# au arepsilon 2asp : Implementing $\mathcal{T} \mathcal{E}$ via Answer Set Programming

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**Abstract.** This paper studies computational issues related to the problem of reasoning about action and change in timed domains by translating it into answer set programming paradigm. Based on this idea, we implement a new action and change reasoning solver -  $\tau \varepsilon 2$ asp with a polynomial translation without increasing its complexity for checking satisfiability and entailment in  $\mathcal{T}\mathcal{E}$  and report some experimental results.

## 1 Introduction

In order to extend the study of reasoning about action and change 1 - 3 to handle timed domains, the so-called timed action language  $\mathcal{A}_{\mathcal{T}}$  4, narrative-based action logic  $AL_{TC}^2$  5, timed action language  $\mathcal{T}_{\mathcal{E}}$  6 are proposed. This paper focuses on providing a computational realization of solutions to reasoning about problems described by  $\mathcal{T}_{\mathcal{E}}$ , regarded as an extension of action language  $\mathcal{E}$  7. Reasoning about action and change in timed domains, is not only considered as toy problems, e.g., timed Yale Shooting, but also studied as real-world applications, e.g., timed automaton 8, assembly plant 8, and rail road crossing control 9. Compared with  $\mathcal{A}_{\mathcal{T}}$  and  $AL_{TC}^2$ ,  $\mathcal{T}_{\mathcal{E}}$  overcomes semantics defect of  $\mathcal{A}_{\mathcal{T}}$  and can be implemented by satisfiability modulo theory (SMT) 10 solvers.

This paper studies a link between timed action language  $\mathcal{TE}$  and declarative logic programming approach of answer set programming (ASP) [11], intending to implement reasoning about action and change in timed domains by translating  $\mathcal{TE}$  into ASP [12], a promising approach implemented by a number of sophisticated solvers [13-16]. A study on encodings of reasoning problems in language  $\mathcal{E}$  was developed by exploiting the relation between  $\mathcal{E}$  and ASP [7].

This translation is not only theoretically interesting but also of practical relevance. Based on the translation, we implement a new action and change reasoning solver, called  $\tau \varepsilon 2$ asp. We report some experimental results, which demonstrate that the performance of  $\tau \varepsilon 2$ asp is rather satisfactory.

The paper is organized as follows. Section 2 recalls some basic notions and definitions in timed action language  $\mathcal{TE}$ . Section 3 presents the translation from  $\mathcal{TE}$  to answer set programming. Section 4 explains the implementation of  $\tau\varepsilon 2$ asp and reports some experiments. Finally, Section 5 concludes the paper.

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## 2 Preliminaries

We consider Shen's timed action language  $\mathcal{TE}[6]$ , in which a  $\mathcal{TE}$  theory should be composed of a 5-tuple and propositions in  $\mathcal{TE}$  are defined as follows:

- C-proposition is either of the form: A initiates F resets  $\lambda$  if C when  $\Psi$ , or A terminates F resets  $\lambda$  if C when  $\Psi$ , means action A can make fluent F hold (or not hold) and reset a set of clocks C;
- H-proposition: A happens-at T means action A happens at time T;
- T-proposition: L holds-at T means fluent L does (or doesn't) hold at T.

C-propositions introduce a new kind of action effect: clock resetting, and action precondition: clock constraint. Resetting a clock is to start counting its ticks and execute an action satisfying related clock constraints besides fluent preconditions. H- and t-propositions remain the same w.r.t.  $\mathcal{E}$  [17]. Model of  $\mathcal{T}\mathcal{E}$  is defined by Definition 2 in [6]. The following is an example [4-6]:

## Example 1. Timed Yale Shooting in $T\mathcal{E}$

Let  $\mathcal{TE} = \langle \mathbf{N}, \{Load, Shoot\}, \{Loaded, Alive\}, \{x\}, \mathcal{B}(\{x\}) \rangle$ , a  $\mathcal{TE}$  theory  $D_{TYS}$  of timed Yale Shooting consists of the following propositions:

- Shoot terminates Alive if  $\{Loaded\}$  when  $\{x < 5\}$
- Load initiates Loaded resets  $\{x\}$
- Alive holds-at 0, ¬Loaded holds-at 0
- Load happens-at 1, Shoot happens-at 3

Based on [6], we implement the translation from  $\mathcal{TE}$  to SMT and can reason by using SMT solver  $\mathbb{Z}3^{\boxed{1}}$ , which tops solvers of kind.

# 3 From $\mathcal{TE}$ to Answer Set Programming

# 3.1 Translation Algorithm

In order to construct translation from  $\mathcal{TE}$  to ASP, we define a  $\mathcal{TE}$  theory D with five sets,  $\alpha, \beta, Hp, Tp, Cp$ , and a N-time sequence, where  $\alpha$  and  $\beta$  is a set composed of all fluent and clocks in D respectively and Hp, Tp, Cp is a set composed of all h-, t-, or c-propositions in D respectively.

The translation from  $\mathcal{TE}$  to ASP can be accomplished by algorithm  $\square$  We can get the facts corresponding to this ASP program, named as  $\Delta$ . To specify the time sequence, a series of propositions denoting time(i.e.  $time(0), time(1), \ldots, time(N-1)$ ) should be taken into  $\Delta$ . Note that we add time(N-1) except time(N) into  $\Delta$  to terminate the reasoning at time N. For every fluent F in D would be expressed in the form of proposition like "fluent(F)." in  $\Delta_{\alpha}$ . Similarly, for every clock C in D would be translated to "clock(C)." in  $\Delta_{\beta}$ .

Assuming that h-propositions  $Hp_1, Hp_2, ..., Hp_p$  comprise of set Hp and an h-proposition  $Hp_i$  is in the form like " $a_i$  happens-at  $t_i$ ". When translated into

 $<sup>^1</sup>$  Z3-3.2 at http://research.microsoft.com/en-us/um/redmond/projects/z3/  $\,$ 

## **Algorithm 1.** Translating $\mathcal{TE}$ to answer set programming

```
input: A \mathcal{TE} theory D = \{\alpha, \beta, Hp, Tp, Cp\} with a N-time sequence
     \mathbf{output} \colon \mathsf{An} \ \mathsf{ASP} \ \mathsf{program} \ \varDelta
 1 \Delta \leftarrow time(0..N-1).
 2 forall the 1 \le i \le l do
                                                                                                                                    // fluent
      \Delta_{\alpha} \leftarrow \Delta_{\alpha} \cup \{fluent(\alpha_i).\}
 4 forall the 1 \le i \le m do
                                                                                                                                    // clocks
       \Delta_{\beta} \leftarrow \Delta_{\beta} \cup \{clock(\beta_i).\}
    for all the 1 \le i \le p do
                                                                                                                       // H-propositions
       \Delta_H \leftarrow \Delta_H \cup \{a_i(t_i).\}
     for all the 1 \le i \le q do
                                                                                                                       // T-propositions
            if f_i = \alpha_j then
              \Delta_T \leftarrow \Delta_T \cup \{fluent(f_i, t_i).\}
10
11
            if f_i = \neg \alpha_j then
              \Delta_T \leftarrow \Delta_T \cup \{-fluent(f_i, t_i).\}
12
13 forall the 1 \le i \le r do
                                                                                                                       // C-propositions
            if initiates fl_i then
14
              \Delta_C \leftarrow \Delta_C \cup \{ac_i(T), Literal(C_i), Number(\Psi_i) \rightarrow ini(fl_i, T+1).\}
15
            if terminates fl_i then
16
               \triangle C \leftarrow \triangle C \cup \{ac_i(T), Literal(C_i), Number(\Psi_i) \rightarrow tmn(fl_i, T+1).\} 
17
            if resets \lambda_i then
18
                  for all the 1 \leq j \leq s do
19
                    \Delta_C \leftarrow \Delta_C \cup \{ac_i(T), Literal(C_i), Number(\Psi_i) \rightarrow rst(\lambda_{ij}, T).\}
20
21 return \Delta \leftarrow \Delta \cup \Delta_{\alpha} \cup \Delta_{\beta} \cup \Delta_{H} \cup \Delta_{T} \cup \Delta_{C}
```

ASP,  $Hp_i$  is switched to a proposition of " $a_i(t_i)$ ." standing for an action occurrence  $a_i$  at time  $t_i$ . A t-proposition like "F holds-at T" means that the positive literal of F at time T is true, while "fluent(F,T)." should be included by the translated ASP program. Correspondingly, " $\neg F$  holds-at T" means the negative one is true and "-fluent(F,T)" should be included. Like set Hp, set Tp is made up of t-propositions  $Tp_1, Tp_2, ..., Tp_q$  and a t-proposition  $Tp_i$  is presented as " $f_i$  holds-at  $f_i$ ". The translation can generate according to the proposition with the different literal of  $f_i$ , positive and negative literal. In order for translation, we introduce two binary functions in syntax: Literal and Number. Literal translates a set of fluent literals into a series of atoms and Number translates a set of temporal constraints into atoms of body in  $\Delta_C$ .

#### 3.2 Reasoning Rules

Besides  $\Delta$  obtained from Algorithm  $\square$  we need 6 reasoning rules, denoted by  $\Gamma$ . Because rules in  $\Gamma$  construct the basic rules of reasoning about  $\mathcal{TE}$  via ASP,  $\Gamma$  can't change with the change of  $\mathcal{TE}$  theory.

Rule 1: generating initial complete knowledge.

$$1\{fluent(F,0), -fluent(F,0)\}1 \leftarrow fluent(F). \tag{1}$$

Rule 2: specifying initial clock value.

$$clock(C, 0, 0) \leftarrow clock(C).$$
 (2)

Rule 3: eliminating inconsistent fluent literals.

$$\leftarrow fluent(F,T), -fluent(F,T).$$
 (3)

Rule 4: specifying fluent persistent.

$$fluent(F, T+1) \leftarrow fluent(F, T), -tmn(F, T+1), time(T).$$
 (4)

$$-fluent(F, T+1) \leftarrow -fluent(F, T), -ini(F, T+1), time(T). \tag{5}$$

$$-ini(F, T+1) \leftarrow not \ ini(F, T+1), time(T), fluent(F). \tag{6}$$

Rule 5: describing initiation and termination effects.

$$fluent(F, T+1) \leftarrow ini(F, T+1).$$
 (7)

$$-fluent(F, T+1) \leftarrow tmn(F, T+1). \tag{8}$$

Rule 6: describing resetting effects.

$$clock(C, XX, T+1) \leftarrow XX := X+1, clock(C, X, T), -rst(C, T), time(T).$$
 (9)

$$clock(C, 1, T + 1) \leftarrow rst(F, T), time(T).$$
 (10)

## 3.3 Temporal Constraints

In addition, an ASP program of rules for defining temporal constraints is necessary, which is called by  $\Lambda$ .

$$eq(C, n, T) \leftarrow X == n, clock(C, X, T). \tag{11}$$

$$eq(C1,C2,n,T) \leftarrow X1 - X2 == n, clock(C1,X1,T), clock(C2,X2,T). \quad (12)$$

Because temporal constraints are different with the change of theory,  $\Lambda$  change accordingly. To reduce the reasoning space, all temporal constraints in theory D are substituted into  $\Lambda$  and those not in D should be removed from  $\Lambda$ .

So far the translation from a  $\mathcal{TE}$  theory to ASP is accomplished and we can obtain ASP program  $\Sigma$  consisting of  $\Delta, \Lambda, \Gamma$ . Observe that  $\Delta$ ,  $\Lambda$  and  $\Delta$  can be translated in polynomial time, so the process is polynomial.

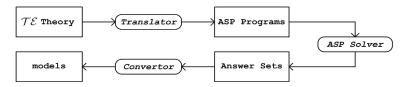
**Theorem 1.** Let D be a  $T\mathcal{E}$  theory whose time is complete and  $\Delta$  be its translated program by Algorithm 1,  $\Lambda$  be its program of temporal constraints rules,  $\Gamma$  be the ASP program of reasoning rules and an ASP program  $\Sigma$  consisting of  $\Delta, \Lambda, \Gamma$ . Then D has a model  $\langle H, K \rangle$  if and only if  $\Sigma$  has an answer set M s.t. every fluent F, every clock C and time T in D,

- 1. H(F,T) = T if and only if  $fluent(F,T) \in M$ ;
- 2.  $H(F,T) = \bot$  if and only if  $-fluent(F,T) \in M$ ;
- 3.  $K(T)(x) = x_T$  if and only if  $clock(x, x_T, T) \in M$ .

# 4 Implementation and Application

## 4.1 Implementation

A new solver is implemented for  $\mathcal{TE}$ , called  $\tau\varepsilon 2asp$  (Fig. 1). An input  $\mathcal{TE}$  theory is firstly translated to an ASP program by the *translator* in  $\tau\varepsilon 2asp$ . Then, an ASP solver Clasp is called to compute answer sets, which will be interpreted to the original  $\mathcal{TE}$  theory by a *convertor*.  $\tau\varepsilon 2asp$  is written in C, running on a machine with 2 processors  $(Intel(R)\ Core(TM) 2\ Duo\ CPU\ T7600)$  under Ubuntu 10.04.



**Fig. 1.** Outline of  $\tau \varepsilon 2$ asp

## 4.2 Experimental Results

Table  $\square$  presents some experimental results of  $\tau \varepsilon 2 \text{asp}$  and compare it with SMT solver Z3 and timed automaton tool  $HyTech^{3}$ , which record computing time of series timed Yale Shooting problem in seconds, taking the average of 50 runs.

Load	Shoot	Bound	auarepsilon 2asp	Z3	HyTech
1	3	8	0.00538	0.03348	0.02668
5	10	15	0.00716	0.04372	0.02142
20	24	30	0.00822	0.05134	0.02822
60	65	80	0.01590	0.11776	0.02108
120	123	150	0.03472	0.23290	0.02732
250	255	300	0.09556	3.85892	0.02076

Table 1. Experimental results of timed Yale Shooting problem

Because HyTech solver can only find traces, a comparator is necessary to confirm whether the trace is consistent with the action sequence. Therefore the data in column HyTech in Table  $\blacksquare$  is the time spent in finding traces to each state, omitting comparison time which is not a polynomial. Because of different computation mechanism, HyTech has little change with the increasing scale of the problem, while time via  $\tau\varepsilon2$ asp and SMT solver Z3 increases radically with the expansion of problem's scale. To sum up, considering the comparison time of Hytech,  $\tau\varepsilon2$ asp solver is the best.

## 5 Conclusions and Future Work

This paper contributes the study of reasoning about action and change in timed domains  $\mathcal{TE}$ . Theoretically,  $\mathcal{TE}$  can be translated into ASP via a polynomial algorithm and a series of rules without increasing its computation complexity.

<sup>&</sup>lt;sup>2</sup> Clasp-2.0.5 at http://www.cs.uni-potsdam.de/clasp/

<sup>&</sup>lt;sup>3</sup> HyTech-1.0.4 at http://embedded.eecs.berkeley.edu/research/hytech/

Practically, a new solver  $\tau \varepsilon 2asp$  is developed. In addition, a series of experiments about the comparison among  $\tau \varepsilon 2asp$ , SMT solver (Z3) and a timed automaton tool (HyTech) have been done. It is a pleasure that  $\tau \varepsilon 2asp$  performs much better than Z3 and HyTech. For future work, it is an important task to extend  $\mathcal{T}\mathcal{E}$  with ramification and qualification and improve  $\tau \varepsilon 2asp$ . Last but not least, to explore more applications of  $\mathcal{T}\mathcal{E}$  by using  $\tau \varepsilon 2asp$  is a job of significance.

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