
Multiview Neural Surface Reconstruction with Implicit Lighting and Material

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Abstract

In this work we address the challenging problem of multiview 3D surface reconstruction. We introduce a neural network architecture that simultaneously learns the unknown geometry, camera parameters, and a neural renderer that approximates the light reflected from the surface towards the camera. The geometry is represented as a zero level-set of a neural network, while the neural renderer, derived from the rendering equation, is capable of (implicitly) modeling a wide set of lighting conditions and materials. We trained our network on real world 2D images of objects with different material properties, lighting conditions, and noisy camera initializations from the DTU MVS dataset. We found our model to produce state of the art 3D surface reconstructions with high fidelity, resolution and detail.

1 Introduction

Learning 3D shapes from 2D images is a fundamental computer vision problem. A recent successful neural network approach to solving this problem involves the use of a (neural) differentiable rendering system along with a choice of (neural) 3D geometry representation. Differential rendering systems are mostly based on ray casting/tracing [50, 40, 28, 30, 46, 31], or rasterization [32, 23, 10, 29, 4], while popular models to represent 3D geometry include point clouds [58], triangle meshes [4], implicit representations defined over volumetric grids [20], and recently also neural implicit representations, namely, zero level sets of neural networks [30, 40].

The main advantage of implicit neural representations is their flexibility in representing surfaces with arbitrary shapes and topologies, as well as being mesh-free (i.e., no fixed a-priori discretization such as a volumetric grid or a triangular mesh). Thus far, differentiable rendering systems with implicit neural representations [30, 31, 40] did not incorporate lighting and reflectance properties required for producing faithful appearance of 3D geometry in images, nor did they deal with trainable camera locations and orientations.

The goal of this paper is to devise an end-to-end neural architecture system that can learn 3D geometries from masked 2D images and rough camera estimates, and requires no additional supervision, see Figure 1. Towards that end we represent the color of a pixel as a differentiable function in the three unknowns of a scene: the geometry, its appearance (i.e., the surface properties and lighting), and the cameras. We call this architecture the Implicit Differentiable Renderer (IDR). We show that IDR is able to approximate the light reflected from a 3D shape represented as the zero level set of a neural network, accounting for its bidirectional reflectance distribution function (BRDF) and the lighting conditions. The approach can handle BRDF and lighting from a certain restricted family, namely, all BRDFs and incoming radiance functions that can be represented as continuous functions of the point on the surface, its normal, and the viewing direction. Furthermore, incorporating a global shape feature vector into IDR increases its ability to handle even more complex appearances.

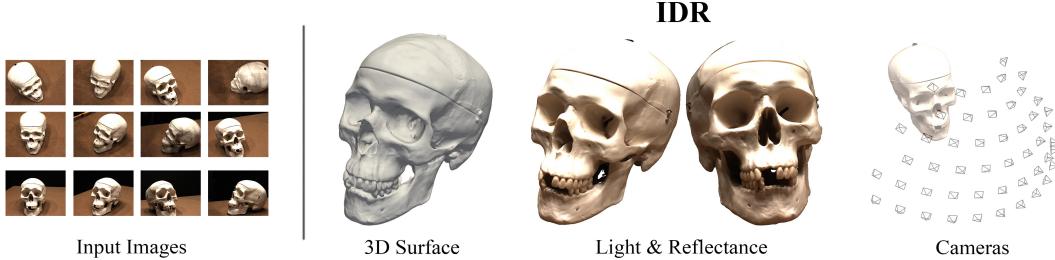


Figure 1: We introduce IDR: end-to-end learning of geometry, appearance and cameras from images.

Most related to our paper is [40], which was first to introduce a fully differentiable renderer for implicit neural occupancy functions [37]. Although their model can represent arbitrary color and texture, it cannot handle general reflectance and lighting effects, nor can it handle unknown, noisy camera locations. For example, we show that the model in [40], as well as several other baselines, fail to generate the Phong reflection model [8]. Moreover, we show experimentally that IDR produces more accurate 3D reconstructions of shapes from 2D images along with accurate camera parameters. Notably, while the baseline often produces shape artifact in specular scenes, IDR is robust to such lighting effects.

To summarize, the key contributions of our approach are:

- End-to-end architecture that handles unknown geometry, reflectance, lighting, and cameras.
- Expressing the dependence of a neural implicit surface on camera parameters.
- Producing state of the art 3D surface reconstructions of different objects with a wide range of appearances, from real-life 2D images, with both exact and noisy camera information.

2 Previous work

Differentiable rendering systems for learning geometry comes (mostly) in two flavors: differentiable rasterization [32, 23, 10, 29, 4], and differentiable ray casting. Since the current work falls into the second category we first concentrate on that branch of works. Then, we will describe related works for multi-view surface reconstruction and neural view synthesis.

Implicit surface differentiable ray casting. Differentiable ray casting is mostly used with *implicit* shape representations such as implicit function defined over a volumetric grid or implicit neural representation, where the implicit function can be the occupancy function [37, 5], signed distance function (SDF) [42] or any other signed implicit [2]. In a related work, [20] use a volumetric grid to represent an SDF and implement a ray casting differentiable renderer. They approximate the SDF value and the surface normals in each volumetric cell. [31] use sphere tracing of pre-trained DeepSDF model [42] and approximates the depth gradients w.r.t. the latent code of the DeepSDF network by differentiating the individual steps of the sphere tracing algorithm; [30] use field probing to facilitate differentiable ray casting. In contrast to these works, IDR utilize exact and differentiable surface point and normal of the implicit surface, and considers a more general lighting and reflectance model, as well as handle noisy cameras.

Multi-view surface reconstruction. During the capturing process of an image, the depth information is lost. Assuming known cameras, classic Multi-View Stereo (MVS) methods [9, 48, 3, 54] try to reproduce the depth information by matching features points across views. However, a post-processing steps of depth fusion [6, 36] followed by the Poisson Surface Reconstruction algorithm [24] are required for producing a valid 3D watertight surface reconstruction. Recent methods use a collection of scenes to train a deep neural models for either sub-tasks of the MVS pipeline, e.g., feature matching [27], or depth fusion [7, 44], or for an End-to-End MVS pipeline [16, 56, 57]. When the camera parameters are unavailable, and given a set of images from a specific scene, Structure From Motion (SFM) methods [51, 47, 22, 19] are applied for reproducing the cameras and a sparse 3D reconstruction. Tang and Tan [53] use a deep neural architecture with an integrated differentiable Bundle Adjustment [55] layer to extract a linear basis for the depth of a reference frame, and features from nearby images and to optimize for the depth and the camera parameters in each forward pass. In contrast to these works, IDR is trained with images from a single target scene, producing an accurate watertight 3D surface reconstruction.

Neural representation for view synthesis. Recent works trained neural networks to predict novel views and some geometric representation of 3D scenes or objects, from a limited set of images with known cameras. [50] encode the scene geometry using an LSTM to simulate the ray marching process. [38] use a neural network to predict volume density and view dependent emitted radiance to synthesis new views from a set of images with known cameras. [41] use a neural network to learns the surface light fields from an input image and geometry and predicting unknown views and/or scene lighting. Differently from IDR, these methods do not produce a 3D surface reconstruction of the scene’s geometry nor handle unknown cameras.

3 Method

Our goal is to reconstruct the geometry of an object from masked 2D images with possibly rough or noisy camera information. We have three unknowns: (i) *geometry*, represented by parameters $\theta \in \mathbb{R}^m$; (ii) *appearance*, represented by $\gamma \in \mathbb{R}^n$; and (iii) *cameras* represented by $\tau \in \mathbb{R}^k$. Notations and setup used in this section are depicted in Figure 2.

We represent the geometry as the zero level set of a neural network (MLP) f ,

$$\mathcal{S}_\theta = \{\mathbf{x} \in \mathbb{R}^3 \mid f(\mathbf{x}; \theta) = 0\}, \quad (1)$$

with learnable parameters $\theta \in \mathbb{R}^m$. To avoid the everywhere 0 solution, f is usually regularized [37, 5]. We opt for f to model a signed distance function (SDF) to its zero level set \mathcal{S}_θ [42]. We enforce the SDF constraint using the implicit geometric regularization (IGR) [11], detailed later. SDF has two benefits in our context: First, it allows an efficient ray casting with the sphere tracing algorithm [12, 20]; and second, IGR enjoys implicit regularization favoring smooth and realistic surfaces.

IDR forward model. Given a pixel, indexed by p , associated with some input image, let $R_p(\tau) = \{c_p + t\mathbf{v}_p \mid t \geq 0\}$ denote the ray through pixel p , where $c_p = c_p(\tau)$ denotes the unknown center of the respective camera and $\mathbf{v}_p = \mathbf{v}_p(\tau)$ the direction of the ray (i.e., the vector pointing from c_p towards pixel p). Let $\hat{\mathbf{x}}_p = \hat{\mathbf{x}}_p(\theta, \tau)$ denote the first intersection of the ray R_p and the surface \mathcal{S}_θ . The incoming radiance along R_p , which determines the rendered color of the pixel $L_p = L_p(\theta, \gamma, \tau)$, is a function of the surface properties at $\hat{\mathbf{x}}_p$, the incoming radiance at $\hat{\mathbf{x}}_p$, and the viewing direction \mathbf{v}_p . In turn, we make the assumptions that the surface property and incoming radiance are functions of the surface point $\hat{\mathbf{x}}_p$, and its corresponding surface normal $\hat{\mathbf{n}}_p = \hat{\mathbf{n}}_p(\theta)$, the viewing direction \mathbf{v}_p , and a global geometry feature vector $\hat{\mathbf{z}}_p = \hat{\mathbf{z}}_p(\hat{\mathbf{x}}_p; \theta)$. The IDR forward model is therefore:

$$L_p(\theta, \gamma, \tau) = M(\hat{\mathbf{x}}_p, \hat{\mathbf{n}}_p, \hat{\mathbf{z}}_p, \mathbf{v}_p; \gamma), \quad (2)$$

where M is a second neural network (MLP). We utilize L_p in a loss comparing L_p and the pixel input color I_p to simultaneously train the model’s parameters θ, τ, γ . We next provide more details on the different components of the model in equation 2.

3.1 Differentiable intersection of viewing direction and geometry

Henceforth (up until section 3.4), we assume a fixed pixel p , and remove the subscript p notation to simplify notation. The first step is to represent the intersection point $\hat{\mathbf{x}}(\theta, \tau)$ as a neural network with parameters θ, τ . We show this can be done with a slight modification to the geometry network f .

Let $\hat{\mathbf{x}}(\theta, \tau) = \mathbf{c} + t(\theta, \mathbf{c}, \mathbf{v})\mathbf{v}$ denote the intersection point. As we are aiming to use $\hat{\mathbf{x}}$ in a gradient descent-like algorithm, all we need to make sure is that our derivations are correct in value and first derivatives at the current parameters, denoted by θ_0, τ_0 ; accordingly we denote $\mathbf{c}_0 = \mathbf{c}(\tau_0)$, $\mathbf{v}_0 = \mathbf{v}(\tau_0)$, $t_0 = t(\theta_0, \mathbf{c}_0, \mathbf{v}_0)$, and $\mathbf{x}_0 = \hat{\mathbf{x}}(\theta_0, \tau_0) = \mathbf{c}_0 + t_0\mathbf{v}_0$.

Lemma 1. *Let \mathcal{S}_θ be defined as in equation 1. The intersection of the ray $R(\tau)$ and the surface \mathcal{S}_θ can be represented by the formula*

$$\hat{\mathbf{x}}(\theta, \tau) = \mathbf{c} + t_0\mathbf{v} - \frac{\mathbf{v}}{\nabla_{\mathbf{x}} f(\mathbf{x}_0; \theta_0) \cdot \mathbf{v}_0} f(\mathbf{c} + t_0\mathbf{v}; \theta), \quad (3)$$

and is exact in value and first derivatives of θ and τ at $\theta = \theta_0$ and $\tau = \tau_0$.

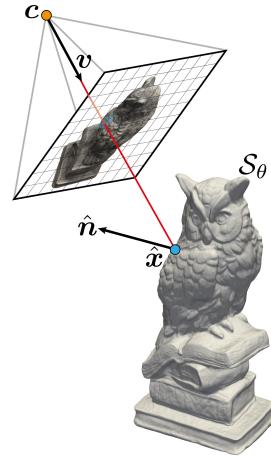


Figure 2: Notations.

To prove this functional dependency of \hat{x} on its parameters, we use implicit differentiation [1, 40], that is, differentiate the equation $f(\hat{x}; \theta) \equiv 0$ w.r.t. v, c, θ and solve for the derivatives of t . Then, it can be checked that the formula in equation 3 possess the correct derivatives. More details are in the supplementary. We implement equation 3 as a neural network, namely, we add two linear layers (with parameters c, v): one before and one after the MLP f . Equation 3 unifies the sample network formula in [1] and the differentiable depth in [40] and generalizes them to account for unknown cameras. The normal vector to S_θ at \hat{x} can be computed by:

$$\hat{n}(\theta, \tau) = \nabla_{\mathbf{x}} f(\hat{x}(\theta, \tau), \theta) / \|\nabla_{\mathbf{x}} f(\hat{x}(\theta, \tau), \theta)\|_2. \quad (4)$$

Note that for SDF the denominator is 1, so can be removed.

3.2 Approximation of the rendering equation

The radiance L is the amount of light reflected from S_θ at \hat{x} in direction $-v$ reaching c . It is determined by two functions: The bidirectional reflectance distribution function (BRDF) describing the reflectance and color properties of the surface, and the light emitted in the scene (i.e., light sources).

The BRDF function $B(x, n, w^o, w^i)$ describes the proportion of reflected radiance (i.e., flux of light) at some wave-length (i.e., color) leaving the surface point x with normal n at direction w^o with respect to the incoming radiance from direction w^i . We let the BRDF depend also on the normal n to the surface at a point. The light sources in the scene are described by a function $L^e(x, w^o)$ measuring the emitted radiance of light at some wave-length at point x in direction w^o . The amount of light reaching c in direction v equals the amount of light reflected from \hat{x} in direction $w^o = -v$ and is described by the so-called rendering equation [21, 17]:

$$L(\hat{x}, w^o) = L^e(\hat{x}, w^o) + \int_{\Omega} B(\hat{x}, \hat{n}, w^i, w^o) L^i(\hat{x}, w^i) (\hat{n} \cdot w^i) dw^i \quad (5)$$

where $L^i(\hat{x}, w^i)$ encodes the incoming radiance at \hat{x} in direction w^i , and the term $\hat{n} \cdot w^i$ compensates for the fact that the light does not hit the surface orthogonally; Ω is the half sphere centered at \hat{n} . This rendering equation holds for every light wave-length; as described later we will use it for the red, green and blue (RGB) wave-lengths.

We restrict our material and lighting settings to those represented by a *continuous* volumetric BRDF function $B : \mathbb{R}^{12} \rightarrow \mathbb{R}$ and light radiance functions $L^e, L^i : \mathbb{R}^6 \rightarrow \mathbb{R}$. We denote the collection of such continuous functions by $\mathcal{P} = \{(B, L^e, L^i)\}$ (see supplementary material for more discussion on \mathcal{P}). In this setting equation 5 can be written as $L(\hat{x}, w^o) = M_0(\hat{x}, \hat{n}, v)$, where M_0 is some continuous function. Replacing M_0 with a (sufficiently large) MLP approximation M provides the radiance approximation:

$$L(\theta, \gamma, \tau) = M(\hat{x}, \hat{n}, v; \gamma). \quad (6)$$

Given an arbitrary choice of BRDF and lighting from \mathcal{P} there exists a choice of weights $\gamma = \gamma_0$ so that M can approximate the correct radiance function M_0 for all x, n, v (in some bounded set). This can be proved using a standard universality theorem for MLPs (details in the supplementary).

The fact that M can learn the correct radiance function M_0 does not mean it is *guaranteed* to learn it during optimization. Indeed, some ambiguity exists such as interpreting shadows on the surface as color of the surface. Nevertheless, being able to approximate M_0 is a *necessary condition* for our framework to separate correctly geometry (via f) and reflectance/lighting (via M). We name this necessary condition \mathcal{P} -*universality*.

Necessity of viewing direction and normal. As it turns out n, v are *necessary* in M in order to achieve a \mathcal{P} -universality. Previous works, e.g., [40], have considered rendering functions of implicit neural representations of the form $L(\theta, \gamma) = M(\hat{x}; \gamma)$. Removing n and/or v from M will result in a non- \mathcal{P} -universal renderer. This can be seen in Figure 3: A renderer M without normal information will produce the same light estimation in cases (a) and (b), while a renderer M without viewing direction will produce the same light estimation in cases (a) and (c). In the supplementary we provide details on how these renderers fail to generate correct radiance under the Phong reflection model [8].

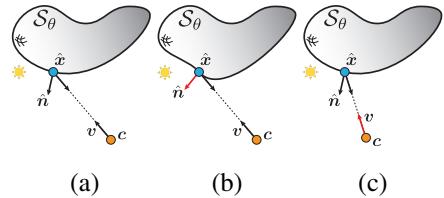


Figure 3: Neural renderers without \hat{n} and/or v are not universal.

Accounting for global light effects. \mathcal{P} -universality is a necessary condition to learn a neural renderer M that can simulate appearance from the collection \mathcal{P} . However, \mathcal{P} does not include global lighting effects such as secondary lighting and self-shadows. We further increase the expressive power of IDR by introducing a global feature vector $\hat{\mathbf{z}}$. This feature vector allows the renderer to reason *globally* about the geometry \mathcal{S}_θ . To produce the vector $\hat{\mathbf{z}}$ we extend the network f as follows: $F(\mathbf{x}; \theta) = [f(\mathbf{x}; \theta), \mathbf{z}(\mathbf{x}; \theta)] \in \mathbb{R} \times \mathbb{R}^\ell$. In general, \mathbf{z} can encode the geometry \mathcal{S}_θ relative to the surface sample \mathbf{x} ; \mathbf{z} is fed into the renderer as $\hat{\mathbf{z}}(\theta, \tau) = \mathbf{z}(\hat{\mathbf{x}}; \theta)$ to take into account the surface sample $\hat{\mathbf{x}}$ relevant for the current pixel of interest p . We have now completed the description of the IDR model, given in equation 2.

3.3 Masked rendering

Another useful type of 2D supervision for reconstructing 3D geometry are masks; masks are binary images indicating, for each pixel p , if the object of interest occupies this pixel. Masks can be provided in the data (as we assume) or computed using, e.g., masking or segmentation algorithms. We would like to consider the following indicator function identifying whether a certain pixel is occupied by the rendered object (remember we assume some fixed pixel p):

$$S(\theta, \tau) = \begin{cases} 1 & R(\tau) \cap \mathcal{S}_\theta \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

Since this function is not differentiable nor continuous in θ, τ we use an almost everywhere differentiable approximation:

$$S_\alpha(\theta, \tau) = \text{sigmoid}\left(-\alpha \min_{t \geq 0} f(\mathbf{c} + t\mathbf{v}; \theta)\right), \quad (7)$$

where $\alpha > 0$ is a parameter. Since, by convention, $f < 0$ inside our geometry and $f > 0$ outside, it can be verified that $S_\alpha(\theta, \tau) \xrightarrow{\alpha \rightarrow \infty} S(\theta, \tau)$. Note that differentiating equation 7 w.r.t. \mathbf{c}, \mathbf{v} can be done using the envelope theorem, namely $\partial_{\mathbf{c}} \min_{t \geq 0} f(\mathbf{c} + t\mathbf{v}; \theta) = \partial_{\mathbf{c}} f(\mathbf{c} + t_* \mathbf{v}; \theta)$, where t_* is an argument achieving the minimum, i.e., $f(\mathbf{c}_0 + t_* \mathbf{v}_0; \theta) = \min_{t \geq 0} f(\mathbf{c}_0 + t\mathbf{v}_0; \theta)$, and similarly for $\partial_{\mathbf{v}}$. We therefore implement S_α as the neural network sigmoid($-\alpha f(\mathbf{c} + t_* \mathbf{v}; \theta)$). Note that this neural network has exact value and first derivatives at $\mathbf{c} = \mathbf{c}_0$, and $\mathbf{v} = \mathbf{v}_0$.

3.4 Loss

Let $I_p \in [0, 1]^3$, $O_p \in \{0, 1\}$ be the RGB and mask values (resp.) corresponding to a pixel p in an image taken with camera $\mathbf{c}_p(\tau)$ and direction $\mathbf{v}_p(\tau)$ where $p \in P$ indexes all pixels in the input collection of images, and $\tau \in \mathbb{R}^k$ represents the parameters of all the cameras in scene. Our loss function has the form:

$$\text{loss}(\theta, \gamma, \tau) = \text{loss}_{\text{RGB}}(\theta, \gamma, \tau) + \rho \text{loss}_{\text{MASK}}(\theta, \tau) + \lambda \text{loss}_{\text{E}}(\theta) \quad (8)$$

We train this loss on mini-batches of pixels in P ; for keeping notations simple we denote by P the current mini-batch. For each $p \in P$ we use the sphere-tracing algorithm [12, 20] to compute the first intersection point, $\mathbf{c}_p + t_{p,0} \mathbf{v}_p$, of the ray $R_p(\tau)$ and \mathcal{S}_θ . Let $P^{\text{in}} \subset P$ be the subset of pixels p where intersection has been found and $O_p = 1$. Let $L_p(\theta, \gamma, \tau) = M(\hat{\mathbf{x}}_p, \hat{\mathbf{n}}_p, \hat{\mathbf{z}}_p, \mathbf{v}_p; \gamma)$, where $\hat{\mathbf{x}}_p, \hat{\mathbf{n}}_p$ is defined as in equations 3 and 4, and $\hat{\mathbf{z}}_p = \hat{\mathbf{z}}(\hat{\mathbf{x}}_p; \theta)$ as in section 3.2 and equation 2. The RGB loss is

$$\text{loss}_{\text{RGB}}(\theta, \gamma, \tau) = \frac{1}{|P|} \sum_{p \in P^{\text{in}}} |I_p - L_p(\theta, \gamma, \tau)|, \quad (9)$$

where $|\cdot|$ represents the L_1 norm. Let $P^{\text{out}} = P \setminus P^{\text{in}}$ denote the indices in the mini-batch for which no ray-geometry intersection exist. The mask loss is

$$\text{loss}_{\text{MASK}}(\theta, \tau) = \frac{1}{\alpha |P|} \sum_{p \in P^{\text{out}}} \text{CE}(O_p, S_{p,\alpha}(\theta, \tau)), \quad (10)$$

where CE is the cross-entropy loss. Lastly, we enforce f to be approximately a signed distance function with Implicit Geometric Regularization (IGR) [11], i.e., incorporating the Eikonal regularization:

$$\text{loss}_{\text{E}}(\theta) = \mathbb{E}_{\mathbf{x}} (\|\nabla_{\mathbf{x}} f(\mathbf{x}; \theta)\| - 1)^2 \quad (11)$$

where \mathbf{x} is distributed uniformly in a bounding box of the scene.

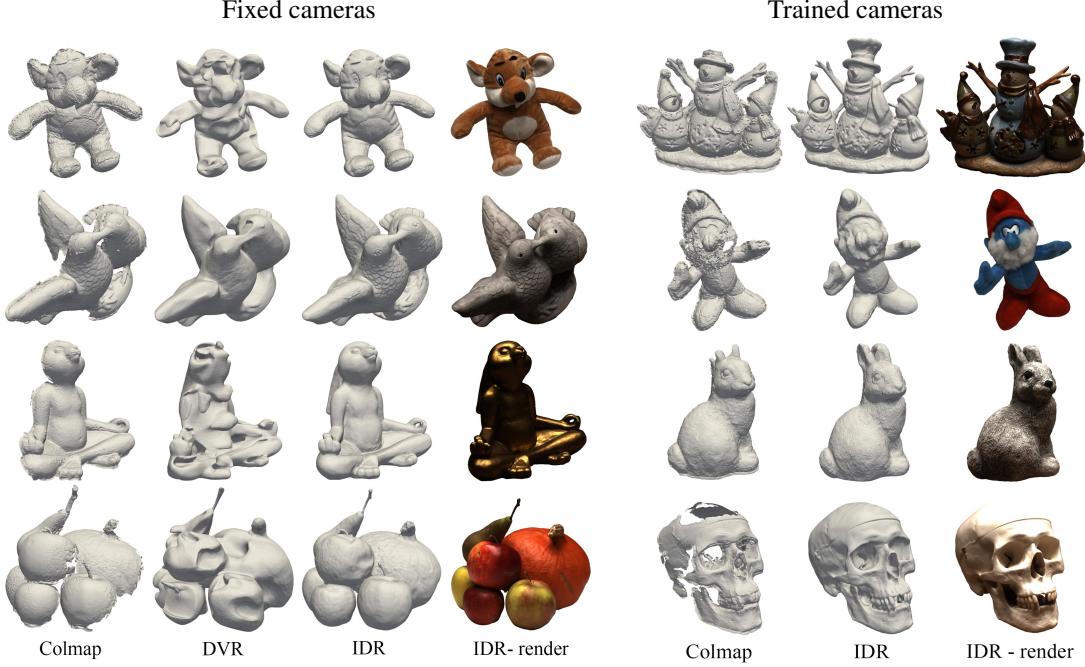


Figure 4: Qualitative results of multiview 3D surface reconstructions. Note the high fidelity of the IDR reconstructions and its realistic rendering.

Implementation details. The MLP $F(\mathbf{x}; \theta) = (f(\mathbf{x}; \theta), \mathbf{z}(\mathbf{x}; \theta)) \in \mathbb{R} \times \mathbb{R}^{256}$ consists of 8 layers with hidden layers of width 512, and a single skip connection from the input to the middle layer as in [42]. We initialize the weights $\theta \in \mathbb{R}^m$ as in [2], so that $f(\mathbf{x}, \theta)$ produces an approximate SDF of a unit sphere. The renderer MLP, $M(\hat{\mathbf{x}}, \hat{\mathbf{n}}, \hat{\mathbf{z}}, \mathbf{v}; \gamma) \in \mathbb{R}^3$, consists of 4 layers, with hidden layers of width 512. We use the non-linear maps of [38] to improve the learning of high-frequencies, which are otherwise difficult to train for due to the inherent low frequency bias of neural networks [45]. Specifically, for a scalar $y \in \mathbb{R}$ we denote by $\delta_k(y) \in \mathbb{R}^{2k}$ the vector of real and imaginary parts of $\exp(i2\pi\omega y)$ with $\omega \in [k]$, and for a vector \mathbf{y} we denote by $\delta_k(\mathbf{y})$ the concatenation of $\delta_k(y_i)$ for all the entries of \mathbf{y} . We redefine F to obtain $\delta_6(\mathbf{x})$ as input, i.e., $F(\delta_6(\mathbf{x}); \theta)$, and likewise we redefine M to receive $\delta_4(\mathbf{v})$, i.e., $M(\hat{\mathbf{x}}, \hat{\mathbf{n}}, \hat{\mathbf{z}}, \delta_4(\mathbf{v}); \gamma)$. For the loss, equation 8, we set $\lambda = 0.1$ and $\rho = 100$. To approximate the indicator function with $S_\alpha(\theta, \tau)$, during training, we gradually increase α and by this constrain the shape boundaries in a coarse to fine manner: we start with $\alpha = 50$ and multiply it by a factor of 2 every 250 epochs (up to a total of 5 multiplications). The gradients in equations (11),(4) are implemented using auto-differentiation. More details are in the supplementary.

4 Experiments

4.1 Multiview 3D reconstruction

We apply our multiview surface reconstruction model to real 2D images from the DTU MVS repository [18]. Our experiments were run on 15 challenging scans, each includes either 49 or 64 high resolution images of objects with a wide variety of materials and shapes. The dataset also contains ground truth 3D geometries and camera poses. We manually annotated binary masks for all 15 scans except for scans 65, 106 and 118 which are supplied by [40].

We used our method to generate 3D reconstructions in two different setups: (1) fixed ground truth cameras, and (2) trainable cameras with noisy initializations obtained with the linear method of [19]. In both cases we re-normalize the cameras so that their visual hulls are contained in the unit sphere. Training each multi-view image collection proceeded iteratively. In each iteration we randomly sampled 2048 pixel from each image and derived their per-pixel information, including (I_p, O_p, c_p, v_p) , $p \in P$. We then optimized the loss in equation 8 to find the geometry \mathcal{S}_θ and renderer network M . After training, we used the Marching Cubes algorithm [33] to retrieve the reconstructed surface from f .

Scan	Trimmed Mesh				Watertight Mesh							
	Colmap _{trim=7}		Furu _{trim=7}		Colmap _{trim=0}		Furu _{trim=0}		DVR [40]		IDR	
	Chamfer	PSNR	Chamfer	PSNR	Chamfer	PSNR	Chamfer	PSNR	Chamfer	PSNR	Chamfer	PSNR
24	0.45	19.8	0.51	19.2	0.81	20.28	0.85	20.35	4.10(4.24)	16.23(15.66)	1.63	23.29
37	0.91	15.49	1.03	14.91	2.05	15.5	1.87	14.86	4.54(4.33)	13.93(14.47)	1.87	21.36
40	0.37	20.48	0.44	19.18	0.73	20.71	0.96	20.46	4.24(3.27)	18.15(19.45)	0.63	24.39
55	0.37	20.18	0.4	20.92	1.22	20.76	1.10	21.36	2.61(0.88)	17.14(18.47)	0.48	22.96
63	0.9	17.05	1.28	15.41	1.79	20.57	2.08	16.75	4.34(3.42)	17.84(18.42)	1.04	23.22
65	1.0	14.98	1.22	13.09	1.58	14.54	2.06	13.53	2.81(1.04)	17.23(20.42)	0.79	23.94
69	0.54	18.56	0.72	18.77	1.02	21.89	1.11	21.62	2.53(1.37)	16.33(16.78)	0.77	20.34
83	1.22	18.91	1.61	16.58	3.05	23.2	2.97	20.06	2.93(2.51)	18.1(19.01)	1.33	21.87
97	1.08	12.18	1.37	12.36	1.4	18.48	1.63	18.32	3.03(2.42)	16.61(16.66)	1.16	22.95
105	0.64	20.48	0.83	19.68	2.05	21.3	1.88	20.21	3.24(2.42)	18.39(19.19)	0.76	22.71
106	0.48	15.76	0.70	16.28	1.0	22.33	1.39	22.64	2.51(1.18)	17.39(18.1)	0.67	22.81
110	0.59	16.71	0.87	16.53	1.32	18.25	1.45	17.88	4.80(4.32)	14.43(15.4)	0.9	21.26
114	0.32	19.9	0.42	19.69	0.49	20.28	0.69	20.09	3.09(1.04)	17.08(20.86)	0.42	25.35
118	0.45	23.21	0.59	24.68	0.78	25.39	1.10	26.02	1.63(0.91)	19.08(19.5)	0.51	23.54
122	0.43	24.48	0.53	25.64	1.17	25.29	1.16	25.95	1.58(0.84)	21.03(22.51)	0.53	27.98
Mean	0.65	18.54	0.84	18.19	1.36	20.58	1.49	20.01	3.20(2.28)	17.26(18.33)	0.9	23.20

Table 1: Multiview 3D reconstruction with *fixed* cameras, quantitative results for DTU dataset. For DVR we also present (in parentheses) the results with a partial set of images (with reduced reflectance) as suggested in [40].

Evaluation. We evaluated the quality of our 3D surface reconstructions using the formal surface evaluation script of the DTU dataset, which measures the standard Chamfer- L_1 distance between the ground truth and the reconstruction. We also report PSNR of train image reconstructions. We note that the ground truth geometry in the dataset has some noise, does not include watertight surfaces, and often suffers from notable missing parts, e.g., Figure 5 and Fig. 7c of [40]. We compare to the following baselines: DVR [40] (for fixed cameras), Colmap [48] (for fixed and trained cameras) and Furu [9] (for fixed cameras). Similar to [40], for a fair comparison we cleaned the point clouds of Colmap and Furu using the input masks before running the Screened Poisson Surface Reconstruction (sPSR) [25] to get a watertight surface reconstruction. For completeness we also report their trimmed reconstructions obtained with the trim7 configuration of sPSR that contain large missing parts (see Fig. 5 middle) but performs well in terms of the Chamfer distance.

Quantitative results of the experiment with known fixed cameras are presented in Table 1, and qualitative results are in Figure 4 (left). Our model outperforms the baselines in the PSNR metric, and in the Chamfer metric, for watertight surface reconstructions. In Table 3 we compare the reconstructions obtained with unknown trained camera. Qualitative results for this setup are shown in Figure 4 (right). The relevant baseline here is the Colmap SFM [47]+MVS[48] pipeline. In Figure 8 we further show the convergence of our cameras (rotation and translation errors sorted from small to large) from the initialization of [19] during training epochs along with Colmap’s cameras. We note that our method simultaneously improves the cameras parameters while reconstructing accurate 3D surfaces, still outperforming the baselines for watertight reconstruction and PSNR in most cases; scan 97 is a failure case of our method. As can be seen in Figure 4, our 3D surface reconstruction are more complete with better signal to noise ratio than the baselines, while our renderings (right column in each part) are close to realistic.

Small number of cameras. We further tested our method on the Fountain-P11 image collections [52] provided with 11 high resolution images with associated GT camera parameters. In Table 2 we show a comparison to Colmap (trim7-sPSR) in a setup of unknown cameras (our method is roughly initialized with [19]). Note the considerable improvement in final camera accuracy over Colmap. Qualitative results are shown in Figure 6.

4.2 Ablation study

We used scan 114 of the DTU dataset to conduct an ablation study, where we removed various components of our renderer $M(\hat{x}, \hat{n}, \hat{z}, v; \gamma)$, including (see Figure 7): (a) the viewing direction v ;

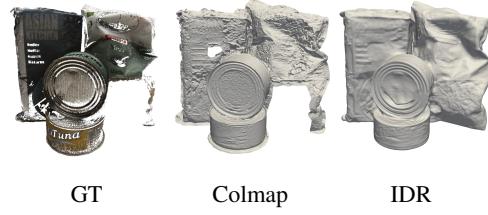


Figure 5: Example of ground truth data (left).

	$R_{\text{error}}(\text{deg})$	$t_{\text{error}}(\text{mm})$	PSNR(dB)
Colmap	0.03	2.86	21.99
IDR	0.02	2.02	26.48

Table 2: Fountain dataset: cameras accuracy and rendering quality.



Figure 6: Qualitative results for Fountain data set.

(b) the normal $\hat{\mathbf{n}}$; and (c) the feature vector, $\hat{\mathbf{z}}$. (d) shows the result with the full blown renderer M , achieving high detailed reconstruction of this marble stone figure (notice the cracks and fine details). In contrast, when the viewing direction, normal, or feature vector are removed the model tends to confuse lighting and geometry, which often leads to a deteriorated reconstruction quality.

In Figure 7 (e) we show the result of IDR training with fixed cameras set to the inaccurate camera initializations obtained with [19]; (f) shows IDR results when camera optimization is turned on. This indicates that the optimization of camera parameters together with the 3D geometry reconstruction is indeed significant.

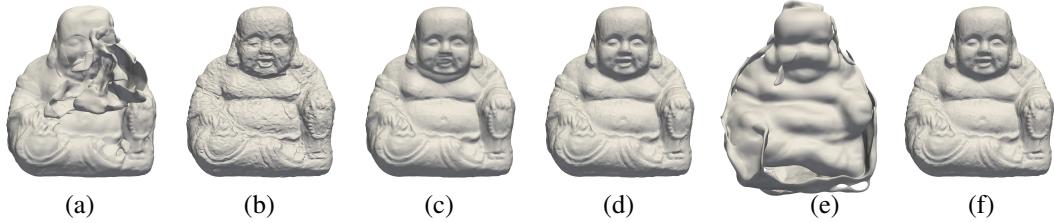


Figure 7: Ablation study, see text for details.

Scan	Trimmed Mesh		Watertight Mesh			
	Colmap _{trim=7}		Colmap _{trim=0}		IDR	
	Chamfer	PSNR	Chamfer	PSNR	Chamfer	PSNR
24	0.38	20.0	0.73	20.46	1.96	23.16
37	0.83	15.5	1.96	15.51	2.92	20.39
40	0.3	20.67	0.67	20.86	0.7	24.45
55	0.39	20.71	1.17	21.22	0.4	23.57
63	0.99	17.37	1.8	20.67	1.19	24.97
65	1.45	15.2	1.61	14.59	0.77	22.6
69	0.55	18.5	1.03	21.93	0.75	22.91
83	1.21	19.08	3.07	23.43	1.42	21.97
97	1.03	12.25	1.37	18.67	-	-
105	0.61	20.38	2.03	21.22	0.96	22.98
106	0.48	15.78	0.93	22.23	0.65	21.18
110	1.33	18.14	1.53	18.28	2.84	18.65
114	0.29	19.83	0.46	20.25	0.51	25.19
118	0.42	23.22	0.74	25.42	0.50	22.58
122	0.4	24.67	1.17	25.44	0.62	24.42
Mean	0.71	18.75	1.35	20.68	1.16	22.79

Table 3: Multiview 3D reconstruction with *trained* cameras, quantitative results for DTU dataset.

5 Conclusions

We have introduced the Implicit Differentiable Renderer (IDR), an end-to-end neural system that can learn 3D geometry, appearance, and cameras from masked 2D images and noisy camera initializations. Considering only rough camera estimates allows for robust 3D reconstruction in realistic scenarios in which exact camera information is not available. The main limitation of our method is that it does require a reasonable camera initialization and cannot work with, say random camera initialization. An interesting future work is to combine IDR with a neural network that will predict camera information directly from the images. Another future work venue is to incorporate IDR in other computer vision and learning applications such as 3D model generation, and learning 3D models from images in the wild.

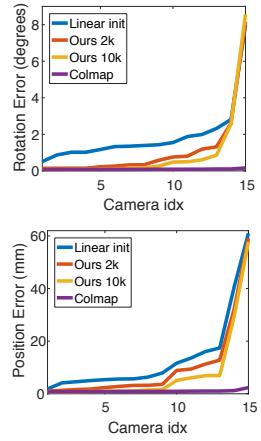


Figure 8: Cameras convergence for the DTU data set.

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A Additional implementation details

A.1 Training details

We trained our networks using the ADAM optimizer [26] with a learning rate of $1e-4$ that we decrease by a factor of 2 at epochs 1000 and 1500. Each model was trained for $2K$ epochs. Training was done on a single Nvidia V-100 GPU, using PYTORCH deep learning framework [43].

A.2 Camera representation and initialization

We represent a camera by $\mathcal{C} = (\mathbf{q}, \mathbf{c}, \mathbf{K})$, where $\mathbf{q} \in \mathbb{R}^4$ is a quaternion vector representing the camera rotation, $\mathbf{c} \in \mathbb{R}^3$ represents the camera position, and $\mathbf{K} \in \mathbb{R}^{3 \times 3}$ is the camera’s intrinsic parameters. Our cameras’ parameters are $\tau = (\mathcal{C}_1, \dots, \mathcal{C}_N)$, where N is the numbers of cameras (also images) in the scene. Let $\mathbf{Q}(\mathbf{q}) \in \mathbb{R}^{3 \times 3}$ denote the rotation matrix corresponding to the quaternion \mathbf{q} . Then, for camera \mathcal{C}_i , $i \in [N]$, and a pixel \mathbf{p} we have:

$$\mathbf{c}_p(\tau) = \mathbf{c}_i \quad (12)$$

$$\mathbf{v}_p(\tau) = \frac{1}{\|\mathbf{K}_i^{-1}\mathbf{p}\|_2} \mathbf{Q}(\mathbf{q}_i) \mathbf{K}_i^{-1} \mathbf{p}, \quad (13)$$

where $\mathbf{p} = (p_x, p_y, 1)^T$ is the pixel p in homogeneous coordinates.

In our experiments with unknown cameras, for each scene (corresponding to one 3D object) we generated relative motions between pairs of cameras using SIFT features matching [34] and robust essential matrix estimation (RANSAC), followed by a decomposition to relative rotation and relative translation [13]. We used these relative motions as inputs to the linear method of [19] to produce noisy cameras initialization for our method.

A.3 Ray marching algorithm

We denote by \mathbf{c} the camera center and by $\mathbf{v} \in \mathcal{S}$ the direction from \mathbf{c} associated with a pixel on the image plane. During our training for each selected pixel, we perform ray tracing along the ray $\{\mathbf{c} + t\mathbf{v} \mid t \geq 0\}$ to find the corresponding first intersection point with \mathcal{S}_θ denoted by \mathbf{x}_0 . As \mathcal{S}_θ is defined by the zero level set of an approximated sign distance field f , we base our tracing method on the sphere tracing algorithm [12]: in each iteration, since the distance along the ray to \mathbf{x}_0 is bounded by the current SDF value $f(\mathbf{c} + t\mathbf{v})$, we march forward by enlarging the current t by $f(\mathbf{c} + t\mathbf{v})$. We continue iterating until convergence, indicated by an SDF value below a threshold of $5e-5$, or divergence, indicated by a step that reaches out of the unit sphere. As we assume the 3D object to be located inside the unit sphere, we start the sphere tracing algorithm at the first intersection point of the ray with the unit sphere as in [31].

We limit the number of sphere tracing steps to 10 in order to prevent long convergence cases e.g., when the ray passes very close to the surface without crossing it. Then, for non convergent rays, we run another 10 sphere tracing iterations starting from the second (i.e., farthest) intersection of the ray with the unit sphere and get \bar{t} . Similar to [40] we sample 100 equal sized steps between t to \bar{t} : $t < t_1 < \dots < t_{100} < \bar{t}$ and find the first pair of consecutive steps t_i, t_{i+1} such that $\text{sign}(f(\mathbf{c} + t_i \mathbf{v})) \neq \text{sign}(f(\mathbf{c} + t_{i+1} \mathbf{v}))$, i.e., a sign transition that describes the first ray crossing of the surface. Then, using t_i and t_j as initial values, we run 8 iterations of the secant algorithm to find an approximation for the intersection point \mathbf{x}_0 . The algorithm is performed in parallel on the GPU for multiple rays associated with multiple pixels.

For all pixels in P^{out} we select a point on the corresponding ray $\mathbf{c} + t_* \mathbf{v}$ for loss_{MASK}, where t_* is approximated by sampling uniformly 100 possible steps $\mathcal{T} = \{t_1, \dots, t_{100}\}$ between the two intersection points of the ray with the unit sphere and taking $t_* = \arg \min_{t_i \in \mathcal{T}} f(\mathbf{c} + t_i \mathbf{v})$.

A.4 Baselines methods running details

In our experiments we compared our method to Colmap [47],[48], DVR [40] and Furu [9]. We next describe implementation details for these baselines.

Colmap. We used the official Colmap implementation [49]. For unknown cameras, only the intrinsic camera parameters are given, and we used the "mapper" module to extract camera poses. For fixed known cameras the GT poses are given as inputs. For both setups we run their "feature_extractor", "exhaustive_matcher", "point_triangulator", "patch_match_stereo" and "stereo_fusion" modules to generate point clouds. We also used their screened Poisson Surface Reconstruction (sPSR) for 3D mesh generation after cleaning the point clouds as described in Section 4.1. For rendering, we used their generated 3D meshes and cameras, and rendered images for each view using the "Pyrender" package [35].

DVR. We run DVR using the official Github code release [39]. In order to be compatible with this implementation, for each scene we used the cameras and the masks to normalize all the cameras such that the object is inside the unit cube. We applied DVR on all DTU scenes using the "ours_rgb" configuration. As mentioned in Table 1 (in the main paper), we reconstructed each model twice: with all the images in the dataset, and with all the images in the dataset that are not in the DVR "ignore list". We run their method for 5000 epochs and took the best model as they suggest. We further used their "generate" and "render" scripts for generating 3D meshes and rendering images respectively.

Furu. The point clouds generated by Furu are supplied by the DTU data set. We used Colmap's sPSR to generate meshes from the cleaned point clouds. As for Colmap we used "Pyrender" [35] to render their images.

A.5 Evaluation details

All the reported Chamfer distances are in millimeters and computed by averaging the accuracy and completeness measures, returned from DTU evaluation script, each represents one sided Chamfer- L_1 distance. The reported PSNR values (in dB) for each scan are averaged over the masked pixels of all the images. Camera accuracy represents the mean accuracy of camera positions (in mm) and camera rotations (in degrees) after L1 alignment with the GT.

B Additional results

In Figure 9 we show a failure case (scan 37 from the DTU dataset). This case presents challenging thin metals parts which our method fails to fully reproduce especially with noisy cameras.

In Figure 10 we present qualitative results for 3D surface reconstruction with fixed GT cameras, whereas Figure 11 further presents qualitative results for the unknown cameras setup. An ablation study for training IDR with fixed noisy cameras compared to IDR's camera training mode initialized with the same noisy cameras is also presented in Figure 11. We refer the reader to a supplementary video named "IDR.mp4" that shows side by side our generated surfaces and rendered images from multiple views in the settings of unknown cameras.

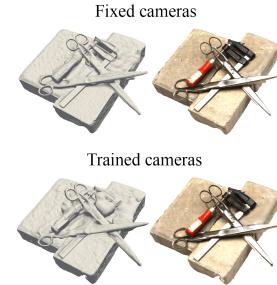


Figure 9: Failure cases.

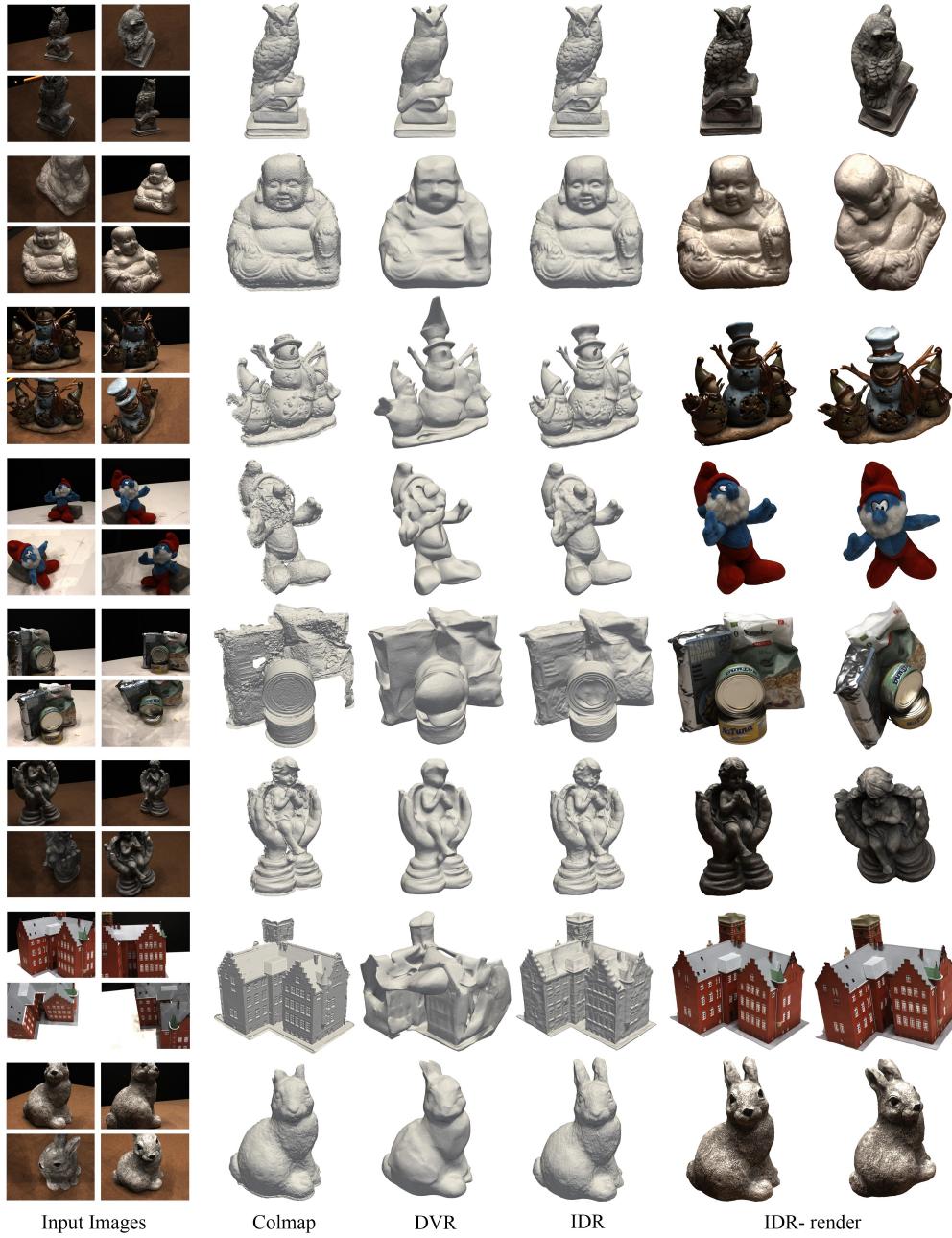


Figure 10: Qualitative results of multiview 3D surface reconstructions with fixed GT cameras for the DTU dataset. We present surface reconstructions generated by our method compared to the baseline methods. Our renderings from two novel views are presented as well.

C Differentiable intersection of viewing direction and geometry (proof of Lemma 1)

We will use the notation of the main paper, repeated here for convenience: $\hat{x}(\theta, \tau) = \mathbf{c}(\tau) + t(\theta, \mathbf{c}(\tau), \mathbf{v}(\tau))\mathbf{v}(\tau)$ denotes the first intersection point of the viewing ray $R_p(\tau)$ for some fixed pixel p and the geometry \mathcal{S}_θ . We denote the current parameters by θ_0, τ_0 ; accordingly we denote $\mathbf{c}_0 = \mathbf{c}(\tau_0)$, $\mathbf{v}_0 = \mathbf{v}(\tau_0)$, and $t_0 = t(\theta_0, \mathbf{c}_0, \mathbf{v}_0)$. We also denote $\mathbf{x}_0 = \hat{x}(\theta_0, \tau_0)$. Note that $\mathbf{x}_0 = \mathbf{c}_0 + t_0 \mathbf{v}_0$ is the intersection point at the current parameters that is $R_p(\tau_0) \cap \mathcal{S}_{\theta_0}$.

To find the function dependence of $\hat{\mathbf{x}}$ on θ, τ we use implicit differentiation [1, 40]. That is we differentiate

$$f(\hat{\mathbf{x}}(\theta, \tau); \theta) \equiv 0 \quad (14)$$

w.r.t. its parameters. Since the functional dependence of \mathbf{c} and \mathbf{v} on τ is known (given in equations 12-13) it is sufficient to consider the derivatives of equation 14 w.r.t. $\theta, \mathbf{c}, \mathbf{v}$. Therefore we consider

$$f(\mathbf{c} + t(\theta, \mathbf{c}, \mathbf{v})\mathbf{v}; \theta) \equiv 0 \quad (15)$$

and differentiate w.r.t. $\theta, \mathbf{c}, \mathbf{v}$. We differentiate w.r.t. \mathbf{c} : (note that we use $\partial f / \partial \mathbf{x}$ equivalently to $\nabla_{\mathbf{x}} f$, which is used in the main paper)

$$\left(\frac{\partial f}{\partial \mathbf{x}} \right)^T \left(I + \mathbf{v} \left(\frac{\partial t}{\partial \mathbf{c}} \right)^T \right) = 0,$$

where $I \in \mathbb{R}^{3 \times 3}$ is the identity matrix, and $\mathbf{v}, \frac{\partial f}{\partial \mathbf{x}}, \frac{\partial t}{\partial \mathbf{c}} \in \mathbb{R}^3$ are column vectors. Rearranging and evaluating at $\theta_0, \mathbf{c}_0, \mathbf{v}_0$ we get

$$\frac{\partial t}{\partial \mathbf{c}}(\theta_0, \mathbf{c}_0, \mathbf{v}_0) = -\frac{1}{\left\langle \frac{\partial f}{\partial \mathbf{x}}(\mathbf{x}_0; \theta_0), \mathbf{v}_0 \right\rangle} \frac{\partial f}{\partial \mathbf{x}}(\mathbf{x}_0; \theta_0). \quad (16)$$

Differentiating w.r.t. \mathbf{v} :

$$\left(\frac{\partial f}{\partial \mathbf{x}} \right)^T \left(tI + \mathbf{v} \left(\frac{\partial t}{\partial \mathbf{v}} \right)^T \right) = 0,$$

Rearranging and evaluating at $\theta_0, \mathbf{c}_0, \mathbf{v}_0$ we get

$$\frac{\partial t}{\partial \mathbf{v}}(\theta_0, \mathbf{c}_0, \mathbf{v}_0) = -\frac{t_0}{\left\langle \frac{\partial f}{\partial \mathbf{x}}(\mathbf{x}_0; \theta_0), \mathbf{v}_0 \right\rangle} \frac{\partial f}{\partial \mathbf{x}}(\mathbf{x}_0; \theta_0). \quad (17)$$

Lastly, the derivatives w.r.t. θ are as in [40] and we give it here for completeness:

$$\left(\frac{\partial f}{\partial \mathbf{x}} \right)^T \mathbf{v} \frac{\partial t}{\partial \theta} + \frac{\partial f}{\partial \theta} = 0.$$

Reordering and evaluating at $\theta_0, \mathbf{v}_0, \mathbf{c}_0$ we get

$$\frac{\partial t}{\partial \theta}(\theta_0, \mathbf{c}_0, \mathbf{v}_0) = -\frac{1}{\left\langle \frac{\partial f}{\partial \mathbf{x}}(\mathbf{x}_0; \theta_0), \mathbf{v}_0 \right\rangle} \frac{\partial f}{\partial \theta}(\mathbf{x}_0; \theta_0) \quad (18)$$

Now consider the formula

$$t(\theta, \mathbf{c}, \mathbf{v}) = t_0 - \frac{1}{\left\langle \frac{\partial f}{\partial \mathbf{x}}(\mathbf{x}_0; \theta_0), \mathbf{v}_0 \right\rangle} f(\mathbf{c} + t_0 \mathbf{v}; \theta). \quad (19)$$

Evaluating at $\theta_0, \mathbf{c}_0, \mathbf{v}_0$ we get $t(\theta_0, \mathbf{c}_0, \mathbf{v}_0) = t_0$ since $f(\mathbf{x}_0; \theta_0) = 0$. Furthermore, differentiating t w.r.t. $\mathbf{c}, \mathbf{v}, \theta$ and evaluating at $\mathbf{c}_0, \mathbf{v}_0, \theta_0$ we get the same derivatives as in equations 16, 17, and 18. Plugging equation 19 into $\hat{\mathbf{x}}(\theta, \mathbf{c}, \mathbf{v}) = \mathbf{c} + t(\theta, \mathbf{c}, \mathbf{v})\mathbf{v}$ we get the formula,

$$\hat{\mathbf{x}}(\theta, \tau) = \mathbf{c} + t_0 \mathbf{v} - \frac{\mathbf{v}}{\left\langle \frac{\partial f}{\partial \mathbf{x}}(\mathbf{x}_0; \theta_0), \mathbf{v}_0 \right\rangle} f(\mathbf{c} + t_0 \mathbf{v}; \theta), \quad (20)$$

that coincide in value and first derivatives with the first intersection points at the current parameters $\theta_0, \mathbf{c}_0, \mathbf{v}_0$; equation 20 can be implemented directly by adding a linear layer in the entrance to the network f and a linear layer at its output.

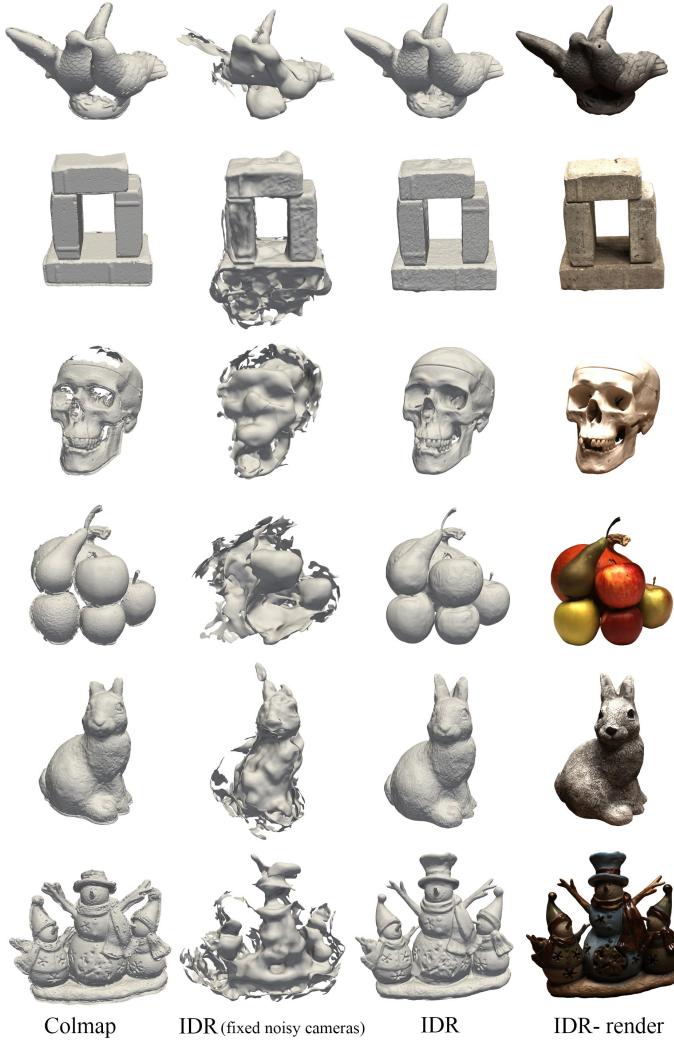


Figure 11: Qualitative results for trained cameras setup. We also compare to IDR with fixed noisy cameras.

D Restricted BRDF and lighting model \mathcal{P}

We restrict our material and lighting settings to those represented by a *continuous* volumetric BRDF function $f^r : \mathbb{R}^3 \times \mathcal{S} \times \mathcal{S} \times \mathcal{S} \rightarrow \mathbb{R}$ and light radiance functions $L^e, L^i : \mathbb{R}^3 \times \mathcal{S} \rightarrow \mathbb{R}$. We denote the collection of such continuous functions by $\mathcal{P} = \{(f^r, L^e, L^i)\}$. This model includes many common materials and lighting conditions such as the popular Phong model [8]:

$$L(\hat{\mathbf{x}}, \mathbf{w}^o) = k_d O_d I_a + k_d O_d I_d \left(\hat{\mathbf{n}} \cdot \frac{\ell - \hat{\mathbf{x}}}{\|\ell - \hat{\mathbf{x}}\|} \right)_+ + k_s O_s I_d \left(\hat{\mathbf{r}} \cdot \frac{\ell - \hat{\mathbf{x}}}{\|\ell - \hat{\mathbf{x}}\|} \right)_+^{n_s}, \quad (21)$$

where $(a)_+ = \max \{a, 0\}$; k_d, k_s are the diffuse and specular coefficients, I_a, I_d are the ambient and point light source colors, O_d, O_s are the diffuse and specular colors of the surface, $\ell \in \mathbb{R}^3$ is the location of a point light source, $\hat{\mathbf{r}} = -(\mathbf{I} - 2\hat{\mathbf{n}}\hat{\mathbf{n}}^T)\mathbf{w}^o$ the reflection of the viewing direction $\mathbf{w}^o = -\mathbf{v}$ with respect to the normal $\hat{\mathbf{n}}$, and n_s is the specular exponent.

Note, that the family \mathcal{P} , however, does not include all possible lighting conditions. It excludes, for example, self-shadows and second order (or higher) light reflections as L^i is independent of the geometry \mathcal{S}_θ .

D.1 \mathcal{P} -Universality of renderer

We consider renderer of the form $M(\mathbf{x}, \mathbf{n}, \mathbf{v}; \gamma)$, where M is an MLP, and show it can approximate the correct radiance function $M_0(\mathbf{x}, \mathbf{n}, \mathbf{v})$ for all $(\mathbf{x}, \mathbf{n}, \mathbf{v})$ contained in some compact set $\mathcal{K} \subset \mathbb{R}^9$. Indeed, for some arbitrary and fixed $\epsilon > 0$ the universal approximation theorem for MLPs (see [15] and [14] for approximation including derivatives) guarantees the existence of parameters $\gamma \in \mathbb{R}^n$ so that $\max_{(\mathbf{x}, \mathbf{n}, \mathbf{v}) \in \mathcal{K}} |M(\mathbf{x}, \mathbf{n}, \mathbf{v}; \gamma) - M_0(\mathbf{x}, \mathbf{n}, \mathbf{v})| < \epsilon$.

D.2 View direction and normal are necessary for \mathcal{P} -universality.

We will next prove that taking out \mathbf{v} or \mathbf{n} from the renderer M will make this model not \mathcal{P} -universal. Assume S_θ is expressive enough to change the normal direction $\hat{\mathbf{n}}$ arbitrarily at a point $\hat{\mathbf{x}}$; for example, even a linear classifier $f(\mathbf{x}; \mathbf{a}, b) = \mathbf{a}^T \mathbf{x} + b$ would do. We will use the Phong model (equation 21) to prove the claims; the Phong model is in \mathcal{P} as discussed above. We first consider the removal of the normal component $\hat{\mathbf{n}}$ from the renderer, i.e., $M(\hat{\mathbf{x}}, \mathbf{v}; \gamma)$ is the approximated radiance arriving at \mathbf{c} in direction \mathbf{v} as predicted by the renderer. Consider the setting shown in Figure 3 (a) and (b) (in the main paper): In both cases $M(\hat{\mathbf{x}}, \mathbf{v}; \gamma)$ yields the same value although changing the normal direction at $\hat{\mathbf{x}}$ will produce, under the Phong model, a different light amount at \mathbf{c} .

Similarly, consider a renderer with the viewing direction, \mathbf{v} , removed, i.e., $M(\hat{\mathbf{x}}, \hat{\mathbf{n}}; \gamma)$, and Figure 3 (a) and (c): In both cases $M(\hat{\mathbf{x}}, \hat{\mathbf{n}}; \gamma)$ produces the same value although, under the Phong model, the reflected light can change when the point of view changes. That is, we can choose light position ℓ in equation 21 so that different amount of light reaches \mathbf{c} at direction \mathbf{v} .

We have shown that renderers with no access to $\hat{\mathbf{n}}$ and/or \mathbf{v} would necessarily fail (at-least in some cases) predicting the correct light amount reaching \mathbf{c} in direction \mathbf{v} when these and the geometry is able to change, hence are not universal.