APPENDIX A
PROOF OF EQUAL TRANSFORMATION

To prove that , we just need to prove that $w_{ikv}(t)$ can replace $f_{iv}^F(t)y_{ik}(t)$.

Variables $w_{ikv}(t)$ subject to the following constraints. $w_{ikv}(t) \le y_{ik}(t), w_{ikv}(t) \le f_{iv}^F(t), w_{ikv}(t) \ge y_{ik}(t) + f_{iv}^F(t) - 1, w_{ikv} \in \{0,1\}$ is equal to $w_{ikv}(t) = f_{iv}^F(t)y_{ik}(t)$. That is to say, we need to prove that at any value of f, y and the above constraints, we have $w_{ikv}(t) = f_{iv}^F(t)y_{ik}(t)$.

When $f_{iv}^F(t) = 0$ or $y_{ik}(t) = 0$, we require $w_{ikv}(t) = 0$. We can see that constraints $w_{ikv}(t) \le f_{iv}^F(t)$ or $w_{ikv}(t) \le y_{ik}(t)$ make sure that $w_{ikv}(t) = 0$. Meanwhile $w_{ikv}(t) = 0$ also satisfy $w_{ikv}(t) \ge y_{ik}(t) + f_{iv}^F(t) - 1$. Therefore we prove that when $f_{iv}^F(t) = 0$ or $y_{ik}(t) = 0$, $w_{ikv}(t) = f_{iv}^F(t)y_{ik}(t)$.

When $f_{iv}^F(t) = 1$ and $y_{ik}(t) = 1$, we need $w_{ikv}(t) = 1$. We can see that constraint $w_{ikv}(t) \ge y_{ik}(t) + f_{iv}^F(t) - 1$ makes sure that $w_{ikv}(t) = 1$. And $w_{ikv}(t) = 1$ does not violate constraints $w_{ikv}(t) \le y_{ik}(t)$and , $w_{ikv}(t) \le f_{iv}^F(t)$. Therefore we prove that when $f_{iv}^F(t) = 0$ or $y_{ik}(t) = 0$, $w_{ikv}(t) = f_{iv}^F(t)y_{ik}(t)$.

Since in all situations, $w_{ikv}(t) = f_{iv}^F(t)y_{ik}(t)$, we complete our proof.

APPENDIX B
PROOF OF LEMMA 1

The dual problem of $P2^r$ is

$$
\begin{aligned}
D2^r : \min & \sum_i \sum_t \alpha_{it} + \sum_i \sum_t \beta_{it} + \sum_k \sum_t \gamma_{kt} U_k \\
& + \sum_i \sum_k \sum_t \lambda_{kt} t_i^{max} + \sum_i \sum_k \sum_v \sum_t \delta_{ikvt} \\
& + a_i \sum_i \sum_t b_i(0) - a_i \sum_i \sum_t \sum_{\tau=1}^{t} g_i(\tau)
\end{aligned}
$$

s.t. $(T-t+1)a_i b_{iv}(t) - \beta_{it}$

$$+ \sum_k \pi_{ikvt} - \sum_k \delta_{ikvt} \le 0, \forall i, k, v, \tag{1a}$$

$$-\alpha_{it} - \frac{e_k}{l_k}(f_i^I(t) + c_i f_i^C) - \gamma_{kt} - \lambda_{ikt}\left(\frac{f_i^I(t)}{l_k} + \frac{c_i f_i^C}{l_k}\right) + \sum_v \theta_{ikvt} - \sum_v \delta_{ikvt} - \epsilon_{ikt} + \epsilon_{ik(t+1)} \le 0, \forall i, k, t, \tag{1b}$$

$$-v d_i^F r_i^F \frac{e_k}{l_k} - \lambda_{ikt} v \frac{d_i^F r_i^F}{l_k} - \theta_{ikvt} - \pi_{ikvt} + \delta_{ikvt} \le 0, \forall i, k, v \backslash \{0\}, t, \tag{1c}$$

$$c_i f_i^C \frac{e_k}{l_k} + \lambda_{ikt} c_i f_i^C \frac{1}{l_k} - \theta_{ik0t} - \pi_{ik0t} + \delta_{ik0t} \le 0, \forall i, k, t, \tag{1d}$$

$$-q_i + \epsilon_{ikt} \le 0, \forall i, k, t, \tag{1e}$$

All the dual variables ≥ 0, except α and β. (1f)

By Algorithm 1, we can calculate the optimal solution of $P_{3,t}^r$ in each time slot. The KKT condition of $P_{3,t}^r$ is

$$(T-t+1)a_i b_{iv}(t) - \widetilde{\beta}_{it} + \sum_k \widetilde{\pi}_{ikvt} - \sum_k \widetilde{\delta}_{ikvt} = 0, \forall i, v, \tag{2a}$$

$$-\widetilde{\alpha}_{it} - \frac{e_k}{l_k}(f_i^I(t) + c_i f_i^C) - \widetilde{\gamma}_{kt} - \widetilde{\lambda}_{ikt}\left(\frac{f_i^I(t)}{l_k} + \frac{c_i f_i^C}{l_k}\right) + \sum_v \widetilde{\theta}_{ikvt} - \sum_v \widetilde{\delta}_{ikvt} - \frac{q_i}{\eta} \ln \frac{\widetilde{y}_{ik}(t) + \xi}{\widetilde{y}_{ik}(t-1) + \xi} = 0, \forall i, k, v, \tag{2b}$$

$$-v d_i^F r_i^F \frac{e_k}{l_k} - \widetilde{\lambda}_{ikt} v d_i^F r_i^F \frac{1}{l_k} - \widetilde{\theta}_{ikvt} - \widetilde{\pi}_{ikvt} + \widetilde{\delta}_{ikvt} = 0, \forall i, k, v \backslash \{0\}, \tag{2c}$$

$$c_i f_i^C \frac{e_k}{l_k} + \widetilde{\lambda}_{ikt} c_i f_i^C \frac{1}{l_k} - \widetilde{\theta}_{ik0t} - \widetilde{\pi}_{ik0t} + \widetilde{\delta}_{ik0t} = 0, \forall i, k, v = 0, \tag{2d}$$

All dual variables ≥ 0, except $\widetilde{\alpha}_{it}$ and $\widetilde{\beta}_{it}$. (2e)

It is easy to verify that all dual variables (except α and β) are non-negative and constraints (1a),(1c),(1d) are satisfied because of KKT condition (2a),(2c),(2d). Then since $\epsilon_{ikt} = \frac{q_i}{\eta} \ln \frac{1+\xi}{\widetilde{y}_{ik}(t-1)+\xi} \le \frac{q_i}{\eta} \ln \frac{1+\xi}{\xi} = \frac{q_i}{\eta}\eta = q_i$, constraint (1e) is also satisfied. Constraint (1b) is satisfied since

$$
\begin{aligned}
& -\alpha_{it} - \frac{e_k}{l_k}(f_i^I(t) + c_i f_i^C) - \gamma_{kt} - \lambda_{ikt}\left(\frac{f_i^I(t)}{l_k} + \frac{c_i f_i^C}{l_k}\right) \\
& \quad + \sum_v \theta_{ikvt} - \sum_v \delta_{ikvt} - \epsilon_{ikt} + \epsilon_{ik(t+1)} \\
& = -\widetilde{\alpha}_{it} - \frac{e_k}{l_k}(f_i^I(t) + c_i f_i^C) - \widetilde{\gamma}_{kt} - \widetilde{\lambda}_{ikt}\left(\frac{f_i^I(t)}{l_k} + \frac{c_i f_i^C}{l_k}\right) \\
& + \sum_v \widetilde{\theta}_{ikvt} - \sum_v \delta_{ikvt} - \frac{q_i}{\eta} \ln \frac{1+\xi}{\widetilde{y}_{ik}(t-1)+\xi} + \frac{q_i}{\eta} \ln \frac{1+\xi}{\widetilde{y}_{ik}(t)+\xi} \\
& = -\widetilde{\alpha}_{it} - \frac{e_k}{l_k}(f_i^I(t) + c_i f_i^C) - \widetilde{\gamma}_{kt} - \widetilde{\lambda}_{ikt}\left(\frac{f_i^I(t)}{l_k} + \frac{c_i f_i^C}{l_k}\right) \\
& \quad + \sum_v \widetilde{\theta}_{ikvt} - \sum_v \delta_{ikvt} - \frac{q_i}{\eta} \ln \frac{y_{ik}(t)+\xi}{y_{ik}(t-1)+\xi} \le 0
\end{aligned}
$$

Therefore we obtain a feasible solution of $D2^r$ by calculate the optimal solution of $P_{3,t}^r$ in each time slot.

Appendix C
Proof of Lemma 2

The non-switching utility $\widetilde{U}_{ns}(t)$ obtained by Algorithm 1 can be decomposed to the following expression

$$\sum_t \widetilde{U}_{ns}(t) = \sum_t \sum_i \sum_v (T-t+1) a_i b_{iv}(t) \widetilde{f}_{iv}^F(t) \tag{3a}$$
$$-\sum_t \sum_i \sum_{v\backslash\{0\}} \sum_k \widetilde{w}_{ikv}(t) v d_i^F r_i^F \frac{e_k}{l_k} \tag{3b}$$
$$+\sum_t \sum_i \sum_k \widetilde{w}_{ik0}(t) c_i f_i^C \frac{e_k}{l_k} \tag{3c}$$
$$-\sum_t \sum_i \sum_k \widetilde{y}_{ik}(t)(f_i^I(t)\frac{e_k}{l_k} + \frac{e_k}{l_k} c_i f_i^C) \tag{3d}$$
$$+\sum_t \sum_i a_i[b_i(0) - \sum_{\tau=1}^{t} g_i(\tau)] \tag{3e}$$

Using KKT condition (2a), we rewrite (3a) as

$$\sum_t \sum_t \sum_v (\widetilde{\beta}_{it} - \sum_k \widetilde{\theta}_{ikvt} + \sum_k \widetilde{\delta}_{ikvt}) \widetilde{f}_{iv}^F(t)$$
$$= \sum_t \sum_i \widetilde{\beta}_{it} - \sum_t \sum_i \sum_v \sum_k \widetilde{\pi}_{ikvt} \widetilde{f}_{iv}^F(t)$$
$$+ \sum_t \sum_i \sum_v \sum_k \widetilde{\delta}_{ikvt} \widetilde{f}_{iv}^F(t). \tag{4}$$

Using KKT condition (2b), we can relax (3d) as

$$\sum_t \sum_i \sum_k \widetilde{y}_{ik}(t)[\widetilde{\alpha}_{it} + \widetilde{\gamma}_{kt} + \lambda_{ikt}(\frac{f_i^I(t)}{l_k} + \frac{c_i f_i^C}{l_k}) - \sum_v \widetilde{\theta}_{ikvt}$$
$$+ \sum_v \widetilde{\delta}_{ikvt} + \frac{q_i}{\eta} \ln \frac{\widetilde{y}_{ik}(t) + \xi}{\widetilde{y}_{ik}(t-1) + \xi}]$$
$$\geq \sum_t \sum_i \widetilde{\alpha}_{it} + \sum_t \sum_k U_k \widetilde{\gamma}_{kt} + \sum_t \sum_i \sum_k \widetilde{\lambda}_{ikt}(\frac{f_i^I(t)}{l_k} + \frac{c_i f_i^C}{l_k}) \widetilde{y}_{ik}(t)$$
$$- \sum_t \sum_i \sum_k \sum_v \widetilde{\theta}_{ikvt} \widetilde{y}_{ik}(t) + \sum_t \sum_i \sum_k \sum_v \widetilde{\delta}_{ikvt} \widetilde{y}_{ik}(t) \tag{5}$$

Using KKT condition (2c), we can rewrite (3b) as

$$\sum_t \sum_i \sum_k \sum_{v\backslash\{0\}} \widetilde{w}_{ikv}(t)(\widetilde{\lambda}_{ikt} v d_i^F r_i^F \frac{1}{l_k} + \widetilde{\theta}_{ikvt} + \widetilde{\pi}_{ikvt} - \widetilde{\delta}_{ikvt}) \tag{6}$$

Using KKT condition (2d), we can rewrite (3c) as

$$\sum_t \sum_i \sum_k \widetilde{w}_{ik0}(t)(-\widetilde{\lambda}_{ikt} c_i f_i^C \frac{1}{l_k} + \widetilde{\theta}_{ik0t} + \widetilde{\pi}_{ik0t} - \widetilde{\delta}_{ik0t}) \tag{7}$$

Combine (6) and (7), we can relax (3b) and (3c) as

$$\sum_t \sum_i \sum_k (\sum_{v\backslash\{0\}} \widetilde{w}_{ikv}(t) \widetilde{\lambda}_{ikt} v d_i^F r_i^F \frac{1}{l_k} - \widetilde{w}_{ik0}(t) \widetilde{\lambda}_{ikt} c_i f_i^C \frac{1}{l_k})$$
$$+ \sum_t \sum_i \sum_k \sum_v \widetilde{w}_{ikv}(t)(\widetilde{\theta}_{ikvt} + \widetilde{\pi}_{ikvt}) - \sum_t \sum_i \sum_k \sum_v \widetilde{w}_{ikv}(t) \widetilde{\delta}_{ikvt}$$
$$\geq \sum_t \sum_i \sum_k \widetilde{\lambda}_{ikt}(t_i^{max} - \widetilde{y}_{ik}(t)(\frac{f_i^I(t)}{l_k} + \frac{c_i f_i^C}{l_k}))$$
$$+ \sum_t \sum_i \sum_k \sum_v \widetilde{w}_{ikv}(t)(\widetilde{\theta}_{ikvt} + \widetilde{\pi}_{ikvt}) - \sum_t \sum_i \sum_k \sum_v \widetilde{w}_{ikv}(t) \widetilde{\delta}_{ikvt}$$
$$\geq \sum_t \sum_i \sum_k \widetilde{\lambda}_{ikt}(t_i^{max} - \widetilde{y}_{ik}(t)(\frac{f_i^I(t)}{l_k} + \frac{c_i f_i^C}{l_k})) +$$
$$\sum_t \sum_i \sum_v \sum_k \widetilde{\theta}_{ikvt} \widetilde{y}_{ik}(t) + \sum_i \sum_k \sum_v \sum_t \widetilde{\pi}_{ikvt} \widetilde{f}_{iv}^F(t)$$
$$- \sum_t \sum_i \sum_v \sum_t \widetilde{\delta}_{ikvt} \widetilde{w}_{ikv}(t)$$
$$\geq \sum_t \sum_i \sum_k \widetilde{\lambda}_{ikt}(t_i^{max} - \widetilde{y}_{ik}(t)(\frac{f_i^I(t)}{l_k} + \frac{c_i f_i^C}{l_k})) +$$
$$\sum_t \sum_i \sum_v \sum_k \widetilde{\theta}_{ikvt} \widetilde{y}_{ik}(t) + \sum_i \sum_k \sum_v \sum_t \widetilde{\pi}_{ikvt} \widetilde{f}_{iv}^F(t)$$
$$- \sum_t \sum_i \sum_k \sum_v \widetilde{\delta}_{ikvt}(\widetilde{y}_{ik}(t) + \widetilde{f}_{iv}^F(t)) + \sum_t \sum_k \sum_v \sum_t \widetilde{\delta}_{ikvt} \tag{8}$$

Combining (4), (5), (8) and (3e), we can relax the non-switching cost as

$$\sum_t U_{ns}(t) \geq \sum_t \sum_i \widetilde{\beta}_{it} - \sum_t \sum_i \sum_v \sum_k \widetilde{\pi}_{ikvt} \widetilde{f}_{iv}^F(t)$$
$$+ \sum_t \sum_i \sum_v \sum_k \widetilde{\pi}_{ikvt} \widetilde{f}_{iv}^F(t) + \sum_t \sum_i \widetilde{\alpha}_{it} + \sum_t \sum_k U_k \widetilde{\gamma}_{kt}$$
$$+ \sum_t \sum_i \sum_k \widetilde{\lambda}_{ikt}(\frac{f_i^I(t)}{l_k} + \frac{c_i f_i^C}{l_k}) \widetilde{y}_{ik}(t)$$
$$- \sum_t \sum_i \sum_k \sum_v (\widetilde{\theta}_{ikvt} - \widetilde{\delta}_{ikvt}) \widetilde{y}_{ik}(t)$$
$$+ \sum_t \sum_i \sum_k \widetilde{\lambda}_{ikt}(t_i^{max} - \widetilde{y}_{ik}(t)(\frac{f_i^I(t)}{l_k} + \frac{c_i f_i^C}{l_k})) +$$
$$\sum_t \sum_i \sum_v \sum_k \widetilde{\theta}_{ikvt} \widetilde{y}_{ik}(t) + \sum_i \sum_k \sum_v \sum_t \widetilde{\pi}_{ikvt} \widetilde{f}_{iv}^F(t)$$
$$- \sum_t \sum_i \sum_k \sum_v \widetilde{\delta}_{ikvt}(\widetilde{y}_{ik}(t) + \widetilde{f}_{iv}^F(t))$$
$$+ \sum_t \sum_k \sum_v \sum_t \widetilde{\delta}_{ikvt} + \sum_t [\sum_i \sum_{\tau=1}^{t} a_i g_i(\tau) + \sum_i a_i b_i(0)]$$
$$= \sum_i \sum_t \widetilde{\alpha}_{it} + \sum_i \sum_t \widetilde{\beta}_{it} + \sum_k \sum_t \widetilde{\gamma}_{kt} U_k + \sum_i \sum_k \sum_t \widetilde{\lambda}_{kt} t_i^{max}$$
$$+ \sum_i \sum_k \sum_v \sum_t \widetilde{\delta}_{ikvt} + a_i \sum_i \sum_t b_i(0) - a_i \sum_i \sum_t \sum_{\tau=1}^{t} g_i(\tau)$$
$$= \widetilde{D}2^r$$

APPENDIX D
PROOF OF THEOREM 1

According to the property of the weak duality, we have

$$\sum_t \widetilde{U}_{ns}(t) \geq \widetilde{D}2^r \geq D2^{r,opt} \geq P2^{r,opt} \geq P1^{opt}$$

Switching cost $\widetilde{C}_s(t)$ obtained by Algorithm 1 can be relaxed to

$$\sum_t \widetilde{C}_s(t) = \sum_t \sum_i \sum_k q_i[\widetilde{y}_{ik}(t) - \widetilde{y}_{ik}(t-1)]^+ \\ \leq \sum_t \sum_i \sum_k q_i \widetilde{y}_{ik}(t) \leq \sum_i T q_i.$$

Therefore we derive the average optimality gap

$$\frac{1}{T}[P1^{opt} - (\sum_t \widetilde{U}_{ns}(t) - \sum_t \widetilde{C}_s(t))] \leq \sum_i q_i.$$

APPENDIX E
PROOF OF THEOREM 2

The expected gap between the fractional solution obtained by Algorithm 1 and integral solution obtained by rounding can be written as follows

$$\sum_t (\widetilde{U}_{ns}(t) - \widetilde{C}_s(t)) - \mathbb{E}\sum_t (\bar{U}_{ns}(t) - \bar{C}_s(t)) \\ = \sum_t (\widetilde{U}_{ns}(t) - \mathbb{E}\bar{U}_{ns}(t)) + \sum_t (-\widetilde{C}_s(t) + \mathbb{E}\bar{C}_s(t))$$

Then the expected gap between the non-switching utility of fractional and integral solution is as follows

$$\sum_t (\widetilde{U}_{ns}(t) - \mathbb{E}\bar{U}_{ns}(t)) = \sum_t [\sum_i \sum_v (T-t+1) a_i b_{iv}(t) \widetilde{f}_{iv}^F(t) \\ - \sum_i \sum_v \sum_k \widetilde{w}_{ikv}(t) v d_i^F r_i^F \frac{e_k}{l_k} \\ + \sum_i \sum_k \widetilde{w}_{ik0}(t) c_i f_i^C \frac{e_k}{l_k} - \sum_i \sum_k \widetilde{y}_{ik}(t)(f_i^I(t) \frac{e_k}{l_k} + \frac{e_k}{l_k} c_i f_i^C)] \\ - \sum_t [\sum_i \sum_v (T-t+1) a_i b_{iv}(t) p_{iv}^f(t) - \sum_i \sum_v \sum_k p_{iv}^f(t) p_{iv}^y(t) v d_i^F r_i^F \frac{e_k}{l_k} \\ + \sum_i \sum_k p_{i0}^f(t) p_{ik}^y(t) c_i f_i^C \frac{e_k}{l_k} - \sum_i \sum_k p_{ik}^y(t)(f_i^I(t) \frac{e_k}{l_k} + \frac{e_k}{l_k} c_i f_i^C)] \\ = \sum_t [\sum_i \sum_v (T-t+1) a_i b_{iv}(t) \widetilde{f}_{iv}^F(t) - \sum_i \sum_v \sum_k \widetilde{w}_{ikv}(t) v d_i^F r_i^F \frac{e_k}{l_k} \\ + \sum_i \sum_k \widetilde{w}_{ik0}(t) c_i f_i^C \frac{e_k}{l_k} - \sum_i \sum_k \widetilde{y}_{ik}(t)(f_i^I(t) \frac{e_k}{l_k} + \frac{e_k}{l_k} c_i f_i^C)] \\ - \sum_t [\sum_i \sum_v (T-t+1) a_i b_{iv}(t) \widetilde{f}_{iv}^F(t) - \sum_i \sum_v \sum_k \widetilde{y}_{ik}(t) \widetilde{f}_{iv}^F(t) v d_i^F r_i^F \frac{e_k}{l_k} \\ + \sum_i \sum_k f_{i0}^F(t) \widetilde{y}_{ik}(t) c_i f_i^C \frac{e_k}{l_k} - \sum_i \sum_k \widetilde{y}_{ik}(t)(f_i^I(t) \frac{e_k}{l_k} + \frac{e_k}{l_k} c_i f_i^C)] \\ \leq \sum_t \sum_i \sum_k (\widetilde{w}_{ik0}(t) c_i f_i^C \frac{e_k}{l_k} + \sum_v \widetilde{y}_{ik}(t) \widetilde{f}_{iv}^F(t) v d_i^F r_i^F \frac{e_k}{l_k}) \\ \leq \sum_t \sum_i \sum_k (\widetilde{y}_{ik} f_i^C \frac{e_k}{l_k} + \sum_v \widetilde{y}_{ik}(t) \widetilde{f}_{iv}^F(t) v d_i^F r_i^F \frac{e_k}{l_k}) \\ \leq T \sum_i (f_i^C \frac{e_{max}}{l_{min}} + v_{max} d_i^F r_i^F \frac{e_{max}}{l_{min}}),$$

where e_{max} represents the highest energy consumption of all the instances and l_{min} represents the lowest computing efficiency of all the instances.

Then the expected gap between the switching cost of fractional and integral solution is as follows

$$\sum_t (-\widetilde{C}_s(t) + \mathbb{E}\bar{C}_s(t)) = -\sum_t \sum_i \sum_k q_i[\widetilde{y}_{ik}(t) - \widetilde{y}_{ik}(t-1)]^+ \\ + \sum_t \sum_i \sum_k q_i p_{ik}^y(t)(1 - p_{ik}^y(t-1))(1-0) = \\ - \sum_t \sum_i \sum_k q_i[\widetilde{y}_{ik}(t) - \widetilde{y}_{ik}(t-1)]^+ + \sum_t \sum_i \sum_k q_i \widetilde{y}_{ik}(t)(1 - \widetilde{y}_{ik}(t-1)) \\ \leq \sum_t \sum_i \sum_k q_i \widetilde{y}_{ik}(t)(1 - \widetilde{y}_{ik}(t-1)) \\ \leq \sum_t \sum_i \sum_k q_i \widetilde{y}_{ik}(t) \leq T \sum_i q_i$$

Combining above two gap and the average gap in Theorem 1, we obtain the final expected average gap between fractional solution and integral solution as follows

$$\mathbb{E}\{\frac{1}{T}[P1^{opt}-(\sum_t \bar{U}_{ns}(t)-\sum_t \bar{C}_s(t))]\}=$$
$$\frac{1}{T}\mathbb{E}\{P1^{opt}-(\sum_t \widetilde{U}_{ns}(t)-\sum_t \widetilde{C}_s(t))$$
$$+(\sum_t \widetilde{U}_{ns}(t)-\sum_t \widetilde{C}_s(t))-(\sum_t \bar{U}_{ns}(t)-\sum_t \bar{C}_s(t))]\}$$
$$\leq \sum_i q_i+\sum_i \sum_k (c_i f_i^C \frac{e_k}{l_k}+\sum_v v d_i^F r_i^F \frac{e_k}{l_k})+\sum_i q_i \leq$$
$$\sum_i (2q_i+f_i^C \frac{e_{max}}{l_{min}}+v_{max} d_i^F r_i^F \frac{e_{max}}{l_{min}})$$