UNIVERSITI SAINS MALAYSIA

River Modeling

- Segmentation and Model Resolution
- Finite Segment Method

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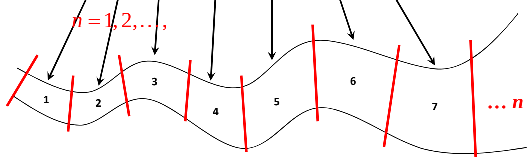
3.1 Segmentation and Model Resolution

- Segmentation = process of dividing space and matter into increments;
- Space: a water body can be ÷ into volumes;
- For which mass balance equations are written.

$$V\frac{dC_n}{dt} = W_n + Q_{in}C_{in} - Q_{out}C_{out} - \gamma V C_n - vA_sC_n$$

$n = 1, 2, \ldots,$



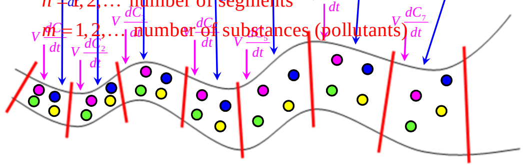
3.1 Segmentation and Model Resolution

- Volume: matter may be ÷ into different chemical and biological forms;
- For which separate eqns would be written;
- n segments + m substances = $m \times n$ mass balance eqns.

$$V\frac{dC_{n,m}}{dt} = W_{n,m} + Q_{in}C_{in} - Q_{out}C_{out} - \gamma V C_{n,m} - vA_sC_{n,m}$$

$n = 1, 2, \ldots$ number of segments

$m = 1, 2, \ldots$ number of substances (pollutants)



3.1 Segmentation and Model Resolution

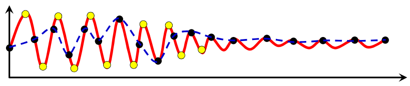
- Aside from space and matter, there is a temporal aspect to segmentation;
- Related to the fact that mass balance defines changes in water body over a finite period of time;
- Model describes additional spatial and material detail by using more segments;
- ↑ temporal focus: shorter "finite period" or ***time step*** for mass balance computation;
- ***Model resolution:*** degree to which space, time and matter are segmented.

Model Resolution

- ≈ photography (Camera's lens focus);
- At times the foreground is important;
- At other times distant details might be of interest.

Significance of Model Resolution for WQ analysis

- Two basic ways:
  1. Fine-scale phenomenon may have a direct, causative influence on predictions made on the coarser scale;

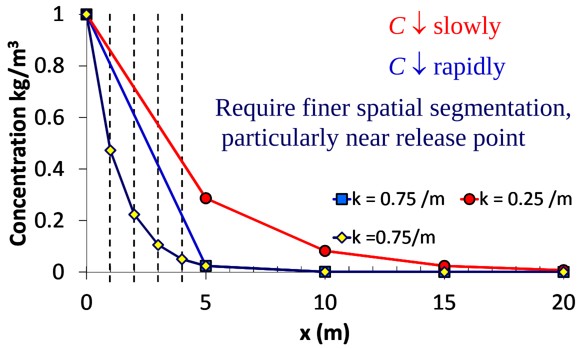


  2. Influence of the problem context on the choice of scales.

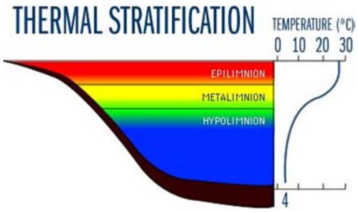
1. Fine scale ⇒ Coarse scale phenomenon

- Primarily a function of substance's properties and physical characteristics of the system;
- E.g. certain pollutants (e.g. enteric bacteria) die rapidly upon entering a water body;
- ∴ typically at ↑ levels near sewage discharge;
- ↓ rapidly ⇒ background levels in open waters;
- Near-shore model of bacterial pollution;
- Require relatively fine spatial and temporal segmentation around sewage outfalls.

1. Fine scale ⇒ Coarse scale phenomenon



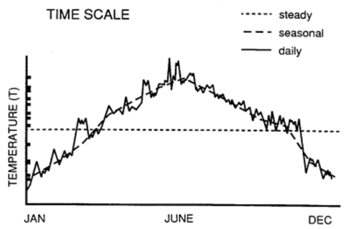
1. Fine scale ⇒ Coarse scale phenomenon



- E.g. dendritic lakes - 1 segment unrealistic;
- Eutrophication - thermal stratification;
  - Need multiple vertical.

2. Influence of prob context on choice of scales

- E.g. phytoplankton go through growth cycles on seasonal time scales and exhibit patchiness on small space scales.
- Steady
- Seasonal
- Monthly
- Daily/Hourly

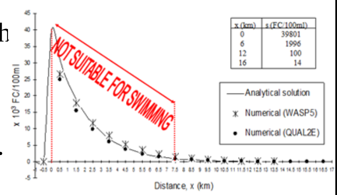


2. Influence of prob context on choice of scales

- But WQ planner might not have the funds to develop models to simulate such short-term variability;
- Often occur when large numbers of small lakes were being evaluated.

2. Influence of prob context on choice of scales

- Spatial and temporal aggregation results in *negligible* loss of relevant planning info;
- Opt for a coarser scale model;
- Bacterial contamination of beaches;
- Finer scale approach is necessary;
- To handle the problem effectively.

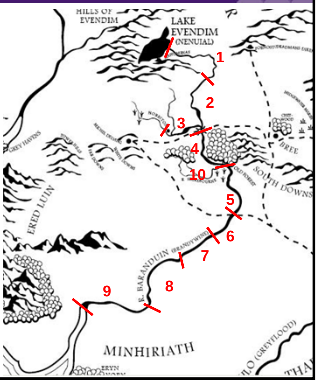


3.1 Segmentation and Model Resolution

- Temporal, spatial and kinetic scales of a problem often are interrelated;
- Fast kinetic processes, e.g. jet mixing of thermal effluents or bacterial die-off ;
- Tend to manifest themselves on local (i.e. small) time and space scales;
- Problems with slow reactions, e.g. decay of persistent contaminants;
- Important on whole-system, long-term basis.

3.2 Finite Segment Method

- Water body is often ÷ into a series of computational elements called segments;
- To account for variations over the water body;
- Levels of substances at various locations;

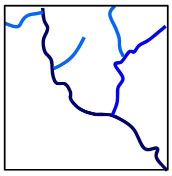


3.2 Finite Segment Method

- Best not to have too many segments than necessary;
- Imply a greater level of detail than is generally possible to parameterize;
- Greater opportunity for error (modify param);
- Comp. less efficient;
- Calibration difficult.

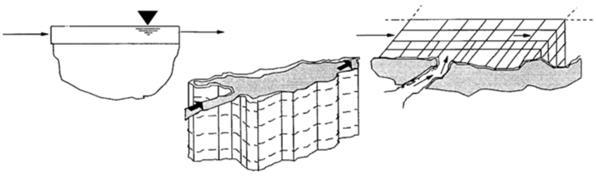
Segmentation Considerations

- Spatial scale of the problem
  - Segment of a water body (e.g., reach, embayment)
  - Whole water body (e.g., main river, lake, estuary)
  - Whole river basin network



Segmentation Considerations

- Dimensionality and spatial discretization of segments
  - Box, 1-, 2-, 3-dimensions
  - Horizontal: tens of meters to tens of kilometers
  - Vertical: whole water column to tens of cm;

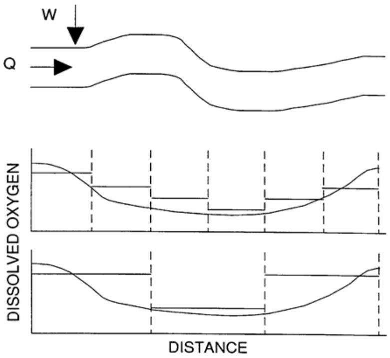


Segmentation Considerations

- Physical components
  - Water column (epilimnion, hypolimnion)
  - Benthic sediments (surface, subsurface)
- Model limitations
  - Maximum # of Segments 3000
  - Maximum # of Time Pairs 4000

Segmentation Considerations

Network scale determines how precisely the model will be able to reproduce observed gradients.

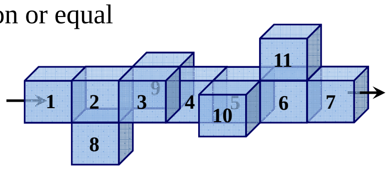


3.2 Finite Segment Method

- FSM - segmentation of model ecosystems;
- Into various “completely mixed” boxes;
- Of known volume and interchange;
- Process known as compartmentalization;
- Popular assumption in fate modelling;
- Because assumption of complete mixing;
- Reduces the set of PDEs (in time and space);
- To one of ODEs (in time only).

3.2 Finite Segment Method

- Nevertheless, it is possible to recover some coarse spatial information;
- By introducing a number of interconnected segments/compartments;
- Interchange between segments is simulated via bulk dispersion or equal counterflows between segments.



3.2 Finite Segment Method

- Estuary width gradually ↑ towards sea;
- Laterally and vertically well-mixed;
- Concentration gradient along the *x*-axis.

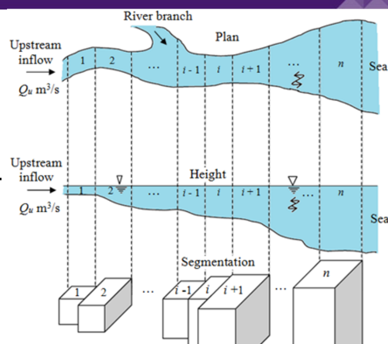


Figure 3.1 Simple representation of an estuary with finite segments

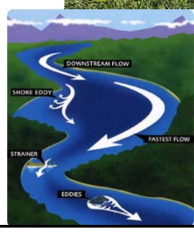
3.2 Finite Segment Method

- A substance reaction in a water body is important aspect of the substance’s fate in the environment;
- But an equally important process has to do with the rate of a substance’s transport in the aquatic environment;
- A substance is transported by water movement and may undergo additional transport processes such as decay or sedimentation.

Transport Processes

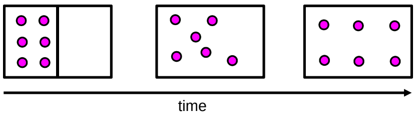
- *Decay* – substance reduced in mass due to decomposition or sedimentation;

- *Advection* – sub movement at current velocity;



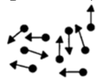
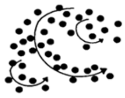
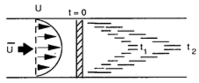
Diffusion and Dispersion

- Diffusion: process where a constituent moves from a higher concentration to a lower concentration.

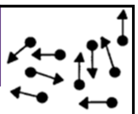


- Dispersion:
  - process by which substance is mixed within water column.
  - mixing caused by physical processes.

Dispersion

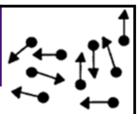
- 3 processes contribute to mixing (dispersion):
  1. Molecular diffusion,
     Random motion of particles
     
  2. Turbulent diffusion, and
     Turbulent mixing of particles
     
  3. Dispersion.
     Mixing caused
     by variations in velocities
     

1. Molecular diffusion



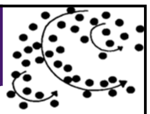
- Mixing of dissolved substances;
- Due to random walk of molecules within fluid;
- Caused by kinetic energies of molecular vibrational, rotational and translational motion;
- In essence, molecular diffusion corresponds to an increase in entropy;
- Move from regions of high concentration to regions of low concentration;
- According to Fick’s laws of diffusion.

1. Molecular diffusion



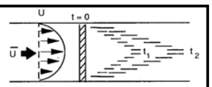
- Generally not an important process;
- In the transport of dissolved substances in natural waters;
- Except relating to transport through thin and stagnant films ;
- At the air-water interface or transport through sediment pore water.

2. Turbulent diffusion



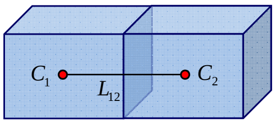
- @ Eddy diffusion;
- Mixing of dissolved substances caused by microscale turbulence;
- Advective process at microscale level caused by eddy fluctuations in turbulent shear flow;
- Shear forces within the body of water are sufficient to cause this form of mixing;
- Several orders of magnitude > mol. diffusion;
- A contributing factor in dispersion.

3. Dispersion



- Interaction of turbulent diffusion with velocity gradients;
- Caused by shear forces in the water body;
- Greater degree of mixing known as *dispersion*;
- Transport of substances in streams and rivers;
- Is predominantly by advection;
- But transport in lakes and estuaries is often dispersion-controlled.

Mathematical Representations



- Diffusion:
  - Fick's First Law of Diffusion

$$\text{Flux} = -D \times (C_2 - C_1)$$

- Dispersion:
  - Analogous to Fick's First Law

$$\text{Exchange} = -\frac{E_{12} \times A_{12}}{L_{12}}(C_2 - C_1)$$

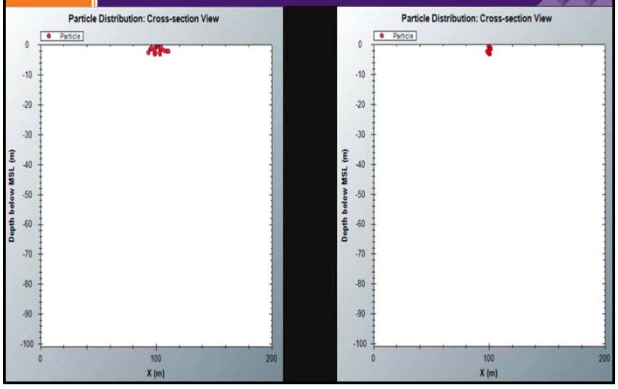
Range of Values for Dispersion

Process	Direction	Typical Range [m^2/s]
Molecular Diffusion	Vertical	10^{-8} to 10^{-9}
	Lateral	10^{-8} to 10^{-9}
	Longitudinal	10^{-8} to 10^{-9}
Turbulent Diffusion	Vertical	10^{-6} to 10^{-2}
	Lateral	10^{-2} to 10^{2}
	Longitudinal	10^{-2} to 10^{2}
Dispersion	Vertical	10^{-3} to 10^{-1}
	Lateral	10^{-2} to 10^{0}
	Longitudinal	10^{-1} to 10^{4}

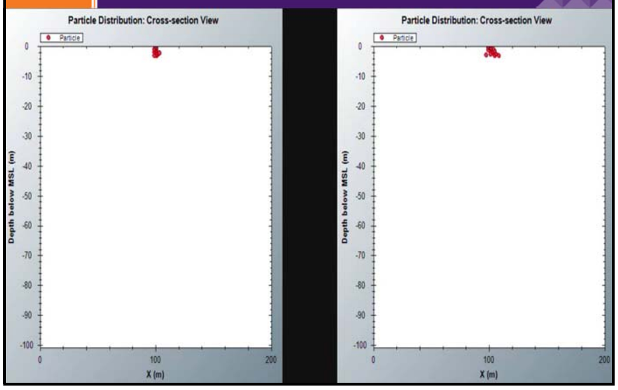
Dispersion Coefficients

Condition	Dispersion Coefficient [m^2/s]
Molecular Diffusion	10^{-9}
Compacted Sediment	10^{-11} to 10^{-9}
Bioturbated Sediment	10^{-9} to 10^{-8}
Lakes – Vertically	10^{-6} to 10^{-4}
Large Rivers – Lateral	10^{-4} to 10^{-3}
Large Rivers – Longitudinal	10^{0} to 10^{2}
Estuaries – Longitudinal	10^{2} to 10^{3}

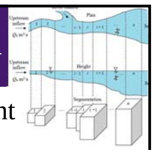
Dispersion



Dispersion + Advection

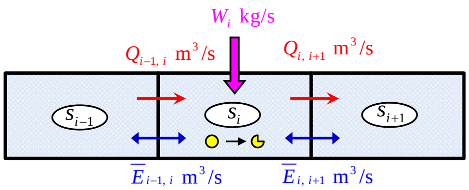


3.2 Finite Segment Method

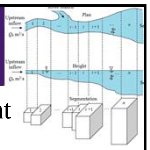


- $\therefore$ mass balance equation at i^{th} segment is influenced by four components;

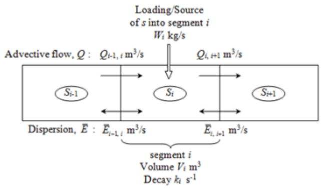
iv. transport of substances due to mixing within the water body (dispersion);



3.2 Finite Segment Method

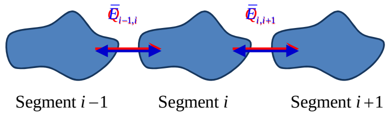


- ∴ mass balance equation at i^{th} segment is influenced by four components;
  - i. transport of the substance s by current of the water (advective flow);
  - ii. transport of due to mixing within the water body (dispersion);
  - iii. loss of mass due to decay process;
  - iv. source of substance s.



3.2 Finite Segment Method

$$V_i \frac{ds_i}{dt} = Q_{i-1,i}\, s_{i-1,i} - Q_{i,i+1}\, s_{i,i+1} - k_i V_i s_i + W_i$$
$$+\bar{E}_{i-1,\,i} \cdot \left(s_{i-1} - s_i\right) \qquad -(\gamma V + vA_s)C$$
$$+\bar{E}_{i,\,i+1} \cdot \left(s_{i+1} - s_i\right) \qquad -kVC$$

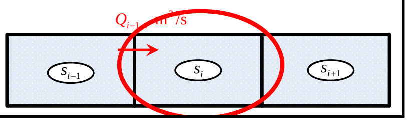


(i) Advective Flow

- Mass per unit time for s that enters and exits segment i due to advective flow Q m^3/s;
- Input of mass kg/s from upstream segment:

$$+Q_{i-1,\,i} \cdot s_{i-1,\,i} = +Q_{i-1,\,i} \cdot s_{i-1} \qquad (3.1)$$

$$\left(\frac{\text{m}^3}{\text{s}}\right) \cdot \left(\frac{\text{kg}}{\text{m}^3}\right) = \left(\frac{\text{kg}}{\text{s}}\right)$$

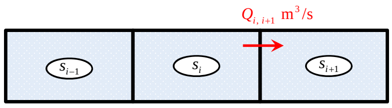


(i) Advective Flow

- Transport of mass by advective flow that exits segment i:

$$-Q_{i,\,i+1} \cdot s_{i,\,i+1} = -Q_{i,\,i+1} \cdot s_i \qquad (3.2)$$

$$\left(\frac{\text{m}^3}{\text{s}}\right) \cdot \left(\frac{\text{kg}}{\text{m}^3}\right) = \left(\frac{\text{kg}}{\text{s}}\right)$$

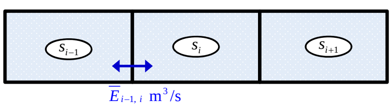


(ii) Dispersion

- Mass change due to diffusion and mixing;
- Transport directly proportional to mass conc difference between two adjacent segments;
- Exchange of mass between segment i and segment i – 1 due to dispersion:

$$+\bar{E}_{i-1,\,i} \cdot \left(s_{i-1} - s_i\right) \qquad (3.3)$$

$$\left(\frac{\text{m}^3}{\text{s}}\right) \cdot \left(\frac{\text{kg}}{\text{m}^3}\right) = \left(\frac{\text{kg}}{\text{s}}\right)$$

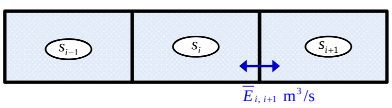


(ii) Dispersion

- Exchange of mass between segment i and segment i + 1 due to dispersion:

$$+\bar{E}_{i,\,i+1} \cdot \left(s_{i+1} - s_i\right) \qquad (3.4)$$

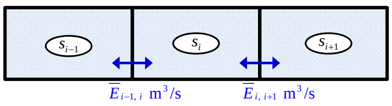
$$\left(\frac{\text{m}^3}{\text{s}}\right) \cdot \left(\frac{\text{kg}}{\text{m}^3}\right) = \left(\frac{\text{kg}}{\text{s}}\right)$$



(ii) Dispersion

- $S_{i-1} > S_i$: mass from Seg *i*-1 to *i*;
- $S_i > S_{i-1}$: mass from Seg *i* to *i*-1;
- Similarly for interface of Seg *i* and *i*+1;
- Bulk dispersion coefficient $\overline{E}$ is related to *E:*

$$\overline{E}_{i-1,\,i} = \frac{E_{i-1,\,i} \cdot A_{i-1,\,i}}{\overline{\Delta x}_{i-1,\,i}} \qquad (3.5)$$



(ii) Dispersion

$$\overline{E}_{i-1,\,i} = \frac{E_{i-1,\,i} \cdot A_{i-1,\,i}}{\overline{\Delta x}_{i-1,\,i}} \qquad (3.5)$$

- $A_{i-1,\,i}$ m^2 = CS area between Seg *i* and *i*-1;
- $\overline{\Delta x}_{i-1,\,i}$ m = mean length between Seg *i* and *i*-1;

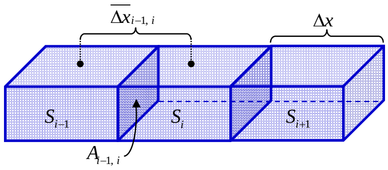


Figure 3.3 Cross sectional area, mean length and segment length

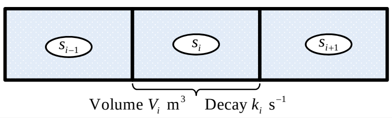
(iii) Decay

- For mass loss due to first-order decay process:

$$-k_i \quad V_i \quad s_i \qquad (3.6)$$

$$\left(\frac{1}{s}\right)\cdot\left(m^3\right)\cdot\left(\frac{kg}{m^3}\right)=\left(\frac{kg}{s}\right)$$

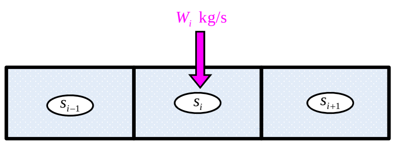
with k_i s^{-1} = decay coefficient of segment *i*.
V_i m^3 = volume of segment *i*.



(iv) Sources and Sinks

- Source or sink of substance *s*:

$$\pm W_i \quad \left(\frac{kg}{s}\right) \qquad (3.7)$$



3.2 Finite Segment Method

- Total quantity of all masses obtained must;
- = Rate of change in mass at segment *i*;

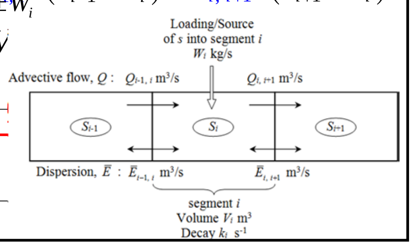
$$\frac{dM_i}{dt} = \frac{d\left(V_i \; s_i\right)}{dt} \approx V_i \frac{ds_i}{dt} \qquad (3.8)$$

- for $M_i = V_i \; s_i$ with V_i constant;
- Good approximation for most estuaries.

Overall Mass Balance at Segment *i*

$$V_i \frac{ds_i}{dt} = Q_{i-1,\,i} \cdot s_{i-1} - Q_{i,\,i+1} \cdot s_i + \overline{E}_{i-1,\,i} \cdot \left(s_{i-1} - s_i\right) + \overline{E}_{i,\,i+1} \cdot \left(s_{i+1} - s_i\right) - k_i \; V_i \; s_i \pm W_i \qquad (3.9)$$

Figure 3.2 Notations for segment *i* illustrating advective flow, dispersion, mixing and decay of substance *s*.



3.2 Finite Segment Method

$$V_i \frac{ds_i}{dt} = Q_{i-1,\,i} \cdot s_{i-1} - Q_{i,\,i+1} \cdot s_i + \bar{E}_{i-1,\,i} \cdot \left(s_{i-1} - s_i\right) + \bar{E}_{i,\,i+1} \cdot \left(s_{i+1} - s_i\right) - k_i\, V_i\, s_i \pm W_i \quad (3.9)$$

- Eqn (3.9) is a numerical approx of

$$\frac{\partial s}{\partial t} = -u \frac{\partial s}{\partial x} + E \frac{\partial^2 s}{\partial x^2} - k\, s + W \quad (3.10)$$

3.2 Finite Segment Method

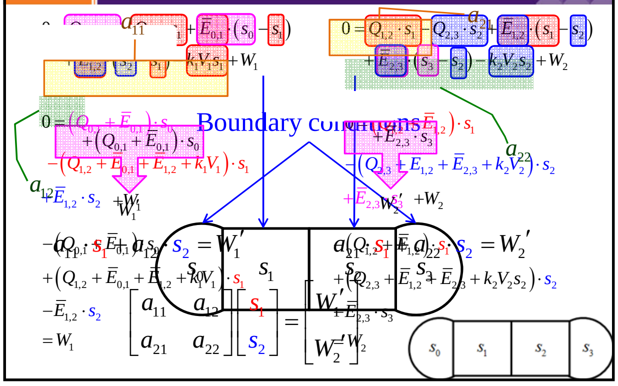
- 1^{st} - order linear ODE, time-dependent;
- Assume steady-state, i.e. input, flow, exchange and reaction rate do not change with time:

$$V_i \frac{ds_i}{dt} = 0 \quad \Rightarrow \text{Eqn (3.9)} = \text{linear algebraic eqn}$$

- n segments $\Rightarrow$ n number of Eqn (3.9).

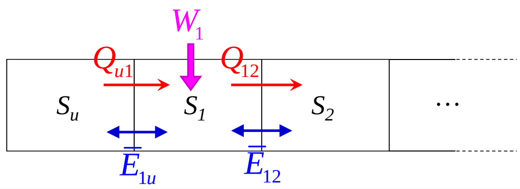
$$0 = Q_{i-1,\,i} \cdot s_{i-1} - Q_{i,\,i+1} \cdot s_i + \bar{E}_{i-1,\,i} \cdot \left(s_{i-1} - s_i\right) + \bar{E}_{i,\,i+1} \cdot \left(s_{i+1} - s_i\right) - k_i\, V_i\, s_i \pm W_i$$

3.2 Finite Segment Method



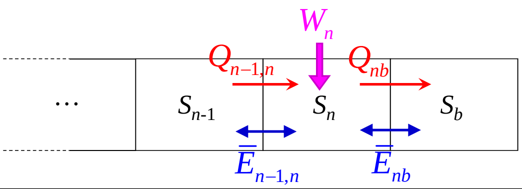
3.2 Finite Segment Method

Segment 1: $0 = Q_{u1} s_u - Q_{12} s_1 + \bar{E}_{1u}\left(s_u - s_1\right) + \bar{E}_{12}\left(s_2 - s_1\right) - k_1 V_1 s_1 \pm W_1 \quad (3.11)$



3.2 Finite Segment Method

Segment n: $0 = Q_{n-1,n}\, s_{n-1} - Q_{nb} s_n + \bar{E}_{n-1,n}\left(s_{n-1} - s_n\right) + \bar{E}_{nb}\left(s_b - s_n\right) - k_n V_n s_n \pm W_n \quad (3.12)$



3.2 Finite Segment Method

$$0 = Q_{i-1,\,i} \cdot s_{i-1} - Q_{i,\,i+1} \cdot s_i + \bar{E}_{i-1,\,i} \cdot \left(s_{i-1} - s_i\right) + \bar{E}_{i,\,i+1} \cdot \left(s_{i+1} - s_i\right) - k_i\, V_i\, s_i \pm W_i$$

- Group to the left all terms with s_{i-1}, s_i, s_{i+1};

$$\left(-Q_{i-1,\,i} - \bar{E}_{i-1,\,i}\right) \cdot s_{i-1} + \left(Q_{i,\,i+1} + \bar{E}_{i-1,\,i} + \bar{E}_{i,\,i+1} + k_i\, V_i\right) s_i + \left(-\bar{E}_{i,\,i+1}\right) s_{i+1} = W_i \quad (3.13)$$

3.2 Finite Segment Method

Let:

$$a_{i,i-1} = -Q_{i-1,\,i} - \bar{E}_{i-1,\,i} \qquad (3.14)$$

$$a_{i,i} = Q_{i,\,i+1} + \bar{E}_{i-1,\,i} + \bar{E}_{i,\,i+1} + k_i\,V_i \qquad (3.15)$$

$$a_{i,i+1} = -\bar{E}_{i,\,i+1} \qquad (3.16)$$

Eqn (3.13) can be simplified to

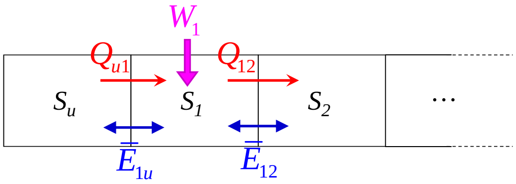
$$a_{i,i-1}\,s_{i-1} + a_{i,i}\,s_i + a_{i,i+1}\,s_{i+1} = W_i \qquad (3.17)$$

3.2 Finite Segment Method

Segment 1: $a_{11}s_1 + a_{12}s_2$

$$= W_1 + Q_{u1}s_u + \bar{E}_{1u}s_u = W_1' \qquad (3.18)$$

$$W_1' = W_1 + Q_{u1}s_u + \bar{E}_{1u}s_u \qquad (3.19)$$

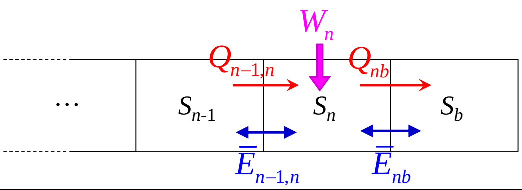


3.2 Finite Segment Method

Segment n: $a_{n,n-1}\,s_{n-1} + a_{n,n}\,s_n$

$$= W_n + \bar{E}_{nb}\,s_b = W_n' \qquad (3.20)$$

$$W_n' = W_n + \bar{E}_{nb}\,s_b \qquad (3.21)$$



3.2 Finite Segment Method

$\therefore$ The complete set of eqns for n segments:

$$\begin{array}{ccccccccccccccc}
a_{11}s_1 & + & a_{12}s_2 & + & 0 & + & \cdots & & \cdots & & & + & 0 & = & W_1' \\
a_{21}s_1 & + & a_{22}s_2 & + & a_{23}s_3 & + & 0 & + & \cdots & & \cdots & + & 0 & = & W_2' \\
0 & + & a_{32}s_2 & + & a_{33}s_3 & + & a_{34}s_4 & + & 0 & + & \cdots & + & 0 & = & W_3' \\
\vdots & & \vdots & & \vdots & & \vdots & & \vdots & & \vdots & & \vdots & & \\
0 & + & \cdots & & \cdots & & \cdots & & \cdots & + & a_{n,n-1}s_{n-1} & + & a_{n,n}s_n & = & W_n'
\end{array}$$

(3.22)

3.2 Finite Segment Method

In matrix form: $[A](s) = (W)$

$$\begin{bmatrix} a_{11} & a_{12} & 0 & \cdots & \cdots & \cdots & 0 \\ a_{21} & a_{22} & a_{23} & 0 & \cdots & \cdots & 0 \\ 0 & a_{32} & a_{33} & a_{34} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & \cdots & \cdots & 0 & a_{n,n-1} & a_{n,n} \end{bmatrix} \begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ \vdots \\ s_n \end{pmatrix} = \begin{pmatrix} W_1' \\ W_2' \\ W_3' \\ \vdots \\ W_n' \end{pmatrix}$$

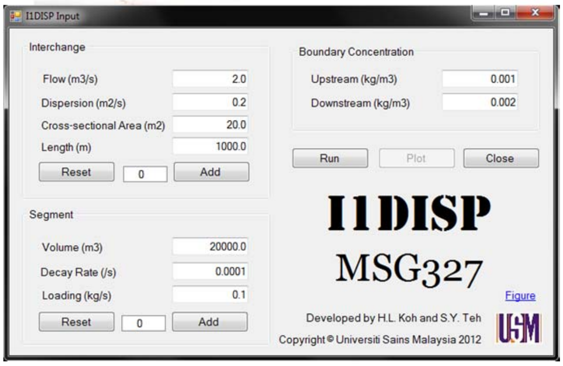
$$(\mathrm{m^3/s})\cdot(\mathrm{kg/m^3}) = (\mathrm{kg/s}) \qquad (3.23)$$

$[A]$ = matrix $n \times n$ $\quad (s)$ and $(W) = n \times 1$ vectors

3.2 Finite Segment Method

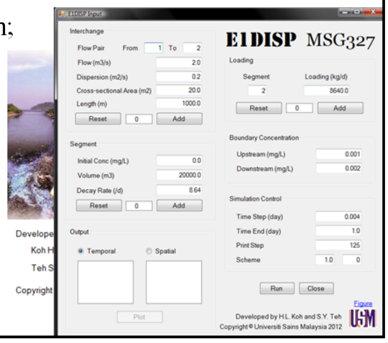
- (3.23) : Boundary value problem-steady-state;
- Boundary conditions: s_u and s_b;
- FSM – can only be solved easily by hand;
- For systems of dimension 3 by 3;
- River pollution - complex matter;
- Requires division of river into many segments;
- For more accurate and reliable solution.
- $\Rightarrow$ system of equations with high dimensions;
- Numerical method - I1DISP (Koh, 2004).

I1DISP – Steady-state Model



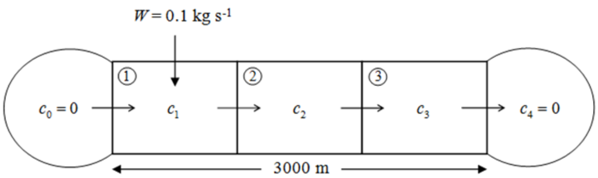
3.2 Finite Segment Method

- Non steady-state, $ds/dt \neq 0$;
- Initial condition;
- Initial value problem;
- Runge Kutta methods;
- E1DISP.



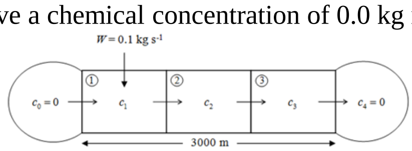
Example 3.1

Consider a uniform river with cross sectional area A = 20 m^2, length L = 3000 m, velocity u = 0.1 m/s and dispersion coefficient E = 0.0 m^2/s. This river is divided into three uniform segments with Δx = 1000 m for each segment.



Example 3.1

A chemical is released into segment 1 of the river at the rate of W = 0.1 kg s^{-1}. This chemical decays at the rate of k = 10^{-4} s^{-1}. The water at the upstream and downstream segments is assumed to have a chemical concentration of 0.0 kg m^{-3}.



Find the concentration for every segment c_1, c_2, c_3 after the steady state is achieved.

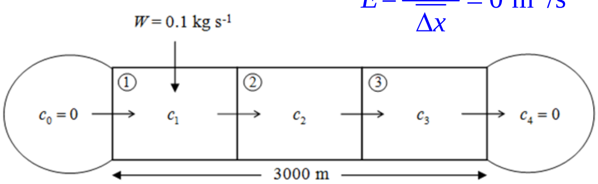
Example 3.1

A = 20 m^2 L = 3000 m u = 0.1 m/s

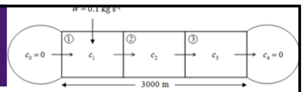
E = 0.0 m^2/s Δx = 1000 m W = 0.1 kg/s

k = 10^{-4} s^{-1} $Q = uA = 2\ \mathrm{m^3/s}$

$$\bar{E} = \frac{E \cdot A}{\Delta x} = 0\ \mathrm{m^3/s}$$



Example 3.1



$kV = (10 \ldots) = 2\ \mathrm{m^3/s}$

Segment 1:

$0 = \boxed{-\ 2 \cdot c_1 \quad 2}\ \ 0.1 \qquad -k_1 \cdot V_1 \cdot c_1 + W_1$

$-\ 4c_1$

Segment 2:

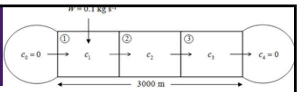
$0 = 2 \cdot c_1 \boxed{-\ 2 \cdot c_2 \quad 2} \qquad -k_2 \cdot V_2 \cdot c_2$

$-\ 4c_2$

Segment 3:

$0 = 2 \cdot c_2 \boxed{-\ 2 \cdot c_3 \quad 2} -\ 4c_3 \qquad -k_3 \cdot V_3 \cdot c_3$

Example 3.1



Segment 1:

$4 \cdot c_1 = \qquad +0.1$

$A\underset{\sim}{c} = \underset{\sim}{w}$

$$\begin{bmatrix} 4 & 0 & 0 \\ -2 & 4 & 0 \\ 0 & -2 & 4 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0 \\ 0 \end{bmatrix}$$

Segment 2:

$-2 \cdot c_1 + 4 \cdot c_2 = 0$

$c_1 = 0.025 \text{ kg/m}^3 = 25 \text{ mg/L}$

$c_2 = 0.0125 \text{ kg/m}^3 = 12.5 \text{ mg/L}$

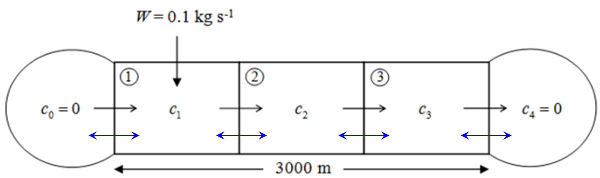
Segment 3:

$-2 \cdot c_2 + 4 \cdot c_3 = 0$

$c_3 = 0.00625 \text{ kg/m}^3 = 6.25 \text{ mg/L}$

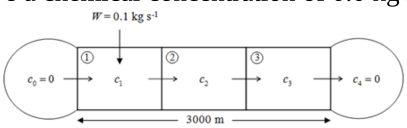
Example 3.2

Consider a uniform river with cross sectional area A = 20 m^2, length L = 3000 m, velocity u = 0.1 m/s and dispersion coefficient ***E* = 0.2 m^2/s**. This river is divided into three uniform segments with Δx = 1000 m for each segment.



Example 3.2

A chemical is released into segment 1 of the river at the rate of W = 0.1 kg s^{-1}. This chemical decays at the rate of k = 10^{-4} s^{-1}. The water at the upstream and downstream segments is assumed to have a chemical concentration of 0.0 kg m^{-3}.



Find the concentration for every segment c_1, c_2, c_3 after the steady state is achieved.

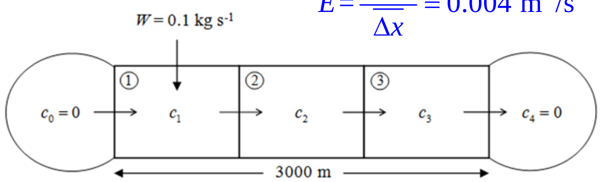
Example 3.2

A = 20 m^2 L = 3000 m u = 0.1 m/s

***E* = 0.2 m^2/s** Δx = 1000 m W = 0.1 kg/s

k = 10^{-4} s^{-1} $Q = uA = 2 \text{ m}^3/\text{s}$

$$\overline{E} = \frac{E \cdot A}{\Delta x} = 0.004 \text{ m}^3/\text{s}$$



Example 3.2 - Solution

$$A\underset{\sim}{s} = \begin{pmatrix} 4.008 & -0.004 & 0 \\ -2.004 & 4.008 & -0.004 \\ 0 & -2.004 & 4.008 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 0.1 \\ 0 \\ 0 \end{pmatrix} = \underset{\sim}{w}$$

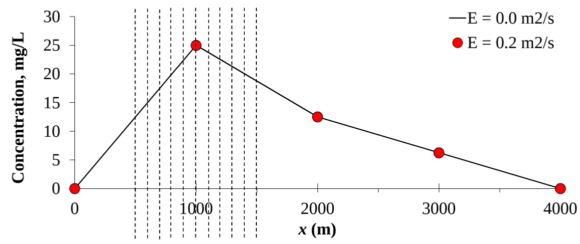
$c_1 = 0.02496 \text{ kg/m}^3$ or $c_1 = 24.96 \text{ mg/L}$

$c_2 = 0.01249 \text{ kg/m}^3$ or $c_2 = 12.49 \text{ mg/L}$

$c_3 = 0.00624 \text{ kg/m}^3$ or $c_3 = 6.24 \text{ mg/L}$

Examples 3.1 & 3.2: Comparison

c (mg/L)	x (m)				
	0	1000	2000	3000	4000
Example 3.1	0	25	12.5	6.25	0
Example 3.2	0	24.96	12.49	6.24	0



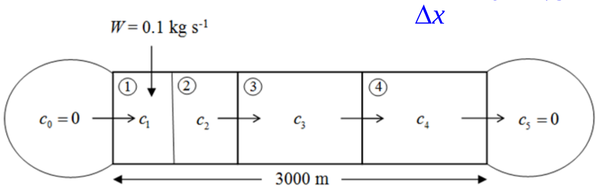
Example 3.3

$A = 20\ \text{m}^2$ L = 3000 m $u = 0.1$ m/s

$E = 0.0\ \text{m}^2/\text{s}$ $\Delta x = 1000$ m $W = 0.1$ kg/s

$k = 10^{-4}\ \text{s}^{-1}$ $Q = uA = 2\ \text{m}^3/\text{s}$

$$\overline{E} = \frac{E \cdot A}{\overline{\Delta x}} = 0\ \text{m}^3/\text{s}$$



Example 3.3 - Solution

$$\underset{\sim}{A}\underset{\sim}{s} = \begin{pmatrix} 3 & 0 & 0 & 0 \\ -2 & 3 & 0 & 0 \\ 0 & -2 & 4 & 0 \\ 0 & 0 & -2 & 4 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} = \begin{pmatrix} 0.1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \underset{\sim}{w}$$

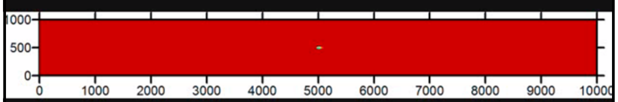
$c_1 = 0.033\ \text{kg/m}^3$ or $c_1 = 33$ mg/L

$c_2 = 0.022\ \text{kg/m}^3$ or $c_2 = 22$ mg/L

$c_3 = 0.011\ \text{kg/m}^3$ or $c_3 = 11$ mg/L

$c_4 = 0.0056\ \text{kg/m}^3$ or $c_3 = 5.6$ mg/L

2D Water Quality



Thank You

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