

APPENDIX A
PROOF OF EQUAL TRANSFORMATION

To prove that , we just need to prove that $w_{ikv}(t)$ can replace $x_{iv}(t)y_{ik}(t)$.

Variables $w_{ikv}(t)$ subject to the following constraints. $w_{ikv}(t) \leq y_{ik}(t)$, $w_{ikv}(t) \leq x_{iv}(t)$, $w_{ikv}(t) \geq y_{ik}(t) + x_{iv}(t) - 1$, $w_{ikv} \in \{0, 1\}$ is equal to $w_{ikv}(t) = x_{iv}(t)y_{ik}(t)$. That is to say, we need to prove that at any value of f, y and the above constraints, we have $w_{ikv}(t) = x_{iv}(t)y_{ik}(t)$.

When $x_{iv}(t) = 0$ or $y_{ik}(t) = 0$, we require $w_{ikv}(t) = 0$. We can see that constraints $w_{ikv}(t) \leq x_{iv}(t)$ or $w_{ikv}(t) \leq y_{ik}(t)$ make sure that $w_{ikv}(t) = 0$. Meanwhile $w_{ikv}(t) = 0$ also satisfy $w_{ikv}(t) \geq y_{ik}(t) + x_{iv}(t) - 1$. Therefore we prove that when $x_{iv}(t) = 0$ or $y_{ik}(t) = 0$, $w_{ikv}(t) = x_{iv}(t)y_{ik}(t)$.

When $x_{iv}(t) = 1$ and $y_{ik}(t) = 1$, we need $w_{ikv}(t) = 1$. We can see that constraint $w_{ikv}(t) \geq y_{ik}(t) + x_{iv}(t) - 1$ makes sure that $w_{ikv}(t) = 1$. And $w_{ikv}(t) = 1$ does not violate constraints $w_{ikv}(t) \leq y_{ik}(t)$ and $w_{ikv}(t) \leq x_{iv}(t)$. Therefore we prove that when $x_{iv}(t) = 1$ and $y_{ik}(t) = 1$, $w_{ikv}(t) = x_{iv}(t)y_{ik}(t)$.

Since in all situations, $w_{ikv}(t) = x_{iv}(t)y_{ik}(t)$, we complete our proof.

APPENDIX B
PROOF OF LEMMA 1

The dual problem of $P2$ is

$$P4 : \min \sum_i \sum_t \alpha_{it} + \sum_i \sum_t \beta_{it} + \sum_k \sum_t \gamma_{kt} U_k + \sum_i \sum_k \sum_t \lambda_{kt} t_i^{max} + \sum_i \sum_k \sum_v \sum_t \delta_{ikvt} + a_i \sum_i \sum_t b_i(0) - a_i \sum_i \sum_t \sum_{\tau=1}^t g_i(\tau)$$

s.t. $(T - t + 1)a_i b_{iv}(t) - \beta_{it} + \sum_k \pi_{ikvt} - \sum_k \delta_{ikvt} \leq 0, \forall i, k, v,$ (1a)

$$- \alpha_{it} - \frac{e_k}{l_k} (f_i^I(t) + c_i f_i^C) - \gamma_{kt} - \lambda_{ikt} \left(\frac{f_i^I(t)}{l_k} + \frac{c_i f_i^C}{l_k} \right) + \sum_v \theta_{ikvt} - \sum_v \delta_{ikvt} - \epsilon_{ikt} + \epsilon_{ik(t+1)} \leq 0, \forall i, k, t, \quad (1b)$$

$$- v d_i^F r_i^F \frac{e_k}{l_k} - \lambda_{ikt} v \frac{d_i^F r_i^F}{l_k} - \theta_{ikvt} - \pi_{ikvt} + \delta_{ikvt} \leq 0, \forall i, k, v \setminus \{0\}, t, \quad (1c)$$

$$c_i f_i^C \frac{e_k}{l_k} + \lambda_{ikt} c_i f_i^C \frac{1}{l_k} - \theta_{ik0t} - \pi_{ik0t} + \delta_{ik0t} \leq 0, \forall i, k, t, \quad (1d)$$

$$- q_i + \epsilon_{ikt} \leq 0, \forall i, k, t, \quad (1e)$$

$$\text{All the dual variables } \geq 0, \text{ except } \alpha \text{ and } \beta. \quad (1f)$$

By Algorithm 1, we can calculate the optimal solution of P_3^t in each time slot. The KKT condition of P_3^t is

$$(T - t + 1)a_i b_{iv}(t) - \tilde{\beta}_{it} + \sum_k \tilde{\pi}_{ikvt} - \sum_k \tilde{\delta}_{ikvt} = 0, \forall i, v, \quad (2a)$$

$$- \tilde{\alpha}_{it} - \frac{e_k}{l_k} (f_i^I(t) + c_i f_i^C) - \tilde{\gamma}_{kt} - \tilde{\lambda}_{ikt} \left(\frac{f_i^I(t)}{l_k} + \frac{c_i f_i^C}{l_k} \right) + \sum_v \tilde{\theta}_{ikvt} - \sum_v \tilde{\delta}_{ikvt} - \frac{q_i}{\eta} \ln \frac{\tilde{y}_{ik}(t) + \xi}{\tilde{y}_{ik}(t-1) + \xi} = 0, \forall i, k, v, \quad (2b)$$

$$- v d_i^F r_i^F \frac{e_k}{l_k} - \tilde{\lambda}_{ikt} v d_i^F r_i^F \frac{1}{l_k} - \tilde{\theta}_{ikvt} - \tilde{\pi}_{ikvt} + \tilde{\delta}_{ikvt} = 0, \forall i, k, v \setminus \{0\}, \quad (2c)$$

$$c_i f_i^C \frac{e_k}{l_k} + \tilde{\lambda}_{ikt} c_i f_i^C \frac{1}{l_k} - \tilde{\theta}_{ik0t} - \tilde{\pi}_{ik0t} + \tilde{\delta}_{ik0t} = 0, \forall i, k, v = 0, \quad (2d)$$

$$\text{All dual variables } \geq 0, \text{ except } \tilde{\alpha}_{it} \text{ and } \tilde{\beta}_{it}. \quad (2e)$$

It is easy to verify that all dual variables (except α and β) are non-negative and constraints (1a),(1c),(1d) are satisfied because of KKT condition (2a),(2c),(2d). Then since $\epsilon_{ikt} = \frac{q_i}{\eta} \ln \frac{1+\xi}{\tilde{y}_{ik}(t-1)+\xi} \leq \frac{q_i}{\eta} \ln \frac{1+\xi}{\xi} = \frac{q_i}{\eta} \eta = q_i$, constraint (1e) is also satisfied. Constraint (1b) is satisfied since

$$- \alpha_{it} - \frac{e_k}{l_k} (f_i^I(t) + c_i f_i^C) - \gamma_{kt} - \lambda_{ikt} \left(\frac{f_i^I(t)}{l_k} + \frac{c_i f_i^C}{l_k} \right) + \sum_v \theta_{ikvt} - \sum_v \delta_{ikvt} - \epsilon_{ikt} + \epsilon_{ik(t+1)} = - \tilde{\alpha}_{it} - \frac{e_k}{l_k} (f_i^I(t) + c_i f_i^C) - \tilde{\gamma}_{kt} - \tilde{\lambda}_{ikt} \left(\frac{f_i^I(t)}{l_k} + \frac{c_i f_i^C}{l_k} \right) + \sum_v \tilde{\theta}_{ikvt} - \sum_v \tilde{\delta}_{ikvt} - \frac{q_i}{\eta} \ln \frac{1+\xi}{\tilde{y}_{ik}(t-1) + \xi} + \frac{q_i}{\eta} \ln \frac{1+\xi}{\tilde{y}_{ik}(t) + \xi} = - \tilde{\alpha}_{it} - \frac{e_k}{l_k} (f_i^I(t) + c_i f_i^C) - \tilde{\gamma}_{kt} - \tilde{\lambda}_{ikt} \left(\frac{f_i^I(t)}{l_k} + \frac{c_i f_i^C}{l_k} \right) + \sum_v \tilde{\theta}_{ikvt} - \sum_v \tilde{\delta}_{ikvt} - \frac{q_i}{\eta} \ln \frac{y_{ik}(t) + \xi}{y_{ik}(t-1) + \xi} \leq 0$$

Therefore we obtain a feasible solution of $P4$ by calculate the optimal solution of P_3^t in each time slot.

APPENDIX C
PROOF OF LEMMA 2

The non-switching utility $U_{ns}^t(\tilde{\mathbf{x}}(t), \tilde{\mathbf{y}}(t))$ obtained by Algorithm 1 can be decomposed to the following expression

$$\sum_t U_{ns}^t(\tilde{\mathbf{x}}(t), \tilde{\mathbf{y}}(t)) = \sum_t \sum_i \sum_v (T - t + 1) a_i b_{iv}(t) \tilde{x}_{iv}(t) \quad (3a)$$

$$- \sum_t \sum_i \sum_{v \setminus \{0\}} \sum_k \tilde{w}_{ikv}(t) v d_i^F r_i^F \frac{e_k}{l_k} \quad (3b)$$

$$+ \sum_t \sum_i \sum_k \tilde{w}_{ik0}(t) c_i f_i^C \frac{e_k}{l_k} \quad (3c)$$

$$- \sum_t \sum_i \sum_k \tilde{y}_{ik}(t) (f_i^I(t) \frac{e_k}{l_k} + \frac{e_k}{l_k} c_i f_i^C) \quad (3d)$$

$$+ \sum_t \sum_i a_i [b_i(0) - \sum_{\tau=1}^t g_i(\tau)] \quad (3e)$$

Using KKT condition (2a), we rewrite (3a) as

$$\begin{aligned}
& \sum_t \sum_i \sum_k (\tilde{\beta}_{it} - \sum_k \tilde{\theta}_{ikvt} + \sum_k \tilde{\delta}_{ikvt}) \tilde{x}_{iv}(t) \\
&= \sum_t \sum_i \tilde{\beta}_{it} - \sum_t \sum_i \sum_k \sum_v \tilde{\pi}_{ikvt} \tilde{x}_{iv}(t) \\
&\quad + \sum_t \sum_i \sum_k \sum_v \tilde{\delta}_{ikvt} \tilde{x}_{iv}(t). \quad (4)
\end{aligned}$$

Using KKT condition (2b), we can relax (3d) as

$$\begin{aligned}
& \sum_t \sum_i \sum_k \tilde{y}_{ik}(t) [\tilde{\alpha}_{it} + \tilde{\gamma}_{kt} + \lambda_{ikt} (\frac{f_i^I(t)}{l_k} + \frac{c_i f_i^C}{l_k}) - \sum_v \tilde{\theta}_{ikvt} \\
&\quad + \sum_v \tilde{\delta}_{ikvt} + \frac{q_i}{\eta} \ln \frac{\tilde{y}_{ik}(t) + \xi}{\tilde{y}_{ik}(t-1) + \xi}] \\
&\geq \sum_t \sum_i \tilde{\alpha}_{it} + \sum_t \sum_k U_k \tilde{\gamma}_{kt} \\
&\quad + \sum_t \sum_i \sum_k \tilde{\lambda}_{ikt} (\frac{f_i^I(t)}{l_k} + \frac{c_i f_i^C}{l_k}) \tilde{y}_{ik}(t) \\
&- \sum_t \sum_i \sum_k \sum_v \tilde{\theta}_{ikvt} \tilde{y}_{ik}(t) + \sum_t \sum_i \sum_k \sum_v \tilde{\delta}_{ikvt} \tilde{y}_{ik}(t) \quad (5)
\end{aligned}$$

Using KKT condition (2c), we can rewrite (3b) as

$$\sum_t \sum_i \sum_k \sum_{v \setminus \{0\}} \tilde{w}_{ikv}(t) (\tilde{\lambda}_{ikt} v d_i^F r_i^F \frac{1}{l_k} + \tilde{\theta}_{ikvt} + \tilde{\pi}_{ikvt} - \tilde{\delta}_{ikvt}) \quad (6)$$

Using KKT condition (2d), we can rewrite (3c) as

$$\sum_t \sum_i \sum_k \tilde{w}_{ik0}(t) (-\tilde{\lambda}_{ikt} c_i f_i^C \frac{1}{l_k} + \tilde{\theta}_{ik0t} + \tilde{\pi}_{ik0t} - \tilde{\delta}_{ik0t}) \quad (7)$$

Combine (6) and (7), we can relax (3b) and (3c) as

$$\begin{aligned}
& \sum_t \sum_i \sum_k (\sum_{v \setminus \{0\}} \tilde{w}_{ikv}(t) \tilde{\lambda}_{ikt} v d_i^F r_i^F \frac{1}{l_k} - \tilde{w}_{ik0}(t) \tilde{\lambda}_{ikt} c_i f_i^C \frac{1}{l_k}) \\
&\quad + \sum_t \sum_i \sum_k \sum_v \tilde{w}_{ikv}(t) (\tilde{\theta}_{ikvt} + \tilde{\pi}_{ikvt} - \tilde{\delta}_{ikvt}) \\
&\geq \sum_t \sum_i \sum_k \tilde{\lambda}_{ikt} (t_i^{max} - \tilde{y}_{ik}(t) (\frac{f_i^I(t)}{l_k} + \frac{c_i f_i^C}{l_k})) \\
&\quad + \sum_t \sum_i \sum_k \sum_v \tilde{w}_{ikv}(t) (\tilde{\theta}_{ikvt} + \tilde{\pi}_{ikvt} - \tilde{\delta}_{ikvt}) \\
&\geq \sum_t \sum_i \sum_k \tilde{\lambda}_{ikt} (t_i^{max} - \tilde{y}_{ik}(t) (\frac{f_i^I(t)}{l_k} + \frac{c_i f_i^C}{l_k})) + \\
&\quad \sum_t \sum_i \sum_k \sum_v \tilde{\theta}_{ikvt} \tilde{y}_{ik}(t) + \sum_t \sum_i \sum_k \sum_v \tilde{\pi}_{ikvt} \tilde{x}_{iv}(t) \\
&\quad - \sum_t \sum_i \sum_k \sum_v \tilde{\delta}_{ikvt} \tilde{w}_{ikv}(t) \\
&\geq \sum_t \sum_i \sum_k \tilde{\lambda}_{ikt} (t_i^{max} - \tilde{y}_{ik}(t) (\frac{f_i^I(t)}{l_k} + \frac{c_i f_i^C}{l_k})) + \\
&\quad \sum_t \sum_i \sum_k \sum_v \tilde{\theta}_{ikvt} \tilde{y}_{ik}(t) + \sum_t \sum_i \sum_k \sum_v \tilde{\pi}_{ikvt} \tilde{x}_{iv}(t) \\
&- \sum_t \sum_i \sum_k \sum_v \tilde{\delta}_{ikvt} (\tilde{y}_{ik}(t) + \tilde{x}_{iv}(t)) + \sum_t \sum_k \sum_v \sum_t \tilde{\delta}_{ikvt} \quad (8)
\end{aligned}$$

Combining (4), (5), (8) and (3e), we can relax the non-switching cost as

$$\begin{aligned}
& \sum_t U_{ns}^t(\tilde{\mathbf{x}}(t), \tilde{\mathbf{y}}(t)) \geq \sum_t \sum_i \tilde{\beta}_{it} - \sum_t \sum_i \sum_k \sum_v \tilde{\pi}_{ikvt} \tilde{x}_{iv}(t) \\
&+ \sum_t \sum_i \sum_k \sum_v \tilde{\pi}_{ikvt} \tilde{x}_{iv}(t) + \sum_t \sum_i \tilde{\alpha}_{it} + \sum_t \sum_k U_k \tilde{\gamma}_{kt} \\
&\quad + \sum_t \sum_i \sum_k \tilde{\lambda}_{ikt} (\frac{f_i^I(t)}{l_k} + \frac{c_i f_i^C}{l_k}) \tilde{y}_{ik}(t) \\
&\quad - \sum_t \sum_i \sum_k \sum_v (\tilde{\theta}_{ikvt} - \tilde{\delta}_{ikvt}) \tilde{y}_{ik}(t) \\
&\quad + \sum_t \sum_i \sum_k \tilde{\lambda}_{ikt} (t_i^{max} - \tilde{y}_{ik}(t) (\frac{f_i^I(t)}{l_k} + \frac{c_i f_i^C}{l_k})) + \\
&\quad \sum_t \sum_i \sum_k \sum_v \tilde{\theta}_{ikvt} \tilde{y}_{ik}(t) + \sum_t \sum_i \sum_k \sum_v \tilde{\pi}_{ikvt} \tilde{x}_{iv}(t) \\
&\quad - \sum_t \sum_i \sum_k \sum_v \tilde{\delta}_{ikvt} (\tilde{y}_{ik}(t) + \tilde{x}_{iv}(t)) \\
&\quad + \sum_t \sum_k \sum_v \sum_t \tilde{\delta}_{ikvt} + \sum_t [\sum_i \sum_{\tau=1}^t a_i g_i(\tau) + \sum_i a_i b_i(0)] \\
&= \sum_i \sum_t \tilde{\alpha}_{it} + \sum_i \sum_t \tilde{\beta}_{it} + \sum_k \sum_t \tilde{\gamma}_{kt} U_k + \sum_i \sum_k \sum_t \tilde{\lambda}_{kt} t_i^{max} \\
&\quad + \sum_i \sum_k \sum_v \sum_t \tilde{\delta}_{ikvt} + a_i \sum_i \sum_t b_i(0) - a_i \sum_i \sum_t \sum_{\tau=1}^t g_i(\tau) \\
&\quad = P4(\mathbf{\Pi})
\end{aligned}$$

APPENDIX D PROOF OF THEOREM 1

According to the property of the weak duality, we have

$$\sum_t U_{ns}^t(\tilde{\mathbf{x}}(t), \tilde{\mathbf{y}}(t)) \geq P4(\mathbf{\Pi}) \geq P4^{opt} \geq P2^{opt} \geq U_{sum}(\mathbf{x}^*, \mathbf{y}^*)$$

Switching cost $C_s^t(\tilde{\mathbf{y}}(t))$ obtained by Algorithm 1 can be relaxed to

$$\begin{aligned}
\sum_t C_s^t(\tilde{\mathbf{y}}(t)) &= \sum_t \sum_i \sum_k q_i [\tilde{y}_{ik}(t) - \tilde{y}_{ik}(t-1)]^+ \\
&\leq \sum_t \sum_i \sum_k q_i \tilde{y}_{ik}(t) \leq \sum_i T q_i.
\end{aligned}$$

Therefore, we derive the average optimality gap

$$\begin{aligned}
& \frac{1}{T} [U_{sum}(\mathbf{x}^*, \mathbf{y}^*) - U_{sum}(\tilde{\mathbf{x}}, \tilde{\mathbf{y}})] = \\
& \frac{1}{T} [U_{sum}(\mathbf{x}^*, \mathbf{y}^*) - \sum_t U_{ns}^t(\tilde{\mathbf{x}}(t), \tilde{\mathbf{y}}(t)) + \sum_t C_s^t(\tilde{\mathbf{y}}(t))] \leq \sum_i q_i.
\end{aligned}$$

APPENDIX E PROOF OF THEOREM 2

The expected gap between the fractional solution obtained by Algorithm 1 and integral solution obtained by rounding can be written as follows

$$\mathbb{E}[U_{sum}(\tilde{\mathbf{x}}, \tilde{\mathbf{y}}) - U_{sum}(\bar{\mathbf{x}}, \bar{\mathbf{y}})]$$

$$\begin{aligned}
&= \sum_t (U_{ns}^t(\tilde{\mathbf{x}}(t), \tilde{\mathbf{y}}(t)) - \mathbb{E}U_{ns}^t(\bar{\mathbf{x}}(t), \bar{\mathbf{y}}(t))) \\
&\quad + \sum_t (-C_s^t(\tilde{\mathbf{y}}(t)) + \mathbb{E}C_s^t(\bar{\mathbf{y}}(t))) \\
&\quad \frac{1}{T} \mathbb{E}\{U_{sum}(\mathbf{x}^*, \mathbf{y}^*) - U_{sum}(\tilde{\mathbf{x}}, \tilde{\mathbf{y}}) + (U_{sum}(\tilde{\mathbf{x}}, \tilde{\mathbf{y}}) - U_{sum}(\bar{\mathbf{x}}, \bar{\mathbf{y}}))\} \\
&\leq \sum_i q_i + \sum_i (f_i^C \frac{e_{max}}{l_{min}} + v_{max} d_i^F r_i^F \frac{e_{max}}{l_{min}}) + \sum_i q_i \\
&\leq \sum_i (2q_i + f_i^C \frac{e_{max}}{l_{min}} + v_{max} d_i^F r_i^F \frac{e_{max}}{l_{min}})
\end{aligned}$$

Then the expected gap between the non-switching utility of fractional and integral solution is as follows

$$\begin{aligned}
&\sum_t (U_{ns}^t(\tilde{\mathbf{x}}(t), \tilde{\mathbf{y}}(t)) - \mathbb{E}U_{ns}^t(\bar{\mathbf{x}}(t), \bar{\mathbf{y}}(t))) = \\
&\sum_t [\sum_i \sum_v (T-t+1) a_i b_{iv}(t) \tilde{x}_{iv}(t) - \sum_i \sum_v \sum_k \tilde{w}_{ikv}(t) v d_i^F r_i^F \frac{e_k}{l_k} \\
&+ \sum_i \sum_k \tilde{w}_{ik0}(t) c_i f_i^C \frac{e_k}{l_k} - \sum_i \sum_k \tilde{y}_{ik}(t) (f_i^I(t) \frac{e_k}{l_k} + \frac{e_k}{l_k} c_i f_i^C)] \\
&- \sum_t [\sum_i \sum_v (T-t+1) a_i b_{iv}(t) p_{iv}^x(t) - \sum_i \sum_v \sum_k p_{iv}^x(t) p_{iv}^y(t) v d_i^F r_i^F \frac{e_k}{l_k} \\
&+ \sum_i \sum_k p_{i0}^x(t) p_{ik}^y(t) c_i f_i^C \frac{e_k}{l_k} - \sum_i \sum_k p_{ik}^y(t) (f_i^I(t) \frac{e_k}{l_k} + \frac{e_k}{l_k} c_i f_i^C)] \\
&= \sum_t [\sum_i \sum_v (T-t+1) a_i b_{iv}(t) \tilde{x}_{iv}(t) - \sum_i \sum_v \sum_k \tilde{w}_{ikv}(t) v d_i^F r_i^F \frac{e_k}{l_k} \\
&+ \sum_i \sum_k \tilde{w}_{ik0}(t) c_i f_i^C \frac{e_k}{l_k} - \sum_i \sum_k \tilde{y}_{ik}(t) (f_i^I(t) \frac{e_k}{l_k} + \frac{e_k}{l_k} c_i f_i^C)] \\
&- \sum_t [\sum_i \sum_v (T-t+1) a_i b_{iv}(t) \tilde{x}_{iv}(t) - \sum_i \sum_v \sum_k \tilde{y}_{ik}(t) \tilde{x}_{iv}(t) v d_i^F r_i^F \frac{e_k}{l_k} \\
&+ \sum_i \sum_k f_{i0}^F(t) \tilde{y}_{ik}(t) c_i f_i^C \frac{e_k}{l_k} - \sum_i \sum_k \tilde{y}_{ik}(t) (f_i^I(t) \frac{e_k}{l_k} + \frac{e_k}{l_k} c_i f_i^C)] \\
&\leq \sum_t \sum_i \sum_k (\tilde{w}_{ik0}(t) c_i f_i^C \frac{e_k}{l_k} + \sum_v \tilde{y}_{ik}(t) \tilde{x}_{iv}(t) v d_i^F r_i^F \frac{e_k}{l_k}) \\
&\leq \sum_t \sum_i \sum_k (\tilde{y}_{ik} f_i^C \frac{e_k}{l_k} + \sum_v \tilde{y}_{ik}(t) \tilde{x}_{iv}(t) v d_i^F r_i^F \frac{e_k}{l_k}) \\
&\leq T \sum_i (f_i^C \frac{e_{max}}{l_{min}} + v_{max} d_i^F r_i^F \frac{e_{max}}{l_{min}}),
\end{aligned}$$

where e_{max} represents the highest energy consumption of all the instances and l_{min} represents the lowest computing efficiency of all the instances.

Then the expected gap between the switching cost of fractional and integral solution is as follows

$$\begin{aligned}
&\sum_t (-C_s^t(\tilde{\mathbf{y}}(t)) + \mathbb{E}C_s^t(\bar{\mathbf{y}}(t))) = - \sum_t \sum_i \sum_k q_i [\tilde{y}_{ik}(t) - \tilde{y}_{ik}(t-1)]^+ \\
&\quad + \sum_t \sum_i \sum_k q_i p_{ik}^y(t) (1 - p_{ik}^y(t-1)) (1 - 0) = \\
&- \sum_t \sum_i \sum_k q_i [\tilde{y}_{ik}(t) - \tilde{y}_{ik}(t-1)]^+ + \sum_t \sum_i \sum_k q_i \tilde{y}_{ik}(t) (1 - \tilde{y}_{ik}(t-1)) \\
&\leq \sum_t \sum_i \sum_k q_i \tilde{y}_{ik}(t) (1 - \tilde{y}_{ik}(t-1)) \\
&\leq \sum_t \sum_i \sum_k q_i \tilde{y}_{ik}(t) \leq T \sum_i q_i
\end{aligned}$$

Combining above two gaps and the average gap in Theorem 1, we obtain the final expected average gap between optimal solution and integral solution as follows

$$\mathbb{E}\{\frac{1}{T} [U_{sum}(\mathbf{x}^*, \mathbf{y}^*) - U_{sum}(\bar{\mathbf{x}}, \bar{\mathbf{y}})]\} =$$