# Legibility of Robot Motion Through Graph Search

Yunong Hao, Yuxin Sun, and Jordan Triming

***Abstract*— The legibility of motion describes how easy it is for an observer to infer its goal. In this paper we propose a greedy, graph-based optimisation approach for legible robot motion stemming from the formulation of Dragan in an attempt to alleviate the concerns regarding the computational resources required for alternative methods. Our results show that this method can generate improved legible trajectories when compared to other trajectory constructions.**

## I. INTRODUCTION

As developments have progressed in the field of robotics and motion planning, Human Robot Collaboration (HRC) has become an increasingly focal discipline of study. Numerous industries, such as factory automation and automated cleaning, have incorporated robots in such a way that requires interaction with humans. Thus, it has become important that, in such environments, it is ensured that these agents perform in a manner that does not compromise the safety and efficiency of the humans.

One central aspect is the notion of legible motion. When humans are required to interact in an environment wherein robotic agents are also present, it would be beneficial if the humans were able to infer the intentions of the robot. This would help improve safety (humans can avoid the robot and its inferred path); and efficiency (if a robot is performing a task, a human need to complete it).

Recent research has pioneered approaches for generating legible motion of robotic agents in the context of HRC. Many of these approaches include computationally expensive operations which can become infeasible for larger and more complicated problem instances. In this paper, we propose an alternative method for achieving a trajectory with increased legibility. Our approach combines the principles of sampling methods with the notion of graph-based optimisation to construct a legible trajectory. This method allows for the implementation in settings wherin obstacles may be present.

## II. PRIOR WORK

While many researchers have discussed contributing factors of legible motion, the formalisation was introduced by Dragan in his thesis and a series of preceding papers in 2013, 2015. In their paper 'Legibility and Predictability of Robot Motion', Dragan et al. defined legible motion as "motion that enables an observer to quickly and confidently infer the correct goal $G$" [1]. This definition is complemented with a mathematical formulation, for a trajectory $\xi$

$$\text{legibility}(\xi) = \frac{\int P(G^*|\xi_{S\to\xi(t)})f(t)dt}{\int f(t)dt} \tag{1}$$

where

$$P(G^*|\xi_{S\to Q}) \propto \frac{\exp(-C(\xi_{S\to Q}) - C(\xi^*_{Q\to G^*}))}{\exp(-C(\xi^*_{S\to G^*}))}P(G) \tag{2}$$

is the probability of inferring goal $G^*$, given the trajectory $\xi_{S\to Q}$. The authors provide the derivation, based on the assumption that the cost we expect the robot to minimise, $C$, can be approximated as a quadratic, yielding a constant Hessian. $\xi^*$ indicates the optimal trajectory with respect to this cost, that is the one that minimises $C$. $P(G)$ denotes the probability that the agent is attempting to reach that goal. The authors allege that, without prior knowledge, it is safe to assume this is uniform. The function $f(t)$ is used to provide greater weighting to the earlier segments of the trajectory, based on the belief that the earlier the goal can be inferred, the better. As an example, the authors propose the use of

$$f(t) = T - t$$

where $T$ is the total duration of the trajectory.

The authors distinguish legible motion with motion that is predictable, i.e. "motion that matches what an observer would expect, given a goal $G$". The authors assert that, mathematically, predictability and legibility are fundamentally different measures [1]. That is, a trajectory being more legible can mean being less predictable, and vice-versa. To explore this hypothesis, a user study was conducted comparing predictable trajectories with those that are legible. The results not only corroborate the authors claim but show that their described legibility measure correlates with human perceived legible motion. From the results the authors conclude that "the probability of making a correct inference is indeed higher for the legible [according to their measure] trajectory at all times".

A valid mathematical formulation for legibility is powerful in that it provides a score that can be optimised for the purpose of generating legible motion. As outlined by Dragan et al., the problem with this optimisation is that it is non-convex and $C$ may not be known. In the paper 'Generating Legible Motion', Dragan and Srinivasa propose a gradient ascent formulation, which is expanded in Dragan's thesis 'Legible Robot Motion Planning' [2][3].

Dragan et al., Dragan and Srinivasa, and Dragan, in his thesis, observes that increasing legibility will often result in increasingly unpredictable trajectories. Dragan and Srinivasa assert that, in some cases "by optimising the legibility functional, one can become arbitrarily unpredictable", with proof [2]. The authors explain that the model used for inference in which the definition of legibility is derived often does not reflect how humans make inferences in

unpredictable situations. That is, if the legibility measure is increased enough, the motion may become unpredictable and thus make it difficult for humans to correctly infer the goal. The former paper briefly mentions the use of a regularising term to alleviate this problem [1]. The latter two papers, however, extend the previously developed gradient method so that the agents motion remains somewhat predictable [2][3]. This is achieved by introducing a trust region. Only trajectories that are above a certain threshold in predictability, or lie within the trust region, will be considered valid. This is enforced by ensuring that the cost of the trajectory lies below a certain value, $\beta$. Dragans thesis, and his paper with Srinivasa provide an adjusted iterative gradient method to solve this constrained optimisation problem. Upon conducting a user study, Dragan and Srinivasa obtained evidence to support their hypothesis that there exists a specific trust region that within which, legibility optimisation can make trajectories significantly more legible, and that improving the legibility score outside of this region no longer actually improves legibility in practice.
Although the theory is rigorously presented, the authors of these papers provide little insight into practical implementation. Each iteration in the gradient ascent approach requires one to calculate the inverse of a (potentially large) matrix. Additionally, the explicit calculation of both the legibility score and its gradient could become computationally expensive, especially as $T$ increases. In their survey of legibility frameworks, Sebastian et al. described Dragan legibility score as exhibiting a high evaluation cost [4]. Furthermore, in these initial works, there is little discussion on the expected number of iterations of gradient update required before reaching a suitable trajectory.

## III. METHODOLOGY

In this section, we outline our methodology for optimising the legibility of trajectories in the context of motion planning. Building upon the insights gained from the reviewed literature, our research aims to address the challenges associated with optimising legibility while considering computational efficiency.

### A. Sampling

To achieve this, we begin by sampling configurations in the space and then employ the Greedy Search algorithm with a specific heuristic function to connect these configurations, forming an effective path from the initial point to the goal exhibiting high legibility.
We choose to partition the space into equal regions and adjust the number of samplings in this region for the next sampling based on the probability of invalid sampling in that region. Thus, when the total number of samplings is fixed, this method increases the probability of valid data collection, thereby enhancing the likelihood of finding a valid path from the initial configuration to the goal configuration.
This method requires constructing a trajectory from a set of valid configurations. To this end, we incorporate random sampling. We first partition the workspace into N equally sized regions and sample $M_i$ configurations within each region $i$, retaining only valid configuration points. This sampling continues until we have a total of $W$ valid configuration points. It's worth noting that, for a particular region $i$, $M_i$ is adjusted based on the probability of encountering invalid configurations in that region during the previous sampling. Specifically

$$M_i^{(t+1)} = M_i^{(t)} \times d_i^{(t+1)} \tag{3}$$

where

$$d_i^{(t+1)} = \begin{cases} d_i^{(t)} + 0.5 & n_i^{(t)} = 0 \\ \exp(-0.2 n_i^{(t)}) & \text{otherwise} \end{cases} \tag{4}$$

with $n_i^{(t)}$ representing the number of invalid configurations sampled from region $i$ during iteration $t$. Note, that initially, $d_i$ is set to 1.

### B. Greedy Search

Once we complete the configuration space sampling, we employ the Greedy Search algorithm to identify a valid and highly legible trajectory from initial to goal among the collected $W$ configuration points. The greedy search utilises, as a heuristic, the probability of inferring goal $G$ given the trajectory defined from the current node to prospective subsequent node. This is calculated as in (2).
The process begins by selecting the initial configuration as the current configuration point $S$ and choosing it as the starting point of the path. Subsequently, we consider all the configuration points sampled during the initial phase, excluding those already integrated into the path as candidate configuration points. We consider each of these candidates as the next configuration $Q$, and evaluate the heuristic value, $P(G|\xi_{S\to Q})$, and sorted accordingly (in descending order). Sequentially, according to the sorting, the validity of the path from $S$ to each of the candiates $Q$ is verified. As soon as a valid path is identified, the corresponding configuration point is selected. This newly selected configuration point is the one with the highest heuristic value under the condition of path validity. This process continues iteratively with the chosen candidate $Q$ becoming $S$ in the next iteration.
It is, however, essential to note the case where no valid path are identified among the candidate configurations. In this case, the current configuration, $S$, is discarded from the path, and we backtrack to the previous configuration point. Then, we continue examining the configuration points in the sorted list associated with the backtracked point, starting after where the last selection was made. The next valid configuration point in the path sequence is selected as the new current configuration point S, integrated into the path, and the search for the next valid configuration point with the highest heuristic value continues. This newly selected configuration point is the one with the subsequent highest

heuristic value under the condition of path validity.
This process continues until it reaches the goal configuration, $G$, or exhaustively explores all possible configurations. This looping process plays a vital role in identifying a highly legible path within the set of feasible trajectories from the initial configuration to the goal configuration.

### C. Heuristic and Cost

The aim of this method is to maximise trajectory legibility in a computationally efficient manner. A natural heuristic is to use the legibility score, (1), of the path between nodes. This score, however, is computationally expensive to evaluate so, as a compromise, we opted for (2), a more computationally efficient calculation. Higher values of $P(G|\xi_{S\to Q})$ indicate greater confidence in the trajectory, implying higher legibility.
This calculation requires knowledge about both the cost function, and the (cost of the) optimal trajectory. In our testing scenarios, all obstacles were polygons, making the visibility graph an ideal choice for finding the optimal trajectory. This is because constructing a visibility graph involves simplifying the edges of obstacles into nodes and edges, and polygon edges are typically composed of straight lines, making it relatively straightforward to construct the visibility graph.
To calculate the expected cost of a trajectory, we choose to use the cost function detailed by Dragan et al [1].

$$C_{\text{approx}} = \sum_t \|\xi(t+1) - \xi(t)\|^2 \tag{5}$$

The authors failed to disclose an appropriate norm for this calculation. For this implementation, we opted for the L2 Norm, with the (approximate) cost evaluating to the squared Euclidean distance between the robot's configuration over the trajectory $\xi$ at time $t$ and the robot's configuration at the next time step $t+1$. It quantifies how much the trajectory changes or deviates from one-time step to the next.
We have further modified this by considering only the time steps associated with each configuration point and ignoring the time steps between configuration points. Thus, we can calculate the cost of a trajectory passing through configurations $A \to B \to C$ as

$$C_{\text{approx}} = \|B - A\|^2 + \|C - B\|^2 \tag{6}$$

## IV. RESULTS AND DISCUSSION

To compare the effectiveness of our approach it was tested against trajectories of robots of different complexity on various problem settings. Three trajectories were derived from the graph of sampled points: a Depth-First-Search (DFS) path, a greedy path minimising the cost, and a greedy path maximising trajectory. The legibility score of each, averaged over 30 different randomisations, were collected and collated in Table 1.

TABLE I
AVERAGE LEGIBILITY SCORE OF TRAJECTORIES

| Obstacles | Joints | DFS | Optimal | Legibile |
|---|---|---|---|---|
| 2 | 4 | 0.09045 | 0.20382 | 0.20481 |
| 2 | 6 | 0.04683 | 0.08971 | 0.09017 |
| 2 | 8 | 0.02704 | 0.04312 | 0.04365 |
| 3 | 4 | 0.08627 | 0.14837 | 0.15094 |
| 3 | 6 | 0.04336 | 0.06836 | 0.07001 |
| 3 | 8 | 0.03585 | 0.04012 | 0.04052 |
| 4 | 4 | 0.08056 | 0.15345 | 0.16907 |
| 4 | 6 | 0.05313 | 0.05832 | 0.06370 |
| 4 | 8 | 0.02572 | 0.03213 | 0.03561 |

From these results, it can be inferred that our approach, the legibility greedy, did improve the legibility score of the trajectory. Using the DFS derived trajectory as a benchmark, the greedy approach improved on the legibility by, on average, 66.45%. The greatest improvements occurred with the simple 4 jointed robot, averaging an improvement of 103.76%. Additionally, in the scenario of the robot avoiding 2 obstacles, the legible greedy trajectory recorded an average improvement in legibility (compared to the DFS) of 93.47%, an increase compared to the 49.82% and 56.07% for the 3 and 4 obstacle cases respectively. These results match our intuition. The complicated scenarios are more constraining and thus more difficult to find feasible trajectories.
The greedy search minimising cost (OG) also had a notably larger legibility score compared to the DFS baseline. The OG generated trajectories exhibited a slightly lower legibility score than the legibility greedy trajectories, with the latter demonstrating an average increase of 1.23% over the former on scenarios with 2 and 3 obstacles. In the case of 4 obstacles, the legibility greedy improved upon the OG by 10.08%. This observation can be attributed to the fact that in these complicated scenarios, it is increasingly difficult to derive an optimal solution.
Fig. 1-3 detail the path of the robot along the trajectory devised by DFS search and the greedily guided legibility implementation. These figures show that when following the legible trajectories, the robot exhibits less randomness in its movements. This is particularly noticeable in the fewer jointed agents. While the 8 jointed robots' motion is improved, randomness is still present. This could be due to the legibility score becoming too large. As outlined in the literature, as the legibility score increases, the predictability of the motion decreases [2][3]. After exceeding a certain threshold, the motion may become exceptionally unpredictable, and in fact difficult for humans to infer the intentions of the robot. To ensure the motion remains within a certain degree of predictability, a regularising term can be incorporated into the score we wish to maximise, yielding

$$L(\xi) = \text{legibility}(\xi) - \lambda C(\xi) \tag{7}$$

where $\lambda \geq 0$ is the parameter controlling the tradeoff between legibility and predictability [1].

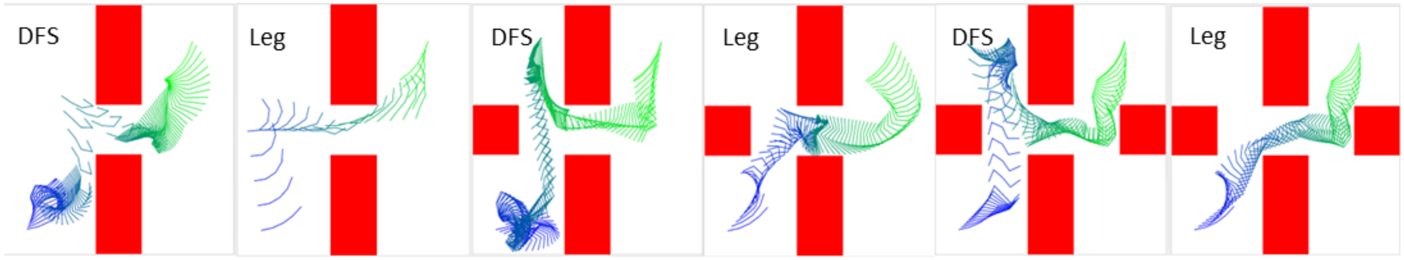


Fig. 1. The Trajectories of the 4 jointed robot, in different scenarios

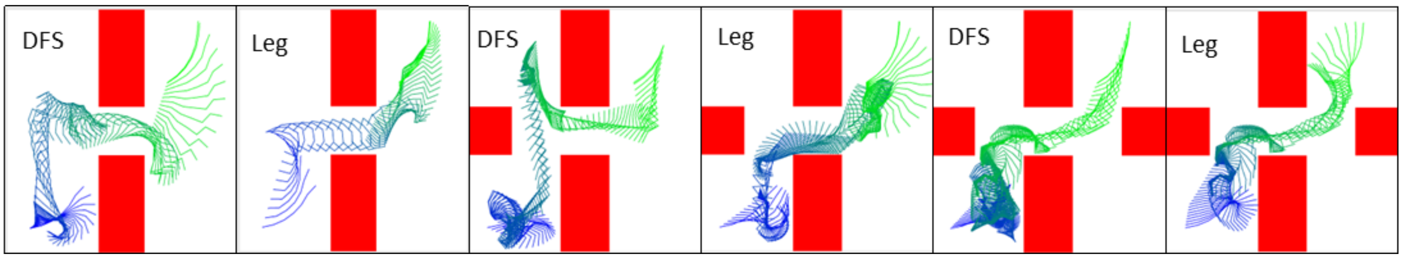


Fig. 2. The Trajectories of the 6 jointed robot, in different scenarios

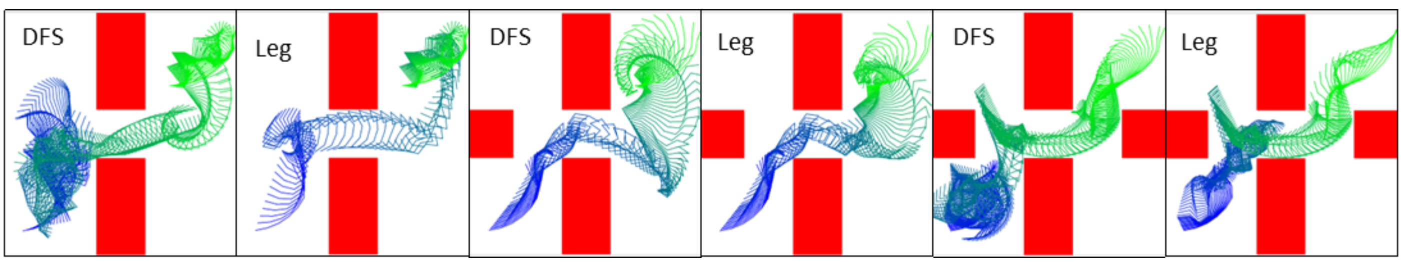


Fig. 3. The Trajectories of the 8 jointed robot, in different scenarios

The implementation relies heavily on the set of points constructed throughout the sampling stage. A thorough covering of the space allows the greedy search to comprehensively explore the workspace, improving the probability of finding a highly legible trajectory. Conversely, if the sampled points insufficiently represent the entire space, the search algorithm is constrained by these potentially limited paths.

## V. CONCLUSION AND FUTURE WORK

As robots are becoming increasingly prevalent in everyday use, the legibility of motion, and human collaboration as a whole, is becoming paramount. Dragan and his contemporaries devised a formulation for legibility in addition to a gradient ascent approach for optimising it. This approach, however, can become quite computationally expensive in complicated problem settings with long trajectories. To reduce computational requirements, we proposed an alternative graph-based approach for generating legible motion. Using the principles of Dragan's score as a heuristic, we create a graph of configurations within the c-space using random sampling and perform a greedy based search [3]. Through comparing this technique to a DFS generated trajectory, as well as a greedy agent attempting to minimise the cost (of the trajectory), we were able to show that our method yielded the trajectory with the highest average legibility score on all test cases. Although the improvements over the greedily devised optimal, our approach improved over the DFS with an average increase of 66.45%, with the maximum recorded increase of 126.43%.

This work can be further extended in numerous ways. As outlined previously, increasing the legibility yields increasingly unpredictable motion which, at a certain point, deviates from what humans would expect [1]. The incorporation of a regularisation term can ensure that the motion remains somewhat predictable, diminishing this effect. While this paper used a sampling method for its configuration graph construction, alternative methods may be considered. Different alternatives may be more applicable for different scenarios. For example, if the robot must pass through a narrow passageway, a sample near obstacles approach can be considered. Our heuristic function is based on Dragans legibility score [3]. There are, however, alternative formulations for legibility. Considering different measures of legibility could result in an improved legible (with respect to human inference) trajectory. As an example, Zhao et al. formulate three different legibility measures during their legibility construction through a POMDP model [5]. Different measures are likely to be useful in different problem settings.

## APPENDIX

## REFERENCES


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