Problem 1 B

Friday, January 27, 2023 11:55 AM

$f_W(X) = ABX$

$1 \times n \quad 1 \times k \quad k \times d \times d \times n$

$\nabla_B \alpha(W) \in k \times d$

$B^{(t+1)} = B^{(t)} - \eta \cdot \nabla_B \alpha(W)$

$\alpha(W) = \frac{1}{2} \underbrace{\| y - f_W(x) \|_2^2}_{\in 1 \times n} \in 1 \times 1$

$= \frac{1}{2} \sum_{i=1}^{n} \left(y^{(i)} - f_W(x)^{(i)} \right)^2$

$\nabla_B \alpha(W) = -\sum_{i=1}^{n} \left(y^{(i)} - f_W(x)^{(i)} \right) \nabla_B f_W(x)^{(i)}$

$k \times d \qquad 1 \times 1 \qquad k \times d$

$\nabla_{B_{ij}} f_W(x)^{(i)} \in 1 \times 1$

$f_W(x)^{(i)} = f_W(x) = ABx \quad (1 \times k \cdot k \times d \cdot d \times 1)$

$= [A_1 \cdots A_k] \begin{bmatrix} B_{11} & \cdots & B_{1d} \\ & \vdots & \\ B_{k1} & \cdots & B_{kd} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_d \end{bmatrix}$ (B_{ij} in the ith row, jth column; $1 \times d$)

$i \in k \quad j \in d$

$\downarrow$ jth

$[\cdots, A_i \cdot B_{ij}, \cdots] \in 1 \times d \begin{bmatrix} x_1 \\ \vdots \\ x_d \end{bmatrix}$

$\Rightarrow \frac{df_W(x)}{dB_{ij}} = A_i \cdot x_j$

$A \in 1 \times k \quad X \in d \times n$

$A^T \in k \times 1 = k$

$X^T \in n \times d$

$A^T \cdot (y - f_W(x)) \cdot X^T$

$k \times 1 \quad 1 \times n \quad n \times d \rightarrow k \times d$

$\therefore B^{(t+1)} = B^{(t)} + \eta A^T (y - f_W(x)) \cdot X^T$