South China University of Technology

The Experiment Report of Machine Learning

SCHOOL: SCHOOL OF SOFTWARE ENGINEERING

SUBJECT: SOFTWARE ENGINEERING

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Linear Regression, Linear Classification and Gradient Descent

Abstract— In this experiment, we aim to further understand the linear regression and gradient descent. We conduct some experiments under small scale data sets, where 'Housing' is used for linear regression and gradient descent and 'australian' is used for linear classification and gradient descent. The best results of linear regression shows that validation loss is near to 11.3 while the learning rate is 0.2. The best results of linear classification shows that validation loss is near to 0.53 and its accuracy reaches 0.85 while the learning rate is 0.21 and regularization parameter is 0.5.

I. Introduction

The motivation of the experiment is further understanding of linear regression, linear classification and gradient descent. Linear regression uses 'Housing' in LIBSVM Data, including 506 samples and each sample has 13 features. Linear classification uses 'australian' in LIBSVM Data, including 690 samples and each sample has 14 features.

All the experiment steps is as follows. After downloading, we divided data sets into training set, validation set firstly. Secondly we initialize linear model parameters. After that we set all parameter into zero and initialize it with normal distribution. Thirdly, we defined the loss function of the linear regression to be least squared loss, and defined the loss function of the linear classification to be hinge loss. Fourthly compute the gradient of the loss function with respect to the weight W and bias b. Fifthly update the parameters W and b.
Then repeat above steps for several times until convergence.

While doing experiment, we could realize the process of optimization and adjust parameters to find the best case.

II. Methods and Theory

A)Linear regression and gradient descent

We defined the loss function of the linear regression to be least squared loss:

$$L = \frac{1}{2n}\sum_{i=1}^{n}\left(y_i - W^T x_i\right)^2$$

The gradient of the loss function is:

$$G = \frac{1}{n}\sum_{i=1}^{n}(-x_i) * \left(y_i - W^T x_i\right)$$

Update the parameter W use:

$$W = W - \eta G$$

Where η is the pre-defined learning rate.

B)Linear classification and gradient descent

We defined the loss function of the linear classification to be Hinge loss:

$$L = \frac{\lambda}{2}||W||^2 + \frac{1}{n}\sum_{i=1}^{n} max\left(0, 1 - y_i(W^T x_i + b)\right)$$

The gradient of the loss function is:

$$G_W = \begin{cases} \lambda W & y_i(W^T x_i + b) \geq 1 \\ \lambda W + \frac{1}{n}\sum_{i=1}^{n} -y_i x_i & y_i(W^T x_i + b) < 1 \end{cases}$$

$$G_b = \begin{cases} 0 & y_i(W^T x_i + b) \geq 1 \\ \frac{1}{n}\sum_{i=1}^{n} -y_i & y_i(W^T x_i + b) < 1 \end{cases}$$

Update the parameters W and b:

$$W = W - \eta G_W$$
$$b = b - \eta G_b$$

Where η is the learning rate, λ is the regularization parameter.

III. Experiment

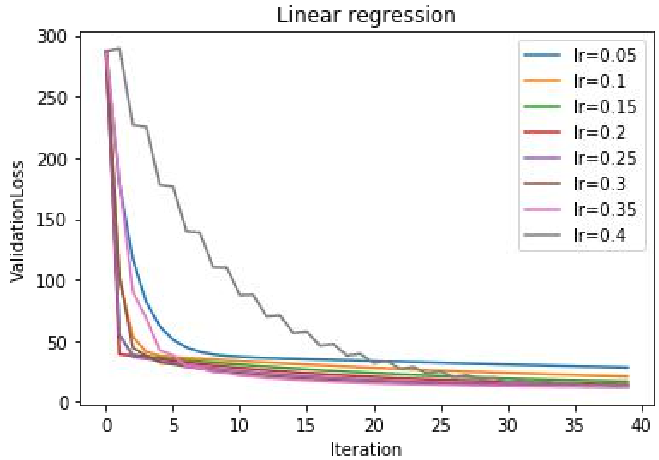
The code is as follows.

A)Linear regression and gradient descent

```
from sklearn.externals.joblib import Memory
from sklearn.datasets import load_svmlight_file
from sklearn.datasets import load_svmlight_file
mem = Memory("./mycache")
```

```
@mem.cache
#load data
def get_data():
    data = load_svmlight_file("housing_scale.txt")
    return data[0], data[1]

X, y = get_data()
X = X.toarray()

#X = [X,1]
import numpy as np
addone= np.ones(X.shape[0])
X= np.column_stack((X,addone))

#divide data to traning part and validation part
from sklearn.model_selection import train_test_split
from numpy import random
X_train, X_validation, y_train, y_validation = train_test_split(X, y, test_size=0.22, random_state=25)
In [251]:
N = X_train.shape[1]
W_zeros = np.zeros(N)
W_random = random.random(size=N)
#use NumPy random.normal fuction to get datas in normal distribution
W_normal = np.random.normal(size=N)
In [252]:
#Choose Least squared loss function
def cal_Loss(X,W,y):
    preY = np.dot(X,W)
    diifY = y - preY
    Loss = np.dot(diifY,diifY.T)/(2 * X.shape[0])
    return Loss
#Calculate the gradient
def cal_G(X,W,y):
    preY = np.dot(X,W)
    diifY = y - preY
    G = - np.dot(diifY,X)/ X.shape[0]
    return G

def draw_plot(Loss_train,Loss_validation):
    plt.plot(Loss_train,label="Loss_train")
plt.plot(Loss_validation,label="Loss_validation")
    plt.legend()
    plt.xlabel("Iteration")
    plt.ylabel("Loss")
    plt.title("Linear regression")
    plt.show()
In [253]:
lr = 0.15
iteration = 100
W = W_normal
Loss_train = np.zeros(iteration)
Loss_validation = np.zeros(iteration)
for j in range(0,iteration):
    #the training loss
    Loss_train[j] = cal_Loss(X_train,W,y_train)
    #the gradient of the loss function
    G = cal_G(X_train,W,y_train)
    #the validation loss
    Loss_validation[j] = cal_Loss(X_validation,W,y_validation)
    #update the parameter W
    W = W - G * lr
#draw the result
draw_plot(Loss_train,Loss_validation)
```

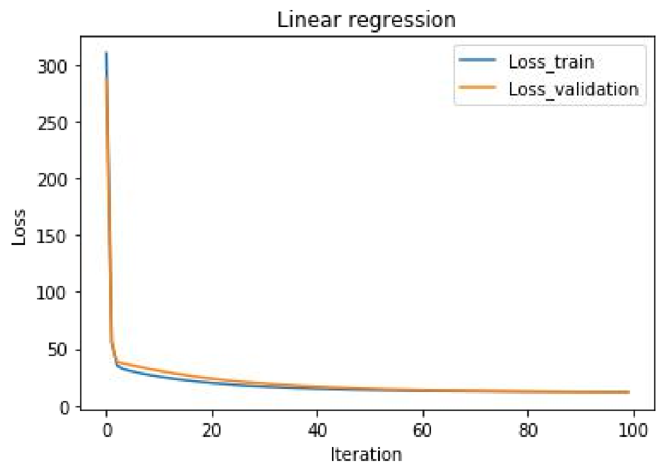
We chose 8 different learning rates [0.05, 0.1, ..., 0.4] to optimize the linear regression algorithm, the results are as follows:



If the learning rate is too small, the decline of the curve is slow and the number of the iteration to reach convergence will be large.If the learning rate is too large, the loss curve will be oscillating.

The best learning rate is 0.2, which is illustrated as the red line.

The best result is shown as follows, with 11.29208 as the validation loss.

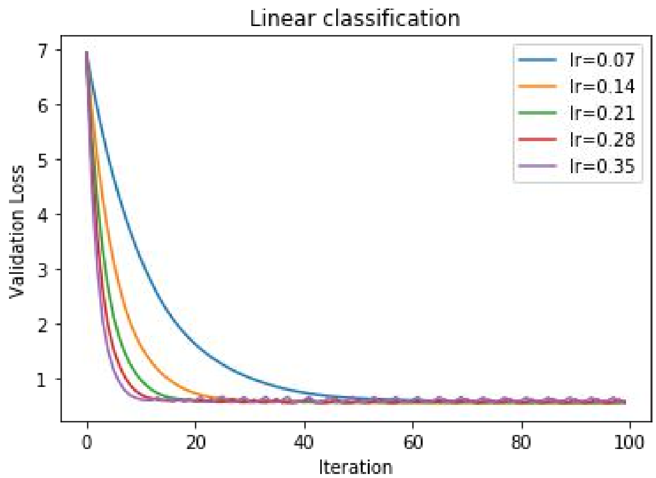


B)Linear classification and gradient descent

```
import numpy as np
import matplotlib.pyplot as plt
from numpy import random
from sklearn.externals.joblib import Memory
from sklearn.datasets import load_svmlight_file
from sklearn.model_selection import
train_test_split
mem = Memory("./mycache2")

@mem.cache
#load data
def get_data():
    data =
load_svmlight_file("australian_scale.txt")
    return data[0], data[1]

X, y = get_data()
X = X.toarray()
#X = [X,1]
addone= np.ones(X.shape[0])
X= np.column_stack((X,addone))
In [154]:
#divide data to traning part and validation part
from sklearn.model_selection import
train_test_split
from numpy import random
X_train, X_validation, y_train, y_validation =
train_test_split(X, y, test_size=0.25,
random_state=25)
In [155]:
N=X_train.shape[1]
#Normal distribution initialized
W_nor = np.random.normal(size=N)
print(W.shape)
print(W)
#calculate the loss
def cal_Loss(X,W,y,lambdal,W_0):
    preY = np.dot(X,W)
    diifY = np.ones(y.shape[0]) - y * preY
    diifY[diifY < 0] =0
    Loss =np.sum(diifY) / X.shape[0] +
np.dot(W_0,W_0.T)/2*lambdal
    return Loss

#calculate the gradient
def cal_G(X,W,y,lambdal,W_0):
    preY = np.dot(X,W)
    diifY = np.ones(y.shape[0]) - y * preY
    y_get = y.copy()
    y_get[diifY <= 0] =0
    G = -np.dot(y_get,X) / X.shape[0] + W_0
*lambdal
    return G
```

```
#calculate the accuracy
def cal_Accuracy(X,W,y):
    preY = np.dot(X,W)
    count = np.sum(preY * y >0)
    Accuracy = count / X.shape[0]
    return Accuracy

def
draw_plot(Loss_train,Loss_validation,Accuracy):
    fig = plt.figure()
    ax1 = fig.add_subplot(111)
    ax1.plot(Loss_train,label="Loss_train")

ax1.plot(Loss_validation,label="Loss_validation")
    ax1.set_ylabel("Loss")
    ax1.set_xlabel("Iteration")
    ax1.legend()
    ax2 = ax1.twinx()
    ax2.plot(Accuracy,label="Accuracy",color =
'r')
    ax2.legend()
    ax2.set_ylabel("Accuracy")
    ax2.set_title("Linear classification")
    plt.show()

plt.close()
lr = 0.21
lambdal = 0.5
iteration = 400
#get different kinds of initial data
（W_zeros,W_random or W_normal）
W = W_nor
Loss_train = np.zeros(iteration)
Loss_validation = np.zeros(iteration)
Accuracy = np.zeros(iteration)
for j in range(0,iteration):
    W_0 = W.copy()
    W_0[N-1]= 0
    #the training loss
    Loss_train[j] =
cal_Loss(X_train,W,y_train,lambdal,W_0)
    #the gradient of the loss function
    G = cal_G(X_train,W,y_train,lambdal,W_0)
    #the validation loss
    Loss_validation[j] =
cal_Loss(X_validation,W,y_validation,lambdal,W_0
)
    #accuracy
    Accuracy[j] =
cal_Accuracy(X_validation,W,y_validation)
    #update the parameter W
    W = W - G * lr
#draw the result
draw_plot(Loss_train,Loss_validation,Accuracy)
plt.close()
```

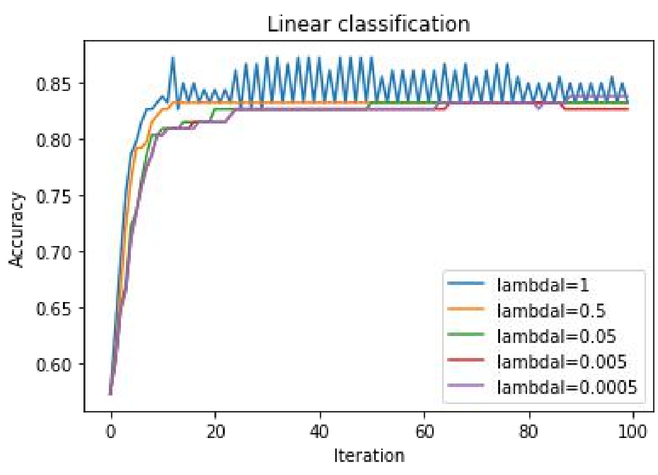
We chose 5 different learning rates [0.07, 0.14, ..., 0.35] to optimize the linear regression algorithm with specific regularization parameter 0.5, the results are shown as follows:



If the learning rate is too small, the decline of the curve is slow and the number of the iteration to reach convergence will be large.If the learning rate is too large, the loss curve will be oscillating.

The best learning rate is 0.21, which is illustrated as the green line.

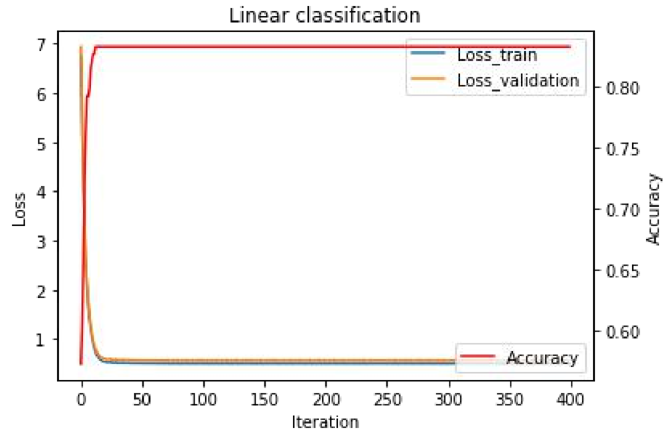
We chose 5 different regularization parameter [1, 0.5, 0.05, 0.005, 0.0005] to optimize the linear regression algorithm, the results are shown as follows:



If the regularization parameter is too small, the model might be over-fitting.If the regularization parameter is too large, the model may be under-fitting.

The best regularization parameter is 0.5, which is illustrated as the yellow line.

The best result is shown as follows, with 0.527841 as the validation loss and 0.84826 as the accuracy.



IV. CONCLUSION

There are two kinds of loss function for the linear regression and classification. Linear regression uses least squared loss, while linear classification updates the parameters by Hinge loss.

For the linear regression, we learn a model to simulate the mapping between input X and output y. We compare the validation loss with the train loss to evaluate the model. We find that the best learning rate is 0.2 where the decline of curve is quickly without oscillating.

For the linear classification task, we find a hyper plane to separate the different target. We evaluate the model by calculating the accuracy and comparing the validation loss with the train loss. We find the best learning rate is 0.21 where the decline of curve is quickly without oscillation. And we also find that the best regularization parameter is 0.5 where the rise of curve is quickly without oscillation.