```
% Consider the set of vectors in R5: B1 = {w1, w2, w3, w4, w5, w6} where
w1 = [-1; 1; 2; 4; 1];
w2 = [-1; 1; 2; 1; 1];
w3 = [3; 1; -1; 2; 0];
w4 = [2; 1; 0; 3; -1];
w5 = [5; 4; 1; 11; 3];
w6 = [1; 0; -1; 2; 1];

% Show B1 is a linearly dependent set.
% The set of vectors in R5: B1 is linearly dependent if there is
% a non-trivial solution to c1w1 + c2w2 + c3w3 + c4w4 + c5w5 + c6w6 = 0

% Create a system of equations using the vectors in B1
B1 = [w1(:), w2(:), w3(:), w4(:), w5(:), w6(:)];
zeroVectorR5 = [0; 0; 0; 0; 0];

% Create an augmented matrix wih the zero vector
B1Aug = cat(2, B1, zeroVectorR5);
% Row reduce B1Aug into RREF
B1RREF = rref(B1Aug)

% The system of equations is not linearly dependent as there is a non-zero
% solution to the system. The columns without leading 1's are linearly
% dependent
sprintf('As seen in the RREF of B1RREF, the system is consistent with a non-
zero solution');
sprintf('There is no leading variables for x5 and x6, they are free
 variables');

% Find a maximal linearly independent set B#1 of vectors from B1
sprintf('The maximal linearly independent set B''1 of vectors is {w1, w2, w3,
 w4}')

% For testing purposes only... making sure that there is a unique solution
B1_maximal = [w1(:), w2(:), w3(:), w4(:)];
B1_maximal_rref = rref(B1_maximal);

% Show that vectors from B1 that are NOT in B#1 set are contained in the span
 of B#1 (and hence, that span B1 = span B#1).
% I am using consistency in w5 = {w1, w2, w3, w4} and w6 = {w1, w2, w3, w4}
B1_w5_Aug = cat(2, [w1(:), w2(:), w3(:), w4(:)], w5);
B1_w5_Rref = rref(B1_w5_Aug);

B1_w6_Aug = cat(2, [w1(:), w2(:), w3(:), w4(:)], w6);
B1_w6_Aug_Rref = rref(B1_w6_Aug);
% Clearly consistent

% What is the dimension of span B1?
sprintf(['The dimension of the span is equal to how many independent vectors
 there are in Span B''1. In this case' ...
    'there are obviously 4 independent vectors. Thus the dimension is 4.'])
```

```
% Consider the set: B2 = {z1, z2, z3, z4, z5} where
z1 = [5; 2; 1; 7; 1];
z2 = [2; -1; 0; 0; 1];
z3 = [1; 2; 1; 1; 0];
z4 = [2; -4; -2; 4; 1];
z5 = [0; 1; 2; 3; -1];

% Create a system of equations using the vectors in B2
B2 = [z1(:), z2(:), z3(:), z4(:), z5(:)];

% Create an augmented matrix wih the zero vector
B2Aug = cat(2, B2, zeroVectorR5);
% Row reduce B1Aug into RREF
B2RREF = rref(B2Aug)
sprintf('As seen in B2RREF, the system is consistent with a non-zero
 solution');
sprintf('There is no leading variables for x4, it is a free variable');

% Find a maximal linearly independent set B#2 of vectors from B2
sprintf('The maximal linearly independent set B''1 of vectors is {z1, z2, z3,
 z5}')

% For testing purposes only... making sure that there is a unique solution
B2_maximal = [z1(:), z2(:), z3(:), z5(:)];
B2_maximal_rref = rref(B2_maximal);

% Find all the vectors in the intersection span B1 INTERSECTION span B2.
% Find the maximal linearly independent set for both!
% To find the intersection of the span, find the vectors in both... Set the
% linearly combination of both spans equal to each other.
B1B2_Maximal_System = [w1(:), w2(:), w3(:) w4(:), -z1(:), -z2(:), -z3(:), -
z5(:)];
B1B2_Aug = cat(2, B1B2_Maximal_System, zeroVectorR5);
B1B2_Rref = rref(B1B2_Aug)

% Show that this is a subspace of R5.
% By definition, any span is a subspace. In addition, the intersection of
% two spans will also be a subspace. However, to be more verbose, a
% subspace is non-empty, closed under addition. and closed under scalar
% multiplcation. It obviously contains the non-zero vector. (Say for marks)

% For

% For closed under addition, if we let w1 and w2 be some vectors in B1 and
% B2, then that means that

%
% Show w1 + w2 is in the span of B1 and B2. if w1 and w2 are
% in span(B2). That implies that both w1 and w2 are in the intereciton of
% B2. that implies that w1 and w2 are in the intersection.
% Similarly for scalar multiplication
```

B1RREF =

```
     1     0     0     0     2     1     0
     0     1     0     0     0    -1     0
     0     0     1     0     3     1     0
     0     0     0     1    -1    -1     0
     0     0     0     0     0     0     0


ans =

    'The maximal linearly independent set B'1 of vectors is {w1, w2, w3, w4}'


ans =

    'The dimension of the span is equal to how many independent vectors there
 are in Span B'1. In this casethere are obviously 4 independent vectors. Thus
 the dimension is 4.'


B2RREF =

     1     0     0     1     0     0
     0     1     0     0     0     0
     0     0     1    -3     0     0
     0     0     0     0     1     0
     0     0     0     0     0     0


ans =

    'The maximal linearly independent set B'1 of vectors is {z1, z2, z3, z5}'


B1B2_Rref =

  Columns 1 through 7

    1.0000         0         0         0         0    0.7778    0.4444
         0    1.0000         0         0         0    0.5556   -1.6111
         0         0    1.0000         0         0    1.3333   -0.9167
         0         0         0    1.0000         0    1.0000   -0.7500
         0         0         0         0    1.0000    1.3333   -0.4167

  Columns 8 through 9

    0.2222         0
   -0.5556         0
    1.1667         0
   -1.5000         0
    0.1667         0
```

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