‘Financial Contagion’

Application of a Structural Model of Credit Risk to the Network of Interbank Loans

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SYSRISK

Conference on Systemic Risk and Financial Stability

The importance of pricing counterparty risk

Under Basel II, the risk of counterparty default and credit migration risk were addressed but mark-to-market losses due to credit valuation adjustments (CVA) were not. During the financial crisis, however, roughly two thirds of losses attributed to counterparty credit risk were due to CVA losses and only about one third were due to actual defaults.

Basel Committee on Banking Supervision, 2011

The problem of pricing risk in a network

The Bank's solvency contagion model examines how deteriorating capital positions lead to revaluation of interbank debt claims, which in turn can affect banks' capital positions further ... Bank staff's judgement is that ... the overall impact on the system via this channel remains immaterial ... See Bank of England, Staff Working Paper No. 662, 'The decline of solvency contagion risk', June 2017.

Bank of England stress testing results, 2017

Outline

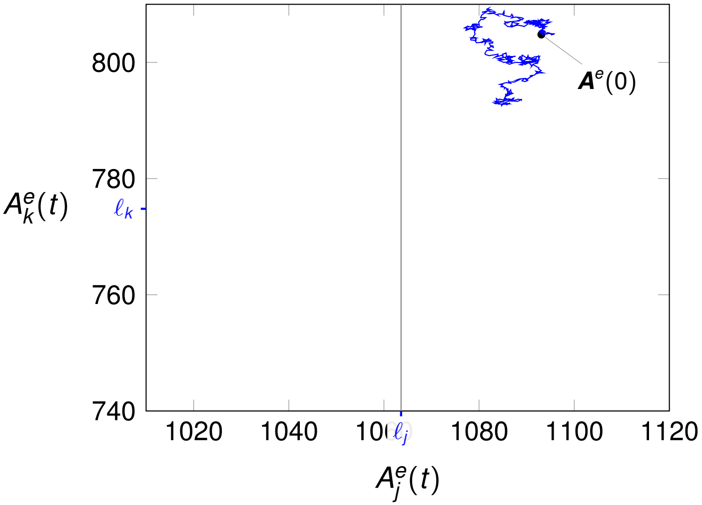
Illustrative plots

- Two banks and zero recovery rate
- The Bank of England model

Development

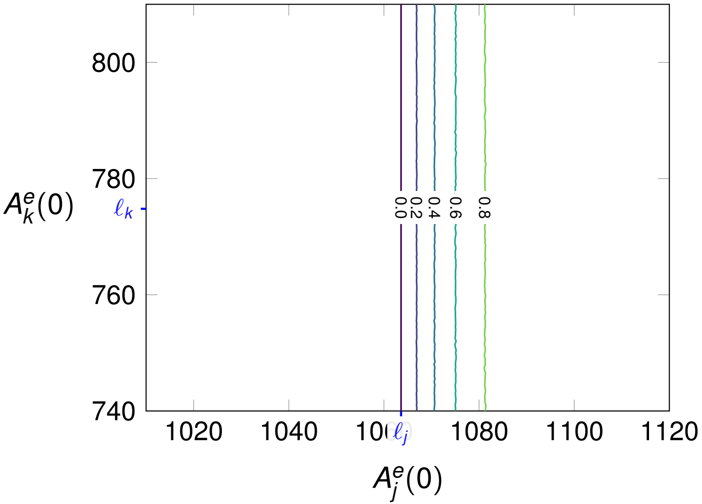
- Alternative maturity profile
- General solution structure for multiple banks
- Nonzero recovery rate

Evolution of the real economy



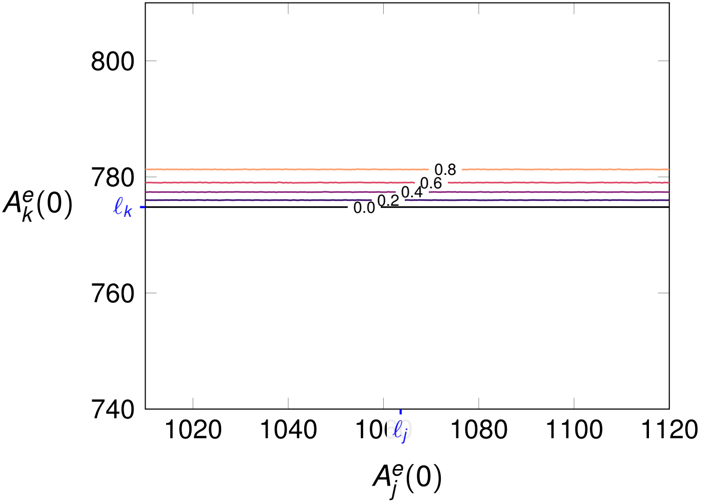
Pricing in the risk of failure

Bank j debt valuation $V_j(0) = \mathbb{E}^{\mathbb{Q}}[\mathbb{1}_{\tau_j>1} | \boldsymbol{A}^e(0)]$



Pricing in the risk of failure

Bank k debt valuation $V_k(0) = \mathbb{E}^{\mathbb{Q}}[\mathbb{1}_{\tau_k > 1} | \boldsymbol{A}^e(0)]$



Balance sheets in the unconnected case

Bank j

Assets	Liabilities
$A_j^e(t)$	ℓ_j
	$E_j(t)$

Bank k

Assets	Liabilities
$A_k^e(t)$	ℓ_k
	$E_k(t)$

Balance sheets with interbank exposures

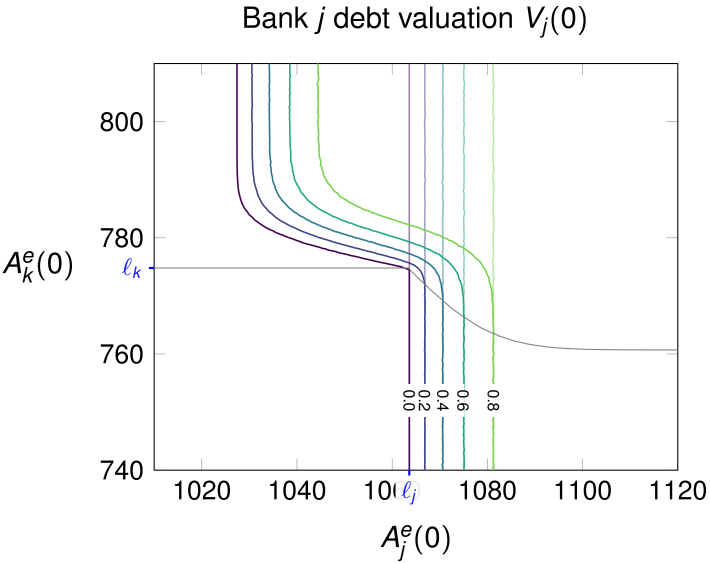
Bank j

Assets	Liabilities
$A_j^e(t)$	ℓ_j
	L_{jk}
$V_k(t)L_{kj}$	$E_j(t)$

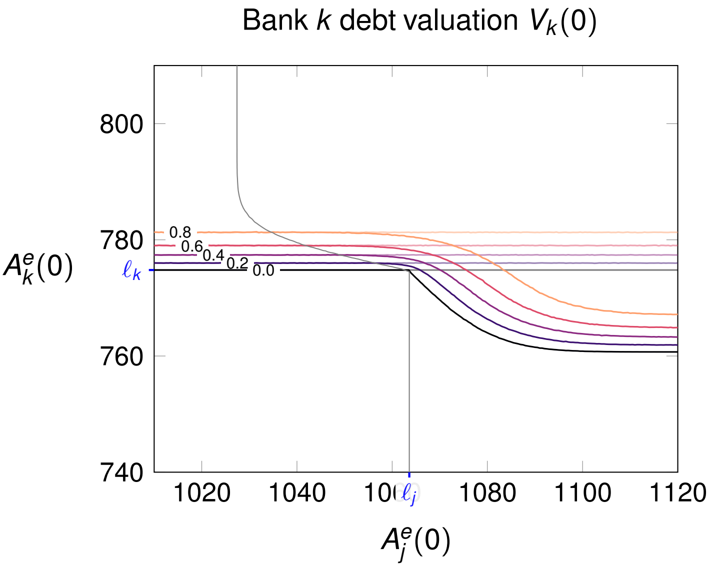
Bank k

Assets	Liabilities
$A_k^e(t)$	ℓ_k
	L_{kj}
$V_j(t)L_{jk}$	$E_k(t)$

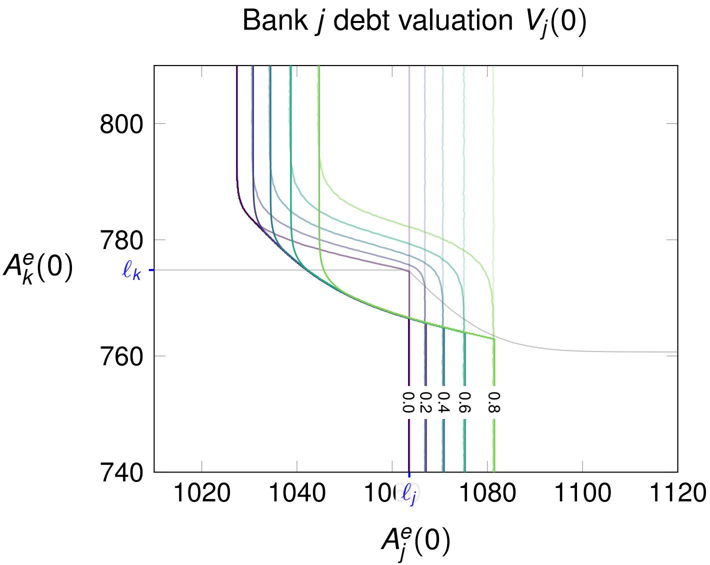
Pricing in the effect of interbank exposures



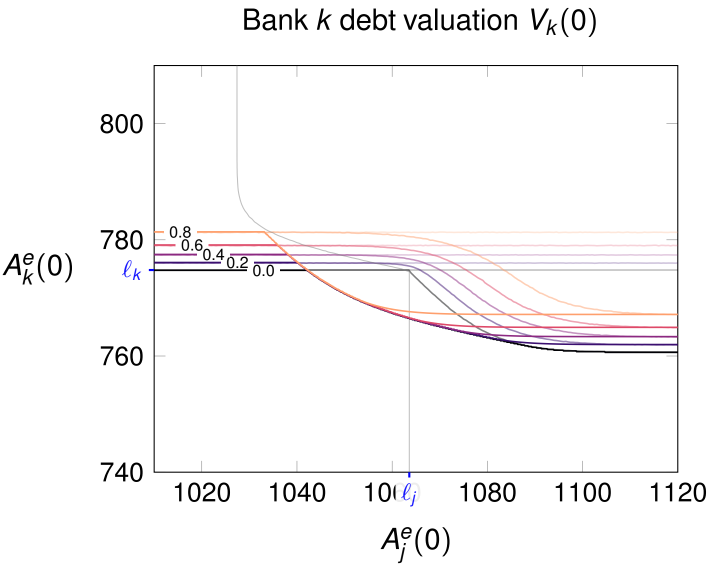
Pricing in the effect of interbank exposures



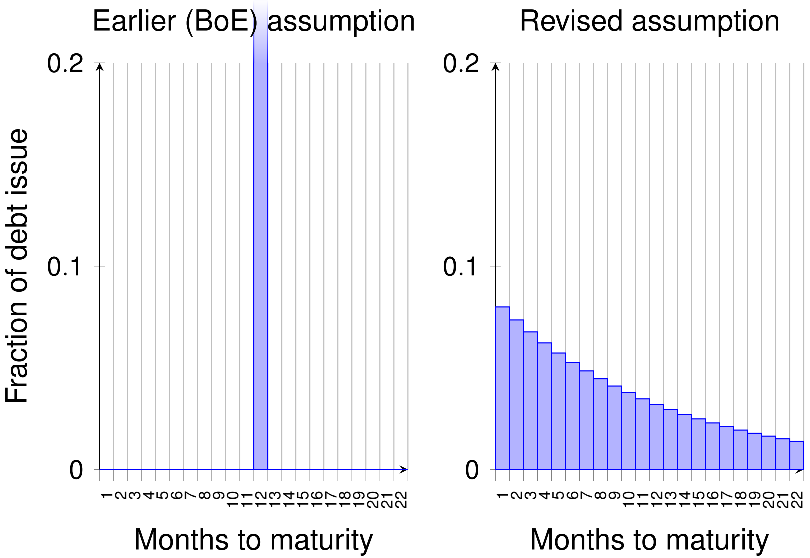
Comparison with the Bank of England model



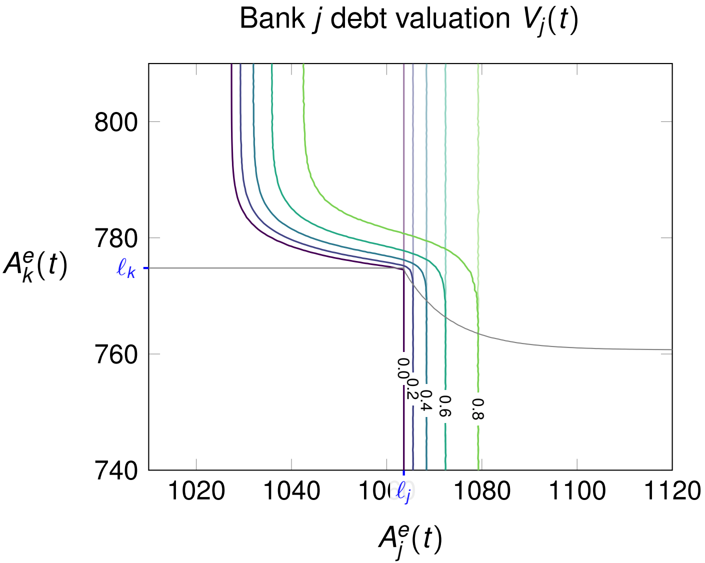
Comparison with the Bank of England model



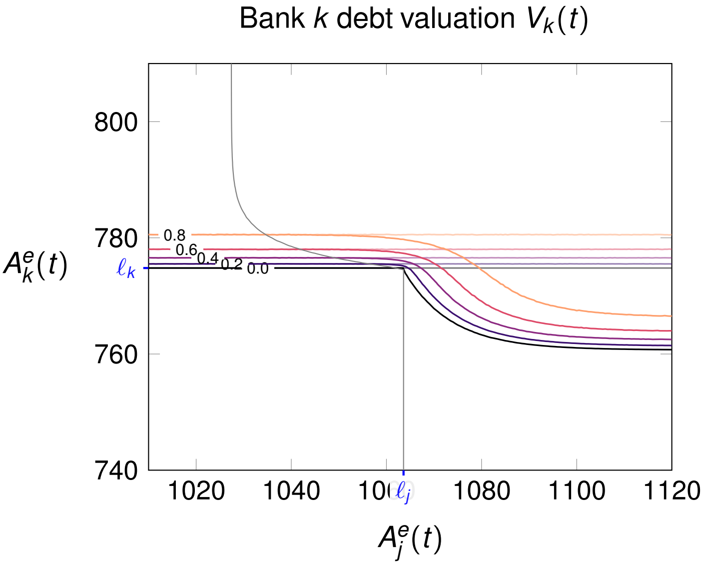
Alternative maturity profile



Pricing with revised maturity profile



Pricing with revised maturity profile



Solution dependency structure in general

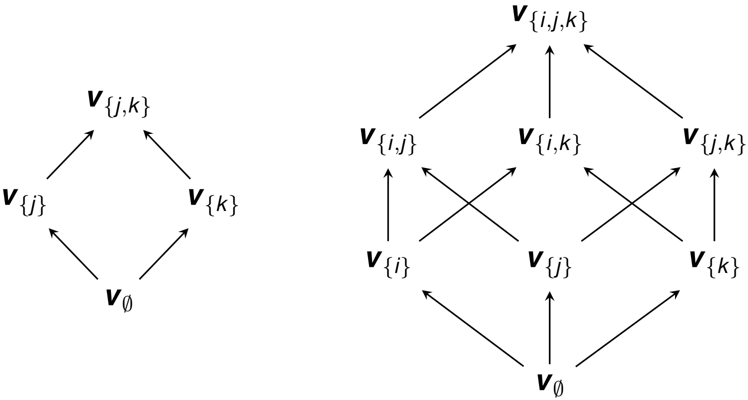


Figure: Progression of the algorithm for a system of two banks and for a system of three.

Some supporting theory

Theorem

For survival sets A *and* B*,*

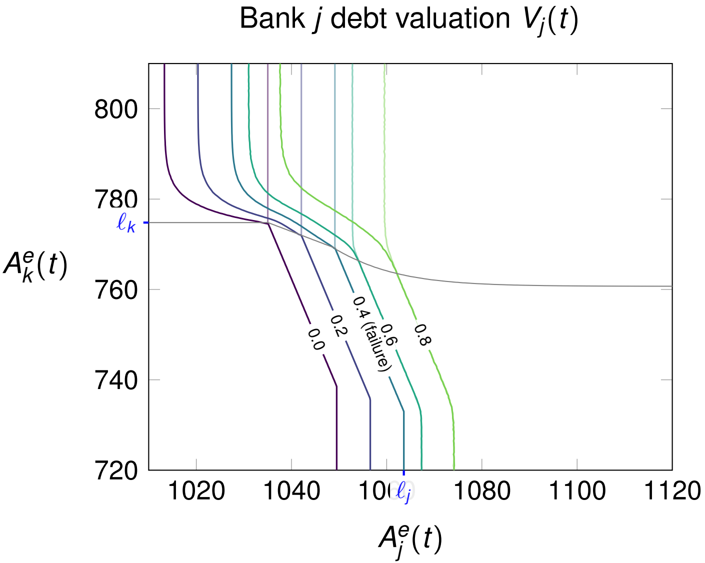
$$\mathrm{B} \subseteq \mathrm{A} \implies \boldsymbol{v}_{\mathrm{B}} \leq \boldsymbol{v}_{\mathrm{A}}.$$

Corollary

With $\boldsymbol{E}^*(\boldsymbol{a}^e, \boldsymbol{u})$ *the equity valuation function indicating solvency,*

$$\begin{aligned} \textit{if} \quad \exists \mathrm{B} \subseteq \mathrm{A} \quad \textit{such that} \quad & \forall i \in \mathrm{A} \quad 0 < E_i^*(\boldsymbol{a}^e, \boldsymbol{v}_{\mathrm{B}}(\boldsymbol{a}^e)) \\ \textit{then} \quad & \forall i \in \mathrm{A} \quad 0 < E_i^*(\boldsymbol{a}^e, \boldsymbol{v}_{\mathrm{A}}(\boldsymbol{a}^e)). \end{aligned}$$

Pricing with recovery rate $\beta = 0.4$



Pricing with recovery rate $\beta = 0.4$