Artificial Intelligence
——Perceptron Learning Algorithm

Fear Surprise Sadness Anger Disgust Joy

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Decision Trees References

- 两个连续变量的线性相关度，用协方差或相关系数来衡量
  - 非线性相关度：Maximal Information Coefficient (MIC). Detecting novel associations in large data sets, *Science*, 2011.
- 两个离散变量的相关度，用互信息度量
  - 互信息倾向于选择取值更多的离散型属性：A framework to adjust dependency measure estimates for chance, *SDM*, 2016.
  - Entropy evaluation based on confidence intervals of frequency estimates: application to the learning of decision trees, *ICML*, 2015.
  - Standardized mutual information for clustering comparisons: one step further in adjustment for chance, *ICML*, 2014.

Regression Review

- Least-squares solutions

$$n^{-1}\sum_{i=1}^{n}(y_i - w_0 - w_1 x_i) = 0$$

$$n^{-1}\sum_{i=1}^{n}x_i(y_i - w_0 - w_1 x_i) = 0$$

$$Q(w_0, w_1) = \min_{w_0, w_1}\sum_{i=1}^{n}(y_i - w_0 - w_1 x_i)^2$$

$$\partial Q(w_0, w_1) / \partial w_0 = 0 \qquad \partial Q(w_0, w_1) / \partial w_1 = 0$$

$$-2\sum_{i=1}^{n}(y_i - w_0 - w_1 x_i) = 0 \qquad -2\sum_{i=1}^{n}x_i(y_i - w_0 - w_1 x_i) = 0$$

Regression Review

- Least-squares solutions

$$w_0 = \bar{y} - w_1\bar{x}$$

$$w_1 = \frac{\sum_{i=1}^{n} x_i(y_i - \bar{y})}{\sum_{i=1}^{n} x_i(x_i - \bar{x})}$$

$$= \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}$$

Perceptron Learning Algorithm

- Dealing with all attributes jointly which are continuous variables
- Discrete random variables can be changed to continuous variables

Perceptron Learning Algorithm

- Dealing with all attributes jointly which are continuous variables
- Discrete random variables can be changed to continuous variables
- For $\mathbf{x}=(x_1,x_2,\ldots,x_d)$ with d features, compute a weighted 'score' and
  predict +1(good) if $\sum_{i=1}^{d} w_i x_i > threshold$
  predict -1(bad) if $\sum_{i=1}^{d} w_i x_i < threshold$
- $\mathbf{y}=\{+1(\text{good}), -1(\text{bad})\}$

$$h(\mathbf{x}) = sign\left(\left(\sum_{i=1}^{d} w_i x_i\right) - threshold\right)$$

Perceptron Learning Algorithm

$$h(\mathbf{x}) = sign\left(\left(\sum_{i=1}^{d} w_i x_i\right) - threshold\right)$$

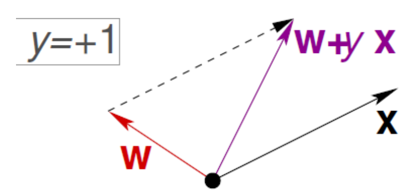
$$= sign\left(\left(\sum_{i=1}^{d} w_i x_i\right) + \underbrace{(-threshold)}_{w_0} \cdot \underbrace{(+1)}_{x_0}\right)$$

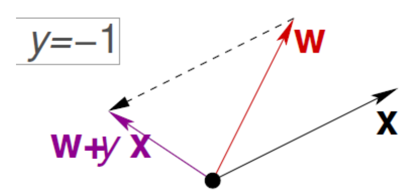
$$= sign\left(\sum_{i=0}^{d} w_i x_i\right)$$

$$= sign\left(\mathbf{W}^{\mathrm{T}} \mathbf{X}\right)$$

Perceptron Learning Algorithm

- Difficult: the set of $h(\mathbf{x})$ is of infinite size
- Idea: start from some initial weight vector $\mathbf{w}_{(0)}$, and "correct" its mistakes on D
- For t = 0, 1,…
  - find a mistake of $\mathbf{w}_{(t)}$ called $(\mathbf{x}_{n(t)}, y_{n(t)})$
    $$sign\left(\mathbf{w}_{(t)}^{\mathrm{T}}\mathbf{x}_{n(t)}\right) \neq y_{n(t)}$$
  - (try to) correct the mistake by
    $$\mathbf{w}_{(t+1)} \leftarrow \mathbf{w}_{(t)} + y_{n(t)}\mathbf{x}_{n(t)}$$
  - until no more mistakes
- Return last **W** (called $\mathbf{W}_{\mathrm{PLA}}$)





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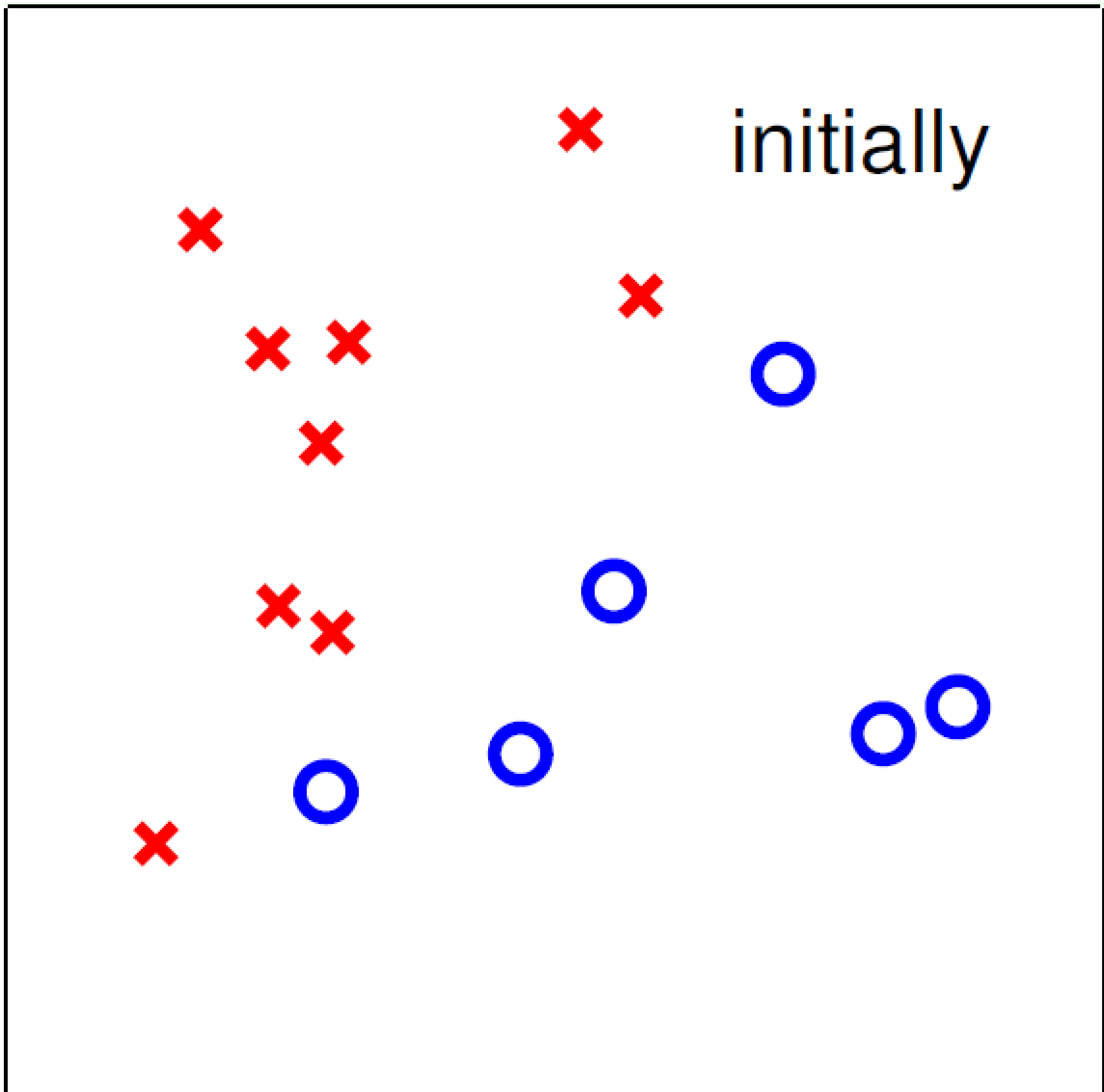

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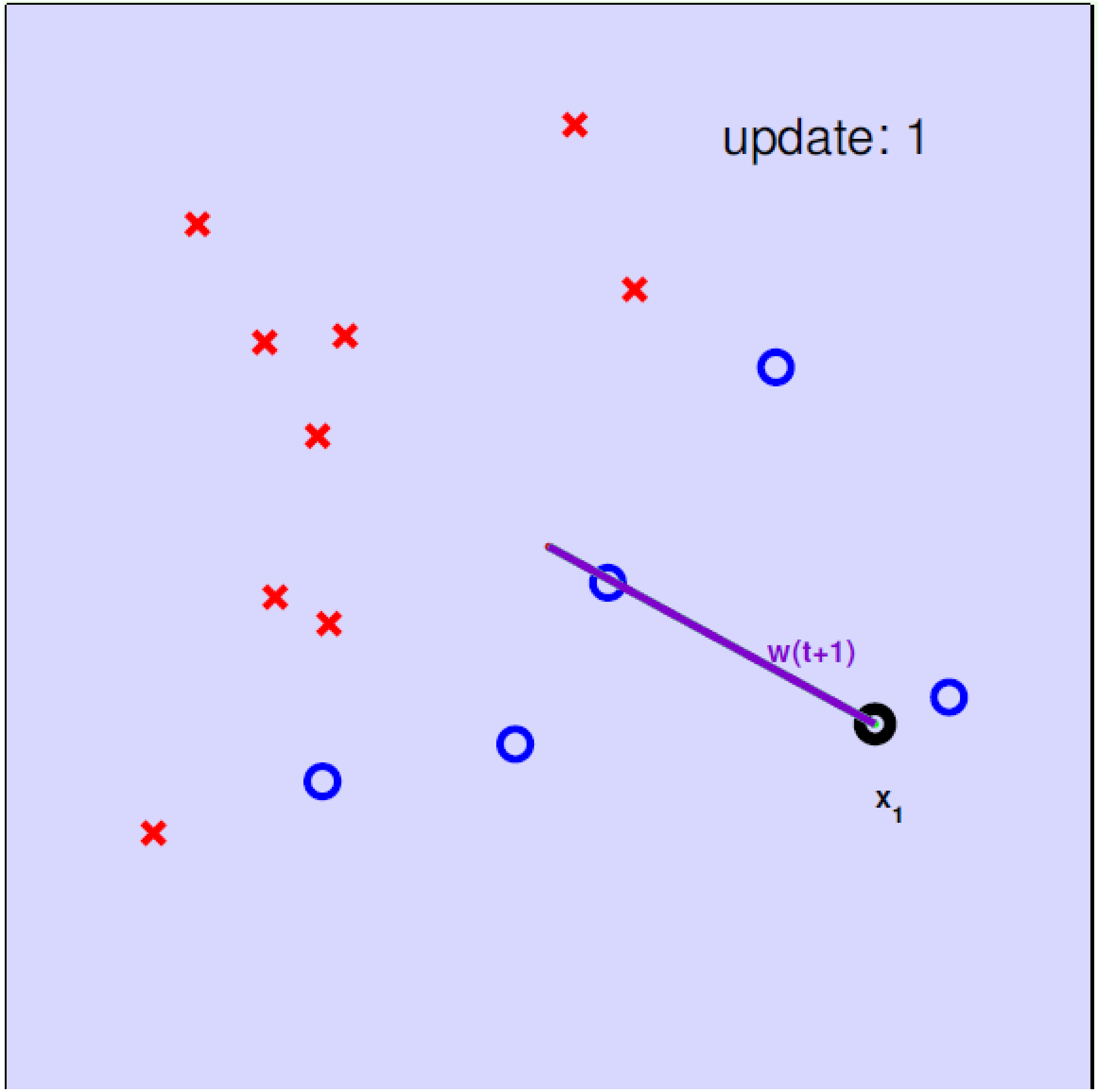

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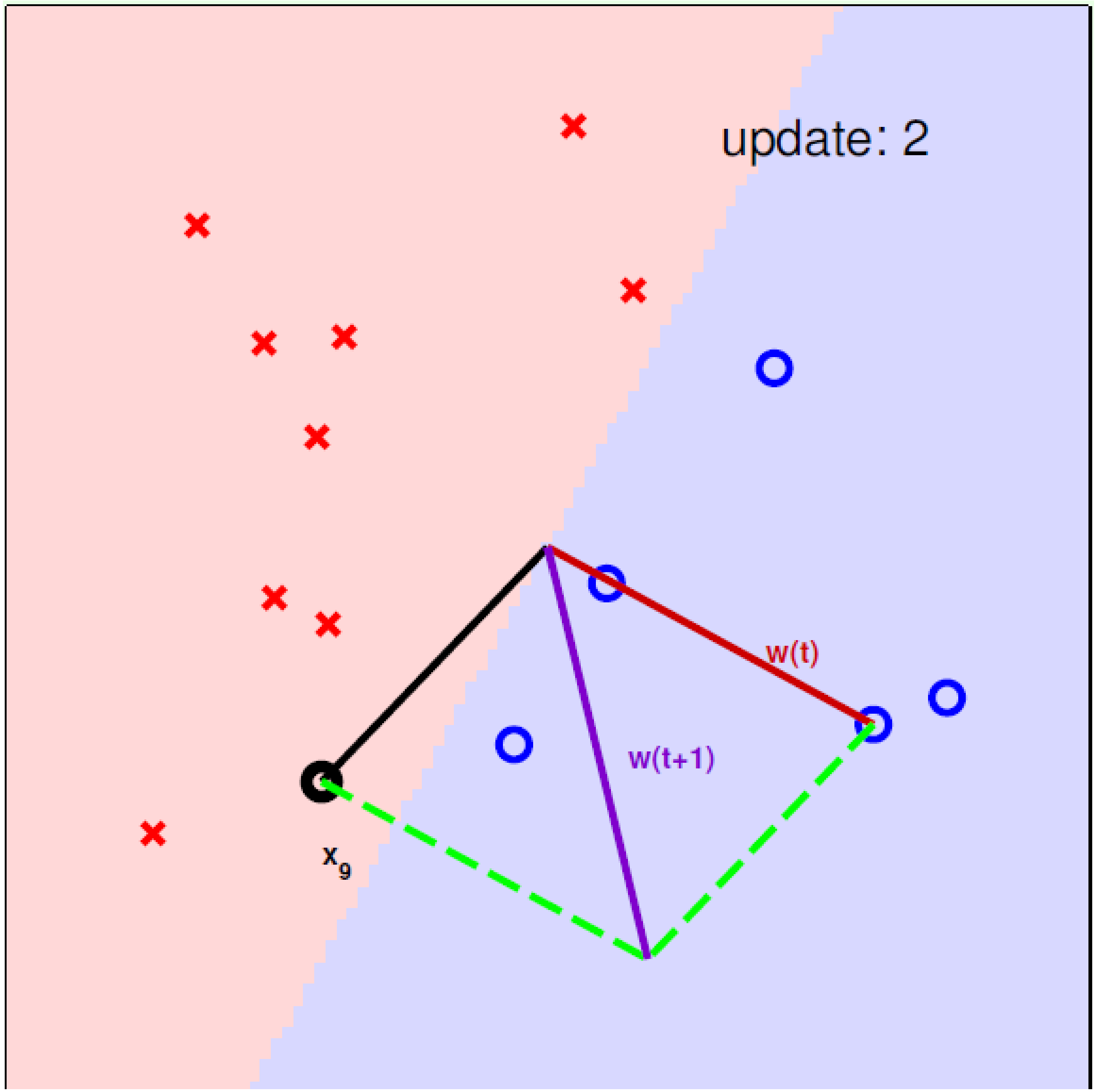

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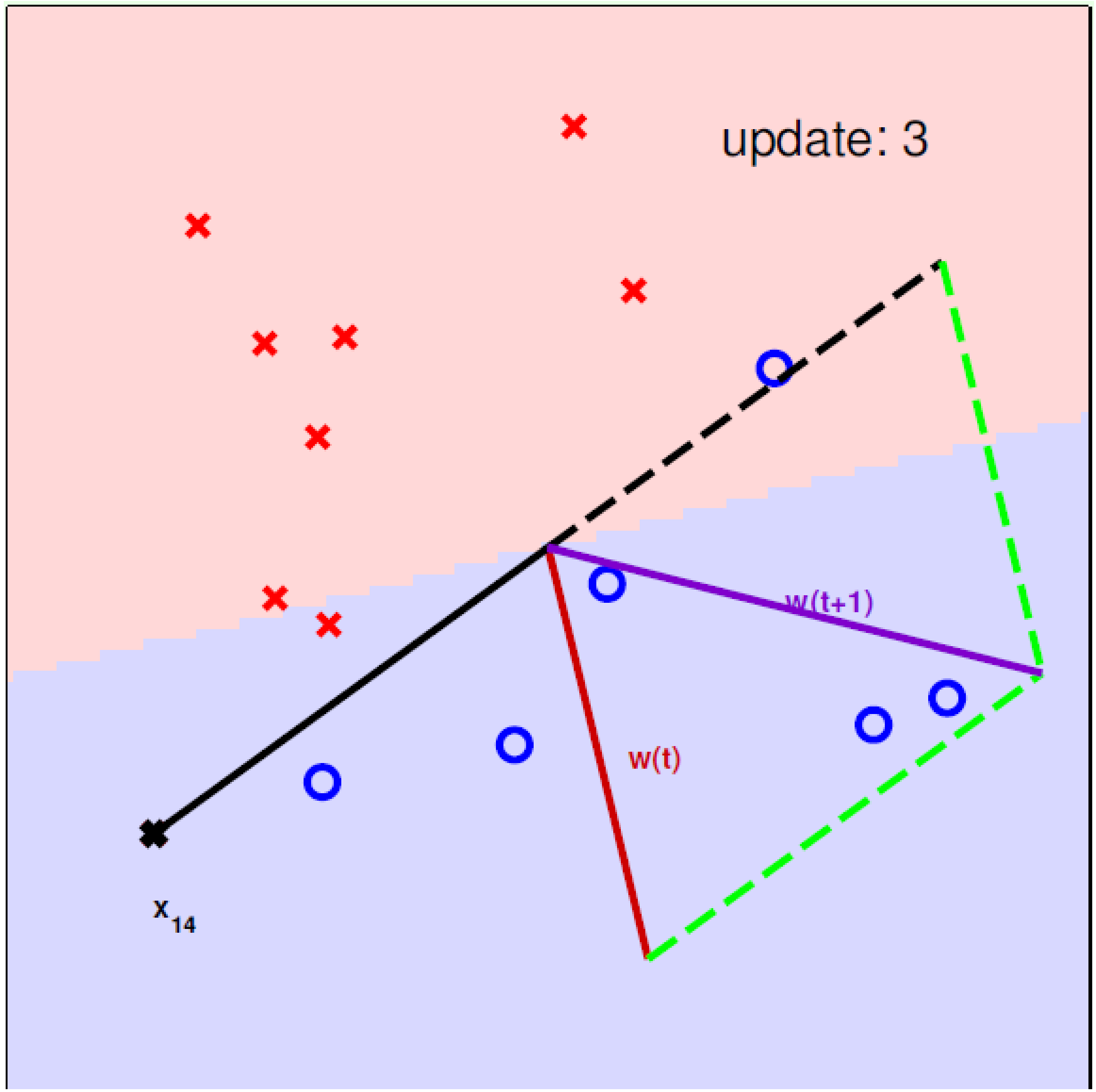

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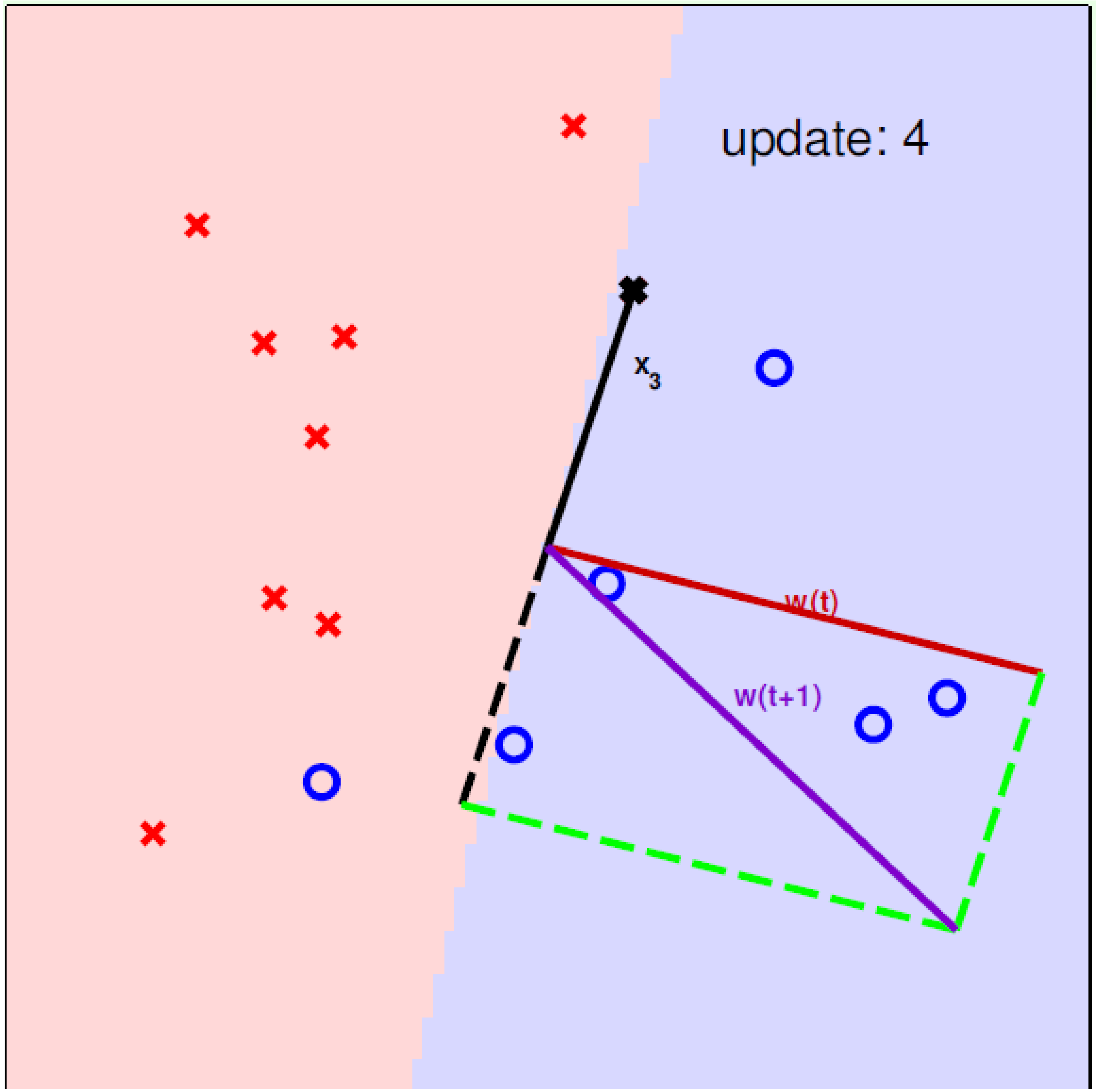

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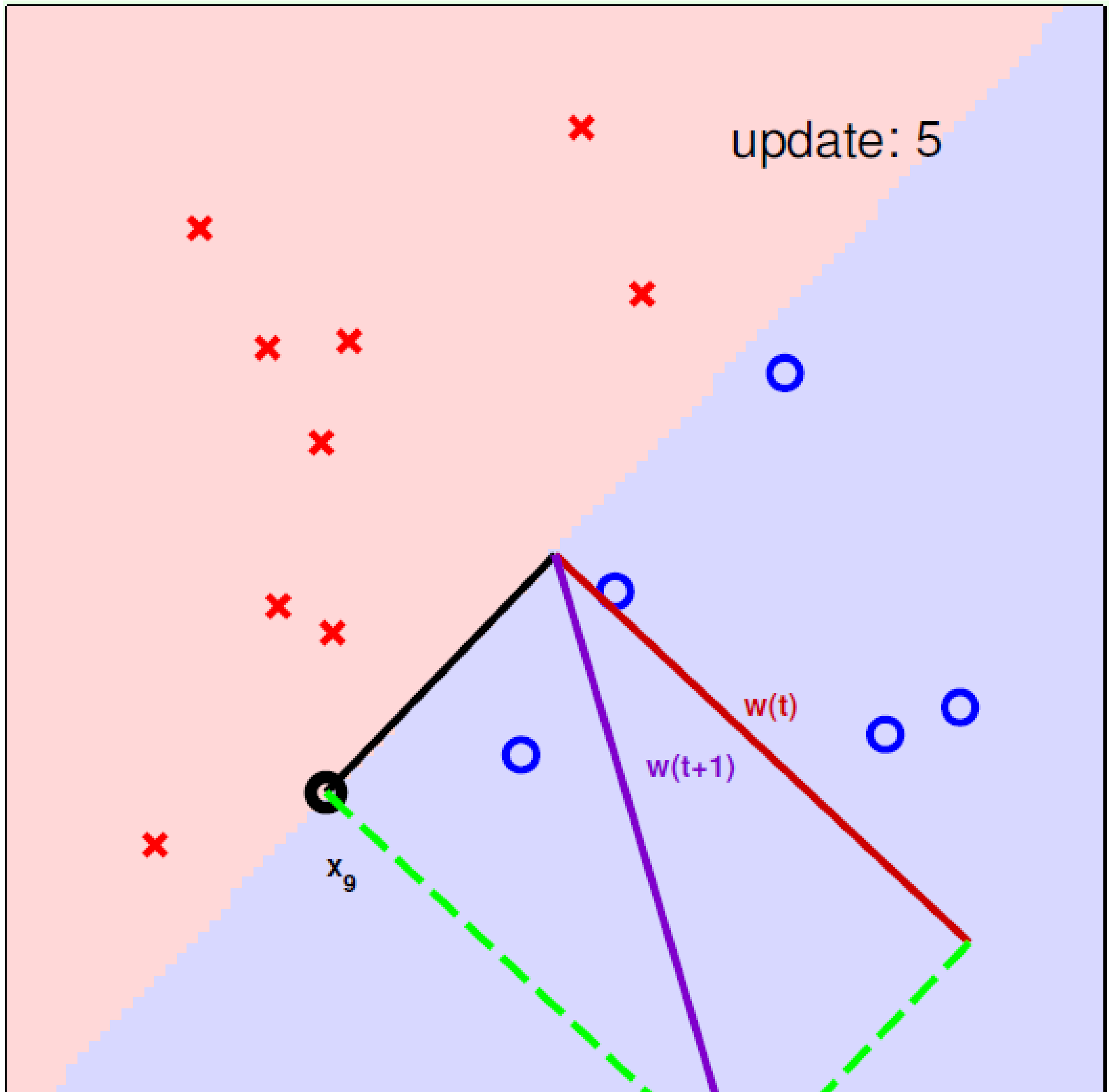

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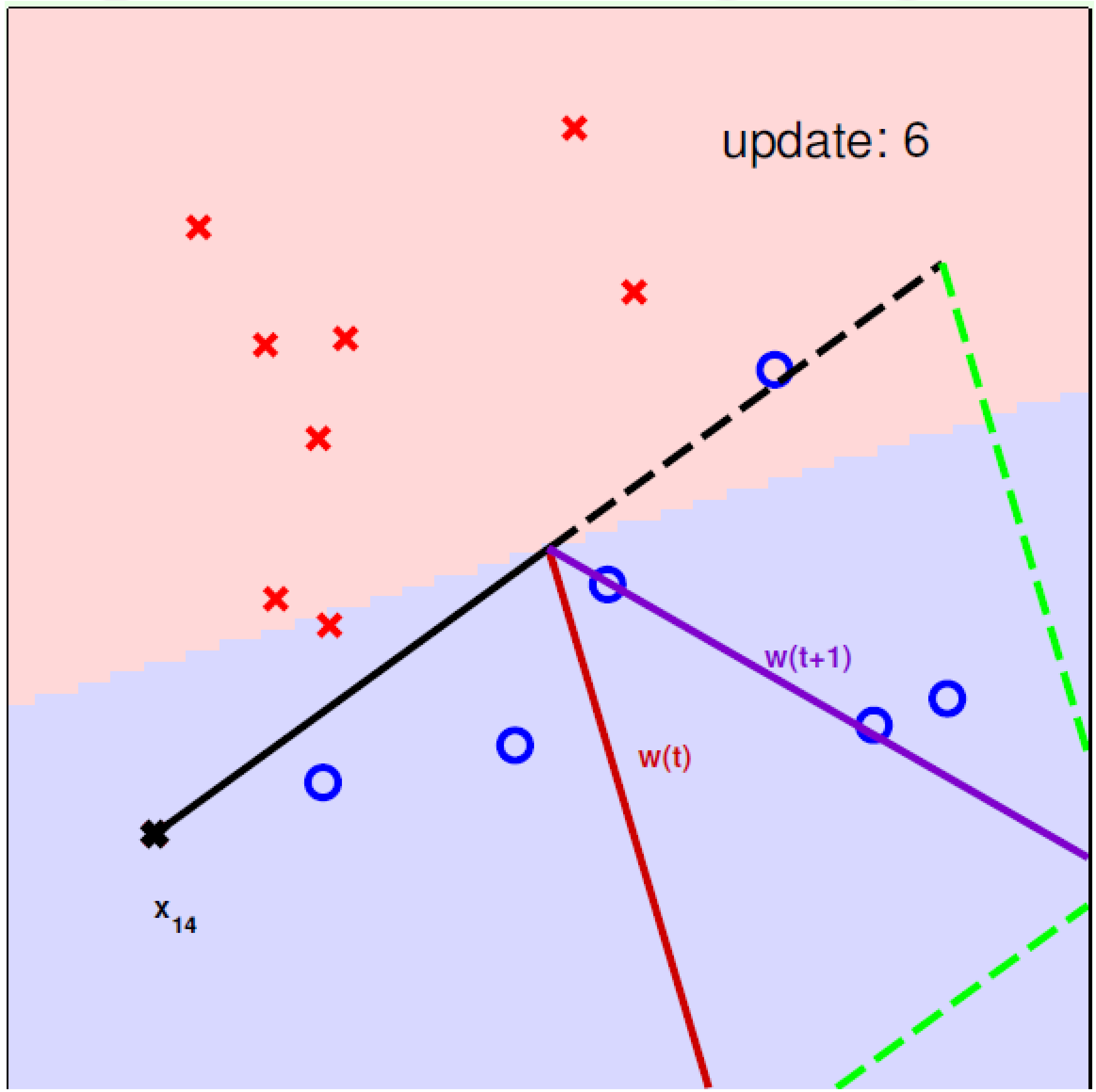

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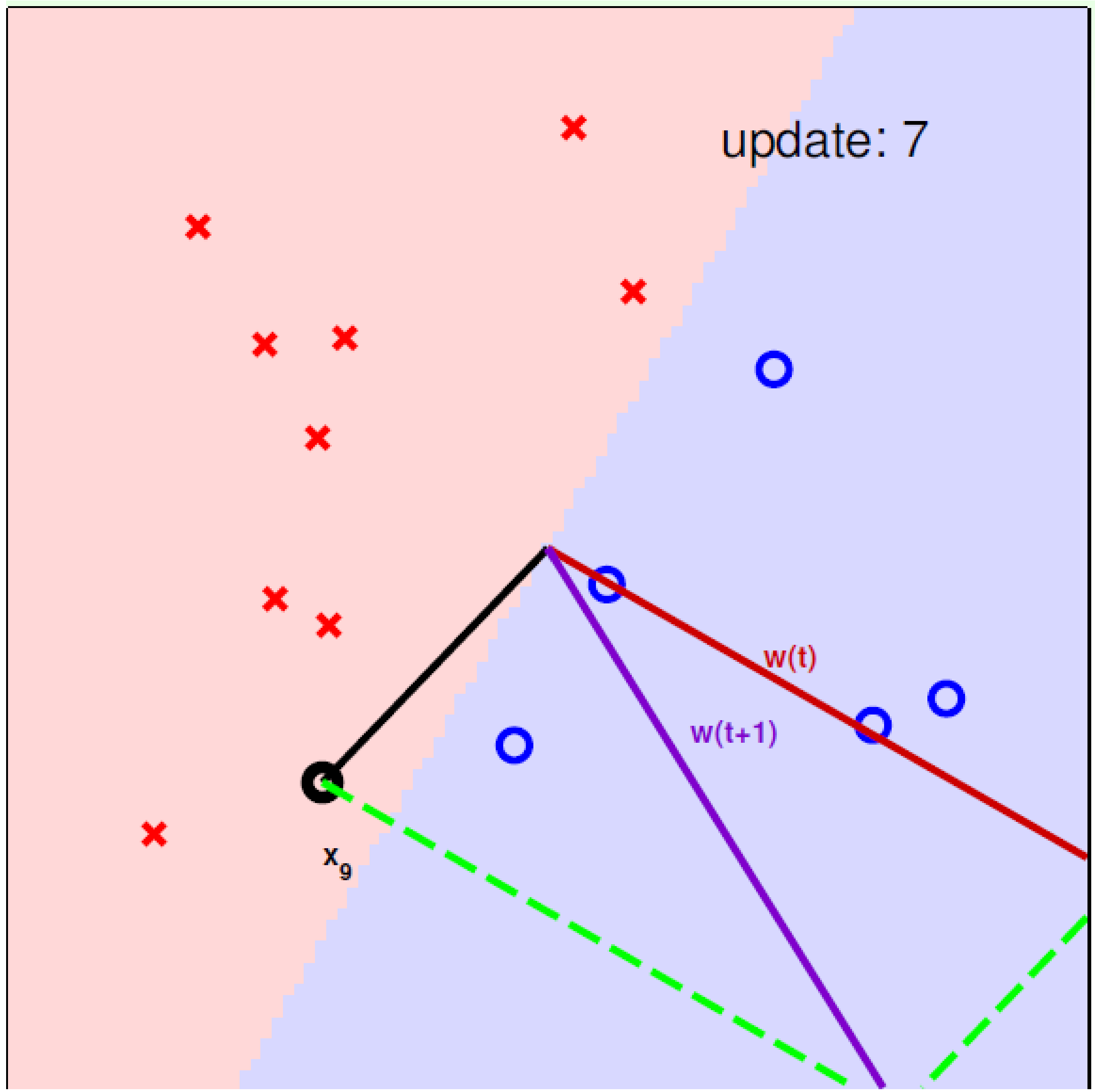

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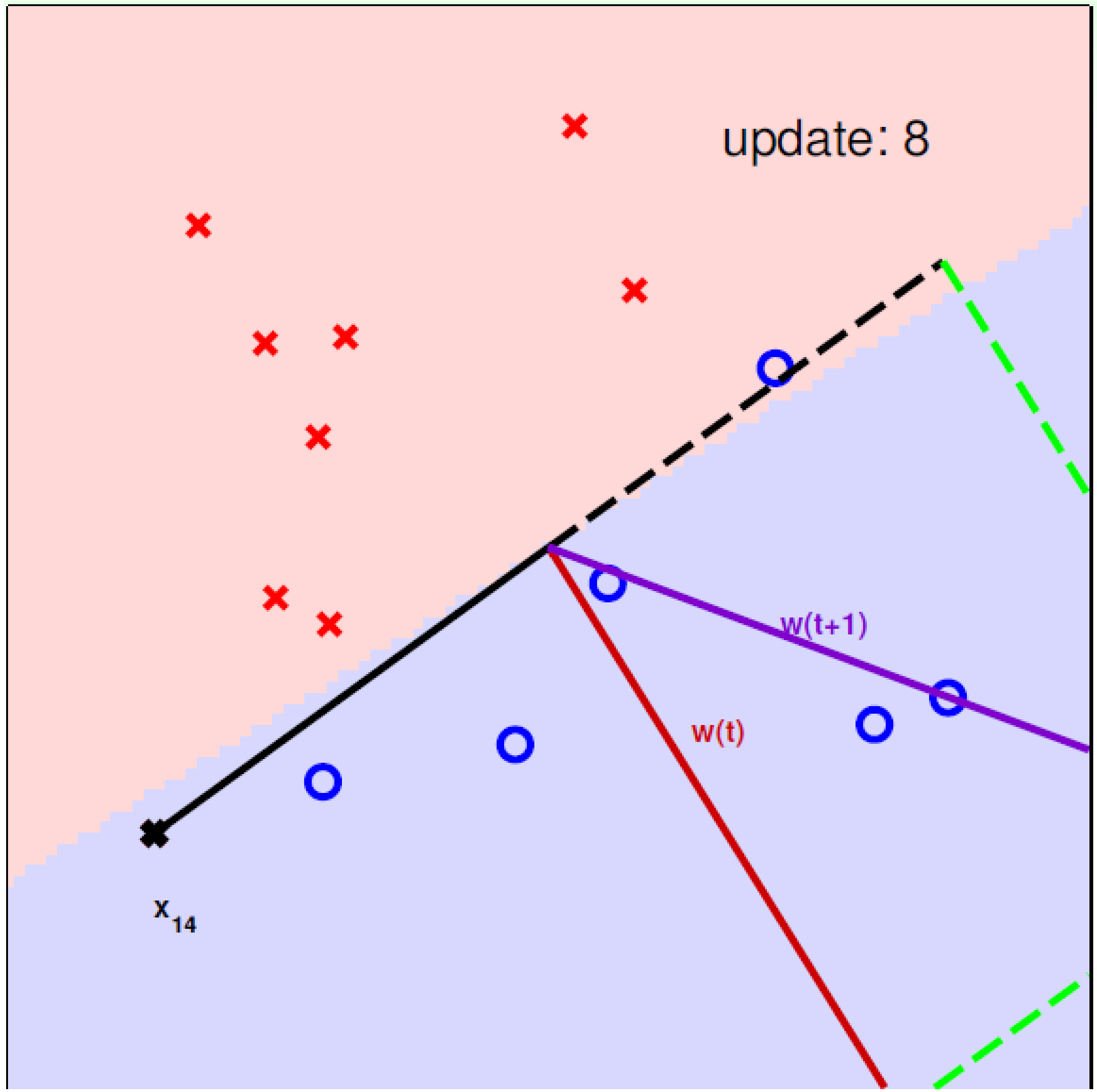

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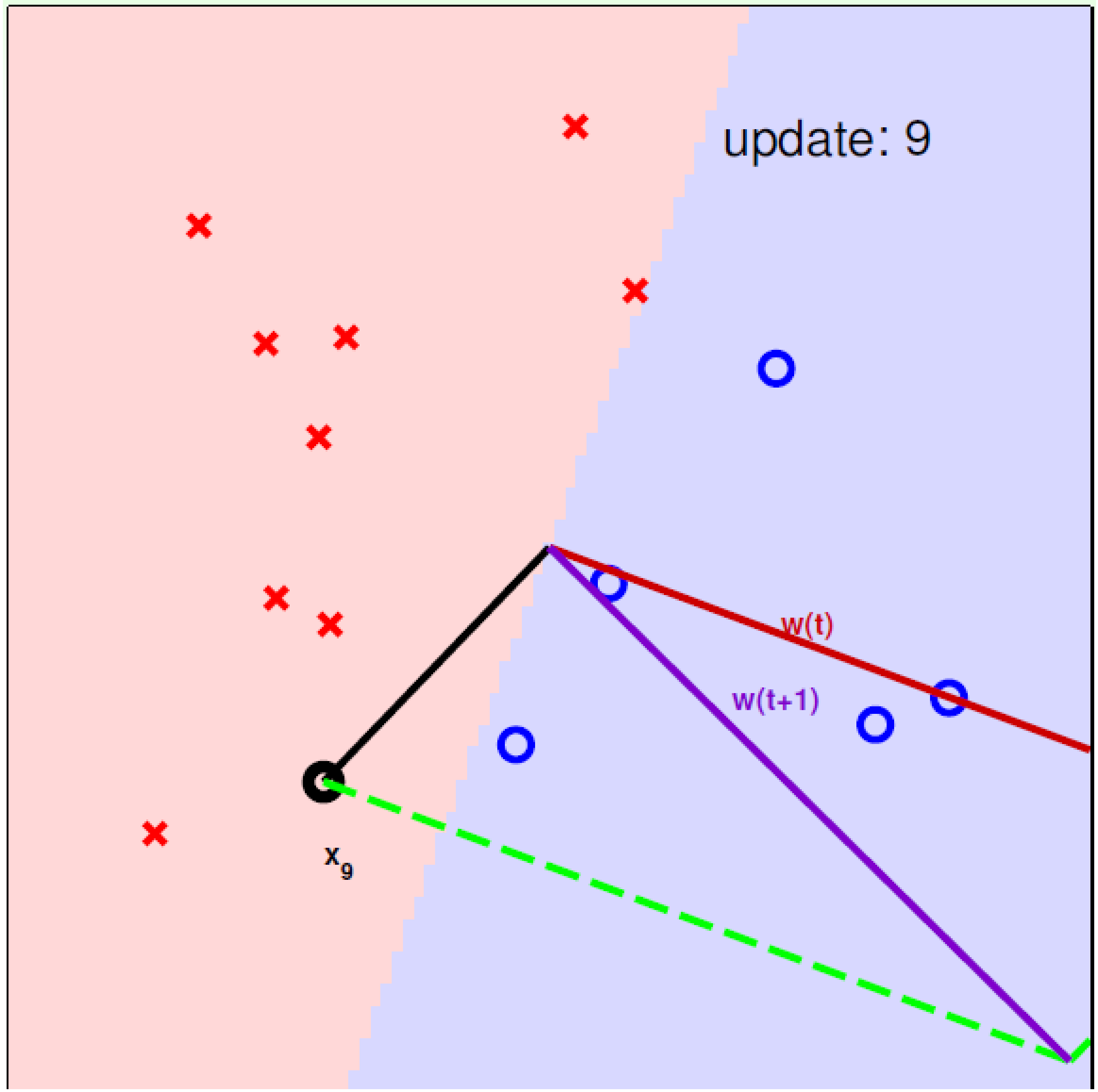

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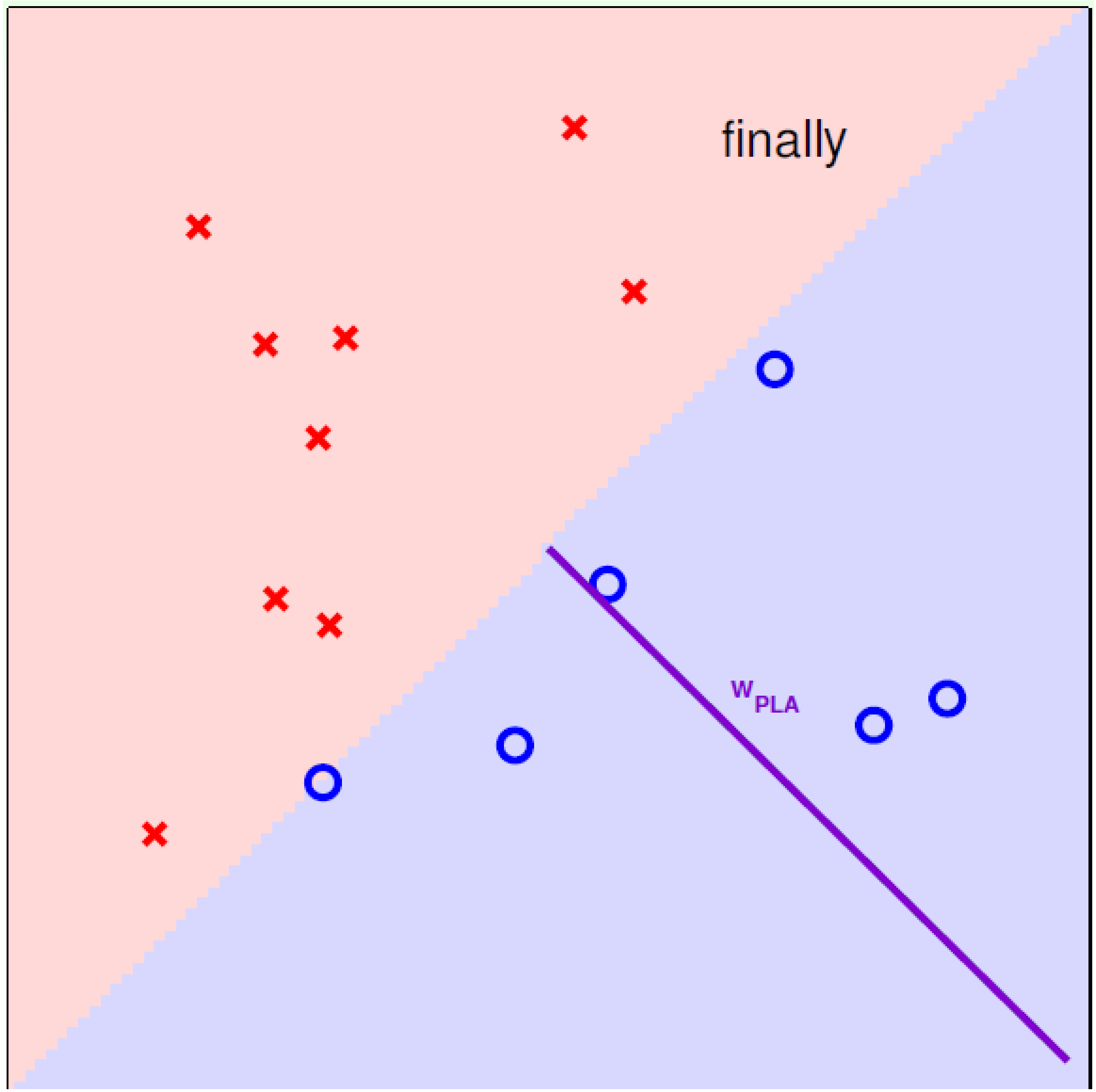

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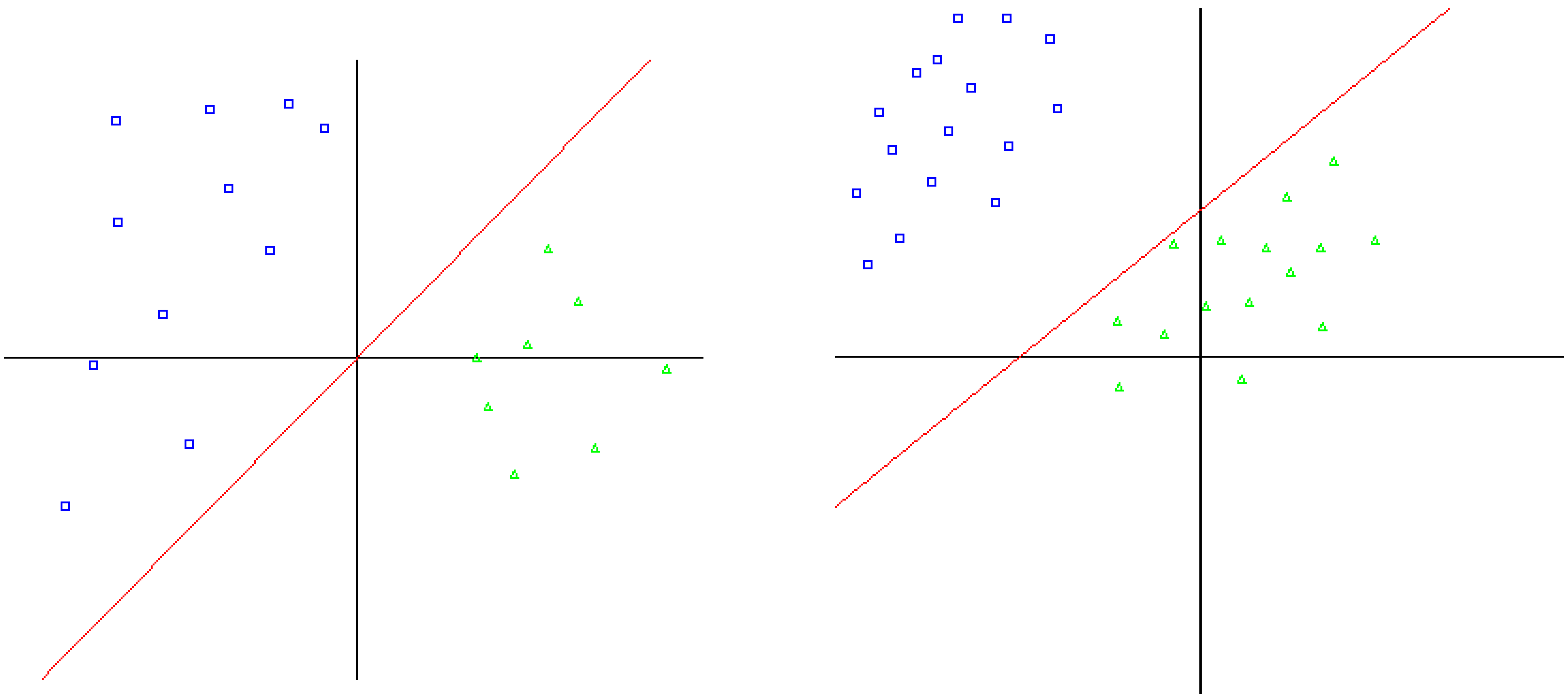

Perceptron Learning Algorithm

- Only if there exists an hyperplane that correctly classifies the data, the Perceptron procedure is guaranteed to converge; furthermore, the algorithm may give different results depending on the order in which the elements are processed, indeed several different solutions exist.

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