Learning Continuous Image Representation with Local Implicit Image Function

Yinbo Chen Sifei Liu Xiaolong Wang UC San Diego NVIDIA UC San Diego

Abstract

How to represent an image? While the visual world is presented in a continuous manner, machines store and see the images in a discrete way with 2D arrays of pixels. In this paper, we seek to learn a continuous representation for images. Inspired by the recent progress in 3D reconstruction with implicit function, we propose Local Implicit Image Function (LIIF), which takes an image coordinate and the 2D deep features around the coordinate as inputs, predicts the RGB value at a given coordinate as an output. Since the coordinates are continuous, LIIF can be presented in an arbitrary resolution. To generate the continuous representation for pixel-based images, we train an encoder and LIIF representation via a self-supervised task with superresolution. The learned continuous representation can be presented in arbitrary resolution even extrapolate to ×30 higher resolution, where the training tasks are not provided. We further show that LIIF representation builds a bridge between discrete and continuous representation in 2D, it naturally supports the learning tasks with size-varied image ground-truths and significantly outperforms the method with resizing the ground-truths. Our project page with code is at https://yinboc.github.io/liif/.

1. Introduction

Our visual world is continuous. However, when a machine tries to process a scene, it will need to first store and represent the images as 2D arrays of pixels, where the trade-off between complexity and precision is controlled by resolution. While the pixel-based representation has been successfully applied in various computer vision tasks, they are also constrained by the resolution of images. For example, a dataset is often presented by images with different resolutions. If we want to train a convolutional neural network, we will usually need to resize the images to the same size, which sacrifices fidelity. Instead of representing an image with a fixed resolution, we propose to study a continuous representation for images. By modeling an image as a function defined in a continuous domain, we can restore and generate the image in arbitrary resolution if needed.

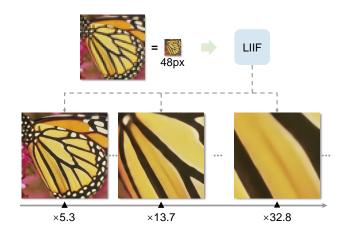


Figure 1: Local Implicit Image Function (LIIF) represents an image in continuous domain, which can be presented in arbitrary high resolution.

How do we represent the image as a continuous function? Our work is inspired by the recent progress in implicit neural representation [31, 24, 6, 34, 35] for 3D shape reconstruction. The idea of implicit function is to represent an object as a function that maps coordinates to corresponding signal (e.g. signed distance to a 3D object surface, RGB value in an image). A neural implicit function is parameterized by a deep neural network. To share knowledge across instances instead of fitting individual implicit function for each object, encoder-based methods [24, 6, 37] are proposed to predict a latent code for each object. Then the implicit function is shared by all the objects while it takes the latent code as an additional input. Despite its success in 3D tasks, previous encoder-based methods of implicit function only succeeded in representing simple images such as digits [6], but failed to represent natural images with high fidelity [37]. We hypothesize the reason is that it is hard to encode all the fine details of a natural image in a simple

In this paper, we propose the Local Implicit Image Function (LIIF) for representing natural and complex images in a continuous manner. Instead of using one latent code to encode the whole image, we represent an image by a set of latent codes distributed in spatial dimensions that each of

them stores information about its local area. Given a coordinate, the local implicit image function takes the coordinate information and queries the local latent codes around the coordinate as inputs, then predicts the RGB value at the given coordinate as the output. Since the coordinate system is continuous, the LIIF representation can be presented in arbitrary resolution.

To generate such continuous representation for pixel-based images, since we hope the generated continuous representation can generalize to higher precision than the input image, we train an encoder with the LIIF representation via a self-supervised task with super-resolution, where the input and ground-truth are provided in continuously changing up-sampling scales. In this task, take a pixel-based image as an input, the encoded LIIF representation is trained to predict its higher resolution counterpart. While previous super-resolution works [9, 20, 21, 19] focus on learning an up-sampling function for specific scales in a convolution-deconvolution framework, our LIIF representation is continuous, and we show it can be presented in an arbitrary high resolution that extrapolates to $\times 30$ higher resolution, where the training tasks are not provided.

We further demonstrate that LIIF builds a bridge between discrete and continuous representation in 2D. In learning tasks with size-varied image ground-truths, LIIF can naturally exploit the information provided in different resolutions. Previous methods with fixed-size output usually need to resize all the ground-truths to the same size for training, which sacrifices data fidelity. The LIIF representation can be presented in arbitrary resolution, therefore it can be trained in an end-to-end manner without resizing ground-truths and achieves significantly better results than the method with resizing the ground-truths.

Our contributions include: (i) A novel local implicit image function for representing natural and complex images continuously; (ii) Our representation allows extrapolation to even $\times 30$ higher resolution which is not presented during training time; (iii) Our representation is effective for the learning tasks with size-varied image ground-truths.

2. Related Work

Implicit neural representation. The implicit neural representation is a continuous and differentiable function. It parameterizes the data samples as multilayer perceptrons (MLPs) that map coordinates to signal. The implicit function has been widely applied in modeling 3D object shapes [6, 25, 2, 11], 3D surfaces of the scene [38, 16, 33, 4] as well as the appearance of the 3D structure [30, 29, 26]. For example, Mildenhall et al. [26] propose to perform novel view synthesis by learning an implicit representation for a specific scene using multiple image views. Comparing to explicit 3D representations such as voxel, point cloud, and mesh, the continuous implicit representation can cap-

ture the very fine details of the shape with a small number of parameters. Its differentiable property also allows back-propagation through the model for neural rendering [38].

Learning implicit function space. Instead of learning an independent implicit neural representation for each object, recent works share a function space for the implicit representations of different objects. Typically, a latent space is defined where each object corresponds to a latent code. The latent code can be obtained by optimization with an auto-decoder [31, 6]. For example, Park et al. [31] propose to learn a Signed Distance Function (SDF) for each object shape and different SDFs can be inferred by changing the input latent codes. Recent work from Sitzmann et al. [36] also proposes a meta-learning-based method for sharing the function space. Instead of using auto-decoder, our work adopts the auto-encoder architecture [24, 6, 34, 35, 43], which gives an efficient and effective manner for sharing knowledge between a large variety of samples. For example, Mescheder et al. [24] propose to estimates a global latent code given an image as input, and use an occupancy function conditioning on this latent code to perform 3D reconstruction for the input object.

Despite the success of implicit neural representation in 3D tasks, the amount of works for 2D tasks is much less. Early works [39, 27] parameterize 2D images with compositional pattern producing networks. Chen et al. [6] explore 2D shape generation from latent space for simple digits. Recently, Sitzmann et al. [37] observe that previous implicit neural representation parameterized by MLP with ReLU [28] is incapable of representing fine details of natural images. They replace ReLU with periodic activation functions (sinusoidal) and demonstrates it can model the natural images in higher quality. However, none of these approaches can represent natural and complex images with high fidelity when sharing the implicit function space, while it is of limited generalization to higher precision if not to share the implicit function space. Instead of using a global shape vector, our work is related to [34] where a spatial image feature map is used for querying the local feature to predict the 3D occupancy.

Image generation and super-resolution. Our work is related to the general image-to-image translation tasks [46, 15, 48, 12, 7], where one image is given as input and it is translated to a different domain or format. For example, Isola et al. [15] propose conditional GANs [10] to perform multiple image translation tasks. Unlike the deconvolution-based approaches, our method can perform realistic and high-resolution image generation by independently querying the pixels at different coordinates from the generated implicit representation. While our method is useful for general purposes, in this paper, we perform our experiments on generating high-resolution images given low-resolution inputs, which is related to the image super-

resolution tasks [5, 42, 40, 8, 20, 21, 19, 47, 45]. For example, Lai et al. [19] propose a Laplacian Pyramid Network to progressively reconstruct the image. While related, we stress that our work on learning continuous representation is different from the traditional super-resolution setting. Previous super-resolution models are designed for upsampling with a specific scale (or a fixed set of scales), while our goal is to learn a continuous representation that can be presented in arbitrary high resolution. On this aspect, our work is more close to recent work by Hu et al. [13] on magnification-arbitrary super-resolution. This method generates a convolutional up-sampling layer by its metanetwork, while it can perform arbitrary up-sampling in its training scales, it has limited performance on generalizing to the larger-scale synthesis that is out of training distribution. Our method, on the other hand, when trained with tasks from $\times 1$ to $\times 4$, can generate $\times 30$ higher resolution image based on the continuous representation in one forward pass.

3. Local Implicit Image Function

In this section, we introduce Local Implicit Image Function (LIIF), our continuous representation for 2D images, as demonstrated in Figure 2. In the following, we introduce our method by first giving a general definition of LIIF, then we provide the detailed designing choices.

In our LIIF representation, each continuous image $I^{(i)}$ is represented by a 2D feature map $M^{(i)} \in \mathbb{R}^{H \times W \times D}$. A neural implicit function f_{θ} (with θ as its parameters) is shared by all the images, it is parameterized as a MLP and takes the form (omit θ for clarity):

$$s = f(z, x), \tag{1}$$

where z is a vector, $x \in \mathcal{X}$ is a 2D coordinate in the continuous image domain, $s \in \mathcal{S}$ is the predicted signal (i.e. the RGB value). With a defined f, each vector z can be considered as representing a function $f(z,\cdot): \mathcal{X} \mapsto \mathcal{S}$. The $f(z,\cdot)$ can be viewed as a continuous image, i.e. a function that maps coordinate to RGB value. We assume the $H \times W$ feature vectors (we call them latent codes from now on) of $M^{(i)}$ are evenly distributed in the 2D space of the continuous image domain of $I^{(i)}$ (i.e. yellow circles in Figure 2), then we assign a 2D coordinate to each of them. For the image $I^{(i)}$, the RGB value at coordinate x_q is defined by:

$$I^{(i)}(x_q) = f(z^*, x_q - v^*), \tag{2}$$

where z^* is the nearest (Euclidean distance) latent code from x_q in $M^{(i)}$, v^* is the coordinate of latent code z^* in the image domain. Take Figure 2 as an example, z_{11}^* is the z^* for x_q in our current definition, while v^* is defined as the coordinate for z_{11}^* .

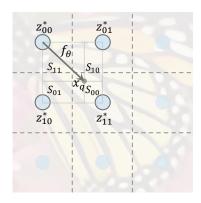


Figure 2: **LIIF representation with local ensemble.** A continuous image is represented by a 2D feature map and a implicit function f_{θ} shared by all the images. The signal is predicted by ensemble of the local implicit functions, which guarantees smooth transition between different areas.

As a summary, with an implicit function f shared by all the images, a continuous image is represented by a 2D feature map $M^{(i)} \in \mathbb{R}^{H \times W \times D}$ which is viewed as $H \times W$ latent codes evenly spread in the 2D domain. Each latent code z in $M^{(i)}$ represents a local piece of the continuous image, it is responsible for predicting the signal of the set of coordinates that are closest to it.

Feature unfolding. To enrich the information contained in each latent code in $M^{(i)}$, we apply feature unfolding to $M^{(i)}$ and get $\hat{M}^{(i)}$. A latent code in $\hat{M}^{(i)}$ is the concatenation of the 3×3 neighboring latent codes in $M^{(i)}$. Formally, the feature unfolding is defined as

$$\hat{M}_{jk}^{(i)} = \text{Concat}(\{M_{j+l,k+m}^{(i)}\}_{l,m\in\{-1,0,1\}}),$$
 (3)

where Concat refers to concatenation of a set of vectors, $M^{(i)}$ is padded by zero-vectors outside its border. After feature unfolding, $\hat{M}^{(i)}$ replaces $M^{(i)}$ for any computation. For simplicity, we will only use the notation $M^{(i)}$ in the following sections regardless of feature unfolding.

Local ensemble. An important issue in Eq 2 is the discontinuous prediction. Specifically, since the signal prediction at x_q is done by querying the nearest latent code z^* in $M^{(i)}$, when x_q moves in the image domain, the selection of z^* can suddenly switch from one to another (i.e. the selection of nearest latent code changes). For example, it happens when x_q crossing the dash lines in Figure 2. Around those coordinates where the selection of z^* switches, the signal of two infinitely close coordinates will be predicted from different latent codes. As long as the learned implicit function f is not perfect, discontinuous patterns would appear at these borders where z^* selection switches.

To address this issue, we propose the local ensemble technique as shown in Figure 2. We enlarge the field that

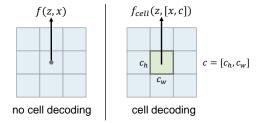


Figure 3: **Cell decoding.** With cell decoding, the implicit function takes the shape of the query pixel as an additional input and predicts the RGB value for the pixel.

each latent code is responsible for, and extend Eq 2 to:

$$I^{(i)}(x_q) = \sum_{t \in \{00,01,10,11\}} \frac{S_t}{S} \cdot f(z_t^*, x_q - v_t^*), \quad (4)$$

where $z_t^*(t \in \{00,01,10,11\})$ refers to the nearest latent code in top-left, top-right, bottom-left, bottom-right subspaces, v_t^* is the coordinate of z_t^* , S_t is the area of the rectangle between x_q and $v_{t'}^*$ where t' is diagonal to t (i.e. 00 to 11, 10 to 01). The weights are normalized by $S = \sum_t S_t$. We consider the feature map $M^{(i)}$ to be mirror-padded outside the borders, so that the formula above also works for coordinates near the borders.

This is to let the local image pieces represented by latent codes overlap with its neighboring pieces so that at each coordinate there are four latent codes for independently predicting the signal. These four predictions are then merged by voting with normalized confidences, which are proportional to the area of the rectangle between the query point and its nearest latent code's diagonal counterpart, thus the confidence gets higher when the query coordinate is closer. One can verify that by having this voting, it achieves continuous transition at coordinates where z^* switches (i.e. dash lines in Figure 2).

Cell decoding. In practice, we want that the LIIF representation can be presented as the pixel-based form in arbitrary resolution. Suppose the desired resolution is given, a straight-forward way is to query the RGB values at the coordinates of pixel centers in the continuous representation $I^{(i)}(x)$. While this can already work well, it may not be optimal since the predicted RGB value of a query pixel is independent of its size, the information in its pixel area is all discarded except the center value.

To address this issue, we add cell decoding as shown in Figure 3. We reformulate f in Eq 1 as f_{cell} with the form:

$$s = f_{cell}(z, [x, c]), \tag{5}$$

where $c = [c_h, c_w]$ contains two values that specify the height and width of the query pixel, [x, c] refers to the concatenation of x and c (i.e. c is an additional input).

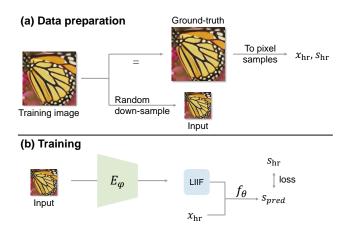


Figure 4: Learning to generate continuous representation for pixel-based images. An encoder is jointly trained with the LIIF representation in a self-supervised superresolution task, in order to encourage the LIIF representation to maintain high fidelity in higher resolution.

The meaning of $f_{cell}(z, [x, c])$ can be interpreted as what the RGB value should be, if we render a pixel centered at coordinate x with shape c (e.g. c is 1/64 of the image width for rendering in 64×64 resolution). As we will show in the experiments section, having an extra input c can be beneficial when presenting the continuous representation in a given resolution. Logically, $f(z,x) = f_{cell}(z, [x, c])$ when $c \to 0$, i.e. a continuous image can be viewed as an image with infinitely small pixels.

4. Learning Continuous Image Representation

In this section, we introduce our method of learning to generate a continuous representation for pixel-based images, an overview is demonstrated in Figure 4. Formally, in this task we have a set of images as the training set, the goal is to generate a continuous representation for an unseen image.

The general idea is to train an encoder E_{φ} (with φ as its parameters) that maps a pixel-based image to a 2D feature map as its LIIF representation, where we jointly learn a neural implicit function f_{θ} . We hope that the generated continuous LIIF representation is not only able to reconstruct its input image, but more importantly, it should maintain high fidelity even when being presented in higher resolution. Therefore, we propose to learn the encoder and the implicit function in a self-supervised task with superresolution.

We first take a single training image as an example, as shown in Figure 4, for a training image, an input is generated by down-sampling the training image with a random scale. A ground-truth is obtained by representing the training image as pixel samples $x_{\rm hr}, s_{\rm hr}$, where $x_{\rm hr}$ are the center

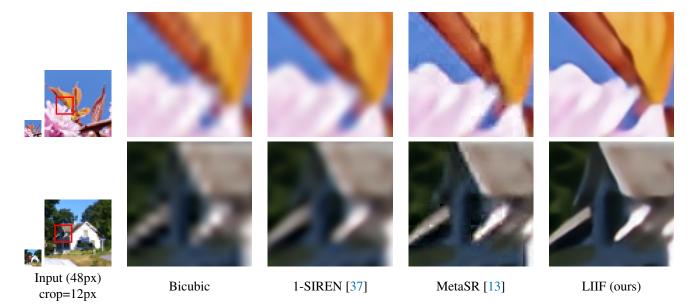


Figure 5: Qualitative comparison of learning continuous representation. The input is a 48×48 patch from images in DIV2K validation set, a red box indicates the crop area for demonstration ($\times 30$). 1-SIREN refers to fitting an independent implicit function for the input image. MetaSR and LIIF are trained for continuous random scales in $\times 1-\times 4$ and tested for $\times 30$ for evaluating the generalization to arbitrary high precision of the continuous representation.

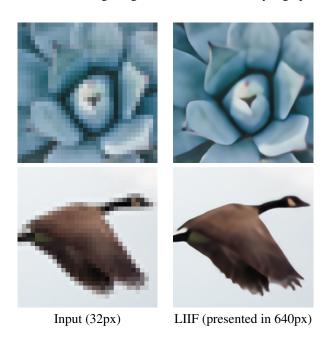


Figure 6: Visualization of generated LIIF representation, presented in scale $\times 20$ to the input.

coordinates of pixels in the image domain, $s_{\rm hr}$ are the corresponding RGB values of the pixels. The encoder E_{φ} maps the input image to a 2D feature map as its LIIF representation. The coordinates $x_{\rm hr}$ are then used to query on the LIIF representation, where f_{θ} predicts the signal (RGB value) for each of these coordinates based on LIIF representation. Let

 s_{pred} denote the predicted signal, a training loss (L1 loss in our experiment) is then computed by s_{pred} and the ground-truth signal s_{hr} . For batch training, we sample batches from the training set where the loss is the average over instances. We replace x with [x, c] when having cell decoding.

5. Experiments

5.1. Learning continuous image representation

Setup. We use DIV2K dataset [1] of NTIRE 2017 Challenge [41] for experiments on learning continuous image representation. It consists of 1000 images in 2K resolution and provides low-resolution counterparts with downsampling scales $\times 2, \times 3, \times 4$, which are generated by imresize function in Matlab with the default setting of bicubic interpolation. We follow the standard split using 800 images in DIV2K for training. For testing, we report the results on the DIV2K validation set with 100 images which follows prior work [21], and on four standard benchmark datasets: Set5 [3], Set14 [44], B100 [23], and Urban100 [14].

The goal is to generate a continuous representation for pixel-based images. A continuous representation is expected to have infinite precision, and it can be presented in arbitrary high resolution while maintaining high fidelity. Therefore, to quantitatively evaluate the effectiveness of the learned continuous representation, besides evaluating the up-sampling tasks of scales that are in training distribution, we propose to also evaluate extremely large up-sampling scales that are out of training distribution. Specif-

Method	In-distribution			Out-of-distribution				
Wethod	×2	$\times 3$	$\times 4$	×6	$\times 12$	$\times 18$	$\times 24$	$\times 30$
Bicubic [21]	31.01	28.22	26.66	24.82	22.27	21.00	20.19	19.59
EDSR-baseline [21]	34.55	30.90	28.94	-	-	-	-	-
EDSR-baseline-MetaSR [‡] [13]	34.64	30.93	28.92	26.61	23.55	22.03	21.06	20.37
EDSR-baseline-LIIF (ours)	34.67	30.96	29.00	26.75	23.71	22.17	21.18	20.48
RDN-MetaSR [‡] [13]	35.00	31.27	29.25	26.88	23.73	22.18	21.17	20.47
RDN-LIIF (ours)	34.99	31.26	29.27	26.99	23.89	22.34	21.31	20.59

Table 1: Quantitative comparison on DIV2K validation set (PSNR (dB)). \sharp indicates ours implementation. The results that surpass others by 0.05 are bolded. EDSR-baseline trains different models for different scales. MetaSR and LIIF use one model for all scales, and are trained with continuous random scales uniformly sampled in $\times 1-\times 4$.

Dataset Method		In-	distribut	ion	Out-of-distribution		
Dataset	Method	$\times 2$	$\times 3$	$\times 4$	×6	$\times 8$	
	RDN [47]	38.24	34.71	32.47	-	-	
Set5	RDN-MetaSR [‡] [13]	38.22	34.63	32.38	29.04	26.96	
	RDN-LIIF (ours)	38.17	34.68	32.50	29.15	27.14	
	RDN [47]	34.01	30.57	28.81	-	-	
Set14	RDN-MetaSR [‡] [13]	33.98	30.54	28.78	26.51	24.97	
	RDN-LIIF (ours)	33.97	30.53	28.80	26.64	25.15	
	RDN [47]	32.34	29.26	27.72	-	-	
B100	RDN-MetaSR [‡] [13]	32.33	29.26	27.71	25.90	24.83	
	RDN-LIIF (ours)	32.32	29.26	27.74	25.98	24.91	
	RDN [47]	32.89	28.80	26.61	-	-	
Urban100	RDN-MetaSR [‡] [13]	32.92	28.82	26.55	23.99	22.59	
	RDN-LIIF (ours)	32.87	28.82	26.68	24.20	22.79	

Table 2: Quantitative comparison on benchmark datasets (PSNR (dB)). \sharp indicates ours implementation. The results that surpass others by 0.05 are bolded. RDN trains different models for different scales. MetaSR and LIIF use one model for all scales, and are trained with continuous random scales uniformly sampled in $\times 1-\times 4$.

ically, in training time, the up-sampling scales are uniformly sampled in $\times 1-\times 4$ (continuous range). During test time, the models are evaluated on unseen images with much higher up-sampling scales, namely $\times 6-\times 30$, that are unseen scales during training. The out-of-distribution tasks evaluate whether the continuous representation can generalize to arbitrary precision.

Implementation details. We follow prior work [21] and use 48×48 patches as the inputs for the encoder. Let B denote the batch size, we first sample B random scales $r_{1\sim B}$ in uniform distribution $\mathcal{U}(1,4)$, then we crop B patches with sizes $\{48r_i \times 48r_i\}_{i=1}^B$ from training images. These images are converted to pixel samples (coordinate-RGB pairs) and we sample 48^2 pixel samples for each of them so that the shapes of ground-truths are the same in a batch.

We use EDSR-baseline [21] or RDN [47] (without its upsampling modules) as the encoder E_{φ} , it generates a feature map with the same size as the input image. The implicit function f_{θ} is a 5-layer MLP with ReLU activation and hidden dimensions of 256. We follow [21] and use L1 loss. For training, we use bicubic resizing in PyTorch [32] to per-

form continuous down-sampling. For evaluation of scales $\times 2, \times 3, \times 4$, we use the low resolution inputs provided in DIV2K (with border-shaving that follows [21]). For evaluation of scales $\times 6-\times 30$ we first crop the ground-truths to make their shapes divisible by the scale, then we generate low-resolution inputs by bicubic down-sampling. We use Adam [18] optimizer with an initial learning rate $1 \cdot 10^{-4}$, the models are trained for 1000 epochs with batch size 16, the learning rate decays by factor 0.5 every 200 epochs. The experimental setting of MetaSR is the same as LIIF, except replacing our LIIF representation with their meta convolutional layer.

Quantitative results. In Table 1 and Table 2, we show a quantitative comparison between our method and: i) EDSR-baseline, RDN: Encoders with up-sampling modules, ii) MetaSR [13]: generating a convolutional layer from the meta-network for up-sampling. EDSR-baseline and RDN reply on up-sampling modules, therefore they have to *train different models for different scales and cannot be tested for out-of-distribution scales*. For in-distribution scales, we observe that our method achieves competitive performance

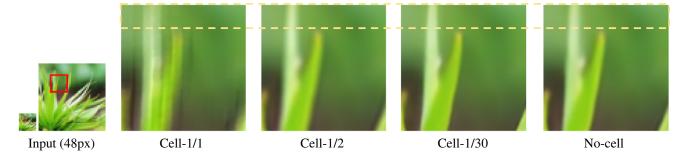


Figure 7: **Qualitative ablation study on cell decoding.** The model is trained for $\times 1-\times 4$ and tested for $\times 30$. The annotation 1/k refers to the cell size is 1/k to a pixel in the input image. The pictures demonstrate that the learned cell generalizes to unseen scales, using a proper cell size (1/30 in this case) is less blurry (e.g. the area inside the dashed line).

	In-distribution			Out-of-distribution					
	×2	$\times 3$	$\times 4$	×6	$\times 12$	$\times 18$	$\times 24$	$\times 30$	
LIIF	34.67	30.96	29.00	26.75	23.71	22.17	21.18	20.48	
LIIF(-c)	34.53	30.92	28.97	26.73	23.72	22.19	21.19	20.51	
LIIF(-u)	34.64	30.94	28.98	26.73	23.69	22.16	21.17	20.47	
LIIF(-e)	34.63	30.95	28.97	26.72	23.66	22.13	21.14	20.45	
LIIF(-d)	34.65	30.94	28.98	26.71	23.64	22.10	21.12	20.42	

Table 3: **Quantitative ablation study on designing choices of LHF.** Evaluated on the DIV2K validation set (PSNR (dB)). -c/u/e correspondingly refers to removing cell decoding, feature unfolding, and local ensemble. -d refers to reducing the depth of neural implicit function.

to prior works. Note that both EDSR-baseline and RDN are trained and evaluated for a specific scale, thus they have more advantages on "overfitting" the task than our method. For out-of-distribution scales, both EDSR and RDN baselines cannot be directly applied. We observe LIIF significantly outperforms MetaSR, which shows the advantage of using implicit representation becomes more obvious when the scale is larger. We will perform more detailed comparisons with qualitative results to explain why our method works better as below.

Qualitative results. We demonstrate a qualitative comparison in Figure 5. In the figure, 1-SIREN refers to independently fitting a SIREN [37] neural implicit function for the test image, i.e., one neural network function for one image without using an image encoder. MetaSR and LIIF are trained with scales $\times 1$ – $\times 4$ and are tested for scale $\times 30$. From the visualization, we observe that LIIF is significantly better than other methods. While 1-SIREN is capable of fitting an image as a neural implicit function, it does not share the knowledge across images, therefore its performance in higher precision is limited. MetaSR shows discontinuity since it generates a convolutional layer for up-sampling. LIIF is capable of learning a continuous representation, it maintains high fidelity even in an extreme $\times 30$ scale that is out of training distribution, and demonstrates visually pleasing results.

5.2. Ablation study

Cell decoding. In cell decoding, we attach the shape of query pixel to the input of implicit function f, which allows the implicit function to predict different values for differently sized pixels at the same location. Intuitively, it tells f to gather local information for predicting the RGB value for a given query pixel.

By comparing LIIF and LIIF(-c) in Table 3, we first observe that using cell decoding improves the performance for scales $\times 2, \times 3, \times 4$, which is expected according to our motivation. However, as the scale goes up, when LIIF is presented in out-of-distribution high resolution, it seems that cell decoding can hurt the performance of the PSNR value. Is this indicating that the learned cell does not generalize to out-of-distribution scales?

To have a closer look, we perform a qualitative study in Figure 7. The pictures are generated in a task of $\times 30$ up-sampling scale, where the model is trained for $\times 1-\times 4$. Cell-1/k refers to using the cell size that is 1/k to a pixel in the input image. Therefore, cell-1/30 is the one that should be used for $\times 30$ representation. From the figure, we can see that cell-1/30 displays much clearer edges than cell 1/1, cell-1/2, and no-cell. This is expected if we make an approximation that we assume the cell query is simply taking the average value of the image function $I^{(i)}(x)$ in the query

	In-distribution			Out-of-distribution				
	$\times 2$	$\times 3$	$\times 4$	×6	$\times 12$	$\times 18$	$\times 24$	$\times 30$
LIIF	34.67	30.96	29.00	26.75	23.71	22.17	21.18	20.48
LIIF ($\times 2$ -only)	34.71	30.72	28.77	26.51	23.54	22.05	21.09	20.40
LIIF (\times 3-only)	34.32	30.98	28.99	26.71	23.68	22.15	21.17	20.48
LIIF ($\times 4$ -only)	34.08	30.87	29.00	26.77	23.74	22.21	21.21	20.51

Table 4: **Quantitative ablation study on training LIIF for a single up-sampling scale.** Evaluated on the DIV2K validation set (PSNR (dB)). $\times k$ -only refers to the model is trained with the sample pairs of up-sampling scale k.

pixel, decoding by a cell that is larger than the actual pixel size is similar to having a "large average kernel" on the image. Similar reasons also apply to no-cell, since it is trained with scales $\times 1-\times 4$, it may implicitly learn a cell size for $\times 1-\times 4$ during training, which causes blurry in $\times 30$.

To summarize our observation, using cell decoding helps representation of in-distribution scales. When the resolution scale goes high that is out-of-distribution, it can hurt PSNR but still improves visual results. We hypothesize this is because, for out-of-distribution large scales, there can be much more uncertainty. The PSNR is computed about the "ground-truth", which may not be the unique optimal solution, especially for large scales. To some extent, being blurry (i.e. taking average around neighbor) may help a bit but not significantly (< 0.05) when uncertainty exists for PSNR computing. Note our results are still significantly better than MetaSR [13] quantitatively and qualitatively on out-of-distribution scales.

Other designing choices. In Table 3 we also have an ablation study on other designing choices. We find feature unfolding mainly helps representation in moderate scales in the comparison between LIIF and LIIF(-u). By comparing LIIF with LIIF(-e), we observe that the local ensemble consistently improves the quality of implicit representation, which demonstrates its effectiveness. To confirm the benefits of using a deep neural implicit function, we compare LIIF to LIIF(-d), which reduces 5 layers to 3 layers. It turns out that having a deep implicit function is beneficial for in-distribution scales and also generalizes better to out-of-distribution scales.

Training with $\times k$ -only. We evaluate training LIIF with the super-resolution task of a fixed up-sampling scale, the results are shown in Table 4. As shown in the table, we observe that while training for a specific scale can achieve a slightly better result for that scale, it performs worse than training with random scales when evaluated by other scales that are in-distribution. For out-of-distribution scales, while training for a specific scale can sometimes achieve slightly better PSNR, our visualization confirmed that this is a similar case as we discussed in the cell decoding ablation study, that training with random scales still displays better visual results. The results demonstrate the benefits of training with

Task	Method	PSNR (dB)
L = 64,	Up-sampling modules [21]	34.78
H = 128	LIIF (ours)	35.96
L = 32,	Up-sampling modules [21]	27.56
H = 256	LIIF (ours)	27.74

Table 5: Comparison on learning with size-varied ground-truths. Evaluated on the CelebAHQ dataset. The task is to map a face image in $L \times L$ resolution to $H \times H$ resolution, where the training images are in resolutions uniformly distributed from $L \times L$ to $H \times H$.

various random scales.

5.3. Learning with size-varied ground-truths

In this section, we introduce further applications of our LIIF continuous representation. Since LIIF representation is resolution-free, it can be compared with *arbitrarily sized* ground-truths. Specifically, in the tasks where the ground-truths are images in different resolutions (which is common in practice), suppose we have a model that generates a feature map with a fixed size, we do not need to resize the size-varied ground-truth to the same size *which sacrifices data fidelity*. LIIF can naturally build the bridge between the fixed-size feature map and the ground-truths in different resolutions. Below we show an image-to-image task as an example of this application.

Setup. We use CelebAHQ [17] dataset, that has 30,000 high-resolution face images selected from the CelebA [22] dataset. We split 25,000 images as the training set, 5,000 images as the test set (where we use 100 images for model selection). The task is to learn a mapping from a face image in $L \times L$ (low) resolution to its counterpart in $H \times H$ (high) resolution. However, the sizes of images in the training set are uniformed distributed from $L \times L$ to $H \times H$, and for every training image, we have its down-sampled counterpart in $L \times L$ resolution (as input).

Note that while this problem is also a super-resolution task, it is essentially different from the super-resolution task in the previous section where we used for learning continuous representation for pixel-based images. In the experiments of previous section, as we assume the dataset contains general natural scenes (not category specific), the

training can be patched-based. For specific up-sampling scale, the input and output size can be fixed for a super-resolution model since we can crop any sizes of patches in an image. However, in the face task, the model needs to take the whole face image in $L \times L$ resolution as input to follow the input distribution during test time, instead of training in a patch-based style. Therefore, we will need to address the challenge of *size-varied ground-truths*. Note that the input can potentially be any other fixed-size information (e.g. natural noise and perturbation) for predicting the output image, we choose the input information as a $L \times L$ low-resolution counterpart for simplicity here. In general, this task is framed as an image-to-image task with size-varied ground-truths.

Methods. We compare two end-to-end learning methods for this task. The first is denoted by "Up-sampling modules", where all the ground-truths are resized to $H \times H$ with bicubic resizing, then an encoder is trained with upsampling modules on the top. The second is to use our LIIF representation, where we train the same encoder that generates a feature map, but we take the feature map as LIIF representation and we jointly train the encoder with the implicit function. In this case, since LIIF representation can be presented in arbitrary resolution, all the ground-truths can keep their original resolution for supervision.

Implementation details. The E_{φ} and f_{θ} are the same to the previous experiment. We follow [21] for up-sampling modules and the training loss is L1 loss. We use Adam [18] optimizer, with initial learning rate $1 \cdot 10^{-4}$, the models are trained for 200 epochs with batch size 16, the learning rate decays by factor 0.1 at epoch 100.

Results. The evaluation results are shown in Table 5. For both tasks of L=64, H=128 and L=32, H=256, we consistently observe that using LIIF representation is significantly better than resizing the ground-truths to the same size and training with classical up-sampling modules. While the resizing operation sacrifices data fidelity, training with LIIF representation can naturally exploit the information provided in ground-truths in different resolutions. The results demonstrate that LIIF provides an effective framework for learning tasks with size-varied ground-truths. Also, note that the learned LIIF is a continuous representation, it can be presented in resolutions other than $H \times H$.

6. Conclusion

In this paper, we proposed the Local Implicit Image Function (LIIF) for continuous image representation. In LIIF representation, each image is represented by a 2D feature map, a neural implicit function is shared by all the images, which outputs the RGB value based on the input coordinate and neighboring feature vectors.

By training an encoder with LIIF representation in a self-supervised task with super-resolution, we can generate continuous LIIF representation for pixel-based images. The continuous representation can be presented in extreme high-resolution, we show that it can generalize to much higher precision than the training scales while maintaining high fidelity. We further demonstrated that LIIF representation builds a bridge between discrete and continuous representation in 2D, it provides a framework that can naturally and effectively exploit the information from image ground-truths in different resolutions.

References

- [1] Eirikur Agustsson and Radu Timofte. Ntire 2017 challenge on single image super-resolution: Dataset and study. In *Pro*ceedings of the IEEE Conference on Computer Vision and Pattern Recognition Workshops, pages 126–135, 2017. 5
- [2] Matan Atzmon and Yaron Lipman. Sal: Sign agnostic learning of shapes from raw data. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pages 2565–2574, 2020. 2
- [3] Marco Bevilacqua, Aline Roumy, Christine Guillemot, and Marie Line Alberi-Morel. Low-complexity single-image super-resolution based on nonnegative neighbor embedding. 2012. 5
- [4] Rohan Chabra, Jan Eric Lenssen, Eddy Ilg, Tanner Schmidt, Julian Straub, Steven Lovegrove, and Richard Newcombe. Deep local shapes: Learning local sdf priors for detailed 3d reconstruction. arXiv preprint arXiv:2003.10983, 2020. 2
- [5] Hong Chang, Dit-Yan Yeung, and Yimin Xiong. Superresolution through neighbor embedding. In *Proceedings of* the 2004 IEEE Computer Society Conference on Computer Vision and Pattern Recognition, 2004. CVPR 2004., volume 1, pages I–I. IEEE, 2004. 3
- [6] Zhiqin Chen and Hao Zhang. Learning implicit fields for generative shape modeling. In *Proceedings of the IEEE Con*ference on Computer Vision and Pattern Recognition, pages 5939–5948, 2019. 1, 2
- [7] Emily L Denton, Soumith Chintala, Rob Fergus, et al. Deep generative image models using a laplacian pyramid of adversarial networks. In *Advances in neural information process*ing systems, pages 1486–1494, 2015. 2
- [8] Chao Dong, Chen Change Loy, Kaiming He, and Xiaoou Tang. Learning a deep convolutional network for image super-resolution. In *European conference on computer vision*, pages 184–199. Springer, 2014. 3
- [9] Chao Dong, Chen Change Loy, Kaiming He, and Xiaoou Tang. Image super-resolution using deep convolutional networks. *IEEE transactions on pattern analysis and machine* intelligence, 38(2):295–307, 2015.
- [10] Ian Goodfellow, Jean Pouget-Abadie, Mehdi Mirza, Bing Xu, David Warde-Farley, Sherjil Ozair, Aaron Courville, and Yoshua Bengio. Generative adversarial nets. In *Advances* in neural information processing systems, pages 2672–2680, 2014. 2

- [11] Amos Gropp, Lior Yariv, Niv Haim, Matan Atzmon, and Yaron Lipman. Implicit geometric regularization for learning shapes. *arXiv preprint arXiv:2002.10099*, 2020. 2
- [12] Judy Hoffman, Eric Tzeng, Taesung Park, Jun-Yan Zhu, Phillip Isola, Kate Saenko, Alexei Efros, and Trevor Darrell. Cycada: Cycle-consistent adversarial domain adaptation. In *International conference on machine learning*, pages 1989– 1998, PMLR, 2018. 2
- [13] Xuecai Hu, Haoyuan Mu, Xiangyu Zhang, Zilei Wang, Tieniu Tan, and Jian Sun. Meta-sr: A magnification-arbitrary network for super-resolution. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, pages 1575–1584, 2019. 3, 5, 6, 8
- [14] Jia-Bin Huang, Abhishek Singh, and Narendra Ahuja. Single image super-resolution from transformed self-exemplars. In Proceedings of the IEEE conference on computer vision and pattern recognition, pages 5197–5206, 2015. 5
- [15] Phillip Isola, Jun-Yan Zhu, Tinghui Zhou, and Alexei A Efros. Image-to-image translation with conditional adversarial networks. CVPR, 2017. 2
- [16] Chiyu Jiang, Avneesh Sud, Ameesh Makadia, Jingwei Huang, Matthias Nießner, and Thomas Funkhouser. Local implicit grid representations for 3d scenes. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pages 6001–6010, 2020. 2
- [17] Tero Karras, Timo Aila, Samuli Laine, and Jaakko Lehtinen. Progressive growing of gans for improved quality, stability, and variation. arXiv preprint arXiv:1710.10196, 2017. 8
- [18] Diederik P Kingma and Jimmy Ba. Adam: A method for stochastic optimization. arXiv preprint arXiv:1412.6980, 2014. 6, 9
- [19] Wei-Sheng Lai, Jia-Bin Huang, Narendra Ahuja, and Ming-Hsuan Yang. Deep laplacian pyramid networks for fast and accurate super-resolution. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pages 624–632, 2017. 2, 3
- [20] Christian Ledig, Lucas Theis, Ferenc Huszár, Jose Caballero, Andrew Cunningham, Alejandro Acosta, Andrew Aitken, Alykhan Tejani, Johannes Totz, Zehan Wang, et al. Photorealistic single image super-resolution using a generative adversarial network. In Proceedings of the IEEE conference on computer vision and pattern recognition, pages 4681–4690, 2017. 2, 3
- [21] Bee Lim, Sanghyun Son, Heewon Kim, Seungjun Nah, and Kyoung Mu Lee. Enhanced deep residual networks for single image super-resolution. In *Proceedings of the IEEE conference on computer vision and pattern recognition workshops*, pages 136–144, 2017. 2, 3, 5, 6, 8, 9
- [22] Ziwei Liu, Ping Luo, Xiaogang Wang, and Xiaoou Tang. Deep learning face attributes in the wild. In *Proceedings of the IEEE international conference on computer vision*, pages 3730–3738, 2015. 8
- [23] David Martin, Charless Fowlkes, Doron Tal, and Jitendra Malik. A database of human segmented natural images and its application to evaluating segmentation algorithms and measuring ecological statistics. In *Proceedings Eighth IEEE International Conference on Computer Vision. ICCV 2001*, volume 2, pages 416–423. IEEE, 2001. 5

- [24] Lars Mescheder, Michael Oechsle, Michael Niemeyer, Sebastian Nowozin, and Andreas Geiger. Occupancy networks: Learning 3d reconstruction in function space. In *Proceed*ings of the IEEE Conference on Computer Vision and Pattern Recognition, pages 4460–4470, 2019. 1, 2
- [25] Mateusz Michalkiewicz, Jhony K Pontes, Dominic Jack, Mahsa Baktashmotlagh, and Anders Eriksson. Implicit surface representations as layers in neural networks. In *Pro*ceedings of the IEEE International Conference on Computer Vision, pages 4743–4752, 2019. 2
- [26] Ben Mildenhall, Pratul P Srinivasan, Matthew Tancik, Jonathan T Barron, Ravi Ramamoorthi, and Ren Ng. Nerf: Representing scenes as neural radiance fields for view synthesis. arXiv preprint arXiv:2003.08934, 2020. 2
- [27] Alexander Mordvintsev, Nicola Pezzotti, Ludwig Schubert, and Chris Olah. Differentiable image parameterizations. *Distill*, 3(7):e12, 2018. 2
- [28] Vinod Nair and Geoffrey E Hinton. Rectified linear units improve restricted boltzmann machines. In ICML, 2010. 2
- [29] Michael Niemeyer, Lars Mescheder, Michael Oechsle, and Andreas Geiger. Differentiable volumetric rendering: Learning implicit 3d representations without 3d supervision. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition, pages 3504–3515, 2020. 2
- [30] Michael Oechsle, Lars Mescheder, Michael Niemeyer, Thilo Strauss, and Andreas Geiger. Texture fields: Learning texture representations in function space. In *Proceedings of the IEEE International Conference on Computer Vision*, pages 4531–4540, 2019. 2
- [31] Jeong Joon Park, Peter Florence, Julian Straub, Richard Newcombe, and Steven Lovegrove. Deepsdf: Learning continuous signed distance functions for shape representation. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, pages 165–174, 2019. 1, 2
- [32] Adam Paszke, Sam Gross, Francisco Massa, Adam Lerer, James Bradbury, Gregory Chanan, Trevor Killeen, Zeming Lin, Natalia Gimelshein, Luca Antiga, Alban Desmaison, Andreas Kopf, Edward Yang, Zachary DeVito, Martin Raison, Alykhan Tejani, Sasank Chilamkurthy, Benoit Steiner, Lu Fang, Junjie Bai, and Soumith Chintala. Pytorch: An imperative style, high-performance deep learning library. In H. Wallach, H. Larochelle, A. Beygelzimer, F. d'Alché-Buc, E. Fox, and R. Garnett, editors, Advances in Neural Information Processing Systems 32, pages 8024–8035. Curran Associates, Inc., 2019. 6
- [33] Songyou Peng, Michael Niemeyer, Lars Mescheder, Marc Pollefeys, and Andreas Geiger. Convolutional occupancy networks. *arXiv preprint arXiv:2003.04618*, 2020. 2
- [34] Shunsuke Saito, Zeng Huang, Ryota Natsume, Shigeo Morishima, Angjoo Kanazawa, and Hao Li. Pifu: Pixel-aligned implicit function for high-resolution clothed human digitization. In *Proceedings of the IEEE International Conference on Computer Vision*, pages 2304–2314, 2019. 1, 2
- [35] Shunsuke Saito, Tomas Simon, Jason Saragih, and Hanbyul Joo. Pifuhd: Multi-level pixel-aligned implicit function for high-resolution 3d human digitization. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pages 84–93, 2020. 1, 2

- [36] Vincent Sitzmann, Eric Chan, Richard Tucker, Noah Snavely, and Gordon Wetzstein. Metasdf: Meta-learning signed distance functions. Advances in Neural Information Processing Systems, 33, 2020. 2
- [37] Vincent Sitzmann, Julien Martel, Alexander Bergman, David Lindell, and Gordon Wetzstein. Implicit neural representations with periodic activation functions. *Advances in Neural Information Processing Systems*, 33, 2020. 1, 2, 5, 7
- [38] Vincent Sitzmann, Michael Zollhöfer, and Gordon Wetzstein. Scene representation networks: Continuous 3d-structure-aware neural scene representations. In Advances in Neural Information Processing Systems, pages 1121–1132, 2019. 2
- [39] Kenneth O Stanley. Compositional pattern producing networks: A novel abstraction of development. *Genetic pro*gramming and evolvable machines, 8(2):131–162, 2007. 2
- [40] Ying Tai, Jian Yang, Xiaoming Liu, and Chunyan Xu. Memnet: A persistent memory network for image restoration. In *Proceedings of the IEEE international conference on computer vision*, pages 4539–4547, 2017. 3
- [41] Radu Timofte, Eirikur Agustsson, Luc Van Gool, Ming-Hsuan Yang, and Lei Zhang. Ntire 2017 challenge on single image super-resolution: Methods and results. In *Proceedings of the IEEE conference on computer vision and pattern recognition workshops*, pages 114–125, 2017. 5
- [42] Radu Timofte, Vincent De Smet, and Luc Van Gool. Anchored neighborhood regression for fast example-based super-resolution. In *Proceedings of the IEEE international* conference on computer vision, pages 1920–1927, 2013. 3
- [43] Qiangeng Xu, Weiyue Wang, Duygu Ceylan, Radomir Mech, and Ulrich Neumann. Disn: Deep implicit surface network for high-quality single-view 3d reconstruction. In Advances in Neural Information Processing Systems, pages 492–502, 2019. 2
- [44] Roman Zeyde, Michael Elad, and Matan Protter. On single image scale-up using sparse-representations. In *International conference on curves and surfaces*, pages 711–730. Springer, 2010. 5
- [45] Kai Zhang, Luc Van Gool, and Radu Timofte. Deep unfolding network for image super-resolution. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pages 3217–3226, 2020. 3
- [46] Richard Zhang, Phillip Isola, and Alexei A Efros. Colorful image colorization. In *European conference on computer* vision, pages 649–666. Springer, 2016. 2
- [47] Yulun Zhang, Yapeng Tian, Yu Kong, Bineng Zhong, and Yun Fu. Residual dense network for image super-resolution. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pages 2472–2481, 2018. 3, 6
- [48] Jun-Yan Zhu, Taesung Park, Phillip Isola, and Alexei A Efros. Unpaired image-to-image translation using cycleconsistent adversarial networks. In *Proceedings of the IEEE* international conference on computer vision, pages 2223– 2232, 2017. 2