

Improved principal component analysis for anomaly detection: Application to an emergency department[☆]



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ABSTRACT

Monitoring of production systems, such as those in hospitals, is primordial for ensuring the best management and maintenance desired product quality. Detection of emergent abnormalities allows preemptive actions that can prevent more serious consequences. Principal component analysis (PCA)-based anomaly-detection approach has been used successfully for monitoring systems with highly correlated variables. However, conventional PCA-based detection indices, such as the Hotelling's T^2 and the Q statistics, are ill suited to detect small abnormalities because they use only information from the most recent observations. Other multivariate statistical metrics, such as the multivariate cumulative sum (MCUSUM) control scheme, are more suitable for detection small anomalies. In this paper, a generic anomaly detection scheme based on PCA is proposed to monitor demands to an emergency department. In such a framework, the MCUSUM control chart is applied to the uncorrelated residuals obtained from the PCA model. The proposed PCA-based MCUSUM anomaly detection strategy is successfully applied to the practical data collected from the database of the pediatric emergency department in the Lille Regional Hospital Centre, France. The detection results evidence that the proposed method is more effective than the conventional PCA-based anomaly-detection methods.

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1. Introduction

In today's competitive atmosphere, there is growing demand for enhanced process safety to maintain the safe and reliable process operations that are required to meet the higher expectations of process performances and product quality. Process monitoring, such as reliable detection and diagnosis of anomalies, is an important element to process safety and ultimately high quality-products. For example, a survey performed by Nimmo (1995) showed that the petrochemical industry in the USA could increase profits up to 10 billion USD per year if anomalies in their monitored process could be suitably detected and diagnosed. When an anomaly occurs in a monitored process, the monitoring process must immediately detect the anomaly and assist in determining if the process can continue to operate normally (Isermann, 2006).

Management and monitoring in hospital emergency department (ED) systems are among the most growing areas of concern

for many countries (Cochran & Broyles, 2010; Aboueljinane, Sahin, & Jemai, 2013). In particular, monitoring patient flow in EDs is a critical issue for many hospital administrations in France and worldwide because often leads to strain situations (Kadri, Chaabane, & Tahon, 2014; Kadri et al., 2013). In France, between 1990 and 1998, the annual number of ED demand increased by 43 (Baubeau et al., 2000), and according to the annual public report of Medical Emergencies (Rapport de la Cour des Comptes, 2006), the 7 million patients that visited EDs in France in 1990 had doubled by 2004. Between 1993 and 2003, the Institute of Medicine of the National Academies (I. of Medicine Committee on the Future of Emergency Care in the US Health System et al., 2006) published a report highlighting a disparity in the US between need and availability of ED facilities: the number of patients who visited EDs increased by approximately 26%, while the number of EDs decreased approximately 9% (Kellermann, 2006). Patient influx can generate strain situations that affect building safety and reliability of EDs (Kadri, Harrou, Chaabane, & Tahon, 2014). Therefore, detecting abnormal demands On EDs will contribute to improving the management of patients and medical resources (human and material). The early detection of abnormal demands in EDs promotes reactive control which can help to prevent strain situations, specifically limit the consequences, and allows efficient resource

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allocation. Thus, the goal of this study is to develop an anomaly-detection strategy that detects abnormal ED demands.

An anomaly is defined as an unpermitted deviation of at least one characteristic property of a variable from its acceptable behavior. Therefore, the anomaly is a state that may lead to a malfunction in the system (Isermann, 2005). Two main kinds of anomalies can be distinguished by the way they affect the monitored system: gradual and abrupt anomalies. In an ED, slow or gradual anomalies usually indicate a slow increasing demand or patient flow, while abrupt anomalies, are characterized by sudden increasing demands (patient flow). Here, we address the problem of detecting abrupt and gradual anomalies encountered by various anomaly-detection techniques that have been developed for the safe operation of systems or processes (Harrou, Fillatre, & Nikiforov, 2014; Hwang, Kim, Kim, & Seah, 2010; Qin, 2012; Isermann, 2006; Venkatasubramanian, Rengaswamy, Kavuri, & Yin, 2003). Model-based methods are implemented by measuring the dissimilarity between measured process variables and information obtained from explicit process models. Unfortunately, building a precise model for a monitored process can be challenging. When there is no process model, multivariate latent variable regression (LVR) methods, such as partial least square (PLS) regression and principal component analysis (PCA), have been used successfully in process monitoring because they can effectively deal with highly correlated process variables (Qin, 2012; Harrou, Nounou, Nounou, & Madakyaru, 2013). A number of the characteristics interest to the operational framework of EDs make it difficult to accurately model their behavior (Kadri et al., 2014; Bhattacharjee & Ray, 2014): (i) they are dynamic and disturbed environments, (ii) some elements that characterize care activity are non-deterministic (e.g. processing time, waiting time, and additional examinations), (iii) each patient requires treatment that is specific to their pathology and involves different routes within the ED, and (iv) no assumptions can be made concerning the types of emergency treatment that patients will require within a given period of time. For these reasons, PCA a well-known multivariate data analysis technique, can be used because it requires no prior knowledge about the process model (MacGregor & Kourti, 1995).

This paper aims to present a statistical anomaly-detection scheme based on a PCA model that can detect abnormal ED demands. Our basis for this approach was conceived by PCA's reputation as a linear dimensionality reduction modeling technique, which is favorable when processing data sets that have a high degree of cross correlation among the variables (Qin, 2012). The basic concept behind PCA is to reduce the dimensionality of highly correlated data, while retaining the maximum possible amount of variability present in the original data set (MacGregor & Kourti, 1995). Detecting an anomaly based on PCA has been widely used in practice because the only information needed is a good historical database describing the normal process operation. In such a framework, PCA and its extensions have successfully been applied for detecting anomalies in various disciplines (Wise & Gallagher, 1996; Simoglou, Martin, & Morris, 1997; Yu, 2011). However, PCA-based monitoring statistics, such as T^2 and Q statistics, are unsuitable for detecting changes resulting from small anomalies (Montgomery, 2005). Unlike PCA-based statistics, multivariate statistical process control charts, such as the multivariate cumulative sum (MCUSUM) (Montgomery, 2005; Bersimis, Psarakis, & Panaretos, 2007; Crosier, 1988), have shown a greater aptitude to detect small anomalies in the process mean. Because the MCUSUM control scheme better detects small faults in the process mean (Montgomery, 2005), the main objective of this paper is to combine the advantages of the MCUSUM and PCA method to enhance their performances and widen their applicability in practice. More specifically, this paper proposes a PCA-based MCUSUM

fault detection methodology for identifying signs of abnormal situations caused by abnormal demand for the Pediatric Emergency Department (PED) in the Lille Regional Hospital Centre, France.

The remainder of this paper is organized as follows. Section 2 briefly describes the PCA theory and how it can be used in anomaly detection, and Section 3 explain the MCUSUM control scheme that is commonly used in quality control. Next, the proposed PCA-based MCUSUM anomaly-detection approach that integrates PCA modeling and MCUSUM control scheme is presented in Section 4. Section 5 presents the application of the proposed methodology in the detection of abnormal situations in the PED in the Lille Regional Hospital Centre, France, and describes the practical data set used in the case study. Section 6 presents results of the proposed PCA-based MCUSUM anomaly-detection methodology and compare them with that of conventional PCA-based anomaly-detection. Finally, Section 7 reviews the main points discussed in this work and concludes the study.

2. PCA based statistical monitoring

PCA has a reputation for its usefulness in multivariate statistical techniques for reducing the dimensionality of the process data. Linear PCAs are valued for their ability to manage collinear data with several variables. In its general form, PCAs find the latent variables (not directly observed or measured) from the process data by capturing the largest variability in the data. In this Section we present the PCA theory and how it can be used in anomaly-detection.

2.1. PCA modeling

Let us consider the following raw data matrix $\mathbf{X} = [x_1^T, \dots, x_n^T]^T \in \mathbb{R}^{n \times m}$ consisting of n observations and m correlated variables, where $x_n \in \mathbb{R}^m$. Before computing the PCA model, the raw data matrix \mathbf{X} is usually pre-processed by scaling every variable to have zero mean and unit variance. This is because variables are measured with various means and standard deviations in different units. This pre-processing step puts all variables on an equal basis for analysis (Ralston, DePuy, & Graham, 2001). Let \mathbf{X}_s denote the standardized matrix \mathbf{X} . By using singular value decomposition (SVD), PCA transforms the data matrix \mathbf{X}_s into a new matrix $\mathbf{T} = [t_1 \ t_2 \ \dots \ t_m] \in \mathbb{R}^{n \times m}$ of uncorrelated variable called score or principal components, $t_1 \in \mathbb{R}^n$. Each new variable is a linear combination of the original variables, so that \mathbf{T} is obtained from \mathbf{X}_s by orthogonal transformations (rotations) designed by $\mathbf{P} = [p_1 \ p_2 \ \dots \ p_m] \in \mathbb{R}^{m \times m}$, which is given as the following:

$$\mathbf{X}_s = \mathbf{TP}^T. \quad (1)$$

The column vectors $p_i \in \mathbb{R}^m$ of the matrix $\mathbf{P} \in \mathbb{R}^{m \times m}$ (also known as the loading vectors) are formed by the eigenvectors associated with the covariance matrix of \mathbf{X}_s , i.e., Σ . The covariance matrix, Σ , is defined as follows:

$$\Sigma = \frac{1}{n-1} \mathbf{X}_s^T \mathbf{X}_s = \mathbf{P} \mathbf{\Lambda} \mathbf{P}^T \quad \text{with} \quad \mathbf{P} \mathbf{P}^T = \mathbf{P}^T \mathbf{P} = \mathbf{I}_n, \quad (2)$$

where $\mathbf{\Lambda} = \text{diag}(\lambda_1, \dots, \lambda_m)$ is a diagonal matrix containing the eigenvalues in a decreasing order ($\lambda_1 > \lambda_2 > \dots > \lambda_m$), and \mathbf{I}_n is the identity matrix (Jackson & Mudholkar, 1979).

For collinear processes, the dimensionality reduction of the m -dimensional space is obtained by retaining only the first (l) largest PCs, which correspond to the largest eigenvalues of the covariance matrix. The first (l) largest PCs normally describe the most of the variance in the data. The smallest PCs are considered noise contributors. An important step in the building of PCA model

is to determine the number of PCs, l , required to adequately capture the major variability in the data sets. Fig. 1 shows how a 3-dimensional collinear data set can be represented in a reduced 2-dimensional space using only two PCs.

2.2. How many principal components should be use?

The goodness of the PCA model depends on a good choice of how many PCs are retained (Qin & Dunia, 2000). If the number of PCs, l , is not correctly estimated, the model may underfit (underestimation) or overfit (overestimation) the data. By overestimating the number of PCs, l , noise can be introduced and the model will fail to capture some of the information. However, by underestimating the number of PCs, important features in the data may not be captured, which degrades the prediction quality of the PCA model. Various techniques have been proposed to select the number of PCs including Scree plot (Zhu & Ghodsi, 2006), cumulative percent variance (CPV), parallel analysis, sequential tests, resampling, profile likelihood (Zhu & Ghodsi, 2006; Jolliffe, 2002), and cross validation (Li, Morris, & Martin, 2002). In this study, the CPV technique will be used to determine the number of PCs for the PCA model. The idea behind the CPV technique is to select those l PCs that capture a certain percentage of the total variance (e.g., 90%) are chosen. The CPV is defined as follows:

$$CPV(l) = \frac{\sum_{i=1}^l \lambda_i}{\text{trace}(\Sigma)} \times 100. \quad (3)$$

Once the number of PCs l is determined, the data matrix \mathbf{X}_s can be represented using PCA as the sum of two orthogonal parts: an approximated data matrix $\hat{\mathbf{X}}$ and a residual data matrix \mathbf{E} , that is

$$\begin{aligned} \mathbf{X}_s &= \begin{bmatrix} \mathbf{T} & \mathbf{P}^T \end{bmatrix} \begin{bmatrix} \hat{\mathbf{T}} & \tilde{\mathbf{T}} \end{bmatrix}^T \\ &= \hat{\mathbf{T}}\hat{\mathbf{T}}^T + \tilde{\mathbf{T}}\tilde{\mathbf{T}}^T \\ &= \underbrace{\hat{\mathbf{X}}}_{\hat{\mathbf{X}}} \underbrace{\hat{\mathbf{P}}\hat{\mathbf{P}}^T}_{\mathbf{E}} + \underbrace{\mathbf{X}_s(\mathbf{I}_m - \hat{\mathbf{P}}\hat{\mathbf{P}}^T)}_{\mathbf{E}} \end{aligned} \quad (4)$$

where $\hat{\mathbf{T}} \in \mathbb{R}^{n \times l}$ and $\tilde{\mathbf{T}} \in \mathbb{R}^{n \times (m-l)}$ are matrices containing the l retained PCs and the $(m-l)$ ignored PCs, respectively, and $\hat{\mathbf{P}} \in \mathbb{R}^{m \times l}$ and $\tilde{\mathbf{P}} \in \mathbb{R}^{m \times (m-l)}$ are matrices containing the l retained eigenvectors and the $(m-l)$ ignored eigenvectors, respectively (Jackson & Mudholkar, 1979).

By doing so, a subspace (process subspace) containing the true (non-random) variation is identified. Complementary to this subspace is the noise subspace, which ideally contains only noise (see Fig. 2).

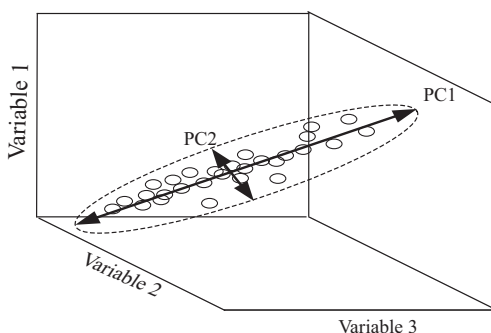


Fig. 1. Dimensionality reduction using PCA.

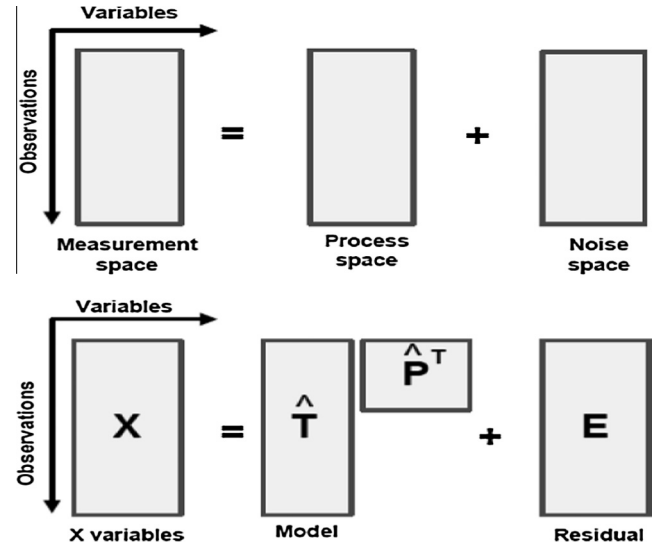


Fig. 2. Decomposition of \mathbf{X} into a process subspace and a residual subspace.

2.3. PCA based monitoring metrics

Once a PCA model has been built using process data obtained in normal operating conditions, it can be used along with one of the detection indices, such as Hotelling's T^2 or Q statistics, to detect faults.

2.3.1. Hotelling's T^2 statistic

The T^2 statistic measures variations in the PCs at different time samples. For anomaly detection, the T^2 statistic based on the first l PCs is defined as Hotelling (1933):

$$T^2 = \mathbf{x}_s^T \hat{\mathbf{P}} \hat{\Lambda}^{-1} \hat{\mathbf{P}}^T \mathbf{x}_s = \sum_{i=1}^l \frac{t_i^2}{\lambda_i} \quad (5)$$

where the matrix $\hat{\Lambda} = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_l)$ is a diagonal matrix containing the eigenvalues associated with the l retained PCs. The T^2 statistic given in (5) can be viewed as an ellipsoid in l dimensional space. For new testing data, when the value of T^2 exceeds a threshold value, $T_{l,n,\alpha}^2$, a fault is declared (Hotelling, 1933). The threshold value used for the T^2 statistic can be computed as follows (Hotelling, 1933):

$$T_{l,n,\alpha}^2 = \frac{l(n-1)}{n-l} F_{l,n-l,\alpha} \quad (6)$$

where n is the number of samples in the data, l is the number of retained PCs, α is the level of significance (α usually takes values between 1% and 5%), and $F_{l,n-l}$ is the Fisher F distribution with l and $n-l$ degrees of freedom.

When the number of observations, n , is rather large, the T^2 statistic threshold can be approximated with a χ^2 distribution with l degrees of freedom, that is

$$T_{\alpha}^2 = \chi_{l,\alpha}^2.$$

These threshold values are computed using fault-free data. For new testing data, when the value of T^2 exceeds the value of the threshold, $T_{l,n,\alpha}^2$ or T_{α}^2 , a fault is declared.

2.3.2. Q statistic

The Q statistic (also referred to as the Squared Prediction Error, SPE), defined as Qin (2003):

$$Q = \left\| (I - \hat{\mathbf{P}}\hat{\mathbf{P}}^T) \mathbf{x}_s \right\|^2 \quad (7)$$

measures the projection of a data sample on the residual subspace, which provides an overall measure of how a data sample fits the PCA model. In other words, a Q -based metric can be viewed as a measure of process model mismatch. When a vector of new data is available, the Q statistic is calculated and compared with the threshold value Q_x given in Jackson and Mudholkar (1979). If the confidence limit is violated (i.e., $Q > Q_x$), then a fault is declared. These confidence limits are calculated based on the assumptions that the measurements are time independent and multivariate normally distributed. The threshold value Q_x is given by Jackson and Mudholkar (1979):

$$Q_x = \varphi_1 \left[\frac{h_0 c_x \sqrt{2\varphi_2}}{\varphi_1} + 1 + \frac{\varphi_2 h_0 (h_0 - 1)}{\varphi_1^2} \right] \quad (8)$$

$$\varphi_i = \sum_{j=l+1}^m \lambda_j^i \quad i = 1, 2, 3$$

$$h_0 = 1 - \frac{2\varphi_1\varphi_3}{3\varphi_2^2},$$

where l is the number of retained PCs and c_x is the value of the normal distribution with α levels of significance. This threshold value is calculated based on the assumptions that the measurements are time independent and multivariate normally distributed. The Q fault detection index is very sensitive to modeling errors and its performance largely depends on the choice of the number of retained principal components, l (Benaicha, Guerfel, Boughila, & Benothman, 2010).

Generally, in PCA-based process monitoring, PCA develops a reference model using the normal data collected from the normal process. The new process behavior can thus be compared with the predefined behavior by the monitoring system to ensure that it remains under normal operating conditions. When a fault occurs, the process moves out of normal operation regions indicating that a change in a process behaviors has occurred. The PCA anomaly-detection algorithm is summarized in Table 1.

The Q statistic is usually preferred over the T^2 statistic for fault detection because it is more sensitive to faults with smaller magnitudes. The T^2 statistic is defined by the Mahalanobis distance whereas the Q statistic is defined by the Euclidean distance to avoid ill conditioning due to small eigenvalues (Kourti & MacGregor, 1995; MacGregor & Kourti, 1995; Qin, 2003). In other words, the T^2 statistic is a measure of the variation in the PCA model and the Q statistic is a measure of the amount of variation not captured by the PCA model. The main disadvantage of using PCs in process monitoring is the lack of physical interpretation (Ranger & Alt, 1996; Kourti & MacGregor, 1996). In addition, in a previous study, Romagnoli and Palazoglu (2006) showed that the T^2 statistic can result in false negatives (missed detection) because latent space can be insensitive to small process upsets, which is because each latent variable is a combination of all process variables. An additional disadvantage of T^2 statistic is that it cannot detect a faults in the process mean that are orthogonal to the first PCs (Mastrangelo, Runger, & Montgomery, 1996). However, the Q statistic is more sensitive to modeling errors, and its performance largely depends on the choice of the number of retained principal components, l , Benaicha et al. (2010). Improper choice of the number of PCs to be used in the model may result in autocorrelated residuals, which will affect the confidence limits for the Q test. It is these disadvantages of T^2 and Q statistics that motivate the use of alternative methods. In this study, we develop an anomaly

Table 1
PCA anomaly detection algorithm.

Step	Action
1:	Given: <ul style="list-style-type: none"> • A training fault-free data set that represents normal process operations and a testing data set (possibly faulty data).
2:	Data preprocessing <ul style="list-style-type: none"> • Scale the data to zero mean and unit variance.
3:	Build the PCA model using the training fault-free data <ul style="list-style-type: none"> • Compute the covariance matrix, Σ, using Eq. (2). • Calculate the eigenvalues and eigenvectors of Σ and sort the eigenvalues in decreasing order. • Determine how many principal components should be used. Many techniques can be used in this regards (see Section 2.2). In this work, the CVP criterion shown in Eq. (3) is used. • Express the data matrix as a sum of approximate and residual matrices as shown in Eq. (4). • Compute the control limits for the statistical model (e.g., the Q_x statistic limits).
4:	Test the new data <ul style="list-style-type: none"> • Scale the new data with the mean and standard deviation obtained from the training data. • Generate a residual vector, \mathbf{e}, using PCA. • Compute the monitoring statistic (Q or T^2 statistics) for the new data using Eq. (5) or (7).
5:	Check for anomalies <ul style="list-style-type: none"> • Declare an anomaly when new data exceeds the control limits (e.g., $Q \geq Q_x$).

detection technique, based on a linear PCA model and a MCUSUM control scheme, to improve detection performance compared to conventional PCA-based anomaly-detection methods. A succinct introduction to the basic ideas behind the MCUSUM monitoring scheme is explained in the subsequent section.

3. A multivariate cumulative sum (MCUSUM) monitoring chart

Several data-based anomaly detection techniques are referenced in the literature, and they can be broadly divided into two main classes: univariate and multivariate techniques (Montgomery, 2005). Univariate statistical monitoring methods, such as the exponentially weighted average (EWMA) and cumulative sum (CUSUM) schemes, are primarily used to monitor only single process variables (Page, 1954; Hawkins & Olwell, 1998; Lucas & Saccucci, 1990). However, production systems often involve a large number of highly cross-correlated process variables, and may thus be unsuitable for monitoring multivariate processes. Moreover, multivariate control charts, such as multivariate Shewhart (Lowry & Montgomery, 1995; Ryan, 2011; Wierda, 1994), multivariate EWMA (MEWMA) (Lowry, Woodall, Champ, & Rigdon, 1992; Rabhu & Runger, 1997) and MCUSUM (Crosier, 1986; Hawkins, 1991; Healy, 1987; Runger & Testik, 2004), have been used successfully to detect faults in multivariate processes with highly cross-correlated variables. The multivariate control charts have the advantage of taking into account existing correlations between variables.

Multivariate process measurement benefits from the use of inherent multivariate methods rather than a collection of univariate charting methods applied to the individual components (Santos-Fernández, 2012). The development of multivariate control charts originates from the work by Hotelling (1947, chap. ii). Multivariate Shewhart control charts use information from only the current sample, and they are relatively insensitive to small and moderate shifts in the mean vector. MCUSUM and MEWMA control charts have been developed to overcome this problem. The MCUSUM is a procedure that uses the cumulative sum of deviations of each random vector previously observed compared to the nominal value to monitor the vector of means of a multivariate

process. This chart was proposed by Crosier (1986) and is an extension of the univariate CUSUM control chart analysis.

The MCUSUM control chart is an extension of the univariate CUSUM chart for the multidimensional cases. Several versions of the MCUSUM chart have been proposed in the literature (Crosier, 1986; Hawkins, 1991; Healy, 1987). Here we used the CUSUM of vectors proposed by Crosier (1986). The vector-valued scheme is devised by replacing the scalar quantities of a univariate CUSUM scheme by vectors and is given by:

$$C_t = \sqrt{L_t^T \Sigma^{-1} L_t} \quad (9)$$

where

$$L_t = \begin{cases} 0 & \text{if } C_t \leq k \\ (L_{t-1} + X_t - \mu_0) \left(1 - \frac{k}{C_t}\right) & \text{otherwise} \end{cases} \quad (10)$$

and $C_t = \sqrt{(L_{t-1} + X_t - \mu_0)^T \Sigma^{-1} (L_{t-1} + X_t - \mu_0)}$.

Because of the fact that the average run length (ARL) performance of this chart depends on the non-centrality parameter, Crosier (1988) recommended that $L_0 = 0$ and $K = \frac{\sqrt{(\mu_1 - \mu_0)^T \Sigma^{-1} (\mu_1 - \mu_0)}}{2}$ and follow the convention of resetting the MCUSUM chart following a signal. Where μ_0 and μ_1 represent the in-control and the out-of-control process mean vectors, respectively. This scheme signals when $C_t > H$, where H is chosen to provide a predefined in-control ARL using simulation.

In the next section, the MCUSUM control scheme is integrated with PCA to improve the ability of PCA to detect small anomalies occurring in the mean of process or system measurements.

4. PCA-based MCUSUM fault detection strategy

In this section, PCA is integrated with MCUSUM to develop a new anomaly detection scheme with a higher sensitivity to small anomalies in the data. Once developed, PCA models can be combined with the MCUSUM control schemes for detecting unusual process conditions. Towards this end, control limits can be placed on the residuals obtained from the PCA model. The general principle of the proposed method is schematically illustrated in Fig. 3. Indeed, the residuals of the PCA model can be used as an indicator of the existence or absence of anomalies (Kinnaert, 2003). When the monitored system is under normal operating conditions (no anomaly), the residual is zero or close to zero in cases of modeling uncertainties and measurement noise. However, when a fault occurs, they tend to largely deviate from zero indicating the

presence of a new condition that is significantly distinguishable from the normal healthy working mode (Kinnaert, 2003). In this paper, MCUSUM is used to enhance process monitoring through its integration with PCA, because its ability to detect small changes in the data.

Thus, this work exploits the advantages of the MCUSUM control scheme to improve anomaly detection over conventional PCA-based methods. More specifically, the MCUSUM control scheme is used to monitor the uncorrelated residuals obtained from the PCA model.

The methodology proposed in this study is summarized in Fig. 4. This methodology consists of two main phases (training phase and testing phase) described as follows:

- 1. The training phase:** This phase is defined as a learning phase on which we can learn from the collected normal operating condition data (training data). The main objective of this phase is to build a PCA reference model based on training data. The following is a summary of this phase:
 - *Step 1:* Collect training data which represents normal process operations. Scale the training data to zero mean and unit variance and obtain the scale parameter vectors μ and σ .
 - *Step 2:* Build the PCA reference model using the training data set. The objective is to find and validate the best descriptive model based on the analysis of the training data. Perform SVD to obtain a PCA model. Determine the number of principal components.
 - *Step 3:* Based on the PCA model, compute the MCUSUM thresholds. In this step, based on the residuals (the ignored PCs) obtained from the PCA reference model, the upper control limits of the control chart can be computed.
- 2. The testing phase:** The training phase is followed by the testing phase, which uses new, unseen data (testing data). The testing data set (anomaly data) may possibly contain abnormalities that correspond to abnormal operations in the monitored system. This phase consists of four steps:
 - *Step 1:* Data pre-treatment, involves the analysis and scaling of the testing data with the mean and standard deviation of the training data.
 - *Step 2:* Generate the residuals based on the reference PCA model constructed in the training phase.
 - *Step 3:* Compute the MCUSUM decision function, C_t . The computation of C_t is summarized in Section 3.
 - *Step 4:* Check the presence of anomalies on the inspected system. If the MCUSUM decision function (C_t) exceeds the predefined threshold (h), an anomaly is declared. In this

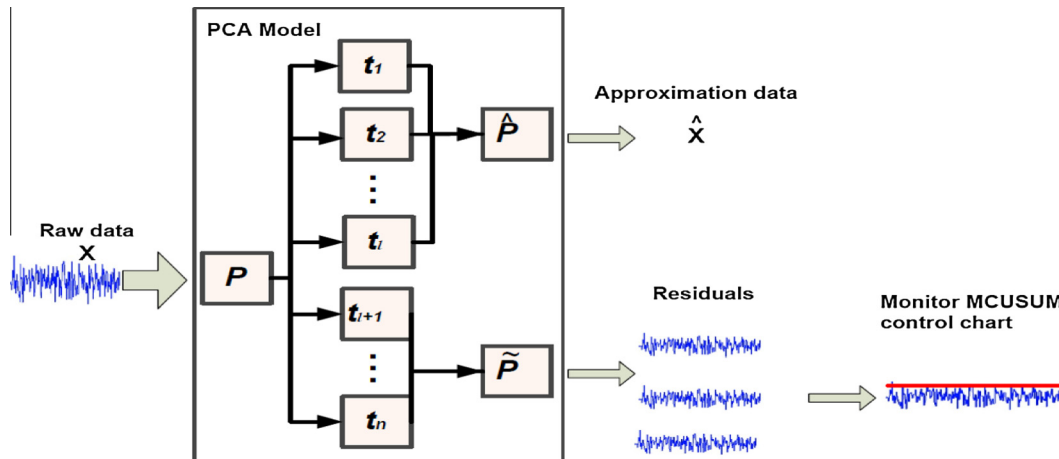


Fig. 3. Principle of PCA-based MCUSUM anomaly detection algorithm.

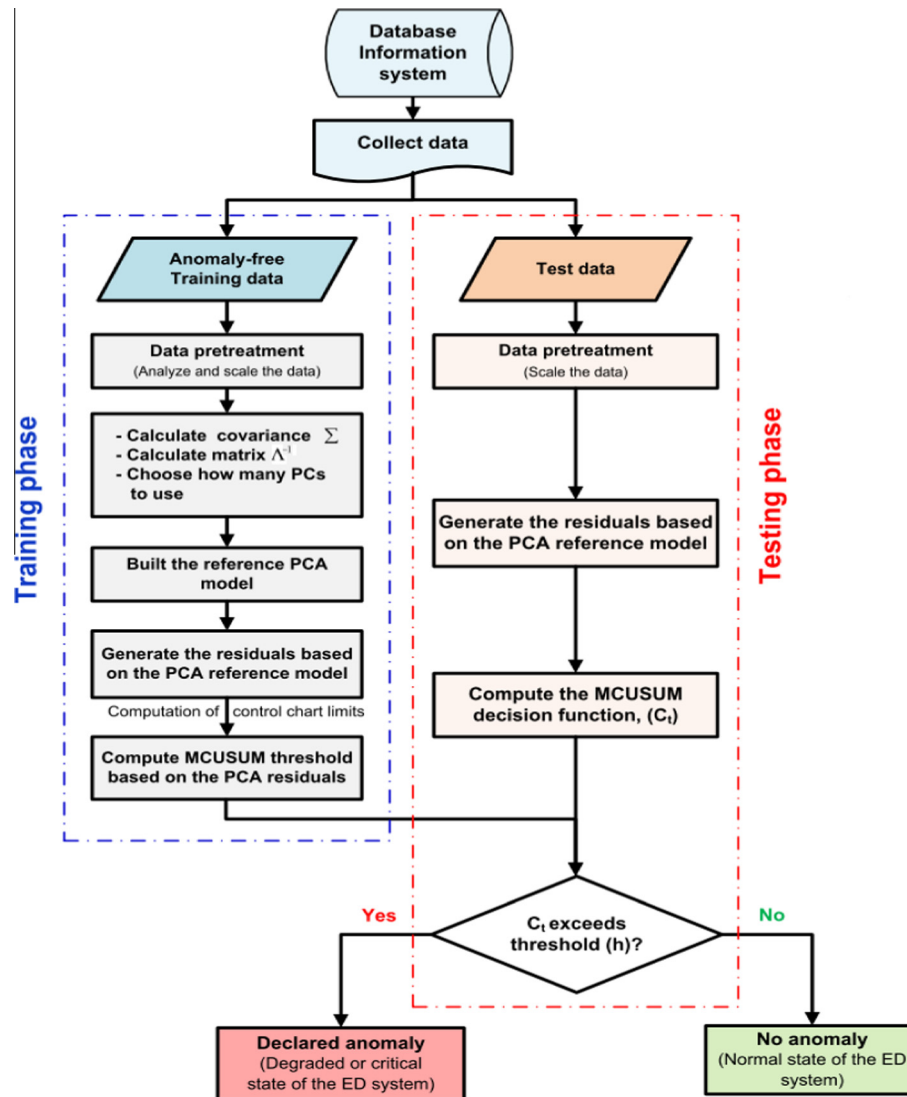


Fig. 4. A block diagram of the PCA-based MCUSUM anomaly detection method.

case, emergency management (corrective actions and emergency measures) is triggered in an attempt to return to an acceptable operating state.

5. Application to an emergency department

The performance of the proposed PCA-based MCUSUM method of anomaly-detection will be assessed in the next section and compared with conventional PCA anomaly detection methods by means of practical data collected from the PED in the Lille Regional Hospital Centre, France. In the next subsections, data source and preliminary descriptive analyses of the data are conducted to identify important features in the data.

5.1. Data source: Pediatric Emergency Department (PED)

In this study, the PED in Lille Regional Hospital Centre (CHRU), France will be monitored. The CHRU serves four million inhabitants in Nord-Pas-de-Calais, a region characterized by one of the largest population densities in France (7% of the French population). The PED is open 24 h a day and receives 23,900 patients a year on average. In addition to their internal capacity, the PED

shares many resources with other hospital departments such as administrative patient registration, clinical laboratory, and blood bank.

The patient treatment process in PED involves five main activities or stages as shown in Fig. 5: (i) patient registration (administrative registration) shared with adult emergency, (ii) patient admission and referral to the PED, (iii) patient admission to the vital emergency room, (iv) patient treatment in outpatient unit, and (v) patient admission to the short-term hospitalization unit.

The simplified care process in the PED, shown in Fig. 6, begins when a patient arrives in the PED and ends when a patient is either discharged from the PED (back home) or admitted to another department in the hospital or to another establishment for further treatment. More specifically, when a non-urgent patient arrives at the entrance of the PED, he is received by an administrative agent (administrative registration), who records the patient's information. A hostess admits the patient to the PED, which involves filing the patient's reason for coming, age, phone number, and health history. A first triage is performed after the consultation with the nurse. Next, based on the urgency of the patient's condition, the individual is directed to the waiting room or to a medical examination room (consultation box). After an examination with the

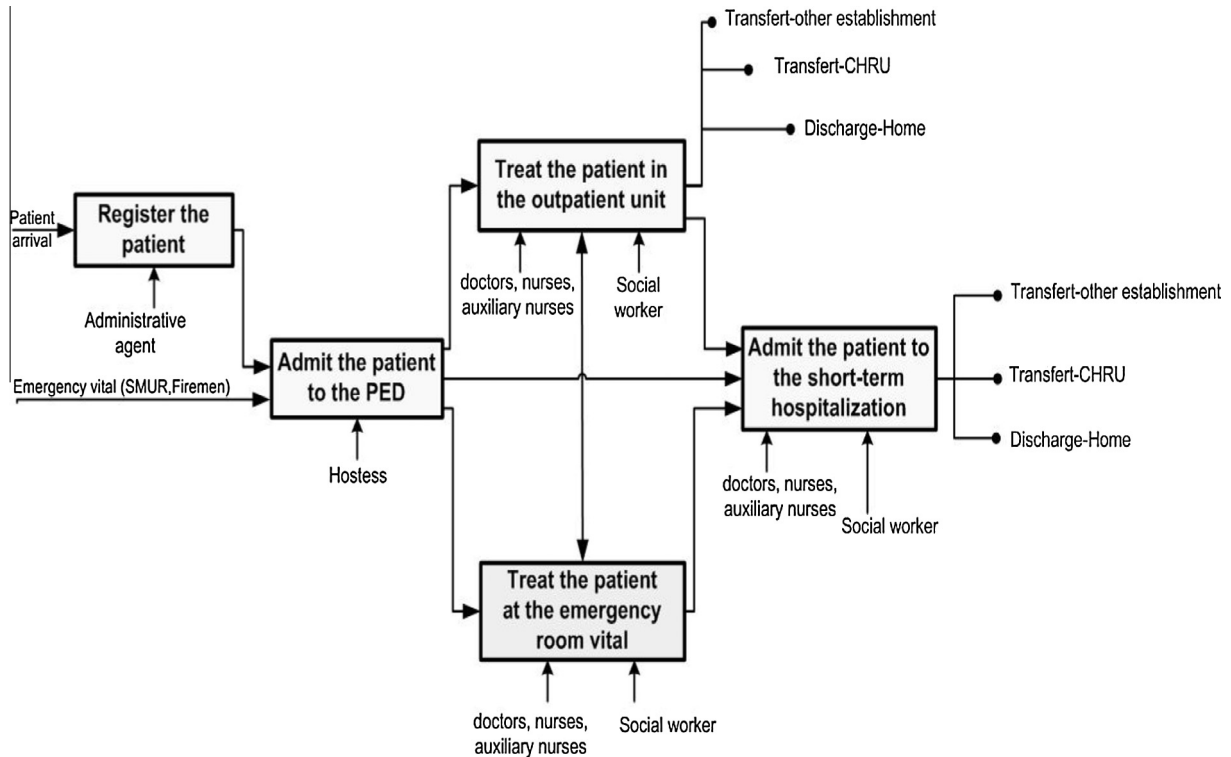


Fig. 5. Five main stages of the patient treatment process in the PED in CHRU-Lille.

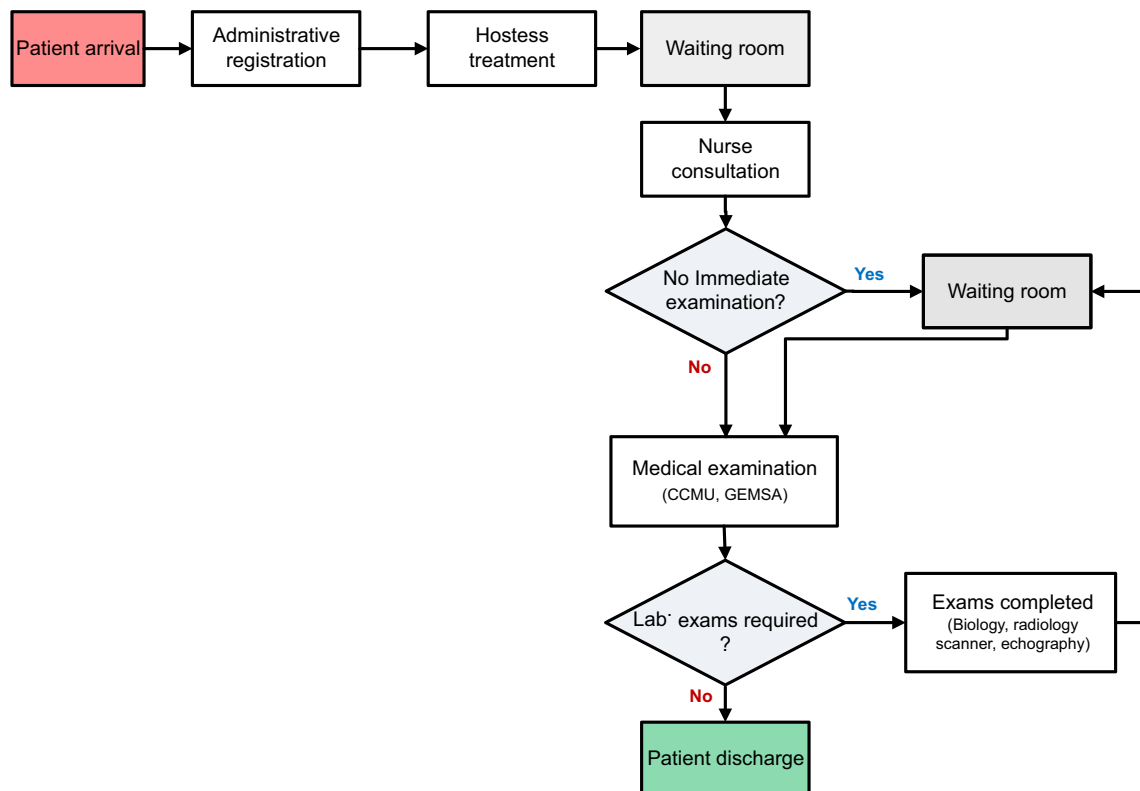


Fig. 6. Simplified care process in the PED.

medical staff (doctors), a second triage is performed. As a result of this triage, the patient is directed to the appropriate examination room (laboratory examination) or discharged from the PED

(home). In the case of a vital emergency, such as a patient was arrived by the mobile emergency and resuscitation service or by the fire and rescue department, is directly admitted to the PED,

and the administrative procedure is completed either in parallel or after treatment.

5.2. Data description

This retrospective study was conducted using a dataset comprising a daily time series extracted from the PED's database collected from January to December 2011 (see Table 2).

5.3. Data analysis

Fig. 7 illustrates the actual total number of patient arrivals per month of the ten time-series variables presented in Table 2.

Table 2
Time series variables collected from the PED database.

Time series variables	Definition
Arrival number (x_1)	Daily number of patient arrivals to the PED
Arrival means (x_2)	Daily number of patient arrivals to the PED using personal means
CCMU1 (x_3)	Daily number of non-urgent arrivals patient to the PED (length of stay for these patients is unpredictable)
CCMU2 (x_4)	Daily number of patient arrivals to the PED with a lesional state and/or stable functional prognosis; possibility of complementary examination and/or therapeutic treatment (plaster, suture) by staff from the PED
GEMSA2 (x_5)	Daily number of unexpected patient arrivals
Radiology (x_6)	Daily number of patient arrivals for radiology
Scanner (x_7)	Daily number of patient arrivals for a scanner
Echography (x_8)	Daily number of patient arrivals for echography
Biology (x_9)	Daily number of patient arrivals for biology
Discharge-home (x_{10})	Daily number of patients returning home after PED cares

Generally, the flow of patients varied between winter (epidemic) (November–March) and other (April–October) periods with fewest arrivals from July to September.

Fig. 8 presents the weekly patient arrival number at the PED of the ten time series variables over the entire period (January–December 2011). The number of patients arriving at the PED varies considerably according to the day of the week, as shown by the height of individual spikes in Fig. 9. Larger number of arrivals of Sundays and Mondays likely correspond with the closure of family clinics and special event days like holidays, sporting events, and festivals in the region. Fig. 9 presents the daily arrivals at the PED of the ten time series variables over the entire period (January–December 2011). Some descriptive characteristics (e.g., minimum, mean, median, quartiles, and maximum) of each time-series variable over the entire period (January–December 2011 day by day) are illustrated in Fig. 10.

6. Modeling the PED data using PCA

This section is devoted to the assessment of the proposed PCA based MCUSUM anomaly detection strategy using practical PED data.

6.1. Training of PCA model

The data used in training includes ten variables ($m = 10$): Arrival number, Arrival means, CCMU 1, CCMU 2, GEMSA 2, Radiology, Scanner, Echography, Biology, and Discharge-home (see Table 2). Thus, the data matrix with 362 rows and 10 columns is used to construct the PCA model after scaling the variables.

The PCA approach assumes that the normal sub-space can be correctly described by the first PCs because they capture the highest level of information. We used CVP method, which is typical for

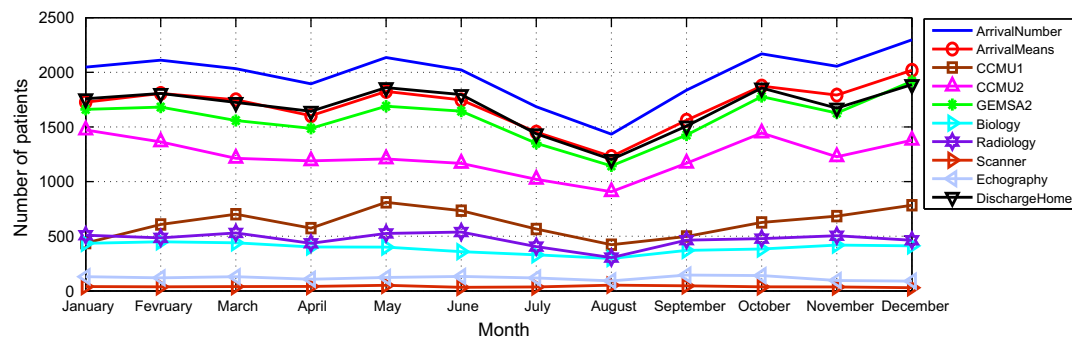


Fig. 7. Actual number of arrivals per month from January 2011 to December 2011.

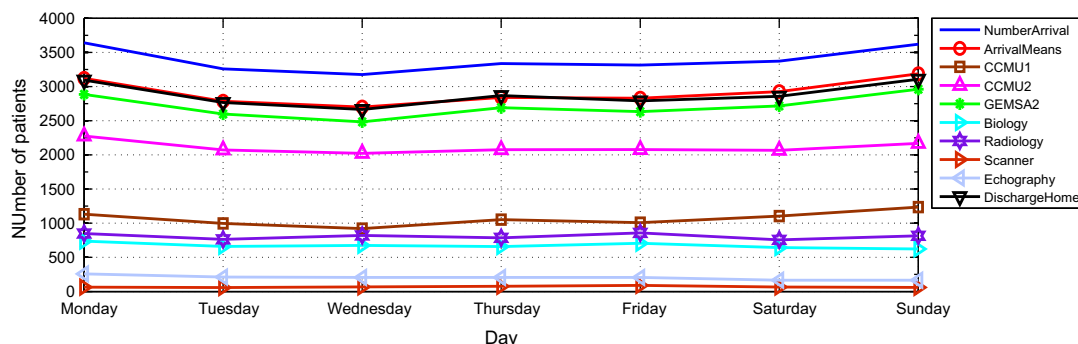


Fig. 8. Weekly paediatric emergency department arrivals from January to December 2011.

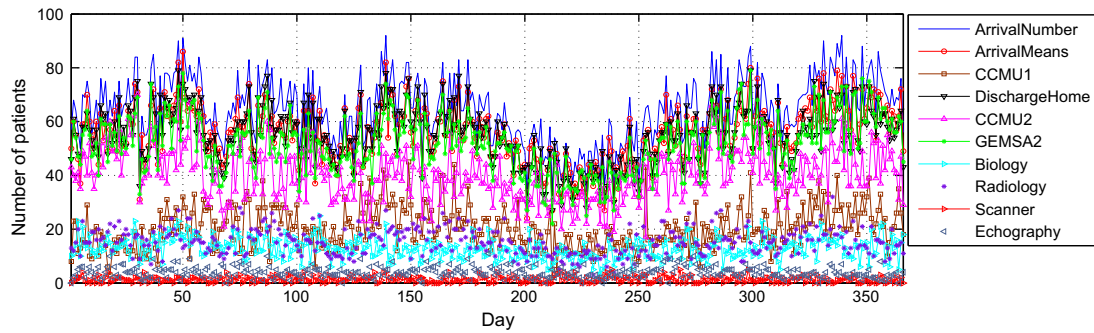


Fig. 9. Daily paediatric emergency department arrivals from January to December 2011.

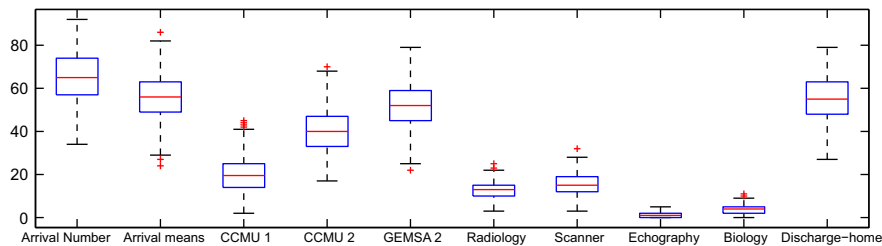


Fig. 10. Box plots of time series variables collected from the PED database.

determining the number of PCs to use. In this study, the threshold value of cumulative variance is 90%. We will keep the first five PCs (which capture 53.51%, 14.84%, 10.27%, 8.13%, and 6.37% of the total variations) because they capture 93.13% of the variation of the given data (see Fig. 11), and use them to build the monitoring model.

Once the number of PCs has been determined, the model prediction capability must be tested for model to be accepted. To illustrate the quality of the constructed PCA model, one common and simple approach is to regress predicted versus observed values (or vice versa). Fig. 12 shows the scatter plot of observed versus predicted values of the testing data set obtained from the selected PCA model, and illustrates that the points follow the diagonal line (predicted = observed) closely with no indication of a curvature or other anomalies. Note that the PCA model provides satisfactory predictions of the PED data considering the complexity of the ED

system; however, some variables, such as Radiology and Scanner, are not as well estimated as others. We now examine the effect of these modeling errors on the detection of abnormal situations in the monitored PED.

Application of these charts requires data with normal distribution and an absence of autocorrelation. Before applying the PCA-based anomaly-detection methods, we need to check whether the residuals follow a Gaussian distribution. By the term residuals we mean the model error matrix, \mathbf{E} , that was not captured by the PCA model. Normality was verified using histograms of the residual vectors that are shown in Fig. 13. Normality was also verified using Mardia's test (Mardia, 1974) as given in Table 3. Next, we check the independence of residuals (specifically, the absence of autocorrelation), which are assumed to be non-autocorrelated. If the assumption is satisfied, the autocorrelation function (ACF) of the residuals have no significant spikes at any non-zero lags. From Fig. 14, the residuals are not significantly correlated, and the distribution is close to normality.

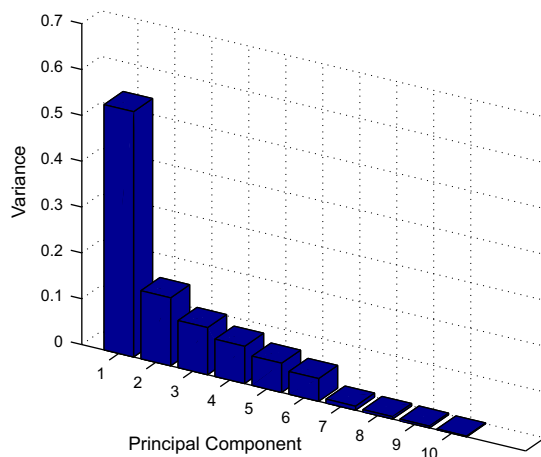


Fig. 11. Variance captured by each principal component.

6.2. Detection results

In this subsection, the PCA model was integrated with the MCUSUM control scheme to monitor the occurrence of strain situations generated by abnormal patient arrivals at the PED. The anomaly detection abilities of the PCA-based MCUSUM strategy are assessed, and compared to the performance of T^2 and Q statistics.

6.2.1. Testing data set with abnormal patient arrivals

In the case of testing data from the PED database that contained no real anomalies, two different anomalies were simulated to assess the performance of the proposed anomaly detection strategy. These simulated anomalies were helpful because they enabled a theoretical comparison between various monitoring charts as the simulated anomalies in these case studies were known. Before pre-processing the data, we injected additive anomalies into the

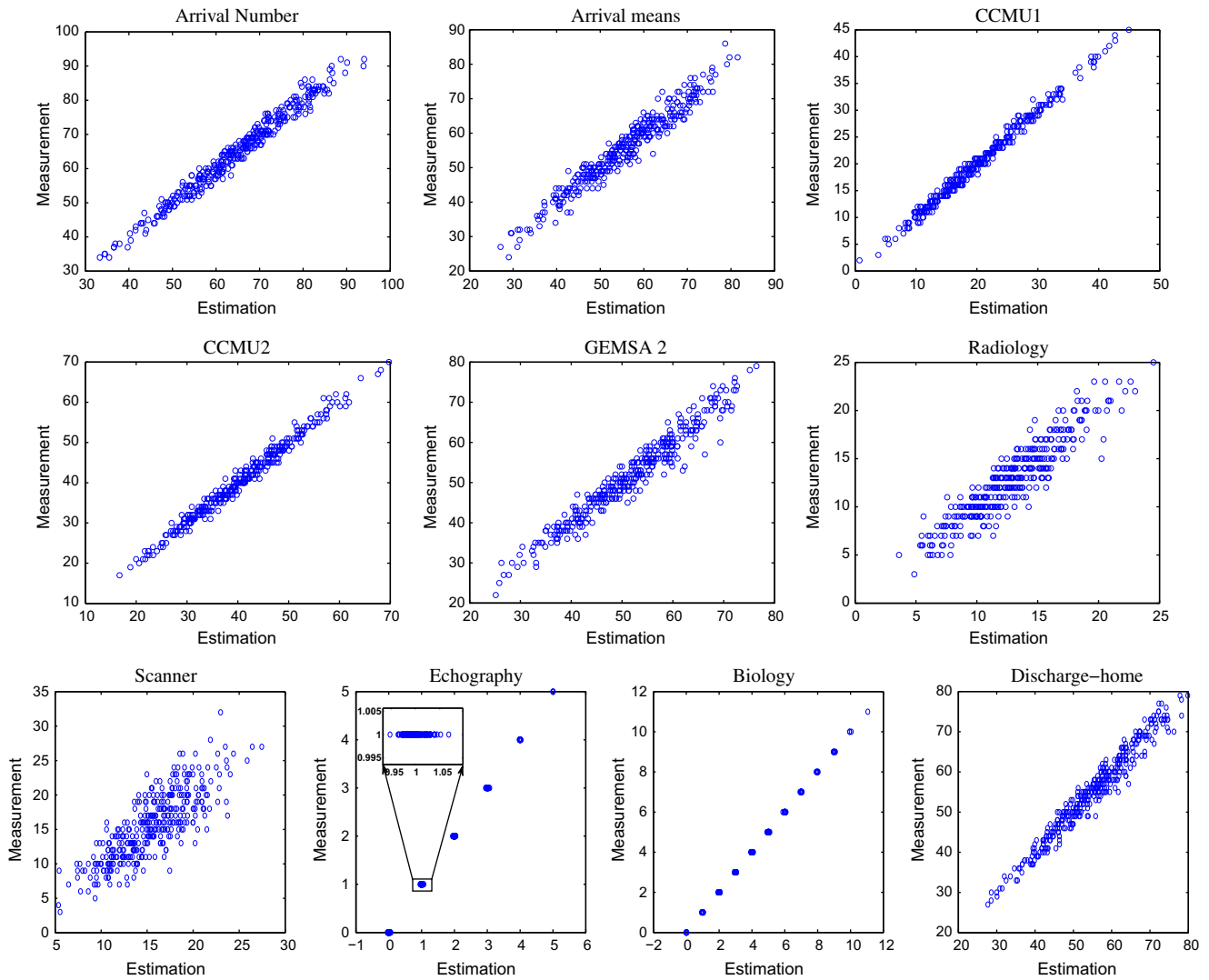


Fig. 12. Measurements and estimations using PCA model.

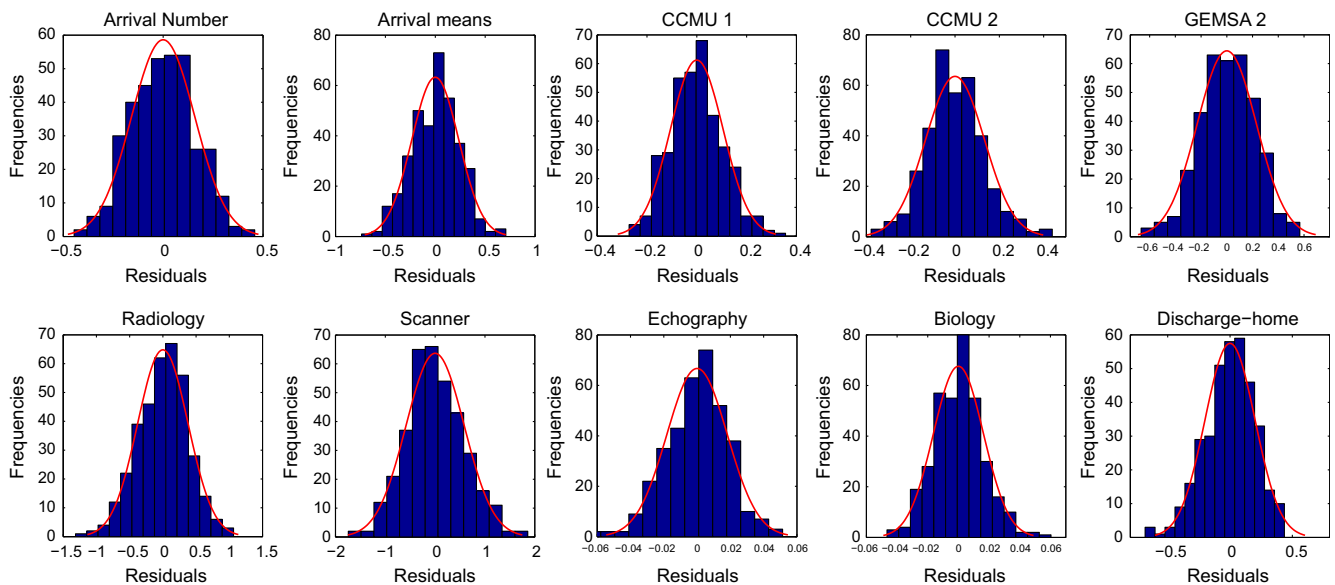


Fig. 13. Histograms showing the normality of the residuals.

Table 3
Mardia's multivariate normality test.

	Statistic	p-value
Skewness	0.002	0.76
Kurtosis	2.70	0.24

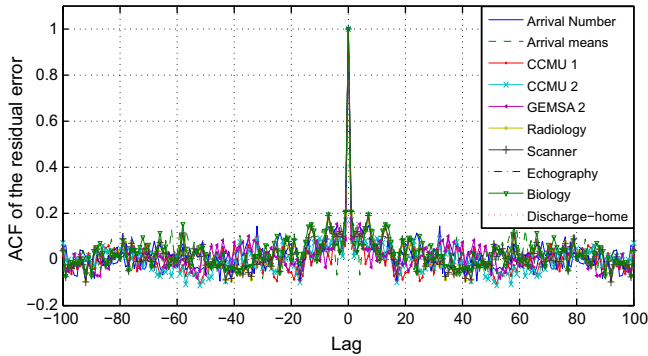


Fig. 14. ACF of residual errors.

Table 4
Anomaly types tested in the simulation study.

Fault type	Description	Parameters
Abrupt anomaly	$\mathbf{x}_i(t) = \mathbf{x}_i(t) + b$	b: bias
Gradual anomaly	$\mathbf{x}_i(t) = \mathbf{x}_i(t) + s(k - k_s)$	s: slope, and k_s : starting point of the gradual anomaly

raw data. We simulated two different types of anomalies: abrupt anomaly (case A) and gradual anomaly (case B). The two different types of anomaly cases are summarized in Table 4.

6.2.2. Case A: Abrupt abnormal patient arrivals

In this case study, we investigated the problem of detecting abrupt abnormal patient arrivals. The testing data set used in this study consists of daily attendance at the PED, collected from January to December 2012, before preprocessing the raw data with

simulated abrupt anomalies. First, we standardized the testing data using the mean and standard deviation computed from the training data, and then we used the PCA model build from these anomaly-free data to detect anomalies in the standardized testing data. Two examples are presented here to illustrate the ability of the proposed PCA-based MCUSUM control scheme to detect abrupt abnormal patient arrivals.

Case A₁: In the first example, multiple-day anomalies are considered. To do that, an additive anomaly is introduced in the variable X_1 from samples 141 to 147. This abnormal situation is represented by a constant bias of amplitude equal to 50% of the total variation in X_1 . The results of Q and T^2 statistics based on the testing data are shown in Figs. 15 and 16, respectively. The Q statistic successfully detected the abnormality by exceeding the threshold value. The alarm condition is triggered at a given sample 141, when the Q index overpasses the control limit. However, Hotelling's T^2 statistic did not recognize the evolution of the given faulty condition, as shown in Fig. 16. Most of the T^2 values in the sampling interval where the anomaly was introduced did not exceed the control limits, evidencing its unreliable anomaly detection. The T^2 statistic provides a measure of the PC deviation that is most important to normal system operation. Therefore, because the normal operating region defined by the T^2 control limits is usually larger than that defined by the Q control limits, abnormalities with moderate magnitudes can easily exceed the Q threshold, but not the T^2 threshold, making the Q statistic typically more sensitive than the T^2 statistic for these types of anomalies. The result of the PCA-based MCUSUM anomaly-detection algorithm based on testing data in Fig. 17, clearly shows the violation of the confidence limits providing the ability of the proposed algorithm to correctly detect this strain situation. For the construction of the MCUSUM, the parameters of MCUSUM are chosen to be $ARL_0 = 200$ (i.e., false alarm rate $\alpha = 0.5\%$) and the reference value $k = 0.5$. The decision limit is $h = 6.885$.

Case A₂: In the second example, a moderate bias level of 25% of the total variation in the testing data, was injected between samples 141 and 147. Q statistic performances are illustrated in Fig. 18, where it is evident that the Q values for the testing data are slightly larger in the region of the bias anomaly, but do not violate the confidence limits. This result shows that the Q statistic is completely unable to detect this moderate simulated anomaly. This is because this monitoring chart only takes into account the

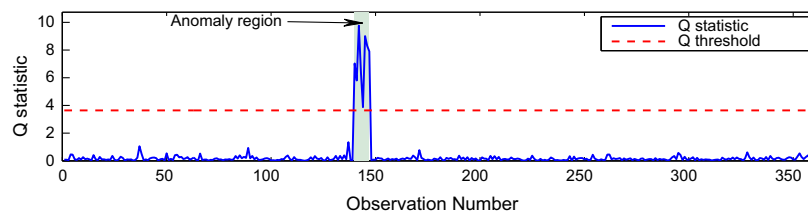


Fig. 15. The time evolution of the Q statistic in the presence of a multiple-day strain situation (Case A₁).

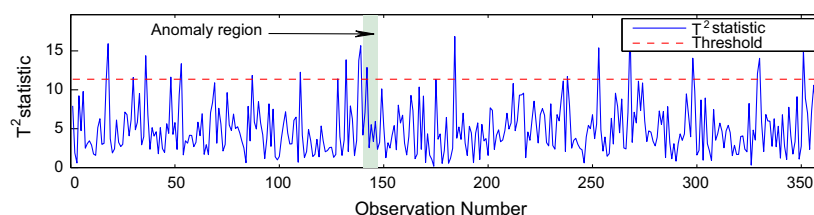


Fig. 16. The time evolution of the T^2 statistic in the presence of a multiple-day strain situation (Case A₁).

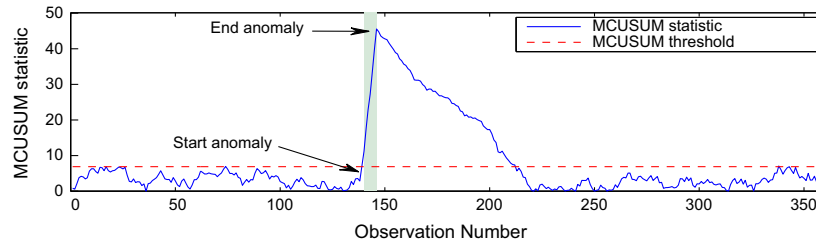


Fig. 17. The time evolution of the MCUSUM statistic in the presence of a multiple-day strain situation (Case A_1).

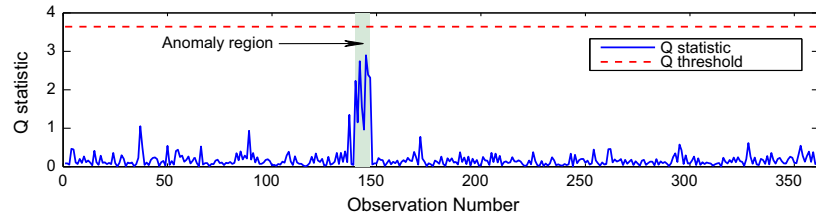


Fig. 18. The time evolution of the Q statistic in the presence of a multiple-day strain situation (Case A_2).

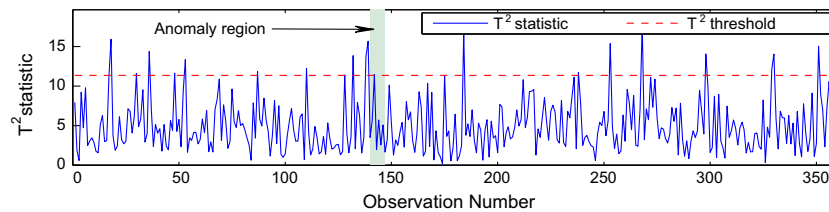


Fig. 19. The time evolution of the T^2 statistic in the presence of a multiple-day strain situation (Case A_2).

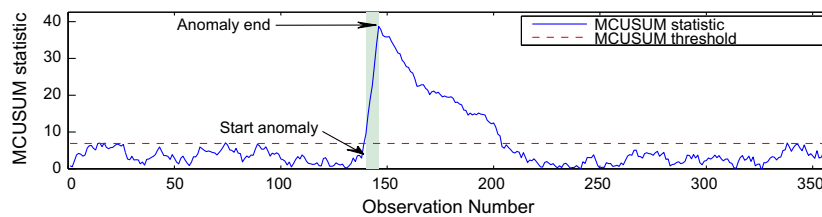


Fig. 20. The time evolution of the MCUSUM statistic in the presence of a multiple-day strain situation (Case A_2).

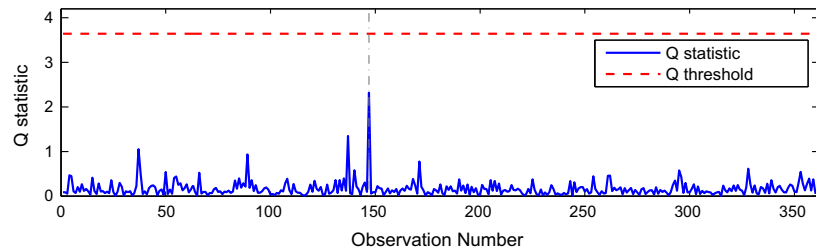


Fig. 21. The time evolution of the Q statistic in the presence of a single-day strain situation.

information provided by the data samples in the decision making process, for which the Q statistic is not powerful enough to detect small changes. From Fig. 19 it is evident that the T^2 statistic detected no anomaly. Fig. 20 presents the results of the PCA-based MCUSUM anomaly-detection algorithm based on the

testing data. This figure clearly shows the violation of the confidence limits and thus, the ability of the proposed PCA-based MCUSUM anomaly-detection algorithm to detect this strain situation correctly (substantial patient flow), making it superior to the conventional PCA-based anomaly-detection methods.

To illustrate the performance of the developed PCA-based MCUSUM method in the case of a single-day strain situation, an additive anomaly was introduced in \mathbf{x}_1 , which consists of an amplitude bias equal to 25% of the total variation in \mathbf{x}_1 at sample number 147 (highlighted by the vertical dashed line). The Q statistic and the T^2 shown in Figs. 21 and 22, respectively, show that this conventional PCA was unable to detect this simulated anomaly. Meanwhile Fig. 23 clearly indicates that the proposed PCA-based MCUSUM anomaly-detection method did detect this simulated anomaly. Therefore, this index is effective for detecting this simulated anomaly, verifying that a PCA-based MCUSUM control scheme can detect small events that are difficult to detect by conventional PCA.

6.2.3. Case B: Gradual abnormal patient arrivals

In this case study, the variable x_1 is contaminated by a slow gradual anomaly starting at sample number 300 with a slope of 0.1. The results of the Q and T^2 statistics, shown in Figs. 24 and 25, respectively, show that anomalies remained undetectable by applying conventional PCA statistics. In contrast Fig. 26, clearly indicates that the proposed PCA-based MCUSUM anomaly-detection algorithm detected all anomalies. We maintained the false alarm rate, the reference value, and the decision limit at $\alpha = 0.5\%$, $k = 0.5$, and $h = 6.885$, respectively. As the anomaly developed, the MCUSUM statistic gradually increased until it violated the control limits such that the magnitude of the anomaly become sufficiently high enough to be detected by the model.

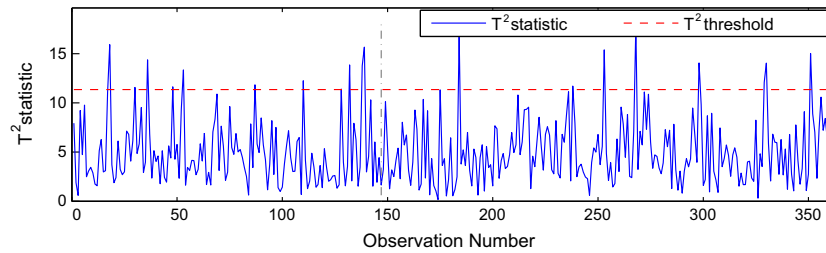


Fig. 22. The time evolution of the T^2 statistic in the presence of a single-day strain situation.

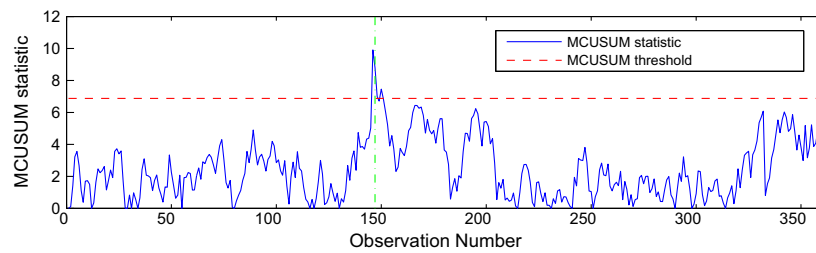


Fig. 23. The time evolution of the MCUSUM statistic in the presence of a single-day strain situation.

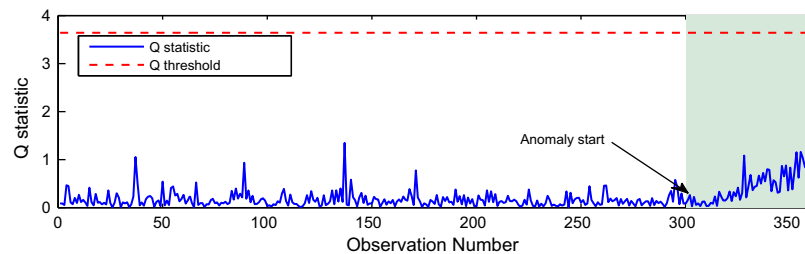


Fig. 24. The time evolution of the Q statistic in the presence of a gradual strain situation.

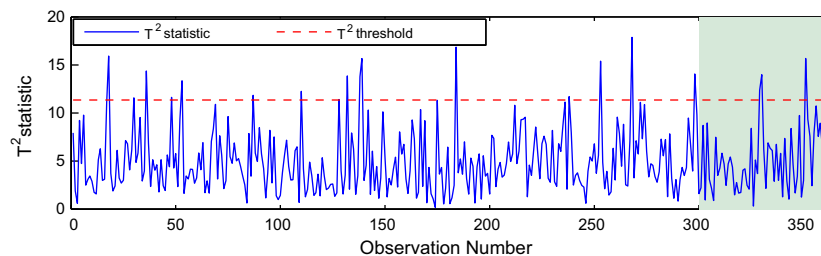


Fig. 25. The time evolution of the T^2 statistic in the presence of a gradual strain situation.

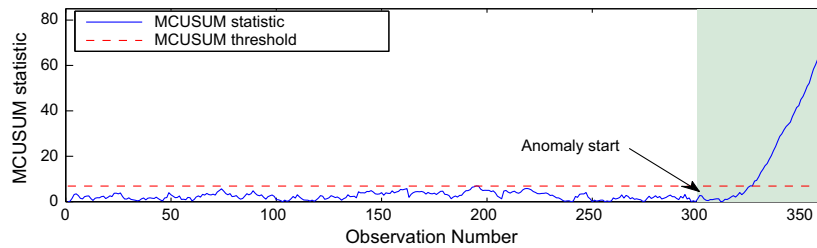


Fig. 26. The time evolution of the MCUSUM statistic in the presence of a gradual strain situation.

This case study clearly shows that the performance of the proposed PCA-based MCUSUM detection method was satisfactory for monitoring the abnormal patient arrivals at the PED in Lille Regional Hospital Centre, France. The results of two simulated case studies (cases A and B) show an even clearer advantage for a PCA-based MCUSUM strategy over conventional PCA approaches, especially in the case of small anomalies.

7. Conclusion

This study reports the development of a PCA-based MCUSUM anomaly-detection methodology. Conventional PCA-based fault detection metrics Q and T^2 have the disadvantage of limited effectiveness in detecting small or moderate faults in the mean of the process. The MCUSUM scheme more effectively detects small faults, making it an attractive alternative to conventional PCA monitoring statistics. The focus of this work was to integrate PCA modeling and the MCUSUM control scheme to improve the detection of strain situation signs in EDs. More specifically, in this developed PCA-based MCUSUM anomaly-detection method, PCA was used in the modeling framework, and then the MCUSUM control was applied to the residuals obtained from the PCA model to detect anomalies when the data did not fit the PCA model. The proposed anomaly-detection algorithm better detected faults than do conventional detection indices Q and T^2 . We applied our developed strategy to a practical data set collected from the PED in the Lille Regional Hospital Centre, France. Results evidence the effectiveness of the developed algorithm compared to conventional PCA statistics, and indicate that a PCA-based MCUSUM anomaly-detection strategy can be used as an automatic tool to detect strain situation signs in EDs.

The detection of significant patient flow (abnormal demand for care) in EDs is beneficial for supporting the reactive control of strain situations (Kadri et al., 2014). This is important not only for managing patients and internal resources, but also for predicting sufficient hospitalization capacity downstream so as to be prepared to absorb the occasional increase in patient flow without reducing the quality of patient care. Therefore, by exploiting the information provided by this anomaly-detection scheme, the performance of the ED can be optimized.

Next, we integrated the developed methodology into a decision support system (DSS) to anticipate and manage the occurrence of strain situations at the ED in Lille, France. This is a step toward real-time monitoring of ED behavior aimed at preventing and predicting abnormal situations and examining the relationship between the indicators of these situations and the corresponding corrective actions (action plans). The DSS must allow for: (i) proactive action (proactive management) so that ED managers can forecast for the short and/or long-term occurrence of strain situations based on the actual behavior of the ED and the changes in ED demands, and to propose effective corrective actions to avoid these situations and (ii) reactive action (reactive management), so that ED managers can act quickly to the occurrence of a strain situation

by the implementation of effective and adequate corrective actions.

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