Signals and Systems

MATLAB Homework 2

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(a) Use the MATLAB function `plot` to plot $x[n]$ vs n, where $x[n]$ is given by

$$x[n] = \frac{\sin 2\pi n T_s}{2\pi n T_s}, n \in \{-500, -499, \dots, 499, 500\}$$

and $T_s = \frac{100}{500} = 0.2$.

The signal is given by Figure 1.

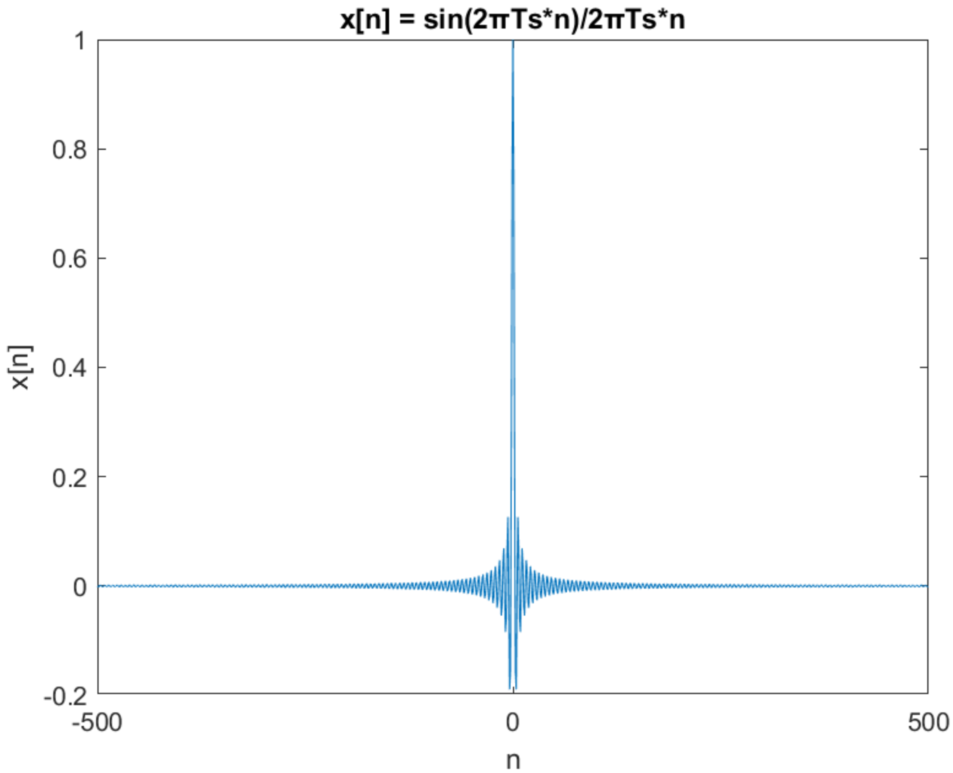

Figure 1. $x[n] = \frac{\sin 2\pi n T_s}{2\pi n T_s}$

We can see more detail of the signal on Figure 2.

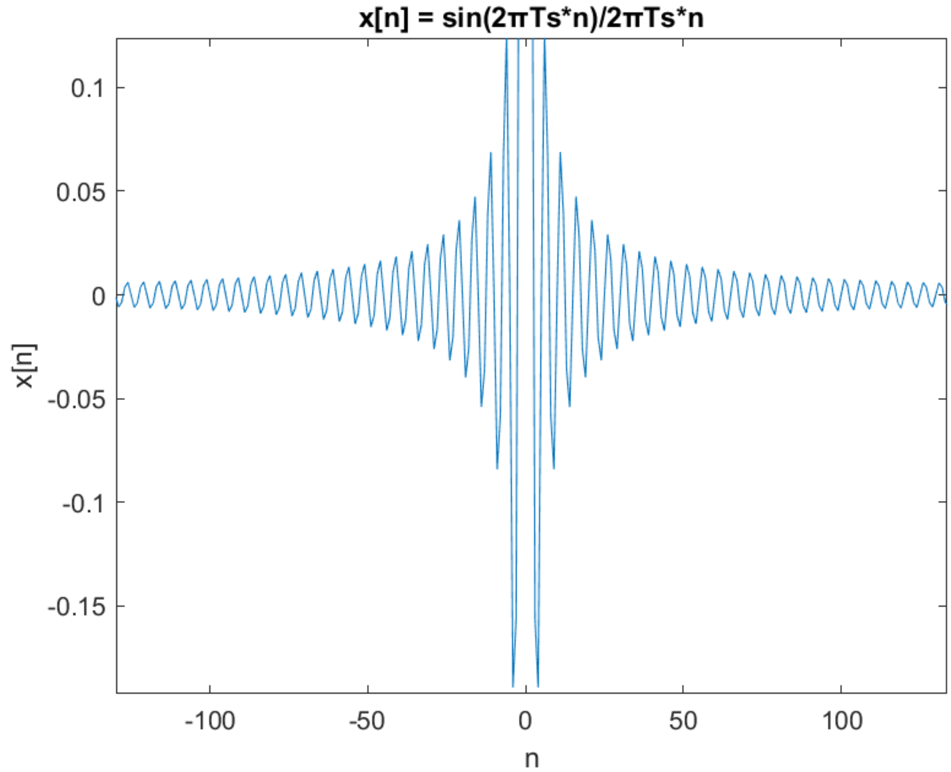

Figure 2. A closer look on $x[n]$

As Figure 2 shows, the plot of $x[n]$ looks like a sinusoidal wave with decreasing amplitude. However, the peaks on the plots are not smooth. Instead, they exhibit sharp turns, since $x[n]$ is a sample of the continuous signal $x(t)$. These peaks contribute to the high-frequency components when we calculate the Fourier transform of $x[n]$.

(b) Use the MATLAB function `fft` directly to compute DFT of $x[n]$, and use the MATLAB function `plot` to plot the magnitude of the `fft` output vs frequency ω. The zero frequency should be centered in your plot. Observe the *Gibbs phenomenon* in (b) and give some explanation for it in your report.

The Fourier transform of x[n] using `fft` function is given in Figure 3.

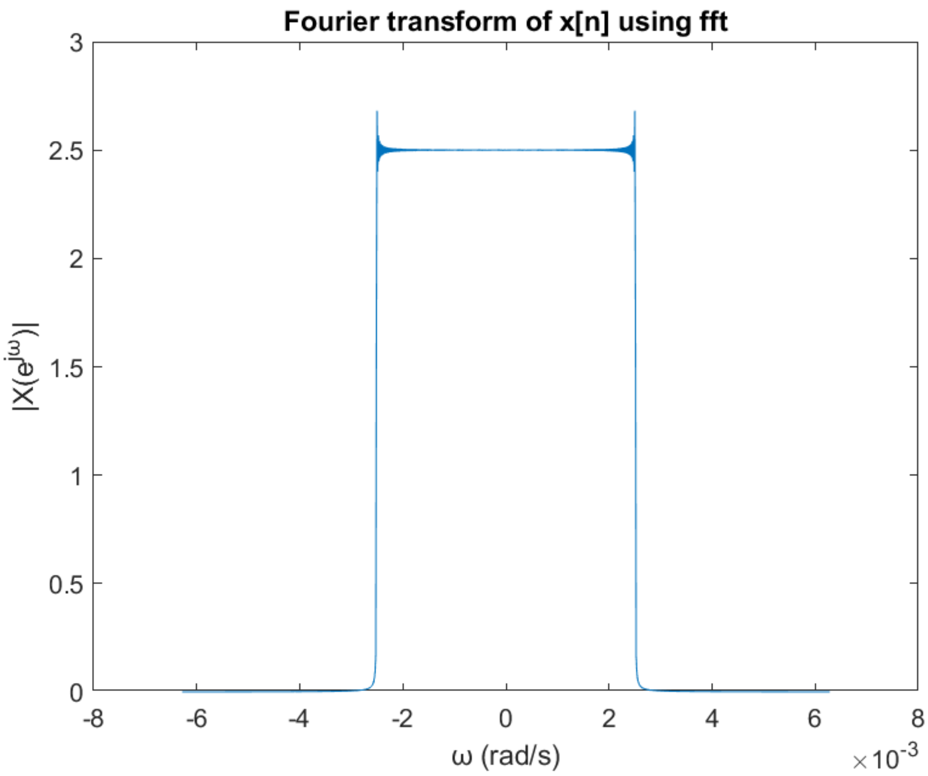

Figure 3. $X(e^{j\omega})$, the Fourier transform of $x[n]$ using `fft`

As we learned from the course, the Fourier transform of an aperiodic square wave is a sinc function. Then by the duality property of Fourier transform, a sinc wave corresponds to a square wave spectrum. However, from the sharp turns of peaks in Figure 2, we require more high-frequency components to make up for such changes. This is why we observe dramatic changes near the highest frequency range in Figure 3. We can see such variations (Gibbs phenomenon) clearer in Figure 4.

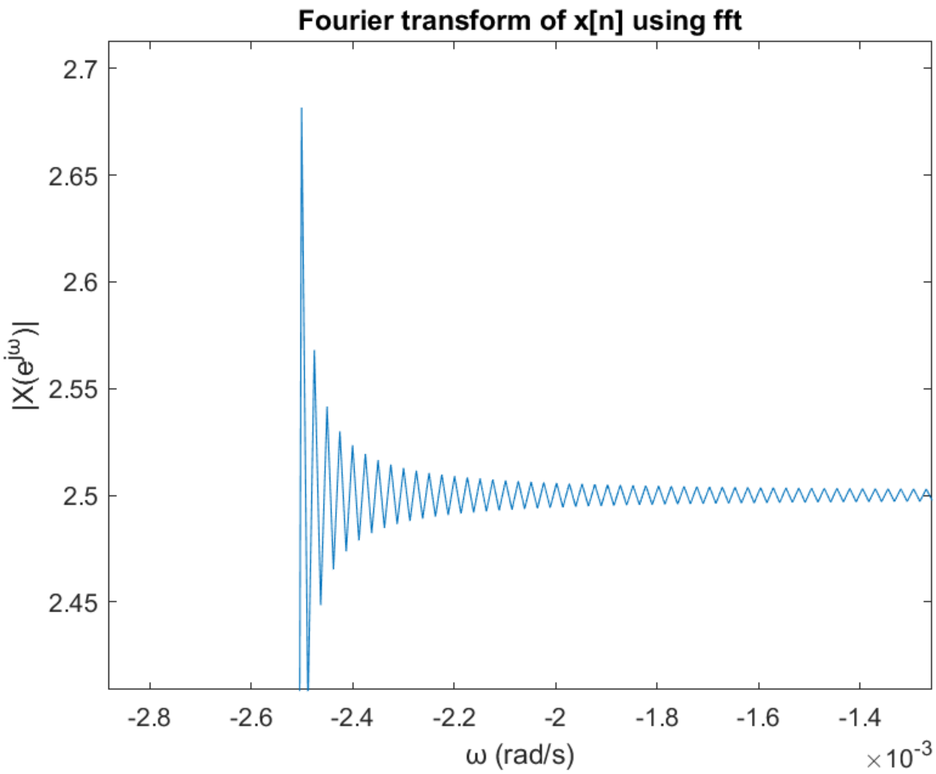

Figure 4. A closer look on $X(e^{j\omega})$ evaluated by `fft`

(c) Create a MATLAB program by yourself to compute $X_k(e^{j\omega})$ of equation (1) and use the MATLAB function `plot` to plot the magnitude of $X_k(e^{j\omega})$ vs frequency ω. You also need to rearrange $X_k(e^{j\omega})$ so that the zero frequency is centered in your plot. Verify whether the answer is the same as Problem (b).

The Fourier transform of x[n] without using the fft function is implemented by the following code, where the array X_s stores all the coefficients calculated directly from the definition of Fourier transform.

```
%% Question (c)
X_s = zeros(1, N); % Implementation of DTFT

for k = -N1 : N1
    for m = -N1 : N1
        X_s(1, k + N1 + 1) = X_s(1, k + N1 + 1) + x(m + N1 + 1) * exp(-1j * k * omega * m);
    end
end


figure % Figure 3 - Fourier transform of x[n], no fft
plot(f, abs(X_s))
title('Fourier transform of x[n], no fft')
xlabel('ω (rad/s)')
ylabel('|X(e^{jω})|')
```

Figure 5. Implementation of Fourier transform of $x[n]$

The plot of this Fourier transform is given in Figure 6.

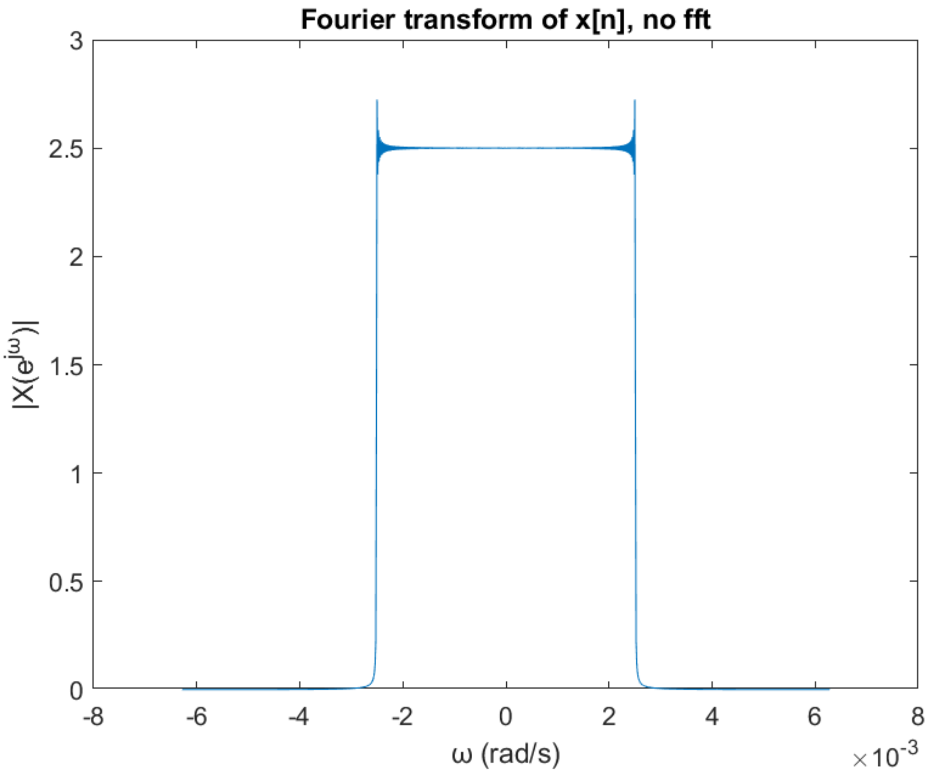

Figure 6. $X(e^{j\omega})$, the Fourier transform of $x[n]$ without `fft`

It looks nearly identical to the result calculated by using the `fft` function. However, we should compare these two results and see whether they are identical. Figure 7 is the comparison of these two Fourier transforms.

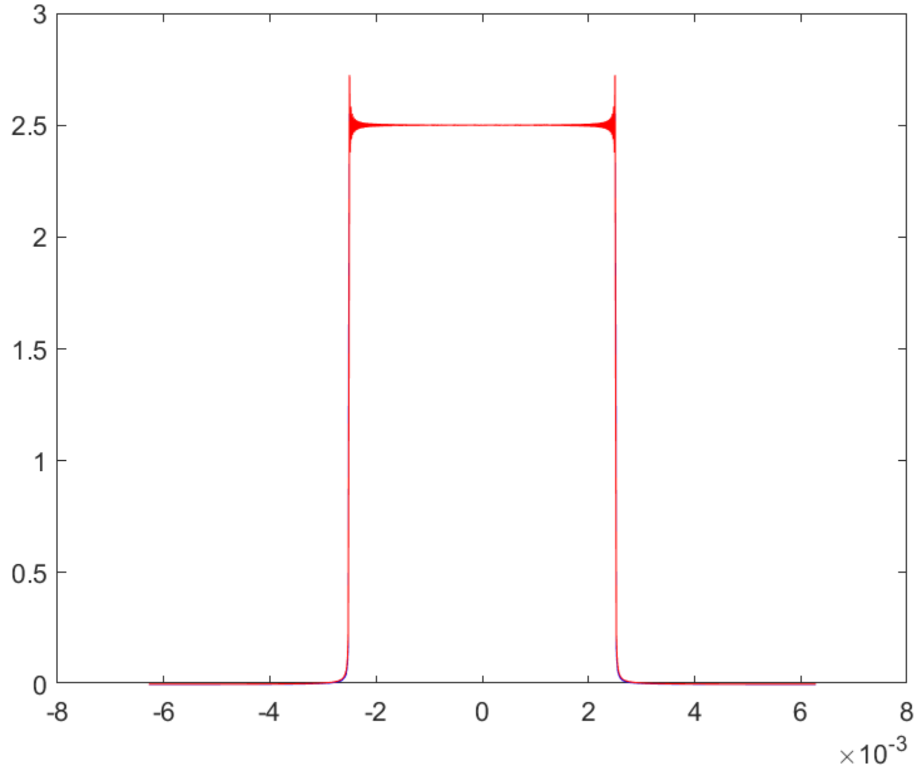

Figure 7. Two Fourier transforms of $x[n]$

The red spectrum is the Fourier transform calculated by using the definition, and the blue spectrum is the Fourier transform calculated by `fft`. Since we still cannot distinguish the spectra, we can look closer at the graph, which is shown in Figure 8:

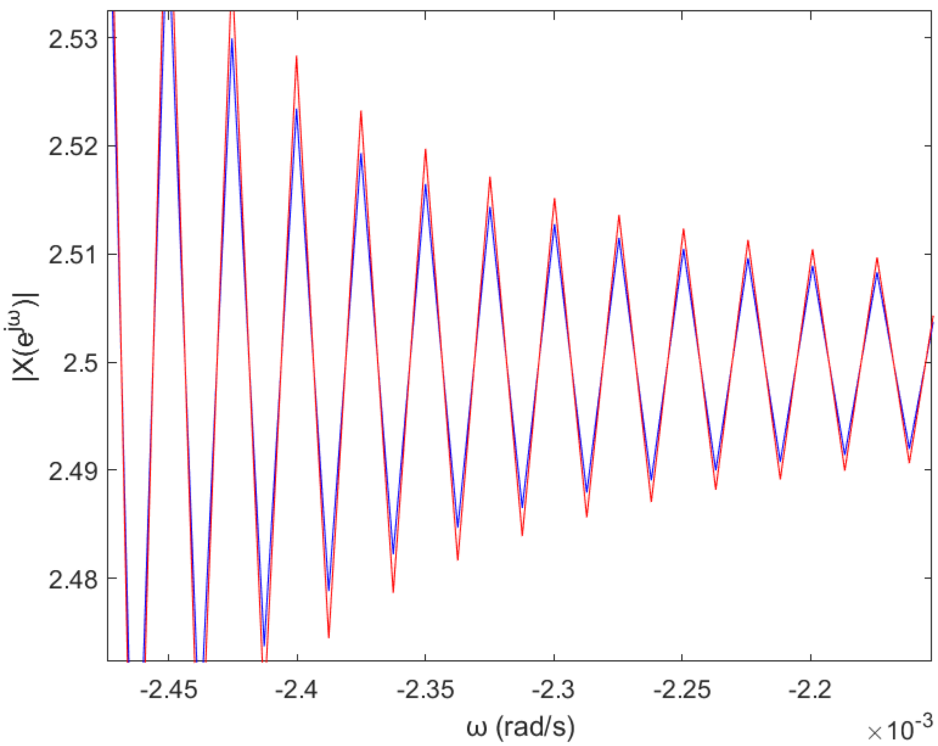

Figure 8. A closer look on Figure 7

The magnitudes of the coefficients evaluated using the definition are slightly larger than those evaluated using the `fft` function. Such difference may be attributed to the calculation of complex numbers. When we calculate the coefficients from the definition, the multiplication and addition of complex numbers are involved. Because the precision of floating-point numbers is limited, the errors in calculation may accumulate. Indeed, the two spectra are not identical, but they don’t differ significantly.

(d) Use the MATLAB function `plot` to plot $w[n]$ vs n, where $w[n]$ is given by

$$w[n] = \begin{cases} \frac{1}{2}\left(1 + \cos\frac{2\pi|nT_s|}{T_w}\right), & |nT_s| \leq \frac{T_w}{2} \\ 0, & \text{otherwise} \end{cases}$$

and $T_w = \frac{T}{2} = 50$.

The signal is given in Figure 9.

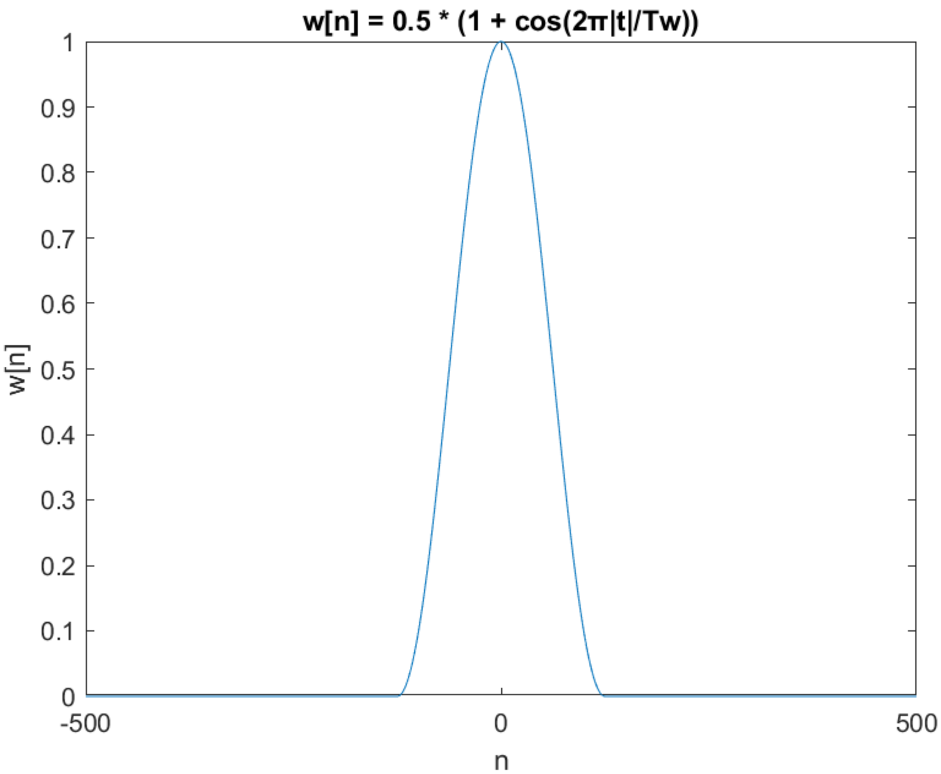

Figure 9. $w[n] = \begin{cases} \frac{1}{2}\left(1 + \cos\frac{2\pi|nT_s|}{T_w}\right), & |nT_s| \leq \frac{T_w}{2} \\ 0, & \text{otherwise} \end{cases}$

(e) Use the MATLAB function `plot` to plot $y[n]$ vs n, where $y[n] = x[n]w[n]$, and $x[n]$ is the signal plotted in (a).

The signal is given in Figure 10.

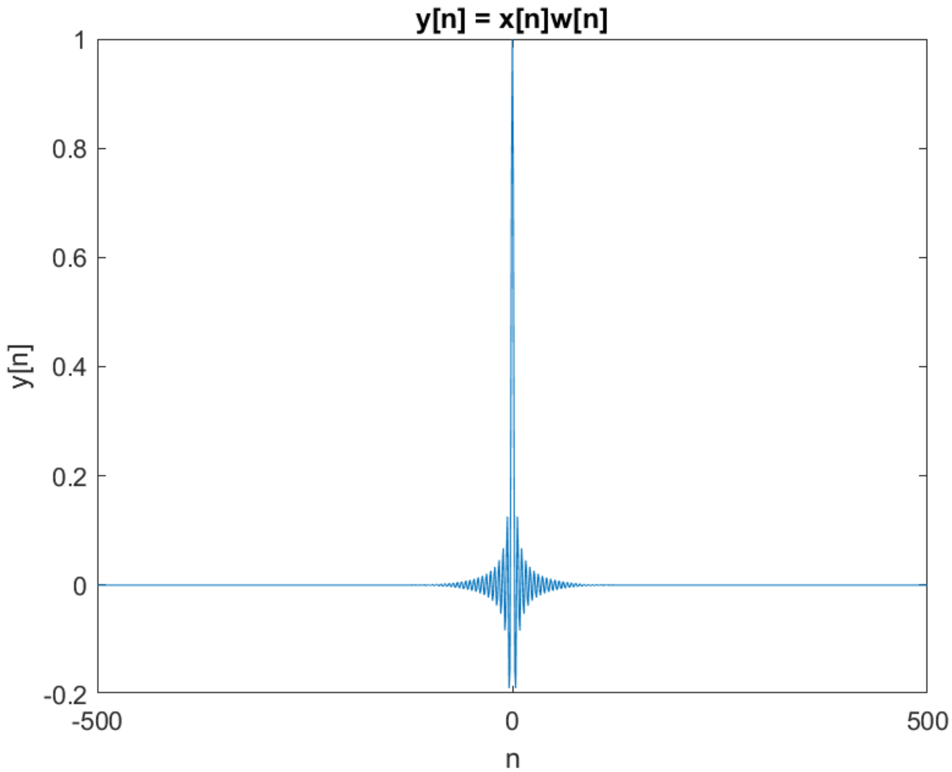

Figure 10. $y[n] = x[n]w[n]$

Figure 10 looks similar to Figure 1, therefore, we have to plot these two signals on the same coordinated in order to discover their differences, which is given in Figure 11.

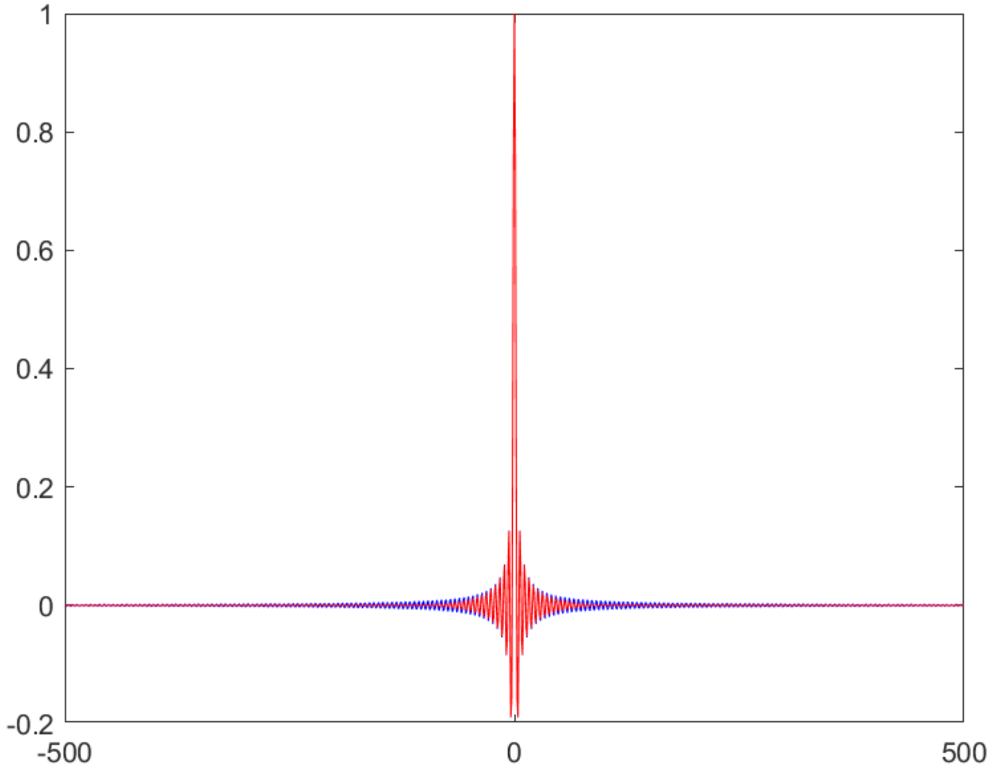

Figure 11. Comparison between $x[n]$ and $y[n] = x[n]w[n]$

The blue signal represents $x[n]$, and the red signal represents $y[n] = x[n]w[n]$. $w[n]$ behaves like a “switch”, which restricts the frame of $x[n]$ to a certain range. In this case, the range of n is all integers from -125 to 125. When n increases, the magnitude of $x[n]$ truncates with the cosine function, and eventually goes to zero. Figure 12 offers a closer look.

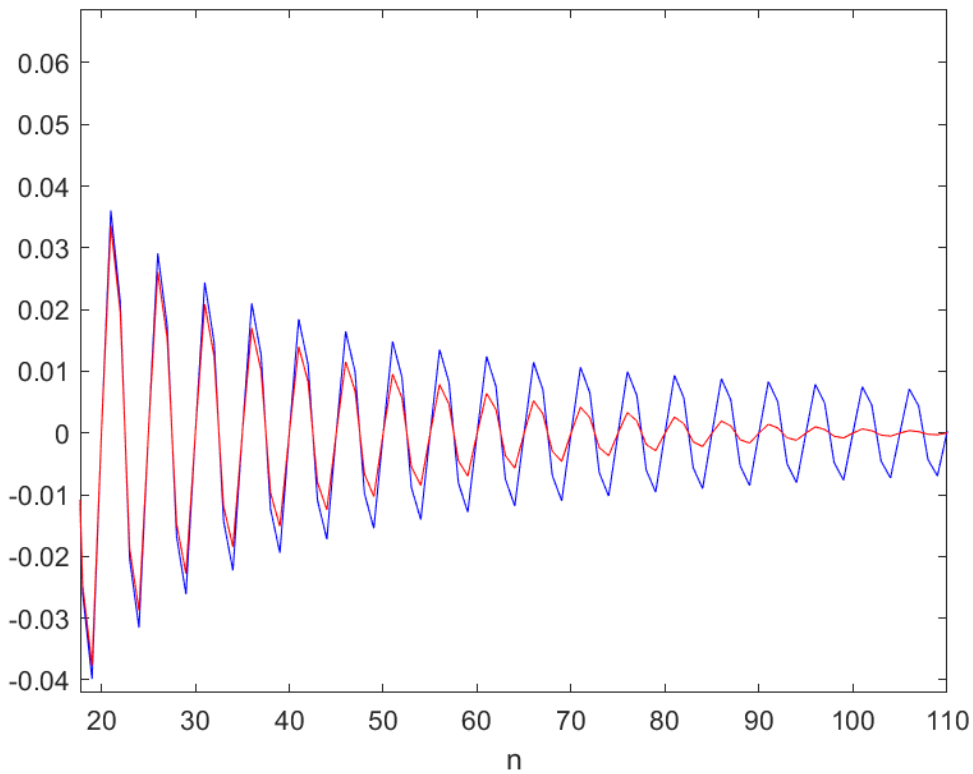

Figure 12. A closer look of Figure 11

(f) Use the MATLAB function `fft` directly to compute DFT of $y[n]$ in (e), and use the MATLAB function `plot` to plot the magnitude of the `fft` output vs frequency ω. The zero frequency should be also centered in your plot. Observe the *Gibbs phenomenon* here and give some explanation for comparison with (b) in your report.

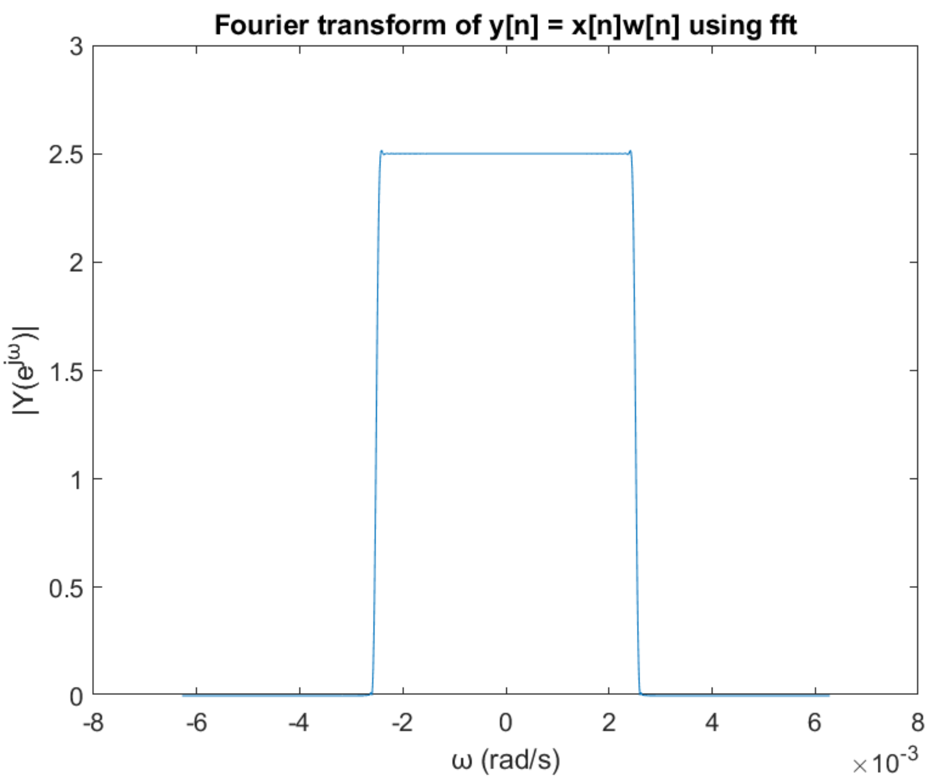

Figure 13. $X(e^{j\omega})$, the Fourier transform of $x[n]$ using `fft`

The difference between Figure 13 and Figure 6 is obvious. As Figure 13 shows, we don't observe dramatic changes of magnitude near the highest frequency range, which means that Gibbs phenomenon is reduced. This is reasonable because of $w[n]$. From Figure 11 and Figure 12, we see that all the sharp turns are ignored when $|n| > 125$. Less sharp turns and changes, less demand for high frequency components. As a result, Gibbs phenomenon is reduced and we see a "good" square wave shape spectrum.