Signals and Systems

MATLAB Homework 4

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(a) Use the MATLAB function `zplane` to plot the locations of poles and zeros of $H(z)$, where $H(z)$ is given by

$$H(z) = \frac{0.09(z-1)^2(z+1)^2}{(z-0.3-0.4i)(z-0.3+0.4i)(z-0.1-0.1i)(z-0.1+0.1i)} \tag{1}$$

Please also state the ROC in your report.

The pole-zero plot is given in Figure 1.

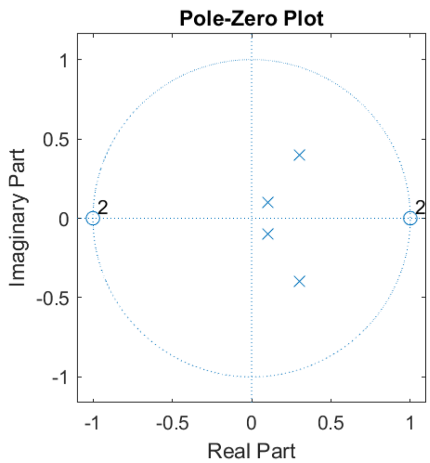

Figure 1. Pole-zero Plot of $H(z)$

The ROC of $H(z)$ is $|z| > 0.5$.

(b) Use the output of the MATLAB function `residuez` to construct the real $h[n]$, where $h[n]$, is the inverse z-transform of $H(z)$. Then, use the MATLAB function `stem` to plot $h[n]$ vs n, for $n = 0 \sim 20$.

Hint: What is the meaning of `r`, `p` and `k` in eq. (2)?

Figure 2 is the plot of $h[n]$, where $n = 0 \sim 20$.

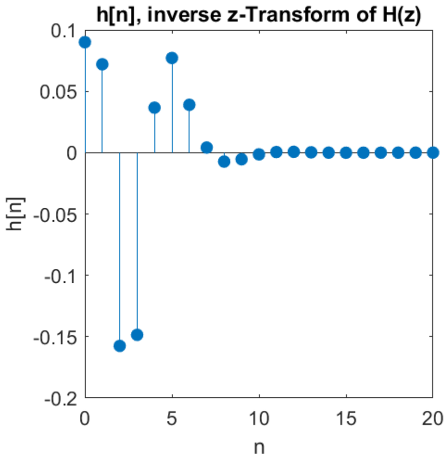

Figure 2. Plot of $h[n]$

(c) Use the MATLAB function `plot` to plot the magnitude and phase response of $H(z)$ vs ω for $z = e^{j\omega}$.

Hint: You may consider the MATLAB function `freqz`.

Figure 3 is the magnitude response of $H(z)$, and Figure 4 is the phase response (in degrees) of $H(z)$.

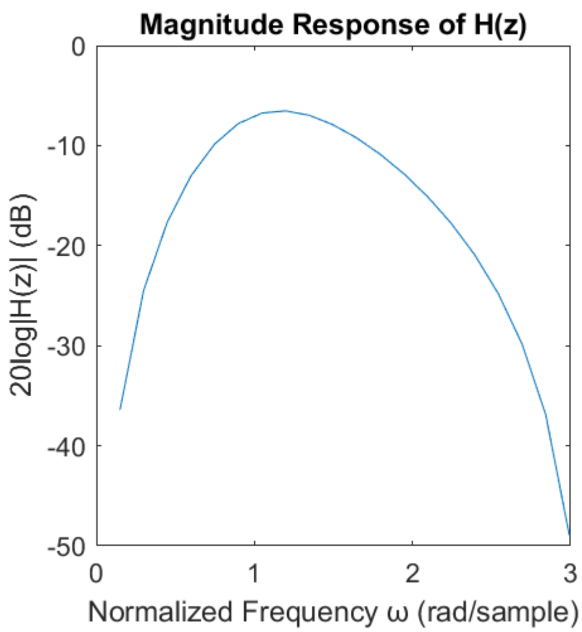

Figure 3. Plot of $20\log|H(z)|$

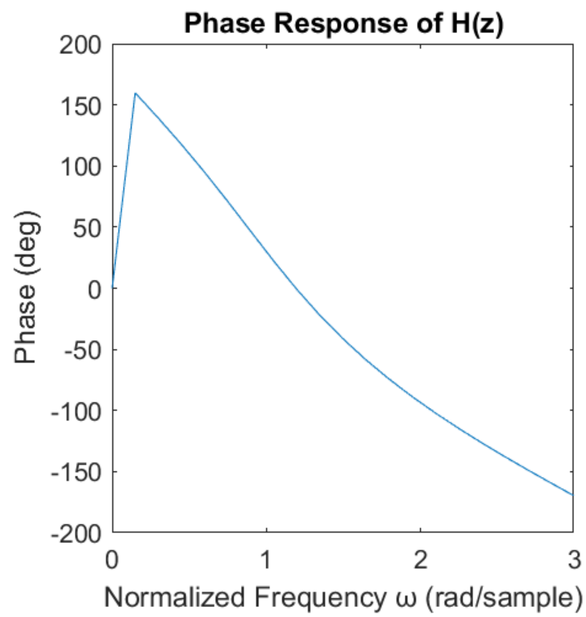

Figure 4. Plot of $\angle H(z)$

(d) Write down a representation of $H(z)$ as a cascade of two second-order systems with real coefficients in your report, that is, $H(z) = H_1(z)H_2(z)$. Hint: You may consider the MATLAB function `zp2sos`.

From the result of zp2sos, we can indeed find $H_1(z)$ and $H_2(z)$ such that $H(z) = H_1(z)H_2(z)$:

$$H_1(z) = \frac{0.09 + 0.18z^{-1} + 0.09z^{-2}}{1 - 0.02z^{-1} + 0.2z^{-2}} \tag{2}$$

$$H_2(z) = \frac{1 - 2z^{-1} + z^{-2}}{1 - 0.6z^{-1} + 0.25z^{-2}} \tag{3}$$

(e) Use the MATLAB function `plot` to plot the magnitude response of each system in (d), i.e., $H_1(z)$ vs ω and $H_2(z)$ vs ω, for $z = e^{j\omega}$. Furthermore, directly plot the multiplication result of the magnitude response $|H_1(z)|$ and $H_2(z)$. Compare the result with (c) in your report.

Figure 5 is the magnitude response of $H_1(z)$, and Figure 6 is the magnitude response of $H_2(z)$.

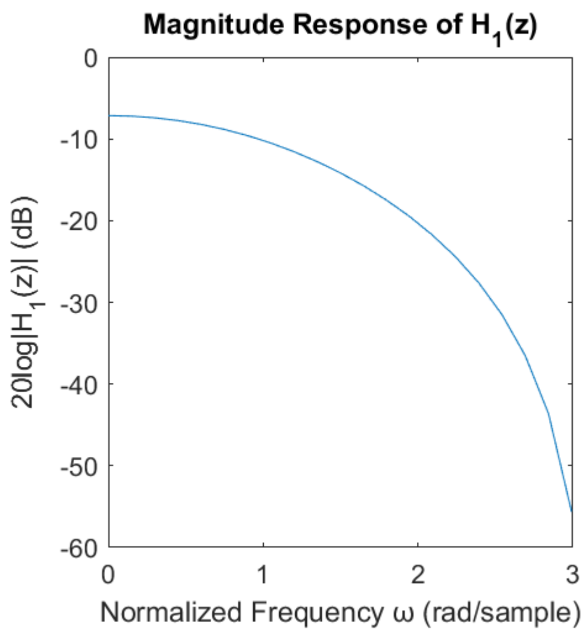

Figure 5. Plot of $20\log|H_1(z)|$

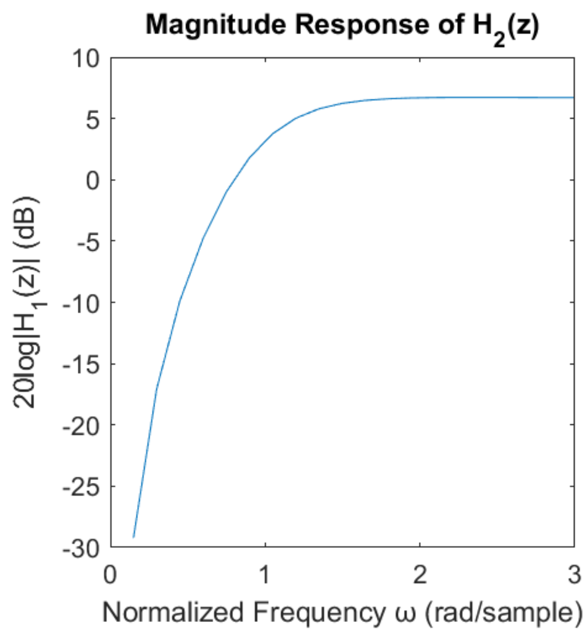

Figure 6. Plot of $20\log|H_2(z)|$

Figure 7 is the magnitude response of $H_1(z)H_2(z)$.

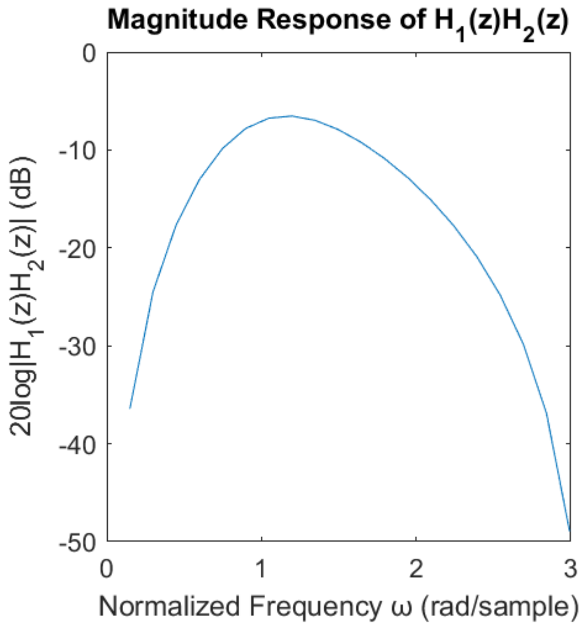

Figure 7. Plot of $20\log|H_1(z)H_2(z)|$

Theoretically, since $H(z) = H_1(z)H_2(z)$, the magnitude response of $H(z)$ and $H_1(z)H_2(z)$ should be equal. We use the following code to plot the two signals on the same graph:

```
figure
plot(w, 20*log10(abs(H)), 'b', w, 20*log10(abs(H1 .* H2)), 'r')
```

After executing the code, Figure 8 is generated. The blue signal is $20\log|H(z)|$, and the red signal is $20\log|H_1(z)H_2(z)|$. From Figure 8, we can conclude that the two signals are indeed identical.

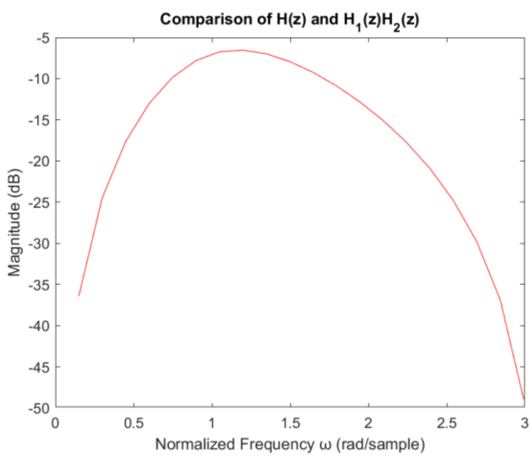

Figure 8. Comparison of $H(z)$ and $H_1(z)H_2(z)$

(f) Use the MATLAB function `filter` to find the real $y[n]$ when an input $x[n] = \delta[n]$ is passed through the system $H(z)$. Then, use the MATLAB function `stem` to plot the impulse response $y[n]$ vs n for $n = 0 \sim 20$, and compare it with the result in (b).

Figure 9 is the plot of $y[n]$.

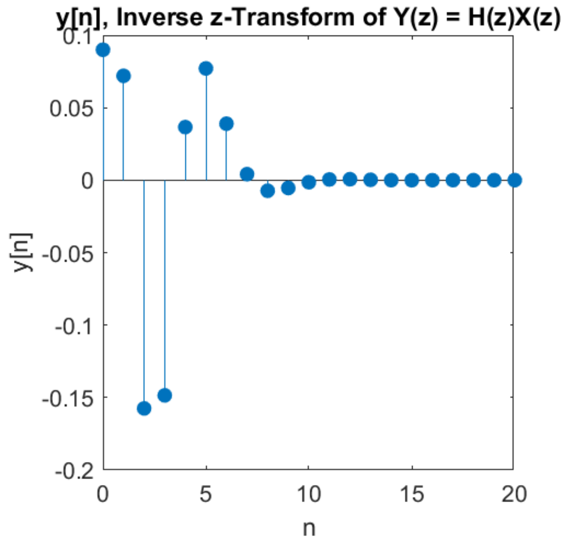

Figure 9. Plot of $y[n]$

The z-transform of $x[n] = \delta[n]$ is $X(z) = 1$. Therefore, $Y(z) = X(z)H(z) = H(z)$ and $y[n]$, the inverse z-transform of $Y(z)$, is identical to $h[n]$.

The following code is executed to plot $h[n]$ and $y[n]$ on the same figure.

```
figure
stem(n, h(n+1), 'filled', 'b')
hold on;
stem(n, y(n+1), 'filled', 'r')
title('Comparison of h[n] and y[n]')
xlabel('n')
ylabel('h[n] (blue) and y[n] (red)')
```

In Figure 10, the two signals are plotted on the same figure. The blue signal is $h[n]$, and the red signal is $y[n]$. From Figure 10, we can conclude that the signals in Figure 2 and Figure 9 are the same.

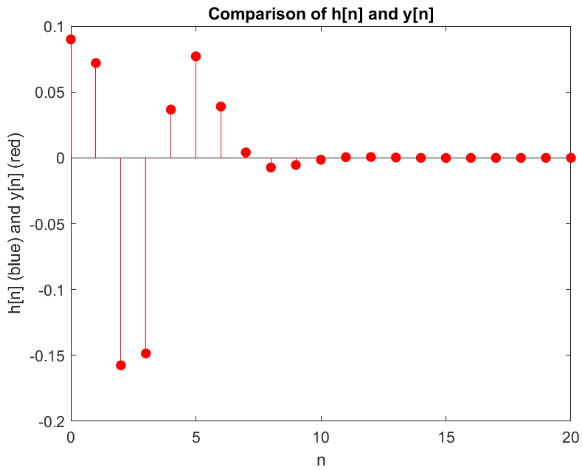

Figure 10. Comparison of $h[n]$ and $y[n]$