# Matematika Peminatan
# MA Husnul Khotimah
# Kelas 12 IPA

Tahun Pembelajaran 2020-2021

# • Cakupan Materi

Semester Ganjil

- Bab 1 : Limit Fungsi Trigonometri
- Bab 2 : Turunan Fungsi Trigonometri
- Bab 3 : Integral Trigonometri

Semester Genap

- Bab 4 : Distribusi dan Peluang Binomial
- Bab 5 : Distribusi dan Peluang Normal

# BAB 1
# Limit Fungsi Trigonometri

1. Limit Fungsi Trigonometri
2. Limit di Ketakberhinggaan Fungsi Trigonometri

# 1.1 Limit di Keberhinggan Fungsi Trigonometri

a. **Substitusi Langsung**
b. **Bentuk tak tentu 0/0 dengan pengubahan bentuk trigonometri**
c. **Bentuk tak tentu 0/0 dengan rumus dasar limit trigonometri**
d. **Bentuk tak tentu 0/0 dengan menggunakan rumus-rumus trigonometri**

# a. Substitusi Langsung

$$\lim_{x \to a} f(x) = f(a)$$

## Contoh 1

*a.* $\lim_{x \to \pi} (\sin x - \cos x) = \sin \pi - \cos \pi = 0 - (-1) = 1$

*b.* $\lim_{x \to \frac{\pi}{4}} \frac{\sin x + \cos x}{\sin x} = \frac{\sin \frac{\pi}{4} + \cos \frac{\pi}{4}}{\sin \frac{\pi}{4}} = \frac{\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 2$

*c.* $\lim_{x \to 0} \frac{\sin x}{\cos x + \sin x} = \frac{sin\ 0}{\cos 0 + \sin 0} = \frac{0}{1+0} = 0$

# b. Bentuk tak tentu 0/0 dengan pengubahan bentuk trigonometri

$sin^2x + cos^2x = 1$

Sin 2x = 2 sin x cos x

Cos 2x = $cos^2x$ - $sin^2x$

Cos 2x = 1 – 2 $sin^2x$

Cos 2x = 2$cos^2x$ – 1

tan x = $\frac{\sin x}{\cos x}$ , cot x = $\frac{1}{\tan x}$

Sec x = $\frac{1}{\cos x}$ , csc x = $\frac{1}{\sin x}$

$Sec^2x = 1 + tan^2x$

$Csc^2x = 1 + cot^2x$

# Contoh 2

a. $\lim\limits_{x \to 0} \dfrac{\sin 2x}{\sin x} = \dfrac{0}{0}$, (kita ubah bentuk sin 2x )

$\Rightarrow \lim\limits_{x \to 0} \dfrac{2 \sin x \cos x}{\sin x} = \lim\limits_{x \to 0} 2 \cos x = 2 \cos 0 = 2$

b. $\lim\limits_{x \to \frac{\pi}{4}} \dfrac{\cos 2x}{\cos x - \sin x} = \dfrac{0}{0}$, (kita ubah bentuk cos 2x)

$\Rightarrow \lim\limits_{x \to \frac{\pi}{4}} \dfrac{cos^2 x - sin^2 x}{\cos x - \sin x} = \lim\limits_{x \to \frac{\pi}{4}} \dfrac{(\cos x + \sin x)(\cos x - \sin x)}{\cos x - \sin x}$

$= \lim\limits_{x \to \frac{\pi}{4}} (\cos x + \sin x) = \cos \dfrac{\pi}{4} + \sin \dfrac{\pi}{4} = \sqrt{2}$

# c. Bentuk tak tentu 0/0 dengan menggunakan rumus dasar

Rumus Dasar
Limit Fungsi Trigonometri.

$$\lim_{x \to 0} \frac{\sin x}{x} = \lim_{x \to 0} \frac{x}{\sin x} = 1$$

$$\lim_{x \to 0} \frac{\tan x}{x} = \lim_{x \to 0} \frac{x}{\tan x} = 1$$

$$\lim_{x \to 0} \frac{\sin x}{\tan x} = \lim_{x \to 0} \frac{\tan x}{\sin x} = 1$$

# Rumus Perluasan 1
# Limit Fungsi Trigonometri

1. $\lim_{x \to 0} \frac{\sin mx}{nx} = \frac{m}{n}$
2. $\lim_{x \to 0} \frac{mx}{\sin nx} = \frac{m}{n}$
3. $\lim_{x \to 0} \frac{\tan mx}{nx} = \frac{m}{n}$
4. $\lim_{x \to 0} \frac{mx}{\tan nx} = \frac{m}{n}$
5. $\lim_{x \to 0} \frac{\sin mx}{\sin nx} = \frac{m}{n}$
6. $\lim_{x \to 0} \frac{\tan mx}{\tan nx} = \frac{m}{n}$
7. $\lim_{x \to 0} \frac{\sin mx}{\tan nx} = \frac{m}{n}$
8. $\lim_{x \to 0} \frac{\tan mx}{\sin nx} = \frac{m}{n}$

# Rumus Perluasan 2
# Limit Fungsi Trigonometri

1. $\lim\limits_{x \to a} \dfrac{\sin m(x-a)}{n(x-a)} = \dfrac{m}{n}$
2. $\lim\limits_{x \to a} \dfrac{m(x-a)}{\sin n(x-a)} = \dfrac{m}{n}$
3. $\lim\limits_{x \to a} \dfrac{\tan m(x-a)}{n(x-a)} = \dfrac{m}{n}$
4. $\lim\limits_{x \to a} \dfrac{m(x-a)}{\tan n(x-a)} = \dfrac{m}{n}$
5. $\lim\limits_{x \to a} \dfrac{\sin m(x-a)}{\sin n(x-a)} = \dfrac{m}{n}$
6. $\lim\limits_{x \to a} \dfrac{\tan m(x-a)}{\tan n(x-a)} = \dfrac{m}{n}$
7. $\lim\limits_{x \to a} \dfrac{\sin m(x-a)}{\tan n(x-a)} = \dfrac{m}{n}$
8. $\lim\limits_{x \to a} \dfrac{\tan m(x-a)}{\sin n(x-a)} = \dfrac{m}{n}$

# contoh 3

1. $\lim_{x \to 0} \frac{\sin 3x}{2x} = \frac{3}{2}$
2. $\lim_{x \to 3} \frac{\tan(x-3)}{2x-6} = \lim_{x \to 3} \frac{\tan(x-3)}{2(x-3)} = \frac{1}{2}$
3. $\lim_{x \to 0} \frac{\sin 2x \cos x}{3 \tan 5x} = \frac{1}{3} \lim_{x \to 0} \frac{\sin 2x}{\tan 5x} . \lim_{x \to 0} \cos x$

   $= \frac{1}{3} . \frac{2}{5} . 1 = \frac{2}{15}$
4. $\lim_{x \to 0} \frac{x - \sin 5x}{2x} = \lim_{x \to 0} \frac{x}{2x} - \lim_{x \to 0} \frac{\sin 5x}{2x}$

   $= \frac{1}{2} - \frac{5}{2} = \frac{-4}{2} = -2$

# d. Bentuk tak tentu 0/0 dengan menggunakan Rumus-Rumus Trigonometri

**Rumus yang sering digunakan :**

$$\cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

$$1 - \cos^2 A = \sin^2 A$$

$$1 - \cos A = 2 \sin^2\left(\frac{A}{2}\right)$$

$$\cos A - 1 = -2 \sin^2\left(\frac{A}{2}\right)$$

# Contoh. 4

*1.* $\lim_{x\to 0} \frac{1-cos^2 x}{x \tan 2x} = \lim_{x\to 0} \frac{sin^2 x}{x \tan 2x} = \lim_{x\to 0} \frac{\sin x}{x} . \lim_{x\to 0} \frac{\sin x}{\tan 2x}$

$= 1. ½ = ½$

2. $\lim_{x\to 0} \frac{\cos 4x - 1}{3x^2} = \lim_{x\to 0} \frac{-2sin^2 2x}{3x^2} = \frac{-2}{3} \lim_{x\to 0} \frac{sin2x}{x} . \lim_{x\to 0} \frac{sin2x}{x}$

$= \frac{-2}{3} . \frac{2}{1} . \frac{2}{1} = \frac{-8}{3}$

3. $\lim_{x\to 0} \frac{\cos 3x - \cos x}{x \tan 5x} = \lim_{x\to 0} \frac{-2 \sin 2x \sin x}{x \tan 5x}$

$= -2 \lim_{x\to 0} \frac{\sin 2x}{x} . \lim_{x\to 0} \frac{\sin x}{\tan 5x} = -2. \frac{2}{1} . \frac{1}{5} = \frac{-4}{5}$

# Latihan Soal

Latihan 1

Latihan 2

Latihan 3

Latihan 4

# Latihan 1

| No | soal | jawaban |
|---|---|---|
| 1 | $\lim_{x\to\frac{\pi}{2}} \sin x$ | 1 |
| 2 | $\lim_{x\to\frac{\pi}{4}} (1+\cot x)$ | 2 |
| 3 | $\lim_{x\to 0} \frac{\tan x - \sin x}{\cos x}$ | 0 |
| 4 | $\lim_{x\to 0} \frac{1+\sin x}{1+\cos x}$ | ½ |
| 5 | $\lim_{x\to\frac{\pi}{4}} \frac{\cos 2x - 1}{2\sin x}$ | $\frac{-1}{2}\sqrt{2}$ |
| 6 | $\lim_{x\to 0} \frac{\cos x}{1 - \cos 2x}$ | 1/0 |
| 7 | $\lim_{x\to\frac{\pi}{4}} \frac{\tan 3x}{4x}$ | $\frac{-1}{\pi}$ |
| 8 | $\lim_{x\to\frac{\pi}{4}} \frac{\tan x}{\sin x}$ | $\sqrt{2}$ |

# Latihan 2

| No | soal | jawaban |
|---|---|---|
| 1 | $\lim_{x\to 0} \frac{\cos 2x + 1}{\cos x}$ | 2 |
| 2 | $\lim_{x\to \frac{\pi}{2}} \frac{1 - cos^2 \mathrm{x}}{2sin^2 x}$ | 1/2 |
| 3 | $\lim_{x\to \frac{\pi}{2}} \frac{1 - cos^3 \mathrm{x}}{sin^2 x}$ | 1 |
| 4 | $\lim_{x\to \frac{\pi}{4}} \frac{csc^2 \mathrm{x} - 2}{\cot x - 1}$ | 2 |
| 5 | $\lim_{x\to \frac{\pi}{4}} \frac{1 - \sin 2x}{cos^2 2x}$ | 1/2 |
| 6 | $\lim_{x\to 0} \frac{\cos 2x - 1}{sin^2 x}$ | -1/2 |

# Latihan 3

1. $\lim_{x \to 0} \frac{\tan 5x}{3x}$
2. $\lim_{x \to 0} \frac{sin^3 2x}{tan^3 \frac{1}{2}x}$
3. $\lim_{x \to 0} \frac{\sin 4x + \tan 3x - \sin 5x}{2x}$
4. $\lim_{x \to 0} \frac{(x-2) \sin 3x}{\tan 2x}$
5. $\lim_{x \to 0} \frac{2 \sin x \cos x}{\tan 5x}$
6. $\lim_{x \to 0} \frac{\sin x + \sin 3x}{x \cos x}$
7. $\lim_{x \to 0} \frac{\tan 5x - x}{\sin x + x}$
8. $\lim_{x \to \frac{\pi}{2}} \frac{sin\,(x - \frac{\pi}{2})}{(x - \frac{\pi}{2})}$
9. $\lim_{x \to 1} \frac{(3x+1) \sin(x-1)}{x^2 + 2x - 3}$
10. $\lim_{x \to 0} \frac{\cot 5x}{\cot 10x}$

# Latihan. 4

1. $\lim_{x \to 0} \frac{1 - \cos 2x}{1 - \cos 4x}$
2. $\lim_{x \to 0} \frac{tan^2 4x}{1 - \cos 6x}$
3. $\lim_{x \to 0} \frac{\cos 8x - 1}{4x^2}$
4. $\lim_{x \to 0} \frac{\cos 2x - \cos x}{x^2}$
5. $\lim_{x \to \frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x}$
6. $\lim_{x \to 0} \frac{\cos 4x - 1}{x \tan 2x}$
7. $\lim_{x \to 0} \frac{1 - cos^2 2x}{x \sin 2x}$
8. $\lim_{x \to 0} \frac{\csc 9x - csc\, 5x}{cot\, 5x - \cot 3x}$
9. $\lim_{x \to 2} \frac{x^2 - 4x + 4}{1 - cos^2 (x-2)}$
10. $\lim_{x \to 0} \frac{\mathrm{x} \tan x}{sin^2 x - \cos 2x + 1}$

# Penutup

Alhamdulillah

Selesai sudah pertemuan pertama

Terus dibaca dan dipahmi

Terus berlatih dan berlatih

Bersiap dengan pertemuan selanjutnya

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