中山大学本科生期末考试
考试科目:《高等量子力学》(A 卷)

学年学期: 2016 学年第 2 学期　　姓名 (Full name): _____

学　　院: 物理学院　　学号 (Student ID #): _____

考试方式: 闭卷　　年级专业: 物理 14 级, 研 17 级等

考试时长: 120 分钟　　班　　别: _____

警示《中山大学授予学士学位工作细则》第八条: "考试作弊者, 不授予学士学位."

————以下为试题区域, 共三道大题, 总分 100 分, 考生请在答题纸上作答————

————*All answers should be written on the answer sheets*————

一、选择与填空题 Multiple choice or completion (共 10 小题, 每小题 5 分, 共 50 分.)

1. Schrödinger 方程 The Schrödinger equation $i\hbar\partial\psi/\partial t = H\psi$　① 只能描述无自旋的粒子. can only describe spinless particles　② 可以描述有自旋的粒子, 只是需要找到适当的 H. can describe particles with spin, as long as a suitable H is available.

2. 一个算符的厄米性 The Hermiticity of an operator　① 只依赖于算符的形式. only depends on the form of the operator.　② 除了与算符的形式有关, 也依赖于其所作用的空间. depends on both the operator itself and the space it acts on.

3. 全同性原理 The principle for identical particles　① 只适用于粒子全为 Bose 子或全为 Fermi 子的多粒子体系. is applicable to many-body systems whose particles are all bosons or all fermions.　② 可适用于同时包含 Bose 子和 Fermi 子的多粒子体系. is applicable to many-body systems consisting of both bosons and fermions.

4. 粒子在势场 $V(\boldsymbol{x}) = ax^2 + by^2 + cz^2$ 中运动, 其中 a、b、c 是常数, 则波包中心的运动规律与经典粒子 For a particle moving in the potential $V(\boldsymbol{x}) = ax^2 + by^2 + cz^2$, where a, b and c are constants, the motion of the center of its wave packet　① 相同. is always the same as a classical particle.　② 不一定相同, 只当 $a, b, c \geq 0$ 时才相同. is the same as a classical particle only when $a, b, c \geq 0$.

5. 设力学量 F 显含 t 且 $[F, H] \neq 0$, 但它满足 $i\hbar\partial F/\partial t + [F, H] = 0$, 则 F 在任一态中的平均值 A dynamical variable F depends on t explicitly and $[F, H] \neq 0$, but it satisfies $i\hbar\partial F/\partial t + [F, H] = 0$, then the mean value of F in an arbitrary state　① 不随时间变化. does not vary with time.　② 一般会随时间变化. in general varies with time.

6. 角动量算符的本征值谱 The spectrum of eigenvalues for an angular momentum　① 只适用于轨道角动量、自旋角动量以及它们的耦合. is only applicable to orbit angular momentum, spin or their coupling.　② 适用于任何与角动量算符具有同样对易关系的算符. is applicable to any vector operator that satisfies the same commutation relations as an angular momentum.

7. 一个两体系统的哈密顿为 A two-body system has the Hamiltonian $H = p_1^2/2m_1 + p_2^2/2m_2 + \frac{1}{2}m_1\omega_1^2 r_1^2 + \frac{1}{2}m_2\omega_2^2 r_2^2 + V(|\boldsymbol{r}_1 - \boldsymbol{r}_2|)$. 它具有下述对称性: It possesses the following symmetries: ① 空间平移. space translation.　② 空间转动. space rotation.　③ 上述两者. both of the above.

8. 电磁场中的 Schrödinger 方程是 The Schrödinger equation in an electromagnetic field is $i\hbar\partial\psi/\partial t = -(\hbar^2/2\mu)(\nabla - iq\boldsymbol{A}/\hbar)^2\psi + q\varphi\psi$. 设 $\psi(\boldsymbol{x}, t)$ 是方程的解. Let $\psi(\boldsymbol{x}, t)$ be a solution

of it. 今对同一电磁场, 改取矢势 Now for the same electromagnetic field, take the new vector potential $\varphi' = \varphi$, $A'_x = A_x + cyz$, $A'_y = A_y + czx$, $A'_z = A_z + cxy$, 其中 a、b、c 是常数. where a, b and c are constants. 则新的解 $\psi'(\boldsymbol{x}, t)$ 与 $\psi(\boldsymbol{x}, t)$ 的关系是 Then the relation between the new solution $\psi'(\boldsymbol{x}, t)$ and the old one $\psi(\boldsymbol{x}, t)$ is

9. 一粒子在外场中运动, 其能级为 $E_n = p_n\epsilon$, 其中 ϵ 是常数, p_n 是第 n 个素数(质数). 设初态波函数平方可积, 则在以后的运动中, A particle moving in an external field has the energy levels $E_n = p_n\epsilon$, where ϵ is a constant, and p_n is the nth prime number. Assume that the wave packet of the initial state is square integrable. At later times, ① 波包宽度不会无限增大. the width of the wave packet cannot go to infinity. ② 波包宽度是否会无限增大取决于初态的具体形式. whether the width will become infinity depends on the specific form of the initial state.

10. 下列哈密顿描述的体系那个具有时间反演不变性? Which system with the following Hamiltonian possess time-reversal invariance? ① $H = \boldsymbol{p}^2/2m + V(r) + \xi(r)\boldsymbol{L}\cdot\boldsymbol{S}$ ② $H = -\mu\boldsymbol{S}\cdot\boldsymbol{B}$ ③ $H = (p_x + qBy)^2/2\mu + p_y^2/2\mu + p_z^2/2\mu$

二、计算题之一 Analysis and calculation 1 (本题 20 分.)

设一维势场中运动的粒子具有哈密顿 A particle moving in a one-dimensional potential has the Hamiltonian $H = p^2/2\mu - \mu\omega^2x^2/2$. 设 x 和 p 在初态中的平均值分别为 x_0 和 p_0, 试求出它们在 t 时刻的平均值, 记作 $\bar{x}(t)$ 和 $\bar{p}(t)$, 用 x_0, p_0 和 t 的显式表示. Assume that the mean values of x and p in the initial state are respectively x_0 and p_0, find their mean values at time t, denoted by $\bar{x}(t)$ and $\bar{p}(t)$, expressed as explicit functions of x_0, p_0 and t. 试求出粒子 (或波包中心) 不会到达无穷远的条件. Find the condition under which the particle (or center of the wave packet) never arrives at infinity.

三、计算题之二 Analysis and calculation 2 (共 4 小题, 各小题分数依次为 5 分、5 分、12 分、8 分, 共 30 分.)

已知 A、B 为力学量, A and B are dynamical variables, $[A, B] = \mathrm{i}K$.

1. 写出 ΔA、ΔB 与 $\langle K\rangle$ 所满足的不确定关系.不必写出推导过程. Give the uncertainty relation among ΔA, ΔB and $\langle K\rangle$. You are not required to prove it.

2. 当 $A = L_x$, $B = L_y$, 写出相应的不确定关系, 其中 L_x 等是轨道角动量算符的分量. Give the uncertainty relation when $A = L_x$, $B = L_y$, where L_x etc are components of the orbit angular momentum.

3. 在 $\{\boldsymbol{L}^2, L_z\}$ 的共同本征态 $|lm\rangle$ 中, 计算上述不确定关系中出现的各量, 验证不确定关系. 对于给定的 l, $\Delta L_x\Delta L_y$ 何时取得最小值? 此时不确定关系取什么形式? In the simultaneous eigenstate $|lm\rangle$ of $\{\boldsymbol{L}^2, L_z\}$, calculate all quantities appearing in the above uncertainty relation and verify it. For a given l, when does $\Delta L_x\Delta L_y$ attain the smallest value? What is the uncertainty relation in this case?

4. 对于上述 $\Delta L_x\Delta L_y$ 取得最小值的态, 求出其在坐标表象中的波函数. For the above states where $\Delta L_x\Delta L_y$ attains the smallest value, find their wave functions in the coordinate representation.

可能有用的公式 Formulae that may be useful:

$$L_\pm|lm\rangle = \sqrt{(l\mp m)(l\pm m+1)}\hbar|l(m\pm 1)\rangle.$$

$$L_x = \mathrm{i}\hbar\left(\sin\phi\frac{\partial}{\partial\theta} + \cot\theta\cos\phi\frac{\partial}{\partial\phi}\right), \quad L_y = -\mathrm{i}\hbar\left(\cos\phi\frac{\partial}{\partial\theta} - \cot\theta\sin\phi\frac{\partial}{\partial\phi}\right), \quad L_z = -\mathrm{i}\hbar\frac{\partial}{\partial\phi}.$$