Appendix A
Proof of Equal Transformation

To prove that , we just need to prove that $w_{ikv}(t)$ can replace $x_{iv}(t)y_{ik}(t)$.

Variables $w_{ikv}(t)$ subject to the following constraints. $w_{ikv}(t) \leq y_{ik}(t), w_{ikv}(t) \leq x_{iv}(t), w_{ikv}(t) \geq y_{ik}(t) + x_{iv}(t) - 1, w_{ikv} \in \{0,1\}$ is equal to $w_{ikv}(t) = x_{iv}(t)y_{ik}(t)$. That is to say, we need to prove that at any value of f, y and the above constraints, we have $w_{ikv}(t) = x_{iv}(t)y_{ik}(t)$.

When $x_{iv}(t) = 0$ or $y_{ik}(t) = 0$, we require $w_{ikv}(t) = 0$. We can see that constraints $w_{ikv}(t) \leq x_{iv}(t)$ or $w_{ikv}(t) \leq y_{ik}(t)$ make sure that $w_{ikv}(t) = 0$. Meanwhile $w_{ikv}(t) = 0$ also satisfy $w_{ikv}(t) \geq y_{ik}(t) + x_{iv}(t) - 1$. Therefore we prove that when $x_{iv}(t) = 0$ or $y_{ik}(t) = 0$, $w_{ikv}(t) = x_{iv}(t)y_{ik}(t)$.

When $x_{iv}(t) = 1$ and $y_{ik}(t) = 1$, we need $w_{ikv}(t) = 1$. We can see that constraint $w_{ikv}(t) \geq y_{ik}(t) + x_{iv}(t) - 1$ makes sure that $w_{ikv}(t) = 1$. And $w_{ikv}(t) = 1$ does not violate constraints $w_{ikv}(t) \leq y_{ik}(t)$and , $w_{ikv}(t) \leq x_{iv}(t)$. Therefore we prove that when $x_{iv}(t) = 0$ or $y_{ik}(t) = 0$, $w_{ikv}(t) = x_{iv}(t)y_{ik}(t)$.

Since in all situations, $w_{ikv}(t) = x_{iv}(t)y_{ik}(t)$, we complete our proof.

Appendix B
Proof of Lemma 1

The dual problem of $P2$ is

$$P4: \min \sum_i \sum_t \alpha_{it} + \sum_i \sum_t \beta_{it} + \sum_k \sum_t \gamma_{kt} U_k + \sum_i \sum_k \sum_t \lambda_{kt} t_i^{max} + \sum_i \sum_k \sum_v \sum_t \delta_{ikvt} + a_i \sum_i \sum_t b_i(0) - a_i \sum_i \sum_t \sum_{\tau=1}^{t} g_i(\tau)$$

$$\text{s.t. } (T-t+1)a_i b_{iv}(t) - \beta_{it} + \sum_k \pi_{ikvt} - \sum_k \delta_{ikvt} \leq 0, \forall i,k,v, \tag{1a}$$

$$-\alpha_{it} - \frac{e_k}{l_k}(f_i^I(t) + c_i f_i^C) - \gamma_{kt} - \lambda_{ikt}(\frac{f_i^I(t)}{l_k} + \frac{c_i f_i^C}{l_k}) + \sum_v \theta_{ikvt} - \sum_v \delta_{ikvt} - \epsilon_{ikt} + \epsilon_{ik(t+1)} \leq 0, \forall i,k,t, \tag{1b}$$

$$-v d_i^F r_i^F \frac{e_k}{l_k} - \lambda_{ikt} v \frac{d_i^F r_i^F}{l_k} - \theta_{ikvt} - \pi_{ikvt} + \delta_{ikvt} \leq 0, \forall i,k,v \backslash \{0\}, t, \tag{1c}$$

$$c_i f_i^C \frac{e_k}{l_k} + \lambda_{ikt} c_i f_i^C \frac{1}{l_k} - \theta_{ik0t} - \pi_{ik0t} + \delta_{ik0t} \leq 0, \forall i,k,t, \tag{1d}$$

$$-q_i + \epsilon_{ikt} \leq 0, \forall i,k,t, \tag{1e}$$

$$\text{All the dual variables } \geq 0, \text{ except } \alpha \text{ and } \beta. \tag{1f}$$

By Algorithm 1, we can calculate the optimal solution of P_3^t in each time slot. The KKT condition of P_3^t is

$$(T-t+1)a_i b_{iv}(t) - \widetilde{\beta}_{it} + \sum_k \widetilde{\pi}_{ikvt} - \sum_k \widetilde{\delta}_{ikvt} = 0, \forall i,v, \tag{2a}$$

$$-\widetilde{\alpha}_{it} - \frac{e_k}{l_k}(f_i^I(t) + c_i f_i^C) - \widetilde{\gamma}_{kt} - \widetilde{\lambda}_{ikt}(\frac{f_i^I(t)}{l_k} + \frac{c_i f_i^C}{l_k}) + \sum_v \widetilde{\theta}_{ikvt} - \sum_v \widetilde{\delta}_{ikvt} - \frac{q_i}{\eta} \ln \frac{\widetilde{y}_{ik}(t) + \xi}{\widetilde{y}_{ik}(t-1) + \xi} = 0, \forall i,k,v, \tag{2b}$$

$$-v d_i^F r_i^F \frac{e_k}{l_k} - \widetilde{\lambda}_{ikt} v d_i^F r_i^F \frac{1}{l_k} - \widetilde{\theta}_{ikvt} - \widetilde{\pi}_{ikvt} + \widetilde{\delta}_{ikvt} = 0, \forall i,k,v \backslash \{0\}, \tag{2c}$$

$$c_i f_i^C \frac{e_k}{l_k} + \widetilde{\lambda}_{ikt} c_i f_i^C \frac{1}{l_k} - \widetilde{\theta}_{ik0t} - \widetilde{\pi}_{ik0t} + \widetilde{\delta}_{ik0t} = 0, \forall i,k,v = 0, \tag{2d}$$

$$\text{All dual variables } \geq 0, \text{ except } \widetilde{\alpha}_{it} \text{ and } \widetilde{\beta}_{it}. \tag{2e}$$

It is easy to verify that all dual variables (except α and β) are non-negative and constraints (1a),(1c),(1d) are satisfied because of KKT condition (2a),(2c),(2d). Then since $\epsilon_{ikt} = \frac{q_i}{\eta} \ln \frac{1+\xi}{\widetilde{y}_{ik}(t-1)+\xi} \leq \frac{q_i}{\eta} \ln \frac{1+\xi}{\xi} = \frac{q_i}{\eta}\eta = q_i$, constraint (1e) is also satisfied. Constraint (1b) is satisfied since

$$\begin{aligned}
&-\alpha_{it} - \frac{e_k}{l_k}(f_i^I(t) + c_i f_i^C) - \gamma_{kt} - \lambda_{ikt}(\frac{f_i^I(t)}{l_k} + \frac{c_i f_i^C}{l_k}) + \sum_v \theta_{ikvt} - \sum_v \delta_{ikvt} - \epsilon_{ikt} + \epsilon_{ik(t+1)} \\
&= -\widetilde{\alpha}_{it} - \frac{e_k}{l_k}(f_i^I(t) + c_i f_i^C) - \widetilde{\gamma}_{kt} - \widetilde{\lambda}_{ikt}(\frac{f_i^I(t)}{l_k} + \frac{c_i f_i^C}{l_k}) \\
&+ \sum_v \widetilde{\theta}_{ikvt} - \sum_v \delta_{ikvt} - \frac{q_i}{\eta} \ln \frac{1+\xi}{\widetilde{y}_{ik}(t-1)+\xi} + \frac{q_i}{\eta} \ln \frac{1+\xi}{\widetilde{y}_{ik}(t)+\xi} \\
&= -\widetilde{\alpha}_{it} - \frac{e_k}{l_k}(f_i^I(t) + c_i f_i^C) - \widetilde{\gamma}_{kt} - \widetilde{\lambda}_{ikt}(\frac{f_i^I(t)}{l_k} + \frac{c_i f_i^C}{l_k}) \\
&+ \sum_v \widetilde{\theta}_{ikvt} - \sum_v \delta_{ikvt} - \frac{q_i}{\eta} \ln \frac{y_{ik}(t)+\xi}{y_{ik}(t-1)+\xi} \leq 0
\end{aligned}$$

Therefore we obtain a feasible solution of $P4$ by calculate the optimal solution of P_3^t in each time slot.

Appendix C
Proof of Lemma 2

The non-switching utility $U_{ns}^t(\widetilde{\boldsymbol{x}}(t), \widetilde{\boldsymbol{y}}(t))$ obtained by Algorithm 1 can be decomposed to the following expression

$$\sum_t U_{ns}^t(\widetilde{\boldsymbol{x}}(t), \widetilde{\boldsymbol{y}}(t)) = \sum_t \sum_i \sum_v (T-t+1)a_i b_{iv}(t)\widetilde{x}_{iv}(t) \tag{3a}$$

$$-\sum_t \sum_i \sum_{v \backslash \{0\}} \sum_k \widetilde{w}_{ikv}(t) v d_i^F r_i^F \frac{e_k}{l_k} \tag{3b}$$

$$+\sum_t \sum_i \sum_k \widetilde{w}_{ik0}(t) c_i f_i^C \frac{e_k}{l_k} \tag{3c}$$

$$-\sum_t \sum_i \sum_k \widetilde{y}_{ik}(t)(f_i^I(t)\frac{e_k}{l_k} + \frac{e_k}{l_k} c_i f_i^C) \tag{3d}$$

$$+\sum_t \sum_i a_i[b_i(0) - \sum_{\tau=1}^{t} g_i(\tau)] \tag{3e}$$

Using KKT condition (2a), we rewrite (3a) as

$$\sum_t\sum_t\sum_v(\widetilde{\beta}_{it}-\sum_k\widetilde{\theta}_{ikvt}+\sum_k\widetilde{\delta}_{ikvt})\widetilde{x}_{iv}(t)$$
$$=\sum_t\sum_i\widetilde{\beta}_{it}-\sum_t\sum_i\sum_v\sum_k\widetilde{\pi}_{ikvt}\widetilde{x}_{iv}(t)$$
$$+\sum_t\sum_i\sum_v\sum_k\widetilde{\delta}_{ikvt}\widetilde{x}_{iv}(t). \quad (4)$$

Using KKT condition (2b), we can relax (3d) as

$$\sum_t\sum_i\sum_k\widetilde{y}_{ik}(t)[\widetilde{\alpha}_{it}+\widetilde{\gamma}_{kt}+\lambda_{ikt}(\frac{f_i^I(t)}{l_k}+\frac{c_if_i^C}{l_k})-\sum_v\widetilde{\theta}_{ikvt}$$
$$+\sum_v\widetilde{\delta}_{ikvt}+\frac{q_i}{\eta}\ln\frac{\widetilde{y}_{ik}(t)+\xi}{\widetilde{y}_{ik}(t-1)+\xi}]$$
$$\geq\sum_t\sum_i\widetilde{\alpha}_{it}+\sum_t\sum_k U_k\widetilde{\gamma}_{kt}$$
$$+\sum_t\sum_i\sum_k\widetilde{\lambda}_{ikt}(\frac{f_i^I(t)}{l_k}+\frac{c_if_i^C}{l_k})\widetilde{y}_{ik}(t)$$
$$-\sum_t\sum_i\sum_k\sum_v\widetilde{\theta}_{ikvt}\widetilde{y}_{ik}(t)+\sum_t\sum_i\sum_k\sum_v\widetilde{\delta}_{ikvt}\widetilde{y}_{ik}(t) \quad (5)$$

Using KKT condition (2c), we can rewrite (3b) as

$$\sum_t\sum_i\sum_k\sum_{v\backslash\{0\}}\widetilde{w}_{ikv}(t)(\widetilde{\lambda}_{ikt}vd_i^Fr_i^F\frac{1}{l_k}+\widetilde{\theta}_{ikvt}+\widetilde{\pi}_{ikvt}-\widetilde{\delta}_{ikvt}) \quad (6)$$

Using KKT condition (2d), we can rewrite (3c) as

$$\sum_t\sum_i\sum_k\widetilde{w}_{ik0}(t)(-\widetilde{\lambda}_{ikt}c_if_i^C\frac{1}{l_k}+\widetilde{\theta}_{ik0t}+\widetilde{\pi}_{ik0t}-\widetilde{\delta}_{ik0t}) \quad (7)$$

Combine (6) and (7), we can relax (3b) and (3c) as

$$\sum_t\sum_i\sum_k(\sum_{v\backslash\{0\}}\widetilde{w}_{ikv}(t)\widetilde{\lambda}_{ikt}vd_i^Fr_i^F\frac{1}{l_k}-\widetilde{w}_{ik0}(t)\widetilde{\lambda}_{ikt}c_if_i^C\frac{1}{l_k})$$
$$+\sum_t\sum_i\sum_k\sum_v\widetilde{w}_{ikv}(t)(\widetilde{\theta}_{ikvt}+\widetilde{\pi}_{ikvt}-\widetilde{\delta}_{ikvt})$$
$$\geq\sum_t\sum_i\sum_k\widetilde{\lambda}_{ikt}(t_i^{max}-\widetilde{y}_{ik}(t)(\frac{f_i^I(t)}{l_k}+\frac{c_if_i^C}{l_k}))$$
$$+\sum_t\sum_i\sum_k\sum_v\widetilde{w}_{ikv}(t)(\widetilde{\theta}_{ikvt}+\widetilde{\pi}_{ikvt}-\widetilde{\delta}_{ikvt})$$
$$\geq\sum_t\sum_i\sum_k\widetilde{\lambda}_{ikt}(t_i^{max}-\widetilde{y}_{ik}(t)(\frac{f_i^I(t)}{l_k}+\frac{c_if_i^C}{l_k}))+$$
$$\sum_t\sum_i\sum_v\sum_k\widetilde{\theta}_{ikvt}\widetilde{y}_{ik}(t)+\sum_i\sum_k\sum_v\sum_t\widetilde{\pi}_{ikvt}\widetilde{x}_{iv}(t)$$
$$-\sum_t\sum_i\sum_v\sum_t\widetilde{\delta}_{ikvt}\widetilde{w}_{ikv}(t)$$
$$\geq\sum_t\sum_i\sum_k\widetilde{\lambda}_{ikt}(t_i^{max}-\widetilde{y}_{ik}(t)(\frac{f_i^I(t)}{l_k}+\frac{c_if_i^C}{l_k}))+$$
$$\sum_t\sum_i\sum_v\sum_k\widetilde{\theta}_{ikvt}\widetilde{y}_{ik}(t)+\sum_i\sum_k\sum_v\sum_t\widetilde{\pi}_{ikvt}\widetilde{x}_{iv}(t)$$
$$-\sum_t\sum_i\sum_k\sum_v\widetilde{\delta}_{ikvt}(\widetilde{y}_{ik}(t)+\widetilde{x}_{iv}(t))+\sum_t\sum_k\sum_v\sum_t\widetilde{\delta}_{ikvt} \quad (8)$$

Combining (4), (5), (8) and (3e), we can relax the non-switching cost as

$$\sum_t\widetilde{U}_{ns}(t)\geq\sum_t\sum_i\widetilde{\beta}_{it}-\sum_t\sum_i\sum_v\sum_k\widetilde{\pi}_{ikvt}\widetilde{x}_{iv}(t)$$
$$+\sum_t\sum_i\sum_v\sum_k\widetilde{\pi}_{ikvt}\widetilde{x}_{iv}(t)+\sum_t\sum_i\widetilde{\alpha}_{it}+\sum_t\sum_k U_k\widetilde{\gamma}_{kt}$$
$$+\sum_t\sum_i\sum_k\widetilde{\lambda}_{ikt}(\frac{f_i^I(t)}{l_k}+\frac{c_if_i^C}{l_k})\widetilde{y}_{ik}(t)$$
$$-\sum_t\sum_i\sum_k\sum_v(\widetilde{\theta}_{ikvt}-\widetilde{\delta}_{ikvt})\widetilde{y}_{ik}(t)$$
$$+\sum_t\sum_i\sum_k\widetilde{\lambda}_{ikt}(t_i^{max}-\widetilde{y}_{ik}(t)(\frac{f_i^I(t)}{l_k}+\frac{c_if_i^C}{l_k}))+$$
$$\sum_t\sum_i\sum_v\sum_k\widetilde{\theta}_{ikvt}\widetilde{y}_{ik}(t)+\sum_i\sum_k\sum_v\sum_t\widetilde{\pi}_{ikvt}\widetilde{x}_{iv}(t)$$
$$-\sum_t\sum_i\sum_k\sum_v\widetilde{\delta}_{ikvt}(\widetilde{y}_{ik}(t)+\widetilde{x}_{iv}(t))$$
$$+\sum_t\sum_k\sum_v\sum_t\widetilde{\delta}_{ikvt}+\sum_t[\sum_i\sum_{\tau=1}^{t}a_ig_i(\tau)+\sum_i a_ib_i(0)]$$
$$=\sum_i\sum_t\widetilde{\alpha}_{it}+\sum_i\sum_t\widetilde{\beta}_{it}+\sum_k\sum_t\widetilde{\gamma}_{kt}U_k+\sum_i\sum_k\sum_t\widetilde{\lambda}_{kt}t_i^{max}$$
$$+\sum_i\sum_k\sum_v\sum_t\widetilde{\delta}_{ikvt}+a_i\sum_i\sum_t b_i(0)-a_i\sum_i\sum_t\sum_{\tau=1}^{t}g_i(\tau)$$
$$=P4(\mathbf{\Pi})$$

APPENDIX D
PROOF OF THEOREM 1

According to the property of the weak duality, we have

$$\sum_t U_{ns}^t(\widetilde{\boldsymbol{x}}(t),\widetilde{\boldsymbol{y}}(t))\geq P4(\mathbf{\Pi})\geq P4^{opt}\geq P2^{opt}\geq U_{sum}(\boldsymbol{x}^*,\boldsymbol{y}^*)$$

Switching cost $C_s^t(\widetilde{\boldsymbol{y}}(t))$ obtained by Algorithm 1 can be relaxed to

$$\sum_t C_s^t(\widetilde{\boldsymbol{y}}(t))=\sum_t\sum_i\sum_k q_i[\widetilde{y}_{ik}(t)-\widetilde{y}_{ik}(t-1)]^+$$
$$\leq\sum_t\sum_i\sum_k q_i\widetilde{y}_{ik}(t)\leq\sum_i Tq_i.$$

Therefore, we derive the average optimality gap

$$\frac{1}{T}[U_{sum}(\boldsymbol{x}^*,\boldsymbol{y}^*)-U_{sum}(\widetilde{\boldsymbol{x}},\widetilde{\boldsymbol{y}})]=$$

$$\frac{1}{T}[U_{sum}(\boldsymbol{x}^*,\boldsymbol{y}^*)-\sum_t U_{ns}^t(\widetilde{\boldsymbol{x}}(t),\widetilde{\boldsymbol{y}}(t))+\sum_t C_s^t(\widetilde{\boldsymbol{y}}(t))]\leq\sum_i q_i.$$

APPENDIX E
PROOF OF THEOREM 2

The expected gap between the fractional solution obtained by Algorithm 1 and integral solution obtained by rounding can be written as follows

$$\mathbb{E}[U_{sum}(\widetilde{\boldsymbol{x}},\widetilde{\boldsymbol{y}})-U_{sum}(\bar{\boldsymbol{x}},\bar{\boldsymbol{y}})]$$

$$= \sum_t (U_{ns}^t(\widetilde{\boldsymbol{x}}(t), \widetilde{\boldsymbol{y}}(t)) - \mathbb{E}U_{ns}^t(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}})) + \sum_t (-C_s^t(\widetilde{\boldsymbol{y}}(t)) + \mathbb{E}C_s^t(\bar{\boldsymbol{y}}(t)))$$

Then the expected gap between the non-switching utility of fractional and integral solution is as follows

$$\begin{aligned}
&\sum_t (U_{ns}^t(\widetilde{\boldsymbol{x}}(t), \widetilde{\boldsymbol{y}}(t)) - \mathbb{E}U_{ns}^t(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}})) = \\
&\sum_t [\sum_i \sum_v (T-t+1) a_i b_{iv}(t) \widetilde{x}_{iv}(t) - \sum_i \sum_v \sum_k \widetilde{w}_{ikv}(t) v d_i^F r_i^F \frac{e_k}{l_k} \\
&+ \sum_i \sum_k \widetilde{w}_{ik0}(t) c_i f_i^C \frac{e_k}{l_k} - \sum_i \sum_k \widetilde{y}_{ik}(t)(f_i^I(t) \frac{e_k}{l_k} + \frac{e_k}{l_k} c_i f_i^C)] \\
&- \sum_t [\sum_i \sum_v (T-t+1) a_i b_{iv}(t) p_{iv}^f(t) - \sum_i \sum_v \sum_k p_{iv}^f(t) p_{iv}^y(t) v d_i^F r_i^F \frac{e_k}{l_k} \\
&+ \sum_i \sum_k p_{i0}^f(t) p_{ik}^y(t) c_i f_i^C \frac{e_k}{l_k} - \sum_i \sum_k p_{ik}^y(t)(f_i^I(t) \frac{e_k}{l_k} + \frac{e_k}{l_k} c_i f_i^C)] \\
&= \sum_t [\sum_i \sum_v (T-t+1) a_i b_{iv}(t) \widetilde{x}_{iv}(t) - \sum_i \sum_v \sum_k \widetilde{w}_{ikv}(t) v d_i^F r_i^F \frac{e_k}{l_k} \\
&+ \sum_i \sum_k \widetilde{w}_{ik0}(t) c_i f_i^C \frac{e_k}{l_k} - \sum_i \sum_k \widetilde{y}_{ik}(t)(f_i^I(t) \frac{e_k}{l_k} + \frac{e_k}{l_k} c_i f_i^C)] \\
&- \sum_t [\sum_i \sum_v (T-t+1) a_i b_{iv}(t) \widetilde{x}_{iv}(t) - \sum_i \sum_v \sum_k \widetilde{y}_{ik}(t) \widetilde{x}_{iv}(t) v d_i^F r_i^F \frac{e_k}{l_k} \\
&+ \sum_i \sum_k f_{i0}^F(t) \widetilde{y}_{ik}(t) c_i f_i^C \frac{e_k}{l_k} - \sum_i \sum_k \widetilde{y}_{ik}(t)(f_i^I(t) \frac{e_k}{l_k} + \frac{e_k}{l_k} c_i f_i^C)] \\
&\le \sum_t \sum_i \sum_k (\widetilde{w}_{ik0}(t) c_i f_i^C \frac{e_k}{l_k} + \sum_v \widetilde{y}_{ik}(t) \widetilde{x}_{iv}(t) v d_i^F r_i^F \frac{e_k}{l_k}) \\
&\le \sum_t \sum_i \sum_k (\widetilde{y}_{ik} f_i^C \frac{e_k}{l_k} + \sum_v \widetilde{y}_{ik}(t) \widetilde{x}_{iv}(t) v d_i^F r_i^F \frac{e_k}{l_k}) \\
&\le T \sum_i (f_i^C \frac{e_{max}}{l_{min}} + v_{max} d_i^F r_i^F \frac{e_{max}}{l_{min}}),
\end{aligned}$$

where e_{max} represents the highest energy consumption of all the instances and l_{min} represents the lowest computing efficiency of all the instances.

Then the expected gap between the switching cost of fractional and integral solution is as follows

$$\begin{aligned}
&\sum_t (-C_s^t(\widetilde{\boldsymbol{y}}(t)) + \mathbb{E}C_s^t(\bar{\boldsymbol{y}}(t))) = -\sum_t \sum_i \sum_k q_i [\widetilde{y}_{ik}(t) - \widetilde{y}_{ik}(t-1)]^+ \\
&+ \sum_t \sum_i \sum_k q_i p_{ik}^y(t)(1 - p_{ik}^y(t-1))(1-0) = \\
&- \sum_t \sum_i \sum_k q_i [\widetilde{y}_{ik}(t) - \widetilde{y}_{ik}(t-1)]^+ + \sum_t \sum_i \sum_k q_i \widetilde{y}_{ik}(t)(1 - \widetilde{y}_{ik}(t-1)) \\
&\le \sum_t \sum_i \sum_k q_i \widetilde{y}_{ik}(t)(1 - \widetilde{y}_{ik}(t-1)) \\
&\le \sum_t \sum_i \sum_k q_i \widetilde{y}_{ik}(t) \le T \sum_i q_i
\end{aligned}$$

Combining above two gaps and the average gap in Theorem 1, we obtain the final expected average gap between optimal solution and integral solution as follows

$$\begin{aligned}
&\mathbb{E}\{\frac{1}{T}[U_{sum}(\boldsymbol{x}^*, \boldsymbol{y}^*) - U_{sum}(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}})]\} = \\
&\frac{1}{T}\mathbb{E}\{U_{sum}(\boldsymbol{x}^*, \boldsymbol{y}^*) - U_{sum}(\widetilde{\boldsymbol{x}}, \widetilde{\boldsymbol{y}}) + (U_{sum}(\widetilde{\boldsymbol{x}}, \widetilde{\boldsymbol{y}}) - U_{sum}(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}})]\} \\
&\le \sum_i q_i + \sum_i \sum_k (c_i f_i^C \frac{e_k}{l_k} + \sum_v v d_i^F r_i^F \frac{e_k}{l_k}) + \sum_i q_i \\
&\le \sum_i (2q_i + f_i^C \frac{e_{max}}{l_{min}} + v_{max} d_i^F r_i^F \frac{e_{max}}{l_{min}})
\end{aligned}$$