Problem 4

In [1]:
```
import torch
import torchvision
import torchvision.transforms as transforms
```

In [2]:
```
transform = transforms.Compose(
    [transforms.ToTensor(),
     transforms.Normalize((0.5, 0.5, 0.5), (0.5, 0.5, 0.5))])

batch_size = 1

trainset = torchvision.datasets.CIFAR10(root='./data', train=True,
                                        download=True, transform=transform)
trainloader = torch.utils.data.DataLoader(trainset, batch_size=batch_size,
                                          shuffle=False, num_workers=2)

testset = torchvision.datasets.CIFAR10(root='./data', train=False,
                                       download=True, transform=transform)
testloader = torch.utils.data.DataLoader(testset, batch_size=batch_size,
                                         shuffle=False, num_workers=2)

classes = ('plane', 'car', 'bird', 'cat',
           'deer', 'dog', 'frog', 'horse', 'ship', 'truck')
```

```
Files already downloaded and verified
Files already downloaded and verified
```

In [3]:
```
trainset
```

Out[3]:
```
Dataset CIFAR10
    Number of datapoints: 50000
    Root location: ./data
    Split: Train
    StandardTransform
Transform: Compose(
               ToTensor()
               Normalize(mean=(0.5, 0.5, 0.5), std=(0.5, 0.5, 0.5))
           )
```

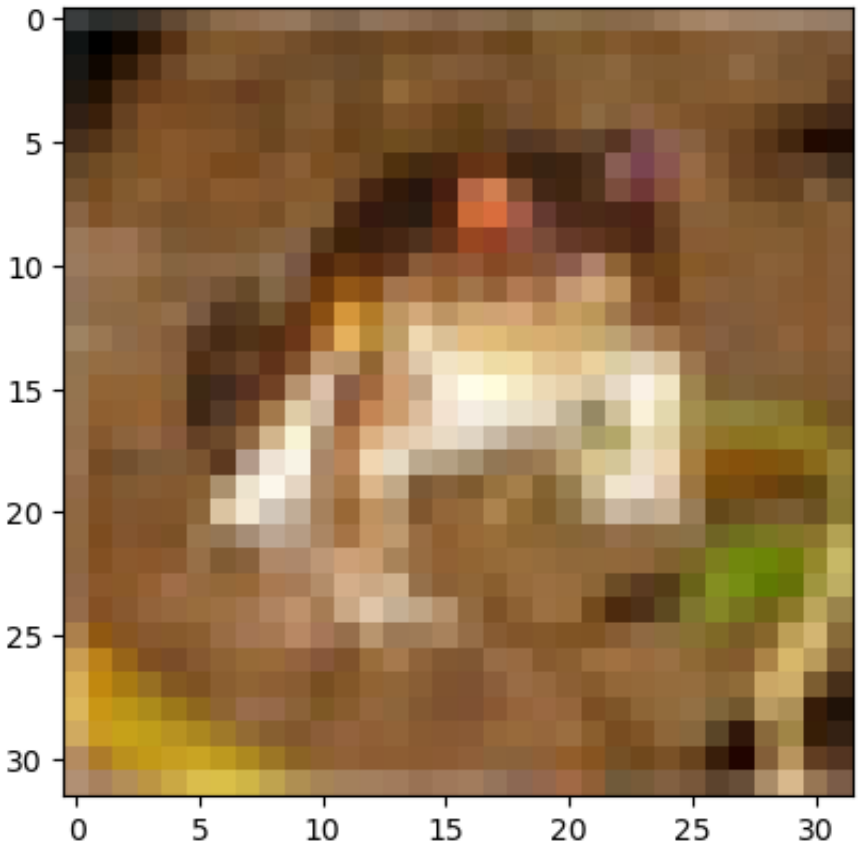
In [4]:
```
import matplotlib.pyplot as plt
import numpy as np

# functions to show an image


def imshow(img):
    img = img / 2 + 0.5     # unnormalize
    npimg = img.numpy()
    plt.imshow(np.transpose(npimg, (1, 2, 0)))
    plt.show()


# get some random training images
dataiter = iter(trainloader)
z1, labels = next(dataiter)

# show images
imshow(torchvision.utils.make_grid(z1))
# print labels
print(' '.join(f'{classes[labels[j]]:5s}' for j in range(batch_size)))
```




```
frog
```

(a) Train an image with an error less than 1e-5

In [5]:
```
import torch.nn as nn
import torch.nn.functional as F

class Net(nn.Module):
    def __init__(self, k):
        super(Net, self).__init__()
        self.k = k
        self.B = nn.Linear(3 * 32 * 32, self.k, bias=False)
        self.A = nn.Linear(self.k, 3 * 32 * 32, bias=False)

    def forward(self, x):
        x = x.view(-1, 3 * 32 * 32) # [batchsize, 3072]
        x = self.B(x)
        x = F.relu(x)
        x = self.A(x)
        return x

f = Net(k=1024)
```

In [6]:
```
# evaluating data points with Mean Square Error (MSE)
def L(x, fx):
    x = x.view(-1, 3 * 32 * 32)
    diff = x - fx
    return 0.5 * (torch.norm(diff, p=2)**2)
```

In [7]:
```
steps = 100
lr = 1e-3

def train(steps, lr):
    losses = []
    for i in range(steps):
        # Generate Prediction
        fx = f(z1)
        # Get the loss and perform backpropagation
        loss = L(z1, fx)
        losses.append(loss)
        loss.backward() # get gradient
        # Let's update the weights
        with torch.no_grad():
            f.A.weight -= lr * f.A.weight.grad
            f.B.weight -= lr * f.B.weight.grad
            # Set the gradients to zero
            f.A.weight.grad.zero_()
            f.B.weight.grad.zero_()
        print(f"step {i}: Loss: {loss}")
    print(f"minimal loss achieved: {min(losses)}")
```

In [8]:
```
train(steps, lr)
```

```
step 0: Loss: 325.329833984375
step 1: Loss: 273.7133483886719
```

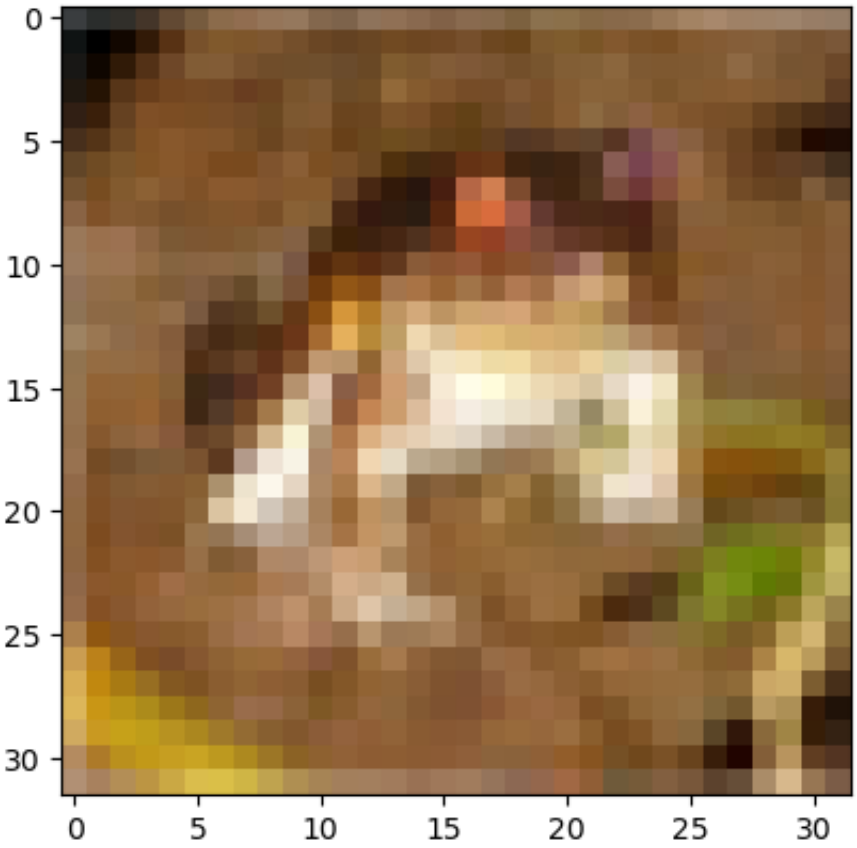
```
step 2: Loss: 244.14212036132812
step 3: Loss: 198.89141845703125
step 4: Loss: 139.93614196777344
step 5: Loss: 81.38546752929688
step 6: Loss: 39.56410598754883
step 7: Loss: 17.088428497314453
step 8: Loss: 6.952592849731445
step 9: Loss: 2.7564098834991455
step 10: Loss: 1.081404447555542
step 11: Loss: 0.4225172996520996
step 12: Loss: 0.1648177206516266
step 13: Loss: 0.06425297260284424
step 14: Loss: 0.025042491033673286
step 15: Loss: 0.009759376756846905
step 16: Loss: 0.003803201951086521
step 17: Loss: 0.0014820826472714543
step 18: Loss: 0.0005775515455752611
step 19: Loss: 0.00022506603272631764
step 20: Loss: 8.770507702138275e-05
step 21: Loss: 3.41781233146321e-05
step 22: Loss: 1.3318700439413078e-05
step 23: Loss: 5.19000332133146e-06
step 24: Loss: 2.0226091237418586e-06
step 25: Loss: 7.881436658863095e-07
step 26: Loss: 3.0718047128175385e-07
step 27: Loss: 1.196830083927125e-07
step 28: Loss: 4.663068509103141e-08
step 29: Loss: 1.8179873606527508e-08
step 30: Loss: 7.100092513923073e-09
step 31: Loss: 2.7733082497150008e-09
step 32: Loss: 1.0935854444227289e-09
step 33: Loss: 4.4747863747751637e-10
step 34: Loss: 2.041107144412635e-10
step 35: Loss: 1.1425811685672471e-10
step 36: Loss: 7.551961139773411e-11
step 37: Loss: 5.577729944583609e-11
step 38: Loss: 4.2543746303636e-11
step 39: Loss: 3.4071839100091594e-11
step 40: Loss: 2.7972523602981525e-11
step 41: Loss: 2.354040053165196e-11
step 42: Loss: 2.025936189642419e-11
step 43: Loss: 1.7818900868715737e-11
step 44: Loss: 1.579360080217196e-11
step 45: Loss: 1.4059483612050006e-11
step 46: Loss: 1.2719950960582427e-11
step 47: Loss: 1.1545031423920715e-11
step 48: Loss: 1.012597037469698e-11
step 49: Loss: 9.23511180722647e-12
step 50: Loss: 8.365031757551211e-12
step 51: Loss: 7.517048615512945e-12
step 52: Loss: 6.829037335620569e-12
step 53: Loss: 6.275987975840058e-12
step 54: Loss: 5.700450528278722e-12
```

```
step 55: Loss: 5.5734878517954556e-12
step 56: Loss: 5.199500986013961e-12
step 57: Loss: 4.980309567526042e-12
step 58: Loss: 4.499964637028064e-12
step 59: Loss: 4.109062048951451e-12
step 60: Loss: 3.924071137473284e-12
step 61: Loss: 3.685220531512989e-12
step 62: Loss: 3.554077388293053e-12
step 63: Loss: 3.2659242064220217e-12
step 64: Loss: 3.1987136463879073e-12
step 65: Loss: 3.0448976673369543e-12
step 66: Loss: 2.988852004434661e-12
step 67: Loss: 3.1237720743426678e-12
step 68: Loss: 2.5516674276998552e-12
step 69: Loss: 2.611591281773129e-12
step 70: Loss: 2.3714569804406116e-12
step 71: Loss: 2.5601135794639518e-12
step 72: Loss: 2.2941386203928493e-12
step 73: Loss: 2.2238042570593697e-12
step 74: Loss: 2.031854068676453e-12
step 75: Loss: 2.0095450910945223e-12
step 76: Loss: 2.1124907378933244e-12
step 77: Loss: 1.995121299072644e-12
step 78: Loss: 1.8691504507367673e-12
step 79: Loss: 1.846523151396995e-12
step 80: Loss: 1.812966443637265e-12
step 81: Loss: 1.6090878629526628e-12
step 82: Loss: 1.6250682440335784e-12
step 83: Loss: 1.5416921211874879e-12
step 84: Loss: 1.535273752746591e-12
step 85: Loss: 1.461442186885542e-12
step 86: Loss: 1.3970768986126814e-12
step 87: Loss: 1.3399627544694037e-12
step 88: Loss: 1.2956529295629626e-12
step 89: Loss: 1.2132395782460392e-12
step 90: Loss: 1.2527132109418915e-12
step 91: Loss: 1.2355256791621305e-12
step 92: Loss: 1.1848438896677749e-12
step 93: Loss: 1.1214987272190058e-12
step 94: Loss: 1.084703615990279e-12
step 95: Loss: 1.192540857730684e-12
step 96: Loss: 1.0977885267493548e-12
step 97: Loss: 9.348216645221896e-13
step 98: Loss: 9.883848297101427e-13
step 99: Loss: 9.945187035009795e-13
minimal loss achieved: 9.348216645221896e-13
```

(b) Visualize

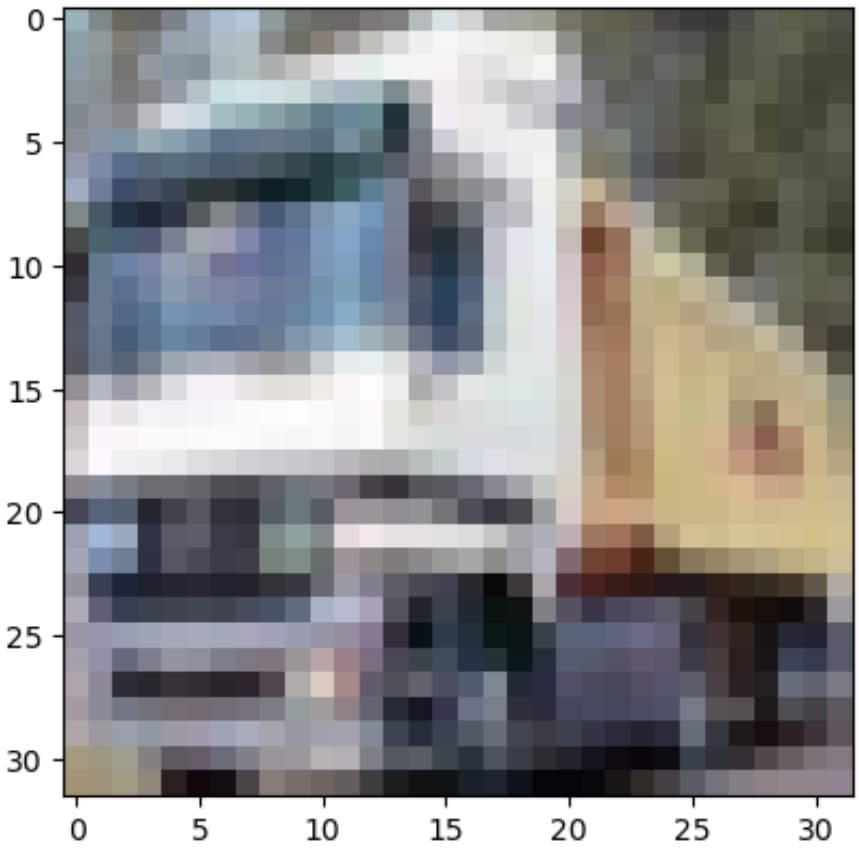
Training image

In [9]:
```
pred = f(z1)
pred = pred.reshape_as(z1)
pred = pred.detach()
imshow(torchvision.utils.make_grid(pred))
```



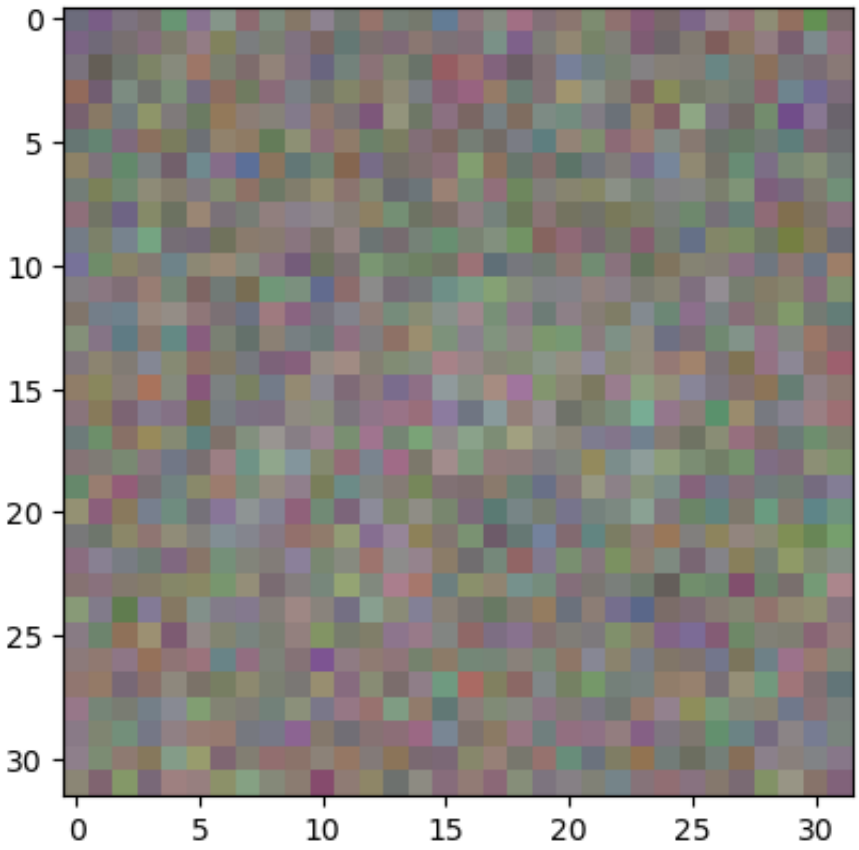
Random training image

In [10]:
```
# Random training image
random_train, labels = next(dataiter)
imshow(torchvision.utils.make_grid(random_train))
print(' '.join(f'{classes[labels[j]]:5s}' for j in range(batch_size)))
```




```
truck
```

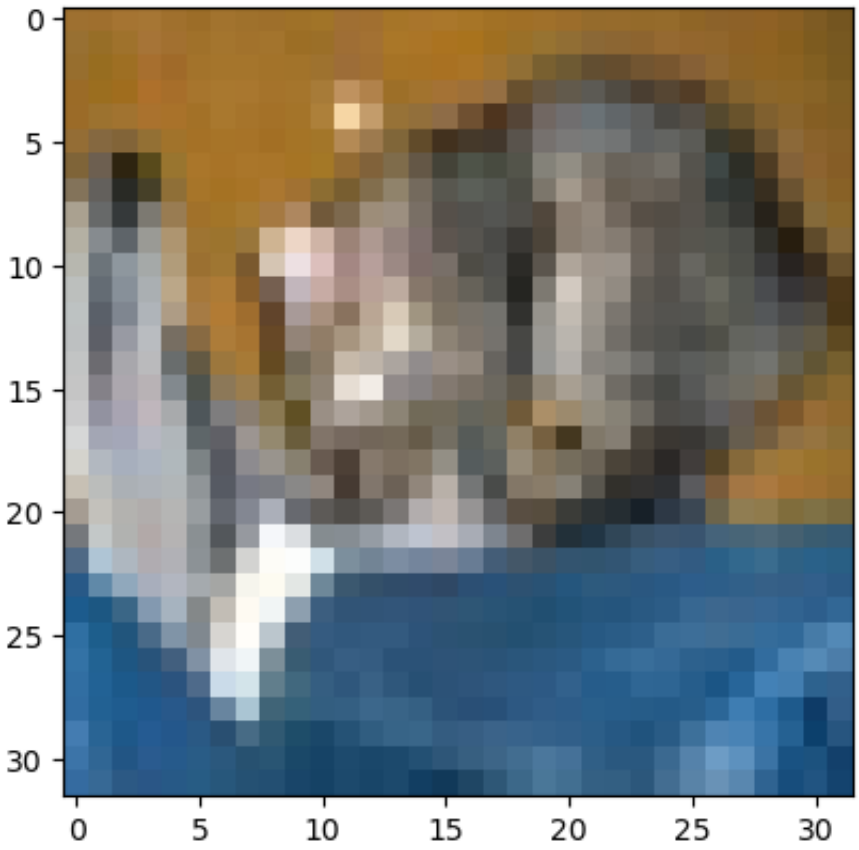
In [11]:
```
pred = f(random_train)
pred = pred.reshape_as(random_train)
pred = pred.detach()
imshow(torchvision.utils.make_grid(pred))
```



Random test image

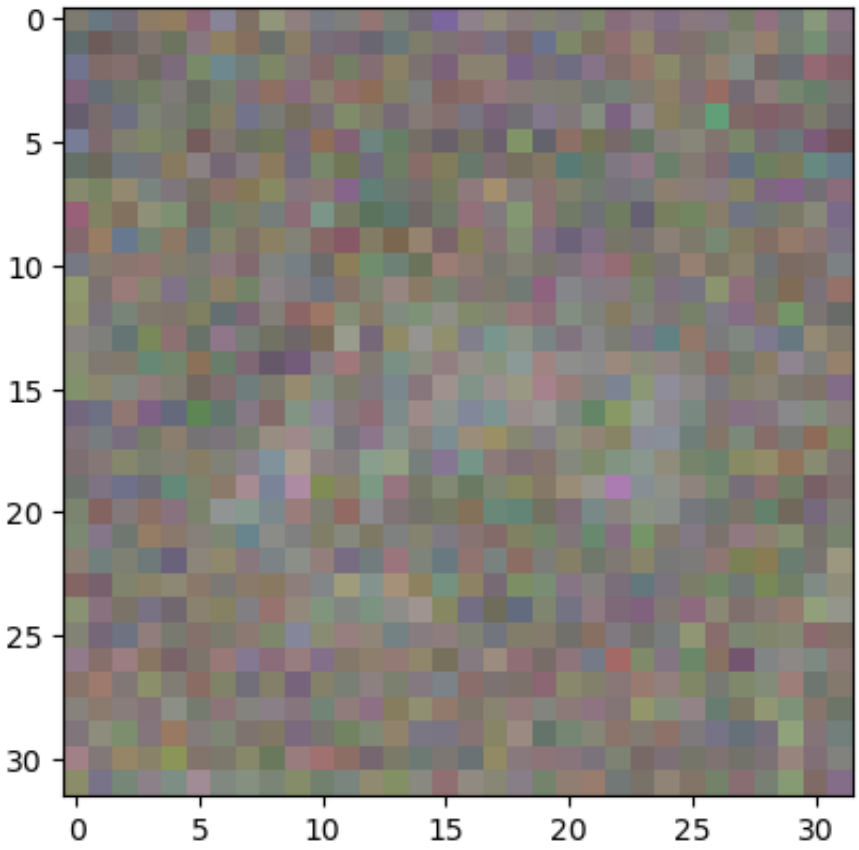
In [12]:
```
# get some random training images
dataiter = iter(testloader)
random_test, labels = next(dataiter)

# show images
imshow(torchvision.utils.make_grid(random_test))
# print labels
print(' '.join(f'{classes[labels[j]]:5s}' for j in range(batch_size)))
```



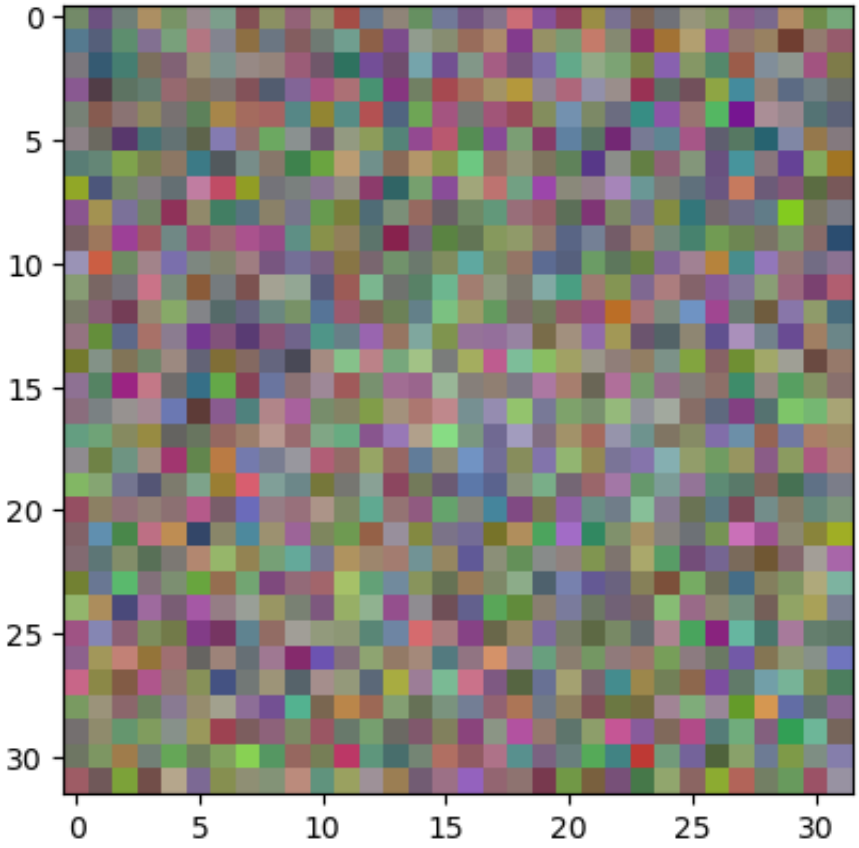
cat

In [13]:
```
pred = f(random_test)
pred = pred.reshape_as(random_test)
pred = pred.detach()
imshow(torchvision.utils.make_grid(pred))
```



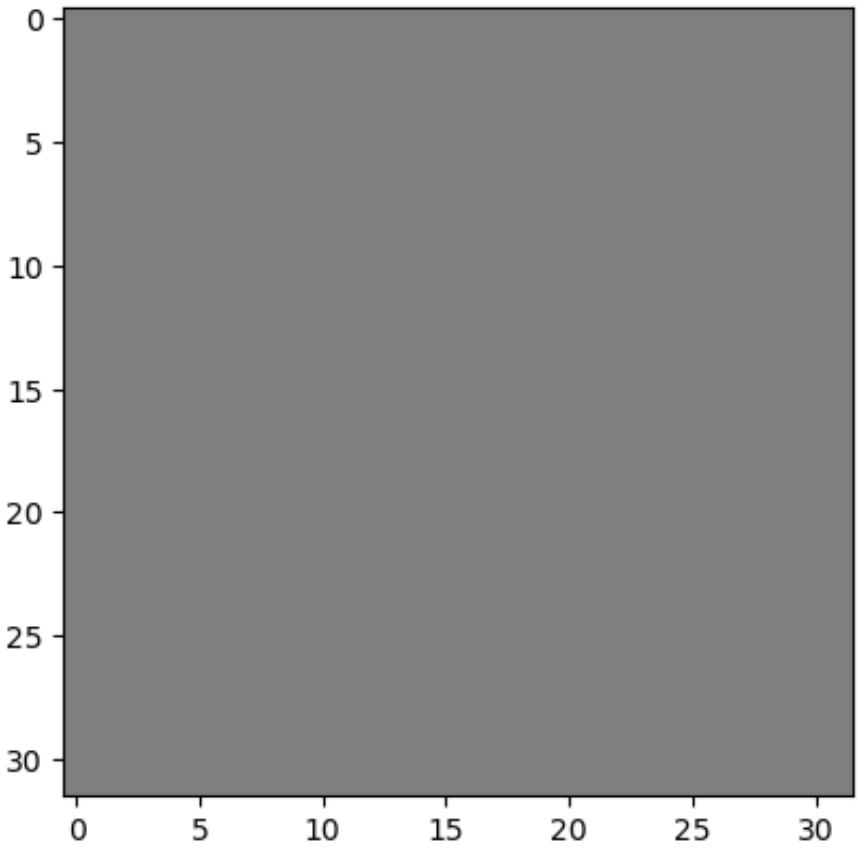
Random noise

In [14]:
```
noise = torch.normal(0, 1 , size=(1, 3, 32, 32), requires_grad=False)
pred = f(noise)
pred = pred.reshape_as(noise)
pred = pred.detach()
imshow(torchvision.utils.make_grid(pred))
```



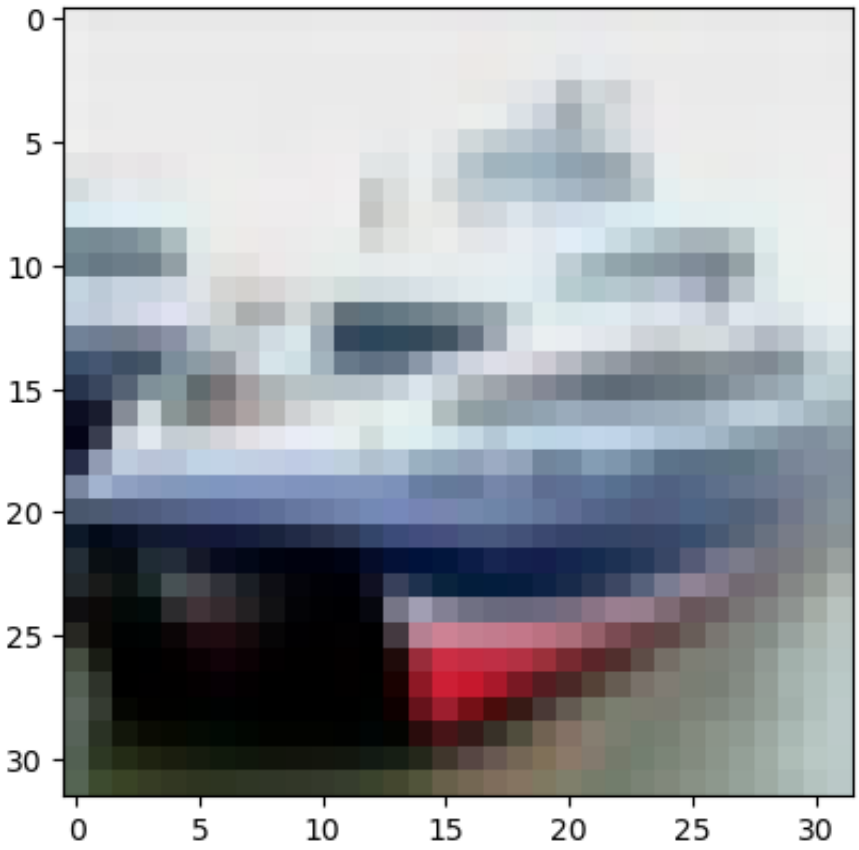
All zeros

In [15]:
```
zeros = torch.normal(0, 0 , size=(1, 3, 32, 32), requires_grad=False)
pred = f(zeros)
pred = pred.reshape_as(zeros)
pred = pred.detach()
imshow(torchvision.utils.make_grid(pred))
```



(c) Training on 2 images

In [16]:
```
z2, labels = next(dataiter)
# show images
imshow(torchvision.utils.make_grid(z2))
# print labels
print(' '.join(f'{classes[labels[j]]:5s}' for j in range(batch_size)))
```




```
ship
```

In [17]:
```
steps = 1000
lr = 1e-4
f = Net(k=1024)

def train2(steps, lr):
    losses = []
    for i in range(steps):
        if i % 2 == 0:
            x = z1
        else:
            x = z2
        # Generate Prediction
        fx = f(x)
        # Get the loss and perform backpropagation
        loss = L(x, fx)
        losses.append(loss)
        loss.backward() # get gradient
        # Let's update the weights
        with torch.no_grad():
            f.A.weight -= lr * f.A.weight.grad
            f.B.weight -= lr * f.B.weight.grad
            # Set the gradients to zero
            f.A.weight.grad.zero_()
            f.B.weight.grad.zero_()
        print(f"step {i}: Loss: {loss}")
    print(f"minimal loss achieved: {min(losses)}")
```

In [18]:
```
train2(steps, lr)
```

```
step 0: Loss: 327.88580322265625
step 1: Loss: 700.64794921875
step 2: Loss: 316.836669921875
step 3: Loss: 652.8576049804688
step 4: Loss: 308.36370849609375
step 5: Loss: 625.0802612304688
step 6: Loss: 301.4964904785156
step 7: Loss: 606.9642333984375
step 8: Loss: 295.74029541015625
step 9: Loss: 592.322998046875
step 10: Loss: 290.8751525878906
step 11: Loss: 578.8724365234375
step 12: Loss: 286.5518493652344
step 13: Loss: 565.206787109375
step 14: Loss: 282.59967041015625
step 15: Loss: 550.6947021484375
step 16: Loss: 278.98480224609375
step 17: Loss: 534.9141235351562
step 18: Loss: 275.56146240234375
step 19: Loss: 517.6426391601562
step 20: Loss: 272.2120361328125
step 21: Loss: 498.6922912597656
step 22: Loss: 268.87908935546875
```

```
step 23: Loss: 478.0023193359375
step 24: Loss: 265.52301025390625
step 25: Loss: 455.56219482421875
step 26: Loss: 262.07733154296875
step 27: Loss: 431.41168212890625
step 28: Loss: 258.5296936035156
step 29: Loss: 405.76983642578125
step 30: Loss: 254.87376403808594
step 31: Loss: 378.8576965332031
step 32: Loss: 251.06134033203125
step 33: Loss: 350.99676513671875
step 34: Loss: 247.087158203125
step 35: Loss: 322.5587158203125
step 36: Loss: 242.93609619140625
step 37: Loss: 293.9726867675781
step 38: Loss: 238.6042022705078
step 39: Loss: 265.6807861328125
step 40: Loss: 234.08609008789062
step 41: Loss: 238.1218719482422
step 42: Loss: 229.37242126464844
step 43: Loss: 211.70025634765625
step 44: Loss: 224.46142578125
step 45: Loss: 186.76055908203125
step 46: Loss: 219.35093688964844
step 47: Loss: 163.57046508789062
step 48: Loss: 214.0438995361328
step 49: Loss: 142.31459045410156
step 50: Loss: 208.54429626464844
step 51: Loss: 123.08268737792969
step 52: Loss: 202.8596954345703
step 53: Loss: 105.88780212402344
step 54: Loss: 196.99981689453125
step 55: Loss: 90.67615509033203
step 56: Loss: 190.97752380371094
step 57: Loss: 77.3438491821289
step 58: Loss: 184.80691528320312
step 59: Loss: 65.7522201538086
step 60: Loss: 178.5053253173828
step 61: Loss: 55.743553161621094
step 62: Loss: 172.09185791015625
step 63: Loss: 47.1514778137207
step 64: Loss: 165.58836364746094
step 65: Loss: 39.80912399291992
step 66: Loss: 159.01678466796875
step 67: Loss: 33.559146881103516
step 68: Loss: 152.40110778808594
step 69: Loss: 28.255844116210938
step 70: Loss: 145.76629638671875
step 71: Loss: 23.767404556274414
step 72: Loss: 139.13754272460938
step 73: Loss: 19.976667404174805
step 74: Loss: 132.54066467285156
step 75: Loss: 16.78044319152832
```

```
step 76: Loss: 126.0013656616211
step 77: Loss: 14.089128494262695
step 78: Loss: 119.54437255859375
step 79: Loss: 11.825434684753418
step 80: Loss: 113.19332122802734
step 81: Loss: 9.922957420349121
step 82: Loss: 106.97029113769531
step 83: Loss: 8.325211524963379
step 84: Loss: 100.89632415771484
step 85: Loss: 6.984189987182617
step 86: Loss: 94.99032592773438
step 87: Loss: 5.859060764312744
step 88: Loss: 89.26852416992188
step 89: Loss: 4.915434837341309
step 90: Loss: 83.74527740478516
step 91: Loss: 4.124213218688965
step 92: Loss: 78.43235778808594
step 93: Loss: 3.460897445678711
step 94: Loss: 73.3391342163086
step 95: Loss: 2.9048776626586914
step 96: Loss: 68.47264862060547
step 97: Loss: 2.4388275146484375
step 98: Loss: 63.83707046508789
step 99: Loss: 2.048197031021118
step 100: Loss: 59.43452072143555
step 101: Loss: 1.7207491397857666
step 102: Loss: 55.265079498291016
step 103: Loss: 1.446230411529541
step 104: Loss: 51.32695388793945
step 105: Loss: 1.2161104679107666
step 106: Loss: 47.61666488647461
step 107: Loss: 1.0231757164001465
step 108: Loss: 44.12925338745117
step 109: Loss: 0.8613846302032471
step 110: Loss: 40.85853958129883
step 111: Loss: 0.7256777286529541
step 112: Loss: 37.797306060791016
step 113: Loss: 0.6118183732032776
step 114: Loss: 34.93754959106445
step 115: Loss: 0.5162590742111206
step 116: Loss: 32.270687103271484
step 117: Loss: 0.43602922558784485
step 118: Loss: 29.787681579589844
step 119: Loss: 0.3686724305152893
step 120: Loss: 27.479257583618164
step 121: Loss: 0.31201332807540894
step 122: Loss: 25.335979461669922
step 123: Loss: 0.26440495252609253
step 124: Loss: 23.348468780517578
step 125: Loss: 0.22436116635799408
step 126: Loss: 21.507457733154297
step 127: Loss: 0.19064068794250488
step 128: Loss: 19.803810119628906
```

```
step 129: Loss: 0.16224084794521332
step 130: Loss: 18.228717803955078
step 131: Loss: 0.13830390572547913
step 132: Loss: 16.77366065979004
step 133: Loss: 0.11809547990560532
step 134: Loss: 15.430447578430176
step 135: Loss: 0.10103265196084976
step 136: Loss: 14.191292762756348
step 137: Loss: 0.08661586046218872
step 138: Loss: 13.048810005187988
step 139: Loss: 0.07439390569925308
step 140: Loss: 11.995981216430664
step 141: Loss: 0.0640515461564064
step 142: Loss: 11.026226043701172
step 143: Loss: 0.055261529982089996
step 144: Loss: 10.133353233337402
step 145: Loss: 0.04779597744345665
step 146: Loss: 9.311551094055176
step 147: Loss: 0.04144347086548805
step 148: Loss: 8.555465698242188
step 149: Loss: 0.036020323634147644
step 150: Loss: 7.859989166259766
step 151: Loss: 0.03139352798461914
step 152: Loss: 7.220425605773926
step 153: Loss: 0.02742904983460903
step 154: Loss: 6.632402420043945
step 155: Loss: 0.024029778316617012
step 156: Loss: 6.091865062713623
step 157: Loss: 0.02110857143998146
step 158: Loss: 5.595061302185059
step 159: Loss: 0.018592441454529762
step 160: Loss: 5.138516426086426
step 161: Loss: 0.01642162911593914
step 162: Loss: 4.719027996063232
step 163: Loss: 0.014540680684149265
step 164: Loss: 4.333626747131348
step 165: Loss: 0.01290850155055523
step 166: Loss: 3.979578971862793
step 167: Loss: 0.011488468386232853
step 168: Loss: 3.654357671737671
step 169: Loss: 0.010249658487737179
step 170: Loss: 3.355639934539795
step 171: Loss: 0.009166031144559383
step 172: Loss: 3.081282615661621
step 173: Loss: 0.00821552611887455
step 174: Loss: 2.829312801361084
step 175: Loss: 0.0073795076459646225
step 176: Loss: 2.597914457321167
step 177: Loss: 0.006642189808189869
step 178: Loss: 2.385416030883789
step 179: Loss: 0.005990123841911554
step 180: Loss: 2.190281391143799
step 181: Loss: 0.005411945749074221
```

```
step 182: Loss: 2.0110950469970703
step 183: Loss: 0.00489794509485364
step 184: Loss: 1.846558690071106
step 185: Loss: 0.004439824260771275
step 186: Loss: 1.6954782009124756
step 187: Loss: 0.004030512645840645
step 188: Loss: 1.5567545890808105
step 189: Loss: 0.0036639769095927477
step 190: Loss: 1.4293785095214844
step 191: Loss: 0.0033349506556987762
step 192: Loss: 1.3124237060546875
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step 643: Loss: 8.478653334975661e-10
step 644: Loss: 1.1443885838957613e-08
step 645: Loss: 8.259004591337771e-10
step 646: Loss: 1.0975965025750156e-08
step 647: Loss: 8.033388954054033e-10
step 648: Loss: 1.0527610783128694e-08
step 649: Loss: 7.817927416553516e-10
step 650: Loss: 1.0103111236503537e-08
step 651: Loss: 7.627348197480899e-10
step 652: Loss: 9.716238480450556e-09
step 653: Loss: 7.438095694922708e-10
step 654: Loss: 9.345063389787356e-09
step 655: Loss: 7.278335156790661e-10
step 656: Loss: 9.007439238928328e-09
step 657: Loss: 7.097888943263797e-10
step 658: Loss: 8.68367333595188e-09
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step 659: Loss: 6.929546936262909e-10
step 660: Loss: 8.377000426662562e-09
step 661: Loss: 6.764513393875404e-10
step 662: Loss: 8.088358427471576e-09
step 663: Loss: 6.597372093075649e-10
step 664: Loss: 7.811876479024704e-09
step 665: Loss: 6.453075296342092e-10
step 666: Loss: 7.550577940662606e-09
step 667: Loss: 6.322387613444391e-10
step 668: Loss: 7.316441674731777e-09
step 669: Loss: 6.185546519432705e-10
step 670: Loss: 7.0736776436319815e-09
step 671: Loss: 6.036963706712584e-10
step 672: Loss: 6.850993994333976e-09
step 673: Loss: 5.909984168717131e-10
step 674: Loss: 6.642418171054487e-09
step 675: Loss: 5.795548485565405e-10
step 676: Loss: 6.430780796762292e-09
step 677: Loss: 5.678383319107638e-10
step 678: Loss: 6.239929906115549e-09
step 679: Loss: 5.556227700154182e-10
step 680: Loss: 6.060871360347164e-09
step 681: Loss: 5.441612715983979e-10
step 682: Loss: 5.884166043301775e-09
step 683: Loss: 5.323366192300227e-10
step 684: Loss: 5.715433015751614e-09
step 685: Loss: 5.215494147670086e-10
step 686: Loss: 5.556566762265902e-09
step 687: Loss: 5.113838796866332e-10
step 688: Loss: 5.403820058091924e-09
step 689: Loss: 5.009138104306032e-10
step 690: Loss: 5.260434310372375e-09
step 691: Loss: 4.904216477363832e-10
step 692: Loss: 5.1260515832041165e-09
step 693: Loss: 4.799644570674388e-10
step 694: Loss: 4.988945701001057e-09
step 695: Loss: 4.706138811982896e-10
step 696: Loss: 4.860537305972912e-09
step 697: Loss: 4.610543336003303e-10
step 698: Loss: 4.739659331676194e-09
step 699: Loss: 4.5348474975170916e-10
step 700: Loss: 4.625064331520434e-09
step 701: Loss: 4.4431339163431005e-10
step 702: Loss: 4.508954543069876e-09
step 703: Loss: 4.364367756082288e-10
step 704: Loss: 4.3998342746931485e-09
step 705: Loss: 4.2864023441779864e-10
step 706: Loss: 4.297135980380062e-09
step 707: Loss: 4.2113348919237126e-10
step 708: Loss: 4.199149028494276e-09
step 709: Loss: 4.1312925302960934e-10
step 710: Loss: 4.1059671218590665e-09
step 711: Loss: 4.0468442485952494e-10
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step 712: Loss: 4.0111944876741745e-09
step 713: Loss: 3.9738526358412685e-10
step 714: Loss: 3.920797020384725e-09
step 715: Loss: 3.8968725468713217e-10
step 716: Loss: 3.8338399122039846e-09
step 717: Loss: 3.832988371144097e-10
step 718: Loss: 3.751178034860914e-09
step 719: Loss: 3.7739347757970165e-10
step 720: Loss: 3.6712408668648777e-09
step 721: Loss: 3.700315609478366e-10
step 722: Loss: 3.5973415357659633e-09
step 723: Loss: 3.639488155293691e-10
step 724: Loss: 3.523079827871811e-09
step 725: Loss: 3.5827737998594955e-10
step 726: Loss: 3.4489147093808015e-09
step 727: Loss: 3.520150559932489e-10
step 728: Loss: 3.3775762187104874e-09
step 729: Loss: 3.459126873828211e-10
step 730: Loss: 3.309648333171822e-09
step 731: Loss: 3.406254722726487e-10
step 732: Loss: 3.242547563786502e-09
step 733: Loss: 3.359446609785266e-10
step 734: Loss: 3.1786715481985084e-09
step 735: Loss: 3.307951967901346e-10
step 736: Loss: 3.115644631179748e-09
step 737: Loss: 3.257223102348661e-10
step 738: Loss: 3.05509928466563e-09
step 739: Loss: 3.2077118738982335e-10
step 740: Loss: 2.9967237580308392e-09
step 741: Loss: 3.151584826444065e-10
step 742: Loss: 2.938932652796211e-09
step 743: Loss: 3.095491363236391e-10
step 744: Loss: 2.878831617536548e-09
step 745: Loss: 3.038793383591809e-10
step 746: Loss: 2.825199185707561e-09
step 747: Loss: 2.998048198588066e-10
step 748: Loss: 2.773005380873883e-09
step 749: Loss: 2.95376417769333e-10
step 750: Loss: 2.7206068509144643e-09
step 751: Loss: 2.9115179711602934e-10
step 752: Loss: 2.6755866411320994e-09
step 753: Loss: 2.871828885808725e-10
step 754: Loss: 2.6238706762882202e-09
step 755: Loss: 2.829842749019207e-10
step 756: Loss: 2.5793627234094174e-09
step 757: Loss: 2.7932511859063425e-10
step 758: Loss: 2.535659682223468e-09
step 759: Loss: 2.756295192085645e-10
step 760: Loss: 2.4893842542894618e-09
step 761: Loss: 2.725680792181606e-10
step 762: Loss: 2.4458082226175293e-09
step 763: Loss: 2.6944887987490063e-10
step 764: Loss: 2.4018502742251258e-09
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step 765: Loss: 2.6562130273077855e-10
step 766: Loss: 2.3579473928947436e-09
step 767: Loss: 2.6083363247053626e-10
step 768: Loss: 2.3208128752116863e-09
step 769: Loss: 2.578496305361e-10
step 770: Loss: 2.2838779756284566e-09
step 771: Loss: 2.5428675831662417e-10
step 772: Loss: 2.2455668435839016e-09
step 773: Loss: 2.503849350077303e-10
step 774: Loss: 2.2076371841706077e-09
step 775: Loss: 2.471831628270138e-10
step 776: Loss: 2.170512214405562e-09
step 777: Loss: 2.4373811302602633e-10
step 778: Loss: 2.1345250011961525e-09
step 779: Loss: 2.4058380287961256e-10
step 780: Loss: 2.098590634602715e-09
step 781: Loss: 2.3703872198410636e-10
step 782: Loss: 2.0650048337955695e-09
step 783: Loss: 2.3372911939212315e-10
step 784: Loss: 2.0304942172089113e-09
step 785: Loss: 2.3082939726304375e-10
step 786: Loss: 2.0007162593316252e-09
step 787: Loss: 2.2742069338832493e-10
step 788: Loss: 1.9673327411595665e-09
step 789: Loss: 2.2497677332200539e-10
step 790: Loss: 1.9367392134483907e-09
step 791: Loss: 2.2186898152032342e-10
step 792: Loss: 1.9098900239100658e-09
step 793: Loss: 2.1978809050526849e-10
step 794: Loss: 1.8807675417065184e-09
step 795: Loss: 2.1648684234154558e-10
step 796: Loss: 1.852985209715996e-09
step 797: Loss: 2.1362611679620613e-10
step 798: Loss: 1.8246666400045797e-09
step 799: Loss: 2.109122737570246e-10
step 800: Loss: 1.798781013029327e-09
step 801: Loss: 2.0829098168473337e-10
step 802: Loss: 1.7714745226271589e-09
step 803: Loss: 2.056037146092038e-10
step 804: Loss: 1.7464593105032122e-09
step 805: Loss: 2.0349133489361293e-10
step 806: Loss: 1.7232218985085979e-09
step 807: Loss: 2.0076050544215462e-10
step 808: Loss: 1.69969127661318e-09
step 809: Loss: 1.9940253614958436e-10
step 810: Loss: 1.6753238796241021e-09
step 811: Loss: 1.9698388753486284e-10
step 812: Loss: 1.6523672430324154e-09
step 813: Loss: 1.9384963079183137e-10
step 814: Loss: 1.6302768024445413e-09
step 815: Loss: 1.922346587468482e-10
step 816: Loss: 1.6067377428541363e-09
step 817: Loss: 1.8958976055749588e-10
```

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step 818: Loss: 1.5840417866286316e-09
step 819: Loss: 1.8788744171605032e-10
step 820: Loss: 1.5624775917544298e-09
step 821: Loss: 1.8560018799629319e-10
step 822: Loss: 1.5413406107001038e-09
step 823: Loss: 1.8320854555664567e-10
step 824: Loss: 1.5226305771776083e-09
step 825: Loss: 1.8075278773732606e-10
step 826: Loss: 1.5031976774437794e-09
step 827: Loss: 1.7858547973759187e-10
step 828: Loss: 1.4846918139355125e-09
step 829: Loss: 1.7684582964694329e-10
step 830: Loss: 1.4672990600317348e-09
step 831: Loss: 1.7509892147327122e-10
step 832: Loss: 1.4485255217522308e-09
step 833: Loss: 1.7245230243823073e-10
step 834: Loss: 1.4301522188731042e-09
step 835: Loss: 1.705292990150653e-10
step 836: Loss: 1.4122194524901488e-09
step 837: Loss: 1.683495565174553e-10
step 838: Loss: 1.3949769117616029e-09
step 839: Loss: 1.6671067304407927e-10
step 840: Loss: 1.377934322199792e-09
step 841: Loss: 1.6535649238758054e-10
step 842: Loss: 1.361845414216134e-09
step 843: Loss: 1.6350860942981882e-10
step 844: Loss: 1.3439531709735775e-09
step 845: Loss: 1.6124612756129864e-10
step 846: Loss: 1.3288741218531186e-09
step 847: Loss: 1.597880439074828e-10
step 848: Loss: 1.3128335085710319e-09
step 849: Loss: 1.5788002849514982e-10
step 850: Loss: 1.2955670980474565e-09
step 851: Loss: 1.5659806784640296e-10
step 852: Loss: 1.2813927696697647e-09
step 853: Loss: 1.5456227964172342e-10
step 854: Loss: 1.266821203493862e-09
step 855: Loss: 1.5318243895556805e-10
step 856: Loss: 1.2525160908438693e-09
step 857: Loss: 1.5127209207488335e-10
step 858: Loss: 1.2380616531970645e-09
step 859: Loss: 1.5010698239947828e-10
step 860: Loss: 1.2237423296923566e-09
step 861: Loss: 1.4889119104299908e-10
step 862: Loss: 1.208742772540461e-09
step 863: Loss: 1.4729602260121766e-10
step 864: Loss: 1.1956509116117786e-09
step 865: Loss: 1.4627594968619206e-10
step 866: Loss: 1.1833711788256096e-09
step 867: Loss: 1.4460918573711012e-10
step 868: Loss: 1.1708878311367243e-09
step 869: Loss: 1.4321072105971666e-10
step 870: Loss: 1.1570890912082632e-09
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step 871: Loss: 1.4143863857896122e-10
step 872: Loss: 1.144060624014287e-09
step 873: Loss: 1.4006437676350458e-10
step 874: Loss: 1.131678861732155e-09
step 875: Loss: 1.3860385061903457e-10
step 876: Loss: 1.1195351312665025e-09
step 877: Loss: 1.381077058271174e-10
step 878: Loss: 1.1067052829716317e-09
step 879: Loss: 1.3579443125522062e-10
step 880: Loss: 1.0951392015456918e-09
step 881: Loss: 1.3427828293721689e-10
step 882: Loss: 1.0838687725112095e-09
step 883: Loss: 1.332560867206567e-10
step 884: Loss: 1.0708081088495192e-09
step 885: Loss: 1.3166320811386356e-10
step 886: Loss: 1.0601662880915796e-09
step 887: Loss: 1.3049752944915838e-10
step 888: Loss: 1.0489218382758736e-09
step 889: Loss: 1.2899152579404216e-10
step 890: Loss: 1.0384839654875577e-09
step 891: Loss: 1.2792759906954387e-10
step 892: Loss: 1.0277464435048955e-09
step 893: Loss: 1.2654555181512706e-10
step 894: Loss: 1.0183758281101518e-09
step 895: Loss: 1.2546420846692996e-10
step 896: Loss: 1.00858466023368e-09
step 897: Loss: 1.2468992505176857e-10
step 898: Loss: 9.97817384273958e-10
step 899: Loss: 1.2358421230818095e-10
step 900: Loss: 9.87902759597148e-10
step 901: Loss: 1.2265369275787918e-10
step 902: Loss: 9.774522302663513e-10
step 903: Loss: 1.2144107941480797e-10
step 904: Loss: 9.688080337966198e-10
step 905: Loss: 1.2083745115631928e-10
step 906: Loss: 9.583053239836659e-10
step 907: Loss: 1.1957614065583044e-10
step 908: Loss: 9.502268971672834e-10
step 909: Loss: 1.1928096010915823e-10
step 910: Loss: 9.411618151489165e-10
step 911: Loss: 1.1790395049171565e-10
step 912: Loss: 9.309939486001895e-10
step 913: Loss: 1.1743810091058293e-10
step 914: Loss: 9.214697338499889e-10
step 915: Loss: 1.1575912450823012e-10
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step 918: Loss: 9.040595494447246e-10
step 919: Loss: 1.1353117751466968e-10
step 920: Loss: 8.957798947051288e-10
step 921: Loss: 1.1309374270407346e-10
step 922: Loss: 8.875238877159575e-10
step 923: Loss: 1.1224709356438822e-10
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step 924: Loss: 8.785547844780695e-10
step 925: Loss: 1.1089622969917556e-10
step 926: Loss: 8.705833276501096e-10
step 927: Loss: 1.1082874895596007e-10
step 928: Loss: 8.608989632286068e-10
step 929: Loss: 1.0949127715598195e-10
step 930: Loss: 8.534465911758105e-10
step 931: Loss: 1.0860879556817693e-10
step 932: Loss: 8.459807299132649e-10
step 933: Loss: 1.0766845748300113e-10
step 934: Loss: 8.377037952200794e-10
step 935: Loss: 1.0687694479427634e-10
step 936: Loss: 8.299750331453026e-10
step 937: Loss: 1.0530438326883385e-10
step 938: Loss: 8.219708247381163e-10
step 939: Loss: 1.0504135755651234e-10
step 940: Loss: 8.149662611423025e-10
step 941: Loss: 1.0332370375953914e-10
step 942: Loss: 8.071616153237926e-10
step 943: Loss: 1.0308844750062107e-10
step 944: Loss: 7.997966733341855e-10
step 945: Loss: 1.0178333176291687e-10
step 946: Loss: 7.925577416578733e-10
step 947: Loss: 1.0072701006613727e-10
step 948: Loss: 7.853360184384428e-10
step 949: Loss: 1.0033505970508116e-10
step 950: Loss: 7.786924993702371e-10
step 951: Loss: 9.909745246616808e-11
step 952: Loss: 7.713554239785481e-10
step 953: Loss: 9.850506521580371e-11
step 954: Loss: 7.645392652300131e-10
step 955: Loss: 9.799422384659806e-11
step 956: Loss: 7.569119220285359e-10
step 957: Loss: 9.727799121783676e-11
step 958: Loss: 7.506116839195442e-10
step 959: Loss: 9.636357684028596e-11
step 960: Loss: 7.441051108614261e-10
step 961: Loss: 9.600853445590474e-11
step 962: Loss: 7.376331212505249e-10
step 963: Loss: 9.480682211515656e-11
step 964: Loss: 7.306243943183688e-10
step 965: Loss: 9.42563527228657e-11
step 966: Loss: 7.241833244187035e-10
step 967: Loss: 9.335840434054887e-11
step 968: Loss: 7.180255279237713e-10
step 969: Loss: 9.303233183821646e-11
step 970: Loss: 7.114914768457936e-10
step 971: Loss: 9.24463422480315e-11
step 972: Loss: 7.054787309890287e-10
step 973: Loss: 9.203812018077073e-11
step 974: Loss: 6.988414846809121e-10
step 975: Loss: 9.142375051451879e-11
step 976: Loss: 6.949807951350806e-10
```

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step 977: Loss: 9.070013490264373e-11
step 978: Loss: 6.883335568197424e-10
step 979: Loss: 8.991090511001332e-11
step 980: Loss: 6.830466747764774e-10
step 981: Loss: 8.92292698062569e-11
step 982: Loss: 6.773600569331961e-10
step 983: Loss: 8.858722783111617e-11
step 984: Loss: 6.717100209385762e-10
step 985: Loss: 8.826587377663841e-11
step 986: Loss: 6.660704765515391e-10
step 987: Loss: 8.751873531442911e-11
step 988: Loss: 6.605767599587864e-10
step 989: Loss: 8.676635104842845e-11
step 990: Loss: 6.551614806227235e-10
step 991: Loss: 8.587763139500382e-11
step 992: Loss: 6.500540661313892e-10
step 993: Loss: 8.575751220263328e-11
step 994: Loss: 6.447729572478522e-10
step 995: Loss: 8.485066815833164e-11
step 996: Loss: 6.389547779761529e-10
step 997: Loss: 8.451319505331512e-11
step 998: Loss: 6.327757207102991e-10
step 999: Loss: 8.342503771130438e-11
minimal loss achieved: 8.342503771130438e-11
```
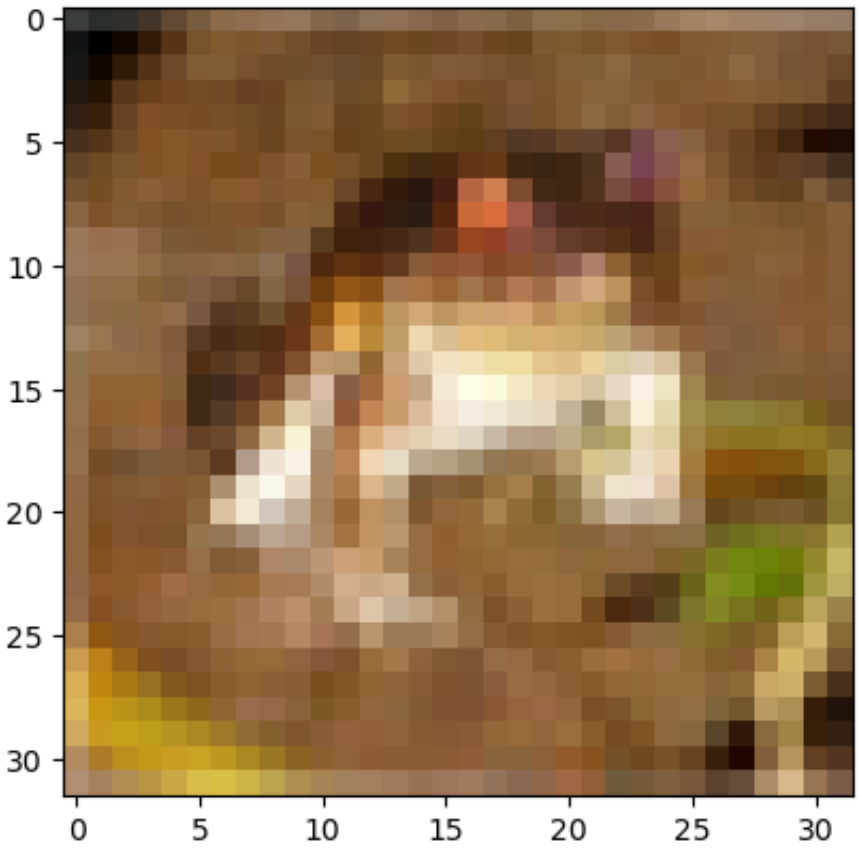
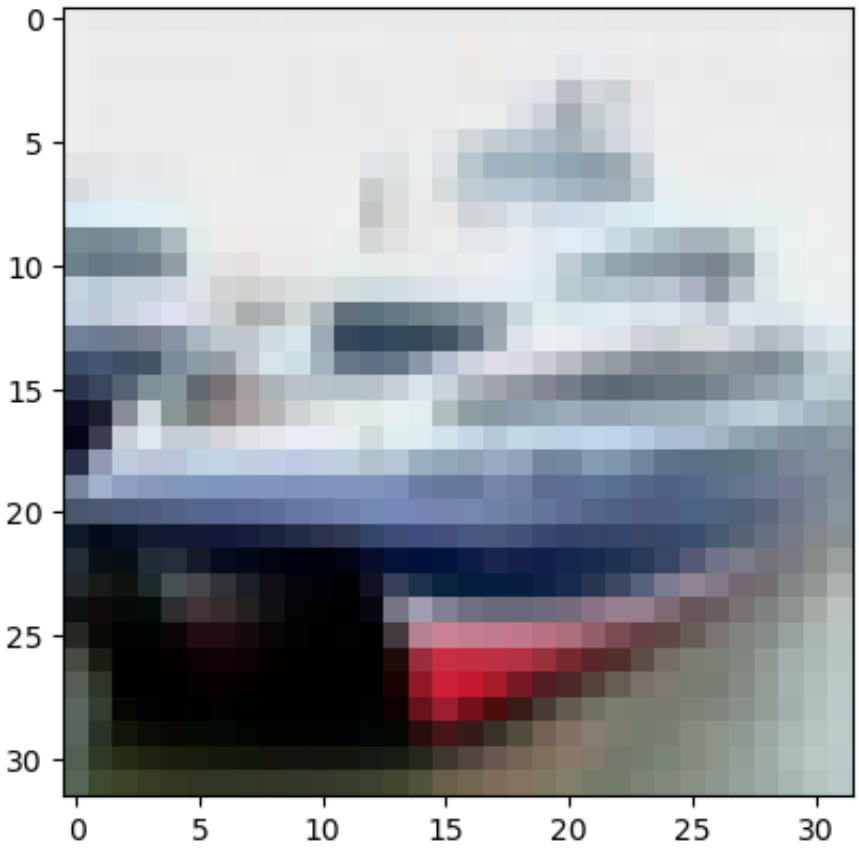

In [19]:
```
pred = f(z1)
pred = pred.reshape_as(z1)
pred = pred.detach()
imshow(torchvision.utils.make_grid(pred))
```

```
Clipping input data to the valid range for imshow with RGB data ([0..1] for
floats or [0..255] for integers).
```

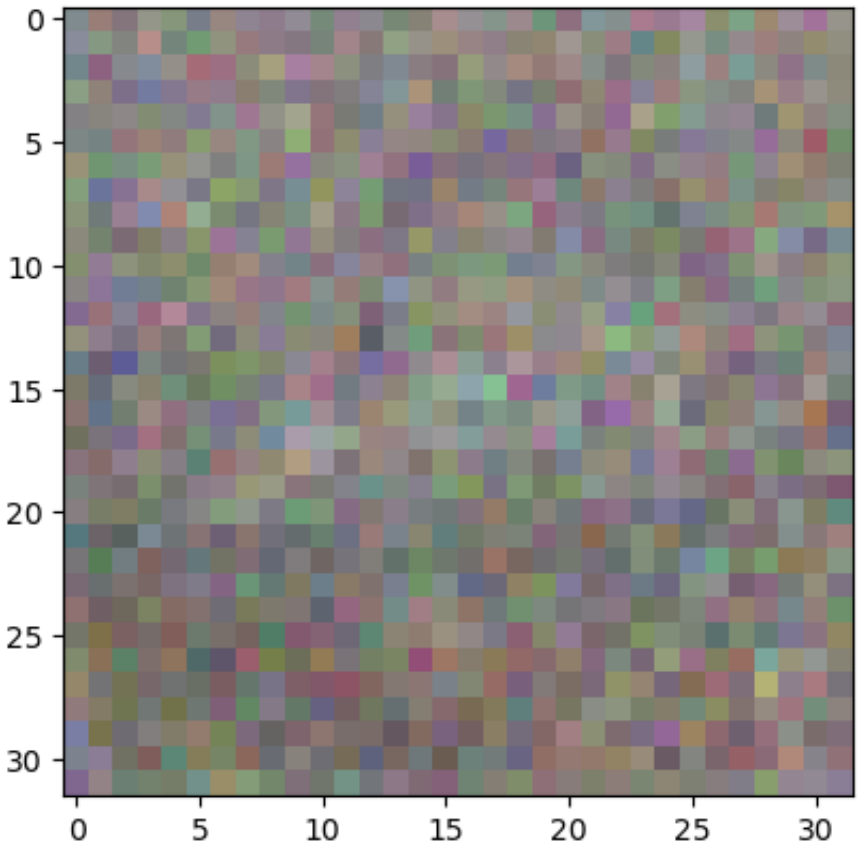


In [20]:
```
pred = f(z2)
pred = pred.reshape_as(z2)
pred = pred.detach()
imshow(torchvision.utils.make_grid(pred))
```

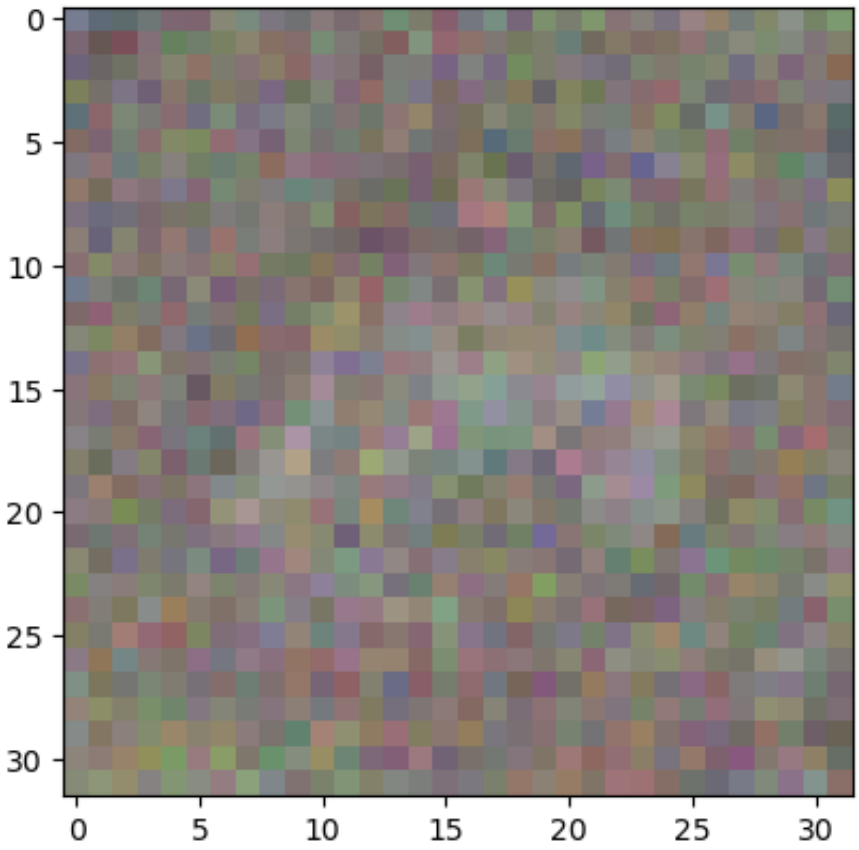
```
Clipping input data to the valid range for imshow with RGB data ([0..1] for
floats or [0..255] for integers).
```



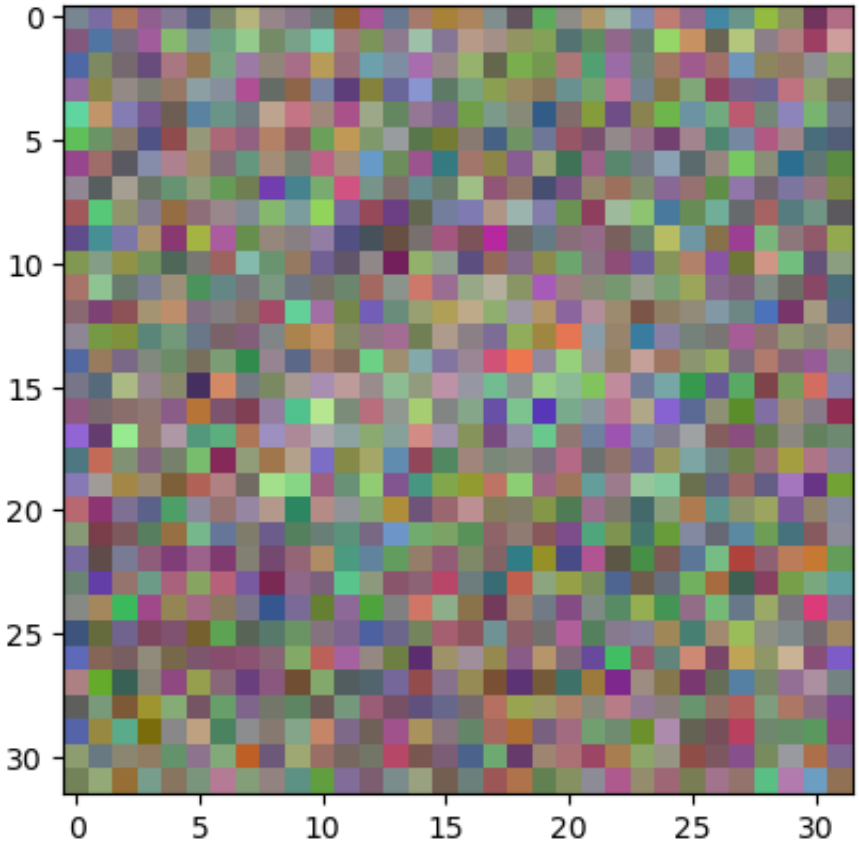
In [21]:
```
pred = f(random_train)
pred = pred.reshape_as(random_train)
pred = pred.detach()
imshow(torchvision.utils.make_grid(pred))
```



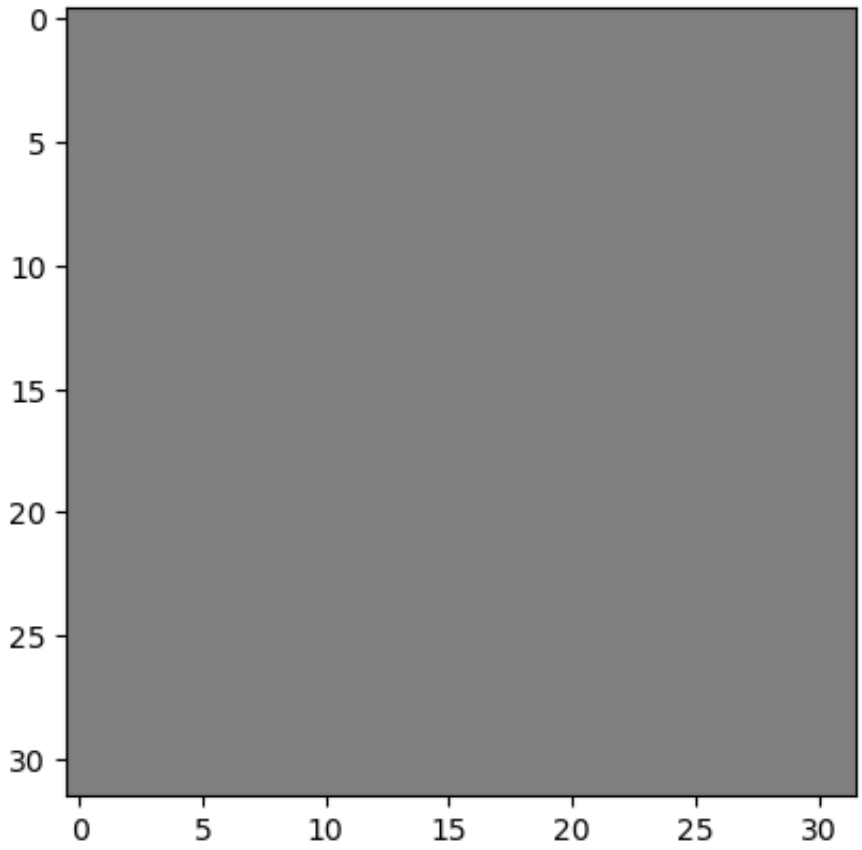
In [22]:
```
pred = f(random_test)
pred = pred.reshape_as(random_train)
pred = pred.detach()
imshow(torchvision.utils.make_grid(pred))
```



In [23]:
```
pred = f(noise)
pred = pred.reshape_as(noise)
pred = pred.detach()
imshow(torchvision.utils.make_grid(pred))
```



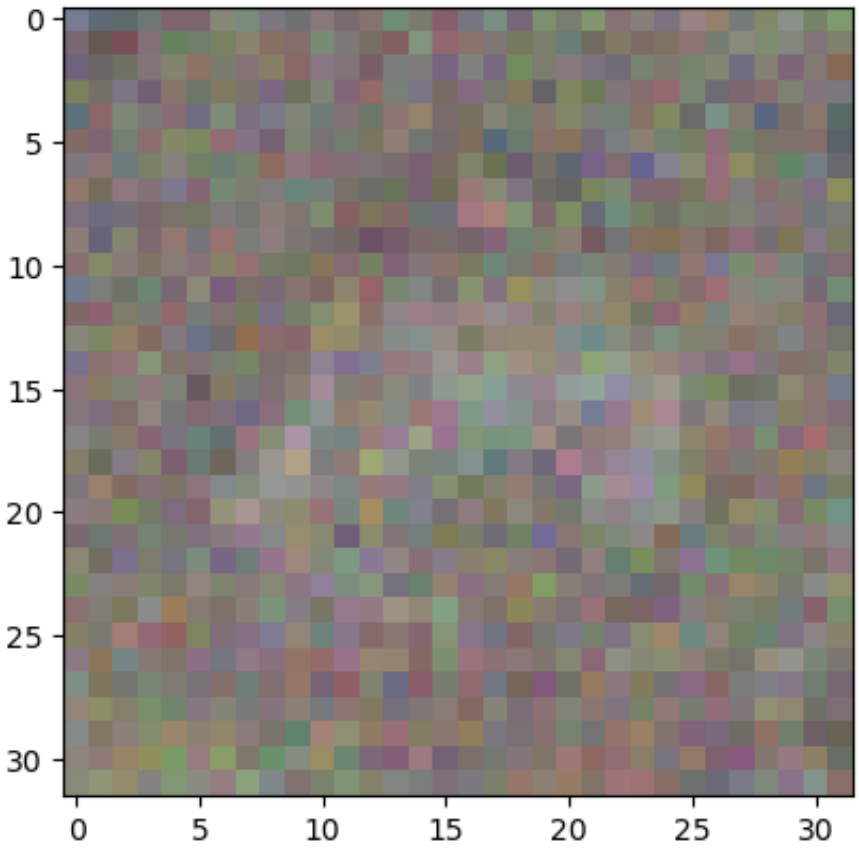
In [24]:
```
pred = f(zeros)
pred = pred.reshape_as(zeros)
pred = pred.detach()
imshow(torchvision.utils.make_grid(pred))
```



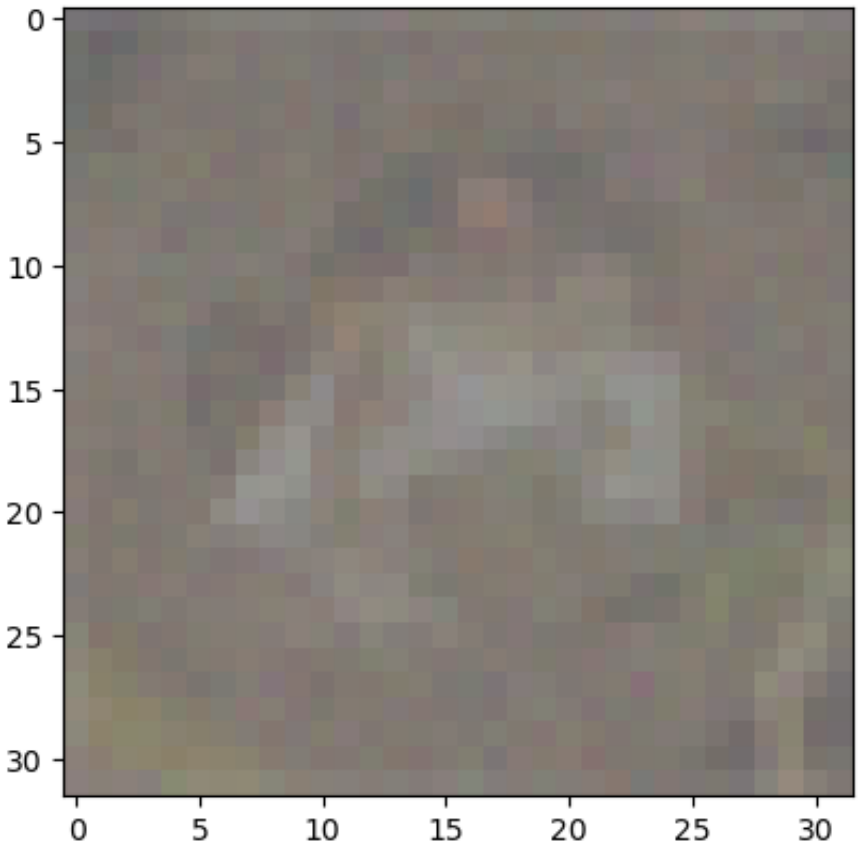
(d) $f(x), f^2(x), f^{20}(x)$

In [25]:
```
def fn(x ,n, f):
    for i in range(n):
        x = f(x)
    return x
```

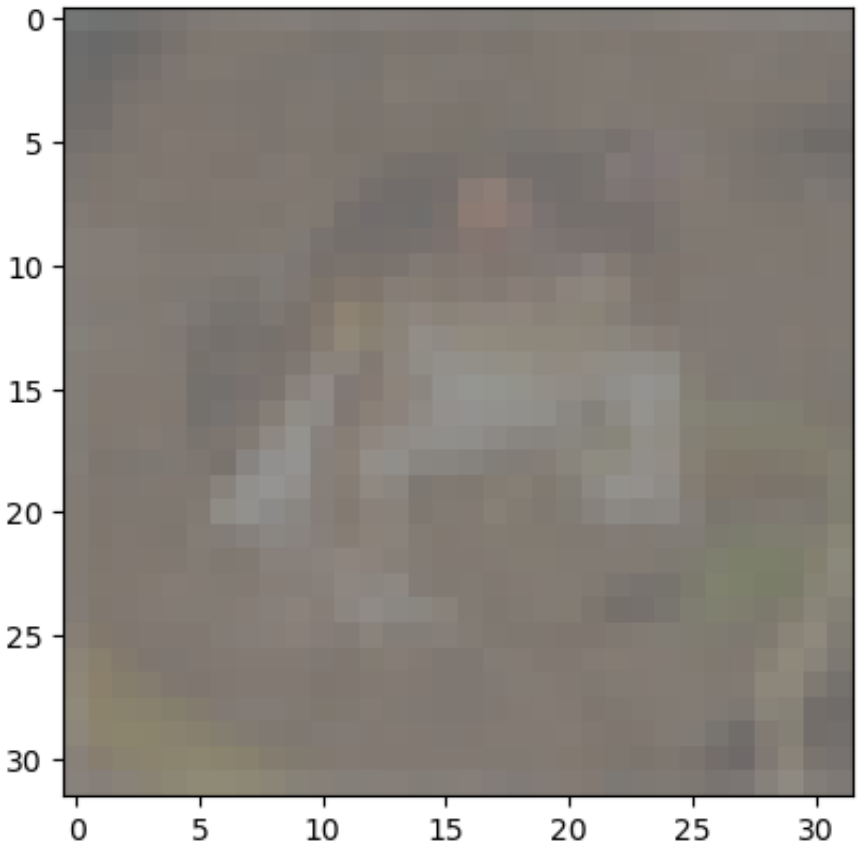
In [26]:
```
pred = fn(random_test, 1, f)
pred = pred.reshape_as(zeros)
pred = pred.detach()
imshow(torchvision.utils.make_grid(pred))
```



In [27]:
```
pred = fn(random_test, 2, f)
pred = pred.reshape_as(zeros)
pred = pred.detach()
imshow(torchvision.utils.make_grid(pred))
```



In [28]:
```
pred = fn(random_test, 20, f)
pred = pred.reshape_as(zeros)
pred = pred.detach()
imshow(torchvision.utils.make_grid(pred))
```



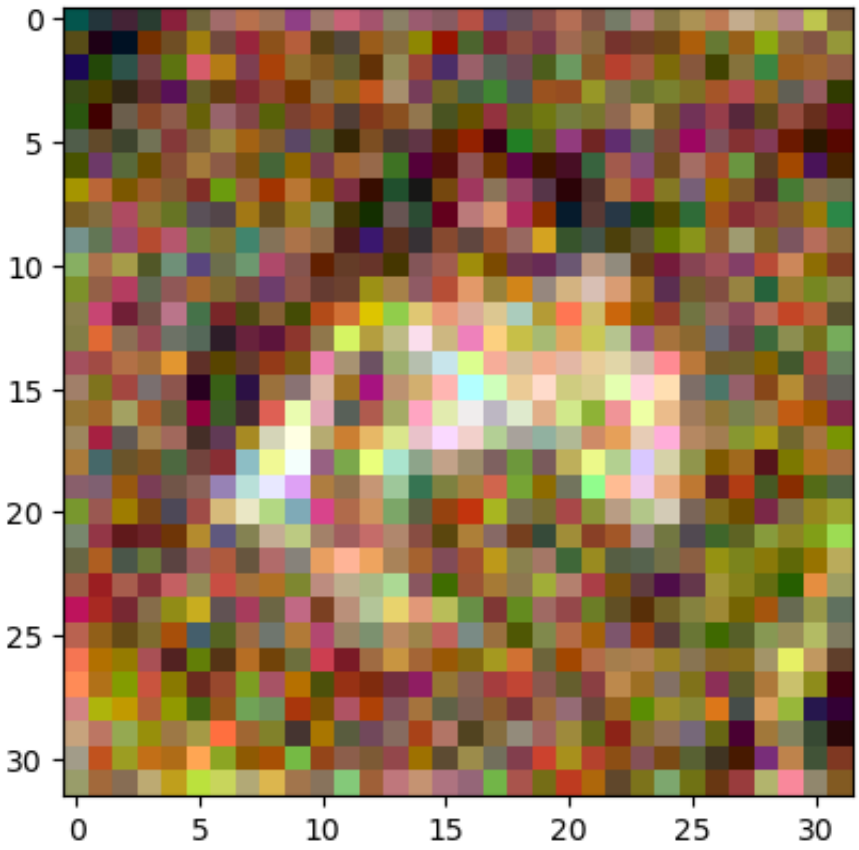
They become more close to the z_1 as n increases (but why not close to z_2?). Honestly, I don't understand why this happens theoretically but want to understand it.

(e)

In [29]:
```
def add_noise(x):
    return x + torch.normal(0, 1 , size=(1, 3, 32, 32), requires_grad=False)
```

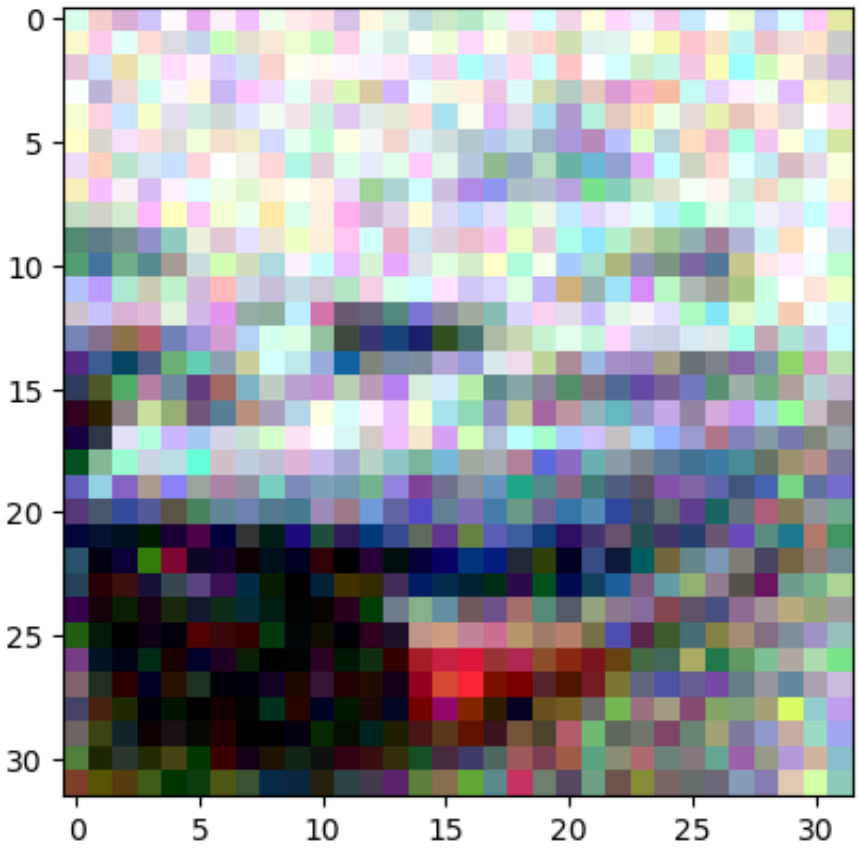
In [30]:
```
pred = fn(add_noise(z1), 1, f)
pred = pred.reshape_as(z1)
pred = pred.detach()
imshow(torchvision.utils.make_grid(pred))
```

```
Clipping input data to the valid range for imshow with RGB data ([0..1] for
floats or [0..255] for integers).
```



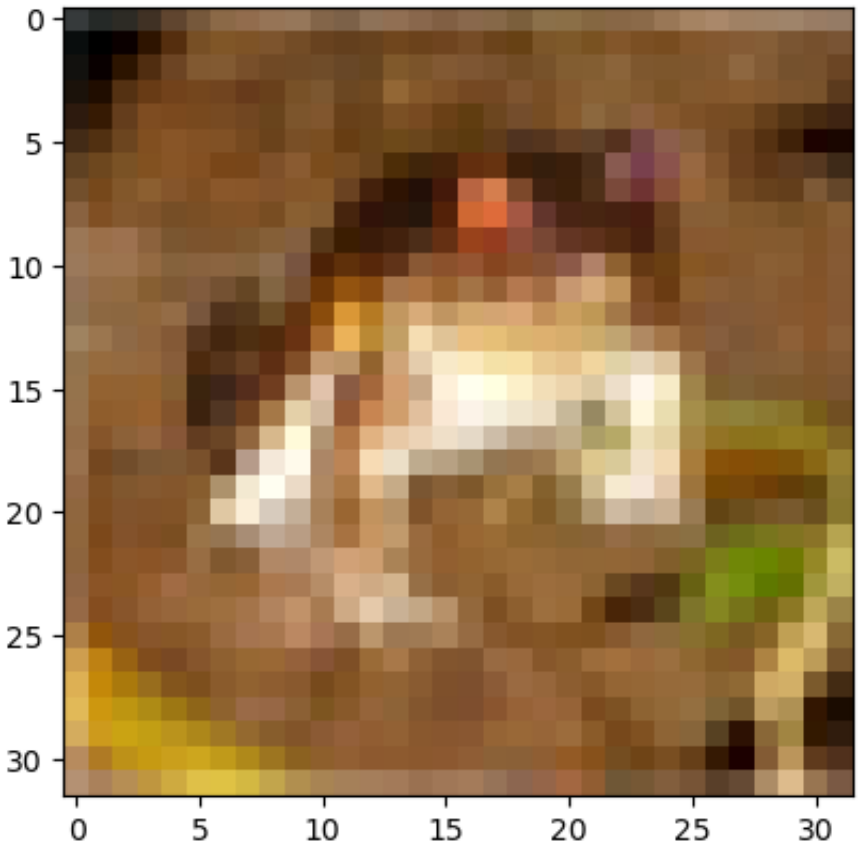
In [31]:
```
pred = fn(add_noise(z2), 1, f)
pred = pred.reshape_as(z1)
pred = pred.detach()
imshow(torchvision.utils.make_grid(pred))
```

```
Clipping input data to the valid range for imshow with RGB data ([0..1] for
floats or [0..255] for integers).
```

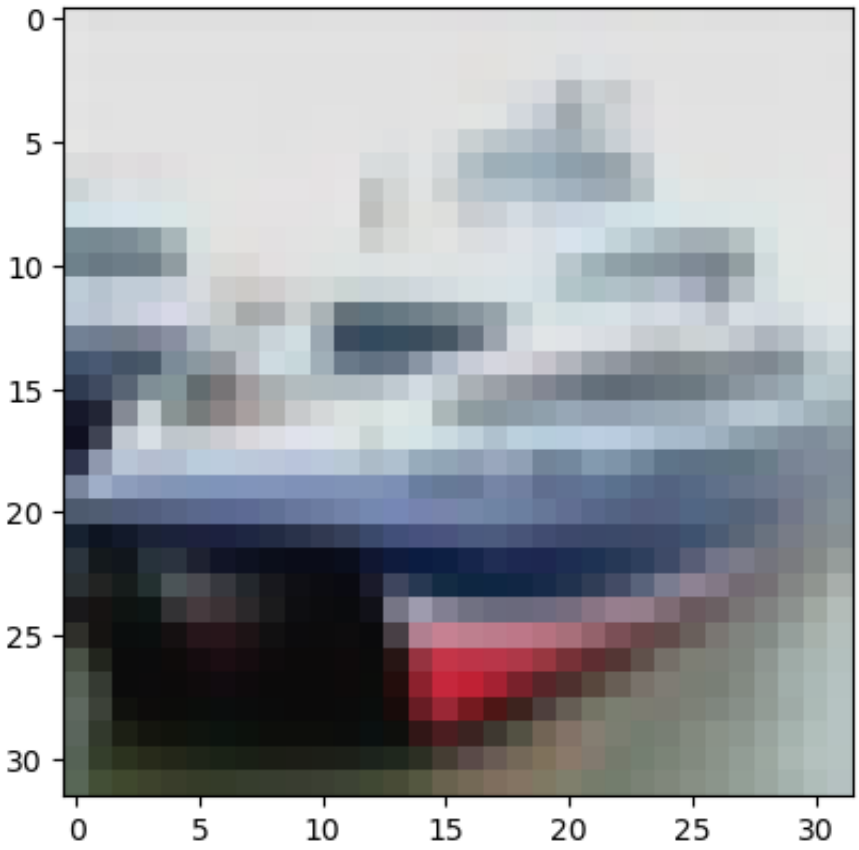


In [32]:
```
pred = fn(add_noise(z1), 10, f)
pred = pred.reshape_as(z1)
pred = pred.detach()
imshow(torchvision.utils.make_grid(pred))
```

```
Clipping input data to the valid range for imshow with RGB data ([0..1] for
floats or [0..255] for integers).
```



In [33]:
```
pred = fn(add_noise(z2), 10, f)
pred = pred.reshape_as(z1)
pred = pred.detach()
imshow(torchvision.utils.make_grid(pred))
```



Noise makes the reconstruction by autoencoder more clear. (but why?)