BOD-DO Model

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5.1 Biochemical Oxygen Demand (BOD)

- BOD measures O_2 consumed by microorganisms;
- In decomposing (in)organic matter in water;
- Wastewater from sewage treatment plants;
  $\Rightarrow$ organic materials decomposition, – O_2;
- Chemical oxidation of inorganic matter;
  $\Rightarrow$ extraction of O_2 via chemical reaction;
- Common, environmental parameter;
- Measures the extent to which oxygen within a sample can support microbial life.

5.1 Biochemical Oxygen Demand (BOD)

- Test: Measure O_2 consumed during a specified period of time (5 days at 20 °C);
- Rate of O_2 consumption is affected by
  - Temperature, pH, microorganisms presence, and organic and inorganic material type.
- BOD directly affects DO in rivers and streams;
- ↑ BOD, more rapid oxygen depletion;
- $\Rightarrow$ < O_2 is available to aquatic life.

5.1 Biochemical Oxygen Demand (BOD)

- Consequences of high BOD = low DO:
  - aquatic organisms stressed, suffocate, and die.
- Sources of BOD:
  - leaves and woody debris;
  - dead plants and animals;
  - animal manure;
  - effluents from pulp and paper mills, wastewater treatment plants, feedlots, and food-processing plants;
  - failing septic systems; and urban storm water runoff.

5.1 Biochemical Oxygen Demand (BOD)

- BOD Test – to determine the relative O_2 requirements of wastewaters, effluents and polluted waters.
- cBOD
  - carbonaceous BOD
  - Inhibits nitrogenous oxygen demand
- BOD_5
  - carbonaceous + nitrogenous oxygen demand

5.1 Biochemical Oxygen Demand (BOD)

- BOD_5 Test
  - Fill airtight bottle
  - Incubate at specific temperature for 5 days.
  - Measure DO before and after incubation.
  - BOD_5 is computed from the difference between initial and final DO.

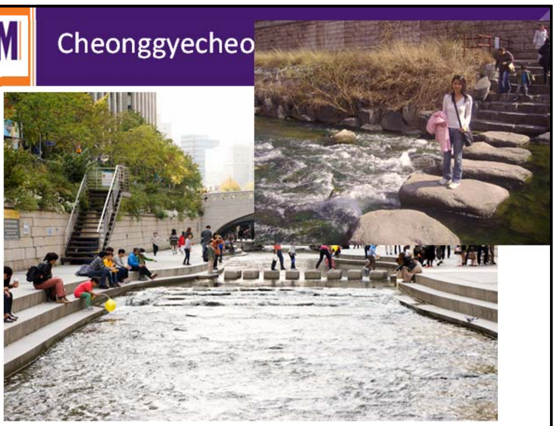
5.2 Dissolved Oxygen (DO)

- Stream both produces and consumes O_2;
- + O_2: atmosphere and plants photosynthesis;
- O_2 measured in its dissolved form DO;
- Best indicators of water ecosystem's health;
- Necessary for good water quality;

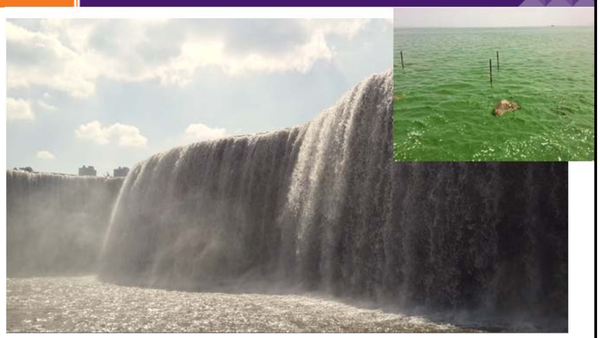
5.2 Dissolved Oxygen (DO)

- DO distribution vary due to hydraulic regimes:
- ⇒ DO reservoirs ≠ rivers;
- >DO in running water than still water.

Cheonggyecheo



Kunming Waterfall Park, China

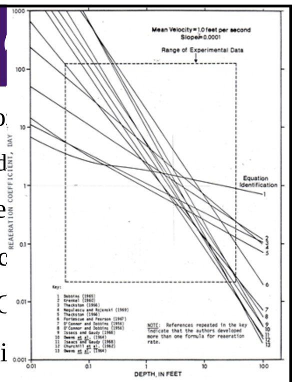


5.2 Dissolved Oxygen (DO)

- – O_2: Respiration, decomposition, and various chemical reactions;
- If > O_2 is consumed than produced;
- DO levels decline;
- Sensitive animals may move away/weaken/die;
- Fluctuate seasonally and over 24-hour period;
- In response to temperature & biological activity;

5.2 Dissolved (

- O_2 solubility is a functio
  - – temperature, salinity, and
- O_2 ↓ with ↑ temperature
- O_2 ↑ with ↑ atmospheric
- Cold water holds more (
- Water holds less O_2 at hi



- Thermal discharges raise the temperature of water and lower its O_2 content.

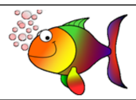
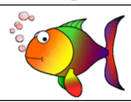
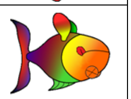
5.2 Dissolved Oxygen (DO)

- ↓ DO = indication of pollutant influx;
- For maintenance of aquatic health, DO should approach ***saturation***;
- Saturation conc is in equilibrium with the partial pressure of atmospheric O_2;
- DO >= 7.0 mg/L ⇒ aquatic ecosystem health;
- DO < 5.0 mg/L ⇒ stress aquatic life;
- ↓ DO ⇒ greater the stress.

5.2 Dissolved Oxygen (DO)

- O_2 remains below 1-2 mg/L for a few hours can result in large fish kills;
- DO can range from 0-18 ppm;
- But most natural water systems require 5-6 ppm to support a diverse population;
- DO level in good fishing waters generally averages about 9.0 ppm.

5.2 Dissolved Oxygen (DO)

DO (ppm = mg/L)	Water Quality	
5-14	**Good. Supports life.**	
3-5	**Poor. Stressful to many organisms.**	
0-3	**Bad (hypoxia). Lethal to many organisms.**	

Primary Production

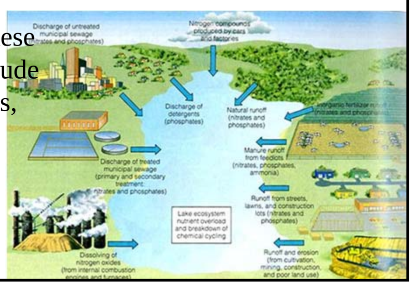
- ± O_2 = primary production (algae);
- Daytime: algal photosynthesis $+O_2$ > Respiration$-O_2$;
- ⇒ DO > saturation level, i.e., super-saturation;
- Night time: ~~photosynthesis~~, algal respiration ↓ DO significantly.

Primary Production

- In biologically productive (eutrophic) lakes, DO can become supersaturated;
- O_2 is produced by algae or rooted aquatic plants more quickly;
- Than it can escape into the atmosphere;
- 100 percent saturation = DO conc in water is in equilibrium with O_2 in the atmosphere;
- When DO conc > 110 percent saturation, harm may come to certain fish.

Eutrophication

- Occurs when large quantities of nutrients such as nitrates and phosphates enter an aquatic environment.
- Sources of these nutrients include animal wastes, agricultural runoff, and sewage.



Eutrophication

- Ecosystem quickly experiences an increase in photosynthetic and blue-green algae, as these organisms thrive in the presence of the added nutrients.
- Eg. Tasik Harapan

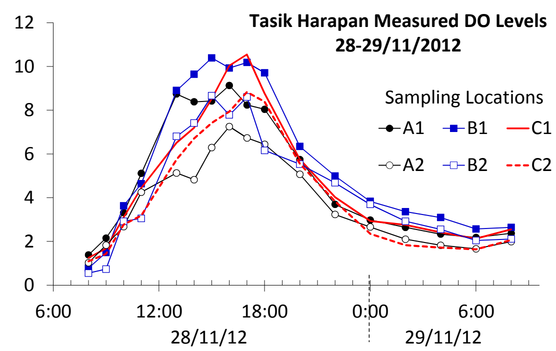
Eutrophication

Eutrophication

- Some fish are unable to survive without this light, but for them an even more serious problem arises when the algae begin to die.
- At this point, oxygen-demanding bacteria take over the ecosystem, decomposing the algae and using up dissolved oxygen in the process.
- These bacteria increase the biological oxygen demand (BOD) of the ecosystem.

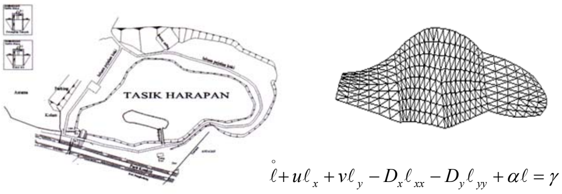
Tasik Harapan 2004-2012

DO in T. Harapan



Mechanical Aerator

Mechanical Aerator

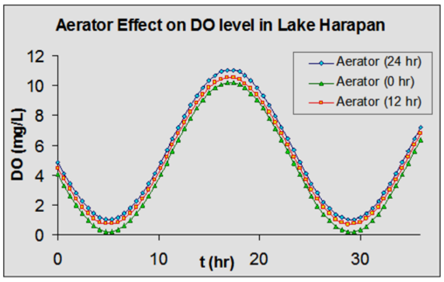


$$\overset{\circ}{\ell} + u\ell_x + v\ell_y - D_x\ell_{xx} - D_y\ell_{yy} + \alpha\ell = \gamma$$

$$\overset{\circ}{c} + uc_x + vc_y - D_x c_{xx} - D_y c_{yy}$$

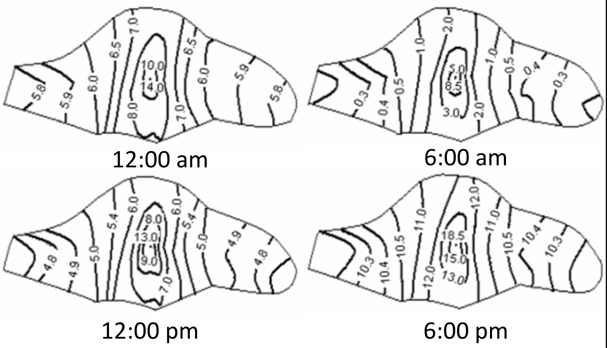
$$+\alpha\ell - \beta(c_s - c) - \delta\sin(\sigma t - 7\pi/12) = \Phi$$

Aerator effect on DO levels in Tasik Harapan

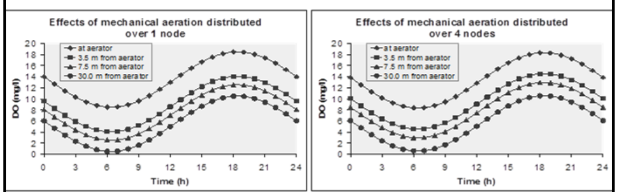


Mechanical Aerator

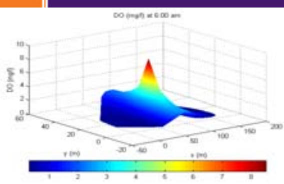
Aerator effect on DO levels in Tasik Harapan



Aerator effect on DO levels in Tasik Harapan

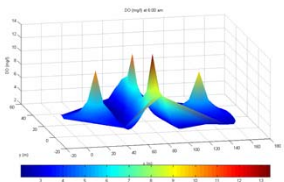


Mechanical Aerator



One aerator

Four aerators



Role of Mudball & EM Solution

Lurling, M., Tolman, Y. and van Oosterhout, F. (2010). *Cyanobacteria blooms* ***cannot*** *be controlled by Effective Microorganisms (EM®) from mud- or Bokashi-balls.* Hydrobiologia 646, 133–143.

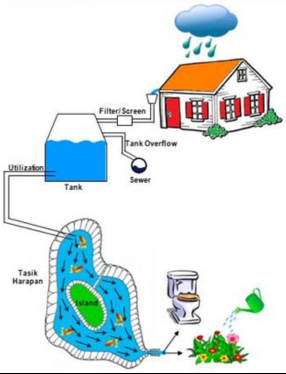
Tasik Harapan

- Tasik Harapan is highly eutrophicated;
- Wild fluctuation of DO over the diurnal cycle;
- Reaching > 18 mg/L in late afternoon;
- Mechanical aerator not effective;
- Does not remove the source of nutrients;
- Adding DO is meaningless in TH;

Tasik Harapan

- Mudball and EM solution did not appear to reduce the degree of eutrophication in TH;
- Addition of mudballs may even ↑ turbidity;
- And add additional nutrients;
- Further complicate eutrophication process;
- Removal of sediment from the lake bottom;
- Viable option that deserves more careful study;

Tasik Harapan



- Sediment removal is sustainable in long run;
- If a source of water can be found;
- In the form of rainwater harvesting;
- To provide flow to Tasik Harapan;
- Should be closely look at in the near future.

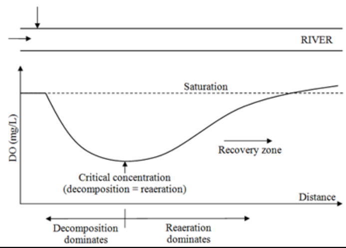
Reaeration

- Another mechanism that may ↑ or ↓ DO is transfer through air-water inte
- Typically, transfer is from atm water (***reaeration***);
- DO in most natural waters is l
- Supersaturated conditions: ne from water body to atmosphe
- Measurement: DO meter;
- Converts signals from a probe into mg/L.

5.2 Dissolved Oxygen (DO)

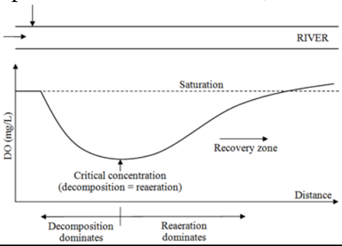
- Introduction of untreated sewage;
- Into a stream originally unpolluted;
- Will deplete the DO levels in the stream.

Figure 5.1 DO sag that occurs below sewage discharges into streams



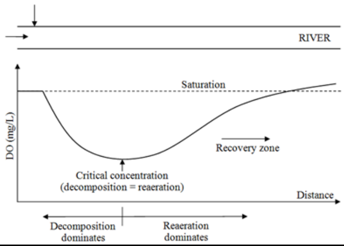
5.2 Dissolved Oxygen (DO)

- As O_2 ↓, atmospheric oxygen enters water;
- To compensate for O_2 deficit;
- Initially, O_2 consumption dwarfs reaeration;
- As the organic matter assimilated and O_2 drop;
- ⇒ Depletion & reaeration in balance.



5.2 Dissolved Oxygen (DO)

- Lowest or critical level of O_2 = ***DO sag***;
- Beyond this point, reaeration dominates;
- O_2 levels begin to rise;
- Location and magnitude of this critical conc;
- Depends on:
  - loading size,
  - stream's flow and morphometry,
  - temperature, etc.



5.3 Streeter-Phelps Eqn

- Streeter-Phelps model: 2 primary mechanisms governing DO in a stream receiving sewage:
  - decomposition of organic matter;
  - O_2 reaeration.
- 1st step in modelling DO sag: characterize the strength of the wastewater;
- Focus on the decomposition process;
- By measuring the amount of oxygen consumed, i.e. BOD.

5.3 Streeter-Phelps Eqn

- BOD oxidation in a uniformly mixed segment is generally written as a first order reaction:

$$\frac{d\ell}{dt} = -\alpha\ell + \gamma \qquad (5.1)$$

where ℓ = BOD concentration, mg/L;
t = time, day;
γ = BOD Loading, mg/L/d;
α = first-order deoxygenation rate constant, d^{-1};

5.3 Streeter-Phelps Eqn

- DO mass balance equation :

$$\frac{dc}{dt} = -\alpha\ell + \beta\left(c_s - c\right) \qquad (5.2)$$

where c = DO concentration, mg/L;
c_s = saturated DO concentration, mg/L;
β = first-order reaeration rate constant, d^{-1};

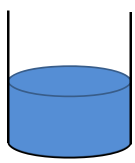
Reaeration

- DO reaeration: $\beta\left(c_s - c\right)$

$$c_s > c \Rightarrow \left(c_s - c\right) > 0$$

$$\beta > 0 \Rightarrow \beta\left(c_s - c\right) > 0$$

$$\frac{dc}{dt} = +\beta\left(c_s - c\right)$$



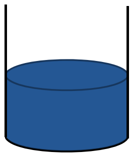
O_2 from atmosphere enters water

Reaeration

- DO reaeration: $\beta(c_s - c)$

$$c_s < c \Rightarrow (c_s - c) < 0$$
$$\beta > 0 \quad \Rightarrow \quad \beta(c_s - c) < 0$$
$$\frac{dc}{dt} = -\beta(c_s - c)$$

O_2 in water escape to atmosphere



5.3 Streeter-Phelps Eqn

Before proceeding to other aspects of BOD-DO modeling, let's review some of the parameters that relate to BOD-DO.

- BOD Decay Rate α
- BOD Loading γ
- DO Saturation c_s
- Reaeration Rate β

5.3.1 BOD Decay Rate α

- BOD bottle decay typical values: 0.05-0.5 d^{-1};
- Mean of about **0.15** d^{-1};
- Info used to estimate a 95 % response time;
- For the bottle test as t_{95} = 3/**0.15** = 20 d;
- Long measurement period is unacceptable;
- Adopt a 5-day BOD (BOD_5) test.

Table 5.1 Typical values of the BOD decomposition rate for various levels of treatment

Treatment	α (d^{-1}) at 20 °C
Untreated	0.35 (0.20 – 0.50)
Primary	0.20 (0.10 – 0.30)
Activated sludge	0.075 (0.05 – 0.10)

- Raw sewage = mixture of compounds;
- From easily decomposable sugars;
- To refractory substances (longer to decay);
- Waste treatment selectively remove former;
- BOD decay rates is lower for treated sewage.

5.3 Streeter-Phelps Eqn

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- BOD Decay Rate α
- BOD Loading γ
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- Reaeration Rate β

5.3.2 BOD Loading γ

- BOD loading (mass/time) = BOD conc of wastewater (effluent) released into a body of water.

Table 5.2 Typical loading rates for untreated domestic sewage

	Per-capita flow rate (m^3 capita^{-1} d^{-1})	Per-capita CBOD (m^3 capita^{-1} d^{-1})	CBOD conc (mg/L)
United States	0.57 (150)*	125 (0.275)**	220
Developing countries	0.19 (50)*	60 (0.132)**	320

Table 5.2 Typical loading rates for untreated domestic sewage

- US flow rate ↑ - higher standard of living;
- Per capita BOD generation rate ↑ - garbage disposals and other accoutrements of a developed economy;

	Per-capita flow rate (m^3 $capita^{-1}$ d^{-1})	Per-capita CBOD (m^3 $capita^{-1}$ d^{-1})	CBOD conc (mg/L)
United States	0.57 (150)*	125 (0.275)**	220
Developing countries	0.19 (50)*	60 (0.132)**	320

Table 5.2 Typical loading rates for untreated domestic sewage

- Average conc in developing countries is generally ↑ - lower water;
- ↓ water use in developing countries outweighs the higher per capita US BOD contribution.

	Per-capita flow rate (m^3 $capita^{-1}$ d^{-1})	Per-capita CBOD (m^3 $capita^{-1}$ d^{-1})	CBOD conc (mg/L)
United States	0.57 (150)*	125 (0.275)**	220
Developing countries	0.19 (50)*	60 (0.132)**	320

5.3.2 BOD Loading γ

- Raw waste from palm oil processing factory;
- Before treatment, can reach 30000 mg/L BOD;
- Proper treatment ⇒ 300 mg/L or lower;
- Department of Environment (DOE);
- Receiving Water Quality Criteria for Malaysia in 1985;
- Aim: Develop a water quality management approach for the long term water quality of the nation's water resources.

5.3.2 BOD Loading γ

- Environment Quality (Sewerage and Industrial Effluents) Regulations 1979;
- Effluent quality: Standards A and B;
- Standard A : ≤ 20 mg/L BOD;
- Standard B : ≤ 50 mg/L BOD;
- Effluent discharged upstream of a water supply intake should meet Standard A;
- Effluent discharged downstream has to meet Standard B.

Table 5.3 Parameter limits of effluent of Standards A and B

	Parameter	Unit	Standard A	Standard B
1	Temperature	°C	40	40
2	pH	–	6.0 – 9.0	5.5 – 9.0
3	**BOD_5 @ 20 °C**	**mg/L**	**20**	**50**
4	COD	mg/L	50	100
5	Suspended Solids	mg/L	50	100
6	Mercury	mg/L	0.005	0.05
7	Cadmium	mg/L	0.01	0.02
8	Chromium, Hexvalent	mg/L	0.05	0.05
9	Arsenic	mg/L	0.05	0.10
10	Cyanide	mg/L	0.05	0.10
11	Lead	mg/L	0.10	0.5
12	Chromium, Trivalent	mg/L	0.20	1.0
13	Copper	mg/L	0.20	1.0
14	Manganese	mg/L	0.2	1.0
15	Nickel	mg/L	0.20	1.0
16	Tin	mg/L	0.20	1.0
17	Zinc	mg/L	1.0	1.0
18	Boron	mg/L	1.0	4.0
19	Iron (Fe)	mg/L	1.0	5.0
20	Phenol	mg/L	0.001	1.0
21	Free Chlorine	mg/L	1.0	2.0
22	Sulphide	mg/L	0.50	0.50
23	Oil and Grease	mg/L	Not detectable	10.0

5.3 Streeter-Phelps Eqn

Before proceeding to other aspects of BOD-DO modelling, let's review some of the parameters that relate to BOD-DO.

- BOD Decay Rate α
- BOD Loading γ
- DO Saturation c_s
- Reaeration Rate β

5.3.3 DO Saturation c_s

- c_s = O_2 saturation constant;
- Highest DO conc achieved under certain circumstances;
- Several env factors affect DO saturation:
  - temperature, salinity and partial pressure variations due to elevation.
- Several empirical derived equations have been developed to predict how these factors influence saturation.

5.3.3 DO Saturation c_s

Temp °C	Salinity (ppt)							
	0	5	10	15	20	25	30	35
0	14.60	14.11	13.64	13.18	12.74	12.31	11.90	11.50
2	13.81	13.36	12.91	12.49	12.07	11.67	11.29	10.91
4	13.09	12.67	12.25	11.85	11.47	11.09	10.73	10.38
6	12.44	12.04	11.65	11.27	10.91	10.56	10.22	9.89
8	11.83	11.46	11.09	10.74	10.4	10.07	9.75	9.44
10	11.28	10.92	10.58	10.25	9.93	9.62	9.32	9.03
12	10.77	10.43	10.11	9.80	9.50	9.21	8.92	8.65
14	10.29	9.98	9.68	9.38	9.1	8.82	8.55	8.29
16	9.86	9.56	9.28	9.00	8.73	8.47	8.21	7.97
18	9.45	9.17	8.90	8.64	8.38	8.14	7.90	7.66
20	9.08	8.81	8.56	8.31	8.06	7.83	7.60	7.38
22	8.73	8.48	8.23	8.00	7.77	7.54	7.33	7.12
24	8.40	8.16	7.93	7.71	7.49	7.28	7.07	6.87
26	8.09	7.87	7.65	7.44	7.23	7.03	6.83	6.64
28	7.81	7.59	7.38	7.18	6.98	6.79	6.61	6.42
30	7.54	7.33	7.14	6.94	6.75	6.57	6.39	6.22

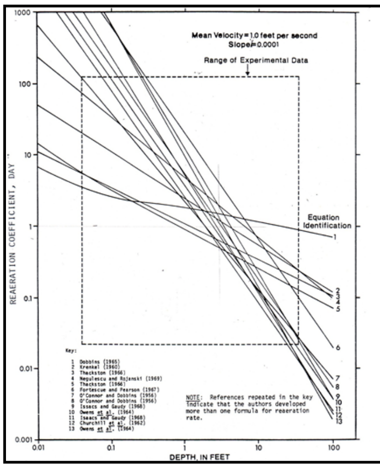
5.3 Streeter-Phelps Eqn

Before proceeding to other aspects of BOD-DO modelling, let's review some of the parameters that relate to BOD-DO.

- BOD Decay Rate α
- BOD Loading γ
- DO Saturation c_s
- Reaeration Rate β

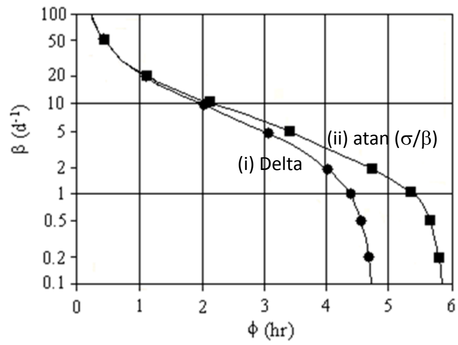
5.3.4 Reaeration Rate β

- Many investigators have developed formulas;
- For predicting reaeration in streams and rivers;
- Comprehensive reviews (Bennett and Rathbun, 1972; Bowie et al., 1985; USEPA, 1985);
- For standing waters, e.g. lakes, impoundments and wide estuaries, wind becomes the predominant factor in causing reaeration.



USEPA Reaeration β Estimation

Delta Method β Estimation



5.4 Interim National River WQ Standards

- WQ data were used to determine WQ status;
- Rivers in Malaysia;
- Status: Clean, slightly polluted or polluted;
- River classification: Class I, II, III, IV or V;
- Based upon Water Quality Index (WQI) and Interim National Water Quality Standards for Malaysia (INWQS).

5.4 Interim National River WQ Standards

- WQI is computed based upon 6 main parameters as follows.
  - **Biochemical Oxygen Demand (BOD)**
  - Chemical Oxygen Demand (COD)
  - Ammoniacal Nitrogen (NH_3N)
  - pH
  - **Dissolved Oxygen (DO)**
  - Suspended Solids (SS)
- Other parameters, e.g. heavy metals and bacteria, according to site requirement.

Table 5.4 Classification of rivers in Malaysia

Class	Use
Class I	Conservation of natural environment, Water Supply I – practically no treatment necessary, Fishery I – very sensitive aquatic species.
Class IIA	Water supply II – conventional treatment required, Fishery II – sensitive aquatic species.
Class IIB	Recreational use with body contact
Class III	Water supply III – extensive treatment required, Fishery III – common, of economic value, and tolerant species livestock drinking
Class IV	Irrigation
Class V	None of the above

Table 5.4 Interim National Water Quality Standards for Malaysia

Parameter	Unit	Classes					
		I	IIA	IIB	III	IV	V
Ammoniacal Nitrogen	mg/L	0.1	0.3	0.3	0.9	2.7	>2.7
BOD	mg/L	1	3	3	6	12	>12
COD	mg/L	10	25	25	50	100	>100
DO	mg/L	7	5-7	5-7	3-5	<3	<1

Parameter	Unit	Classes					
		I	IIA	IIB	III	IV	V
BOD	mg/L	1	3	3	6	12	>12
DO	mg/L	7	5-7	5-7	3-5	<3	<1
Faecal Coliform**	counts/100mL	10	100	400	5000 (20000)[a]	5000 (20000)[a]	–
Total Coliform	counts/100mL	100	5000	5000	50000	50000	>50000

5.5 BOD-DO Dynamics

- BOD-DO model formed based upon mass balance principles;
- As a result of various processes involved;
- And sources and sinks that influence conc;
- BOD sources: industrial waste, domestic sewage and runoff from agriculture or rain;
- DO loss is caused by various processes;
- BOD oxidation and aquatic plant respiration.

5.5 BOD-DO Dynamics

- BOD and DO conc are also influenced by hydraulic processes;
- E.g. advective flow and dispersion;
- Simplification: Photosynthesis and respiration processes assumed to cancel out each other;
- Hence, both processes are omitted from the mass balance equation.

BOD-DO Model

BOD : $\frac{d\ell}{dt} = -\alpha\ell + \gamma$ (5.1)

DO : $\frac{dc}{dt} = -\alpha\ell + \beta(c_s - c)$ (5.2)

5.5.1 Closed Bottle

- Consider BOD oxidation in a closed bottle:

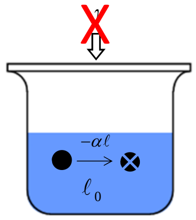


Figure 5.2 A closed bottle

BOD:

$$\frac{d\ell}{dt} = \boxed{-\alpha\ell} + \cancel{\gamma} \quad (5.1a)$$

Solution:

$$\ell(t) = \ell_0 e^{-\alpha t} \quad (5.3)$$

5.5.1 Closed Bottle

- BOD oxidizes $\Rightarrow$ DO will be reduced;
- Deoxygenate 1 mg BOD=Eliminate 1 mg DO.

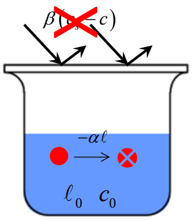


Figure 5.2 A closed bottle

DO: $\ell(t) = \ell_0 e^{-\alpha t}$

$$\frac{dc}{dt} = \boxed{-\alpha\ell} + \boxed{\beta(\cancel{c_s} - c)} \quad (5.2a)$$

Solution:

$$c(t) = (c_0 - \ell_0) + \ell_0 e^{-\alpha t}$$

Figure 5.3 BOD and DO levels in a closed bottle

- Case (a) : Initial DO > initial BOD.

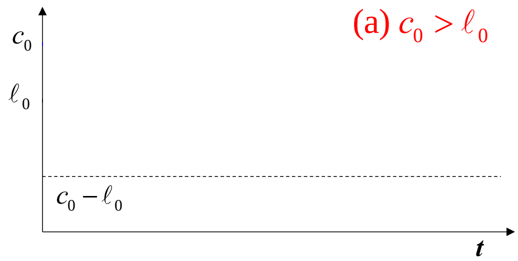
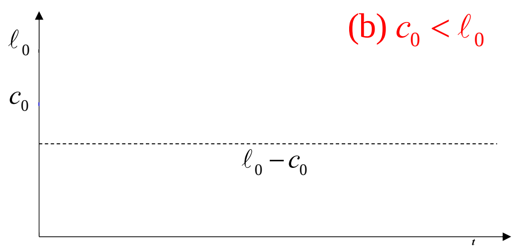


Figure 5.3 BOD and DO levels in a closed bottle

- Case (b) : Initial DO < initial BOD.



5.5.1 Opened Bottle

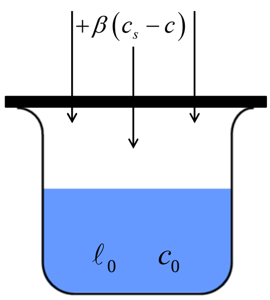


Figure 5.4 An opened bottle

$$\frac{d\ell}{dt} = -\alpha\ell \quad (5.1a)$$

$$\frac{dc}{dt} = -\alpha\ell \quad (5.2a)$$
$$+\beta(c_s - c) \quad (5.2)$$

$$D = c_s - c \quad (5.5)$$

D = ***DO deficit***

5.5.1 Opened Bottle

$$D = c_s - c \quad (5.5)$$

$$\frac{dc}{dt} = -\alpha\ell + \beta(c_s - c) \quad (5.2)$$

Differentiate with respect to t

$$\frac{dD}{dt} = -\frac{dc}{dt} \quad (5.6)$$

Substitute into (5.2)

$$\frac{d\ell}{dt} = -\alpha\ell \quad (5.1a)$$

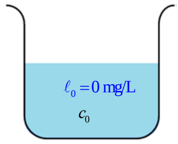
$$-\frac{dD}{dt} = -\alpha\ell + \beta D \quad (5.7)$$

- Eqns (5.1a) and (5.7): change in BOD and DO (deficit) in the opened bottle;
- Use of deficit simplifies differential equation.

Zero Initial BOD

$\ell_0 = 0$ mg/L

- Clean water;
- Subject to only reaeration.



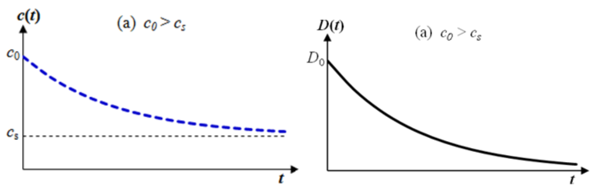
DO Deficit: $\frac{dD}{dt} = -\beta D \quad (5.7a)$

Solution: $D(t) = D_0 e^{-\beta t}$ with $D_0 = c_s - c_0 \quad (5.8)$

$$\Rightarrow c(t) = c_s - (c_s - c_0)e^{-\beta t} \quad (5.9)$$

Figure 5.5 DO concentration and DO deficit in an opened bottle

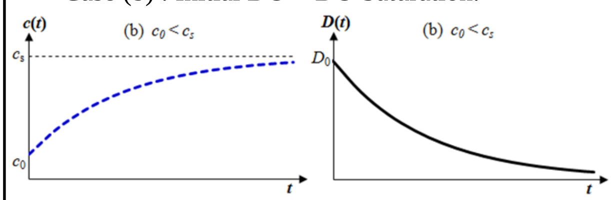
- Case (a) : Initial DO > DO Saturation.



- DO Deficit increases exponentially to c_s;
- DO decreases exponentially to c_s;
- $t \to \infty \Rightarrow c(t) \to c_s$, $D(t) \to 0$;

Figure 5.5 DO concentration and DO deficit in an opened bottle

- Case (b) : Initial DO < DO Saturation.

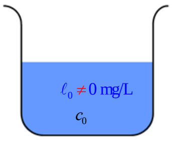


- DO Deficit decreases exponentially to zero;
- DO increases exponentially to c_s;
- $t \to \infty \Rightarrow c(t) \to c_s$, $D(t) \to 0$;

Non-zero Initial BOD

$\ell_0 \neq 0$ mg/L

- Polluted water;
- DO: ↓BOD decay + ↑reaeration.



$$-\frac{dD}{dt} = -\alpha\ell + \beta D \quad (5.7)$$

$$\ell(t) = \ell_0 e^{-\alpha t}$$

$$c_0 = c(0)$$

Solution:

$$D(t) = \frac{\alpha\ell_0}{\beta - \alpha}\left(e^{-\alpha t} - e^{-\beta t}\right) + D_0 e^{-\beta t} \quad (5.10)$$

$$\Rightarrow c(t) = c_s - \frac{\alpha\ell_0}{\beta - \alpha}\left(e^{-\alpha t} - e^{-\beta t}\right) - (c_s - c_0)e^{-\beta t} \quad (5.11)$$

Non-zero Initial BOD

$$\ell(t) = \ell_0 e^{-\alpha t} \quad (5.3)$$

$$D(t) = \frac{\alpha\ell_0}{\beta - \alpha}\left(e^{-\alpha t} - e^{-\beta t}\right) + D_0 e^{-\beta t} \quad (5.10)$$

or

$$c(t) = c_s - \frac{\alpha\ell_0}{\beta - \alpha}\left(e^{-\alpha t} - e^{-\beta t}\right) - (c_s - c_0)e^{-\beta t} \quad (5.11)$$

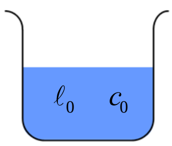
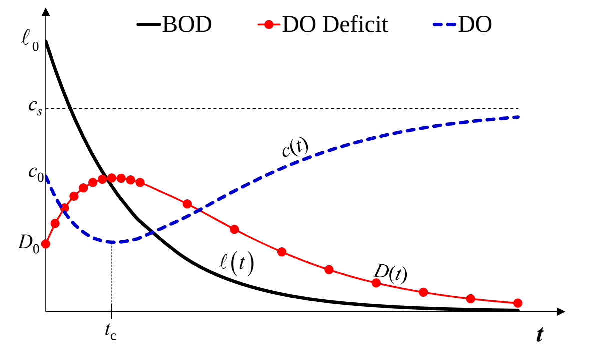


Figure 5.6 BOD, DO and DO deficit in the opened bottle



Critical Time and Value

- *Critical value* c_c : minimum DO value; maximum DO deficit D_c ;
- *Critical time* t_c : time when DO is minimum.

$$D(t)=\frac{\alpha\ell_0}{\beta-\alpha}\left(e^{-\alpha t}-e^{-\beta t}\right)+D_0e^{-\beta t} \quad (5.10)$$

$$\frac{dD}{dt}=-\alpha\ell+\beta D \quad (5.7)$$

Differentiate

Set $dD/dt = 0$

$$t_c=\left(\frac{1}{\beta-\alpha}\right)\ln\left[\frac{\beta}{\alpha}\left(1-\frac{D_0(\beta-\alpha)}{\alpha\ell_0}\right)\right] \quad (5.12)$$

Set $dD/dt = 0$

$$D_c=\left(\frac{\alpha}{\beta}\right)\ell_0e^{-\alpha t_c} \quad (5.13)$$

Critical Time

$$0 = \frac{\alpha\ell_0}{\beta-\alpha}\left(-\alpha e^{-\alpha t}+\beta e^{-\beta t}\right)-\beta D_0e^{-\beta t} \quad (5.10)$$

Differentiate $D(t)$ with respect to t

Set $dD/dt = 0$

$$t_c=\frac{1}{\beta-\alpha}\ln\left(\frac{\beta}{\alpha}\left(1-\frac{D_0(\beta-\alpha)}{\alpha\ell_0}\right)\right) \quad (5.13)$$

Critical Value

$$\frac{dD}{dt}=-\alpha\ell+\beta D \quad (5.7)$$

Set $dD/dt = 0$

$$0 = -\alpha\ell+\beta D_c$$

$$D_c=\left(\frac{\alpha}{\beta}\right)\ell_0e^{-\alpha t_c} \quad (5.13)$$

Example 5.1

A bottle opened to reaeration and filled with polluted water has the following characteristics:

$\ell_0 = 17.98$ mg/L $\quad c_0 = 6.681$ mg/L $\quad c_s = 8.418$ mg/L

$\beta = 0.97$ d^{-1} $\quad \alpha = 0.40$ d^{-1}

Find $\ell(t)$ and $c(t)$ after

(a) 1/3 × 10^4 s; (b) 5/3 × 10^4 s; (c) 1/3 × 10^5 s.

Also, find the critical time t_c, critical DO deficit D_c and critical DO c_c.

Example 5.1 – Solution

First, list the relevant equations:

$$\ell(t)=\ell_0e^{-\alpha t}$$

$$D(t)=D_0e^{-\beta t}+\frac{\alpha\ell_0}{\beta-\alpha}\left(e^{-\alpha t}-e^{-\beta t}\right)$$

$$=D_1+D_2$$

with $D_1=D_0e^{-\beta t}$ and $D_2=\frac{\alpha\ell_0}{\beta-\alpha}\left(e^{-\alpha t}-e^{-\beta t}\right)$

$$c(t)=c_s-D(t)$$

$$t_c=\left(\frac{1}{\beta-\alpha}\right)\ln\left[\frac{\beta}{\alpha}\left(1-\frac{D_0(\beta-\alpha)}{\alpha\ell_0}\right)\right]$$

$$D_c=\left(\frac{\alpha}{\beta}\right)\ell_0e^{-\alpha t_c}\ ,\quad c_c=c_s-D_c$$

Find $\ell(t)$ and $c(t)$ after
(a) 1/3 × 10^4 s;
(b) 5/3 × 10^4 s;
(c) 1/3 × 10^5 s.
Also, find the critical time t_c, critical DO deficit D_c and critical DO c_c.

Example 5.1 – Solution

$D_0 = c_s - c_0 = 8.418 - 6.681 = 1.737$ mg/L

$\beta - \alpha = 0.97 - 0.40 = 0.57$ d^{-1}

(a) After 1/3 × 10^4 s

$$\ell = 17.98 \times \exp\left(\frac{-0.4 \times 1/3 \times 10^4}{86400}\right) = 17.70 \text{ mg/L}$$

$$D_1 = 1.737 \times \exp\left(\frac{-0.97 \times 1/3 \times 10^4}{86400}\right) = 1.673 \text{ mg/L}$$

$$D_2 = 17.98 \times \left(\frac{0.4}{0.57}\right) \times \left[\exp\left(\frac{-0.4 \times 1/3 \times 10^4}{86400}\right) - \exp\left(\frac{-0.97 \times 1/3 \times 10^4}{86400}\right)\right]$$

$$= 0.2697 \text{ mg/L}$$

$\therefore\ D = D_1 + D_2 = 1.673 + 0.2697 = 1.943$ mg/L

and $c = 8.418 - 1.943 = 6.475$ mg/L

Example 5.1 – Solution

(b) After 5/3 × 10^4 s

$\ell = 16.64$ mg/L

$D = 2.66$ mg/L

$c = 5.76$ mg/L

(c) After 1/3 × 10^5 s

$\ell = 15.41$ mg/L

$D = 3.33$ mg/L

$c = 5.09$ mg/L

Equation (5.12) :

$t_c = 1.294$ day

Equation (5.13) :

$$D_c = \left(\frac{0.4}{0.97}\right)(17.98)e^{(-0.4 \times 1.294)}$$

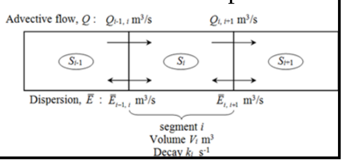
$$= 4.418 \text{ mg/L}$$

$$c_c = c_s - D_c = 8.418 - 4.418$$

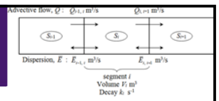
$$= 4.0 \text{ mg/L}$$

5.6 BOD-DO Model for River

- Determined by considering hydrological processes;
- E.g. advective flow and dispersion;
- In addition to deoxygenation and reaeration;
- Finite segment method discussed in Chapter 3 can be employed for this purpose.



5.6 BOD-DO Model for River



- Mass balance eqn for BOD in segment i :

$$V_i \frac{d\ell}{dt} = \boxed{Q_{i-1,i} \cdot \ell_{i-1} - Q_{i,i+1} \cdot \ell_i} \quad \textit{Advective flow}$$
$$\boxed{+\bar{E}_{i-1,i}\left(\ell_{i-1} - \ell_i\right) + \bar{E}_{i,i+1}\left(\ell_{i+1} - \ell_i\right)} \quad \textit{Dispersion}$$
$$\boxed{-K_r V_i \ell_i \pm W_i} \quad \textit{Reaction} \qquad (5.14)$$

- Mass balance eqn for DO in segment i :

$$V_i \frac{dc}{dt} = \boxed{Q_{i-1,i} \cdot c_{i-1} - Q_{i,i+1} \cdot c_i} \quad \textit{Advective flow}$$
$$\boxed{+\bar{E}_{i-1,i}\left(c_{i-1} - c_i\right) + \bar{E}_{i,i+1}\left(c_{i+1} - c_i\right)} \quad \textit{Dispersion}$$
$$\boxed{-K_r V_i \ell_i + K_a\left(c_s - c_i\right)V_i} \quad \textit{Reaction} \qquad (5.15)$$

5.6 BOD-DO Model for River

$$Q = Au \qquad \bar{E} = \frac{EA}{\Delta x} \qquad (5.16)$$

Q m³/s = advective flow;

A m² = cross sectional area;

u m/s = velocity;

E m²/s = dispersion coefficient;

V m³ = segment volume;

Δx m = segment length;

$\bar{E}$ m³/s = bulk dispersion coefficient;

c kg/m³ = DO concentration;

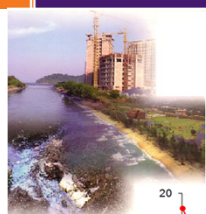
c_s kg/m³ = DO saturation level;

ℓ kg/m³ = BOD concentration;

K_r s⁻¹ = deoxygenation (decay) rate;

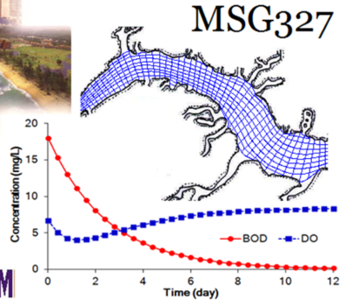
K_a s⁻¹ = reaeration rate.

5.6 BOD-DO Model for River



E2DISP

MSG327



Developed by

Koh Hock Lye

Teh Su Yean

5.6 BOD-DO Model for River

- When $\Delta x \to 0$; Assume $E = 0$, $W = 0$
- Eqns (5.14) and (5.15) become:

$$0 = -u\frac{\partial \ell}{\partial x} - K_r\ell \qquad (5.17)$$

Assume steady state

$$\frac{d\ell}{dx} = -\left(\frac{K_r}{u}\right)\ell \qquad (5.19)$$

$$0 = -u\frac{\partial c}{\partial x} - K_r\ell + K_a(c_s - c) \qquad (5.18)$$

$$\frac{dc}{dx} = -\left(\frac{K_r}{u}\right)\ell + \left(\frac{K_a}{u}\right)(c_s - c) \qquad (5.20)$$

5.6 BOD-DO Model for River

$$\frac{d\ell}{dx} = -\left(\frac{K_r}{u}\right)\ell \qquad (5.19)$$

$$\frac{dc}{dx} = -\left(\frac{K_r}{u}\right)\ell + \left(\frac{K_a}{u}\right)(c_s - c) \qquad (5.20)$$

Let $x = ut$ or $t = x/u$. (5.21)

$$\frac{d\ell}{dt} = -K_r\ell \ , \quad \ell(0) = \ell_0 \qquad (5.22)$$

$$\frac{dc}{dt} = -K_r\ell + K_a(c_s - c) \ , \quad c(0) = c_0 \qquad (5.23)$$

Example 5.2

A uniform river has a velocity of u = 0.3 m/s and other characteristics as follows:

$c_s = 8.418$ mg/L $\quad K_a = 0.97$ d^{-1} $\quad K_r = 0.40$ d^{-1}

At x = 0 km, BOD and DO concentrations are:

$\ell_0 = 17.98$ mg/L $\quad c_0 = 6.681$ mg/L

Find $\ell(x)$ and $c(x)$ at (a) x = 1 km; (b) x = 5 km; (c) x = 10 km. Also, find the critical time t_c, critical DO deficit D_c and critical DO c_c. Sketch the graph of $\ell(x)$ and $c(x)$.

Example 5.2 – Solution

The relation $x = ut$ with u = 0.3 m/s are used here. Hence, x = 1 km is equivalent to $t = 1/3 \times 10^4$ s. Thus, all equations used in Example 5.1 are used here with $x = 0.3t$.

This means that the answers here are similar to the answers in Example 5.1 with $x = ut$ and u = 0.3 m/s .

Example 5.2 – Solution

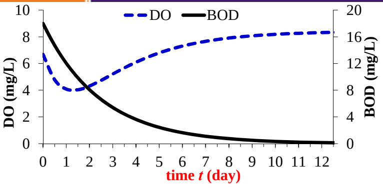


Figure 5.7 BOD and DO concentrations versus time for Example 5.1

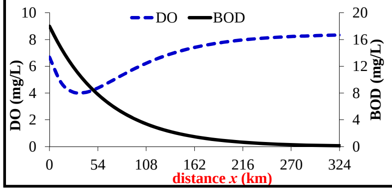


Figure 5.8 BOD and DO concentrations versus distance for Example 5.2

Thank You

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