## KINEMATICS

1. A stone is dropped from the top of a vertical cliff. At time $t$ seconds after the stone has been dropped, the height, $h$ metres, of the stone above the ground is given by $h = 125 - 5t^2$, $(t \geq 0)$.

(a) Write down the height of the cliff.

(b) Find the value of $t$ when the stone hits the ground.

(c) Find the speed of the stone when $t = 2$. **[May 15R/P1/Q22]**

2. A particle, $P$, is moving along a straight line. At time $t$ seconds, the distance $s$ metres of $P$ from a fixed point $O$ of the line is given by $s = kt^2 - 6t + 3$, where $k$ is a constant and $t \geq 0$.

(a) Given that at $t = 1$, $P$ is momentarily at rest, find the value of $k$.

(b) Find the distance moved in the 3rd second. **[May 13/P1/Q28]**

3. A particle is moving in a straight line through $O$. The displacement, $s$ metres, of the particle from $O$ at time $t$ seconds is given by $s = 3t^2 - 4t + 10$, $t \geq 0$.

(a) Find an expression, in terms of $t$, for the velocity, $v$ m/s, of the particle at time $t$ seconds.

(b) Calculate the value of $t$ when the particle is instantaneously at rest.

(c) Calculate the distance, in metres, travelled by the particle in the fifth second. **[May 14/P2/Q3]**

4. A particle, $P$, is moving along a straight line so that, at time $t$ seconds, the displacement, $s$ metres, of $P$ from a fixed point $O$ of the line is given by $s = 4t^3 - 26t^2 + 40t$. The particle starts at the point $O$ when $t = 0$.

(a) Write down the values of $t$ when $P$ passes through $O$.

(b) Find an expression for the velocity, $v$ m/s, of $P$ at time $t$.

(c) Find the values of $t$ when the velocity of $P$ is zero.

(d) Find the acceleration of $P$ when $t = 3$. **[Jan 15R/P2/Q11]**

5. A particle $P$ is moving along a straight line. The displacement, $s$ metres, of $P$ from a fixed point $O$ on the line at time $t$ seconds is given by $s = 4 + 12t - t^3$, $t \geq 0$.

(a) Write down the distance, in m, of the particle from $O$ at time $t = 0$.

Particle $P$ comes to instantaneous rest at the point $A$.

(b) Find the value of $t$ when $P$ is at $A$.

(c) Find the acceleration, in m/s$^2$, of $P$ when $P$ is at $A$. **[May 14R/P1/Q29]**

6. A particle $P$ is moving along a straight line. At time $t$ seconds, the displacement, $x$ metres, of $P$ from a fixed point $O$ on the line is given by $x = -3t^3 + 6t^2 + kt + 4$, $t \geq 0$. At time $t$ seconds, the velocity of $P$ is $v$ m/s such that $v = 9$ when $t = 2$.

(a) Show that $k = 21$.

Particle $P$ comes to instantaneous rest at the point $A$.

(b) Using $k = 21$, find the value of $t$ when $P$ is at $A$.

(c) Find, to the nearest meter, the distance $OA$. **[May 19/P1/Q28]**

7. A particle $P$ is moving along a straight line. At time $t$ seconds, the displacement, $s$ metres, of $P$ from a fixed point $O$ of the line is given by $s = 6t^3 - t^4$. At time $t$ seconds, the velocity of $P$ is $v$ m/s.

(a) Find an expression for $v$ in terms of $t$.

For $t > 0$, the particle comes to instantaneous rest at the point $A$.

(b) Find the distance, in metres to 3 significant figures, of $A$ from $O$. **[May 16R/P1/Q25]**

8. A particle $P$ is moving along a straight line. At time $t$ seconds, $(t \geq 1)$, the displacement, $s$ metres, of $P$ from a fixed point $O$ of the line is given by $s = t + \frac{4}{t}$. The particle comes to instantaneous rest at the point $A$.

(a) Find the value of $t$ for which $P$ is at $A$.

When $t = 8$, $P$ is at the point $B$.

(b) Find, in metres, the distance $AB$. **[Jan 18R/P1/Q20]**

9. A particle $P$ is moving along a straight line. At time $t$ seconds, the displacement, $x$ metres, of $P$ from a fixed point $O$ on the line is given by $x = 4 + 7t - 2t^2$, $t \geq 0$. At time $t$ seconds, the velocity of $P$ is $v$ m/s .

(a) Find an expression for $v$ in terms of $t$.

In the interval $0 \leq t \leq 4$, $P$ is furthest away from $O$ when $P$ is at the point $A$ on the line.

(b) Find the value of $t$ when $P$ is at the point $A$.

(c) Find the distance, in metres, of $A$ from $O$.

(d) Find the total distance, in metres, travelled by $P$ in the interval $0 \leq t \leq 4$. **[Jan 19R/P1/Q26]**

10. A particle $P$ is moving along a straight line through the fixed point $O$. The displacement, $s$ metres, of $P$ from $O$ at time $t$ seconds is given by $s = t^3 - 27t + 55$, $t \geq 0$.

(a) Write down the distance, in metres, of $P$ from $O$ when $t = 0$.

(b) Find an expression, in terms of $t$, for the velocity, $v$ m/s, of $P$ at time $t$ seconds.

(c) Find the value of $t$ when $P$ is closest to $O$.

(d) Find the distance, in metres, of $P$ from $O$ when $P$ is closest to $O$.

(e) Find the distance, in metres, travelled by $P$ in the interval $0 \leq t \leq 5$. **[Jan 17R/P2/Q4]**

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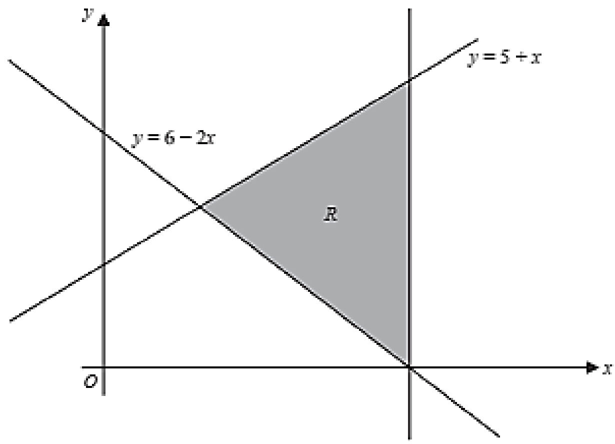
**Answers:**

**[1]** (a) $125$ m, (b) $5$ s, (c) $-20$ m/s, **[2]** (a) $3$, (b) $9$ m, **[3]** (a) $v = 6t - 4$, (b) $\frac{4}{6}$ s, (c) $23$ m, **[4]** (a) $2.5$ s, $4$ s, (b) $v = 12t^2 - 52t + 40$, (c) $\frac{10}{3}$ s, $1$ s, (d) $20$ m/s², **[5]** (a) $4$ m, (b) $2$ s, (c) $-12$ m/s², **[6]** (b) $\frac{7}{3}$ s, (c) 48 m, **[7]** (a) $v = 18t^2 - 4t^3$, (b) $136.69$ m, **[8]** (a) $2$ s, (b) $4.5$ m, **[9]** (a) $v = 7 - 4t$, (b) $\frac{7}{4}$ s, (c) $\frac{81}{8}$ m, (d) $\frac{65}{4}$ m, **[10]** (a) $55$ m, (b) $3t^2 - 27$, (c) $3$ s, (d) $1$ m, (e) $98$ m.

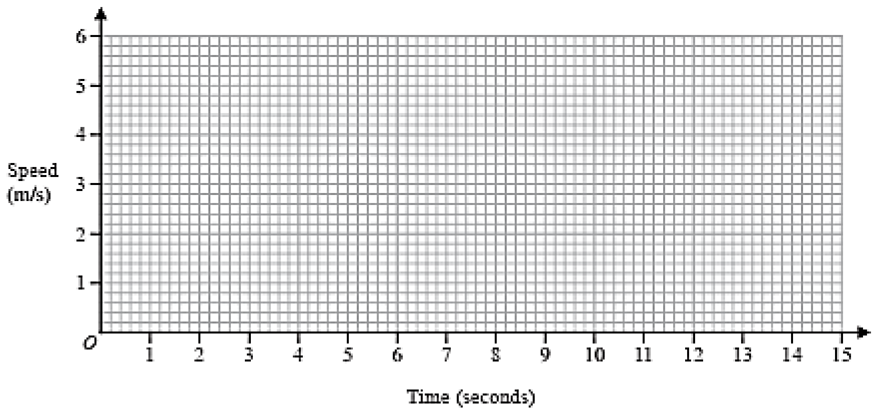
## EQUATION GRAPH

1. The diagram shows the shaded region $R$, which is bounded by three straight lines, one of which is parallel to the $y$-axis. One vertex of $R$ lies on the $x$-axis.

Find three inequalities that define $R$. **[Jan 20/P1/Q15]**




2. At time $t = 0$ seconds, a cyclist passed the point $P$ on a straight horizontal road. The cyclist was moving with a constant speed of $5$ m/s. The cyclist travelled a distance of $35$ m at this speed to the point $Q$ on the road. On reaching $Q$, the cyclist decelerated at a constant rate, coming to rest at the point $R$ on the road such that $PQR$ is a straight line and $QR = 10$ m.




Represent, on the grid, the information for the journey of the cyclist from $P$ to $R$ as a speed-time graph.

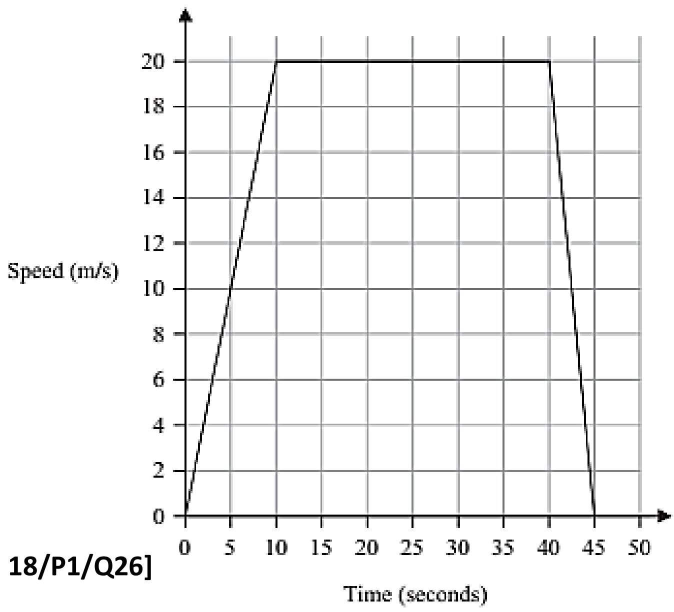
**[Jan 20R/P1/Q27]**

3. A car travels from rest between two sets of traffic lights in $45$ seconds. The speed-time graph below gives information about this journey.

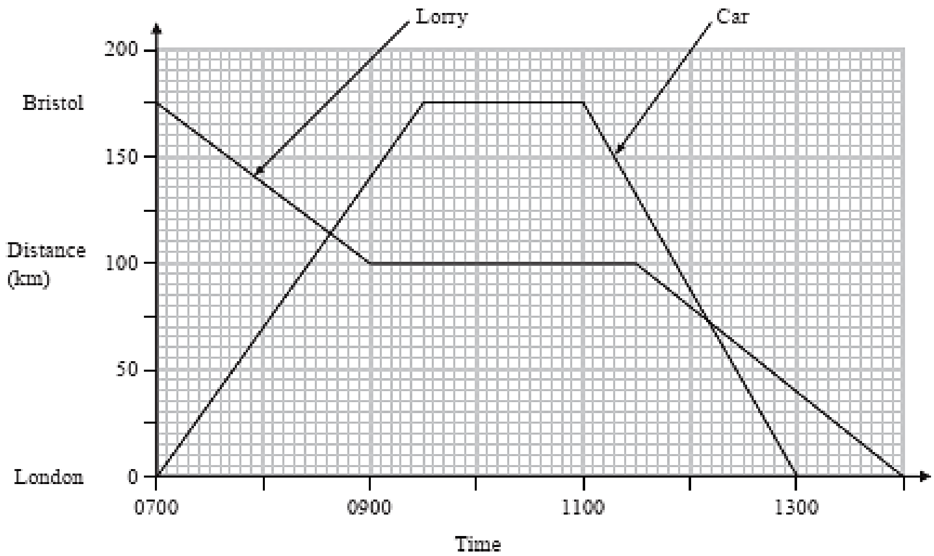
(a) Calculate the acceleration of the car during the first $10$ seconds of its journey.

(b) Find the total distance travelled by the car between the two sets of traffic lights.

(c) Find the average speed of the car on its journey between the two sets of traffic lights. **[Jan 18/P1/Q26]**



4. The distance-time graph for the journey of a car between London and Bristol and the distance-time graph for the journey of a lorry travelling from Bristol to London are shown on the grid. The car and the lorry travel along the same roads.




(a) For how long was the car stationary in Bristol?

(b) Calculate the average speed, in km/h, of the car as it travelled back from Bristol to London.

(c) At what time did the car overtake the lorry when they were both travelling in the same direction?

**[Jan 15/P1/Q18]**

5. A train, starting from rest, accelerates at a constant rate and attains a speed of 30 m/s after 30 seconds. The train then travels at this speed for 5 minutes. The train then slows down at a constant rate and comes to rest in 1 minute.

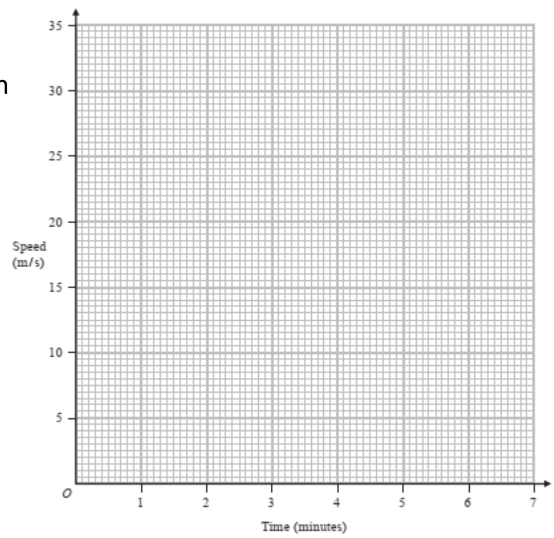
(a) Represent this information on a speed-time graph.

(b) Find the total distance, in metres, travelled by the train.

Find:

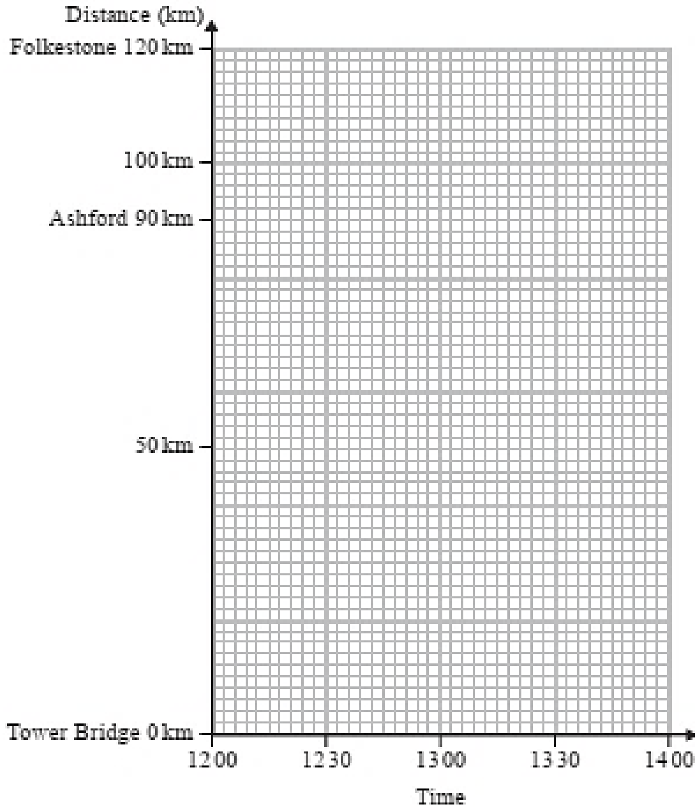
(c) the average speed, in m/s, of the train,

(d) the rate, in $m/s^2$, at which the train slows down.




**[Jan 16/P1/Q28]**

6. A lorry leaves Tower Bridge at 12 00 and is driven at a constant speed for 90 km, arriving in Ashford at 13 15. The lorry is parked in Ashford for 15 minutes. The lorry is then driven at a constant speed to Folkestone. The lorry arrives in Folkestone at 14 00, having been driven a total distance of 120 km.




(a) Represent, on the grid, the journey of the lorry as a distance-time graph. Label your graph clearly.

A car is driven from Folkestone to Tower Bridge through Ashford, along the same roads as the lorry.

The car leaves Folkestone at 1200 and is driven at a constant speed to Tower Bridge, arriving at 13 30.

(b) Represent on the same grid the journey of the car as a distance-time graph. Label your graph clearly.

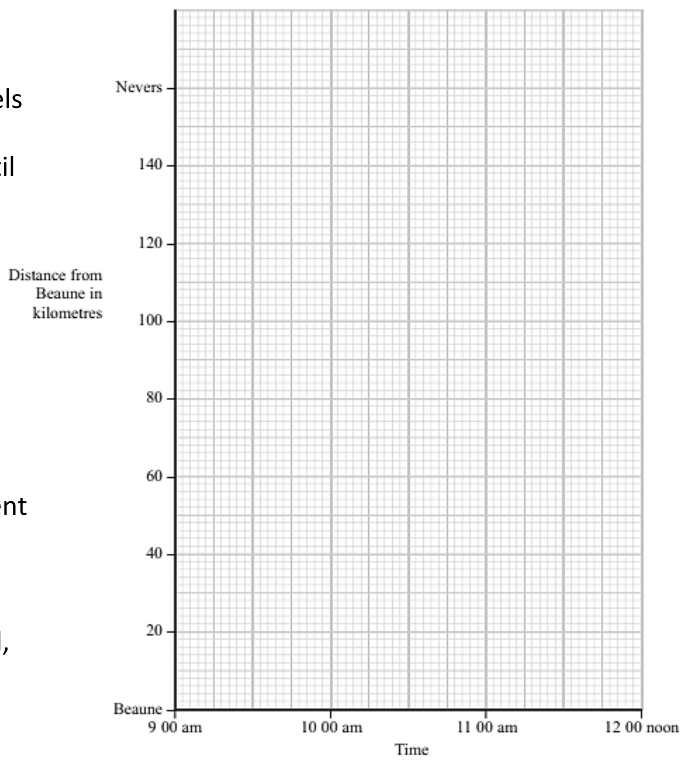
(c) Calculate the speed, in km/h, of the lorry from Tower Bridge to Ashford.

(d) Use your graph to write down the distance between the lorry and the car at 12 30. **[May 16R/P1/Q26]**

7. The distance from Beaune to Nevers is 160 km.

A motorist starts from Beaune at 9 00 am and travels towards Nevers at a constant speed of 64 km/h until he arrives at Autun, which is 48 km from Beaune.

At Autun he rests for 24 minutes before continuing his journey at a constant speed to arrive at Nevers at 11 45 am.




(a) Using the grid opposite, draw a graph to represent the motorist's journey.

(b) Using your graph, calculate the motorist's speed, in km/h, for his journey from Autun to Nevers.

At 9 30 am a second motorist starts from Nevers to journey to Beaune on the same road as the first motorist.

The second motorist travels at a constant speed of 80 km/h.

(c) Draw, on the same graph, a straight line to represent the second motorist's journey.

(d) Using your graph, write down:

(i) the time that the two motorists meet.

(ii) how far both motorists are from Autun when they meet. **[May 15R/P2/Q6]**

8. $f(x) = 4x^3 - 13x - 6$.

(a) Use the factor theorem to show that $(2x + 1)$ is a factor of $f(x)$.

(b) Hence factorize $f(x)$ fully.

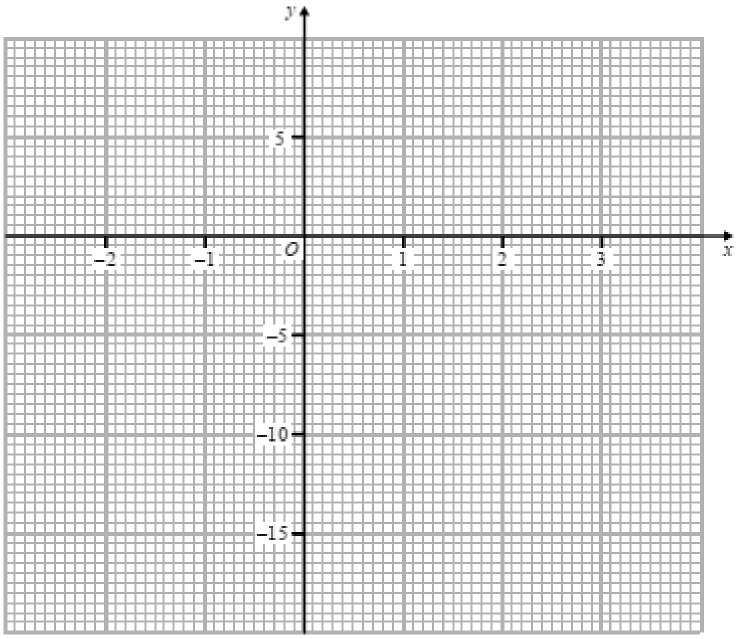
The curve $C$ has equation $y = f(x)$.

(c) Find the coordinates of the points of intersection of $C$ with the $x$-axis.

(d) Find the coordinates, to 2 decimal places, of the turning points of $C$.

The table below gives the coordinates of three points on $C$.

| $x$ | $-2$ | $0.5$ | $1.5$ |
|---|---|---|---|
| $y$ | $-12$ | $-12$ | $-12$ |

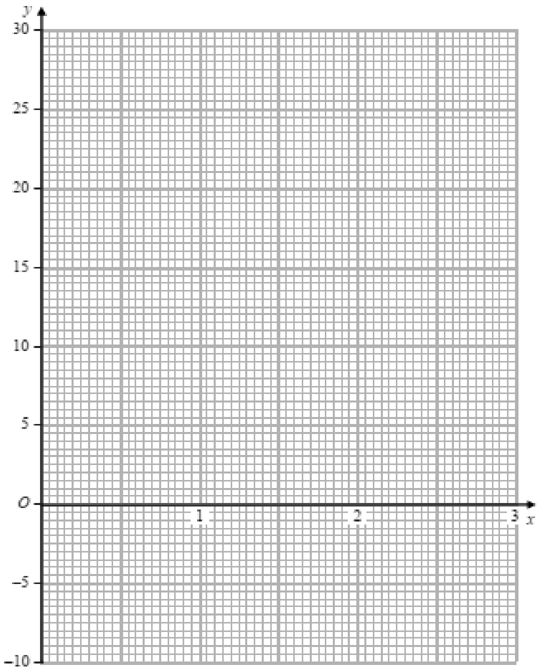



(e) On the grid opposite, draw the curve $C$ for $-2 \le x \le 2$. Clearly label the coordinates of the turning points of $C$ and the coordinates of the points of intersection with the $x$-axis and the $y$-axis.

**[Jan 19/P2/Q11]**

9. (a) Complete the following table of values for $y = 2x^3 - x^2 - 6x$.

| $x$ | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | 0 | | $-5$ | | 0 | 10 | |




(b) On the grid, plot the points from your completed table and join them to form a smooth curve.

(c) Using your graph, find an estimate, to 1 decimal place, of the minimum value of $2x^3 - x^2 - 6x$ in the interval $0 \leq x \leq 3$.

(d) On your grid, draw the straight line with equation $y = 4x - 7$.

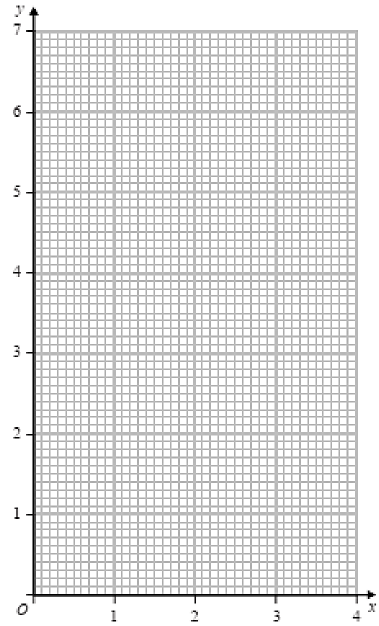
(e) Use your graphs to find the range of values, to 1 decimal place, of $x$ in $0 \leq x \leq 3$ for which $2x^3 - x^2 - 10x + 7 < 0$. **[Jan 19/P2/Q11]**

10. $y = \frac{9}{x^2} + 2x - \frac{8}{x}$

(a) Complete the table of values for $y = \frac{9}{x^2} + 2x - \frac{8}{x}$ giving your values of $y$ to 2 decimal places.

| $x$ | 0.75 | 1 | 1.25 | 1.75 | 2 | 2.5 | 3 | 3.5 |
|---|---|---|---|---|---|---|---|---|
| $y$ | 6.83 | 3 | | | 2.25 | | 4.33 | 5.45 |

(b) On the grid, plot the points from your completed table and join them to form a smooth curve.




(c) Using your curve, write down an estimate, to 2 decimal places, of the value of $x$ for which $\frac{9}{x^2} + 2x - \frac{8}{x}$ has a minimum value in the interval $0.75 \leq x \leq 3.5$.

(d) Show that $4x^3 - 6x^2 - 8x + 9 = 0$ can be written in the form $\frac{9}{x^2} + 2x - \frac{8}{x} = ax + b$ where $a$ and $b$ are integers. Give the value of $a$ and the value of $b$.

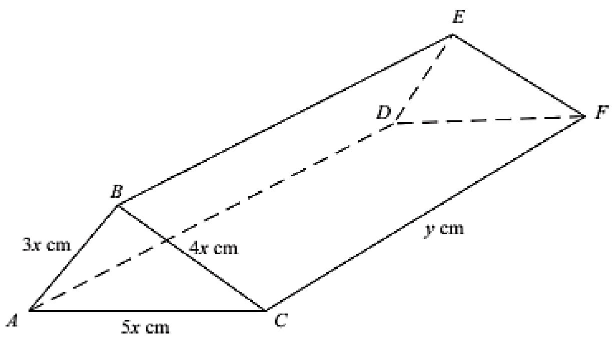
(e) Hence, by drawing a suitable straight line on the grid, find estimates, to 2 decimal places, of the solutions of the equation $4x^3 - 6x^2 - 8x + 9 = 0$ in the interval $0.75 \leq x \leq 3.5$. **[Jan 19R/P2/Q10]**

11. $ABCFED$ is a prism with triangular cross section in which $CF = AD = BE = y$ cm, $AB = DE = 3x$ cm, $BC = EF = 4x$ cm and $AC = DF = 5x$ cm. Given that the total surface area of the prism is $S$ cm$^2$.

(a) (i) Write down the size, in degrees, of $\angle ABC$,

(ii) Show that the area of $\Delta ABC$ is $6x^2$ cm$^2$,

(iii) Find an expression for $S$ in terms of $x$ and $y$.



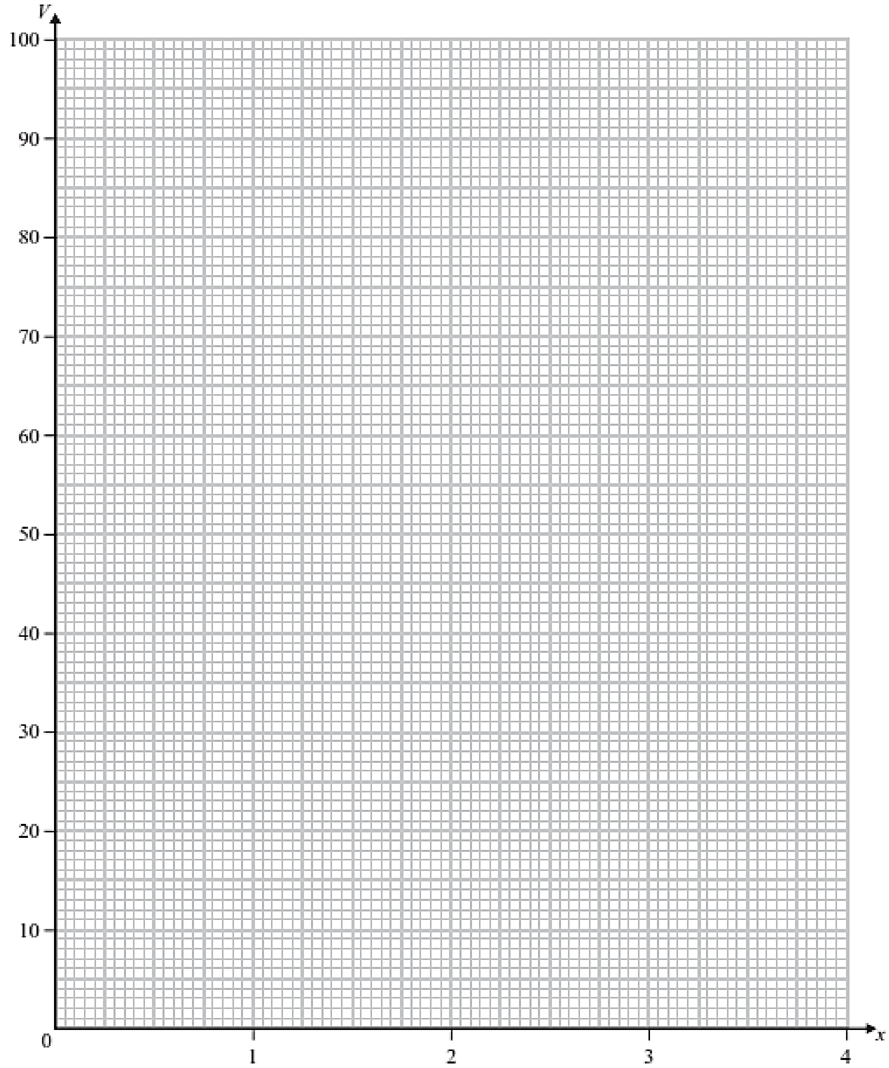

(b) Given also that $S = 144$, show that $y = \frac{12-x^2}{x}$

The volume of the prism is $V$ cm$^3$.

(c) Show that $V = 6x(12 - x^2)$.

(d) Complete the table for $V = 6x(12 - x^2)$, giving the values of $V$ to 1 decimal place where necessary.

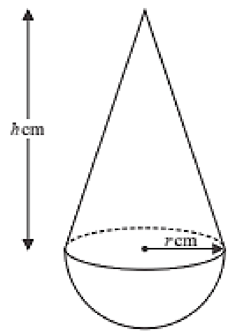
| $x$ | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 | 3.4 |
|---|---|---|---|---|---|---|---|---|
| $V$ | 0 | | 66 | | | | 54 | 9.0 |




(e) On the graph paper, plot the points from your completed table and join them to form a smooth curve.

(f) Using your graph, write down the maximum value of $V$. **[Jan 13/P2/Q9]**

12. A child's toy is made by fixing a solid right circular cone, with base radius $r$cm and height $h$ cm, on the flat circular face of a solid hemisphere of radius $r$cm. The centre of the base of the cone coincides with the centre of the hemisphere, as shown in figure.




Given that $h + 6r = 15$.

(a) Find the upper bound for the value of $r$. Give a reason for your answer.

The volume of the toy is $V$ cm$^3$.

(b) Show that $V = \frac{1}{3}\pi r^2(15 - 4r)$.

(c) Complete, to 1 decimal place, the table of values for $V = \frac{1}{3}\pi r^2(15 - 4r)$.

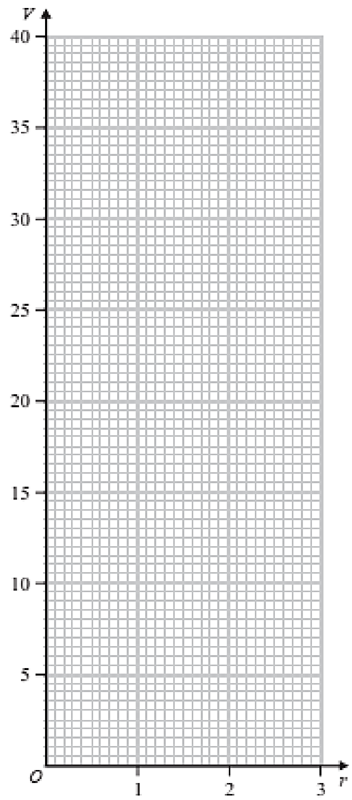
| $r$ | 0 | 0.4 | 0.8 | 1.2 | 1.6 | 2.0 | 2.4 |
|---|---|---|---|---|---|---|---|
| $V$ | 0 | | 7.9 | | | 29.3 | 32.6 |

(d) On the grid, plot the points from your completed table and join them to form a smooth curve.

The volume of one particular toy is 26 cm$^3$.

(e) Use your curve to find, to one decimal place, the value of $r$.

The manufacturer of the toy decides that the value of $V$ should be twice the value of $h$.




(f) By drawing a suitable straight line on the grid, find an estimate, to the nearest integer, for the value of $V$.

**[May 19/P2/Q11]**

**Answers:**

**[1]** $x \leq 3, y \leq 5 + x, y \geq 6 - 2x$, **[3]** (a) 2 m/s$^2$, (b) 750 m, (c) $\frac{50}{3}$ m/s, **[4]** (a) 90 mins, (b) 87.5 km/h, (c) $12:12$, **[5]** (b) $10350$ m, (c) $27$ m/s, (d) $\frac{1}{2}$ m/s$^2$, **[6]** (c) 72 km/h, (d) 44 km, **[7]** (b) 70 km/h, (d) (i) $10:33$, (ii) 28 km, **[8]** (a) $(2x+1)$, (b) $(2x+1)(2x+3)(x-2)$, (c) $\left(-\frac{1}{2}, 0\right)$, $\left(-\frac{3}{2}, 0\right)$, $(2, 0)$, (d) $(1.04, -15.02)$, $(-1.04, 3.02)$, **[9]** (a) $-3, -4.5, 27$, (c) $-5.0$, (e) $x > 0.7$, $0.8 < x < 2.1$, **[10]** (a) $1.86, 1.87, 3.24$, (c) $1.46$, (d) $a = -2, b = 6$, (e) $0.88, 1.93$, **[11]** (a) (i) $90°$, (ii) $S = 12x^2 + 12xy$, (d) $35.3, 87.8, 96, 86.3$, (f) 96 cm$^3$, **[12]** (a) 2.5 cm, (b) $V = \frac{1}{3}\pi r^2(15 - 4r)$, (c) 2.2, 15.4, 23.1, (f) 16 cm$^3$.

# KINEMATICS

1) $h = 125 - 5t^2 \qquad (t \geq 0)$

a) $h = 125$ m high

b) $h = 0$,

$$125 - 5t^2 = 0$$

$$t = +5\text{ s} \quad (t \geq 0)$$

c) $v = \dfrac{dh}{dt} = -10t$

at $t = 2$ s,

$$v = -10(2) = -20 \text{ m/s}$$

2) $s = kt^2 - 6t + 3$ for $t \geq 0$;

a) $v = \frac{ds}{dt} = 2kt - 6$

at $t = 1s$, $v = 0$;

$\Rightarrow 2k(1) - 6 = 0$

$\therefore k = 3$

b) distance moved in the third second $= s_3 - s_2$

$= [3(3)^2 - 6(3) + 3] - [3(2)^2 - 6(2) + 3]$

$= 9m$

3) $s = 3t^2 - 4t + 10$ for $t \geq 0$;

a) $v = \frac{ds}{dt} = 6t - 4$ m/s

b) at instantaneous rest, $v = 0$;

$\Rightarrow 6t \quad 4 = 0$

$\therefore t = 2/3$ s

c) distance travelled in

the fifth second $= s_5 - s_4$

$= \left[3(5)^2 - 4(5) + 10\right] - \left[3(4)^2 - 4(4) + 10\right]$

$= 23$ m

4) $s = 4t^3 - 26t^2 + 40t$

a) at 0, $s = 0$,

$\Rightarrow 2t(2t^2 - 13t + 20) = 0$

$\Rightarrow 2t(2t-5)(t-4) = 0$

$\therefore t = 0s, 2.5s, 4s$.

b) $v = \frac{ds}{dt} = 12t^2 - 52t + 40$ m/s

c) $v = 0$,

$\Rightarrow 12t^2 - 52t + 40 = 0$

$\Rightarrow 4(3t^2 - 13t + 10) = 0$

$\Rightarrow 4(3t-10)(t-1) = 0$

$\therefore t = 1s, 10/3 s$

d) $a = \frac{dv}{dt} = 24t - 52$ m/s$^2$

at $t = 3$,

$a = 24(3) - 52$

$= 20$ m/s$^2$

5) $s = 4 + 12t - t^3$ for $t \geqslant 0$

a) at $t = 0$,

$s = 4\,m$

b) at instantaneous rest, $v = 0$;

$$v = \frac{ds}{dt} = 12 - 3t^2$$

$\Rightarrow 12 - 3t^2 = 0$

$\therefore t = +2s \quad (t \geqslant 0)$

c) $$a = \frac{dv}{dt} = -6t$$

at $t = 2s$,

$a = -6(2) = -12\ m/s^2$

6) $x = -3t^3 + 6t^2 + kt + 4$ for $t \geqslant 0$

a) $v = \frac{dx}{dt} = -9t^2 + 12t + k$

given that $v = 9$ at $t = 2$,

$\Rightarrow 9 = -9(2)^2 + 12(2) + k$

$\Rightarrow k = 21$ (shown)

b) $v = -9t^2 + 12t + 21$

at instantaneous rest,

$v = 0$;

$-9t^2 + 12t + 21 = 0$

$-3[3t^2 - 4t - 7] = 0$

$-3(3t-7)(t+1) = 0$

$t = 7/3\,s$, $\underline{\underline{-1\,s}}$ (N/A)

c) $OA = [-3(7/3)^3 + 6(7/3)^2 + 21(7/3) + 4]$

$= 47.6\,m$

$\simeq 48\,m$

7) $s = 6t^3 - t^4$

a) $v = \frac{ds}{dt} = 18t^2 - 4t^3$ m/s

b) at instantaneous rest, $v = 0$;

$\Rightarrow 18t^2 - 4t^3 = 0$

$\Rightarrow 2t^2(9 - 2t) = 0$

$\Rightarrow t \neq 0,\ t = 9/2$ s

at $t = 9/2$ s,

$s = 6(9/2)^3 - (9/2)^4$

$= 137$ m (3 s.f.)

8) $s = t + \frac{4}{t}$ for $t \geq 1$

a) $v = \frac{ds}{dt} = 1 - \frac{4}{t^2}$

at instantaneous rest, $v = 0$;

$1 - \frac{4}{t^2} = 0$

$\Rightarrow t = +2s \quad (t \geq 1)$

b) at $t = 8$,

$s_8 = 8 + \frac{4}{8} = 8.5\,m$ (point B from O)

at $t = 2$,

$s_2 = 2 + \frac{4}{2} = 4m$ (point A from O)

$\therefore AB = s_8 - s_2$

$= 8.5 - 4$

$= 4.5\,m$

9) $x = 4 + 7t - 2t^2$ for $t \geqslant 0$

a) $v = \dfrac{dx}{dt} = 7 - 4t$ m/s

b) for furthest distance reached,

$v = 0$

$\Rightarrow 7 - 4t = 0$

$t = 7/4$ s at point A

c) at $t = 7/4$ s,

$x = 4 + 7(7/4) - 2(7/4)^2$

$= \dfrac{81}{8}$ m from O.

d) total distance $= \left[ x_t \right]_0^{7/4} + \left[ x_t \right]_4^{7/4}$

$= \left[ 4 + 7t - 2t^2 \right]_0^{7/4} + \left[ 4 + 7t - 2t^2 \right]_4^{7/4}$

$= \left[ \dfrac{81}{8} - 4 \right] + \left[ \dfrac{81}{8} - 0 \right]$

$= \dfrac{65}{4}$ m

10) $s = t^3 - 27t + 55$ for $t \geq 0$

a) at $t = 0$,

$s = (0)^3 - 27(0) + 55 = 55\,m$

b) $v = \frac{ds}{dt} = 3t^2 - 27\ m/s$

c) for closest distance, $v = 0$,

$\Rightarrow 3t^2 - 27 = 0$

$\Rightarrow t^2 = 9$

$\therefore t = +3s\ (t \geq 0)$

d) at $t = 3$,

$s = (3)^3 - 27(3) + 55$

$= 1\,m$

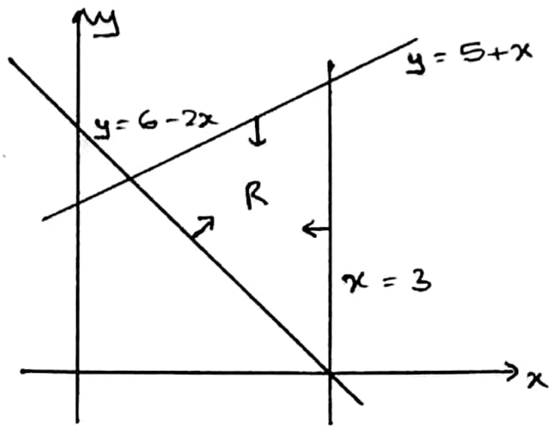
e) total distance $= [s]_0^3 + [s]_5^3$

$= |[1 - 55]| + |[1 - 45]|$

$= 98\,m$

## EQUATION GRAPH

1) $y = 6 - 2x$ — ①
$y = 5 + x$ — ②




for ①, at x-axis, $y = 0$

$\Rightarrow 6 - 2x = 0$

$\therefore x = 3$

the point is $(3, 0)$

R is defined by -

$y \leq 5 + x$ , $y \geq 6 - 2x$ , $x \leq 3$

3) a) slope is the acceleration;

$\Rightarrow a = \dfrac{20}{10} = 2 \, m/s^2$

b) total distance travelled = Area under graph

$= \frac{1}{2} \times (30 + 45) \times 20$

$= 750m$

c) average speed

$= \dfrac{\text{total distance travelled}}{\text{total time taken}} = \dfrac{750}{45} = \dfrac{50}{3} \, m/s$

4) a) from 9:30 to 11:00 $\Rightarrow$ 90 min , or 1.5 hrs

b) average speed $= \dfrac{175 km}{2 hrs}$

$= 87.5 \, km/h$

c) 5 small gaps $\longrightarrow$ 30 min
2 " " $\longrightarrow \dfrac{30}{5} \times 2 = 12 min$

$\therefore$ at 12:12

2)

time taken (P to Q) $= \frac{35}{5} = 7s$

let $t_2$ be time taken to travel from Q to R. (constant deceleration)

$$\Rightarrow \frac{1}{2} \times 5 \times t_2 = 10$$

$$t_2 = 4s$$

5) a) graph

b) total distance

= area under graph

$= \frac{1}{2} \times (5+6.5) \times 60 \times 30$

$= 10350 m$

c) average speed

$= \frac{\text{total distance travelled}}{\text{total time taken}}$

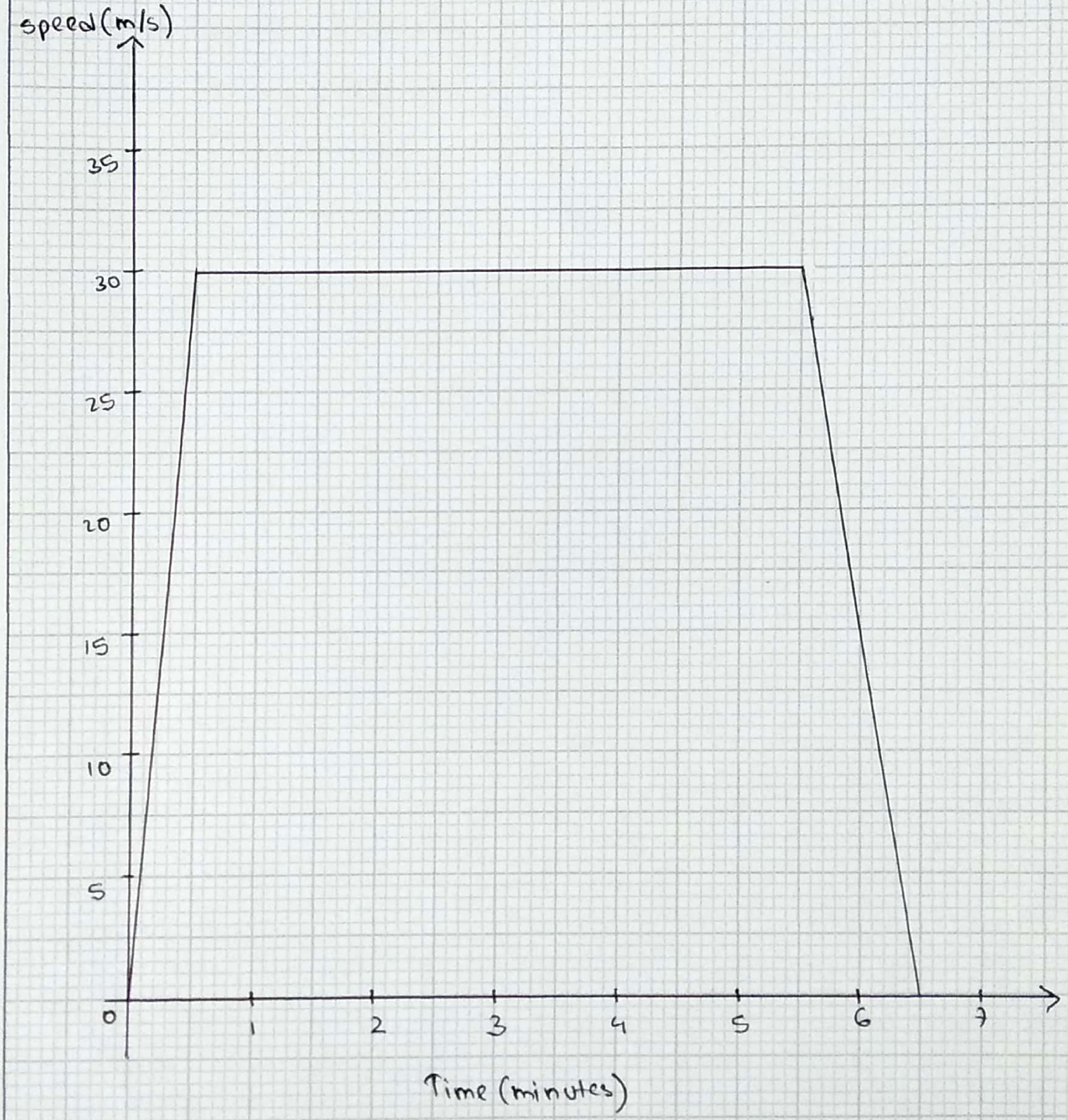
$= \frac{10350}{6.5 \times 60}$

$= 26.53 m/s$

d) deceleration $= \frac{0-30}{1 \times 60}$

$= 0.5 m/s^2$

5) a)

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6) a) graph

b) graph

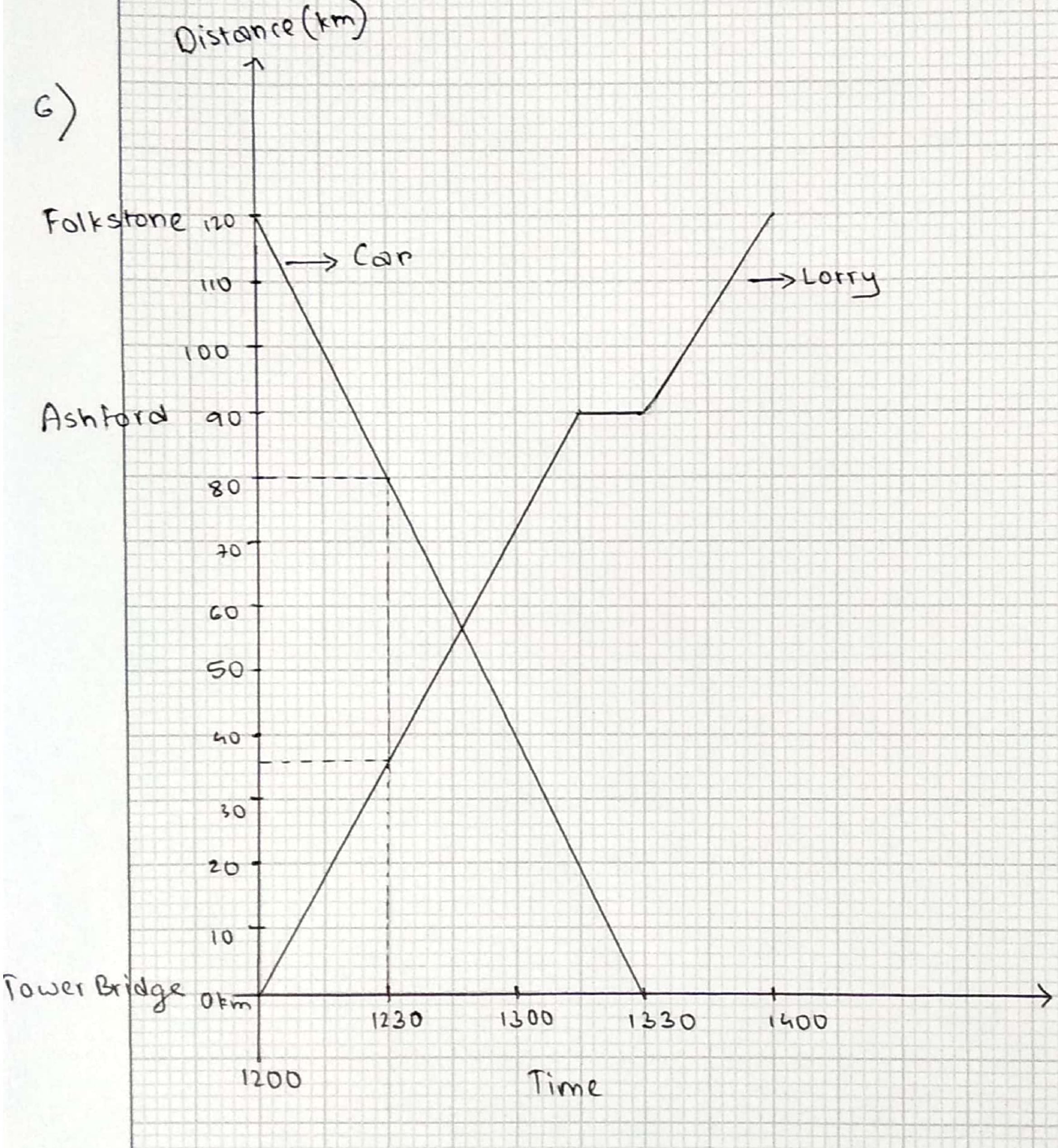
c) speed $= \dfrac{90\ km}{1.25\ hr}$

$= 72\ km/h$

d) distance $= 80 - 36$

$= 44\ km$

6)



Distance (km)
Folkstone
Ashford
Tower Bridge
120
110
100
90
80
70
60
50
40
30
20
10
0km
→ Car
→ Lorry
1200
1230
1300
1330
1400
Time


18 cm x 26 cm

7) a) graph

b) distance = 160 - 48

= 112 km

time : 10:09 to 11:45

$\Rightarrow$ 10:09 to 11:00 + 11:00 to 11:45

$\downarrow$ $\downarrow$

51/60 45/60

= 0.85 hr + = 0.75 hr

= 1.6 hr

speed = $\frac{112}{1.6}$

= 70 km/h

c) graph

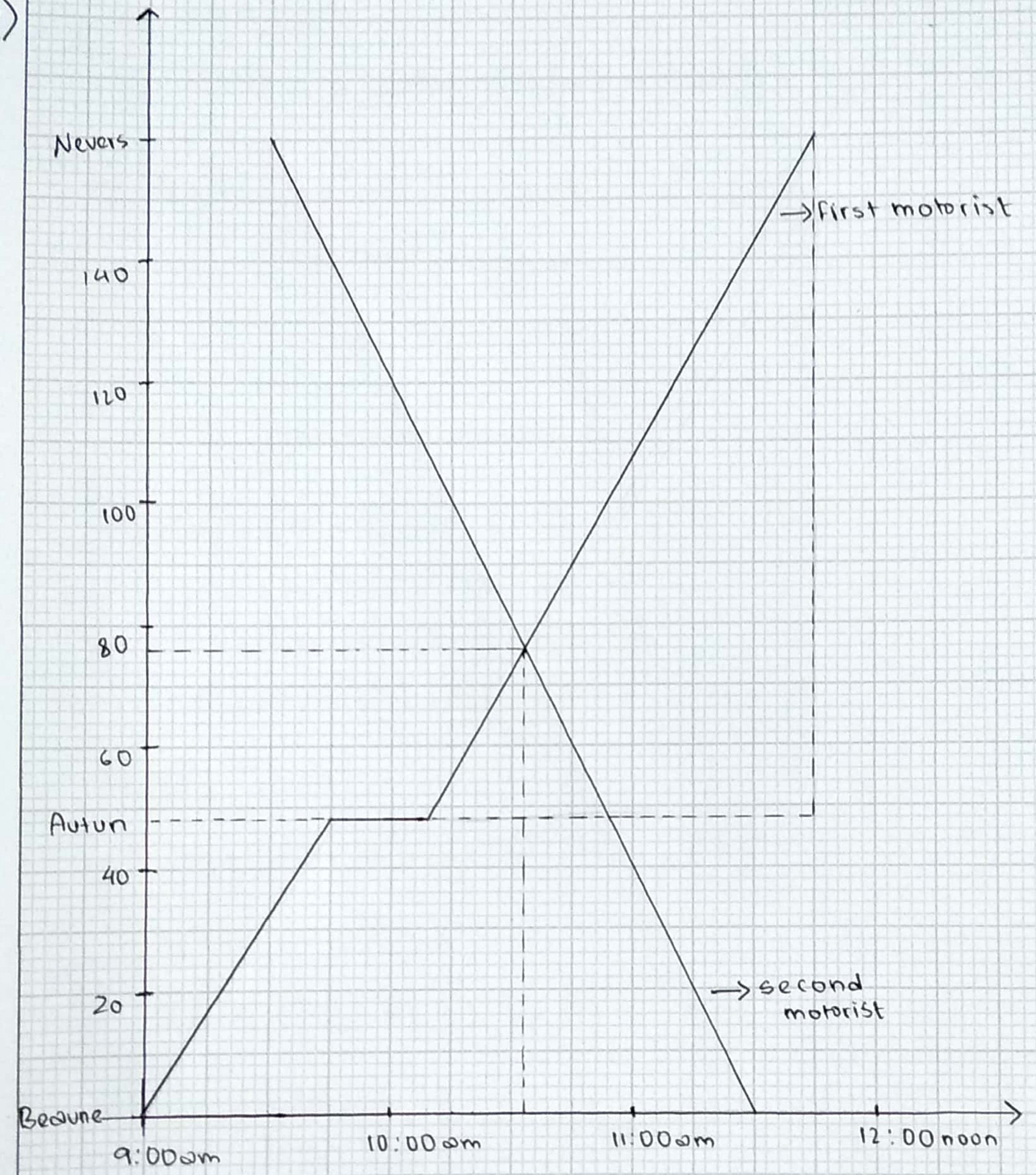
d) from graph,

i) 10:33

ii) distance = 76 - 48

= 28 km

7)




for second motorist:

$\Rightarrow$ time $= \frac{160}{80} = 2$ hrs

8) $f(x) = 4x^3 - 13x - 6$

a) $f(-1/2) = 4(-1/2)^3 - 13(-1/2) - 6$

$\therefore = 0$

$\therefore (2x+1)$ is a factor of $f(x)$. (shown)

b) $2x+1 \,|\, 4x^3 + 0x^2 - 13x - 6 \,|\, 2x^2 - x - 6$

$\underline{\overset{(-)}{}4x^3 \overset{(-)}{+} 2x^2}$

$-2x^2 - 13x$

$\underline{\overset{(+)}{-}2x^2 \overset{(+)}{-} x}$

$-12x - 6$

$\underline{\overset{(+)}{-}12x \overset{(+)}{-} 6}$

$0$

$$\therefore f(x) = (2x+1)(2x^2 - x - 6)$$
$$= (2x+1)(2x^2 - 4x + 3x - 6)$$
$$= (2x+1)(2x(x-2) + 3(x-2))$$
$$= (2x+1)(2x+3)(x-2)$$

c) $f(x) = y = 0$;

$\Rightarrow (2x+1)(2x+3)(x-2) = 0$

$\therefore x = -1/2, -3/2, 2$

$\Rightarrow (-1/2, 0), (-3/2, 0), (2, 0)$

d) $\frac{dy}{dx} = 12x^2 - 13 = 0$ at turning point.

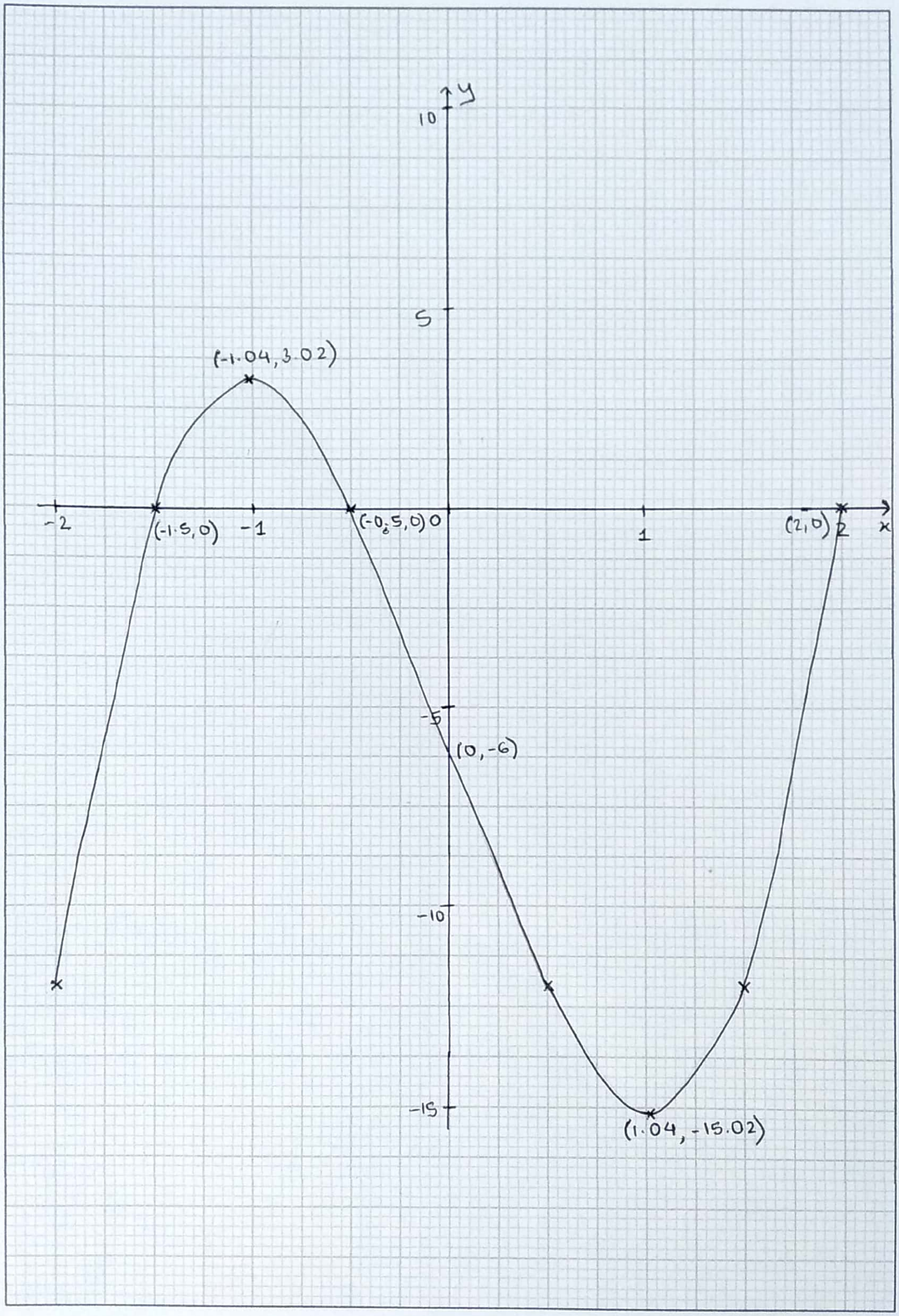
$\Rightarrow x = \pm\sqrt{13/12} = \pm 1.04$

$y_1 = 4(1.04)^3 - 13(1.04) - 6 =$

$y_2 = 4(-1.04)^3 - 13(-1.04) - 6 = 3.02$

$\therefore (1.04, -15.02), (-1.04, 3.02)$

8)

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9) a) $y = 2(0.5)^3 - (0.5)^2 - 6(0.5)$

$= -3$

$y = 2(1.5)^3 - (1.5)^2 - 6(1.5)$

$= -4.5$

$y = 2(3)^3 - (3)^2 - 6(3)$

$= 27$

b) graph

c) from graph, $y = -5.1$

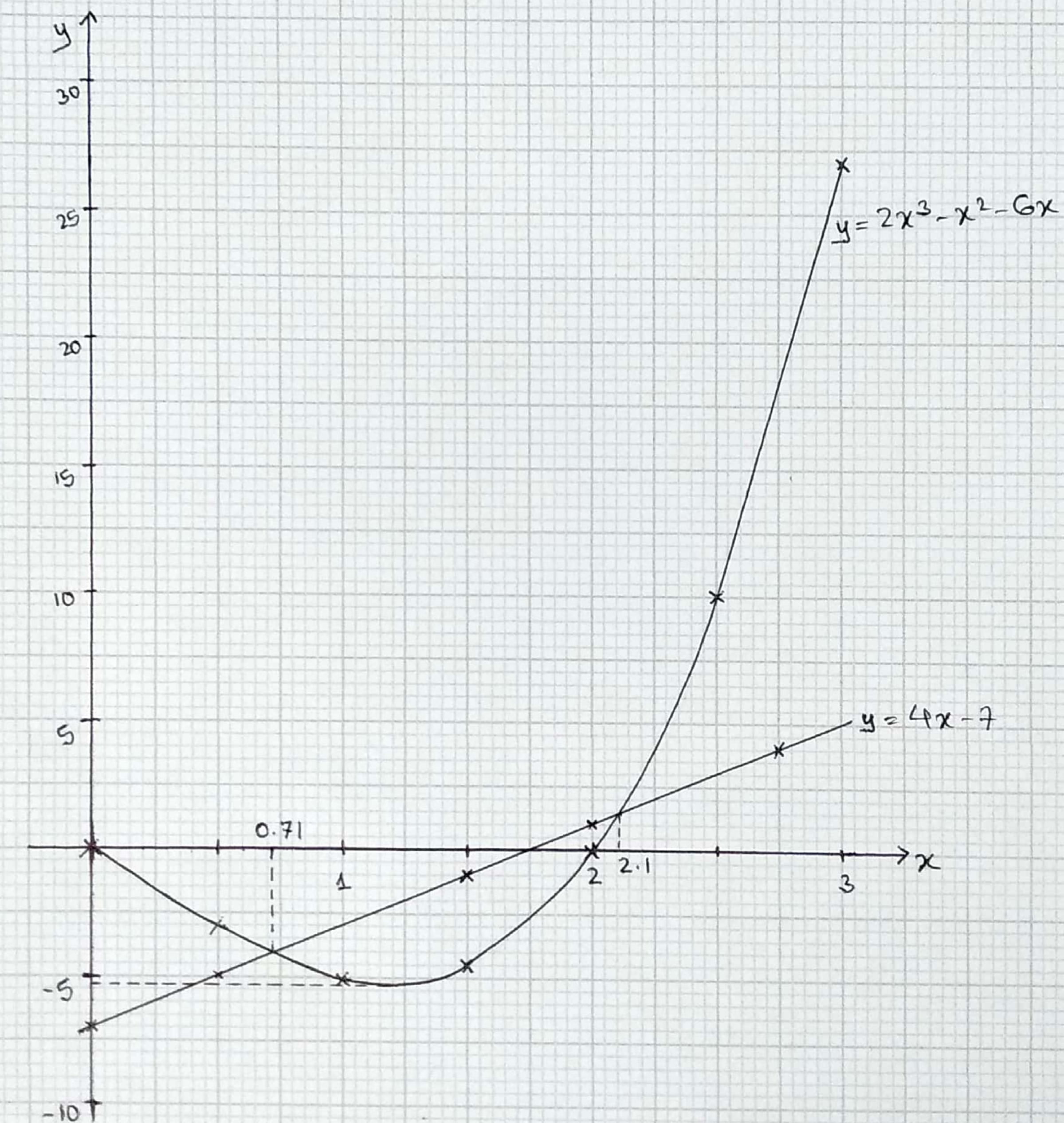
d)

| $x$ | 0 | 0.5 | 1.5 | 2 | 2.5 |
|---|---|---|---|---|---|
| $y$ | 0 | -5 | -1 | 1 | 3 |

e) from graph,

$0.7 < x < 2.1$

9)

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10) a) $y = \frac{9}{(1.25)^2} + 2(1.25) - \frac{8}{(1.25)}$

$= 1.86$

$y = \frac{9}{(1.75)^2} + 2(1.75) - \frac{8}{1.75}$

$= 1.87$

$y = \frac{9}{(2.5)^2} + 2(2.5) - \frac{8}{2.5}$

$= 3.24$

b) graph

c) from graph, $x = 1.46$ at min. pt.

d) $4x^3 - 6x^2 - 8x + 9 = 0$

$\Rightarrow 4x - 6 - \frac{8}{x} + \frac{9}{x^2} = 0$ [dividing with $x^2$]

$\Rightarrow \frac{9}{x^2} - \frac{8}{x} = -4x + 6$

$\Rightarrow \frac{9}{x^2} + 2x - \frac{8}{x} = -4x + 2x + 6$ [adding $2x$ to both sides]

$= \underset{\downarrow}{-2}x + \underset{\downarrow}{6}$

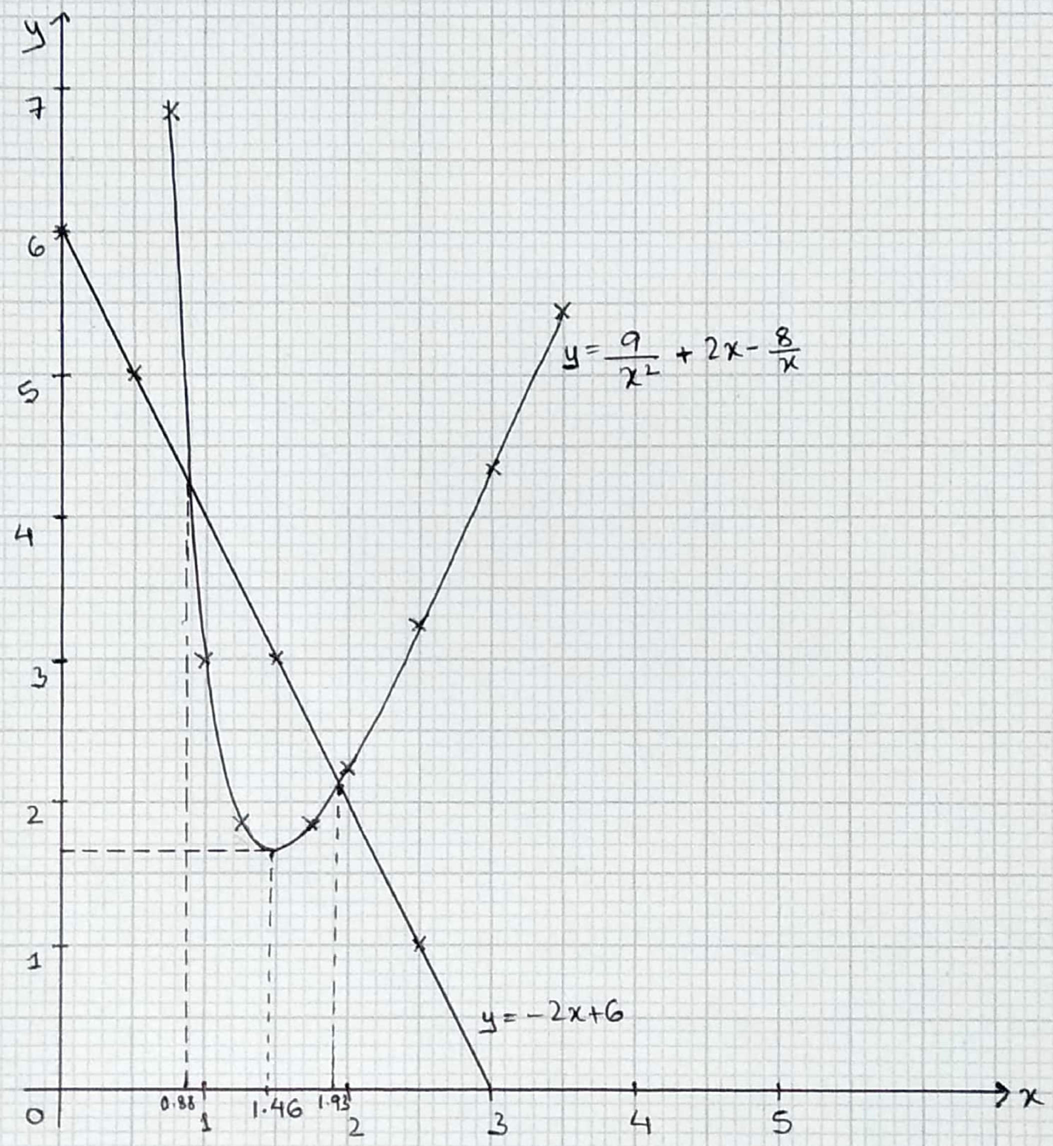
$\quad a \quad\quad b$

e)

| $x$ | 0 | 0.5 | 1.5 | 2.5 | 3 |
|---|---|---|---|---|---|
| $y$ | 6 | 5 | 3 | 1 | 0 |

$x = 0.88, 1.93$ (from graph)

10)

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11) a) i) $AB^2 + BC^2 = AC^2$

$\therefore \angle ABC = 90°$

ii) Area $= \frac{1}{2} \times 3x \times 4x = 6x^2$ cm$^2$ (shown)

iii) $S = ABDE + BCEF + \Delta ABC \times 2 + ACDF$

$= 3xy + 4xy + 6x^2 \times 2 + 5xy$

$= 12x^2 + 12xy$

b) $S = 144$,

$\Rightarrow 144 = 12x^2 + 12xy$

$\Rightarrow 12xy = 144 - 12x^2$

$\therefore y = \frac{144 - 12x^2}{12x} = \frac{12 - x^2}{x}$ (shown)

c) Volm $=$ C.S.A $\times$ length

$= 6x^2 y$

$= 6x^2 \left(\frac{12 - x^2}{x}\right)$

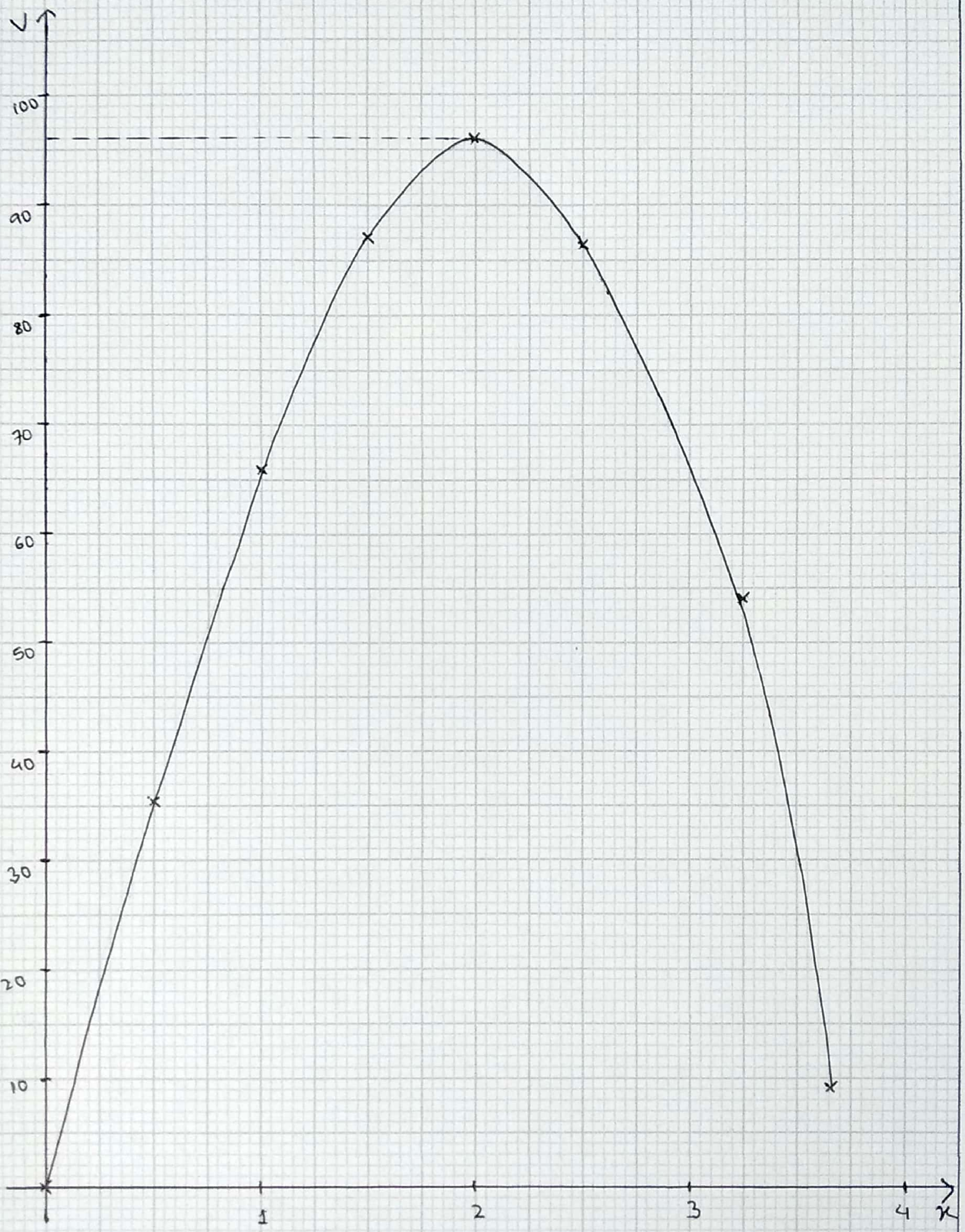
$= 6x(12 - x^2)$ (shown)

d) 35.3, 87.8, 96, 86.3

e) graph

f) from graph, $V_{max} = 96$ cm$^3$

11)




18 cm x 26 cm

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12) a) $h + 6r = 15$

$\Rightarrow h = 15 - 6r$

$\Rightarrow h > 0$

$\Rightarrow 15 - 6r > 0$

$r < 2.5$

$\therefore$ upper bound value of $r = 2.5$

b) $V$ = volume of hemisphere + volume of cone

$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$

$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 (15 - 6r)$

$= \frac{2}{3}\pi r^3 + 5\pi r^2 - 2\pi r^3$

$= -\frac{4}{3}\pi r^3 + 5\pi r^2$

$= \frac{1}{3}\pi r^2 (15 - 4r)$ (shown)

c) 2.2, 15.4, 23.1

d) graph

e) from graph, $r = 1.76$ cm

$\simeq 1.8$ cm (1 d.p.)

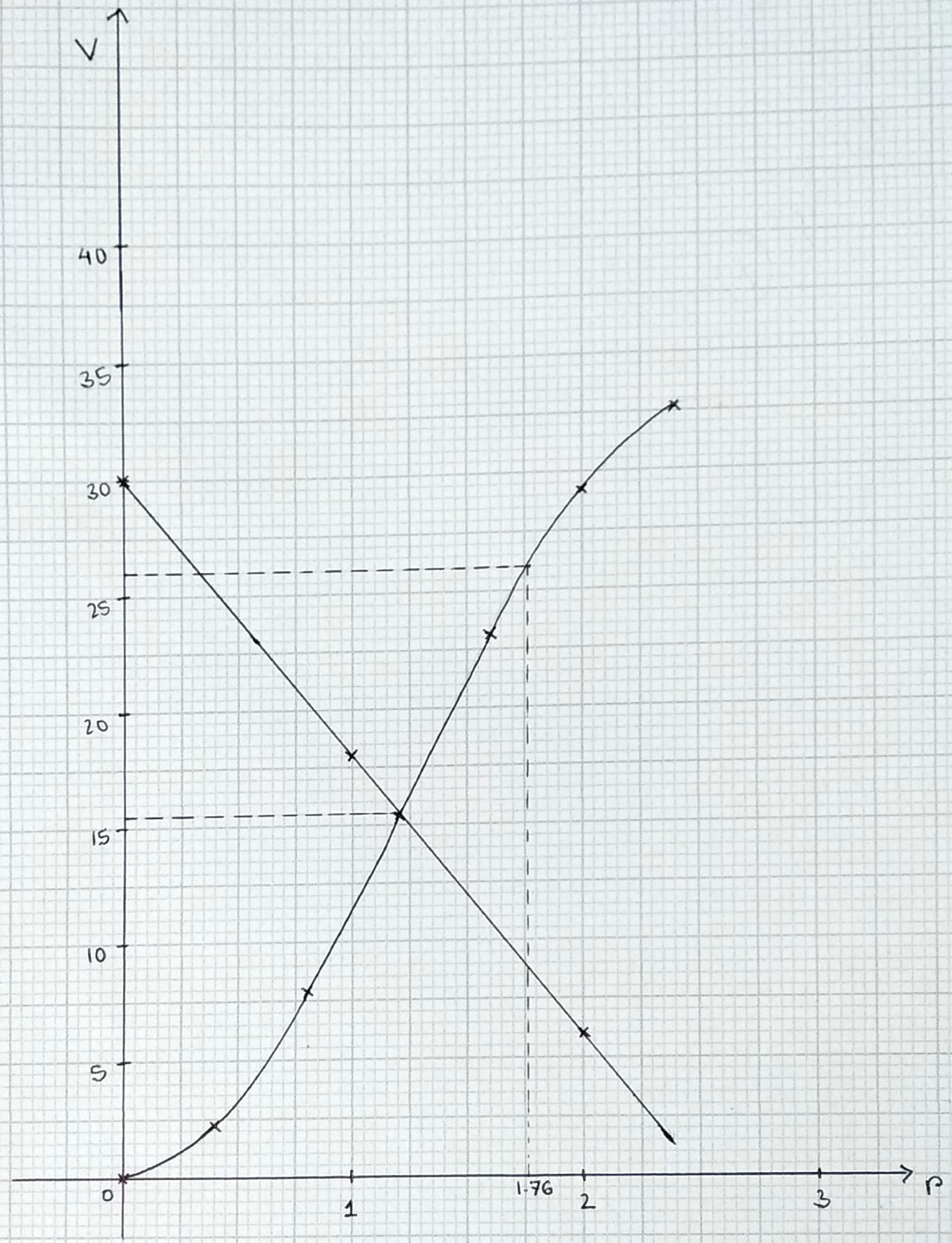
f) $V = 30 - 12r$

| $r$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $V$ | 30 | 18 | 6 | -6 |

$V = 15.5$

$\simeq 16$ (from graph)

12)




18 cm x 26 cm

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