ZNOTES.ORG

UPDATED TO 2020 SYLLABUS

IB MIDDLE YEARS PROGRAM
MATHS

SUMMARIZED NOTES ON THE THEORY SYLLABUS

1. Number Systems

1.1. Forms of Numbers

- Irrational number - a real number which cannot be written in the form p/q, where p and q are integers and q is not 0
- Rational number - number that can be expressed as the quotient or fraction p/q of two integers, a numerator p and a non-zero denominator q
  - Since q may equal 1, every integer is rational
- Recurring decimals - to simplify a recurring decimal

$$x = 0.7777...$$
$$10x = 7.7777...$$
$$x = 0.7777...$$
$$9x = 7 \qquad x = \frac{7}{9}$$

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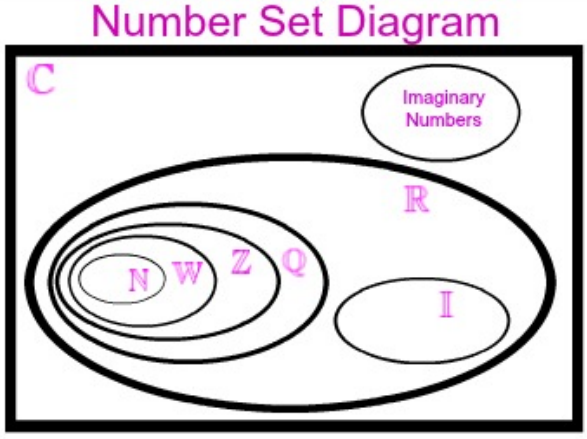
- Real numbers - a number which can be expressed on the number line



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- Radical - an expression with the radical symbol
  - A surd is an irrational radical
    - If the number under the radical sign can be written as a perfect square, then the radical is rational
  - Radicand = number under the radical
  - Principle square root = positive square root
- A radical is in simplest form when the number under the radical sign is the smallest possible integer
- The radical conjugate of a + bc is a - bc
- Know how to divide a quantity into a ratio
- Finding percentage change

Percent Change

$$\text{Percent Change} = \frac{\text{New Value} - \text{Old Value}}{\text{Old Value}} \times 100\%$$

If the result is positive, it is an increase.
If the result is negative, it is a decrease.

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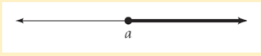
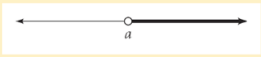
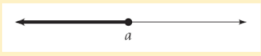
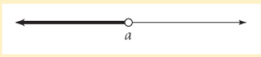
1.2. Sets and Venn Diagrams

- Set - a collection of distinct numbers or objects
  - Element - a number, letter, point, object, etc... contained in a set
  - Subset () - A is a subset of B, if every element of A is also an element of B
    - Proper subset () - all elements of B are in A, but A contains at least one elements that is not contained in B
  - Universal set - set of all numbers under consideration
  - Complement of A is A' - contains all elements of U that are not in A
    - {x | xA, xU}
- Set notation -
  - If set E = {even numbers} = (2, 4, 6, 8, ...}
  - If set V = {vowels} = {a, e, i, o, u}
  - Finite vs. infinite set
  - Empty set - { } contains no elements
- Intersection - contains elements in A and B
- Union - contains elements which are in A or B
- Two sets are disjoint or mutually exclusive if that have no elements in common; AB=
- Venn diagrams - an illustration of the relationships between and among sets, groups of objects that share something in common
  - When representing a set on a venn diagram, if we are only interested in the number of elements in a set, we write the number or elements in each section in brackets
    - When writing this out, we would write n(A) = x
- Set identities

name	*rule*
Commutative laws	$A \cap B = B \cap A \qquad A \cup B = B \cup A$
Associative laws	$A \cap (B \cap C) = (A \cap B) \cap C$ $A \cup (B \cup C) = (A \cup B) \cup C$
Distributive laws	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
DeMorgan's laws	$\overline{A \cap B} = \overline{A} \cup \overline{B} \qquad \overline{A \cup B} = \overline{A} \cap \overline{B}$
Complement laws	$A \cap \overline{A} = \emptyset \qquad A \cup \overline{A} = U$
Double complement law	$\overline{\overline{A}} = A$
Idempotent laws	$A \cap A = A \qquad A \cup A = A$
Absorption laws	$A \cap (A \cup B) = A \qquad A \cup (A \cap B) = A$
Dominance laws	$A \cap \emptyset = \emptyset \qquad A \cup U = U$
Identity laws	$A \cup \emptyset = A \qquad A \cap U = A$

- Interval notation

Infinite Intervals

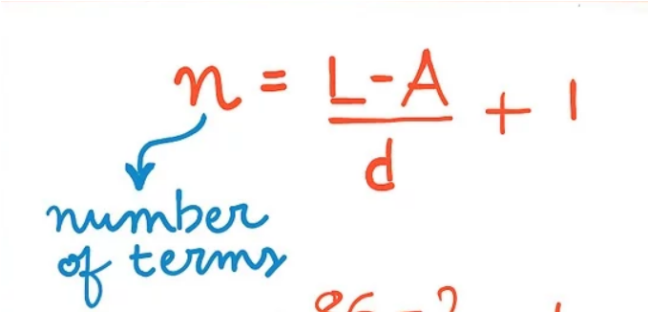
Interval Notation	Set Notation	Graph	Type
$[a, \infty)$	$\{x \mid x \geq a\}$		Closed
(a, ∞)	$\{x \mid x > a\}$		Open
$(-\infty, a]$	$\{x \mid x \leq a\}$		Closed
$(-\infty, a)$	$\{x \mid x < a\}$		Open

1.3. Direct and Inverse Proportion

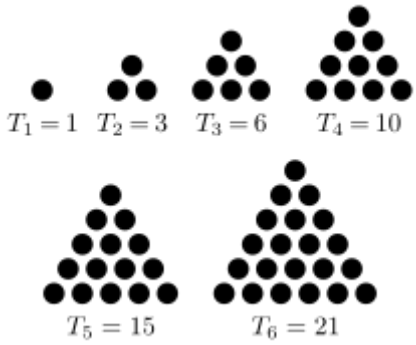
- Two variables are directly proportional if multiplying one of them by a number results in the other one being multiplied by the same number
  - y x→ y=kx
    - k is the proportionality constant
    - When y is graphed against x, the graph is a straight line with gradient k, which passes through the origin
- Two variables are inversely proportional if, when one is multiplied by a constant, the other is divided by the same constant
  - y 1x→ y=kx
    - The graph forms part of a hyperbola

1.4. Number Sequences

- Sum of first "n" terms in an arithmetic sequence
  - s_n = $\frac{n}{2}$(a1-an)
  - Where a_n = a_1 + (n-1)d



- Know how to solve linear, quadratic, triangular and exponential sequences
  - To prove a rule for a sequence, just plug in a value into the rule and cross check
- Triangular sequence



The Fibonacci Sequence

1,1,2,3,5,8,13,21,34,55,89,144,233,377...

1+1=2	13+21=34
1+2=3	21+34=55
2+3=5	34+55=89
3+5=8	55+89=144
5+8=13	89+144=233
8+13=21	144+233=377

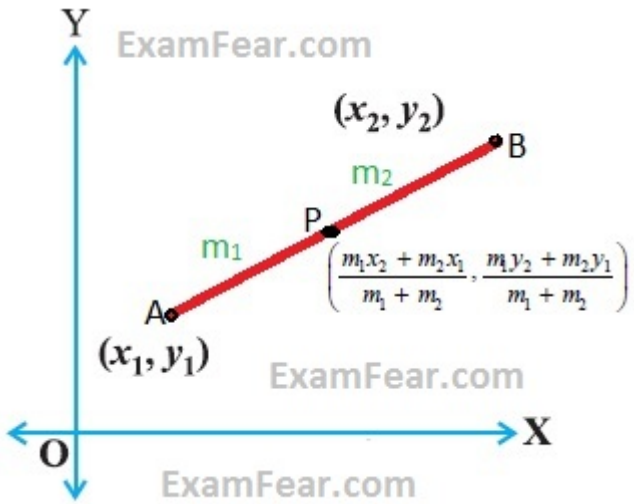
Property Name	Definition	Example
Product of Powers	$a^m \cdot a^n = a^{m+n}$	$5^{1/2} \cdot 5^{3/2} = 5^{(1/2+3/2)}$
Power of a Power	$(a^m)^n = a^{mn}$	$(3^{5/2})^2 = 3^{(5/2 \cdot 2)}$
Power of a Product	$(ab)^n = a^n b^n$	$(16 \cdot 9)^{1/2} = 16^{1/2} \cdot 9^{1/2}$
Negative Exponent	$a^{-m} = \frac{1}{a^m}, a \neq 0$	$36^{-1/2} = \frac{1}{36^{1/2}}$
Zero Exponent	$a^0 = 1, a \neq 0$	$213^0 = 1$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{4^{5/2}}{4^{1/2}} = 4^{(5/2-1/2)}$
Power of a Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$	$\left(\frac{27}{64}\right)^{1/3} = \frac{27^{1/3}}{64^{1/3}}$

2. Geometry

2.1. Distances

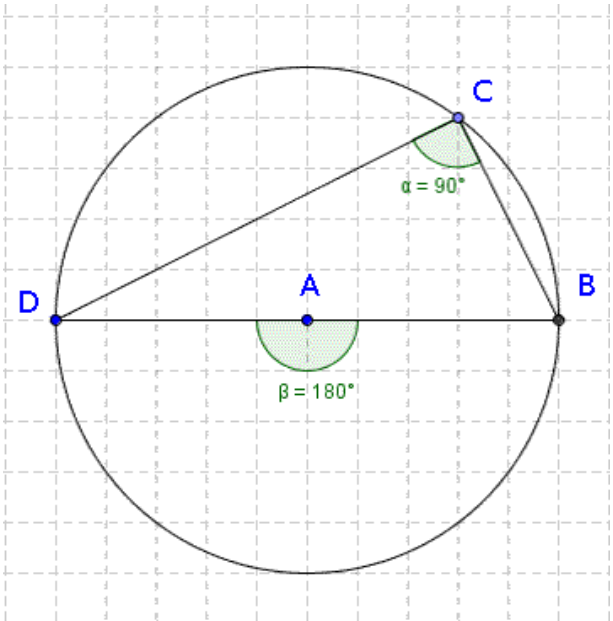
- The distance formula - the distance between 2 points, A (x_1, y_1) and B (x_2, y_2) is given by: $AB = \sqrt{(x_2 - x_1) + (y_2 - y_1)}$
- The midpoint formula - If A (x_1, y_1) and B (x_2, y_2) are two points, then the coordinates of the midpoint of AB will be ($\frac{x_2+x_1}{2}$, $\frac{y_2+y_1}{2}$)
- Gradient - the measure of steepness of a line m is given by the formula $m = \frac{y_2-y_1}{x_2-x_1}$
  - 2 parallel lines will always have the same slope/gradient
  - If 2 lines are perpendicular, then their slopes are negative reciprocals
    - ex: $m_1 = 4$ and $m_2 = -\frac{1}{4}$
- Equation of a line passing through 2 points (x1, y1) and (x2, y2) where m = $\frac{y_2-y_1}{x_2-x_1}$ is given by $y - y_1 = m(x -$

$x_1)$
  - $y - b = m(x - a)$
- Gradient intercept form - where m = slope and c = y-intercept, $y = mx + c$
- General Form - $Ax + By = C$
- 3 or more points are collinear if they lie on the same straight line
- Distance from a point to a line - the distance from a point (m,n) to the line Ax + By + C = 0 is given by $d = \frac{Am+Bn+C}{\sqrt{(a^2+b^2)}}$
- section formula - gives the coordinates of a point which divides the line joining two points in a ratio
  - $(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2})$

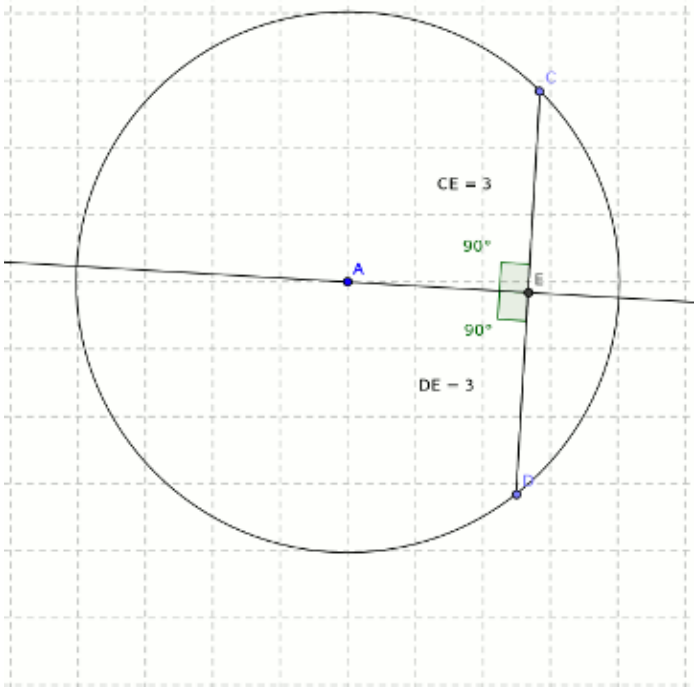


2.2. Circle Theorems and Angle Properties

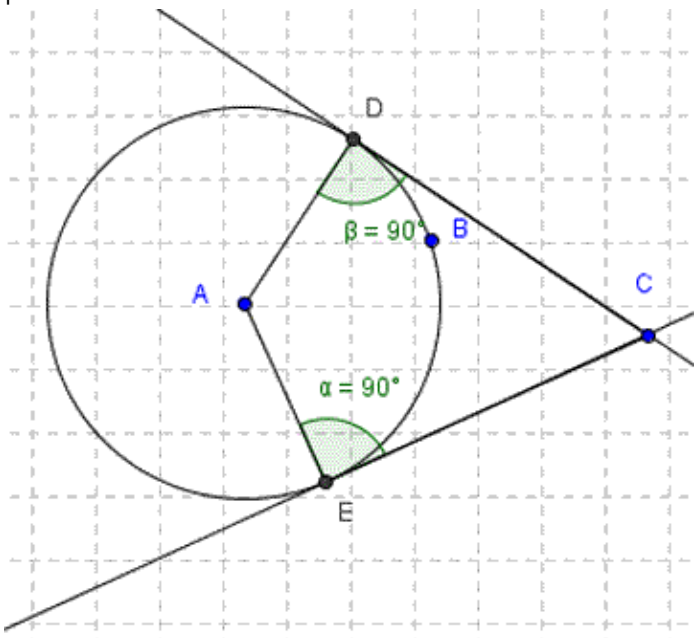
1. The angle inside a semicircle is a right angle



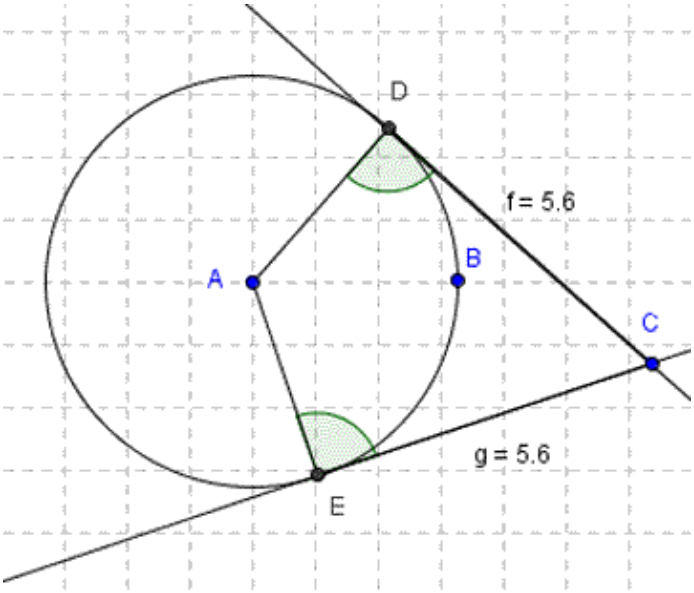
2. The perpendicular from the center of a circle to a chord bisects the chord



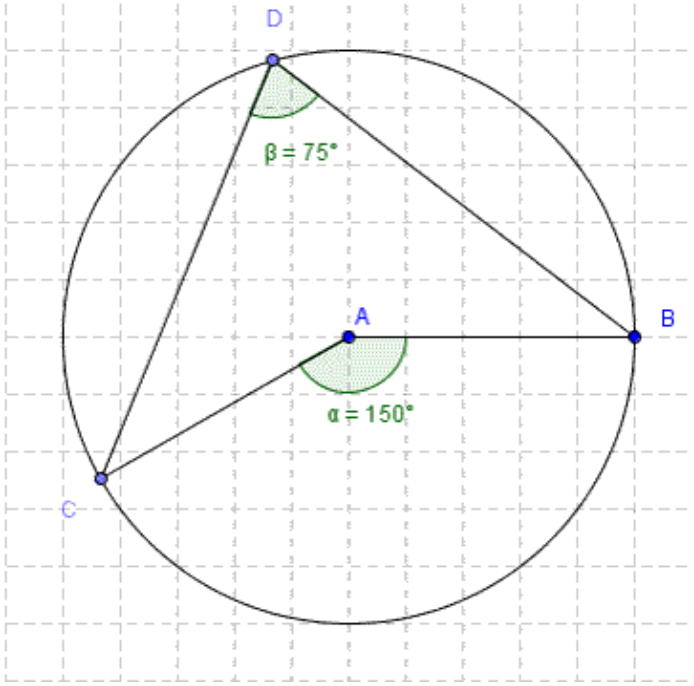
3. The tangent to a circle is perpendicular to the radius at the point of chord



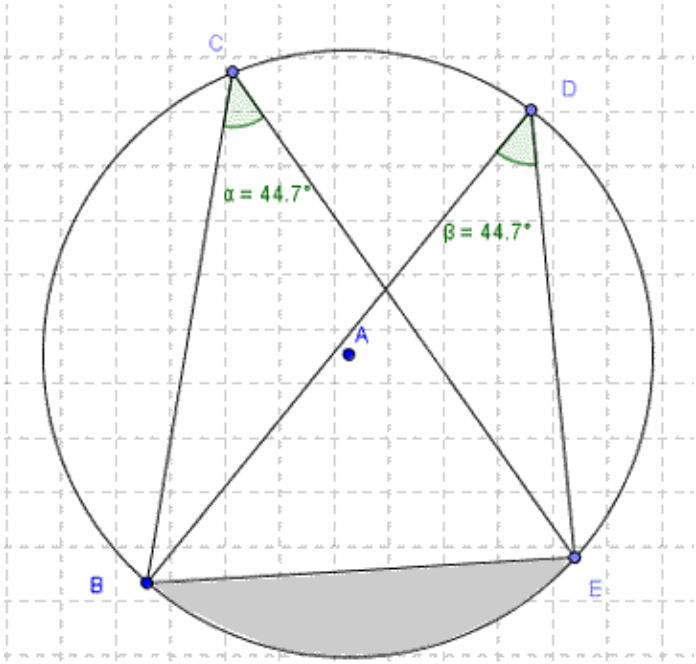
4. Tangents from an external point are equal in length



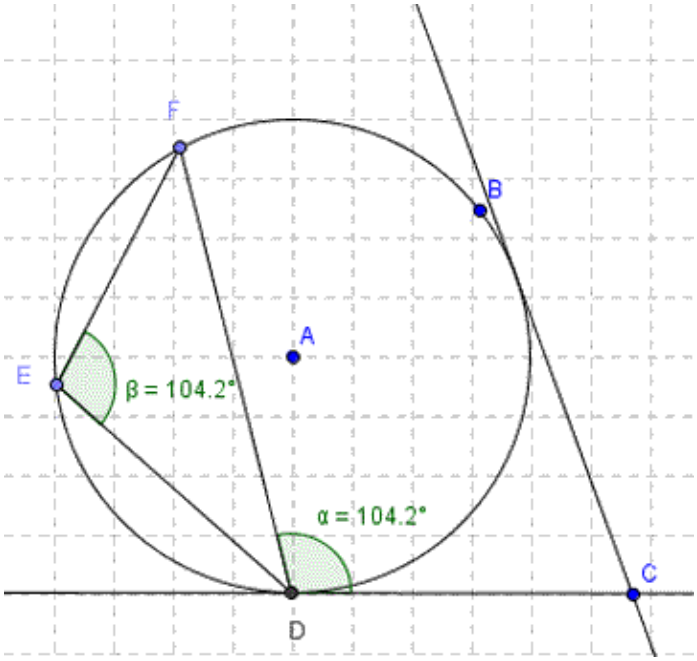
5. The angle at the center of a circle is twice the angle of the circumference subtended by the same arc



6. Angles subtended by an arc on the circumference are equal in size

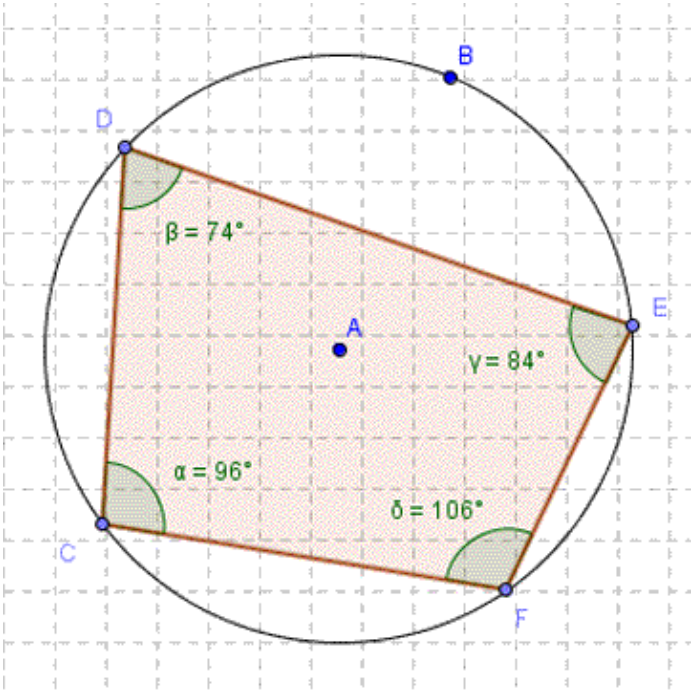


7. The angle between a tangent and a chord at the point of contact equals an angle on the circumference subtended by the same chord. This is known as the alternate segment theorem.



8. Cyclic quadrilaterals - 4 points are said to be concyclic if a circle can be drawn through them. If 4 concyclic points are joined to form a convex quadrilateral then it's called a cyclic quadrilateral. The opposite angles of a cyclic quadrilateral are

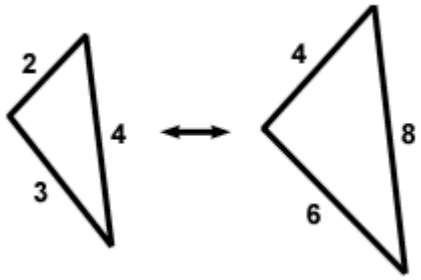
supplementary.



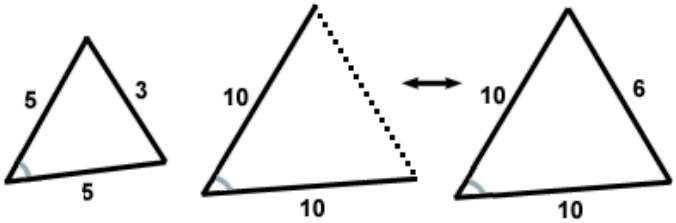
2.3. Similarity and Congruence

Congruence Criteria for Triangles

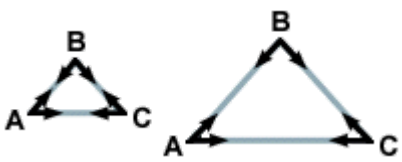
1. SSS



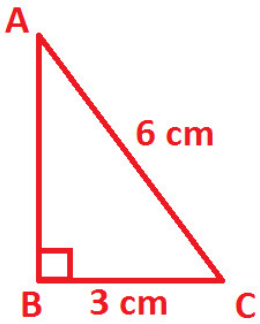
1. SAS

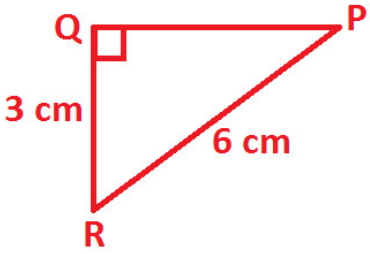


1. ASA



1. RHS





Similarity of Two Triangles

1. Two figures are similar if one is an enlargement of the other. They have the same shape but different size.
2. Two figures are similar if
    1. They are equiangular AND
    2. The corresponding side lengths are in the same ratio
3. Two triangles are similar if
    1. They are equiangular OR
    2. The corresponding side lengths are in the same ratio

Similarity Postulates

1. Two triangles are similar if -
    1. They are equiangular; the angles of both the triangles are equal OR
    2. The corresponding sides of both triangles have the same ratio OR
    3. Two pairs of corresponding sides have the same ratio and the included angle is equal

Scale Factors

- SCALE FACTOR IS ALWAYS $\frac{New}{Old}$
- For a side or a line, the new length is k old length
- If a figure is enlarged with the scale factor k, to produce a similar figure, the the new area is K^2 old area
- Similarly, for a 3D figure, the new volume is k^3 * old volume

3. Trigonometry

3.1. Sine, Cosine and Tangent

- The hypotenuse is the longest side of the triangle, the opposite is the side opposite to theta and the adjacent is the side that touches theta that is not the hypotenuse.
- sin = $\frac{opposite}{hypotenuse}$
- cos = $\frac{adjacent}{hypotenuse}$
- tan = $\frac{opposite}{adjacent}$

3.2. Trigonometric Identities

1. cscθ = $\frac{1}{sin\theta}$
2. secθ = $\frac{1}{cos\theta}$
3. cotθ = $\frac{1}{tan\theta}$
4. tanθ = $\frac{sin\theta}{cos\theta}$
5. $sin^2\theta + cos^2\theta$ = 1
6. $sec^2\theta + tan^2\theta$ = 1
7. $scs^2\theta + cot^2\theta$ = 1
8. sin (90 - A) = cos A and cos(90 - A) = sin A
9. tan(90 - A) = cot A and cot(90 - A) = tan A
10. sec(90 - A) = cosec A and cosec(90 - A) = sec A

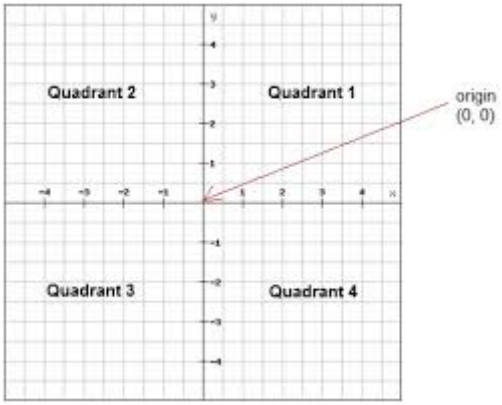
3.3. Values of Sin, Cos, and Tan For Standard Angles

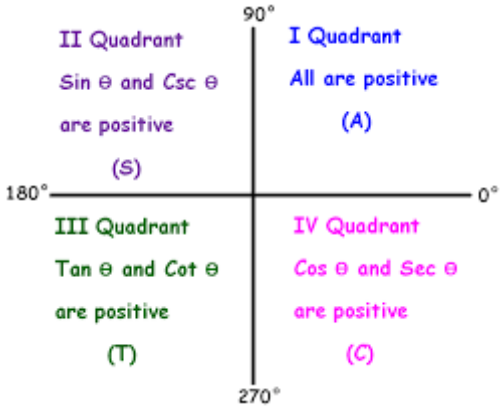
Trigonometry Table

	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec θ	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot θ	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

3.4. The Unit Circle

- Radius = 1
- Coordinates for any point on the circle = (cos A, sin A)
  - A is positive for anticlockwise and negative for clockwise rotations
- All Silver Tea Cups





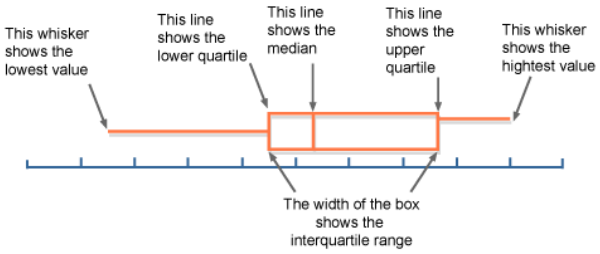
- the complement of A is (90 - A) \n the supplement of A is (180 - A)
- for any angle A
  - cos(90 - A) = sin A \n sin(90 - A) = cos A
  - cos(180 - A) = -cos A \n sin(180 - A) = sin A
  - cos(-A) = cos A \n sin(-A) = -sin A
- cos^2 A + sin^2 A = 1
- sec^2 A - tan^2 A = 1
- csc^2 A - cot^2 A = 1
- trigonometric function - a function which involves one of the trig ratios
  - sin graph starts at 0
  - cos graph starts at 1
  - period, amplitude, max point, min point, principal axis/mean line
- properties of a sine function: y = A sin Bx + C
  - the graph starts at 0
  - the amplitude is | A |
  - the period is 360/B for B>0
    - in radians, the period = 2pi/|B|
  - the principal axis is y = C
- cosine function: y = A cos Bx + C
  - the graph starts at 1
- 1 pi radian = 180 degrees \n 2 pi radians = 360 degrees

4. Statistics

4.1. Box and Whisker Plots

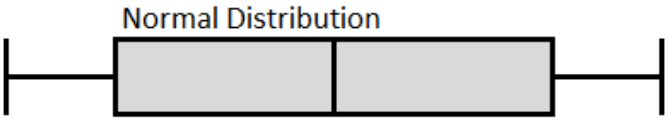
- A box and whisker plot displays a data set along a number line

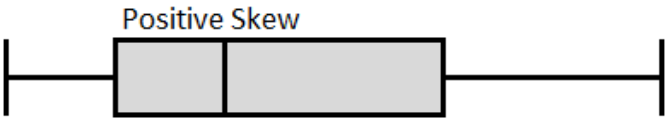
- It summarizes data using the median, upper quartile, lower quartile, and the range of values of the data set
- Quartiles divide the data set into two halves
  - the median of the lower half is the first quartile and the median of the upper half is the third quartile
- 5 point summary - median, Q1, Q3, IQ, range

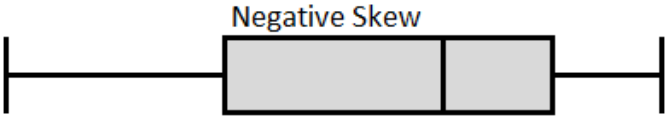


Skewed Data

- When data are skewed, the majority of the data are located on the high or low side of the graph
- Skewness indicates that the data isn't normally distributed
- If the distribution is normal, the mean will be the same as the median and the box plot will look symmetric
- If the distribution is skewed to the right, most of the values are small, but there are a few exceptionally large one. These will impact the mean and pull it to the right so that mean > median.
- If the distribution is skewed to the left, most values are large, but there are a few exceptionally small ones. These impact the mean and move it to the left, so that mean < median.





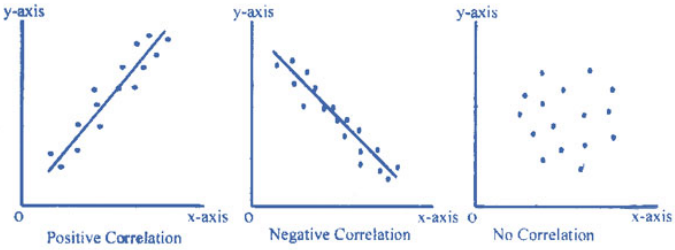


Outliers

- Many data sets contain values that are either extremely high or extremely low compared to the rest of the data values. These values are called the outliers.
- A value is considered an outlier if it is either:
  - less than Q1- (1.5IQR) OR
  - greater than Q3+ (1.5IQR)
  - IQR stands for the interquartile range

4.2. Scatter Plots

- A graph that relates 2 sets of data where the x coordinate is the independent variable and the y coordinate is the dependent variable
- When 2 variables are so related that a change in one is accompanied by a change in the other, they are said to be correlated
- Correlation is a measure of how closely related the 2 sets of data are. The higher the correlation, the closer the data sets are to forming a straight line
- *r is the correlation coefficient which varies between +1.00 and -1.00. The closer r is to 1, the stronger the correlation*
- A correlation of +.1.00 ar -1.00 means that the data forms a perfectly straight line. If *r* is positive, the data has a positive slope and if *r* is negative, the data has a negative slope. If $r > 0.8$ or $r < -0.8$ the correlation is said to be strong
- A line of best fit is a straight line that best represents the data on a scatter plot. This line may pass through some of the points, none of the points or all of the points. It is used to predict data values for a given value of x or y.
- Positive correlation - both variables increase or decrease
- Negative correlation - one increases and the other decreases and vice versa
- No correlation - variables aren't related

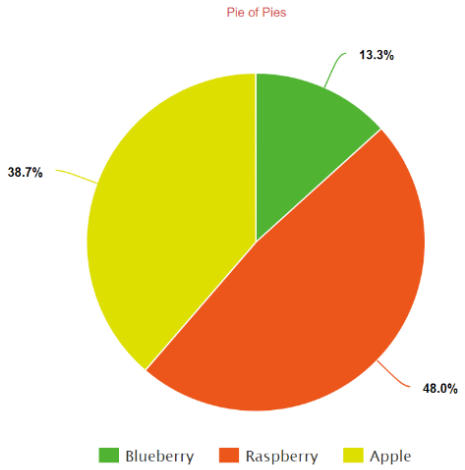


Drawing the Line of Best Fit

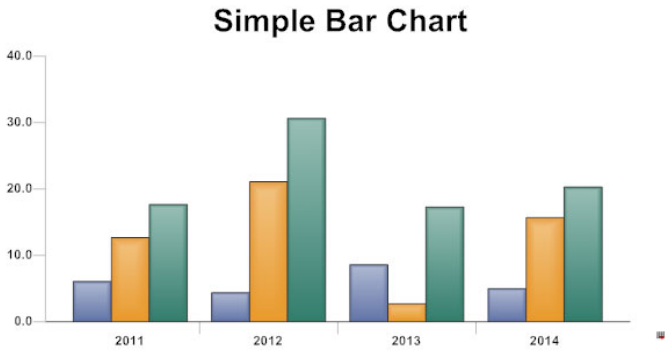
1. Calculate the mean of the x values (represented as x with a line drawn on top) and the mean value of the y values (represented as y with a line drawn on top)
2. Draw the mean point (x,y) on the scatter diagram
3. Draw a line through the mean point such that about the same number of data points are above the line as below it

Other Types of Charts and Graphs

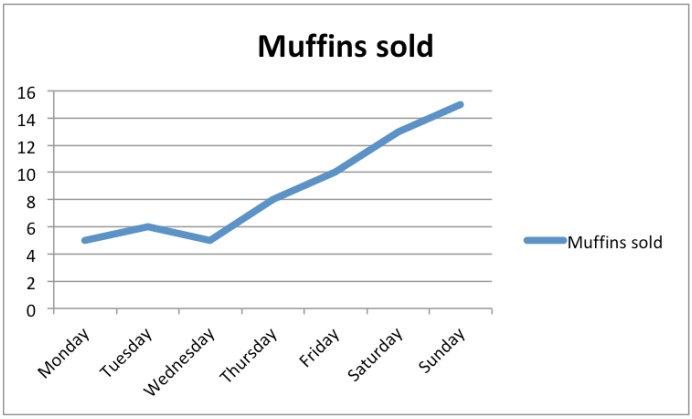
- Pie chart - to construct a pie chart, find the sum of the data, and then divide each section by the total and multiply by 360 to find the degrees that each section should take up



- Bar graph - used to compare data among categories

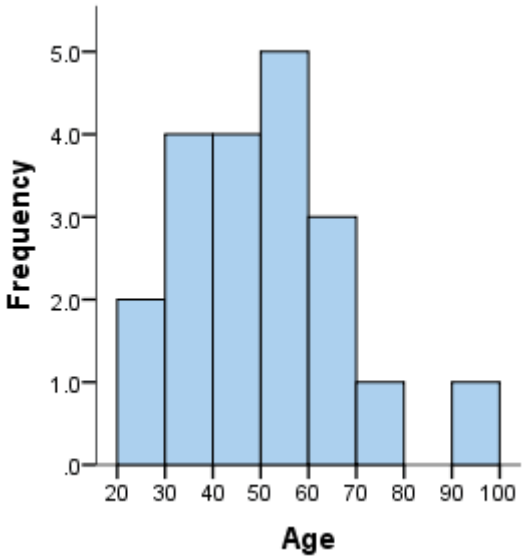


- Line graph - used to visualize the value of something over time



Continuous Data

- Continuous variable - takes values within a certain range
- Data is organized in class intervals
- Frequency histogram is used to display the data



- Modal class - the class of values that appears most often
  - Tallest bar on histogram** \n **

Bimodal - if a data set has two modes, then it is bimodal

Data Collection and Population Sampling

- Data - information recorded for statistical purposes
  - Qualitative data - data that can only be written in words not numbers
  - Quantitative data - data that can be written in numbers
  - Discrete data - numerical data that can't be shown in decimals
  - Continuous data - numerical data that can be shown in decimals
  - Primary data - data that has been collected from the original source for a specific purpose
  - Secondary data - data that is not originally collected by a group for a specific purpose
- Sampling - a selection of data that is used to judge a larger scenario
  - Should only be used when there is a large population
  - Using the entire population
    - All data points are accounted for and results are more reliable
    - Takes longer and can be more expensive
  - Using a sample
    - Quick and cost-effective
    - Can lead to bias, only a small set of data points are considered and different samples may produce different results
  - Random sampling - each data point in the population is equally likely to be selected
    - Can use a random number generator, random number button on a scientific calculator or roll a die
- Stratified sampling - when the groups are of different sizes, the number of items selected from each group will be proportional to the number of items in that group

$$Number\ selected\ from\ each\ strata = \left(\frac{strata\ size}{total\ population}\right) \times sample\ size$$

- Strata size = number of people in each group
- Questionnaires - may give a lot of well-understood information but can be costly and time-consuming
  - Should be careful to avoid wording that may cause bias
  - A data collection sheet can be used to make the data collected easier to read and understand
- Two-way tables - ex:

	Owns a pet	Does not own a pet	Total
Boys	9	2	11
Girls	4	5	9
Total	13	7	20

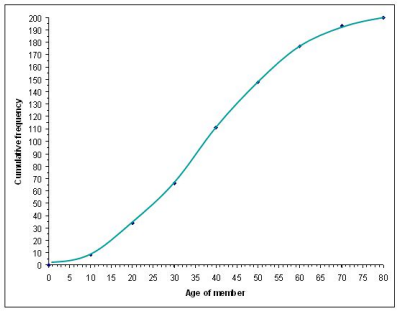
Measures of Central Tendency

- Mean - the arithmetic average; x=xn
- Median - middle value of an ordered set
  - Splits the ordered data set in half - half of the data are less than or equal to the median, and half are greater than or equal to the median
  - If there are n data, the median is n+12
- Mode - most frequently occuring value in the data set
  - If there are two values that occur most frequently, the data set is bimodal
  - If there are more than two modes, we do not use mode as a measure of central tendency
- Percentile - a measure used in statistics indicating the value below which a given percentage of observations in a group of observations falls
- To calculate percentile

1. Order all the values in the data set from smallest to largest.
2. Multiply k percent by the total number of values, n.
   This number is called the index.
3. If the index obtained in Step 2 is not a whole number, round it up to the nearest whole number and go to Step 4a. If the index obtained in Step 2 is a whole number, go to Step 4b.
4. 4a.Count the values in your data set from left to right (from the smallest to the largest value) until you reach the number indicated by Step 3.
   The corresponding value in your data set is the k^{th} percentile.
5. 4b.Count the values in your data set from left to right until you reach the number indicated by Step 2.
   The k^{th} percentile is the average of that corresponding value in your data set and the value that directly follows it.

- Cumulative frequency - the running total of frequencies

We can now use this to draw a cumulative frequency graph.



Age (a years)	Cumulative frequency
a < 10	8
a < 20	34
a < 30	66
a < 40	111
a < 50	148
a < 60	177
a < 70	193
a < 80	200

- Mean of grouped data - we can't find the exact mean, but we can use the midpoint of an interval to represent all of the data points within the interval

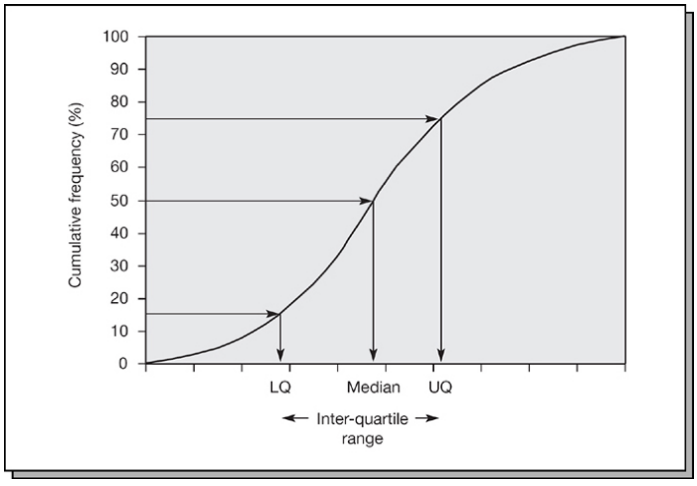
Finding the Mean

Time (minutes)	Midpoint t	Frequency f	t x f
17 ≤ t < 18	17.5	4	17.5 x 4 = 70
18 ≤ t < 19	18.5	7	18.5 x 7 = 129.5
19 ≤ t < 20	19.5	8	19.5 x 8 = 156
20 ≤ t < 21	20.5	13	20.5 x 13 = 266.5
21 ≤ t < 22	21.5	12	21.5 x 12 = 256
22 ≤ t < 23	22.5	9	22.5 x 9 = 202.5
23 ≤ t < 24	23.5	7	23.5 x 7 = 164.5
		Total = 60	Total = 1247

$$mean = \frac{1247}{60}$$

$$= 20.8\ (3s.f.)$$

- Mode of grouped data - whichever class has the highest frequency
- Median and quartiles of grouped data



- When to use each measure of central tendency
  - Median

    - If there are a few extreme scores in the distribution
    - The data set is ordinal
        - Ordinal data is a categorical, statistical data type where the variables have natural, ordered categories and the distances between the categories is not known
- Mean - the data is not skewed
- Mode - the data is nominal (the measurement scale is not numerical)

5. Probability

5.1. Introduction

- Impossible event - 0% chance of happening
- Certain event - 100% chance of happening
- Very likely vs very unlikely

5.2. Experimental Probability

- Number of trials - total number of times that the experiment is repeated
- Outcomes - different results possible for one trial of the experiment
- Frequency - number of times an outcome is observed
- Relative frequency - frequency of outcome divided by total number of trials
    - Relative frequency = $\frac{frequency}{number of trials}$
    - This is the experimental probability
- Two-way tables - tables comparing two variables
- Sample space - set of all possible outcomes of an event
    - Methods of representation of sample space: list, 2-D grid, a tree diagram

5.3. Theoretical Probability

- Event - an outcome with a particular feature or property
    - P(E) = $\frac{number of outcomes corresponding to E}{number of outcomes in sample space}$
    - Complement of event E is the event that E does not occur
        - Denoted as E'
        - For any event E, P(E') = 1-P(E)

5.4. Compound Events

- Independent events - the occurrence of each event does not affect the occurence of the other
    - P(A and B) = P(A) * P(B)
    - P(A or B) = P(A) + P(B)
- Dependent events - the occurrence of one event affects the occurrence of the other
    - P(A and B) = P(A) * P(B given that A has occurred)
    - P(A or B) = P(A) + P(B given that A has occurred)
- Problems may have events that either have replacement or no replacement
    - Without replacement - dependent event
    - With replacement - independent event

5.5. Mutually Exclusive and Independent Events

- Mutually exclusive/disjoint events - have no common outcomes
    - P(A and B) = 0
    - P(A or B) = P(A) + P(B)
- Not mutually exclusive events - have common outcomes/a common outcome
    - P(A or B) =P(A) +P(B)-P(A and B)

6. Algebra

6.1. Algebra Basics

- Factorization or linear and quadratic expressions (take common values)
- Substitution and changing the subject of a formula
- Solving equations involving algebraic fractions
- Solving linear, quadratic, and simultaneous equations, both algebraically and graphically

6.2. Quadratics

- Vertex form: y = $a(x-h)^2+k$
    - Vertx = (h,k)
- y = ax^2+bx+c
    - Midpoint: x = -b/2a = h; y → plug in the value of x
    - Axis of symmetry: x = h
- Intercept form: (x-a)(x-b)
- The graph of a quadratic function has a minimum point if it opens upwards, and a maximum point if it opens downwards
- There are three ways to solve a quadratic equation
    - Splitting the middle term
    - Completing the square
    - Using the quadratic formula
- The quadratic formula

$$ax^2+bx+c=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

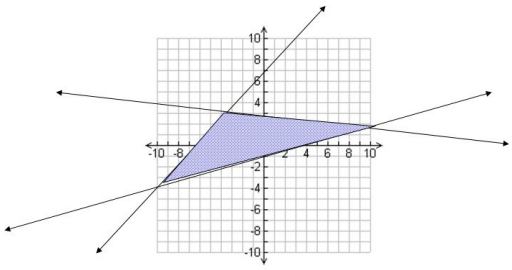
Calcworkshop.com

6.3. Inequalities

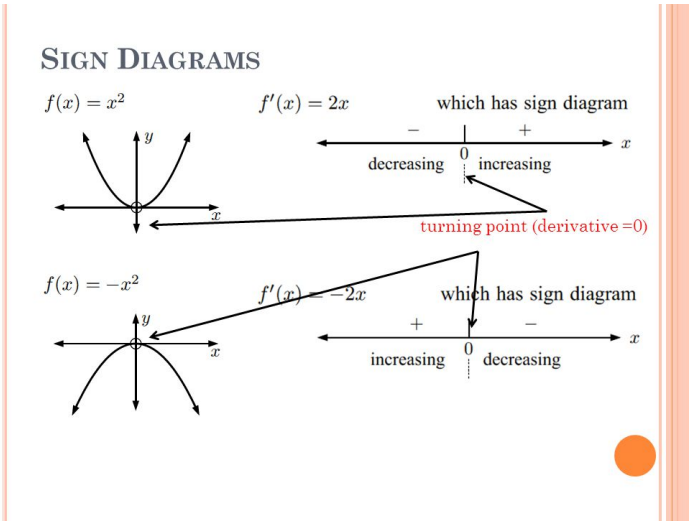
- Square bracket notation
  - Endpoints of an interval are written in square brackets
  - The bracket is reversed if the endpoint is not included
  - The is used to replace “or”
- Rules for solving linear inequalities
  - If we exchange the LHS and RHS, we must flip the sign
  - If we add or subtract the same number from both sides, the inequality is maintained
  - If we multiply or divide both sides by a positive number, the inequality is maintained
  - If we multiply or divide both sides by a positive number, the sign must be flipped
- Feasible region

Feasible Region

The area on the graph where all the answers of the system are graphed. This a bounded region.

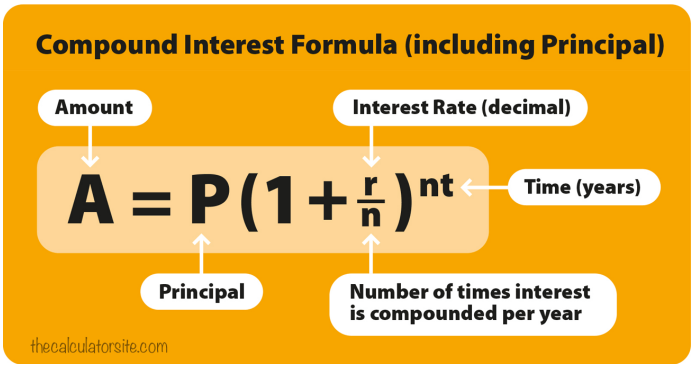


- Also known as the simplex
- Each corner of the simplex is known as a vertex
- Linear programming - a method for finding the optimal value of a linear expression whose variables are contained within a simplex
  - The optimum value of a linear expression over a simplex occurs at a vertex of the simplex
  - If the optimal solution occurs at two of the vertices, the problem has multiple solutions
  - Objective function: x+y = k
    - Multiple solutions exist when the objective function is parallel to one of the constraint lines
- Sign diagrams
  - Consists of
    - A horizontal line representing the x axis
    - Positive (+) and negative (-) signs indicating where the graph is above and below the x axis
    - Critical values which are the graph’s x intercepts, or where it is undefined



6.4. Functions

- Relation - any set of points which connects two variables
  - Domain: set of all possible x values
  - Range: set of all possible y values
- Function - a relation in which no two different ordered pairs have the same first member
  - To find the domain of a function we need to consider what values of x make the function undefined
- Modulus
- Exponential Functions - a function in which the variable occurs as a part of the index or exponent; quantity increases or decreases by a fixed percentage
  - Horizontal asymptote
  - If a>1, then the graph is increasing (growth) \n If a<1 and a>0, then the graph is decreasing (decay)
  - If y = -a^x, then the graph is reflected across the x-axis \n If y = a^-x, then the graph is reflected across the y-axis
  - If y = a^x - k, then the graph moves downwards \n If y = a^x + k, then the graph moves upwards
  - If y = a^(x-k), then the graph moves to the right \n If y = a^(x+k), then the graph moves to the left
  - Graphs of the form k*a^x are vertical dilations
    - The y intercept changes
    - If k>1, the graph moves away from the x-axis
    - If 0<k<1, the graph moves towards the x-axis
  - Graphs of the form y = a^(x/k) are horizontal dilations
    - If k>1, the graph moves away from the y-axis
    - If 0<k<1, the graph moves towards the y-axis
  - If a^x = a^k, then x=k
  - Compound interest:



Rules for Transformation of Linear Functions		
Transformation	**Function**	**Description**
Horizontal Shift	f(x + *h*)	Shift left *h* units
	f(x - *h*)	Shift right *h* units
Vertical Shift	f(x) + *k*	Shift up *k* units
	f(x) - *k*	Shift down *k* units
Reflection	–f(x)	Reflect across x-axis
	f(–x)	Reflect across y-axis
Vertical Stretch/Compress	*a* f(x), *a* > 1	Stretch vertically by a factor of *a*
	a f(x), 0 < *a* < 1	Compress vertically by a factor of *a*
Horizontal Stretch/Compress	f(*a*x), *a* > 1	Compress horizontally by a factor of $\frac{1}{a}$
	f(*a*x), 0 < *a* < 1	Stretch horizontally by a factor of $\frac{1}{a}$

Transformations of Exponential Functions			
Transformation	***f*(*x*) Notation**	**Examples**	
Vertical translation	$f(x) + k$	$y = 2^x + 3$ $y = 2^x - 6$	3 units up 6 units down
Horizontal translation	$f(x - h)$	$y = 2^{x-2}$ $y = 2^{x+1}$	2 units right 1 unit left
Vertical stretch or compression	$af(x)$	$y = 6(2^x)$ $y = \frac{1}{2}(2^x)$	stretch by 6 compression by $\frac{1}{2}$
Horizontal stretch or compression	$f\left(\frac{1}{b}x\right)$	$y = 2^{\left(\frac{1}{5}x\right)}$ $y = 2^{3x}$	stretch by 5 compression by $\frac{1}{3}$
Reflection	$-f(x)$ $f(-x)$	$y = -2^x$ $y = 2^{-x}$	across *x*-axis across *y*-axis

6.5. Algebra Formulae

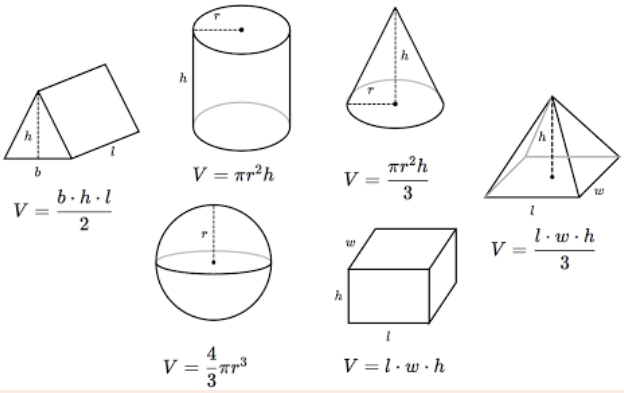
1. $(a+b)^2 = a^2 + 2ab + b^2;\ a^2 + b^2 = (a+b)^2 - 2ab$
2. $(a-b)^2 = a^2 - 2ab + b^2;\ a^2 + b^2 = (a-b)^2 + 2ab$
3. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$
4. $(a+b)^3 = a^3 + b^3 + 3ab(a+b);\ a^3 + b^3 = (a+b)^3 - 3ab(a+b)$
5. $(a-b)^3 = a^3 - b^3 - 3ab(a-b);\ a^3 - b^3 = (a-b)^3 + 3ab(a-b)$
6. $a^2 - b^2 = (a+b)(a-b)$ **Basic Algebraic**
7. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
8. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ **formulas**
9. $a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \cdots + b^{n-1})$
10. $a^n = a.a.a \ldots n$ times

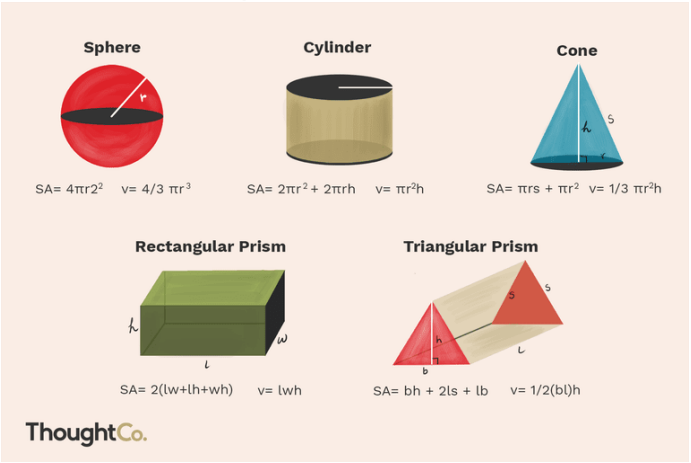
$$
\begin{aligned}
&a^3+b^3+c^3-3abc \\
&=[a^3+b^3]+c^3-3abc \\
&=[(a+b)^3-3ab(a+b)]+c^3-3abc \qquad [Put\ (a+b)=x] \\
&=x^3-3abx+c^3-3abc \\
&=x^3+c^3-3abx-3abc \\
&=(x+c)(x^2+c^2-cx)-3ab(x+c) \\
&=(x+c)[(x^2+c^2-cx-3ab] \qquad [Replace\ x=a+b] \\
&=(a+b+c)[(a+b)^2+c^2-c(a+b)-3ab] \\
&=(a+b+c)[a^2+b^2+2ab+c^2-ca-bc)-3ab] \\
&=(a+b+c)[a^2+b^2+c^2-ab-bc-ca]
\end{aligned}
$$

7. Perimeter, Area and Volume

7.1. Important Formulae

1. Volume of a prism = area of cross section * height
2. Area of triangle = $\frac{1}{2}$ * product of 2 sides * sin of the angle between them
3. Area of a sector = $\frac{\theta}{360} * \pi r^2$
4. Perimeter of a sector = $\frac{\theta}{360}$ * 2πr

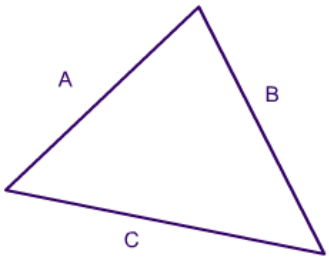




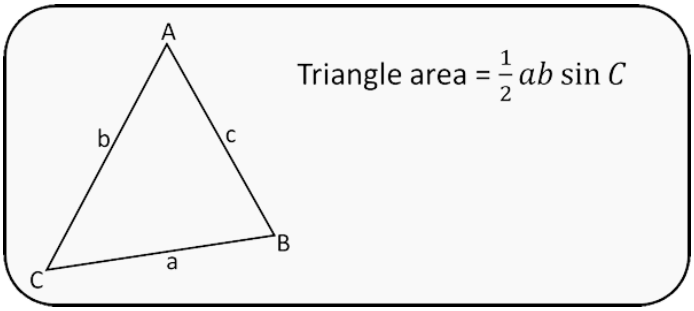
Heron's Formula

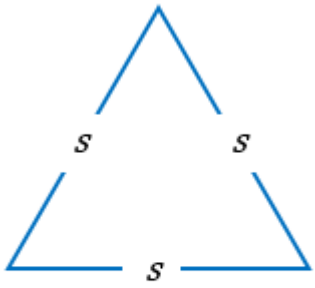
mathwarehouse.com

$$S = \frac{A + B + C}{2}$$



$$\text{Area} = \sqrt{S(S - A)(S - B)(S - C)}$$

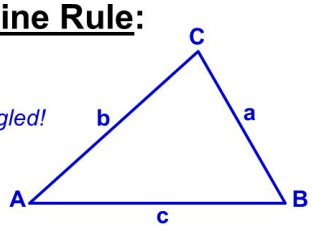




$$A = \frac{\sqrt{3}}{4} s^2$$

The Sine Rule:

Not right-angled!



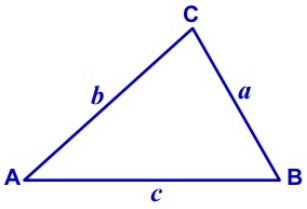
In any triangle *ABC* $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

or $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

- the cosine rule

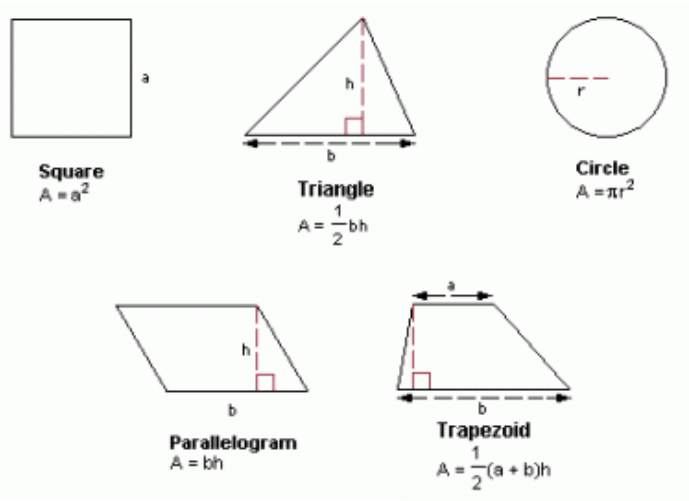
The Cosine Rule:

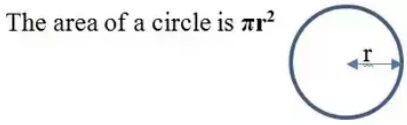
If the triangle is not right-angled, and there is not a matching pair, you will need then Cosine Rule.

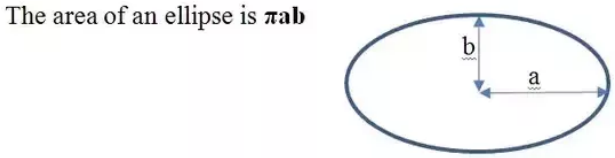


In any triangle *ABC* $a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = b^2 + c^2 - 2bc\cos A$	$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$
$b^2 = a^2 + c^2 - 2ac\cos B$	$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$
$c^2 = a^2 + b^2 - 2ab\cos C$	$\cos C = \frac{a^2 + b^2 - c}{2ab}$







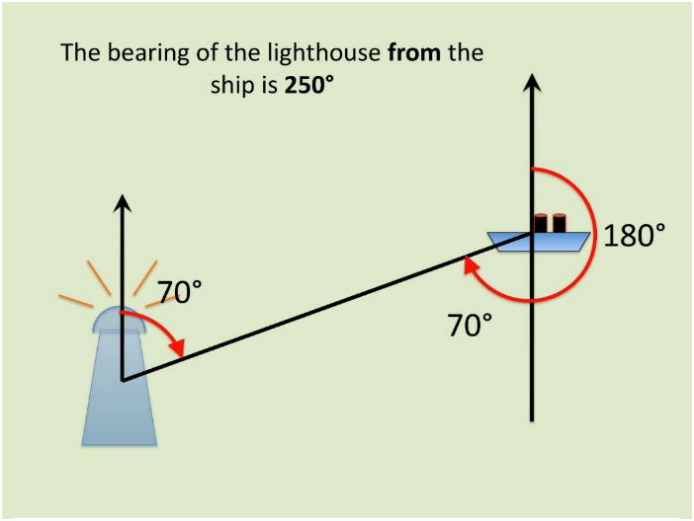
7.2. Frustum

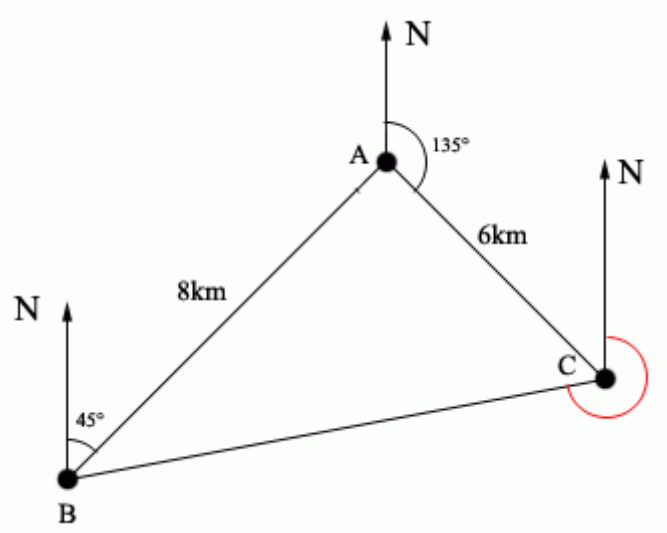
- A Frustum is a portion of a cone or a pyramid which remains after its upper part is cut off by a plane parallel to its base
- VFrustum = VCone(l) - VCone(s)
- CSAFrustum = CSACone(l) - CSACone(s)
- TSAFrustum = TSACone(l) - TSACone(s)

7.3. Pythagoras' Theorem

- In any right angled triangle,the area of the square of the hypotenuse is equal to the sum of the areas of the squares of the other two sides (legs)
- Converse - if a triangle has sides of length a, b and c units where a2+b2=c2, then the triangle is right angled

7.4. Bearings

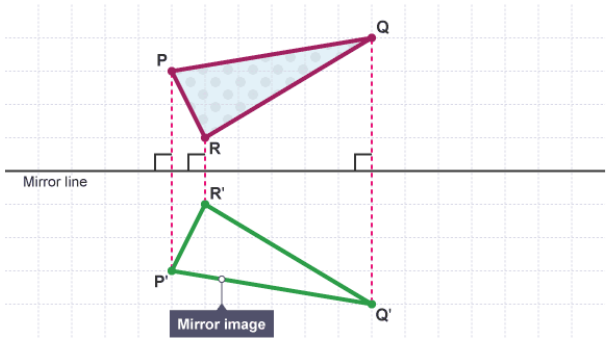




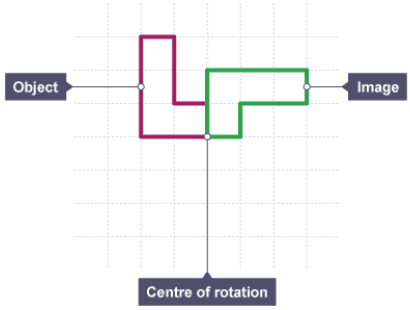
- Bearings are always written in 3 digits; ex: 007, 365, 027

7.5. Transformations of Shapes

- Reflection - line joining new shape and original shape is perpendicular to the mirror line



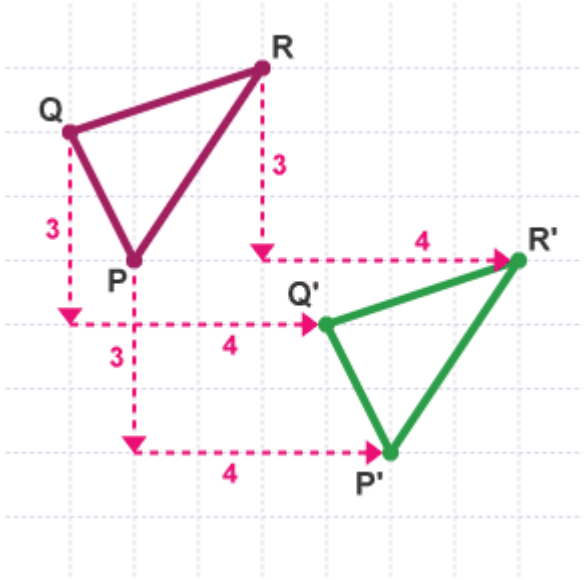
- Rotation - turns a shape around a fixed point known as the center of rotation
  - You need to know the center of rotation, angle of rotation and direction of rotation



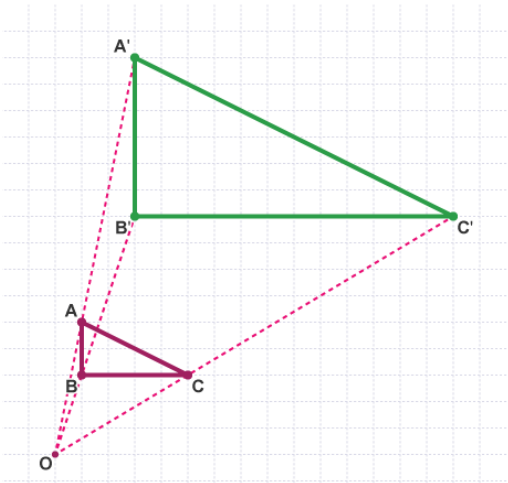
- Each vertex of the new shape is the same distance from the center of rotation as the corresponding vertex of the original shape

- Translation - moves the position of the shape up, down, left, or right
  - Every point is translated the same distance in the same direction
  - Column vector: → $\begin{bmatrix}

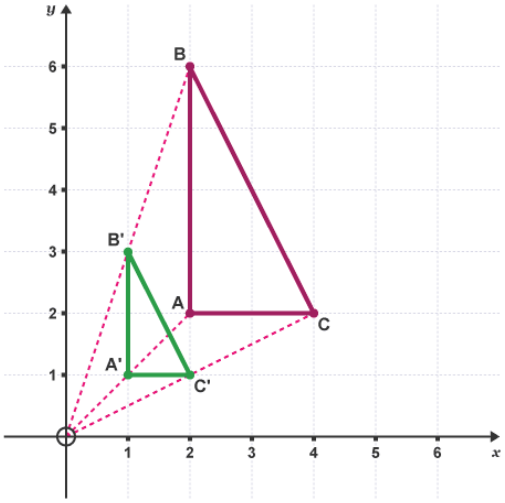
x
y
\end{bmatrix}$ x is horizontal movement, and y is vertical movement



- Enlargement - enlarge the sides of a shape by a scale factor with reference to a center of enlargement

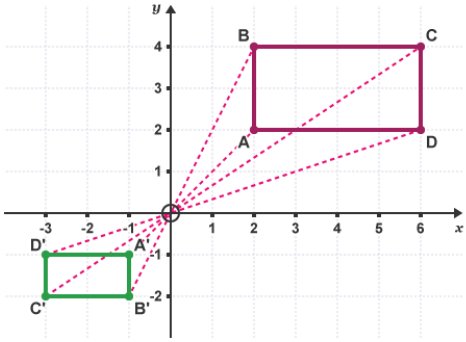


* The lengths in triangle A'B'C' are three times as long as triangle ABC. The distance from O to triangle A'B'C' is three times the distance from O to ABC



* If the scale factor is a fraction, the size will decrease

- Negative enlargements - produce an image on the other side of the center of enlargement



  - Ex: scale factor of -½
  - Scale factor of -1 is the same as a 180 degree rotation

IB MIDDLE YEARS PROGRAM
Maths