A Covariance Absolute Value Detection Algorithm Exploiting Generalized Stochastic Resonance

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***Abstract*—As an essential aspect of cognitive radio (CR), spectrum sensing has always been a research hotspot. In a multi-antenna scenario, the received signals from each antenna exhibit strong spatial correlation. Therefore, the covariance absolute value (CAV) detection algorithm is commonly employed, although its performance at low signal-to-noise ratio (SNR) needs improvement. This paper proposes an improved CAV detection algorithm that leverages generalized stochastic resonance (GSR) in multi-antenna scenarios. The research demonstrates that the performance of the CAV algorithm can be enhanced by introducing an appropriate direct current signal. By maximizing the probability of detection for a fixed probability of false alarm, the optimal amplitude of the additional direct current signal can be determined. Unlike previous work, this paper derives more exact formulas and considers a more general random signal model and a multi-antenna scenario. Theoretical analysis and simulation results confirm that the proposed method outperforms traditional CAV detection methods, particularly under low SNR conditions.**

***Index Terms*—cognitive radio, spectrum sensing, generalized stochastic resonance, covariance matrix**

I. Introduction

With the rapid development of wireless communication technology, the demand for spectrum resources has significantly increased. However, the efficient utilization of spectrum resources remains a challenge, often resulting in underutilization of available spectrum. To address this issue, cognitive radio (CR) has been proposed as a solution [1]. CR technology enables the sensing of surrounding spectrum resources and facilitates their optimal utilization. It allows secondary users (SUs) to opportunistically access idle frequency bands in both time and space, without causing interference to primary users (PUs). Spectrum sensing technology plays a crucial role in CR by determining the presence of PU signals in specific frequency bands. It serves as the fundamental basis for other components within the CR system, making it a vital area of research and development.

At present, there are several common spectrum sensing methods, including matched filtering (MF) detection [2], cyclostationary detection [3], energy detection (ED) [4] and covariance matrix-based detection [5], [6]. MF detection is considered the optimal spectrum sensing method under white Gaussian noise (WGN). However, it requires prior knowledge of the PU signal, such as modulation mode, amplitude, phase, etc. Obtaining these information in practical scenarios can be challenging. Cyclostationary detection takes advantage of the cyclic stationary characteristics of the PU signal to achieve excellent performance, even at low signal-to-noise ratio (SNR). However, similar to MF detection, it also relies on prior knowledge of the PU signal. Additionally, cyclostationary detection has long sensing times and high computational complexity [7]. ED is a common blind spectrum sensing method that is applicable in many scenarios due to its low complexity. However, its performance becomes limited at low SNR and it is susceptible to the effects of noise uncertainty. Covariance matrix-based detection utilizes the sample covariance matrix of the received signal to make a detection. It performs well when the PU signal exhibits a strong autocorrelation property. Moreover, it can be applied in multi-antenna scenarios where the received signals of each antenna have strong spatial correlation. However, its performance deteriorates at low SNR.

In 1981, the concept of stochastic resonance (SR) was first introduced to address the issue of periodic climate variations in ancient times [8]. It indicates that appropriate noise can enhance the signal in some nonlinear systems. To improve the performance of spectrum sensing algorithms under low SNR, some researchers attempted to apply SR to spectrum sensing [9], [10] and achieved significant improvements. Unfortunately, these approaches require a high sampling rate, resulting in high hardware implementation costs. Subsequently, some scholars proposed the concept of generalized stochastic resonance (GSR) [11], which expands upon the original concept of SR. They demonstrated the benefits of noise in certain nonlinear detectors. The relevant research on SR noise-enhanced spectrum sensing algorithm has been published [12], effectively improving the performance of the ED algorithm. Besides, SR enhanced covariance matrix detection has also been studied [13], but the considered case is extreme and has limited applicability.

In this paper, we propose a covariance absolute value (CAV) detection algorithm exploiting GSR for multi-antenna spectrum sensing scenarios. Unlike other studies, our model assumes the PU signal to be a random signal rather than a direct current signal. In addition, we provide a detailed theoretical analysis. We apply the CAV algorithm based on GSR to the multi-antenna spectrum sensing model and achieve improved performance by adding a suitable direct current signal. In the meantime, the computational complexity does not increase significantly. Both theoretical and simulation

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analysis confirm the superiority of our proposed method.

The rest of this paper is organized as follows: Section II introduces the system model, including the spectrum sensing model and the CAV detection algorithm. In Section III, GSR based CAV detection algorithm is proposed and analyzed theoretically, then simulation results and discussions are presented in Section IV. Finally, we draw a conclusion in Section V.

II. System Model

A. Spectrum Sensing Model

In this study, a universal multi-antenna spectrum sensing scenario is considered. We assume that there are L antenna and N sampling of the PU signal. In this case, the spectrum sensing problem can be formulated as a binary hypothesis testing problem:

$$\begin{aligned} &H_0 : x_i(n) = \eta_i(n), \\ &H_1 : x_i(n) = s(n) + \eta_i(n), \end{aligned} \tag{1}$$

where hypotheses H_0 and H_1 represent the absence and presence of the PU signal, respectively. Thus, $s(n)$ and $\eta_i(n)$ denote the sampling of the signal transmitted by PU and WGN with zero-mean and σ_n^2-variance, respectively. Note that the PU signal $s(n)$ and the noise $\eta_i(n)$ are independent of each other. Term $i = 1, 2, \ldots, L$ and $n = 0, 1, \ldots, N-1$ denote the antenna index and sampling index, respectively.

For convenience, the received signal sample matrix can be defined as

$$\mathbf{X} = [\mathbf{x}(0), \mathbf{x}(1), \cdots, \mathbf{x}(N-1)], \tag{2}$$

where $\mathbf{x}(n) = [x_1(n), x_2(n), \cdots, x_L(n)]^T (n = 0, 1, \ldots, N-1)$ represents the observation vector of signal received at n-th sampling time. Then we can obtain the statistical covariance matrix $\mathbf{R}_x = \mathbb{E}[\mathbf{x}(n)\mathbf{x}^T(n)]$, where symbol $\mathbb{E}[\cdot]$ represents the statistical expectation. Correspondingly, $\mathbf{R}_n = \sigma_n^2 \mathbf{I}_L$ is the statistical covariance matrix of noise vector, where $\mathbf{I}_L$ represents the identity matrix with order L. Therefore, the binary hypothesis testing problem is rewritten as

$$\begin{aligned} &H_0 : \mathbf{R}_x = \sigma_n^2 \mathbf{I}_L, \\ &H_1 : \mathbf{R}_x = \mathbf{R}_s + \sigma_n^2 \mathbf{I}_L, \end{aligned} \tag{3}$$

where $\mathbf{R}_s$ denotes the statistical covariance matrix of the PU signal. In this study, $s(n)$ is assumed as a Gaussian random signal with μ-mean and σ_s^2-variance. Then we have $\mathbf{R}_s = (\sigma_s^2 + \mu^2)\mathbf{1}$, where $\mathbf{1}$ is a $L \times L$ matrix whose elements are all ones.

In practice, however, the number of available samples is finite and we can replace the statistical covariance matrix with the sample covariance matrix which is defined as

$$\begin{aligned} \mathbf{R}_x(N) &= \frac{1}{N}\sum_{n=0}^{N-1} \mathbf{x}(n)\mathbf{x}^T(n) \\ &= \frac{1}{N}\mathbf{X}\mathbf{X}^T. \end{aligned} \tag{4}$$

Hence, $\mathbf{R}_x$ can be approximated by $\mathbf{R}_x(N)$ as N is large enough.

B. CAV Detection

In CAV detection algorithm, let

$$T_1 = \sum_{i=1}^{L}\sum_{j=1}^{L} |r_{ij}|, \tag{5}$$

$$T_2 = \sum_{i=1}^{L} |r_{ii}|, \tag{6}$$

where r_{ij} denotes the $(i,j)^{th}$ element of $\mathbf{R}_x$, which is derived as

$$r_{ij} = \frac{1}{N}\sum_{n=0}^{N-1} x_i(n)x_j(n). \tag{7}$$

Then the test statistic is defined as

$$T = \frac{T_1}{T_2}. \tag{8}$$

According to (3), if the PU signal does not exist, the off-diagonal elements are all zero and $T = 1$. If the PU signal is present, the off-diagonal elements are non-zero and $T > 1$. The probability of false alarm is derived as [5]

$$\begin{aligned} P_{fa} &= P(T > \gamma_{th} \mid H_0) \\ &= P(T_1 > \gamma_{th} T_2 \mid H_0) \\ &\approx P(T_2 < \frac{1}{\gamma_{th}}\mathbb{E}[T_1] \mid H_0) \\ &= 1 - Q(\frac{\frac{1}{\gamma_{th}}(1 + (L-1)\sqrt{\frac{2}{\pi N}}) - 1}{\sqrt{\frac{2}{N}}}), \end{aligned} \tag{9}$$

where $Q(\cdot)$ is Q-function, which is defined as

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{+\infty} e^{-u^2/2} du. \tag{10}$$

In the above derivation, T_1 is replaced by $\mathbb{E}[T_1]$, which may result in the inaccuracy of the probability of false alarm P_{fa}. And if P_{fa} is given, the threshold can be obtained as

$$\gamma_{th} = \frac{1 + (L-1)\sqrt{\frac{2}{\pi N}}}{1 - Q^{-1}(P_{fa})\sqrt{\frac{2}{N}}}. \tag{11}$$

III. Proposed Algorithm and Performance Analysis

In this section, GSR based CAV algorithm is proposed. Then accurate formulas for the probability of false alarm and detection are derived. Finally, the optimal additional direct signal is obtained by numerical simulation according to the formulation of an optimization problem.

A. GSR-based CAV Algorithm

According to the theory of GSR, it is demonstrated that suitable additional noise can improve the performance of some nonlinear detectors, such as the ED detector and the CAV detector. In [11] the probability density function (PDF) of the best GSR noise is derived as

$$p_n^{opt}(k) = \lambda\delta(k - k_1) + (1-\lambda)\delta(k - k_2), \tag{12}$$

where $0 < \lambda < 1$ and $k_1 \neq k_2$. It represents that k_1 and k_2 are selected with probability λ and $1-\lambda$, respectively.

However, it is difficult to obtain the optimal noise of (12), which has three parameters λ, k_1 and k_2 to be optimized. To simplify the analysis, suboptimal GSR noise is selected [12], which is expressed by

$$p_n(k) = \delta(k-d). \tag{13}$$

Note thas d is actually a direct current signal and we denote it by $\mathbf{u}(n) = d$. Moreover, based on the above discussion, GSR based CAV (GSR-CAV) detector is proposed, which is shown in Fig. 1. Firstly, add optimal direct current signal $\mathbf{u}(n)$ to

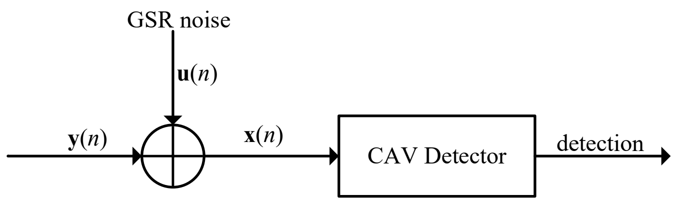

Fig. 1: GSR based CAV detector

received signal $\mathbf{y}(n)$ to obtain $\mathbf{x}(n)$. Then calculate the test statistic according to (4) – (8). Finally, make a decision by comparing it with the threshold of the GSR-CAV detector. The optimal d and threshold will be derived later.

B. Performance Analysis and Optimal Direct Current Signal

Recall that in (11) the threshold γ_{th} is approximated because of the replacement in (9). Motivated by [14], we use the similar method to get a more accurate threshold, which will be detailed in the following.

The test statistic defined in (8) can be rewritten as

$$T = 1 + 2T_0, \tag{14}$$

where

$$T_0 = \sum_{1\le i<j\le L} \left(\frac{|r_{ij}|}{\sum_{i=1}^{L} r_{ii}}\right). \tag{15}$$

When the PU signal exists, from (7) we have

$$\mathbb{E}[r_{ij}; H_1] = (\mu+d)^2 + \sigma_s^2, \tag{16}$$

$$\operatorname{var}(r_{ij}; H_1) = \frac{1}{N}[(\sigma_n^2+\sigma_s^2)^2 + \sigma_s^4 + 2(\mu+d)^2(2\sigma_s^2+\sigma_n^2)], \tag{17}$$

$$\mathbb{E}[r_{ii}; H_1] = (\mu+d)^2 + \sigma_s^2 + \sigma_n^2, \tag{18}$$

$$\operatorname{var}(r_{ii}; H_1) = \frac{2}{N}(\sigma_n^2+\sigma_s^2)[\sigma_n^2+\sigma_s^2+2(\mu+d)^2]. \tag{19}$$

Let N be large, then r_{ii} can be replaced with $\mathbb{E}[r_{ii}; H_1]$ approximately. For convenience, we denote $\mathbb{E}[r_{ij}; H_1]$ by Θ_1 and $\operatorname{var}(r_{ij}; H_1)$ by Δ_1. The numerator $|r_{ij}|$ in (15) follows half-normal distribution, whose mean and variance are derived as

$$\mathbb{E}[|r_{ij}|; H_1] = \sqrt{\frac{2\Delta_1}{\pi}} e^{-\frac{\Theta_1^2}{2\Delta_1}} + \Theta_1[2Q(-\frac{\Theta_1}{\sqrt{\Delta_1}}) - 1], \tag{20}$$

$$\operatorname{var}(|r_{ij}|; H_1) = \mathbb{E}[|r_{ij}|^2; H_1] - \mathbb{E}^2[|r_{ij}|; H_1], \tag{21}$$

where

$$\mathbb{E}[|r_{ij}|^2; H_1] = [(\mu+d)^2+\sigma_s^2]^2 + \frac{(\sigma_s^2+\sigma_n^2)^2 + \sigma_s^4 + 2(\mu+d)^2(2\sigma_s^2+\sigma_n^2)}{N}. \tag{22}$$

Consequently, being sum of such half-normal random variables, T_0 is approximated by the tail of Gaussian distribution $\mathcal{N}(\mu_1, \sigma_1^2)$, where

$$\mu_1 = \frac{(L-1)\mathbb{E}[|r_{ij}|; H_1]}{2\mathbb{E}[r_{ii}; H_1]}, \tag{23}$$

$$\sigma_1^2 = \frac{(L-1)\operatorname{var}(|r_{ij}|; H_1)}{2L\mathbb{E}^2[r_{ii}; H_1]}. \tag{24}$$

When the PU signal does not exist, we have

$$\mathbb{E}[r_{ij}; H_0] = d^2, \tag{25}$$

$$\operatorname{var}(r_{ij}; H_0) = \frac{1}{N}(\sigma_n^4 + 2d^2\sigma_n^2), \tag{26}$$

$$\mathbb{E}[r_{ii}; H_0] = d^2 + \sigma_n^2, \tag{27}$$

$$\operatorname{var}(r_{ii}; H_0) = \frac{2}{N}\sigma_n^2(\sigma_n^2 + 2d^2). \tag{28}$$

Denote $\mathbb{E}[r_{ij}; H_0]$ and $\operatorname{var}(r_{ij}; H_0)$ by Θ_0 and Δ_0, respectively. Similar to (20), we have

$$\mathbb{E}[|r_{ij}|; H_0] = \sqrt{\frac{2\Delta_0}{\pi}} e^{-\frac{\Theta_0^2}{2\Delta_0}} + \Theta_0[2Q(-\frac{\Theta_0}{\sqrt{\Delta_0}}) - 1], \tag{29}$$

$$\operatorname{var}(|r_{ij}|; H_0) = \mathbb{E}[|r_{ij}|^2; H_0] - \mathbb{E}^2[|r_{ij}|; H_0], \tag{30}$$

where

$$\mathbb{E}[|r_{ij}|^2; H_0] = d^4 + \frac{1}{N}(\sigma_n^4 + 2d^2\sigma_n^2). \tag{31}$$

Finally, the mean and variance of the Gaussian distribution $\mathcal{N}(\mu_0, \sigma_0^2)$ can be obtained as

$$\mu_0 = \frac{(L-1)\mathbb{E}[|r_{ij}|; H_0]}{2\mathbb{E}[r_{ii}; H_0]}, \tag{32}$$

$$\sigma_0^2 = \frac{(L-1)\operatorname{var}(|r_{ij}|; H_0)}{2L\mathbb{E}^2[r_{ii}; H_0]}. \tag{33}$$

Note that the probability of false alarm and probability of detection are as follows:

$$\begin{aligned} P_{fa} &= P(T > \gamma_{th} \mid H_0) \\ &= P(T_0 > \gamma_0 \mid H_0) \\ &= Q(\frac{\gamma_0 - \mu_0}{\sigma_0}), \end{aligned} \tag{34}$$

$$\begin{aligned} P_d &= P(T > \gamma_{th} \mid H_1) \\ &= P(T_0 > \gamma_0 \mid H_1) \\ &= Q(\frac{\gamma_0 - \mu_1}{\sigma_1}), \end{aligned} \tag{35}$$

where $\gamma_{th} = 1 + 2\gamma_0$. As a result, for a given P_{fa}, from (34) the threshold γ_0 is derived as

$$\gamma_0 = \mu_0 + Q^{-1}(P_{fa})\sigma_0. \tag{36}$$

Furthermore, the optimal d can be obtained from an optimization problem

$$d_{opt} = \arg\max_{d} P_d = \arg\max_{d} Q(\frac{\mu_0 - \mu_1 + Q^{-1}(P_{fa})\sigma_0}{\sigma_1}). \quad (37)$$

However, it is hard to find the optimal value because of $Q(\cdot)$ and the exponential term. In this study, we find the optimal d in the way of numerical calculation simulation, which will be detailed in section IV. Obviously, it needs some prior knowledge, i.e. $\mu, \sigma_s^2, \sigma_n^2$. Finally, we can get the threshold γ_0 by (36).

C. Complexity Analysis

The comparison of computational complexity of CAV and GSR-CAV algorithm is shown in Table I. Here we use formula (14) to obtain the test statistic T for both algorithms. In CAV algorithm, there are NL^2 multiplications and $(N-1)L^2$ additions when doing matrix manipulation to obtain the sample covariance matrix, 2 multiplication and $(L^2 + L - 2)/2$ additions when calculating the test statistic. Compared with the traditional CAV, the proposed GSR-CAV algorithm has one more step adding direct current signal. Therefore, GSR-CAV has more NL additions.

TABLE I: Comparison of Computational Complexity

Algorithm	Multiplication	Addition
CAV	$NL^2 + 2$	$NL^2 + (L^2 - L - 2)/2$
GSR-CAV	$NL^2 + 2$	$N(L^2 + L) + (L^2 - L - 2)/2$

IV. Simulation Results

In this section, necessary simulation results and analysis are displayed to verify the effectiveness of the proposed GSR-CAV detection algorithm. Unless stated especially, the simulation parameters are uniformly set as $\mu = 0.5, \sigma_s^2 = 1, N = 500, L = 8, P_{fa} = 0.1$. The results are obtained by averaging 10000 Monte Carlo simulations.

A. Optimal Direct Current Signal

As mentioned in section III-B, numerical calculation simulation is used to find the optimal d in this section. Firstly, set the interval and step of d for optimization problem (37). Then calculate P_d for each d. Finally, search for the optimal d corresponding to the maximum P_d and save it. Fig. 2 shows the numerical calculation curve of P_d as a function of d when SNR = -30 dB, -20 dB, and -10 dB, respectively. It can be seen that there is an optimal d under every SNR such that P_d takes the maximum value, which demonstrates that adding a suitable direct current signal can improve the detection probability of the CAV algorithm. Note that there may be multiple optimal d, and we select the smallest non-negative value. Furthermore, we can get the optimal d with respect to each SNR by the numerical simulations.

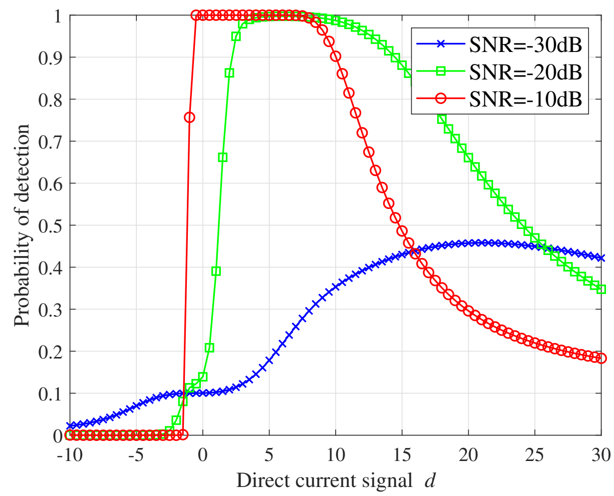

Fig. 2: The relationship between probability of detection and direct current signal d

B. PDF of test statistic

Fig. 3 shows the PDF of the test statistic T for SNR =-18 dB without and with exploiting GSR, respectively. It shows that it is hard to distinguish the two hypotheses H_0 and H_1 for the CAV algorithm. It is obvious that the distance of PDF between H_0 and H_1 in the GSR-CAV algorithm is greater than it is in the CAV algorithm, which makes it easier to distinguish the two hypotheses. It indicates that a suitable direct signal can improve the performance of the CAV algorithm in our method.

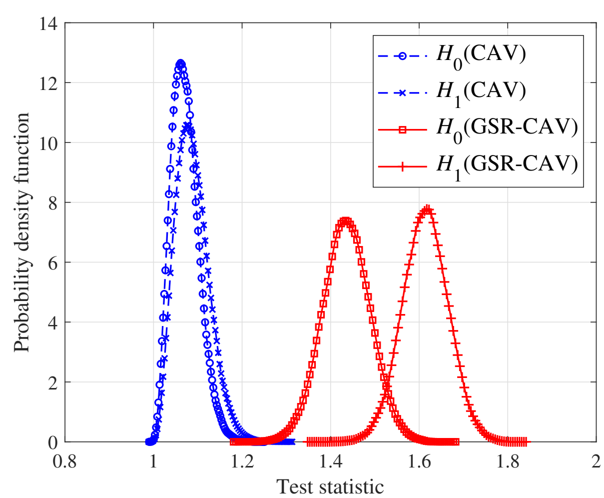

Fig. 3: The probability density function of test statistic

C. Detection Performance

Fig. 4 shows the comparison of the detection probability of GSR-CAV and the conventional algorithms ED, CAV and

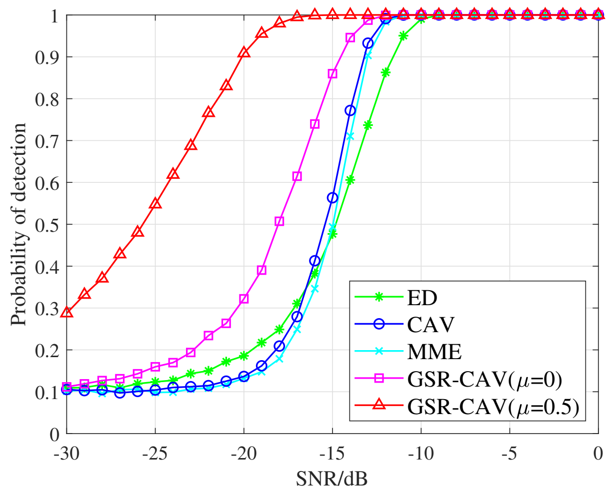

Fig. 4: Probability of detection with different schemes under different SNR.

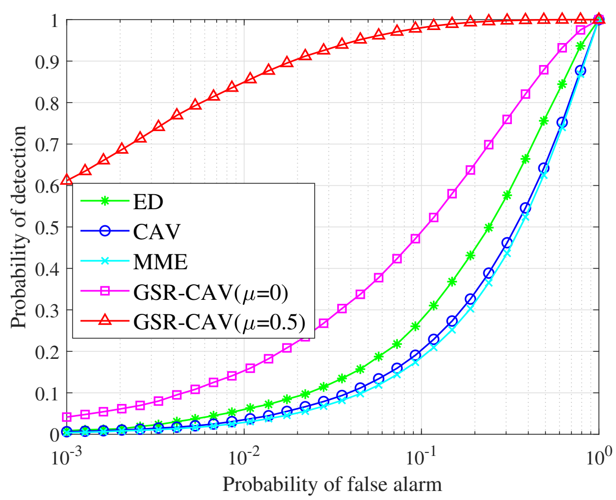

Fig. 5: ROC curve with different schemes at SNR = -18 dB.

maximum-minimum eigenvalue (MME) detection for $P_{fa} = 0.1$. It can be found that the proposed GSR-CAV algorithm has better performance than conventional CAV algorithm, which indicates that a suitable additional direct current signal could improve the performance of the CAV algorithm in practice. Besides, the proposed GSR-CAV algorithm is sensitive to mean μ, while the conventional algorithms is not. The larger the mean μ, the more significant the improvements. No matter what the mean value is, the proposed GSR-CAV always has the best detection performance compared with other classical algorithms.

The Receiver Operating Characteristics (ROC) curves of different schemes for fixed SNR = -18 dB are displayed in Fig. 5, which presents the detection probability values versus false alarm probability values. The larger the Area Under Curve (AUC), the better the performance. It is clear that the AUC of the GSR-CAV algorithm is larger, so it has better performance regardless of the mean μ. Furthermore, as aforementioned, the larger the parameter μ, the better the performance of the GSR-CAV algorithm.

V. Conclusion

In this paper, we propose a CAV spectrum sensing algorithm exploiting GSR in multi-antenna scenarios. It is shown that a suitable direct current signal can effectively increase the distance of test statistics under two hypotheses, resulting in the performance improvements of the CAV algorithm. By maximizing the probability of detection for a fixed probability of false alarm, we can obtain the optimal direct current signal through numerical calculation, which requires some prior knowledge of the PU signal. Theoretical analysis and simulation results indicate that our proposed method outperforms the traditional CAV algorithm and other conventional algorithms while maintaining reasonable computational complexity. Additionally, our proposed GSR-CAV algorithm's performance is affected by the mean of the PU signal, making it particularly applicable to non-zero mean signals such as pulse amplitude modulation signals.

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