Given the following three-dimensional points and their actual labels:

$$\mathbf{x}_A = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \quad , \quad y_A = -1$$

默认是列向量

$$\mathbf{x}_B = \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix} \quad , \quad y_B = +1$$

$$\mathbf{x}_C = \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix} \quad , \quad y_C = +1$$

If we initial the vector of weights for each dimension (including w_0) as

$\tilde{\mathbf{w}} = \begin{pmatrix} -3 \\ 2 \\ 2 \\ 0 \end{pmatrix}$. What's the vector of weights using PLA until convergence?

Answer:

Based on the initial weights, we can predict the label of each point as

$$\begin{aligned} \hat{y}_A &= sign\left(\tilde{\mathbf{w}}^{\mathrm{T}}\tilde{\mathbf{x}}_A\right) \\ &= sign\left(\begin{pmatrix} -3 \\ 2 \\ 2 \\ 0 \end{pmatrix}^{\mathrm{T}} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}\right) \\ &= sign\left(-3\times1+2\times1+2\times1+0\times0\right) \\ &= +1 \end{aligned}$$

, where $\hat{y}_A$ means the predicted label of A.

Since the actual label of A is $y_A = -1$, so the label of A is predicted incorrectly by the initial weights, and PLA uses the input vector of A and its actual label to adjust the vector of weights, as follows:

$$\tilde{\mathbf{w}}_{new} = \tilde{\mathbf{w}}_{old} + y_A \tilde{\mathbf{x}}_A$$

$$= \begin{pmatrix} -3 \\ 2 \\ 2 \\ 0 \end{pmatrix} + (-1) \times \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}.$$

$$= \begin{pmatrix} -4 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

Based on the new weights, we can predict the label of each point as

$$\hat{y}_A = sign(-4 \times 1 + 1 \times 1 + 1 \times 1 + 0 \times 0) = -1,$$
$$\hat{y}_B = sign(-4 \times 1 + 1 \times 3 + 1 \times 3 + 0 \times 1) = +1,$$
$$\hat{y}_C = sign(-4 \times 1 + 1 \times 4 + 1 \times 3 - 0 \times 1) = +1.$$

The vector of weights stops updating since the labels of all points are predicted correctly.