# Optimal Bayesian Kalman Filtering with Prior Update

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# Overview

1. What's this paper?
2. Overview Kalman Filter
   - Basic Idea
   - Algorithm
   - Problem of Kalman Filter
   - Two Solutions: Empirical and Bayesian
3. A Bayesian Solution: IBR Kalman Filter
   - Overview: IBR Filter
4. Prior update: Optimal Bayesian Kalman Filter
   - Overview: OBKF
   - Details
   - Results
   - Future Works
5. Conclusion

# What's this Paper?

- Introduce a new Kalman Filter: Optimal Bayesian Kalman Filter(OBKF)(Dehghannasiri et al. 2018)
- OBKF is a advanced Kalman Filter exploiting Bayesian Approach
- OBKF solves the problem "Uncertainty of Prior Distribution"

# Kalman Filter: Basic idea

- Provides the optimal estimation of the state of a process
- Exploits both the prediction based on the model and the measurement
- Application Example: Robot Localization

# Kalman Filter: Algorithm

Kalman Filter works for a linear system:

$$
\begin{aligned}
x_t &= A_t x_{t-1} + B_t u_t + \epsilon_t \\
z_t &= C_t x_t + \delta_t
\end{aligned}
$$

# Kalman Filter: Algorithm

Kalman Filter works for a linear system:

$$x_t = A_t x_{t-1} + B_t u_t + \epsilon_t$$
$$z_t = C_t x_t + \delta_t$$

**Algorithm Kalman Filter($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$):**

$$\bar{\mu}_t = A_t \mu_{t-1} + B u_t$$
$$\bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$$
$$K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$$
$$\mu_t = \hat{\mu}_t + K_t (z_t - C_t \bar{\mu}_t)$$
$$\Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$$

# Problems of Kalman Filter

- The performance is sensitive to the accuracy of $R_t$ and $Q_t$
- It's impossible to obtain the exact value of $R_t$ and $Q_t$

# Two Solutions: Emprirical and Bayesian

- Empirical Approach: Adaptive Kalman Filter
- Bayesian Approach: IBR Kalman Filter

# Overview: IBR Filter

- Provide the optimal filter relative to an uncertain class of processes
- IBR Kalman Filter is found by Dehghannasiri et al. 2017

# Optimal Bayesian Kalman Filter

- Using measured data $Y_k$ to estimate the unknown value $\theta$
- Applying IBRKF relative to the posterior distribution $f(Y_k|\theta)$

# Details

- Factor Graph
- Metropolis-Heisting
- IBR Kalman Filter

- If the prior distribution doesn't include the true value, the performance will be poor.
- MCMC is slow. This can be handle.
- Factor Graph is redundant. This can be handle.

# References I

Dehghannasiri, R., Esfahani, M. S., Qian, X., and Dougherty, E. R. (2018).
Optimal Bayesian Kalman Filtering with Prior Update.
*IEEE Transactions on Signal Processing*, 66(8):1982–1996.