Optimal Bayesian Kalman Filtering with Prior Update

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February 26, 2019

Overview

1. What's this paper?
2. Overview Kalman Filter
   - Basic Idea
   - Algorithm
   - Problem of Kalman Filter
   - Solutions of the Problem
3. A Bayesian Solution: IBR Kalman Filter
   - Baysian Setup
   - Algorithm: IBR Kalman Filter
4. Prior update: Optimal Bayesian Kalman Filter
   - Overview: OBKF
   - Details
   - Results
   - Future Works
5. Conclusion

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What's this Paper?

- Introduce a new Kalman Filter: Optimal Bayesian Kalman Filter(OBKF)[Dehghannasiri et al., 2018]
- OBKF is a advanced Kalman Filter of IBR Kalman Filter[Dehghannasiri et al., 2017]
- OBKF exploit the measured data to estimate the true unknown value

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Kalman Filter: Basic idea

- Outputs the optimal estimation of the state of a dynamic system
- Exploits both the prediction based on the model and the measurement
- Application Example: Robot Localization

Kalman Filter: Algorithm I

Kalman Filter works for linear dynamic systems:

Linear Dynamic System Example

$$\mathbf{x}_{k+1} = \mathbf{\Phi}_k \mathbf{x}_k + \mathbf{\Gamma}_k \mathbf{u}_k$$
$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{v}_k$$

- $\mathbf{x}_k$: *state vector*
- $\mathbf{z}_k$: *observation vector*
- $\mathbf{u}_k$, $\mathbf{v}_k$: *zero-mean noise vector*
- $\mathbf{\Phi}_k$, $\mathbf{\Gamma}_k$, $\mathbf{H}_k$: *transition matrix*

Kalman Filter: Algorithm II

Algorithm 1 Classic Kalman Filter

Input: $\hat{\mathbf{x}}_k$, ${\mathbf{P}^{\mathbf{x}}}_k$, $\mathbf{y}_k$

1: $\tilde{\mathbf{z}}_k = \mathbf{y}_k - \mathbf{H}_k \hat{\mathbf{x}}_k$

2: $\mathbf{K}_k = {\mathbf{P}^{\mathbf{x}}}_k \mathbf{H}_k^T (\mathbf{H}_k {\mathbf{P}^{\mathbf{x}}}_k \mathbf{H}_k^T + \mathbf{R})^{-1}$

3: $\hat{\mathbf{x}}_{k+1} = \mathbf{\Phi}_k \hat{\mathbf{x}}_k + \mathbf{\Phi}_k \mathbf{K}_k \tilde{\mathbf{z}}_k$

4: ${\mathbf{P}^{\mathbf{x}}}_{k+1} = \mathbf{\Phi}_k (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) {\mathbf{P}^{\mathbf{x}}}_k \mathbf{\Phi}_k^T + \mathbf{\Gamma}_k \mathbf{Q} \mathbf{\Gamma}_k^T$

Output: $\hat{\mathbf{x}}_{k+1}$, ${\mathbf{P}^{\mathbf{x}}}_{k+1}$

- $\hat{\mathbf{x}}_k$: *estimated mean*
- ${\mathbf{P}^{\mathbf{x}}}_k$: *estimated covariance*
- $\mathbf{R}$: *Noise covariance of* $\mathbf{u}_k$
- $\mathbf{Q}$: *Noise covariance of* $\mathbf{v}_k$

Problems of Kalman Filter

- The performance is sensitive to the accuracy of the noise covariance matrixes: $\mathbf{Q}$ and $\mathbf{R}$[Sangsuk-Iam and Bullock, 1990]

Solutions of the Problem

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  - Doesn't require any prior knowledge
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- Bayesian Approach: IBR Kalman Filter
  - Require prior knowledge
  - Robust: Guarantee the best average performance relative to the prior distribution
- Bayesian Approach: Optimal Bayesian Kalman Filter
  - Require prior knowledge
  - Utilize measured data to estimate unknown parameters
  - Optimal over the posterior distribution obtained from measured data

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Bayesian Setup

Using $\theta = [\theta_1, \theta_2]$, assume the covariance matrices of the noise are parameterized as:

$$E[\mathbf{u}_k^{\theta_1}(\mathbf{u}_l^{\theta_1})^T] = \mathbf{Q}^{\theta_1}\delta_{kl}$$
$$E[\mathbf{v}_k^{\theta_2}(\mathbf{v}_l^{\theta_2})^T] = \mathbf{R}^{\theta_2}\delta_{kl}$$

And assume the prior distribution $\pi(\theta)$

Algorithm: IBR Kalman Filter

Algorithm 2 IBR Kalman Filter

Input: $\hat{\mathbf{x}}_k^\theta$, $E_\theta[\mathbf{P}^{\mathbf{x},\theta}{}_k]$, $\mathbf{y}_k^\theta$

1: $\tilde{\mathbf{z}}_k^\theta = \mathbf{y}_k^\theta - \mathbf{H}_k \hat{\mathbf{x}}_k^\theta$

2: $\mathbf{K}_k^{\boldsymbol{\Theta}} = E_\theta[\mathbf{P}^{\mathbf{x},\theta}{}_k]\mathbf{H}_k^T E_\theta^{-1}[\mathbf{H}_k \mathbf{P}^{\mathbf{x},\theta}{}_k \mathbf{H}_k^T + \mathbf{R}^{\theta_2}]$

3: $\hat{\mathbf{x}}_{k+1}^\theta = \boldsymbol{\Phi}_k \hat{\mathbf{x}}_k^\theta + \boldsymbol{\Phi}_k \mathbf{K}_k^{\boldsymbol{\Theta}} \tilde{\mathbf{z}}_k^\theta$

4: $E_\theta[\mathbf{P}^{\mathbf{x},\theta}{}_{k+1}] = \boldsymbol{\Phi}_k(\mathbf{I} - \mathbf{K}_k^{\boldsymbol{\Theta}}\mathbf{H}_k)E_\theta[\mathbf{P}^{\mathbf{x},\theta}{}_k]\boldsymbol{\Phi}_k^T + \boldsymbol{\Gamma}_k E_{\theta_1}[\mathbf{Q}^{\theta_1}{}_k]\boldsymbol{\Gamma}_k^T$

Output: $\hat{\mathbf{x}}_{k+1}^\theta$, $E_\theta[\mathbf{P}^{\mathbf{x},\theta}{}_{k+1}]$

Optimal Bayesian Kalman Filter

- Using measured data Y_k to estimate the unknown value θ
- Applying IBRKF relative to the posterior distribution $f(Y_k|\theta)$

Details

- Factor Graph
- Metropolis-Heisting
- IBR Kalman Filter

- If the prior distribution doesn't include the true value, the performance will be poor.
- MCMC is slow. This can be handle.
- Factor Graph is redundant. This can be handle.

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