p>	<p>Only</p> <p>tiers ≤ 12</p>

 <p>0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30</p> <p>30 m</p>	Only Gaps left and right ≥ 2
 <p>0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30</p> <p>30 m</p>	Only Gaps between sections ≥ 1.5 Note that the gap on the right is >2 but the gap on the left no. So not accepded

		Answers	Notes	Total
8	a	.1 Correctly describe one pattern in words .2 Correctly describe a second pattern in words	<p>ACCEPT complete terminology only, for example (below are four different descriptions)</p> <p>(T) goes up by 4, increases by 4, moves up by 4, adds 4</p> <p>They are odd numbers</p> <p>Linear with difference 4, arithmetic with difference 4, Constant difference 4</p> <p>Second difference is zero</p> <p>DO NOT ACCEPT incomplete terminology, for example: Arithmetic, linear, increasing by a constant, constant difference, the odd numbers</p> <p>DO NOT ACCEPT the rule in words, for example: 4 times n then subtract 3 The difference between $4n$ and 3</p> <p>DO NOT ACCEPT n goes up by 1 It is increasing general rules in terms of n, example: $T = 4n - 3$</p> <p>More than two different patterns, all correct award (2 marks) Ex: adds 4 and Second difference is zero and it is 4 times n minus 3</p> <p>More than two different patterns, with any incorrect award (1 mark) Ex: adds 4 and Second difference is zero and it is 4 times n</p>	2

b	<ul style="list-style-type: none"> ·1 The correct general rule ·2 The correct simplified general rule with correct notation 	<ul style="list-style-type: none"> ·1 $4n - 3$ or $T=4*n-3$ or $u_n = 4n - 3$ or $t=4n-3$ or $T=2n+2n-3$ or $T=4x-3$ ·2 $T = 4n - 3$ or $T_n = 4n-3$ <p>DO NOT ACCEPT description in words</p>	2														
c	<ul style="list-style-type: none"> ·1 Correctly substitute $n \geq 5$ into their general rule ·2 Correctly calculate their value of T after substituting $n \geq 5$ ·3 Recognize that their result is the same as when continuing the correct pattern. See table below <table border="1" data-bbox="316 584 1073 906" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th style="text-align: center;">Stage (n)</th> <th style="text-align: center;">Number of triangles (T)</th> </tr> </thead> <tbody> <tr><td style="text-align: center;">1</td><td style="text-align: center;">1</td></tr> <tr><td style="text-align: center;">2</td><td style="text-align: center;">5</td></tr> <tr><td style="text-align: center;">3</td><td style="text-align: center;">9</td></tr> <tr><td style="text-align: center;">4</td><td style="text-align: center;">13</td></tr> <tr><td style="text-align: center;">5</td><td style="text-align: center;">17</td></tr> <tr><td style="text-align: center;">6</td><td style="text-align: center;">21</td></tr> </tbody> </table>	Stage (n)	Number of triangles (T)	1	1	2	5	3	9	4	13	5	17	6	21	<ul style="list-style-type: none"> ·1 Ex: $4 \times 5 - 3$ ·2 Ex: 17 (for the $n = 5$) ·3 Same as value I predicted in table (and we find the candidate has 17 in the table for $n = 5$) OR same as when we continue the pattern and explains how 17 is obtained from pattern of adding 4 to 13 ·3 ACCEPT seeing the 17 in the table and seeing their calculated $T = 17$ when $n = 5$ 	3
Stage (n)	Number of triangles (T)																
1	1																
2	5																
3	9																
4	13																
5	17																
6	21																
d	<p>AM1</p> <ul style="list-style-type: none"> ·1 Correctly substitute into trig ratio OR correctly substitute 4 and 8 into Pythagoras OR Correctly recognize ratio of 30-60-90 triangle of 1: $\sqrt{3}$: 2. ·2 Correctly calculate height of triangle h ·3 Correctly substitute values in area of triangle formula OR Correctly calculate area of triangle using decimals AND show that $16\sqrt{3}$ equals the same, using decimals. <p>$16\sqrt{3}$ AG</p>	<p>AM1</p> <ul style="list-style-type: none"> ·1 $\sin 60 = \frac{h}{8}$ OR $\cos 30 = \frac{h}{8}$ OR $h = \sqrt{8^2 - 4^2}$ ACCEPT $\tan 60 = \frac{h}{4}$ Accept not seeing this step. ·2 $\frac{8\sqrt{3}}{2}$ or 6.93 OE ACCEPT $8 \sin 60$ ·3 $\frac{1}{2} \times 8 \times \frac{8\sqrt{3}}{2}$ OE <p>OR Both ($A=$) 27.71(281...) AND $16\sqrt{3} = 27.71(281...)$</p> <p style="color: red; font-weight: bold;">AM2 next page</p>	3														

		<p>AM2 using $\frac{1}{2}ab \sin C$</p> <p>.1 Correctly substitute into the area of triangle formula using sin .2 Correctly calculate area of triangle .3 Show that $16\sqrt{3}$ equals the same</p> <p>$16\sqrt{3}$</p>	<p>AM2 using $\frac{1}{2}ab \sin C$</p> <p>.1 $(A =) \frac{1}{2} \times 8 \times 8 \times \sin 60$.2 $(A =) 27.71(28\dots)$.3 $16\sqrt{3} = 27.71(28\dots)$</p> <p>Showing $16\sqrt{3}$ using other values in the table or using the geometric progression, award 0 marks.</p>	
8	e	Scroll down for the marking grid		23

Mark	1	2	3	4	5
Predictions (P)	<p>Correctly predict one value for A OR two values for L</p>	<p>Correctly predict two values for A OR one value for A and two values for L</p>	<p>Correctly predict two values for A AND two values for L</p>		

Stage (n)	Side length of outer triangle (L)	Area (A)	Area (A)
1	1	$\frac{\sqrt{3}}{4}$	0.4330127019
2	2	$\sqrt{3}$	1.732050808
3	4	$4\sqrt{3}$	6.92820323
4	8	$16\sqrt{3}$	27.71281292
5	16	$64\sqrt{3}$	110.8512517
6	32	$256\sqrt{3}$	443.4050067
7	64	$1024\sqrt{3}$	1773.620027
8	128	$4096\sqrt{3}$	7094.480108

Description (D)	<p>Correctly describe a pattern in words (for L or A)</p> <p>Examples for L : Value doubles WTTE The difference doubles Increasing by multiplying by 2 All even except 1 It is exponential or geometric Add 1 then add 2 then add 4,...etc OE</p> <p>DO NOT ACCEPT L is increasing n goes up by 1</p> <p>Examples for A : Value quadruples WTTE The difference quadruples It is exponential or geometric All multiples of $\sqrt{3}$</p> <p>DO NOT ACCEPT A is increasing n goes up by 1 $\sqrt{3}$ is always there Length squared times $\sqrt{3}$ There is a common ratio</p> <p>OR</p> <p>Valid attempt to write down general rule for L in terms of n Ex.: $L = 2^n$ DO NOT ACCEPT $L=2n$</p> <p>Ignore additional incorrect patterns (for Notation see N)</p>	<p>Correctly describe a pattern in words for L and for A</p> <p>See patterns in D1</p> <p>OR</p> <p>Correctly describe a pattern in words (for L or A) and valid attempt to write down general rule for L in terms of n</p>	<p>Correctly describe a pattern in words for L and A and valid attempt to write down general rule for L in terms of n</p> <p>OR</p> <p>Correctly write down the general rule for L in terms of n $L = 2^{n-1}$</p> <p>OR</p> <p>Valid attempt to write down general rule for A in terms of n</p> <p>Examples: seeing in their rule 4^n added or multiplied by something</p> <p>$4^n \times \sqrt{3}$</p> <p>$A = \frac{1}{2} \times 2^n \times 2^n \times \frac{\sqrt{3}}{2}$</p> <p>$A = \sqrt{3}L$</p> <p>$A = u_1 \times 4^{n-1}$</p>	<p>Correctly describe a pattern in words (for L or A) and correctly write down the general rule for L</p> <p>OR</p> <p>Correctly describe a pattern in words (for L or A) and valid attempt to write down the general rule for A in terms of n</p> <p>OR</p> <p>Correctly write down the general rule for A in terms of n</p> <p>$A = 2^{2n-4} \sqrt{3}$ or</p> <p>$A = \frac{4^n}{16} \sqrt{3}$ or</p> <p>$A = 4^{n-1} \frac{\sqrt{3}}{4}$ or</p> <p>$A = 4^{(n-2)} \sqrt{3}$</p>	<p>Correctly describe a pattern in words for L and for A</p> <p>AND</p> <p>Correctly write down the general rule for A in terms of n</p> <p>Ignore additional incorrect patterns (for Notation see N)</p>
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Testing (T)	<p>Attempt to test their general rule for A using $n \leq 4$</p> <p>Correctly substitute in their general rule for A value of $n \leq 4$</p> <p>OR</p> <p>Correctly test their general rule for L or described pattern or recursive rule</p> <p>OR</p> <p>Correctly test their rule for A or described pattern or recursive rule</p> <p>ACCEPT testing their general rule for A in terms of L</p>	<p>Correctly test their general rule for A only in terms of n using $n \leq 4$</p> <p>Ex:</p> <p>Correctly calculate their value for A in their general rule using $n \leq 4$</p> <p>AND</p> <p>Recognize that their correctly calculated value for A is the same as the given value.</p>	ACCEPT seeing their correctly calculated value for A and the given value in the table being equal		
Verifying (V)	<p>Attempt to verify their general rule for A using $n \geq 5$</p> <p>Ex: correctly substitute in their general rule value of $n \geq 5$</p> <p>OR</p> <p>Correctly verify their described pattern or their rule (e.g. recursive rule)</p> <p>ACCEPT verifying their rule for A in terms of L</p>	<p>Correctly calculate their value for A in their general rule only in terms of n using their $n \geq 5$</p>	<p>Correctly calculate their value for A in their general rule only in terms of using $n \geq 5$</p> <p>AND</p> <p>Recognize that their correctly calculated value for A is the same as their predicted value obtained by continuing the pattern</p> <p>ACCEPT seeing their correctly calculated value for A and their predicted value in the table being equal</p>		

Justify/prove (J)	<p>Attempt to justify a correct described pattern or their general rule</p> <p>Substitute at least two other values of n in A and say they are the same or the rule works</p> <p>OR</p> <p>Seeing $A = 0.5 \times L \times h$ OE</p> <p>DO NOT ACCEPT $A = 0.5 \times b \times h$</p> <p>OR</p> <p>Recognize it is geometric progression and define first term $\frac{\sqrt{3}}{4}$ or ratio 4, WTTE</p> <p>DO NOT ACCEPT if D1 not achieved</p>	<p>Correctly justify the general rule for A in relation to geometry in terms of L or height of triangle (h)</p> $A = \frac{1}{2}L \times L \times \frac{\sqrt{3}}{2}$ <p>or</p> $A = 0.5 \times L \times L \times \sin 60 \text{ OE}$ <p>or</p> $A = 0.5 \times 2^{n-1} \times h$ <p>OR</p> <p>Recognize it is geometric progression and define first term $\frac{\sqrt{3}}{4}$ and ratio 4, WTTE</p> <p>DO NOT ACCEPT if D3 not achieved</p>	<p>Attempt to justify the general rule for A in relation to geometry in terms of n</p> <p>Examples using area of triangle but including errors :</p> $\frac{2^{n-1} \times 2^{n-1} \sqrt{3}}{2}$ $\frac{2^{n-2} \times 2^{n-2} \sqrt{3}}{2}$ $\frac{2^n \times 2^n \sqrt{3}}{2}$ <p>ACCEPT seeing the justification inside their work and not separate at the end</p> <p>DO NOT ACCEPT if D3 not achieved</p>	<p>Correctly justify the correct general rule for A geometrically in terms of n</p> <p>Using the area of a triangle</p> $A = \frac{B \times h}{2} \text{ or } \frac{a \times b \times \sin C}{2}$ $\frac{2^{n-1} \times 2^{n-2} \sqrt{3}}{2}$ <p>or</p> $\frac{2^{n-1} \times 2^{n-1} \times \sqrt{3}}{2}$ <p>ACCEPT seeing the justification inside their work and not separate at the end</p> <p>DO NOT ACCEPT if D4 not achieved</p>	
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Communication criteria

Mark	1	2	3
Notation and terminology (N)	<p>Correct notation of <u>their general rule</u></p> <p>OR</p> <p>Correct terminology describing a pattern</p> <p>DO NOT ACCEPT if they don't have any rules and they don't describe any patterns correct</p>	<p>Correct notation of <u>the general rule</u> for L or A</p> <p>OR</p> <p>The notation of <u>the general rule</u> for L or A includes errors AND Correct terminology describing a pattern in words</p> <p>DO NOT ACCEPT if they don't have the general rule</p> <p>For notation of the general rule for L, $L = 2^{n-1}$</p> <p>ACCEPT using U_n instead of L only if they mention that $L = U_n$</p> <p>ACCEPT non simplified rules ex: $L = \frac{2^n}{2}$ OE</p> <p>DO NOT ACCEPT Using * for multiplication Using / for division Using ^ for power Using x instead of n “the rule for L is:” instead of “$L =$”</p>	<p>Correct notation of <u>the general rule</u> for A</p> <p>AND</p> <p>Correct terminology describing pattern in words for A</p> <p>DO NOT ACCEPT if they don't have the general rule for A</p> <p>For notation of the general rule for A</p> <p>$A = 4^{(n-2)}\sqrt{3}$ or $A = 4^{(n-2)} \times \sqrt{3}$</p> <p>ACCEPT using U_n instead of A only if they mention that $A = U_n$</p> <p>ACCEPT non simplified rules ex: $A = \frac{1}{2} \times 2^{2n-2} \times \frac{\sqrt{3}}{2}$ or $A = 4^{n-1} \frac{\sqrt{3}}{4}$ OE</p> <p>DO NOT ACCEPT The Using * for multiplication Using / for division Using ^ for power Using x instead of n “the rule for A is:” instead of “$A =$”</p>

Continued on next page

Mark	1	2	3
<p>Communication (L)</p> <p>Organisation and coherence</p> <p>Can be awarded even if there are errors in their descriptions and working.</p> <p>Different items can be considered seen (or identified for coherence) if they include errors but not if awarded 0 marks</p>	<p>At least three from the following items are seen:</p> <ul style="list-style-type: none"> · describe a pattern in words · write a rule · test their general rule or rule or recursive rule or pattern · verify their general rule or rule or recursive rule or pattern · justify their general rule or rule or recursive rule or pattern 	<p>At least four of the following items are seen:</p> <ul style="list-style-type: none"> · describe a pattern in words (for L or A) · write a general rule (for L or A) · test their general rule (for L or A) · verify their general rule (for L or A) · justify their general rule (for L or A) <p>AND For coherence, they identify the processes correctly. At least one from the following:</p> <ul style="list-style-type: none"> · test · verify · justify <p>Ex:</p> <ul style="list-style-type: none"> → For test: they say "test" and they test using value(s) of $n \leq 4$ only → For verify: they say "verify" and they verify using value(s) of $n \geq 5$ only → For test and for verify: they say 'test and verify' and they test using value(s) of $n \leq 4$ and then verify using value(s) of $n \geq 5$ → For justify: they say "justify" or "my rule works because" WTTE and their justification is seen → For justify: they substitute at least two values of n and say "the rule justified" or "it works" WTTE → For justify: They justify <u>the general rule for A</u> geometrically 	<p>DO NOT ACCEPT if D4 not awarded</p> <p>At least four of the following items are seen:</p> <ul style="list-style-type: none"> · describe a pattern in words · write <u>the general rule</u> for A · test <u>the general rule</u> for A · verify <u>the general rule</u> for A · justify <u>the general rule</u> for A <p>AND For coherence, they identify the processes correctly. At least two from the following:</p> <ul style="list-style-type: none"> · test · verify · justify <p>Ex:</p> <ul style="list-style-type: none"> → For test: they say "test" and they test using value(s) of $n \leq 4$ only → For verify: they say "verify" and they verify using value(s) of $n \geq 5$ only → For test and for verify: they say 'test and verify' and they test using value(s) of $n \leq 4$ and then verify using value(s) of $n \geq 5$ → For justify: they say "justify" or "my rule works because" WTTE and their justification is seen → For justify: they substitute at least two values of n and say "the rule justified" or "it works" WTTE → For justify: They justify <u>the general rule for A</u> geometrically

Markscheme

May 2021

Extended Mathematics

On-screen examination

27 pages

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The markscheme may make use of the following abbreviations:

OE or equivalent

WTTE or words to that effect or accept incomplete calculator display

AG Answer given

- Bullet notation means award 1 mark – see example below

Example

- ¹ mark awarded and corresponding notes are aligned

b	<ul style="list-style-type: none">•¹ Show clear line of reasoning in the method•² their correct result	<ul style="list-style-type: none">•¹ 45 & 49 seen OE eg, $49 = 45 + x$•² their 4	2
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Error Carried Forward (ECF) marks

Errors made at any step of a solution affect all working that follows. In general, **Error Carried Forward (ECF)** marks are awarded after an error.

- a) **ECF** applies from one part of a question to a subsequent part of the question and also applies within the same part.
- b) If an answer resulting from **ECF** is inappropriate (eg, negative distances or $\sin x > 1$) then subsequent marks should not be awarded.
- c) If a question is transformed by an error into a **simpler question** then **ECF** may not be fully awarded.
- d) To award **ECF** marks for a question part, **there must be working present for that part**.
- e) **ECF** is only applied to working which is correct. This means that all working subsequent to an error must be checked for accuracy.
- f) A misread (**MR**) is an error. **ECF** is normally awarded.

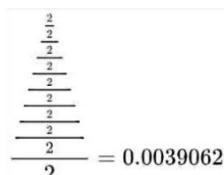
General points

- a) As this is an international examination, accept all **alternative forms of notation**, for example 1,9 as 1.9 ; 12,000 or 12 000 as 12000
- b) Accept notation errors in intermediate steps.
- c) Ignore further working after a correct answer **unless** noted otherwise.
- d) In the case when a correct result is obtained using incorrect seen method, do not award the mark for the result.
- e) Where candidates have written two solutions to a question, mark the first solution.
- f) In the markscheme, equivalent examples of **numerical** and **algebraic** forms or **simplified** answers will generally be written in the notes preceded by **OE** (Or Equivalent) e.g. $\frac{1}{2}$ OE means we accept 1/2 or 0.5 or $2 \div 4$ or 2^{-1} ; $\frac{x}{2}$ OE means we accept $x / 2$ or $x \div 2$ or $0.5x$; 0.23 OE means we accept 23%
- g) In the markscheme, information provided in brackets indicate detail that may be seen in a candidate response but is not necessary to award the marks.
- h) Special case marks **SC** can be allocated instead of but not in addition to the marks prescribed in the markscheme.
- i) Accept seeing equation not in-line.
- j) Calculator screenshots are accepted as working steps. And when a calculator screenshot is taken, accept not seeing the whole operation.
- k) In task 2 and 3 where the markscheme is set out in a table then, unless noted otherwise, awarding the highest mark in a category includes all the lower marks in that category. It is probably best to look for the top category mark answer and if you don't find it look at the next mark down.
- l) ACCEPT using the correct values regardless their previous result.

Question		Answers	Notes	Total
1	a	$\frac{4}{7}$ OE	Only accept $\frac{4}{7}$ OE as the final answer.	1
	b	.1 multiply correct probabilities for first selected numbers without replacement .2 multiply correct probabilities for second selected numbers without replacement .3 correctly add their multiplied probabilities	.1 $\frac{4}{10} \times \frac{6}{9}$ OE ACCEPT 0,266(66...) or 0,267 .2 $\frac{6}{10} \times \frac{4}{9}$ OE ACCEPT 0,266(66...) or 0,267 .3 their $\frac{8}{15}$ OE , ACCEPT their 0.533(33...)	

Question		Answers	Notes	Total
2	a	.1 Correctly write a in a simplified exact form	.1 $\log\left(\frac{1}{2}\right)$ or $\log(1/2)$ or $\log(0.5)$ or $-\log 2$ seen	3
		.2 Correctly write b in a simplified exact form	.2 $\log 12$ seen DO NOT ACCEPT $\log(6x2)$ as their answer	
		.3 Correctly write c in a simplified exact form	.3 $\log 32$ or $5\log 2$ seen ACCEPT $\log 2^5$ DO NOT ACCEPT $\log(16x2)$ as their answer	
	b	.1 Correctly write a in a simplified index form	.1 $2x^{-\frac{1}{2}}y^2$ or $\frac{2y^2}{x^{\frac{1}{2}}}$ or $2x^{-1/2}y^2$ seen DO NOT ACCEPT y^2 written as y^{2} and DO NOT ACCEPT $x^{\frac{1}{2}}$ written as \sqrt{x} or $\text{sqrt}(x)$ or $x^{-1/2}$	3
		.2 Correctly write b in a simplified index form	.2 $2xy$ seen	
		.3 Correctly write c in a simplified index form	.3 $\frac{1}{2}$ or $1/2$ or 0.5 or 2^{-1} seen	

Question		Answers	Notes	Total
3	a	.1 Equate $f(x)$ and 2 .2 Correct value of a	.1 $4^x = 2$ or $4^a = 2$ ACCEPT $x = \log_4 2$ or $x \log 4 = \log 2$.2 ($a =$) 0.5 OE DO NOT ACCEPT 0.5 written in log form as final answer ACCEPT as coordinates (0.5, 2) or (0.5,0)	2
	b	AM1 .1 Equate $g(x)$ and 2 .2 Correctly apply log rule .3 Correctly rearrange for their x or their $-x$.4 The correct value of x AM2 .1 Equate $g(x)$ and 2 .2 Correctly apply laws of exponents .3 Correctly rearrange for their x .4 The correct value of x	AM1 .1 $3^{5-x} = 2$ or $\log 3^{5-x} = \log 2$ ACCEPT not seeing this step .2 $\log_3 2 = 5-x$ or $(5-x)\log 3 = \log 2$ OE ACCEPT $5-x = 0.63(09\dots)$.3 $(x =) 5 - \frac{\log 2}{\log 3}$ or $\frac{5\log 3 - \log 2}{\log 3}$ or $5 - \log_3 2$ or $-x = \frac{\log 2}{\log 3} - 5$ or $-x = \frac{\log(\frac{2}{243})}{\log 3}$ The correct .3 implies .2 and .1 .4 ($x =$) 4.369(070....) ACCEPT 4.37 or 4.4 or as coordinates (4.37,2) or (4.37,0) with or without brackets AM2 .1 $3^{5-x} = 2$ ACCEPT not seeing this step .2 $\frac{3^5}{3^x} = 2$.3 $3^x = \frac{3^5}{2}$ or $(x =) \log_3(\frac{3^5}{2})$ The correct .3 implies .2 and .1 .4 ($x =$) 4.369(070....) ACCEPT 4.37 or 4.4 or as coordinates (4.37,2) or (4.37,0) with or without brackets In all .1, .2,.3, .4 ACCEPT using any symbol x or b ,..etc	4
	c	Correctly subtract their a from their b	($x =$) their 3.869(070....) ACCEPT 3.869 or 3.87 or 3.9 DO NOT ACCEPT their 3.869(070...) written in log form	1

Question	Answers	Notes	Total
4 a	Correctly write the value of r.	$\frac{1}{\sqrt{2}}$ OE DO NOT ACCEPT in words ACCEPT 0.707(10.6...) or 0.71 or 0.7 DO NOT ACCEPT $\div\sqrt{2}$	1
b	.1 correctly write at least two more terms .2 correct value of n	.1 $(4, 2\sqrt{2}, 2,)\sqrt{2}, 1, (\frac{\sqrt{2}}{2}$ OE) Equation with their $\frac{1}{\sqrt{2}}$ doesn't get .1 .2 $(n =)6$ ACCEPT U_6	2
c	.1 correctly substitute 4 and 21 and their r into the nth term of G.S formula .2 at least one correct intermediate step for their k using any method .3 the correct value of k from their .1	.1 $(U_{21} =)4 \times \text{their}(\frac{1}{\sqrt{2}})^{21-1}$ OE ACCEPT their 0.0039(...) .1 DO NOT ACCEPT if their $r \geq 1$.2 Examples of correct intermediate steps : Using powers of 2 : $\frac{2^2}{\text{their } 2^{10}}$ or $\frac{2^2}{\text{their } 2^{(\frac{1}{2})^{20}}}$ or $2^2 \times \text{their } 2^{-10}$ Using logs : $\log_2(4 \times \text{their}(\frac{1}{\sqrt{2}})^{21-1}) = k$ OE ACCEPT not seeing k Listing powers of 2 : see image  $\frac{2^0}{2^1} \frac{2^1}{2^2} \frac{2^2}{2^3} \frac{2^3}{2^4} \frac{2^4}{2^5} \frac{2^5}{2^6} \frac{2^6}{2^7} \frac{2^7}{2^8} \frac{2^8}{2^9} \frac{2^9}{2^{10}} \frac{2^{10}}{2^{20}} = 0.00390625$	3

4	d	<p>.1 substitute 4 and their r into the sum to infinity formula</p> <p>.2 correctly write their answer as one radical fraction</p> <p>.3 correctly rationalize their denominator</p> <p>.4 correct simplified sum to infinity</p>	<p>.1 $\frac{4}{1 - \text{their} \frac{1}{\sqrt{2}}}$ OE</p> <p>.2 their $\frac{4\sqrt{2}}{\sqrt{2}-1}$ OE, DO NOT AWARD if their r is $\sqrt{2}$</p> <p>.3 their $\frac{4\sqrt{2}(\sqrt{2}+1)}{1}$</p> <p>.4 $8 + 4\sqrt{2}$ ACCEPT only if their 8 and their 4 $\in \mathbb{N}$</p>	4
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Question	Answers	Notes	Total
5 a	<p>AM1 (using sin or cos ratios) .1 correctly divide by two the 150 AND 5.4</p> <p>.2 correctly substitute into trig ratio</p> <p>.3 correct value of r before rounding AG 2.80</p> <p>AM2 (using sine rule) .1 correctly substitute into sine rule</p> <p>.2 correctly rearrange for r on one side</p> <p>.3 correct value of r before rounding AG 2.80</p> <p>AM3 (using cos rule) .1 correctly substitute into cosine rule</p> <p>.2 correctly rearrange for r^2 on one side</p> <p>.3 correct value of r before rounding AG 2.80</p> <p>AM4 (using tan ratio) .1 correctly calculate the angle and correctly divide 5.4 by two</p> <p>.2 correctly substitute tan15 or tan75 ratio into Pythagoras</p> <p>.3 correct value of r before rounding AG 2.80 Award any VALID method using same marking principles</p>	<p>AM1 (using sin or cos ratios) .1 75 AND 2.7 seen. ACCEPT 15 AND 2.7 seen</p> <p>.2 $\sin 75 = \frac{2.7}{r}$ OR $\cos 15 = \frac{2.7}{r}$ OE ACCEPT $\frac{2.7}{\sin 75}$ OR $\frac{2.7}{\cos 15}$ seen</p> <p>.3 ($r =$)2.79(52....)</p> <p>.3 ACCEPT their correct r due to earlier rounding provided it rounds to 2.8</p> <p>AM2 (using sine rule) .1 $\frac{5.4}{\sin 150} = \frac{r}{\sin 15}$ OE ACCEPT not seeing this step</p> <p>.2 $(r =) \frac{5.4 \times \sin 15}{\sin 150}$ OE</p> <p>.3 ($r =$)2.79(52....)</p> <p>.3 ACCEPT their correct r due to earlier rounding provided it rounds to 2.8</p> <p>AM3 (using cos rule) .1 $5.4^2 = 2r^2 - 2r^2 \cos 150$ OE ACCEPT not seeing this step</p> <p>.2 $(r^2 =) \frac{5.4^2}{2 - 2 \cos 150}$ OE , or 7.81(33..) seen or $2r^2 = 15.62(68...)$</p> <p>.3 ($r =$)2.79(52....)</p> <p>.3 ACCEPT their correct r due to earlier rounding provided it rounds to 2.8</p> <p>AM4 (using tan ratio) .1 15 and 2.7 or 15 and 2.7 seen</p> <p>.2 $r^2 = 2.7^2 + 2.7^2 \tan^2 15$ or $(r^2 =) 2.7^2 + \frac{2.7^2}{\tan^2 75}$ OE or 7.81(33..) seen</p> <p>.3 ($r =$)2.79(52....)</p> <p>.3 ACCEPT their correct r due to earlier rounding provided it rounds to 2.8</p>	3

5	b	<p>.1 correct trig ratio using R and h</p> <p>.2 correctly write R in terms of h OR correctly write h in terms of R</p> <p>.3 substitute 2.8 into volume of sphere formula OR substitute .2 into volume of cone formula</p> <p>.4 evidence of equating their two volumes</p> <p>.5 correctly rearrange their equated volumes to have h or h^3 on one side OR to have R^3 of R on one side</p> <p>.6 correctly calculate their h after their rearrangement of their equated volumes</p>	<p>.1 $\tan 15 = \frac{R}{h}$ or $\tan 75 = \frac{h}{R}$ or $\frac{R}{\sin 15} = \frac{h}{\sin 75}$</p> <p>.2 $R = h \tan 15$ or $R = \frac{h}{\tan 75}$ or $h = \frac{R}{\tan 15}$ or $h = R \tan 75$ or $R = \frac{h \sin 15}{\sin 75}$ or $h = \frac{R \sin 75}{\sin 15}$ ACCEPT $h=3.73R$ or $R=0.27h$</p> <p>.2 implies .1</p> <p>.3 $(V =) \frac{4\pi 2.8^3}{3}$ OR $\frac{\pi \text{their}(h \tan 15)^2 h}{3}$ or $\frac{\pi R^2 \frac{\text{their}R}{\tan 15}}{3}$ or any from their .2 OE ACCEPT if $r=2.795$ for the sphere ACCEPT $91.95(23\dots)$</p> <p>.4 $\frac{4\pi 2.8^3}{3} = \frac{\pi \text{their}(h \tan 15)^2 h}{3}$ OE or $\frac{4\pi 2.8^3}{3} = \frac{\pi \text{their}R^3}{3 \tan 15}$ or any from their .3 OE</p> <p>.5 $(h^3 =) \frac{4\pi 2.8^3}{\pi (\tan 15)^2}$ OE OR $(R^3 =) 4(2.8)^3 (\tan 15)$ OE .5 implies .4</p> <p>.6 $(h =) 10.69(407\dots)$ DO NOT AWARD the last mark if their $R=2.8$ or their $R=2.7$</p>	6
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Question	Answers	Notes	Total
6	a 8 to 10 hours within the interval 6 pm to 6 am	Ex: 8pm to 5am ACCEPT correct 24-hour format ignoring am/pm	1
b	AM1 (using the 12-hour clock) .1 Maximum at 12:00 pm .2 Minimum at 12:00 am AM2 (using the 24-hour clock) .1 Maximum at 12:00 (am/pm) .2 Minimum at 24:00 (am/pm)	AM1 (using the 12-hour clock) .2 ACCEPT 0:00 am AM2 (using the 24-hour clock) .2 ACCEPT 0:00	2
c	.1 Amplitude 0.5 OE .2 Period 24	.1 ACCEPT .5 DO NOT ACCEPT -0.5	2
d	.1 Maximum 37 .2 Minimum 36		2

6	e	<p>.1 evidence of substituting 7.25 into the formula</p> <p>.2 the correct value of B</p> <p>.3 correctly round their value of B in .2 to 1 dp</p>	<p>.1 $(B =) -0.5 \cos\left(\frac{\pi}{12} \times 7.25\right) + 36.5$ ACCEPT $(B =) -0.5 \cos(15 \times 7.25) + 36.5$</p> <p>.2 36.66(071973....) DO NOT ACCEPT the using degrees answer (36.00027433)</p> <p>.3 their 36.7</p>	3
	f	.1 correctly write the equation modelling Ray's temperature	<p>.1 $R = -0.5 \cos\frac{\pi}{12}t + 36.75$ using R or any other letter</p> <p>ACCEPT $B = -0.5 \cos\frac{\pi}{12}t + 36.75$</p> <p>ACCEPT $-0.5 \cos\frac{\pi}{12}t + 36.75$</p>	
6	g	<p>.1 evidence of correctly equating their expression in terms of t with 36.5</p> <p>.2 correctly rearrange for their $\cos\frac{\pi}{12}t$ on one side</p> <p>.3 correctly inverse their cosine in radians</p> <p>.4 correct value of <u>their first</u> t after correctly inverse their cosine</p>	<p>.1 $36.5 = \text{their } -0.5 \cos\frac{\pi}{12}t + 36.75$ or $36.5 = -0.5 \cos\frac{\pi}{12}t + \text{their } 36.75$ ACCEPT using x instead of t</p> <p>.2 $\cos\frac{\pi}{12}t = \frac{\text{their}(-0.25)}{-0.5}$ OE</p> <p>.2 ACCEPT correctly rearrange linear equation for t but DO NOT AWARD .3 and .4 e.g $(t =) \text{their} \frac{36.5 - 36.75}{-0.5 \cos\left(\frac{\pi}{12}\right)}$ but do not award .3 and .4</p> <p>.3 $\text{their} \frac{\pi}{12}t = 1.047(197551...) \text{ OE}$ or $\frac{\pi}{12}t = \frac{\pi}{3}$ ACCEPT not seeing this step</p> <p>.4 $(t =) \text{their } 4 \text{ (am)}$ or 04:00 OE ignore incorrect time of day after seeing their 4</p>	4

Question	Answers	Notes	Total
7	a .1 mode 0.78 .2 median 0.77		2
	b .1 add the product of grade and frequency .2 divide the sum of products by 20 0.77 AG	<p>.1 $4 \times 0.75 + 3 \times 0.76 + 5 \times 0.77 + 6 \times 0.78 + 1 \times 0.79 + 1 \times 0.8$ OE .1 ACCEPT 15.4 seen .1 ACCEPT not seeing the whole operation from calculator screenshot provided it shows at least 4 correct products. Ex:</p> <p style="margin-left: 40px;">Screenshot $(4 \times 0.75 + 3 \times 0.76 + 5 \times 0.77 + 6 \times 0.78 + 1 \times 0.$</p> <p>Or</p> <p style="margin-left: 40px;">Screenshot $3 \times 0.76 + 5 \times 0.77 + 6 \times 0.78 + 1 \times 0.79 + 1 \times 0.8$</p> <p>.2 $\frac{4 \times 0.75 + 3 \times 0.76 + 5 \times 0.77 + 6 \times 0.78 + 1 \times 0.79 + 1 \times 0.8}{4 + 3 + 5 + 6 + 1 + 1}$ ACCEPT $\frac{15.4}{20}$ seen .2 ACCEPT not seeing the whole operation from calculator screenshot Ex:</p> <p style="margin-left: 40px;">Screenshot $(4 \times 0.75 + 3 \times 0.76 + 5 \times 0.77 + 6 \times 0.78 + 1 \times 0.$</p> <p style="margin-left: 40px;">Screenshot $\frac{\text{ans}}{20} = 0.77$</p>	2

<p>7</p> <p>c</p> <p>.1 any two from</p> <ul style="list-style-type: none"> i. line within the zone ii. fairly passing through points Ex: at least two points above and two points below the line iii. line domain at least [5.5.10.5] <p>.2 the third from</p> <ul style="list-style-type: none"> i. line within the zone ii. fairly passing through points. Ex: at least two points above and two points below the line iii. line domain at least [5.5.10.5] <p>DO NOT award any marks for horizontal line</p> <p>DO NOT award any marks if they have more than one line drawn</p> <p>DO NOT award any marks if their line has positive gradient</p>	<p>The first graph shows a yellow shaded zone between two parallel lines. A red line passes through several points within this zone. The text 'Line must be in the zone indicated' is present.</p> <p>The second graph shows a red line passing through most of the data points, which are scattered around it. The text 'Fairly passing between points - at least two points above and below the line' is present.</p> <p>The third graph shows a horizontal blue line with a tick mark indicating its domain from 5.5 to 10.5. The text 'Line domain [5.5, 10.5]' is present.</p> <p>Ex:</p> <p>A scatter plot with 'Number of hours sleep (h)' on the x-axis (ranging from 0 to 12) and 'Mean reaction time in seconds (r)' on the y-axis (ranging from 0.0 to 1.2). Data points are plotted, and a red line of best fit shows a negative linear trend.</p>	<p>2</p>
<p>Examples acceptable for (2 marks)</p> <p>Three separate scatter plots, each with a red line of best fit. The first shows a line with a negative gradient passing through most points. The second shows a horizontal line. The third shows a line with a negative gradient passing through most points.</p>		

Further examples of different scenarios on the next page

7	c	<p>Examples acceptable for (1 mark)</p> <p>line within the zone and line domain at least [5.5,10.5], but the line is not fairly passing through points. Award 1 mark</p> <p>fairly passing through points and domain at least [5.5,10.5], but the line is not within the zone. Award 1 mark</p> <p>line is within the zone and fairly passing through points but domain not at least [5.5,10.5] Award 1 mark</p> <p>Examples not acceptable (0 marks)</p> <p>Line is not within the zone and domain is not [5.5,10.5] Award 0 marks</p> <p>more than one line drawn. Award 0 marks</p> <p>their line has positive gradient. Award 0 marks</p>
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7	d	.1 correct value of their r for $h = 4$.2 correct value of their r for $h = 7.5$.1 ACCEPT error ± 0.02 DO NOT ACCEPT if $h=4$ is not on their line .2 ACCEPT error ± 0.02 DO NOT ACCEPT if $h=7.5$ is not on their line	2
	e	.1 correctly substitute 0.77 into the formula .2 correct value of w .3 correctly round their w to 2 sf	.1 $(w =) 24(100)^{-0.77}$.2 0.692(1675608...) ACCEPT not seeing this step .3 their 0.69 OE	3

Question 7f 8 marks			
Mark	1	2	3
Factors (F)	The Two factors below identified explicitly Reaction time AND Sleeping time WTTE DO NOT ACCEPT factors embedded in working		
Calculate (C)	one correct w value from their r without working OR At least two incorrect w values from their r with working seen ACCEPT $w > 1$ ACCEPT w and its corresponding r value seen in the table or in the response box ACCEPT their rounding of w provided it correctly rounds to 1 d.p Ex: $w=0.6867$ and they write 0.68 DO NOT ACCEPT w for $r=0.77$	Two correct w values from their r without working ACCEPT $w > 1$ ACCEPT w and its corresponding r value seen in the table or in the response box ACCEPT their rounding of w provided it correctly rounds to 1 d.p Ex: $w=0.6867$ and they write 0.68 DO NOT ACCEPT w for $r=0.77$	Three correct w values from their r without working ACCEPT $w > 1$ ACCEPT w and its corresponding r value seen in the table or in the response box ACCEPT their rounding of w provided it correctly rounds to 1 d.p Ex: $w=0.6867$ and they write 0.68 DO NOT ACCEPT w for $r=0.77$
Comment (J)	Correct comment on the positive relationship between probability of winning and sleeping. Ex: WTTE When sleeping duration increases the probability of winning increases ACCEPT: -Good sleep increases probability of winning -Bad sleep decreases probability of winning -reaction time is better when sleeping well so probability of winning increases -sleeping 10h has probability 0.8, sleeping 4h has probability 0.2 and 0.8 is more than 0.2 -sleeping 10h has $w=0.8$ while sleeping 4h has $w=0.1$ only DO NOT ACCEPT : -comment involving only reaction time and sleeping. -sleeping 10h has probability 0.8 compared to sleeping 4h has probability 0.1	Additional correct comment realizing that probability of winning depends on more than sleeping. Ex: WTTE -Chance of winning will not just keep increasing when the number of hours of sleep increases. -Sleeping h hours does not automatically mean that the sprinter will win or will not win -Winning also depends on other factors ACCEPT Wining also depends on any of: talent or endurance or fitness or training or experience OE DO NOT ACCEPT J2 if J1 not awarded	

Mark	1	2	3
Justify degree of Accuracy (A)	<p>Weak justification inaccurate with weak justification Ex: -inaccurate since I used line of best fit that has approximations -inaccurate as values given are not exact -inaccurate as sample size small or only 20 -accurate, however I used rounding -accurate to a certain extent, as data given was not exact -using rounding decreased the accuracy</p> <p>OR Accurate with acceptable justification Ex: -accurate since numbers I used in my calculations are rounded 2 s.f. -the accuracy of my findings comes from rounding I used which is 2 s.f.</p> <p>ACCEPT accurate due to rounding only if they mention the degree of accuracy of their rounding</p> <p>DO NOT ACCEPT -accurate because I used my line of best fit or data given or graph OE -accurate because I used exact values or because I didn't round my results OE</p> <p>DO NOT ACCEPT just seeing their values rounded correctly</p>	<p>Inaccurate with good justification inaccurate AND state that the relation between reaction time and sleeping cannot be linear WTTE</p> <p>OR inaccurate AND state that the line equation may not be valid beyond the data given WTTE</p>	

Question	Answers	Notes	Total
8 a	correctly place 100 and 144		1
b	.1 correctly describe one pattern for V in words with correct terminology .2 correctly describe a second pattern for V in words with correct terminology	<p>ACCEPT complete terminology only, for example (below are different descriptions): DO NOT ACCEPT two from the same description</p> <ul style="list-style-type: none"> - The increase is increasing by a constant, the number you add increases constantly, the increase goes up by a constant, second difference is constant, the difference is in pattern 12, 20, 28..., V goes up by 12, 20, 28... - Quadratic - Square numbers, square of even numbers - Multiples of 4, divisible by 4 <p>DO NOT ACCEPT, for example: Arithmetic, increasing, increasing by a constant Even numbers, the square numbers, the multiples of 4</p> <p>DO NOT ACCEPT The rule in words, for example: 2 times n squared, n multiplied by 2 squared, double of n squared, twice stage number squared, the square of n times 2 and product of n with 2</p> <p>Note: More than two different patterns, all correct award (2 marks) Ex: multiples of 4, square numbers and it is 2 times n squared</p> <p>More than two different patterns, with any incorrect award (1 mark) Ex: multiples of 4, second difference is constant and it is 3 times n</p>	2
c	.1 the correct general rule .2 the correct simplified general rule with correct notation for V in terms of n	<p>.1 ($V =$) $4n^2$ or ($V =$) $4n^2$ or ($V =$) $4 \cdot n \cdot n$ or ($V =$) $(2 \times n)^2$ or $V = 4 \times n^2$ ACCEPT $V=4x^2$</p> <p>.2 $V = 4n^2$ or $V=(2n)^2$ ACCEPT $V_n = 4n^2$ or $V(n)=4n^2$ or use v for V</p> <p>DO NOT ACCEPT description in words</p> <p>SC for 1 mark if NR in 8c and correct general rule seen in 8b condone incorrect notation award 1 mark</p>	2

8	d	<p>.1 correctly substitute $n \geq 5$ into their general rule (from 8c or 8b)</p> <p>.2 correctly calculate their value of V after substituting $n \geq 5$</p> <p>.3 recognise that their correctly calculated value of V is the same as their predicted value</p> <p>.1 Ex: 4×5^2</p> <p>.2 Ex: 100 (for $n = 5$)</p> <p>.3 same as when candidate explains how the pattern continues Ex: how 100 is obtained by adding 36 to 64</p> <p>.3 ACCEPT seeing the value in the table in 8a and seeing their matching calculated V using $n \geq 5$ Ex: we find the candidate has 100 in the table for $n = 5$</p>	3
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Question 8e 22 marks

Mark	1	2	3	4
Predictions (P)	Correctly predict two terms for H or A ACCEPT whether in the table or in the response box.	Correctly predict at least two terms in H and one term in A ACCEPT whether in the table or in the response box.	Correctly predict at least two terms in H and two terms in A ACCEPT whether in the table or in the response box.	
Description (D)	<p>Correctly describe a pattern in words for A Ex for patterns: Multiples of 4 / Divisible by 4 It's a cubic sequence The first difference is quadratic The difference of the difference increases The second difference is linear (increasing by 24) The second difference is in arithmetic sequence The third difference is constant (24)</p> <p>DO NOT ACCEPT The multiples of 4, exponential sequence, the increase increases more, even numbers, V is square of H</p> <p>OR Attempt to describe a general rule for A in terms of n Ex: Rule in words Four times the cube of n Four n cubed Incorrect rule $A = 8n^3$, $A = (4n)^3$</p> <p>OR Correctly describe a general rule for A in terms of V or H Ex $A = V \times n$ or $A = V \times \frac{h}{2}$ or $A = 2H \times n^2$ ACCEPT non-simplified and ignore incorrect notation</p>	<p>Correctly describe a pattern in words for A AND Attempt to describe a general rule for A in terms of n or correctly describe a general rule for A in terms of V or H</p> <p>ACCEPT non-simplified and ignore incorrect notation</p>	<p>Correctly describe a general rule for A in terms of n</p> <p>Rule: $A = 4n^3$</p> <p>ACCEPT the rule is $4n^3$</p> <p>ACCEPT non-simplified and ignore incorrect notation</p> <p>Ex $\frac{1}{2} \times 4n^2 \times 2n$</p>	<p>Correctly describe a pattern in words for A AND Correctly describe a general rule for A in terms of n</p> <p>Rule: $A = 4n^3$</p> <p>ACCEPT the rule is $4n^3$</p> <p>ACCEPT non-simplified and ignore incorrect notation</p> <p>Ex $\frac{1}{2} \times 4n^2 \times 2n$</p>

Mark	1	2	3	4
Testing (T)	<p>Attempt to test their general rule for A using $n \leq 4$ Ex: Correctly substitute in their general rule value of $n \leq 4$</p> <p>OR</p> <p>Correctly test their described pattern or their rule (e.g. recursive rule)</p> <p>OR</p> <p>Correctly test their general rule for A in terms of V or H (that may include n)</p>	<p>Correctly test their general rule for A only in terms of n using $n \leq 4$ Ex: Correctly calculate their value for A in their general rule using $n \leq 4$</p> <p>AND</p> <p>Recognise that their correctly calculated value for A is the same as the given value.</p> <p>ACCEPT seeing their correctly calculated value for A and the given value in the table being equal</p>		
Verifying (V)	<p>Attempt to verify their general rule for A using $n \geq 5$ Ex: Correctly substitute in their general rule value of $n \geq 5$</p> <p>OR</p> <p>Correctly verify their described pattern or their rule (e.g. recursive rule)</p> <p>OR</p> <p>Correctly verify their general rule for A in terms of V or H (that may include n)</p>	<p>Correctly calculate their value for A in their general rule only in terms of n using $n \geq 5$</p>	<p>Correctly calculate their value for A in their general rule only in terms of n using $n \geq 5$</p> <p>AND</p> <p>Recognise that their correctly calculated value for A is the same as their predicted value obtained by continuing the pattern</p> <p>ACCEPT seeing their correctly calculated value for A and their predicted value in the table being equal</p>	

Mark	1	2	3	4
Justify (J)	<p>Attempt to justify their described pattern or their general rule Ex: Substitute at least two other values of n in A and say they are the same or the rule works</p> <p>OR Cubic model and valid attempt to find coefficients using any method</p> <p>OR Substitute at least two other values of V and H into area of rhombus formula and say they are the same or the rule works</p> <p>OR Substitute into area of rhombus formula at least one from $V=4n^2$ and $H=2n$ and simplify correctly Ex: $A = \frac{4n^2 \times H}{2} = 2n^2H$</p> <p>OR Spotting the general rule from the table seeing $A = (4n^2)n = 4n^3$</p> <p>DO NOT ACCEPT only saying ($A=$) Vn or ($A=$) $\frac{V \times H}{2}$</p>	<p>Justify their general rule arithmetically</p> <p>Cubic model and get correct values of coefficients using any method</p> <p>OR</p> <p>Compare values they obtain using the general rule with values they obtain using the area of rhombus or triangles formula</p> <p>OR</p> <p>Weak attempt to justify the general rule for A geometrically by using correct general rules for V and H in terms of n seeing $\frac{V \times H}{2} = \frac{4n^2 \times 2n}{2}$</p>	<p>Attempt to justify the general rule for A geometrically by using correct general rules for V and H in terms of n seeing $\frac{V \times H}{2} = \frac{4n^2 \times 2n}{2}$</p> <p>OR seeing $\frac{4n^2 \times 2n}{2} = 4n^3$</p> <p>ACCEPT all of the above in a correct description in words.</p> <p>ACCEPT the $4n^3$ as $n(4n^2)$ or $(2n)^2$</p>	<p>Correctly justify the general rule for A geometrically by using correct general rules for V and H in terms of n <u>and related to the Area of a rhombus</u></p> <p>seeing all of:</p> <ul style="list-style-type: none"> • <u>"area of rhombus"</u> WTTE • $\frac{V \times H}{2} = \frac{4n^2 \times 2n}{2} = 4n^3$ <p>ACCEPT all of the above in a correct description in words.</p> <p>ACCEPT the $4n^3$ as $n(4n^2)$ or $n(2n)^2$</p>

Communication criteria

Mark	1	2	3
Notation and terminology (N)	<p>Correct notation of <u>their general rule</u> Ex: $A = Vn$, $A = \frac{VH}{2}$, $A = 8n^3$</p> <p>OR The notation of <u>the general rule</u> includes errors, ex: $A = 4n^3$, $A = 4 * n^3$, $A = 4 \times n^3$ The rule for A is $4n^3$</p> <p>or non-simplified general rule $A = \frac{1}{2} \times 4n^2 \times 2n$</p> <p>OR Correctly describe at least one pattern in words for A</p> <p>DO NOT ACCEPT if they don't have any rules and they don't describe any patterns</p>	<p>Correct notation of <u>the general rule</u> for A $A = 4n^3$ ACCEPT $A_n = 4n^3$ or $A(n) = 4n^3$</p> <p>OR The notation of <u>the general rule</u> includes errors (see examples in N1) AND Correctly describe at least one pattern in words for A</p> <p>DO NOT ACCEPT if they don't have a general rule</p>	<p>Correct notation of <u>the general rule</u> for A AND Correctly describe at least one pattern in words for A</p>

Continued on next page

Mark	1	2	3
Communication (L) Organisation and coherence Can be awarded even there are errors in their descriptions and working.	<p>At least three from the following are seen:</p> <ul style="list-style-type: none"> • describe a pattern or rule in words • write a general rule • test their general rule or pattern • verify their general rule or pattern • justify their general rule or pattern 	<p>At least four of the following are seen:</p> <ul style="list-style-type: none"> • describe a pattern or rule in words • write a general rule • test their general rule or pattern • verify their general rule or pattern • justify their general rule or pattern <p>AND For coherence, they identify the processes correctly. At least one from the following:</p> <ul style="list-style-type: none"> • test • verify • justify <p>Ex:</p> <ul style="list-style-type: none"> • For test: they say "test" and they test using value(s) of $n \leq 4$ only • For verify: they say "verify" and they verify using value(s) of $n \geq 5$ only • For test and for verify: they say 'test and verify' and they test using value(s) of $n \leq 4$ and then verify using value(s) of $n \geq 5$ • For justify: they say "justify" or "my rule works because" WTTE and their justification is seen • For justify: they substitute at least two values of n and say "the rule justified" or "it works" WTTE • For justify: They justify <u>the general rule</u> for A geometrically 	<p>DO NOT ACCEPT if D3 and J2 not awarded</p> <p>At least four of the following are seen:</p> <ul style="list-style-type: none"> • describe a pattern or rule in words • write <u>the general rule</u> • test <u>the general rule</u> • verify <u>the general rule</u> • justify <u>the general rule</u> <p>AND For coherence, they identify the processes correctly. At least two from the following:</p> <ul style="list-style-type: none"> • test • verify • justify <p>Ex:</p> <ul style="list-style-type: none"> • For test: they say "test" and they test using value(s) of $n \leq 4$ only • For verify: they say "verify" and they verify using value(s) of $n \geq 5$ only • For test and for verify: they say 'test and verify' and they test using value(s) of $n \leq 4$ and then verify using value(s) of $n \geq 5$ • For justify: they say "justify" or "my rule works because" WTTE and their justification is seen • For justify: they substitute at least two values of n and say "the rule justified" or "it works" WTTE • For justify: They justify <u>the general rule</u> for A geometrically

n	Vertical length (V)	Horizontal length (H)	Area of rhombus (A)
1	4	2	4
2	16	4	32
3	36	6	108
4	64	8	256
5	100	10	500
6	144	12	864
7	196	14	1372
n	$4n^2$	$2n$	$4n^3$

Markscheme

November 2021

Extended mathematics

On-screen examination

25 pages

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- Bullet notation means award 1 mark – see example below

Example 1
.1 mark awarded and corresponding notes are aligned

b	<p>.1 Show clear line of reasoning in the method</p> <p>.2 4</p>	<p>.1 45 & 49 seen OE eg, $49 = 45 + x$</p> <p>.2 Accept $45 + X/10 = 4.9$ and Ans 4</p>	2
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Error Carried Forward (ECF) marks

Errors made at any step of a solution affect all working that follows. In general, **Error Carried Forward (ECF)** marks are awarded after an error.

- a) **ECF** applies from one part of a question to a subsequent part of the question and also applies within the same part.
- b) If an answer resulting from **ECF** is inappropriate (eg, negative distances or $\sin x > 1$) then subsequent marks should not be awarded.
- c) If a question is transformed by an error into a **simpler question** then **ECF** may not be fully awarded.
- d) To award **ECF** marks for a question part, **there must be working present for that part**.
- e) **ECF** is only applied to working which is correct. This means that all working subsequent to an error must be checked for accuracy.
- f) A misread (**MR**) is an error. **ECF** is normally awarded.

General points

- a) As this is an international examination, accept all alternative forms of **notation**, for example 1.9 and 1,9 or 1 000 or 1.000. However **DO NOT ACCEPT** incorrect mathematical notation e.g x^2 for x^2 unless noted otherwise in the MS.
- b) Accept notation errors in intermediate steps.
- c) Ignore further working after a correct answer **unless** it indicates a lack of mathematical understanding **i.e. if the further working contradicts the correct answer**, then the last mark cannot be awarded.
- d) In the case when a correct result is obtained using incorrect seen method, do not award the mark for the result.
- e) Where candidates have written two solutions to a question, mark the first solution.
- f) In the markscheme, equivalent examples of **numerical** and **algebraic** forms or **simplified** answers will generally be written in the notes preceded by **OE** (or equivalent) e.g. $\frac{1}{2}$ **OR** $1/2$ **OR** $1 \div 2$ and $\frac{x}{2}$ **OR** $x/2$ **OR** $x \div 2$
- g) In the markscheme, information provided in brackets indicate detail that may be seen in a candidate response but is not necessary to award the marks.
- h) Special case marks **SC** can be allocated instead of but not in addition to the marks prescribed in the markscheme.
- i) Accept seeing equation not in-line.
- j) Calculator screenshots are accepted as working steps. And when a calculator screenshot is taken, accept not seeing the whole operation.
- k) In task 2 and 3 where the markscheme is set out in a table then, unless noted otherwise, awarding the highest mark in a category includes all the lower marks in that category. It is probably best to look for the top category mark answer and if you don't find it look at the next mark down.
- l) **ACCEPT** using the correct values regardless their previous result

Question 1		Answers	Notes	Total
	a	<p>.1 add the correct vectors</p> <p>.2 correctly calculate both components</p>	<p>.1 $-\begin{pmatrix} 4 \\ -0.5 \end{pmatrix} + \begin{pmatrix} 6 \\ 3 \end{pmatrix}$ or $\begin{pmatrix} -4 \\ 0.5 \end{pmatrix} + \begin{pmatrix} 6 \\ 3 \end{pmatrix}$ or $\begin{pmatrix} 6 \\ 3 \end{pmatrix} - \begin{pmatrix} -4 \\ 0.5 \end{pmatrix}$; ACCEPT BA+AC or AC-AB.</p> <p>.2 $\begin{pmatrix} 2 \\ 3.5 \end{pmatrix}$</p> <p>.2 DO NOT ACCEPT if not in column vector form</p>	2
	b	<p>AM1</p> <p>.1 correctly double the vector AB</p> <p>.2 add their correct vectors</p> <p>.3 correctly calculate both components</p> <p>AM2</p> <p>.1 correctly double the vector AB OR route seen</p> <p>.2 correct horizontal component seen in column form</p> <p>.3 correct vertical component seen in column form</p>	<p>AM1</p> <p>.1 $\begin{pmatrix} 8 \\ -1 \end{pmatrix}$ seen</p> <p>.2 $\begin{pmatrix} 6 \\ 3 \end{pmatrix} - \text{their} \begin{pmatrix} 8 \\ -1 \end{pmatrix}$ or $\begin{pmatrix} 4 \\ -0.5 \end{pmatrix} + \text{their} \begin{pmatrix} 2 \\ 3.5 \end{pmatrix} - \text{their} \begin{pmatrix} 8 \\ -1 \end{pmatrix}$; ACCEPT AC+CD or AC-DC or AB+BC+CD or AC-2AB</p> <p>.3 $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$ ACCEPT $\begin{pmatrix} \text{their} -2 \\ \text{their} 4 \end{pmatrix}$ only if .2 is awarded</p> <p>.3 DO NOT ACCEPT if not in column vector form</p> <p>AM2</p> <p>.1 $\begin{pmatrix} 8 \\ -1 \end{pmatrix}$ seen OR AC+CD or AC-DC or AB+BC+CD or AC-2AB</p> <p>.2 $\begin{pmatrix} -2 \\ \text{their} 4 \end{pmatrix}$</p> <p>.3 $\begin{pmatrix} \text{their} -2 \\ 4 \end{pmatrix}$</p> <p>ACCEPT the working being not in column form in any AM</p>	3

	c	<p>.1 evidence of using the dot product between the correct vectors .2 correct operation for the dot product of their vectors .3 correctly calculate the dot product AG they are perpendicular</p>	<p>.1 $\begin{pmatrix} 6 \\ 3 \end{pmatrix} \bullet \text{their} \begin{pmatrix} -2 \\ 4 \end{pmatrix}$; ACCEPT $\mathbf{AC} \bullet \mathbf{AD}$.1 ACCEPT using x instead of \bullet or just not putting any sign between multiplied vectors .2 Their($6x - 2 + 3x 4$), ACCEPT their($-12 + 12$) .3 ($6x - 2 + 3x 4 = 0$) or ($-12 + 12 = 0$) (and hence perpendicular) .3 DO NOT ACCEPT if their dot product is not equal to 0</p>	3
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Question	Answers	Notes	Total
2	<p>AM 1 (using symbols implies the use of AM1)</p> <p>.1 correctly write one equation</p> <p>.2 correctly write the other equation</p> <p>.3 correctly step towards solving the two equations</p> <p>.4 correctly reduce to one equation in one unknown</p> <p>.5 correctly identify the x value being 30</p> <p>AM 2 (Using values instead of symbols wasn't used by any student so can be removed)</p> <p>.1 Seeing three numbers that fit the first equation</p> <p>.2 equate the correct operation with 160</p> <p>.3 seeing three numbers that fit the second equation</p> <p>.4 equate the correct operation with 70</p> <p>.5 correctly identify the x value being 30</p>	<p>AM 1</p> <p>.1 $t + 2c + b = 160$</p> <p>.2 $t + b - c = 70$</p> <p>.3 Attempt to subtract equations or add •¹ to double of •² equation OR substitute c in terms of t and b into the other equation OR double equation of .2</p> <p>.4 $3c = 90$ OE ACCEPT $3(t + b) = 300$</p> <p>.5 $c = 30$, DO NOT ACCEPT their c value</p> <p>AM 2</p> <p>.1 Ex: 100 and 10 and 40</p> <p>.2 Ex: $100 + 2 \times 10 + 40 = 160$</p> <p>.3 Ex: 90 and 40 and 60</p> <p>.4 Ex: $90 + 40 - 60 = 70$</p> <p>.5 $c = 30$, DO NOT ACCEPT their c value</p> <p>ACCEPT using the word "carton" or any symbol to represent it</p>	5

Question	Answers	Notes	Total
	Note: If their probability used is greater than 1 then do not award the bullet point		
3	a • $\frac{3}{5}$ or 0.6 OE		1
	b correctly write $\frac{3}{5}$ and $\frac{3}{10}$ and $\frac{2}{3}$ in the appropriate place		1
	c .1 multiply $\frac{2}{5}$ by their Not Lost .2 multiply their Not shortest by their Not Lost .3 correctly calculate their result after adding their multiplied fractions	.1 $\frac{2}{5} \times$ their $\frac{3}{10}$ or 0.12 OE .2 their $\frac{3}{5} \times$ their $\frac{2}{3}$.3 $(\frac{2}{5} \times$ their $\frac{3}{10} +$ their $\frac{3}{5} \times$ their $\frac{2}{3}) =$ their $\frac{13}{25}$ OE , DO NOT ACCEPT 0.5	3
d	.1 correctly probability of shortest and not lost explicitly seen OR divide a probability by their $\frac{13}{25}$.2 correctly calculate their result after dividing their correct probability of shortest and not lost by their $\frac{13}{25}$.1 $\frac{2}{5} \times$ their $\frac{3}{10}$ or 0.12 OE OR $\frac{\text{a probability}}{\text{their } \frac{13}{25}}$.2 their $\frac{3}{13}$ or their 0.23(07...) OE DO NOT ACCEPT their $\frac{3}{13}$ without working	2

	e	<p>.1 correct working for probability of Shortest and Not Lost OR Not shortest and lost</p> <p>.2 correctly place their probability of Shortest and Not Lost on the diagram</p> <p>.3 correctly place their probability of Not Shortest and Lost on the diagram</p>	<p>Shortest Lost</p> <p>$\frac{6}{15}$</p> <hr/> <p>.1 $\frac{2}{5} \times \text{their } \frac{3}{10}$ or $\text{their } \frac{13}{25} - \frac{6}{15}$ OR $\text{their } \frac{3}{5} \times \frac{1}{3}$</p> <p>.2 their $\frac{3}{25}$ OE correctly placed on the diagram</p> <p>.3 their $\frac{1}{5}$ OE correctly placed on the diagram</p> <p>.3 DO NOT award unless the total of probabilities is 1</p>	3
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Question		Answers	Notes	Total
4	a	.1 correctly substitute 10 and -2 .2 correctly use log laws to show that $k=-1$ $(k =)-1$ AG	.1 $-2 = k \log_3(10 - 1)$ ACCEPT $-2 = -1 \times \log_3(10 - 1)$.2 $\frac{1}{9} = 9^k$ or $-2 = 2k$ seen after correct steps ACCEPT $-2 = \log_3 9^k$	2
	b	.1 correct expression for the distance .2 correctly apply addition law for logs .3 correctly expand	.1 $\log_3(5x+7) - \log_3(x-1)$ ACCEPT $f(x) - g(x)$ or $ g(x)-f(x) $.2 ($d =) \log_3(5x+7)(x-1)$.3 ($d =) \log_3(5x^2+2x-7)$	3
	c	AM1 .1 correctly substitute 4 into their $\log_3(5x^2+2x-7)$.2 correctly calculate their result after using log AM2 .1 correctly calculate $f(4)$ and $g(4)$.2 correctly determine 4 as the vertical distance between the two y-coordinates	AM1 .1 ($d =) \log_3(5x^2+2x-7)$ OE .2 their 4 ACCEPT their 4 only if their $\log_3(5x^2+2x-7)$ includes log base 3 of a quadratic AM2 (ACCEPTED even though it is not hence because this is a determine question but note that .2 has to be 4 not their 4) .1 $f(4)=3$ and $g(4)=-1$.2 4 DO NOT ACCEPT 4 without working	2
	d	.1 correctly eliminate the log .2 correctly substitute their coefficients into quadratic formula .3 correctly calculate their positive root .4 correctly write their positive answer only to 1dp	.1 1 = their $5x^2+2x-7$ DO NOT ACCEPT if their $\log_3(5x^2+2x-7)$ doesn't include log .2 their $\frac{-2 \pm \sqrt{2^2 - 4(5)(-8)}}{2(5)}$ ACCEPT even if their $\log_3(5x^2+2x-7)$ doesn't include log .3 ($x =) \text{their } 1.0806\dots$.4 ($x =) \text{their } 1.1$ DO NOT ACCEPT if their $1.0806\dots$ is a whole number	4

Question	Answers	Notes	Total
5 a	.1 correctly substitute in distance formula OR add distances .2 correct distance before rounding 1170 AG	.1 $(D =) 500 \times \frac{140}{60}$ OE OR $500 + 500 + \frac{20}{60} \times 500$ OE .1 ACCEPT $(D =) 500 \times 2.3(\dots)$.1 ACCEPT (speed =) $\frac{1170}{(140 / 60)}$ or $\frac{1170}{2.3(\dots)}$.2 1166.66 ACCEPT [1165,1167]	2
b	.1 correctly divide 5760 by 900 .2 correctly convert their time to minutes	.1 6.4 .2 $(\text{their } 6.4 \times 60) = \text{their } 384$	2
c	AM1 using distance as unknown .1 correctly write one time in terms of x (distance) .2 correctly write the other time in terms of x (distance) .3 correctly calculate the value of x (distance) .4 correctly calculate the time in hours .5 correctly write their time after 7:00 AM2 using time as unknown .1 correctly write one distance expression .2 correctly write the other distance expression .3 equate the correct expressions .4 correctly calculate the time in hours .5 correctly write their time after 7:00 AM3 (trial and improvement) .1 correctly calculate time for plane from Seoul to Tokyo or Tokyo to Seoul until the same value of x .2 correctly calculate more times for plane from Seoul to Tokyo until same values of x .3 correctly reach the value of x (distance) .4 correctly calculate the time in hours .5 correctly write their time after 7:00	AM1 .1 $x/500$ or $(1170-x)/436$ ACCEPT a number/500 or a number/ 436 .2 $x/500$ and $(1170-x)/436$.3 $(x =) 625$.4 $(625/500 =) 1.25$ OE .5 their 8 :15 AM2 .1 $(d =) 436t$ or $(d =) 500t$ ACCEPT $436x$ a number or $500x$ a number .2 $1170 - 500t$ or $1170 - 436t$.3 $436t = 1170 - 500t$ or $500t = 1170 - 436t$.4 $(t =) 1.25$ OE .5 their 8 :15 AM3 (trial and improvement) .1 Ex: time at $x= 585$ or at 170 and 1000 ACCEPT only if both distances sum is 1170 .2 ACCEPT averaging the times they obtain .3 $(x=)625$.4 $(t =) 1.25$ OE .5 their 8 :15	5

	d	.1 evidence of correct gradient .2 correctly substitute (80,2400) into $h(x) = 60x + c$.3 correct $h(x)$.1 60 seen as gradient .2 $2400=60(80)+c$.3 $h(x) = 60x-2400$	3
	e	.1 correctly substitute 700 into $q(x)$.2 correct $p(x)$.3 correct deduction after subtracting their 11580 from 12000	.1 $(q(x)=) -40(700) +39580$.2 $(p(x)=) 11580$.3 (Safe) because their 420 > 300. WTTE .3 ACCEPT their 420 being the result of any calculation they make DO NOT ACCEPT their 420 if less than 300	3

Question		Answers	Notes	Total
6	a	.1 correctly place two inequalities .2 correctly place the third inequality and region.	<p>DO ACCEPT ECF for their region from their constraints DO NOT ACCEPT their region placed in-between regions</p>	2
	b	.1 select 10 and 20 .2 correctly substitute their 10 and their 20 .3 correctly calculate their maximum weight.	.1 10 and 20 seen .2 their 10x(30)+their 20x(44) .2 ACCEPT their 10 and their 20 only if they are whole numbers or they are (10.17 , 20.34) .3 Their 1180 (lb) ACCEPT their 1180 only if less than 1200	3
	c	AM1 .1 evidence of substituting into the cosine rule .2 correctly substitute into the cosine rule .3 correctly calculate their BC^2 from cosine rule .4 correctly calculate their BC after square root	AM1 .1 Ex: substitute incorrectly into cosine rule .2 $(BC^2 =) 1^2 + 1.5^2 - 2 \times 1 \times 1.5 \times \cos 120$ OE .3 $(BC^2 =) \text{their } 4.75$.4 their [2.179..., 2.2]	4

	d	correctly add their BC to 1.5	Their [3.6794...,3.7]	1
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e	Mark	1	2	3	4	
	Identify factors (F)	<p>Explicitly state two factors from:</p> <ul style="list-style-type: none"> - Length of route - Number of days for the trip - Availability of food and water - Amount of goods they are able to carry for trading - Terrain features (mountain or crossing river,..etc) <p>Ignore additional irrelevant factors DO NOT ACCEPT factors embedded in working</p>	Explicitly state three factors			
	Calculations (L)	<p>Correct two values 7 days for $4 \leq D < 8$</p> <p>Modal class ACCEPT $8 < D < 12$ or 8 to 12 or 8-12</p> <p>ACCEPT median [9,10[</p> <p>Estimate mean =8.769.. ACCEPT 8.8</p> <p>Total number of days = 26</p> <p>Estimate for total distance travelled = 228</p> <p>In all, allow ecf from their number of days for $4 \leq D < 8$</p>	Correct three values	Correct four values	Correct six values	10

	Comaprison (C)	<p>Compare statistical values: Correctly compare at least two statistical values Example: Mean and number of days and total distance are less in Cimarron route MUST compare using a word like less, more, on the other hand, while,..etc OR State at least three statistical values for each route without explicit word for comparison</p> <p>OR</p> <p>Correctly compare nature of the two routes Example: comparing the roughness of the two routes OR Realise that the objective of the journey is to trade and the mountain route allows more space for trading goods</p>	<p>Compare statistical values: Correctly compare at least two statistical values Example: Mean and number of days and total distance are less in Cimarron route MUST compare using a word like less, more, on the other hand, while,..etc OR State at least three statistical values for each route without explicit word for comparison</p> <p>AND</p> <p>Correctly compare nature of the two routes Example: comparing the roughness of the two routes OR Realise that the objective of the journey is to trade and the mountain route allows more space for trading goods</p>		
	Justify accuracy (A)	<p>inaccurate with weak justification</p> <p>Concerning the maths Inaccurate because rounding used OR these are approximate calculations and not accurate OR mean and median are just estimates</p> <p>OR</p>	<p>inaccurate with good justification</p> <p>Concerning the maths The mean and total distance travelled are just estimates <u>since we are using mid-interval class.</u> OR The <u>use of mid-class</u> in calculations makes it an estimate</p> <p>OR</p>		

			Concerning the context Anything related to hazards or things unaccounted for that families may face OR because families cannot report exact distances every day OR we cannot know for sure how they measure their distances travelled DO NOT ACCEPT: my results are accurate with any reason WTTE	Concerning the maths: mean and median are just estimates AND Concerning the context Anything related to hazards or things unaccounted for that families may face OR because families cannot report exact distances every day OR we cannot know for sure how they measure their distances travelled			
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Distance travelled (D) in miles		Number of days (N)		Measures of central tendency for the distance travelled by Family Fry			Total number of days	Estimate for the total distance travelled
Modal class	Estimate for the median	Estimate for the mean						
0 ≤ D < 4	3							
4 ≤ D < 8	7							
8 ≤ D < 12	12							
12 ≤ D < 16	3							
16 ≤ D < 20	1							

Mountain Route

Measures of central tendency for the distance travelled by Family Kane			Total number of days	Estimate for the total distance travelled
Modal class	Estimate for the median	Estimate for the mean		
12 ≤ D < 16	13.5	12.87	39	502

Question	Answers		Notes	Total														
7	a	correctly place 68π and 76π OE	<table border="1"> <thead> <tr> <th>Ring (n)</th> <th>Area of the ring (R)</th> </tr> </thead> <tbody> <tr><td>1</td><td>36π</td></tr> <tr><td>2</td><td>44π</td></tr> <tr><td>3</td><td>52π</td></tr> <tr><td>4</td><td>60π</td></tr> <tr><td>5</td><td>68π</td></tr> <tr><td>6</td><td>76π</td></tr> </tbody> </table>	Ring (n)	Area of the ring (R)	1	36π	2	44π	3	52π	4	60π	5	68π	6	76π	1
Ring (n)	Area of the ring (R)																	
1	36π																	
2	44π																	
3	52π																	
4	60π																	
5	68π																	
6	76π																	
	b	.1 correctly describe one pattern for A only in words with acceptable terminology	<p>Examples of suitable patterns and acceptable terminology: Increases by 8π WTTE</p> <p>DO NOT ACCEPT Increasing by 8 Multiples of 8 even numbers</p>	1														
	c	.1 the correct general rule .2 the correct simplified general rule with correct notation	<ul style="list-style-type: none"> •¹ ($R = 28\pi + 8n\pi$) ACCEPT ($R = 28\pi + 8 \times n\pi$) ACCEPT $R = 28 + 8n$ Accept one incorrect coefficient e.g. ($A = 48\pi + 8n\pi$) •² $R = 28\pi + 8n\pi$ ACCEPT $R = 36\pi + (n - 1)8\pi$ DO NOT ACCEPT description in words DO NOT ACCEPT incorrect notation on its own 	2														
	d	.1 correctly substitute $n \geq 5$ into their general rule .2 correctly calculate their value of C after substituting $n \geq 5$.3 recognize that their correctly calculated value of C is the same as their predicted value	<p>.1 Ex : $28\pi + 8 \times 5\pi$</p> <p>.2 68π (for $n = 5$)</p> <p>.3 Same as value I predicted in table (and we find the candidate has 68π in the table for $n = 5$) OR same as when we continue the pattern and explains how 68π is obtained from pattern of adding 8π to 60π</p>	3														

	e	<p>.1 Set equation for area ring = 6x area pink tile + 12 x area blue tile OR 24 seen in denominator</p> <p>.2 divide 36π by 24</p> <p>AG $\frac{3}{2}\pi$</p>	<p>.1 $36\pi = 6(2x) + 12(x)$ or $36\pi = 24x$ OE ACCEPT not seeing this step</p> <p>.2 $\frac{36\pi}{24}$</p> <p>ACCEPT using decimals provided they write $\frac{3}{2}\pi = 4.71(2....)$</p>	2
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7	f	QIG8			22	
Mark		1	2	3	4	5
Predictions (P)		<p>Correctly predict three terms for P or A</p> <p>ACCEPT whether in the table or in the response box</p> <p>ACCEPT typing errors like seeing $5/3\pi$ or using pi instead of π or missing the pi</p>	<p>Correctly predict four terms for P and A</p> <p>ACCEPT whether in the table or in the response box</p> <p>ACCEPT typing errors like seeing $5/3\pi$ or using pi instead of π or missing the pi</p>			
Description (D)		<p>Attempt to describe a pattern in words for A</p> <p>Ex: numerators and denominator are even numbers The up number increases by 8 or 8π The lower/bottom number increases by 6 The denominator increases by 6π The denominator is the number of pink tiles Linear sequence for numerator DO NOT ACCEPT A is linear</p> <p>OR</p> <p>Attempt to describe a rule in words for A</p> <p>OR</p> <p>Attempt to describe a pattern in words for A and</p>	<p>Attempt to describe pattern for A as general rule</p> <p>Ex: Correct general rule for numerator ($8n+10$) Or Correct general rule for denominator ($6n$)</p> <p>OR</p> <p>One correct pattern described in words for A</p> <p>Ex: Numerator increases by 8 or 8π Denominator increases by 6</p> <p>OR</p> <p>Attempt to describe a pattern in words for A and correct rule for A not in terms of n only Ex: $A=(R-12B)/P$</p>	<p>Correctly describe the pattern for A as a general rule</p> <p>Rule: $A = \frac{8n+10}{6n} \pi$ OE</p> <p>ACCEPT if the π is missing and penalize in notation</p> <p>ACCEPT rule for numerator=$8n+10$ and rule for denominator $6n$ and penalize in notation</p> <p>OR</p> <p>Two correct patterns described in words for A (one for numerator and the other for denominator)</p> <p>OR</p> <p>Attempt to describe pattern for A as general rule AND two correct patterns described in words for A (one for numerator and the other for denominator)</p>	<p>Correctly describe the pattern for A as a general rule AND one correct pattern described in words for A</p> <p>ACCEPT if the π is missing and penalize in notation</p> <p>ACCEPT rule for numerator=$8n+10$ and rule for denominator $6n$ and penalize in notation</p> <p>OR</p> <p>Attempt to describe pattern for A as general rule AND two correct patterns described in words for A (one for numerator and the other for denominator)</p>	

	<p>correct rule for A not in terms of n only Ex: $A=(R-12B)/P$</p> <p>OR</p> <p>A correct pattern described as general rule for P</p> <p>DO NOT ACCEPT A is increasing DO NOT ACCEPT any description for P in words</p>		<p>Correct general rule for numerator AND Two attempts to describe pattern in words OR</p> <p>Attempt to describe pattern for A as general rule AND one correct pattern described in words for A</p>		
Testing (T)	<p>Attempt to test their general rule for A using $n \leq 4$</p> <p>Ex: Substitute in their general rule value of $n \leq 4$</p> <p>OR</p> <p>Correctly test their described pattern or their rule (e.g. recursive rule)</p>	<p>Correctly test their general rule for A only in terms of n using $n \leq 4$</p> <p>Ex: Correctly calculate their value for A in their general rule using $n \leq 4$</p> <p>AND</p> <p>Recognise that their correctly calculated value for A is the same as the given value.</p> <p>ACCEPT seeing their correctly calculated value for A and the given value in the table being equal</p>			
Verifying (V)	<p>Attempt to verify their general rule for A using $n \geq 5$</p> <p>Ex:</p>	<p>Correctly calculate their value for A in their general rule only in terms of n using $n \geq 5$</p>	<p>Correctly calculate their value for A in their general rule only in terms of n using $n \geq 5$</p> <p>AND</p> <p>Recognise that their correctly calculated value</p>		

	<p>substitute in their general rule value of $n \geq 5$</p> <p>OR</p> <p>Correctly verify their described pattern or their rule (e.g. recursive rule)</p>		<p>for A is the same as their predicted value obtained by continuing the pattern</p> <p>ACCEPT seeing their correctly calculated value for A and their predicted value in the table being equal</p>		
Justify/proof (J)	<p>Attempt to justify any of their described patterns or their general rule</p> <p>Ex: Attempt to use the arithmetic sequence OR Substitute at least two other values of n in A and say they are the same or the rule works (regardless the π)</p>	<p>Justify their general rule correctly</p> <p>Ex: Use the arithmetic sequence for the numerator to show the rule.</p> <p>OR</p> <p>Weak attempt at geometrical justification</p> <p>Ex: Attempt to equate <u>their R rule</u> (area of ring) to the sum of <u>their areas</u> of pink and blue tiles</p> <p>Justify the general rule of A geometrically not in terms of n only</p>	<p>Attempt to justify geometrically the general rule</p> <p>Ex: Attempt to equate the correct R rule (area of ring) to the sum of correct areas of blue and pink tiles</p> $8\pi n + 28\pi = 12\left(\frac{3}{2}\pi\right) + 6n \times A$ <p>OE</p> <p>ACCEPT with our without π</p>	<p>Correctly justify geometrically the general rule</p> <p>Ex: Show that the correct general rule for A is equal The correct R rule – area of blue tiles and divide by the correct rule of number of pink tiles</p> $\frac{8\pi n + 28\pi - 12\left(\frac{3}{2}\pi\right)}{6n}$ <p>DO NOT ACCEPT without π</p>	
Notation and terminology (N)	<p>Correct notation of <u>their</u> general rule</p> <p>Ex: rule for numerator $A = (4n + 5)\pi$</p>	<p>Correct notation of <u>the</u> <u>general</u> rule for A</p> $A = \frac{(8n + 10)\pi}{6n} \text{ OE}$	<p>Correct notation of <u>the</u> <u>general</u> rule for A</p> <p>AND</p>		

	<p>OR The notation of <u>the general rule</u> includes errors Ex: the rule is $\frac{8n+10}{6n}$ or $A = \frac{4x+5}{3x} \pi$ or $\frac{8n+10}{6n} \pi$ or The rule for numerator is 8n+10 and rule for denominator is 6n $A_n = \frac{18 + (n-1)8}{6 + (n-1)6} \pi$</p> <p>OR Correctly describe a pattern in words for A</p> <p>DO NOT ACCEPT if they don't have any rules and they don't describe any patterns</p>	<p>OR The notation of <u>the general rule</u> includes errors AND Correctly describe a pattern in words for A</p> <p>DO NOT ACCEPT if they don't have a rule</p>	Correctly describe a pattern in words for A		
Communication (L) can be awarded even there are errors in their descriptions and working	<p>At least three from the following are seen:</p> <ul style="list-style-type: none"> - describe a pattern or rule in words - write a general rule - test their general rule or pattern - verify their general rule or pattern - justify their general rule or pattern 	<p>At least four of the following are seen:</p> <ul style="list-style-type: none"> - describe a pattern or rule in words - write a general rule - test their general rule or pattern - verify their general rule or pattern - justify their general rule or pattern <p>AND</p>	<p>DO NOT ACCEPT if D3 and J2 not awarded</p> <p>At least four of the following are seen:</p> <ul style="list-style-type: none"> - describe a pattern or rule in words - write <u>the general rule</u> - test <u>the general rule</u> - verify <u>the general rule</u> - justify <u>the general rule</u> <p>AND For coherence, they identify the processes</p>		

		<p>For coherence, they identify the processes correctly. At least one from the following:</p> <ul style="list-style-type: none"> - test - verify - justify <p>Ex: -For test: they say "test" and they test using value(s) of $n \leq 4$ only</p> <p>-For verify: they say "verify" and they verify using value(s) of $n \geq 5$ only</p> <p>-For test and for verify: they say 'test and verify' and they test using value(s) of $n \leq 4$ and then verify using value(s) of $n \geq 5$</p> <p>-For justify: they say "justify" or "my rule works because" WTTE and their justification is seen</p> <p>-For justify: they substitute at least two values of n and say "the rule justified" or "it works" WTTE</p> <p>- For justify:</p>	<p>correctly. At least two from the following:</p> <ul style="list-style-type: none"> - test - verify - justify <p>Ex: -For test: they say "test" and they test using value(s) of $n \leq 4$ only</p> <p>-For verify: they say "verify" and they verify using value(s) of $n \geq 5$ only</p> <p>-For test and for verify: they say 'test and verify' and they test using value(s) of $n \leq 4$ and then verify using value(s) of $n \geq 5$</p> <p>-For justify: they say "justify" or "my rule works because" WTTE and their justification is seen</p> <p>- For justify: They assume quadratic model (or 2nd diff 8 OE) and get values of coefficient(s) using any method</p> <p>- For justify: They justify <u>the general rule</u> for A geometrically</p>	
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		<p>They assume quadratic model (or 2nd diff 8 OE) and get values of coefficient(s) using any method</p> <p>- For justify: They justify <u>the general rule</u> for A geometrically</p>		
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Ring (n)	Area of the ring (R)	Number of pink tiles (P)	Area of a pink tile (A)
1	36π	6	$\frac{18}{6}\pi$
2	44π	12	$\frac{26}{12}\pi$
3	52π	18	$\frac{34}{18}\pi$
4	60π	24	$\frac{42}{24}\pi$
5	68π	30	$\frac{50}{30}\pi$ or $\frac{5}{3}\pi$ OE
6	76π	36	$\frac{58}{36}\pi$ or $\frac{29}{18}\pi$ OE

Markscheme

November 2020

Extended mathematics

On-screen examination

23 pages

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The markscheme may make use of the following abbreviations:

ECF Marks that can be awarded as **error carried forward** from previous results in the question

BOD Benefit of the doubt

MR misread

NWS no working shown

SC special case

OE or equivalent

WTTE or words to that effect

AG Answer given

- Bullet notation means award 1 mark – see example 1 below

Example

•¹ mark awarded and corresponding notes are aligned

b	• ¹ Show clear line of reasoning in the method • ² their correct result	• ¹ 45 & 49 seen OE eg, $49 = 45 + x$ • ² their 4	2
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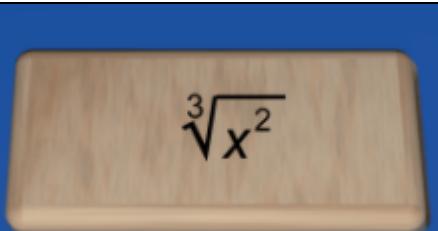
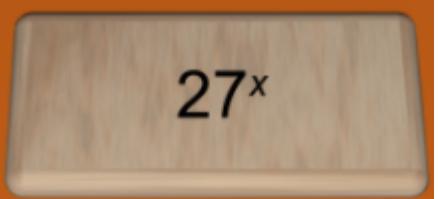
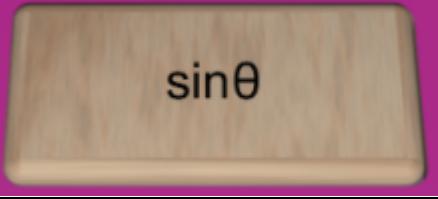
Error Carried Forward (ECF) marks

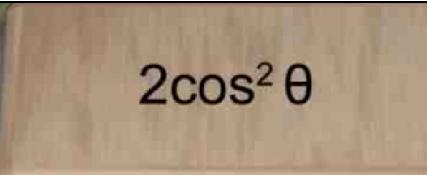
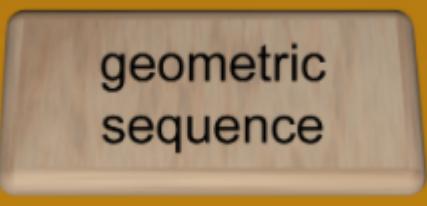
Errors made at any step of a solution affect all working that follows. In general, **Error Carried Forward (ECF)** marks are awarded after an error.

- a) **ECF** applies from one part of a question to a subsequent part of the question and also applies within the same part.
- b) If an answer resulting from **ECF** is inappropriate (eg, negative distances or $\sin x > 1$) then subsequent marks should not be awarded.
- c) If a question is transformed by an error into a **simpler question** then **ECF** may not be fully awarded.
- d) To award **ECF** marks for a question part, **there must be working present for that part**.
- e) **ECF** is only applied to working which is correct. This means that all working subsequent to an error must be checked for accuracy.
- f) A misread (**MR**) is an error. **ECF** is normally awarded.

General points

- a) As this is an international examination, accept all alternative forms of **notation**, for example 1.9 and 1,9 or 1 000 or 1.000. However, **DO NOT ACCEPT** incorrect mathematical notation e.g x^2 for x^2 in final answers unless noted otherwise in the MS.
- b) Accept notation errors in intermediate steps.
- c) Ignore further working after a correct answer **unless** it indicates a lack of mathematical understanding **i.e. if the further working contradicts the correct answer**, then the last mark cannot be awarded.
- d) In the case when a correct result is obtained by coincidence based on incorrect seen method, do not award the mark for the result.
- e) Where candidates have written two solutions to a question, mark the response that deserves more marks.
- f) In the markscheme, equivalent examples of **numerical** and **algebraic** forms or **simplified** answers will generally be written in the notes preceded by **OE** or equivalent e.g. $\frac{1}{2}$ or $1/2$ or $1 \div 2$ and $\frac{x}{2}$ or $x / 2$ or $x \div 2$
- g) In the markscheme, information provided in brackets indicate detail that may be seen in a candidate response but is not necessary to award the marks.
- h) Special case marks **SC** can be allocated instead of but not in addition to the marks prescribed in the markscheme..
- i) When a calculator screenshot is taken, accept not seeing the whole operation.
- j) Accept seeing an equation not in-line

Question	Answers		Notes	Total
1 a				1
b				1
c				1
d				1
e				1

	f				1
	g				1

Question		Answers	Notes	Total
2	a	$2\ln(x - 6)$ OE		1
	b	<ul style="list-style-type: none"> •1 ($s =$) 2 •2 ($t =$) 6 		2
	c	<ul style="list-style-type: none"> •1 correctly eliminate ln •2 correctly expand their quadratic •3 correctly factorise their quadratic or substitute into quadratic formula •4 correctly select their single result after factorisation or using quadratic formula 	<ul style="list-style-type: none"> •1 $(x - 6)^2 = x$ •2 their $x^2 - 12x + 36 = x$ or their $x^2 - 13x + 36 = 0$ •3 their $(x - 4)(x - 9)$ •4 their $(x =) 9$ only, Accept only if their $x > 6$ 	4

Standard (9 marks)

Question		Answers	Notes	Total
3	a	<ul style="list-style-type: none"> •1 seeing 40(%) •2 multiply their 40 % by 25 % AG 0.1 	<ul style="list-style-type: none"> •1 0.4, ACCEPT 40(%) on the diagram •2 0.4×0.25 OE 	2
	b	Correctly write 3 as the answer		1
	c	<ul style="list-style-type: none"> •1 correctly place their 3 and their 27 in 'First donor' •2 correct probabilities for their O- second branches •3 correct probabilities for their Not O- second branches 	<p style="text-align: center;">First donor</p> <p style="text-align: right;">3</p> <p>.1 ACCEPT their 3 only if positive integer $0 < \text{their3} < 30$.2 or .3 ACCEPT only $0 < \text{their probability} < 1$</p>	3

	d	<ul style="list-style-type: none"> •1 correctly calculate the probability first O- and second O- from their tree diagram •2 correctly calculate the probability first not O- and second O- from their tree diagram •3 correctly substitute their values into the correct conditional probability formula •4 correctly calculate their probability after applying the formula 	<ul style="list-style-type: none"> •1 $\frac{\text{their3}}{30} \times \frac{\text{their2}}{\text{their29}}$ or $\frac{1}{145}$ OE •2 $\frac{\text{their27}}{30} \times \frac{\text{their3}}{\text{their29}}$ or $\frac{27}{290}$ OE •3 $\frac{\text{their } \bullet 2}{\text{their } \bullet 1 + \text{their } \bullet 2}$ OE •4 $\frac{\text{their27}}{\text{their29}}$ OE 	4
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Question	Answers	Notes	Total
4 a	<ul style="list-style-type: none"> •1 correctly write 5 hours 15 mins as 5.25 (h) •2 correctly calculate their volume in cubic metres •3 correctly set their equation of v •4 correctly calculate their r^2 •5 correctly calculate their r before rounding AG 4 m 	<ul style="list-style-type: none"> •1 ACCEPT not seeing this step •2 5.25×11.2 or 58.8 or $58800/1000$ seen •3 $\pi r^2 \times 1.17$ = their 58.8 •4 $\frac{58.8}{\pi \times 1.17}$ or 15.997(...) or 15.779(....) •5 3.99(...) or 3.97(...) Accept only if their answer rounds to 4 For .4 and .5 DO NOT ACCEPT substituting $r = 4$ and calculating volume 	5
b	<ul style="list-style-type: none"> •1 correctly substitute 4 into the correct area of circle formula •2 correctly substitute 4 into the correct circumference of a circle formula •3 multiply their circumference of circle by 0.3 •4 correctly add their •1 and •3 •5 correctly round their .4 to the nearest square metre 	<ul style="list-style-type: none"> •1 $\pi \times 16$ or $\pi \times 4^2$ or 50.27 or 50.24 (using 3.14) •2 $\pi \times 8$ or $2 \times \pi \times 4$ or 25.1(327...) •3 their $\pi \times 8 \times 0.3$ or their $2 \times \pi \times 4 \times 0.3$ or 7.5(398...) •4 Their 57.8(05...) •5 their 58 ACCEPT earlier all correct rounding 	5
c	correctly multiply their 57.8(05...) or 58 by 3.4	<p>Their (\$196.52 or (\$197.2</p> <p>ACCEPT their 196.52 without working DO NOT ACCEPT their 57.8(05...) x 3.4 incorrectly rounded in c)</p>	1

Question	Answers	Notes	Total
5 a	<ul style="list-style-type: none"> •1 correctly determine 1.95 •2 correct trig ratio used •3 correctly write their answer before rounding or correct inverse trig ratio AG 70° 	<ul style="list-style-type: none"> •1 $12.55 - 10.6$ •2 $\cos y = \frac{1.95}{5.7}$ accept not seeing this step •3 $y = \cos^{-1} \frac{1.95}{5.7}$ or $69.99(480991)$ Accept only if their answer rounds to 70 	3
b	<p>AM1</p> <ul style="list-style-type: none"> •1 correctly convert 70 from degrees to radians •2 subtract double of y from π <p>AM2</p> <ul style="list-style-type: none"> •1 subtract double of y from 180 •2 correctly convert 40 from degrees to radians <p>AG $\frac{2}{9}\pi$</p>	<p>AM1</p> <ul style="list-style-type: none"> •1 $70 \times \frac{\pi}{180}$ or $\frac{7\pi}{18}$ seen •2 $\pi - 2 \times \frac{7\pi}{18}$ <p>AM2</p> <ul style="list-style-type: none"> •1 $180 - 2 \times 70$ or 40 seen . ACCEPT seeing 40 degrees on the canvas in part (a) •2 $40 \times \frac{\pi}{180}$ 	2
c	<p>AM1</p> <ul style="list-style-type: none"> •1 correct trig ratio used with 10.6 OR correctly substitute into sine rule •2 correctly rearrange their trig ratio for v •3 correctly calculate their v after dividing by their trig ratio 	<p>AM1</p> <ul style="list-style-type: none"> •1 $\cos \frac{7\pi}{18} = \frac{10.6}{v}$ or $\sin \frac{\pi}{9} = \frac{10.6}{v}$ OE OR $\frac{v}{\sin \frac{7\pi}{18}} = \frac{21.2}{\sin \frac{2\pi}{9}}$ OE <p>ACCEPT using degrees: $\cos 70 = \frac{10.6}{v}$ OE or $\sin 20 = \frac{10.6}{v}$ or</p> $\frac{v}{\sin 70} = \frac{21.2}{\sin 40} \quad \text{OE}$ <ul style="list-style-type: none"> •1 ACCEPT 10.6 and $\cos \frac{7\pi}{18}$ or $\sin \frac{\pi}{9}$ seen •2 their ($v = \frac{10.6}{\cos 70}$) or their ($v = \frac{10.6}{\sin 20}$) or their ($v = \frac{21.2 \sin 70}{\sin 40}$) OE 	3

	<p>AM2</p> <ul style="list-style-type: none"> •1 correct trig ratio used with 12.55 OR correctly substitute into sine rule •2 correctly calculate their hypotenuse •3 correctly calculate their v after subtracting 5.7 from their calculated hypotenuse <p>AM3</p> <ul style="list-style-type: none"> •1 seeing evidence of using similar triangles correctly •2 correctly rearrange for v OR correctly apply their scale factor •3 correctly calculate their v after dividing their product 	<ul style="list-style-type: none"> •3 their 30.99(23....) or 31 or 31.2(11..) ACCEPT only if .1 or .2 is awarded <p>AM2</p> <ul style="list-style-type: none"> •1 $\cos \frac{7\pi}{18}$ or $\sin \frac{\pi}{9} = \frac{12.55}{hyp}$ OR $\frac{a}{\sin \frac{7\pi}{18}} = \frac{25.1}{\sin \frac{2\pi}{9}}$ OE •2 their 36.69(37...) ACCEPT (hyp=) $\frac{12.55}{\cos 70}$ OE •3 their 30.99(23....) or 31 or 31.2(11..) ACCEPT only if .1 or .2 is awarded <p>AM3</p> <ul style="list-style-type: none"> •1 Seeing $\frac{12.55}{1.95}$ or $\frac{10.6}{12.55}$ OE or $\frac{v}{10.6} = \frac{v+5.7}{12.55}$ OE •2 $10.6 \times \frac{5.7}{1.95}$ OR $1.95v = 60.42$ •3 their 30.99(23....) or 31 or 31.2(11..) ACCEPT only if .1 or .2 is awarded
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d	<p>AM1</p> <ul style="list-style-type: none"> •1 correctly substitute $\frac{2}{9}\pi$ and their v or their v+5.7 into correct sector formula •2 subtract their areas of sectors •3 correctly calculate their area after subtracting their areas of sectors <p>AM 2</p> <ul style="list-style-type: none"> •1 correctly substitute their v or their v+5.7 into correct area of circle formula •2 subtract their areas of circles or sectors •3 correctly calculate their result after multiplying their difference of areas by their 40/360 or x/(2 π) 	<p>AM1</p> <ul style="list-style-type: none"> •1 $(A =) \frac{1}{2} \text{their } 36.7^2 \times \frac{2}{9}\pi$ or $(A =) \frac{1}{2} \text{their } 31^2 \times \frac{2}{9}\pi$ •2 $(A =) \frac{1}{2} \text{their } 36.7^2 \times \frac{2}{9}\pi - \frac{1}{2} \text{their } 31^2 \times \frac{2}{9}\pi$ •3 their 134.6(...) or 134.7(...) or 135 <p>AM 2</p> <ul style="list-style-type: none"> •1 $\pi \times \text{their } 36.7^2$ or $\pi \times \text{their } 31^2$ seen •2 $\pi \times \text{their } 36.7^2 - \pi \times \text{their } 31^2$ OE •3 their 134.6(...) or 134.7(...) or 135 	3
e	<ul style="list-style-type: none"> •1 correctly calculate the material wasted •2 correctly write their fraction of material wasted •3 correctly calculate their percentage <p>AM1</p> <ul style="list-style-type: none"> •1 correctly write their fraction of material used •2 Subtract their fraction from 1 •3 correctly calculate their percentage 	<ul style="list-style-type: none"> •1 $(25.1 \times 7.6 - \text{their } 134.6 =) \text{their } 56.16$ •2 $\frac{\text{their } 56.16}{\text{their } 25.1 \times 7.6}$ •3 their 29(....) (%) <p>AM1</p> <ul style="list-style-type: none"> •1 $\frac{\text{their } 134.6}{\text{their } 25.1 \times 7.6}$ •2 $1 - \text{their } 0.705(...)$ •3 their 29(....) (%) 	3

Question	Answers	Notes	Total														
6 a	<ul style="list-style-type: none"> •1 correctly drag three heights •2 correctly drag the other three heights 	<table border="1"> <caption>Data for Histogram</caption> <thead> <tr> <th>Annual exposure / (E)</th> <th>Number of European countries</th> </tr> </thead> <tbody> <tr><td>0 - 10</td><td>3</td></tr> <tr><td>10 - 20</td><td>17</td></tr> <tr><td>20 - 30</td><td>6</td></tr> <tr><td>30 - 40</td><td>9</td></tr> <tr><td>40 - 50</td><td>3</td></tr> <tr><td>50 - 60</td><td>1</td></tr> </tbody> </table> <p>SC: All six bars correct and thinner award 1 mark</p>	Annual exposure / (E)	Number of European countries	0 - 10	3	10 - 20	17	20 - 30	6	30 - 40	9	40 - 50	3	50 - 60	1	2
Annual exposure / (E)	Number of European countries																
0 - 10	3																
10 - 20	17																
20 - 30	6																
30 - 40	9																
40 - 50	3																
50 - 60	1																
b	A valid statement including both 1990 and 2016	<p>DO NOT ACCEPT a comment on only one interval Ex: in 1990, 3 countries had E between 0 and 10 while in 2016 there are 8 Ex: Highest in 1990 is 17 while highest in 2016 is 24</p> <p>ACCEPT Overall correct comment ex: Less particulates or less exposure in the air in 2016 than 1990 OE Comments including more than one interval. Ex: In 2016 no countries had E more than 40 but in 1990 there were</p>	1														
c	<ul style="list-style-type: none"> •1 seeing two correct midpoints and two correct frequencies •2 Add their four products of midpoints and frequencies •3 Divide the sum of their products by 39 •4 Correctly write their mean before rounding AG 15.3 ($\mu\text{g}/\text{m}^3$) 	<ul style="list-style-type: none"> •1 Any two of 5, 15, 25, 35 AND any two of 8, 24, 5, 2 seen •2 $5 \times 8 + 15 \times 24 + 25 \times 5 + 35 \times 2$ OR 595 •3 <i>their</i> $\frac{595}{39}$ •4 15.25(6....). ACCEPT only if their answer rounds to 15.3 	4														
d	<ul style="list-style-type: none"> •1 $(a =) -1/5$ OE •2 $(b =) 18$ 	<ul style="list-style-type: none"> •1 ACCEPT $-0.21 < a < -0.14$ •2 ACCEPT correctly calculated b from their correct a 	2														

6	e	Mark	1	2	3	
		F Identification of Factors	<p>One factor mentioned from: The model/equation used to predict. or Whether the data will follow same trend or not or Acknowledge the E is decreasing or The fact that these are models based on scatter plots. ACCEPT factors affecting the trend like more data in the last 5 years collected or using electric cars or new technology that affects the environment or using solar power instead of petrol or air pollution or control of particular matter DO NOT ACCEPT only saying many factors</p>			
		PL (Prediction from line)	<p>Attempt to calculate the E in 2030 using their line equation Ex: Substitute 40 in the equation and make incorrect calculation OR Substitute 2030 into the equation and make correct calculation OR Attempt to use the drop every 5 years to predict the drop in 2030 from the graph OR Describe the trend in words and predict it will reach below 13</p>	<p>Correctly calculate the E in 2030 using their line equation Ex: their $(-0.15(40) + 18 =)12$ OR correctly use the drop every 5 years to predict the drop in 2030 from the graph</p>		10

	PE (Prediction from Exponential)	<p>Attempt to calculate the E in 2030 using the exponential equation</p> <p>Ex: Substitute 40 in the equation $(6 \times 0.91^{40} + 13.8)$ and make incorrect calculation</p> <p>OR</p> <p>Substitute 2030 into the equation and make correct calculation</p> <p>OR</p> <p>Attempt to use the drop every 5 years to predict the drop in 2030 from the graph</p> <p>OR</p> <p>Describe the trend in words and predict it will not reach 13</p>	<p>Correctly calculate the E in 2030 using the exponential equation $(6 \times 0.91^{40} + 13.8 =) 13.9(379)$, accept 14</p> <p>OR</p> <p>use the drop every 5 years to predict the drop in 2030 from the graph</p> <p>ACCEPT if they mention that the exponential will never reach 13 and refer to horizontal asymptote being E=13.8</p>		
	D degree of accuracy	<p>Inaccurate with weak reason (Interpolating)</p> <p>Example: inaccurate because I used line of best fit</p> <p>Inaccurate because of sudden increase of pollution happening</p> <p>OR</p> <p>Accurate with valid reason</p> <p>Example: accurate because I used my line of best fit to estimate and made correct calculations</p> <p>OR</p> <p>Rounding to a whole number used for their estimated year</p> <p>Ex: 14 for the E from exponential DO NOT ACCEPT if they just write down a year without any reference or calculations</p> <p>DO NOT ACCEPT accurate or inaccurate without reason</p>	<p>Inaccurate with a valid reason related to variables affecting the future (extrapolating)</p> <p>Ex:</p> <p>The prediction I made not very accurate because many factors may vary in the future</p> <p>or</p> <p>predictions using line of best fit for the future not guaranteed</p> <p>or</p> <p>The prediction using the line equation not very accurate because it is taking only a window or isolated time</p> <p>or</p> <p>The prediction using the line equation not very accurate because it assumes the future follows same pattern</p> <p>or</p> <p>My predictions not very accurate because a disaster may happen and affect pollution</p> <p>DO NOT ACCEPT if they did not make a prediction</p>		

		J Justifying the model	<p>Selecting the Exponential model (even implicitly ex: substituting only in exponential model) without justification OR Select the line with acceptable justification Ex: it passes through more points OR comparison comment without selection Ex: Line will decrease in a short time while exponential will take longer OE</p>	<p>Selecting the Exponential model (even implicitly) with general justification (no explicit data) Ex: I advise using graph 2 because it takes most of the points into account or Line will decrease in a short time while exponential will take longer or exponential more fitting</p>	<p>Selecting the Exponential model (even implicitly) With good justification involving data in the last years Ex: graph 2 is better because in the last years, the E did not vary much or it is nearly constant in the last years and it takes this into account while graph 1 doesn't or the decrease in the line is constant while E is not is not and the exponential does not have constant decrease rate</p>	
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Answers			Notes	Total
7 a	<p>AM1</p> <ul style="list-style-type: none"> •1 evidence of using Pythagoras correctly for diagonal of small square •2 correctly rearrange their equation to have y^2 on one side •3 square root the diagonal <p>AM2</p> <ul style="list-style-type: none"> •1 evidence of using Pythagoras correctly in the right angled triangle •2 correctly rearrange their equation to have x^2 on one side •3 doubling the value of the side of triangle <p>AM3</p> <ul style="list-style-type: none"> •1 evidence of using $\sin 45$ or $\cos 45$ •2 correctly rearrange their ratio to have unknown on one side •3 doubling the value of side of triangle <p>AG $3\sqrt{2}$</p>	<p>AM1</p> <ul style="list-style-type: none"> •1 $3^2 + 3^2 = y^2$ •2 $18 = y^2$ •3 $\sqrt{18}$ <p>AM2</p> <ul style="list-style-type: none"> •1 $x^2 + x^2 = 3^2$ •2 $x^2 = \frac{9}{2}$ •3 $2 \times \frac{3}{\sqrt{2}}$ <p>AM3</p> <ul style="list-style-type: none"> •1 $\sin 45 = \frac{x}{3}$ or $\cos 45 = \frac{x}{3}$ or $x\sqrt{2} = 3$ OE •2 $(x =) 3\sin 45$ or $(x =) 3\cos 45$ or $(x =) \frac{3}{\sqrt{2}}$ OE •3 $2 \times 3\sin 45$ or $2 \times 3\cos 45$ or $(x =) \frac{2 \times 3}{\sqrt{2}}$ OE <p>DO NOT AWARD •3 unless their result simplifies to $3\sqrt{2}$</p> <p>SC Using $(\frac{3\sqrt{2}}{2})^2 + (\frac{3\sqrt{2}}{2})^2 = 9$ and $\sqrt{9} = 3$ award 2 marks.</p>	3	
b	•1 correctly place 144 •2 correctly place 288			2

c	<ul style="list-style-type: none"> •1 correctly describe one pattern in words •2 correctly describe a second pattern in words <p>Allow ECF from their table in 8b</p>	<p>ACCEPT Multiply by 2 every time ...WTTE All even numbers except first (9) or after stage 1 The ratio is constant It is geometric Multiplies of 9 Starting from 9, every second number is a square number ACCEPT if they correctly describe in words their pattern</p> <p>DO NOT ACCEPT It is the square of L $A = L^2$ It is increasing All A are even numbers general rules in terms of n, example: $A = 9 \times 2^{n-1}$</p>	2
d	<ul style="list-style-type: none"> •1 The correct general rule •2 The correct general rule with correct notation 	<ul style="list-style-type: none"> •1 $9 \times 2^{n-1}$ OE •2 $A = 9 \times 2^{n-1}$ OE 	2
e	<ul style="list-style-type: none"> •1 correctly substitute $n \geq 5$ into their rule •2 correctly calculate their value of P after substitution $n \geq 5$ •3 recognize that their result is the same as their predicted value 	<ul style="list-style-type: none"> •1 Ex: 9×2^4 •2 Ex: 144 (for the $n = 5$) •3 Same as value I predicted in table (and we find the candidate has 144 in the table for $n = 5$) OR same as when we continue the pattern and explains how 144 is obtained from pattern of multiplying 72 by 2 •3 ACCEPT seeing the 144 in the table and seeing their calculated $A=144$ when $n = 5$ <p>SC for 1 mark Correctly test by applying the steps of verification mentioned in the left column with a value of $n \leq 4$</p> <p>SC for 1 mark Correctly verify their described pattern or rule (e.g. recursive rule)</p>	3

7	f	Mark	1	2	3	4	
		Predictions (P)	Correctly predict one value for P	Correctly predict two values for P			
		Description (D)	<p>Attempt to describe a pattern in words or to write a rule Ex: number beside $\sqrt{2}$ multiplies by 2 every other time $\sqrt{2}$ is there every other time OR Correctly describe one pattern in words for L OR Correctly describe in words their pattern for P OR Attempt to write down a general rule for L, example: $(\sqrt{2})^n$</p> <p>OR Weak attempt to write down a general rule for P, example $(\sqrt{2})^{n-1}$</p> <p>DO NOT ACCEPT L or P is increasing n goes up by 1</p>	<p>Correctly describe one pattern in words for P Ex: P multiplies by $\sqrt{2}$ P is a geometric sequence P is exponential function DO NOT ACCEPT the general rule in words OR Correctly write down general rule for L in terms of n. Rule: $(L =) 3 \times (\sqrt{2})^{n-1}$ OR Acceptable attempt to write down a general rule for P, example: $12(\sqrt{2})^n$ or recursive rule</p> <p>For notation see N</p>	<p>Correctly describe one pattern in words for P AND Acceptable attempt to write down a general rule for P OR Correctly write down general rule for P in terms of n. $12(\sqrt{2})^{n-1}$OE OR Correctly describe one pattern in words for P AND correctly write down general rule for L</p> <p>For notation see N</p>	<p>Correctly describe one pattern in words for P AND Correctly write down the general rule for P in terms of n</p> <p>For notation see N</p>	
		Testing (T)	<p>Attempt to test their general rule for P using $n \leq 4$ Ex: correctly substitute in their general rule value of $n \leq 4$ Ex:</p>	<p>Correctly test their general rule for P using $n \leq 4$ Ex: Correctly calculate their value for P in their general</p>			20

		<p>substitute in their general rule value of $n \leq 4$</p> <p>OR</p> <p>Correctly test their described pattern or their rule (e.g. recursive rule)</p>	<p>rule using $n \leq 4$ AND Recognise that their correctly calculated value for P is the same as the given value.</p> <p>ACCEPT seeing their correctly calculated value for P and the given value in the table being equal</p>		
	Verifying (V)	<p>Attempt to verify their general rule for P using $n \geq 5$ Ex: correctly substitute in their general rule value of $n \geq 5$</p> <p>OR</p> <p>Correctly verify their described pattern or their rule (e.g. recursive rule)</p>	<p>Correctly calculate their value of P using their $n \geq 5$ in their general rule</p>	<p>Correctly calculate their value for P in their general rule using $n \geq 5$ AND Recognise that their correctly calculated value for P is the same as their predicted value obtained by continuing the pattern</p> <p>ACCEPT seeing their correctly calculated value for P and their predicted value in the table being equal</p>	
	Justify/proof (J)	<p>Weak attempt to justify their described pattern or their general rule</p> <p>Examples: trying at least two more values and arguing as justification that they are the same or rule works OR Just seeing their rule as $12 \times (\sqrt{2})^{n-1}$ without any explanation OR Assuming geometric sequence and valid attempt to find U1 and r</p>	<p>Good attempt to justify their general rule for P</p> <p>Examples: Recognise it is a geometric sequence and correctly write values of U1 and r OR multiplying by 4 incorrect rule for L</p>	<p>correctly justify the correct general rule for P in relation to geometry</p> <p>Examples: Writing in words OE that multiplying side by 4 gets perimeter if their L rule is $3 \times (\sqrt{2})^{n-1}$ and their general rule is $12 \times (\sqrt{2})^{n-1}$</p> <p>J3 automatically gains T2 and V3</p>	

		DO NOT ACCEPT if D2 not achieved				
	Notation and terminology (N)	<p>Correct notation of <u>their</u> rule OR Correct terminology describing a pattern</p> <p>DO NOT ACCEPT if they don't have any rules and they don't describe any patterns</p>	<p>Correct notation of <u>the general</u> rule for P OR The notation of <u>the general</u> rule includes errors AND Correct terminology describing pattern in words for P</p> <p>DO NOT ACCEPT if they don't have a general rule</p>	<p>Correct notation of <u>the general</u> rule for P AND Correct terminology describing pattern in words for P</p> <p>ACCEPT using U_n instead of P only if they mention that $P = U_n$</p> <p>For notation of the general rule, DO NOT ACCEPT Ex: $12(\sqrt{2})^{(n-1)}$ Or the rule is: $12(\sqrt{2})^{n-1}$ Or non-simplified rules</p>		
	Communication (L)	<p>Very weak communication</p> <p>More than two lines of communication that lacks organisation and coherence OR Only calculations or algebraic steps</p>	<p>Weak communication</p> <p>At least three lines of communication with an attempt of organisation but lacks coherence</p>	<p>Good communication</p> <p>More than three lines of communication with acceptable organisation and coherence</p> <p>Can be awarded only if J2 is achieved</p>		

Stage number (n)	Side length (L)	Perimeter of square (P)
1	3	12
2	$3\sqrt{2}$	$12\sqrt{2}$
3	6	24
4	$6\sqrt{2}$	$24\sqrt{2}$
5	12	48
6	$12\sqrt{2}$	$48\sqrt{2}$
7	24	96
8		

General rules:

$$L = 3 \times (\sqrt{2})^{n-1} \text{ or } 3 \times 2^{\frac{n-1}{2}}$$

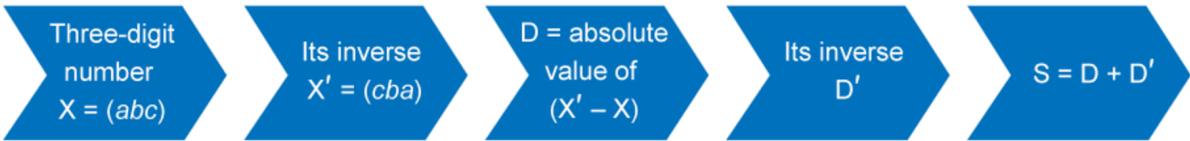
$$P = 12 \times (\sqrt{2})^{n-1} \text{ or } 3 \times 2^{\frac{n+3}{2}}$$

Question 1 (7 marks)

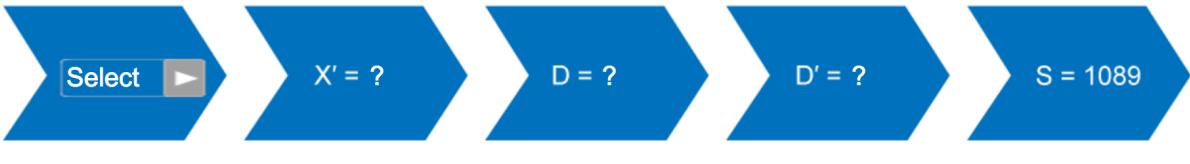
Question 1a (3 marks)

In this question, we will discover an interesting and magical property of three-digit numbers using a special algorithm, illustrated in the algorithm flow diagram below.

Algorithm flow diagram



Here is a simulator for the algorithm flow diagram which provides some examples of how this algorithm affects three-digit numbers. Select a number and see what happens.



Reset

Apply the same algorithm to 437 to complete the missing values.



Question 1b (2 marks)

A three-digit number can be written in terms of sum of multiples of its digits. For example, 437 can be written in the format shown below.



X is a three-digit number abc . Write down X and X' as a sum of multiples of their digits.

$X =$

$X' =$



Question 1c (2 marks)

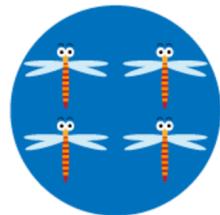
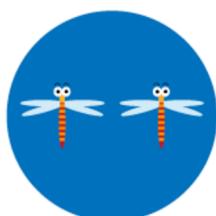
Using your answer from part (b), **determine** the difference D in terms of a and c .

B I | ← → | U x_a x^a $\frac{!}{!}$ Σ Styles





Question 2 (6 marks)



©

Find the value of X for the following expression.



B I ← → U x₂ x² $\frac{1}{x}$ $\frac{d}{dx}$ Ω Σ Styles

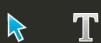
**Question 3 (8 marks)****Question 3a (4 marks)**

To raise money for their graduation party, senior students organize a cookie and muffin sale. x represents the number of cookies and y represents the number of muffins. The amount of cookies and muffins sold are represented by the shaded region in the diagram below.

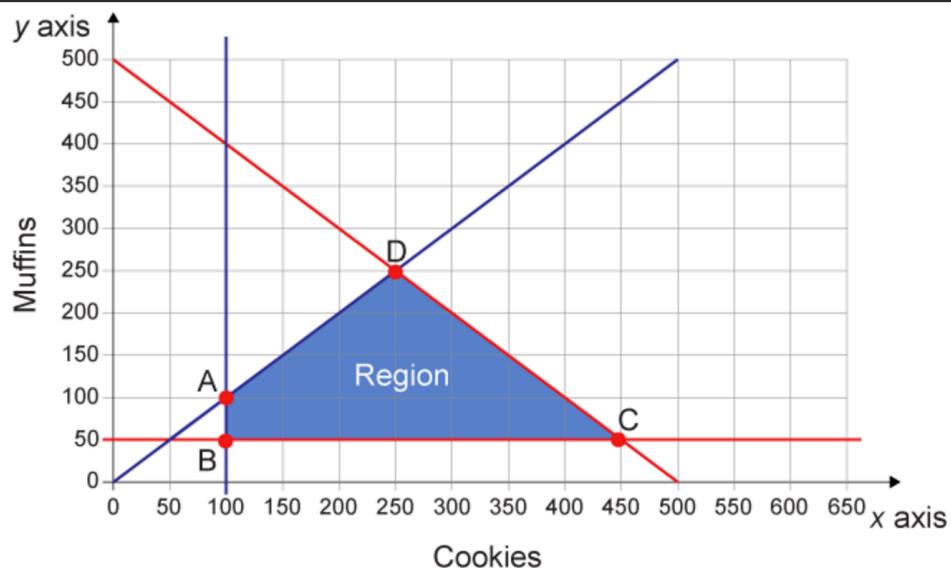
Using the information provided in the diagram below:

- **Identify** the shaded region by completing the inequalities below.
- The first constraint is that they must sell 100 cookies or more. **State** the other three constraints in the spaces provided.

Draggable inequalities	Inequalities	Constraints
\geq	$x \geq 100$	The number of cookies is at least 100
\leq	$y \leq 50$	
	$y \leq x$	
	$x + y \leq 500$	

Diagram

T



**Question 3b (1 mark)**

The profit P , in Canadian dollars (CAD), for the cookies and muffins sale is calculated using the formula $P = x + 1.5y$.

Interpret the meaning of the coefficients of x and y in the profit formula above.

**Question 3c (3 marks)**

The table below shows the profit at vertex A and vertex B.

Vertices (x, y)	$P = x + 1.5y$ (CAD)
A (100, 100)	250
B (100, 50)	175
C	
D	

Find the number of cookies and the number of muffins that the students must sell in order to maximize their profit.

**Question 4 (6 marks)****Question 4a (2 marks)**

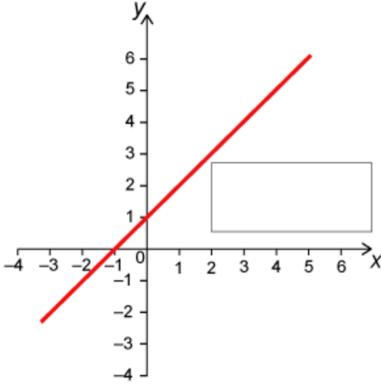
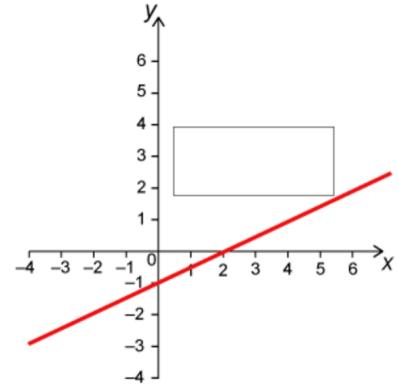
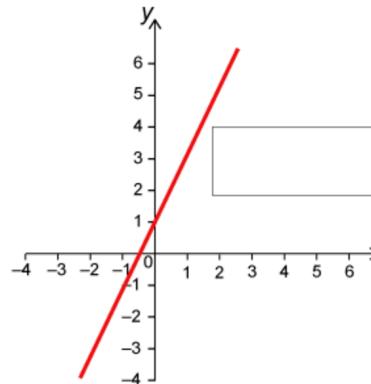
Select the line equations and place them with the corresponding graphs.

Draggable:

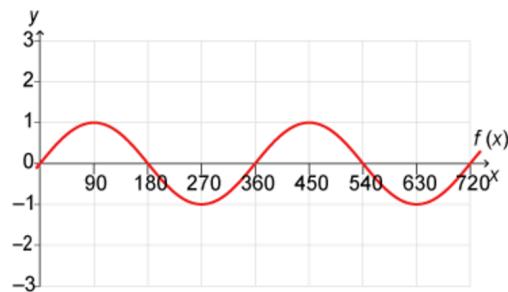
$y = x + 1$

$y = 2x + 1$

$y = \frac{1}{2}x - 1$

**Question 4b (4 marks)**

The graph below represents the function $f(x)$. Transformations of $f(x)$ are shown in the following graphs.



Select the equations and place them with the corresponding graphs.

Key :

$f(x)$

Transformation of $f(x)$

Draggable:

$$f(x) + 2$$

$$f(x) - 2$$

$$2f(x)$$

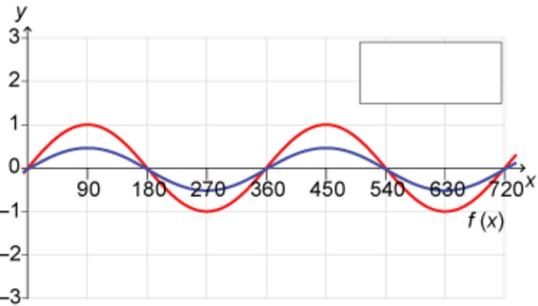
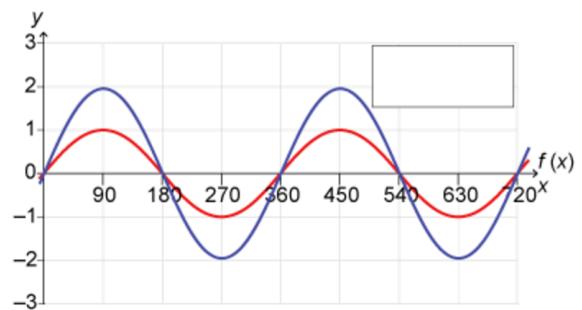
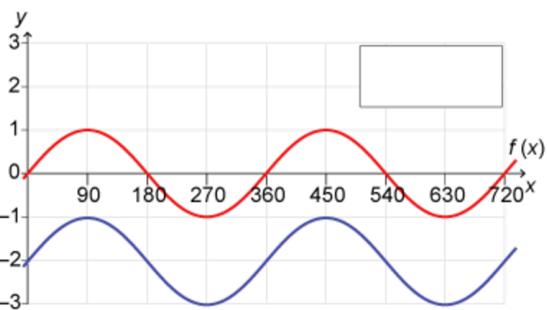
$$-f(x)$$

$$\frac{1}{2}f(x)$$

$$f(-x)$$

$$f(2x)$$

$$f\left(\frac{1}{2}x\right)$$





Question 5 (8 marks)



The animation illustrates the effect of the river current (\vec{C}) on the resultant direction (\vec{R}) of the boat.

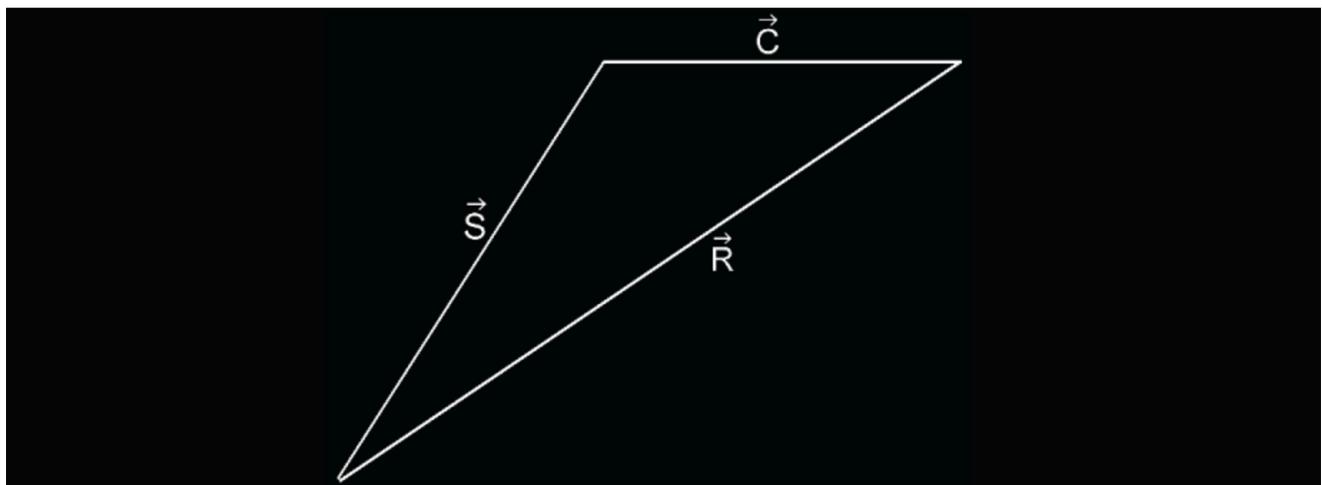
The boat must adjust its sailing direction (\vec{S}) in order to arrive at its destination.

This media contains no audio



A boat sails with velocity vector $\vec{S} = \begin{pmatrix} 5 \\ 12 \end{pmatrix}$, which is given in component form.

Diagram not to scale





Question 5a (2 marks)

The speed is the magnitude of the velocity vector. **Determine** the speed of the boat.

B I | ← → U x_e x^e = Ω Σ Styles



Question 5b (1 mark)

A current flows with velocity vector $\vec{C} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$, **Show that** the resultant velocity vector \vec{R} is $\begin{pmatrix} 8 \\ 12 \end{pmatrix}$ in component form.

B I | ← → U x_e x^e = Ω Σ Styles



Question 5c (5 marks)

Find the angle between the resultant vector \vec{R} and the vector \vec{S} .

B I | ← → U x_e x^e = Ω Σ Styles

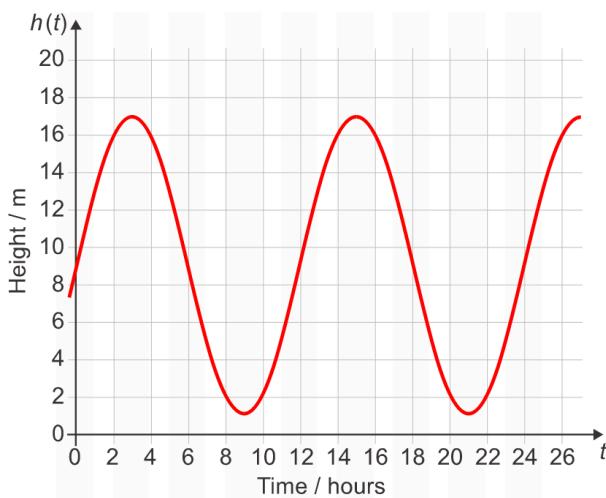
**Question 6 (10 marks)**

The following video illustrates how tidal range can be modelled over time by a sine function.



Below is the sine curve modelling the tide in Saint-Malo on a day in November 2017.

$h(t)$ is the height in metres (m) of water in the harbour and t is the number of hours after midnight.

**Question 6a (2 marks)**

Determine the tidal range which is the difference between the height of the low and high tides in the harbour.



Question 6c (6 marks)

The behaviour of this tide can be modelled by the function,

$$h(t) = 8\sin\left(\frac{\pi}{6}t\right) + 9$$

By substituting into the equation,

$$h(t) = 8\sin\left(\frac{\pi}{6}t\right) + 9,$$

calculate at what times, during the first 12 hours after midnight, the height of water in the harbour is 13 m.



Question 6b (2 marks)

The period can be measured as the time difference between two consecutive high tides. **Determine** the period of this tide.

B I | ← → U x_a x^a 1= :≡ Ω Σ
Styles

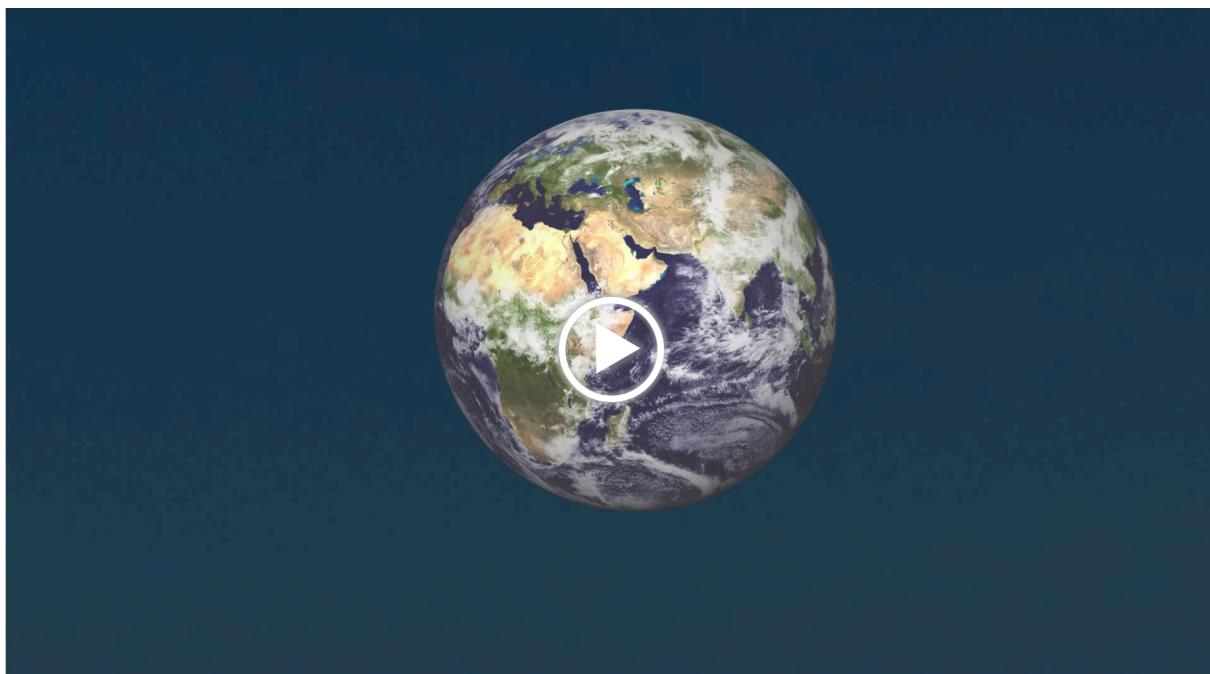
B I | ← → U x_a x^a 1= :≡ Ω Σ
Styles



Question 7 (23 marks)



The following video shows how mathematics can be used to make predictions for population growth over time and space.



Tab 1

Tab 2

Tab 1 Tab 2

The table below shows the age distribution of the population of Australia (percentage to nearest 1 %, data correct as at 2015).

Age (A)	Percentage
$0 < A \leq 20$	23
$20 < A \leq 40$	32
$40 < A \leq 60$	27
$60 < A \leq 80$	17
$80 < A \leq 100$	1

The table below shows the country of birth of immigrants to Australia (data correct as at 2015, top ten countries only).

Country of birth	Number of immigrants
United Kingdom	1 207 000
New Zealand	611 400
China	481 800
India	432 700
Philippines	236 400
Vietnam	230 200
Italy	198 200
South Africa	178 700
Malaysia	156 500
Germany	125 900

**Question 7a (1 mark)**

A person was selected at random from the population of Australia in 2015. Using the data in Tab 1, **show that** the probability of this person being over 60 years old is 0.18.

Calculator tools:

B I | ← → U x₂ x² $\frac{1}{x}$ $x^{\frac{1}{2}}$ Ω Σ
Styles

Large empty text area for working space.

**Question 7b (2 marks)**

The total population of Australia in 2015 was estimated to be 23 858 000 to the nearest 1000. Using the data in Tab 2, **show that** 5 % of the Australian population were immigrants from the United Kingdom (UK), to the nearest percent.

Calculator tools:

B I | ← → U x₂ x² $\frac{1}{x}$ $x^{\frac{1}{2}}$ Ω Σ
Styles

Large empty text area for working space.

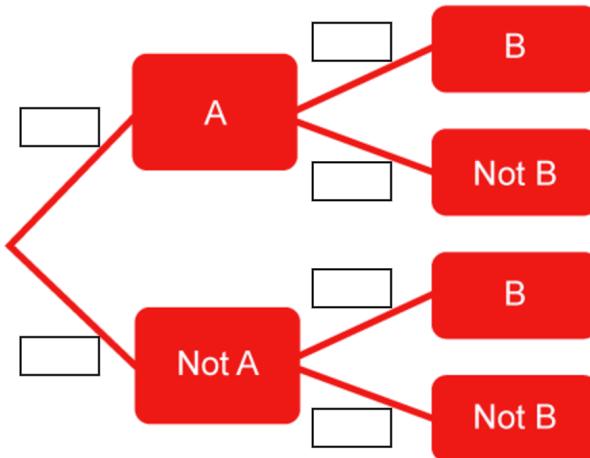
**Question 7c (2 marks)**

A person was selected at random from the Australian population in 2015.

Event A represents: The person is an immigrant from the UK

Event B represents: The person is over 60 years old.

Write down the missing probabilities in the tree diagram below to the nearest 2 decimal places.



**Question 7d (3 marks)**

A person was selected at random from the population of Australia in 2015.

Using your tree diagram, **find** the probability that this person was over 60 years old.

(This is a text input field for the answer to Question 7d.)

**Question 7e (2 marks)**

Given that this person was over 60 years old, **determine** the probability that the person was an immigrant from the UK.

(This is a text input field for the answer to Question 7e.)

**Question 7f (1 mark)**

Using parts (b) and (e), or otherwise, **state** the type of relationship between events A and B.

(This is a text input field for the answer to Question 7f.)

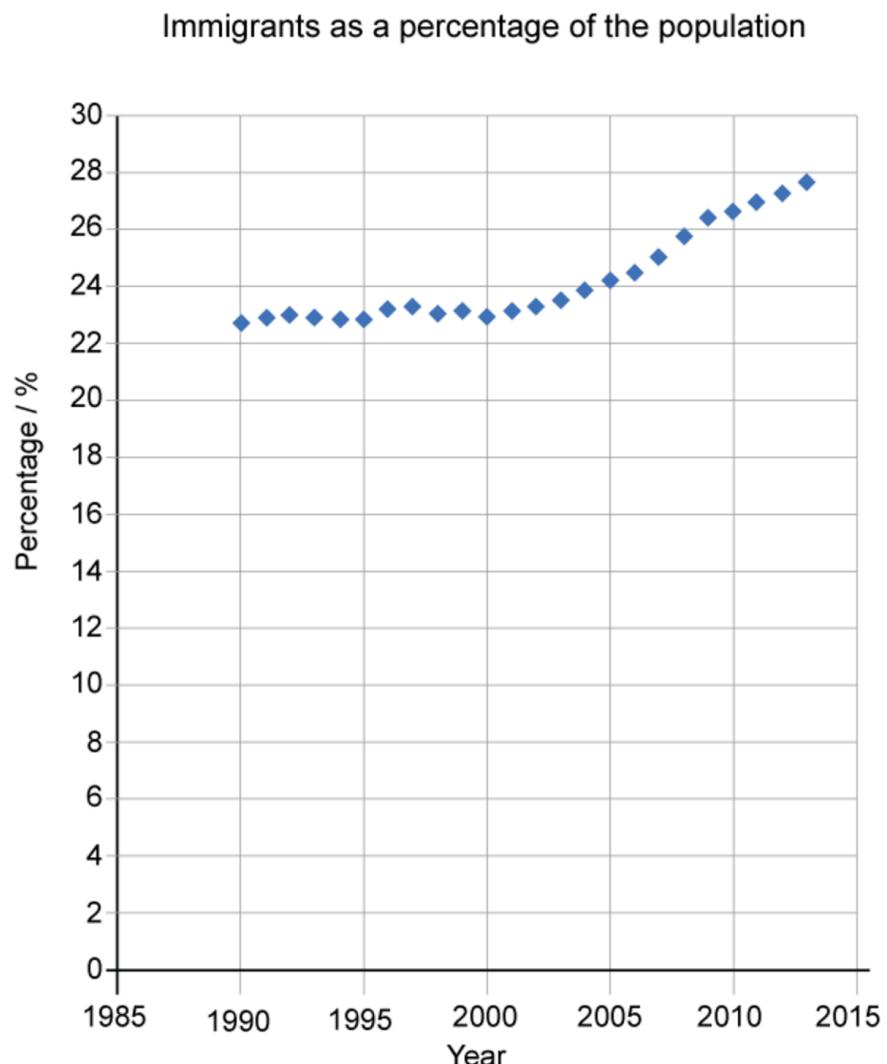


Question 7g (2 marks)

The graph below shows immigrants as a percentage of the population of Australia from 1990 to 2013.

Draw a line of best fit on the graph.

Draggable:



Scroll down to continue



Question 7h (10 marks)

AUSTRALIAN NEWS

IMMIGRANTS IN AUSTRALIA WILL EXCEED THE NATIVE POPULATION BY 2050



Analyse the information provided to comment on this news headline.

In your answer, you should:

- identify the factors to be considered when making your prediction
- estimate in what year the percentage of immigrant population in Australia will exceed 50 %
- use a suitable degree of accuracy for your results
- justify whether the year you estimated makes sense in the context of the problem
- comment on the news headline.

Tab 1

Tab 2

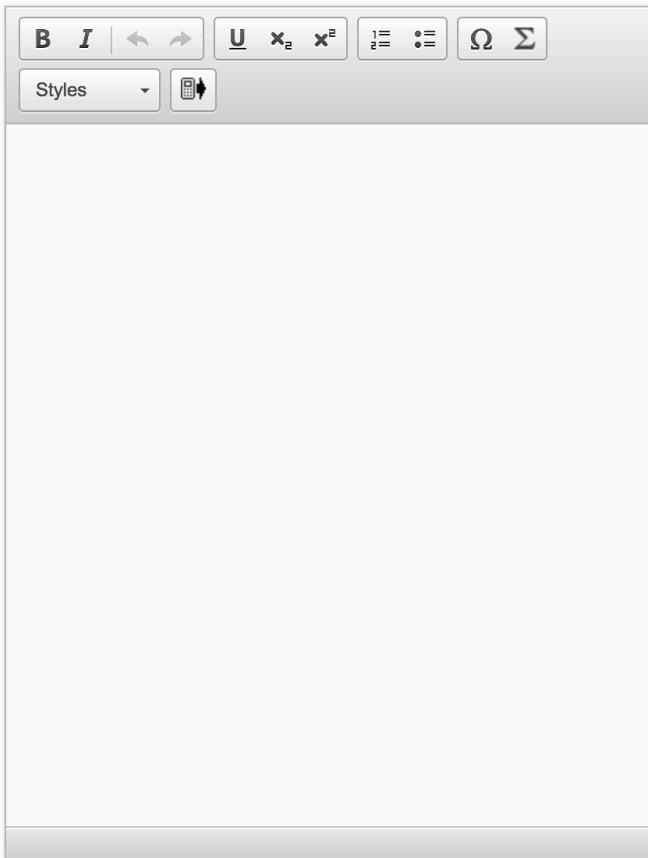
Tab 1 Tab 2

The table below shows the age distribution of the population of Australia (percentage to nearest 1 %, data correct as at 2015).

Age (A)	Percentage
$0 < A \leq 20$	23
$20 < A \leq 40$	32
$40 < A \leq 60$	27
$60 < A \leq 80$	17
$80 < A \leq 100$	1

The table below shows the country of birth of immigrants to Australia (data correct as at 2015, top ten countries only).

Country of birth	Number of immigrants
United Kingdom	1 207 000
New Zealand	611 400
China	481 800
India	432 700
Philippines	236 400
Vietnam	230 200
Italy	198 200
South Africa	178 700
Malaysia	156 500
Germany	125 900





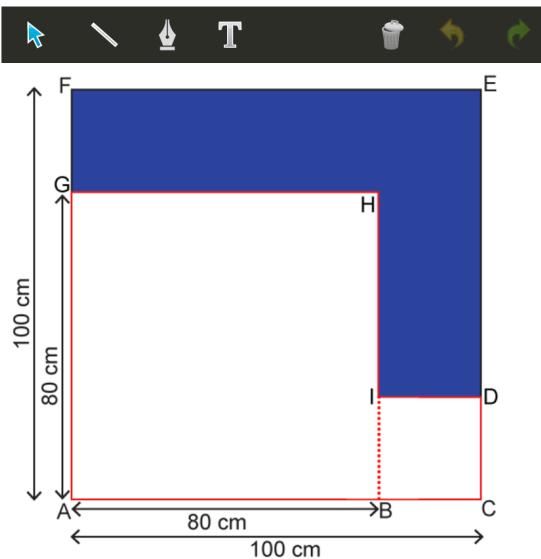
Question 8 (32 marks)



Question 8a (2 marks)

The diagram below shows squares ACEF, ABHG and BCDI.

Diagram not to scale



The large square shown has side $AC = 100 \text{ cm}$. AB has length 80 cm .

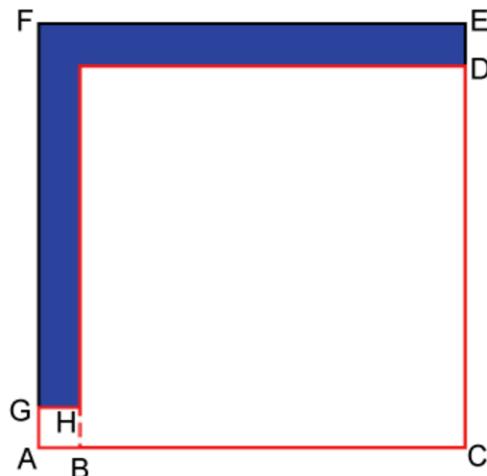
Show that the shaded area is 3200 cm^2 .

A screenshot of a digital math editor interface. At the top is a toolbar with icons for selection, text, drawing, trash, undo, redo, and other functions. Below the toolbar is a 'Styles' dropdown menu. The main area is a large white space for writing or drawing mathematical content. In the bottom right corner of the main area, there is a small blue bookmark icon.

On the simulator below, use the slider to change the length of AB and see the corresponding values of the shaded area when AC is 100 cm .



Drag the slider right and left



Length of AB	Shaded area / cm^2
10	1800

**Question 8b (1 mark)**

Write down the missing values in the table.

Length of AC in cm	Length of AB (L) in cm	Length of BC in cm	Shaded area (A) in cm^2
100	20	80	3200
100	30	70	4200
100	40		
100	50	50	5000
100		40	
100	70	30	4200
100	80	20	3200

Reset**Question 8c (1 mark)**

Write down, in words, a pattern from the table for the shaded area (A).

**Question 8d (2 marks)**

Determine a general rule for (A), the shaded area, in terms of (L), the length of AB.

**Question 8e (3 marks)**

Verify your general rule.



Question 8f (3 marks)

Traditional shoemakers used a tool as shown below.

Click on "Start" to illustrate the shapes studied by ancient Greeks based on the shoemaker tool.

Start



Diagram not to scale

Traditional shoemakers tool

When diameter AB is 100 cm and diameter AC is 40 cm, the area of the semi-circle with diameter AB is 1250π cm².

Show that the shaded area is 600π cm².



Question 8g (20 marks)

On the simulator below, use the slider to change the diameter AC and see the corresponding shaded area.

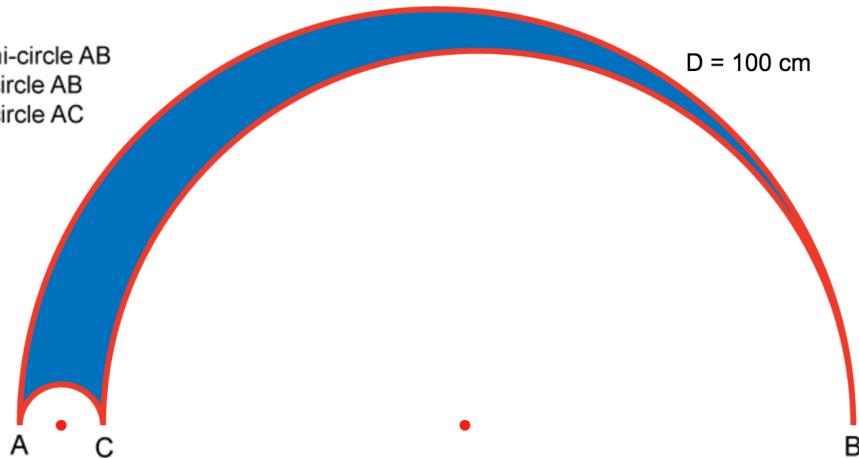
Key:

D: diameter of the semi-circle AB

R: radius of the semi-circle AB

r: radius of the semi-circle AC

A: shaded shape



AB (cm)	R (cm)	AC (cm)	r (cm)	A (cm)
100	50	10	5	225π



Drag the slider right and left

Investigate the relationship between the radius (r) and the shaded area (A) for this semi-circle where $D = 100$ cm. You may use the simulator above, which is for $D = 100$ cm. In your answer, you should:

- predict more values for A
- write down, in words, any patterns you see for A
- find a general rule for the shaded area (A) in terms of (r)
- test your general rule
- prove or verify and justify your general rule
- ensure that you communicate the above appropriately.

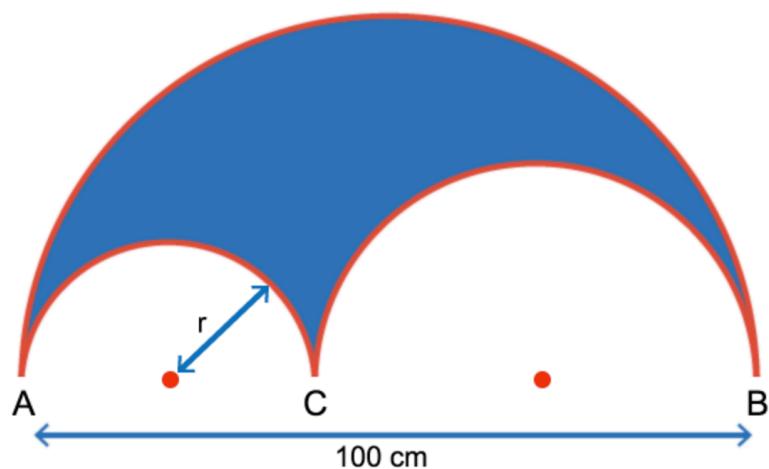
Text input area with rich text toolbar:

B I ← → U x_e x^a i≡ :≡ Ω Σ Styles

Large text input area for writing the investigation answer.

To support your investigation, you may record data in the table below.

D	R	r		A
100	50	5		225π
100	50	10		400π
100	50	15		525π
100	50	20		600π
100	50	25		$\square\pi$
100	50	30		$\square\pi$
100	50	\square		$\square\pi$
100	50	\square		$\square\pi$
\square	\square	\square		$\square\pi$
\square	\square	\square		$\square\pi$





Markscheme

May 2018

Extended mathematics

On-screen examination

This markscheme is **confidential** and for the exclusive use of examiners in this examination session.

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The following are the annotations available to use when marking responses.

Annotation	Explication	Annotation	Explication	Shortcut
	Unclear		Award 0 marks	Alt+0
	Special case		Award 1 mark	Alt+1
	Misread		Award 2 marks	Alt+2
	No working shown		Award 3 marks	Alt+3
	Error carried forward		Award 4 marks	Alt+4
	Words to that effect		Award 5 marks	Alt+5
	Benefit of the doubt		Award 6 marks	Alt+6
	Answer Given		Award 7 marks	Alt+7
	Highlight tool		Award 8 marks	Alt+8
	Ellipse tool		Award 9 marks	Alt+9
	On page comment tool		Award 10 marks	
	Seen		Award 11 marks	
	Caret - Omission		Award 12 marks	
	Wavy underline tool			

The markscheme may make use of the following abbreviations:

RM Assessor has the following annotations that should be used to award marks:

A0 only use to award a zero mark for an answer that has no merit eg, awarded for the candidate that has a wrong answer with no working

NR only use when the candidate has not made any response also stamp the response with

SEEN

Marks awarded by stamping the tick

SEEN Seen; must be stamped on all blank response areas and on concatenated responses

? unclear

- Bullet notation means award 1 mark – see example 1 below

BOD Benefit of the doubt

MR misread

NWS no working shown

SC special case

OE or equivalent

WTTE or words to that effect

AG Answer given

Example 1
• 1 mark awarded and corresponding notes are aligned

b	<ul style="list-style-type: none">• Show clear line of reasoning in the method	45 & 49 seen OE eg, $49 = 45 + x$	2
	<ul style="list-style-type: none">• 4	ACCEPT $45 + X/10 = 4.9$ <u>and</u> Ans 4	

Error Carried Forward (ECF) marks

Errors made at any step of a solution affect all working that follows. In general, **Error Carried Forward (ECF)** marks are awarded after an error.

- a) **ECF** applies from one part of a question to a subsequent part of the question and also applies within the same part.
- b) If an answer resulting from **ECF** is inappropriate (e.g. negative distances or $\sin x > 1$) then subsequent marks should not be awarded.
- c) If a question is transformed by an error into a simpler question then **ECF** may not be fully awarded.
- d) To award **ECF** marks for a question part, there must be working present for that part.
- e) **ECF** is only applied to working which is correct. This means that all working subsequent to an error must be checked for accuracy.
- f) A misread (**MR**) is an error. **ECF** is normally awarded.

General points

- a) As this is an international examination, ACCEPT all alternative forms of **notation**, for example 1.9 and 1,9 or 1 000 or 1.000. However DO NOT ACCEPT incorrect mathematical notation e.g x^2 for x^2 in final answers unless noted otherwise in the MS.
- b) Ignore further working after a correct answer **unless** it indicates a lack of mathematical understanding i.e. if the further working contradicts the correct answer, then the last mark cannot be awarded.
- c) Where candidates have written two solutions to a question, mark the response that deserves more marks.
- d) In the markscheme, equivalent examples of **numerical** and **algebraic** forms or **simplified** answers will generally be written in the notes preceded by **OE** or equivalent
 - e.g. $\frac{1}{2}$ or $1/2$ or $1 \div 2$ and $\frac{x}{2}$ or $x/2$ or $x \div 2$
- e) In the markscheme, information provided in brackets indicate detail that may be seen in a candidate response but is not necessary to award the marks.
- f) Special case marks **SC** can be allocated instead of but not in addition to the marks prescribed in the markscheme.
- g) ACCEPT notation errors in intermediate steps.
- h) When a calculator screenshot is taken, ACCEPT not seeing the whole operation.
- i) ACCEPT seeing an equation not in-line.

General note for marking open-ended response questions:

In cases in Task 2 and 3 where the mark scheme is set out in a table then awarding the highest box includes all the lower boxes. So if for example you see at J4 in Task 3 that they have correctly proved their general rule, then J4 is awarded. That is the 4 (full) marks. You don't need to look at the other J criteria. It is probably best to look for the top box answer and if you don't find it look at the next box down. So if they don't get D4 then look at their attempt at a general rule and they might gain D3. If you don't see that look to see if they described a pattern and so on.

Question	Answers	Notes	Total
1 a	X  X'  D  D'  D+D' 	•1 734 and their 297 correctly reversed •2 their D correct •3 their D and D' correctly added	3
b	•1 expressing X correctly in terms of multiples of a, b and c •2 expressing X' correctly in terms of multiples of a, b and c •3 subtracting and reversing all signs of their X OR their X' •4 their answer correct in terms of a and c only	ACCEPT inappropriate notation provided it reads correctly. Ex: $100*a+10*b+1*c$ DO NOT ACCEPT incorrectly read expressions. Ex: $100c+10(b)+a$ •2 $100c + 10b + (1)a$ ACCEPT not seeing this step •3 $100c+10b+1a - 100a - 10b-1c$ OE If their X and X' are numbers, ACCEPT their X- their X' only if their result is positive •4 $99c - 99a$ or $99(c - a)$ or $99(a - c)$ ACCEPT absolute value of $99(c - a)$ OE	4

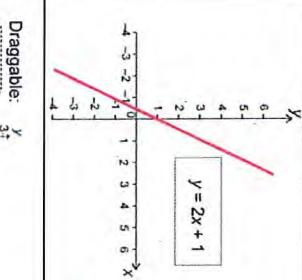
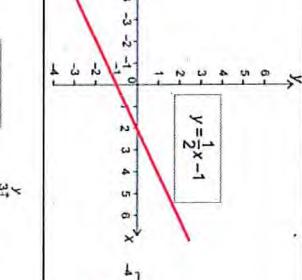
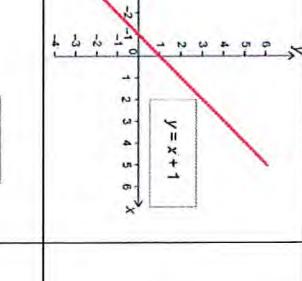
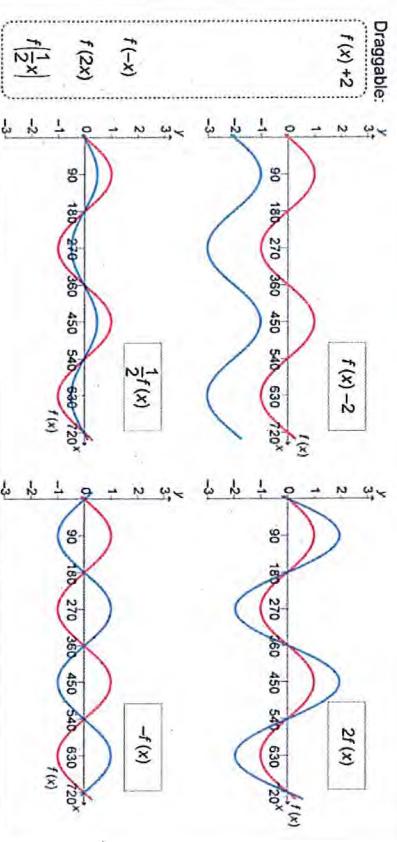
Notes continued below

1	b
	<p>99c-99a or $99(c-a)$ without working : award 3 marks 99c - 99a or $99(c-a)$ with one step of working: award 4 marks</p> <p>*⁴ DO NOT ACCEPT unless it is in terms of a and c</p> <p>ACCEPT not seeing absolute value</p> <p>SC 1 mark 100a, 10b, c and 100c, 10b, a</p>

Question	Answers	Notes	Total
2	<p>Candidates may use any appropriate algebraic notation to represent the frog and dragonfly here we have used x and y</p> <ul style="list-style-type: none"> •₁ setting both equations correctly •₂ one correct step to eliminate one variable OR for substitution •₃ correctly eliminate one variable •₄ Their y correct •₅ Their x correct •₆ their x and their y added correctly <p>Alternative method for trial and error responses</p> <ul style="list-style-type: none"> •₁ seeing two numbers with sum 22 OR seeing two numbers with difference 12 •₂ seeing values of x and y that satisfy $3x + 2y = 22$ •₃ seeing values of x and y that satisfy $4y - 2x = 12$ •₄ x or y correct •₅ x and y correct •₆ their values added correctly 	<ul style="list-style-type: none"> •₁ $3x + 2y = 22$ and $4y - 2x = 12$ OE •₂ for example, $6x + 4y = 44$ and $12y - 6x = 36$ OR $y = (22 - 3x)/2$ •₃ 80 seen or 10 seen or $16y$ or $8x$ OE •₄ ($y =$) 5 •₅ ($x =$) 4 ECF substituting their incorrect y in any of their equations •₆ ($5 + 4 =$) 9 <p>9 without working from •₁ to •₃ : award 4 marks 9 with one of •₁ to •₃ : award 5 marks 9 with two of •₁ to •₃ : award 6 marks</p> <p>Alternative method trial and error response</p> <ul style="list-style-type: none"> •₁ e.g. $12 + 10 = 22$ OR $20 - 8 = 12$ •₂ e.g. $3 \times 6 + 2 \times 2 = 22$ •₃ e.g. $4 \times 4 - 2 \times 2 = 12$ •₄ 4 or 5 •₅ 4 and 5 •₆ their 9 <p>9 without working from •₁ to •₃ : award 4 marks 9 with one of •₁ to •₃ : award 5 marks 9 with two of •₁ to •₃ : award 6 marks</p>	6

Question	Answers	Notes	Total								
3 a	<ul style="list-style-type: none"> •1 two inequalities correct •2 The third inequality correct •3 one correct constraint OR three correct constraints not including "equal to" •4 the second and third constraint correct <table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th>Inequalities</th> <th>Constraints</th> </tr> </thead> <tbody> <tr> <td>$x + y \leq 500$</td> <td>(Number of) cookies and muffins is less than or equal 500 <i>WTTE cannot exceed, at most</i></td> </tr> <tr> <td>$y \geq 50$</td> <td>(number of) muffins more than or equal 50 WTTE exceeds or equal 50, at least 50, a minimum of 50, not less than 50 ACCEPT between 50 and 250 OR bigger than 49 OR can exceed or equal to 50</td> </tr> <tr> <td>$y \leq x$</td> <td>(number of) cookies more than or equal (the number) of muffins WTTE exceeds or equal, at least, minimum as, not less than ACCEPT x can exceed or equal y OR y cannot exceed</td> </tr> </tbody> </table>	Inequalities	Constraints	$x + y \leq 500$	(Number of) cookies and muffins is less than or equal 500 <i>WTTE cannot exceed, at most</i>	$y \geq 50$	(number of) muffins more than or equal 50 WTTE exceeds or equal 50, at least 50, a minimum of 50, not less than 50 ACCEPT between 50 and 250 OR bigger than 49 OR can exceed or equal to 50	$y \leq x$	(number of) cookies more than or equal (the number) of muffins WTTE exceeds or equal, at least, minimum as, not less than ACCEPT x can exceed or equal y OR y cannot exceed	<p>ACCEPT if they use x and y instead of cookies and muffins</p> <p>DO NOT ACCEPT cost or price or value of cookie/muffin</p> <p>³ An example for "OR" Cookies exceeds 100 and muffins exceeds 50 and cookies bigger than muffins ³ DO NOT ACCEPT ECF from the inequality</p> <p>4</p>	
Inequalities	Constraints										
$x + y \leq 500$	(Number of) cookies and muffins is less than or equal 500 <i>WTTE cannot exceed, at most</i>										
$y \geq 50$	(number of) muffins more than or equal 50 WTTE exceeds or equal 50, at least 50, a minimum of 50, not less than 50 ACCEPT between 50 and 250 OR bigger than 49 OR can exceed or equal to 50										
$y \leq x$	(number of) cookies more than or equal (the number) of muffins WTTE exceeds or equal, at least, minimum as, not less than ACCEPT x can exceed or equal y OR y cannot exceed										
b	<p>profit (on/of) one (cookie and/or muffin) OR profit per cookie and per muffin</p> <p>Profit and one or per must be seen or implied</p> <p>ACCEPT amount of money gained to express profit</p> <p>DO NOT ACCEPT cost or price DO NOT ACCEPT the profit of a muffin is 1.5 times the profit of a cookie OE</p>	1									

3	C	<ul style="list-style-type: none">•¹ evidence of working•² 250 (cookies)•³ 250 (muffins)	<p>•¹ ($P =$) $(1x)450 + 1.50 \times 50$ OR $(P =)(1x)250 + 1.5 \times 250$ OR 525 OR 625 Incorrect substitution and correct calculation: award only 1 mark</p> <p>•² 250 (cookies) without working: award 1 mark</p> <p>•³ 250 (muffins) without working: award 1 mark</p> <p>250 must be seen twice to gain •² and •³</p> <p>SC 2 marks 525 and 625 seen or 725 and 50 cookies and 450 muffins</p> <p>SC 1 mark 725 and 50 and 450</p>	3
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Question	Answers	Notes	Total
4 a	<ul style="list-style-type: none"> •1 one equation placed correctly •2 the other two equations placed correctly 	  	2
b	<ul style="list-style-type: none"> •1 1st equation placed correctly •2 2nd equation placed correctly •3 3rd equation placed correctly •4 4th equation placed correctly 	<p>Draggable:</p> 	4

DO NOT ACCEPT $f(-x)$

Do not award the mark if two functions are dragged on the same graph

Question	Answers	Notes	Total
5 a	<ul style="list-style-type: none"> •¹ substitutes into magnitude formula •² (speed) = 13 	<ul style="list-style-type: none"> •¹ square root of $5^2 + 12^2$ 13 without working: award 2 marks 	2
b	<p>adds vector S and C</p> $\begin{pmatrix} 8 \\ 12 \end{pmatrix} \text{ AG}$	$\begin{pmatrix} 5 \\ 12 \end{pmatrix} + \begin{pmatrix} 3 \\ 0 \end{pmatrix} \text{ OR } 5+3 \text{ and } 12+0. \text{ ACCEPT } 5+3 \text{ and } 12$	1
c	<ul style="list-style-type: none"> •¹ evidence of use of dot product •² dot product correctly calculated •³ magnitude of vector R correctly calculated •⁴ evidence of substituting into the correct formula •⁵ their angle correct 	<ul style="list-style-type: none"> •¹ $(5 \times 8 + 12 \times 12)$ may be implied •² 184 •³ $4\sqrt{13}$ or $2\sqrt{52}$ or $\sqrt{208}$ or 14.4(22) ACCEPT $\sqrt{8^2 + 12^2}$ for $4\sqrt{13}$ in calculator display only •⁴ $(\cos\theta =) 0.9813 \dots$ or $46/(13\sqrt{13})$ or $\frac{\text{their13}}{\text{their4}\sqrt{13}}$ their 11° (.0702 ...) must come from $-1 \leq \cos\theta \leq 1$ 	
	<p>SC 5 marks $\theta = \cos^{-1} \left(\frac{5 \times 8 + (12 \times 12)}{\sqrt{5^2 + 12^2} \times \sqrt{8^2 + 12^2}} \right) = 11^\circ$</p> <p>Alternative method</p> <ul style="list-style-type: none"> •¹ using the tan ratio twice •² applying inverse tan for one •³ applying inverse tan for the second •⁴ subtracting their angles •⁵ angle correct 	<p>5</p>	
	<p>Another alternative method below</p>		

5 C Alternative method	Alternative method <ul style="list-style-type: none"> •¹ magnitude of vector R correctly calculated •² magnitude of vector C correctly calculated •³ evidence of substituting their R, C and S into the cosine rule •⁴ their cosθ correct •⁵ their angle correct 	<ul style="list-style-type: none"> •¹ $4\sqrt{13}$ or $2\sqrt{52}$ or $\sqrt{208}$ ACCEPT $\sqrt{8^2 + 12^2}$ for $4\sqrt{13}$ in calculator display only •² 3 •³ $\text{their}3^2 = \text{their}13^2 + (\text{their}4\sqrt{13})^2 - 2(\text{their}13)(\text{their}4\sqrt{13})\cos\theta$ may be implied •⁴ $(\cos\theta) = 0.9813 \dots$ or $46/(13\sqrt{13})$ or $\frac{\text{their}(13^2 + (4\sqrt{13})^2 - 9)}{2(\text{their}13)(\text{their}4\sqrt{13})}$ •⁵ their $11^\circ (.0702 \dots)$ must come from $-1 \leq \cos\theta \leq 1$ 	5
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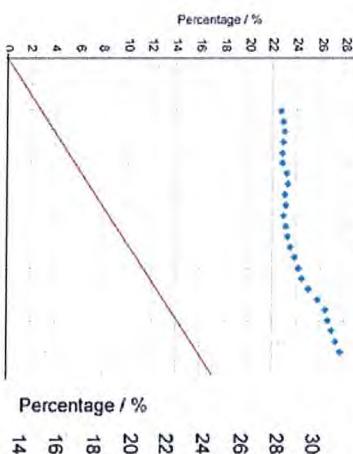
Question	Answers	Notes	Total
6 a	<ul style="list-style-type: none"> •¹ (Min) 1 and (Max) 17 •² correct subtraction to get their 16 	<ul style="list-style-type: none"> •¹ 1 and 17 •² 16 	2
b	<ul style="list-style-type: none"> •¹ identifying any two times 12 hours apart •² 12 (hours) 	<ul style="list-style-type: none"> •¹ ACCEPT seeing two times which are 12 hours apart •¹ ACCEPT recognizing that there are two cycles in 24 hours OE •² DO NOT ACCEPT any other value but 12 <p>16 without working: award 2 marks</p>	2
c	<ul style="list-style-type: none"> •¹ correct substitution •² one correct algebraic step •³ two correct algebraic steps •⁴ correct use of their inverse sin •⁵ correct step using π or 180 •⁶ their times both correct 	<p>SC 1 mark Seeing only (6, 16) or -16</p> <ul style="list-style-type: none"> •¹ 3 and 15. •¹ ACCEPT seeing two points with x-coordinates 12 hours apart •¹ ACCEPT seeing two times which are 12 hours apart •¹ ACCEPT recognizing that there are two cycles in 24 hours OE •² DO NOT ACCEPT any other value but 12 <p>12 without working: award 2 marks</p> <p>ACCEPT work in degrees instead of radians</p> <ul style="list-style-type: none"> •¹ $13 = 8 \sin(\pi t/6) + 9$ •² $4 = 8 \sin(\pi t/6)$ •³ $\frac{1}{2} = \sin(\pi t/6)$ ACCEPT seeing •² or •³ for both marks •² and •³ •⁴ $\pi t/6 = \pi/6$. ACCEPT using $\pi = 180$ in their calculations and writing $30t = 30$ or $\pi t/6 = 30$ OE ACCEPT $t = 1$ or $t = 57(.29..)$ •⁵ $\pi t/6 = \pi -$ their $\pi/6$ ACCEPT using $\pi = 180$ in their calculations and writing $30t = 180 -$ their 30 or $\pi t/6 = 180 -$ their 30 •⁶ their $t = 1$ (am) and their $t = 5$ (am) <p>Attempts to work with e.g. $\pi t/6 = 30$ cannot be awarded •⁶</p>	6

Question	Answers	Notes	Total
7 a	$\frac{17+1}{100}$	OE	1
b	<ul style="list-style-type: none"> •1 dividing 1207000 by 23858000 •2 0.05059... or 0.0506 or 5.059... (%) or 5.06(%) 	<ul style="list-style-type: none"> •1 ACCEPT 1207/23858 •2 ACCEPT 0.0505 or 5.05(%) 	
	5 (%) AG	ACCEPT seeing cross multiplication process for 2 marks Examples: 23858 – 100% 1207 – 5.059%	
c	<ul style="list-style-type: none"> •1 0.82 written on both branches •2 0.05 and 0.95 	SC 1 mark $0.05 \times 23858000 = 1192900$ or ≈ 1207000 OR $1207000 / 0.05 = 24140000$ or ≈ 23858000	2
d	<ul style="list-style-type: none"> •1 calculating probability from one branch •2 calculating probability from the other branch •3 adding correctly their probabilities of their branches 	<ul style="list-style-type: none"> •1 0.05×0.18 •2 0.95×0.18 •3 $0.05 \times 0.18 + 0.95 \times 0.18 = 0.18$ <p>0.18 without working: award 1 mark ACCEPT 5%, 5/100 for 0.05 OE</p>	3

7	e	<ul style="list-style-type: none"> •1 dividing their probability of first branch by their result in (d) •2 their result correctly after division 	$\frac{0.05 \times 0.18}{0.18}$ •2 0.05 2
	f	<ul style="list-style-type: none"> •1 independent (events) 	0.05 without working: award 2 marks ACCEPT incorrect spellings of "independent" OE ACCEPT independent even if answers for b) and e) are not correct
	g	<ul style="list-style-type: none"> •1 the line is crossing twice or touching at least two points •2 domain covering at least year 1994 to 2010 and the line is within the range shown below 	<ul style="list-style-type: none"> •1 see examples below •2 see examples below 1

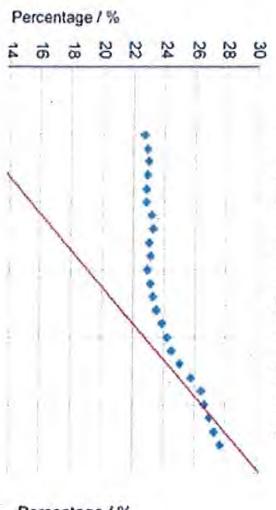
7 g

Examples not acceptable (0 marks): Not crossing twice, Not touching two points and not in range

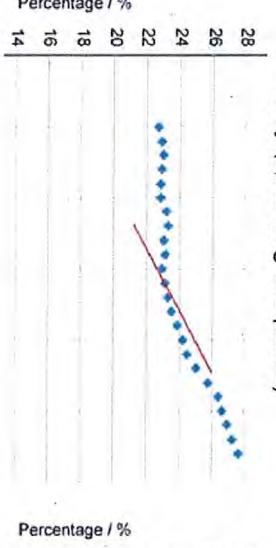


Examples acceptable for •1 or •2 only (1 mark)

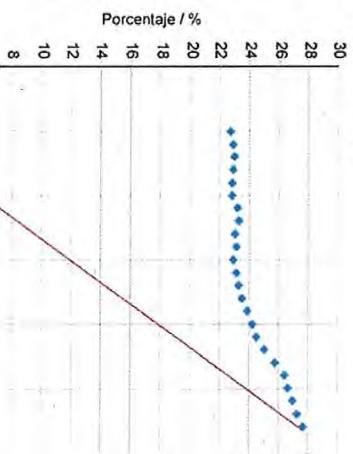
•1 only (Touching two points)



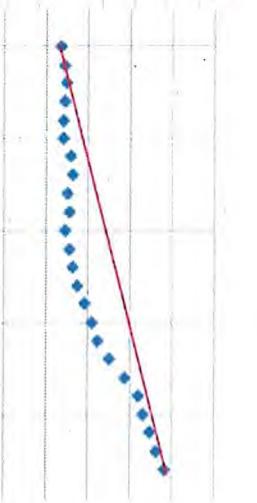
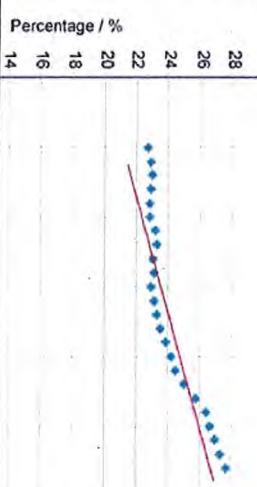
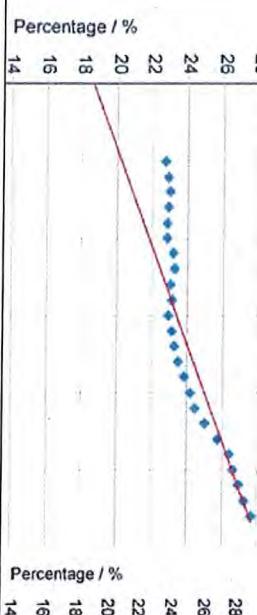
•1 only (Touching two points)



•2 only (In range but not touching two points)



Examples acceptable for •1 and •2 (2 marks)



7	h	Mark	1	2	3	4
		F: Identification of Factors	<p>One factor mentioned from:</p> <ul style="list-style-type: none"> - Nature of the increase - Life expectancy - Birth rate or mortality - Rules of immigration - Gradient or rate of line or Percentage increase of immigrants seen - Percentage of immigrants - Change in population distribution (human landscape) - Economy status - Imagined factor like disaster happening in Australia DO NOT ACCEPT only saying many factors 	<p>Two factors mentioned from:</p> <ul style="list-style-type: none"> - Nature of the increase - Life expectancy - Birth rate or mortality - Rules of immigration - Gradient or rate of line or Percentage increase of immigrants seen - Percentage of immigrants - Change in population distribution (human landscape) - Economy status - Imagined factor like disaster happening in Australia DO NOT ACCEPT only saying many factors 		
		E: Estimation	<p>Estimation of year in range (2055 to 2300) or (in 42 to 287 years) without showing calculations</p> <p>OR</p> <p>Estimation not in range using their incorrect line equation or incorrect proportions or percentages</p>	<p>Estimation of year in range (2055 to 2300) using: equation of line not matching their line of best fit</p> <p>OR</p> <p>Incorrect proportion or percentage increase of immigration Ex: Estimate 2080 using 6% in 23 years, or estimate 2150 using 0.3% every year, or 2070 using 1% every 5 years</p>	<p>Estimation of year in range (2055 to 2300) using: equation of line matching their line of best fit but incorrect result Ex: reaching estimation 2100 from $y=0.2x+20$</p> <p>OR</p> <p>Using appropriate proportions Ex: Estimate 2180 using 6% in 23 years, or estimate 2090 using 0.3% every year, or 2070 using 2% every 5 years</p>	

7	h	Mark	1	2	3	4
		D: Degree of Accuracy	<p>Suitable rounding used for their estimated year</p> <p>DO NOT ACCEPT if they just write down a year without any reference or calculations</p> <p>SC: if they use their line equation or proportions correctly and their estimated year does not need rounding: award D1</p>	<p>Inaccurate with weak reason</p> <p>Examples:</p> <ol style="list-style-type: none"> 1. Inaccurate because I used line of best fit. 2. Inaccurate because I used % which are not accurate <p>inaccurate because the difference between what I estimated and news headline is very big</p>	<p>Inaccurate with a valid reason related to variables affecting the future</p> <p>Examples:</p> <ol style="list-style-type: none"> 1. The prediction I made not very accurate because many factors may vary 2. Predictions using line of best fit for the future not guaranteed 3. The prediction using the line equation not very accurate because it is taking only a window or isolated time 4. The prediction using the line equation not very accurate because it assumes the future follows same pattern 5. My predictions not very accurate because the population (human landscape) can change in Australia 	
		OR	<p>Accurate with valid reason</p> <p>Examples:</p> <ol style="list-style-type: none"> 1. Accurate because I used my line of best fit to estimate 2. Accurate because I used average % of increase 3. Accurate because I considered all data given <p>DO NOT ACCEPT My prediction is accurate because I made the calculations</p> <p>DO NOT ACCEPT accurate or inaccurate without reason</p>	<p>ACCEPT Calculating using their line equation the % immigration in 2050 and showing it is not 50%</p>		
			<p>Can be awarded only if E1 is achieved</p>			

7	h	Mark	1	2	3	4
		N: Comment on News headline	Comment on the news headline, with a supporting reason, seen anywhere Example: 1. Not accurate because I estimated my calculation 2300 2. It can be correct as my value estimated is close 3. Many factors affect it they cant say for sure DO NOT ACCEPT right or wrong headline without reason			

10 marks

Question	Answers	Notes	Total																																
8 a	<ul style="list-style-type: none"> •¹ two correct area values •² third correct value and correct subtraction to get 3200 AG <p>OR finding the area of shaded</p>	Values 10000, 6400, 400 e.g. $10\ 000 - 6\ 400 = 400$	2																																
b	<ul style="list-style-type: none"> •¹ correct area for one shaded rectangle •² second correct area for shaded rectangle and addition seen 60 and 4800 twice 	<p>OR</p> <ul style="list-style-type: none"> •¹ For example: 1600 OR 2000 •² 2×1600 OR $2000 + 1200$ 																																	
c	a suitable pattern described in appropriate terminology	<table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Length of AC In cm</th> <th>Length of AB In cm</th> <th>Length of BC In cm</th> <th>Shaded area (A) In cm²</th> </tr> </thead> <tbody> <tr><td>100</td><td>20</td><td>80</td><td>3200</td></tr> <tr><td>100</td><td>30</td><td>70</td><td>4200</td></tr> <tr><td>100</td><td>40</td><td>60</td><td>5000</td></tr> <tr><td>100</td><td>50</td><td>50</td><td>5000</td></tr> <tr><td>100</td><td>60</td><td>40</td><td>4200</td></tr> <tr><td>100</td><td>70</td><td>30</td><td>3200</td></tr> <tr><td>100</td><td>80</td><td>20</td><td>2000</td></tr> </tbody> </table> <p>Some examples: it's quadratic It increases and then decreases Maximum at 50 Maximum 5000</p> <p>Symmetry or symmetry about 5000 or 50 $AB \times BC \times 2$ or WTE i.e Second column \times third column \times 2 ACCEPT: They are all multiples of 100</p>	Length of AC In cm	Length of AB In cm	Length of BC In cm	Shaded area (A) In cm ²	100	20	80	3200	100	30	70	4200	100	40	60	5000	100	50	50	5000	100	60	40	4200	100	70	30	3200	100	80	20	2000	1
Length of AC In cm	Length of AB In cm	Length of BC In cm	Shaded area (A) In cm ²																																
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100	30	70	4200																																
100	40	60	5000																																
100	50	50	5000																																
100	60	40	4200																																
100	70	30	3200																																
100	80	20	2000																																
d	<ul style="list-style-type: none"> •¹ attempt to express the general rule in terms of their L •² correct general rule for A in terms of L <p>SC 1 mark</p> <p>seeing $(100 - L)$ or seeing $2L^2$ or $2L$ or seeing $AB \times (100 - AB) \times 2$</p>	<ul style="list-style-type: none"> •¹ $(A =) 200L - 2L^2$ OR $2L(100 - L)$, ACCEPT $(A =) 200 \times L - 2 \times L^2$ or $2L \times (100 - L)$ •² $(A =) 200L - 2L^2$ OR $2L(100 - L)$, ACCEPT $(A =) 200 \times L - 2 \times L^2$ or $2L \times (100 - L)$ 	2																																

8	e	<ul style="list-style-type: none"> •¹ substitute L different than (20, 30, 40, 50, 60, 70, 80) into their rule •² correctly calculate their value of A after substitution of L different than (20, 30, 40, 50, 60, 70, 80) •³ recognizing that their result is the same as their predicted value 	<ul style="list-style-type: none"> •¹ Example: use $L = 10$ or 90 $200(10) - 2(10)^2$ or their own value of L •² Example: 1800 (for the $L = 10$) •³ Same as value I predicted in table (and we find the candidate has 1800 in the table for $L = 10$) OR same as when we continue the pattern and explains how 1800 is obtained from pattern of constant second difference OR the same value given by the simulator <p>•³ ACCEPT seeing the 1800 in the table and seeing their calculated $A = 1800$ when $L = 10$</p> <p>SC 1 mark if "tested" correctly with a value from the table. for example: tests with $L = 50$ to get $P = 5000$ and recognizing their result is same as table</p>	3
	f	<ul style="list-style-type: none"> •¹ correctly substituting in area formula •² evidence of subtracting their two areas from the whole •³ dividing by 2 <p>600π AG</p>	<ul style="list-style-type: none"> •¹ $\pi 30^2$ or $\pi 20^2$ •² $2500\pi - 900\pi - 400\pi$ •³ $(A =)1250\pi - 450\pi - 200\pi$ <p>ACCEPT •² and •³ in any order</p> <p>•³ ACCEPT dividing by 2 seen in formula step for •³</p> <p>•³ ACCEPT approximated answers leading to $1884 = 600\pi$</p> <p>Seeing only $1250\pi - 450\pi - 200\pi$: award 3 marks Seeing only $1250\pi + 650\pi$: award 2 marks 600 π without working : award 0 marks</p>	3

8	g	Mark	1	2	3	4
		Predictions (P)	Predict correctly one value of A Examples: $r = 25, A = 625\pi$	Predict correctly three values of A Examples: $r = 25, A = 625\pi$ $r = 30, A = 600\pi$ $r = 35, A = 525\pi$		
		OR	Predict correctly three values of A without corresponding r	DO NOT ACCEPT if corresponding r not mentioned		
		Description (D)	Attempt to describe a pattern in words. Examples 1. Increasing then deceasing 2. It is quadratic or parabola EX. Symmetrical	Describe correctly pattern in words Example: A are multiples of 5 OR	Attempt to describe pattern for A as general rule. Examples: 1. $(A =) 50^2 - r^2 - (50 - r)^2$ 2. $(A =) 50^2 - AC^2 - (50 - AC)^2$	Correctly describe pattern as general rule for A Examples: 1. $A = \pi r (50 - r)$ 2. $A = \pi (50r - r^2)$ 3. $A = \frac{2500\pi}{2} - \frac{\pi r^2}{2} - \frac{\pi(50-r)^2}{2}$
		Testing (T)	Attempt to use r from {5, 10, 15, 20} in their described pattern or general rule	Attempts to describe radius or diameter of the semicircles as a general rule Ex: $100 - 2r$ or $50 - r$ could be seen in the response, table or diagram	Evidence of subtracting two small semicircles/circles from a larger one.	OR the below but award N1 $(A =) \pi r (50 - r)$ $(A =) \pi (50r - r^2)$ the general rule is $\pi r (50 - r)$
		Verifying (V)	Attempt to use r other than 5, 10, 15, 20 in their described pattern or general rule	Recognizing that their result is the same as value in table ACCEPT seeing their calculated A value from their general rule and the value in the table being equal even without π	Recognizing that their result is the same as their predicted value ACCEPT seeing their calculated A value from their general rule and their predicted value in the table being equal provided their calculated value includes π	

8	g	Mark	1	2	3	4
		Justify/proof (J)	Attempt to justify their described pattern or rule or general rule Examples: 1. Trying at least two values and justifying that they are the same or rule works 2. We always multiply r by the difference between 50 and r 3. The second difference is constant	Good attempt to Justify their general rule Examples: 1. The second difference is constantly 50 2. The equation is quadratic with - ve coefficient of r^2 so increasing then decreasing	Attempt to prove the general rule for the area A Example: Attempt to add areas in terms of r	Correctly prove the general rule for the Area A Example: Adding the areas in terms of r correctly J4 automatically gains T2 and V3
		Notation and terminology (N)	Notation or terminology is correct OR the notation and terminology have significant errors for example 3.14 for pi OR The general rule is correct but not in correct notation $A = 50^2 - r^2 - (50 - r)^2/2$ Ex: The general rule is $\pi(50r - r^2)$	The notation and terminology are correct Note: One minor error, not in general rule, can be overlooked Can be awarded only if they have a general rule ACCEPT the use of x or * for multiplication	-	-
		Communication (L)	Very weak communication OR Only calculations or algebraic steps	Weak communication OR More than three lines of communication but lack coherence Can be awarded only if J2 is achieved	Good communication More than three lines of coherent communication	20 marks

Markscheme

May 2017

Extended mathematics

On-screen examination

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The annotation **OE** meaning Or Equivalent is used in the mark scheme.

Error Carried Forward (ECF) marks

Errors made at any step of a solution affect all working that follows. In general, **Error Carried Forward (ECF)** marks are awarded after an error.

- a) ECF applies from one part of a question to a subsequent part of the question and also applies within the same part.
- b) If an answer resulting from ECF is inappropriate (eg, negative distances or $\sin x > 1$) then subsequent marks should not be awarded.
- c) If a question is transformed by an error into a **simpler question** then ECF may not be fully awarded.
- d) To award ECF marks for a question part, **there must be working present for that part**.
- e) ECF is only applied to working which is correct. This means that all working subsequent to an error must be checked for accuracy.
- f) A misread (**MR**) is an error. ECF is normally awarded.

General points

- a) As this is an international examination, accept all alternative forms of **notation**, for example 1.9 and 1,9 or 1 000 or 1.000. However **DO NOT ACCEPT** incorrect mathematical notation e.g x^2 for x^2 unless noted otherwise in the markscheme.
- b) Ignore further working after a correct answer **unless** it indicates a lack of mathematical understanding **i.e. if the further working contradicts the correct answer**, then the last mark cannot be awarded.
- c) Where candidates have written two solutions to a question, mark the response that deserves more marks.
- d) In the markscheme, equivalent examples of **numerical** and **algebraic** forms or **simplified** answers will generally be written in the notes preceded by **OE** or equivalent e.g.

$$\frac{1}{2} \text{ OR } 1/2 \text{ OR } 1 \div 2 \quad \text{and} \quad \frac{x}{2} \text{ OR } x/2 \text{ OR } x \div 2$$

- e) In the markscheme, information provided in brackets indicate detail that may be seen in a candidate response but is not necessary to award the marks.
- f) Special case marks SC can be allocated instead of but not in addition to the marks prescribed in the markscheme.
- g) Accept seeing equations not in-line.
- h) Accept notation errors in intermediate steps.
- i) When a calculator screenshot is taken, accept not seeing the whole operation.

Example 1

- 1 mark awarded and corresponding notes are aligned

b	• ¹ Show clear line of reasoning in the method • ² 4	• ¹ 45 & 49 seen OE eg, $49 = 45 + x$ • ² ACCEPT $4.5 + X/10 = 4.9$ and Ans 4	2
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Task 1

Answers			Notes	Total
1	a	$A \cap B \cap C$	1 mark ACCEPT $(A \cap B) \cap C$ ACCEPT $(A \cap B) \cap (B \cap C) \cap (C \cap A)$ or similar	1
	b	{5, 6}		1
	c	<ul style="list-style-type: none"> •¹ For $A \cap (B \cup C) = \{2, 5, 6\}$ •² For $(A \cap B) \cup (A \cap C) = \{2, 5, 6\}$ 	Correct answers might be seen in part (d) Note: The name of the set must be seen to get the mark. for {2, 5, 6} seen without being named: Award 1 mark for {2, 5, 6} seen twice but without any name: Award 1 mark	2
	d	They are the same (or equal)	ACCEPT: It illustrates the distributive law (for sets, for \cap over \cup) or $A \cap (B \cup C) = A \cap B \cup (A \cap C)$ ACCEPT "both contain 2,5,6 WTTE, ECF from (c) for example {2, 5, 6} and {2, 5, 7} and response in (d) - "2 and 5 are in both sets" or similar	1
	e	<ul style="list-style-type: none"> •¹ {1, 2, 5, 6} Alternative method using set algebra •¹ simplifies eg $A \cap (B \cup B')$ using rule from b) •² (It is the same as) A. 	1 mark for set Do not ACCEPT partial answers for • ¹ eg {2, 5, } \cup {1, 6} • ² WTTE.	2

Answers			Notes	Total
2	a	<ul style="list-style-type: none"> •¹ (The medians) 57 and 52 seen •² This means the average age of medicine is older/bigger than physics WTTE •³ (The interquartile ranges) 20 and 15 •⁴ This means the winners in physics are more close/less spread in age. This comment must refer to spread. 	<ul style="list-style-type: none"> •¹ Is for mathematical observation for example: (Median Q2 medicine =) 57 (Median Q2 physics =) 52 •¹ for both values seen and no comment award 1 mark •¹ ACCEPT 5 seen for 1 mark •¹ and •² "in general medicine is older by 5 years" WTTE award 2 marks •³ Is for mathematical observation for example: (IQR for medicine 67-47=) 20 (IQR for physics 62-47 =) 15 •³ award for both values seen and no comment, accept differences 67-47; 62-47 •³ DO NOT ACCEPT 'IQR is between 47 and 67' or WTTE •³ DO NOT ACCEPT 'the majority is between 47 and 67' or WTTE •³ must refer to IQR not range in general •⁴ Do not accept "less skewed" •⁴ ACCEPT 50% (in this range) 	4
	b	<ul style="list-style-type: none"> •¹ 155 seen •² $\frac{\text{their } 155}{210}$ 	<ul style="list-style-type: none"> •² OE for example 0.7 OR 0.738 OR 0.74 OR 73.8 % OR 74 % OR better: 	2
	c	<ul style="list-style-type: none"> •¹ 11 seen •² $\frac{11}{\text{their } 155}$ 	<ul style="list-style-type: none"> •¹ 11 can be seen anywhere in <i>their</i> working •² OE for example 0.071 OR 0.07 OR 7.1 % OR 7% OR better: 	2

Answers			Notes	Total
2	d	<ul style="list-style-type: none">•¹ Applying the 20% to 210•² Read off the graph the corresponding age	<ul style="list-style-type: none">•¹ $\frac{20}{100} \times 210$ OR 42 OR $\frac{80}{100} \times 210$ OR 168	2

Answers			Notes	Total
3	a	<p>Step a: 14,3 . Step c: 5,7, 3</p> <ul style="list-style-type: none"> •¹ All step a correct •² All step c correct 	<ul style="list-style-type: none"> •¹ 14, 3 •² their 5, 7, their 3 <p>Allow ECF for from their step a</p>	2
	b	<ul style="list-style-type: none"> •¹ Correct sum of their numbers •² Not divisible by 10 or Not a multiple of 10 	<ul style="list-style-type: none"> •¹ their 83 •² Accept 'must be a multiple of 10' <p>Sum their values correctly to a number divisible by 10 and saying it is valid because sum is divisible by 10 award •¹ only</p>	2
	c	<ul style="list-style-type: none"> •¹ $(61 + X =) 70$ •² $(X =) 9$ 	9 seen no working: Award 2 marks	2

Answers			Notes	Total
3	d	<ul style="list-style-type: none"> •¹ Instruction F correct •² Instruction G correct •³ Instruction H correct 	<p>•¹ (If the doubled digit is a two-digit number) add the two digits together Accept “add the two numbers together” OR WTTE</p> <p>•² Add the numbers Accept “plus” the numbers OR “sum” the numbers OR WTTE</p> <p>•³ If the sum is divisible by 10 then it is valid OR if the sum is not divisible by 10 then not valid OR check if (make sure) the number is divisible by 10 OR WTTE The instruction has to be in the correct place to award the mark</p> <p>SC: Step G adding and if sum divisible by 10 valid. Step H adding and if sum not divisible by 10 invalid Award only 1 mark for •² and •³. Award •¹ as appropriate</p> <p>SC: Step G adding and if sum divisible by 10 valid Step H if the sum not divisible by 10 invalid Award 2 marks for •² and •³. Award •¹ as appropriate</p>	3

Answers			Notes	Total
4	a	<ul style="list-style-type: none"> •¹ $2f(x-2) = 6$ •² $f(x-2) = 3$ •³ $x-2 = 6$ OR $f(6) = 3$ •⁴ $x = 8$ 	Marks awarded for <ul style="list-style-type: none"> •¹ first algebra step, number to other side •² second algebra step, divide by 2 •³ refers back to table •⁴ algebra step 	4
	b	<ul style="list-style-type: none"> •¹ 0 •² 2 		2
	c	<ul style="list-style-type: none"> •¹ $f(g(0)) = f(g(4)) (= -4)$ •² The inverse function of -4 has two different values 	<ul style="list-style-type: none"> •¹ May be implied <p>Accept for 2 marks</p> <ul style="list-style-type: none"> • 0 and 4 both go to -4 • $f(g(x))$ is not a one to one function • $f(g(x))$ is a two or many – one function • Horizontal line test for $f(g(x))$ gives two values • Vertical line test for the inverse of $f(g(x))$ gives two values • The inverse function has two different values for the same value • for a single value of x there are two values of y • $f(g(x))$ has several values in x for the same value in y, so the reciprocal/inverse will have several values in y for the same value in x <p>Accept for 1 mark The inverse function has two different values</p>	2

Task 2

Answers			Notes	Total
5	a	(BCA) = 57		1
	b	<p>Alternative method 1</p> <ul style="list-style-type: none"> •¹ Use of sine rule •² $\frac{100}{\sin(\text{their } 57)} = \frac{R}{\sin 3}$ •³ Attempt to solve for R •⁴ (R=) 6.240347.... (m) <p>6.24 AG</p> <p>Alternative method 2</p> <ul style="list-style-type: none"> •¹ $\sin 30 = \frac{BE}{100}$ OR $\cos 30 = \frac{AE}{100}$ •² $\tan 33 = \frac{\text{their BE} + R}{\text{their AE}}$ •³ $\tan 33 = \frac{\text{correct BE} + R}{\text{correct AE}}$ •⁴ (R=) 6.2403478...(m) <p>6.24 AG</p> <p>Alternative method 3</p> <ul style="list-style-type: none"> •¹ $\cos 30 = \frac{AE}{100}$ •² $\cos 33 = \frac{\text{their AE}}{AC}$ •³ $\text{their } 03.3^2 = (50\sqrt{3})^2 + EC^2$ and $EC = 56.2403478...$ •⁴ (R=) 6.2403478...(m) <p>AG 6.24m</p> <div style="border: 1px solid black; padding: 10px; margin-left: 20px;"> <p>Note: Some candidates notice that ABE is a 30,60,90 triangle and can write down without working that BE is 50 and AE is $50\sqrt{3}$</p> </div> <div style="border: 1px solid black; padding: 10px; margin-left: 20px;"> <p>BE = 50 AE = 86.6025... AC = 103.26... EC = 56.2403</p> </div>	<p>Alternative method 1</p> <ul style="list-style-type: none"> •¹ 1 mark for use of sine rule •² 1 mark for correct substitution in sine rule •³ 1 mark for attempt to solve (do not have to see this step) •⁴ 1 mark for showing the answer before approximations <p>Alternative method 2</p> <ul style="list-style-type: none"> •¹ 1 mark for using correctly sin or cos •² 1 mark for using tan •³ 1 mark for correct values used with tan •⁴ 1 mark for showing the answer before approximation (may be seen as 56.24034.. earlier) <p>Alternative method 3</p> <ul style="list-style-type: none"> •¹ 1 mark for finding AE, $\cos 30 = AE/100$ OR by inspection $50\sqrt{3}$ •² 1 mark for finding AC = 103.3 •³ 1 mark for correct values in Pythagoras and find EC •⁴ 1 mark for showing the answer before approximation (may be seen as 56.24034.. earlier) 	4

Marks			Notes	Total
5	b	<p>Alternative method 4</p> <ul style="list-style-type: none"> •³ $\sin 33^\circ = EC/\text{their AC and their EC}$ •⁴ $(R=) 6.2403478\ldots(m)$ <p>AG 6.24 m</p> <p>Alternative method 5 Cos rule</p> <ul style="list-style-type: none"> •³ $R = \sqrt{100^2 + 103.26^2 - (2 \times 100 \times 103.26 \times \cos 3^\circ)}$ •⁴ $(R=) 6.2403478\ldots(m)$ 	<p>Alternative method 4</p> <ul style="list-style-type: none"> •³ 1 mark for their values in SOH and find EC •⁴ 1 mark for showing the answer before approximation (may be seen as 56.24034.. earlier) <p>Alternative method 5</p> <ul style="list-style-type: none"> •³ 1 mark for correct substitution in cosine rule •⁴ 1 mark for showing the answer before approximation 	
	c	<ul style="list-style-type: none"> •¹ Substituting 6.24 correctly into volume formula •² Volume calculated correctly using <i>their</i> radius •³ Their value correctly approximated to nearest m^3 	<ul style="list-style-type: none"> •¹ $\frac{4}{3}\pi(6.24)^3$ Accept: $4/3\pi6.24^3$ •² <i>their</i>1017.7529..(using π) OR <i>their</i>1017.237.. (using 3.14) OR 323.96π OE •³ 1018 (m^3) OR 1017 (m^3) 	3

		Answers	Notes	Total
5	d	<ul style="list-style-type: none"> •¹ A valid mathematical reason •² Another valid reason (mathematical or not) 	<p>Mathematical reasons:</p> <ul style="list-style-type: none"> (i) Referring to rounding of measurements OR calculations (ii) Referring to inner and outer diameter OR thickness of sphere <p>Examples of real-life reasons:</p> <ul style="list-style-type: none"> (i) Referring to shape not a perfect sphere (ii) Referring to the sphere is not totally filled with water (regardless of the reason) (iii) Expansion of water <p>DO NOT ACCEPT the vertical/pipes post may include water as well DO NOT ACCEPT referring to errors in measurements given Do NOT ACCEPT “because maybe I made mistakes”</p>	2

5	e	Marks	1	2	10
		(H) Calculate total water consumption for the households in the community	<p>Two attempts from any of the calculations below:</p> <p>Dividing 300000 by 4 to find the number of households $300\ 000/4 (= 75\ 000)$ households</p> <p>OR</p> <p>Multiplying their 366 by their 75000 to find the amount of water consumed per day by households $their366 * their75000 (= 27\ 450\ 000)$ litres per day</p> <p>OR</p> <p>Dividing their 366 by 24 to find the amount of water in litres per hour for each household $their366/24(=15.25)$ l/h</p> <p>OR</p> <p>Calculating how many litres for every household during the 4 hours outage time</p> <p>OR</p> <p>Acceptable estimation for the amount of water used by household per hour (between 1100 000 and 1200000) without calculations</p>	<p>Calculate correctly the amount of water used by households per hour: $(27\ 450\ 000/24 =) 1\ 143\ 750$ litres per hour</p> <p>OR</p> <p>$(15.25*75000=)1143750$ litres per hour</p>	
		(L) Calculate total water consumption for individuals in the community	<p>One attempt from the calculations below:</p> <p>Multiplying 300000 by their 51 to find the amount of water consumed per day by the individuals $Ex: 300\ 000 \times their51 (=15\ 300\ 000)$ litres per day</p> <p>OR</p> <p>Dividing by 24 to find amount of water consumed per individual per hour $Their51/24(=2.125)$ l/h</p> <p>OR</p> <p>Calculating how much litres for every individual during the 4 hours outage time</p> <p>OR</p> <p>Acceptable estimation for the amount of water used by individuals per hour (between 630000 and 650000 l/h) without calculations</p>	<p>Calculate correctly the amount of water used by individuals per hour: $Ex: (15300000/24=)637500$ litres per hour</p> <p>OR</p> <p>$(2.125*300000=)637500$ litres per hour</p>	

5	e	Marks	1	2	
		(T) Estimate amount of time before the water runs out	<p>Attempt to calculate estimate for time: Household $\frac{950\ 000}{their\ 1\ 143\ 750}$ ($\approx 0.830....$ hrs ≈ 50 mins)</p> <p>OR Individual $\frac{950\ 000}{their\ 637\ 500}$ ($\approx 1.49....$ hrs ≥ 89 mins)</p> <p>OR attempt for household and individuals but wrong result $\frac{950\ 000}{their\ 1\ 143\ 750 + their\ 637\ 500}$ ($\approx not\ 0.5$)</p>	<p>Calculating their estimate combining both household and individual information correctly</p> <p>$\frac{950\ 000}{their\ 1\ 143\ 750 + their\ 637\ 500}$ $\approx 0.5(33....)$ hrs ≈ 30 to 35 mins</p>	
		(A) Advice for the community	<p>Advice related to minimizing the use in general</p> <p>OR Advise related to minimize/stop one specific activity</p>	<p>Advice requesting clearly to stop/reduce at least two of the following activities that consume too much water: Showers, wash dishes, washing clothes, or garden watering</p>	
		(J) Justification of degree of accuracy	<p>Weak justification not supported</p> <p>Examples: This is just an estimate</p> <p>OR We never know for sure the actual amount</p> <p>OR This is just an average</p> <p>OR Correct and sensible rounding for their value(s) of time Example 30 min or 0.5 hours</p> <p>Do not accept: "my time is very accurate because I made the correct calculations"</p>	<p>Good justification supported</p> <p>Examples: Comment that the time calculated is if all population do not respond to advice and that the estimate of time can be more if they respond to advice</p> <p>OR Calculate time if they respond to advice</p> <p>OR the number of persons per household is an average so not accurate</p> <p>OR Referring to fact that during the specific 4 hours of outage not all activities considered in calculations are actually performed</p>	
ATTENTION: Seeing the total consumption for households and individuals ($1\ 143\ 750 + 637\ 500 =$) $1\ 781\ 250$ l/h allows the award of H2 and L2					

Notes Ignore incorrect units Apply ECF in E and J				Total 10 marks
6	Marks	1	2	
	C	Calculates the area of the Circle $\pi \times 10^2$ method/formula eg finds area of circle. Seen or implied at T		
	T	Calculation for the equilateral Triangle $\frac{1}{6}$ of circle seen OR $3 \times \frac{1}{6}$ of circle seen OR $\frac{1}{2}$ of circle seen	Their area of three sectors (50π) OR their 157 (.0796327)	
	A	Correct Area for the triangle without a signal OR 15.9(...) OR 16 OR answer to 173 - their 157 (.0796327)		
	S	Calculation for the Square without a signal OR $4 \times \frac{1}{4}$ of circle seen OR 100π	400 – $\pi \times 10^2$ sets up subtraction OR 85.8(...) OR 86	
	E	Comparison of the Efficiency of the two ways the transmitters have been positioned OR Triangle has a smaller area than the square (Their 85.8 – their 15.9)	Triangle is smaller in area than the square AND Triangle has a smaller area without signal than the square (Their 85.8 – their 15.9)	
	J	Justification of the most efficient layout <ul style="list-style-type: none">• Triangle has 9.2% not covered• Square has 21.45% not covered• Double the triangle to 346 and its 31.86 not covered• Doubling the triangle shows that there is a lot smaller area not covered than the square	Two numerical statement/comparison from eg <ul style="list-style-type: none">• Triangle has 9.2% not covered• Square has 21.45% not covered• Double the triangle to 346 and its 31.86 not covered• Doubling the triangle shows that there is a lot smaller area not covered than the square	

Task 3

Answers			Notes						Total
7	a	<ul style="list-style-type: none"> •¹ Any three correct: award 1 mark •² All correct 							
	b	$y_c = -\frac{1}{4k}$							1
	c	<ul style="list-style-type: none"> •¹ Completing the pattern for another value not in the table $k \geq 6$ adding 4 to denominator ($20+4=24$ seen) •² Calculating $y_c = -\frac{1}{4(6)} = -\frac{1}{24}$ •³ Comment that they are equal 	Award 1 mark for testing a value from the table $k \leq 5$ but only if the three verify steps are seen <ul style="list-style-type: none"> • value from the table • calculate from the rule • say they are the same/values are correct. 					3	
	d	<p>Alternative method 1</p> <ul style="list-style-type: none"> •¹ $\frac{y_A - y_C}{x_A - x_C} = 1$ and $\frac{y_A - y_C}{\frac{1}{2k} - 0} = 1$ •² $\frac{k(x_A)^2 - y_C}{\frac{1}{2k} - 0} = 1$ OR $\frac{k\left(\frac{1}{2k}\right)^2 - y_C}{\frac{1}{2k} - 0} = 1$ •³ $k(x_A)^2 - y_C = \frac{1}{2k}$ OR $k\left(\frac{1}{2k}\right)^2 - y_C = \frac{1}{2k}$ $y_C = -\frac{1}{4k} \text{ AG}$	<ul style="list-style-type: none"> •¹ Gradient formula = 1 and correct substitution for x_A •² Correctly substitutes $y_A = k(x_A)^2$ or $y_A = k\left(\frac{1}{2k}\right)^2$ •³ further working to $y_C = -\frac{1}{4k}$ $y_C = -\frac{1}{4k}$ is seen at part b) and here it is the same as AG 					3	

Answers			Notes	Total
7 d	<p>Alternative method 2</p> <p>•¹ $\frac{k(x_A)^2 - y_C}{\frac{1}{2k} - 0} = 1$</p> <p>•² $k(x_A)^2 - y_C = \frac{1}{2k}$</p> <p>•³ $k\left(\frac{1}{2k}\right)^2 - y_C = \frac{1}{2k}$</p> $-y_C = \frac{1}{2k} - k\left(\frac{1}{2k}\right)^2$ $-\frac{1}{4k} = y_C$ <p>AG</p>	<p>•¹ Gradient formula = 1 and correct substitution for x_A and $y_A = k(x_A)^2$</p> <p>•² Cross multiplies correctly</p> <p>•³ Substitutes $k\left(\frac{1}{2k}\right)^2$ for $k(x_A)^2$ and further working to $y_C = -\frac{1}{4k}$</p> <p>$y_C = -\frac{1}{4k}$ is seen at part b) and here it is the same as AG</p> <p>SC</p> <p>Award 2 mark for a "proof" similar to example below. It is based on $y_A = -y_C$ which has been found by inspection.</p> $\frac{y_A - y_C}{x_A - x_C} = 1$ $\frac{-y_C - y_C}{\frac{1}{2k} - 0} = 1$ $-2y_C = \frac{1}{2k}$ $-\frac{1}{4k} = y_C$		

Answers			Notes	Total
7	e	x_c has the same value as p	$x_c = p$	1
	f	<p>Alternative method 1</p> <ul style="list-style-type: none"> •¹ The denominators of y_c are always 8 •² The numerator of y_c goes up by 8 <p>Alternative method 2</p> <ul style="list-style-type: none"> •¹ It is an arithmetic progression •² The common difference is 1 	ACCEPT: The values of y_c go up by 1 : award 2 marks	2
	g	<ul style="list-style-type: none"> •¹ The rule can be obtained by substituting $k = 2$ into $-1/4k$ and then testing/trial and error OR by inspection $0 - 1/8; 1 - 1/8$ etc •² $y_c = \frac{8q-1}{8}$ OR $y_c = q - \frac{1}{8}$ OR If no working seen award 1 for each part 	<ul style="list-style-type: none"> •¹ for suitable working •² working can be implied by a correct answer. 	2

7	h	Mark	Predictions P	Description D	Testing T	Verifying V	Justify/proof J	Notation and terminology N	Communication C	
1		Attempted to make predictions	Attempted to describe a pattern	Attempted to test their described pattern OR general rule	Attempted to verify their described pattern or general rule	Attempted to justify their described pattern OR general rule	The notation OR terminology are not always correct	No communication. Only calculations or algebraic steps		
2		Correctly predicted one term	Correctly described one pattern OR a simple pattern	Tested correctly their described pattern	Verified correctly their described pattern OR general rule	Justified their general rule correctly	The notation and terminology are correct Award only if D3 is awarded	Some coherent communication		22
3		Correctly predicted more than one term Accept predictions with $p = 0$ OR $q = 0$	Correctly described more than one pattern OR one complex pattern	Uses a correct general rule to generate a value given in the table Award only if D5 is awarded		Attempted to prove their general rule		Good coherent communication Award only if J2 is achieved		
4			Attempted to describe pattern as general rule			Correctly proved their general rule				
5			Correctly described pattern as general rule							

Exemplification

-Prediction: Makes entries in the table. Accept predictions with $p = 0$ or $q = 0$

-Description of the pattern: $x_c = p$
 y_c denominator multiple of 4 ($4k$), numerator $4kq - 1$ $y_c \uparrow q$

-General rule : $x_c = p$ and $y_c = -1/4k + q$ OR $y_c = \frac{-1+4kq}{4k}$ OR $y_c = q - \frac{1}{4k}$

- Testing the rule using values given in the table

Ex. $k=2$, $p=2$, $q=1$ $x_c = 2$ $y_c = 7/8 \Rightarrow x_c = p = 2$ and this agrees with table $y_c = q - \frac{1}{4k} \quad y_c = 1 - \frac{1}{4(2)} = \frac{7}{8}$ and same as the table

- Verifying the rule by one value not given in the table

Ex. $k = 4$, $p = 2$, $q = 1$, $x_c = 2$ $y_c = 15/16 \Rightarrow x_c = p = 2$ and this agrees my table $y_c = q - \frac{1}{4k} \quad y_c = 1 - \frac{1}{4(4)} = \frac{15}{16}$ and this agrees with the table

- Proving

$$C(0, y_c) \text{ where } y_c = -1/4k \text{ so } C(0, -1/4k)$$

After (p, q) is applied $\Rightarrow C$ become $C(0 + p, -1/4k + q)$

$$\text{So } x_c = p \text{ and } y_c = -1/k + q \text{ OR } y_c = \frac{-1+4kq}{4k}$$