

(1) For any two atomic sentences P and Q, is the following sentence true?

Why?

$$(P \Rightarrow Q) \Leftrightarrow (\neg P \vee Q)$$

Yes, the above sentence is true. For the sentence $P \Rightarrow Q$, if P is true, then $P \Rightarrow Q$ is true if Q is true, and $P \Rightarrow Q$ is false if Q is false. Since now P is true, $\neg P \vee Q$ is true if Q is true, and $\neg P \vee Q$ is false if Q is false. On the other hand, for the sentence $P \Rightarrow Q$, if P is false, then $P \Rightarrow Q$ is true always. Since now P is false, $\neg P \vee Q$ is also true always.

(2) 用一阶谓词逻辑解答如下问题。已知：张三在哪里，李四也总跟到哪里。现在知道张三在公园，问李四在哪里？

用一阶谓词逻辑表达，上述陈述可以写为：

$$\forall x (At(张三, x) \Rightarrow At(李四, x)) = \text{True}$$

$$At(张三, 公园) = \text{True}$$

问：At(李四, ?)

答：

因为 $(P \Rightarrow Q) \Leftrightarrow (\neg P \vee Q)$ ，所以 $\forall x (At(张三, x) \Rightarrow At(李四, x)) = \text{True}$

等价于 $\forall x (\neg At(张三, x) \vee At(李四, x)) = \text{True}$ 。

令 $x = \text{公园}$ ，则 $\neg At(张三, 公园) \vee At(李四, 公园) = \text{True}$ 。

又因为 $At(张三, 公园) = \text{True}$ ，所以 $At(李四, 公园) = \text{True}$ 。