Assignment 1

1a.) I generated 100 random $n \times n$ sized matrices with $8 \le n \le 20$ of real numbers; checking whether their RREF's was equal to the n identity matrix. Of 100 test cases, 100/100 evaluated true with an unique solution.

$$\text{Exception: } \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 2 & 3 & 2 \\ 1 & 2 & 3 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

1.b) Similar methodology from Part(A) with the following test for inconsistency: If the rank of the coefficient matrix is less than the rank of the augmented matrix, the system of equations is inconsistent. Of 100 test cases, 100/100 evaluated true with no solutions.

$$\text{Exception: } \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 6 & 2 \\ 3 & 3 & 4 & 3 \\ 10 & 10 & 12 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 2.5 \\ 0 & 1 & 0 & -7.5 \\ 0 & 0 & 1 & 4.5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

1.c) Similar methodology from Part(B) with the following test for many solutions: If the system was consistent, then I verified that $n-$ rank(coefficient) > 0 which meant many solutions exist. Of 100 test cases, 100/100 evaluated true with many solutions.

$$\text{Exception: } \begin{bmatrix} 1 & 2 & 3 & 4 & 1 \\ 5 & 6 & 7 & 8 & 2 \\ 9 & 10 & 11 & 12 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -1 & -2 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

2.a) **Show B_1 is a linearly dependent set and Theorem 1.2.2:** The set of vectors in B_1 is linearly dependent if there is a non-trivial solution to $c_1w_1 + c_2w_2 + c_3w_3 + c_4w_4 + c_5w_5 + c_6w_6 = 0$. I expressed these equations in c_is into an augmented matrix form and solved it. The resulting RREF indicates that w_5 and w_6 are linearly dependent as leading 1's do not exist for those columns.

$$B_1RREF: \begin{bmatrix} 1 & 0 & 0 & 0 & 2 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2.b) **Find a maximal linearly independent set B_1 of vectors from B_1, and show that vectors from B_1 that are NOT in B_1 set are contained in the span of B_1 (and hence, that span B_1 = span B_1).**

Theorem 1.2.2 tells us that a set is linearly independent IFF none of the vectors in B can be written as a linear combination of the others. Using MATLAB, I checked that the system of $c_1w_1 + c_2w_2 + c_3w_3 + c_4w_4 = w_5$ OR w_6 is consistent. Therefore, we can conclude that this is the maximal independent set $B_1' = \{w_1, w_2, w_3, w_4\}$

2.c) **What is the dimension of Span B_1?** The dimension of the Span B_1 is the number of vectors in the maximal linearly independent set B_1'. In this case, there are 4 independent vectors in B_1' and the dimension is 4.

2.d) **Find all the vectors in the intersection Span B1 $\cap$ Span B2.** I subtracted the first equation from the second equation and concatenated them into an augmented matrix. After doing a row reduction, I was able to determine that the system was consistent. Now, substituting the RREF values back into B_1', I was able to get the following general form of the intersection:

$$B_1' \cap B_2' = s\begin{bmatrix} -7/9 \\ -5/9 \\ -4/3 \\ -1 \\ -4/3 \end{bmatrix} + t\begin{bmatrix} -4/9 \\ 29/18 \\ 11/12 \\ 3/4 \\ 5/12 \end{bmatrix} + p\begin{bmatrix} -2/9 \\ 5/9 \\ -7/6 \\ 3/2 \\ -1/6 \end{bmatrix}$$

2.e) **Find the dimension of the intersection and show that this is a subspace of R5.** The dimension of the intersection is 3. The maximal linearly independent set of vectors B_1' and B_2' are both subsets of the spans B_1 and B_2. In other words, the spans of $B_1 \cap B_2 = B_1' \cap B_2'$. By definition, spans are all linear combinations of a set of vectors and constitute a subspace because they contain the zero-vector, closed under addition, and closed under scalar multiplication. Because the intersection of $B_1' \cap B_2'$ is a span of vectors, it is a subspace.