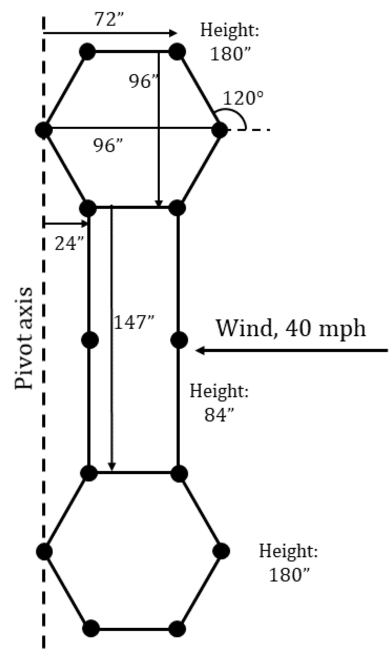
Structural Integrity Analysis
Next Big Thing 2020

Overturning Analysis

In this section, we will show that the design of the structure allows it to withstand a 40 mph gust of wind with a safety factor of 1.63, greater than the required 1.5.

1. Torque provided by wind

As a worst-case-scenario for the torque provided on wind by the structure, consider a 40 mph wind parallel from the ground, directed on the wide side of the structure, as shown in the diagram below. The black circles represent foundation holes.



To calculate the wind torque on a piece of the structure, consider slices of area with width w and height dh. The force provided by the wind will be

$$dF = \frac{1}{2} C_d v^2 \rho w dh$$

and the torque will be $d\tau = h dF$. The total torque will therefore be

$$\tau = \int_0^H \frac{1}{2} C_d v^2 \rho w h dh = \frac{1}{4} C_d v^2 \rho w H^2$$

where C_d is the coefficient of drag of the surface of the structure, w and H are the width and height respectively, $v = 17.9$ m/s is the speed of the wind, and $\rho = 1.23$ kg/m^3 is the density of air at sea level.

We break the structure into two parts: the middle and the towers. The middle section has height $H = 2.1336$ m and width $w = 3.7338$ m. It consists of a flat plate, which has $C_d = 1.3$. Hence, the torque on the middle part is $\tau = 2,177$ Nm.

The height and width of the towers are $H = 4.57$ m and $w = 2.44$ m. The coefficient of drag for an angled plate like those that make up the tower is $C_d = 1.3 \sin\alpha$, where $\alpha = 120^\circ$ is the angle from the plate to the wind. Hence, $C_d = 1.13$, and the total torque on one tower is $\tau = 5,652.5$ Nm. Adding the torques from each tower and the middle section gives

$$\tau_{wind} = 13,400 \text{ Nm}.$$

2. Torque provided by gravity

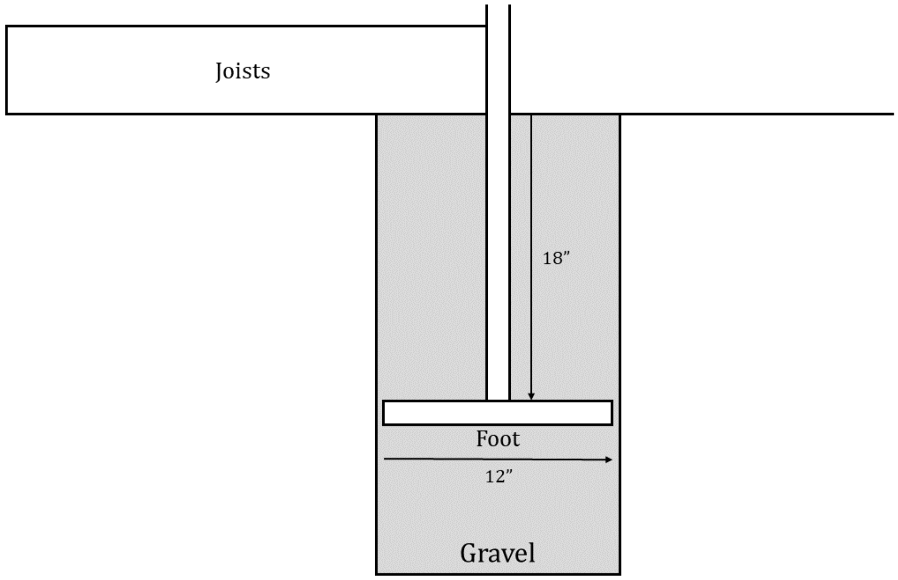
The counteracting torques are provided by the gravity of the structure and the foundation holding the structure to the ground. The torque provided by gravity is simple to calculate. The diagram above shows that the pivot axis of the structure is 4 feet away from the center, so the force of gravity has a lever arm of $L = 1.2192$ m which is 4 feet. Multiplying by the weight of the structure will give us the torque of gravity. The structure is built of 78 2x4x8s, 22 2x4x12s, 24 2x4x16s, 40 2x6x8s, and 24 pieces of 0.75” 4x8 foot OSB. It also contains screws, paint, activities, and potentially people, all of which we will neglect as a worst-case scenario. This amounts to 1272 feet of 2x4 and 320 feet of 2x6, or 111,816 cubic inches of wood, which is 1.83 cubic meters. Spruce has a density of 450 kg/m^3, so the lumber masses at 824 kg. We also have 82,944 cubic inches of OSB which has a density of 600 kg/m^3, massing at 816 kg. Thus, the total weight is 16,100 N which contributes a torque of

$$\tau_{gravity} = 19,600 \text{ Nm}.$$

It is reasonable to use the finished mass of the structure instead of a partially-built mass because wind will be a concern most when the siding is installed, which is the last step in the building process. Until it is installed, the foundation will protect the structure from gusts.

3. Torque provided by the foundation

The foundation consists of fourteen 2x4s attached firmly to the structure which are buried 18 inches deep in gravel as shown below. They are screwed into a 12” foot, also made of 2x4.



In order for the structure to tip, the foundations must either snap at the point where they meet the structure or be pried out of the ground. The seven foundations closer to the pivot axes are more likely to snap, whereas the seven farther away are merely experiencing an upward force. Consider these farther foundations only. They can fail in three ways:

1. The whole foundation can come out of the hole, pushing the gravel aside.
2. They can shear off the structure
3. The foot can come off the vertical piece inside the gravel, allowing the vertical piece to slide out easily

Consider the first failure case. If the whole structure emerges from the hole, it will have to push all the gravel on top of the foot aside. Gravel interlocks with itself, which is why we are using it. But for simplicity, we will assume it does not, and therefore the foot only needs to lift the gravel directly above it. The volume of this gravel is the area of the foot (3.5 inches times 10.5 inches) times 18 inches, which is 0.0108 cubic meters. The density of gravel is about 1346 kg/m^3, and therefore the gravel weighs 143 N. Applying this force upward will pull the foundation out of the ground. This force is minuscule compared to the force that even one screw can withstand, so we will consider this failure case only.

There are two foundation pieces that are 8 feet away from the pivot line, and they can provide a torque of 2 times 8 feet times 143 N, which is 698 Nm. There are also five which are 6 feet away from the pivot line, which provide a torque of 1571 Nm. Altogether, this is a torque of

$$\tau_{foundation} = 2,270 \text{ Nm}$$

to resist the wind. This still doesn't take the near-side foundation into account, nor the packing of the gravel, so it is a significant underestimate.

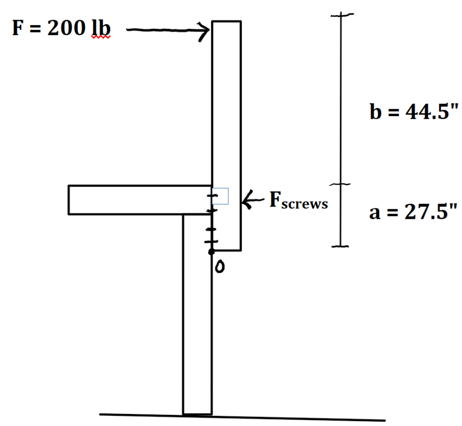
4. Conclusion & safety factor

We have found that the wind can produce a torque of 13,400 Nm and that gravity and the foundation can resist it with a combined torque of 21,870 Nm. This gives us a safety factor of 1.63, greater than the safety factor of 1.5 that was required. Wind gusts of 40 mph are therefore not threatening to the structure.

Guardrail Analysis

In this section, we will demonstrate that any individual beam on the guardrail is able to withstand a 200 lb concentrated load simply by means of attachment to the ground frame. This is an analysis of a worst case scenario -- in actuality, the guardrail beams will be supported by other guardrail beams, connected by horizontal cross bars. If an individual beam is able to support 200 lbs of concentrated load, the whole structure will be able to support more than this.

FBD of one beam and fastening mechanism:



We will start by calculating writing down the sum of moments about point O indicated on the above diagram, in order to solve for the forces needed to be exerted by the screws (assuming they will all be placed in the location indicated by F_{screws}).

Sum of Moments:

$$\sum_{M_O} = \; -F(a+b) + F_{screws}(a) \; = \; 0$$

We can then use this equation to calculate the force required to be exerted by the screws to maintain equilibrium when the concentrated load is applied.

$$F_{screws} = F(a+b)\,/\,a$$

This can then be easily calculated, by substituting the values. For reference, here are the values of F, a, and b in metric units: F = 889.64 N, a = 0.6985 m, b = 1.13 m.

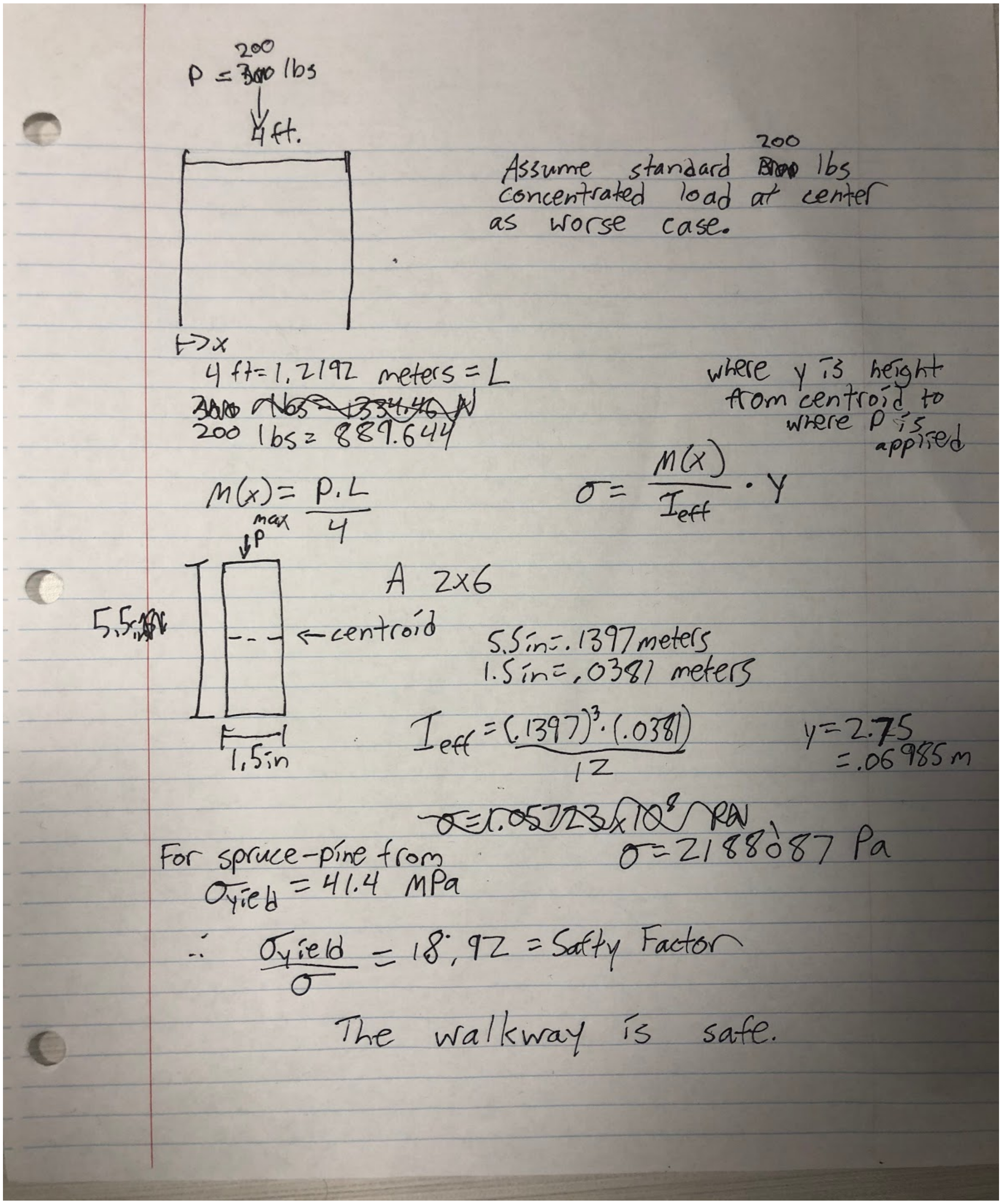
$$F_{screws} = (889.64\ N)(0.6985\ m + 1.13\ m)/(0.6985\ m) = 2328.9\ N$$

This value for F_{screws} is easily achievable, using about 3 screws. According to our estimates*, 1 wood screw should be able to provide around 996.4 N of connection capacity. If this assumption is valid then 3 screws should provide $996.4(3) = 2989.2\ N$ of connection capacity, which is more than enough to counteract the concentrated load.

*According to the American Wood Council connection capacity online calculator (https://www.awc.org/codes-standards/calculators-software/connectioncalc). We estimated this value inputting the following parameters into the online calculator: LRFD model, withdrawal loading, wood screws, Sitka Spruce members, 2.5 in long screws, main member = 1.5 in, side member = 1in, screw number = 8, other settings default.

Roof Floor Beam Calculation

In this section, we will demonstrate that the flooring on the platform of the structure connecting the two towers can support a concentrated load of 200 lbs. The platform contains joists with a horizontal span of 4 ft with 1 ft spacing.



$P = \cancel{300}\ 200$ lbs

4 ft.

Assume standard ~~300~~ 200 lbs concentrated load at center as worse case.

$\mapsto x$

4 ft = 1.2192 meters = L

~~300 lbs = 1334.46 N~~

200 lbs = 889.644

where y is height from centroid to where P is applied

$$M(x)_{max} = \frac{P \cdot L}{4} \qquad \sigma = \frac{M(x)}{I_{eff}} \cdot y$$

A 2x6

5.5 in

← centroid

5.5 in = .1397 meters

1.5 in = .0381 meters

1.5 in

$$I_{eff} = \frac{(.1397)^3 \cdot (.0381)}{12}$$

y = 2.75 = .06985 m

~~$\sigma = 1.05723 \times 10^8$ Pa~~

$\sigma = 2188087$ Pa

For spruce-pine from

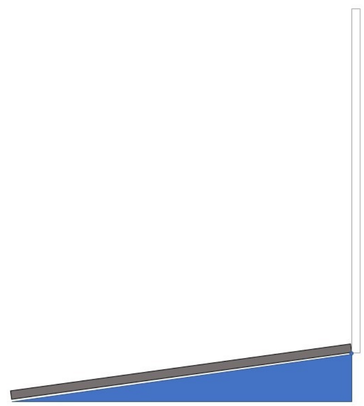
$\sigma_{yield} = 41.4$ MPa

$$\therefore \frac{\sigma_{yield}}{\sigma} = 18.92 = \text{Safty Factor}$$

The walkway is safe.

Drawbridge

The drawbridge will consist of 3’ 7.5” x 6’ OSB that will be attached by hinges inset so that the drawbridge rests on a 2x6 when in the upright as shown below.



When the drawbridge is closed, it will be latched into place to prevent it from falling open. A rope will run from the drawbridge, through holes in the ceiling, and out the back of the structure.

The drawbridge will be opened by two operators slowly lowering the OSB and one in the back of the structure pulling the rope taut. When the drawbridge is down, it will be resting on a ramp as shown above, which will support its weight. The ramp consists of a 2x6 wooden shell and is packed with soil. Therefore, there are no extraneous forces exerted on the structure by the drawbridge.