

## 1.1 Analyze the total time required to push $n$ elements to the Stack and express it using the Big O notation

To provide the necessary context to address this question, the **Stack ADT class** implemented in Question 1 is defined by the following characteristics:

- The underlying data structure is a singly headed singly linked list (SHSL)
- The stack must be maintained in Last-In-First-Out (LIFO) order
- The location of the Stack's top begins at the back (Last node of the linked list)

Since we are working with a SHSL list (Single reference to the head node), we must traverse through the entire list of  $n$  elements before adding the next node to the Stack. In other words, the time complexity of `push()` is  $O(n)$  as it is dependent on the size of the linked list.

In the scenario of pushing  $n$  elements to an arbitrary  $m$  sized linked list, there are two scenarios. First suppose our linked list is empty ( $m = 0$ ), then we just need to create a new Node, set it's data value, and make the head point to this Node. Second, suppose our linked list has non-zero elements.

Iterations Required	Elements in Linked List	Elements To Add
0	$m$	$n$
$m$	$m+1$	$n-1$
$m+1$	$m+2$	$n-2$
$m+2$	$m+3$	$n-3$
.....	.....	.....
$m+n-1$	$m+n-1$	1
$m+n$	$m+n$	0

We need to also create a new Node like the first case, but we need to traverse from the head all the way to the address of the last Node ( $m \geq 1$ ); setting its next pointer to our new Node. Each successive push to the list will require an additional iteration because  $m$  has increased by 1 in the previous push operation. This is formally defined in a mathematical definition below:

$$\begin{aligned}
 &= (m+1) + (m+2) + \dots + (m+n-1) + (m+n) \\
 &= m * n + \frac{(1+n)n}{2} \\
 &= m * n + \frac{n}{2} + \frac{n^2}{2} \\
 &= n + \frac{n}{2} + \frac{n^2}{2}
 \end{aligned}$$

**Therefore:** Pushing  $n$  elements has an overall time complexity of  $O(n^2)$

## 1.2 Analyze the total time required to pop those $n$ elements from the Stack and express it using the Big O notation

Similarly to the execution of `push`, the `pop()` method requires us to traverse through the linked list in order to remove the "top" element. In other words, the time complexity of `pop()` is

$O(n)$  as it is also dependent on the number of elements in the Stack.

In the scenario of popping our  $n$  elements from an arbitrary  $m$  sized linked list, we need to traverse from head to the last Node in the linked list.

Iterations	Elements of a linked list	The element to be remove
0	m	n
m	m-1	n-1
m-1	m-2	n-2
m-2	m-3	n-3
.....	.....	.....
2	1	1
1	0	0(this case m = n)

To remove the first element, we need to loop  $m$  times to reach the last node from "top" and remove it. To remove the second element, we need to loop  $m - 1$  times to reach our new "top" element. This pattern continues until the linked list as only one element remaining in the linked list as defined in the mathematical definition below:

$$\begin{aligned}
 &= (m - 1) + (m - 2) + (m - 3) + \dots + (m - n + 1) + (m - n - 1) \\
 &= m * n - (1 + 2 + 3 + \dots + n) \\
 &= n^2 - \frac{(1 + n) * n}{2} \\
 &= n^2 - \frac{n}{2} - \frac{n^2}{2}
 \end{aligned}$$

**Therefore:** Popping  $n$  elements has an overall time complexity of  $O(n^2)$