

Query Answering with Inconsistent Existential Rules under Stable Model Semantics

Hai Wan¹, Heng Zhang^{2,*}, Peng Xiao¹, Haoran Huang³, and Yan Zhang⁴

¹School of Data and Computer Science, Sun Yat-sen University, Guangzhou, China

wanhai@mail.sysu.edu.cn

²School of Computer Science and Technology, Huazhong University of Science and Technology, Wuhan, China

hengzhang@hust.edu.cn

³School of Computer Science, Fudan University, Shanghai, China

⁴School of Computing, Engineering and Mathematics, Western Sydney University, Sydney, Australia

Abstract

Classical inconsistency-tolerant query answering relies on selecting maximal components of an ABox/database which are consistent with the ontology. However, some rules in ontologies might be unreliable if they are extracted from ontology learning or written by unskillful knowledge engineers. In this paper we present a framework of handling inconsistent existential rules under stable model semantics, which is defined by a notion called rule repairs to select maximal components of the existential rules. Surprisingly, for R-acyclic existential rules with R-stratified or guarded existential rules with stratified negations, both the data complexity and combined complexity of query answering under the rule repair semantics remain the same as that under the conventional query answering semantics. This leads us to propose several approaches to handle the rule repair semantics by calling answer set programming solvers. An experimental evaluation shows that these approaches have good scalability of query answering under rule repairs on realistic cases.

Introduction

Querying inconsistent ontologies is an intriguing new problem that gives rise to a flourishing research activity in the description logic (DL) and existential rules community. Consistent query answering, first developed for relational databases (Arenas, Bertossi, and Chomicki 1999; Chomicki 2007) and then generalized as the AR and IAR semantics for several DLs (Lembo et al. 2010), is the most widely recognized semantics for inconsistency-tolerant query answering. These two traditional semantics are based upon the notion of *repair*, defined as an inclusion-maximal subset of the ABox consistent with the TBox. Du, Qi, and Shen (2013) studied query answering under weight-based AR semantics for DL *SHIQ*. Bienvenu, Bourgaux, and Goasdoué (2014) studied variants of AR and IAR semantics for DL-Lite_R obtained by replacing classical repairs with various preferred repairs. Existential rules (also known as Datalog[±]) are set to play a central role in the context of query answering and information extraction for the Semantic Web. Lukasiewicz et al.

(2012; 2013; 2015) studied the data complexity and combined complexity of AR semantics under the main decidable classes of existential rules enriched with negative constraints.

However, observe that some rules might be unreliable if they are extracted from ontology learning or written by unskillful knowledge engineer (Lehmann et al. 2011). Meyer et al. (2006) proposed a tableau-like algorithm which yields EXPTIME as upper bound for finding maximally concept-satisfiable terminologies represented in *ALC*. Kalyanpur et al. (2006) provided solutions on repairing unsatisfiable concepts in a consistent OWL ontology. Furthermore, usually there exist preferences between rules, and rules with negation are often considered less preferred than rules without negation. Scharrenbach et al. (2010) proposed that the original axioms must be preserved in the knowledge base under certain conditions and requires changing the underlying logics for repair. Wang et al. (2014) proposed that when new facts are added that contradict to the ontology, it is often desirable to revise the ontology according to the added data. Therefore, this motivates us to consider another repair that selects maximal components of the existential rules. We illustrate the motivation via the following example.

Example 1. Let $D = \{Bat(a), Mammal(a)\}$ be a database and let Σ be the following rule set expressing that each bat can fly and has at least one cave to live in; and if one creature lives in cave then it is a troglodene; and if we do not know one mammal can fly then it can not fly; if one creature can fly then it is a bird; additionally a bird can not be a troglodene at the same time; similarly a bird can not be a mammal meanwhile.

$$Bat(x) \rightarrow CanFly(x), \quad (1)$$

$$Bat(x) \rightarrow \exists y LiveIn(x, y), Cave(y), \quad (2)$$

$$LiveIn(x, y), Cave(y) \rightarrow Troglodene(x), \quad (3)$$

$$Mammal(x), \text{not } CanFly(x) \rightarrow CanNotFly(x), \quad (4)$$

$$CanFly(x) \rightarrow Bird(x), \quad (5)$$

$$Bird(x), Troglodene(x) \rightarrow \perp, \quad (6)$$

$$Bird(x), Mammal(x) \rightarrow \perp. \quad (7)$$

Clearly $\langle \Sigma, D \rangle$ is inconsistent under stable model semantics. We assume $P_1 = \{(1), (2), (3)\}$ is more reliable (or preferred) than $P_2 = \{(4), (5), (6), (7)\}$. Then we can delete

*Corresponding author.

(6) and (7), or (5) in P_2 to restore the consistency, and get inclusion-maximal preferred consistent rule sets w.r.t. D :

$$\{(1), (2), (3), (4), (6), (7)\}, \quad \{(1), (2), (3), (4), (5)\}.$$

We will focus on the case where the database is reliable but rules are not. Our main goal is to present a framework of handling inconsistent existential rules under stable model semantics. We define a notion called *rule repairs* to select maximal components of the rules, the philosophy behind that is to trust the rules as many as possible. Our second goal is to perform an in-depth analysis of the data and combined complexity of inconsistency-tolerant query answering under rule repair semantics. Let us recall some previous work on existential rules under stable model semantics. Magka, Krötzsch, and Horrocks (2013) presented R-acyclic and R-stratified normal rule sets each of which always admits at most one finite stable models. Zhang, Zhang, and You (2015) implicitly showed that the R-acyclicity is enough to capture all negation-free rule sets with finite stable models. Gottlob et al. (2014) proved the decidability of query answering under stable model semantics for guarded existential rules. Alviano and Pieris (2015) extended the stickiness notion to normal rule sets and showed that it assures the decidability for well-founded semantics rather than stable model semantics. We will focus on R-acyclic rule sets with R-stratified or full negations and guarded existential rules with stratified or full negations.

Our main contributions are briefly summarized as follows. We define rule repair semantics to handle inconsistent existential rules under stable model semantics. We consider rule repairs w.r.t. inclusion-maximal subset or cardinality, and that with preference. We obtain a (nearly) complete picture of the data and combined complexity of inconsistency-tolerant query answering under rule repair semantics (Table 1). Surprisingly, for R-acyclic existential rules with R-stratified or guarded existential rules with stratified negations, both the data complexity and combined complexity of query answering under the rule repair semantics remain the same as that under the conventional query answering semantics. Interestingly, the data complexity based upon weak-acyclic or guarded existential rules with stratified negation is PTIME-complete. This leads us to propose several approaches to handle the rule repair semantics by calling answer set programming (ASP) solvers. An experimental evaluation shows that these approaches have good scalability of query answering rule repairs on realistic cases.

Preliminaries

We consider a standard first-order language. We use $\text{Var}(\varepsilon)$ to denote the variables appearing in an expression ε .

Databases. We assume an infinite set Δ of (data) constants, an infinite set Δ_n of (labeled) nulls (used as fresh Skolem terms), and an infinite set Δ_v of variables. A term t is a constant, a null, or a variable. We denote by \mathbf{x} a sequence of variables x_1, \dots, x_k with $k \geq 0$. An atom α has the form $R(t_1, \dots, t_n)$, where R is an n -ary relation symbol, and t_1, \dots, t_n are terms. A conjunction of atoms is often identified with the set of all its atoms. We assume a relational schema \mathcal{R} , which is a finite set of relation symbols.

An instance I is a (possibly infinite) set of facts $p(\mathbf{t})$, i.e., atoms without involving variables, where \mathbf{t} is a tuple of constants and nulls. A database D over a relational schema \mathcal{R} is a finite instance with relation symbols from \mathcal{R} and with arguments only from Δ (i.e., without involving nulls).

Normal Logic Programs and Stable Models. Each normal (logic) program is a finite set of NLP rules of the form

$$\alpha \leftarrow \beta_1, \dots, \beta_n, \text{not } \beta_{n+1}, \dots, \text{not } \beta_m \quad (1)$$

where $\alpha, \beta_1, \dots, \beta_m$ are atoms and $m \geq n \geq 0$. Given a rule r of the above form, let $\text{head}(r) = \alpha$, let $\text{body}^+(r) = \{\beta_1, \dots, \beta_n\}$, and let $\text{body}^-(r) = \{\beta_{n+1}, \dots, \beta_m\}$.

Let Π be a normal program. The Herbrand universe and Herbrand base of Π are denoted by $HU(\Pi)$ and $HB(\Pi)$, respectively. A variable-free rule r' is called an instance of some rule $r \in \Pi$ if there is a substitution $\theta : \Delta_v \rightarrow HU(\Pi)$ such that $r\theta = r'$. Let $\text{ground}(\Pi)$, the grounding of Π , be the set of all instances of r for all $r \in \Pi$.

The Gelfond-Lifschitz reduct of a normal program Π w.r.t. a set $M \subseteq HB(\Pi)$, denoted Π^M , is the (possibly infinite) ground positive program obtained from $\text{ground}(\Pi)$ by

- deleting every rule r such that $\text{body}^-(r) \cap M \neq \emptyset$, and
- deleting all negative literals from each remaining rule.

A subset M of $HB(\Pi)$ is called a *stable model* of Π if it is the least model of $\text{ground}(\Pi^M)$. For more about stable model semantics, refer to (Gelfond and Lifschitz 1988; Ferraris, Lee, and Lifschitz 2011).

Normal Existential Rules. Every normal (existential) rule is a first-order sentence of the form $\forall \mathbf{x} \forall \mathbf{y} \varphi(\mathbf{x}, \mathbf{y}) \rightarrow \exists \mathbf{z} \psi(\mathbf{x}, \mathbf{z})$, where φ is a conjunction of literals, i.e., atoms or negated atoms (of the form $\neg \alpha$ where α is atomic), ψ is a conjunction of atoms, and each universally quantified variable appears in at least one positive conjunct of φ . In the above normal rule, φ is called its *body*, and ψ its *head*. A normal rule is called a *constraint* if its head is the “false” \perp . For simplicity, when writing a rule, we often omit the universal quantifiers; by a *normal rule set*, we always mean a finite number of normal existential rules.

Let r be a normal rule $\varphi(\mathbf{x}, \mathbf{y}) \rightarrow \exists \mathbf{z} \psi(\mathbf{x}, \mathbf{z})$. For each variable $z \in \mathbf{z}$, we introduce an n -ary fresh function symbol f_z^r where $n = |\mathbf{x}|$. The skolemization of r , denoted $\text{sk}(r)$, is the rule obtained from r by substituting $f_z^r(\mathbf{x})$ for $z \in \mathbf{z}$, followed by substituting “not” for \neg . Let Σ be a normal rule set. We define $\text{sk}(\Sigma)$ to be the set of rules $\text{sk}(r)$ for all $r \in \Sigma$. Clearly, $\text{sk}(\Sigma)$ can be regarded as a normal program in an obvious way. Given any database D , an instance is called a *stable model* of $D \cup \Sigma$ if it is a stable model of $D \cup \text{sk}(\Sigma)$.

A normal rule r is called *guarded* if there is a positive conjunct in the body of r that contains all the universally quantified variable of r , and a normal rule set is called *guarded* if every rule in it is guarded.

A normal rule set Σ is *stratified* if there is a function ℓ that maps relation symbols to integers such that for all $r \in \Sigma$:

- for all relation symbols R occurring in the head and S positively occurring in the body, $\ell(R) \geq \ell(S)$, and
- for all relation symbols R occurring in the head and S negatively occurring in the body, $\ell(R) > \ell(S)$.

Sometimes, the negations that occur in a stratified normal rule set are called *stratified negations*, and those in a non-stratified normal rule set are called *full negations*.

Let r_1 and r_2 be two normal rules, and let B_i^+ (resp., B_i^- and H_i) be the set of atoms positively (resp., negatively and positively) occurring in the body (resp., body and head) of r_i . W.l.o.g., assume that no variable occurs in both r_1 and r_2 . Rule r_2 *positively relies* on r_1 , written $r_1 \rightarrow^+ r_2$, if there exist a database D and a substitution θ such that $B_1^+ \theta \subseteq D$, $B_1^- \theta \cap D = \emptyset$, $B_2^+ \theta \subseteq D \cup H_1 \theta$, $B_2^- \theta \cap (D \cup H_1 \theta) = \emptyset$, $B_2^+ \theta \not\subseteq D$ and $H_2 \theta \not\subseteq D \cup H_1 \theta$. Rule r_2 *negatively relies* on r_1 , written $r_1 \rightarrow^- r_2$, if there exist a database D and a substitution θ such that $B_1^+ \theta \subseteq D$, $B_1^- \theta \cap D = \emptyset$, $B_2^+ \theta \subseteq D$, $B_2^- \theta \cap H_1 \theta \neq \emptyset$ and $B_2^- \theta \cap D = \emptyset$. A normal rule set P is called *R-acyclic* if there is no cycle of positive reliances $r_1 \rightarrow^+ \dots \rightarrow^+ r_n \rightarrow^+ r_1$ that involves a rule with an existential quantifier, and P is called *R-stratified* if there is a partition $\{P_1, \dots, P_n\}$ of P such that, for every two normal rule sets P_i, P_j and rules $r_1 \in P_i$ and $r_2 \in P_j$, if $r_1 \rightarrow^+ r_2$ then $i \leq j$ and if $r_1 \rightarrow^- r_2$ then $i < j$.

Classical Boolean Query Answering. A normal Boolean conjunctive query (NBCQ) Q is an existentially closed conjunction of atoms and negated atoms involving no null. Let Q^+ (respectively, Q^-) be the set of atoms positively (respectively, negatively) occurring in Q . An NBCQ is called *safe* if every variable in an atom from Q^- has at least one occurrence in Q^+ ; it is *covered* if for every atom α in Q^- , there is an atom in Q^+ that contains all arguments of α .

Given a database D and an NBCQ Q , we write $D \models Q$ if there exists an assignment h (that is, a function that maps each variable to a variable-free term) such that $h(Q^+) \subseteq D$ and $h(Q^-) \cap D = \emptyset$. Furthermore, given a database D , a normal rule set Σ and an NBCQ Q , we write $D \cup \Sigma \models_s Q$ if, for each stable model M of $D \cup \Sigma$, we have that $M \models Q$.

Complexity Classes. We assume that the reader is familiar with the complexity theory. Given a unary function T on natural numbers, by $\text{DTIME}(T(n))$ ($\text{NTIME}(T(n))$, respectively) we mean the class of languages decidable in time $T(n)$ by a deterministic (nondeterministic, respectively) Turing machine. Besides the well-known complexity classes such as $(\text{co})\text{NPTIME}$ and $(\text{co})\text{N2EXPTIME}$, we will also use several unusual classes as follows. By notation $\Delta_2\text{-2EXPTIME}$ we mean the class of all languages decidable in exponential time by a deterministic Turing machine with an oracle for some N2EXPTIME -complete problem. The Boolean hierarchy (BH) is defined as follows: $\text{BH}(1)$ is NPTIME ; for $k \geq 1$, $\text{BH}(2k)$ ($\text{BH}(2k+1)$) is the class of languages each of which is the intersection (union, respectively) of a language in $\text{BH}(2k-1)$ ($\text{BH}(2k)$, respectively) and a language in coNPTIME (NPTIME , respectively); BH is then the union of $\text{BH}(n)$ for all $n \geq 1$. Note that DP , the class for *difference polynomial time*, is exactly the class $\text{BH}(2)$; $\text{BH}(2k)$ is actually the class of languages each of which is the union of k languages in DP ; and BH is closed under complement. It was shown by (Chang and Kadin 1996) that a collapse of the Boolean hierarchy implies a collapse of the polynomial hierarchy; thus it seems impossible to find a BH -complete problem.

Existential Rule Repair Semantics

In this section, we propose several semantics to handle inconsistency in ontological knowledge base. Different from many existing works, we will focus on the case where the database is reliable but rules are not. Similar to the data repair semantics, see (Lembo et al. 2010), our inconsistency-tolerant semantics will rely on a notion called *rule repairs*.

To define rule repairs, we arm every rule set with a preference. Such rule sets are called preference-based ontologies.

Definition 1. Each preference-based ontology is an ordered pair (Σ, \preceq) , where Σ is a finite set of normal rules, and \preceq is a preorder (i.e., a reflexive and transitive binary relation) on $\mathcal{P}(\Sigma)$ (i.e., the power set of Σ). We call \preceq a preference.

Now, we are in the position to define rule repairs.

Definition 2. Let O be a preference-based ontology (Σ, \preceq) and D a database. A subset S of Σ is called a (preferred rule) repair of Σ w.r.t. \preceq and D (or simply a repair w.r.t. \preceq if Σ and D are clear from the context) if $D \cup S$ has at least one stable model, and for all subsets S' of Σ with $S \prec S'$ (i.e., $S \preceq S'$ but $S' \not\preceq S$), $D \cup S'$ has no stable model.

Intuitively, a preferred rule repair is a maximal component of the rule set which is consistent with the current database. The philosophy behind it is to trust the rules as many as possible. Note that the number of repairs are normally more than one. To avoid a choice among them, we follow the spirit of “certain” query answering. The semantics is then as follows.

Definition 3. Let O be a preference-based ontology (Σ, \preceq) where Σ is a finite set of normal rules, and let D be a database and Q an NBCQ. Then we write $\langle D, O \rangle \models Q$ if, for all preferred rule repairs S of Σ w.r.t. \preceq and D , we have $D \cup S \models_s Q$.

The following proposition shows us that our semantics for inconsistency-tolerant query answering will coincide with the classical semantics for query answering if the ontological knowledge base is consistent, which is clearly important.

Proposition 1. Let O be a preference-based ontology (Σ, \preceq) and let D be a database. If $\Sigma \cup D$ has a stable model, then $\langle D, O \rangle \models Q$ iff $\Sigma \cup D \models_s Q$ for any NBCQ Q .

With the above definitions, we then have a framework to define semantics for rule-based inconsistency-tolerant query answering. To define concrete semantics, we need to find preferences which will be useful in real-world applications. Besides the preference based on the set inclusion \subseteq , similar to (Bienvenu, Bourgaux, and Goasdoué 2014), we will consider other four kinds of preferences over subsets, which were first proposed by (Eiter and Gottlob 1995) to study logic-based abduction.

Cardinality (\leq). Given any $S, S' \subseteq \Sigma$, we write $S \leq S'$ if $|S| \leq |S'|$. The intuition of using this preference is that we always prefer the rule set with the maximum number of rules which are most likely to be correct.

Priority Levels (\subseteq_P, \leq_P). Every prioritization P of Σ is a tuple $\langle P_1, \dots, P_n \rangle$ where $\{P_1, \dots, P_n\}$ is a partition of Σ . Given a prioritization $P = \langle P_1, \dots, P_n \rangle$ of Σ , the preferences \subseteq_P and \leq_P can be defined as follows:

- **Prioritized set inclusion (\subseteq_P):** Given $S, S' \subseteq \Sigma$, we write $S \subseteq_P S'$ if $S \cap P_i = S' \cap P_i$ for every $1 \leq i \leq n$, or there is some $1 \leq i \leq n$ such that $S \cap P_i \subsetneq S' \cap P_i$ and for all $1 \leq j < i$, $S \cap P_j = S' \cap P_j$.
- **Prioritized cardinality (\leq_P):** Given $S, S' \subseteq \Sigma$, we write $S \leq_P S'$ if $|S \cap P_i| = |S' \cap P_i|$ for every $1 \leq i \leq n$, or there is some $1 \leq i \leq n$ such that $|S \cap P_i| < |S' \cap P_i|$ and for all $1 \leq j < i$, $|S \cap P_j| = |S' \cap P_j|$.

Weights (\leq_w). A *weight assignment* is a function $w : \Sigma \rightarrow \mathbb{N}$. Given two sets $S, S' \subseteq \Sigma$ and a weight assignment w , we write $S \leq_w S'$ if $\sum_{r \in S} w(r) \leq \sum_{r \in S'} w(r)$.

In the rest of this paper, we will fix P as a prioritization and w as a weight assignment unless otherwise noted.

Example 2 (Example 1 continued). *Let Σ and D be the same as in Example 1. Then the repairs w.r.t. \subseteq and D are:*

$$\begin{aligned} &\{(1), (3), (4), (5), (6)\}, & \{(1), (2), (3), (4), (5)\}, \\ &\{(1), (2), (4), (5), (6)\}, & \{(1), (2), (3), (4), (6), (7)\}, \\ &\{(2), (3), (4), (5), (6), (7)\}. \end{aligned}$$

The repairs w.r.t. \leq and D include:

$$\{(1), (2), (3), (4), (6), (7)\}, \quad \{(2), (3), (4), (5), (6), (7)\}.$$

Let $P = \langle P_1, P_2 \rangle$ where P_1, P_2 are the same as in Example 1. Then the repairs w.r.t. \subseteq_P and D are shown in Example 1, and the repairs w.r.t. \leq_P and D are:

$$\{(1), (2), (3), (4), (6), (7)\}.$$

Let w be the weight assignment that maps each rule to its index. Then the only repair w.r.t. \leq_w and D is:

$$\{(2), (3), (4), (5), (6), (7)\}.$$

Let Q_a be query “Mammal(a)” and Q_b be query “Bird(a)”, then we have $\langle D, (\Sigma, \subseteq) \rangle \models Q_a$ and $\langle D, (\Sigma, \subseteq_P) \rangle \models Q_a$, but $\langle D, (\Sigma, \subseteq) \rangle \not\models Q_b$ and $\langle D, (\Sigma, \subseteq_P) \rangle \not\models Q_b$.

We find that repairs under \subseteq_P , \leq , \leq_P , and \leq_w are the subset of the inclusion-maximal repairs.

Theorem 1. *The repairs under \subseteq_P , \leq , \leq_P , \leq_w are the subset of the repairs under \subseteq .*

Proof. Let S be the set of repairs under \subseteq , S_P be the set of repairs under \subseteq_P , we prove that $S_P \subseteq S$. Suppose for contradiction that $S_P \not\subseteq S$, then there exists a repair R , $R \in S_P$ and $R \notin S$. Because the repairs in S are inclusion-maximal, we have $R \subset R'$ for some $R' \in S$. It is clear that $R \subsetneq_P R'$, then R is not a \subseteq_P repair which contradict our assumption.

The rest semantics can be proved similarly. \square

Complexity Results

In this section, we study the data and combined complexity for query entailment under our rule repair semantics. In particular, we focus on the following decision problems:

- **Data complexity:** Fixing a preference-based ontology O and an NBCQ Q , given any database D as input, deciding whether $\langle D, O \rangle \models Q$.
- **Combined complexity:** Given any preference-based ontology O , any NBCQ Q and any database D as input, deciding whether $\langle D, O \rangle \models Q$.

To measure the size of input, we fix a natural way to represent a database D , a normal rule set Σ , an NBCQ Q , a prioritization P and a weight assigning function w , and let $\|D\|, \|\Sigma\|, \|Q\|, \|P\|, \|w\|$ denote the sizes of D, Σ, Q, P, w , respectively, w.r.t. the fixed representing approach. Given a preference-based ontology $O = (\Sigma, \preceq)$, we define

$$\|O\| := \begin{cases} \|\Sigma\| & \text{if } \preceq \in \{\subseteq, \leq\}, \\ \|\Sigma\| + \|P\| & \text{if } \preceq \in \{\subseteq_P, \leq_P\}, \\ \|\Sigma\| + \|w\| & \text{if } \preceq = \leq_w. \end{cases}$$

By properly representing, we can have that $\|O\| = \|\Sigma\|^{O(1)}$.

The following result is obvious.

Proposition 2. *Let O be a preference-based ontology (Σ, \preceq) , where $\preceq \in \{\subseteq, \leq, \subseteq_P, \leq_P, \leq_w\}$. Then, given any subsets $S, S' \subseteq \Sigma$, deciding whether $S \prec S'$ is in $\text{DTIME}(\|O\|^{O(1)})$.*

Now, let us consider the complexity of query answering for R-acyclic and R-stratified rule sets under our semantics.

Algorithm 1: PRQA(D, O, Q)

Input : a database D , a preference-based ontology $O = (\Sigma, \preceq)$, and a Boolean query Q

Output: *true* if $\langle D, O \rangle \models Q$, and *false* otherwise

```

1 foreach  $S \subseteq \Sigma$  do
2   if  $D \cup S$  has at least one stable model then
3      $isRepair := true$ ;
4     foreach  $S' \subseteq \Sigma$  with  $S \prec S'$  do
5       if  $D \cup S'$  has at least one stable model then
6          $isRepair := false$ ;
7         break;
8   if  $isRepair$  and  $D \cup S \not\models_s Q$  then
9     return false;
10 return true;

```

Theorem 2. *Let O be a preference-based ontology (Σ, \preceq) , where Σ is R-acyclic and R-stratified, and $\preceq \in \{\subseteq, \leq, \subseteq_P, \leq_P, \leq_w\}$. Given a database D and a safe NBCQ Q , deciding whether $\langle D, O \rangle \models Q$ is PTIME-complete for data complexity, and 2EXPTIME-complete for combined complexity.*

Proof. Let D be a database and Q be a safe NBCQ. By the definition of semantics, it is easy to verify that the problem of deciding whether $\langle D, O \rangle \models Q$ can be solved by Alg. 1.

First, we consider the data complexity. In Alg. 1, let us fix a preference-based ontology $O = (\Sigma, \preceq)$ as defined in this theorem, fix a safe NBCQ Q , and let D be the only input. As Σ is R-acyclic and R-stratified, by Theorem 5 in (Magka, Krötzsch, and Horrocks 2013), it is clear that the body of the second loop (the inside one) in Alg. 1 is computable in PTIME w.r.t. D . (Note that the existence of stable models can be reduced to the query answering problem in a routine way.) Since the second loop will be repeated a constant times, and by Proposition 2 the loop condition can be checked in a constant time. (Note that the rule set Σ is fixed now.) Thus, the second loop can be computed in PTIME w.r.t. the size of D . By a similar argument, we can show that Alg. 1 can be implemented in PTIME w.r.t. D .

This then completes the proof of membership. The hardness follows from the PTIME-hardness of Datalog for data complexity, see, e.g., (Dantsin et al. 2001).

Next, we prove the combined complexity. Again, first address the membership. Let n be the number of rules in Σ . Clearly, the body of the second loop will be repeated at most 2^n times. By Theorem 9 in (Magka, Krötzsch, and Horrocks 2013), it is computable in $\text{DTIME}(2^{2^{\|\Sigma\|^{O(1)}}})$. By Proposition 2, it is also clear that the loop condition can be checked in $\text{DTIME}(\|O\|^{O(1)})$. So, the second loop is computable in $\text{DTIME}(2^{2^{\|O\|^{O(1)}}})$ since $n \leq \|\Sigma\| \leq \|O\|$. By a similar evaluation, we know that the algorithm is implementable in $\text{DTIME}(2^{2^{\|O\|^{O(1)}}})$. Thus, the combined complexity is in 2EXPTIME. And the hardness follows from the 2EXPTIME-hardness of query answering of the R-acyclic language (Magka, Krötzsch, and Horrocks 2013) and the fact that $D \cup \Sigma \models_s Q$ iff $\langle D, (\Sigma^*, \preceq) \rangle \models q$, where Σ^* is $\Sigma \cup \{Q \rightarrow q\}$ and q a fresh 0-ary relational symbol. \square

Theorem 3. *Let O be a preference-based ontology (Σ, \preceq) , where Σ is R-acyclic with full negations and $\preceq \in \{\subseteq, \leq, \subseteq_P, \leq_P, \leq_w\}$. Then, given a database D and a safe NBCQ Q , deciding whether $\langle D, O \rangle \models Q$ is in BH for data complexity and in $\Delta_2\text{-2EXPTIME}$ for combined complexity.*

Proof. We first prove the data complexity. To do this, we need to define some notations. Let \mathcal{R} be the schema of Σ . Given any subset X of Σ , let L^X be the set of all \mathcal{R} -databases D such that

1. $D \cup X$ has at least one stable model, and
2. $D \cup X \models_s Q$ does not hold, and
3. for all $Y \subseteq \Sigma$ with $X \prec Y$, $D \cup Y$ has no stable model.

Let L denote the union of L^X for all subsets X of Σ . By the definition of the rule repair semantics, it is easy to see that $\langle D, O \rangle \models Q$ iff there is no $X \subseteq \Sigma$ such that $D \in L^X$, iff D does not belong to L . Thus, if the following claim is true, by the definition of BH we then have the desired result. Notice that the complexity class BH is closed under complement.

Claim. Given any subset X of Σ , it is in DP (*w.r.t.* the size of input database D) to determine whether $D \in L^X$.

Now, it remains to show the claim. Fix a subset $X \subseteq \Sigma$. Let L_1 denote the set of all \mathcal{R} -databases such that conditions 1 and 2 hold, and let L_2 denote the set of all \mathcal{R} -databases such that the condition 3 holds. According to Theorem 2 in (Magka, Krötzsch, and Horrocks 2013), L_1 is in NPTIME and L_2 in coNPTIME . (Note that, as Σ and X are fixed, the number of subsets Y is independent on the size of input database; thus L_2 should be in coNPTIME .) By definition, $L^X = L_1 \cap L_2$ is in DP. This proves the data complexity.

Next, we show the combined complexity. It is clear that $\langle D, O \rangle \models Q$ holds iff there does not exist $S \subseteq \Sigma$ such that

1. $D \cup S$ has at least one stable model, and
2. $D \cup S \models_s Q$ does not hold, and
3. for all $S' \subseteq \Sigma$ with $S \prec S'$, $D \cup S'$ has no stable models.

By Theorem 2 in (Magka, Krötzsch, and Horrocks 2013) and an analysis similar to that in Theorem 2 (for combined complexity), it is not difficult to see that, fixing $S \subseteq \Sigma$, both conditions 1 and 2 are in coN2EXPTIME , and condition 3 is in N2EXPTIME. For “there does not exist $S \subseteq \Sigma$ ”, we can simply enumerate all subsets S , which can be done in $2^{|\Sigma|}$ times. Therefore, query answering under the mentioned semantics must be in $\Delta_2\text{-2EXPTIME}$ for combined complexity, which is as desired. \square

Now let us focus on guarded rules. The proof of the following is similar to that of Theorem 2, but employs the complexity results in (Cali, Gottlob, and Lukasiewicz 2012). The only thing we should be careful about is the constraints.

Theorem 4. *Let O be a preference-based ontology (Σ, \preceq) , where Σ is guarded and stratified, and $\preceq \in \{\subseteq, \leq, \subseteq_P, \leq_P, \leq_w\}$. Given a database D and a covered NBCQ Q , deciding whether $\langle D, O \rangle \models Q$ is PTIME-complete for data complexity, and 2EXPTIME-complete for combined complexity.*

For guarded rules with full negations, we have some results as below, where the proof for data complexity is similar to that in Theorem 3, and the proof for combined complexity is similar to that in Theorem 2. Both results rely on the corresponding complexity results in (Gottlob et al. 2014).

Theorem 5. *Let O be a preference-based ontology (Σ, \preceq) , where Σ is guarded, and $\preceq \in \{\subseteq, \leq, \subseteq_P, \leq_P, \leq_w\}$. Then, given a database D and a covered NBCQ Q , deciding whether $\langle D, O \rangle \models Q$ is in BH for data complexity and 2EXPTIME-complete for combined complexity.*

Finally, we conclude the results of this section as follows:

	Data complexity	Combined complexity
RA + RS	PTIME-complete	2EXPTIME-complete
RA + Full	in BH	in $\Delta_2\text{-2EXPTIME}$
G + Stra	PTIME-complete	2EXPTIME-complete
G + Full	in BH	2EXPTIME-complete

Table 1: The data and combined complexity of Boolean query answering over normal rule sets under preference-based semantics for 5 types of preferred rule repairs, including $\subseteq, \leq, \subseteq_P, \leq_P$, and \leq_w . Here, “RA” means “R-acyclic rule sets”, “G” means “guarded rule sets”, “RS” means “with R-stratified negations”, “Stra” means “with stratified negations”, and “Full” means “with full negations”.

Experimental Evaluation

To demonstrate the effectiveness, we have implemented a prototype system for query answering of R-acyclic rule languages under the rule-repair semantics *w.r.t.* $\leq, \subseteq_P, \leq_P$ and \leq_w , by calling a state-of-the-art ASP solver.

From Query Answering to ASP

To improve the efficiency, we adopt particular algorithm for each rule-repair semantics. The algorithms are all based on breadth-first search. Finding rule repairs *w.r.t.* \subseteq uses the basic process illustrated in Alg. 1, and exponential checking will be conducted during the process. For rule repairs *w.r.t.* \leq , though it works better than \subseteq for the reason that there

is no need to search the rest levels once it finds consistent sets. As for rule repairs *w.r.t.* \subseteq_P , we design an algorithm which iterates over the rules from low to high prioritization. Once finding consistent results in the rules with lower prioritization, the searching stops. It's known that \leq_P can be translated into \leq_w , but not vice versa. As for \leq_w , we search by deleting rules from the lowest weight to the greatest.

Experiments

We developed a prototype system QAIER¹ (Query Answering with Inconsistent Existential Rules) in C++. QAIER can answer queries with inconsistent R-acyclic rule sets. When it needs to check the existence of stable models, QAIER invokes an ASP solver clingo-4.4.0².

Benchmarks. To estimate the performance of QAIER in a view of data complexity, we use the modified LUBM³ as a benchmark. Because LUBM is not R-acyclic, we modified LUBM by changing atoms and deleting rules to make sure that modified LUBM is R-acyclic. We use Hermit⁴ to transform the modified LUBM ontology into DL-clauses, and replace at-least number restrictions in head atoms with existential quantification, then get 127 rules. Next we add default negations or constraints, and introduce the prioritization and weight under rule repair semantics. Considering that the number of default negations or constraints would not be very large, we introduce 9-11 for each instance. The introduced prioritization or weight depends on the reliability of the rules. We use the EUDG⁵ to generate a database. By $dXtY$ (Table 2) we mean that the instance involves X thousands facts and Y unreliable rules. For the performance in the view of combined complexity, we use the modified ChEBI (Magka, Krötzsch, and Horrocks 2013) as a benchmark. By $cXtY$ (Table 3) we mean that the instance involves X molecules and chemical classes and Y unreliable rules.

Experimental results. Table 2 (Table 3 respectively)⁶ shows the data (combined, respectively) complexity performance among rule repairs scale up, when $\#facts$ and $\#negs$ ($\#rules$ and $\#negs$, respectively) grow. t_{\subseteq} , t_{\leq} , t_{\subseteq_P} , t_{\leq_P} , or t_{\leq_w} records the queries answering time. Each instance is computed three times and taken the average. Because QAIER computes all the stable models, the sizes or the types of queries are not the important issues. Clearly, rule repairs *w.r.t.* \subseteq_P , \leq_P , and \leq_w have better performances than those of \subseteq and \leq , which is due to the few number of unreliable rules. This condition can be easily found in realistic cases because most of the rules are reliable, while the latest learned rules considered unreliable are few.

¹<http://ss.sysu.edu.cn/%7ewh/qaier.html>

²clingo-4.4.0. sourceforge.net/projects/potassco/files/clingo/

³LUBM. <http://swat.cse.lehigh.edu/projects/lubm/>

⁴Hermit. <http://www.hermit-reasoner.com/>

⁵EUDG. <http://www.informatik.uni-bremen.de/~clu/combined/>

⁶All experiments run in Linux Ubuntu 14.04.1 LTS on a HP compaq 8200 elite with a 3.4GHz Intel Core i7 and 4G 1333 MHz memory. Real numbers in the tables figure the run time (in seconds) of query answering. If the time exceeds 1800 seconds, we write it as “—”. $\#facts$, $\#negs$, and $\#rules$ means the number of facts, default negations and constraints, and rules respectively.

id	#facts	#negs	t_{\subseteq}	t_{\leq}	t_{\subseteq_P}	t_{\leq_P}	t_{\leq_w}
d6t3	6000	9	1757.4	956.7	11.4	12.6	17.5
d6t5	6000	11	—	968.9	19.1	32.5	47.5
d12t5	12000	11	—	1743.3	35.7	76.9	50.2
d30t5	30000	11	—	—	81.8	187.9	124.6
d110t5	110449	11	—	—	365.4	267.5	149.6
d252t3	252498	9	—	—	278.4	466.6	147.2
d252t5	252498	11	—	—	843.7	579.1	186.4
d500t3	500000	9	—	—	308.6	605.5	168.9
d500t5	500000	11	—	—	1464.9	619.3	200.2
d686t3	686028	9	—	—	410.8	615.2	230.5
d686t5	686028	11	—	—	—	—	247.2
d1236t5	1236999	11	—	—	—	—	432.4

Table 2: Experiments for the Modified LUBM

id	#rules	#negs	t_{\subseteq}	t_{\leq}	t_{\subseteq_P}	t_{\leq_P}	t_{\leq_w}
c1t1	170	9	470.1	9.3	0.9	0.9	0.3
c1t3	170	10	909.1	723.2	7.1	6.6	4.3
c1t5	170	12	911.2	735.2	28.9	28.3	12.3
c2t1	253	9	1155.2	19.4	8.2	7.6	0.8
c2t3	253	10	1171.9	1282.6	32.8	32.8	49.8
c2t5	253	12	1136.9	1253.3	127.8	131.4	169.4
c3t1	361	9	—	—	1423.2	1291.4	35.4
c3t3	361	10	—	—	—	—	—

Table 3: Experiments for the Modified ChEBI

Related Work and Conclusions

In terms of changing the rule set/Tbox for repair, Meyer et al. (2006) proposed an algorithm running in EXPTIME that finds maximally concept-satisfiable terminologies in \mathcal{ALC} . Scharrenbach et al. (2010) showed that probabilistic description logics can be used to resolve conflicts and receive a consistent knowledge base from which inferences can be drawn again. Also Qi and Du (2009) proposed model-based revision operators for terminologies in DL, and Wang et al. (2014) introduced a model-theoretic approach to ontology revision. In order to address uncertainty arising from inconsistency, Gottlob et al. (2013) extended the Datalog[±] with probabilistic uncertainty based on Markov logic networks. Several works have focused on reasoning with inconsistent ontologies, see (Huang, van Harmelen, and ten Teije 2005; Haase et al. 2005) and references therein. This paper shows that for R-acyclic existential rules with R-stratified or guarded existential rules with stratified negations both the data complexity and combined complexity of query answering under the rule repair semantics do not increase.

We have developed a general framework to handle inconsistent existential rules with default negations. Within this framework, we analyzed the data and combined complexity of inconsistency-tolerant query answering under rule repair semantics. We proposed approaches simulating queries answering under rule repairs with calling ASP solvers and developed a prototype system called QAIER. Our experiments show that QAIER can scale up to large databases under rule repairs in practice. Future work will focus on identifying first order rewritable classes under rule repair semantics.

Acknowledgments

We thank the reviewers for their comments and suggestions for improving the paper. The authors would like to thank Yongmei Liu and her research group for their helpful and informative discussions. Hai Wan's research was in part supported by the National Natural Science Foundation of China under grant 61573386 and 61463044, Natural Science Foundation of Guangdong Province of China under grant S2012010009836, and Guangzhou Science and Technology Project under grant 2013J4100058.

References

- Alviano, M., and Pieris, A. 2015. Default negation for non-guarded existential rules. In *Proceedings of the 34th ACM Symposium on Principles of Database Systems, PODS 2015, Melbourne, Australia, May 31 - June 4, 2015*, 79–90.
- Arenas, M.; Bertossi, L. E.; and Chomicki, J. 1999. Consistent query answers in inconsistent databases. In *Proceedings of the Eighteenth ACM SIGACT-SIGMOD-SIGART Symposium on Principles of Database Systems, May 31 - June 2, 1999, Philadelphia, Pennsylvania, USA*, 68–79.
- Bienvenu, M.; Bourgaux, C.; and Goasdoué, F. 2014. Querying inconsistent description logic knowledge bases under preferred repair semantics. In *Proceedings of the Twenty-Eighth AAAI Conference on Artificial Intelligence, July 27 -31, 2014, Québec City, Québec, Canada*, 996–1002.
- Calì, A.; Gottlob, G.; and Lukasiewicz, T. 2012. A general datalog-based framework for tractable query answering over ontologies. *Journal Web Semantics* 14:57–83.
- Chang, R., and Kadin, J. 1996. The boolean hierarchy and the polynomial hierarchy: A closer connection. *SIAM Journal on Computing* 25(2):340–354.
- Chomicki, J. 2007. Consistent query answering: Five easy pieces. In *Proceedings of 11th International Conference, Database Theory - ICDT 2007, Barcelona, Spain, January 10-12, 2007*, 1–17.
- Dantsin, E.; Eiter, T.; Gottlob, G.; and Voronkov, A. 2001. Complexity and expressive power of logic programming. *ACM Computing Surveys* 33(3):374–425.
- Du, J.; Qi, G.; and Shen, Y. 2013. Weight-based consistent query answering over inconsistent *SHIQ* knowledge bases. *Knowledge Information System* 34(2):335–371.
- Eiter, T., and Gottlob, G. 1995. The complexity of logic-based abduction. *Journal of the ACM* 42(1):3–42.
- Ferraris, P.; Lee, J.; and Lifschitz, V. 2011. Stable models and circumscription. *Artificial Intelligence* 175(1):236–263.
- Gelfond, M., and Lifschitz, V. 1988. The stable model semantics for logic programming. In *Proceedings of the Fifth International Conference and Symposium Logic Programming, Seattle, Washington, August 15-19, 1988 (2 Volumes)*, 1070–1080.
- Gottlob, G.; Lukasiewicz, T.; Martinez, M. V.; and Simari, G. I. 2013. Query answering under probabilistic uncertainty in datalog+/- ontologies. *Annals of Mathematics and Artificial Intelligence* 69(1):37–72.
- Gottlob, G.; Hernich, A.; Kupke, C.; and Lukasiewicz, T. 2014. Stable model semantics for guarded existential rules and description logics. In *Proceedings of the Fourteenth International Conference Principles of Knowledge Representation and Reasoning, KR 2014, Vienna, Austria, July 20-24, 2014*, 258–267.
- Haase, P.; van Harmelen, F.; Huang, Z.; Stuckenschmidt, H.; and Sure, Y. 2005. A framework for handling inconsistency in changing ontologies. In *Proceedings of The Semantic Web - ISWC 2005, 4th International Semantic Web Conference, ISWC 2005, Ireland, November 6-10, 2005*, 353–367.
- Huang, Z.; van Harmelen, F.; and ten Teije, A. 2005. Reasoning with inconsistent ontologies. In *Proceedings of the Nineteenth International Joint Conference on Artificial Intelligence, IJCAI 2005, Edinburgh, Scotland, UK, July 30-August 5, 2005*, 454–459.
- Kalyanpur, A.; Parsia, B.; Sirin, E.; and Grau, B. C. 2006. Repairing unsatisfiable concepts in OWL ontologies. In *Proceedings of the Semantic Web: Research and Applications, 3rd European Semantic Web Conference, ESWC 2006, Budva, Montenegro, June 11-14, 2006*, 170–184.
- Lehmann, J.; Auer, S.; Bühmann, L.; and Tramp, S. 2011. Class expression learning for ontology engineering. *Journal Web Semantics* 9(1):71–81.
- Lembo, D.; Lenzerini, M.; Rosati, R.; Ruzzi, M.; and Savo, D. F. 2010. Inconsistency-tolerant semantics for description logics. In *Proceedings of Web Reasoning and Rule Systems - Fourth International Conference, RR 2010, Bressanone/Brixen, Italy, September 22-24, 2010*, 103–117.
- Lukasiewicz, T.; Martinez, M. V.; Pieris, A.; and Simari, G. I. 2015. From classical to consistent query answering under existential rules. In *Proceedings of the Twenty-Ninth AAAI Conference on Artificial Intelligence, January 25-30, 2015, Austin, USA*.
- Lukasiewicz, T.; Martinez, M. V.; and Simari, G. I. 2012. Inconsistency handling in datalog+/- ontologies. In *Proceedings of 20th European Conference on Artificial Intelligence, ECAI 2012 Including Prestigious Applications of Artificial Intelligence (PAIS-2012) System Demonstrations Track, Montpellier, France, August 27-31, 2012*, 558–563.
- Lukasiewicz, T.; Martinez, M. V.; and Simari, G. I. 2013. Complexity of inconsistency-tolerant query answering in datalog+/- . In *Informal Proceedings of the 26th International Workshop on Description Logics, Ulm, Germany, July 23 - 26, 2013*, 488–500.
- Magka, D.; Krötzsch, M.; and Horrocks, I. 2013. Computing stable models for nonmonotonic existential rules. In *Proceedings of the 23rd International Joint Conference on Artificial Intelligence, IJCAI 2013, Beijing, China, August 3-9, 2013*, 1031–1038.
- Meyer, T. A.; Lee, K.; Booth, R.; and Pan, J. Z. 2006. Finding maximally satisfiable terminologies for the description logic ALC. In *Proceedings of the Twenty-First National Conference on Artificial Intelligence and the Eighteenth Innovative Applications of Artificial Intelligence Conference, July 16-20, 2006, Boston, Massachusetts, USA*, 269–274.
- Qi, G., and Du, J. 2009. Model-based revision operators for terminologies in description logics. In *Proceedings of the 21st International Joint Conference on Artificial Intelligence IJCAI 2009, Pasadena, California, USA, July 11-17, 2009*, 891–897.
- Scharrenbach, T.; Grütter, R.; Waldvogel, B.; and Bernstein, A. 2010. Structure preserving tbox repair using defaults. In *Proceedings of the 23rd International Workshop on Description Logics (DL 2010), Waterloo, Ontario, Canada, May 4-7, 2010*, 384–395.
- Wang, Z.; Wang, K.; Qi, G.; Zhuang, Z.; and Li, Y. 2014. Instance-driven tbox revision in dl-lite. In *Informal Proceedings of the 27th International Workshop on Description Logics, Vienna, Austria, July 17-20, 2014*, 734–745.
- Zhang, H.; Zhang, Y.; and You, J.-H. 2015. Existential rule languages with finite chase: Complexity and expressiveness. In *Proceedings of the Twenty-Ninth AAAI Conference on Artificial Intelligence, January 25-30, 2015, Austin, Texas, USA*.