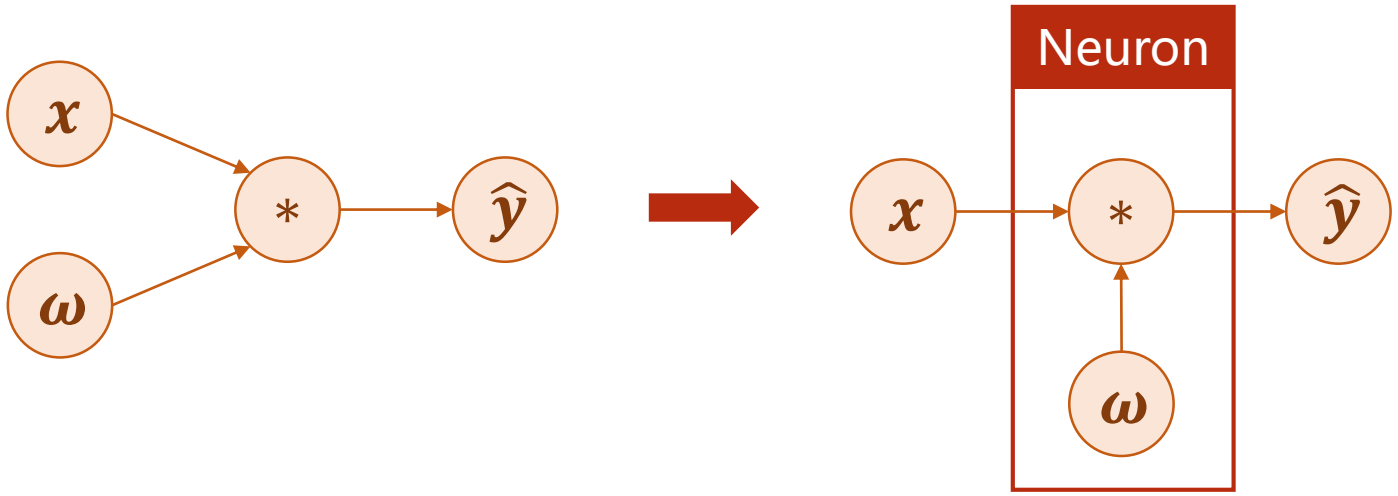
PyTorch Tutorial

04. Back Propagation

Compute gradient in simple network

Linear Model

$$\hat{y} = x * \omega$$



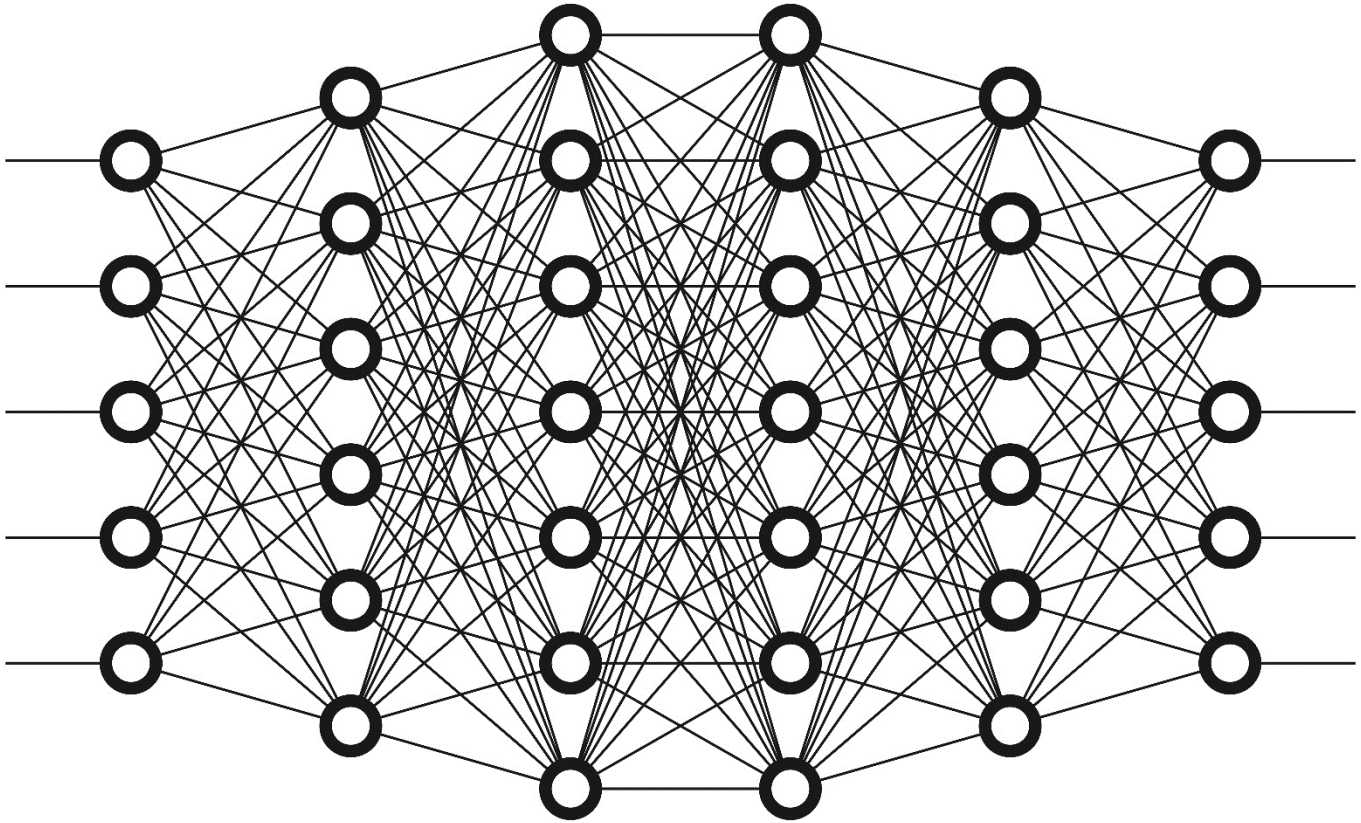
Stochastic Gradient Descent

$$\omega = \omega - \alpha \frac{\partial loss}{\partial \omega}$$

Derivative of Loss Function

$$\frac{\partial loss_n}{\partial \omega} = 2 \cdot x_n \cdot (x_n \cdot \omega - y_n)$$

What about the complicated network?



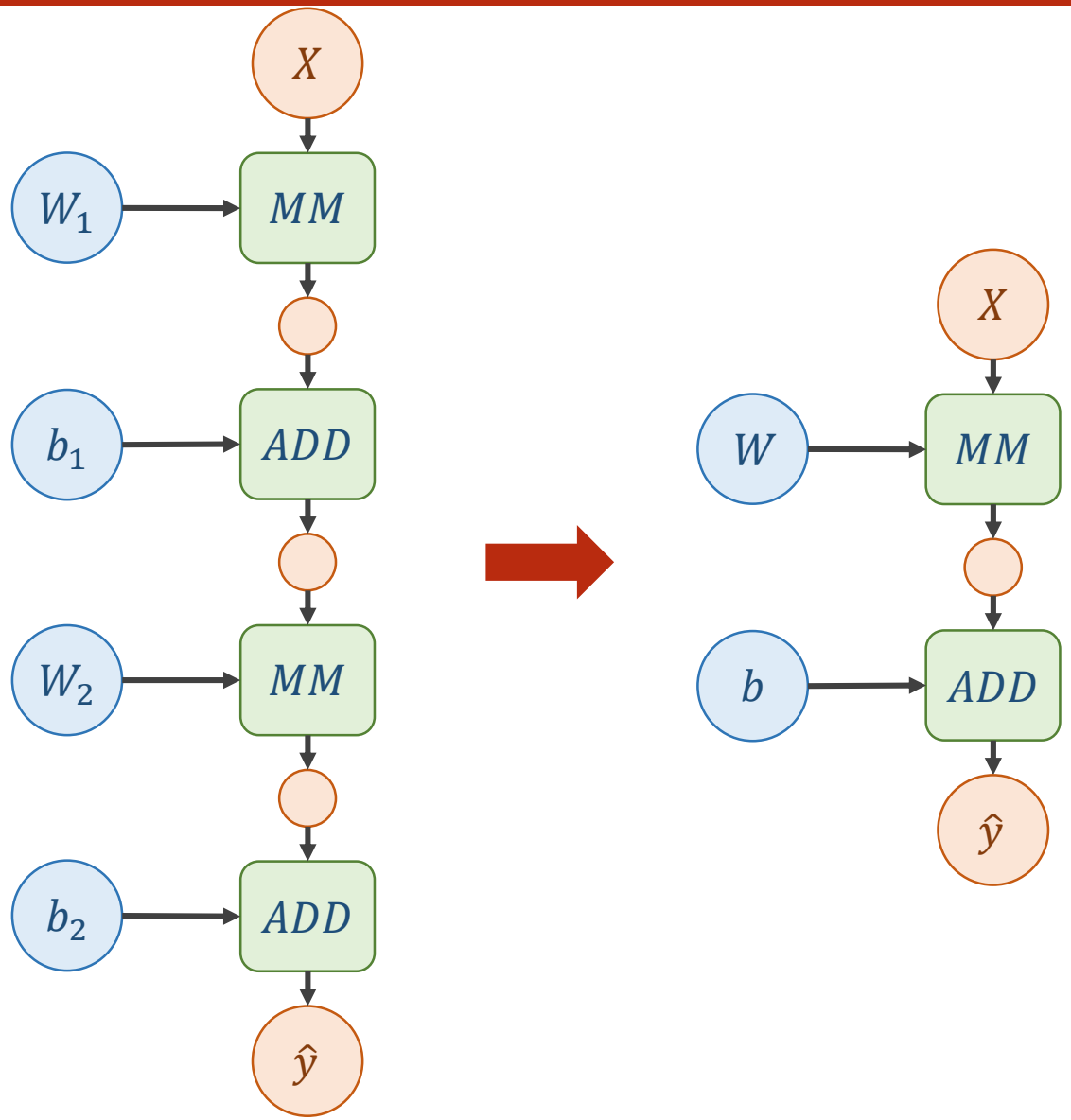
Gradient

$$\frac{\partial loss}{\partial \omega} = ?$$

What problem about this two layer neural network?

A two layer neural network

$$\hat{y} = W_2(W_1 \cdot X + b_1) + b_2$$
$$= W_2 \cdot W_1 \cdot X + (W_2 b_1 + b_2)$$
$$= W \cdot X + b$$



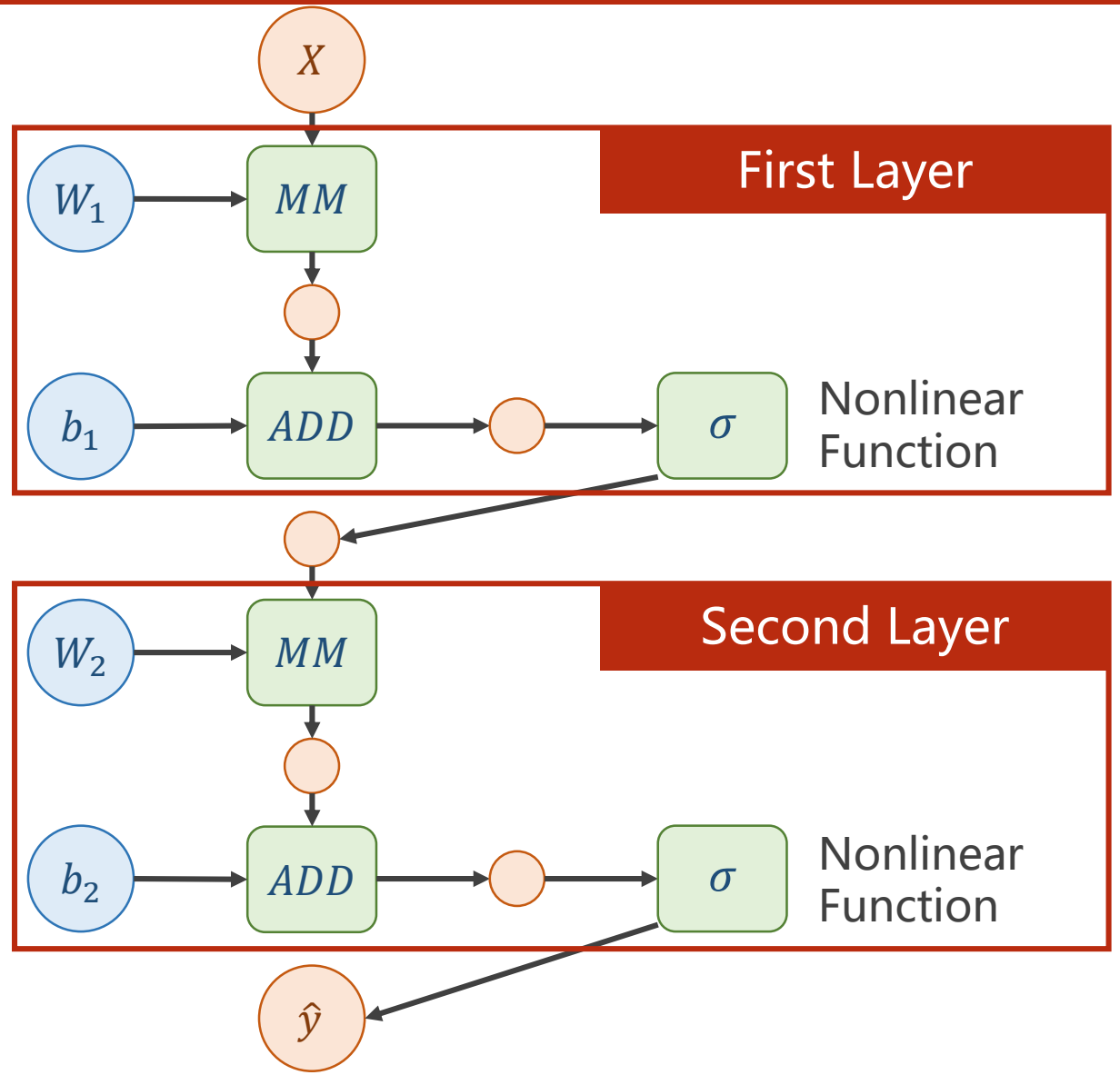
What problem about this two layer neural network?

A two layer neural network

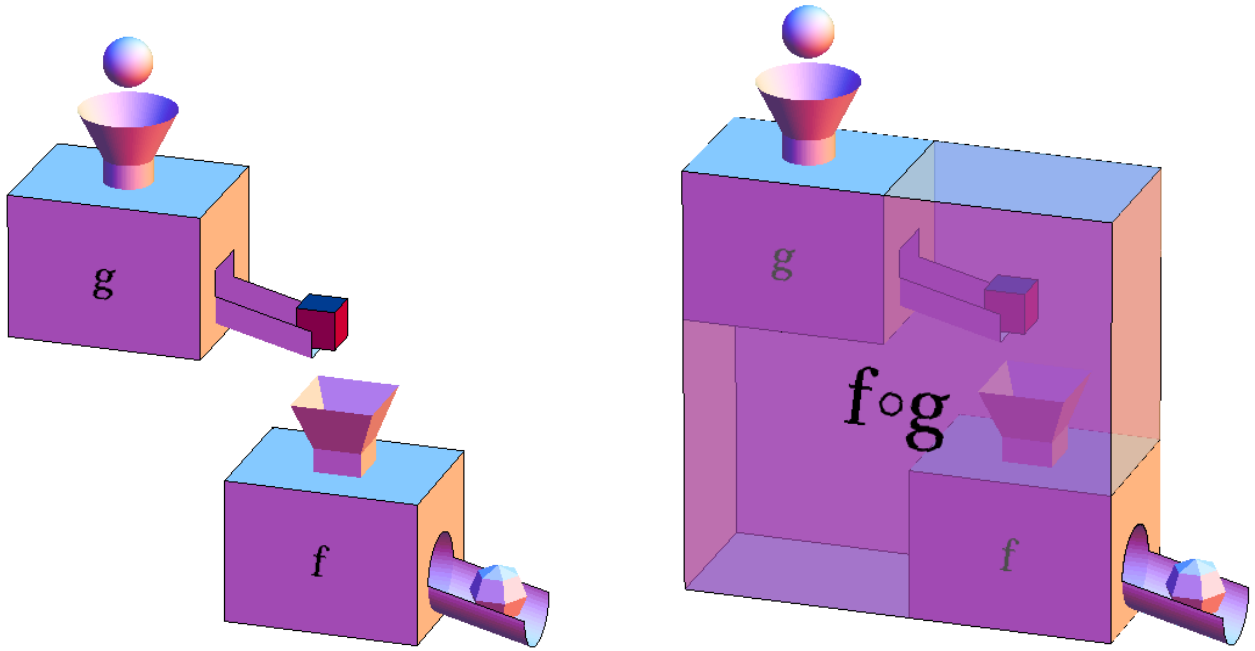
$$\hat{y} = W_2(W_1 \cdot X + b_1) + b_2$$
$$= W_2 \cdot W_1 \cdot X + (W_2 b_1 + b_2)$$
$$= W \cdot X + b$$

A nonlinear function is required by each layer.

We shall talk about this later.

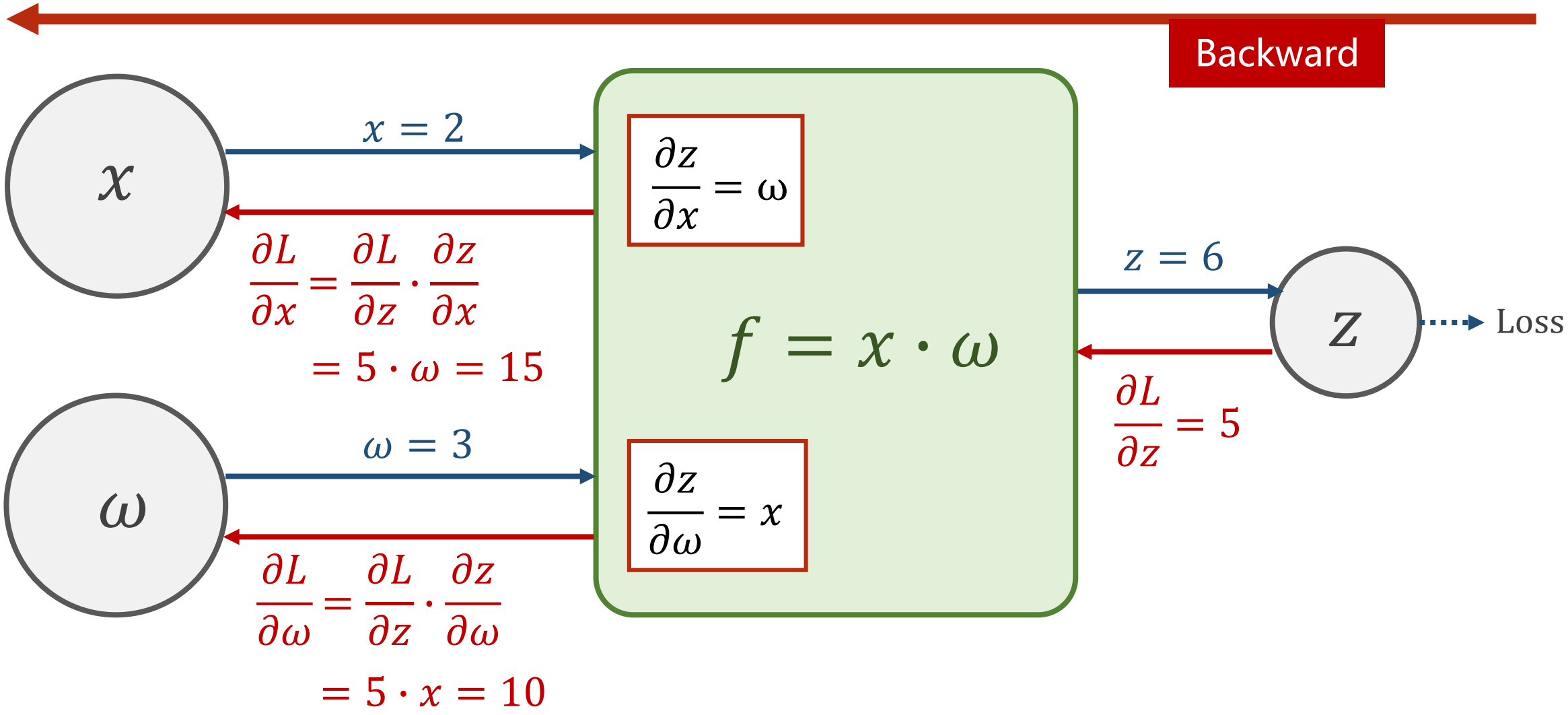


The composition of functions and Chain Rule



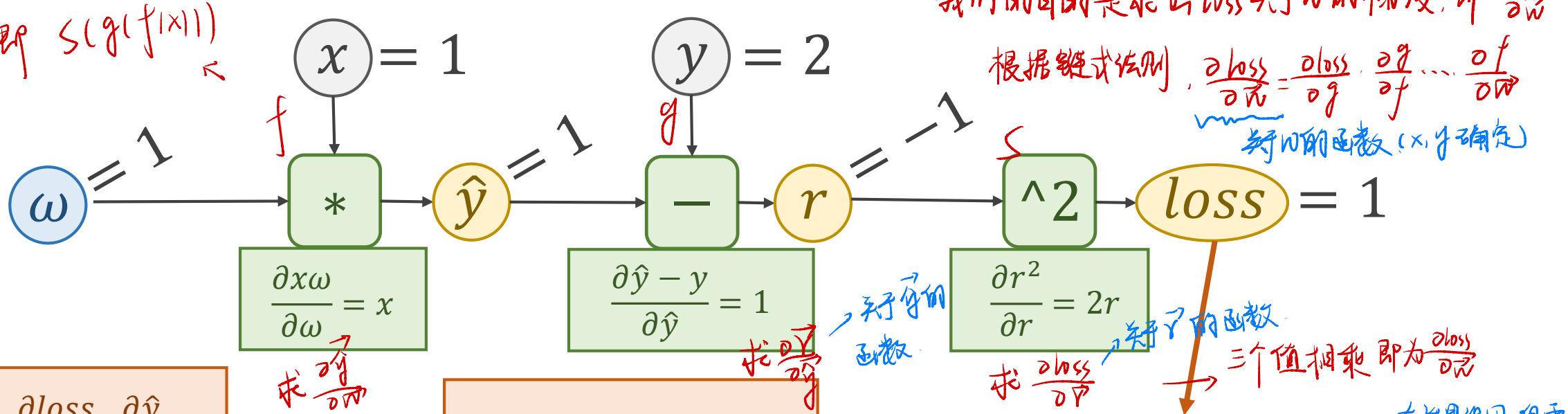
$$\frac{d\,\bigcirc}{d\,\bigcirc} = \frac{d\,\bigcirc}{d\,\square} \times \frac{d\,\square}{d\,\bigcirc}$$

Example: Backward



Computational Graph of Linear Model

Linear Model	Loss Function
$\hat{y} = x * \omega$	$loss = (\hat{y} - y)^2 = (x \cdot \omega - y)^2$



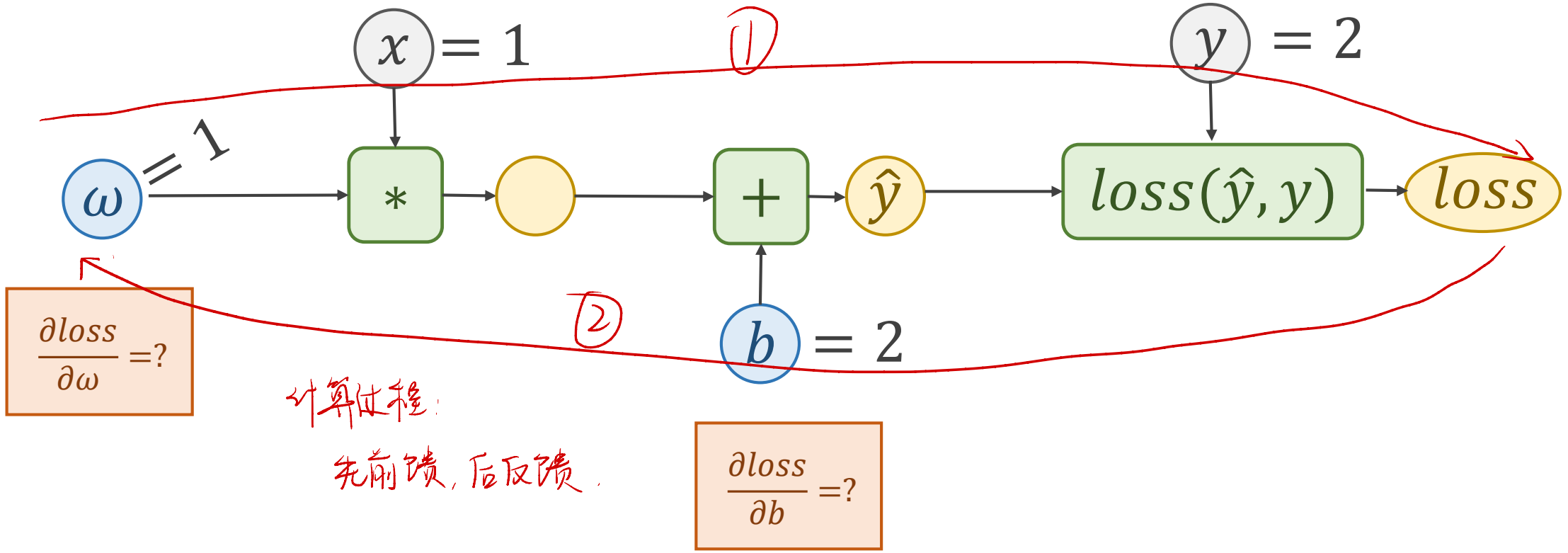
$$\frac{\partial loss}{\partial \omega} = \frac{\partial loss}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial \omega} = -2 \cdot x = -2 \cdot 1 = -2$$

$$\frac{\partial loss}{\partial \hat{y}} = \frac{\partial loss}{\partial r} \cdot \frac{\partial r}{\partial \hat{y}} = -2 \cdot 1 = -2$$

$$\frac{\partial loss}{\partial r} = 2r = -2$$

Exercise 4-2: Compute gradient of Affine model

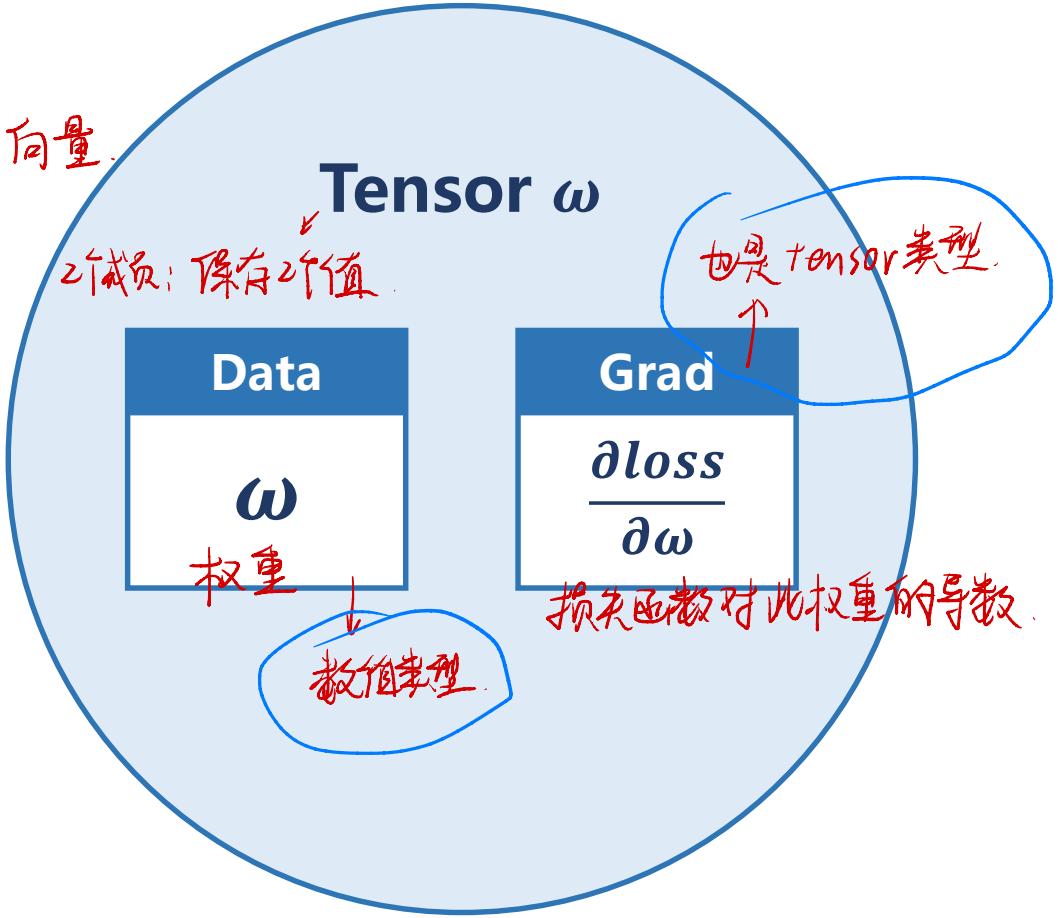
Affine Model	Loss Function
$\hat{y} = x * \omega + b$	$loss = (\hat{y} - y)^2 = (x \cdot \omega - y)^2$



Tensor in PyTorch

In PyTorch, **Tensor** is the important component in constructing dynamic computational graph.

It contains **data** and **grad**, which storage the value of node and gradient w.r.t loss respectively.



Implementation of linear model with PyTorch

```
import torch

x_data = [1.0, 2.0, 3.0]
y_data = [2.0, 4.0, 6.0]

w = torch.Tensor([1.0])
w.requires_grad = True
```

If **autograd mechanics** are required, the element variable **requires_grad** of **Tensor** has to be set to **True**.

Implementation of linear model with PyTorch

```
def forward(x):
    return x * w

def loss(x, y):
    y_pred = forward(x)
    return (y_pred - y) ** 2
```

已经被重载

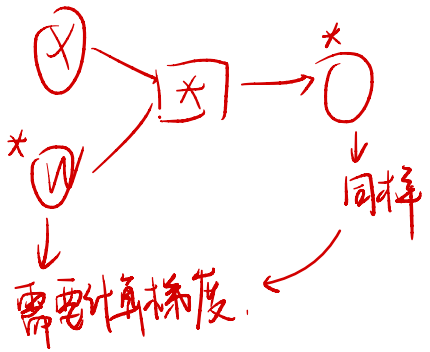
是一个 Tensor

x不一定是Tensor，所以要自动类型转换为Tensor

Define the linear model:

Linear Model

$$\hat{y} = x * \omega$$



即，若一个节点的输入值有要计算梯度的，则这个节点也要计算梯度（链式法则）

Implementation of linear model with PyTorch

构建计算图的过程，直接使用张量(tensor)进行计算，而不用 w.data，X.data

Define the loss function:

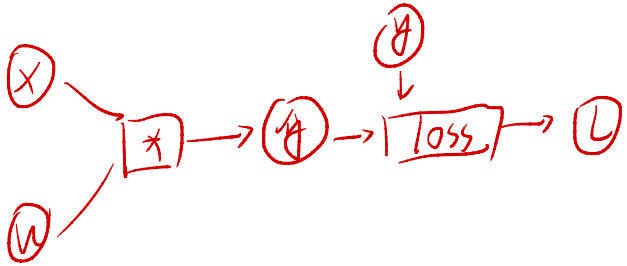
返回值是关于x和w的计算图，而不是数值量

```
def forward(x):
    return x * w

def loss(x, y):
    y_pred = forward(x)
    return (y_pred - y) ** 2
```

Loss Function

$$loss = (\hat{y} - y)^2 = (x \cdot \omega - y)^2$$



Implementation of linear model with PyTorch

应用反向传播训练的代码：

```
print("predict (before training)",  4, forward(4).item())
for epoch in range(100):
    for x, y in zip(x_data, y_data):
        l = loss(x, y)
        l.backward()
        print('\tgrad:', x, y, w.grad.item())
        w.data = w.data - 0.01 * w.grad.data
        w.grad.data.zero_()

    print("progress:", epoch, l.item())

print("predict (after training)", 4, forward(4).item())
```

Forward, compute the loss.

Implementation of linear model with PyTorch

```
print("predict (before training)",  4, forward(4).item())

for epoch in range(100):
    for x, y in zip(x_data, y_data):
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        w.data = w.data - 0.01 * w.grad.data

        w.grad.data.zero_()

    print("progress:", epoch, l.item())

print("predict (after training)", 4, forward(4).item())
```

Backward, compute grad for Tensor whose **requires_grad** set to True

Implementation of linear model with PyTorch

```
print("predict (before training)",  4, forward(4).item())

for epoch in range(100):
    for x, y in zip(x_data, y_data):
        l = loss(x, y)
        l.backward()
        print('\tgrad:', x, y, w.grad.item())
        w.data = w.data - 0.01 * w.grad.data

        w.grad.data.zero_()

    print("progress:", epoch, l.item())

print("predict (after training)", 4, forward(4).item())
```

The **grad** is utilized to update weight.

Implementation of linear model with PyTorch

```
print("predict (before training)",  4, forward(4).item())

for epoch in range(100):
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        l = loss(x, y)
        l.backward()
        print('\tgrad:', x, y, w.grad.item())
        w.data = w.data - 0.01 * w.grad.data

        w.grad.data.zero_()

    print("progress:", epoch, l.item())

print("predict (after training)", 4, forward(4).i
```

NOTICE:

The grad computed by *.backward()* will be **accumulated**.

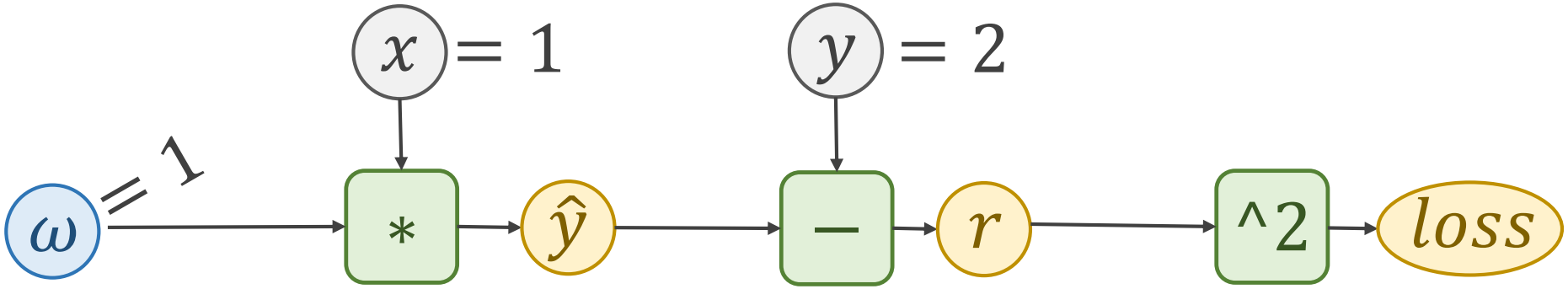
So after update, remember set the grad to **ZERO**!!!

Implementation of linear model with PyTorch

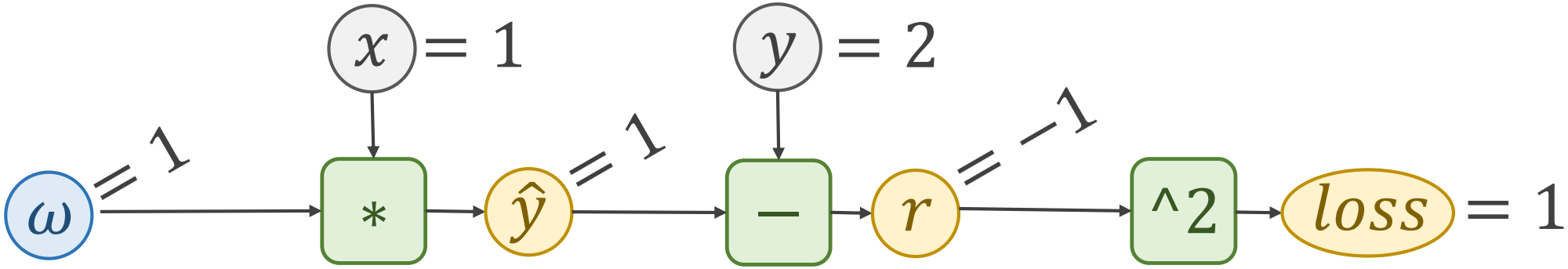
```
print("predict (before training)",  4, forward(4).item())

for epoch in range(100):
    for x, y in zip(x_data, y_data):
        l = loss(x, y)
        l.backward()
        print('\tgrad:', x, y, w.grad.item())
        w.data = w.data - 0.01 * w.grad.data

        w.grad.data.zero_()

    print("progress:", epoch, l.item())

print("predict (after training)", 4, forward(4).item())
```

```
predict (before training) 4 4.0
        grad: 1.0 2.0 -2.0
        grad: 2.0 4.0 -7.840000152587891
        grad: 3.0 6.0 -16.228801727294922
progress: 0 7.315943717956543
        grad: 1.0 2.0 -1.478623867034912
        grad: 2.0 4.0 -5.796205520629883
        grad: 3.0 6.0 -11.998146057128906
progress: 1 3.9987640380859375
        grad: 1.0 2.0 -1.0931644439697266
        grad: 2.0 4.0 -4.285204887390137
        grad: 3.0 6.0 -8.870372772216797
progress: 2 2.1856532096862793
        grad: 1.0 2.0 -0.8081896305084229
        grad: 2.0 4.0 -3.1681032180786133
        grad: 3.0 6.0 -6.557973861694336
progress: 3 1.1946394443511963
        grad: 1.0 2.0 -0.5975041389465332
        grad: 2.0 4.0 -2.3422164916992188
        grad: 3.0 6.0 -4.848389625549316
progress: 4 0.6529689431190491
        grad: 1.0 2.0 -0.4417421817779541
        grad: 2.0 4.0 -1.7316293716430664
        grad: 3.0 6.0 -3.58447265625
progress: 5 0.3569012284278696
        grad: 1.0 2.0 -0.3265852928161621
        grad: 2.0 4.0 -1.2802143096923828
        grad: 3.0 6.0 -2.650045394897461
```

Forward/Backward in PyTorch

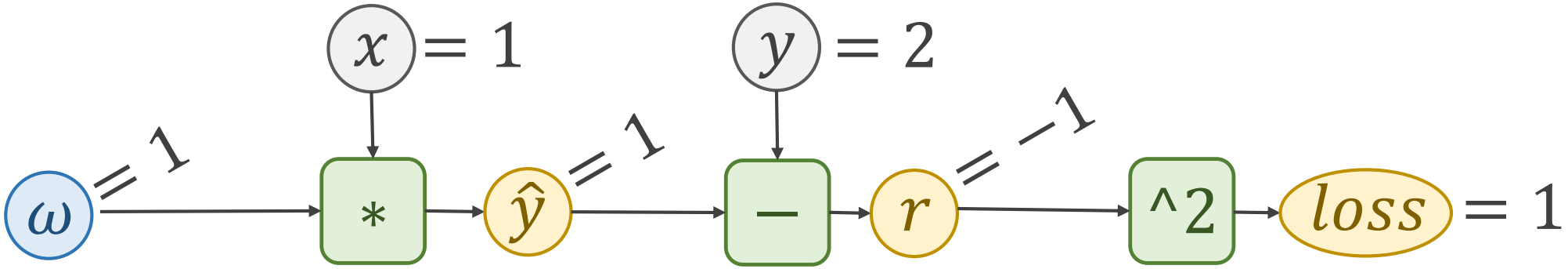


Forward in PyTorch




```
w = torch.Tensor([1.0])
w.requires_grad = True

l = loss(x, y)
```

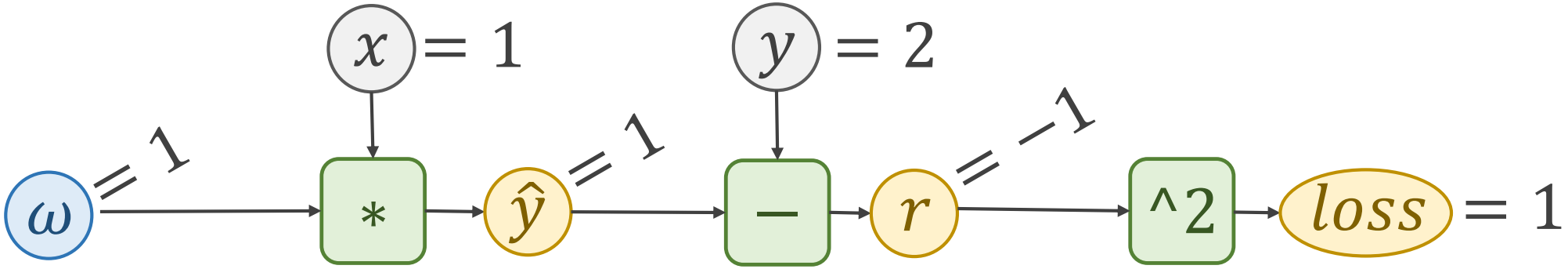
Backward in PyTorch

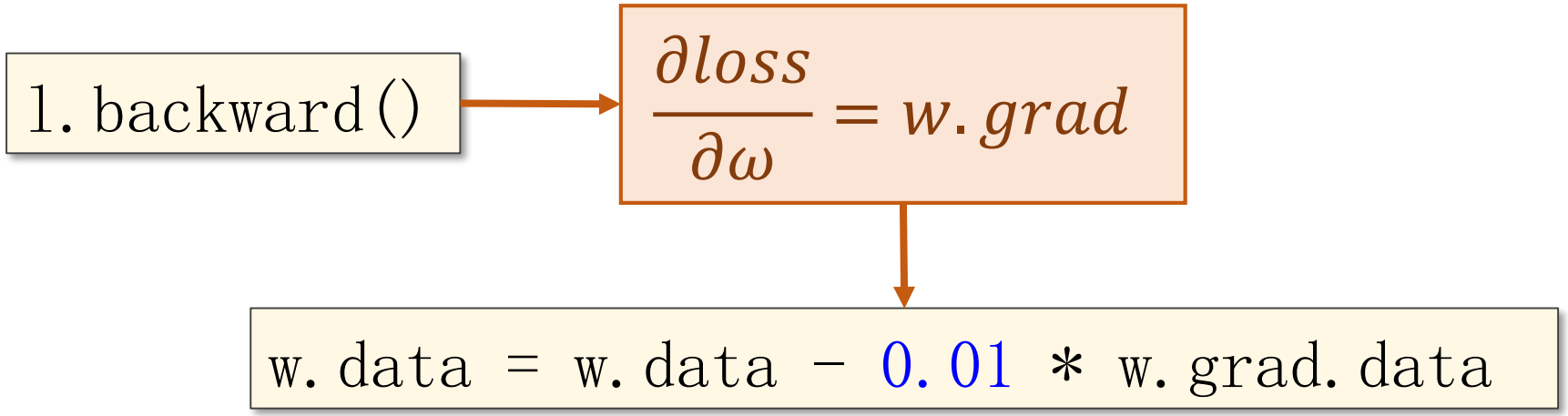


1. backward()

$$\frac{\partial loss}{\partial \omega} = w.grad$$

Update weight in PyTorch





Exercise 4-3: Compute gradients using computational graph

Quadratic Model

$$\hat{y} = \omega_1 x^2 + \omega_2 x + b$$

Loss Function

$$loss = (\hat{y} - y)^2 = (x \cdot \omega - y)^2$$

$$\frac{\partial loss}{\partial \omega_1} = ?$$

$$\frac{\partial loss}{\partial \omega_2} = ?$$

$$\frac{\partial loss}{\partial b} = ?$$

Exercise 4-4: Compute gradients using PyTorch

Quadratic Model

$$\hat{y} = \omega_1 x^2 + \omega_2 x + b$$

Loss Function

$$loss = (\hat{y} - y)^2 = (x \cdot \omega - y)^2$$

$$\frac{\partial loss}{\partial \omega_1} = ?$$

$$\frac{\partial loss}{\partial \omega_2} = ?$$

$$\frac{\partial loss}{\partial b} = ?$$

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