UNIVERSITI SAINS MALAYSIA

Lake Modeling

- Stagnant Lake, Flowing Lake
- Simple Model at Steady State

www.usm.my

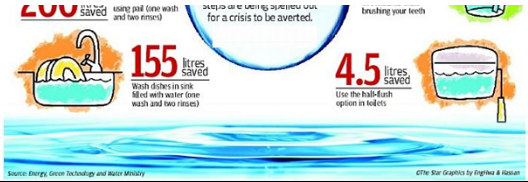
2.1 Water Quality Modeling

- Water is a natural resource vital for survival;
- Human beings who need water in daily lives;
- Increasingly becoming scarce due to reduced quantity and declined quality;

- ↑ demand and ↑ wastage + climate change ⇒ decreasing water sources.

Simple Ways to Conserve H_2O

NST
April 5, 2017
Penangites urged to reduce water usage, taps could run dry in the future



Water Waste = Water Shortage

The recent oil spillage at Sungai Selangor which resulted in a major water disruption in the Klang Valley has shown just how vulnerable and exposed our water sources are.

World Water Prices
(Source: International Water Resources Association, Vol. 1, No. 3, 2001)

Country	Price of water per m³ (US$)	Country	Price of water per m³ (US$)
Germany	1.78	Denmark	1.72
United Kingdom	1.23	The Netherlands	1.13
France	1.08	Belgium	1.01
Singapore	0.66	Italy	0.72
Spain	0.71	Finland	0.64
United States	0.54	Sweden	0.61
Australia	0.54	Canada	0.37
South Africa	0.42	Malaysia	0.09

Domestic Water Tariffs in Malaysia (RM)
Source: Malaysia Water Industry Guide, 2016

State	Domestic Rate (Average for 1st 35m³)	Industrial Rate (Average for 1st 35m³)
Sabah	0.73	1.60
Sarawak	0.62	1.13
Selangor	0.77	2.07
Perak	0.73	1.43
Sibu	0.62	1.06
Kuching	0.62	1.06
Melaka	0.75	2.00
Perlis	0.57	1.30

Domestic Water Tariffs in Malaysia (RM)
Source: Malaysia Water Industry Guide, 2016

The Star
Friday August 23, 2013
Penang to double water conservation surcharge from Sept

Saturday August 24, 2013
Surcharge hike for water wasters

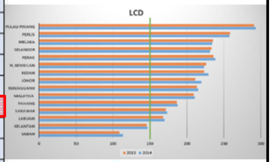
Friday March 20, 2015
Penang water rates to go up in bid to reduce consumption

State	Domestic Rate	Industrial Rate
Penang	0.32	0.94

Average domestic consumption in Penang in 2016 = 286 liter/person/day
National average = 209 liter (2016)

Domestic Consumption 2014-2015
Source: National Water Services Commission, 2016

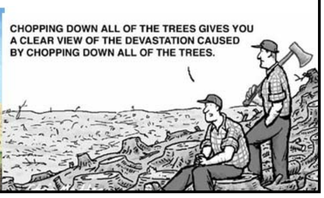
State	Consumption Per Capita Per Day l/cap/d	
	2014	2015
Johor	220	211
Kedah	229	223
Kelantan	147	146
Labuan	170	168
Melaka	234	235
N. Sembilan	223	226
Pulau Pinang	293	291
Pahang	187	187
Perak	239	236
Perlis	258	249
Sabah	114	109
Sarawak	173	172
Selangor	231	234
Terengganu	216	214
MALAYSIA	211	209



Ulu Muda, Kedah

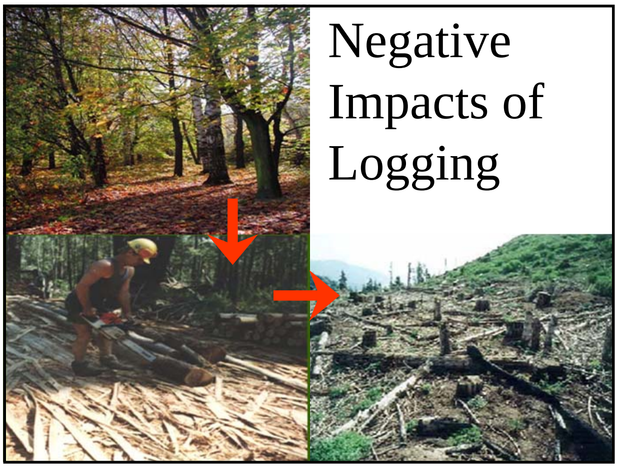
- Penang is "water-stressed";
- Supply from its main source 80%:
  Ulu Muda catchment area.

- Ulu Muda being threatened by illegal logging.



Forest as Sediment Filter

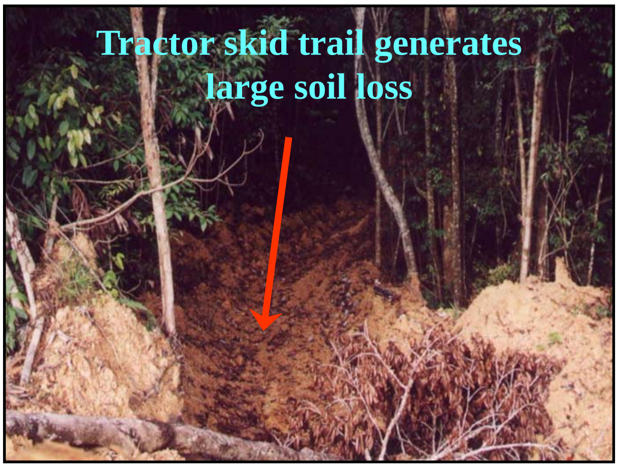
- Sediment settles as speed of flow is reduced by forest floor;
- Sediment is filtered out as sediment loaded water percolates into porous forest floor.

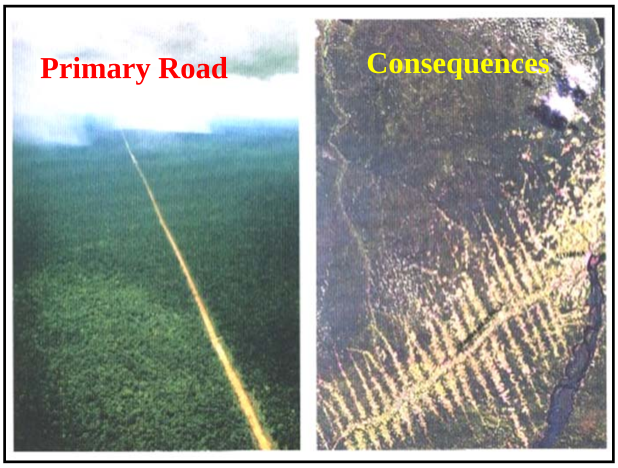


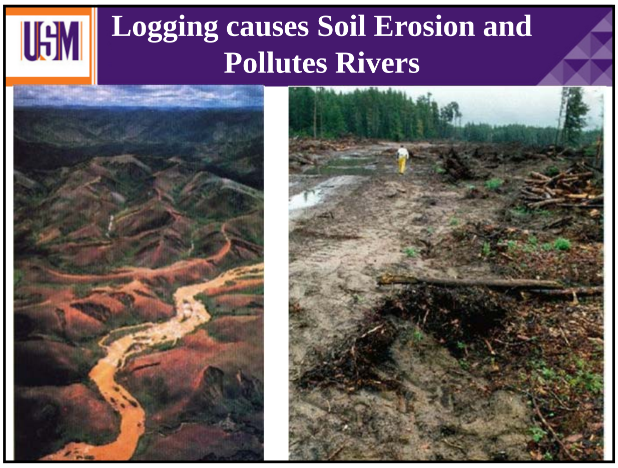
Negative Impacts of Logging

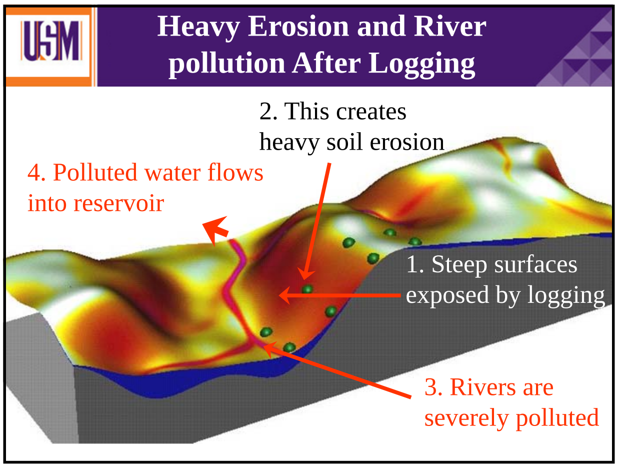
Adverse Impact of Logging

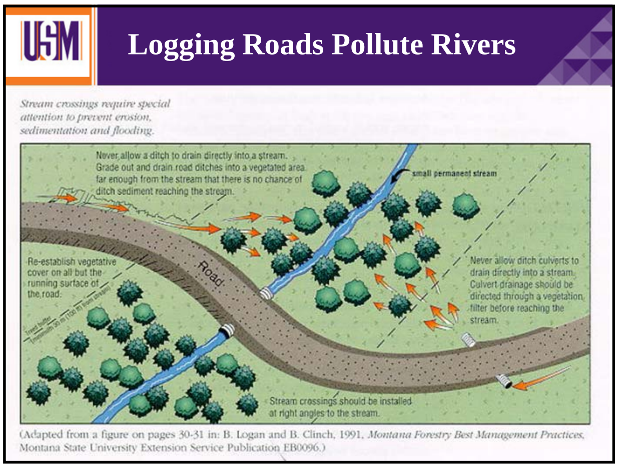

Logging roads generate large soil loss


Tractor skid trail generates large soil loss

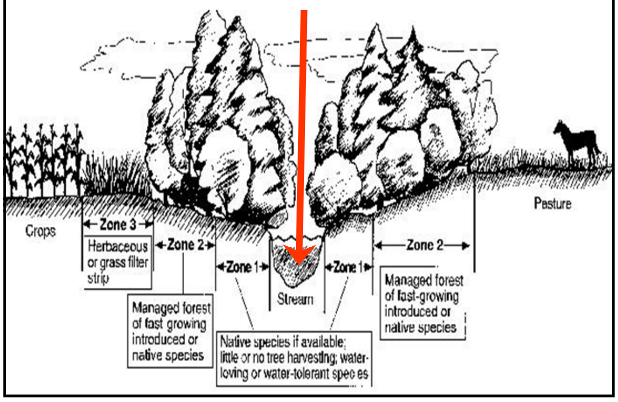

Primary Road
Consequences


USM
Logging causes Soil Erosion and Pollutes Rivers


USM
Heavy Erosion and River pollution After Logging
2. This creates heavy soil erosion
4. Polluted water flows into reservoir
1. Steep surfaces exposed by logging
3. Rivers are severely polluted

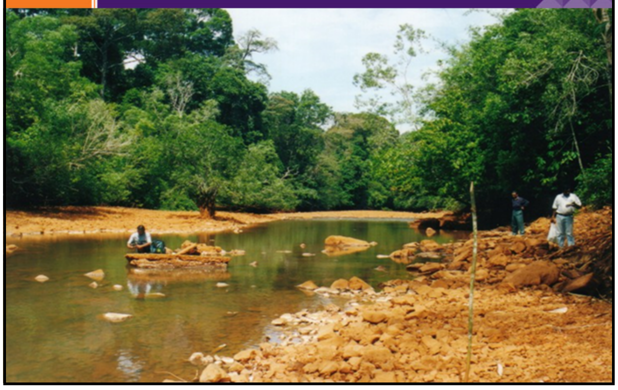

USM
Logging Roads Pollute Rivers
Stream crossings require special attention to prevent erosion, sedimentation and flooding.
Never allow a ditch to drain directly into a stream. Grade out and drain road ditches into a vegetated area far enough from the stream that there is no chance of ditch sediment reaching the stream.
small permanent stream
Re-establish vegetative cover on all but the running surface of the road.
Road
Never allow ditch culverts to drain directly into a stream. Culvert drainage should be directed through a vegetation filter before reaching the stream.
Stream crossings should be installed at right angles to the stream.
(Adapted from a figure on pages 30-31 in: B. Logan and B. Clinch, 1991, Montana Forestry Best Management Practices, Montana State University Extension Service Publication EB0096.)

Logging destroys Riparian Buffer Zones and Pollutes streams



Logging intensifies runoff and generates soil loss

Logging Contaminates Streams

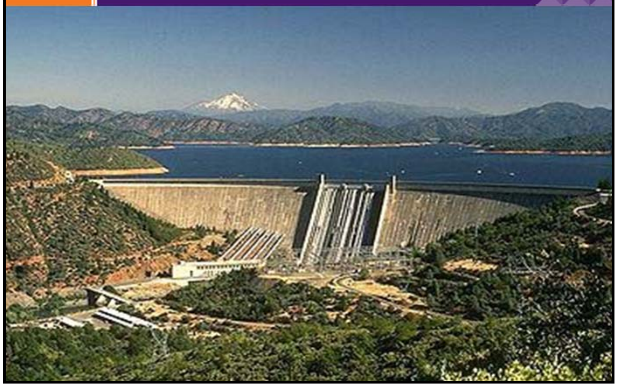


Logging Contaminates Rivers

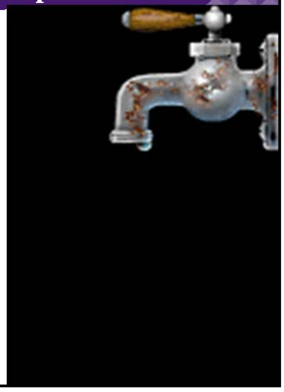
Logging Contaminates Water Supply

Logging Contaminates Reservoir

Logging Contaminates Reservoir



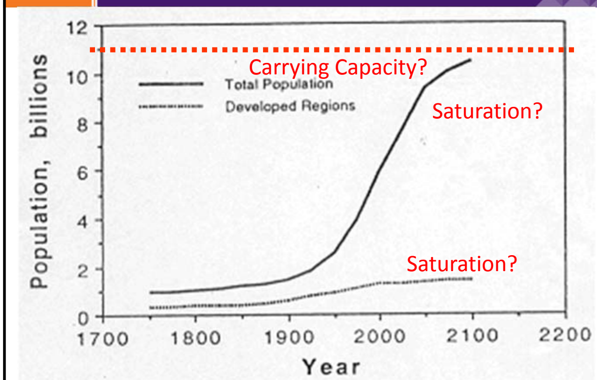
Contaminated Reservoir leads to Water Disruption



Portable (Drinking) Water

- Water is Essential;
- For how long can we endure without WATER?
  - **1 day?**
  - **1 week?**
  - **2 weeks?**
  - **1 month?**

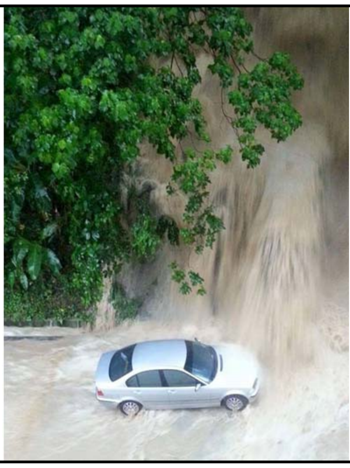
Population Growth



Use Water Wisely

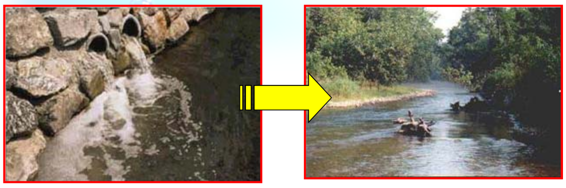
- Recycle;
- Reuse;
- Reduce;
- Reserve Forests;

Paya Terubong (2015)



2.1 Water Quality Modeling

- Water resources are polluted by effluent from industry, housing estates and others;
- ↓ WQ hinders recreational use and disrupts aquatic ecological balance.



Effluents discharge   Impact on Receiving Water

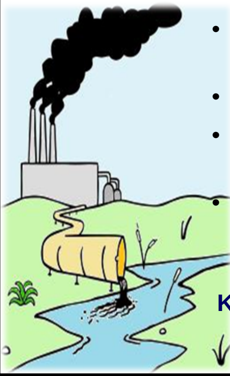
2.1 Water Quality Modeling

- ∴ To help preserve quality of water resources, following steps have to be taken seriously.
  - Determine WQ criteria for each use.
  - Determine WQ standards. *(enforced legally to limit effluent release by industry, municipal, housing,etc)*
  - Determine if the release of standard-abiding effluent will affect use of water according to classification in that area.

2.1 Water Quality Modeling

- Math models - to determine the concentration of dissolved substances in water bodies so that the pollution level can be predicted;
- Predicted water quality for mitigation measures planning;
- MSG427: focus on modelling WQ of lakes and rivers.

2.2 Reaction Kinetics



- A number of things can happen to a pollutant once it enters a water body;
- Some of these are related to transport;
- Pollutant might be changed via chemical and biochemical reactions;
- Kinetics or rate of such reactions can be expressed quantitatively by law of mass action: $\frac{dc}{dt} \propto c$

Kinetic rate is proportional to the concentration of the reactants

2.2 Reaction Kinetics

- In this section, we focus on a single reactant, of which the reaction rate is given as

$$\frac{dc}{dt} = -kc^n$$

- where c = concentration of the single reactant, n = order, t = time and k = kinetic rate;
- The most commonly employed reactions are those of zero-, first- and second-order.

2.3 Zero-order Reaction

- For a zero-order model, the eqn to integrate is

$$\frac{dc}{dt} = -k$$

- The equation can be integrated by separation of variables to yield

$$c = c_0 - kt \qquad \text{IC: } c(0) = c_0$$

- Specifies constant rate of depletion per unit time.

2.3 Zero-order Reaction

- Thus, if a plot of concentration versus time yields a straight line, we can infer that the reaction is zero-order.

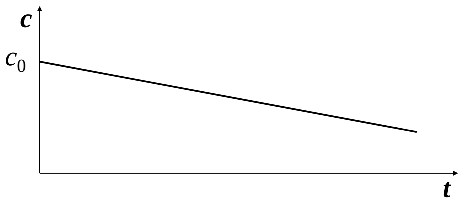


Figure 2.1 Plot of concentration vs. time for a zero-order reaction

2.4 First-order Reaction

- For a first-order model, the eqn to integrate is

$$\frac{dc}{dt} = -kc$$

- The equation can be integrated by separation of variables to yield

$$c = c_0 e^{-kt} \qquad \text{IC: } c(0) = c_0$$

- Specifies an exponential depletion.

2.3 First-order Reaction

- Thus, the concentration curve asymptotically approaches zero with time.

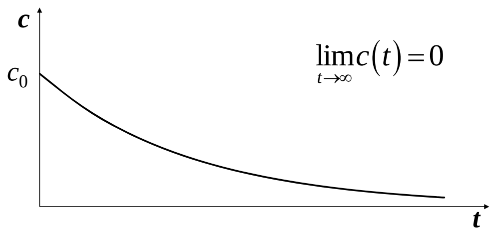


Figure 2.2 Plot of concentration vs. time for a first-order reaction

2.5 Water Quality in Lake

- Formulation of differential equations that describe a pollutant reaction in a lake system;
- Several lake systems are considered here with various flow characteristics;
- Lake models described here represent the *simplest, most ideal* mixing conditions;
- May be quite useful in checking more complicated numerical results;
- And in gaining insight to the dynamics of chemical movement in the environment.

How Complex Should Final Computational Model Be?

- **Proper model complexity is driven by:**
  - Complexity of the environmental system.
  - Complexity of the pollutants of concern.
  - Management questions and related need for accuracy.

How Complex Should Final Computational Model Be?

- **Consequences for overly simple model:**
  - Miss key processes and extrapolate inaccurately.
  - May not address relevant management questions.
  - May not be defensible to adversarial review.
  - Insufficiently adaptable to changing management requirements.

How Complex Should Final Computational Model Be?

- **Consequences for overly complex model:**
  - Adds unnecessary data collection and computational burdens.
  - Adds to uncertainty.
  - Shifts focus away from problem solutions to endless analysis.

How Complex Should Final Computational Model Be?

"Make things as simple as possible, but not any simpler."

Albert Einstein

2.5 Water Quality in Lake

- Major assumptions :
  - Chemical concentration is uniform (completely mixed);
  - Lake outlet has a concentration similar to the in-lake concentration *C*.
- Real world: volume *V*, inflow Q_{in}, outflow Q_{out} and inflow concentration C_{in} can be time-dependent variables.

2.5 Water Quality in Lake

- For the convenience of the discussion;
- In addition to completely mixed assumption;
- Following assumptions are also made:
  - Inflow concentration C_{in} is constant;
  - Volumetric flow rate into and out of the lake is constant ($Q_{in} = Q_{out} = Q$ = constant) and lake volume is constant ($dV/dt = 0$);
  - Rate of change in concentration is governed by a first-order reaction ($dC/dt = -kC$).

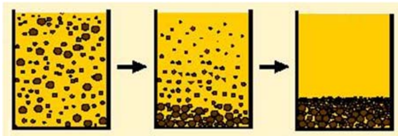
Some Related Terms

- Dynamic – flows change over time;
- Steady State – flows do not change over time; the system is in equilibrium;
- Conservative pollutants – the pollutant does not change form over time; no reactions;
- Non-conservative pollutant – the pollutant changes form over time due to chemical, physical, or biological reactions.

Some Related Terms

- Decay – substance reduced in mass due to decomposition or sedimentation;

- Sedimentation – Settlement of suspended particles.



2.5.1 Stagnant Lake

- Consider a small lake with water volume of V m^3 and initial chemical conc $C(0) = C_0$ kg/m^3;
- Chemical in lake decays at the rate of γ s^{-1};
- Chemical conc is uniform throughout the lake.

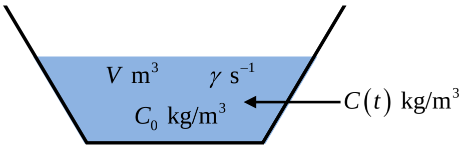
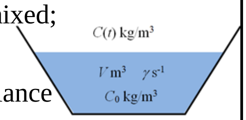


Figure 2.3 A small uniformly-mixed lake

2.5.1 Stagnant Lake

- Lake content = uniformly mixed;
- $\Rightarrow$ lake = single segment;
- By considering the mass balance in a short time step Δt;
- Following equation (in kg) is obtained.



$$\Delta M = [-\gamma M]\Delta t \text{ , kg}$$

Constant Volume $M = CV$

$$\Rightarrow \quad V\Delta C = -\gamma [CV]\Delta t$$

2.5.1 Stagnant Lake

- Lake content = uniformly mixed;
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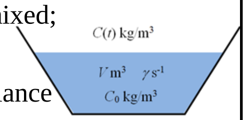
- Following equation (in kg) is obtained.



$$V\Delta C = \boxed{[-\gamma VC]}\Delta t \text{ , kg}$$

Describe chemical's decay process
γ = reaction rate;
C = conc. at time t
V = volume of system modeled

$$\Rightarrow \quad \frac{\Delta C}{\Delta t} = -\gamma C \text{ , kg m}^{-3}\text{s}^{-1}$$

2.5.1 Stagnant Lake

- Let $\Delta t \to 0$, differential equation for C is

$$\frac{\Delta C}{\Delta t} = -\gamma C \qquad (2.1)$$

- A first-order reaction model;
- Solving this DE by separation of variables with IC $C(0) = C_0$ yield the solution

$$C(t) = C_0 e^{-\gamma t}$$

Example 2.1

Consider Lake Harapan in USM with a surface area of 10000 m^2 and a mean depth of 1 m. This small lake has a uniformly mixed content containing 80000 g of pollutant that decays at the rate of 0.5 d^{-1}. The concentration of the pollutant in the lake at time t is denoted by $C(t)$.

a) Form and solve the differential equation for $C(t)$ in mg/L.

b) Sketch the graph of $C(t)$ when $t \to \infty$.

Example 2.1(a) – Solution

a) Form & solve differential equation for $C(t)$ in mg/L.

The change in concentration in the lake with respect to time t is given by the following differential equation.

$$\frac{dC}{dt} = -\gamma C$$

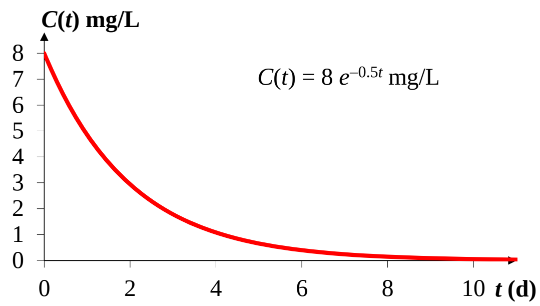
Solving the differential equation yield $C(t) = C_0 e^{-\gamma t}$.
Given that

$$C_0 = \frac{80000 \text{ g}}{10000 \times 1 \text{ m}^3} = \frac{8\times10^4 \text{ g}}{10^4 \text{ m}^3}\cdot\left(\frac{10^3 \text{ mg/g}}{10^3 \text{ L/m}^3}\right) = 8 \text{ mg/L}$$

and $\gamma = 0.5$ d^{-1}, we obtain $C(t) = 8e^{-0.5t}$ mg/L

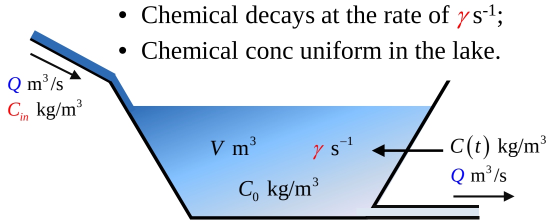
Example 2.1(b) – Solution

b) Sketch the graph of $C(t)$ when $t \to \infty$.



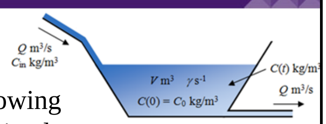
2.5.2 Flowing Lake

- Consider a small lake with V m^3 of water and initial chemical conc $C(0) = C_0$ kg/m^3;
- Water flows in and out at same rate of Q m^3/s;
- Inflow contains C_{in} kg/m^3 of chemical;
  - Chemical decays at the rate of γ s^{-1};
  - Chemical conc uniform in the lake.



2.5.2 Flowing Lake

- By considering mass balance in a short time step Δt, following equation (in kg) is obtained,



$$V\Delta C = \Big[\underbrace{QC_{in}}_{\text{Mass input via inflow}} \underbrace{- QC(t)}_{\text{Mass output via outflow}} \underbrace{- \gamma VC(t)}_{\text{Mass output via decay}}\Big]\Delta t, \text{ kg}$$

$$\Rightarrow \frac{\Delta C}{\Delta t} = \frac{QC_{in}}{V} - \frac{QC(t)}{V} - \gamma C(t), \text{ kg m}^{-3}\text{s}^{-1}$$

2.5.2 Flowing Lake

- Let $\Delta t \to 0$, then the DE for C is

$$\frac{dC}{dt} = \underbrace{\frac{QC_{in}}{V}}_{\beta} \underbrace{-\left(\underbrace{\frac{Q}{V} + \gamma}_{\alpha}\right)C}_{-\left(\frac{Q}{V}+\gamma\right)C} \qquad (2.2)$$

$$\Rightarrow \frac{dC}{dt} = \beta - \cancel{\beta}\alpha C$$

with

$$\beta = \frac{QC_{in}}{V} \quad \text{and} \quad \alpha = \frac{Q}{V} + \gamma$$

2.5.2 Flowing Lake

- Rewriting the first-order linear differential $\frac{dC}{dt} = \beta - \alpha C$, we obtain

$$\frac{dC}{dt} + \alpha C = \beta, \; C(0) = C_0 \text{ kg/m}^3$$

- Solve by integrating factor yield

$$C(t) = k_1 e^{-\alpha t} + k_2$$

- Substitution gives us $k_2 = \beta/\alpha$;
- From the IC $C(0) = C_0$, $k_1 = C_0 - \beta/\alpha$.

2.5.2 Flowing Lake

- ∴ Solution is

$$C(t) = \boxed{\left(C_0 - \frac{\beta}{\alpha}\right) \boxed{e^{-\alpha t}}} + \frac{\beta}{\alpha} \qquad (2.3)$$

0

- Observe that when $t \to \infty$, $e^{-\alpha t} \to 0$, such that $C \to \beta/\alpha$;
- Meaning when $t \to \infty$, $C(t)$ approaches β/α kgm^{-3}.

2.5.2 Flowing Lake

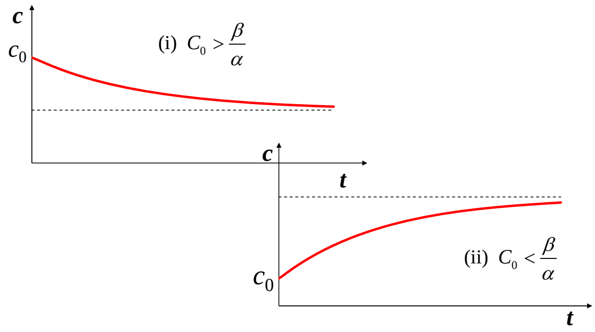
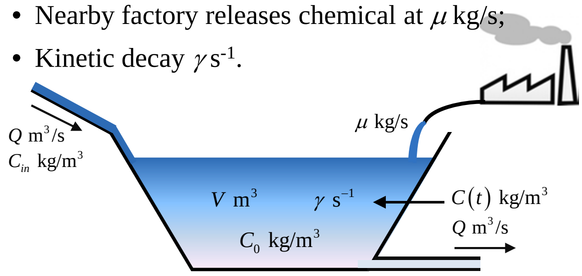


Figure 2.5 Graph for of Equation (2.3) when (i) $C_0 > \beta/\alpha$ and (ii) $C_0 < \beta/\alpha$

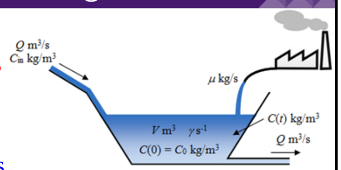
2.5.3 Flowing Lake with Loading

- Similar lake: V m^3 and $C(0) = C_0$ kg/m^3;
- Inflow: Q m^3/s with C_{in} kg/m^3 of chemical;
- Nearby factory releases chemical at μ kg/s;
- Kinetic decay γ s^{-1}.



2.5.3 Flowing Lake with Loading

- Mass **source**:
  - Inflow = $+QCin$ kg/s
  - Load = $+\mu$ kg/s
- Mass **sink**:
  - Outflow = $-QC$ kg/s
  - Decay = $-\gamma VC$ kg/s



- Mass balance in a short time step Δt:

$$V\Delta C = \left[QC_{in} - QC(t) - \gamma VC(t) + \mu\right]\Delta t, \text{ kg}$$

$$\Rightarrow \quad \frac{\Delta C}{\Delta t} = \frac{QC_{in}}{V} - \frac{QC(t)}{V} - \gamma C(t) + \frac{\mu}{V}, \text{ kg m}^{-3}\text{s}^{-1}$$

2.5.3 Flowing Lake with Loading

- Let $\Delta t \to 0$, then the DE for C is

$$-\left(\frac{Q}{V} + \gamma\right)C$$

$$\frac{dC}{dt} = \underbrace{\boxed{\frac{QC_{in}}{V} + \frac{\mu}{V}}}_{\beta} - \underbrace{\boxed{\left(\frac{Q}{V} + \gamma\right)}}_{\alpha} C \qquad (2.4)$$

$$\Rightarrow \quad \frac{dC}{dt} = \beta - \alpha C$$

with $\beta = \frac{QC_{in}}{V} + \frac{\mu}{V}$ and $\alpha = \frac{Q}{V} + \gamma$.

2.5.3 Flowing Lake with Loading

- Rewriting the first-order linear DE $\frac{dC}{dt} = \beta - \alpha C$

$$\frac{dC}{dt} + \alpha C = \beta, \; C(0) = C_0 \text{ kg/m}^3$$

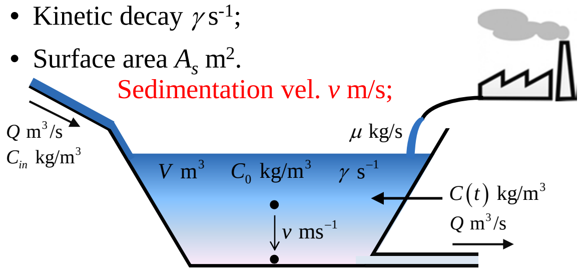
- $\therefore$ The solution is

$$C(t) = \left(C_0 - \frac{\beta}{\alpha}\right)e^{-\alpha t} + \frac{\beta}{\alpha} \qquad (2.5)$$

2.5.4 Flowing Lake with Loading and Sedimentation

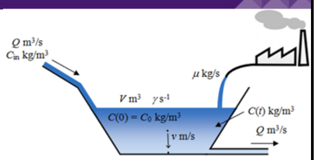
- Similar lake; Wastewater Inflow
- Catchment area wastewater at μ kg/s;
- Kinetic decay γ s^{-1};
- Surface area A_s m^2.

Sedimentation vel. v m/s;



2.5.4 Flowing Lake with Loading and Sedimentation

- Mass **source**:
  - Inflow $= +QC_{in}$ kg/s
  - Load $= +\mu$ kg/s
- Mass **sink**:
  - Outflow $= -QC$ kg/s
  - Decay $= -\gamma VC$ kg/s
  - Sedimentation $= -vA_sC$ kg/s



- Mass balance in a short time step Δt:

$$V\Delta C = \left[QC_{in} - QC(t) - \gamma VC(t) + \mu \qquad \right]\Delta t, \text{ kg}$$

$$\Rightarrow \frac{\Delta C}{\Delta t} = \frac{QC_{in}}{V} - \frac{QC(t)}{V} - \gamma C(t) + \frac{\mu}{V} - \frac{vA_sC(t)}{V}, \text{ kg m}^{-3}\text{s}^{-1}$$

2.5.4 Flowing Lake with Loading and Sedimentation

- Let $\Delta t \to 0$, then the DE for C is

$$\frac{dC}{dt} = \left(\frac{QC_{in}}{V} + \frac{\mu}{V}\right) - \left(\frac{Q}{V} + \gamma + \frac{vA_s}{V}\right)C = \beta - \alpha C \quad (2.6)$$

with $\beta = \frac{QC_{in}}{V} + \frac{\mu}{V}$ and $\alpha = \frac{Q}{V} + \gamma + \frac{vA_s}{V}$.

2.5.4 Flowing Lake with Loading and Sedimentation

- Rewriting the first-order linear DE (2.6):

$$\frac{dC}{dt} + \alpha C = \beta, \; C(0) = C_0 \text{ kg/m}^3$$

- $\therefore$ The solution is

$$C(t) = \left(C_0 - \frac{\beta}{\alpha}\right)e^{-\alpha t} + \frac{\beta}{\alpha} \qquad (2.7)$$

Lake Model – Summary

- Flowing Lake with Loading & Sedimentation:

$$\frac{dC}{dt} = -\gamma C \qquad + \frac{\mu}{V} - \frac{vA_s}{V}C$$

$$C(t) = C_0 e^{-\gamma t}$$

$$\beta = \frac{QC_{in}}{V} + \frac{\mu}{V} \qquad \alpha = \frac{Q}{V} + \gamma + \frac{vA_s}{V}$$

2.5.5 Simple Model at Steady-state

$$V\frac{dC}{dt} = +\mu + Q_{in}C_{in} - Q_{out}C - \gamma VC - vA_sC$$

Mass loading | Mass loading by inflow | Mass loss by outflow | Mass loss by decay | Mass loss by sedimentation

$$V\frac{dC}{dt} = \mu + Q_{in}C_{in} - Q_{out}C - \gamma VC - vA_sC \qquad (2.8)$$

2.5.5 Simple Model at Steady-state

$$V\frac{dC}{dt} = \mu + Q_{in}C_{in} - Q_{out}C - \gamma VC - vA_sC$$

Here,

C = concentration, kg/m^3;

t = time, s; μ = loading, kg/s;

C_{in} = inflow concentration, kg/m^3;

Q_{in} = inflow, m^3/s; V = volume, m^3;

Q_{out} = outflow, m^3/s; A_s = surface area, m^2;

γ = decay rate, s^{-1}; v = settling velocity, m/s.

2.5.5 Simple Model at Steady-state

- If the system is subject to a constant loading μ for a sufficient time;
- It will attain a dynamic equilibrium condition called ***steady-state***;
- In mathematical terms this means that accumulation is zero, that is

$$\frac{dC}{dt} = 0$$

2.5.5 Simple Model at Steady-state

At steady-state $\frac{dC}{dt} = 0$,

$$0 = \mu + Q_{in}C_{in} - (Q_{out} + \gamma V + vA_s)C$$

$$\Rightarrow \quad \mu + Q_{in}C_{in} = (Q_{out} + \gamma V + vA_s)C$$

$$-(Q_{out} + \gamma V + vA_s)C$$

$$\Rightarrow \quad \frac{\mu + Q_{in}C_{in}}{(Q_{out} + \gamma V + vA_s)} = C$$

2.5.5 Simple Model at Steady-state

- If $dC/dt = 0$ and $Q = Q_{in} = Q_{out}$, concentration at steady state is

$$C = \frac{\mu + QC_{in}}{Q + \gamma V + vA_s}, \quad \text{kg/m}^3 \qquad (2.9)$$

with $\overline{W} = \mu + QC_{in}$, kg/s

and $a = Q + \gamma V + vA_s$, m^3/s. (2.10)

2.5.5 Simple Model at Steady-state

- C = f (μ, physics, chemistry, biology)

$$C = \frac{\overline{W}}{a}$$

CONCENTRATION
How polluted the system is

LOADING
The rate of pollutant discharge

ASSIMILATION FACTOR
How physics, chemistry and biology convert the loading rate into concentration

2.5.5 Simple Model at Steady-state

- The number $a = Q + \gamma V + vA_s$ with unit m^3/s is called the **assimilation factor**;
- If the function $\overline{W}$ consists of QC_{in} only, i.e. $\mu = 0$,

  then

$$C = \frac{QC_{in}}{Q + \gamma V + vA_s} \text{ or } \frac{C}{C_{in}} = \frac{Q}{Q + \gamma V + vA_s}.$$

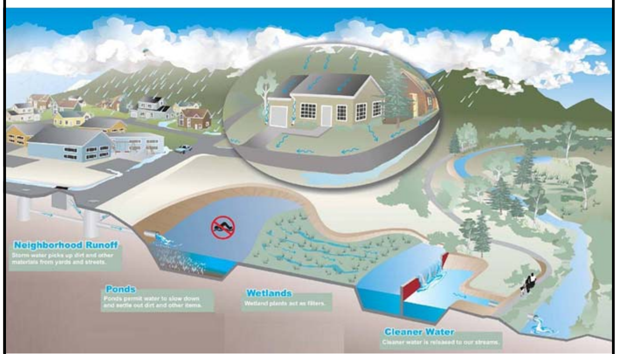
2.5.5 Simple Model at Steady-state

- Ratio $\frac{C}{C_{in}} = \frac{Q}{Q + \gamma V + vA_s}$ reflects treatment level achieved after wastewater flowed thru the lake;
- $\therefore$ the term

$$r = \frac{C}{C_{in}} = \frac{Q}{Q + \gamma V + vA_s}$$

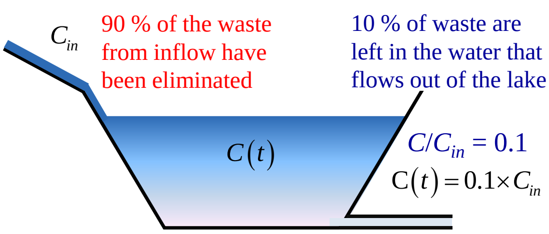
is coined treatment factor (transfer coefficient).

Treatment Pond



2.5.5 Simple Model at Steady-state

- Treatment level achieved after wastewater flowed thru the lake.



Treatment factor/ Transfer coefficient

$$r = \frac{C}{C_{in}} = \frac{Q}{Q + \gamma V + vA_s} \qquad (2.11)$$

- specifies how the system input C_{in} is transformed or "transferred" to an output (as represented by C);
- r is a number without any unit or dimension;
- Numerator = denominator = m^3/s.

Treatment factor/ Transfer coefficient

$$r = \frac{Q}{Q + \gamma V + vA_s} \qquad (2.11)$$

- r << 1: lake's removal mechanisms will greatly reduce lake's pollutant level, i.e. great assimilative capacity;
- $r \rightarrow 1$: lake's removal mechanisms are weak relative to its supply mechanism, i.e. assimilative capacity is minimal.

Treatment factor/ Transfer coefficient $r = \dfrac{Q}{Q + \gamma V + vA_s}$

- Evaluate lake's assimilative capacity;
- Assimilation ↑ for large values of reaction rate, settling velocity, volume, and area;
- Flow in both the numerator and the denominator acts to both ↑ and ↓ assimilation;
- ↑ assimilation = flushing of pollutant through the lake's outlet;
- ↓ assimilation = delivery of pollutant via lake's inflow.

Water Residence Time τ_w

$$\tau_w = \frac{V}{Q} \qquad (2.12)$$

- amount of time required for the outflow to replace the quantity of water in the lake;
- measure of the lake's flushing rate;
- Large volume + small flow = long residence time, i.e. slow flusher;
- Small volume + large flow = short residence time, i.e. fast flusher.

Pollutant Residence Time τ_c

$$\tau_c = \frac{V}{Q + \gamma V + vA_s} \qquad (2.13)$$

- amount of time that the pollutant would stay or "reside" in a system;
- τ_c is affected by reactions and settling in addition to the outflow.

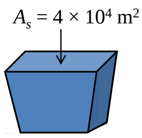
Example 2.2

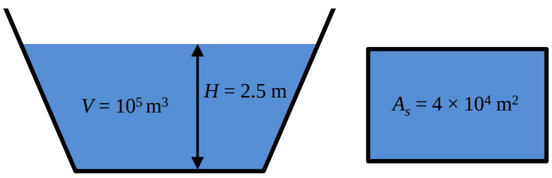
A lake has the following characteristics:

(I) volume $V = 10^5$ m^3,

(II) surface area $A_s = 4 \times 10^4$ m^2 and

(III) mean depth $H = 2.5$ m.

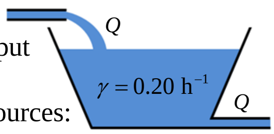




Example 2.2

Inflow and outflow are measured as $Q = 10^4$ m^3/h.

This lake receives waste input that does not undergo sedimentation from three sources:



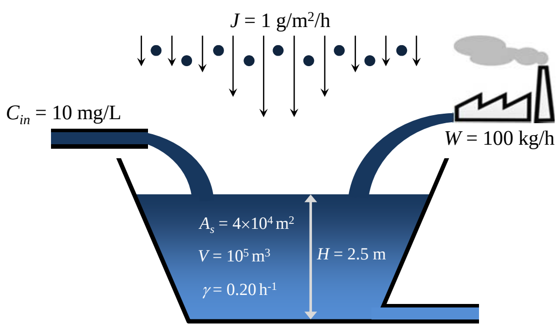
(i) industrial waste at rate of 100 kg/h,

(ii) inflow with concentration 10 mg/L and

(iii) atmospheric flux $J = 1$ g/m^2/h.

This waste decays at the rate of 0.20 h^{-1}.

Example 2.2



Example 2.2

a) Find the assimilation and treatment factors.

b) Determine the waste concentration in the lake at steady state.

c) Find the rate of change in mass for each process and shows the mass balance.

Example 2.2(a) – Solution

a) Find the assimilation and treatment factors.

The assimilation factor (Equation 2.10) is

$$a = Q + \gamma V$$

$Q = 10^4\ \text{m}^3\text{h}^{-1}$ $\quad \gamma = 0.2\ \text{h}^{-1}$ $\quad V = 10^5\ \text{m}^3$

This means that one-third of the inflow waste concentration will flow out of the lake.

Example 2.2(b) – Solution

b) Determine the waste concentration in the lake at steady state.

The concentration steady state (Equation 2.9) is

$$C = \frac{\bar{W}}{a}$$

$a = 3\times10^4\ \text{m}^3\text{h}^{-1}$

$\bar{W} = ?$

$$\bar{W} = \begin{cases} 1)\ \text{Industrial input, } W_1 \\ 2)\ \text{Inflow input, } W_2 \\ 3)\ \text{Atmospheric flux, } W_3 \end{cases}$$

Total input, $\bar{W} = W_1 + W_2 + W_3$ kg h^{-1}

Example 2.2(b) – Solution

b) Determine the waste concentration in the lake at steady state.

1) Industrial input, W_1 kgh^{-1}

$W_1 = 100\ \text{kgh}^{-1}$

2) Inflow input, W_2 kgh^{-1}

$W_2 = QC_{in}$

3) Atmospheric flux, W_3 kgh^{-1}

$W_3 = JA_s$

$Q = 10^4\ \text{m}^3\text{h}^{-1}$

$C_{in} = 10\ \text{mg/L} = 10\times10^{-3}\ \text{kg/m}^3$

$J = 1\ \text{g/m}^2\text{/h} = 1\times10^{-3}\ \text{kg/m}^2\text{/h}$

$A_s = 4\times10^4\ \text{m}^2$

Example 2.2(b) – Solution

b) Determine the waste concentration in the lake at steady state.

Industrial input, $W_1 = 100\ \text{kgh}^{-1}$

Inflow input, $W_2 = 100\ \text{kgh}^{-1}$

$\therefore$ The concentration steady state (Equation 2.9) is

Atmospheric flux, $W_3 = 40\ \text{kgh}^{-1}$

$$C = \frac{\bar{W}}{a}$$

Total input, $\bar{W} = W_1 + W_2 + W_3 = 240\ \text{kgh}^{-1}$

Example 2.2(c) – Solution

c) Find the rate of change in mass for each process and shows the mass balance.

$$\frac{V\,\Delta C}{\Delta t} = -QC - \gamma VC + \bar{W},\ \ \text{kg/h}$$

Rate of mass change from loading, $\bar{W}$ kg/h

Rate of mass change from outflow, QC kg/h

Rate of mass change from kinetic decay, $-\gamma VC$ kg/h

Example 2.2(c) – Solution

c) Find the rate of change in mass for each process and shows the mass balance.

$$\frac{V\Delta C}{\Delta t} = -QC - \gamma VC + \overline{W}, \text{ kg/h}$$

$$\overline{W} = 240 \text{ kg/h}$$

$$-QC = \left(10^4 \text{ m}^3\text{h}^{-1}\right)\times\left(8\times10^{-3} \text{ kgm}^{-3}\right) = 80 \text{ kg/h}$$

$$-\gamma VC = \left(0.2 \text{ h}^{-1}\right)\times\left(10^5 \text{ m}^3\right)\times\left(8\times10^{-3} \text{ kg/m}^3\right)$$

$$= 160 \text{ kg/h}$$

Example 2.2(c) – Solution

c) Find the rate of change in mass for each process and shows the mass balance.

Rate of mass change from loading,

$$\overline{W} = 240 \text{ kg/h}$$

Rate of mass change from outflow,

$$-QC = -80 \text{ kg/h}$$

Rate of mass change from kinetic decay,

$$-\gamma VC = -160 \text{ kg/h}$$

∴ Total rate of change in mass 80+160 = 240 kgh^{-1} balances the total mass input 240 kgh^{-1}

Thank You

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