Signals and Systems

MATLAB Homework 1

B11901110 陳璿吉

(a) Use the MATLAB function `stem` to plot $x_1[n]$ and $x_2[n]$ vs n, where

$$x_1[n] = \begin{cases} n, & 1 \le n \le 20 \\ 40 - n, & 21 \le n \le 39 \\ 0, & \text{otherwise} \end{cases}$$

$$x_2[n] = u[n-1] - u[n-11]$$

The two signals are given in the following figures.

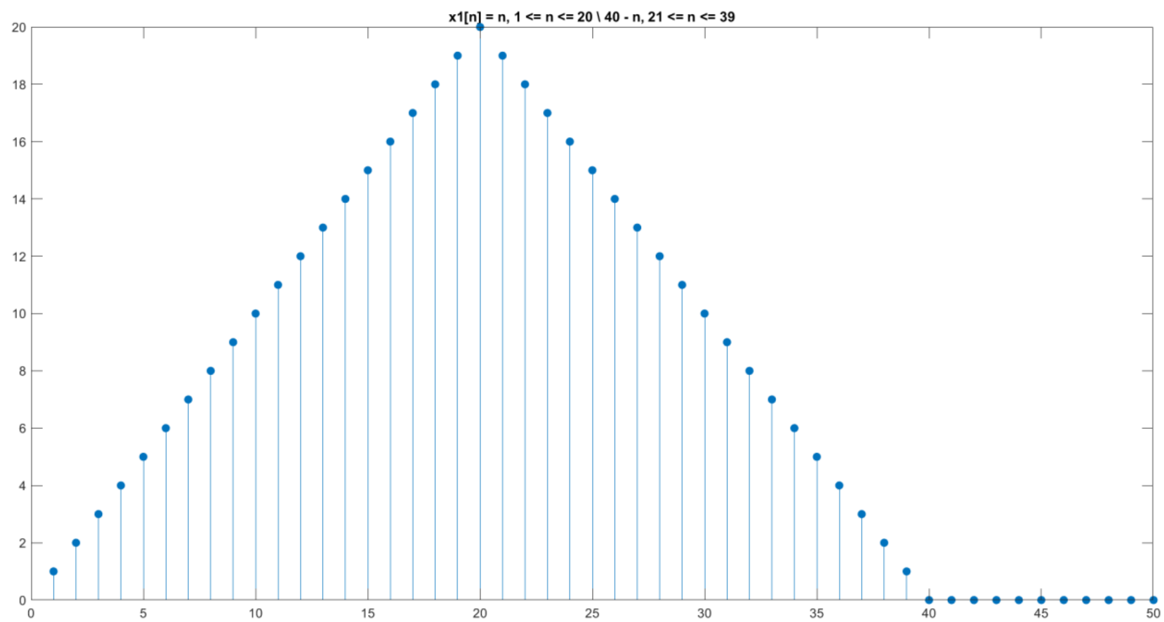

Figure 1. $x_1[n] = \begin{cases} n, & 1 \le n \le 20 \\ 40 - n, & 21 \le n \le 39 \\ 0, & \text{otherwise} \end{cases}$

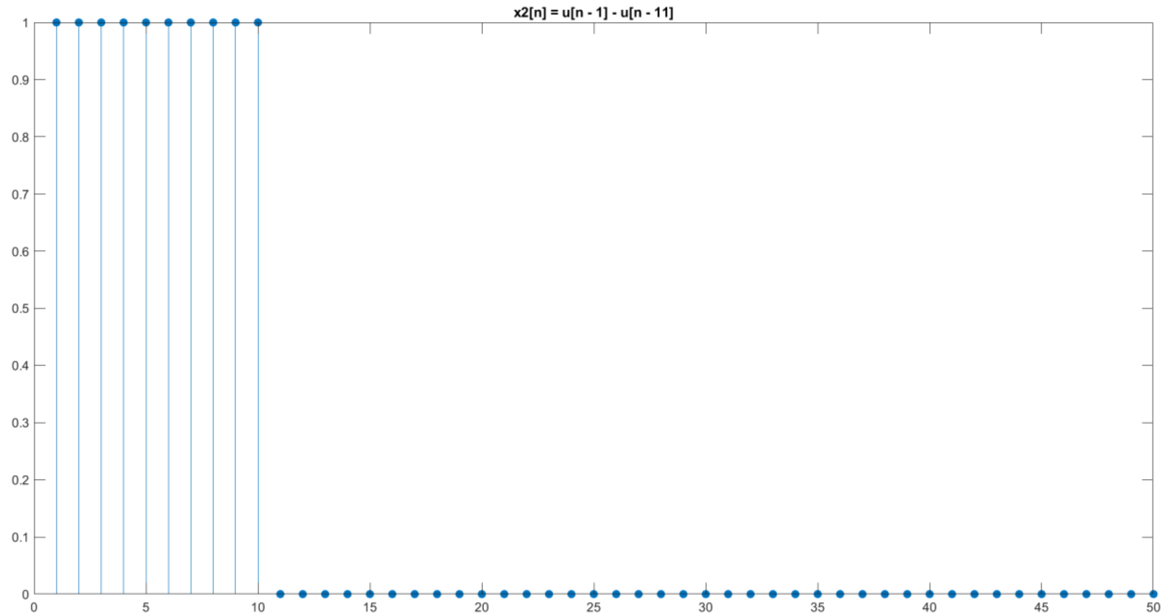

Figure 2. $x_2[n] = u[n-1] - u[n-11]$

(b) Use the MATLAB function `conv` directly to compute equation (1) and use `stem` to plot the output $y[n]$ vs n.

The output of $y[n] = \text{conv}(x_1, x_2)$ is given by the following figure:

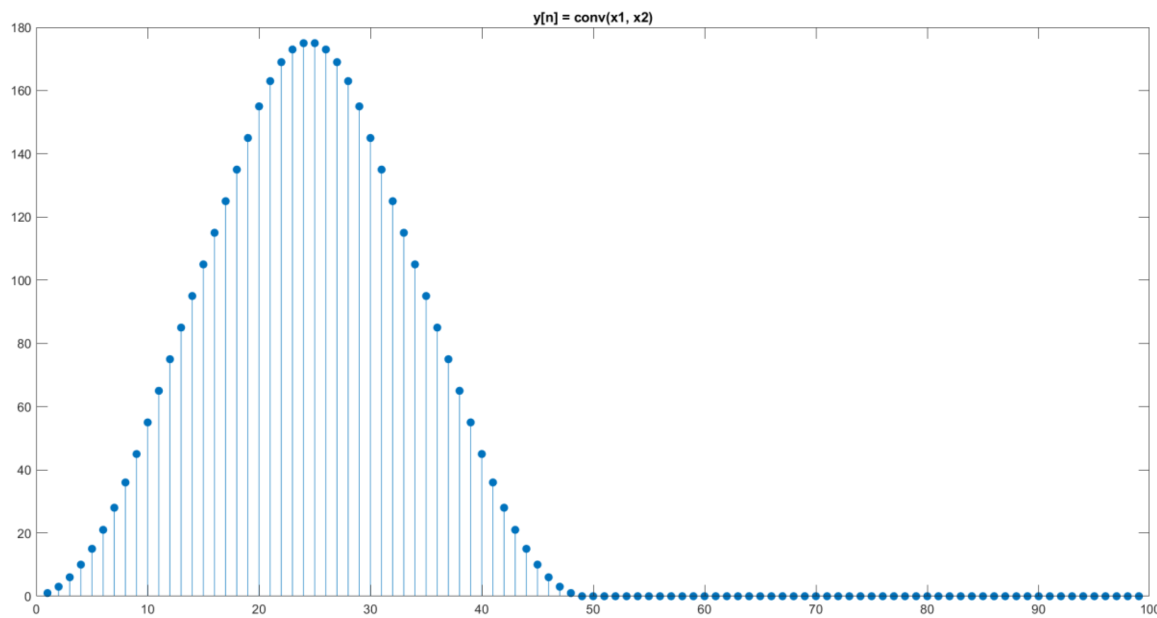

Figure 3. $y[n] = \text{conv}(x_1, x_2)$

(c) Create a MATLAB program by yourself to compute equation (1) by using equation (3) matrix form and use `stem` to plot the output $y[n]$ vs n. You should verify whether the answer is the same as Problem (b).

$$\begin{bmatrix} y[2] \\ \vdots \\ y[N_1+N_2] \end{bmatrix} = \begin{bmatrix} x_1[1] & 0 & \cdots & 0 \\ x_1[2] & x_1[1] & \ddots & \vdots \\ \vdots & x_1[2] & \ddots & 0 \\ x_1[N_1] & \vdots & \ddots & x_1[1] \\ 0 & x_1[N_1] & \ddots & x_1[2] \\ \vdots & 0 & \ddots & \vdots \\ 0 & \cdots & 0 & x_1[N_1] \end{bmatrix} \begin{bmatrix} x_2[1] \\ x_2[2] \\ \vdots \\ x_2[N_2] \end{bmatrix}. \tag{3}$$

Implementation of `myconv` function:

```
%% Question (c)

y_matrix = zeros(n1 + n2, 1);
x1_matrix = zeros(n1 + n2 - 1, n1);
x2_matrix = zeros(n2, 1);

for n = 1:n1
    for m = n:(n + n1 - 1)
        x1_matrix(m, n) = x1(m - n + 1);
    end
end

for m = 1:n2
    x2_matrix(m, 1) = x2(m);
end

y_matrix(2:n1 + n2, 1) = x1_matrix * x2_matrix;

% Plot myconv(x1, x2) = x1[n] * x2[n]
stem(y_matrix, 'filled')
title('y[n] = myconv(x1, x2) = x1[n] * x2[n]')
figure;
```

Figure 4. Implementation of myconv(x_1, x_2)

The output of $y[n] = \text{myconv}(x_1, x_2)$ is given by the following figure:

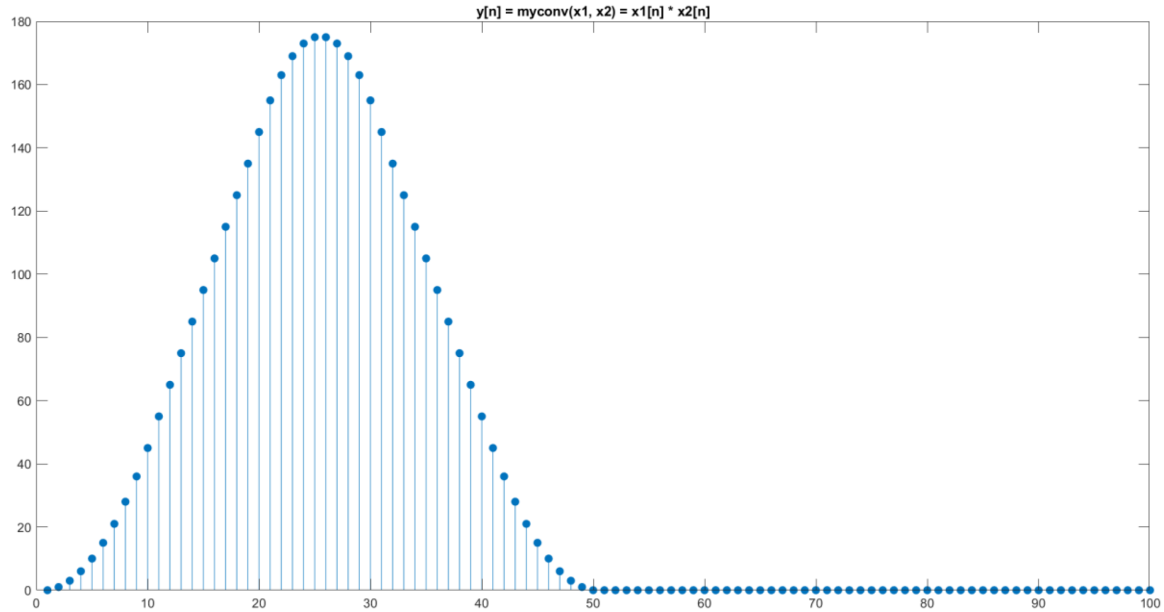

Figure 5. $y[n] = \text{myconv}(x_1, x_2)$

Comparison between $\text{conv}(x_1, x_2)$ (blue) and $\text{myconv}(x_1, x_2)$ (orange) on the same figure:

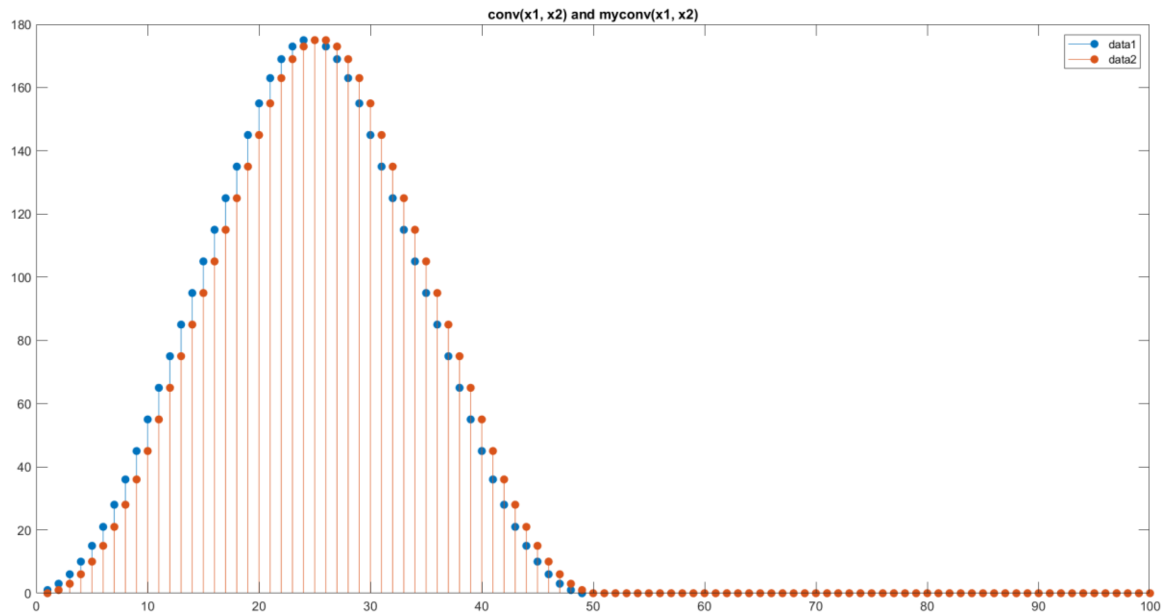

Figure 6. $y[n] = \text{myconv}(x_1, x_2)$ versus $y[n] = \text{conv}(x_1, x_2)$

The blue signal is $\mathrm{conv}(x_1, x_2)$, while the orange signal is $\mathrm{myconv}(x_1, x_2)$. As we can see, the two convolutions differ by a time shift. This is caused by the definition of $\mathrm{conv}(x_1, x_2)$ in MATLAB. In MATLAB, the definition of convolution is

$$x_1[n] * x_2[n] = \sum_{k=1}^{N} x_1[k]x_2[n-k+1]$$

Because MATLAB arrays start from index 1. Due to such definition, MATLAB convolution is shifted 1 unit left from the convolution defined in the course. To address such problem, we can impose a time shift on $x_2[n]$ and define a signal $x_{2,\mathrm{modified}}[n] = x_2[n-1]$, this cancels the +1 term in the MATLAB convolution, and we can obtain the same signal as $\mathrm{myconv}(x_1, x_2)$. The following figures are the implementation and the result.

```
% Supp 2. Shift x2 to obtain correct convolution
% x2_shifted = zeros(n2, 1);
% for n = 2:n2
%     x2_shifted(n) = x2(n - 1);
% end
% y_modified = conv(x1, x2_shifted);
% tiledlayout(2, 1)
% nexttile
% stem(y_modified, 'filled','r')
% title('y[n] = conv(x1, x2__modified)')
```

Figure 7. Implementation of $x_2[n-1]$

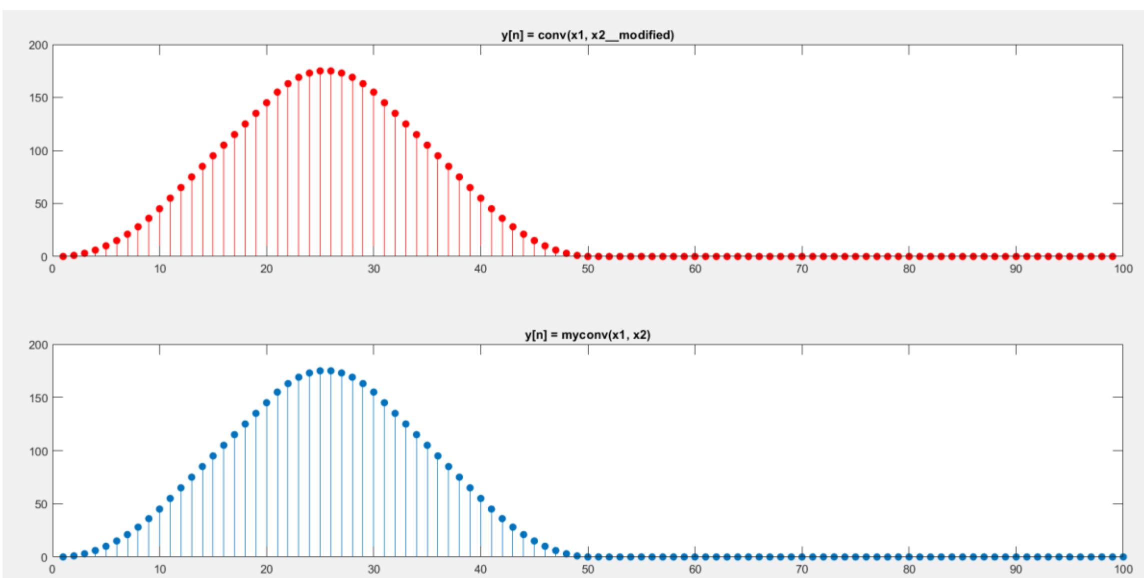

Figure 8. $\mathrm{conv}(x_1, x_2[n-1])$ versus $\mathrm{myconv}(x_1, x_2)$

(d) Repeat Problems (a) to (c) again, but $x_1[n]$ and $x_2[n]$ are changed to the following:

$$x_1[n] = \begin{cases} 3^n, & 1 \le n \le 3 \\ 0, & \text{otherwise} \end{cases}$$

$$x_2[n] = \begin{cases} 2^n, & 1 \le n \le 5 \\ 0, & \text{otherwise} \end{cases}$$

The graphs of the two input signals are given in the following figures:

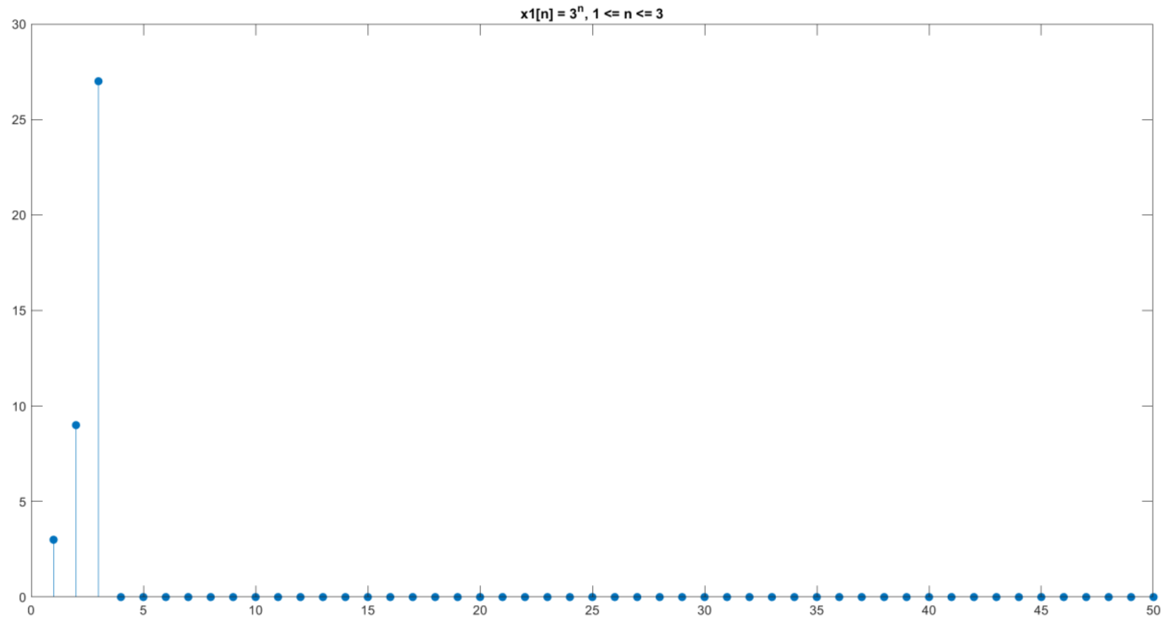

Figure 9. $x_1[n] = \begin{cases} 3^n, & 1 \le n \le 3 \\ 0, & \text{otherwise} \end{cases}$

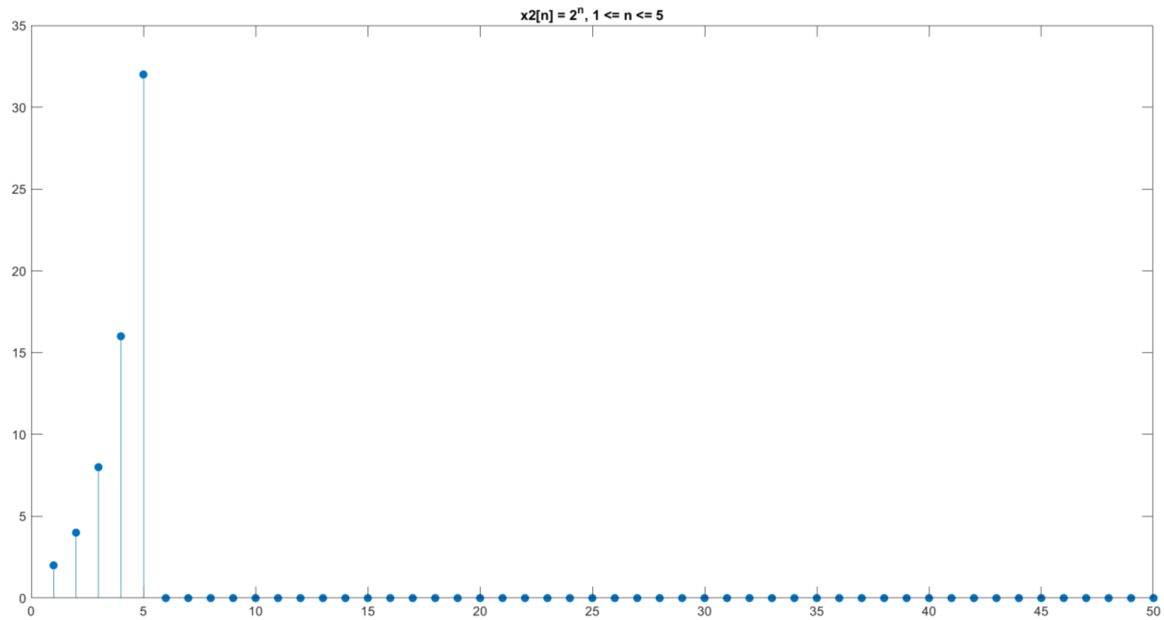

Figure 10. $x_2[n] = \begin{cases} 2^n, & 1 \leq n \leq 5 \\ 0, & \text{otherwise} \end{cases}$

The output of $y[n] = \text{conv}(x_1, x_2)$ is given by the following figure:

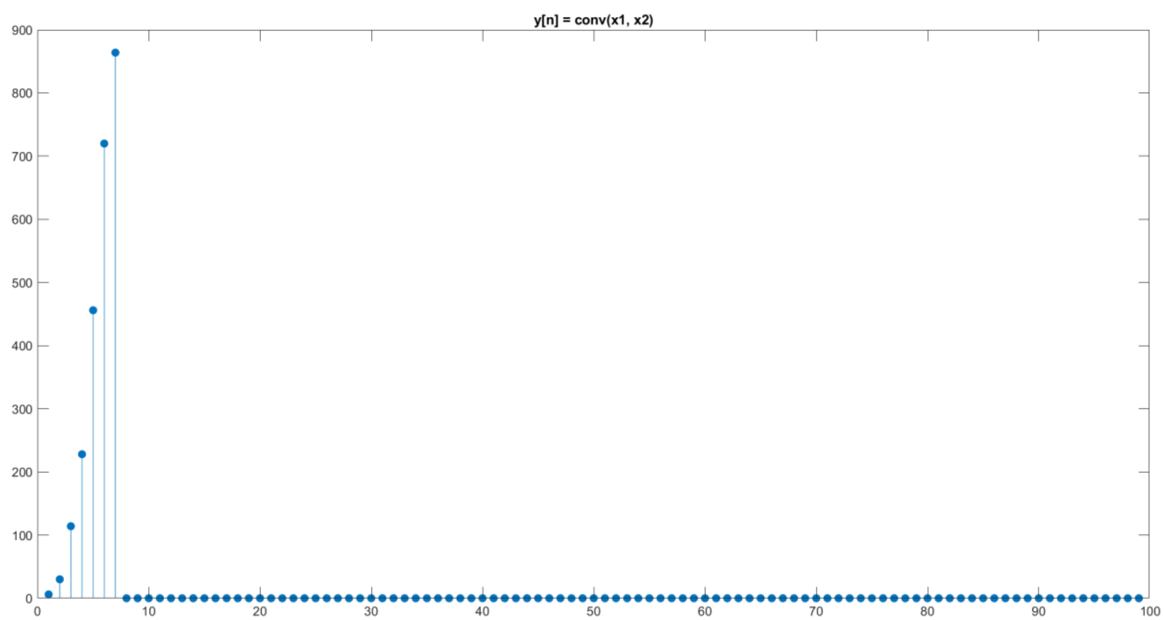

Figure 11. $y[n] = \text{conv}(x_1, x_2)$

The output of $y[n] = \text{myconv}(x_1, x_2)$ is given by the following figure:

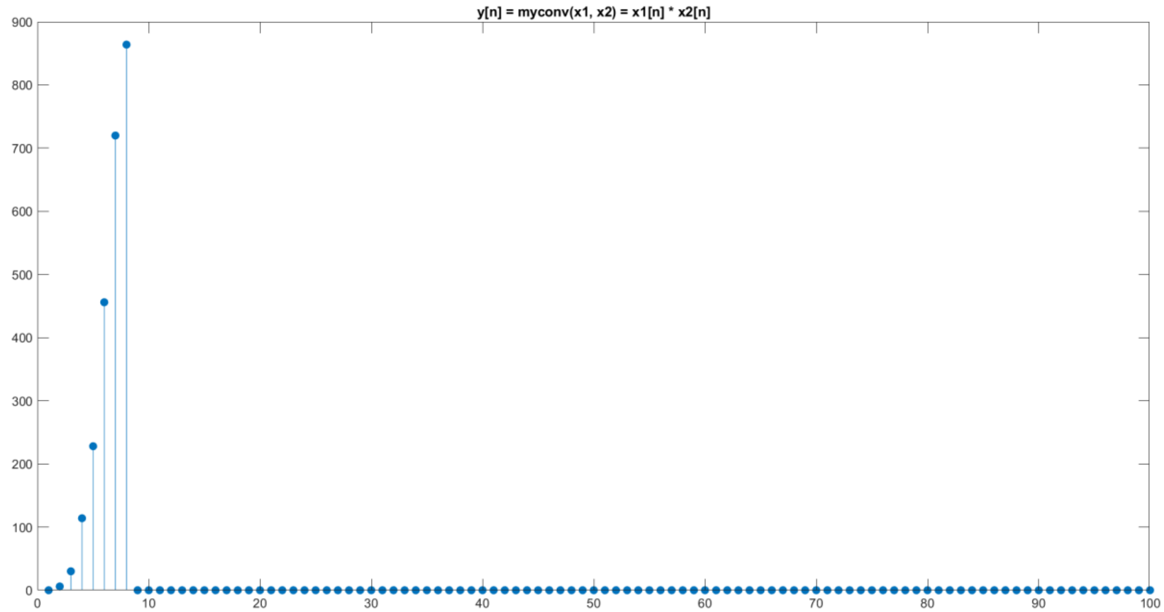

Figure 12. $y[n] = \text{myconv}(x_1, x_2)$

Comparison between $\text{conv}(x_1, x_2)$ (blue) and $\text{myconv}(x_1, x_2)$ (orange) on the same figure:

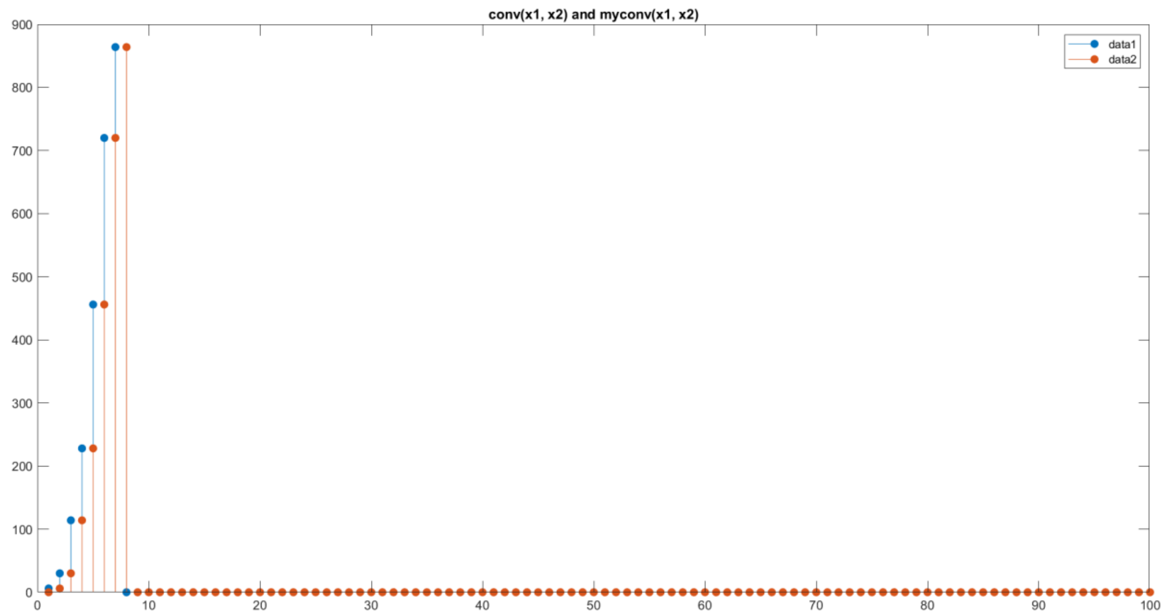

Figure 13. $y[n] = \text{myconv}(x_1, x_2)$ versus $y[n] = \text{conv}(x_1, x_2)$

Using the same shifting technique described in Question (c), we can obtain $\mathrm{myconv}(x_1, x_2)$ by $\mathrm{conv}(x_1[n], x_2[n-1])$:

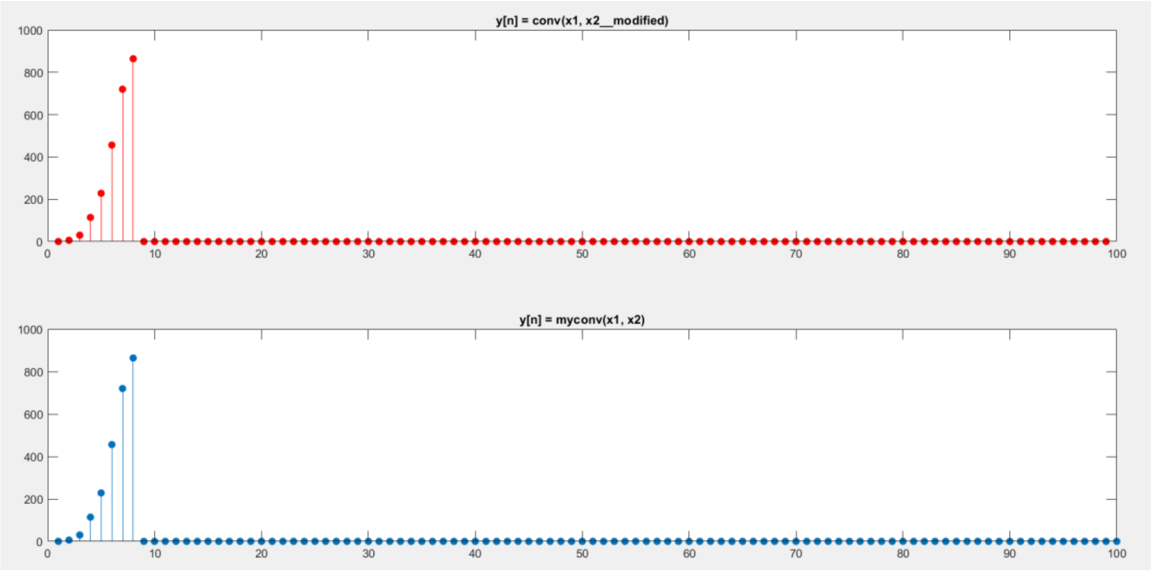

Figure 14. $\mathrm{conv}(x_1, x_2[n-1])$ versus $\mathrm{myconv}(x_1, x_2)$