Artificial Intelligence ——kNN and NB

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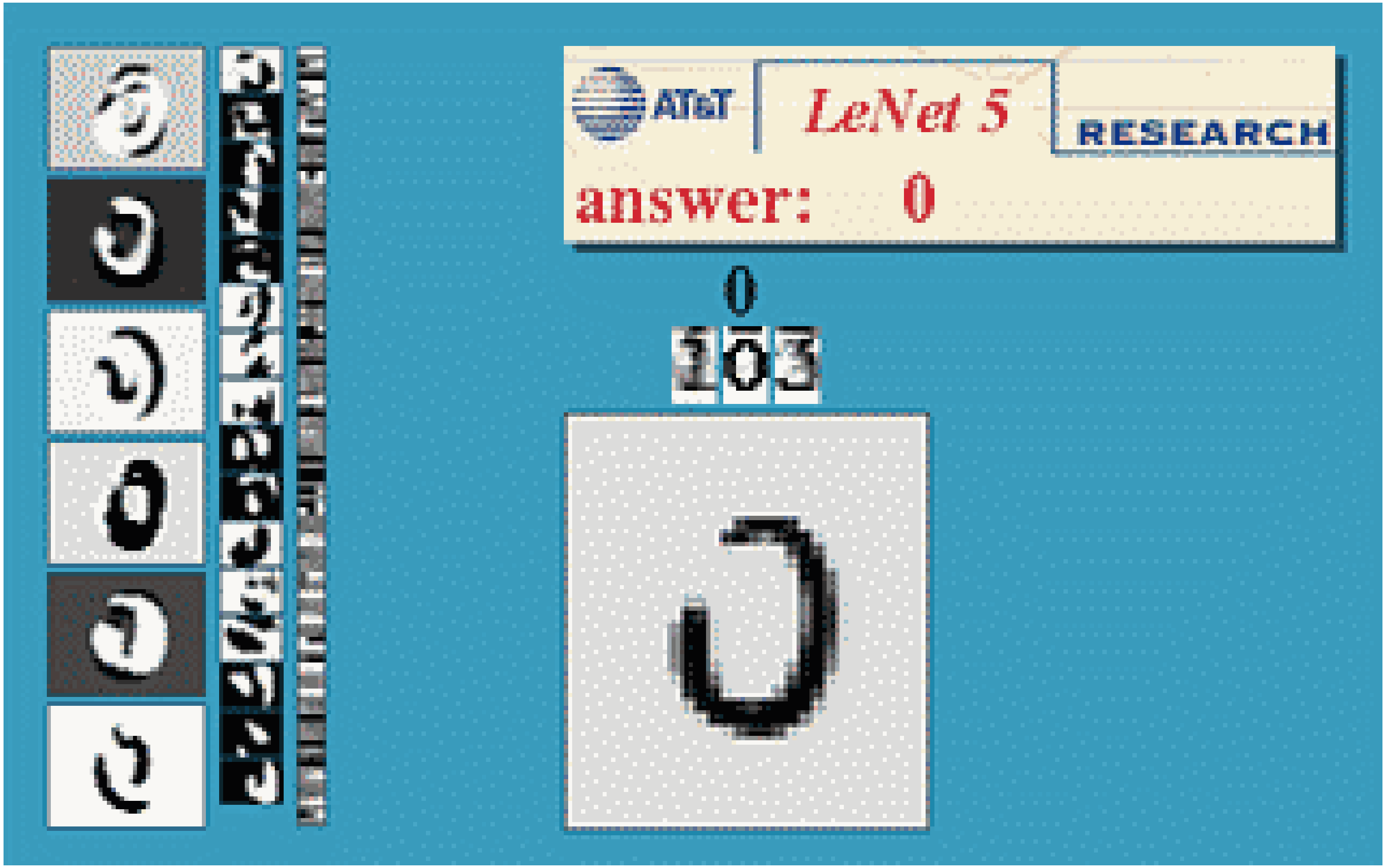
Regression vs Classification

DocumentID	Words (split by space)	joy
train1	sheva delight us	0.6
train2	goal delight for sheva	0.7
test1	sheva goal	?

DocumentID	Words (split by space)	emotion
train1	sheva know us	not joy
train2	goal delight for sheva	joy
test1	sheva goal	?

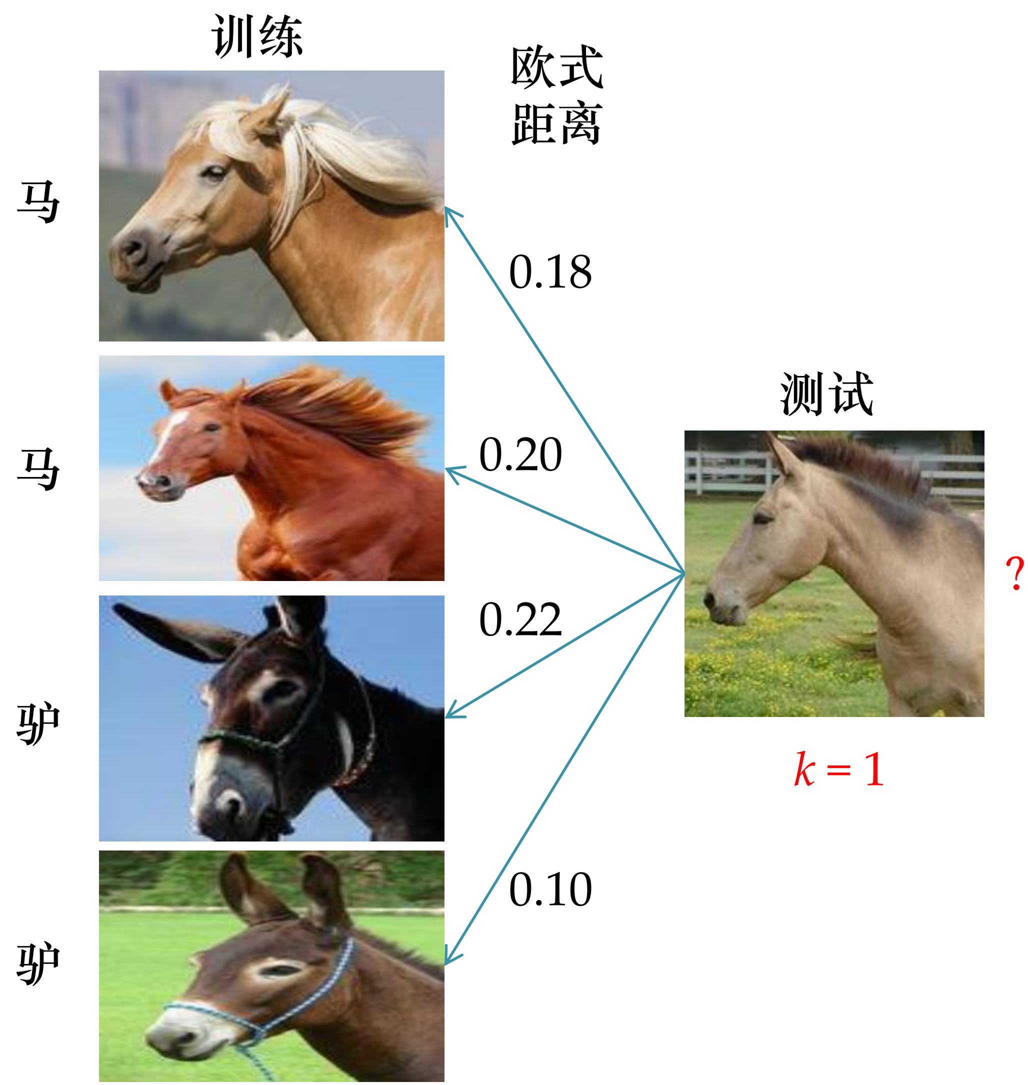
Regression vs Classification



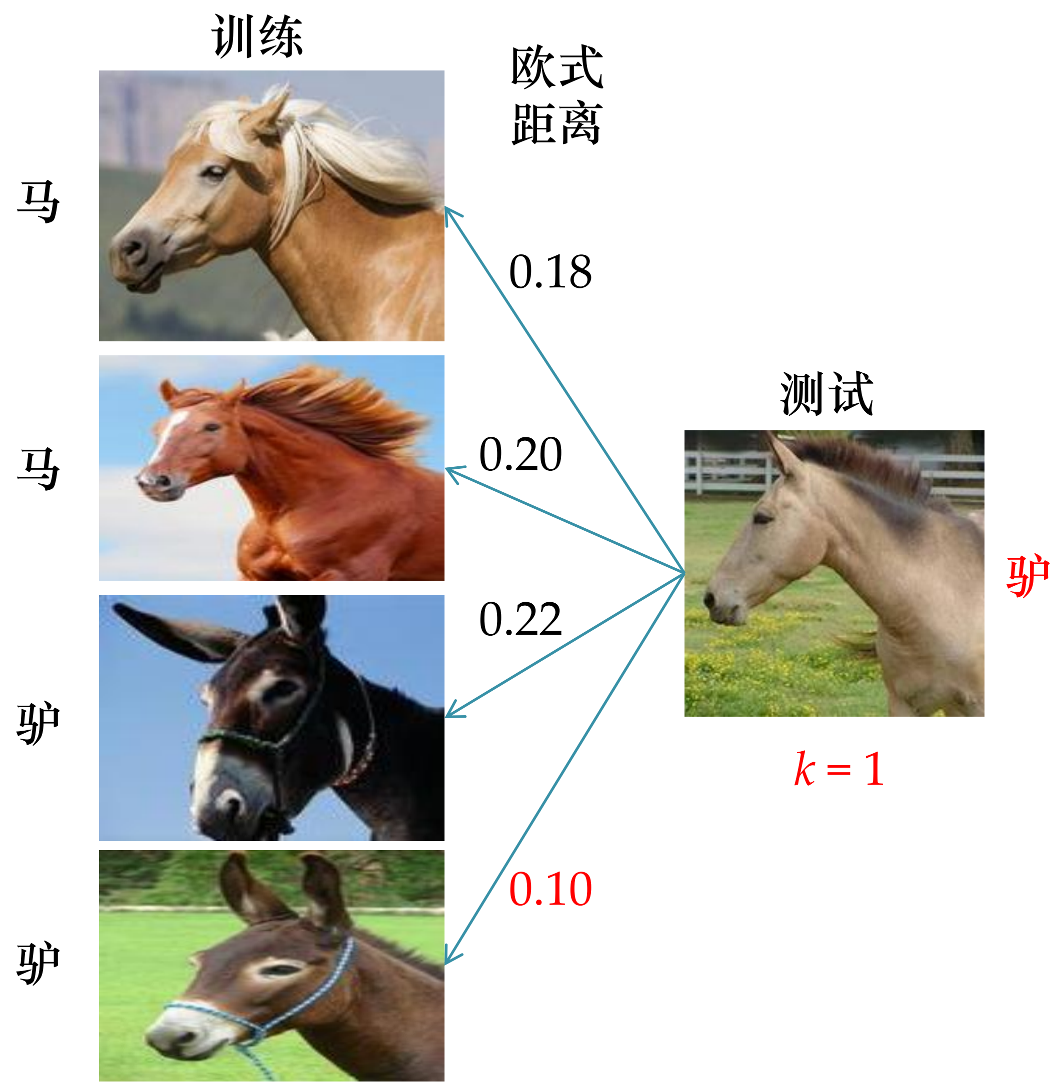
k-Nearest Neighbor

- All data/objects correspond to vectors in the n-D space (n维空间的向量)
- The nearest neighbor could be defined in terms of Euclidean distance, etc.
- Target (目标) vector could be discrete- or real- valued
- For discrete-valued, *k*-NN returns the **most common** value (众数) among the *k* training examples nearest to X(测试)

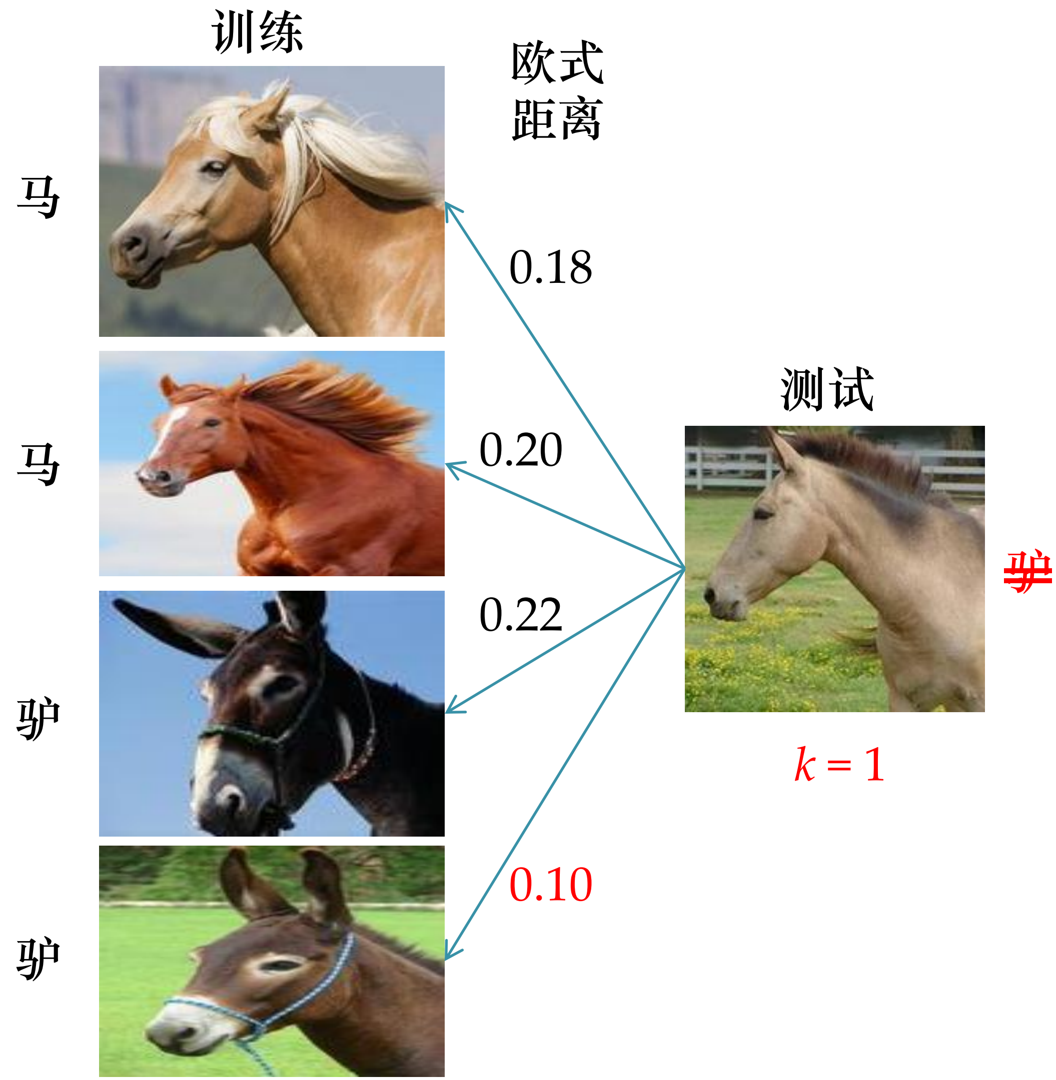
k-Nearest Neighbor



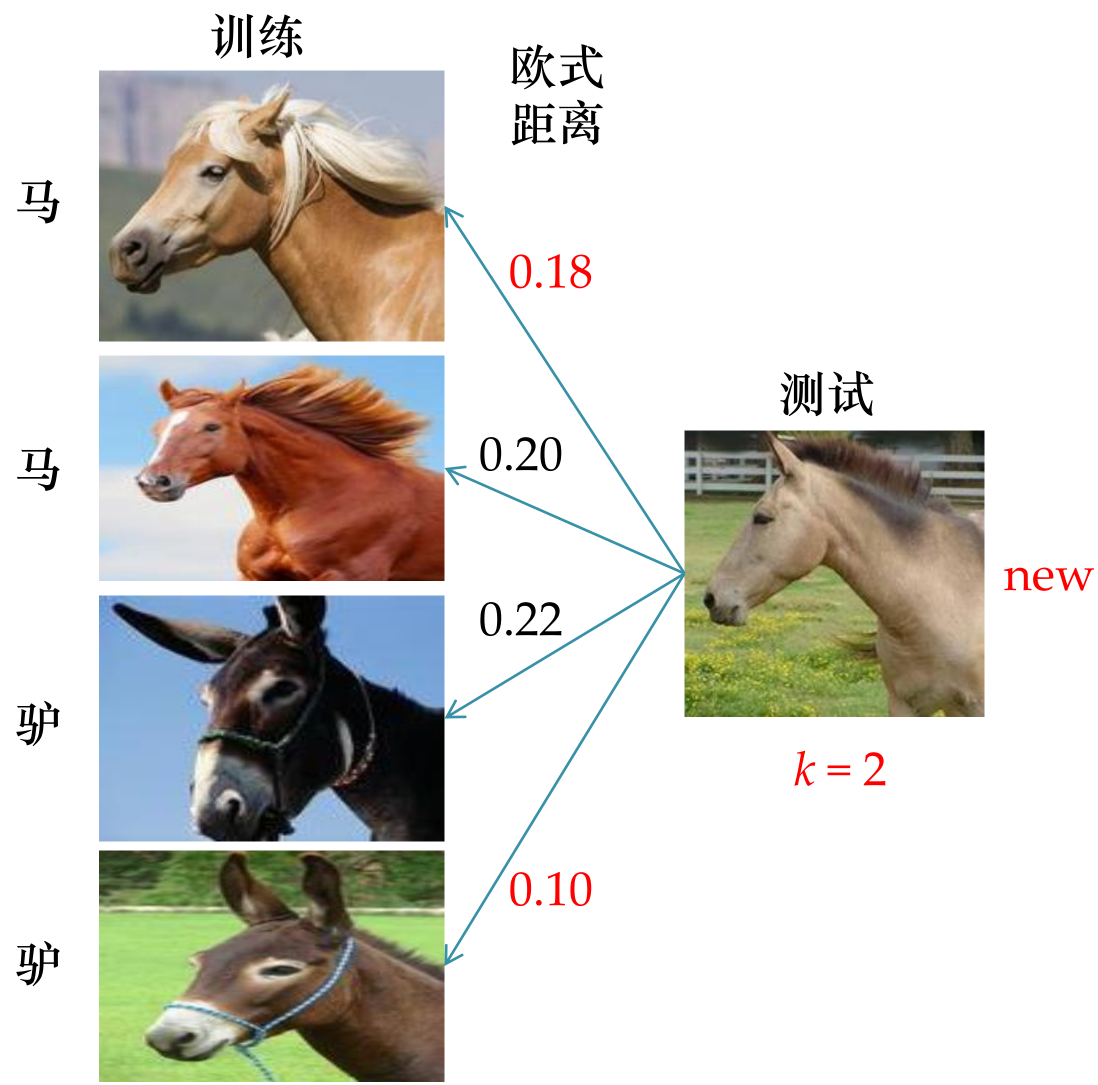
k-Nearest Neighbor



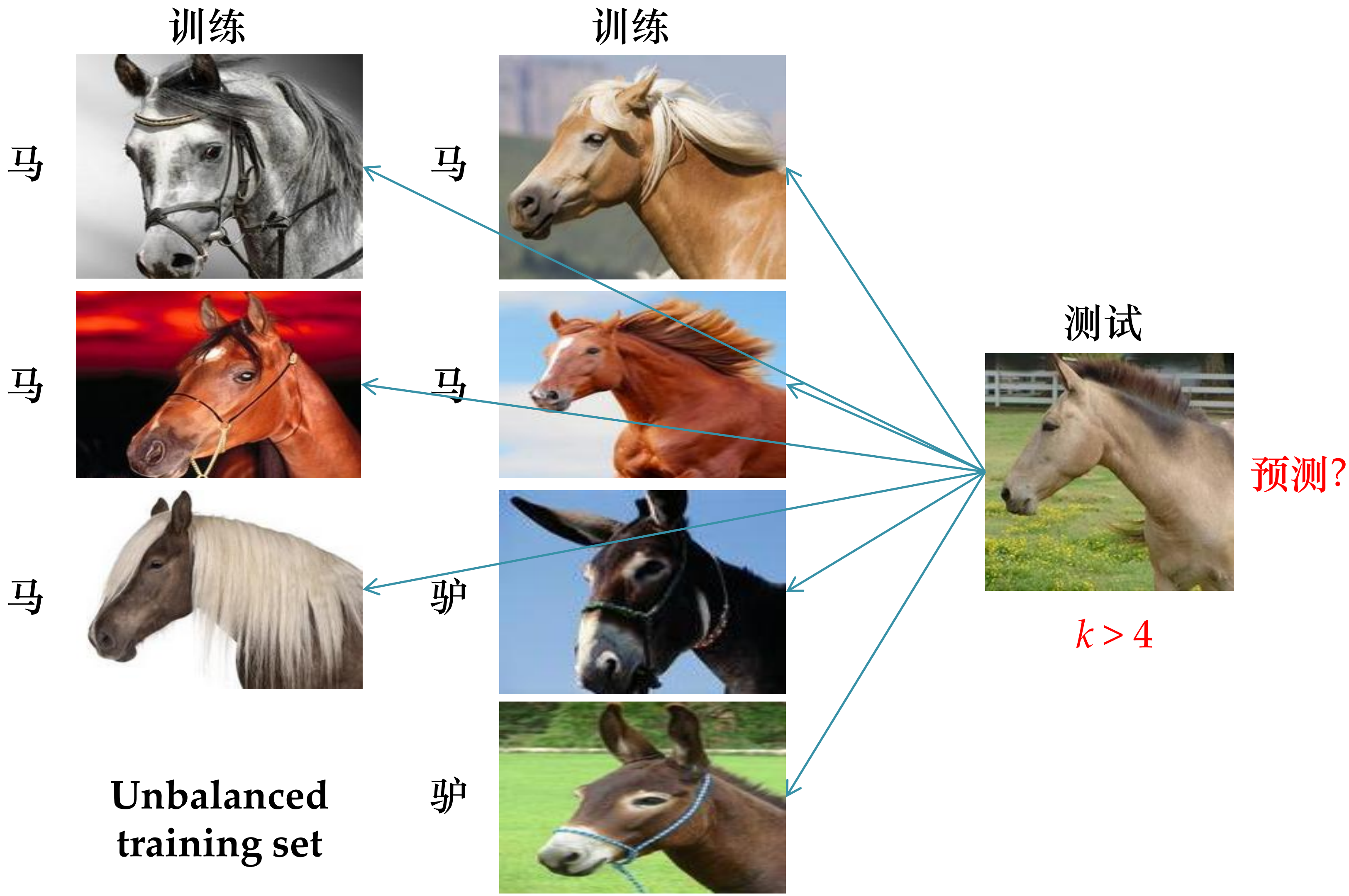
k-Nearest Neighbor



k-Nearest Neighbor



k-Nearest Neighbor

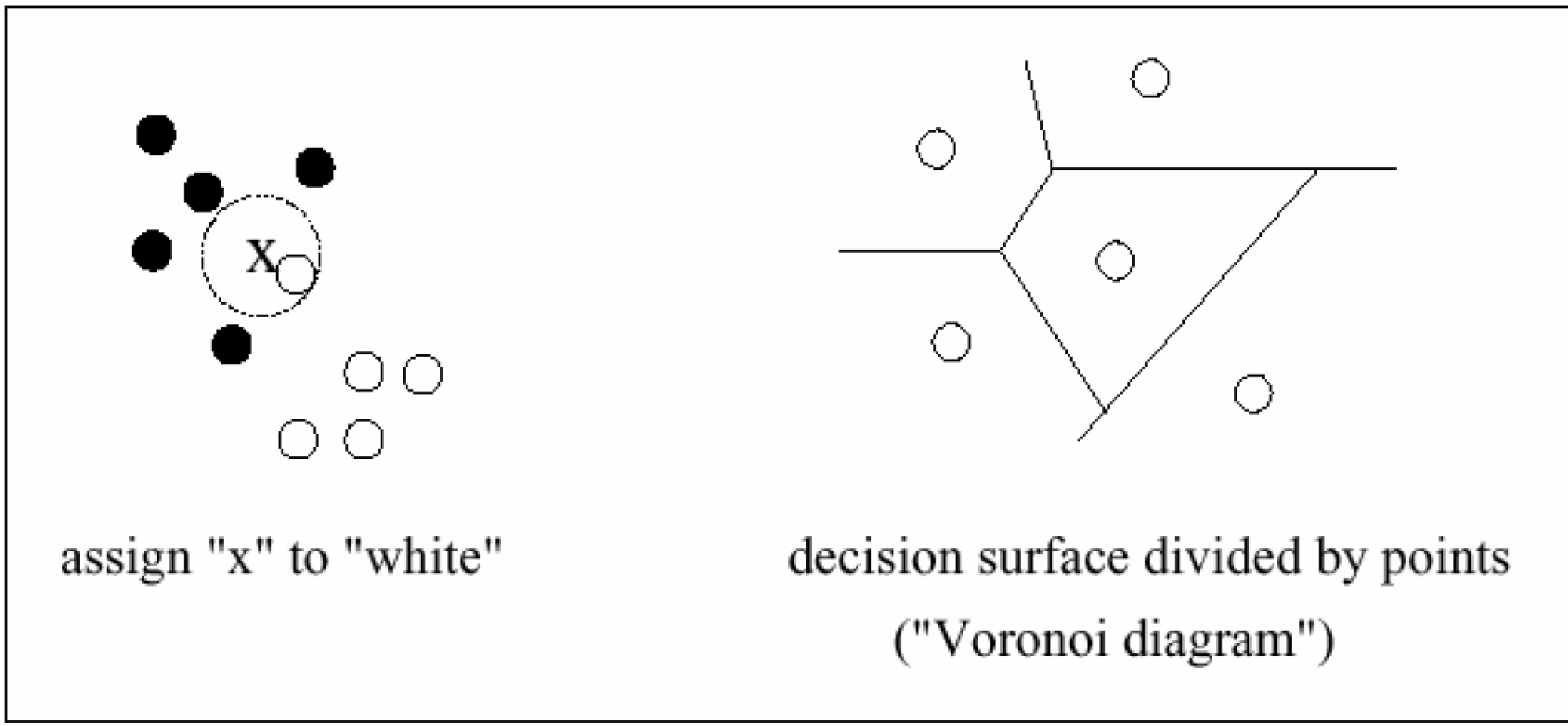


k-Nearest Neighbor

- *k*-NN for real-valued prediction for a given unknown *X* (测试)
  - Returns the mean / median gold values of the *k* nearest neighbors
- Instance-based learning/Lazy-learning
  - initially by Fix and Hodges (1951)
  - theoretical error bound analysis by Duda & Hart (1957)
  - store all the training samples
  - high computational cost for each new object if using the original *k*-NN algorithm

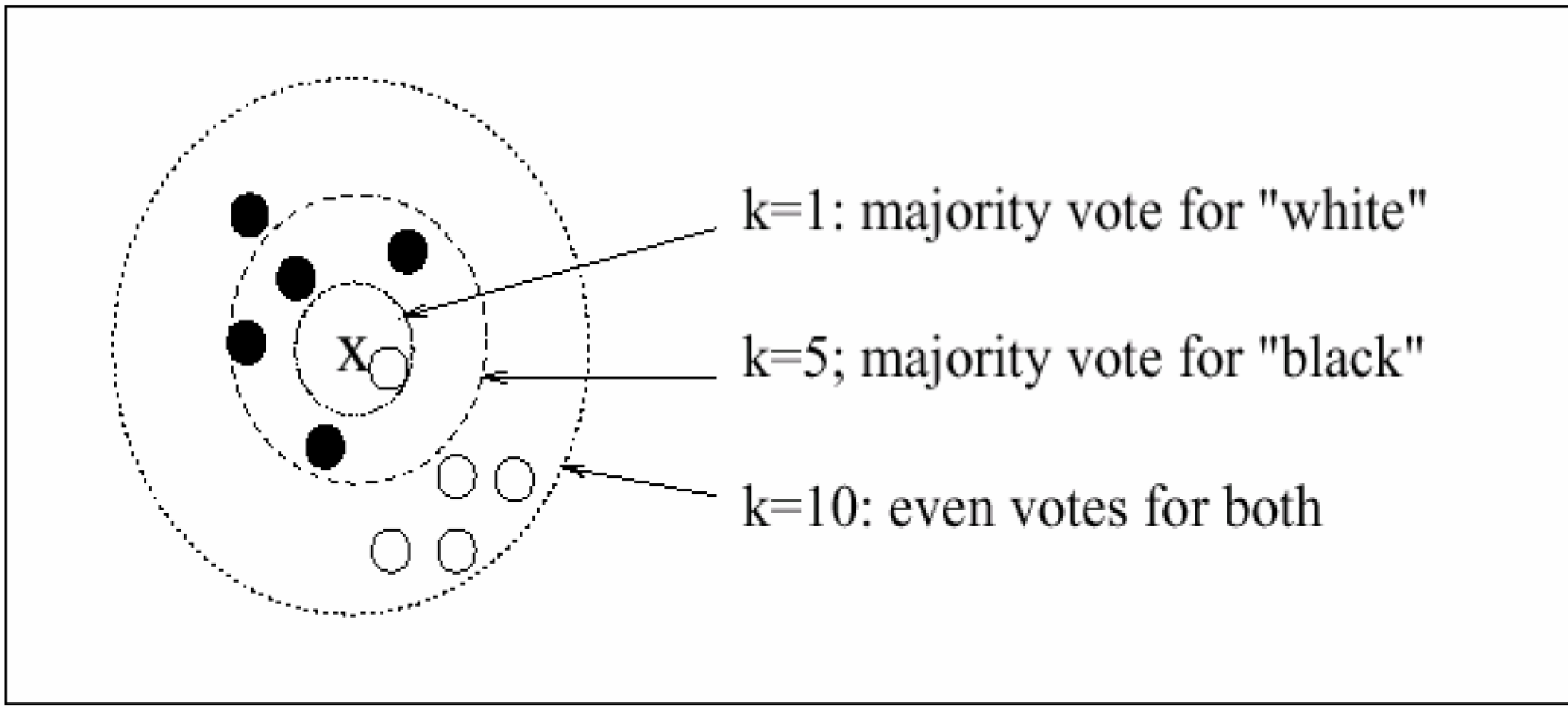
k-Nearest Neighbor

1-NN: assign "x" (new point) to the class of it nearest neighbor



k-Nearest Neighbor

k-NN using a majority voting scheme



k-Nearest Neighbor

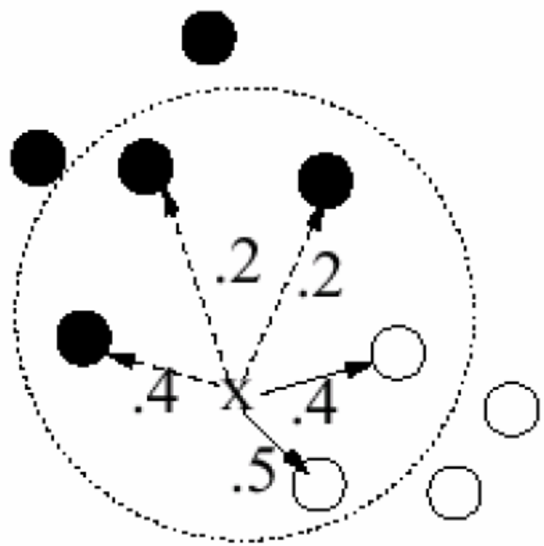
- Key aspects (影响因素) of *k*-NN

k-Nearest Neighbor

- Key aspects (影响因素) of *k*-NN
  - Similar item: We need a functional definition of similarity if we want to apply this automatically.
  - How many neighbors? (the value of *k*)
  - Does each neighbor get the same weight?
  - Count all classes for all neighbors? Or, use the frequently-occurred classes to make decisions?

k-Nearest Neighbor

k-NN using a weighted-sum voting scheme



kNN (k = 5)

Assign "white" to x because the weighted sum of "whites" is larger then the sum of "blacks".

Each neighbor is given a weight according to its nearness.

Naïve Bayesian

- A statistical model
  - Use Bayes' (贝叶斯) Theorem to perform probabilistic prediction, e.g., predict class membership probabilities
- Assumption
  - The effect of an attribute on a given class is independent of other attributes
- Performance
  - Comparable with decision trees (决策树) and selected neural network classifiers

Naïve Bayesian Classifier

- Given a training set of attributes and their associated class labels, and each object is represented by a *n*-D vector (*n*维向量) $\mathbf{X} = (x_1, x_2, \ldots, x_n)$
- Suppose there are in total *m* classes, *i.e.*, $C_1, C_2, \ldots, C_m$.
- Naïve Bayesian Classifier is to derive the maximum posteriori (后验概率), *i.e.*, the maximal $P(C_i|\mathbf{X})$

Naïve Bayesian Classifier

- This can be derived from Bayes' theorem

$$P(C_i \mid \mathbf{X}) = \frac{P(\mathbf{X} \mid C_i)P(C_i)}{P(\mathbf{X})}$$

- Since $P(\mathbf{X})$ is constant for all classes, only

$$P(C_i \mid \mathbf{X}) \propto P(\mathbf{X} \mid C_i)P(C_i)$$

  needs to be maximized
- $P(C_i)$ can be obtained from training set s_i/s

Derivation 推导

- **Assumption**: attributes are conditionally independent (i.e., no dependence relation between attributes): $$P(\mathbf{X} \mid C_i) = \prod_{k=1}^{n} P(x_k \mid C_i)$$
- This greatly reduces the computation cost: Only counts the class distribution
- If A_k is categorical, $P(x_k|C_i) = s_{ik}/s_i$, count the distribution
- If A_k is continuous-valued, $P(x_k|C_i)$ can be computed based on Gaussian distribution

Example

age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31…40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31…40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31…40	medium	no	excellent	yes
31…40	high	yes	fair	yes
>40	medium	no	excellent	no
<=30	medium	yes	fair	?

Example

- $P(C_i)$: P(buys_computer = "yes") = 9/14 = 0.643
  P(buys_computer = "no") = 5/14= 0.357

- Compute $P(X|C_i)$ for each class
  P(age = "<=30" | buys_computer = "yes") = 2/9 = 0.222
  P(age = "<= 30" | buys_computer = "no") = 3/5 = 0.6
  P(income = "medium" | buys_computer = "yes") = 4/9 = 0.444
  P(income = "medium" | buys_computer = "no") = 2/5 = 0.4
  P(student = "yes" | buys_computer = "yes) = 6/9 = 0.667
  P(student = "yes" | buys_computer = "no") = 1/5 = 0.2
  P(credit_rating = "fair" | buys_computer = "yes") = 6/9 = 0.667
  P(credit_rating = "fair" | buys_computer = "no") = 2/5 = 0.4

- **X = (age <= 30 , income = medium, student = yes, credit_rating = fair)**

$P(X|C_i)$: P(X|buys_computer = "yes") = 0.222 x 0.444 x 0.667 x 0.667 = 0.044
P(X|buys_computer = "no") = 0.6 x 0.4 x 0.2 x 0.4 = 0.019

$P(X|C_i)*P(C_i)$: P(X|buys_computer = "yes") * P(buys_computer = "yes") = 0.028
P(X|buys_computer = "no") * P(buys_computer = "no") = 0.007

Example

age	income	student	credit_rating	buys_computer
10	high	no	fair	no
15	high	no	excellent	no
35	high	no	fair	yes
45	medium	no	fair	yes
42	low	yes	fair	yes
42	low	yes	excellent	no
32	low	yes	excellent	yes
30	medium	no	fair	no
28	low	yes	fair	yes
42	medium	yes	fair	yes
30	medium	yes	excellent	yes
36	medium	no	excellent	yes
38	high	yes	fair	yes
42	medium	no	excellent	no
29	medium	yes	fair	?

Example

- The attribute "age" is continuous-valued

$$P(x_k \mid C_i) = \frac{1}{\sqrt{2\pi}\sigma_{C_i,k}} \exp\left(-\frac{(x_k - \mu_{C_i,k})^2}{2\sigma^2_{C_i,k}}\right)$$

$$P(age = 29 \mid buys\_computer = "yes") = ?$$

$$P(age = 29 \mid buys\_computer = "no") = ?$$

Comments

- **Advantages**
  - Easy to implement
  - Good results obtained in most of the cases
- **Disadvantages**
  - Assumption: class conditional independence (给定每个类别, 条件独立), therefore loss of accuracy
    - Practically, dependencies do exist among variables, e.g., Symptoms: fever, cough, etc.
    - Dependencies among these cannot be modeled by Naïve Bayesian Classifier
  - 0 probability value: for example, new words in the testing document. Laplace smoothing factor?