Math 232 – Computing Assignment 1

Due Date: October 14th, at 10:55pm.

You must upload to Crowdmark both your code (i.e., the Matlab or Python or other programming app commands you used) and your report (i.e, answers to the problems here). The assignment is due at 10:55pm. I have set the due time in Crowdmark to 11:00pm and if Crowdmark indicates that you submitted late, you will be given 0 on the assignment.

- Please read the **Guidelines for Computing Assignments in Canvas** first.
- Keep in mind that Canvas discussions are open forums.
- Acknowledge any collaborations and assistance from colleagues/TAs/instructor.

Programming Preamble:

Matlab: `R=rand(5,7)` produces a 5x7 matrix with random entries.

Matlab: `A'` produces A transpose. Changes column vectors to row vectors.

Matlab: `rref(R)`produces the reduced row echelon form of the matrix.

Matlab: `cat(2,A,B)` concatenation of A and B (use to produce an augmented matrix).

python:
```
import numpy
R=numpy.random.rand(5,7)
```

Computing Assignment

Required submission: 1 page PDF document with your answers to the problems here, and 1 page PDF document with your Matlab or Python code, both uploaded to Crowdmark (so, upload 2 pages).

1. **Part 1 - Solutions of systems of linear equations**
   - By considering random matrices of appropriate sizes, find "emperical evidence" that substantiate the following statements. (In this part, m and n are both integers and both are greater than 6, so 7 or larger).
     (a) A system of n linear equations in n unknowns typically has a unique solution.
     (b) A system of m linear equations in n unknowns, where $m > n$, typically has no solution.
     (c) A system of m linear equations in n unknowns, where $m < n$, typically has many solutions.
   - Give examples of exceptions for each case (but for this part, you can use integers m and n that need only be larger than 2, so 3 or larger).

2. **Part 2 - Linear independence, Intersection of subspaces**

- Consider the set of vectors in $\mathbb{R}^5$;

$$B_1 \quad = \quad \{w_1, w_2, w_3, w_4, w_5, w_6\}$$

  where

$$w_1 = \begin{bmatrix} -1 \\ 1 \\ 2 \\ 4 \\ 1 \end{bmatrix}, \quad w_2 = \begin{bmatrix} -1 \\ 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \quad w_3 = \begin{bmatrix} 3 \\ 1 \\ -1 \\ 2 \\ 0 \end{bmatrix}, \quad w_4 = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 3 \\ -1 \end{bmatrix}, \quad w_5 = \begin{bmatrix} 5 \\ 4 \\ 1 \\ 11 \\ 3 \end{bmatrix}, \quad w_6 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 2 \\ 1 \end{bmatrix}$$

  Show B_1 is a linearly <u>dependent</u> set. Then, demonstrate the conclusion of Theorem 1.2.2: Find a maximal linearly <u>independent</u> set B_1' of vectors from B_1, and show that the vectors from B_1 that are NOT in B_1' set are contained in the span of B_1' (and hence, that span $B_1 =$ span B_1').
  What is the dimension of span B_1?

- Consider the set

$$B_2 \quad = \quad \{z_1, z_2, z_3, z_4, z_5\}$$

  where

$$z_1 = \begin{bmatrix} 5 \\ 2 \\ 1 \\ 7 \\ 1 \end{bmatrix}, \quad z_2 = \begin{bmatrix} 2 \\ -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad z_3 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \quad z_4 = \begin{bmatrix} 2 \\ -4 \\ -2 \\ 4 \\ 1 \end{bmatrix}, \quad z_5 = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ -1 \end{bmatrix}$$

- Find <u>all</u> the vectors in the intersection span $B_1 \cap$ span B_2. Show that this is a subspace.