Assignment 2

1A.) Let $x_1 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$ and $x_2 = \begin{bmatrix} 8 \\ 13 \end{bmatrix}$ be vectors representing a rectangle's terminal points. I have elected to apply a stretch in the x-direction by 5, and a reflection about the y-axis (A_2). These geometric transformations can be represented by the following matrices respectively:

$$A_1 = \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Applying our transformation matrices, we get the results in Figure 1 where black is our original rectangle, red is our x-direction stretch, and magenta is our y-axis reflection.

1B.) If we let V be the matrix operator $V = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$ and S be the matrix operator $S = \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}$, then we can see that the composition of V and S does not commute. Let us consider the unit square with corners at $(1, 1), (2, 1), (2, 2), (1, 2)$. Applying the composition of V and S in both orders, Figure 2 shows that the resulting shape depends on the order in which the operations are applied as evident by the red and blue shapes.

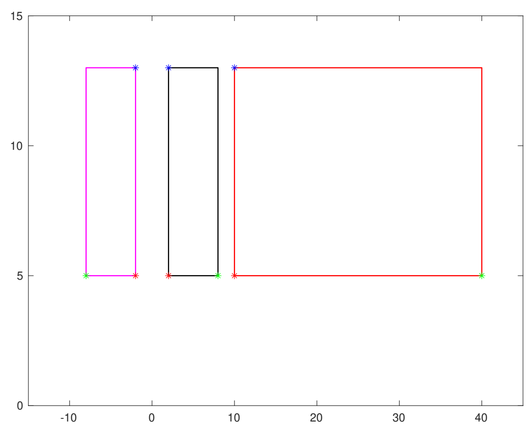

Figure 1: Rectangles in 1A

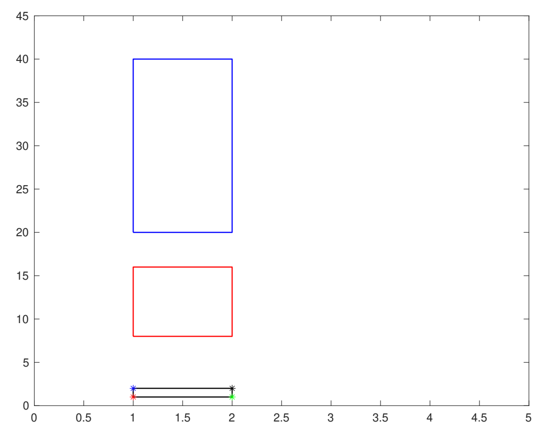

Figure 2: Transformations in 1B

This is corroborated by Theorem 3.2.5 of the text which states that the composition of two linear transformation is given by the matrix product of the two transformations. As seen in the following result, the composition of V and S is not equal to the composition of S and V. Therefore, it is not commutative.

$$[V][S] = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix} \neq \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} = [S][V]$$

$$\begin{bmatrix} 1 & 0 \\ 3 & 5 \end{bmatrix} \neq \begin{bmatrix} 1 & 0 \\ 15 & 5 \end{bmatrix}$$

2) To transform the black rectangle into the red rectangle, we need to apply a rotation R of 180 degrees followed by a horizontal stretch S with a factor of 7. Taking the composition of these two transformations, we obtain the following transformation matrix A:

$$A = [R \circ S] = \begin{bmatrix} cos(\pi) & -sin(\pi) \\ sin(\pi) & cos(\pi) \end{bmatrix} \begin{bmatrix} 7 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 0 \\ 0 & -1 \end{bmatrix}.$$

I also experimentally verified the correctness of transformation matrix A by plotting the black rectangles, multiplying the vertices, and plotting them to ensure that they match the red rectangle plot.